A Study of the Marton Electron Interferometer.

Lehigh University Bethlehem, Pa.

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A STUDY OF THE MARTON ELECTRON INTERFEROMETER

by

John Arol Simpson

A DISSERTATION

Presented to the Graduate Faculty

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CONTENTS

I. INTRODUCTION

Ever since De Broglie, from theoretical considerations, formulated the wave theory of matter there have been continuous efforts to study its experimental implications. The first experiments giving direct verification of the De Broglie[1] hypothesis were those of Davisson-Germer[2] and Thomson and Reid[3] who demonstrated that electrons are scattered from crystal lattices in a manner that is best explained as diffraction of a "matter wave."

1 L. de Broglie, Phil. Mag. 47, 446 (1924).
2 C. J. Davisson and L. H. Germer, Nature 119, 558 (1927).
3 G. P. Thomson and A. Reid, Nature 119, 890 (1927).

As geometrical electron optics developed it was realized that it would be desirable to attempt to duplicate the classic experiments of Young, Fresnel and others upon which the wave optics of light was founded. There is one fundamental difficulty that hampered the work. In order to obtain sufficient penetration and freedom from stray magnetic fields, electron optical studies are usually carried out using electron energies of about 60 kev. At this energy the wave mechanical calculation yields the value .04865 A for the wave length of an electron. This extremely small wave length means that all slits and "point" sources must be scaled down by a factor of almost one hundred thousand from those of visible light. Moreover the interference patterns will be reduced in size by the same factor.

Despite this handicap some interesting and significant work was done by Boersch[4] in 1940 when, by the use of the

4 H. Boersch, Naturwiss 28, 711 (1940).

newly developed electron microscope, he studied the Fresnel fringes about the edges of defocused images of opaque and semi-opaque objects. He observed these fringes up to about the 20th order. This type of investigation was carried further by Hillier and Ramberg[5] in 1947. They showed that the

5 J. Hillier and E. G. Ramberg, J. Appl. Phys. 18, 46 (1947).

patterns about the edge of collodion films could be fitted rather well to Fresnel's formula if it were assumed that the collodion introduced a path difference of one quarter of a wave length. This is not an unreasonable assumption but it necessitates the introduction of an inner potential of the order of ten volts. Even with this assumption and allowing for the inaccuracies of measuring the focal planes of an infinitely variable magnetic lens, there are some unexplained systematic deviations from theory. Hillier and Ramberg's work was done with path differences of less than five wave lengths.

At this point the work remained until increased resolution of the microscope permitted the observation of small scale phenomena when Uyeda[6] and coworkers, followed in close succession by Rees[7] and by Hillier,[8] observed still

6 T. Mitsuishi, H. Nagasaki and R. Uyeda, Proc. Japan Acad. 27, 86 (1951).
7 A. L. G. Rees, Private communication.
8 J. Hillier, Proc. of NBS Symposium in Electron Physics (in press).

other phenomena in micrographs of plate-like crystals. These fringes, which they interpreted as arising from interference effects between displaced lattice planes in overlying crystals, permitted no detailed analysis since they occurred only in some natural crystals and permitted no control of the geometry involved.

This type of investigation was carried further by Rang,[9] who in an ingenious set of experiments investigated

9 O. Rang, Thesis Tech. Hochschule Darmstadt (June 1952).

the interference effects in lead mono-iodide (PbI). This substance has the property of forming blisters when heated and these blisters then form interference patterns in a manner somewhat analogous to Newton's rings. The geometry of these blisters was measured by rotating them about an axis and viewing them in dark field illumination while recording the angles of maximum intensity. This process yields information on the derivative of the blister profile. This derivative was integrated graphically to give the profile of the blisters. To within the accuracy of this process, some 15%, this profile agreed with the one determined by the fringe pattern, but more precise data were unobtainable and only low order interference could be studied.

In view of the continuing interest shown in the detailed nature of these phenomena, it appeared desirable to try to construct an interferometer where all the geometry was known and preferably controllable. Apart from the general interest of such an instrument it should be of value in determining more exactly some of the important physical constants, provide a means of extending the lower limit of length measurements, be an extremely sensitive detector of field gradients, and find use as an interferometer spectrograph for electrons and beta rays. Although no doubt many people, including Marton, had considered this problem for many years a practical instrument was never built until recently when a suggestion by Marton[10] led to the work

10 L. Marton, Phys. Rev. 85, 1057 (1952).

described in this paper.

II. INSTRUMENT PRINCIPLES

If we examine the problem of forming interference fringes under controlled conditions the difficulties appear enormous. The first method, which appeals because of its apparent simplicity, is the method of Young using a double slit. It is possible by the use of compound demagnification to obtain a source small enough to make such an experiment possible, since it has been shown that only the gaussian image of the emitter need be considered in calculations of coherence.[11] Some preliminary experiments carried out in

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11 D. Gabor, Proc. Phys. Soc. B 64, 462 (1951).

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the early phases of our work showed that, while difficult, it would be possible to obtain double slits of appropriate size and spacing. These values must be less than about 200 A and 5000 A respectively in order for the resulting fringes to be resolved by the electron microscope. The technique tried was evaporation of a heavy metal, gold-manganin or palladium from two positions while the slits were shadowed by a fine wire stretched just above the formvar substrate. It was also discovered that good natural slits for this type of work appear in relatively heavy foils of gold-manganin alloy. These attempts were soon discontinued when the instrument as a whole was considered in more detail. In order to achieve any sizable number of fringes the diameter of the source must be of the order of a few hundred wave lengths and must remain fixed in size and position within this limit for the entire exposure. To form a virtual source of this size from a thermonic cathode would require compound demagnification, corresponding to a high resolution, inverted electron microscope. The viewing instrument would have to be another high resolution instrument working in the normal manner. Both these instruments, if magnetic, would require lens currents not varying to more than one part in twenty thousand during the exposure. Intensity considerations lead to exposure times in excess of ten minutes. The experience of Haine[12] working with the diffraction microscope of Gabor,[13] where a similar situation prevails, is that existing techniques of electronic stabilization are not equal to the task. Moreover the problem of achieving the necessary mechanical rigidity of the instrument is an almost impossible task.

12 M. E. Haine, Private communication.
13 D. Gabor, Proc. Roy. Soc. A 197, 454 (1949).

It was by this time clearly apparent that only an instrument of the amplitude-splitting type offered chance of success. Typical members of this family of interferometers are the Michelson[14] and Mach-Zehnder.[15]

14 A. A. Michelson, Phil. Mag. 13, 236 (1882).
15 A good treatment of this instrument appears in E. Mach, PRINCIPLES OF PHYSICAL OPTICS (Methuen, London, 1926). Chap. X.

The amplitude-splitting instruments have the advantage that the component parts (including the source) need not be of a size comparable with the wave length of the illumination. Hence there is no need for extreme constancy of source, size or position and both the stability and rigidity requirements are greatly relaxed. The situation is further improved by the reduction in exposure time to be achieved by the use of larger sources.

The lack of electron mirrors and more especially "half silvered" mirrors seems to doom any attempt to construct a conventional amplitude-splitting instrument. Marton then suggested the use of diffraction from crystals as a means of obtaining both the required mirrors and the beam splitter. The use in light optics of diffraction gratings as interferometric beam splitters, as found later, is not new. Carl Barus[16] used them in his now forgotten

16 C. Barus, Carnegie Institute of Washington Publication, No. 149 (Washington, D. C., Part I 1911, Part II 1922).

work forty years ago. It is rather strange that no reference to these lengthy (although poorly organized and obscurely worded) reports was found in the extensive literature of interferometry.

Almost simultaneously with the Marton[17] paper, Kraushaar[18] suggested the use of diffraction beam splitters

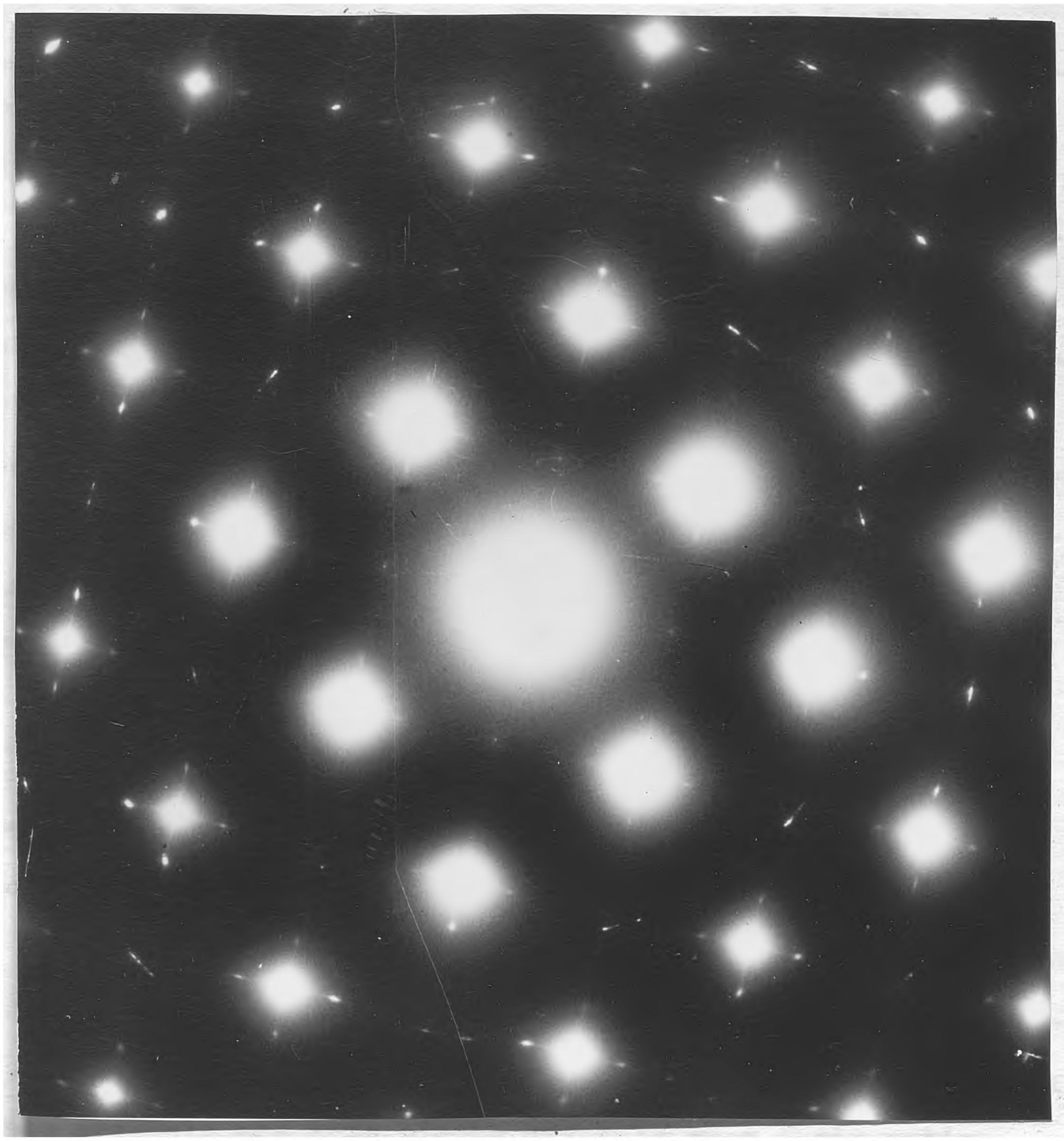

Fig. 1 Diffraction pattern of 100 Å gold single crystal taken with 60 kev electrons.

17 L. Marton, loc. cit.
18 R. Kraushaar, J. Opt. Soc. Am. 40, 480 (1950).

in wind tunnel work, but the use of gratings instead of mirrors seems to be new.

From considerations of intensity and ease of mechanical adjustment it appeared that diffraction from very thin single crystals in transmission would be the most desirable arrangement. If these crystals are only a few hundred atoms thick, while many million atoms in transverse dimensions, the diffraction pattern is that of a cross grating and not that of a Bragg three-dimensional array, since the stringency of each Laue condition is in direct proportion to the number of atom planes involved. The energy of the first order is then concentrated in discrete spots instead of the ring of the Debye-Sherrer pattern of a polycrystal, thus a much greater portion of the initial beam is available.

That single crystals of suitable size can be grown was demonstrated by the preparation of crystals giving transmission patterns as shown in Fig. I. It is not known whether these crystals are true single crystals over their entire area. Micrographs show a mosaic structure that makes this improbable; if they are not, the diffraction pattern shows that each element of the mosaic must be individually oriented and this suffices for our purpose.

The geometry adopted for the instrument is shown

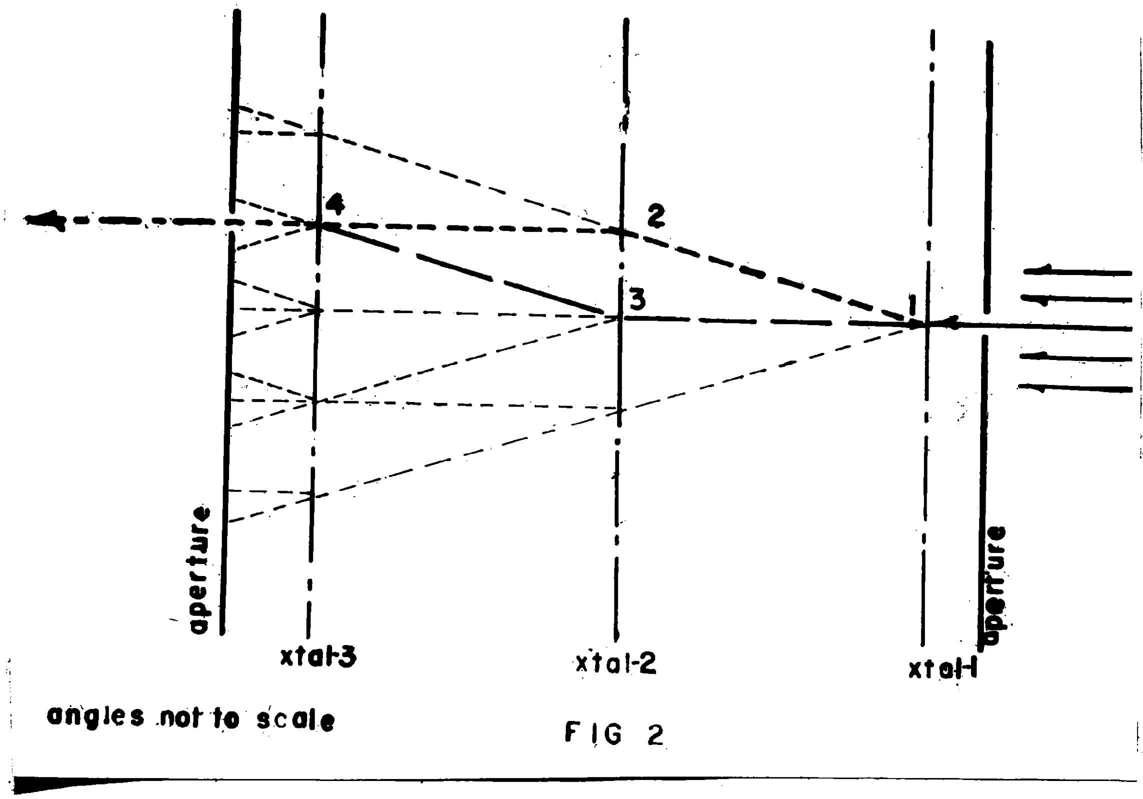

FIG 2

Simplified ray diagram of Merton electron interferometer.

in Fig. 2, where for clarity only the first order of diffraction is shown. Any sketch of the instrument is badly distorted since, for electrons in the usual electron optical range of 50 kev, the angle between the beams for most metallic crystals is about $2x10^{-2}$ radians. The small size of the angle limits the separation of the beams, for an instrument of reasonable size, to the order of one mm. It will be seen the geometry is that of a highly skewed Mach-Zehnder instrument, but using diffraction instead of reflection at points 1, 2, 3 and 4.

III. THEORY OF THE OPTICAL ANALOGUE

The detailed geometry of amplitude-splitting interferometers for the most general type of illumination is highly involved. The theory of the Mach-Zehnder instrument with non-parallel illumination has only recently been given by Bennett[19] and even in this rather long and difficult treatment he was forced to make a number of approximations in order to draw useful conclusions. The Marton instrument is even more intractable. In systems using mirrors, such as the Mach-Zehnder, each change of direction accomplished by reflection obeys the simple and compact vector equation

$$\bar{r}_j = \bar{r}_{j-1} - 2(\bar{r}_{j-1} \cdot \bar{n}_j)\bar{n}_j \quad .$$

Note that this equation involves, besides the unit vector of the incoming and outgoing rays, $\bar{r}_{j-1}$ and $\bar{r}_j$, only the unit normal of the mirror $\bar{n}_j$. In an instrument using diffraction gratings at each change of direction the equations, some of which are developed in this Section, cannot be expressed in such a simple form since they involve not only the grating normal but also a vector parallel to the rulings in complex trigonometric relationships.

In the case of interest to electron optics however,

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19 F. D. Bennett, J. Appl. Phys. 22, 184 (1951).

there is one important simplification which arises as a result of the very small angles of illumination and observation that are used. These angles must be less than 10^{-3} radians to minimize the high spherical aberration of electron lenses.

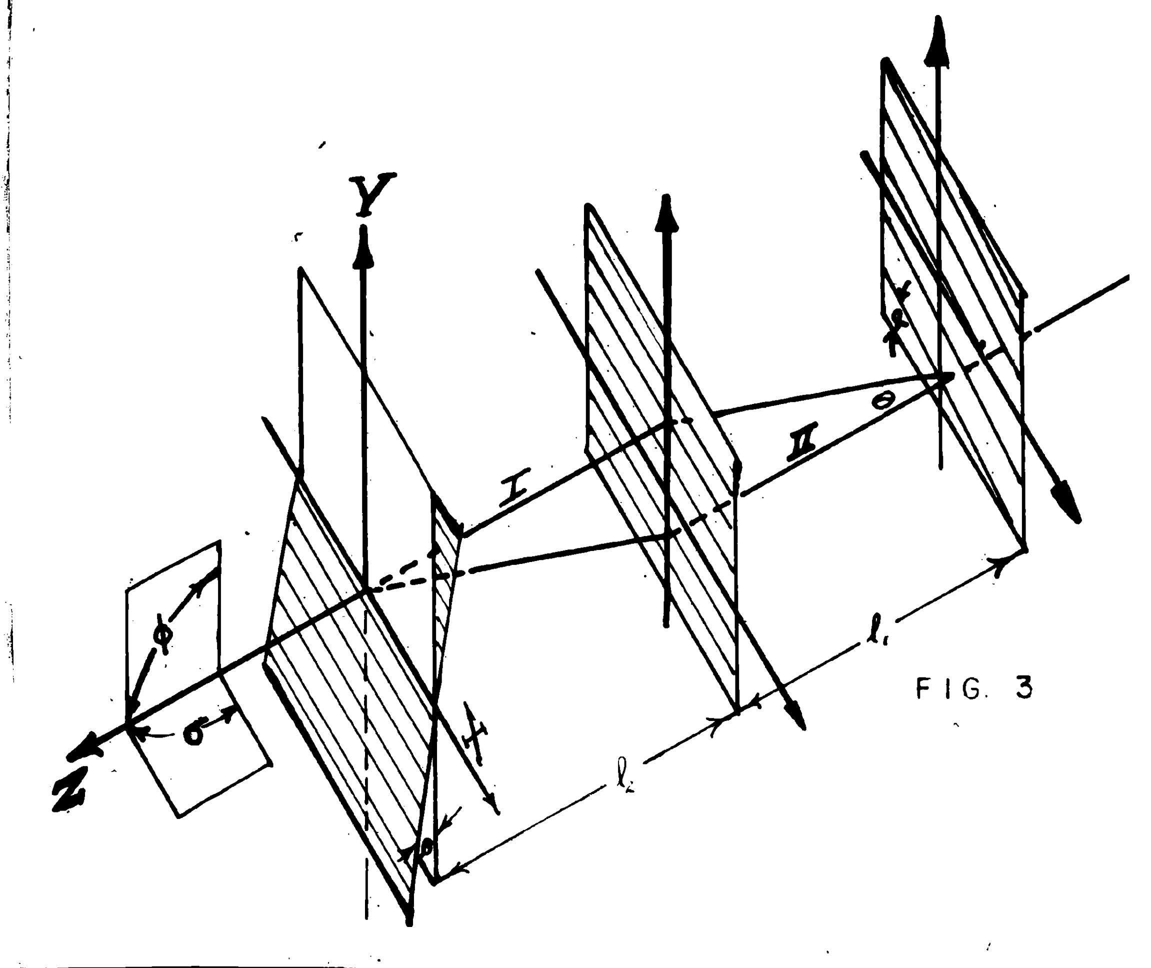
With this restriction on aperture the important formulas giving the characteristics of amplitude splitters are greatly simplified. In this case the wave fronts of the two interfering beams may be considered to be straight and to be viewed along a line almost normal to both wave fronts. The fringe spacing Υ is then given by

$$\Upsilon = \frac{\lambda}{\Phi} , \qquad (3.1)$$

where λ is the wave length of the illumination and Φ the angle between the interfering wave fronts. The direction of the fringes is parallel to the line of intersection of the wave fronts.

The theory of an instrument using diffraction gratings, which we were forced to develop, divides rather naturally into two parts; first, the effect of instrument parameters upon the two wave fronts and second, the resulting effect on fringes caused by these changes in wave front direction.

In order to study the first of these parts one must begin by defining a standard interferometer configuration,



Definition of standard configuration and
instrument parameters.

(Fig. 3), and then indicating deviations from this standard. Let the instrument be placed so that the initial beam falls parallel to the Z axis, with the two beams defining the ZY plane. The angle of the diffracted beam with the Z axis is θ, which in this case is equal to the diffraction angle ψ. The diffraction gratings are considered to be parallel to the XY plane with their rulings parallel to the X axis. The distance between the first and second shall be l_1 and that between the second and third grating l_2. These distances will be equal for standard configuration.

Deviations from standard configuration will be designated as follows. If the plane of the nth grating does not coincide with the XY plane the angle between them shall be called β_n, with the subscript denoting the grating involved. The angle α is defined as the angle between the rulings and the X axis if the gratings are rotated about the Z axis. The difference between l_1 and l_2 shall be called δl.

The effects of these changes on the beams will be designated as follows. The rotation of the nth beam around the X axis shall be designated as ϕ_n. The rotation of the nth beam around the Y axis shall be designated as σ_n.

Bars will be placed over the symbols if it is necessary to distinguish the beam directions after hitting a grating from those before the grating.

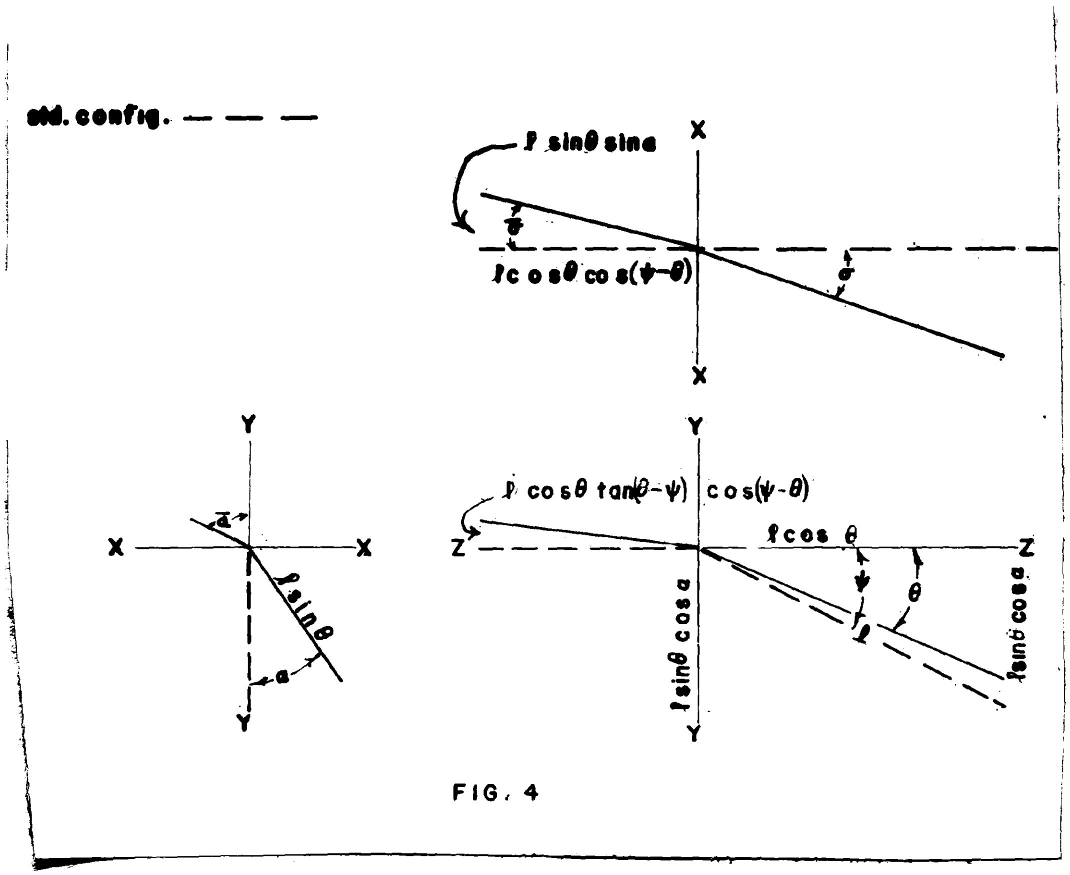


FIG. 4

Detailed geometry at each diffraction showing effects of misalignments in rotation.

The effects of β will be dealt with first. The first effect of a rotation β will be to introduce a difference δl between l_1 and l_2. The magnitude of this distance will depend on the location of the axis of rotation. We shall consider the difference introduced in this manner to be combined with any other inequality of the l's. The second effect of the rotation β will be to change the incident angle causing it to differ from the standard angle θ_{ST}. The equation satisfied at each grating is the well known expression

$$\sin \gamma + \sin \bar{\gamma} = \frac{n\lambda}{d} = \sin \Psi ,$$

where n is an integer(in this work always equal to 1)and d is the grating constant. γ equals the angle between beam and grating normal.

We introduce a rotation β by

$$\gamma = \theta + \beta$$

$$\bar{\gamma} = \bar{\theta} - \beta \qquad (3.2)$$

$$\sin(\theta + \beta) + \sin(\bar{\theta} - \beta) = \sin \Psi .$$

If the angles involved are small and the sines may be replaced by their angle it will be seen that β has no effect on θ since

$$\theta + \bar{\theta} = \sin \Psi = \theta_{ST} + \bar{\theta}_{ST} .$$

This is true to the order of $\beta\theta^2$.

The effect of the rotation of the gratings about the Z axis is more complicated and Fig. 4 shows an orthographic projection of the geometry. The notation is that just described with bars denoting the angles after diffraction. From the diagram Fig. 4 one sees by inspection that

$$\tan\bar{\alpha} = \frac{\sin\phi\sin\alpha}{\cos\phi\sin(\phi-\psi)} \tag{3.3}$$

But $\sin(\phi-\psi) = \sin\phi\cos\psi - \cos\phi\sin\psi$

and

$$\sin\phi = \frac{\sin\theta\cos\alpha}{(\sin^2\theta\cos^2\alpha + \cos^2\theta)^{1/2}}$$

$$\cos\phi = \frac{\cos\theta}{(\sin^2\theta\cos^2\alpha + \cos^2\theta)^{1/2}} \tag{3.4}$$

so that to the approximation $\theta=\psi$ (which is true to some parts per thousand)

$$\tan\bar{\alpha} = \frac{\sin\alpha(\sin^2\theta\cos^2\alpha + \cos^2\theta)^{1/2}}{\cos^2\theta(\cos\alpha - 1)} \ . \tag{3.5}$$

This becomes, as $\alpha \to 0$,

$$\tan\bar{\alpha} = -\cot\frac{\alpha}{2}\sec^2\theta \to -\infty \tag{3.6}$$

This equation (3.6) shows that for small α the only motion of the emerging beam is in the ZY plane, i.e. a rotation about the Y axis. The angle in the XZ plane is σ and is given by

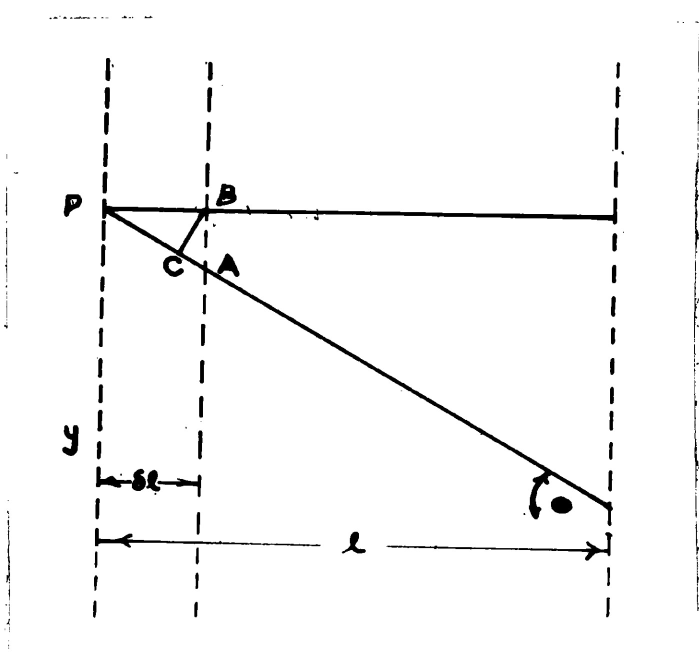


Fig. 5 Detailed geometry at third crystal showing effects of axial displacement .

$$\tan \sigma = \frac{\sin\theta \sin\alpha}{\cos\theta \cos(\theta-\psi)} \tag{3.7}$$

But $\cos(\theta-\psi) = \cos\theta \cos\psi + \sin\theta \sin\psi$

and $\theta \approx \psi$

so that

$$\tan \sigma = \tan\theta \sin\alpha \quad , \tag{3.8}$$

which for small angles becomes

$$\sigma = \theta\alpha \quad .$$

One can see from Fig. 3 that if one assumes that the second crystal defines the YX plane then rotation of the first crystal affects only beam I and rotation of the third crystal affects only beam II.

The effect of inequality of grating spacing will be considered next. This inequality may be considered as arising either from a direct displacement of the first or third grating from the standard configuration or from the rotation β_n about some axis not coincident with the beam. Consider the difference in path between beam I and II at P (Fig. 5) given by

$$\Delta = AP - BP \quad . \tag{3.9}$$

If one drops a perpendicular from B to AP intersecting AP at C, then one has, to terms of higher order,

$$\Delta = \frac{AC}{2} = \frac{1}{2}\delta l \sin\theta \tan\theta \quad . \tag{3.10}$$

From the diffraction equation one sees that

$$\sin\theta = \sin\psi = \frac{\lambda}{d} ,$$

where d equals the grating spacing and λ equals the wave length.

Then from the diagram

$$\tan\theta = \frac{y}{\ell} \tag{3.11}$$

One substitutes into (3.10) and

$$\Delta = \frac{1}{2}\frac{\delta\ell}{\ell}\ \frac{\lambda}{d}\ y . \tag{3.12}$$

The condition for a bright point at P is

$$\Delta/\lambda = n = \frac{\delta\ell}{2\ell}\ \frac{y}{d} ,$$

where n is an integer.

Then, solving for y , one obtains

$$y = \frac{2nd\ \ell}{\delta\ell} . \tag{3.13}$$

The fringe spacing Υ can be expressed as $\frac{\delta y}{\delta n}$ and thus

$$\Upsilon = \frac{2\ell}{\delta\ell} d . \tag{3.14}$$

Equation (3.14) can be expressed in terms of the angle between planes of constant phase by

$$\Upsilon = \frac{\lambda}{\sin\Phi} \approx \frac{\lambda}{\Phi} = \frac{2\ell}{\delta\ell} d , \tag{3.15}$$

hence,

$$\Phi \approx \frac{\lambda}{2d}\ \frac{\delta\ell}{\ell} ; \tag{3.16}$$

The extremely small angle of illumination and viewing necessitates that Φ be small if one is to receive both beams in the viewing instrument. θ is small with usual wave lengths and grating constants.

One is now in a position to find the effect of changes in the instrument parameters on the fringe spacing and fringe orientation. To do this one must first write the equations of the two beams in standard form. As we have defined our coordinate system in Fig. 3 these are

Beam I

$$-\frac{x \sin \sigma_I}{A} \qquad\qquad + \frac{z \cos \sigma_I}{A} = 0$$

Beam II (3.17)

$$-\frac{x \sin \sigma_{II}}{B} + \frac{y \tan \phi_I}{B} + \frac{z \cos \sigma_{II}}{B} = 0$$

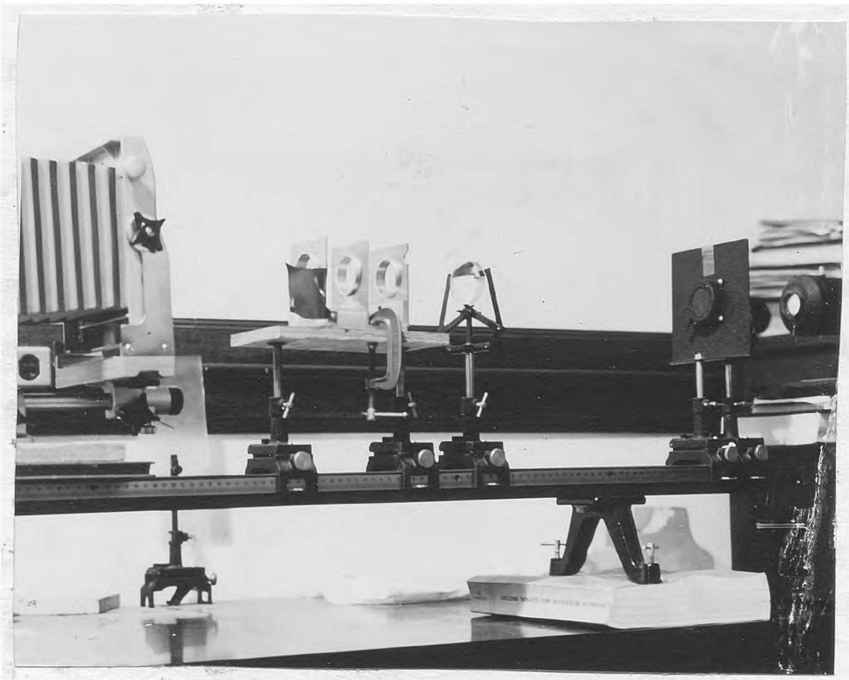
$$A \equiv (\sin^2 \sigma_I + \cos^2 \sigma_I)^{1/2}, \quad B \equiv (\sin^2 \sigma_{II} + \tan^2 \phi_I + \cos \sigma_{II})^{1/2} .$$

Since these beams are normals to the wave fronts one may find the dihedral angle by forming the dot product between the beams. After simplification this gives

$$\cos \Phi = (\sin \sigma_I \sin \sigma_{II} + \cos \sigma_I \cos \sigma_{II}) \cos \phi_I \qquad (3.18)$$

$$= \cos(\sigma_I - \sigma_{II}) \cos \phi_I .$$

One remembers that since we have assumed that each wave front is a plane the fringes run parallel to the line of intersection between these planes. One may obtain this intersection by eliminating z from equations (3.17). The slope



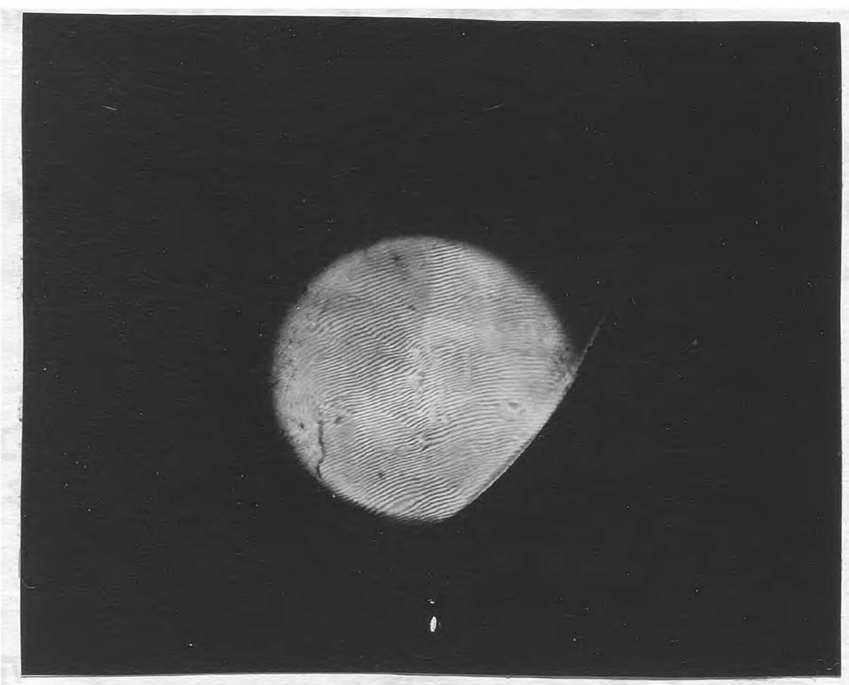


Fig. 6a and 6b Optical analog and typical fringe system obtained with it .

of the fringes with the x axis is then given directly by

$$\tan \gamma = \frac{x}{y} = \left(\frac{\tan \sigma_I \cos \sigma_{II} - \sin \sigma_{II}}{\tan \phi_I} \right) . \qquad (3.19)$$

Now substituting the small angle approximations of equations (3.8) and (3.6) in (3.19) one obtains for the fringe spacing and orientations in terms of instrumental parameters

$$\tan \gamma = (\theta_1 \alpha_1 \cos(\theta_3 \alpha_g) - \theta_3 \alpha_3) \frac{\ell}{\delta\ell} \frac{d}{\lambda} \qquad (3.20)$$

$$\cos \Phi = \cos(\theta_1 \alpha_1 - \theta_3 \alpha_3) \cos\left(\frac{\lambda}{d} \frac{\delta\ell}{\ell} \right) . \qquad (3.21)$$

To apply these results it is desirable to have the derivatives of these expressions with respect to the third grating rotation and with respect to change in spacing between gratings. These are

$$\frac{d\gamma}{d\alpha_3} = \{\theta_1 \alpha_1 \sin(\theta_3 \alpha_3) - 1\} \theta_3 \frac{\ell}{\delta\ell} \frac{d}{\lambda} \qquad (3.22)$$

and

$$\frac{d\Phi}{d(\delta\ell)} = \{\cos(\theta_1 \alpha_1 - \theta_3 \alpha_3)\}\left[\sin\left(\frac{\delta\ell}{\ell} \frac{\lambda}{d} \right)\right] \frac{\lambda}{\ell d} . \qquad (3.23)$$

As an experimental check on these derivations a small light optical instrument was constructed. Fig. 6a shows a picture of the instrument mounted on an optical bench. The mounts for the three gratings, each of 7890 lines per centimeter, may be seen readily. Due to the enormous difference between the wave lengths of visible light and of electrons no attempt could be made to scale the instrument and its dimensions are merely a matter of convenience. Therefore ℓ was taken equal to approximately 10 cm. Fig. 6b is a photograph of a typical fringe system. The irregularities in the fringes arise from the inhomogeneities of the microscope slides upon which the replica gratings were mounted and from ripples in the gratings themselves.

An accident to the mercury arc light source first used in these experiments required the use of filtered white light and it was found that the instrument gave just as many clear fringes with the less pure spectral source. In fact it was found that one could remove the filter and use white light without loss of fringe contrast. In this latter case the fringes were black superimposed on a field that showed the spectral variation from red to violet. The lack of influence of the wave length of the illumination, it was realized later, is implicit in equation (3.14) which shows the fringe spacing to be independent of wave length. It was also learned later that Barus[20] had made the same observation

TABLE I a

$$\delta \Upsilon_{(CALC.)} = \frac{2\Upsilon^2}{\ell d}\,\delta(\delta \ell)_{\alpha_1 = \alpha_3 = 0}$$

Υ	$\delta\Upsilon$	$\delta(\delta\ell)$	$\delta\Upsilon_{(CALC.)}$
.025"		.0455"	
	.005"		.028
.020"		.0502"	
	.010"		.010
.010"		.0560"	

TABLE I b

Υ	λ/Υ	$\delta\alpha_3$	γ	$\delta\gamma/\delta\alpha$	$\delta\gamma/\delta\alpha$ (CALC)
.015"	.00142	.00582	0	3.1	3.5
.02"	.001065	.00291	0	6.5	5.4
.03"	.000718	.00145	0	13.2	8.8
.006"	.003	.00436	1.44	1.66	1.7

and since our experiments Sterrett and Erwin[21] in continuing the work of Kraushaar[22] have reported the same effect.

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20 C. Barus, op. cit.
21 J. R. Sterrett and J. R. Erwin, Tech. Note 2827 (National Advisory Committee for Aeronautics, Washington, D. C., 1952).
22 R. Kraushaar, op. cit.

The existence of high-order interference in non-monochromatic light posed problems of interpretation of the exact significance of the quantity appearing under the name coherence. This quantity, which plays a large part in writings on interferometry, is usually defined only by denotation and hence remains a rather indefinite concept. It was hoped that the Marton interferometer could be used to measure this quantity for electrons and hence a discussion of this concept is given at some length in Appendix I.

Table I a shows the test of equation (3.22) and Table I b the tests of equation (3.23). The equations have been expressed in terms of fringe spacing for purposes of convenience. In these tests the 5460 A green line of mercury was used for a source of monochromatic light. The fringe spacing was measured by a steel scale calibrated to .01 in. which was held in the focal plane of the viewing magnifier. The direction of the fringes was measured by visually setting the fringes parallel to a protractor held next to the interferometer. The estimated error from both these procedures is probably greater than 10%.

The qualitative behavior is as predicted. The quantitative disagreements may be explained in large part by the inaccuracy of the instrument, which was never designed for quantitative work, and by the influence of the glass upon which the replicas are mounted. The effect of refraction at the interfaces was found to be by no means negligible. All attempts to use unmounted replicas failed due to uneven shrinkage of the unsupported replicas and to their vibration due to mechanical causes or stray air currents. Both these effects caused the fringes to be unsteady and of low contrast. The apparent systematic deviation in Table I b suggests that σ was not strictly equal to zero.

IV. APPLICATION OF THE THEORY TO THE ELECTRON INSTRUMENT

The fundamental assumption was made at the beginning that the theory just derived was applicable to electrons without any change other than a change of wave length and the substitution of thin single crystals for the gratings. This latter substitution introduced no change of importance since it was intended to use a diffraction order where one of the Laue indices is zero. When one Laue index is zero no loss of generality is incurred by treating the cross gratings as simple gratings due to the high resolution derived from the enormous number of crystal planes involved.

One question to be answered was: Can an instrument be constructed so that the important parameters may be adjusted with close enough tolerances that the fringes will be large enough to be resolved? It was planned from the conception of the instrument to use a compound electron microscope as a viewing magnifier. It was thought that mechanical difficulties would probably necessitate the use of a longer focal length objective than is usual in electron microscopy with the consequent reduction of resolution. It was estimated that the least resolved distance of the microscope used as proposed would be about 100 A instead of the 20 A obtained in normal use. A safety factor of two was adopted and all calculations assumed that the least resolved distance was twice this value. For purposes of calculation nominal values of the important constants were used: the wave length was taken as .05 A, the distance between crystals 5 cm and the angle between the beams was taken to be equal to the diffraction angle of .02 radians.

Under these conditions equation (3.1) gives the maximum permissible value of the angle between the wave fronts. Since

$$\Phi \approx \sin\Phi = \frac{\lambda}{\Upsilon} \leqq \frac{.05}{2 \times 10^3}\,, \tag{4.1}$$

we obtain $\Phi \leqq 2.5 \times 10^{-5}$ RAD.

Equation (3.2) leads to the restriction that must be met by the tilt of the crystals that

$$\theta^2\beta \leqq 2.5 \times 10^{-5} \quad \text{or} \quad \beta \leqq .5 \text{ RAD.} \tag{4.2}$$

This is so weak a restriction that no adjustment was provided to correct it.

A restriction on either first or third crystal rotation is obtained from equation (3.7). We have

$$\tan\sigma = \tan\theta\sin\alpha \approx \theta\alpha \leqq 2.5 \times 10^{-5}$$
$$\text{or } \alpha \leqq 1.2 \times 10^{-3} \text{ RAD.} \tag{4.3}$$

The restrictions on the equality of crystal spacings may be obtained from equation (3.16). Since

$$\Phi \approx \frac{\lambda}{2d}\frac{\delta\ell}{\ell} \leqq 2.5 \times 10^{-5} \tag{4.4}$$

we obtain $\delta\ell \leqq 12 \times 10^{-3}$ cm.

There is another possible restriction on this parameter which arises from questions of the "coherence length" of the electron and considerable thought was devoted to this problem. It is discussed in some detail in Appendix I. If we deal with only the destruction of the fringes by finite source angle and assume that the fringes become invisible if the path difference across them exceeds one-tenth of a wave length (a rather conservative figure) we can apply a formula derived by Bennett[23] for the number of clear fringes N.

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23 F. D. Bennett, loc. cit.

We have

$$N = 2\left(\frac{\delta\lambda}{\lambda}\right)/\eta \;, \qquad (4.5)$$

where $\delta\lambda$ is the allowable path difference across the fringe and η is the source angle. One sees that for a source angle of 10^{-3} radians one can expect to see interference over path differences of 2×10^{5} wave lengths or approximately one micron. This can be translated into instrumental precision by equation (3.10). As

$$\Delta = \tfrac{1}{2}\delta l \sin\theta \tan\theta \approx \tfrac{1}{2}\delta l\, \theta^2 \leq 10^{-3}\ \text{cm} \qquad (4.6)$$

we obtain

$$\delta l \leq 20\ \text{cm} \;.$$

This is no restriction at all.

It appeared possible to meet all these requirements in the proposed instrument easily and thus construction was begun.

It should be noted that the comparatively lax requirements on the precision of the instrument are a result of two circumstances. The first is that the instrument is very highly skewed due to the smallness of the diffraction angle involved, and the second is the use of diffraction for beam splitters and deflectors. The form of the diffraction equation reduces the stringency of the restriction on crystal tilt. As mirror instruments do not have this advantage, it would be extremely difficult to build one for electrons.

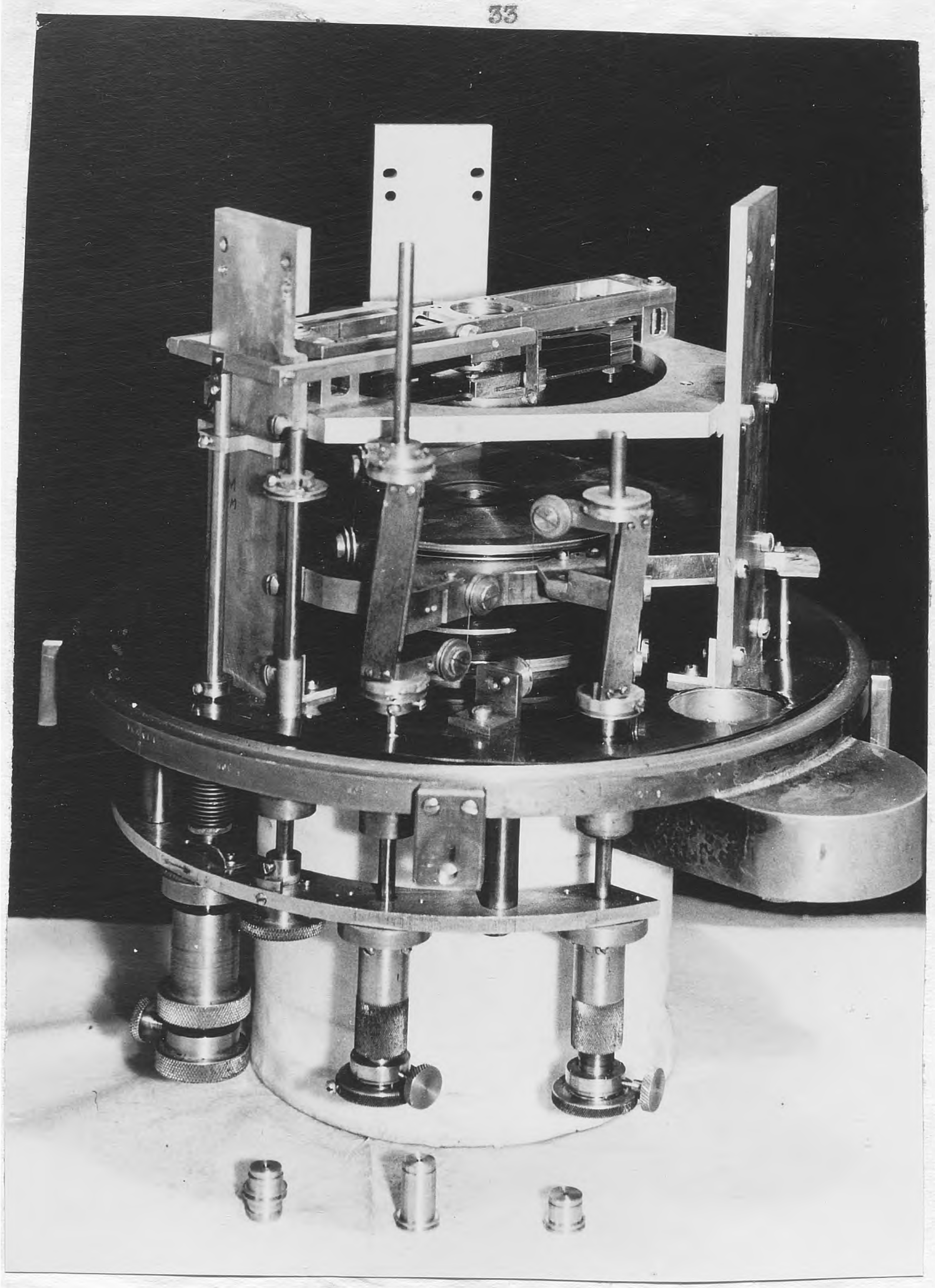

Fig. 7 View of electron interferometer with vacuum case removed.

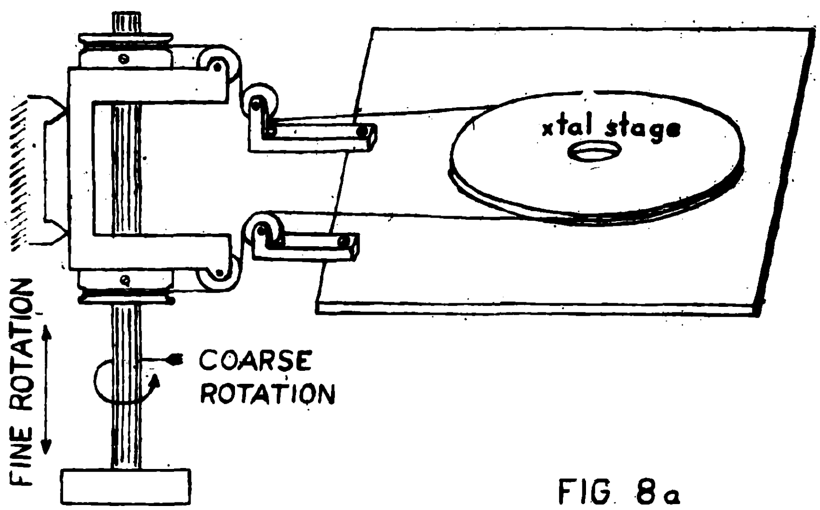

FIG 8a

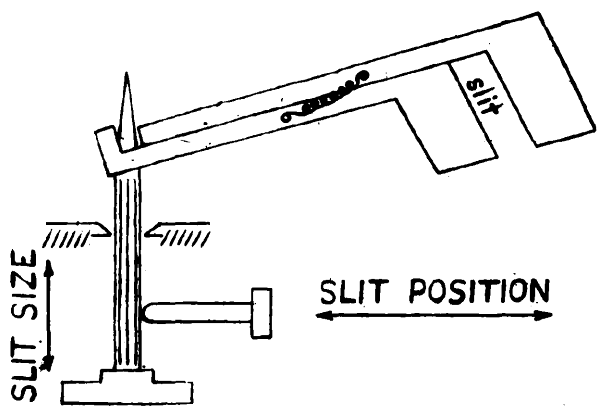

FIG 8b

Mechanical details of crystal stages and slit control.

V. INSTRUMENT DESIGN

Fig. 7 is a photograph of the instrument in its final form. All three crystal stages may be rotated, with the second and third fitted with a fine drive developed from a tangent screw. The mechanism is complicated, as are all electron optical devices, by the necessity of vacuum-wall sealing while still permitting mechanical disassembly for repair and replacement of parts. A schematic of the fine drive which may be set to less than .0001 radians is shown in Fig. 8a. The control rod passes the vacuum wall through a "Wilson" seal which permits it to be rotated or translated axially. The rotary motion, which winds the phosphor bronze belt off and on the two drums, is used for a coarse control and permits about 120° of crystal rotation. The fine control is achieved by locking the rotation of the control rod by use of a set screw, and translating it axially by a micrometer screw. As can be seen by Fig. 8a this movement results in a slow rotation of the stage, as the belts tighten and loosen on opposite sides of the stages. In order to avoid any possibility of the stage rotating due to thermal expansion of the control rod, back lash is deliberately introduced into the system by allowing a small amount of slack in the belt.

The rotating stages are mounted on beryllium copper ball bearings with phosphor bronze friction members that serve both to preload the bearing and to prevent any creep due to external vibrations.

The first crystal is also fitted with a device for moving it parallel to the optic axis. This mechanism consists of a control rod sealed to the vacuum wall by means of a metallic bellows to permit axial displacement. The rod actuates a rocker assembly to reverse the direction of motion and to transfer it to the crystal stage. The rod is moved by means of a differential screw which gives the state either of two rates of travel, approximately .025 mm or .0025 mm per revolution of the control. To maintain parallelism the stage is supported by a double parallelogram of phosphor bronze members which deform elastically during the motion of the stage. The elastic deformation "slide" holds the assembly parallel to within one micron in the 1 mm of travel. It was found necessary to add damping to this mechanism since it is essentially frictionless. This damping was added by paralleling the bronze leaves with lead members so proportioned that they suffer plastic deformation. The final assembly is capable of being set to .0005 mm and in the presence of external vibration the instrument itself vibrates as a unit as far as can be detected.

Below the second crystal may be seen the variable aperture which is so arranged that it may be positioned and its size varied by means of only two controls coming through the vacuum wall on metallic bellows. Fig. 8b shows a sketch of this mechanism. The movable vacuum seal is achieved by means of another metallic bellows on each of the two controls, which differ only in being displaced 90° around the axis of the instrument. Each control can, by means of appropriate screws, be translated axially or pivoted about the knife edge fulcrum. The width of each of the crossed slits making up the aperture is controlled by the axial motion which forces the conical tip of the rod up through the spring loaded jaws of the slit. The position of the entire slit assembly, jaws, spring etc., is controlled by the position of the conical tip which in turn is fixed by the position of the control rod as it is rotated about the fulcrum. The aperture was designed to permit either of the two beams to be blanked out in order that interference phenomena could be demonstrated. The crystals, about 3 mm (1/8 in.) in diameter, are carried in cartridges in the three stages. The center of each stage is displaced slightly off the optic axis so that by rotating the cartridges 90° a different suitable spot on the crystal is exposed to the beam and hence the life of the crystals increased fourfold.

The instrument remained essentially unchanged during the course of the investigation except for reducing the crystal spacings from 50 to 34.9 mm and the changes necessary to bring the last crystal closer to the objective

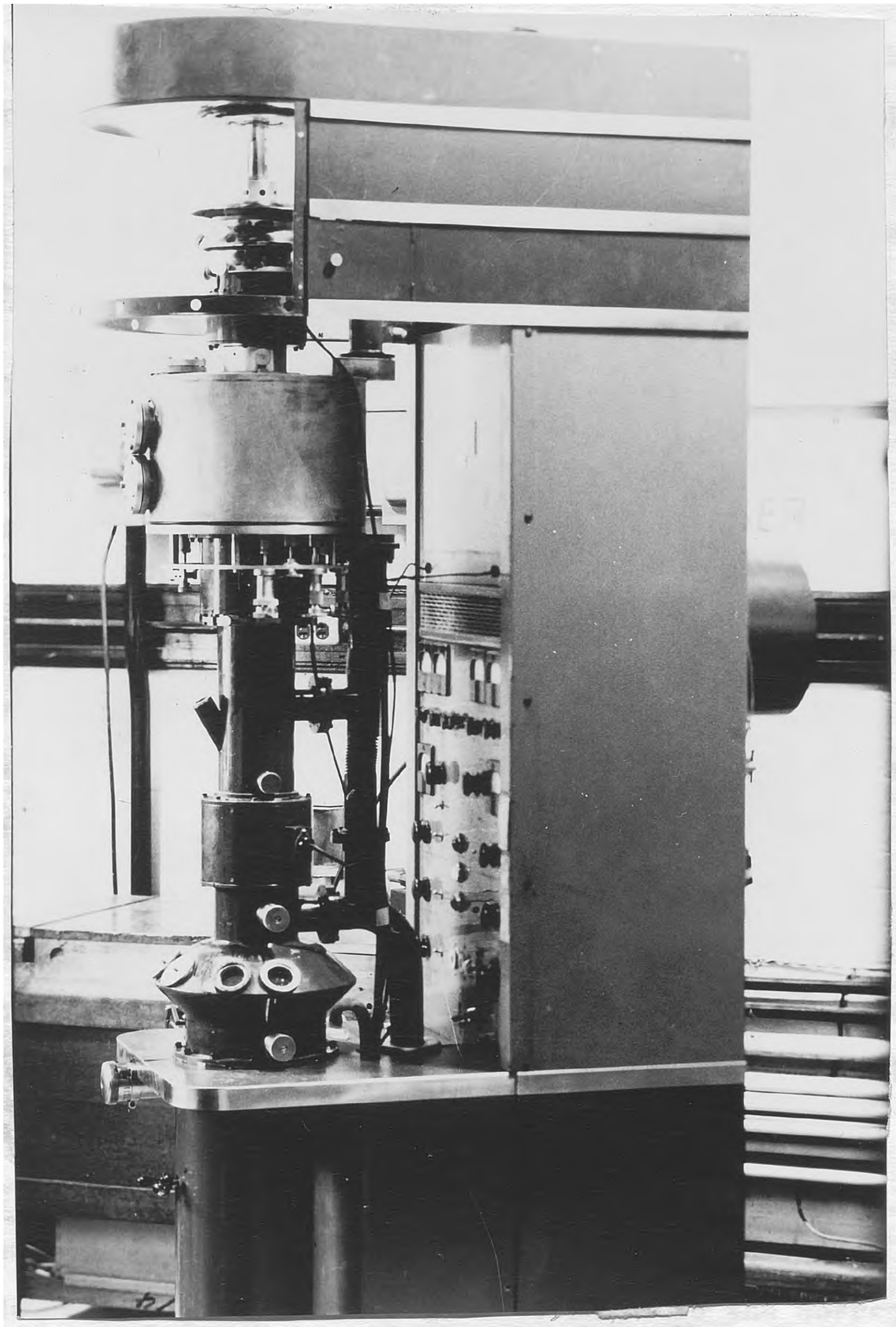

Fig. 9 View of the electron interferometer installed in the electron microscope.

lens. These changes were made to decrease scatter from the cartridge walls and to increase the resolution available in the plane of the third crystal by permitting the use of a shorter focal length objective. Shorter focal length electron lenses suffer less from spherical aberration in addition to permitting high initial magnification.

The vacuum chamber is in the form of a metallic bell jar which may be lifted off for maintanance of the assembly, while the crystals may be changed or removed through suitable ports without disturbing any other alignments. This vacuum chamber supports the gun and condenser assembly and is provided with a micrometer screw so that the illuminating beam may be offset the required amount by the distortion of a metallic bellows.

The interferometer is mounted in place of the object chamber of a magnetic electron microscope which provides both the beam and the magnification necessary for viewing the fringes. Fig. 9 shows it in position. The microscope is otherwise unmodified except for minor circuit changes that allow a higher gun filament temperature, at the expense of filament life, and a pole piece of conventional design in the condenser lens to increase the correction of this lens and thus reduce the cross section of the beam. The electron gun was usually operated at a potential difference of 60 kv with self-biased saturated emission of

300 microamperes.

In order to reduce the intensity of inelastically scattered electrons the objective was fitted with an aperture of 75 microns while a matching condenser aperture of 260 microns was used. The condenser aperture provided critical illumination, thus achieving a maximum usable intensity for a minimum amount of thermal loading on the crystals.

VI. EXPERIMENTAL TECHNIQUES

The crystals were produced by oriented growth from the vapor phase on a crystalline substrate at elevated temperatures. This process, known as epitaxy, was already under study in the Electron Physics Section of the National Bureau of Standards for the production of single crystals for scattering experiments and a tentative theory had been worked out.[23] Additional studies were made to determine the best

23 O. G. Engel, J. of Chem. Physics 20, 1174 (1952) and J. of Research Nat'l. Bur. Standards (in press).

materials for application to the electron interferometer. The epitaxial temperatures were discovered empirically by experiments made around the predicted temperature. The metal was evaporated from wolfram filament or a molybdenum "boat," while the freshly cleaved rock salt substrate was heated in a resistance furnace.

This furnace was in the form of a porcelain tube some 5 cm in diameter and about 12 cm long which was wound with #23 nichrome wire. It was designed to be used with its axis vertical and at its mid-point was suspended a nickel shelf or bulkhead. This bulkhead, to which an iron constantan thermocouple was attached, served to support the substrate during evaporation. The metal was evaporated upwards through a series of 3 mm holes drilled in the nickel plate. These holes were positioned so that five metallic disks are deposited on each of three (1 cm x 1 cm x .1 cm) cleaved pieces of rock salt. When in use the furnace is supported on a small tripod stand about 10 cm above the evaporating filament or "boat." The entire set-up is within the bell jar of a commercial vacuum evaporator. With this geometry the possibility of metal "splashing" on to the substrate is minimized and the heat of the evaporation process does not appreciably raise the substrate temperature. This temperature for most common metals is between 100° and 400° C.

Since contamination deposited by the beam limited the crystal life to about twenty hours of operation,we were restricted to metals which were easily evaporated. This requirement eliminated the platinum family as well as any of the extremely high-melting point metals. Some metals which were otherwise satisfactory could not be completely oriented while others like silver oriented sufficiently but were full of small holes. Among the metals tried only gold, nickel and copper, in order of increasing virtue, were found practical. The thickness of the crystals was calculated from the mass of metal evaporated and the evaporation geometry, after experiments with a Tolansky[24] interferometer

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24 S. Tolansky, MULTIPLE BEAM INTERFEROMETRY (Oxford Univ. Press, London, 1948).

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had shown that this method was accurate enough for our needs (about 25%). The degree of orientation was tested by taking a diffraction pattern of one crystal of each evaporation.

The mounting technique consisted of floating the crystal off the cleavage face onto the surface of the water and then "scooping" the crystal on to a 3 mm mount. Even with the noble metals this must be done as soon as possible after the crystals are grown, it was found that otherwise the crystals would not float off the rock salt. Some batches of crystals would not float off under any conditions. Very little is known about the nature of the epitaxial bond and only about 25% of the crystals grown were usable.

The mounts were one of two designs tried during the course of the experiments; the first consisted of a 3 mm disk of 20 mesh/mm electromesh, the second of a 3 mm disk of copper with a 1 mm hole. This latter consisted of a very carefully polished copper disk about .5 mm thick with a hole in its center. Contrary to expectations it was found that these crystals, so thin as to be barely grey under transmitted light, would support themselves over this area if the edge of the hole was very sharp.

Considerable stress is applied to the crystal by the surface tension of the water as the surface between the mesh wire or across the hole ruptures in the drying process. Attempts to lower the surface tension by additives to the water or the use of liquids other than water failed either by contaminating the crystals or by dissolving them. The latter was true of all organic solvents even when specially purified. For some reason not understood the surface water that had been exposed to the laboratory atmosphere for several weeks was the best of all.

The crystals as grown seem to be in a state of strain as they tend to ripple on the surface of the water. Attempts to improve this situation by evaporating a heavy frame around the crystals only made this rippling worse. It seems surprising that the films remained single crystals during the stress of mounting. It is thought that the crystals are below the thickness where crystal dislocations can be supported, and the diffraction patterns support this belief. Great difficulty was experienced in obtaining films of less than 150 A that would survive the mounting technique and the overall productivity of the process was about 10%. Thicker crystals were useless due to extensive beam attenuation and inelastic scattering. When the crystals were on the mounts they were placed in the cartridges and were ready for use.

For alignment purposes the instrument was very carefully adjusted by the usual electron microscope techniques[25]

25 W. G. Wyckoff, ELECTRON MICROSCOPY (Interscience Press, New York, 1949).

so that the zeroth diffraction order of the first crystal fell along the optic axis. The second crystal was then inserted and the gun condenser assembly offset the amount calculated from the diffraction angle and the intercrystalline distance in use. This offset amounted to about .75 mm for copper, when the crystals were spaced 34.9 mm apart. Under these conditions as the first crystal is rotated the first order beam will generate a right circular cone with its base in the plane of the second crystal and its vertex at the first crystal. The offset is so chosen that the optic axis of the microscope intersects the cone at its base. When the beam generating the cone passes through this intersection there are enough electrons inelastically scattered parallel to the optic axis to be detected. The appearance of this weak beam was the criterion for the adjustment of the first crystal. If the second crystal is now rotated, at the position when its crystalline planes are parallel to those of the first crystal the electrons will not only be scattered inelastically parallel to the optic axis, but also diffracted in the same direction. The intensity of the emerging beam is thus greatly increased at this position. The adjustment of the first two crystals consists of maximizing this beam while the offset is carefully adjusted to place the beam in coincidence with the optic axis.

The third crystal was now inserted and rotated until its crystalline planes were aligned with those of the second as evidenced by the appearance of a second beam in close proximity to the first. The two beams were then caused to overlap by fine adjustments of the second two crystal rotations and the height of the first crystal. To achieve the highest angular resolution the microscope objective was focused as far from the third crystal as the available intensity permitted. The condenser was adjusted for maximum brightness of image and the projector set at the desired magnification. This alignment procedure took about three hours, including the pump down times of the instrument after each crystal was inserted.

As additional crystals were inserted the intensity of the image fell drastically. With the third crystal in place and a magnification of 500 x, the maximum intensity available, using a tungsten cathode with a life of 10 hours, gave beams that were just visible to the dark-adapted eye.

This low intensity required that the fringes be sought photographically. Exposure of approximately six minutes on Kodak Medium Lantern Slides gave usable images after development in D 76 for five minutes. The search for the exact adjustment of the two rotations, the first crystal height, and the correct focus proved to be a time-consuming and rather frustrating procedure. The fringes were first recognized on the 1,231st exposure.

The plates,when dry,were examined under a hand lens and those showing areas of apparent interest were enlarged optically 10 to 15 x. The fringes were difficult to recognize due to the high level of "noise" caused by inelastically scattered electrons, faults in the Na Cl cleavage planes and bent crystal interferences of the type recently studied by Heidenreich.[26] A great deal of effort was put into reducing

26 R. D. Heidenreich, Bell System Tech. J. 30, 867 (1951).

this "noise." The aperture of the instrument was repeatedly reduced by the use of appropriate stops, the crystal holders were modified several times in hopes of reducing scatter from their walls, and the cleavage planes of the rock salt were carefully selected. For some reason not clearly understood the perfection of these planes varied greatly from crystal to crystal,although all were of the highest optical quality and obtained from the same source. It was hoped that if the crystals could be mounted without the supporting mesh the "noise" would be greatly reduced. Unfortunately, when this was accomplished it was found that although fewer electrons were inelastically scattered ,the bent crystal interferences were even more prominent.

Once the electron interference fringes were recognized, they were found on a number of succeeding plates. However, difficulty was experienced in maintaining them for a series of six consecutive exposures. When such a series was obtained it consisted of single beam exposures and a set of double beam exposures with controlled changes of one variable, usually the third crystal rotation.

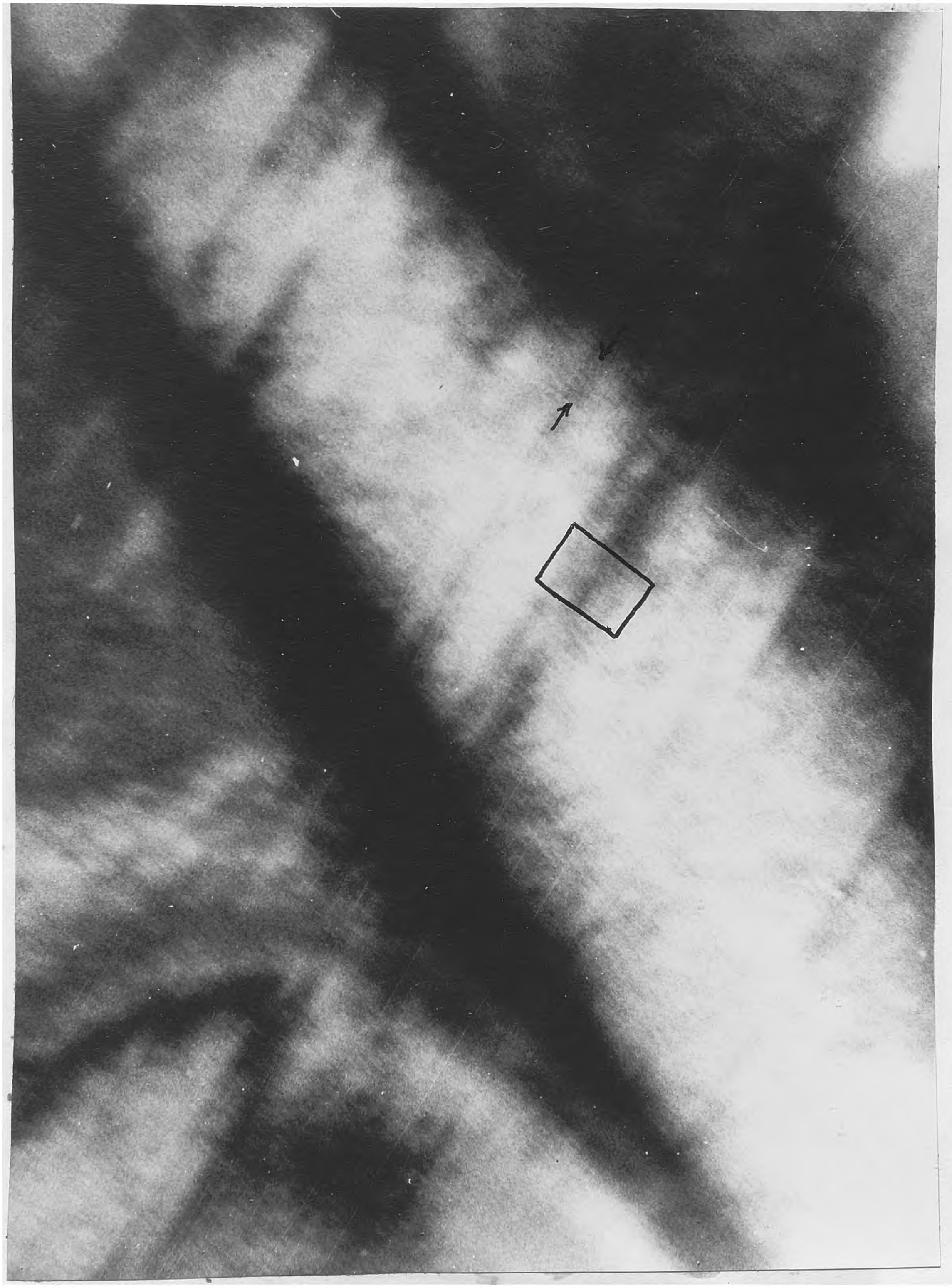

Fig. 10 Interferometer field of view with enlarged portion and densitometer path indicated. Both beams and fringes are visible.

Fig. 11 Same field of view as Fig. 10, but only
Beam I is present, fringes not visible.

Fig. 12 Same field of view as Fig. 10, but only Beam II is present, fringes not visible.

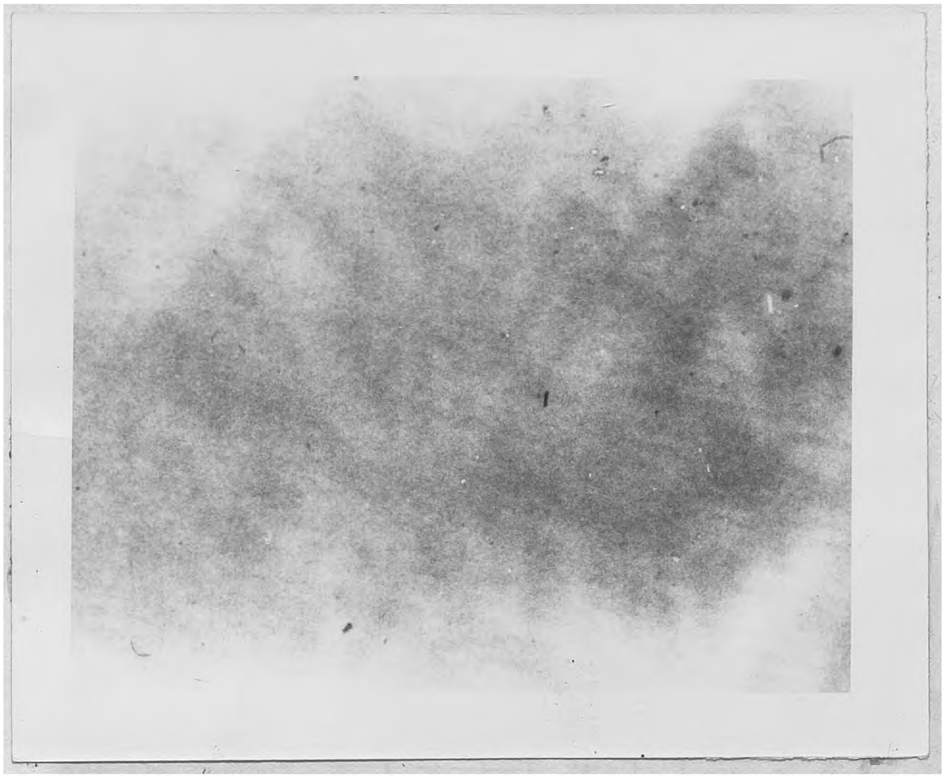

Fig 13 Enlargement of central field of view with contrast increased by multiple printing.

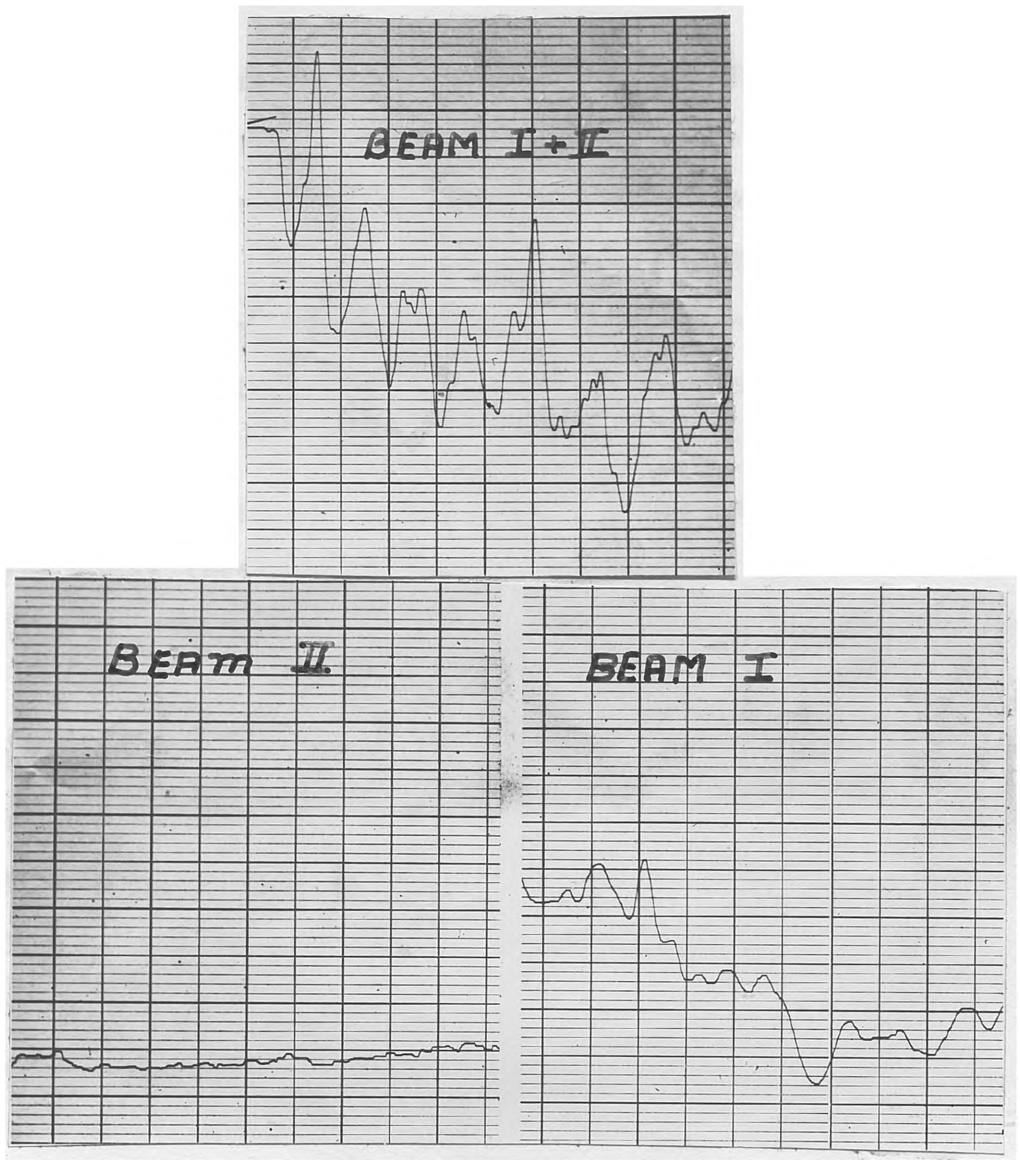

Fig. 14 Densitometer traces showing the necessity of both beams for the production of interference.

VII. RESULTS

The first unambiguous fringes obtained are shown in Figs. 10, 11 and 12. Fig. 10 shows the appearance of the whole field of view at a magnification of about 6,000 diameters. The large scale structure is the dark-field reproduction of the ripples in the crystal; the sharp lines running from lower left to upper right are faults in the cleavage planes of the rock salt upon which the copper crystal was grown. The fringes themselves are much smaller in scale and run from upper left to lower right portion of the field. Figs. 11 and 12 show the same field of view with alternate beams blocked off by the movable aperture. The fringes are clearly missing in either beam. The central area of Fig. 10 is shown enlarged and with the contrast increased by multiple printing in Fig. 13, where the fringes are more easily seen. Fig. 14 shows the microdensitometer tracings between the arrows on Fig. 10 with alternate interfering beams blanked out and with both beams present.

In the field of view of the original plate 154 fringes were counted with an average spacing of 1650A. The direction of the fringes was approximately parallel to the plane of the interfering beams. Due to the rotation introduced by the magnetic objective and projector lens this orientation could not be determined with precision with the instrument as now constructed.

Plate #587 Table II

crystals: copper $d = 1.804$ Å
beam voltage: 60 kev nominal $\lambda =$ .048
diffraction angle: .0269 radians

Exposure	Angle of fringe relative to plate	$\delta\gamma$	$\delta\gamma/\delta\alpha$	Calculated γ	Measured γ
3	83° 50'	8°00'	3.56		
4	75° 50'	8°50'	3.66	85° 37'	90° ± 10°
5	67° 00'				

A series of exposures was taken rotating the third crystal in approximately two degree steps and the orientation of the fringes measured with respect to the crystal fault of Fig. 10.

The results of this series of measurements are summarized in Table II.

To the accuracy with which we were able to determine the fringe orientation the value of τ, calculated from equation (3.18), assuming $\sigma_{II} = 0$, is in agreement with observation. In view of the strong τ dependence of equation (3.18) closer agreement was not expected. It will be noted in column (4) of Table II, however, that the derivative $\frac{d}{d\tau}\left(\delta\tau/\delta\alpha\right)$ is negative as the theory predicts.

Continuing with the assumption that $\sigma_{II} = 0$, which is in agreement with the observation that unless this adjustment is made with great care the beams are vastly different in intensity, we can use equations (3.14) and (3.9) to calculate the mean path difference in the fringe system. We obtain

$$\Delta = \frac{\ell d\theta^2}{\Upsilon} = 3.49 \times 10^8 \times \frac{1.804\,A}{1650\,A}(.026)^2 = 277.5\,A\,.$$

The order of interference, Δ/λ, is therefore approximately 5800. This sets a lower possible limit to the coherence length on the electron. This value greatly exceeds any previous experimental determination.

VIII. FUTURE DEVELOPMENT

It would appear to be well worth the effort to carry this work further. There are some interesting questions that may be answered if new techniques for the production and mounting of crystals can be perfected, so that the instrument becomes more easily lined up and the fringes more clearly resolved. It is believed that if a technique were developed for mounting the crystals without the large scale ripples in them, it would be possible to increase greatly the angular aperture used as well as the intensity and fringe contrast. The same effect could be achieved if the crystal thickness were reduced considerably below 100 Å.[27]

27 R. D. Heidenreich, loc. cit.

The dependence of fringe spacing on geometrical constants and on beam composition would then be studied in detail. This should be coupled with an extensive quantum mechanical study of the phenomena treated classically in Appendix I. For this work the instrument should be reengineered to provide means for centering each crystal individually and to permit measuring of angles of rotation and changes in crystal spacing, preferably by light-interferometric methods. The viewing instrument's resolution should be further improved.

As it has now been shown possible to carry on interferometric calculations classically with results of sufficient accuracy to permit a successful instrument to be constructed, a further attempt should be made to construct an interferometric spectrograph for "coherence" measurements on electrons.

IX. CONCLUSIONS

The electron interferometer proposed by Marton is a realizable instrument which will form predictable and controlled fringes with path differences of thousands of wave lengths. This particular instrument belongs to an unused, but not unknown, class of instruments which form high order fringes in "white" radiation. Unfortunately it can thus throw little light on the problem of electron coherence. This characteristic rules out therefore any possible application of the instrument as an interference spectrometer for the determination of band structure of inner potential of solids as had been suggested.

The production of electron interference fringes with controlled geometry, whose behavior may be predicted by classical theory, shows that the analogy between photon and electron optics extends much further than has generally been assumed. Its validity in the field of interferometry raises questions as to the ultimate limit of the analogy. In Appendix I we will deal with a further step in the attempt to push this analogue to its limit, when we consider some experiments which might throw some light on the rather thorny subject of the significance of an electron's phase velocity. Perhaps the late E. J. Williams was not exactly correct when he thus answered his own question "Is the electron a wave or a particle? It is of course a particle. The wave properties of the elec-

"tron are not properties of the electron, but properties of quantum mechanics."

APPENDIX I

In almost any text in wave optics one of the first statements made when dealing with interference is that the two sources must be coherent. It is usually pointed out that this means that there must be a fixed and unambiguous phase relation between the two wave trains. With this statement no fault can be found. However, the next topic discussed is the size of the monochromatic source that may be used to obtain n fringes in Young's double slit experiment. It is at this point that a minor confusion arises since now the concept is transferred from one concerning the interaction between two beams to the properties of a single beam arising from a single source. These are usually referred to as the coherence properties of the source or more correctly the coherence of the beam. The properties of a beam early attracted attention since they are important in the theory of microscope image formation. The partisans of Rayleigh and Abbe publicly debated for half a century different views on image formation. Zernike[28] believes he has put in the last word and essentially

28 F. Zernike, Proc. Phys. Soc. 61, 147 (1948).

rationalized the conflicting views on this subject. In another paper Zernike[29] treats the matter of coherence in a

29 F. Zernike, Physica 5, 785 (1938).

simpler and more straightforward manner than most of the other authors who have written at length on the matter. A short bibliography is attached at the end of this section for the interested reader.

Zernike defines simply the coherence properties of the beam in terms of the ability of any two points p and p' in the wave field to interfere. He adopts Michelson's[30] visi-

30 A. A. Michelson, Phil. Mag. 31, 338 (1891).

bility index K as a measure of this ability, which is defined by

$$K = (I_{max} - I_{min})/(I_{max} + I_{min}) . \quad (10.1)$$

K can therefore vary between 1, complete coherence, and 0, complete incoherence. Zernike shows that this index bears a simple relationship to the coherence of the two points.

The coherence thus defined depends on a number of properties of the source, one being the angular aperture of the source when viewed from the points in question. This dependence is the basis of Michelson's stellar "interferometer."[31] Gabor[32] extends the analysis of this type of coherence to the

31 A. A. Michelson, Phil. Mag. 30, 1 (1890).
F. Zernike, loc. cit.
32 D. Gabor, Proc. Phy. Soc. B 64, 462 (1951).

case of three-dimensional sources which are avoided in most light applications by the use of apertures. In any event effects depending on source geometry will be called the geometrical coherence of the source.

The second principal type of coherence, which we shall call chromatic coherence, depends on the spectral purity of the source. This type of coherence is not so generally discussed as the geometrical one but is of greater interest for its practical applications. This quantity may be used to determine the spectral distribution of the source, as Michelson[33] did in his pioneer work in hyperfine optical spectroscopy. Although, as Rayleigh[34] pointed out, certain assumptions must be made about the symmetry of line shape, Michelson's method appears an attractive one for application to electron spectroscopy.

33 A. A. Michelson, Phil. Mag. 34, 280 (1892).
34 Lord Rayleigh, Phil. Mag. 34, 407 (1892).

There is however one serious difficulty with our discussion so far, a difficulty that exists throughout the literature. This difficulty is that in order to measure K we must have an instrument of some kind capable of selecting the points p and p' and causing radiation from them to overlap and form interference fringes, i.e. an interferometer. In brief we must have an operational definition for the concept of coherence of a beam. It is at this point where confusion arises in great abundance, for unfortunately K then becomes a function of this instrument and of the way it is used to form the fringes.

Consider geometric coherence first. A finite monochromatic source will give few fringes with a Fresnel mirror or biprism, will give more with Lloyd's mirror (a little known fact) and an unlimited number with a Michelson instrument. Michelson implicitly assumed complete geometric coherence in his spectroscopic work since he assumed K to be a function of spectral purity alone. Bennett[35] has recently

35 F. D. Bennett, J. Appl. Phys. 22, 776 (1951).

worked out a special case of the more general condition where K is a combined function of geometrical coherence and spectral coherence. The number of fringes will be greatly reduced if the Michelson instrument is being used in the usual Twyman-Greene modification[36] with a condensing lens and a telescope

36 L. Twyman, Phil. Mag. 35, 49 (1918).

for viewing. The discussion of this phenomenon is extensively given in the more comprehensive texts[37] on physical optics.

37 G. Bruhat, COURS d'OPTIQUE (Masson, Paris, 1935), Chap. IV as an example.

This apparent increase in K arises simply from the fact that amplitude-dividing instruments usually choose p and p' to be very close together.

The analogous behavior relative to chromatic coherence is less widely known, probably because the interferometers in use do not have this property. However, there are at least two distinct types of interference fringes which are visible to high order in non-monochromatic light. The first of these has long been known under the name "spectra cannelle"[38] and arises essentially from a conventional interferometer followed by a spectroscope. The second type is the "achromatic" fringes studied by Rayleigh[39] which arise from a spectroscope

38 See H. Bousse and Z. Carriere, INTERFERENCES (Delagrave, Paris, 1923), Chap. VI.
39 Lord Rayleigh, Phil. Mag. 28, 77 (1889).

followed by an interferometer. Both these arrangements avoid the usual gradual loss of fringe contrast due to the overlapping of the fringe systems formed by different wave lengths.

If the interferometer contains dispersive elements such as prisms or gratings, both these types of phenomena can occur in the instrument itself. The previously mentioned papers of Barus describe an extensive set of non-systematic studies made of this type of interferometer. Both types of instruments give visibility curves which do not depend on the spectral purity of the source, but only on the resolution of the dispersive elements in the instruments. Around the turn of the century there was considerable controversy about the significance of coherence under this condition. This controversy, which involved among others Gouy, Rayleigh, Shuster and Poincaré, is summarized by Wood.[40] Unfortunately, before any

40 R. W. Wood, PHYSICAL OPTICS (Macmillan, New York, 1905), Chap. XXI.

firm conclusions were arrived at, it evolved into a discussion of the mechanism of the emission, before quantum mechanics, and is thus not very informative.

The interferometer used to measure the coherence may not only increase the apparent coherence of the beam in the manner just discussed but, if the instrument has polarization properties or filter properties, it may destroy the coherence as in the polarization experiments of Fresnel and Arago. This is a case of destroying the coherence by reducing the indeterminacy of the problem upon which, in the quantum view, interference depends. It has been suggested that this type of destruction of coherence, after the beam has been divided, can best be dealt with in quantum mechanical language by means of a density matrix to describe the states of the particle in the interferometer.

There are some added difficulties if the medium through which the radiation travels is such that the group velocity and phase velocity are not equal for all frequencies. In this case, where the medium is dispersive in the usual optical sense, a complex wave packet spreads out as it travels through space. This condition is true for waves spreading over the surface of water, which have been analyzed in detail.[41]

41 H. Lamb, HYDRODYNAMICS (Cambridge Univ. Press, Cambridge, 1924), p. 373 et seq.

The result of this analysis is that the longer the motion continues the more nearly monochromatic the groups become, and the greater is the number of waves which succeed each other with less than some assigned change in wave length.

This behavior would seem to imply that an interferometer operating in a dispersive medium would form a greater number of clear fringes than the same instrument in a non-dispersive medium. That this may indeed be the case for light can be shown in a fairly straightforward manner. The calculation is given in some detail because of its interesting results if one tries to apply it to electrons.

Consider an interferometer submerged in a medium of refractive index μ, where μ is a function of wave length. Let it be forming parallel fringes such that $y = bz$, where y is distance from zero fringe z is the difference of two paths and b is assumed to be a constant.

The optical path difference at a point x is then given by

$$\Delta = \left(\frac{y}{b}\right)\mu \,. \tag{10.2}$$

The positions of the bright fringe is given by the solutions for all integral n's of

$$\Delta/\lambda = y/b \; \mu/\lambda = n \quad \text{or}$$

$$y = \frac{n\lambda b}{\mu} \,. \tag{10.3}$$

The width of the fringe Y is then $\delta y/\delta n$ or

$$Y = \frac{\lambda b}{\mu} \,. \tag{10.4}$$

If the fringe is to be sharp, we must have

$$\delta y / Y \ll 1 ,$$

where δy is the change in y due to a change $\delta\lambda$ in wave length

Differentiating equation (10.3) with respect to wave length and substituting from equation (10.4) one obtains

$$\frac{\delta y}{Y} = n\frac{\delta\lambda}{\lambda} - \frac{n}{\mu}\left(\frac{\delta\mu}{\delta\lambda}\right)\delta\lambda . \qquad (10.5)$$

Rearranging equation (10.5) one obtains

$$\frac{\delta y}{Y} = \frac{n\delta\lambda}{\mu\lambda}\left(\mu - \lambda\frac{\delta\mu}{\delta\lambda}\right) . \qquad (10.6)$$

If we introduce the group velocity defined by

$$U = \frac{d(kV)}{dk}$$

where k is the wave number,

$$k = \frac{2\pi\mu}{\lambda} = \frac{2\pi}{\lambda}\frac{V_o}{V} ,$$

and where V_o is the free space velocity, one gets

$$\frac{V_o}{U} = \frac{V_o\, dk}{d(kV)} = \frac{d(\mu/\lambda)}{d(1/\lambda)} = \mu - \lambda\frac{\delta\mu}{\delta\lambda} . \qquad (10.7)$$

Substituting into equation (10.6) one obtains

$$\frac{\delta y}{Y} = \frac{n\delta\lambda}{\lambda\mu}\left(\frac{V_o}{U}\right) . \qquad (10.8)$$

It is interesting to calculate the ratio of fringe visibility with and without dispersion.

Without dispersion equation (10.8) becomes

$$\frac{\delta y}{Y} = \frac{n_o \delta \lambda}{\lambda \mu}\left(\frac{V_o}{V}\right). \tag{10.9}$$

Therefore

$$\frac{n}{n_o} = \mu U/V_o = \frac{U}{V}. \tag{10.10}$$

Before attempting to apply this analysis to the electron, we must look more closely at the wave mechanics of the free electron. The usual treatment of electron optics deals only with the Newtonian mechanics of a beam of charged particles, or expressed in wave language, the behavior of a monochromatic plane wave. To close the gap between the viewpoints it is only necessary to introduce an index of refraction depending on the electron's energy E and on its potential energy in space F such that

$$\mu^* = \sqrt{\frac{E-F}{E}}$$

Thereupon the entire structure of geometrical optics is applicable without change to electron beams.[42]

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[42] V. K. Zworykin et al, ELECTRON OPTICS AND THE ELECTRON MICROSCOPE (John Wiley and Sons, New York, 1945), Chap. X.
or
V. E. Cosslett, INTRODUCTION TO ELECTRON OPTICS (Oxford Univ. Press, London, 1946), Chap. I.
or
W. Glaser, GRUNDLAGEN DER ELEKTRONENOPTIK (Springer-Verlag, Vienna, 1952), for a truly exhaustive treatment.

In our case, where the important thing is the nature of each individual electron, this analysis is insufficient and one must return to more fundamental considerations. These are given in detail by Glaser and Thomson[43] whose

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43 W. Glaser, op. cit.
G. P. Thomson, THE WAVE MECHANICS OF THE FREE ELECTRON (McGraw-Hill Book Co., New York, 1930).

---

treatment we will follow rather closely.

If one writes for the wave describing a free electron, as given by De Broglie,

$$\psi = \frac{a \sin 2\pi\nu_0\left(t - \frac{ux}{c^2}\right)}{\left(1 - \frac{u^2}{c^2}\right)^{1/2}} , \tag{10.11}$$

where $\nu = mc^2/h$,

one sees upon comparison with the standard expression a traveling wave,

$$\phi = 2\pi\nu\left(t - \frac{x}{V}\right) , \tag{10.12}$$

that for an electron wave, the phase velocity V is given by

$$V = c^2/u . \tag{10.13}$$

For a general wave, the group velocity is the relationship between x and t when the argument of the sine is an extreme for variation of frequency (for an electron this means the variation of u).

Thus if one lets

$$\left(1-\frac{u^2}{c^2}\right)^{-1/2}=\beta$$

and takes the derivative of the argument of the sine in equation (10.11) and sets it equal to zero one obtains

$$t\,\frac{d\beta}{du}-\frac{x}{c^2}\,\frac{d(\beta u)}{du}=0\,. \qquad (10.14)$$

Therefore, the group velocity equals

$$U=\frac{x}{t}=c^2\,\frac{\dfrac{d\beta}{du}}{\beta+\dfrac{d\beta}{du}}\,, \qquad (10.15)$$

but

$$\frac{d\beta}{du}=\frac{\beta u}{c^2-u^2}\,.$$

Hence we obtain

$$U=u$$

giving, as we expected, that the group velocity is equal to the particle velocity. These results could also have been achieved by considering an interrupted stream of electrons, as is done by Mott and Massey.[44]

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44 N. F. Mott and H. S. W. Massey, THE THEORY OF ATOMIC COLLISIONS (Oxford Univ. Press, London, 1950), Chap. I.

To show that the electron waves are dispersive in the usual optical sense we remember that for plane waves the wave length

$$\lambda = \frac{V}{\nu} = \frac{c^2}{u\nu} = \frac{mc^2}{mu\nu} = \frac{h}{mu} . \qquad (10.17)$$

If one eliminates u between equations (10.17) and (10.13) one obtains

$$V^2 = c^2 + \frac{\lambda^2 m^2 c^4}{h^2} . \qquad (10.18)$$

This expression shows the normal wave length dependence of the phase velocity in a dispersive medium. That dispersion results in a vast spreading of the electron wave packet was demonstrated by a calculation of Darwin[45]

45 G. G. Darwin, Proc. Roy. Soc. 117, 258 (1927).

many years ago. He gives as the uncertainty of the position of an electron at a time t about a position x + vt

$$\Delta x = \sqrt{\Delta x_0^2 + \left(\frac{ht}{2\pi m \Delta x_0}\right)} .$$

This uncertainty is usually construed to be the effective length of the wave packet.

To obtain an idea of the magnitude involved in this spreading in a practical case we consider an electron with an energy of 50 kev $\pm$ 5 ev, which corresponds to a Δp of 1.8×10^{-19} gm cm/sec. Making use of the Heisenberg relationship we obtain $\Delta x_0 = 5.5 \times 10^{-6}$ cm. From a knowledge of the velocity corresponding to 50 kev one can calculate the length of time for the particle to cover 100 cm. Then applying Darwin's formula one finds that the new uncertainty is 1.4×10^{-1} cm, an increase by a factor of 10^5.

A difficulty arises if we use the non-relativistic Schrödinger equation instead of the relativistic approach of De Broglie. The fundamental equation is

$$\nabla^2 \psi + \frac{8\pi^2 m_0}{h^2}(E - F)\psi = 0, \tag{10.19}$$

where E is the energy and F the potential energy. In free space where $F = 0$ the equation for a one-dimensional wave becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m_0}{h^2}\left(\tfrac{1}{2} m_0 u^2\right)\psi = 0. \tag{10.20}$$

If this equation is solved by the usual methods and compared with equation (10.12) one finds that the phase velocity is given by

$$V = \tfrac{1}{2} U. \tag{10.21}$$

If we compare equations (10.21) with (10.13) we see that our two approaches have yielded different results. However if we had introduced the rest energy of the electron into equation (10.19) we would have arrived at a result similar to equation (10.13). The group velocity remains unchanged by this substitution if the relativistic value of the particle velocity is used.

The usual view of this conflict of values for phase velocity is that given by Mott and Massey: "However the value of this constant" (the rest energy of the electron) "does not affect any experimental results."[46]

46 N. F. Mott and H. S. W. Massey. op. cit., p. 13.

If we attempt uncritically to apply equation (10.10) to the electron the results are not independent of this constant. It would appear interesting either to analyze in detail the derivation leading to equation (10.10) from a quantum mechanical viewpoint and ascertain the exact point where the analogy between light and electrons breaks down, or alternately, to derive an equivalent equation for electrons. Either procedure is beyond the scope of this paper.

Regardless of the outcome of this analysis it is obvious that, although intuitively coherence is an attractive concept, it is one that must be applied with great care. In any working interferometer it is a complex function of the source, the instrument and the medium.

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In the course of time he was assigned to the Signal Corps and saw service in Europe until VE day and in Japan until 1946. Upon his discharge he reentered Lehigh June 1946 and received his B. S. in Engineering Physics in October of the same year. He remained at Lehigh as a graduate assistant until February of 1948 when upon receiving his M. S. in Physics, he accepted an appointment as physicist at the National Bureau of Standards, working under Dr. L. Marton. Except for the academic year of 1950-51, when he returned to Lehigh to complete the course requirement for the degree of Doctor of Philosophy, he has remained at the Bureau doing research in Electron Physics.

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