The Thermal Conductivity of Metals at High Temperature.

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THE THERMAL
CONDUCTIVITY OF METALS AT
HIGH TEMPERATURE

by

Clarence Lester Hogan

A DISSERTATION

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TABLE OF CONTENTS

The Thermal Conductivity of Metals

Abstract

The Forbes bar method has been considerably modified and adapted to the problem of measuring the thermal conductivity of metals and alloys in the temperature range from 0° C to 1000° C. A complete mathematical analysis of the heat flow problem existing in the furnace is presented, which substantiates the approximations which have been made by several authors in the past employing this basic technique. The electrical and thermal conductivities of nickel, inconel, several stainless steel alloys, and 99.99% pure aluminum are reported.

Introduction

Since Weidemann and Franz first noticed in 1853 that the ratio of thermal to electrical conductivity of most metals was nearly constant at a given temperature, it has been the attempt of several theories of conductivity to explain this phenomenon. Because much of the data which has been published on the thermal conductivity of metals at high temperature has apparently been quite erroneous, there exists today some confusion as to the applicability of the Wiedemann-Franz-Lorentz law. $(k/\sigma T = L)$. It is well to review the present state of the theory.

Since insulators can conduct heat, it is evident that the lattice vibrations in metals must play some part in the thermal conductivity even though the contribution to the thermal conductivity by the so called free electrons is usually the more important factor.

Since the total heat conducted per unit time is the sum of the heat transported by the electrons and by the lattice we should designate the total thermal conductivity k as the sum of two terms.

$$k = k_e + k_g$$

However, as pointed out by Makinson, (1) we cannot simply add a lattice conductivity similar to that of an insulating crystal to the electronic conductivity, because in a metal the interaction of the lattice waves with the conduction electrons reduces the lattice conductivity.

Experiments have been performed which attempted to determine k_e and k_g separately.

Gruneisen and Reddemann $^{(2)}$ assumed that the total thermal conductivity could be represented by

$$k = L_e \, \sigma \, T + k_g$$

where L_e has the constant value $\frac{\pi^2}{3} \left(\frac{k}{e}\right)^2$. Actually, modern theory indicates that $L_e = L_0 = \frac{\pi^2}{3} \left(\frac{k}{e}\right)^2$ only at temperatures above the Debye characteristic temperature θ, and that L_e can vary considerably from this value for $T \ll \Theta$

Eucken and Neumann $^{(3)}$ assumed that grain size affected k_g much more than k_e and that

$$k_g = \frac{a}{b+n}$$

holds at constant temperature, where n is the average number of grain boundaries per unit length in the metal in the direction of flow, and a and b are constants.

From these assumptions k_g can be determined from the measured values of k for various samples which differ only in grain size. Obviously the assumptions in this method are at best rather crude approximations and so the results cannot be relied on for accuracy.

Reddemann (4) used a novel method which is applicable to metals such as bismuth whose electrical conductivity is greatly reduced in a strong magnetic field (magneto-resistance effect). If one assumes that k_e and σ are reduced by the same ratio and that k_g is unaltered, then by measuring the change in k and σ due to the magnetic field, k_g is easily determined. This method offers the most straightforward method of determining k_g. However, as was stated above, since k_g is determined largely by the scattering of the lattice waves by the conducting electrons it is not evident that the magnetic field does not also affect k_g.

Conduction of heat by the electrons in a metal is a second order effect and it is quite difficult to obtain an expression which gives the thermal conductivity over a wide range of temperature. To date, the most successful attempt has been made by Wilson (5). We shall give only the major results of Wilson's paper and also those of Makinson.

The original concept of electrons moving in an electrostatic field with the period of the lattice was introduced by Bloch (6). He has shown that electrons can move unimpeded through a perfectly periodic lattice. Electrical resistance is caused by the departure of the lattice from perfect regularity. This irregularity can arise from the imperfections caused by cracks, strains, and the presence of foreign atoms or from the thermal motion of the lattice. Imperfections act as scattering centers which scatter the free electrons and thus create resistance. The problem of computing the electrical conductivity (or resistivity) of a crystal consists of computing the probability of an electron being scattered. Since the total probability of an electron being scattered is the sum of the probabilities of its being scattered by the two mechanisms above (i.e. imperfections and thermal vibrations) it is possible to write

$$\frac{1}{\sigma} = \frac{1}{\sigma_o} + \frac{1}{\sigma_i}$$

where:

$\frac{1}{\sigma_o}$ is the "impurity" resistivity

$\frac{1}{\sigma_i}$ is the ideal resistivity due to thermal motion of the lattice

This equation essentially expresses Matthiesson's rule which was deduced empirically by him in 1864.(7)

The free path for collisions of electrons with the imperfections is of the order of the average distance between the imperfections and is therefore independent of the temperature of the crystal. The corresponding resistance is also independent of the temperature and varies from specimen to specimen. The ideal resistance on the other hand increases with increasing temperature and tends to zero as the temperature tends to zero. Thus the total resistance at low temperatures tends to a limiting value which is entirely due to the presence of impurities and imperfections. For this reason the "impurity resistance" is called the residual resistance.

Since the electronic heat conductivity depends upon the total probability of an electron being scattered, it also can be written as the sum of two terms.

$$\frac{1}{K_e} = \frac{1}{K_o} + \frac{1}{K_i}$$

where:

$\frac{1}{K_o}$ is the thermal resistance caused by "impurity" scattering,

$\frac{1}{K_i}$ is the ideal thermal resistance due to scattering by the lattice vibrations.

Now the Wiedemann—Franz—Lorentz law should hold for the part of the thermal resistance caused by "impurity" scattering and hence:

$$\frac{1}{K_e} = \frac{1}{L_0 \sigma_0 T} + \frac{1}{K_i}$$

where:

$$L_0 \equiv \frac{\pi^2}{3} \left(\frac{k}{e}\right)^2$$

If the Wiedemann—Franz—Lorentz law also held for the ideal thermal resistance it would obviously hold for the total electronic thermal resistance for then:

$$\frac{1}{K_e} = \frac{1}{L_0 \sigma_0 T} + \frac{1}{L_0 \sigma_i T} = \frac{1}{L_0 T}\left[\frac{1}{\sigma_0} + \frac{1}{\sigma_i}\right] = \frac{1}{L_0 \sigma T}$$

However, the Wiedemann—Franz—Lorentz law does not hold in all temperature regions for the ideal thermal resistance. Wilson shows that;

$$\frac{1}{K_i} = \frac{27\, h}{8\pi^3\, k^2\, \varepsilon_0^2\, \Lambda\, \Theta} \left(\frac{T}{\Theta}\right)^2 \left\{ J_5 + \frac{D}{\varepsilon_0}\left(\frac{T}{\Theta}\right)^2 \left(\frac{2\pi^2}{3} J_5 - \frac{1}{3} J_7\right)\right\}$$

where:

$$\Lambda = \left(\frac{4\pi}{3}\right)^{1/3} \frac{4 M a k \Theta}{3 h^2 C^2}$$

M = mass of an atom

a = lattice constant

C = Constant which gives the interaction between the electrons and the lattice.

$$J_n \equiv \int_0^{\Theta/T} \frac{z^n\, dz}{(e^z - 1)(1 - e^{-z})}$$

$$= \frac{1}{n-1} \left(\frac{\Theta}{T}\right)^{n-1} n-1 - \frac{1}{12(n+1)} \left(\frac{\Theta}{T}\right)^{n+1} \cancel{n+1} + \ldots$$

$$= \left(\frac{1}{n-1}\right)\left(\frac{\Theta}{T}\right)^{n-1} \qquad \text{as } T/\Theta \to \infty, \text{ and}$$

$$= n! \sum_{r=1}^{\infty} \frac{1}{r^n} \qquad \text{as } T/\Theta \to 0$$

$$D = \frac{(6\pi^2)^{2/3} h^2}{16 \pi^2 m a^2}$$

$$\mathcal{E}_o = \frac{h^2}{2m}\left[\frac{(3 n_o)}{(8\pi)}\right]^{2/3} = \text{Fermi energy of electrons.}$$

In general

$$\frac{D}{\mathcal{E}_o} = \frac{1}{2^{1/3} N_a^{2/3}} = \frac{0.794}{N_a^{2/3}}$$

N_a = number of conduction electrons per atom.

Hence the curve of k_i versus (T/Θ) depends upon the value of $D/\mathcal{E}_o$. For the monovalent metals there is approximately one conduction electron per atom. Nickel has approximately 0.6, iron has 0.2, and bismuth has 1.8×10^{-2}. The corresponding values od $D/\mathcal{E}_o$ are:

Monovalent metals = 0.794

Ni = 1.11

Bi = 11.6

Fe = 2.33

The total electronic thermal conductivity can now be written

$$k_e = \frac{\Theta}{4A} \; \frac{L_o \, T/\Theta}{\frac{\rho_o}{4A} + \chi(T/\Theta, D/\mathcal{E}_o)}$$

where:

$$A = \frac{3\pi h D}{16 e^2 \mathcal{E}_o^3 \Lambda}$$

or if $T/\Theta > 0.6$

$$A = \frac{1}{\delta I}\left(\Theta/T\right)$$

and

$$\chi(T/\theta, D/\varepsilon_0) = \left(\frac{T}{\theta}\right)^5 \left[J_5 \left\{ 1 + \frac{3}{2\pi^2}\frac{\varepsilon_0}{D}\left(\frac{\theta}{T}\right)^2 \right\} - \frac{1}{2\pi^2} J_7 \right]$$

The table below shows χ for a monovalent metal such as copper, for nickel, iron, and bismuth. From these values are computed values of $\frac{4\, A\, k_e}{\theta\, L_0}$ which appear in the next table for $\rho_0 = 0$.*

T/θ ↓	Copper $D/\varepsilon_0 = 0.794$	Nickel $D/\varepsilon_0 = 1.11$	Iron $D/\varepsilon_0 = 2.33$	Bismuth $D/\varepsilon_0 = 11.6$
0.0625	0.567×10^{-2}	0.40×10^{-2}	0.184×10^{-2}	0.275×10^{-2}
0.1	2.14	1.56	0.675	
0.125	3.68		1.05	0.14
0.167	6.35			
0.25	10.35		5.12	2.42
0.50	16.6			9.23
0.667	19.9	18.0	15.5	14.0
1.0	27.7	27.2	24.0	23.2
2.0	50.0	50.0	50.0	49.0

$4\, A\, k_e/L_0\theta \quad (\rho_0 = 0)$

T/θ ↓	Copper	Nickel	Iron	Bismuth
0.0625	11.0	15.6	34.0	227
0.1	4.67	6.42	14.8	
0.125	3.40			
0.167	2.63			
0.25	2.42			10.3
0.50	3.02			5.42
0.667	3.35	3.70	4.30	4.76
1.0	3.61	3.68	4.17	4.31
2.0	4.0	4.0	4.05	4.08

* $\rho_0 = 0$ means a pure metal with no lattice defects.

Figures (1), (2), and (3) show the trend of values for three of the metals under consideration. Figure (2) shows the effect of adding impurity to copper and figure (3) shows the same effect for bismuth.

Wilson also shows in the same paper that:

$$\frac{1}{\sigma_i} = \frac{3\pi h D}{4 e^2 \varepsilon_0^3} \left(\frac{T}{\Theta}\right)^5 J_5 \left\{ 1 - \frac{1}{8} \frac{\varepsilon_0}{D} \left(\frac{k\Theta}{\varepsilon_0}\right)^2 \right\}$$

Thus we can compute the so called Lorentz constant for the electronic contribution to the thermal conductivity.

$$\frac{k_e}{\sigma T} = L_e = \frac{\left[\rho_0/4A + (T/\theta)^5 J_5\right] L_0}{\rho_0/4A + \chi\left(T/\theta, D/\epsilon_0\right)}$$

Figures (4), (5), and (6) show this function for the various metals which we are interested in here. The curve indicates very clearly the dependence of L_e upon N_a for low values of T/θ. (i.e. $T/\theta < 1$).

The experimental data which has been taken for copper checks the above results extremely well. However, for bismuth the experimental data which exists gives k and L values which are much higher than predicted. This might be accounted for by crystal lattice conduction.

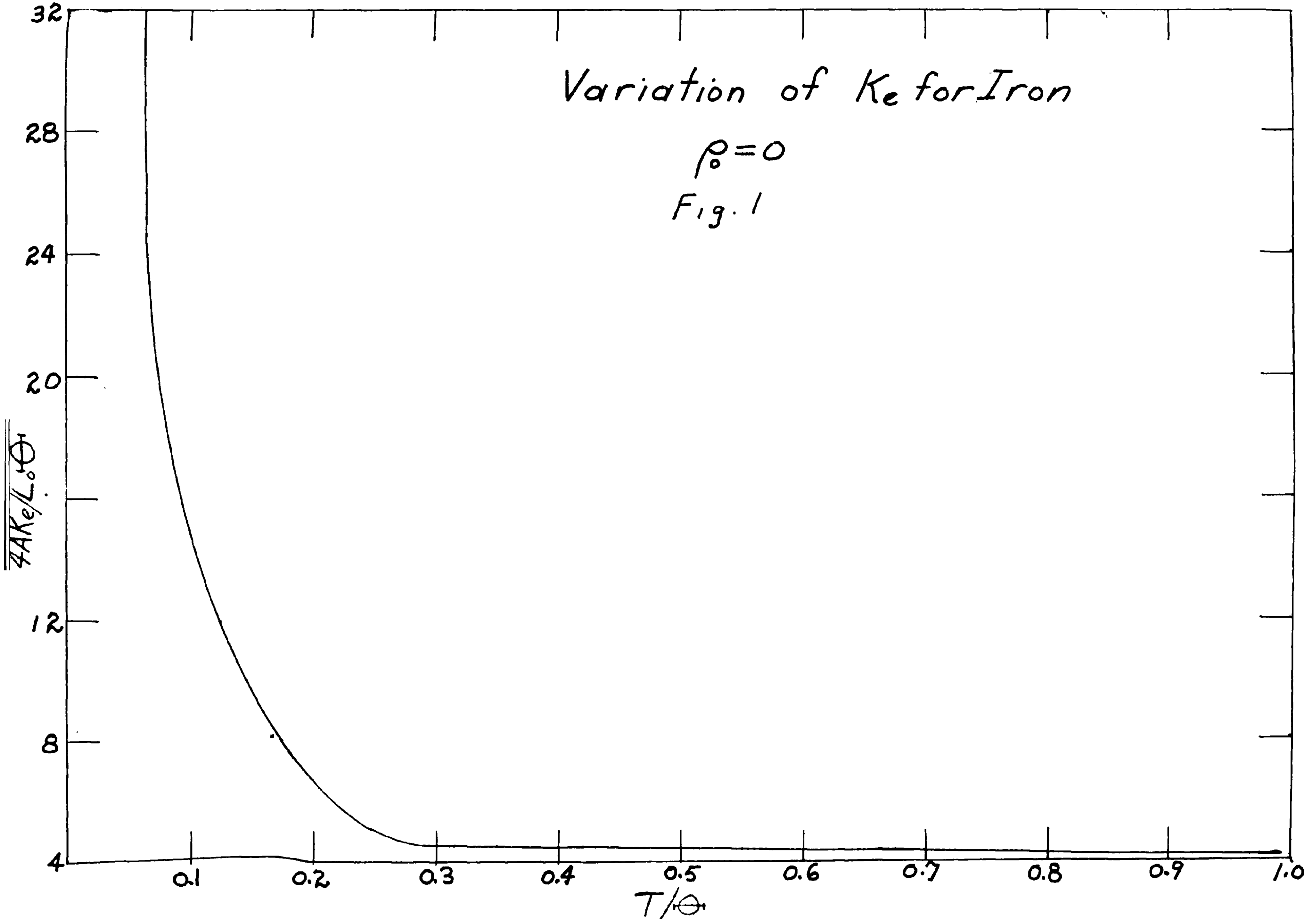
Crystal Lattice Heat Conduction

Thermal resistance of the lattice is created by the scattering of the lattice waves. The principal factors which scatter the lattice waves are:

(1) interaction with conduction electrons

(2) lattice defects

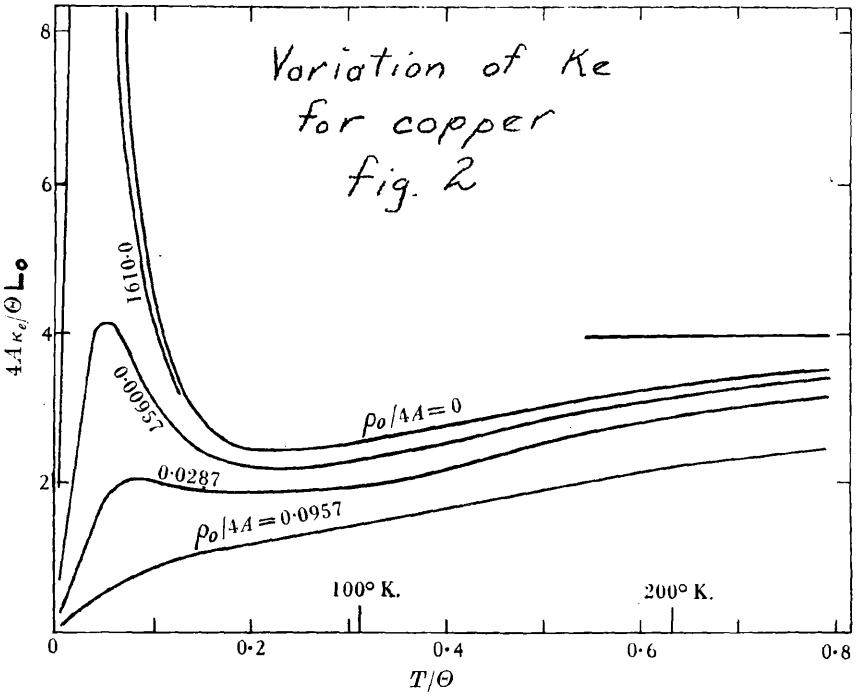
(3) scattering of the waves by each other



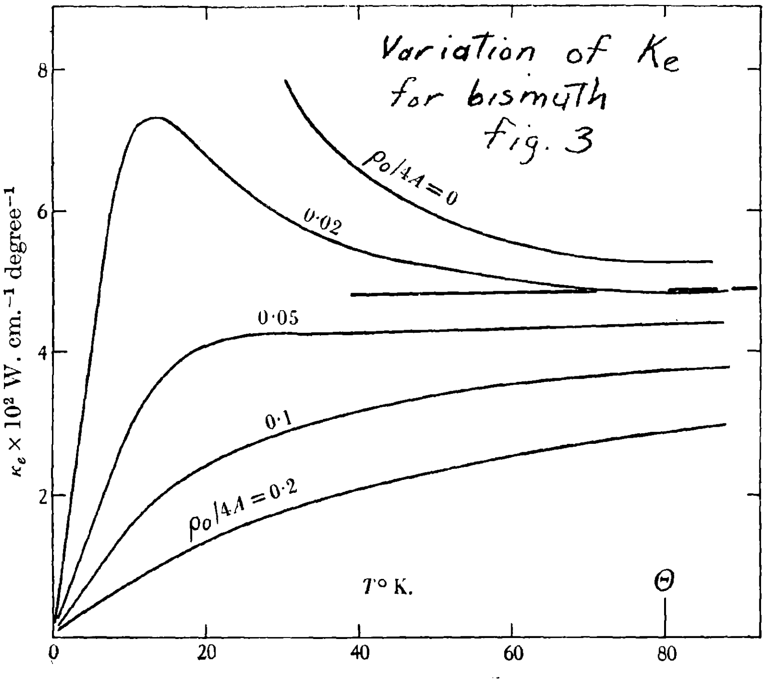
Variation of K_e for Iron

$\rho_0 = 0$

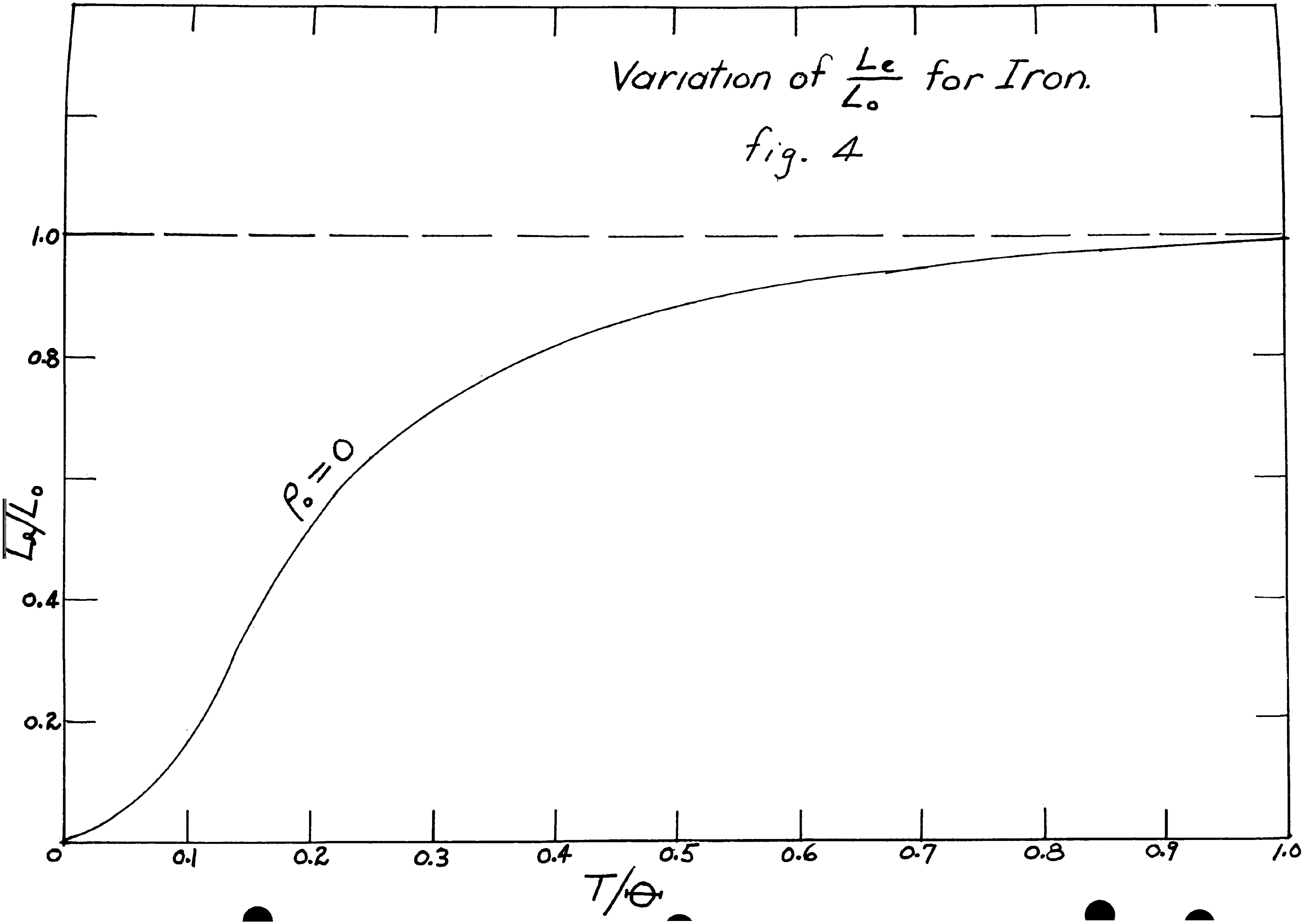
Fig. 1



Variation of Ke for copper fig. 2

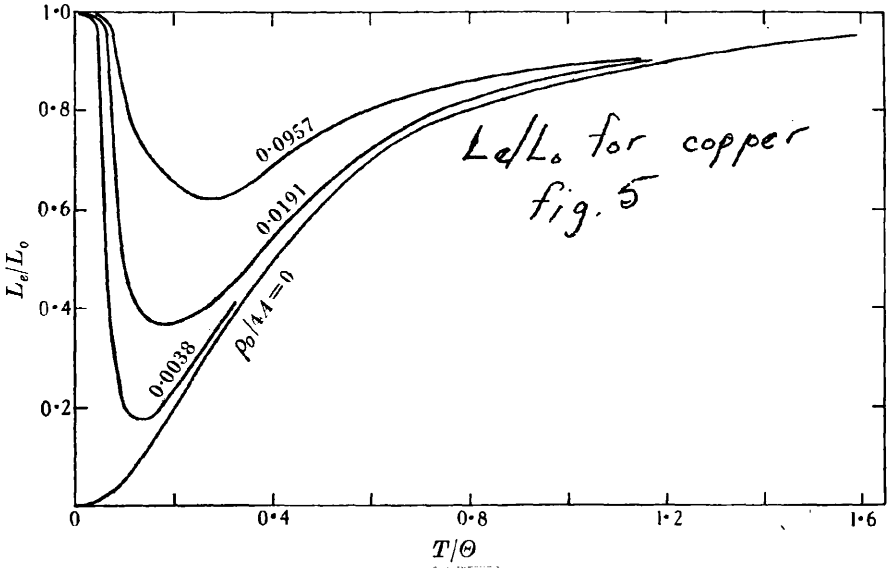


Variation of Ke for bismuth fig. 3

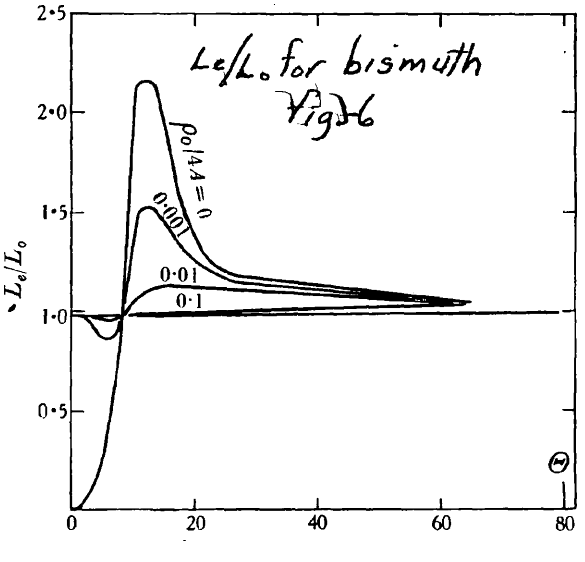


Variation of $\frac{L_e}{L_0}$ for Iron.

fig. 4



L_e/L_0 for copper
fig. 5



L_e/L_0 for bismuth
fig. 6

Peierls (8) and Makinson (9) have derived general relations for k_g, the lattice heat conductivity, but the relations involve functions which are not known and hence are not very useful for general application. However, they indicate that the lattice heat conductivity should vary approximately as figure (7) indicates. The maximum of this curve occurs at approximately $T/\theta = 0.6$.

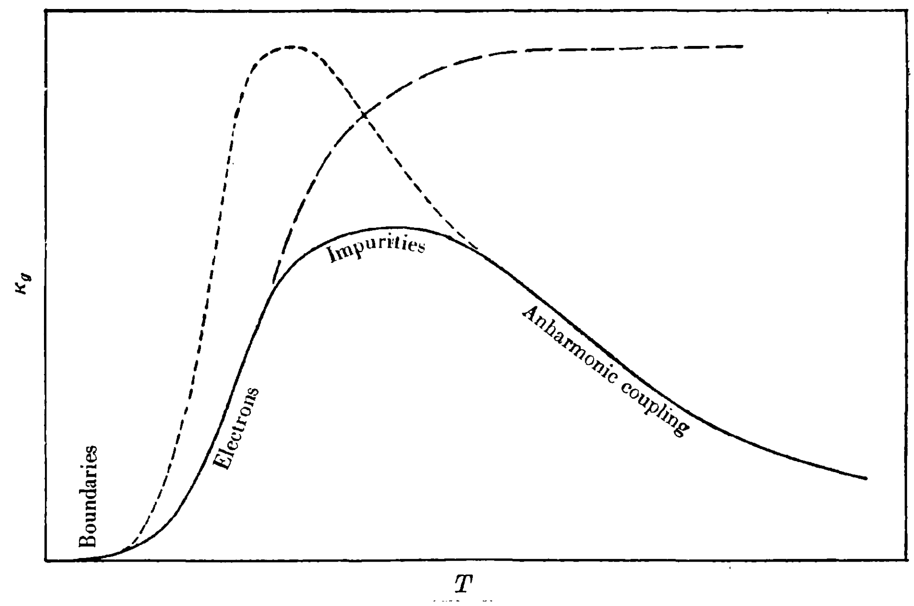
Makinson also shows that:

$$k_g/ k_e < \frac{0.342}{N_a^{2}}$$

at the point where k_g has its maximum. Although this relation is of little value in actually computing the lattice conductivity, it shows that the lattice heat conductivity is very dependent on the number of free or conduction electrons which a metal has.

Apart from the practical importance which thermal conductivity data has, then, it is of extreme importance in the theory of the solid state. Hence the purpose of this dissertation is two fold. The first purpose, and probably the most important purpose was to devise an experimental method by means of which accurate and reliable data could be obtained. The second purpose was to use these data to check the above theories of metallic conduction.

Measurement of thermal conductivity by means of heat flow data introduces many difficulties in instrumentation and control. The main difficulty arises from the lack of a perfect insulator and consequent inability to prevent heat flow in all directions. It is beyound the scope of this dissertation to discuss the various methods which have been tried. A complete picture will be obtained by referring to the bibliography. (10) (11) (12) (13) (14) (15) (16) (17) (18) (20)



κ_g
Boundaries
Electrons
Impurities
Anharmonic coupling
T

fig. 7

In general the data obtained are conflicting. Of all methods discussed in the references above, the Forbes bar method is one of the most reliable. There are few inherent sources of error in this method. Contrary to most methods which attempt to confine heat flow to one dimension, the Forbes bar method allows heat to flow in all three dimensions. Hence the basic weakness of most other heat flow methods is immediately removed.

Experimental Procedure

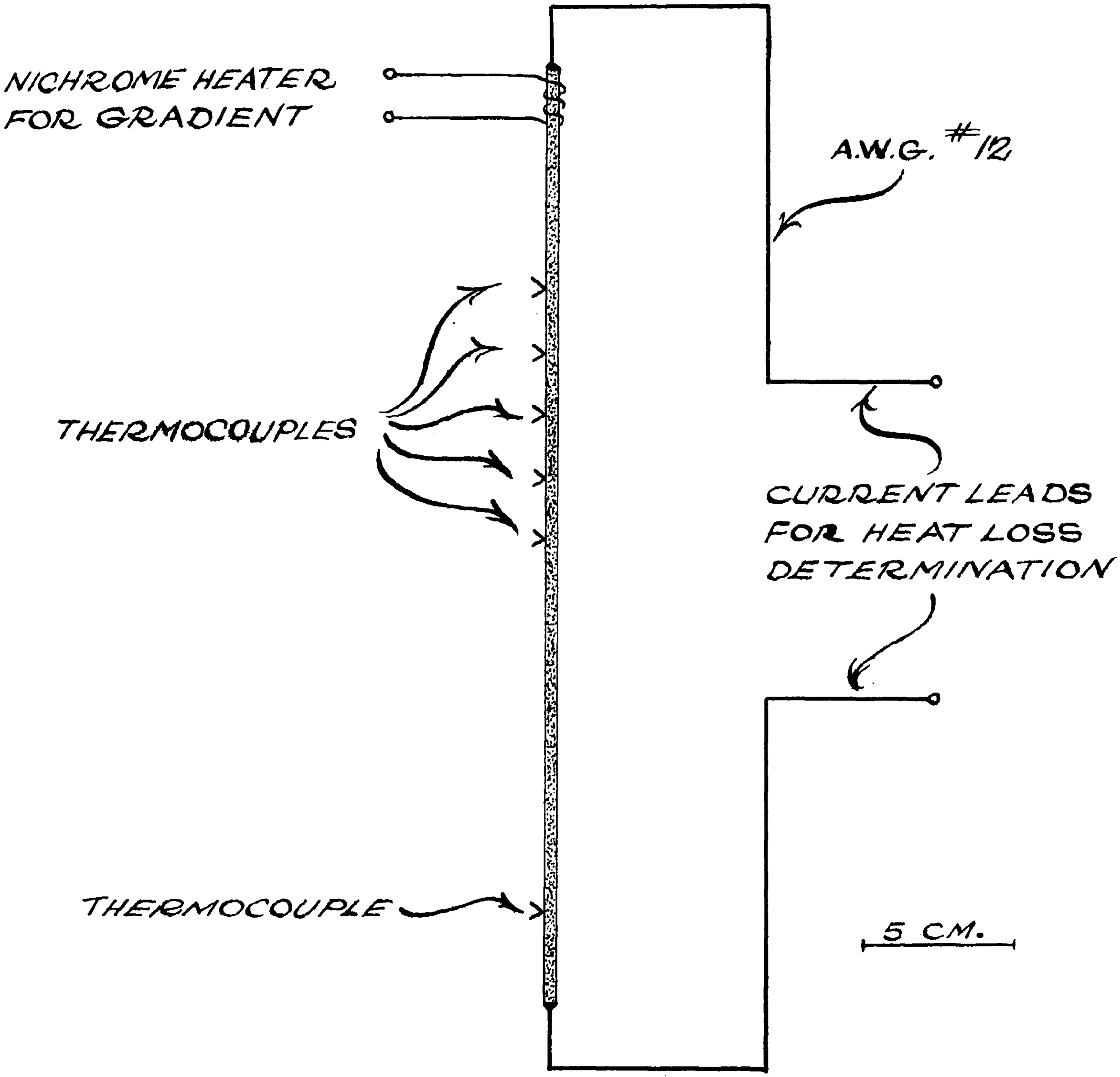
The sample whose thermal conductivity is to be measured is prepared as a cylindrical sample 0.375 cm in radius and 30 cm. long. Chromel-alumel thermocouples of A. W. G. No. 24 wire are tightly peened into small holes at 2 cm intervals along the sample. A small heater (about 2 cm axial length) is would on the top end of the specimen using A. W. G. No. 26 asbestos covered Nichrome wire and No. 12 copper wires are brazed to each end of the sample. Figure (8) indicates the position of these various wires. Before placing the sample in the furnace, the thermocouple wires are placed in fine quartz tubes and the entire sample is then covered with a thin layer of Alundum cement. The sample is then centered in a cylindrical furnace as shown in figure (9). The thermocouple wires are lead out of the bottom of the furnace and the Nichrome heater wires out the top. A reference thermocouple is placed in the furnace, below the sample and usually in contact with the heavy walled copper tube which forms the inside lining of the furnace. The furnace tube is then packed with screened Sil-O-Cel.

All Thermocouples are connected to a switch board so that each thermocouple on the sample can be connected in turn differentially with the reference and the resulting emf connected across a Leeds and Northrup type R galvanometer.

The deflection of the galvanometer is thus a measure of the difference in temperature between the reference thermocouple and the particular location on the sample where the second thermocouple is attached.

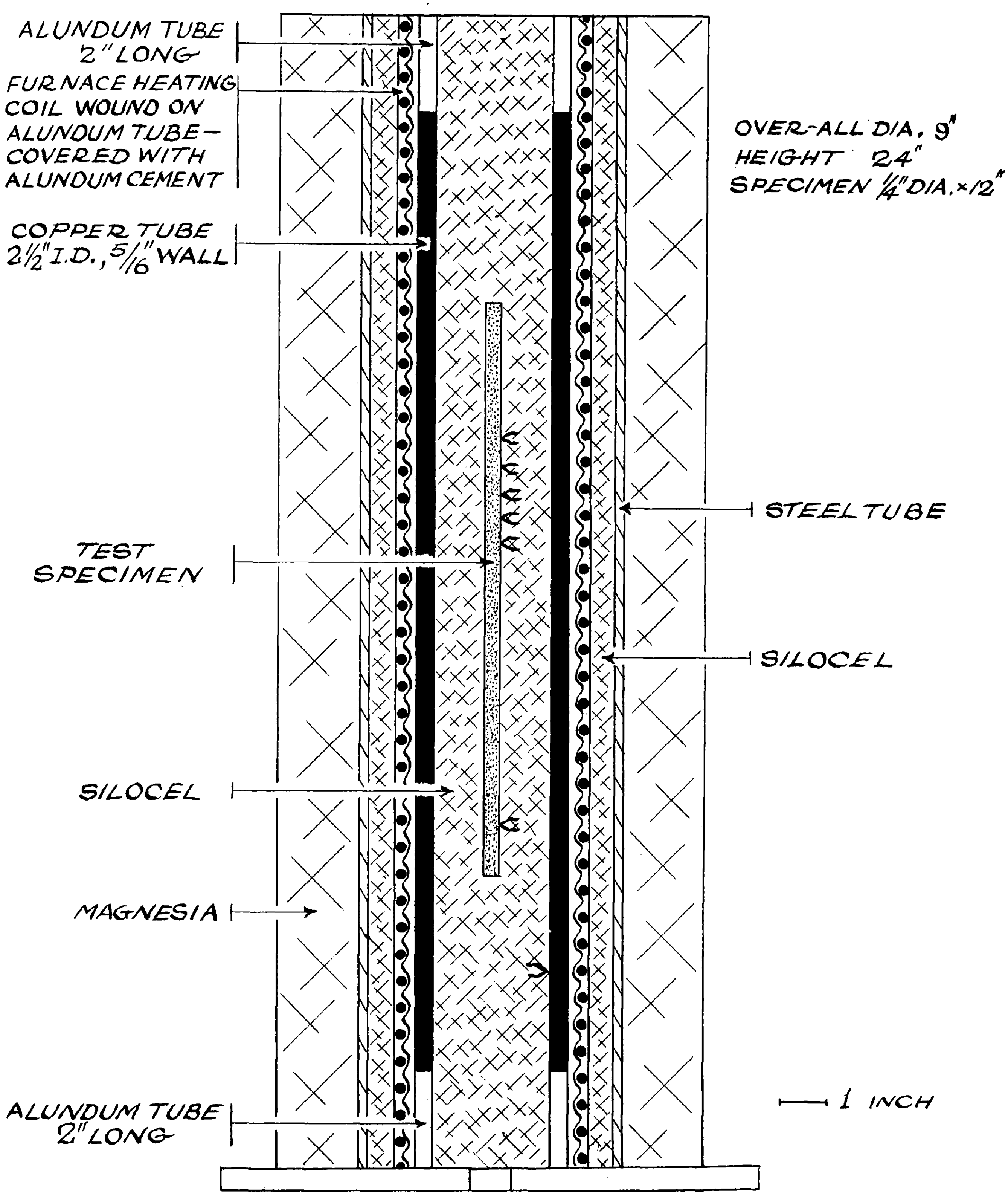
The furnace construction is clearly indicated in figure. (9).

The furnace is brought to any desired temperature by means of the Chromel furnace winding. The furnace voltage is controlled by means of



SPECIMEN SHOWING
LOCATION OF HEATING
COILS & THERMOCOUPLES

fig. 8



GENERAL CONSTRUCTION OF FURNACE SHOWING LOCATION OF SPECIMEN

fig. 9

two Sola constant voltage transformers. When a steady state condition is reached, a current of 5-10 amperes D. C. is passed through the specimen by means of the copper wires brazed to each end. The potential drop per centimeter is measured by connecting the chromel-p wires of each pair of thermocouples in turn to a potentiometer. When the specimen comes to equilibrium with the furnace the thermocouple in the center of the 30 cm long specimen is connected in differential with the reference and the galvanometer is connected in series with them. The galvanometer deflection is then proportional to the temperature difference between the specimen and the furnace lining. The specimen is raised to a temperature of about 1° C above the furnace temperature in this part of the experiment.

The power input per centimeter along the sample is given by:

$$\frac{VI}{JL} = \frac{2\pi k_s \Delta T}{\ln \frac{r_2}{r_1}}$$

where:

V = potential drop (volts) along rod between points L cm apart.

I = current in rod (amperes)

J = Joules constant

k_s = thermal conductivity of furnace packing

ΔT = temperature difference between specimen And copper furnace lining when specimen is in equilibrium carrying a current of I amperes. (° C)

r_2 = radius of copper tube lining furnace

r_1 = radius of specimen

It is not necessary that all these constants be measured. If we let.

$$\frac{k_s}{\ln \frac{r_2}{r_1}} = r_1 H$$

Then H will have the dimensions $\left(\text{calories per second per cm}^2\right)$ and the significance of a heat transfer coefficient, and the equation becomes

$$\frac{VI}{JI} = 2\pi r_1 H \Delta T$$

Further if we let:

$$R = \frac{2H}{r_1}$$

then:

$$R = \frac{VI}{LJ\pi r_1^2 \Delta T}$$

Since both I and ΔT are measured by means of galvanometer deflections, it is not necessary to know the galvanometer constant. If the deflections are sensibly the same in each case it would not even be necessary for the galvanometer to be linear.

After the above data $(V/I, I, \Delta T)$ are taken, the current through the cylindrical sample is turned off and the sample again returns to furnace temperature. At this time a current of approximately 0.5 amperes is passed through the Nichrome heater at the top of the sample. When the steady state is attained, the temperature gradient existing along the sample is measured by the method indicated above. The top of the sample is never raised to a temperature of more than 5°C above the bottom during this part of the experiment.

The temperature (recorded as cm. of galvanometer deflection) of each thermocouple is then plotted against distance down the sample. On semi-log paper this plot is a straight line. If we denote the slope of this line as m, then the thermal donductivity of the sample is:

$$k = \frac{R}{m^2}$$

MATHEMATICAL ANALYSIS OF EXPERIMENT

The problem is essentially that of determining the steady state temperature in a semi-infinite rod if one end of the rod is maintained at temperature T_o and the rod loses heat through its surface according to Newton's law of cooling. Since in the experimental method the temperature of the rod rarely exceeds furnace (environment) temperature by more than 4°C, Newton's law of cooling would be expected to hold extremely well, and this expectation has been substantiated by experimental data. Mathematically our problem is:

1. $$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial Z^2} = 0$$

2. $u(r,o) = T_o$

3. $\lim\limits_{Z \to \infty} u(r,Z) = 0$

4. $$2\pi c k \frac{\partial u(c,z)}{\partial r} = -2\pi c H u$$

$$c \frac{\partial u(c,Z)}{\partial r} = \frac{-cH}{k} u(c,Z)$$

or:

$$c \frac{\partial u(c,Z)}{\partial r} = -h\, u(c,Z)$$

$$h = \frac{cH}{k}$$

where:
c = radius of rod
k = thermal conductivity
H = heat transfer coefficient
$u(r,Z)$ = temperature functions
T_o = temperature at $Z = 0$

This problem can be easily solved.(21) The solution is:

$$u\ (r,Z) = \sum_{j=1}^{\infty} A_j \ e^{-a_jZ} \ J_o\ (a_jr)$$

where:

$$A_j = \frac{2a_jCT_o \ J_1\ (a_jC)}{\left(a_j^2C^2 + h^2\right)\left[J_o\ (a_jC)\right]^2}$$

and a_j are the positive roots of

$$Ca_j\ J_o'\ (a_jC) = -h\ J_o\ (a_jC) \qquad (5)$$

To evaluate the roots of equation (5), h must be determined.

For a cylinder initially above environmental temperature, Newton's law of cooling can be written as:

$$\sigma \rho \pi C^2 \ dZ \ \frac{\partial u}{\partial t} = 2\pi C \ dZ \ H \ u$$

or

$$\frac{\sigma \rho}{u} \ \frac{\partial u}{\partial t} = \frac{2H}{C}$$

where: σ = specific heat

ρ = density

u = temperature of section under consideration

C = radius of cylinder

Now the quantity $\frac{\sigma \rho}{u} \frac{\partial u}{\partial t}$ has been determined for the furnace used. Obviously this quantity would depend upon the thermal conductivity of the material packing the furnace (Sil-o-cel) and would be the same for all materials in the furnace if they are machined to the same shape and size. It has been found that this quantity has a value of approximately:

$$R = 0.004 \ \frac{\text{cal}}{\text{sec cm}^3\ ^\circ\text{C}}$$

Now: $h = \frac{C\,H}{k}$

and: $H = \frac{C\,R}{2}$

So: $h = \frac{C^2R}{2k} = \frac{3.024 \times 10^{-4}}{k}$

Thus equation (5) becomes

$$0.3175 \quad a_j \, J_0' \; (a_jC) = \frac{-3.024 \times 10^{-4}}{k} \quad J_o \; (a_jC)$$

But: $J_o' \; (X) = -J_1 \; (X)$

So: $0.3175 \quad a_j \; J_1 \; (0.3175\, a_j) = \frac{3.024 \times 10^{-4}}{k} \; J_o \; (0.3175\, a_j)$

This equation can be solved exactly for any given value of k. If the thermal conductivity is taken as $0.03 \frac{\text{cal}}{\text{sec cm } ^\circ\text{C}}$, the first four roots of this equation are found to be:

$a_1 = 0.4467$ $\qquad$ $a_3 = 22.11$

$a_2 = 12.07$ $\qquad$ $a_4 = 32.05$

There are an infinite number of roots.

Now at $r = c$ (outside surface of cylinder) the temperature function is:

$$u\,(C,Z) = \sum_{j=1}^{\infty} \frac{2a_jC\,T_o\,J_1\,(a_jC)\,e^{-a_jZ}}{(a_j^2C^2 + h^2)\left[J_o\,(a_jC)\right]}$$

But from (5) above:

$$Ca_j = \frac{h\,J_o\;(a_jC)}{J_1\;(a_jC)}$$

Hence:

$$u\,(C,Z) = \sum_{j=1}^{\infty} \frac{2\,h\,T_o\,e^{-a_jZ}}{(a_j^2C^2 + h^2)}$$

If $K = 0.03$ then $h = 1.01X10^{-2}$ and the series becomes:

$$u\,(0.3175, Z) = 0.997\, T_o\, e^{-.4467Z} + \sum_{2}^{\infty} \frac{R}{K\, d_j^2}\, T_o\, e^{-d_j Z}$$

This series converges extremely rapidly. In fact if only the first term of the series is used the maximum error involved in the determination of temperature would be at $Z=0$ where the error is obviously:

$$\frac{1 - .997}{1}\, 100 = 0.3\%$$

Thus within the accuracy of the experimental method employed the temperature at the outside (at $r = c$) of a semi-infinite cylinder can be written as:

$$u\,(0.3175, Z) \approx T_o\, e^{-.4467Z} = T_o\, e^{-d_1 Z} \quad (6)$$

where k, h, c have the values indicated above.

At: $Z = 30$

$$u\,(o, 30) = T_o\, e^{-13.4} = 0.0000015\, T_o$$

Thus if the top of the rod is heated to a temperature equivalent to 100 cm on the galvanometer (4°C) the bottom of the rod ($Z = 30$ cm) would change by 0.0015 mm in the steady state condition. Thus within the accuracy of the experiment the rod is semi-infinite and does not require a heat sink.

Since the above series converges so rapidly that only the first term is necessary, (i.e. only the first root of equation (5) is required) it is possible to find an approximate expression for this root in terms of the thermal conductivity. Rewriting equation (5) in the final form we had above:

$$0.3175\, a_j \quad J_1\,(0.3175\, a_j) = \frac{3.024X10^{-4}}{K} \quad J_o\,(0.3175\, a_j)$$

Let: $0.3175\, a_j = X$

Thus we must find the roots of

$$X_j \; J_1 \; (X_j) = \frac{3.024X10^{-4}}{K} \quad J_o \; (X_j)$$

But: $J_o \, (X_j) = 1 - \frac{X_j2}{4} + \frac{X_j^4}{64} \ldots\ldots$

Since from the exact solution above we know the first root of the above equation is approximately:

$$a_1 = 0.4467 \text{ or } X_1 = 0.1418$$

we can obviously neglect the third term in the above alternating series. Even for $X = 0.2$ the series is:

$$J_o \; (0.2) = |-.01 + .000025 - + \ldots\ldots$$

Likewise:

$$J_1 \; (X_j) = \frac{X}{2} - \frac{X^3}{16} + \ldots\ldots$$

Substituting these series into equation (5) and rearranging we get:

$$X^2 \left[\frac{1}{2} + \frac{h}{4} \right] = h$$

neglecting h/4 with respect to ½, we get:

$$X^2 = \frac{h}{0.5} = \frac{C^2R}{K}$$

Hence: $X = C \sqrt{\frac{R}{K}}$

and $a_1 = \sqrt{\frac{R}{K}}$

Thus from equation (6) above:

$$u \; (0.3174,Z) = T_o \, e^{-\sqrt{R/k}} \qquad (7)$$

which is the exact solution of a thin wire radiating to its surroundings with one end at temperature T_o and whose isotherms are planes perpendicular to the axis of the wires.

It is the approximate formula for the temperature of a rod 0.3175 cm in radius and whose thermal conductivity is between 0.01 and 1.0.

It has been shown in the previous analysis that when a gradient is applied to a long rod under the furnace conditions with which we work, the steady state temperature of the rod can be expressed as:

$$T(Z) = T_o e^{-\sqrt{\frac{R}{K}} Z} \qquad (1)$$

where:

$T(Z)$ = temperature at any cross section Z centimeters from top

T_o = temperature at section $Z = 0$

K = thermal conductivity

$R = \frac{2H}{C} \quad \frac{\text{calories}}{(\text{cm}^3 \ {}^\circ\text{C sec})}$

H = heat transfer coefficient between rod and surroundings $\frac{\text{calories}}{(\text{cm}^2 \ {}^\circ\text{C sec})}$

C = radius of rod (cm)

Taking the logarithm of both sides of the above equation:

$$\ln T(Z) = \ln T_o - \sqrt{\frac{R}{K}} Z \qquad (2)$$

which is the equation of a straight line on semi-log paper.

The slope (m) of the straight line is:

$$m = \sqrt{\frac{R}{K}}$$

Thus:

$$K = \frac{R}{m^2}$$

Hence, in order to determine the thermal conductivity of a sample we need only plot the temperature of the rod as function of (distance down the rod) and determine the slope (m) of the straight line. If R, which is a function of the rate of heat loss, is known then K is immediately determined.

Obviously F should be a function of the apparent conductivity of the SilO-Cel powder which packs the furnace. Maxwell has calculated the apparent conductivity of a substance composed of spheres of one medium dispersed in another medium and has arrived at the equation. (22), (23), (24)

$$K_p = K_m \frac{\left[2K_d + K_m + P_d \left(K_d - K_m\right)\right]}{\left[2K_d + K_m - 2P_d \left(K_d - K_m\right)\right]}$$

where:

K_p = conductivity of aggregate

K_M = conductivity of matrix

K_D = conductivity of dispersed material

P_D = volume fraction of the dispersed phase in the mixture

The Silocel powder used in packing the sample in the furnace consists of solid particles of diatamaceous silica dispersed in air. Since the conductivity of the air is negligibly small compared to the conductivity of the solid particles, Maxwell's equation can be reduced to:

$$K_p = K_A \left(\frac{3}{2P_A} - \frac{1}{2} \right)$$

where:

K_p = conductivity of powdered Silocel

K_A = conductivity of air

P_A = fraction of total volume occupied by air

(It should be noted that there is confusion in all three of the above references when they attempt to apply Maxwell's equation to the apparent conductivity of a powder. This confusion seems to stem from Eucken's original application in 1940. Apparently, the other two author's are following Eucken. Eucken makes no distinction between a substance composed of solid particles dispersed in air, such as powdered Silocel, and air dispersed in a solid, such as Styrofoam. He treats a powder as if it consisted of air particles dispersed in a solid, and hence arrives at a different formula than we do here.)

Since R is defined as:

$$R = \frac{2H}{C} = \frac{Q}{\pi C^2 \Delta T}$$

Since:

$$K_p = \frac{Q}{2\pi \Delta T} \ln \frac{r_2}{c}$$

where:

r_2 = radius of copper tube used as furnace lining.

ΔT = temperature difference between sample and furnace lining in steady state when sample is losing Q calories /sec/ cm length.

We can write:

$$R = \frac{2K_p}{c^2} \Big/ \ln \frac{r_2}{c}$$

Since:

$C = 0.3175$ cm

$r_2 = 1\frac{1}{4}'' = \overset{3.81}{\cancel{3.175}}$ cm

$R = \cancel{45.7\,K_p} = 7.93\,K_p$

The Sil-o-Cel as packed in the furnace is about 82% air by volume. Hence:

$$K_p = K_a \left(\frac{3}{1.64} - \frac{1}{2} \right) = 1.330\,K_a$$

but at room temperature:

$K_a = .000057$

So: $K_p = .000076$

And: $R = \overset{7.93}{\cancel{45.7}} K_p = \overset{0.00062}{\cancel{0.0035}} \; \frac{\text{calories}}{\text{cm}^3 \, {}^\circ\text{c sec}}$

This theoretically computed value of R compares unfavorably with the experimentally measured values of about 0.004. We should expect somewhat higher values experimentally since K_a is really an apparent conductivity itself and includes the influence of radiation from one solid particle to the next across the air gap. (Convection in the air pockets will probably be negligible since they are so small.) Hence in computing R above we should not have used the true value of the conductivity of air, but something slightly higher. Since P_a varies only from 89% to 82% for loosely packed or tightly packed Sil-o-Cel, our computed value of R could only very from .0031 to.0035. Actually such extreme packing conditions are impossible in the furnace and it is doubtful if R could vary more than 7% from one furnace to another. Hence R values should be consistent from one furnace to another. Experimentally, this fact has been upheld although in our original work extremely erratic values of R were obtained since it is very sensitive to the measurement of the temperature difference between the sample and the furnace and great care must be taken in order to obtain consistent values.

Now, for any particular furnace packing P_a is a constant and hence from above:

$$K_p = AK_a$$

or:

$$R = 45.7AK_a$$

where the constant A is given by :

$$A = \left(\frac{3}{2P_a} - \frac{1}{2}\right)$$

Hence, the temperature coefficient of R should be the same as the temperature coefficient of K_a. Since, however, K_a is an apparent conductivity of air it is difficult to compare the experimentally measured value, with the actual temperature coefficient for K_a. Nevertheless, it has been experimentally determined that:

$$R \approx R_o \;(1 + .0009t)$$

R_o = (extrapolated) value of R at 0° C

t = temperature of furnace $^\circ$C.

Hence from our basic equation for computing conductivity, i.e.

$$k = \frac{R}{m^2}$$

we can write

$$\frac{k_t}{k_0} = \frac{(1 + .0009t)\, m_0^2}{m_t^2}$$

Thus relative values of conductivity can be obtained for a particular furnace without actually measuring R absolutely. These values will be known as accurately as the temperature coefficient (.0009) is known. It is felt that this has been determined to at least ± 5%.

It should be pointed out that the greatest error is introduced into the experiment through the measurement of R. If accurate values of R were known, the thermal conductivity could probably be determined with an error of about ± 1%, which is introduced in the determination of m. This author, however, in early work, measured R values which later work showed were in error by almost 100%. It is necessary that the furnace be in an extremely steady state before an attempt is made to determine R. Whatever voltage control is required for the furnace must thus be used. The author found Sola transformers adequate.

In addition to adequate voltage control, another precaution is to make certain that the reference thermocouple actually measures the temperature of the copper tube furnace lining. The best location for the reference is indicated in figure (9).

The author believes that the R values computed from the above simple theory will lead to more accurate results than experimentally measured values of R unless very great care is taken and unless adequate voltage control is supplied and ample time is allowed for the furnace to come to the steady state. It is impossible to make more than one determination per 24 hours with a furnace of the above design simply because it is so important to wait until the furnace is steady at the new temperature before data is taken.

METHOD OF DATA ANALYSIS

In order to illustrate the present method of data analysis, a data sheet for the determination of the thermal conductivity of stainless steel type 446 is presented and the calculations are made in detail. Furnace temperature for this run was 25°C.

	Thermo C. #1	T.C. #2	T.C. #3	T.C. #4	T.C. #5	T.C. #6	Galv. Zero
Dist. from Therm. C# 1	0cm	2 cm	4 cm	6 cm	8 cm	20 cm	
Zero's	94.9	95.0	95.2	95.2	95.3	95.5	96.8
Resistance	20.43	20.43	20.43	20.43	20.43	20.43	20.43

Heater on at 7:20:00 at 0.6 amps

The following "gradient" was recorded at 9:20:00.

	27.8	56.0	72.5	81.8	87.6	95.5	96.8

For R determination

Current passed thru rod at 4:35:00

Rod in steady state at 6:35:00

IR drop in rod:(between T. C. # 1 and T. C. # 8)

Gmv	2.03mv	4.10mv	6.22mv	8.19mv	20.0mv

T between T. C. # 5 and copper tube = 14.1 cm. (galv. defl.)

Galv. deflection when placed across 0.001 shunt carrying current which is heating sample = 59.8 cm.

Now it was shown previously in this report that:

$$R = \frac{VI}{1\,A\,J\Delta T}$$

But:

$$T = \frac{K\rho d_1}{C}$$

where:

C = thermoelectric power of thermocouple (volts/ °C)

ρ = resistance of galvanometer and thermocouple circuits (ohms)

d_1 = galvanometer deflection (cm)

K = galvanometer constant (amps/cm.)

Thus:

$$R = \frac{\nu I C}{A J K d_1}$$

where:

$$\nu = \frac{V}{I} \quad \text{(volts/cm.)}$$

But:

$$0.001 I = K d_2 \rho_g$$

where:

ρ_g = resistance of galvanometer plus any resistance in series with it. (ohms)

d_2 = deflection of galvanometer when placed across 0.001 ohm shunt. (cm)

So:

$$R = \frac{v \; d_2 \; \rho_g \; C \times 10^3}{A \; J \; \rho \; d_1}$$

For a rod ¼" dia. $A = 0.317 \text{ cm}^2$

Also:

$$\rho_g = 629.2$$

and at 25° C

$$C = 0.0404 \times 10^{-3} \text{ volts/oC}$$

$$R = \frac{0.0192 \quad d_2 \, v \times 10^3}{\rho \; d_1}$$

Now, from the data sheet

d_1 = 14.1 cm.

d_2 = 59.8 cm.

$v = 1.02 \times 10^{-3}$ volts/ cm (average)

$\rho =$ 20.43 ohms (data sheet includes resistance of galvanometer)

So:

$$R = 0.0040 \ \frac{\text{calories}}{\text{cm}^3 \ \text{sec} \ {}^\circ\text{C}}$$

Now if we subtract the galvanometer readings when a gradient existed down the rod from those which occured when no gradient existed we get the "temperature" of the rod at the various points where thermocouples are attached.

Thus:

94.9	95.0	95.2	95.2	95.3	95.5
-27.8	-56.0	-72.5	-81.8	87.6	95.5
67.1	39.0	22.7	13.4	7.7	0

These numbers are plotted in figure (10). The slope of this straight line is the quantity m.

In this case:

$m = 0.272$

$m^2 = 0.0740$

$$k = \frac{.0040}{.0.074} = \frac{0.40}{7.4} = 0.054 \ \frac{\text{cal.}}{\text{cm. sec. } {}^\circ\text{C}}$$

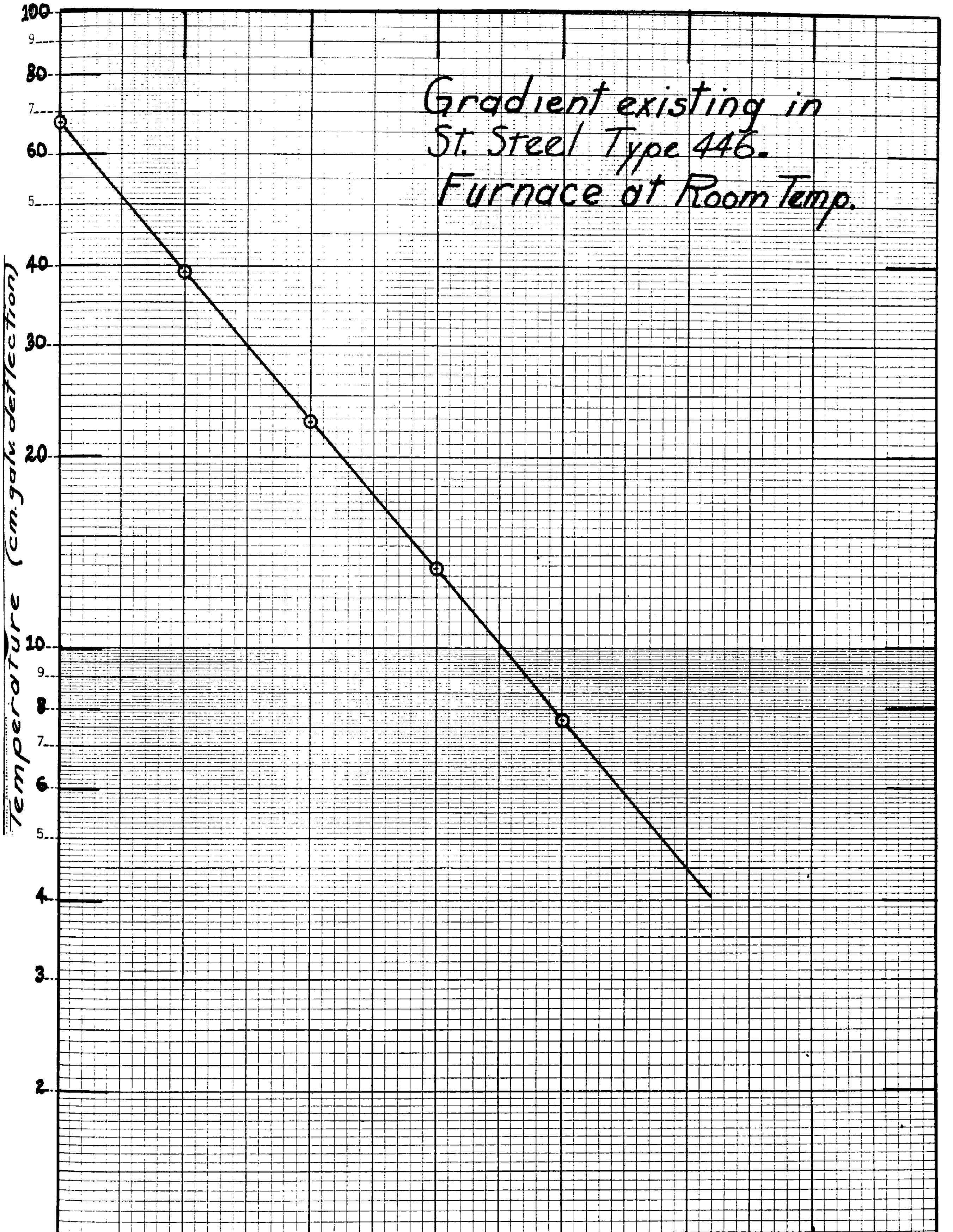

Gradient existing in
St. Steel Type 446.
Furnace at Room Temp.
Temperature (cm. galv. deflection)
100
9
80
7
60
5
40
30
20
10
9
8
7
6
5
4
3
2
1
2
4
6
8
10
Distance Down Rod (cm.)

fig. 10

RESULTS

There are reported herewith, the values of the thermal conductivity and electrical conductivity of Inconel, Commercial "A" Nickel, Stainless Steel Alloys type 303, type 310, type 347, and type 446.

The analyses of these specimens are as follows:

	C	Mn	Si	Ni	Cr	P	S	Cu	N	Al	Cb	Fe
Inconel	0.03	0.47	0.39	73.19	14.38	—	0.007	0.03	—	0.83	—	6.99
Nickel	0.06	0.22	0.02	99.48	—	—	0.005	0.05	—	—	—	0.14
Type 303	0.17	0.61	0.51	8.97	18.42	—	—	—	—	—	—	Bal.
Type 310	0.10	1.83	0.84	20.68	25.54	0.025	0.005	—	—	—	—	50.98
Type 347	0.069	1.80	0.70	11.20	18.10	0.021	0.007	—	—	—	0.77	Bal.
Type 446	0.13	0.56	0.50	0.10	26.00	0.012	0.007	—	0.14	—	—	Bal.

The values of the thermal conductivity of these alloys are tabulated below.

THERMAL CONDUCTIVITY $\left(\frac{\text{calories}}{\text{cm. sec. }^\circ\text{C}}\right)$

Temperature °C	Inconel	Nickel	303	310	347	446
0	0.032	0.174	0.034	0.031	0.046	0.054
100	0.038	0.156	0.036	0.034	0.049	0.057
200	0.043	0.138	0.038	0.039	0.053	0.061
300	0.048	0.120	0.042	0.044	0.056	0.065
400	0.054	0.114	0.047	0.048	0.061	0.068
500	0.060	0.121	0.051	0.054	0.066	0.072
600	0.065	0.128	0.055	0.060	0.070	0.076
700	0.073	0.136	0.059	0.066	0.075	0.080
800	0.081	0.143	0.064	0.072	0.081	0.084
900	0.089	0.150	0.068	0.078	0.087	0.087

The values of the electrical conductivity of these alloys are listed below.

ELECTRICAL CONDUCTIVITY (mho/cm.) x 10^{-4}

Temperature °C	Inconel	Nickel	303	310	347	446
0		11.3	1.38		1.27	1.75
100	0.814	7.56	1.25	0.999	1.18	1.44
200	0.804	5.12	1.13	0.951	1.11	1.25
300	0.794	3.87	1.05	0.908	1.07	1.12
400	0.784	2.87	0.988	0.870	1.03	1.02
500	0.774	2.62	0.940	0.841	1.00	0.940
600	0.764	2.40	0.900	0.818	0.984	0.880
700	0.770	2.25	0.870	0.800	0.967	0.840
800	0.782	2.10	0.846	0.785	0.950	0.820
900	0.795	2.02	0.824	0.771	0.933	0.810

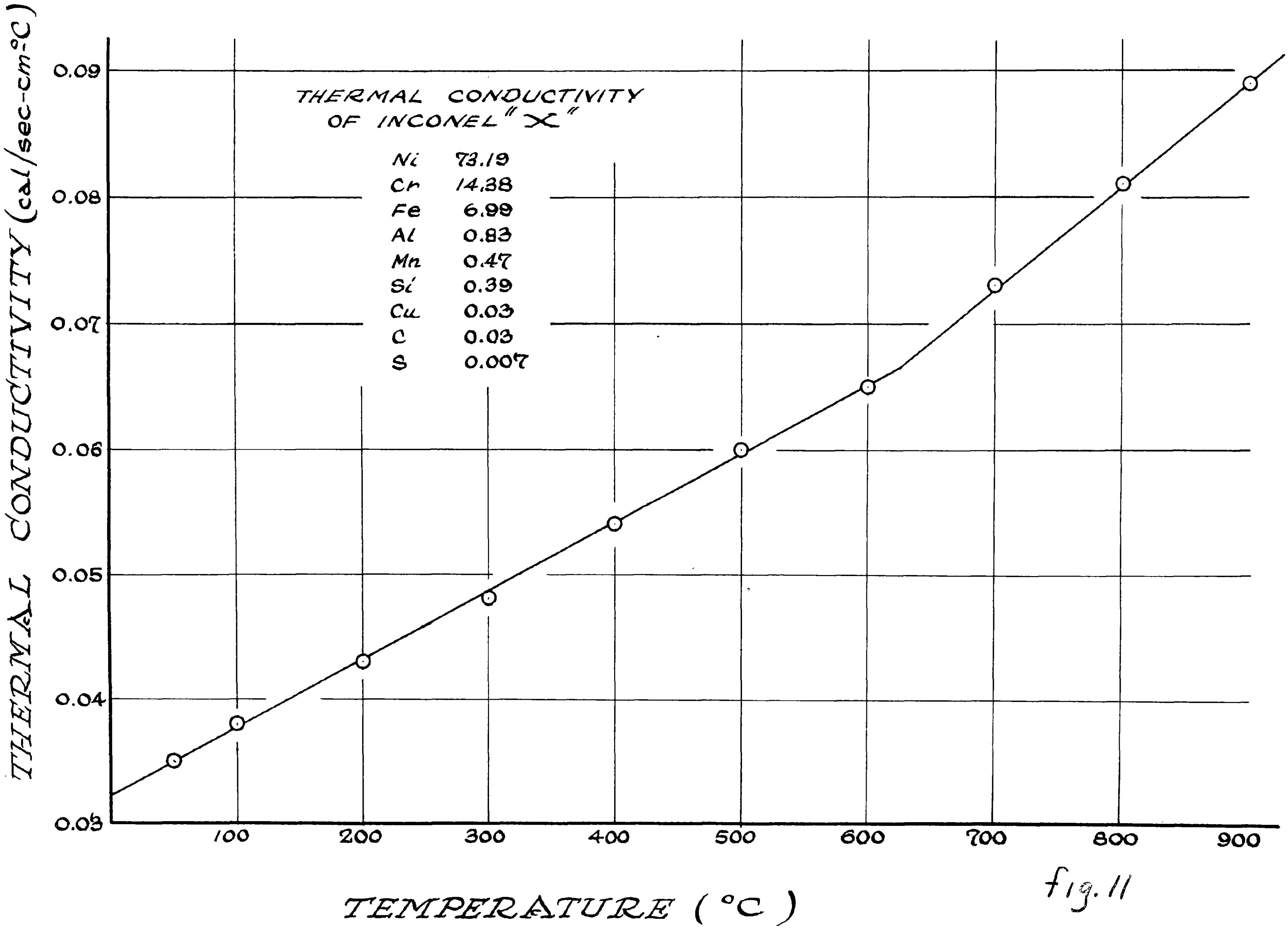

THERMAL CONDUCTIVITY
OF INCONEL "X"
Ni 73.19
Cr 14.38
Fe 6.99
Al 0.83
Mn 0.47
Si 0.39
Cu 0.03
C 0.03
S 0.007
THERMAL CONDUCTIVITY (cal/sec-cm-°C)
0.09
0.08
0.07
0.06
0.05
0.04
0.03
100
200
300
400
500
600
700
800
900
TEMPERATURE (°C)

fig. 11

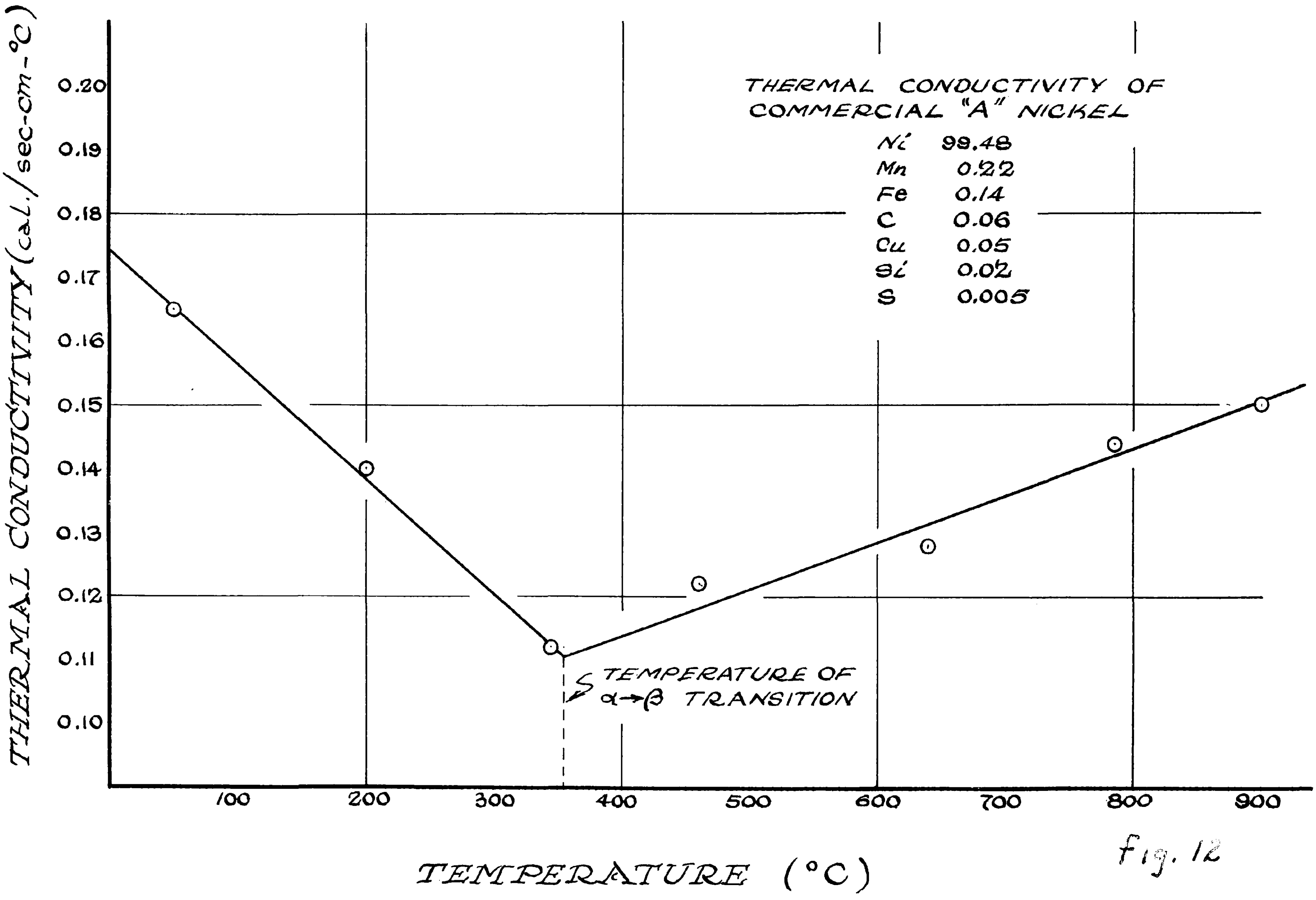

THERMAL CONDUCTIVITY OF
COMMERCIAL "A" NICKEL
Ni 99.48
Mn 0.22
Fe 0.14
C 0.06
Cu 0.05
Si 0.02
S 0.005
TEMPERATURE OF
α→β TRANSITION
THERMAL CONDUCTIVITY (cal./sec-cm-°C)
0.20
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.10
100
200
300
400
500
600
700
800
900
TEMPERATURE (°C)

Fig. 12

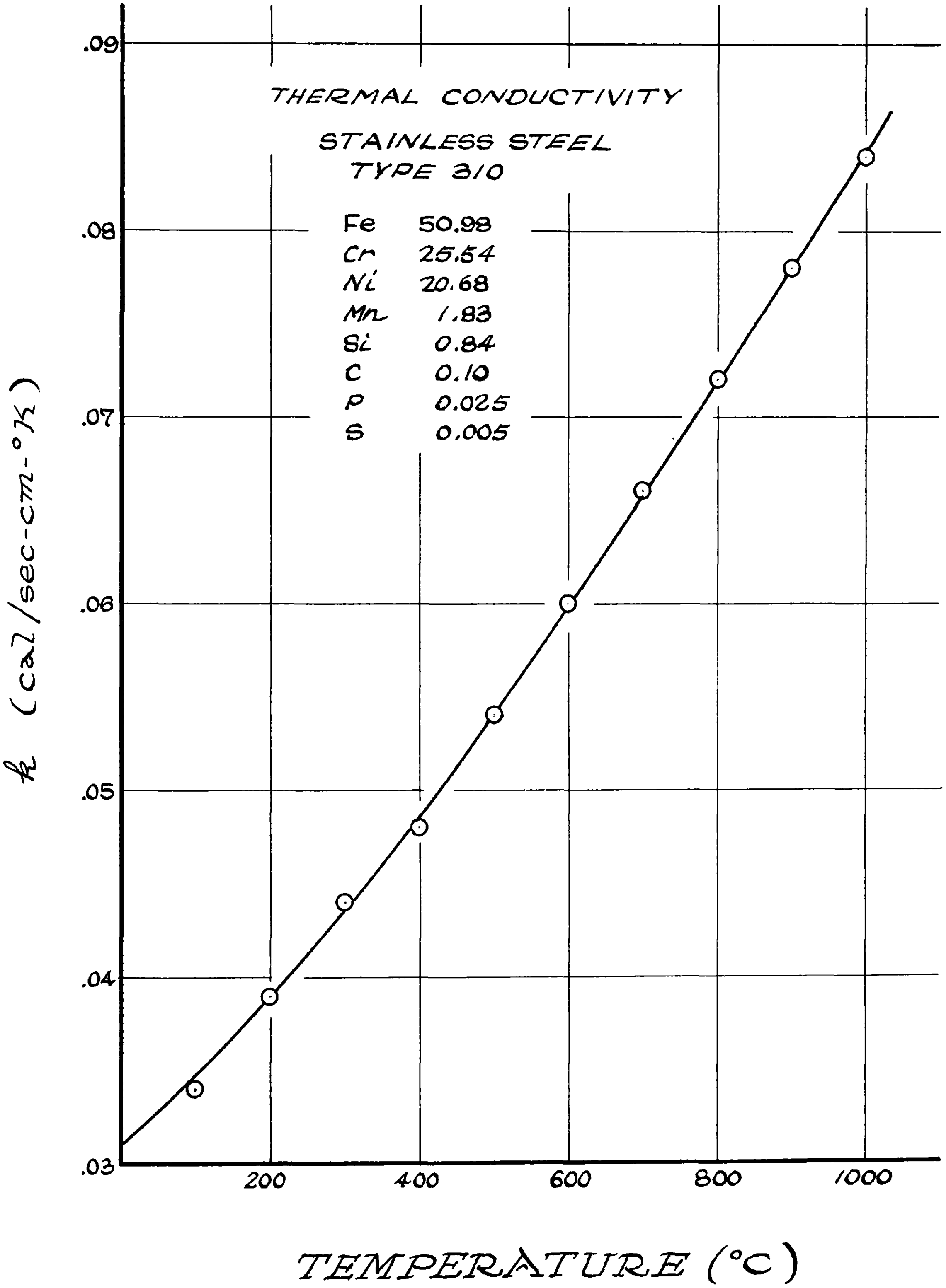

THERMAL CONDUCTIVITY
STAINLESS STEEL
TYPE 310
Fe 50.98
Cr 25.54
Ni 20.68
Mn 1.83
Si 0.84
C 0.10
P 0.025
S 0.005
k (cal/sec-cm-°K)
.09
.08
.07
.06
.05
.04
.03
200
400
600
800
1000
TEMPERATURE (°C)

fig. 12

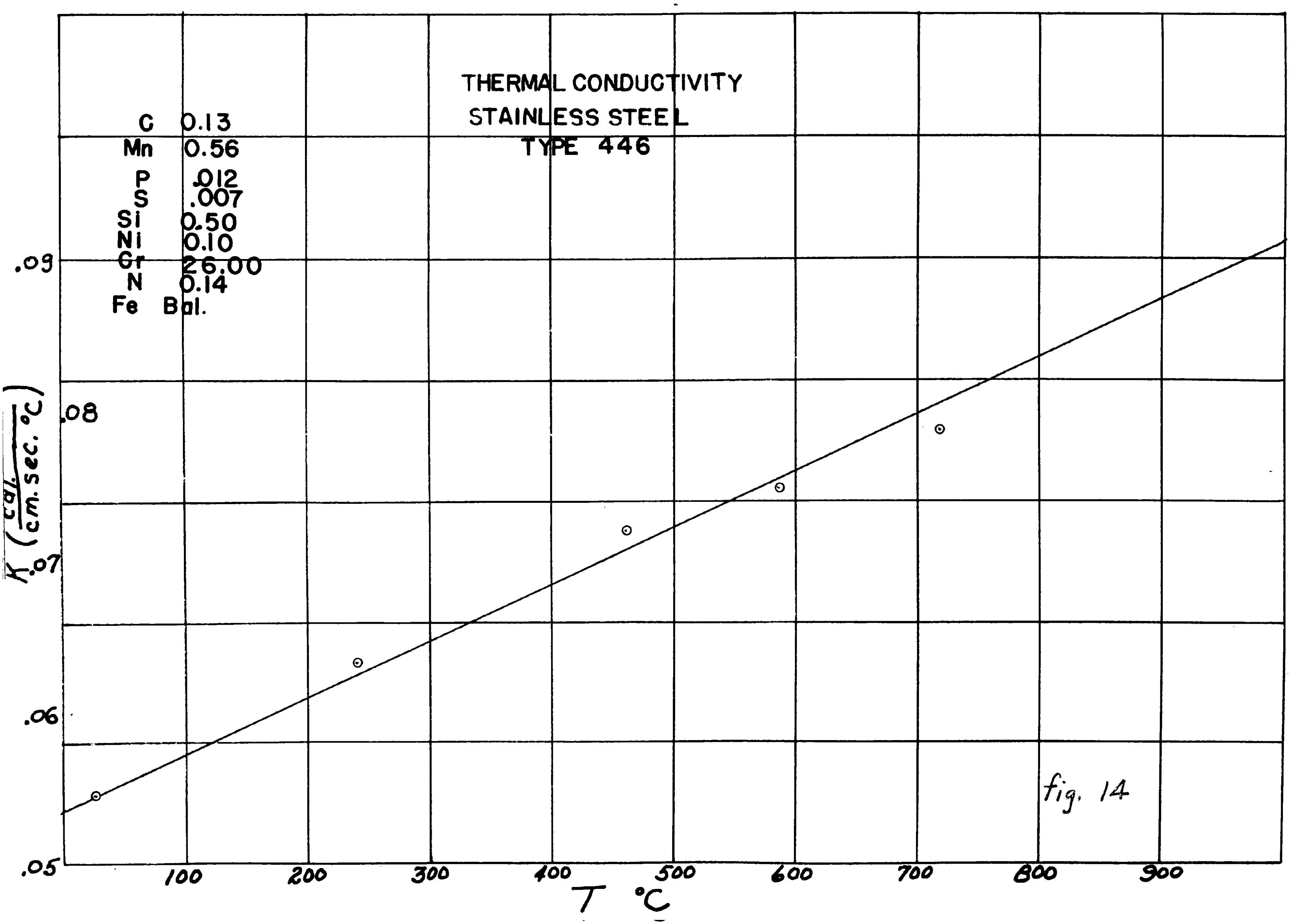

THERMAL CONDUCTIVITY
STAINLESS STEEL
TYPE 446
C 0.13
Mn 0.56
P .012
S .007
Si 0.50
Ni 0.10
Cr 26.00
N 0.14
Fe Bal.
K (cal. / cm. sec. °C)
.09
.08
.07
.06
.05
100
200
300
400
500
600
700
800
900
T °C

fig. 14

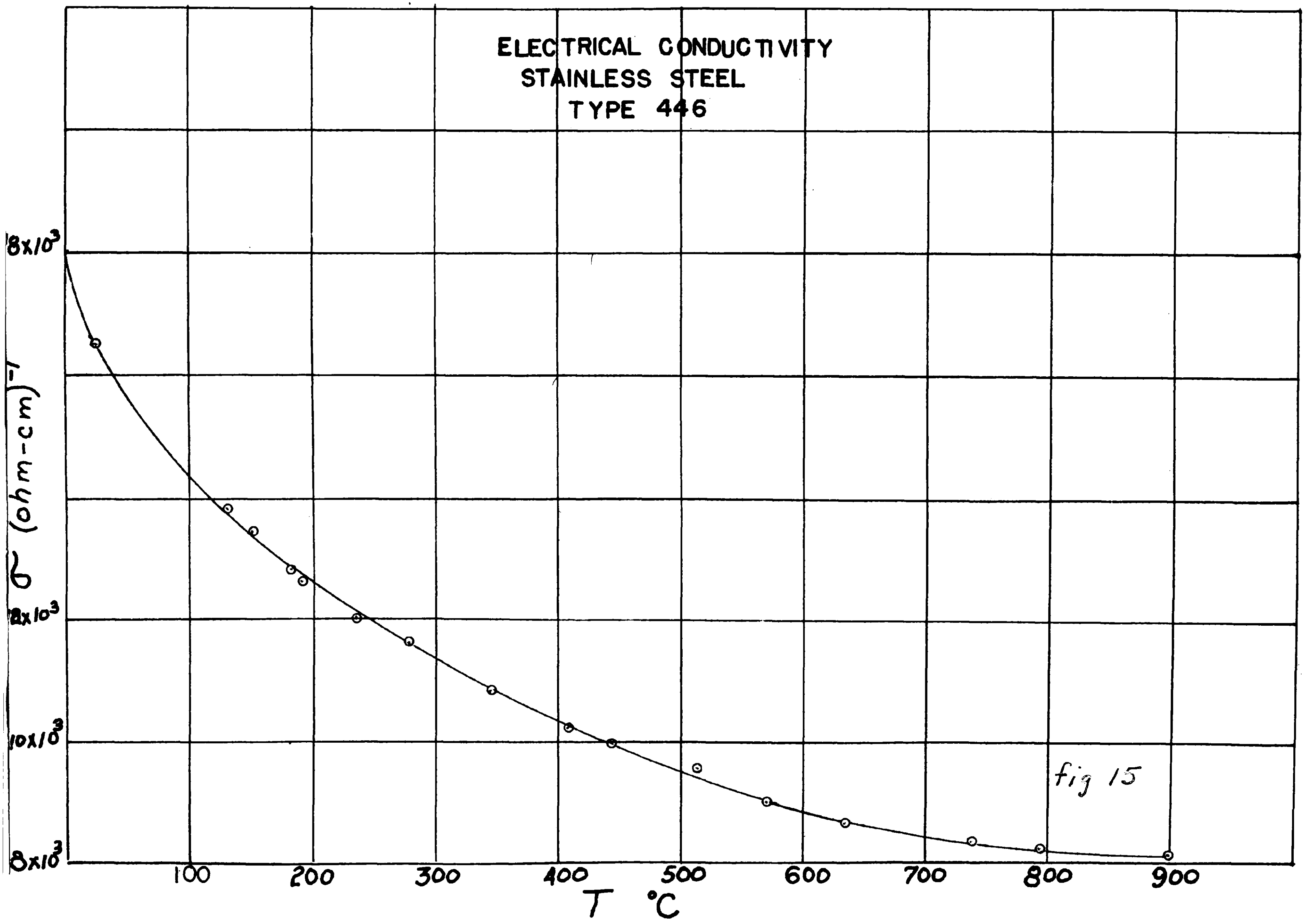

ELECTRICAL CONDUCTIVITY
STAINLESS STEEL
TYPE 446

fig 15

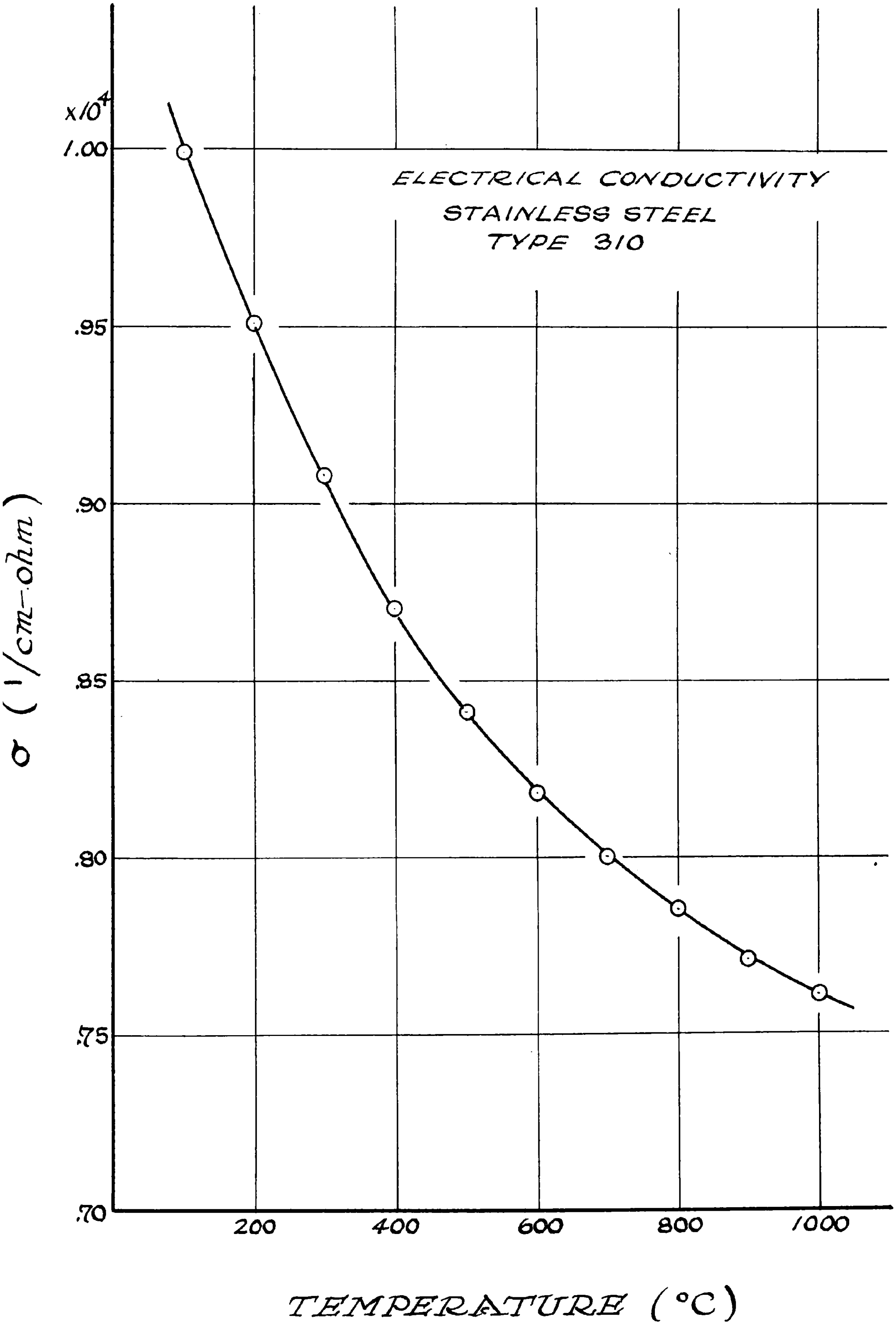

ELECTRICAL CONDUCTIVITY
STAINLESS STEEL
TYPE 310
x10^4
1.00
.95
.90
.85
.80
.75
.70
σ (1/cm-ohm)
200
400
600
800
1000
TEMPERATURE (°C)

fig 16

Some of the values in the above two tables are presented graphically in figures 11, 12, 13, 14, 15, and 16, ~~and 17~~.

Smith and Palmer (25), after having investigated the electrical and thermal conductivities of a number of copper alloys, found that the thermal conductivity could be represented by a straight line relationship of the form

$$k = L\sigma T + a$$

where L and a were constants for a particular set of alloys which had similar physical properties. This relation has been found to hold for all the metals reported on in this paper. Figures 17, 18, 19, and 20, ~~and 21~~ show graphically the result obtained by plotting k versus the product σT for inconel, nickel, and type 310 and type 446 stainless steels. In all cases, the points fall very nearly on a straight line. The experimentally determined values for L and a can be obtained from the slope and intercept of the straight line.

The values obtained are as follows: (Lorentz constant = 0.585 x 10^{-8})

	L	a	K at 0°C
Inconel	0.810 x 10^{-8}	0.012	0.032
Type 303	0.71 x 10^{-8}	0	0.034
Type 310	0.85 x 10^{-8}	0	0.031
Type 347	———	——	0.046
Type 446	0.73 x 10^{-8}	0.018	0.054
Nickel	0.56 x 10^{-8}	0	0.174
Aluminum	0.502 x 10^{-8}	0.03	0.570

It should be pointed out that considerable experimental error exists in determining either of these constants. This is so because

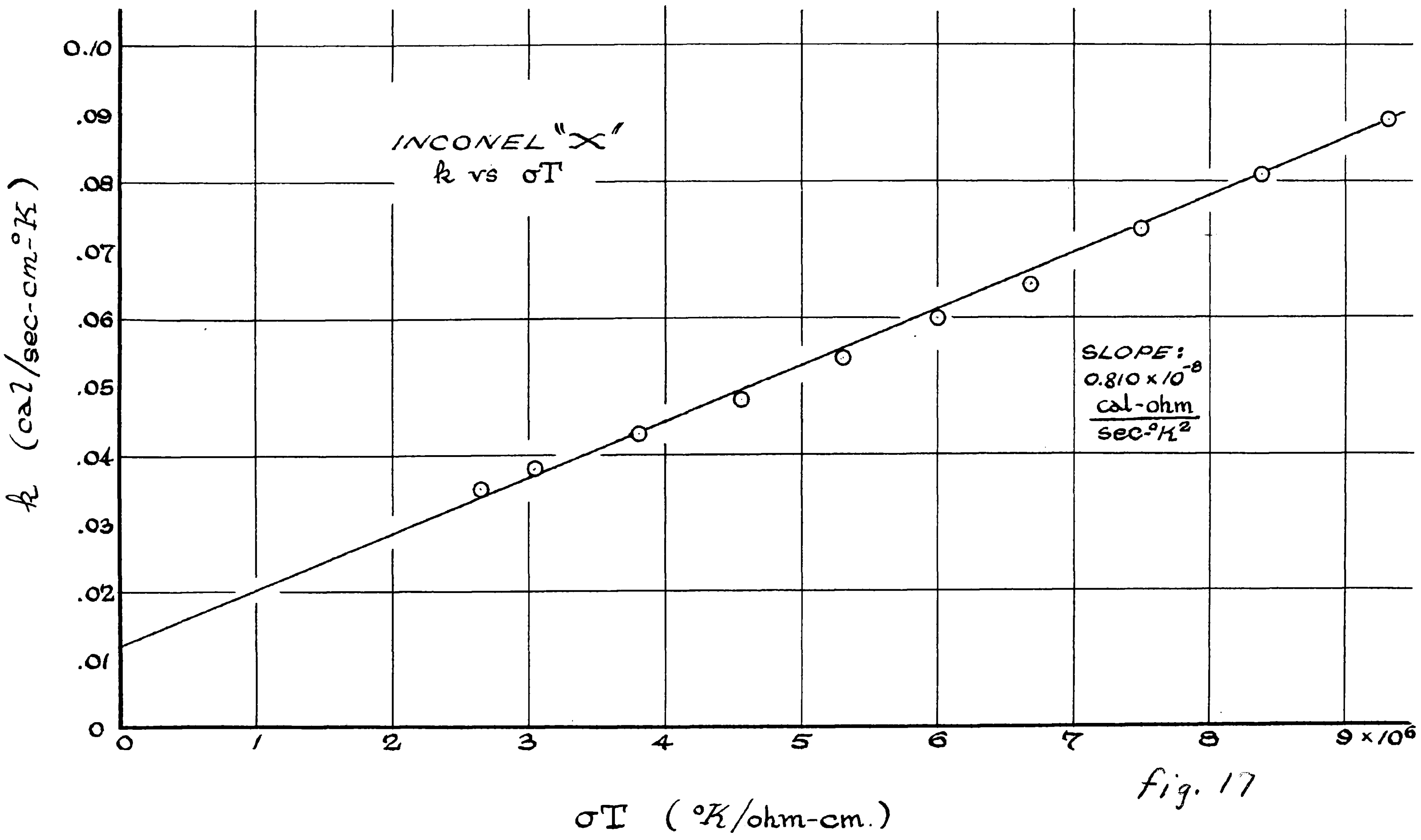

INCONEL "X"
k vs σT
SLOPE:
0.810 × 10^-8 cal-ohm / sec-°K²
k (cal/sec-cm-°K)
0.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
0
0
1
2
3
4
5
6
7
8
9 × 10⁶
σT (°K/ohm-cm.)

fig. 17

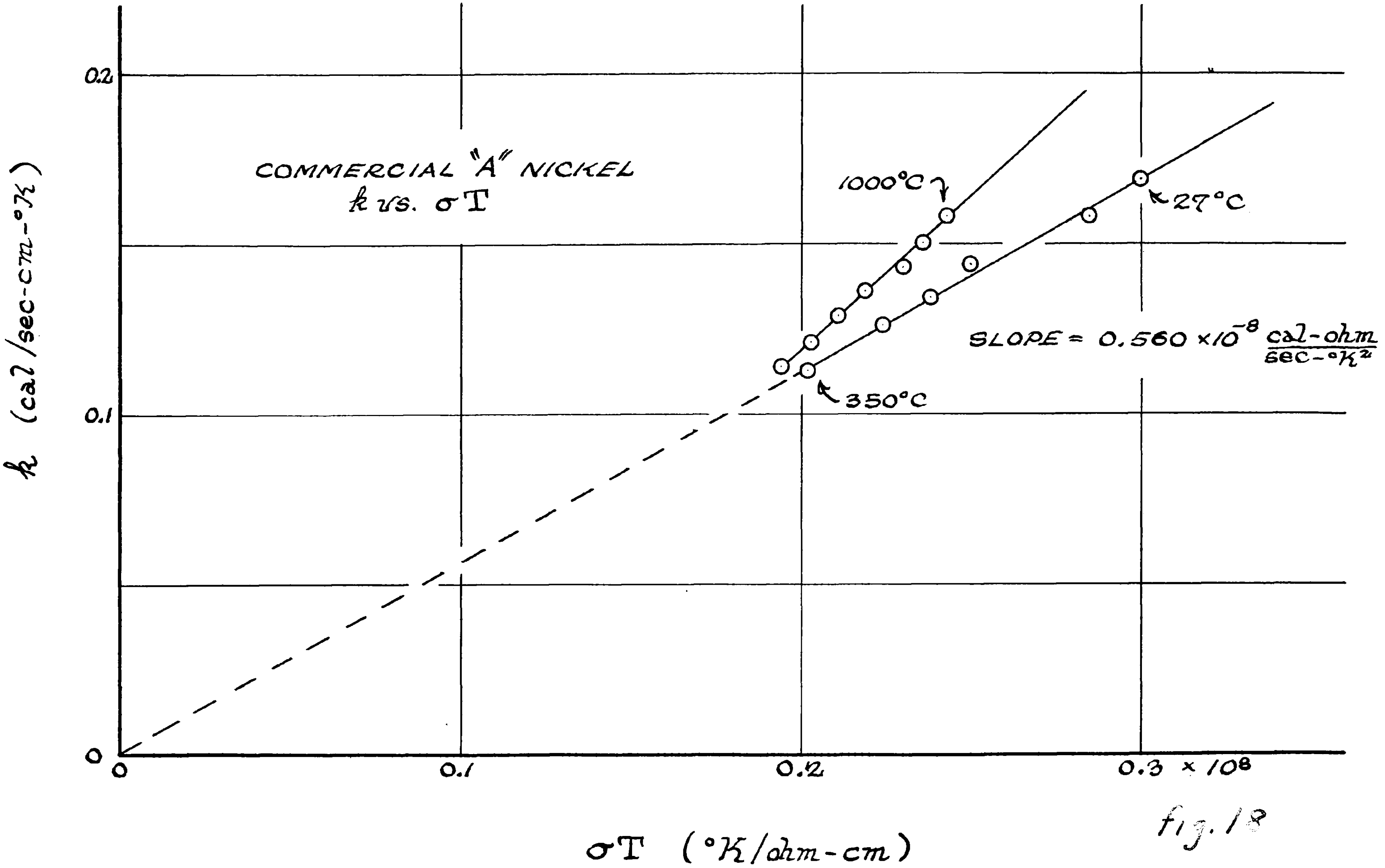

COMMERCIAL "A" NICKEL
k vs. σT
1000°C
27°C
350°C
SLOPE = 0.560 × 10⁻⁸ cal-ohm / sec-°K²
0.2
0.1
0
0
0.1
0.2
0.3 × 10⁸
k (cal/sec-cm-°K)
σT (°K/ohm-cm)

fig. 18

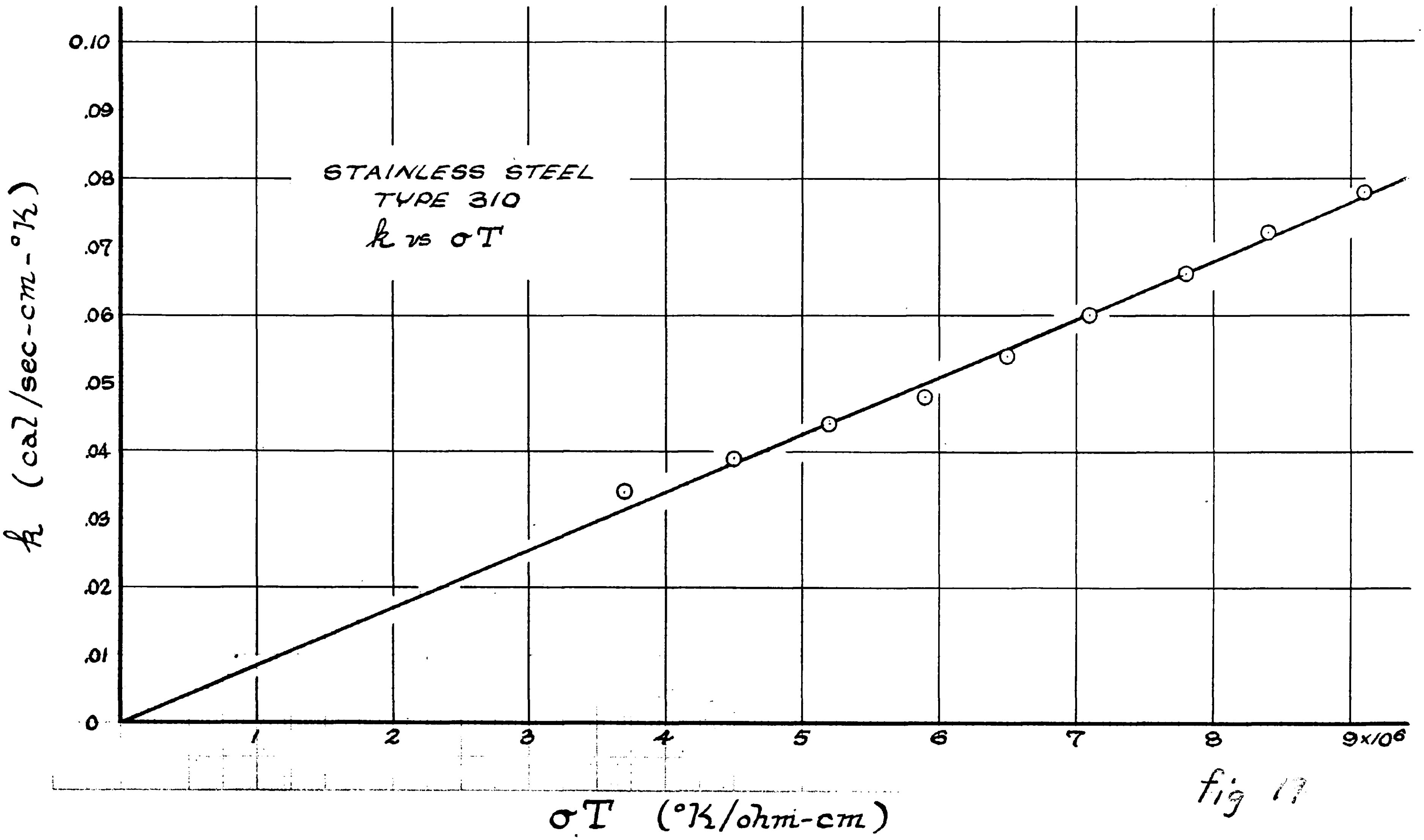
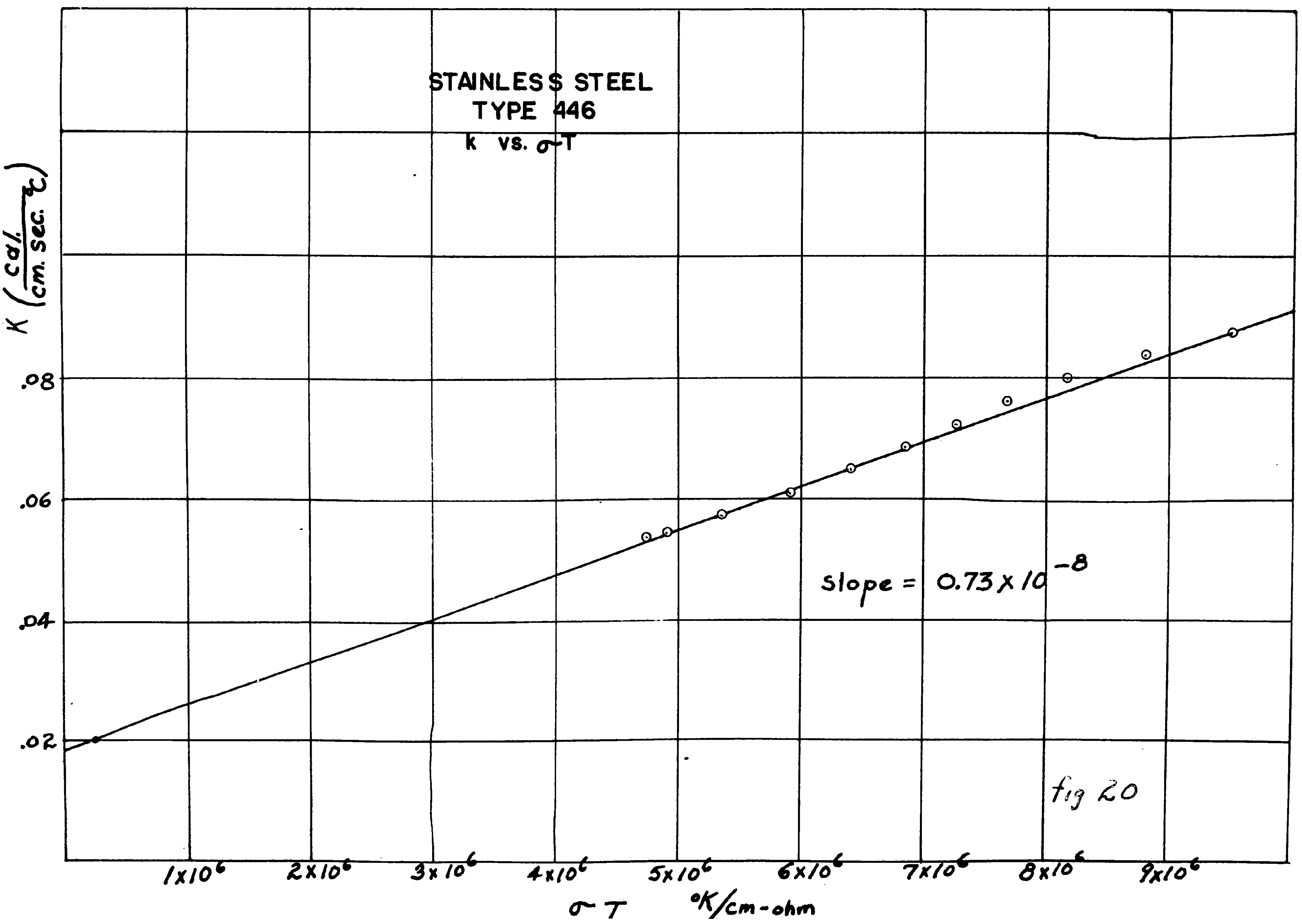

STAINLESS STEEL
TYPE 310
k vs σT
k (cal/sec-cm-°K)
0.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
0
1
2
3
4
5
6
7
8
9×10⁶
σT (°K/ohm-cm)

fig 17



STAINLESS STEEL
TYPE 446
k vs. σT
K (cal./cm. sec. °C)
.08
.06
.04
.02
Slope = 0.73 x 10^-8
1x10^6
2x10^6
3x10^6
4x10^6
5x10^6
6x10^6
7x10^6
8x10^6
9x10^6
σT °K/cm-ohm

fig 20

the relative change in thermal conductivity over the range from 0°C to 1000°C is small for some of the alloys reported and an error of 5% in the thermal conductivities can introduce errors of 20% or more in the slope of the so-called "Lorentz" plot. Hence the actual value of the above constants are in considerable doubt. Nevertheless, a rather interesting a significant trend is evident in the above listed values.

It is evident that the metals which are good conductors such as aluminum (or copper) give a value of L which is less than the classical Lorentz constant $\left(\pi^2/_3 \;\; (k/_e)^2 \right)$. Metals which are intermediate conductors such as nickel give a value of L which is very nearly equal to the Lorentz constant and poor conductors give increasingly large values for L. Thus type 446 stainless which has a thermal conductivity of 0.054 at 0°C gives a value of L equal to 0.73×10^{-8} while inconel which is a much poorer conductor gives a value of L equal to 0.810×10^{-8}. This trend is apparent in much of the work which appears in the literature.

In the past, the relation

$$k = L\sigma T + a$$

has been interpreted to show that the thermal conductivity of metals is equal to the sum of two terms; the first term, $L\sigma T$, is supposed to represent the contribution of the conduction electrons, and the second term, which is the intercept of the straight line, is supposed to represent the contribution of the lattice. There are many objections to this interpretation. One objection is that the lattice contribution would have to be a constant at all temperatures. Another objection is that even with this interpretation, the constant L turns out to be too high. As we have shown in the introduction, the constant $L_e = \frac{k_e}{\sigma T}$

for iron should be less than $L_0 = \frac{\pi^2}{3}\left(\frac{k}{e}\right)^2$ at all temperatures and should approach this value for $T/\theta > 1$. However the Lorentz plot for iron gives a value of $L = 0.625 \times 10^{-8}$ which is higher than L_0. Thus one runs into trouble if he tries to identify L as L_0.

Another very serious objection arises with the attempt to identify the intercept a, of the Lorentz plot with the lattice conductivity. As pointed out in the introduction, it is expected that the contribution of the lattice to the thermal conductivity of copper, aluminum or other good conductor should be negligibly small since the free electrons in these metals scatter the lattice waves. On the other hand, metals such as iron, inconel, etc. which have only a fraction of a conduction electron per atom should have correspondingly larger lattice contributions. Experimentally, it is found that aluminum gives an intercept of 0.03 on the Lorentz plot which is higher than that of any metal or alloy measured. Thus the above interpretation leads to a grave discrepancy with regard to the order of magnitude of the lattice contribution.

The author believes that the fact that the thermal and electrical conductivity of almost all metals and alloys can be represented by a straight line on the Lorentz plot is a coincidence as far as the present day theory is concerned. This arises from the fact that for all the alloys tested a graph of σT versus T is also a straight line. Why this should be so is not known. However, it is to be expected for pure metals that the product σT is practically constant at all temperatures when $T/\theta > 1$

Thus for aluminum it has been found experimentally that

$$k = 0.502 \times 10^{-8} \sigma T + 0.03$$

and we expect (see introduction) a relation such as follows to be valid:

$$k = k_e + k_g = L_e \sigma T + k_g$$

It was also shown in the introduction that for $T/\theta \approx 1$ we would expect for aluminum that:

$$L_e \approx 0.9L = 0.9\ (.585 \times 10^{-8}) = 0.526 \times 10^{-8}$$

Hence for temperatures near the Debye temperature (395°K for Aluminum) we should expect the following relation to be true for aluminum.

$$k = 0.526 \times 10^{-8}\ \sigma T + k_g$$

and empirically we find:

$$k = 0.526 \times 10^{-8}\ \sigma T - 0.024 \times 10^{-8}\ \sigma T + 0.03$$

But for aluminum (99.996%) the product σT is practically constant and equal approximately to 1×10^{8}. Thus we can write the empirical relation above in the form.

$$k = 0.526 \times 10^{-8}\ \sigma T - 0.024 + 0.03$$

$$k = 0.526 \times 10^{-8}\ \sigma T + 0.006$$

where the first term represents the contribution of the conduction electrons and the second term represents the lattice contribution. This relation is, of course, only valid in the region where $T/\theta \approx 1$ since for values of the temperature much lower than this L_e cannot be expected to be constant.

For Nickel it was found experimentally that

$$k = 0.56 \times 10^{-8}\ \sigma T$$

This would indicate from previous interpretations that the lattice contribution is zero. However, from calculations identical with those in the introduction it can be shown that for nickel,

$$L_e = 0.88 L_0 = 0.51 \times 10^{-8}$$

$$\text{for } T/\theta = 1 \ (\theta = 472°K)$$

Also, the product σT for nickel is approximately 0.2×10^{-8} and while it is not as constant as for aluminum we can nevertheless write:

$$k = 0.51 \times 10^{-8} \ \sigma T + 0.01$$

where the second term (0.01) which represents the lattice contribution is not really constant, since the product σT is not constant but nevertheless will be of the order of magnitude shown.

In the two examples above, we have written the total thermal conductivity of the metals as the sum of two terms, one a lattice contribution and the other a free electron contribution. Both free electron terms were computed by means of Wilson's equations (see introduction). The resulting lattice contributions are not completely temperature independent, they are of the correct order of magnitude for lattice conductivity, and the nickel sample shows a larger lattice conductivity than aluminum, which, according to the introduction, is expected.

Since for the stainless steel alloys it is impossible to compute the value of L_e from theoretical considerations we can only inspect the data obtained from these alloys more or less qualitatively. However, as shown in the introduction, the presence of impurities in a metal tends to make the value of L_e approach L_0 more rapidly. Thus it seems probable that the value of L_e for the stainless steel alloys will be very close to L_0.

If we average the values of L and a which are found empirically and listed above for inconel and the three stainless steel alloys we get the following average values.

$$L = 0.775 \times 10^{-8}$$

$$a = 0.007$$

Thus we can write

$$k = 0.775 \times 10^{-8} \sigma T + 0.007$$

or:

$$k = 0.585 \times 10^{-8} \sigma T + 0.190 \times 10^{-8} \sigma T + .007$$

But the product T can be represented empirically by the equation

$$\sigma T = A T + B$$

which is valid from $0^{\circ}C$ to $1000^{\circ}C$

The constants A and B vary somewhat from one alloy to the next but since we are here only interested in qualitative results we can represent all alloys by the average equation:

$$\sigma T = 1.2 \times 10^{6} + 0.66 \times 10^{4} T$$

Putting this into the above equation for thermal conductivity leads to the expression:

$$k = 0.585 \times 10^{-8} \sigma T + 0.93 \times 10^{-2} + 0.13 \times 10^{-4} T$$

where:

$$k_e = 0.585 \times 10^{-8} \sigma T$$

$$k_g = 0.009 + 0.13 \times 10^{-4} T$$

at 400° K this gives for k_g

$$k_g = 0.015$$

This gives a value of k_g which is larger than the value obtained for nickel, which is as we expect. In addition a lattice conductivity is obtained which is not temperature independent.

THERMAL CONDUCTIVITY OF ALUMINUM
IN
THE SOLID AND LIQUID STATES

METHOD

In order to determine the thermal conductivity of aluminum in the liquid state, the sample was prepared in the following manner. An alundum tube 12 inches long and with a bore of ¼" manufactured of Norton's RA 98 material was used. This particular material was chosen because it drills quite easily. Holes were drilled in the Alundum tube, into which the chromel-alumel thermocouples could be placed. Thermocouples were inserted into the tube (projecting inside) at two centimeter intervals down the tube. The thermocouples were then cemented into the holes with Alundum cement and the whole tube was covered with a thin coat of Alundum cement which covered the thermocouple wires completely. This cylindrical crucible was then dried and baked at 1100°C to set the cement. Thin aluminum rods were melted in the crucible until the crucible was full and then the aluminum was allowed to freeze from the bottom up so that no voids would occur. The sample was then ready to be placed into the furnace.

The thermal conductivity of the aluminum was measured in exactly the same manner as the thermal conductivity of the metals which have been previously discussed with the one exception that the R values required for the determination of thermal conductivity in the liquid state were extrapolated from the R values which were measured in the solid state. All these R values were in agreement with the values of R calculated from the thermal conductivity of air as discussed above.

The values obtained for the thermal conductivity of aluminum (99.996%) are as follows: (melting point 658°C.)

0°C	0.570	calories/sec. cm °C
100°	0.560	
200°	0.544	
300°	0.528	
400°	0.513	
500°	0.497	
600°	0.480	
700°	0.247	
790°	0.290	

These values are consistent with the following equation.

$$k = 0.502 \times 10^{-8} \sigma T + 0.03$$

CONCLUSIONS

The need for dependable data on the thermal conductivity of metals over a wide temperature range cannot be overemphasized. The range over which this data is required for engineering application is being enlarged rapidly today due to the increased interest in power plants of all types which operate at high temperatures. In addition to this practical application, knowledge of thermal and electrical conductivities is of fundamental importance in the modern theory of the metallic state. There is good prospect even, that reliable data on the thermal conductivity of molten metals would give some insight into the structure of the liquid state.

Much data occurs in the literature which is obviously in error. Even more misleading, however, is the data which exists in which the errors are much larger than is claimed by the author, and which are not completely obvious to the reader. For this reason, the complete mathematical analysis of the Forbes bar technique was made. This alone, indicates that the basic assumptions usually made in this method lead to an error of about 1% in the determination of thermal conductivity. It is believed that experimental errors, mostly in the determination of R, the rate of heat loss, lead to a total error of ± 5% in the determination of the thermal conductivity. In order to reduce these errors to a minimum, a completely new method of treating the data is introduced here. The major improvements consist of eliminating the need for knowledge of the specific heat and the elimination of the usual graphical analysis which required the measurement of the slopes of arbitrary curves. It is hoped that these improvements will give to the physicist another dependable tool with which he can check the present day theory of metals.

In most respects then the basic theory of Wilson and Makinson which is presented in the introduction of this dissertation is found to be valid. Essentially this states that the total thermal conductivity of a metal can be represented by the sum of two terms. The first term, that due to electron conduction can be computed theoretically. The second term, or that due to crystal lattice conduction, should be less than the conductivity of a similar crystal which is an electrical insulator and it should be less for metals with a large number of free electrons than for those with a small number, because the free electrons scatter the lattice waves and thus reduce the lattice conductivity.

BIBLIOGRAPHY

References to Text

(1.) Makinson, Proc. Cambr. Phil. Soc. 34, 474 (1938)

(2.) Gruneisen and Reddemann, Ann. d. Physik 20, 843 (1934)

(3.) Eucken and Neumann, Zeit. Phys. Chem. 111, 431 (1924)

(4.) Reddeman, Ann. d. Physik 20, 441 (1934)

(5.) Wilson, Proc. Cambr. Phil. Soc. 33, 371 (1936)

(6.) Bloch, Zeit f. Phys. 52, 555 (1928) and 59, 208 (1929)

(7.) Hume - Rothery, "The Metallic State" p. 51 Oxford (1931)

(8.) Peierls, Ann. d. Physik, 4, 121 (1930; 5, 244 (1930); and 12, 154 (1932)

(9.) See (1) above

(10.) Konno, Phil. Mag. 40, 542 (1920)

(11.) Hering, Trans. AIEE, 29, 485 (1910)

(12.) Angell, Phys. Rev., 33, 421 (1911)

(13.) Jaeger and Diesselhorst, Wissenschaftliche Abhandlungender Physikalisch - Technischan Reichsanstalt, 3, 269 (1900)

(14.) Lees, Phil. Trans Roy. Soc. of London A 208, 381 (1908)

(15.) Powell and Hickman, second report of Alloy Steels Committee, London (Iron and Steel Institute) page 242 (1939)

(16.) Hase, Peenemunde Archive No. 47/lg (1939)

(17.) Bureau of Standards Journal of Research 12, 435 (1934)

(18.) Bidwell and Hogan, Journ. Appl. Phys. 18, 876 (1947)

(19.) FIAT, Rev. German Science (1939-1946) "Physics of Solids, I", 212 - 20 (1947)

(20.) Armstrong and Dauphinee, Canadian Jour. Res., A, 25, 357 (1947)

(21.) Churchill, "Fourier Series and Boundary Value Problems," Chap VIII McGraw - Hill.

(22.) Jacob, "Heat Transfer" (1949) John-Wiley and Sons.

(23.) Austin, "Flow of Heat in Metals" (1942) Amer. Soc. for Metals

(24.) Eucken, Forschung a. d. Geb d. Ingenieurwes 11, 6 (1940)

(25.) Smith and Palmer, Metals Technology Vol. 2, No. 6 (1935)

GENERAL REFERENCES RELATED TO SUBJECT (not listed above)

Theory of Conductivity:

(1) Seitz, "Modern Theory of Solids" (1940) McGraw Hill

(2) Wilson, "Metals and Semiconductors" (1939) Cambridge

(3) Wilson, "The Theory of Metals" (1936) Cambridge

(4) Mott and Jones, "Properties of Metals and Alloys" (1936) Oxford

(5) Mott and Sneddon, "Wave Mechanics and it's applications" (1949) Oxford

Additional references pertaining to crystal lattice conductivity:

(6) Debye, "Vortraege uber die kinetische Theorie der Materie und Elektrizitaet" Leipzig and Berlin (1914)

(7) Eucken, Annalen d. Physik 34, 185 (1911)

(8) Peierls, Annalen d. Physik 3, 1055 (1929)

(9) Papapettu, Physik Zeitschr. 35, 527 (1934)

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Publications: "Thermal Conductivity of Aluminum; Solid and Liquid States" by C. C. Bidwell and C. L. Hogan, Journal of Applied Physics 18, 776 (1947)

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