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<u>Via: Dr. H. A. Neville</u>

Dear Mr. Mack:

Attached are two copies of a report by Dr. F. K. Chang and myself which are abstracts of Dr. Chang's dissertation. Since numerous revisions have been made in part of the content of the dissertation I suggest these abstracts be attached as appendices to the dissertation and included on loan whenever the dissertation is called for.

These are being transmitted via Dr. Neville so he will be aware of the fact that the abstract has been completed as promised earlier.

Very truly yours,

Bruce Johnston

Bruce G. Johnston
Director

BGJ:fs

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TORSION OF PLATE GIRDERS#

By

F. K. Chang* and Bruce G. Johnston**

SYNOPSIS

This is to report## on the analytical and experimental analysis of the stress distribution, stiffness, and strength of bolted riveted, and welded plate girders under uniform twisting moment. Seventy-two tests were made on fifteen different test specimens.

Formulas are proposed for calculating rivet or bolt pitch and size of welds for the bolted, riveted, and welded plate girders, or, when rivet pitch or weld size are given, the torsional stiffness and strength can be predicted.

Eleven of the specimens were finally tested into the plastic range. Except for initial pilot tests of small specimens, all tests were made on the 2,000,000 in-lb. torsion testing machine(1) especially built for this program of research. Tests included variable number of cover plates, variable rivet pitch, with or without stiffeners, variable tension in bolts, and a variety of girder cross-section sizes up to four feet in depth.

\# Presented at the January 1950 Meeting of the A.S.C.E.

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\## Based on Ph. D. dissertation by F. K. Chang - Reference 14.

1) Numbers refer to list of references at end of report.

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INTRODUCTION

It should be recognized at the outset that the problem of torsion in the case of an "open" section, such as the I-beam, if it exists at all, is usually a secondary matter, the primary problem being the bending for which this type section is especially effective. If torsional loads are of promary concern, a "closed" box or tube section is recommended.

A knowledge of the torsional behavior of built-up I-section girders is especially needed in considering problems involving lateral buckling, as corroborated by recent investigators in this field. de Vries(7), p. 1250, states, "Torsion formulas for plate girders do not seem to be available ...". Winter(8), p. 1341, in a similar paper comments, "The amount of experimental evidence on the torsional behavior of built-up members cannot yet be regarded as sufficient."

The general mathematical treatment of the torsion problem was first made by Saint-Venant(2) in 1855. He also presented particular solutions for constant twisting moment applied to shafts of various cross-sections. His solution for the rectangle is directly applicable to the plate girder, which consists principally of an assemblage of narrow rectangles.

Space will not permit a review of general torsion theory and the reader may review this subject in references 2 or 5. In the case of a straight shaft of uniform, circular, cross-section, the relationship between torsional moment (M) and angle of twist per unit length (θ) is given by:

$$M = JG\theta \tag{1}$$

G is the shearing modulus of rigidity and J is the polar moment of inertia. In the case of the circular section, plane cross-sections before twist remain plane after twist and the resultant shear stress at any point is proportional to the distance from the central axis of the shaft. Neither of these assumptions hold for the non-circular sections, but the relationship given by Eq. (1) may be modified to

$$M = KG\theta \tag{2}$$

where "K" is simply defined as a "torsion constant"(3) determined by the shape and dimensions of the cross-section. Timoshenko(5,6) uses the notation C = KG, C being termed the "torsional rigidity" of the particular section.

In the case of the narrow rectangle having a width "w" more than three times the thickness "t", the solution by Saint-Venant reduces to:

$$K = \frac{wt^3}{3} - 0.21t^4 \tag{3}$$

In the case of the circular shaft the resultant shear stress is given by:

$$\tau = \frac{Mr}{J} \tag{4}$$

where r is the radial distance from the central axis of the shaft.

In the case of a rectangular plate, except near the narrow edges, the resultant shear stress in the cross-section will be parallel in direction to the wide side and will be proportional in magnitude to the normal distance from the middle plane. Away from the narrow edges, at the surface of the broad sides, the maximum shear stress in the narrow rectangle is closely approximated by the following formula:

$$\tau_{max} = \frac{Mt}{K} \tag{5}$$

The maximum shear stress parallel with the narrow surface at the ends of the section is always less in magnitude than the maximum stress in the broad sides, hence Eq. 3 gives the maximum stress in the section(2). When w is more than four times t, as in the case of most components of structural sections, the error in Eq. 3 is negligible and approaches zero as w gets very large in comparison with "t".

Since the early work of Saint-Venant, there have been many investigations of torsion problems, few of which are on the subject of built-up girders, the primary concern of this report.

The torsion of rolled structural I-beam and wide flange sections had been the subject of an earlier investigation(3) at Lehigh University. Formulas were developed for evaluating the torsional constant, K, for rolled sections, but the application of these formulas to built-up members, such as riveted, bolted, or welded plate girders was open to question. The various components of a built-up member may act together as an equivalent solid section or, if not joined, may act separately. The torsional stiffnesses evaluated for those extreme assumptions will be very different. In some practical cases, the equivalent solid section torsion constant, K, (the measure of torsional stiffness) is as much as fifteen times as large as the "K" for separate action. In order to determine "K" for built-up sections, an extensive experimental investigation was considered necessary. In another Lehigh University investigation I. Madson$^{(4)}$ had made a preliminary study showing that the torsional constants of small models of riveted and welded built-up I-beams were somewhat less than that of the equivalent solid section.

Because small size specimens with cold driven rivets were used in Madson's tests, it was suggested in this investigation to test larger girders and to make an extensive study considering various factors which affect the torsional behavior of built-up members, such as pitch of rivets and bolts, size of weld, tension in bolts, stiffeners, etc.

BUILT-UP PLATES

The rectangular section of built-up plates will be treated initially because they are the basic component of the built-up plate girder.

If a stack of "n" plates, each of equal thickness "t", are bolted or riveted together and transmit a torsional moment, the limits of behavior will be that of a solid piece with the thickness "nt" as one extreme and that of n plates twisted separately as the other. The torsional constant of a plate varies as the cube of its thickness; built-up plates with equivalent solid section behavior therefore are much stronger and stiffer than built-up plates with separate action, as indicated in Table I.

TABLE I

Number of Plates	Ratio of M_{solid} to $M_{sep.}$ for same $\tau_{max.}$	Ratio of M_{solid} to $M_{sep.}$ for same angle of twist
n	n	n^2

For example: A stack of 4 plates, riveted together, will twist 16 times more and be stressed four times more if its plates act separately than if it behaves as an equivalent solid section. In actual cases, the built-up plates behave somewhere between these two extremes depending principally on the pitch, tightness, and location of gauge lines of bolts or rivets. This intermediate behavior will be termed "integral behavior" in this report.

Slip Between Different Components:

If loosely bolted together, the different components of the built-up plate slip longitudinally with respect to each other during twist. If this slip can be prevented, integral behavior will be obtained. The longitudinal slip between plates (assuming no interaction) may be considered as arising from two causes, (1) that due to the longitudinal warping of each individual cross-section, (independent of the location of the center of twist) and (2) longitudinal displacements resulting from rigid body rotations. These are dependent on the location of the center of twist and an arbitrary location of zero displacement. The shearing stress distribution in the component rectangular sections is independent of the second type of displacement.

Longidutinal warping of the individual cross-section, (1 above), may be closely approximated for the narrow rectangular shape of thickness "t" and width "w" (See Fig. 1-a), as follows. At section x-x, a short distance from the narrow edge, the resultant shear will act nearly parallel with the wide side and the component of shear stress acting parallel to the plane x-x will be of negligible magnitude. Hence, if unit length of section is twisted through angle θ, with center of twist at "O" section x-x must rotate as a rigid body through angle θx, as shown in Fig. 1-b. For small angles of twist there will be negligible longitudinal displacement of the middle plane of the rectangle, except for regions very near the narrow edges. Hence, from Fig. 1-b, there will be longitudinal "warping" displacements of magnitude $\frac{\theta xt}{2}$ at the surface of the wide face. Near the narrow edges this simple relationship will no longer hold and a more rigorous approach is required for precise evaluation$^{(2)}$. Nevertheless, the warping displacement at the extreme corners will be closely approximated for the very narrow rectangle by letting $x = \frac{w}{2}$ and will therefore be about $\frac{\theta wt}{4}$.

The longitudinal displacements when the center of twist is not at the centroidal axis, but at any location "O", as shown in Fig. 2, is next considered. Assume that point "b" does not move longitudinally and let r be the distance from O to any point "a" in the middle plane of the rectangle, the angle between r and x being denoted as ϕ. During twist θ per unit length, any point "a" will be displaced transversely a distance $r\theta$ to a' and the component of this displacement in the middle plane of the rectangle will be $r\theta\sin\phi = y\theta =$ a constant. Since there is zero shear stress in the middle plane of the rectangle (except near the narrow edges) constant lateral displacement $y\theta$ can take place in this plane only by rotation through angle $y\theta$ as shown in Fig. 2-b, resulting in longitudinal displacements in the middle plane of magnitude $xy\theta$ relative to "b" as shown in Fig. 2-b. The total longitudinal displacemtn at any point will be the displacement due to warping added to the displacement due to rotation of the middle plane. In the case of two rectangular plates side by side as shown in Fig. 3, $y = \frac{t}{2}$, hence, at the narrow edge, where $x = \frac{w}{2}$, the maximum longitudinal displacement of the middle plane $xy\theta = \frac{\theta wt}{4}$ and the relative displacement of the two middle planes is $\frac{\theta wt}{2}$. If the relative displacement due to warping at point "c" of Fig. 3 is added to that just obtained for the middle planes the amount of total slip between the two plates is determined, as follows:

$$\text{Slip} = \frac{\theta wt}{2}\ \text{(due to warping)} + \frac{\theta wt}{2}\ \begin{pmatrix}\text{due to rotation}\\ \text{of middle plane}\end{pmatrix} = \theta wt \qquad (6)$$

This expression applies to any number of plates as long as they are of the same thickness. If the plates are of different thickness, "t" in Eq. (6) should be the average thickness of the two adjacent plates in question.

Pitch of Rivets or Bolts to Develop Integral Action:

If it is possible to eliminate the relative slipping of several plates when bolted or riveted together, the assemblage of plates would then act integrally. The required shearing forces transmitted by the interfaces between such plates would have to be furnished in large part by friction developed as a result of the clamping action of the bolts or rivets.

A study of the approximate stress distribution within a solid rectangular section under torsion will permit the determination of the pitch, bolt or rivet size, etc. required to transmit the necessary shear forces. Let Fig. 4-a represent an imaginary section normal to the axis of a rectangular bar of width w and thickness t, transmitting torsional moment M. Away from the narrow sides, as at section a-a, the resultant shear stress is proportional to the distance from the middle plane. Let S_1 represent the resultant shear force per unit width of the shear stresses in one direction acting over one-half thickness at section a-a. S_1 must diminish near the narrow edges to zero but in a thin rectangular section it will be nearly constant over most of the width. Near the narrow edges, the shear stress distribution is complex, but the <u>average</u> resultant force per unit distance parallel to the narrow side is represented in Fig. 4-a by $\bar{S}_2$ acting near the narrow sides.

From Fig. 4-a, it is obvious that:

$$S_1 = \frac{\tau_m t}{4} \qquad (7)$$

Now imagine the upper left corner of the section shown in Fig. 4-a to be isolated by longitudinal sections along b-b and c-c, as shown in Fig. 4-b. Along the section at c-c, from the equivalence of shear stresses acting on planes that are 90° apart, longitudinal shear resultants of magnitude S_1 per unit length must act. In the isolated corner section, equilibrium of forces in the longitudinal direction requires that the shear stress resultant along the middle plane section through b-b must also have an intensity S_1 per unit length.

One-half of the resultant torsional couple is supplied by S_1 forces, the other half by $\bar{S}_2$, as will be shown. In Fig. 4b, summing moments about axis d-d:

$$\frac{\bar{S}_2 tp}{2} = \frac{S_1 pt}{3} \text{ or } \bar{S}_2 = \tfrac{2}{3} S_1$$

From Fig. 4a, the torque supplied by $\bar{S}_2$ is seen to be a little less than $2\bar{S}_2 wt$ and that supplied by the distributed S_1 forces is a little less than $\frac{2S_1 wt}{3}$. Hence, introducing the relationship $\bar{S}_2 = \frac{2}{3}S_1$, it is seen that the torques supplied by S_1 and $\bar{S}_2$ are at least approximately equal.

If the rectangular section were split into two plates along its middle plane, b-b of Fig. 4a, and the two portions bolted or riveted together at the narrow edges with bolt pitch "p", the distributed forces S_1 in the middle plane section through b-b over distance p (Fig. 4b) might be assumed to be replaced by concentrated rivet or bolt shear R, or:

$$R = S_1 p$$

Substituting S_1 from Eq. (7) into the foregoing and denoting the combined thickness of the two plates by T.

$$R = \frac{\tau_m Tp}{4} \tag{8}$$

A given thickness "T" and pitch "p" determine a definite ratio between rivet shear "R" and maximum torsional shear stress "τ_m". Let "p_r" be the rivet pitch that will produce rivet slip and shear yield, R' and τ_y, respectively, at the same time. From Eq. 8,

$$p_r = \frac{4R'}{\tau_y T} \tag{9a}$$

If the pitch is greater than p_r, rivet slip will occur before yielding of the material; if the pitch is less than p_r, the material will yield prior to rivet slip. The initial linear relationship between torsional moment and angle of twist per unit length will be limited by whichever occurs first, slip or yield. If the working allowable values of rivet shear and shear stress imply identical factors of safety with respect to slip and yield, respectively, the pitch to produce these values simultaneously will be p_r as given by Eq. 9a. Although these factors of safety usually are not the same, the accepted allowable rivet and shear stress values will be used in discussing the design of the specimens tested in this investigation. The rivet pitch for balanced design in pure torsion will be assumed as:

$$p_r = \frac{4R_w}{\tau_w T} \tag{9b}$$

where R_w is the allowable single shear value of the rivet, τ_w the permissible shear stress for the material. To assure that the distributed shear force S_1, along the middle plane section through b-b (Fig. 4b) within the rivet or bolt pitch, p, will be replaced by the rivet or bolt shear, the effective rivet or bolt clamping distance along the length of the plate must be evaluated. In Ref. 5, p. 48, the solution of the problem of a rectangular strip loaded on opposite sides by two opposed forces in the same line and normal to the surface, shows that the compression in the middle plane falls away rapidly and changes to tension at a distance from the line of action of the forces slightly greater than half the plate thickness. The present problem is somewhat different since it is three-dimensional instead of two-dimensional and since no tension can be developed between the plates. However, it seems reasonable to assume that the effective clamping distance per rivet is equal to the sum of the diameter of rivet (or bolt) head, A, and the overall thickness of the assembly T. Therefore, the rivet or bolt pitch should not be larger than a critical value $A + T$, or, denoting this critical pitch by p',

$$p' = A + T \tag{10}$$

The effect of making p larger than p' will be treated in a later section.

When there are more than two plates, Eq. 8 and 9 still apply, as the force per unit length (S_1) to be transmitted between the two plates nearest the center will still be given by Eq. (7) (if there are an even number of plates) with the substitution of overall thickness T in place of t. If there are an odd number of plates, the force to be transmitted will be somewhat less than S_1 and the procedure will err slightly on the conservative side.

Evaluation of the Torsion Constant:

In the case of several plates riveted or bolted together as shown in Fig. 5a, it is obvious that the outer rows of rivets or bolts may be too far from the edges of the plates to produce the necessary pressure required to develop friction near the edges sufficient to replace lomgitudinal S_1 forces shown in Fig. 4b.

Mr. de Vries(7) assumed that the assemblage between the outer rows of rivets or bolts acts as a solid section, and beyond the line of the assumed solid section, only the contribution of the individual plates would be counted upon as shown in Fig. 5b. This assumption seems very reasonable, simple, and on the conservative side. The integral action torsion constant K_I can be obtained by modifying Eq. (3) according to the foregoing assumptions. Using the notation in Fig. (6a) and (6b), the torsion constant for "n" plates of equal thickness is

$$K_I = \frac{bT^3}{3} + \frac{n2ct^3}{3} - 0.21T^4 \qquad (11A)$$

or, if the edge effect is neglected,

$$K_I = \frac{bT^3}{3} + \frac{n2ct^3}{3} \qquad (11B)$$

If the plates are not of equal thickness, the second term in Eqs. 11A and 11 B should be changed to $\frac{2}{3}c\sum_{n} t^3$.

On the basis of the assumed stress distribution in Fig. 5b the rivet pitch formula may be derived directly from the equilibrium of moments acting on the half section of a combination of two bolted or riveted plates of pitch length p, as shown in Fig. 5c. The resultant shear stress on one plate over length b (refer to previous discussion regarding Fig. 4) will be $\frac{\tau_m Tb}{4}$, developing a couple about a normal to the plate interface equal to $\frac{\tau_m Tbp}{4}$. This couple is opposed by the couple produced by rivet shear "R", with moment arm "b". Equating the two couples:

$$p = \frac{4R}{\tau_m T}$$

which is identical with Eq. (9a) as previously derived.

When more than two plates of equal thickness are bolted or riveted together as in Fig. 6a or 6b, the single shear force on the rivets at the center plane of the combined unit will be the same as in the case of two plates (provided the total number is even) and Eq. (9) for rivet pitch will be unchanged. If there are an odd number of plates of equal thickness, the interface between the central plate and the adjacent plate will not be at the center of the stack and Eq. (9) will be slightly too conservative - the error being about 11% for the case of 3 plates and 4% for 5 plates. In the interest of simplification for actual design it seems desirable to neglect this discrepancy.

If there are four rows of rivets, as shown in Fig. 6b, it might be presumed that the two inner rows would not be as effective as the outer rows. Possibly the rivet value of the inner rows should be discounted slightly, or solid section behavior assumed only to the mid-distance between the outer pairs of rows. Nevertheless, as will be shown later, tests are in good agreement with the assumption that solid section behavior is maintained between the outermost rows, the innermost rows being assumed equally effective. This procedure should not be extended beyond the limits of the present test program and should be limited, therefore, to a maximum of four rows with the inner rows as near to the outer ones as the minimum permissible gage "g" will allow.

For n plates loosely bolted together or not bolted or riveted at all, each plate would act separately and the torsional constant for separate action would be

$$K_s = \frac{n}{3} w t^3 \qquad (12)$$

If the pitch is larger than the critical pitch, p' (Eq. 10), there will be a tendency for the plates to develop separate action in the spaces between the rivet lines. This question also arises in the case of a welded girder using intermittent welds to connect the component parts in which case no clamping effect should be assumed between the welds and p' is simply the intermittent weld length.

In Fig. (7a), along the "clamped" portion p', the integral torsion constant, K_I, will be used. But along the portion p - p', each plate will be assumed to act separately with both "ends" partially restrained. By "end" is meant the edge of the shaded portion in Fig. 11a, at which location integral action is assumed to begin. For a solid rectangular plate with middle cross-section remaining plane during twist, Timoshenko$^{(9)}$ found that the decrease of ϕ, total angle of twist, resulting from the local constraint is the same as that corresponding to the diminution of the length by the quantity $\delta \ell$. $\delta \ell$ is independent of the length and the value varies from 0.212w to 0.195w according to the w/t ratio. The present problem is somewhat different since, although partially restrained, the "ends" of the component parts do not remain completely plane, but would have approximately the degree of warping present in the equivalent solid section. However, as an approximation, the transition between K_I and K_S will be assumed as shown in Fig. (7), thereby compensating for the partial restraint effect.

In view of the approximations involved, the value of $\delta \ell$ in the present case will be assumed as 0.2b. The equivalent torsional constant along the length p - p' (Fig. 7b) will be

$$K_{SE} = \frac{\delta \ell K_I + (p-p' - \delta \ell)K_S}{p-p'} = \frac{0.2bK_I + (p-p' - 0.2b)K_S}{p-p'} \qquad (13A)$$

for $p-p' \geq 0.4b$

If p-p' is smaller than 0.4b, the torsion constant will be assumed to vary as shown in Fig. 7c and the equivalent torsion constant will be

$$K_{SE} = \frac{\frac{p-p'}{2}K_S + \left[0.4b - \frac{(p-p')}{2}\right]K_I}{0.4b} \tag{13B}$$

for $p-p' < 0.4b$

The overall effective torsion constant K_{eff} along the length p can be found by using Fig. 7d.

Let ϕ_p = total angle of twist over the length p

ϕ_1 = total angle of twist over the length p'

ϕ_2 = total angle of twist over the length p-p'

By using Eq. (2), we have

$$\phi_1 = \frac{M}{K_I G} p'$$

$$\phi_2 = \frac{M}{K_{SE} G}(p-p')$$

Then

$$\phi_p = \phi_1 + \phi_2 = \frac{M}{K_I G} p' + \frac{M}{K_{SE} G}(p-p')$$

The average twist per unit length θ_{avg} will be

$$\theta_{avg} = \frac{M}{pG}\left[\left(\frac{1}{K_I} - \frac{1}{K_{SE}}\right) p' + \frac{p}{K_{SE}}\right]$$

$$K_{eff} = \frac{M}{G\theta_{avg}}$$

$$K_{eff} = \frac{K_{SE}}{1 - \left(1 - \frac{K_{SE}}{K_I}\right)\frac{p'}{p}} \tag{14}$$

If p is greater than p', the force distribution shown in Fig. 5c holds only over the distance p', wherein integral behavior is assumed, and the maximum shear stress in this region for a given rivet stress R is, from Eq. 8:

$$\tau_I = \frac{4R}{p'T}$$

If the rivet pitch exceeds p', the calculation of maximum shear stress must also be modified outside of the clamped region in riveted or bolted members, or between intermittent welds in welded members. Within the clamped or welded zone the maximum shear stress may be assumed as

$$\tau_I = \frac{MT}{K_I} \tag{15a}$$

If p - p' is greater than 0.4b, completely separate action midway between rivets or welds implies that the maximum shear stress at this location is

$$\tau_S = \frac{Mt}{K_S} \tag{15b}$$

If the plates are not all of the same thickness, t should be taken as the thickest of the individual plates.

If p - p' is less than 0.4b the maximum shear stress will be intermediate between τ_I and τ_S. Assuming a linear relationship

$$\tau_{IS} = M\left[\frac{T}{K_I}\left(1 - \left(\frac{p-p'}{0.4b}\right)\right) + \frac{t}{K_S}\left(\frac{p-p'}{0.4b}\right)\right] \tag{15c}$$

Eq. 15 is advanced tentatively and it is believed it will give conservative strength estimates. Insufficient stress measurements were made regarding the change in stress along the length of the girder to investigate this item in detail.

Effect of Transverse Holes and Stress Concentration:

If a plate containing several open holes is twisted there will be localized stress concentrations near the holes that can be calculated approximately in the plastic range by available formulas (Reference 6, page 317). If the plate is one of several comprising a built-up bolted or riveted girder, the clamping action of the bolts and rivets, as well as the stress carried through the heads by plate friction, will alter the local stress condition and tend to eliminate the effect of the hole. Furthermore, as in the case of bending of plate girders, initial local yielding in the steel will permit stress redistribution and the holes therefore will have little effect upon the general behavior of the member as a whole.

In the tests to be reported upon, as a limiting case, some of the plate assemblages were tested with zero or nearly zero bolt tension. In such cases, the holes would have their maximum effect upon the torsional stiffness. If was found that a "reduced effective width" gave good agreement with test results. The "reduced effective width" is the average width of a solid plate of identical thickness that has the same net volume of material as is in the actual plate with holes deducted. For example, in the case of a plate of width "w", having "n" rows of holes of diameter "d" and pitch "p", the effective width would be

$$w_{eff} = w - \frac{n\pi d^2}{4p}$$

BOLTED, RIVETED AND WELDED PLATE GIRDERS

As in the case of built-up plate sections, when bolted, riveted, or welded plate girders are twisted, slip between different parts may occur. Therefore, the fabricated girders may not be the equivalent of solid sections either in stiffness or in strength. However, if sufficient rivets, or bolts having sufficient tension, or adequate welds, are used, it is possible to hold the slippage to a minimum and develop "intergral" behavior intermediate between complete "solid section" and "separate" behavior.

Slip Between Different Components:

The maximum slip between different components of fabricated plate girders, if the bolts or rivets are loose, is of comparable magnitude to that between any two plates of similar size, as given previously by Eq. (6). In a girder of I-section, if there are two or more cover plates, the maximum slip between any two cover plates would be given directly by Eq. (6). In the case of an angle connecting flange to web of a riveted plate girder, the longitudinal displacement of the middle plane of the angle is determined by the integrated effect of the friction of the component parts adjacent to it. The determination of such slip on the basis of arbitrary hypotheses will not be treated herein as it is of academic interest and the obvious objective is to eliminate as much slip as possible by means of bolts, rivets, or welds.

Evaluation of Torsion Constant:

The torsion constant, K, may be determined for rolled sections by formulas and procedures given in Reference 3. In studying the test results of built-up girders, it is of interest to compute K values for the two extreme conditions of "equivalent solid section behavior" and "separate action behavior" between which the actual "integral behavior" will lie.

The procedure proposed herein for built-up girders parallels that discussed previously for the case of built-up plates. Figs. 8a and 8b indicate the nomenclature for girder sections with two and four rows of rivets in the flanges, respectively. The effective flange width assumed as solid section is the same as that previously discussed in the case of an assemblage of plates, as shown by the shaded areas in Figs. 8a and 8b.

As noted in the case of the single rectangular plate (Eq. 11a) there is an "edge loss" - a negative term - in the equation for the torsion constant. As the width/thickness ratio increases this edge loss factor becomes a relatively small item and may be neglected without much error. When two or more rectangles are joined together to form a T or I-section, the torsion constant may be obtained by summing the contributions of the separate rectangles, using Eqs. 11a or 11b, but there is an additional contribution at the juncture or junctures of the various parts which has been called the "hump effect" (Ref. 3) and which may be evaluated as a function of D^4 (Figs. 8 and 9). In the case of the rolled section there is some justification for including both the "edge loss" and "hump" effects in evaluating the torsion constant, as has been done for the rolled I and WF shapes(3,15), even though they tend to offset each other. In the case of riveted or bolted girders however, there will always be some uncertainty as to the behavior, dependent as it is on the clamping action of the bolts or rivets and such refinements are not justified. It is also desirable to have as simple a formula as will give an approximation satisfactory for design purposes. This is more important in the case of the plate girder than for the rolled section as in the latter instance sizes are standardized and tables(15) of K values are available. In view of these facts both the "hump" and "edge" effects will be neglected, and the torsion constant, referring to Fig. 8 and letting n = cover plates in each flange, will be:

$$K_I = \frac{2}{3}bT_F^3 + \frac{2}{3}eT_W^3 + \frac{n4ct_F^3}{3} + \frac{4(m+f)\,t_A^3}{3} + \frac{1}{3}at_W^3 \qquad (16)$$

If different parts act separately, the torsion constant, neglecting edge effects will be

$$K_S = \frac{1}{3}\left[2nwt_F^3 + ht_W^3 + 4\left(f + o + m + \frac{(b-t_W)}{2}\right)t_A^3\right] \qquad (17)$$

For welded girders, Fig. 9, the integral torsion constant can be derived in a similar manner

$$K_I = \frac{2}{3}\,bT_F^3 + \frac{1}{3}at_W^3 \qquad (18)$$

If different parts acted separately, with n plates in each flange, the separate action torsion constant, K_S, neglecting the edge effects, would be

$$K_S = \frac{2nbt_F^3}{3} + \frac{1}{3}at_w^3 \qquad (19)$$

Rivet or bolt pitch, p_r, may be determined by Eq. 9 as developed for the case of built-up plates. Then the pitch required in the flange to simultaneously develop allowable working rivet shear (R_W) and maximum shear stress due to torsion (τ_m) will be

$$p_r(\text{flange}) = \frac{4R_W}{\tau_W T_F} \qquad (20a)$$

if the permissible shear stress is 12 ksi, Eq. 20a becomes:

$$p_r(\text{flange}) = \frac{R_W}{3T_F} \qquad (20b)$$

with the clamping distance requirement (Eq. 10) that

$$p'(\text{flange}) = A + T_F$$

The maximum torsional shear stress in the vertical legs of the angles that connect the web plate to the flange will be less than that in the flanges by the factor T_W/T_F, hence the pitch required in these legs is

$$p_r(\text{web}) = \frac{4R_W}{\tau(\text{web})T_W} = \frac{4T_F R_W}{\tau_m T^2_W} \qquad (21a)$$

again, if the permissible shear stress is 12 ksi,

$$p_r(\text{web}) = \frac{T_F R_W}{3T_W^2} \qquad (21b)$$

with the clamping distance requirement that

$$p'_{(web)} = A + T_w$$

If the rivet or bolt pitch is larger than the critical value p' the torsion constant should be reduced. Eqs. (13A), (13B), and (14) for built-up plate case can be applied.

No tests were made in combined bending and torsion, although a subsequent investigation on this subject is now in progress. Presumably, in combined bending and torsion, the rivet shear produced by torsion, as given by Eq. 8, could be combined with the rivet shear produced by the overall shear resultant of the vertical loads. The combined rivet shear should not exceed the permissible rivet value.

For welded plate girders, the sizes of welds should be such as to sustain the maximum shearing stress developed in the welds for solid section behavior. For example, if the size of welds to connect the flange cover plates is desired, a study of the free body diagram of Fig. 4b will indicate that the strength of the welds required is S_1 lbs per lineal inch, or $\frac{\tau_m T_F}{4}$ lbs per lineal inch (from Eq. 7). If a weld of the size $\frac{S}{8}$ in. is assumed to have an allowable strength of 1.2S kips per inch, corresponding to a shear stress of 13.6 ksi through the minimum weld section, the size of weld can then be determined,

$$S = \frac{\tau_m T_F}{4.8} \qquad (22a)$$

For maximum shear stress due to torsion of $\tau_m = 12$ ksi,

$$S = 2.5T_F \qquad (22b)$$

The required size of weld is $\frac{S}{8}$ inches. If welds connecting the various cover plates are intermittent rather than continuous, Eq. 13a or 13b and Eq. 14 may be applied to approximate the effective loss in torsional rigidity, in which case p = weld spacing and p' = length of weld.

Strength:

When a rolled section is under torsion, the critical shearing stress will occur along the outer surface of the beam and along the fillets where the material is thickest, but in built-up structural members, the maximum shear stress may occur somewhere else, especially around the rivet or bolt heads. However, neglecting these local stresses, the approximate shear stress can be determined by Eq. 5, with the change of t to the total thickness T.

The shearing stress is a function of the thickness of the material and the maximum stress in the flange τ_F and in web τ_W will be approximately

$$\tau_F = \frac{MT_F}{K} \tag{23A}$$

$$\tau_W = \frac{MT_W}{K} \tag{23B}$$

provided the bolt or rivet pitch be less than p' for longitudinal continuity or in case of a welded girder, provided that welds are continuous.

If the pitch exceeds p', or if intermittent welds are used in a welded girder, the maximum shear stress in the flange will occur midway between the lines of rivets or between the intermittent welds, as the case may be, and may be calculated by Eq. 15. Eq. 15 probably gives too conservative an approximation of the strength but the problem will be an unusual one because continuous welds should be used if torsional strength is important.

A plate girder of I-beam shape will not be desirable if torsional loads are the primary design factor. The usual need for torsional information will be (1) in connection with lateral instability, where the torsional and lateral bending stiffnesses are of primary concern, and (2) in cases where the load on the girder primarily causes bending but is eccentrically applied, causing torsion. In the case of lateral instability, the only stress due to torsion at loads below the critical will be those unpredictable amounts caused by unavoidable eccentricities in load and the maximum shear stress to be expected is considerably less than 12 ksi. Eqs. 20b and 21b provide an unduly severe rivet pitch requirement in such a case and should be modified by introducing in Eq. 9 whatever value of τ_m is required by the actual situation. In cases of combined bending and torsion, as stated before, the rivet pitch would need to be determined so as to properly transmit the combined shear caused by both bending and torsion.

Illustrative Calculation of Torsion Constant and Strength of Plate Girder Sections:

To illustrate the application of the foregoing equations the plate girder sections shown in Fig. 12 and 14 of riveted and welded members, respectively, will be used as examples. The numerical results will not agree exactly with those reported in comparison with actual tests as the latter are based on measured dimensions of specimens whereas the following examples will make use of the nominal dimensions.

Illustrative Example A: Riveted Plate Girder (Fig. 12)

Using the notation of Fig. 8, the following dimensions are obtained from Fig. 12:

$a = 39.00$ inches
$b = 10.00$ "
$c = 2.00$ "
$d = 0.875$ "
$e = 4.125$ "
$f = 1.250$ "
$h = 48.50$ "

$t_A = 0.625$ inches
$t_F = 0.625$ "
$t_W = 0.500$ "
$T_W = 1.750$ "
$T_F = 1.850$ "
$n = 2$ cover plates in each flange.

The rivet pitch in the flange is 2.625 in. The maximum permissible pitch p' for longitudinal continuity is assumed to be $A + T_F = 1.344 + 1.875 = 3.219$, hence no reduction in torsion constant is called for. If the working value of a 7/8 rivet is taken as the AISC permissible of 9.02 kips (single shear at 15 ksi), Eq. 20b gives the required flange rivet pitch to develop 12 ksi shear stress:

$$P_r(\text{flange}) = \frac{9.02}{3 \text{x} 1.875} = 1.60 \text{ in.}$$

Since this is less than the actual pitch of 2.625 in. it is obvious that 12 ksi shear stress in flange material will not be reached without exceeding the permissible rivet value in single shear. The maximum shear stress in the flange when the permissible rivet value is reached will be $\frac{1.60 x 12}{2.625} = 7.31$ ksi.

Whether or not this is too low will depend upon the cause and nature of the torsional loads that are expected and the degree to which these are combined with other loads causing bending. If the problem is simply one of stability during erection there will be no calculable amount of torsional shear and the pitch is probably adequate.

By Eq. 21b, the pitch required in the connection between flange angles and web plate is

$$p_{r(web)} = \frac{1.875 x 9.02}{3 x 1.75^2} = 1.85 \text{ in.}$$

This also is smaller than the pitch of 2.25 in. that was used, but it is obvious that the pitch used in the flange is relatively more out of line with optimum requirements and is, therefore, the critical pitch.

The torsion constant for integral behavior is now computed by substitution in Eq. 16.

$$K_I = \frac{2}{3} x 10 x (1.875)^3 + \frac{2}{3} x 4.125 x (1.750)^3 + 2 x 4 x 2 x (0.625)^3$$

$$+ \frac{4(1.25 + 1.25)(0.625)^3}{3} + \frac{39 x (0.500)^3}{3} = 62.43 \text{ in.}^4$$

This is about six times the torsion constant for separate behavior of component parts, calculated by Eq. 17, as follows:

$$K_S = \frac{1}{3}\left[2x2x14(0.625)^3 + 48.50x(0.500)^3 + 4(5.375+6)(0.625)^3\right] = 10.28 \text{ in.}^4$$

(K_S is calculated for illustrative purposes and is not needed for design when p is less than p').

The torsional moment capacity of the girder to develop the calculated maximum shear stress of 7.31 ksi that occurs in the flange when the rivets reach their working value in single shear can be calculated by solving for M in Eq. 23a

$$M = \frac{7.31 \times 62.43}{1.875} = 243 \text{ kip-inches}$$

The test results confirm that very nearly linear behavior in torsion will be obtained up to torsional moments greater than the calculated value of 243 kip-inches.

Illustrative Example B: Welded Plate Girder. (Fig. 14)

Using the dimensions obtained from Fig. 14, and the notation of Fig. 9,

a = 45.75 inches

$b_{(avg.)}$ = 13.50 "

t_F = 0.625 "

T_F = 1.875 "

t_W = 0.500 "

The torsion constant for integral behavior is obtained by substitution of the foregoing in Eq. 18:

$$K_I = \frac{2}{3} \times 13.50 \times 1.875^3 + \frac{1}{3} \times 45.75 \times 0.500^3 = 61.23 \text{ in.}^4$$

Intermittent welds were used in this girder, hence the effective torsion constant must be calculated by Eq. 14; also the calculation of K_S, by Eq. 19, is needed:

$$K_S = \frac{1}{3} \times 2 \times 3 \times 0.625^3 + \frac{1}{3} \times 45.75 \times 0.500^3 = 8.50 \text{ in.}^4$$

The distance center to center of welds, or "weld pitch", p, is 12 inches, and the average length of flange cover plate welds, equivalent to rivet clamping distance, p', is 6.5 inches. p - p' = 5.5 inches, which is greater than 0.4b = 5.4 inches, hence Eq. 13a applies and the equivalent torsion constant along distance p-p' is

$$K_{SE} = \frac{2.7 \times 61.23 + (5.5 - 2.7) \times 8.50}{5.5} = 34.39 \text{ in.}^4$$

Then, by Eq. 14, the average effective torsion constant is determined:

$$K_{eff} = \frac{34.39}{1 - (1 - \frac{34.39}{61.23}) \times \frac{5.5}{12}} = 43.03 \text{ in.}^4$$

This example illustrates the value of using continuous welds if torsional rigidity of welded girders is of importance. Continuity would effect an increase of more than 40% in the present example.

By Eq. 22b, the required continuous fillet weld size for balanced torsional strength is $\frac{2.5 \times 1.875}{8} = \frac{5}{8}$ to the nearest eighth inch.

The fillet welds connecting the edges of the two top cover plates are of 3/16 inch size, 6 inches long, 12 inches c-c hence do not meet these requirements. The welds will be assumed as having a permissible shear value in kips per linear inch of 1.2S = 1.2x1.5, "S" being the weld size in eighths. Within the welded zone, where integral behavior is assumed, the maximum shear stress in the flange material can be found by solving Eq. 22a for τ_m:

$$\tau_m = \frac{4.8S}{T_F} = \frac{4.8x1.5}{1.875} = 3.84 \text{ ksi.}$$

The torsional moment corresponding to the foregoing flange shear stress is found by solving Eq. 15a for M,

$$M = \frac{\tau_m K_I}{T_F} = \frac{3.84x61.23}{1.875} = 125.5 \text{ kip-inches}$$

Since p' is greater than 0.4b, the maximum shear stress in the flange, midway between the welded zones, is given by Eq. 15b*.

$$\tau_S = \frac{125.5x0.625}{8.50} = 9.21^* \text{ ksi.}$$

Although this is 2.4 times the maximum shear stress in the welded zone, it is less than the allowable value of 12 ksi that has been assumed in these examples. Hence, the most critically stressed parts of the girder are the welds, on the basis of which the torsional moment of 125.5 kip-inches was evaluated. The test girder of which

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* Limited stress measurements indicate that the actual stress between the welds is lower than as given by Eq. 15b. However, the equation is conservative and may be used for design purposes.

this was an illustrative example showed linear behavior up to torsional moments of more than twice this amount.

TORSIONAL BEHAVIOR ABOVE THE ELASTIC RANGE

If the torsional moment exceeds the value causing initial yielding of the material, there is a gradual increase of the yielded zone and the amount of twist per unit increment of torsional moment progressively increases. The elastic-plastic behavior of structural members in torsion has been treated by Nadai[10], Shaw[11], Hodge[12], and others, usually on the basis of no strain-hardening. Chang[14] has discussed the plastic behavior of the specimens tested in this investigation. Because of the very large twisting deformations of girder sections at torsional moments causing initial inelastic behavior, a presentation of this theory is not of practical importance to the design engineer.

When the angle of twist becomes large, longitudinal fibers away from the axis of twist become helices and tend to shorten in proportion to the distance from the twist axis. Hence compressive forces are induced at the center of the section and tensile forces at the sections farthest removed from the twist axis. The tensile forces, acting at the largest angle to the twist axis, have a torsional component about the axis of twist resisting the applied torsional moment. The effect is negligible for small twist angles but becomes considerable in the early stages of plastic yielding; as a result, torsional moments for complete plastic yielding as computed by the sandheap analogy(10) give too low a value of torsional moment strength for the usual structural section. Timoshenko(6) gives formulas for the direct stresses induced by uniform torsion for the thin rectangular section. Cullimore(13)* and Chang(14) have derived formulas for the direct stresses induced during large twist of I-beam sections and Chang presents experimentally determined stresses for the specimens tested for this investigation. These results are omitted from this report as they are not of great importance to the structural design engineer.

TEST PROGRAM, SPECIMENS AND TEST APPARATUS

As the scale factor in bolted, riveted, and welded girders may be important, experimental work on full size specimens was necessary to verify formulas. Table II gives a list of the specimens tested and the complete test program. For bolted types: different bolt tension, pitch, and gage lines were used. The details of all the specimens are shown in Figs. 10 to 14 inclusive. Space limitations permit presentation of only a part of the test results. For additional data see Reference (14).

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* Reference 13 came to the author's attention at the completion of the present investigation.

TABLE II

TEST PROGRAM

Specimen	Designation	Description	Rivet or Bolt Pitch	Rivet or Bolt Gauge Lines	Bolt Torque (ft.lb.)	Test Range E-Elastic P-Plastic
6"x1/4" pls. Fig. 10A	P-1-1	Single plate, no holes	-	-	-	E
	P-1-2	Single plate with holes	4 1/2"	-	-	E
	P-1-3	2 plates bolted together	"	-	0	E
	P-1-4	" " " "	"	-	30	E
	P-1-5	" " " "	"	-	70	E
	P-1-6	" " " "	"	-	130	E
	P-1-7	" " " "	"	-	170	E
	P-1-8	4 plates, no holes	-	-	-	E
	P-1-9	4 plates with holes	4 1/2"	-	-	E
	P-1-10	4 plates bolted together	"	-	190	E
	P-1-11	4 plates, with holes	2 1/4"	-	-	E
	P-1-12	4 plates bolted together	"	-	40	E
	P-1-13	" " " "	"	-	100	E
	P-1-14	" " " "	"	-	150	E
	P-1-15	" " " "	"	-	190	E
	P-1-16	4 plates riveted together	"	-	-	E and P
20"x5/8" pls. Fig. 10B	P-2-1	Single plate, no holes	-	-	-	E and P
	P-2-2	Single plate with holes	2 1/4" alt.	-	-	E
	P-2-3	2 plates bolted together	"	4 outer lines	0	E
	P-2-4	" " " "	"	"	150	E
	P-2-5	" " " "	"	"	300	E
	P-2-6	" " " "	"	4 inner lines	300	E
	P-2-7	" " " "	6 3/4"alt at center 2 1/4"alt at ends	"	300	E
	P-2-8	" " " "	6 3/4"alt	"	300	E
	P-2-9	4 plates bolted together	2 1/4"alt	4 outer lines	0	E
	P-2-10	" " " "	"	"	15	E
	P-2-11	" " " "	"	"	150	E
	P-2-12	" " " "	"	"	300	E
	P-2-13	" " " "	"	4 inner lines	150	E
	P-2-14	" " " "	"	"	300	E
	P-2-15	" " " "	6 3/4"alt at center 2 1/4"alt at ends	"	300	E

TABLE II CONTINUED

Specimen	Designation	Description	Rivet or Bolt Pitch	Rivet or Bolt Gauge Lines	Bolt Torque (ft.lb)	Test Range E-Elastic P-Plastic
20"x5/8" pls. Fig. 10B	P-2-16	4 plates bolted together	6 3/4"alt	4 inner lines	300	E
	P-2-17	" " " "	2 1/4"alt	"	5	E
	P-2-18	" " " "	"	"	0	E
	P-2-19	4 plates riveted together	"	"	-	E and P
20"Bolted Girder Fig.11 no cov.pls.	T-1A-1	Bolted	2 3/4"	-	0	E
	T-1A-2	"	"	-	100	E
	T-1A-3	"	"	-	200	E
	T-1A-4	"	"	-	300	E
	T-1A-5	"	5 1/2"	-	300	E
	T-1A-6	"	8 1/4"	-	300	E
	T-1A-7	"	8 1/4"at center 5 1/2"at ends	-	300	E
	T-1A-8	"	5 1/2"at center 2 3/4"at ends		300	E
20"Bolted Girder Fig.11 one cov.pl.	T-2A-1	Bolted	2 3/4"	-	15	E
	T-2A-2	"	"	-	150	E
	T-2A-3	"	"	-	300	E
	T-2A-4	"	5 1/2"at center 2 3/4"at ends	-	300	E
	T-2A-5	"	8 1/4" flange 5 1/2" web	-	300	E
	T-2A-6	"	8 1/4"	-	300	E
20"Bolted Girder Fig.11 2 cov.pls.	T-3A-1	Bolted	2 3/4"	-	100	E
	T-3A-2	"	"	-	300	E and P
20"Bolted Girder Fig.11 3 cov.pls.	T-5A-1	Bolted	2 3/4"	-	30	E
	T-5A-2	"	"	-	150	E
	T-5A-3	"	"	-	300	E

TABLE II CONTINUED

Specimen	Designation	Description	Rivet or Bolt Pitch	Rivet or Bolt Gauge Lines	Bolt Torque (ft.lb)	Test Range E-Elastic P-Plastic
20"Bolted Girder Fig. 11 3 cov.pls.	T-5A-4	Bolted	2 3/4" web 8 1/2"at mid third flange	-	300	E
	T-5A-5	"	2 3/4" web 8 1/2"flange	-	300	E
50"Bolted Girder (Fig.12)	T-6A-1	Bolted, no stiffeners	See Fig.12	-	100	E
	T-6A-2	" " "	"	-	300	E
	T-6A-3	Bolted, with stiffeners	"	-	100	E
	T-6A-4	" " "	"	-	300	E
20"Riveted Girder (Fig.11)	T-1B-1	Riveted	See Fig.11	-		E and P
	T-2B-1	"	"	-		E and P
	T-4B-1	"	"	-		E and P
	T-5B-1	"	"	-		E and P
50"Riveted Girders Fig.12	T-6B-1	Riveted, with stiffeners	See Fig.12	-		E and P
20"Welded Girder Fig. 13	T-7-1	Welded, no cover plate	-	-		E
	T-7-2	Welded, one cover plate	-	-		E and P
50"Welded Fig. 14	T-8-A1	Welded, no cover plate, no stiffeners	-	-		E
	T-8-A2	Welded, no cover plate, with stiffeners	-	-		E
	T-8-B1	Welded, 2 cover plates with stiffeners	-	-		E and P
Rolled Sections	T9	Beth. Manual, 18"WF77	-	-		E and P

Design of Specimens:

In general, the specimens were not specifically designed for torsion, but rather were intended to have proportions usual for plate girders as designed for normal vertical loads causing bending in the plane of the maximum moment of inertia. Nearly minimum bolt or rivet pitch was used in many cases, but tests were also made with alternate lines of bolts removed. Space does not permit a detailed analysis of the predicted torsional properties of all the various specimen combinations that were tested and are as listed in Table II. However, the results of many such analyses are shown by the plotted "theoretical" torque-twist curves in the graphically presented test results. The procedures of analysis have been outlined in detail in the two illustrative examples which are based on test Specimens T6B and T8B.

The AISC allowable single shear value for rivets was used in the analysis of both the riveted and bolted specimens. The test results indicated that the high tensile strength bolts, as used, when tightened to 300 ft-lb. torque, very nearly simulated the behavior of rivets in the elastic range and in the early stages of yielding.

Specimens P1 and P2 (Figs. 10a and 10b) were used to study the behavior of built-up plates. These plates simulate the flange of a heavy girder, from whence comes most of the girder's stiffness and strength in torsion.

With one exception, as indicated in Table II, (Test T3A2) the bolted girders were tested only in the elastic range, in order to study various combinations of plates, rivet pitch, line and row spacing, etc. to the best advantage. Then, after the bolted tests, the specimens were returned to the shop for riveting and final testing to failure.

Although the rivet pitch in Specimens P1, P2, and T1 to T6 (Figs. 1 and 12) is in many cases less than that for full torsional <u>strength</u> requirements, the pitch is usually adequate or nearly adequate to provide longitudinal continuity. In such a case full torsional rigidity should be obtained at low torsional loads but general inelastic behavior due to slip, yield, or both, will start at lower levels than for a balanced design.

The rivets were driven in the following manner:

1) Equipment used: 50 ton Bull riveter
2) Air pressure used in driving: 100 lbs. gauge pressure
3) Approximate temperature of rivets when driving: 1650°F, (Cherry red)
4) Approximate driving time: 0.05 minutes per rivet

The two welded girders Specimens T7 and T8, are shown in Figs. 13 and 14 and have welds ample for shear stresses that would result from vertical loads causing bending. As shown in Illustrative Example B, these specimens were underdesigned for pure torsion both for strength and longitudinal continuity.

Test Apparatus:

The torsion testing machine, Fig. 15 (showing a test in progress), was designed and constructed at Fritz Laboratory especially for this project and has been described elsewhere(1). The torque is applied by turning an 88" diameter rotating head by means of a 4" by 1/4" flat wire rope, using an old model standard Riehle testing machine as the power source. To measure the applied torque accurately, calibrated aluminum torque tubes of various capacity mounted with SR-4 gages are inserted at one end of the stationary head. The stationary head, in turn, while resisting the applied torque, rests on rollers that permit longitudinal movement when the specimen shortens during the twist.

Measurements of deformation were made by the following means: a) strain with SR-4 gages, b) the angle of twist with level bars, c) lengthening and shortening of specimens during twist with Ames Dials and d) the slip between different components of the specimens with Whittemore gages.

Adjustable level bar seats were placed at several stations along the length of each girder. The difference of tilt angle between two level bar stations divided by the distance between them gives the angle of twist per unit length.

In all tests reported herein, approximately free-end conditions were obtained. Torque is applied by end fixture plates as shown in Fig. 15. The ends are free to warp except for friction forces, which however, are small in proportion to the forces required to fully restrain the section against warping. Local end effects due to the manner of appliecation of the torque taper off very rapidly. The center portion of the girder, therefore, was in a state of uniform torsion.

TEST RESULTS

Physical Tests of Materials in Test Specimens:

The tensile properties in every part of each specimen were determined. Results are detailed in Reference 14. In general the materi met the ASTM Specification requirements for A-7 steel for bridges or buildings. In computing theoretical twists per unit length per unit torsional moment, a representative value of shear modulus (G) was taken as 11,450 kips per in.^{2}, to correspond with $E = 29,500$ kips per in.^{2} and Poisson's ratio of 0.29.

Bolt and Rivet Tests:

The tension in the bolts was determined as a function of applied bolt torque by mounting SR-4 strain gages on the shank of bolts in a special test shown in Fig. 16. By this means, the strain in the bolt can be measured and the tensile force developed in the bolt can be computed for various applied torques. A curve typical of these test results is shown in Fig. 17.

From the known stresses developed in the bolts due to different wrench torques, the effective coefficient of friction between the plates or between the bolt head, nut, or washer and plate was found approximately by using a set-up similar to that shown in Fig. 18 for a riveted specimen. This coefficient depends on the amount, composition, and the condition of dryness of the paint, the roughness of the surface, etc. For ordinary unpainted structural steel, an average value of 0.25 was obtained. In Table III the test results are summarized.

Table III - Effective Coefficient of Friction

Torque applied by the torque-wrench ft.-lbs.	Unit stress developed in bolts, psi	Total stress in bolts, lbs.	Average load, to start slip (double shear bolt value for friction) lbs.	Effective coefficient of friction
100	17,000	10,000	5,100	0.24
200	35,000	21,000	10,900	0.26
300	50,000	30,000	16,900	0.28

Tests were made to determine the single shear rivet friction values for conditions typical of those used in manufacturing the test specimens (Fig. 18). The test results for the various conditions of equipment, air pressure, temperature, driving time, etc. are summarized in Table IV. These specimens allowed 1/2" slip before coming to bearing, thus differentiating clearly between the two effects, namely friction and bearing resistance.

Table IV - Rivet Tests

Specimen Driving Method	S_1	S_2	S_3	S_4	S_5
Equipment Used	50 ton bull riveter	hand gun	50 ton bull riveter	50 ton bull riveter	50 ton bull riveter
Air pressure used in driving	100 psi	100 psi	80 psi	100 psi	100 psi
Approx. Temp. of rivet when driving	Cherry red 1650°F	Cherry red 1650°F	Cherry red 1650°F	Bright cherry red 1900°F	Cherry red 1650°F
Approximate driving time in minutes	0.05	0.05	0.05	0.05	0.25
Load, to start slip (double shear)	32,000* 20,000**	19,000*	20,000*	30,500*	35,000*

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* Rivets for specimen T6.

** Rivets for specimens T1 to T5.

Plate Specimens:

These were a preliminary to girder tests and only a representative selection of test results are presented.

6" x 1/4" Built-Up Plates (Specimen P1, Fig. 10)

The 6" x 1/4" plates were pilot tests in a standard 24,000 in-lbs. torsion testing machine; all other specimens were tested by using the 2,000,000 in-lbs. machine. The effect of bolt tension on both the strength and stiffness of a stack of four plates is shown in Fig. 19, with the riveted case included for comparison.

The test result of two plates bolted together is almost identical to that of the four plate case and need not be included.

20" x 5/8" Built-Up Blates (Specimen P2, Fig. 10)

Four 20" x 5/8" plates bolted together were twisted in the elastic range only. The effects of bolt torque and gage line location are shown in Fig. 20.

The longitudinal slip between plates during twist was also measured(14) and was found to depend largely on the bolt torque. The longitudinal slippage between loosely bolted plates checked very well with theoretical curves determined by Eq. 6.

Four plates riveted together were tested to destruction. Strain lines appeared on the rivet heads at 240,000 in-lbs. on the plate near the rivet heads at 301,000 in;lbs. and in the portion between the two outer rows of rivets at 326,500 in-lbs. The torque-twist and torque-slip curves of the test are shown in Fig. 21.

Rolled Section (Specimen T9)

One rolled section was tested in order to compare with the bolted, riveted and welded types and correlate this program with the earlier Lehigh investigation(3), with which the test results are in good agreement. Strain lines first appeared along the fillets at 75,000 in-lbs. where stress concentrations occurred due to the sharp curvature of the fillet. At 86,000 in-lbs. strain lines appeared along the center line of the flange where the largest inscribed circle touches the boundary. The torque-twist curve is shown in Fig. 22. The moment for the completely plastic state, assuming no strain-hardening, is also shown, but it is noted that even at low unit twist angle the beam offers a much higher torsional resistance due to the development of longitudinal normal stresses(14).

Bolted Plate Girders

All bolted specimens except T3 were tested in the elastic range only. Different bolt pitches and bolt torques were used in these tests to study their effects on the stiffness of bolted girders. In test T3A2 strain lines appeared around the bolt heads at 117,000 in-lbs. and on both the flange and web at 210,000 in-lbs. (Fig. 23). At a torque of about 340,000 in-lbs. buckling of the web became noticeable, due to the flange shortening effect. The torque-twist curve for specimen T3A2 is compared with that of specimen T4B1 which has the same dimensions as T3A2 except that it is riveted instead of bolted, (Fig. 11). It may be noticed that the bolted (300 ft-lbs. bolt torque) and riveted specimens behave more or less the same in the lower load range, but that the riveted specimens were relatively stronger in the inelastic range, after initial slip. In order to accurately determine the torsion constant "K" of this and other test specimens the elastic range torque-twist curves were drawn to a larger scale than that of Fig. 23.

From Eq. 2, K is the slope of the torque-twist curve (in the elastic range) divided by G.

The effect of stiffeners on the stiffness of bolted girders was studied, Fig. 24, in the test of Specimen T6, (Fig. 12). The shearing stress distribution in the bolted specimens was similar to that shown for the riveted girders in Fig. 29.

20" Riveted Plate Girders Tests T1B to T5B, Fig. 11

20" deep riveted plate girders were tested to failure. The torque-twist curve for Test T1B with no cover plates is shown in Fig. 25.

The torque-twist curve for T2B (one cover plate) is shown in Fig. 26, Strain lines first appeared on the flange near the riveted heads at a load of 111,000 in-lbs. at 202,500 in-lbs. strain lines started to appear on the web.

The torque-twist curve of T4B with two cover plates is plotted in Fig. 23 in comparison with that of a similar bolted girder (Specimen T3A2). The sequence of appearance of strain lines on Specimen T4B was similar to that of Specimen T2B

As with all other bolted and riveted specimens, the first strain lines in Test T5B, with 3 cover plates, appeared on the flange near the rivet heads. At a load of 240,200 in-lbs. the strain lines started to appear on the riveted heads themselves. Strain lines appeared on the web when the load reached 298,000 in-lbs, as shown in Fig. 27. The torque-twist curve is shown in Fig. 28.

Shear stress distribution at certain loads is shown for all the 20" riveted girders (T1B, T2B, T4B and T5B) in Fig. 29 and longitudinal slip in Fig. 30.

50" Riveted Girder with Two Cover Plates (Specimen T6B, Fig. 12)

The 50" deep riveted plate girder with stiffeners was tested to failure, the torque-twist curve being shown in Fig. 31. The first strain line appeared at the flange near the rivet heads at 750,000 in-lbs. and then progressed along the flange between the two rows of rivets. This agrees with the assumption made in evaluating the integral action torsion constant, i.e., that the portion between rivet lines acts as a solid section. Fig. 32 shows the strain line pattern after twist, a maximum load of 1,340,000 in-lbs. having been applied to the specimen. The strain lines on the web indicate the buckling that occurred. Fig. 33 shows the shear stress distribution at 750,000 in-lbs. torque.

20" Welded Girder (Specimen T7-2, Fig. 20)

This specimen was first tested in the elastic range with no cover plate (T7-1), then tested to destruction with one cover plate on each flange (T7-2). Fig. 34 is the torque-twist curve for the latter test. The shear stress distribution at 82,000 in-lbs. is shown in Fig. 35.

50" Welded Plate Girder (Specimen T8-3, Fig. 14)

A 50" welded plate girder with no cover plate was first tested in the elastic range with and without stiffeners (Tests T8-1 and T8-2).

The girder with stiffeners and with two cover plates added to each flange was then tested to failure, the torque-twist curve, being shown in Fig. 36. Good agreement is noted between the theoretical K value (K_{eff} by Eq. 14) and the test results. The maximum load applied was 940,000 in-lbs. At 519,000 in-lbs. strain lines first appeared along the inward side of the flange. At 639,000 in-lbs. load, strain lines appeared on both the flange and the web. The buckling of the web became very apparent at a load of 710,000 in-lbs. At 930,000 in-lbs. load, the welds connecting the cover plates started to break with a corresponding rapid increase in angle of twist.

The shear stress distribution at 452,000 in-lbs. is shown in Fig. 35 for a location within the zone of the intermittent welds. The flange stresses are lower than the theoretical while the web stresses are higher, indicating the possibility that in this region of the girder the shape of the section was not being maintained and the flanges were twisting less than the web. Several shear stress measurements were made between the intermittent welds, but not exactly midway. These stresses averaged about 1.5 times the stress in the welded zone. The approximate stress midway between welds by the tentatively proposed Eq. 15 would be 2.4 times the stress in the welded zone (See Illustrative Example B).

DISCUSSION OF TEST RESULTS

Stiffness of Built-Up Plates and Plate Girders:

The test results are compared with the proposed torsion constant formula in Table V. The equation for K_I, neglecting both the hump and edge effects, gives reasonably good agreement with test results in both the bolted and riveted cases. For Specimen T1B, $K_{(S)}$ (separate action) is the same as $K_{(I)}$ (integral action) because this girder is without cover plate and has only one row of rivets connecting the flange and the web.

As shown in Col. 6, in the case of the riveted or welded girders, the K evaluated by formula was never more than 9% less than that determined by actual test.

Strength of Built-Up Plates and Plate Girders:

For structural members used in common practice, a unit twist of about 0.00006 radians per inch might be allowable. The torsional moments which will cause a permanent set of 0.00006 radians per inch are tabulated in Col. 7 of Table VI, defining thereby the approximate range of linear behavior and affording an arbitrary basis of strength comparison.

The torque-twist curves in the riveted and bolted girder tests start to bend before τ_{max} reaches the yield point in pure shear, the reverse being true for the rolled section and the welded girders.

The torque-twist curves for unloading are nearly straight lines approximately parallel to the initial straight portion of the torque-twist curve.

In Table VI, Col. 3 gives the shear yield of the material as calculated from tensile coupon tests for the various specimens.

TABLE - V

TORSION CONSTANT "K" BY EXPERIMENT AND FORMULA

(1)	(2)	(3)	(4)	(5)	(6)
Specimens	Designation	K_S Separate Action	K_I	K By Test	$\frac{(5)}{(4)}$
Bolted Specimens (7/8" Bolts at 300 ft-lbs. Bolt Torque)					
Two 20"x5/8"pls.	P2-5	3.13	11.28	10.70	0.95
Four 20"x5/8"pls.	P2-12	6.27	85.60	83.80	0.98
20"x9"Girder, 1 cov.pl.	T2A-3	1.94	5.75	7.42	1.29
20"x9"Girder, 2 cov.pls.	T3A-2	2.28	10.36	10.65	1.03
20"x9"Girder, 3 cov.pls.	T5A-3	2.43	18.65	15.43	0.83
50"x14"Girder 2 cov.pls.	T6A-4	10.22	64.92	59.60	0.92
Riveted Specimens (7/8" Diameter Rivets)					
Four 20"x5/8"pls.	P2-19	6.27	67.25	61.10	0.91
20"x9"Girder, no cov.pl.	T1B-1	1.56	1.56	1.86	1.19
20"x9"Girder, 1 cov.pl.	T2B-1	1.94	5.75	6.61	1.15
20"x9"Girder, 2 cov.pl.	T4B-1	2.28	10.36	12.20	1.18
20"x9"Girder, 3 cov.pl.	T5B-1	2.43	18.65	17.40	0.93
50"x14"Girder, 2 cov.pl.	T6B-1	10.22	64.65	73.10	1.13
Welded Girders and Rolled Section					
20"x9"Girder	T7B-1	2.34	5.40	6.35	1.18
50"x14"Girder	T8B-1	8.01	42.47	43.40	1.02
18WF77	T9-1	-	4.01*	3.83	0.96

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* Includes "hump" and "end-loss" effects(3).

TABLE - VI

TORSIONAL STRENGTH OF PLATE GIRDERS

(1)	(2)	(3)	(4)	(5)	(6)	(7)		(8)	(9)	(10)
Specimens	Designations	τ_y* ksi	Initial Yield Moment in-kips by Eq.5 $M_y = \frac{K\tau_y^*}{T}$	by Strain Lines	by Strain Gages	Moment in-kips at 0.00006 Rad/in permanent set	Ratio	$\frac{(5)}{(4)}$	$\frac{(6)}{(4)}$	$\frac{(7)}{(4)}$
Bolted (7/8" Bolts at 300 ft-lbs. bolt torque)										
20"x9"Girder, 2 cov.pl.	T3A-2	20.9	174	117	118	60		0.67	0.68	0.35
Riveted (7/8" rivets)										
20"x9"Girder, no cov.pl.	T1B-1	20.7	65	82	70	—		1.26	1.08	-
20"x9"Girder, 1 cov.pl.	T2B-1	21.1	140	111	124	56		0.79	0.89	0.40
20"x9"Girder, 2 cov.pl.	T4B-1	20.8	173	158	157	60		0.91	0.91	0.35
20"x9"Girder, 3 cov.pl.	T5B-1	21.1	242	168	198	100		0.69	0.82	0.41
50"x14"Girder, 2 cov.pl.	T6B-1	21.7	738	750	750	530		1.02	1.02	0.72
Welded										
20"x9" Girder	T7B-1	19.8	77	138	138	100		1.79	1.79	1.30
50"x14"Girder	T8B-1	20.5	266	519	581	480		1.95	2.18	1.80
Rolled										
18"WF77 lbs.	T9-1	19.4	86	86	86	98		1.00	1.00	1.14

* $\tau_y = 0.58\sigma_y$ (σ_y based on tensile tests of actual material.)

By using Eq. 5 and the material coupon strength, the torsional moment at initial yield can be predicted ($M = \frac{K\tau_Y}{T}$). In Table VI, Col. 4, these moments for all the plate girders are listed. The moments at the initial yield as indicated by strain lines (Col. 5) and as indicated by Strain gages (Col. 6) are also tabulated for comparison. Cols. 8 and 9 show the relationship of these moments to those predicted by the tensile coupon tests and Eq. 5. The initial yield torsional moments by strain lines and strain gages are almost the same in most of the tests.

Col. 7 lists the torsional moment at an arbitrary degree of permanent twist, 0.00006 radians per inch. By Col. 10 it is seen that for bolted and riveted girders this amount of set develops at 35 to 72 percent of the moment predicted by Eq. 5 for initial yield, as listed in Col. 4. This is due to two factors; (1) the rivet or bolt pitch used was usually greater than that required by Eq. 9, and (2) the working value of rivets as permitted by AISC is very close to the rivet loads that cause initial slip. Cols. 8 and 9 show that (omitting Test TlB-1) actual yield occurred at moment loads varying from 67 to 102 percent of that predicted by Eq. 5, early rivet or bolt slip undoubtedly being a contributing factor to the higher than predicted shear stresses. Test TlB-1 showed different behavior from the other riveted or bolted specimens, the slope of the torque twist curve increasing initialy with increase of moment. Apparently behavior is initially very nearly that of separate action, as assumed, but the interfaces between flange angles and web plate may develop greater friction due to binding as twist develops, thus approaching partial integral behavior.

The welded girders exhibited quite different behavior from the riveted or bolted specimens, as shown by the comparisons in Cols. 8, 9 and 10 of Table VI. The calculated values of moment at initial yield, Col. 4, are based on separate action, because of the fact that the welds are intermittent and spaced at a distance of more than 0.4b apart (See Illustrative Example B). This is probably an over-conservative approximation. Better agreement between theory and test would probably have been obtained had fully continuous welds been used and these should be called for if maximum available torsional strength is required. The behavior of these welded girders, underdesigned as they were for torsion, shows their marked superiority to similar riveted girders.

The rolled section exhibited excellent agreement between theory and test, as might be expected, since in this case "integral behavior" is inherent in the section and not something that is approximately brought about by rivets or welds. Similar agreement should be obtained for welded girders with fully continuous welds of adequate strength interconnecting the edges of all cover plates.

For the shear stress distribution on surface of flange and web, the test results show that the integral torsion constant K_I, neglecting both the hump and edge effects, used in Eq. 5 gives fairly good agreement (Figs. 29, 33 and 35). See also "Test Results", final paragraph, for a discussion of the shear distribution in Specimen T8B-1

VARIOUS FACTORS AFFECTING THE TORSIONAL BEHAVIOR OF BUILT-UP STRUCTURAL MEMBERS

Tension in Bolts:

From Table III, it is seen that the bolt tension directly affects the bolt values in friction. The bolt values in friction, in turn are required to supply the longitudinal shearing stresses in the interface Fig. 5c, over the length p. Slip between different components will occur if the resultant of the latter stresses is larger than the bolt value in friction. Therefore, in a given design, the tension in bolts will determine at what torsional moment appreciable slip will occur and the corresponding departure from straight-line relationship in the torque-twist curve, i.e. See Fig. 20.

Method of Driving Rivets:

The rivet value in friction varies with the method of driving, (Table IV). If this value is too low, slip may occur very early. Test results indicate that the 7/8" rivets driven by ordinary shop practice correspond to 7/8" high strength bolts tightened to 300 ft-lbs.

Gage Line Locations of Bolts and Rivets:

In evaluating the integral action torsion constant, the assemblage between the outer rows of rivets or bolts is assumed to act as an equivalent solid section. This assumption agrees very well with the test results. Therefore, the gage line location of bolts and rivets affects both the strength and stiffness of built-up members. If possible, the rivet (or bolt) lines should be located near the edge of the assemblage in order to obtain a stronger and stiffer member. Fig. 20 gives test results on the effect of location of bolt or rivet gage lines.

Pitch of Rivets or Bolts:

The effect of rivet and bolt pitch on the stiffness and strength of built-up structural members has been approximately evaluated. If the pitch is larger than $p' = A \div T$, the pitch required for longitudinal continuity, the torsional constant K will be reduced according to Eqs. 13 and 14 and the maximum shear stress will be increased in comparison with full integral behavior. Likewise, the torsion constant is reduced and the shear stress raised if intermittent welds are used in welded girders.

Stiffeners:

Stiffeners have little effect on the strength and stiffness of built-up members under uniform torsion in the elastic range. The riveted girders with stiffeners are somewhat stiffer than those without stiffeners, (Fig. 24). The effect of stiffeners on welded girders is not appreciable. An important function of stiffeners is to tie the flange and web together in deep girders to assure that they (the flange and the web) will twist through the same angle. In other words, the stiffeners serve to maintain the shape of the cross-section an important function at points of torsional load application.

SUMMARY

1. Equations for evaluating the torsion constant of built-up plates and plate girders are developed and illustrative examples presented. Eqs. 11, 16 and 18 for built-up plates, riveted and bolted plate girders, and welded plate girders, respectively, are recommended for practical design purposes.
2. The pitch of rivets or bolts affects both the stiffness and strength of built-up structural members in torsion. For strength the pitch should be designed by using Eq. (9) and for longitudinal continuity the pitch should not be greater than $p' = A + T$ (Eq. 10). If larger pitch than p' is used, the torsion constant should be calculated by Eqs. 13 and 14 and shear stress computed by Eq. 15.
3. The reduced strength and stiffness of welded girders with intermittent welds may be evaluated in a manner similar to that for riveted girders having a pitch greater than p' (See item 2), p' in this case corresponding to the length of the individual weld.
4. The typical shear stress distributions in bolted and riveted plate girders under torsion are shown in Figs. 29, 33, and 35. The maximum shear stress in the girder sections tested occurs in the fillet between flange and web, the next highest in the flange near the rivet (or bolt) head. The stress concentrations in the fillets have little effect on the overall torsional behavior. Eq. 5 is in good agreement with measured shear stresses in the flange and web away from the fillets.

5. The slip between different components of the built-up members under torsion is discussed. Eq. 6 determines the amount of slip between the corners of loosely bolted built-up plates.
6. The tightness of bolts or the method of driving rivets determines at what torsional moment slip will occur and the torque-twist curve depart from the straight line relationship.
7. Stiffeners in deep riveted plate girders increase the torsional stiffness somewhat and may be required in deep girders to maintain the shape of the cross-section.

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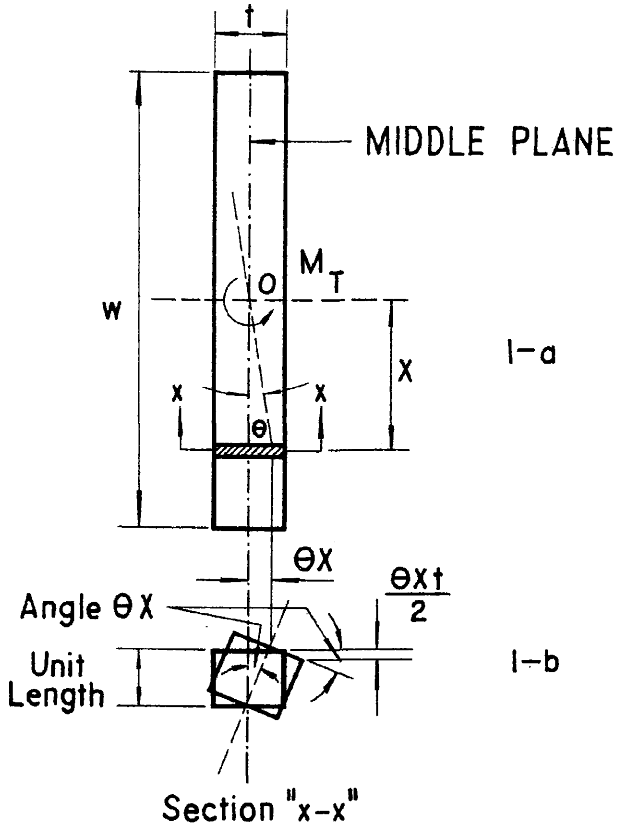


FIG.-1
RECTANGLE WARPING

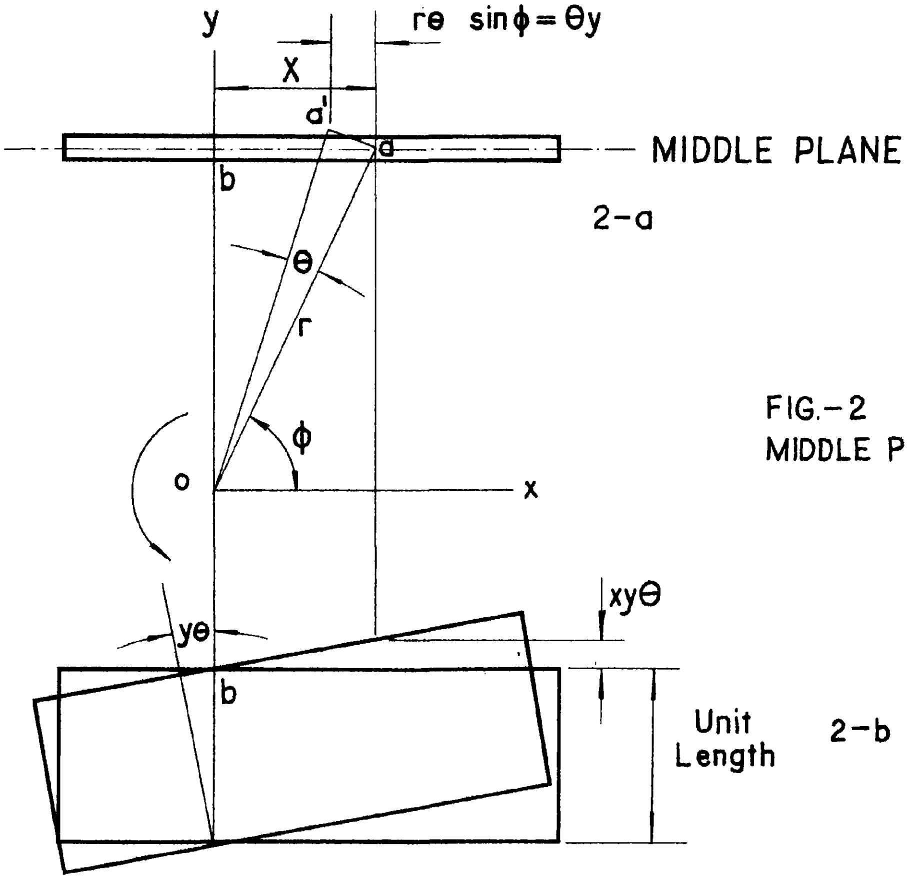


FIG.-2
MIDDLE PLANE ROTATION

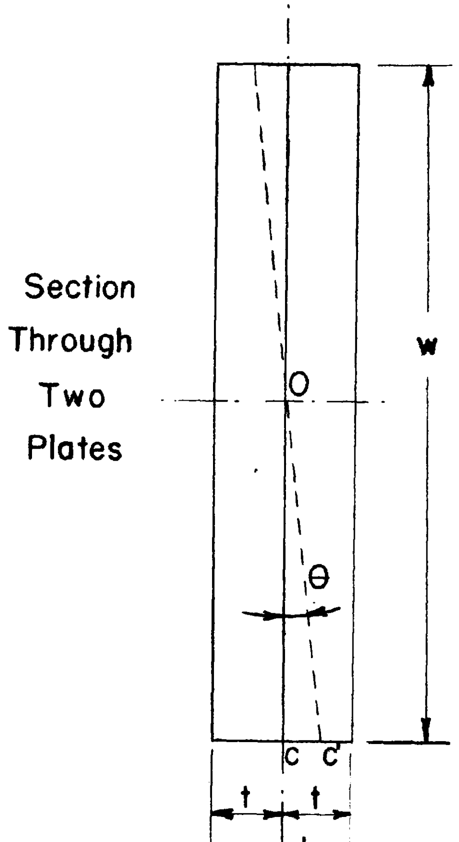


Fig. 3(a)
TWO SECTIONS IN TWIST

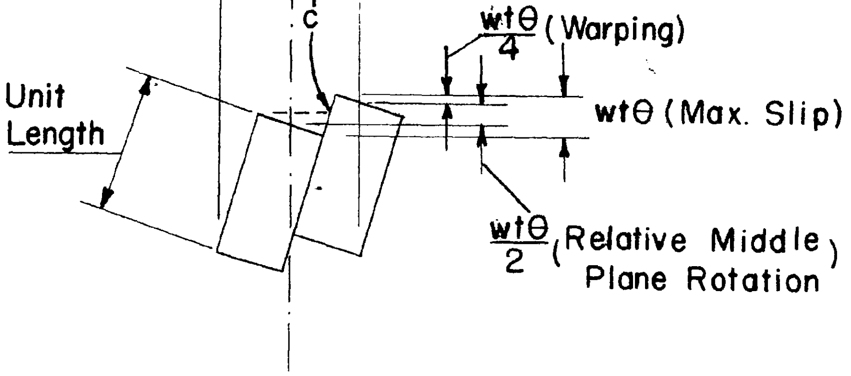


Fig. 3(b)

(UNIT LENGTH)

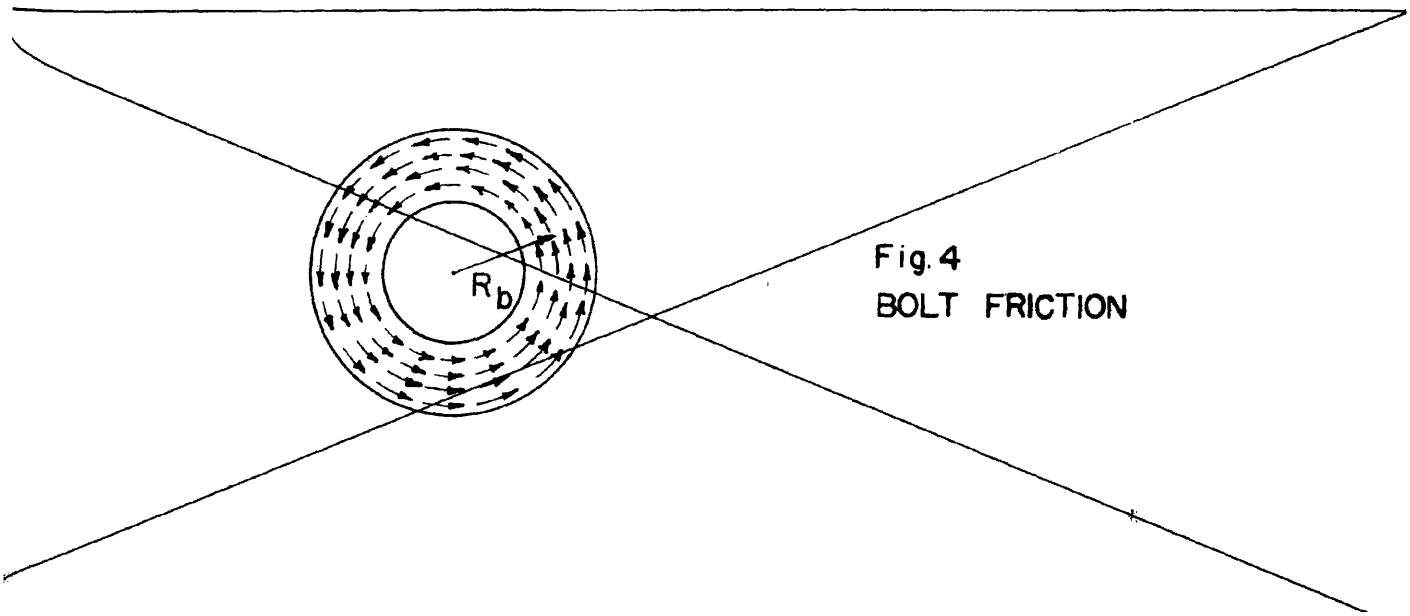


Fig. 4
BOLT FRICTION

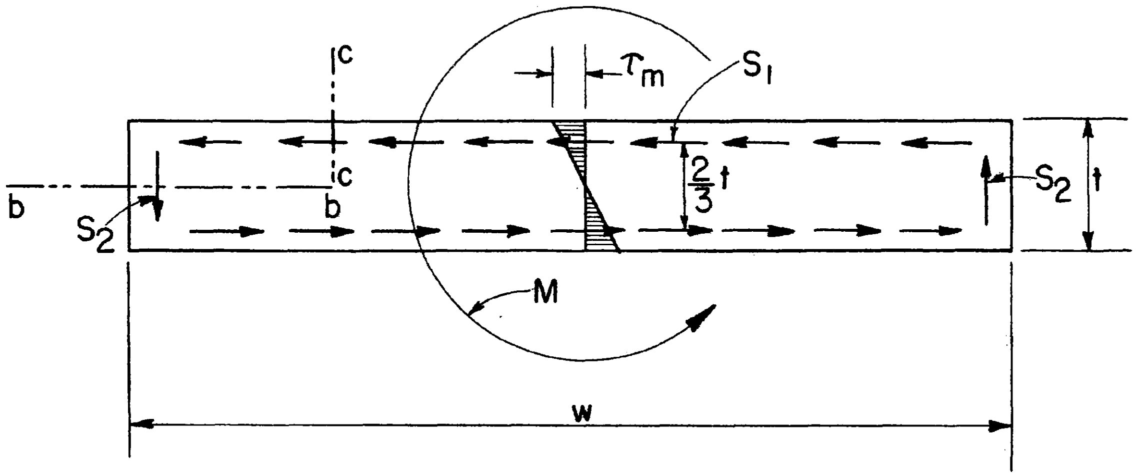


FIG.4a. RESULTANT SHEAR FORCE DISTRIBUTION IN NARROW RECTANGULAR PLATE UNDER TORSION

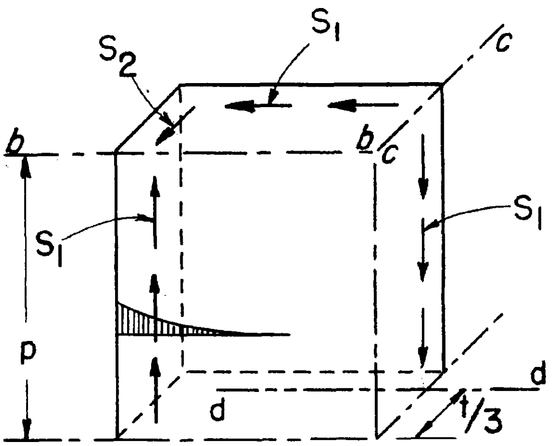


FIG.4b. FREE BODY DIAGRAM OF CORNER SEGMENT

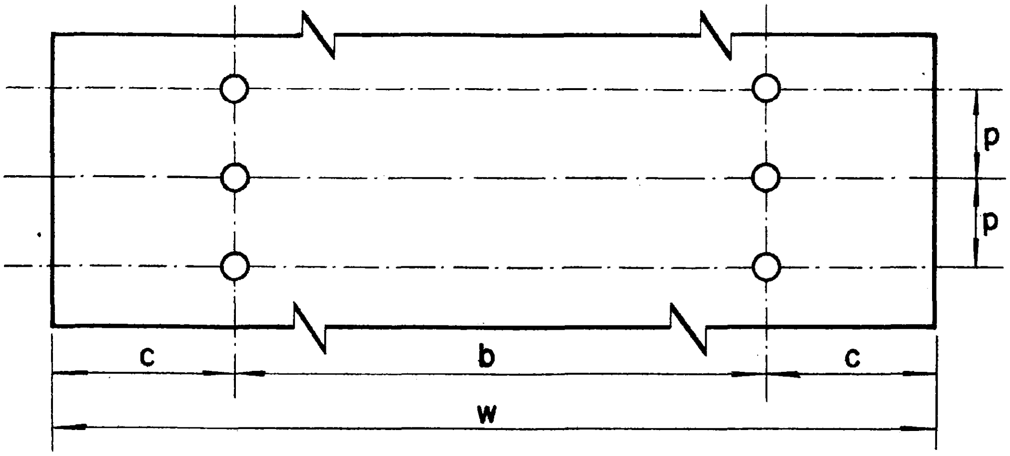


FIG.5a. BUILT-UP PLATE (BOLTED OR RIVETED)

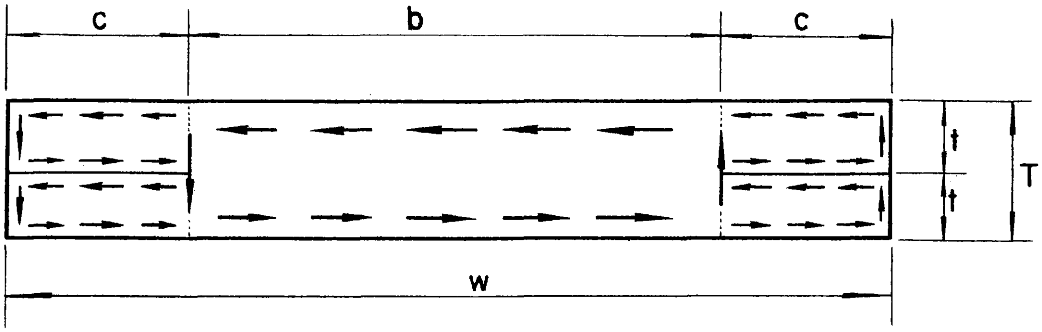


FIG.5b. ASSUMED SHEAR DISTRIBUTION

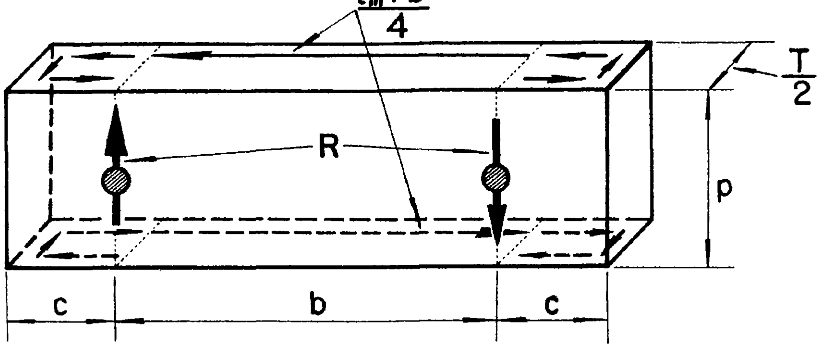


FIG. 5c. STRESSES ON HALF SECTION OF TWO BOLTED OR RIVETED PLATES

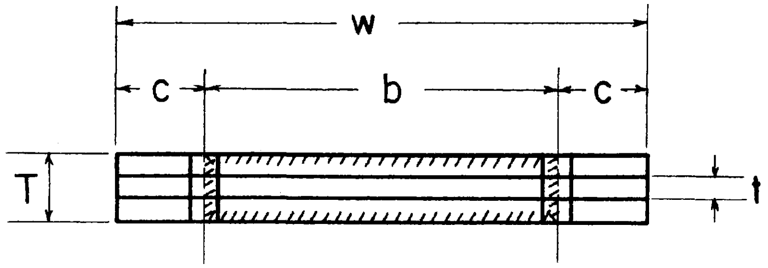


FIG- 6 a

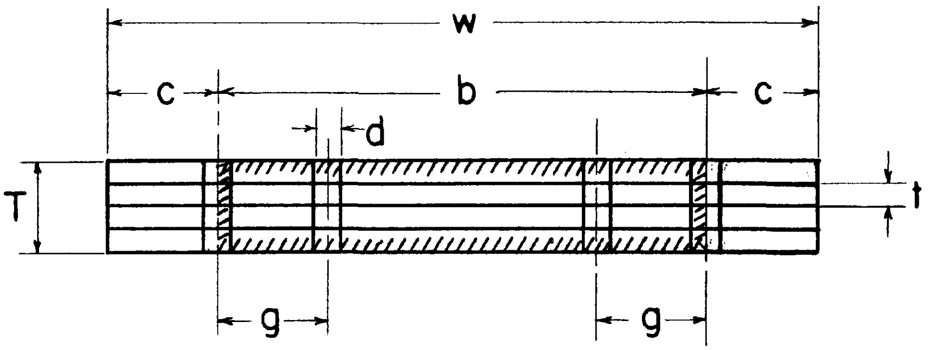


FIG- 6 b

BOLTED OR RIVETED PLATES

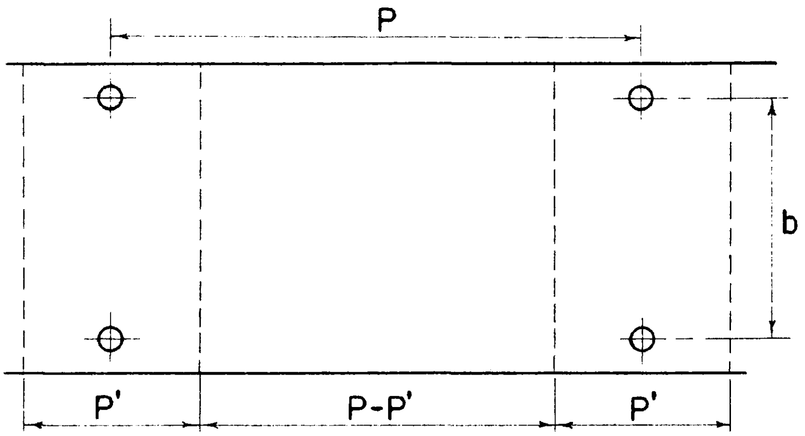


FIG. 7a

BOLTED OR
RIVETED
BUILT-UP
PLATE

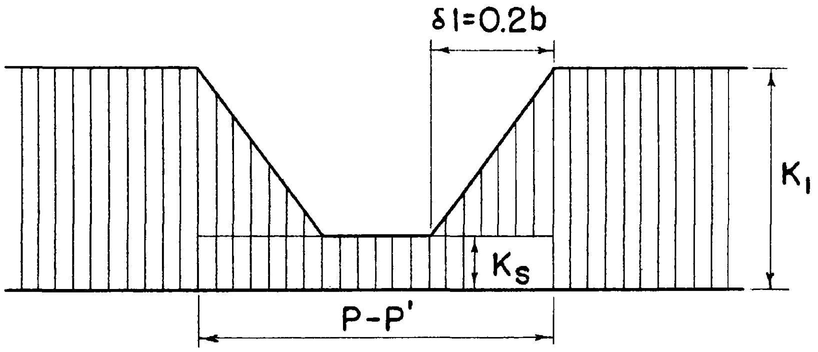


FIG. 7 b

TORSION
CONSTANT
VARIATION
FOR P-P' $\geqq$ .4b

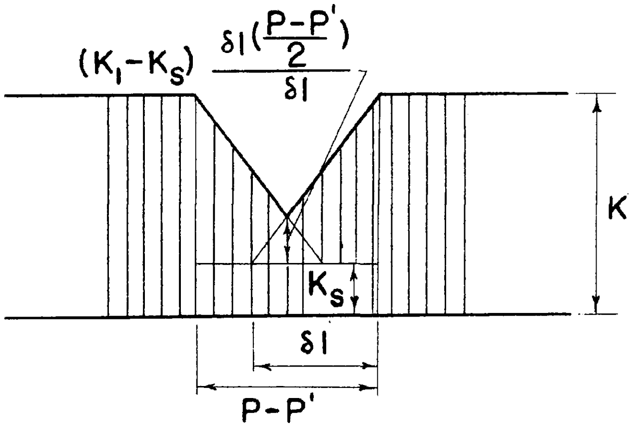


FIG. 7 c

TORSION
CONSTANT
VARIATION
FOR P-P' < .4b

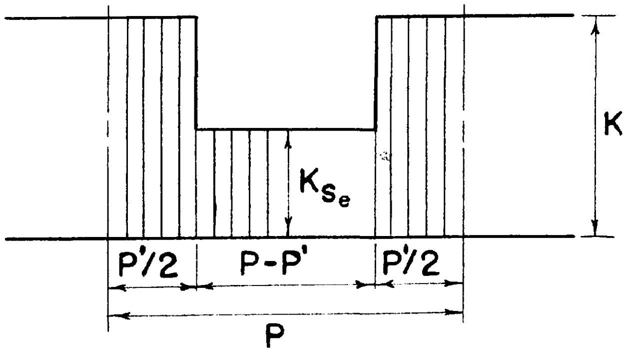


FIG. 7 d

EQUIVALENT
TORSION
CONSTANT

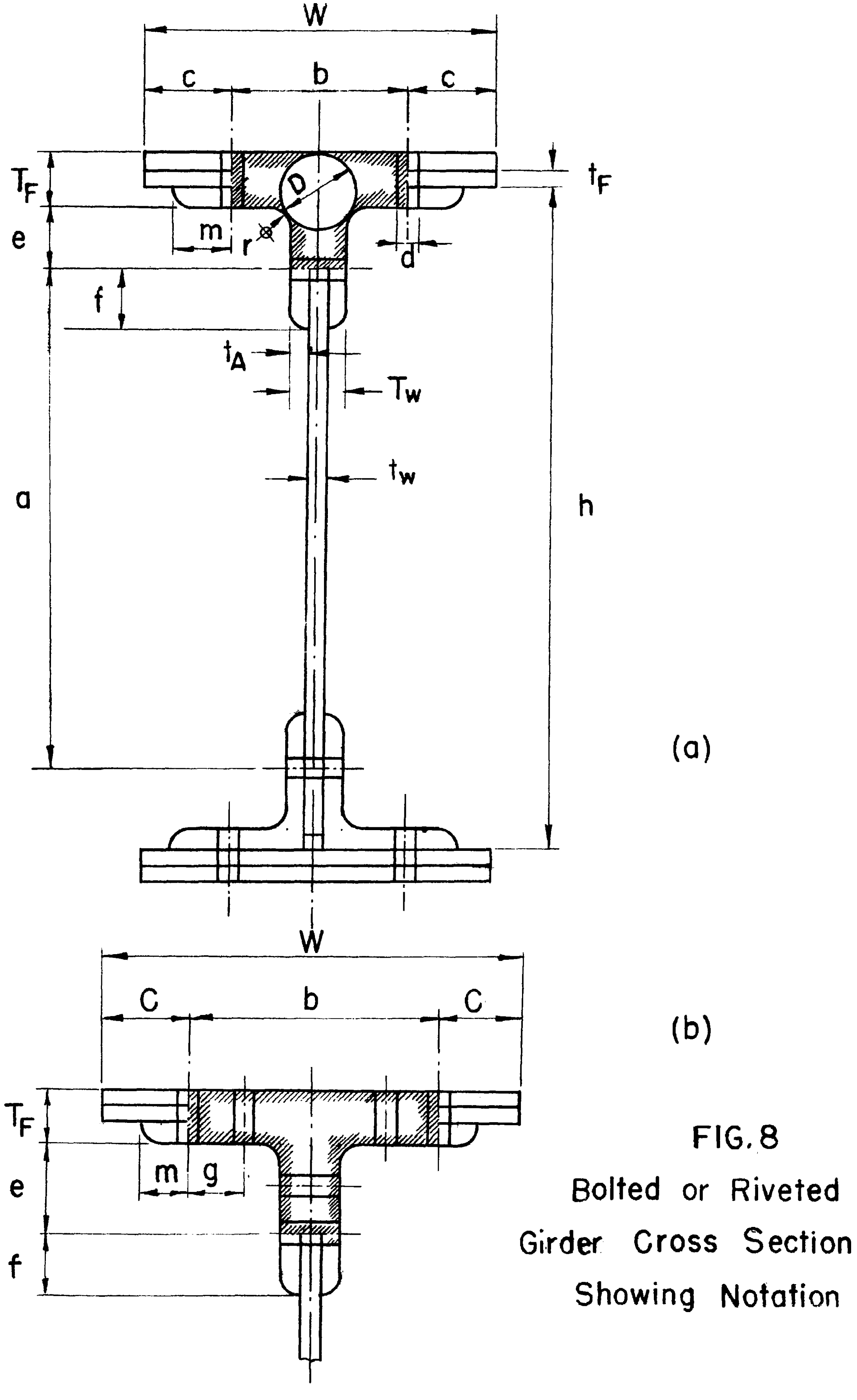
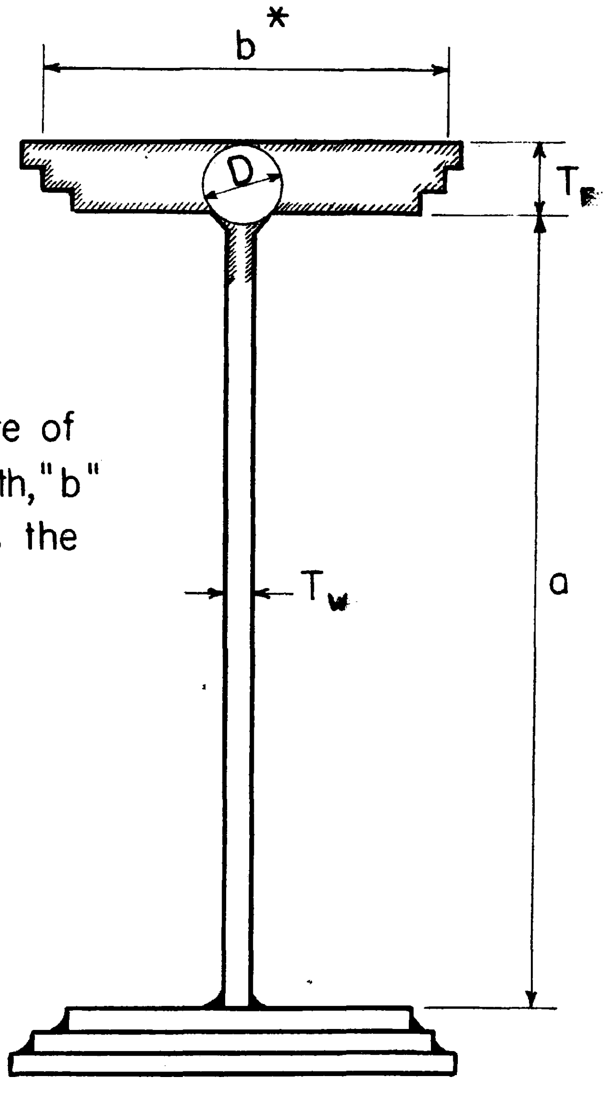
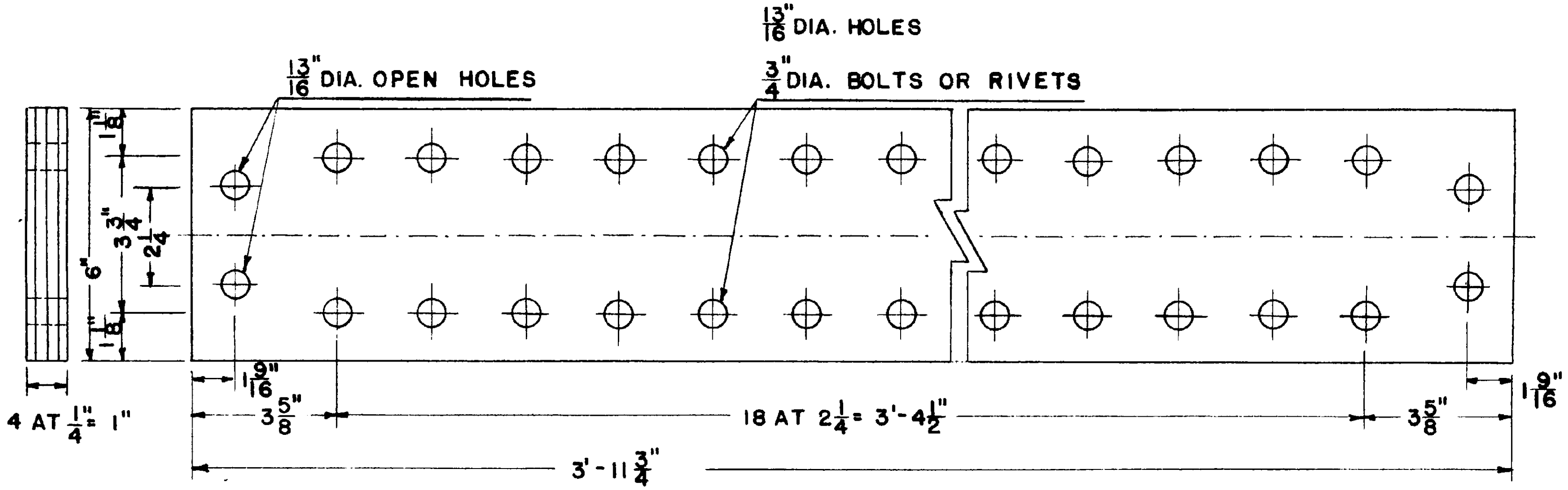


FIG. 8 Bolted or Riveted Girder Cross Section Showing Notation

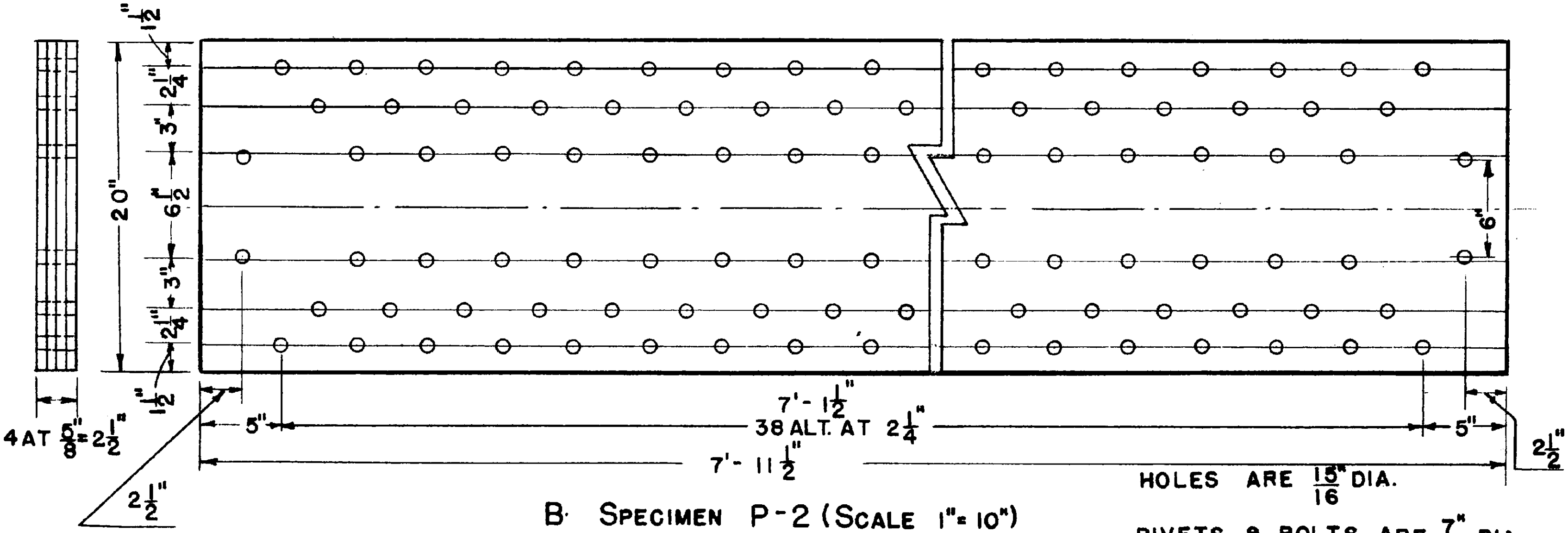


* If cover plates are of nearly same width, "b" may be taken as the average.

FIG. 9. WELDED GIRDER CROSS SECTION SHOWING NOTATION



A. Specimen P-1 (Scale 1" = 4")

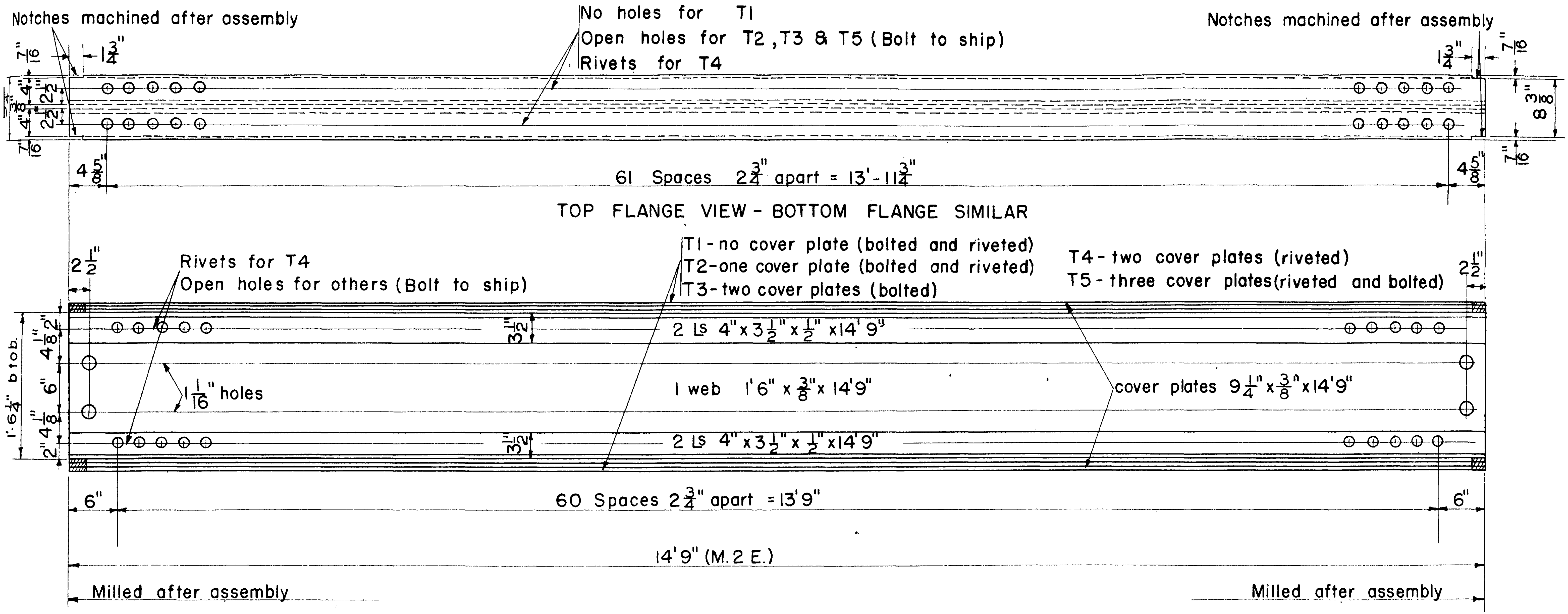


HOLES ARE $\frac{15}{16}$" DIA.

RIVETS & BOLTS ARE $\frac{7}{8}$" DIA.

B. Specimen P-2 (Scale 1" = 10")

Fig. 10. Test Specimens P-1 & P-2



A - Notes:

Material: Structural carbon steel ASTM-A7. All plates of one thickness to be made from the same rolling. The materials for the angles and plates must have nearly the same physical properties.

Holes: 15/16" φ, subpunched and reamed, except as noted.

Bolts: 7/8" φ, high strength, yield point 70,000 psi. minimum, with hexagonal nuts.

Rivets: 7/8" φ, report the temperature of the rivets when driven and the manner of driving.

FIG. 11. TEST SPECIMENS - T1, T2, T3, T4, and T5. (one each) - BUILT UP I BEAMS

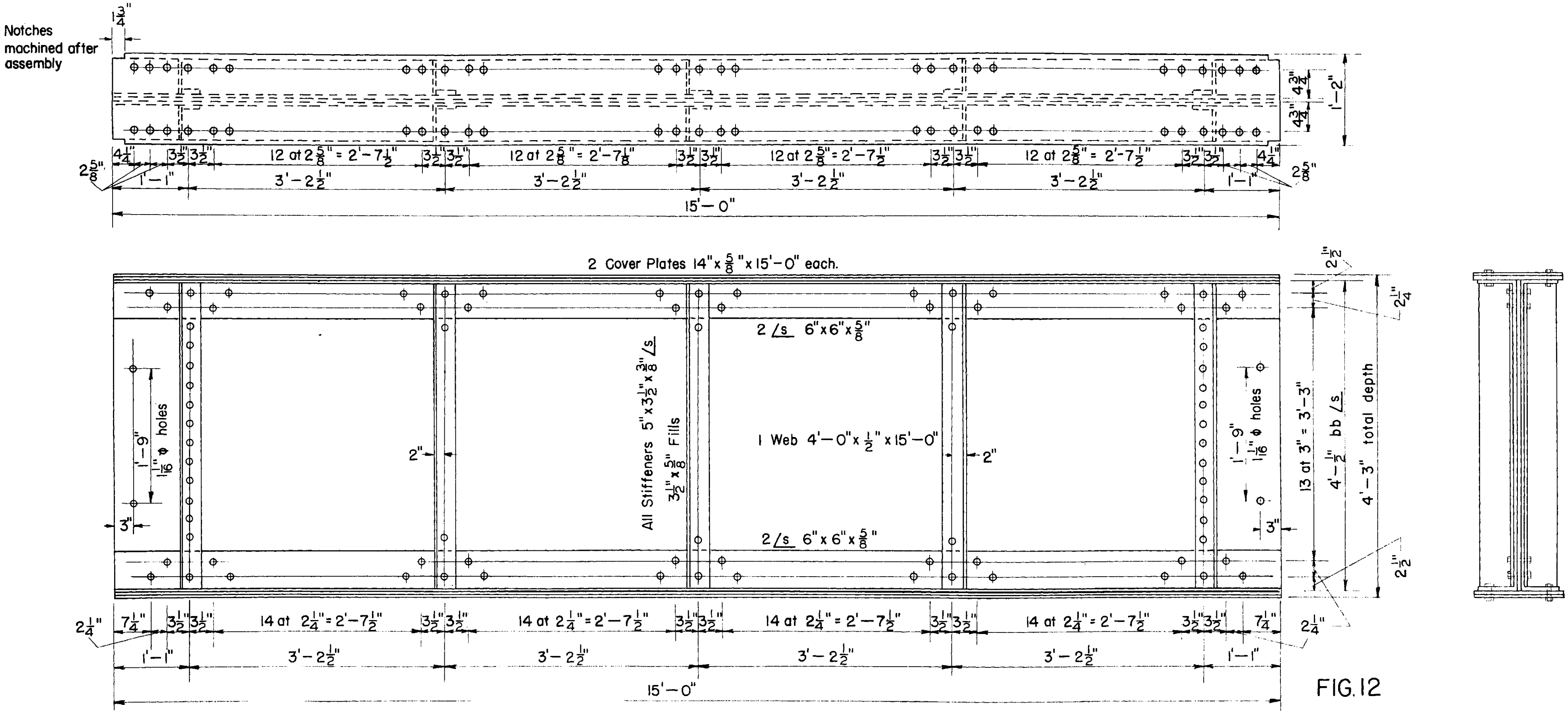


FIG. 12

TEST SPECIMEN T-6

PLATE GIRDER

Holes: 15/16" ϕ, subpunched and reamed, except as noted.

Rivets: 7/8" ϕ.

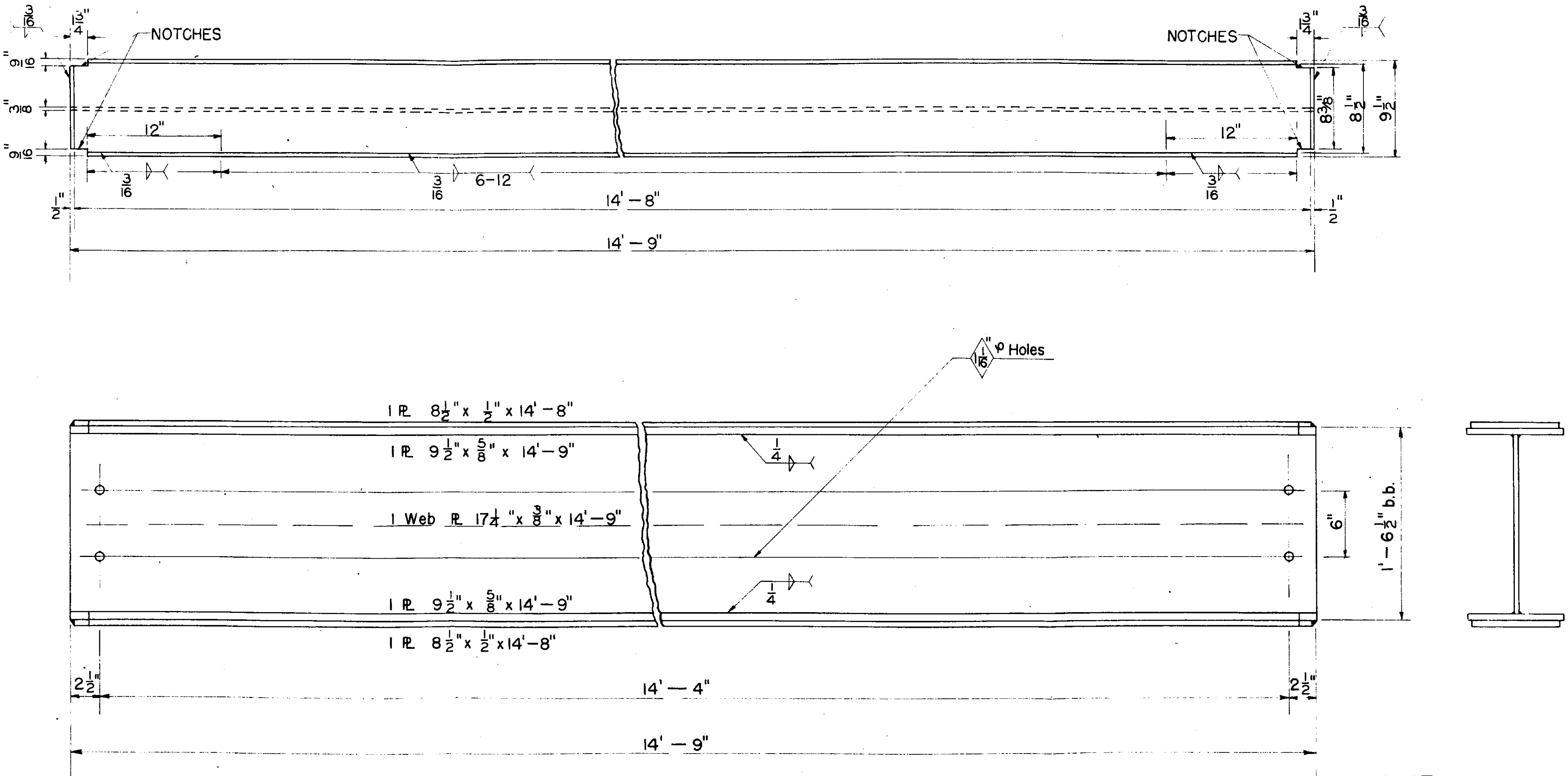


FIG. 13.

TEST SPECIMEN T-7

WELDED PLATE GIRDER

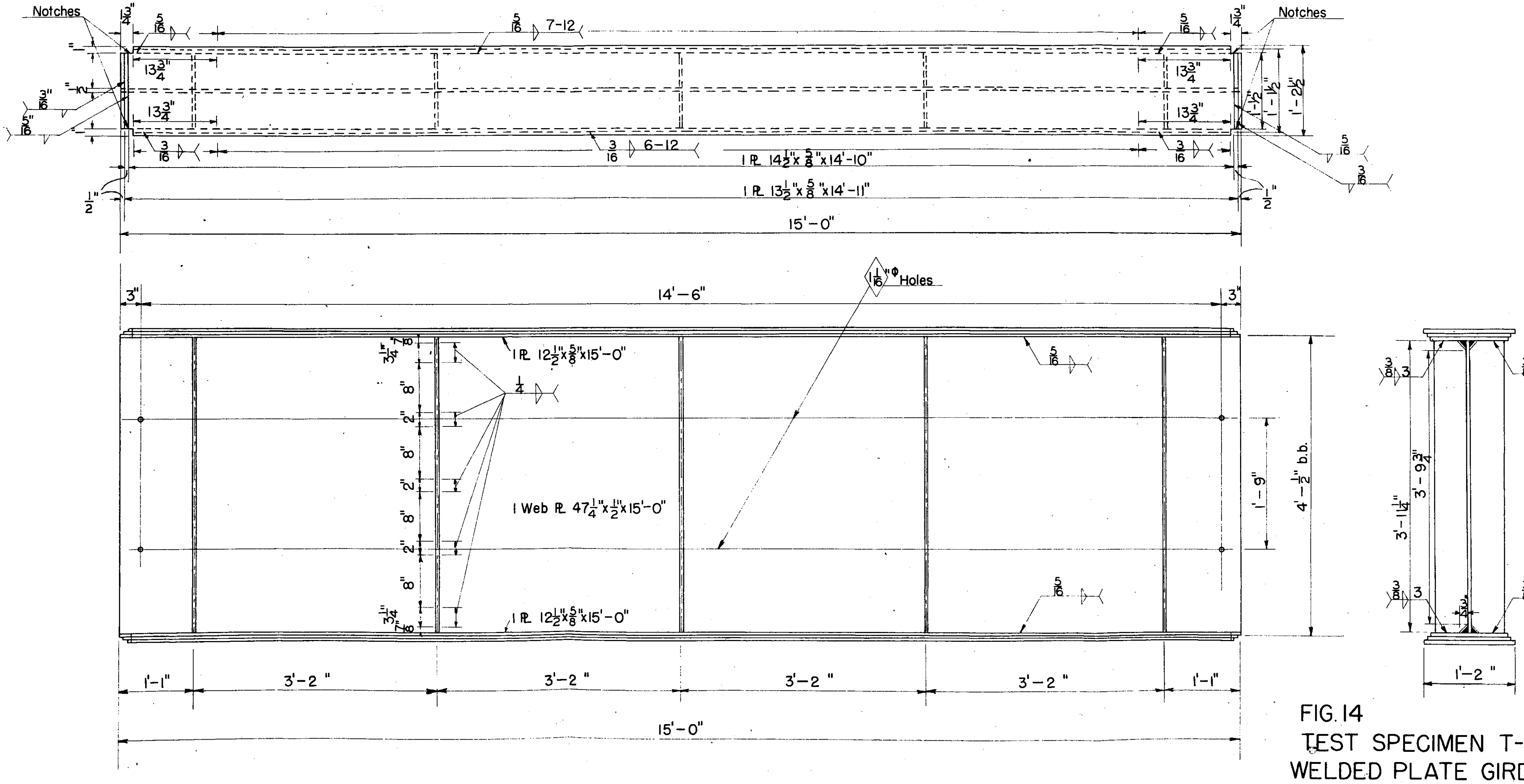


FIG. 14
TEST SPECIMEN T-8
WELDED PLATE GIRDER

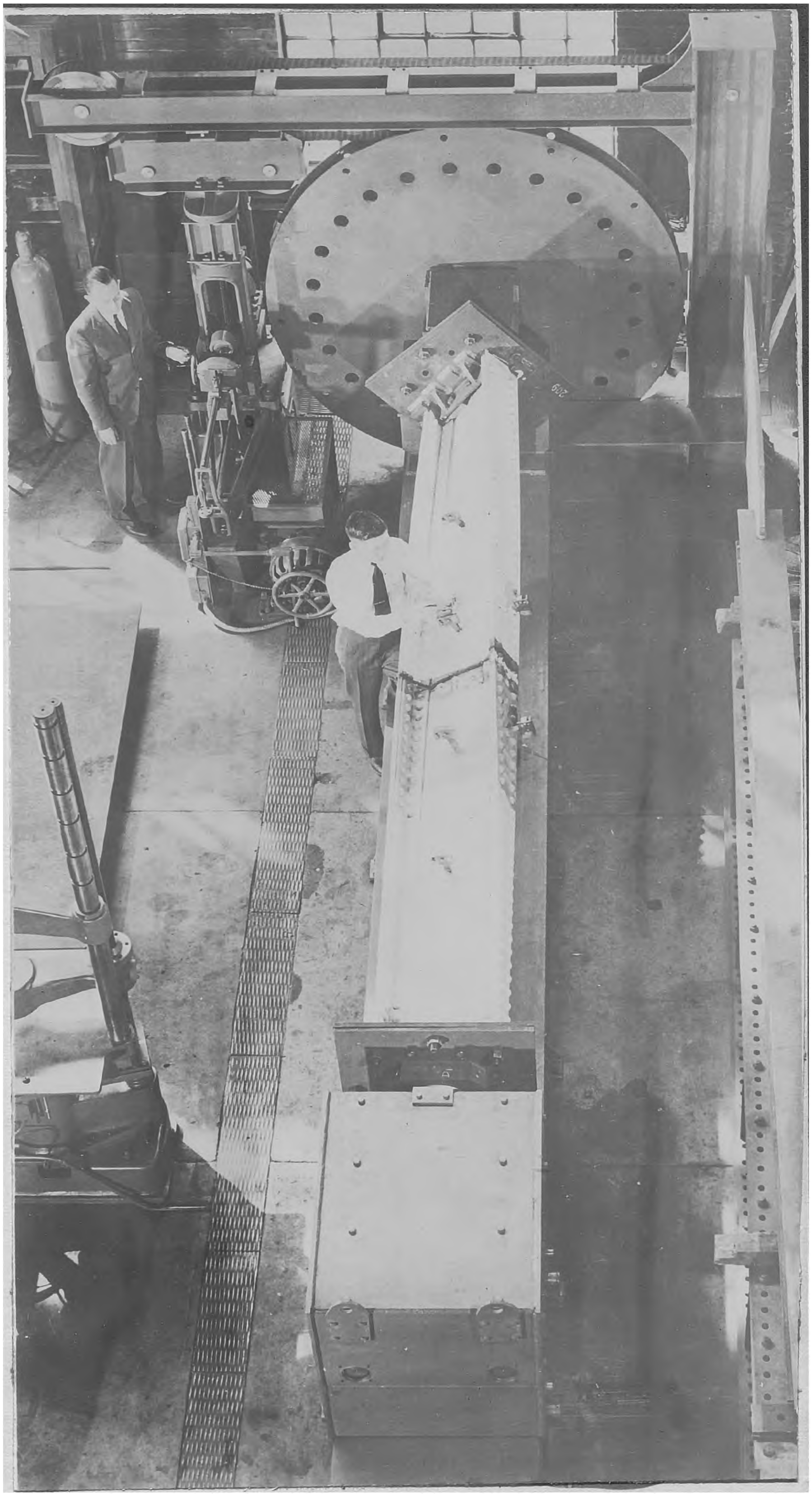


Fig. 15 General View of the 2,000,000 In-Lb. Torsion Testing Machine

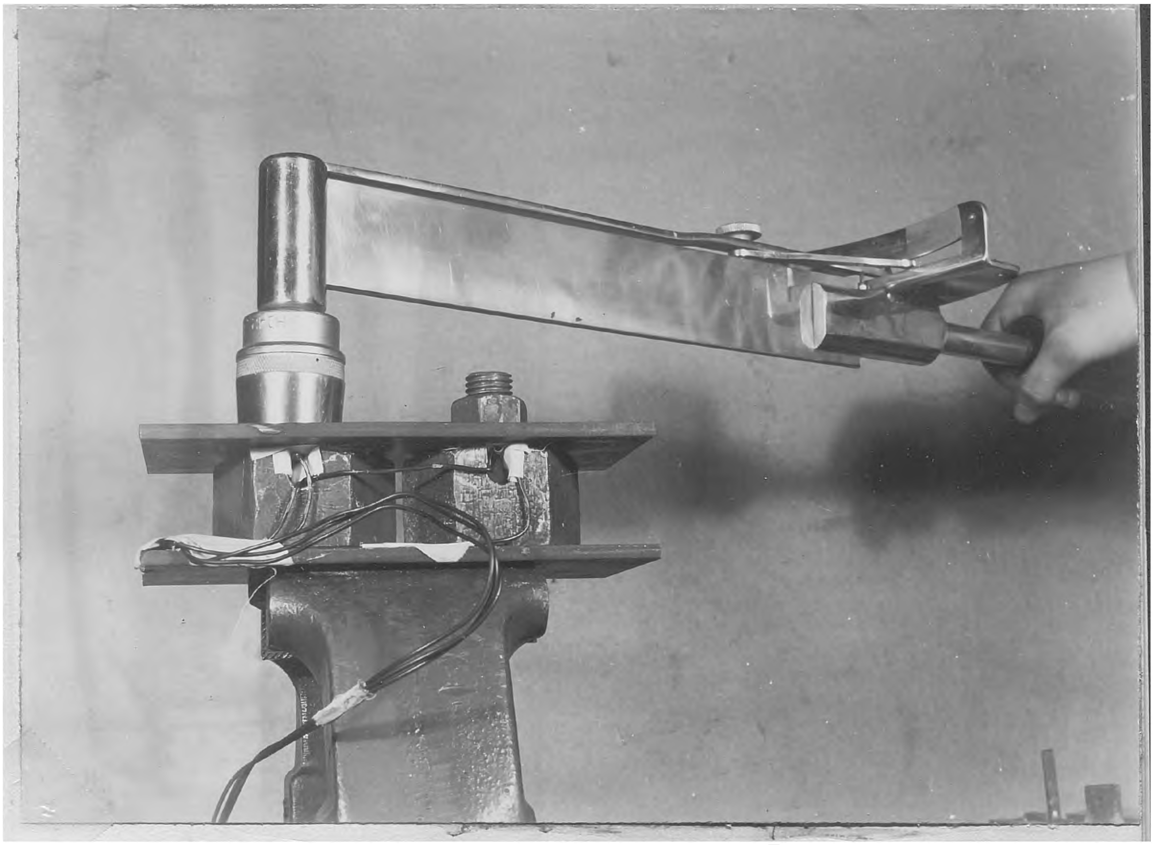


Fig. 16 Set-up for Measuring the Tensions Developed in Bolts by Using the Torque Wrench.

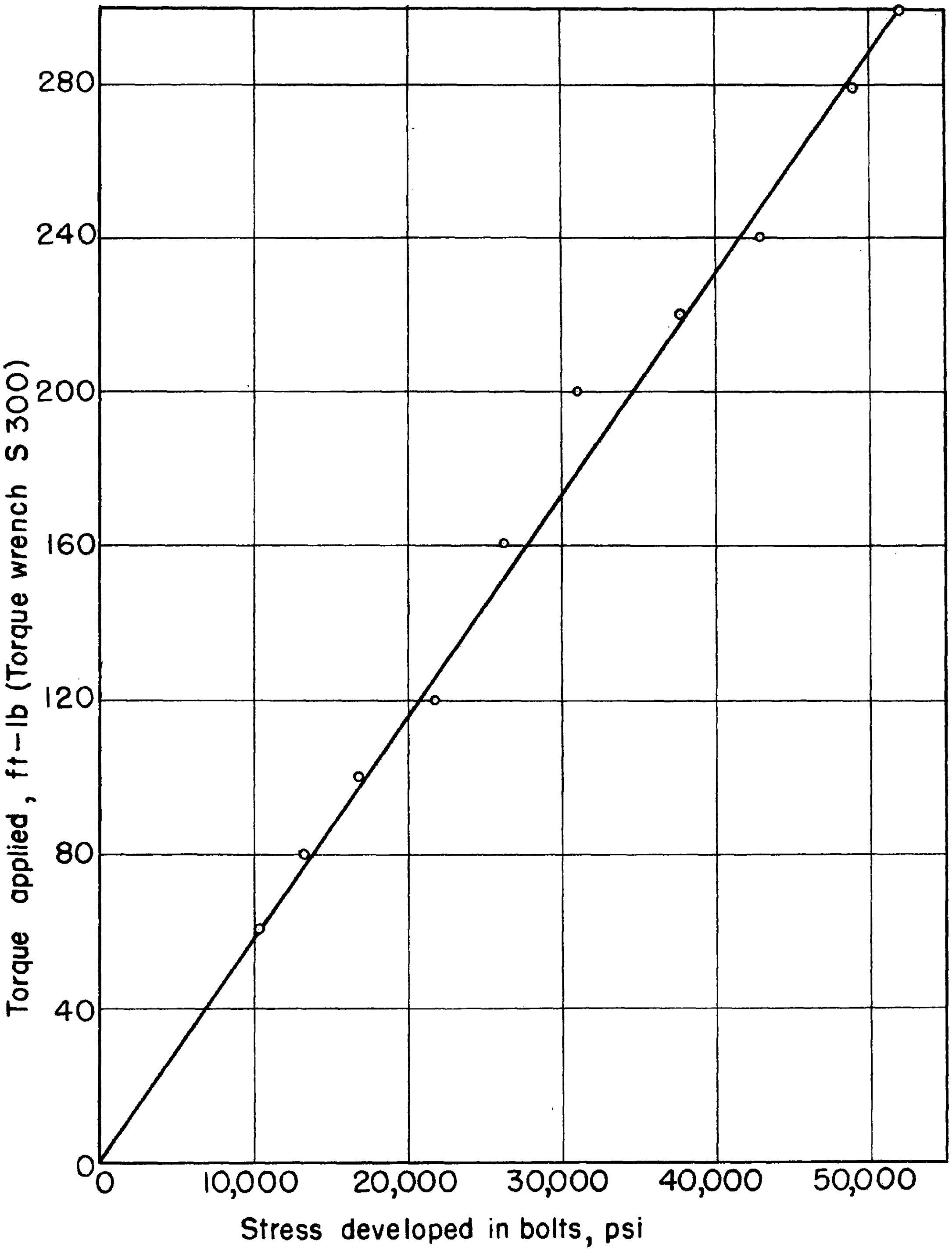


FIG 17 TORQUE-TENSION RELATIONSHIP OF 7/8" HIGH STRENGTH BOLTS

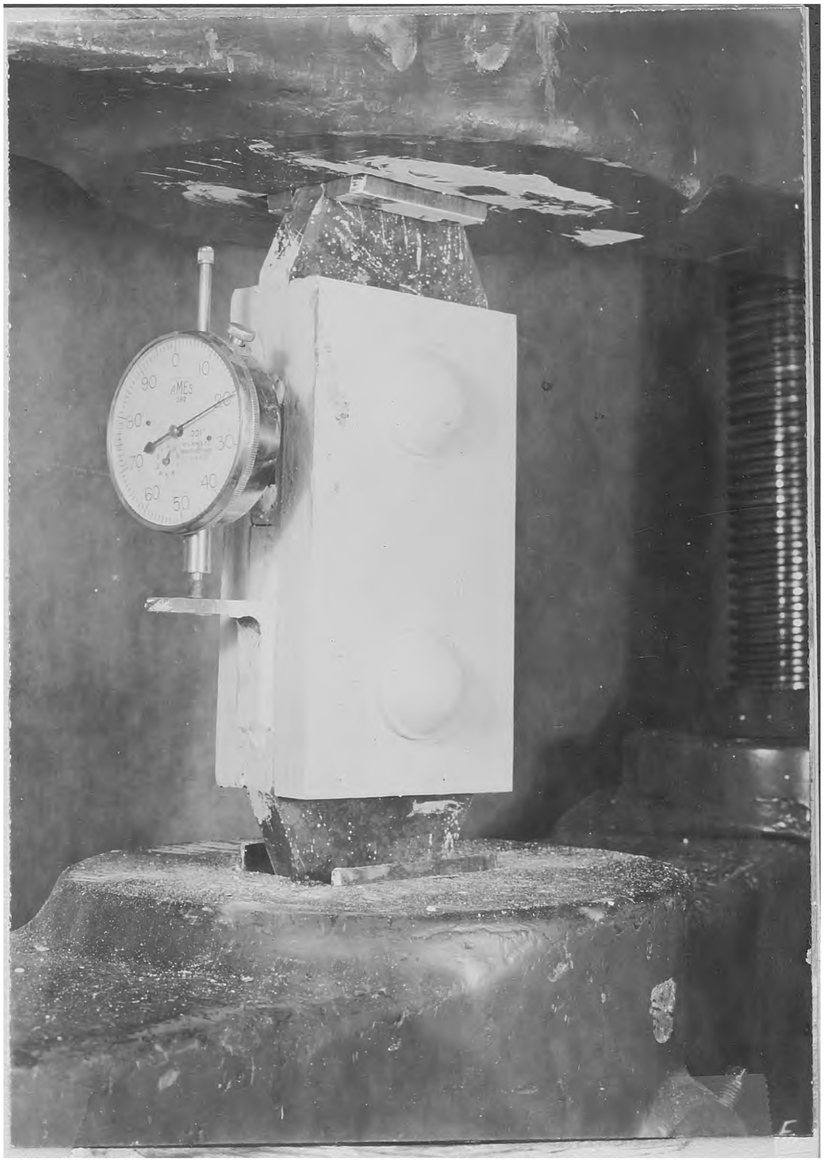


Fig. 18 Test of Riveted Joint

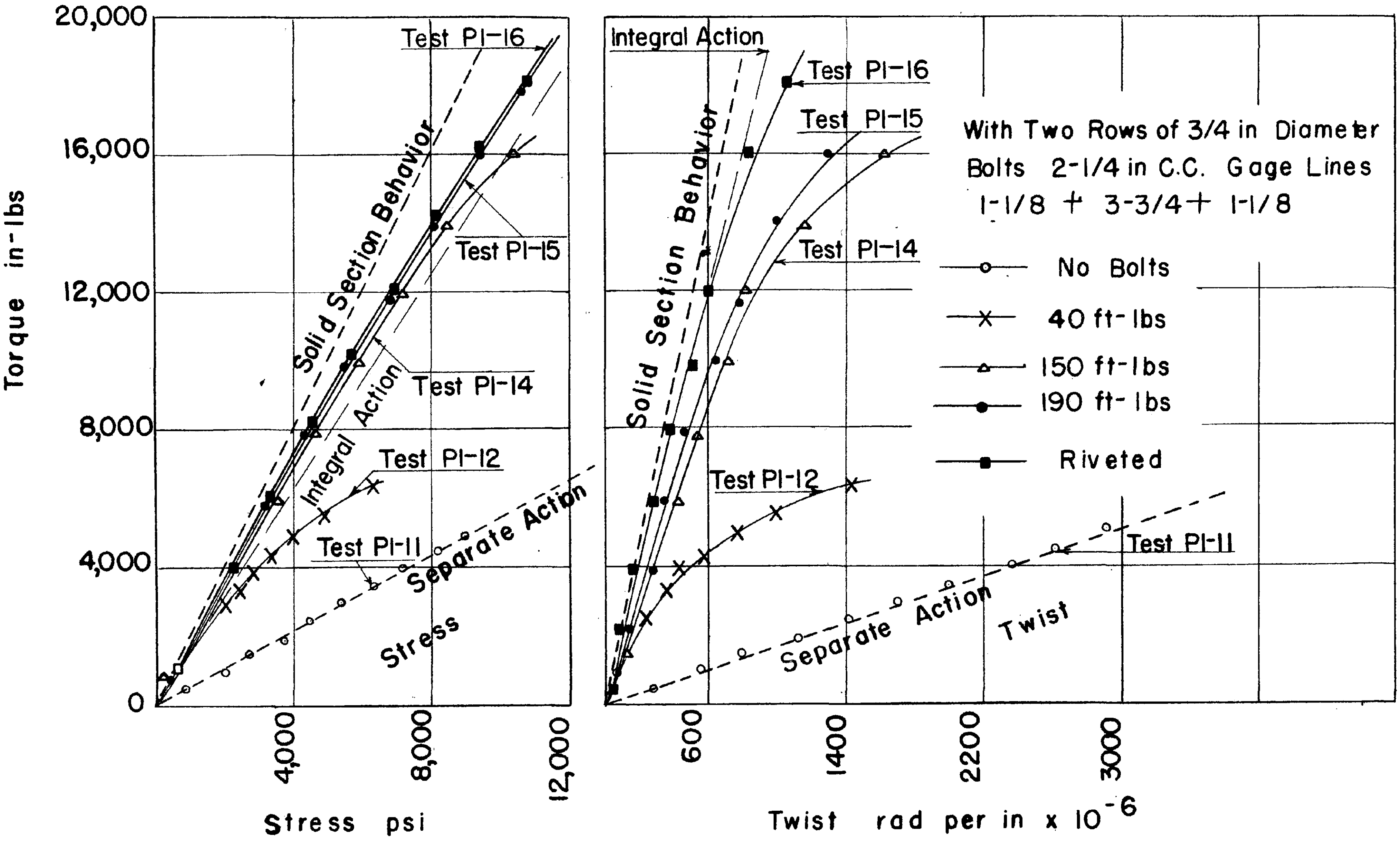


Fig. 19. Effect of Tension of Bolts on Four 6 in x 1/4 in Plates Bolted Together Under Torsion (Specimen P-1)

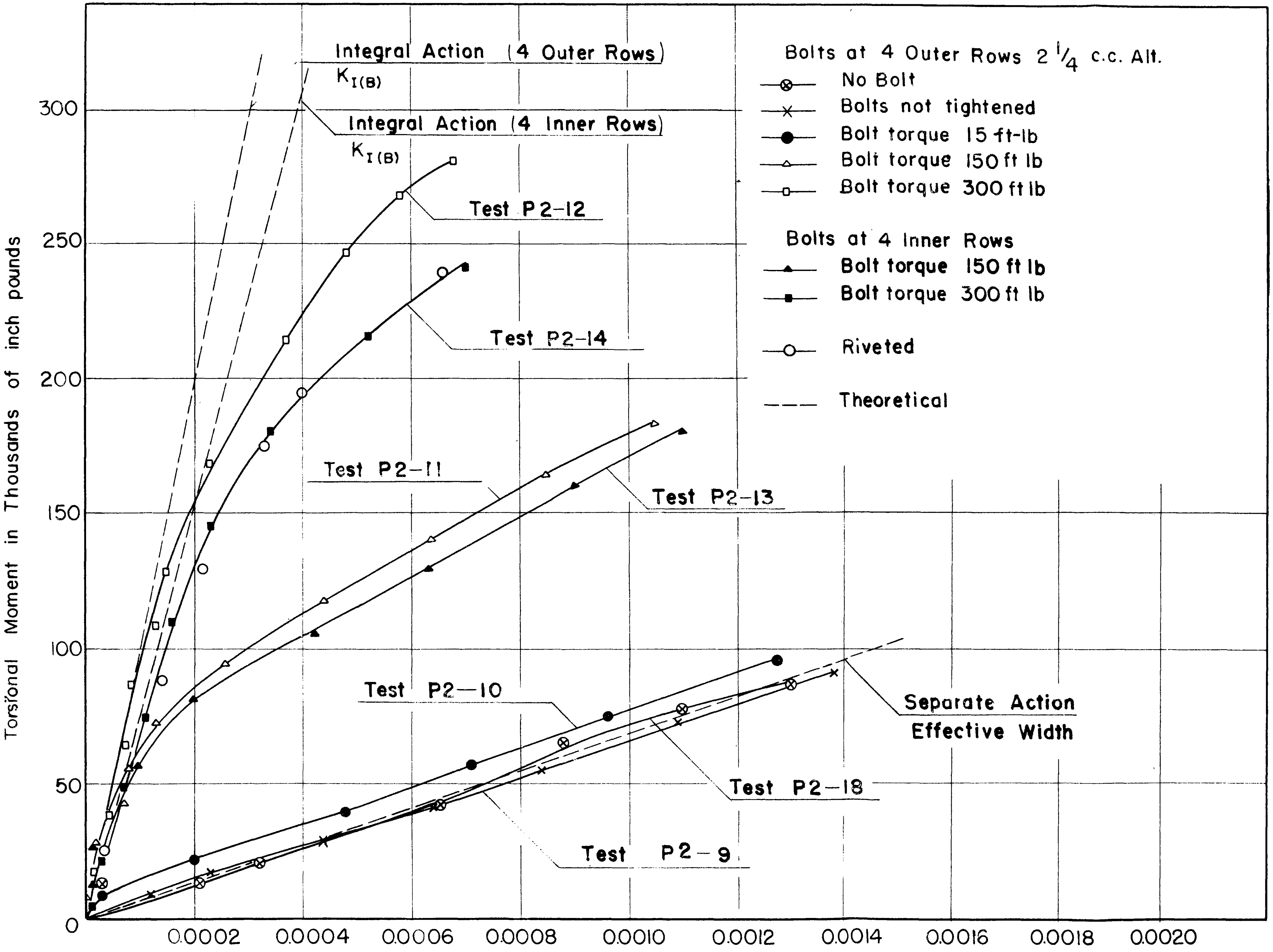


FIG. 20. EFFECT OF BOLT TENSION & GAGE LINE LOCATIONS ON FOUR 20×5/8 PLATES

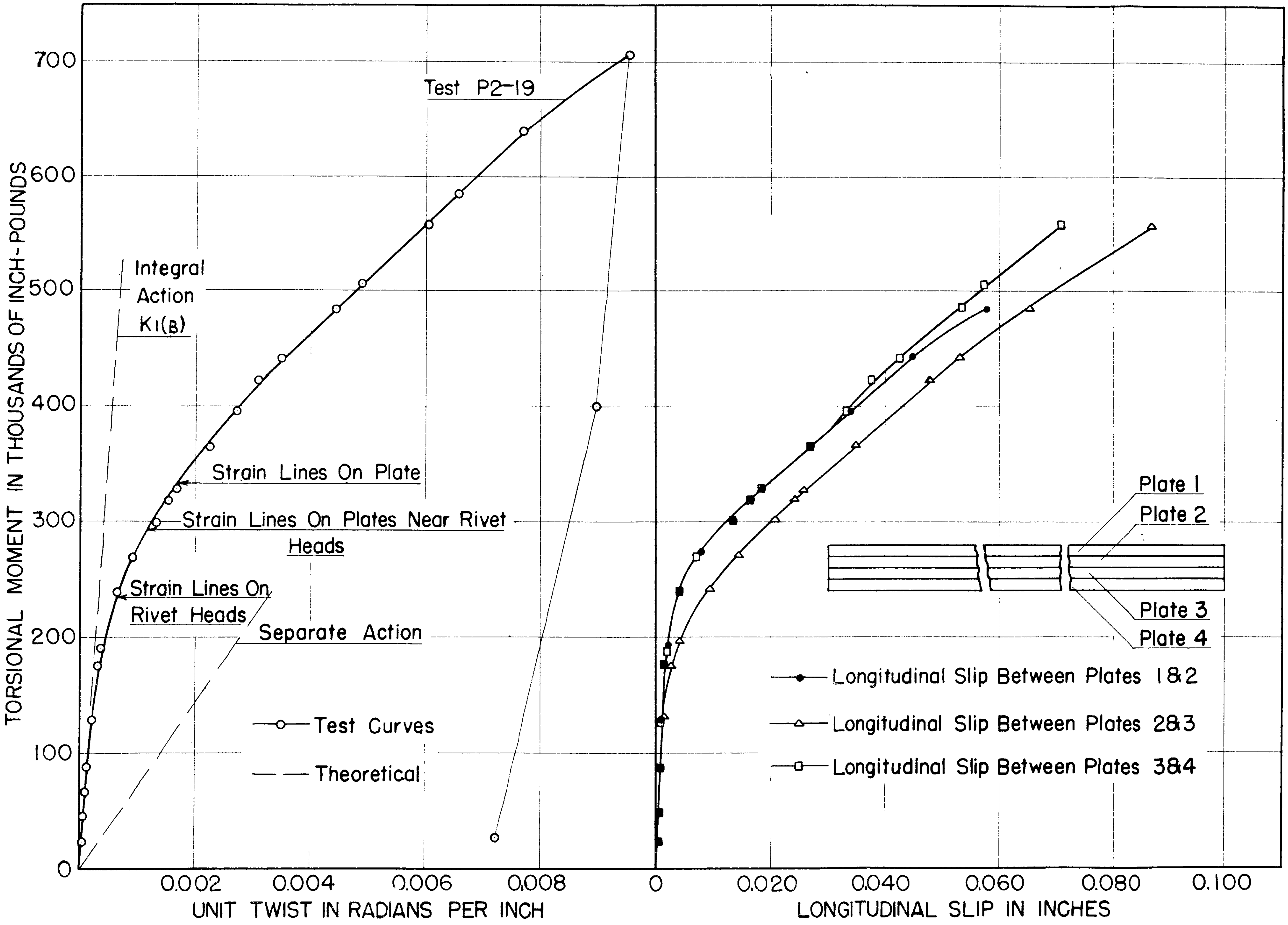


FIG. 21. TORQUE TWIST AND TORQUE SLIP CURVES ON FOUR 20" X $\frac{5}{8}$" PLATES RIVETED TOGETHER UNDER TORSION (SPECIMEN P-2)

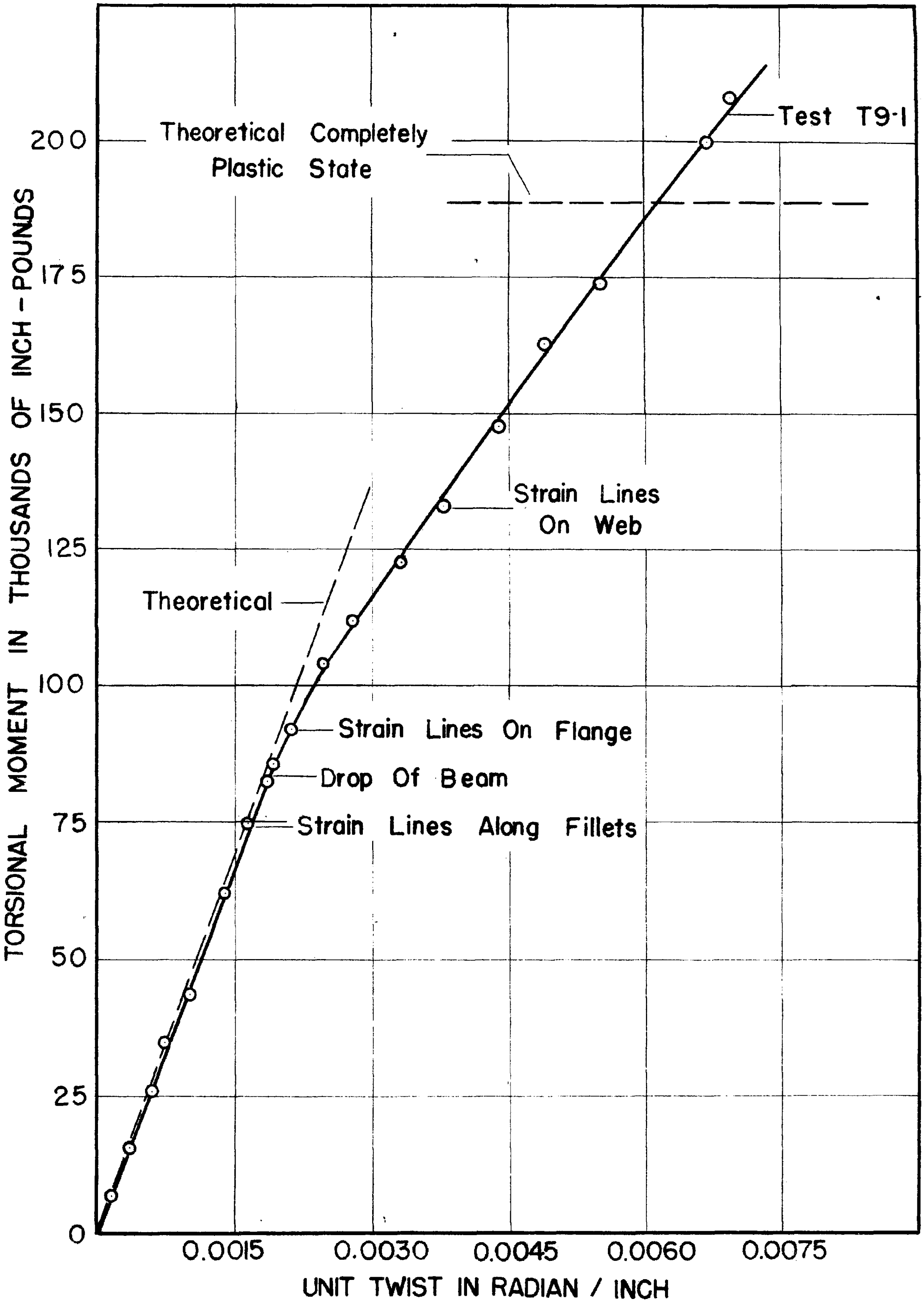


FIG. 22. TORQUE - TWIST CURVE FOR ROLLED SECTION
(SPECIMEN T 9)

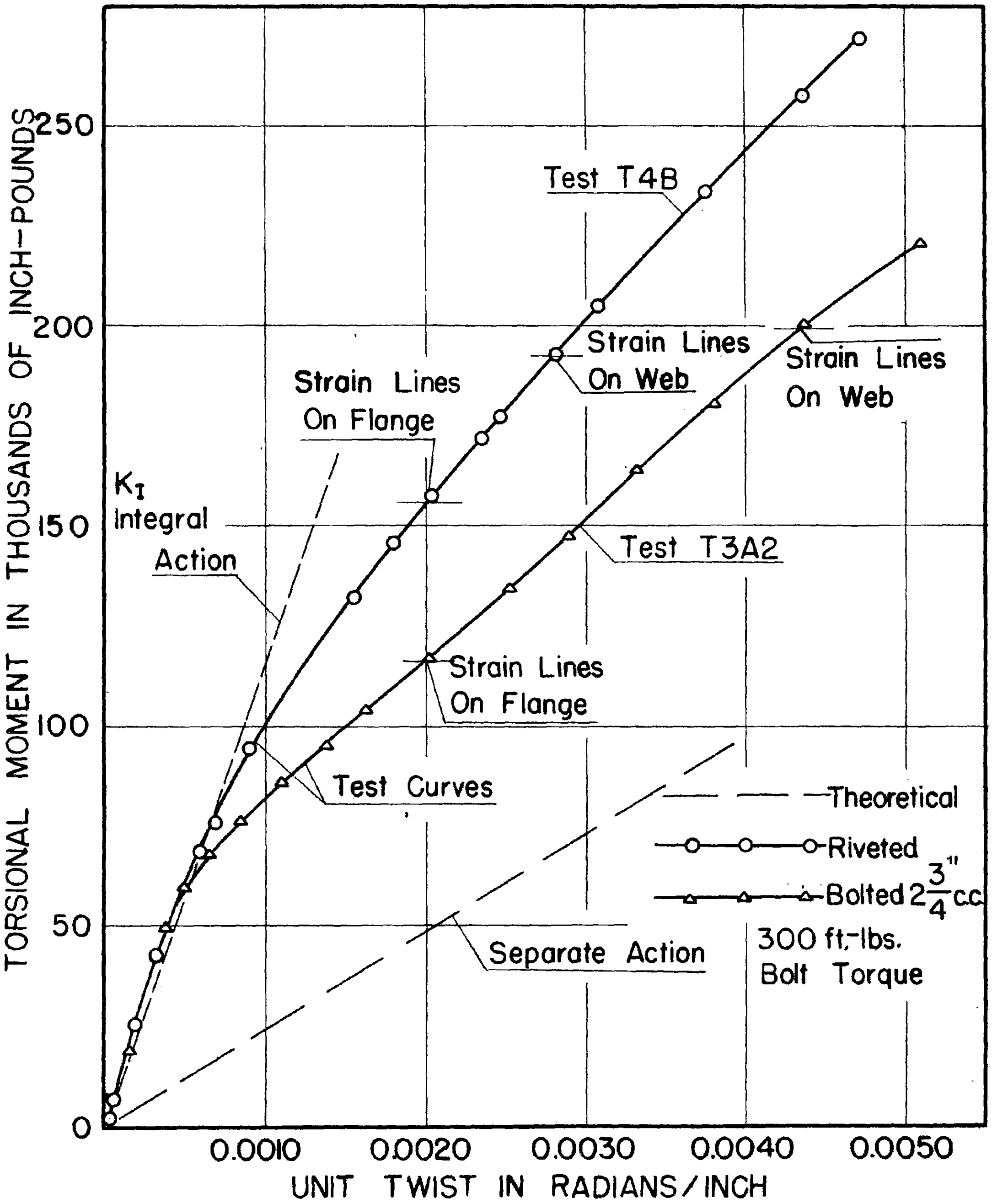


FIG.- 23 TORQUE TWIST CURVE FOR 20" X 9" GIRDERS (SPECIMENS T3 AND T4)

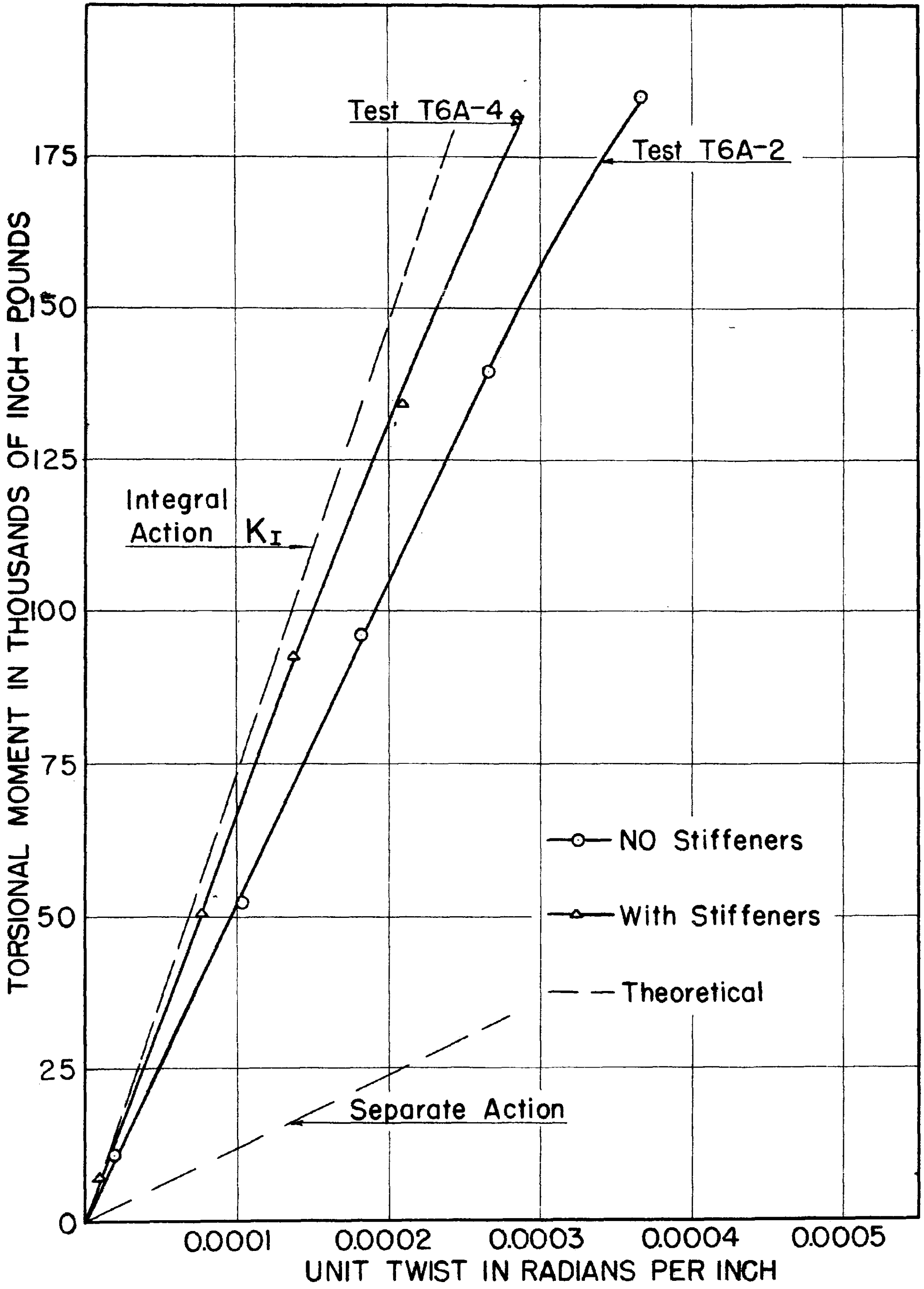


FIG. 24 EFFECT OF STIFFENERS IN BOLTED GIRDER
(SPECIMEN T-6)

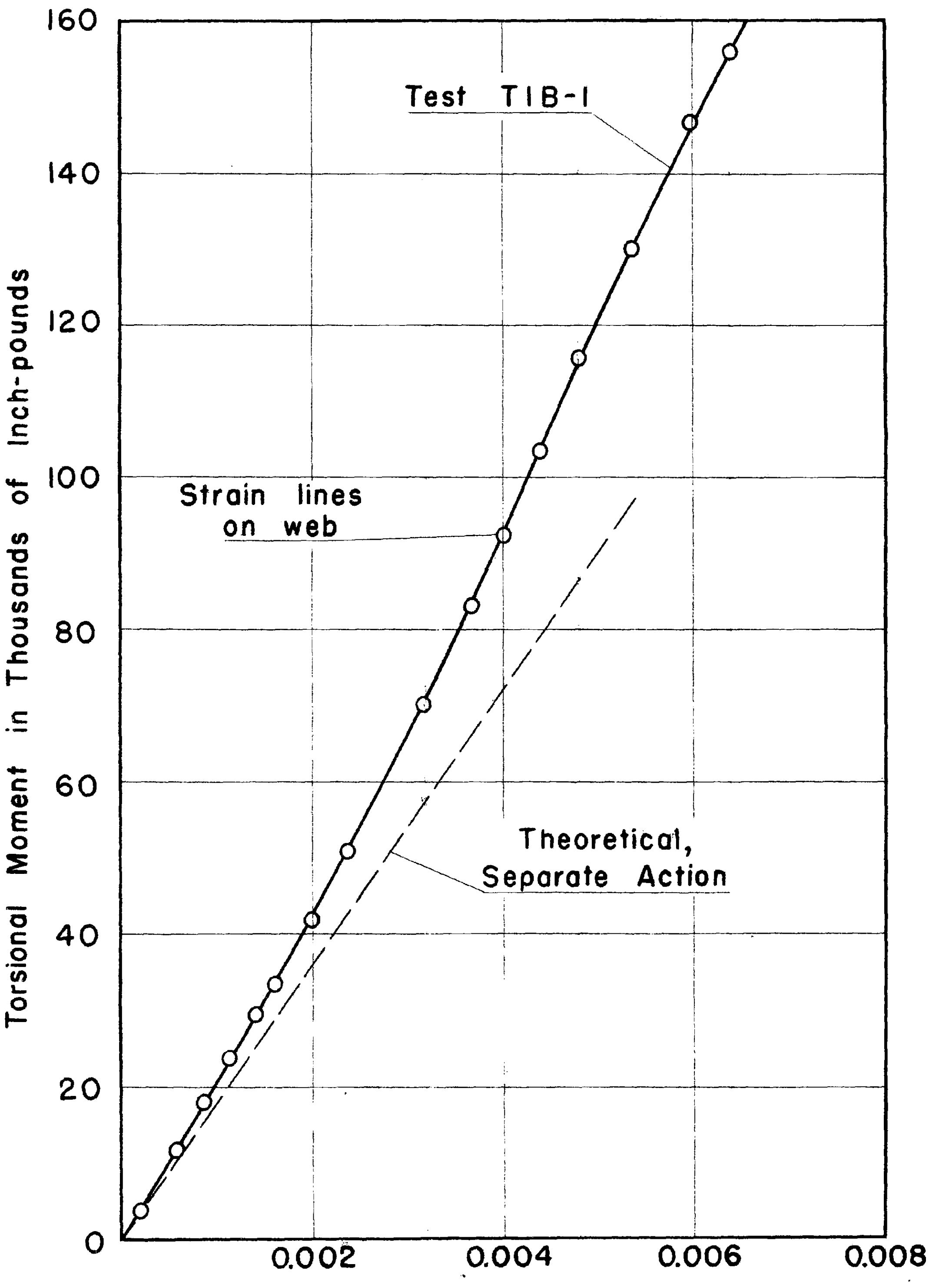


FIG.25. TORQUE-TWIST CURVES FOR 20" X 9" RIVETED GIRDER, (SPECIMEN T-1)

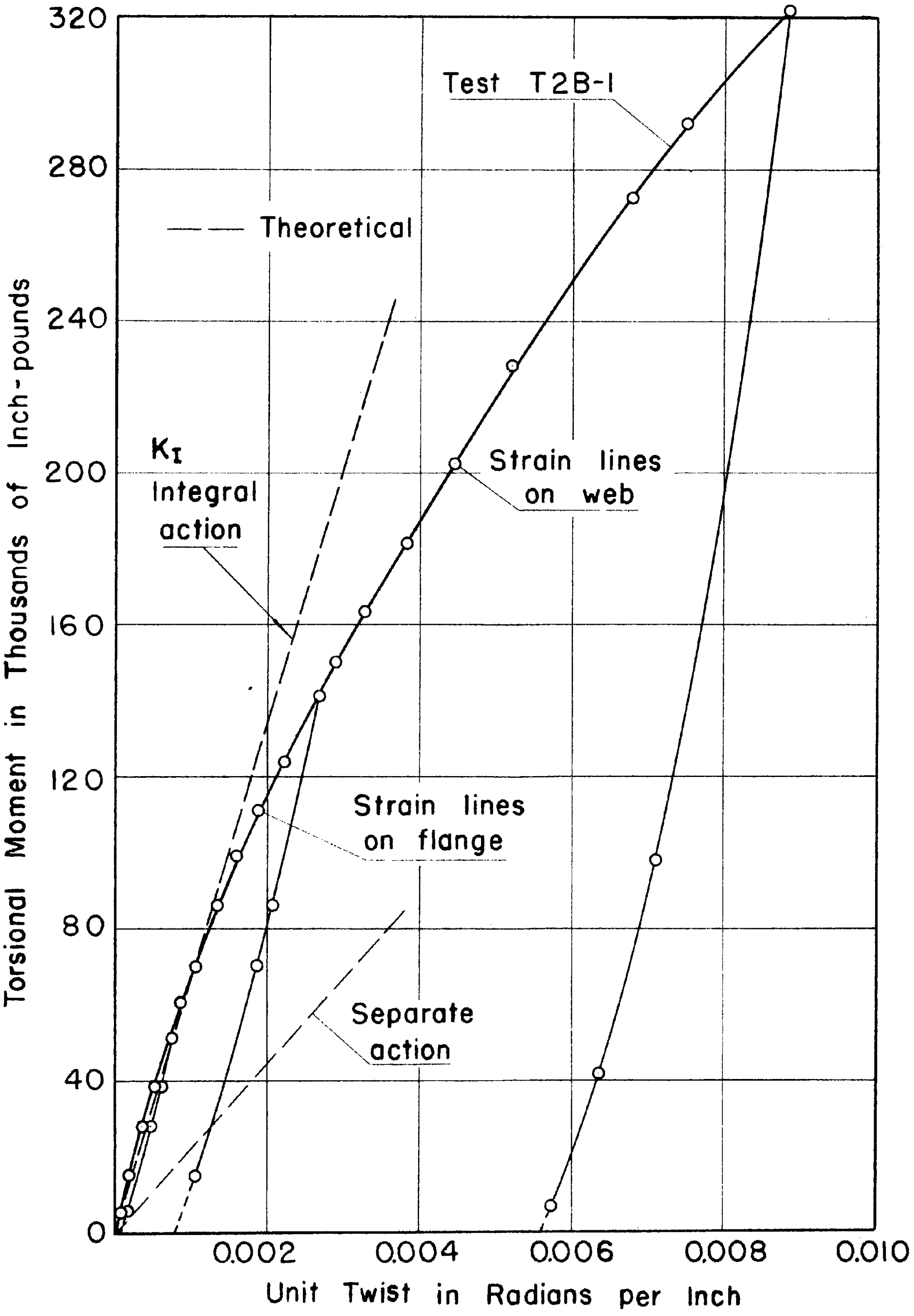


FIG. 26 TORQUE-TWIST CURVES FOR 20" X 9" RIVETED GIRDER (SPECIMEN T-2)

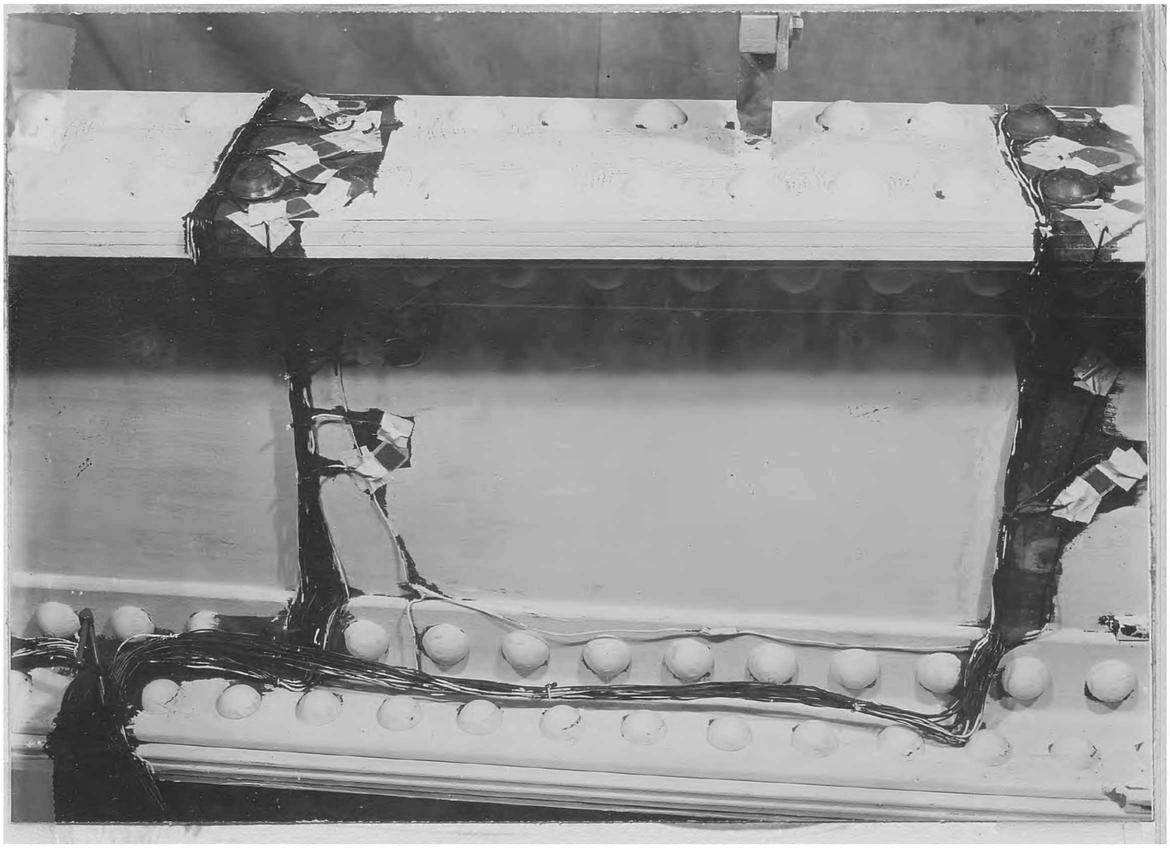


Fig. 27 Strain Line Pattern (Specimen T5)

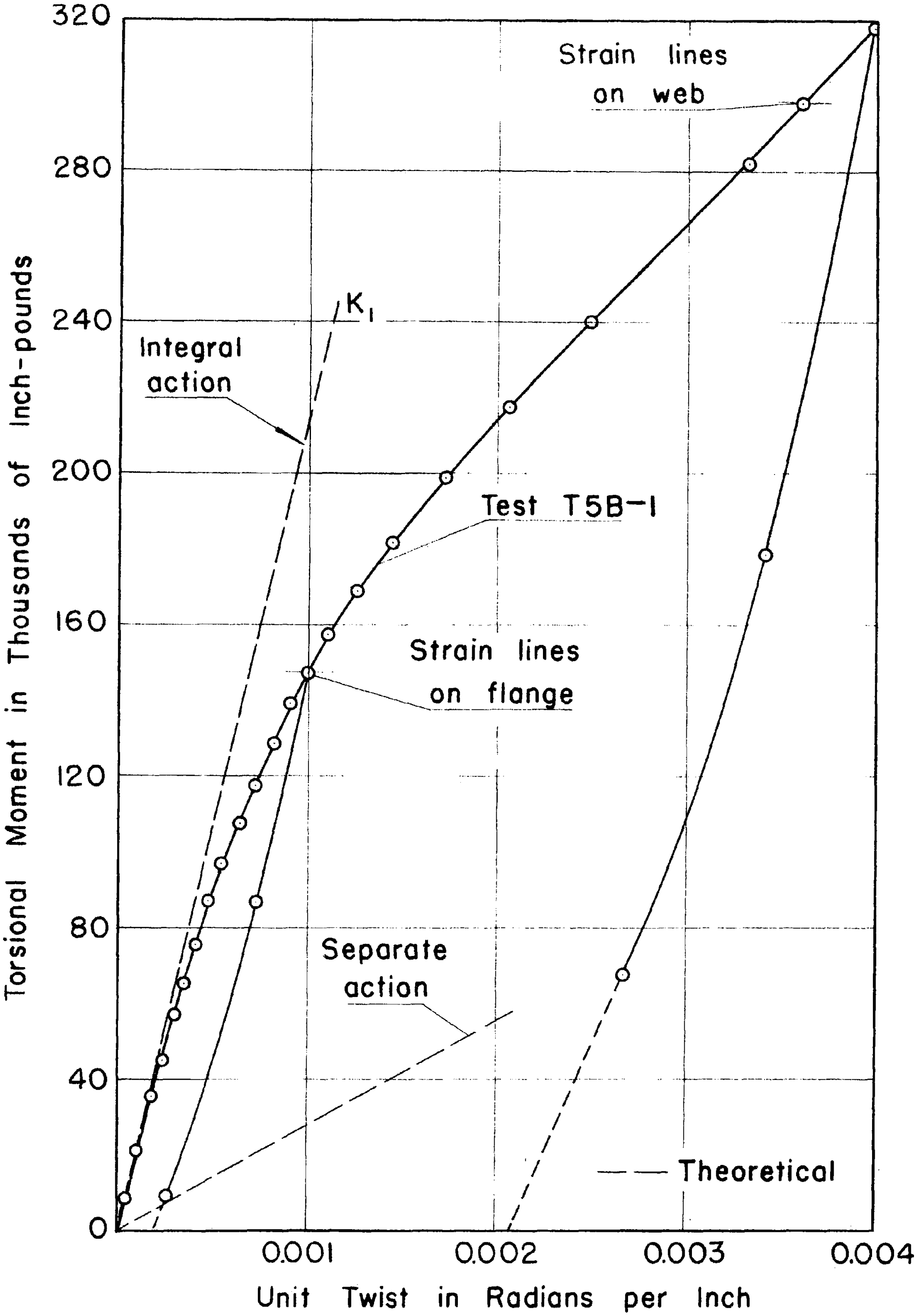
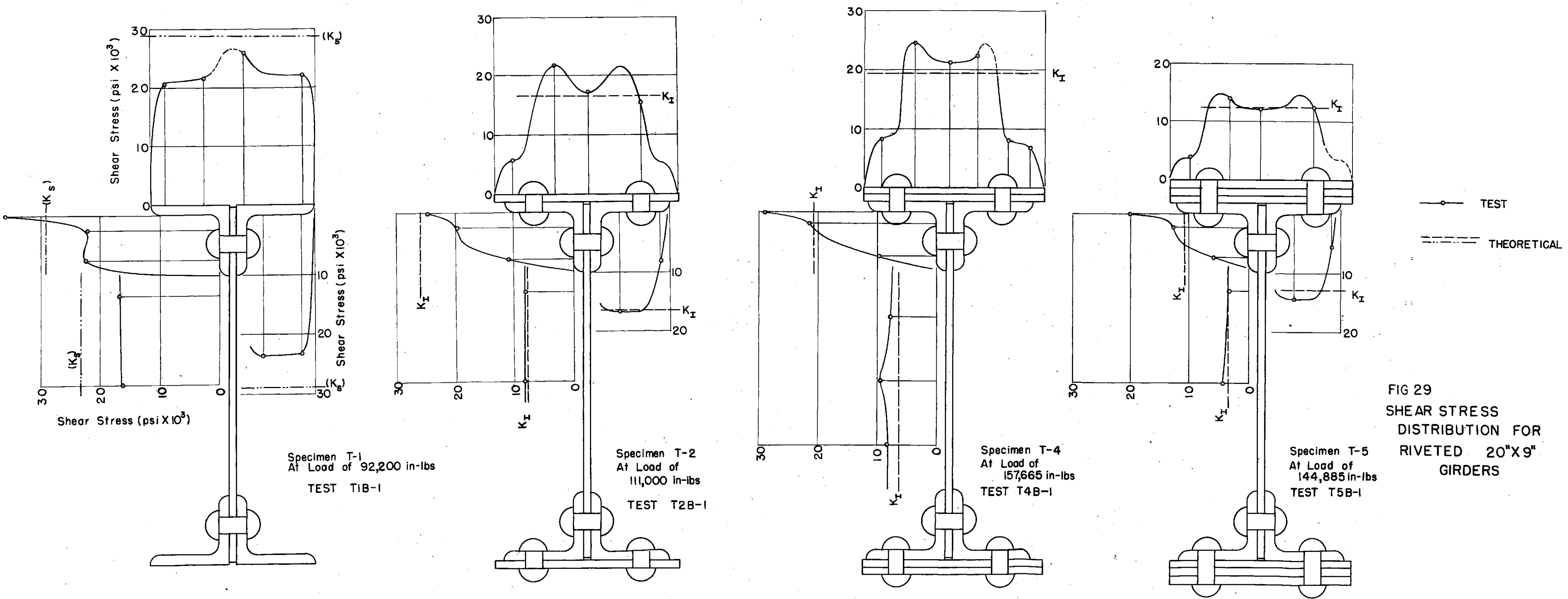


FIG.28 TORQUE-TWIST CURVES FOR 20" X 9"

RIVETED GIRDER (SPECIMEN T-5)



Specimen T-1
At Load of 92,200 in-lbs
TEST T1B-1

Specimen T-2
At Load of
111,000 in-lbs
TEST T2B-1

Specimen T-4
At Load of
157,665 in-lbs
TEST T4B-1

Specimen T-5
At Load of
144,885 in-lbs
TEST T5B-1

FIG 29
SHEAR STRESS
DISTRIBUTION FOR
RIVETED 20"X 9"
GIRDERS

SHEAR STRESS DISTRIBUTION FOR RIVETED 20" X 9" GIRDERS

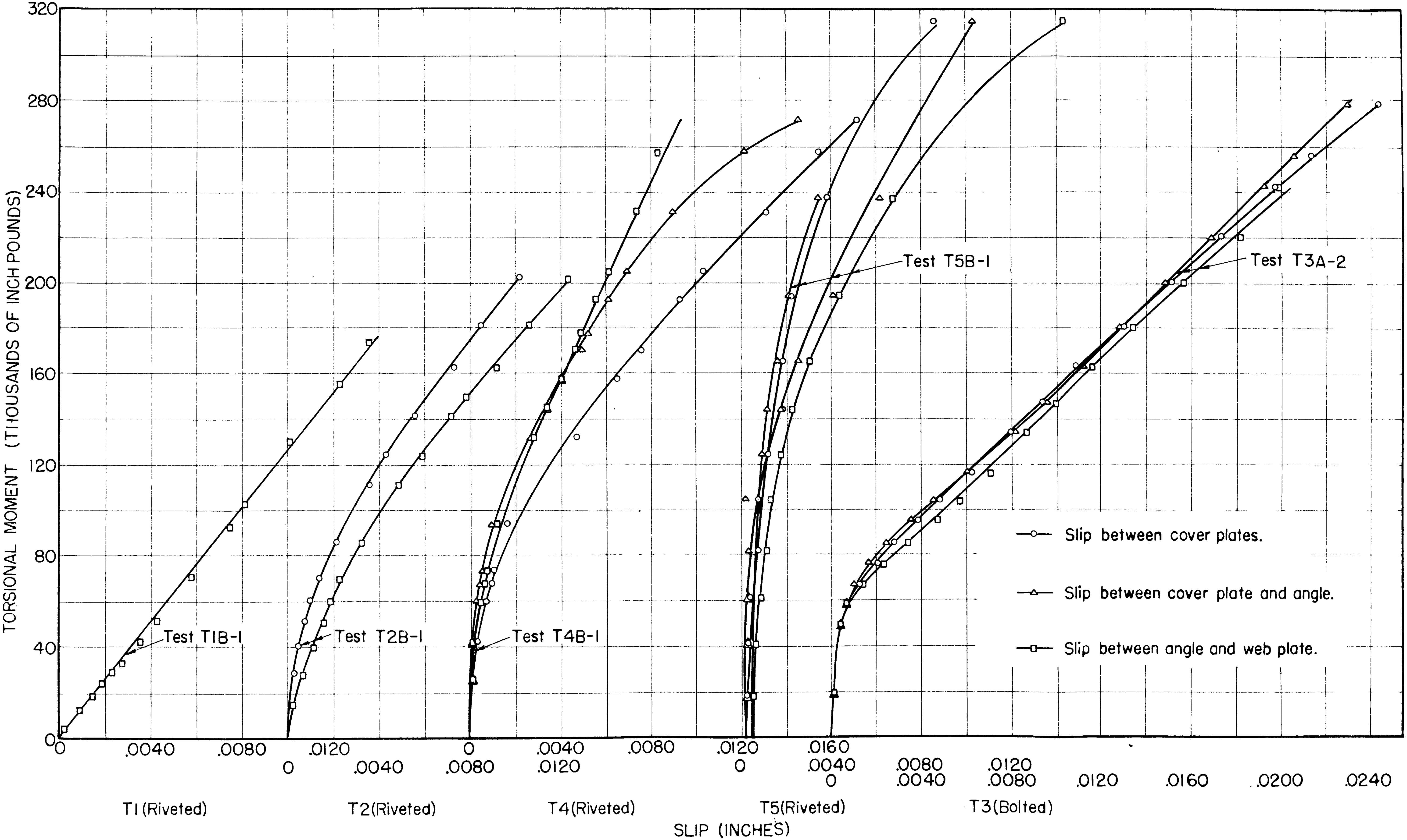


FIG.30. TORQUE-LONGITUDINAL SLIP CURVES FOR 20" RIVETED AND BOLTED GIRDERS (SPECIMENS T1 TO T5).

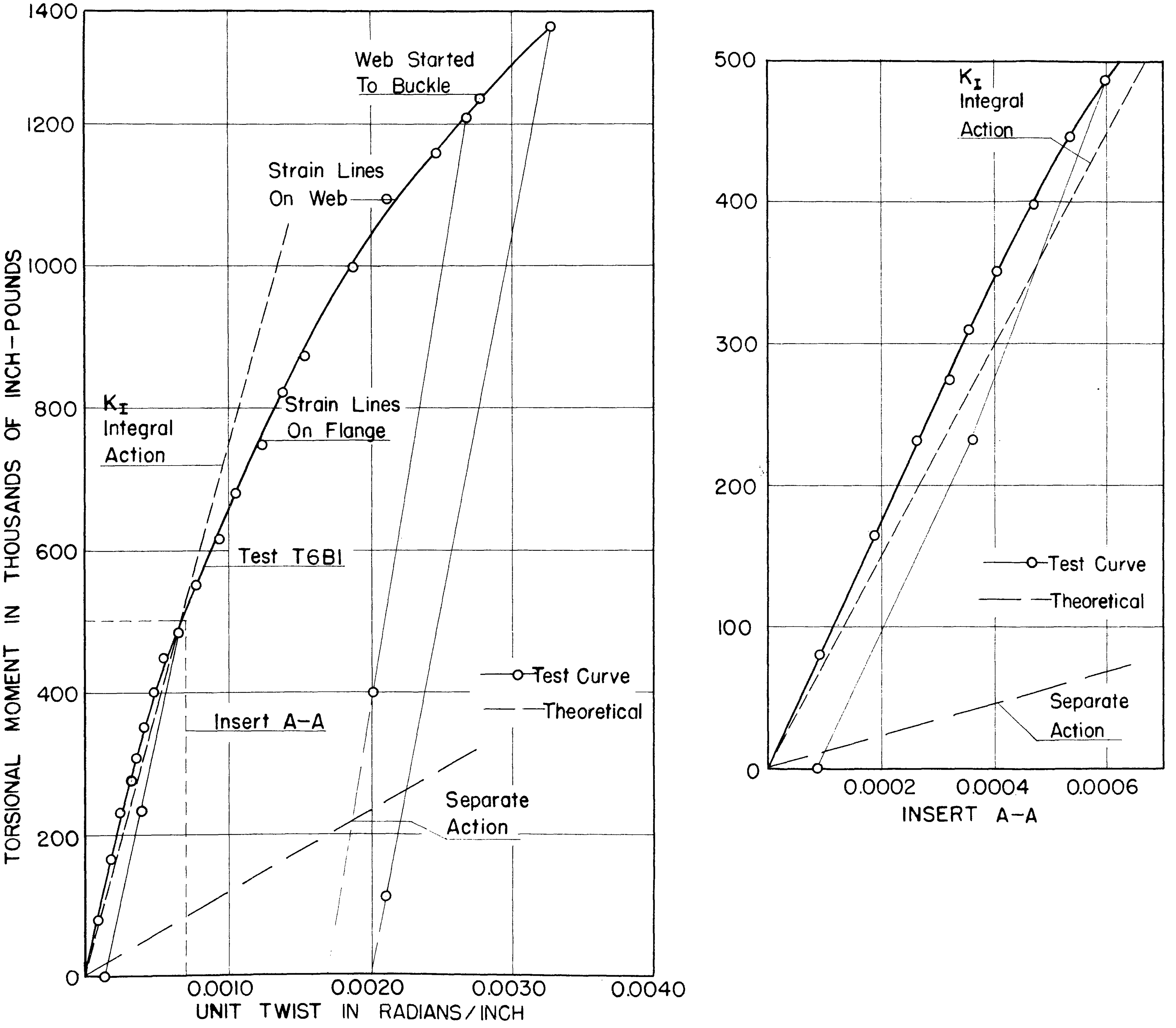


FIG.-31 TORQUE TWIST CURVE FOR 50"X14" RIVETED GIRDER (SPECIMEN T-6)

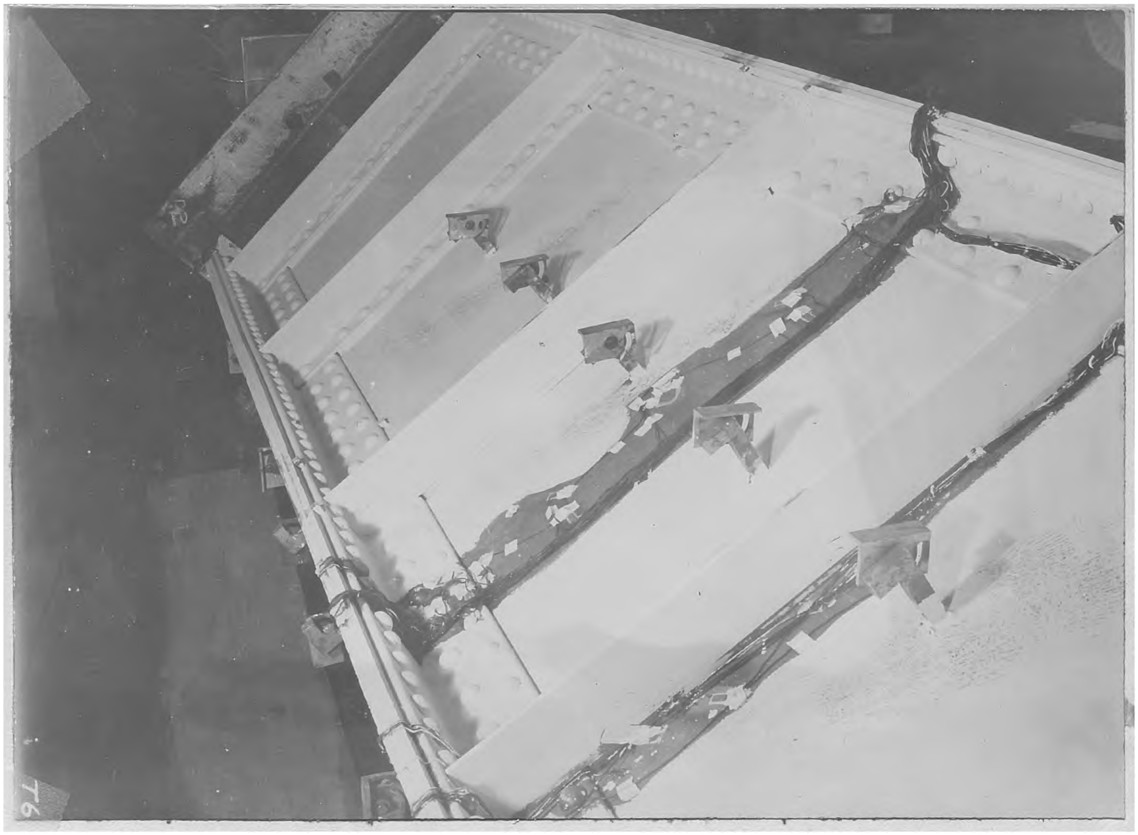


Fig. 32 Strain Line Pattern (Specimen T6)

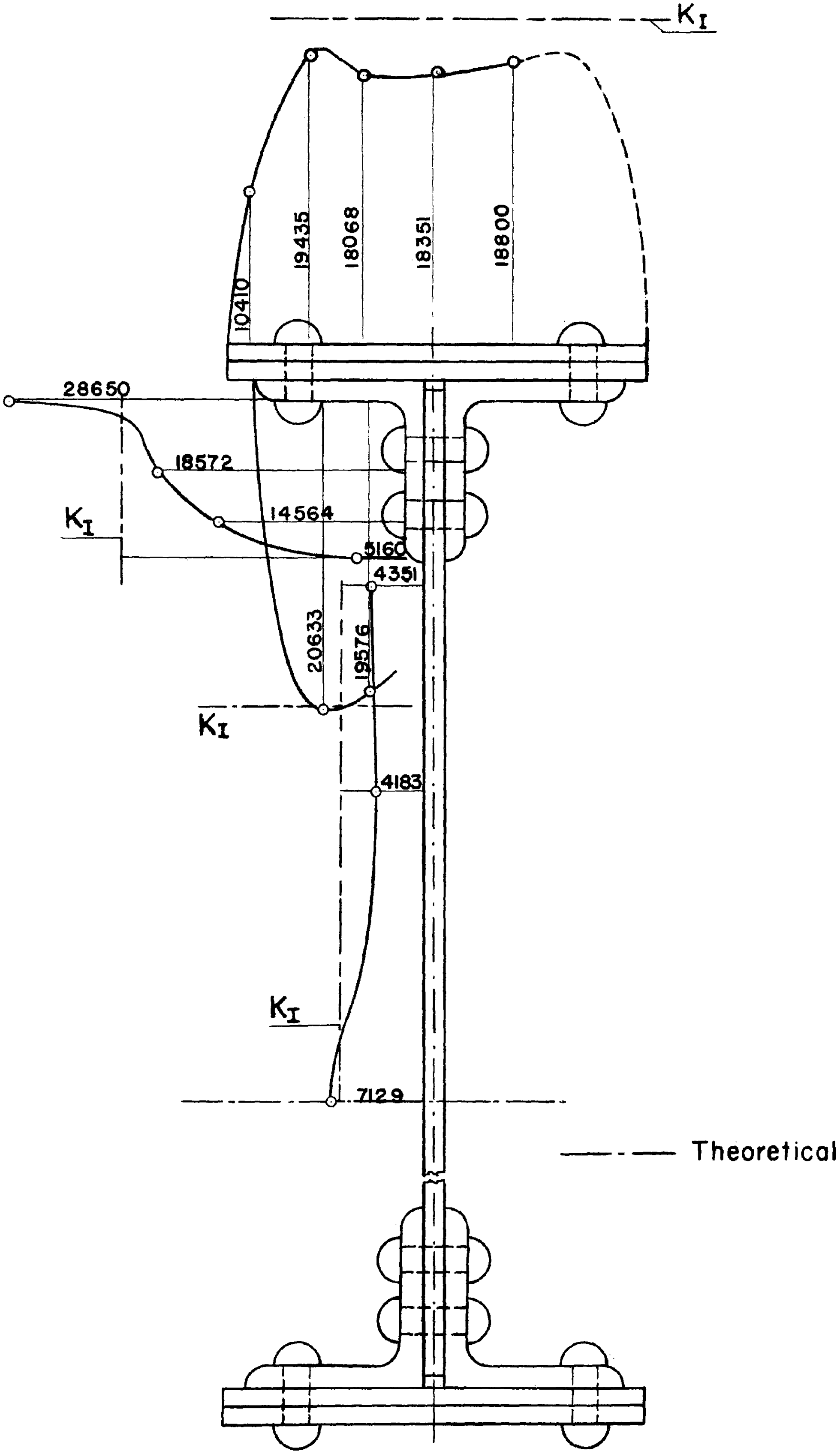


FIG.33 SHEARING STRESS DISTRIBUTION AT LOAD OF 750,000 LB-IN 50" x 14" GIRDER (TEST - T6B1)

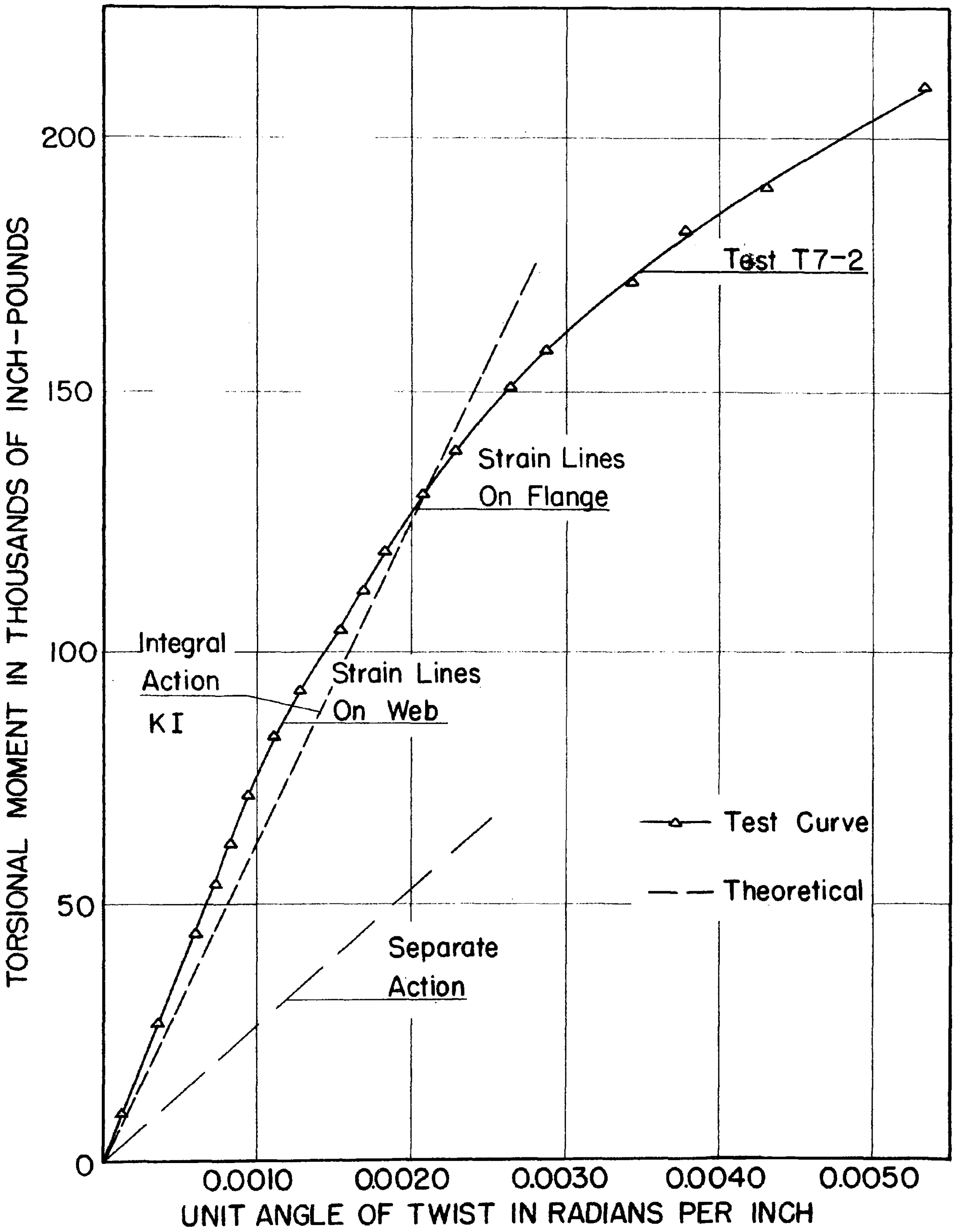
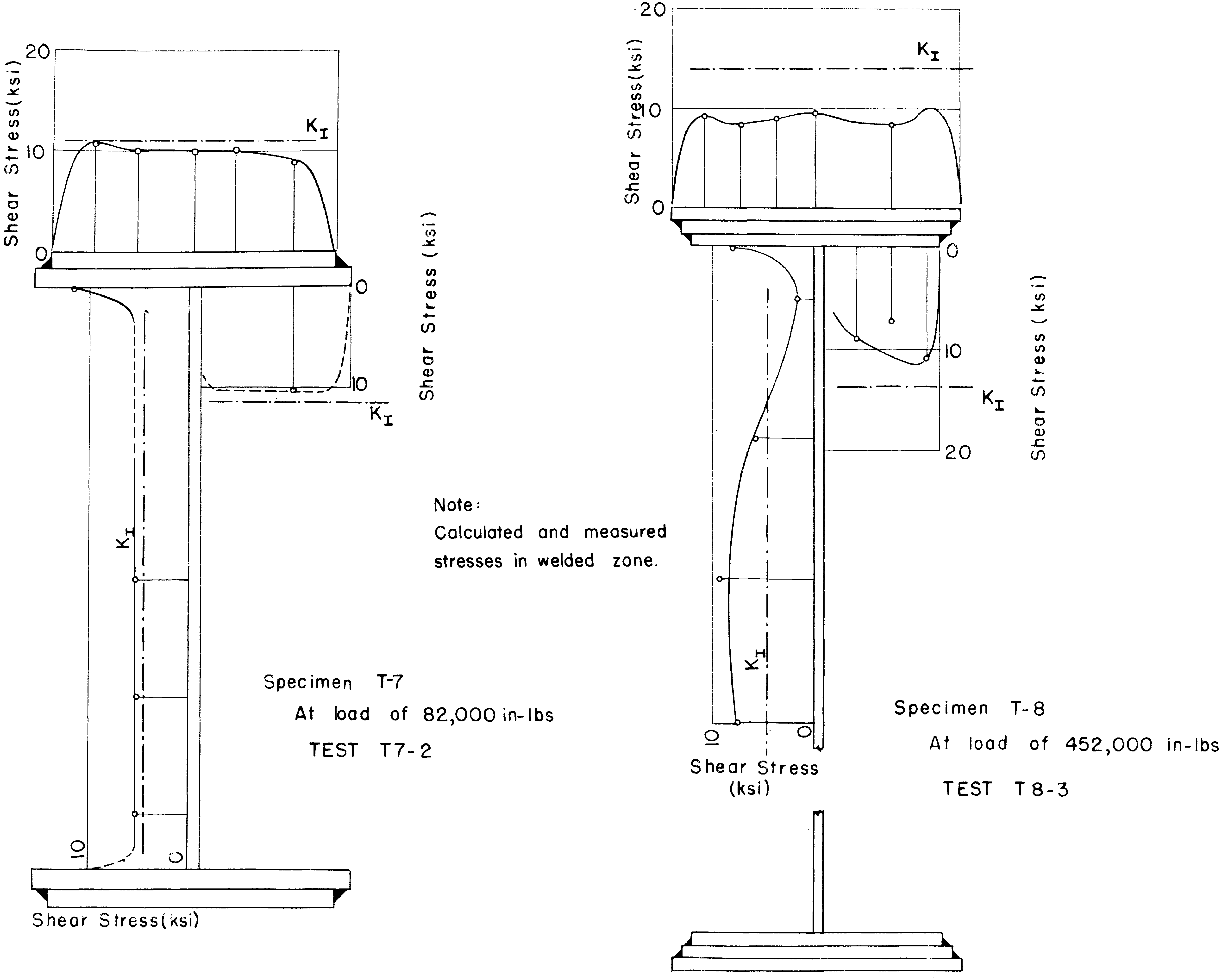


FIG.34 TORQUE TWIST CURVE FOR 20" WELDED GIRDER
(SPECIMEN T-7)



Note:
Calculated and measured
stresses in welded zone.

Specimen T-7
At load of 82,000 in-lbs
TEST T7-2

Specimen T-8
At load of 452,000 in-lbs
TEST T8-3

FIG.35. SHEAR STRESS DISTRIBUTION FOR WELDED GIRDERS

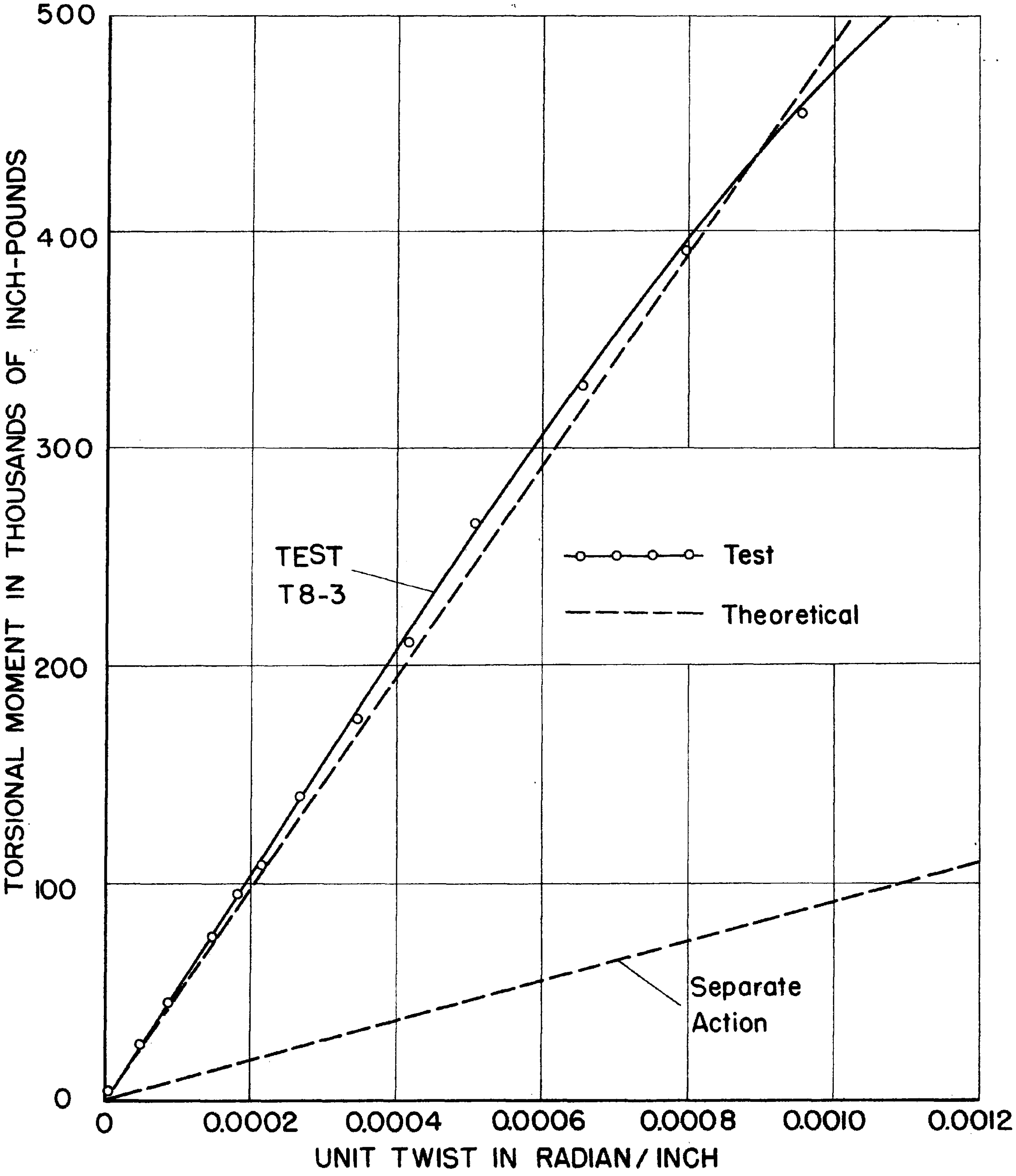


FIG. 36. TORQUE - TWIST CURVE FOR 50"X14" WELDED GIRDER

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