The Growth Of Cracks Due To Variations In Load.

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THE GROWTH OF CRACKS DUE TO VARIATIONS IN LOAD

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This dissertation is dedicated to Mr. Harold Hayden and Mr. Howard Smith, as engineers, in recognition of their unique ability to combine the best of engineering judgment and academic thought without a loss of sense of values or sense of humor.

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Abstract

The main objective of this work is to formulate an analysis of the growth of fatigue cracks employing crack-tip stress-intensity-factors (which are in essence the strength of elastic stress singularities). It is found that identical stress-intensity-factor time-histories at crack tips result in identical crack growth rates in a given material. This result allows the direct comparison of crack growth rates in bodies of a given material but with different configurations provided that the wave form of the load time-histories imposed are equivalent.

It is found that an empirical relationship expressing the proportionality of the crack growth rate to the fourth power of the amplitude of variation of stress-intensity-factors is in general agreement with the broad trend of experimental data on various materials.

Finally two models of crack growth based on considerations at a continuum level are devised. The results of the analyses of these models are shown to be more general than previous investigations and in better agreement with existing test results. However, the test data available to date are insufficient to examine the models in full detail and some inadequacies are suspected.

CHAPTER I - INTRODUCTORY CONSIDERATIONS

1.1 The Role of Crack Growth in Fatigue Phenomena:

The phenomenon of fatigue in general is well known to all structural engineers, and the "model" of fatigue usually employed for engineering purposes is simply that a given material will withstand a certain number of stress cycles of a particular intensity prior to complete failure. However, in the past decade research in the fatigue area has continuously increased attention to the study of fatigue as three separate phenomena. The three subdivisions of fatigue are:

(1) The initiation of cracks.

(2) The growth of cracks.

(3) The rapid propagation of cracks in the final load cycle (i.e. crack growth under static loading).

The first phase of fatigue, initiation, is associated with slip within the crystals of a metal. Perhaps, as many physical-metallurgists have proposed, this slip is actually dislocation movements which cause dislocations to pile up and form a crack.

The mechanism of microscopic crack formation is at least not yet fully understood.

The initiation of cracks, itself, is not within the scope of this dissertation. However, a question remains to be resolved inquiring as to when the first phase, initiation of cracks, ends and the second phase, crack growth, begins. Many investigators in the past have chosen the appearance of cracks visible to the naked eye as the criterion. Such a choice is arbitrary and does not depend on a change in the physical character of the phenomena involved. Furthermore, some recent studies which attempt to microscopically determine the appearance of cracks in polished specimens find that cracks most often appear in the first 5% of the fatigue life if not earlier. Thus crack growth is now quite widely accepted as comprising the major portion of fatigue life in carefully prepared laboratory specimens.

In typical structures which contain fabrication imperfections, such as tool marks or burrs on machined surfaces as well as exposure to corrosive environments, cracks are in effect always present in the structure or appear in the first few cycles of loading. A study by Quist (reported in Ref. 1) finds that in typical aircraft component tests over 90% of the failures are initiated by crack-like imperfections, or gouges caused by the rubbing and corrosion of contact surfaces of adjoining parts. In the type of component studied it must be concluded that from a conservative engineering point of view, the crack growth period comprises the whole of structural life.

Therefore an investigation of crack growth, which is the topic of this dissertation, is of considerable interest to the general understanding of fatigue, in addition to being a pertinent topic for other engineering reasons. For example, cracks are sometimes found during the inspection of structures, whereupon a judgement must be made on the remaining life of the structure. A knowledge of the characteristics of crack growth and final failure is clearly necessary and sufficient in such cases.

The propagation of cracks under static loading, i.e. the third and final phase of fatigue failure, has been thoroughly investigated by Irwin (Ref. 2) and others. In essence Irwin's analysis is the successful application of a macroscopic elastic continuum approach to predict the static strength of bodies containing cracks.

In this dissertation the method of analysis of crack growth under fluctuating loads will also employ in part the elastic continuum model in such a way that final failure in fatigue may be included as part of the crack growth phase. That is to say that phases two and three will be regarded as capable of being treated by similar analyses.

1.2 Initial Observations on the Growth of Cracks:

The growth of cracks is a highly discontinuous process. First, it is self evident that extension of a crack subjected to a cyclic loading occurs only during a portion of each load cycle under the most ideal conditions. Moreover, several experimenters observe that cracks often remain dormant for several cycles of loading and suddenly sustain an increment of extension during part of a cycle, which repeatedly occurs to form the full picture of crack growth (see for example Ref. 3).

In analyzing crack growth for its own sake, it would of course be important to observe the growth process in detail in order to obtain an accurate description. However, the final evaluation of accuracy here should be based on the overall objective of making life predictions for structural members. Consistent with that objective, it may be argued that a knowledge of the average rate of crack growth is sufficient. Therefore the approach employed will most often views crack growth as a continuous process. That assumption will require some attention to insure its justification. (But since the study of dynamics of falling bodies does not usually begin with a discourse on the molecular motion of particles within the body, the assumption of continuous growth of cracks does not seem out of place in mechanics).

1.3 Experimental Data Available on Crack Growth:

Only a few investigators have published quantitative data on crack growth rates to date (Refs. 4, 5, 6, 7, 8, and 9). All of their results have been compiled and arranged in convenient tabular form in a single source (Ref. 1). Since the data compiled in that source will be employed to guide and verify the analysis and results to follow, it seems appropriate to comment on that data in general at this point.

First, all the data available has been produced using a sinusoidal variation of load in time with a mean load superimposed. This is a severe restriction on the wave form of the loading. Moreover, the specimens tested were all wide plates with transverse cracks in order to observe the growth of the cracks directly on the surface of the plate. As will be noted later the use of plates in tests is not a serious restriction.

In all cases the crack's length has been measured periodically at intervals of several hundred to several thousand cycles during the tests, therefore only the average rate of growth over many cycles is available. (No data exists which extensively investigates the discontinuous nature of crack growth quantitatively). Even though the data is averaged in this way, many of the test results show considerable tendency to scatter, (i.e. the crack length vs. cycle number curve of the data is not smooth).

This scatter has often been attributed to the discontinuous nature of crack growth. However, the fact that some investigators have collected data with a small amount of scatter, indicates that test technique is the cause of some of the scatter. However, no tests remove all of the scatter, no matter how carefully performed. Perhaps a certain amount of natural scatter does exist due to the discontinuous growth of cracks and/or inhomogeneity of the material. But the fact remains that some of the experimental results are clearly superior to others for the purpose of analyzing crack growth.

For the above reason the data from various investigators has been subjected to preliminary examination (see also the data plots in Ref. 1) in order to determine the best sample for use herein. The result of this examination is that McEvily and Illg's data (Ref. 4) on 7075-T6 aluminum alloy is most appropriate. Henceforth it will be employed and supplemented with other data only when necessary to clarify a point.

Finally, for those who may be interested in collecting further data in the future, some comments on test technique are pertinent, though not necessary to this discussion. First, the rate of crack growth is rather sensitive to load level as may be seen from available data. Moreover, changes in load level during crack growth tests result in delays in the growth (Ref. 10). Hence it is imperative to maintain accurate load control in crack growth experiments. Second, it is difficult to determine the precise location of the tip of a crack during a test. Accurate means of measuring crack length must be given special attention to obtain suitable results. A few other factors such as misalignment of test jigs, buckling of the test panel, and carefully controlled atmospheric environment warrant additional careful consideration. However, many items which have received wide attention in the past, such as the frequency of loading, appear to be of secondary effect relative to other considerations.

1.4 The Classification of Analyses of Crack Growth Caused by Fluctuating Loads:

As mentioned earlier, the existing data on the growth of cracks has all been collected for loadings of sinusoidal wave form with a mean load present. Some, but only a few percent, of the structures in which fatigue is a problem are subjected to purely sinusoidal loadings of a stationary character. Other types of stationary loading may be encountered such as periodic loading of arbitrary wave form, stationary random loads, or superimposed combinations of these. Moreover, non-stationary loadings (i.e. wave forms whose character changes with time) are also often encountered with both slowly changing and abruptly changing loading character. Therefore some investigation of the effect of loadings which are other than sinusoidal and stationary seems appropriate. The analysis of crack propagation in this dissertation will be subdivided (for reasons which will become obvious later in the work) into three separate problems. They are:

1) The analysis of crack growth for a single type of stationary loading (of which sinusoidal loading is an example).

2) The comparison of crack growth rates between the various types of stationary loadings.

3) The analysis of crack growth for non-stationary loading.

Historically, attempts to date to analyze both crack growth and fatigue have been almost entirely restricted to the first of the above categories. The attacks which have been made on the second and third problems, often termed accumulation of damage studies, are at most naive and inadequate in relation to their importance in engineering. In the past these attacks did not employ the crack growth concept of behavior which may in part be the reason for their lack of success.

The approach in this dissertation will be confined to crack growth behavior and the concepts from its analysis which may provide a new model for fatigue. The contribution to be attempted here is to provide an analysis of crack propagation which will satisfy the first category of problems. In addition some theoretical results will be derived as an indication of possible approaches to the second and third categories.

CHAPTER II - PREVIOUS INVESTIGATIONS

2.1 The Form of Crack Propagation Laws:

In general a crack propagation law may be defined as an expression which relates the rate of increase in crack size to the intensity of the load, the wave form of the load (variation in time), the configuration of the body (including the crack and means of load application) and material constants. It is usual to consider that environmental factors such as temperature, corrosive atmosphere, etc. are reflected in a crack propagation law through alteration of the material constants. An assumption of this nature seems justified if the application of the law is to be restricted to materials which are metallurgically stable within the range of environment to be considered. Such a restriction is usually regarded as a necessity in structural design.

In order to begin development of a crack propagation law it is necessary for the sake of simplicity to place further restrictions on the initial objective. Therefore previous investigators have attempted to derive crack propagation laws for only the simplest conceivable special case. With few exceptions they consider a stationary sinusoidal stress variation, $\Delta\sigma$, with a mean stress,

σ_{mean}, superimposed and a special configuration which is an infinite plate of constant thickness with a crack of length, 2a, to which the stress is uniformly applied at infinity perpendicular to the direction of the crack. This configuration is shown in Figure 2.1.

Therefore, considering the number of cycles of variation in stress to be denoted by N, the form of all previous crack propagation laws may be expressed as:

$$\frac{da}{dN} = f(\Delta\sigma, \ \sigma_{mean}, \ a, \ C_i) \qquad 2.1$$

where C_i are material constants for a given material and environment.

This form will be used in order to compare existing crack propagation laws and examine their validity in the light of available data. It must be recalled during this discussion that a restricted form such as Eq. 2.1 is not the ultimate aim in developing a universally useful law but merely a sensible basis to begin the effort.

2.2. Head's Crack Propagation Law:

The first law which received wide attention was derived by Head (Ref. 11) in 1953. He also made some further comments on fatigue crack propagation in a later paper (Ref. 12). His work is regarded in the literature as the first fundamental effort to analyze crack propagation in a

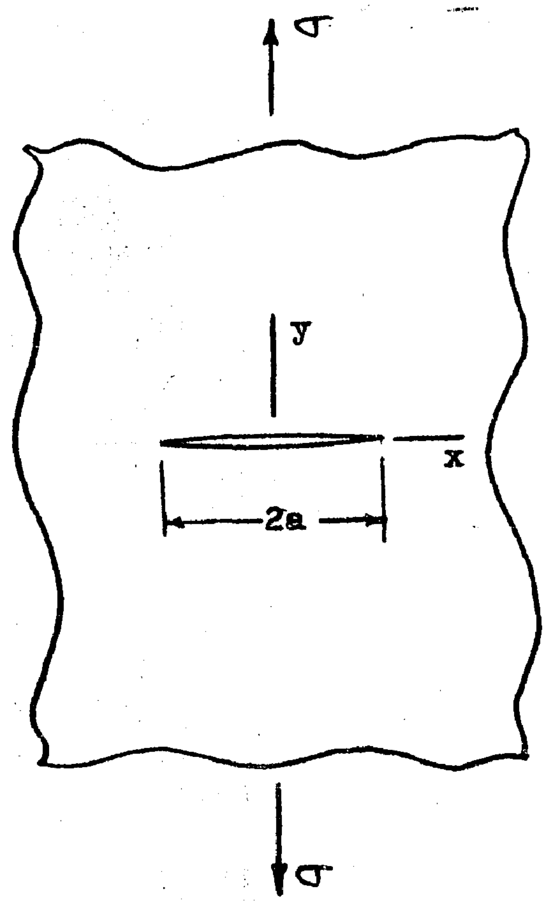

Fig. 2.1

An Infinite Sheet containing a Crack

deductive manner.

Head's approach centered around a model analysis of crack propagation in which an infinite sheet with a central crack (see Fig. 2.1) was regarded as a network of elements. The network is shown in part in Figure 2.2 In that figure elements A and C are purely elastic elements which transmit tension and shear respectively and are located everywhere in the sheet except directly ahead of the crack. Elements B are considered to be rigid-plastic elements with linear work hardening both in tension and compression and are located only along the line of extension of the crack. Further, it is presumed that elements B possess some constant fracture strength and the element directly ahead of the crack is work hardened up to that fracture stress level in order to extend the crack through that element, and so on through the next element.

Upon analyzing this network, which incidently requires considerable algebraic computation, and expressing the results of the analysis in a manner which restores the continuum concept, Head arrives at his law, which is:

$$\frac{da}{dN} = \frac{C_1 \sigma^3 a^{3/2}}{(C_2 - \sigma)\, w^{1/2}} \qquad 2.2$$

where C_1 depends upon the strain hardening modulus, the modulus of elasticity, the yield stress and the fracture stress of the material and C_2 is the yield strength of the

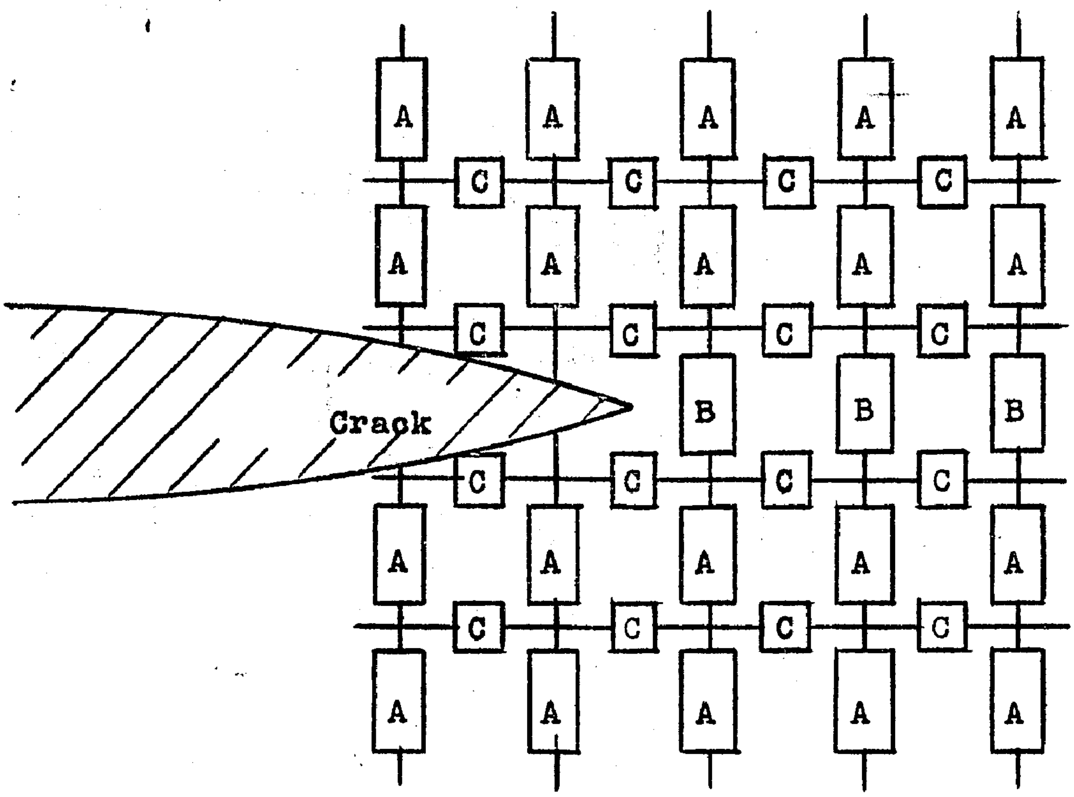

Fig. 2.2

Head's Model of a Crack in a Sheet

material. In addition Head defined w as the width of the zone of plasticity ahead of the crack, which he assumed would be a constant throughout the propagation of a crack subjected to a stationary sinusoidal loading. Moreover, he assumed that the loading would be one which has a mean load of half the double amplitude of the sinusoidal loading so that the minimum load is zero and the maximum is σ.

Interpreting Eq. 2.2 in the form of Eq. 2.1 requires substituting $\Delta\sigma$ for σ and the result is restricted to cases where σ_{mean} is equal to half of $\Delta\sigma$. Therefore Head's law may be written:

$$\frac{da}{dN} = \frac{C_1(\Delta\sigma)^3 \, a^{3/2}}{(C_2 - \Delta\sigma)\, w^{1/2}} \qquad 2.3$$

for

$$\sigma_{mean} = \frac{\Delta\sigma}{2}$$

(as shall be shown later the restriction on the value σ_{mean} in Eq. 2.3 may be ignored since mean stress has very little influence on the rate of crack propagation compared to variation in stress).

Head integrated Eq. 2.2 and obtained:

$$-\frac{1}{a^{1/2}} = D_1(\sigma)\, N + \text{const} \qquad 2.4$$

Noting that $D_1(\sigma)$ is a constant during a test employing a stationary loading, he suggests plotting the data from tests using coordinates of $1/\sqrt{a}$ and N in which case Eq. 2.4 predicts a straight line relationship for each individual test. He implied that obtaining a straight line would verify his law but this method of verification will be discussed and seriously opened to question in the last section of this chapter.

At this point it is sufficient to leave the discussion of Head's work and give some attention to that of his successors who in fact have noted some flaws in Head's arguments.

2.3 The Work of Frost and Dugdale:

Head's work remained unique and unquestioned in the field of derived relationships describing crack propagation until in 1958, Frost and Dugdale (Ref. 13) presented some evidence that questioned Head's assumption of a constant plastic zone size, w. They observed in some of their experiments, (using the configuration shown in Fig. 2.1 and stationary sinusoidal loading), that the plastic zone size in tests seemed to vary linearly with the crack length. That is:

$$w \approx a \qquad 2.5$$

They noted that substituting Eq. 2.5 into Eq. 2.2 or 2.3 results in a crack propagation rate which depends linearly on the crack length instead of the 3/2 power. Their experimental observations of plastic zone size were not convincing enough alone to allow a direct modification of Head's Law. However, they proceeded to obtain similar results employing other arguments.

Frost and Dugdale concluded that from dimensional considerations, referring to Figure 2.1, any increment of crack extension, da, will be proportional to the crack length, a, for a given incremental number of cycles, dN, during a particular test employing stationary loading. The result is:

$$\frac{da}{dN} = D_2 (\sigma) a \qquad 2.6$$

where $D_2 (\sigma)$ depends on the intensity of the stress, σ, and material constants.

Further, Frost and Dugdale found in their tests on aluminum alloys with a mean load equal to the amplitude of an imposed stationary sinusoidal load that the influence of the intensity of loading can be empirically approximated by:

$$D_2 (\sigma) = C_3 \sigma^3 \qquad 2.7$$

Substituting Eq. 2.7 into Eq. 2.6 leads to Frost and Dugdale's crack propagation law, i.e.

$$\frac{da}{dN} = C_3 \sigma^3 a \qquad 2.8$$

Eq. 2.8 may be interpreted in terms of stress variation as was done for Eq. 2.3, which results in the form suggested by Eq. 2.1 or

$$\frac{da}{dN} = C_3 \, (\Delta\sigma)^3 \quad a$$

for $$\sigma_{mean} = \frac{\Delta\sigma}{2} \qquad 2.9$$

Again it should be noted that the mean stress restriction is relatively unimportant.

Frost and Dugdale's law in its initial form, Eq. 2.6 may be integrated, which gives:

$$\ln a = D_2(\sigma) \quad N + \text{const.} \qquad 2.10$$

Observing this relationship, Eq. 2.10, they suggested plotting test results using the coordinates, ln a and N, and implied that if each of the tests plots as a straight line their relationship is verified. This implication will be discussed in the last section of this chapter.

2.4 Liu's Analysis of Crack Propagation:

Somewhat later Liu (Refs. 5 and 14) reviewed Frost and Dugdale's dimensional arguments and restated them in a much more elegant form. He argued on the basis of "homologous stress distributions" near the tip of a crack as it grows in a plate subjected to a stationary sinusoidal loading and derived the expression,

$$\frac{da}{dN} = D_3 \; (\sigma) \quad a \qquad 2.11$$

which is the same as Frost and Dugdale's results, Eq. 2.6. He did not determine the functional nature of D_3 (σ) at the time but experimentally noted its values for a variety of mean stress levels and intensities of stress variations in the aluminum alloy, 2024T-3.

In a later paper, (Ref. 15), Liu attempted to deduce the nature of $D_3(\sigma)$. First, he noticed that mean stress had a minor influence compared to the range of stress variation. Then from an argument based on the hypothesis of constant hysteresis energy absorption to failure, he concluded that:

$$D_3 \; (\sigma) = C_4 \; (\Delta\sigma)^2 \qquad 2.12$$

Finally, he again reopened the dimensional arguments and reasoned that the increment of crack extension, da, for a given number of cycles dN should be proportional to the plastic zone size, or

$$\frac{da}{dN} = C_5 \; w \qquad 2.13$$

Irwin (Ref. 16) had previously noted that for the configuration shown in Fig. 2.1, the plastic zone size for each reversal in loading may be estimated by:

$$w = \frac{(\Delta\sigma)^2 a}{2\ \sigma_{y.p.}^2} \qquad 2.14$$

if the mean load is neglected. Substituting this result into Eq. 2.13 Liu observed:

$$\frac{da}{dN} = C_5\ (\Delta\sigma)^2\ a \qquad 2.15$$

where C_5 has been readjusted. He noticed that Eq. 2.15 is the same as the combination of Eqs. 2.11 and 2.12 and concluded that this coincidence reinforces his arguments and provides a further interpretation of his earlier results.

Again, as in Frost and Dugdale's work, Liu's expressions may be integrated; they then become identical to Eq. 2.10. Liu also concludes that plotting ln a vs. N and obtaining straight lines for each test implies verification of his results. As mentioned with each of the previous laws the matter of verification will be discussed later.

2.5 A Note on the Validity of Dimensional Arguments:

In the two preceding sections it was noted that some previous investigations have employed a dimensional analysis of the configuration shown in Fig. 2.1 as a basis for crack propagation laws. Moreover, their results have been adopted by subsequent investigators without serious question (see for example Ref. 3). However, these dimensional arguments may be open to serious objections.

Both Eqs. 2.6 and 2.11 are the result of assuming that under a stationary loading the extension of a crack, da, is proportional to the crack length, a, for a given number of cycles, dN. But it is well known that in the final cycle of any crack propagation test an infinite extension, da, will occur even though the preceding crack length was finite (see for example Ref. 2). The dimensional arguments do not exclude the final cycle, even if, as stated by Liu, they are based on the concept that the rate of crack extension expected should be proportional to the plastic zone size. Therefore it is concluded here that the previously mentioned investigations still remain open to question.

2.6 NASA's Method of Crack Propagation Analysis:

Rather than attempt to fully determine the functional form of a crack propagation law, Eq. 2.1, McEvily and Illg (Ref. 4) noticed that much could be gained by determining the interrelationship of the arguments in Eq. 2.1. That is to say that it is possible to find more significant parameters which are combinations of the arguments, $\Delta\sigma$,

σ_{mean} and a, in Eq. 2.1. Since this will permit a decrease in the number of independent variables, it provides additional insight in understanding crack propagation.

McEvily and Illg began by adopting a previously known method of stress analysis of cracks devised to predict the static strength of plates containing cracks. A sharp crack results in a mathematically infinite stress at the crack tip if elastic analysis is employed. Therefore they assumed that real crack tips though sharp may be represented as having a finite tip radius, ρ , if elastic analysis is to be employed. The ficticious tip radius, ρ , is considered to be a material constant.

Then they regard the maximum stress in the vicinity of the crack tip as the most significant local stress and denote it by σ_o. In addition they reason that if the material element, at which the significant local stress, σ_o, occurs, is a linearly strain hardening element possessing a given fracture strength, then the crack growth rate should be a function of the mean value, σ_o mean, and the range of variation, $\Delta\sigma_o$, of this significant stress. By this means they observe that the rate of crack propagation should have the functional form:

$$\frac{da}{dN} = F_1(\Delta\sigma_o, \sigma_o \text{ mean}, c_i) \qquad 2.16$$

which has one less variable than Eq. 2.1.

They go on to note that for the configuration shown in Figure 2.1, with a crack of tip radius, ρ , the solution for elastic stresses gives:

$$\sigma_{max} = \sigma_o = (1 + 2\sqrt{\frac{a}{\rho}}) \qquad 2.17$$

Hence the significant local stress, σ_o, can be computed directly from the remote nominal stress, σ, and the crack length, a, provided that the material constant ρ is known. McEvily and Illg give experimentally determined values for ρ from about 0.001 to 0.005 inches for aluminum alloys and show that for a given stress ratio, i.e. in effect the ratio of $\Delta\sigma$ to σ_{mean}, the crack extension rate data for a particular material is correlated into a single curve on a $\Delta\sigma_o$ vs. $\frac{da}{dN}$ diagram. This amounts to observing experimental verification of Eq. 2.16. They also give an empirically chosen function which agrees quite well with the data they obtain, i.e.

$$\log_{10}\frac{da}{dN} = 0.00509\,\Delta\sigma_o - 5.472 - \frac{34}{\Delta\sigma_o - 34} \qquad 2.18$$

But this relationship is of little interest here due to its rather cumbersome form and empirical nature.

Most important in their work is the fact that they have shown that the local stress level near a crack tip is important in analyzing crack propagation phenomena. However, their assumptions of a fictitious crack tip radius ρ and a work hardening material possessing a local ultimate strength may be regarded as somewhat clouding the nature of their results.

It will be shown in a subsequent chapter that similar results may be deduced by other means without the need for such assumptions. Therefore a full discussion of the significance of local stress parameters will follow later in this work.

2.7 Other Pertinent Investigations on Fatigue Crack Propagation:

Some other investigators have attempted to relate the rate of fatigue crack growth to microscopic material parameters as well as macroscopic continuum variables. The most pertinent results have been obtained by Valluri (Ref. 3) and McClintock (Ref. 17).

Valluri combined many microscopic concepts such as dislocation movement stress, grain size effects, etc. with many of the macroscopic approaches such as making use of the concepts of strain hardening of materials, notch stress analysis, a modified Griffith theory of final failure, etc. into a complicated law of crack propagation involving some 10 parameters. However, one of his basic assumptions is a dimensional argument that the crack growth rate should be proportional to the crack length which is questioned herein. Moreover, his many sweeping and admittedly unverified assumptions in other phases of his derivations reduce his result to little better than speculation. Nevertheless, his results stand as a monumental effort to combine a great deal of the detailed knowledge of crack propagation into a single analysis, regardless of whether his assumptions are valid or not. It is at least worth mentioning here that his results are in quite wide disagreement with both the existing experimental data on crack propagation and many of the fundamental conclusions to follow in this work.

McClintock and his colleague Hult (Refs. 17, 18, and 19) have taken advantage of the fact that the plasticity problem of a crack subjected to shear parallel to its leading edge could be solved analytically. They provided the solution and have incorporated that result in a continued effort to synthesize fatigue crack growth. However, they have not been able to extend their results to the more important case of extensional loading perpendicular to cracks. Moreover, as will be shown, many basic relationships can be obtained without the benefit of an elastic-plastic analysis of stresses surrounding cracks. Hence their results remain unique and unincorporated in the work of other investigators at this time.

2.8 The Validity of Existing Crack Propagation Laws:

Head, Frost and Dugdale, and Liu (in Refs. 11, 12, 13, 14 and 15) have suggested that comparing their laws of crack propagation after integrating, Eqs. 2.4 and 2.10 with data from single tests under stationary sinusoidal loading will provide suitable verification. Though these laws, Eqs. 2.4 and 2.10, are in conflict with each other, this method of verification results in the claim that both are valid. Since the method of verification produces an essential contradiction, it is of importance to compare these laws with the experimental evidence in a more fundamental way (Refs. 20 and 21).

Rather than rely on the comparison of laws to individual test results, it seems more appropriate to compare the laws with the broad trend of a whole series of tests. The law which agrees with the broad trend has the obvious advantage of greater generality. The data provided by McEvily and Illg (Ref. 4), which has been chosen as the best available data in section 1.3, is most suitable for the purpose at hand.

First, this data has already been correlated successfully by McEvily and Illg themselves using their parameter, σ_o. Noting that the data was obtained for crack lengths, 2a, of the order of an inch long, whereas the crack tip radius, ρ , is of the order of one thousandth of an inch, the unity within the parenthesis in Eq. 2.17 may be neglected, or

$$\sigma_o = \frac{2}{\sqrt{\rho}} \sigma \sqrt{a} \qquad 2.19$$

Since ρ is presumably a material constant the parameter $\sigma \sqrt{a}$, is suggested by this result as parameter controlling the rate of crack extension for the configuration shown in Figure 2.1. (Later it will be noted by other considerations that this parameter is of further fundamental interest in describing the intensity of the crack tip stress field).

In the test series to be employed in this discussion the mean was always approximately half of the double amplitude of the superimposed sinusoidal variation in stress, or $\Delta\sigma$ may be substituted for σ as a measure of stress intensity. Consequently, the parameter, $\wp$, may be regarded as:

$$\wp = \Delta \sigma \sqrt{a} \qquad 2.20$$

One may then note that Liu's result, Eq. 2.15 may be rewritten as:

$$\frac{da}{dN} = C_5 \wp^2 \qquad 2.21$$

Note also that Head's result, Eq. 2.3, is nearly the same as Eq. 2.21 with the exponent changed to 3. Eq. 2.21 suggests that the data might well be plotted on a (log ρ) vs. (log $\frac{da}{dN}$) diagram which according to Liu or Head should result in a slope of 2 or 3. In order to eliminate finite width effects only the 12" wide plate tests of McEvily and Illg are shown plotted accordingly on Figure 2.3. The resulting slope is clearly close to 4 (not 2 or 3). The slope of 4 indicates that the broad trend of the data is followed by the relationship

$$\frac{da}{dN} = C_6 \rho^4 = C_6 (\Delta\sigma)^4 a^2 \qquad 2.22$$

which might be suggested as a new empirical crack propagation law for the configuration shown in Figure 2.1. However, a more general law of a similar nature will be proposed later, as the result of a less empirical approach.

At any rate, Figure 2.3, and its apparent trend, Eq. 2.22 show that the laws proposed by Head, Frost and Dugdale, and Liu do not seem to agree with the data trend. Moreover, the trap into which these previous investigators have fallen is clear on further inspection of Figure 2.3. Any single test covers less than a log-cycle of crack extension rates, $\frac{da}{dN}$, on that figure. It is evident that the slope of the general trend of data cannot be accurately determined from such a short range of data. Therefore the use of data from single tests to verify their laws is in error.

In conclusion it has been shown in this chapter that the laws of crack propagation derived by previous investigators are not rigorously founded. They are also shown to disagree with the broad trend of the best available data. The only method which remains unquestioned is that of employing local crack tip stress parameters in analyzing crack propagation. However, the methods employed by McEvily and Illg contain some doubtful assumptions though they sucessfully describe the functional form of the data.

Therefore, the way is cleared for further analyses of crack propagation in which local stress parameters play an important role. But it is mandatory that few assumptions be made that cloud the issues if the analysis is to be fruitful in providing some insight into crack propagation behavior in general for a wide variety of configurations and types of load-time histories. For that reason the discussion to follow will begin with a fundamental description of local crack tip stress fields and from that description an attempt will be made to derive an analysis of crack propagation of broad applicability.

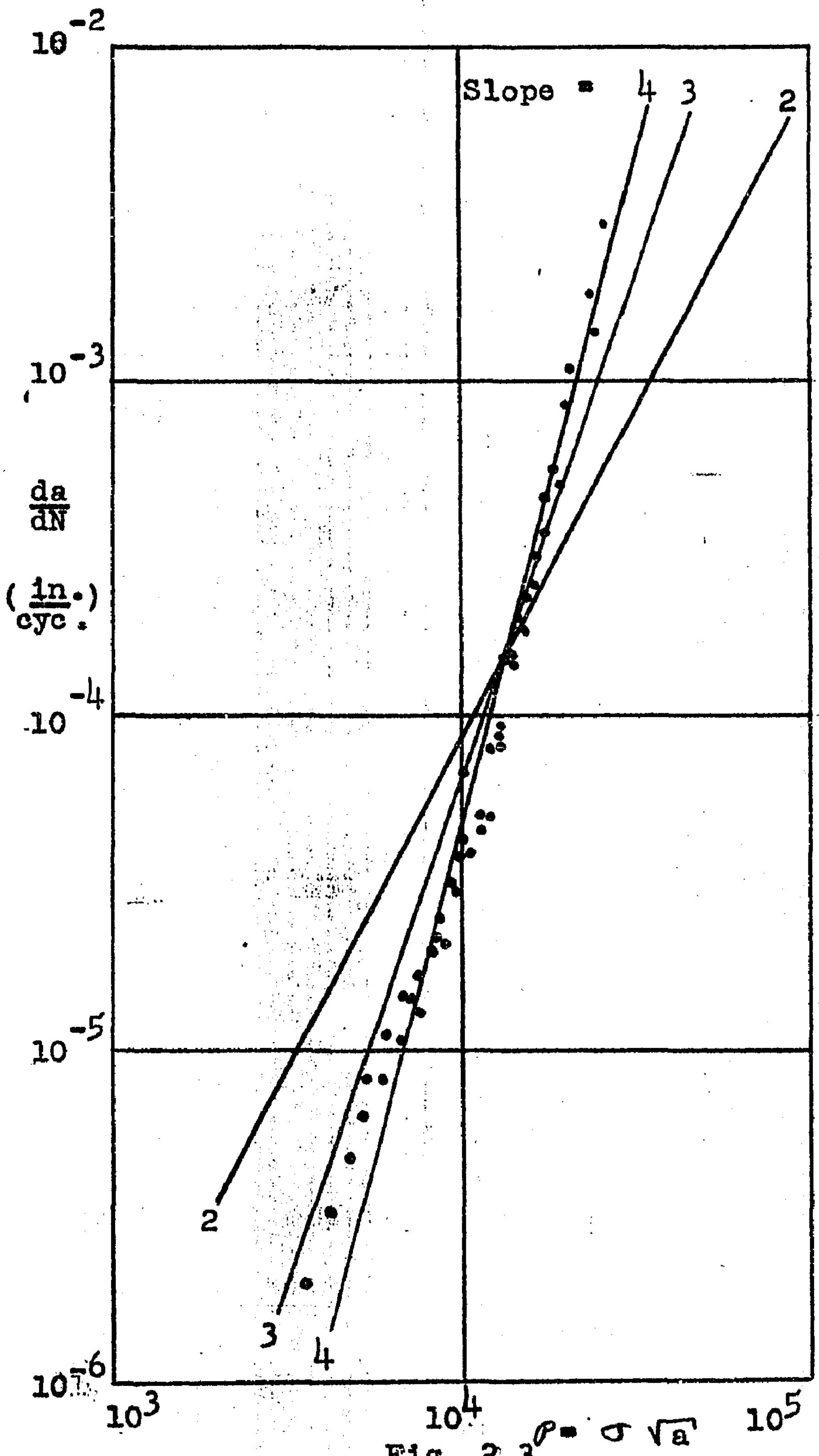

Fig. 2.3

Crack Growth Rate Data on 7075 T6 Aluminum Alloy from Wide Plate Tests

CHAPTER III - THE STRESS ANALYSIS OF CRACKS

3.1 The Stress-Intensity-Factor Concept:

It is informative to consider the boundary value problem of the extension of a plate containing a through-the-thickness crack. This problem has been analyzed elastically by both Irwin (Ref. 22) and Williams (Ref. 23). Some similar results of a less general nature were presented earlier by Sneddon (Ref. 24). The method and notation of Irwin will be adopted here with no loss in generality.

The presence of the adjacent crack surfaces which are free from stress has a dominating influence on the distribution of stress near a crack tip. The effect of that influence becomes clear through the following analysis.

Let the tip of a semi-infinite crack be located at the origin as shown in Figure 3.1. Presuming that the stress distribution will be symmetric with respect to the x-axis for the present allows the use of Westergaard's elastic stress analysis (Ref. 25). Westergaard defines an Airy's stress function by

$$\phi = \mathrm{Re}\,\bar{\bar{Z}} + y\,\mathrm{Im}\,\bar{Z} \qquad 3.1$$

As a consequence the stresses may be computed from

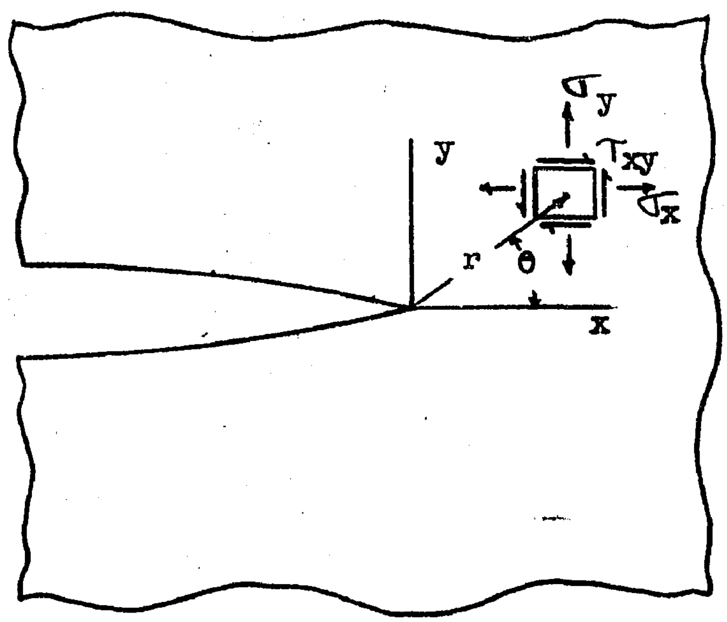

Fig. 3.1

Coordinate and Stress Component Notation Near a Crack Tip

$$\sigma_y = \text{Re } Z + y \text{ Im } Z'$$

$$\sigma_x = \text{Re } Z - y \text{ Im } Z' \qquad 3.2$$

$$\tau_{xy} = - y \text{ Re } Z'$$

where

$$\bar{Z} = \frac{d\bar{\bar{Z}}}{d\zeta}, \qquad Z = \frac{d\bar{Z}}{d\zeta}, \qquad Z' = \frac{dZ}{d\zeta}$$

and

$$\zeta = x + i y = re^{i\theta}$$

Any stress function, Z, which is sectionally holomorphic statisfies equilibrium and compatibility, since ϕ will be biharmonic, so that chosing a stress function which satisfies the boundary conditions is the only necessary condition remaining. Irwin (Ref. 2) noticed that a stress function of the form,

$$Z = \frac{g(\zeta)}{\sqrt{2\zeta}} \qquad 3.3$$

where $g(\zeta)$ is well behaved everywhere near the origin will satisfy the boundary conditions on the crack surface provided that

$$\text{Im } g(x) = 0 \qquad (x < 0) \qquad 3.4$$

Hence, in the region of the origin $g(\zeta)$ can be approximated by a real number K_1 or in the region near the crack tip the stress function is:

$$Z = \frac{K_1}{\sqrt{2z}} \qquad 3.5$$

Substituting Eq. 3.5 into Eqs. 3.2 gives the crack tip stress distribution, which is:

$$\sigma_y = \frac{K_1}{\sqrt{2r}} \cos\frac{\theta}{2} \left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\sigma_x = \frac{K_1}{\sqrt{2r}} \cos\frac{\theta}{2} \left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \qquad 3.6$$

$$\tau_{xy} = \frac{K_1}{\sqrt{2r}} \sin\frac{\theta}{2} \cos\frac{\theta}{2} \cos\frac{3\theta}{2}$$

This local solution for the stresses near a crack tip in symmetric problems is valid regardless of the length of crack and other boundary conditions of the problem provided that the crack surfaces are free from stress adjacent to the crack tip and no further discontinuities in loading or configuration occur in that vicinity. The parameter, K_1, namely the stress-intensity-factor, reflects the influence of the other boundary conditions on the intensity of the stress distribution at the crack tip. In other works K_1 is a local stress parameter which reflects the stress intensity at all points surrounding the crack tip.

Stress-intensity-factors may be defined for problems other than the extension of plates which are symmetric with respect to the crack (Refs. 2 & 26). However, the majority of fatigue cracks seem to tend to form and propagate on planes of symmetry (at least local symmetry) in bodies. For example, in a bar subjected to tension cracks invariably form and grow perpendicular to the tension direction. In such a case it is readily observed that the crack tip stress field is symmetrical. Similar observations may be made in more complicated situations. Therefore the attention here shall be directed toward the analysis of crack growth where the crack tip stress field is characterized by Eqs. 3.6.

3.2 The Determination of Stress-Intensity-Factors for Particular Configurations:

Basically, the stress-intensity-factor may be determined from the stress solution to any particular boundary value problem by expanding the stresses in the vicinity of the crack tip and comparing the result with Eqs. 3.6. For example, the Westergaard stress function for the configuration shown in Figure 2.1 is (see Ref. 25):

$$Z = \frac{\sigma \zeta}{\sqrt{\zeta^2 - a^2}} \qquad 3.7$$

On the line ahead of the crack, (i.e. the x-axis or $\theta = 0$),
the stress component σ_y is:

$$\sigma_y = \text{Re } Z + y \text{ Im } Z' = \text{Re } Z$$

where $$\zeta = a + r \qquad 3.8$$

which when combined with Eq. 3.7 leads to:

$$\sigma_y = \frac{\sigma\sqrt{a}}{\sqrt{2r}} \qquad 3.9$$

upon neglecting terms of the order of r/a compared to 1. Comparing Eq. 3.9 to Eq. 3.6 with θ set equal to zero results in

$$K_1 = \sigma\sqrt{a} \qquad 3.10$$

This result is analogous to the parameter, ρ, defined in Eq. 2.20 and used in plotting Fig. 2.3. Thus its significance may already be anticipated.

Other direct and indirect methods have been developed for determining stress-intensity-factors (see Refs. 26 and 27). It suffices to add that the stress-intensity-factors for over 60 configurations have been computed covering a variety of problems involving extension, bending, and torsion of plates and three-dimensional bodies, as well as some limited results on thermal stress and curved cracks.

Many of these stress-intensity-factors are tabulated in Ref. 28.

Though stress-intensity-factors will be used throughout the discussion to follow, it is not the purpose of this dissertation to present methods of computing them. Subsequently, the stress-intensity-factors for configurations of interest will be simply stated as results taken from the literature.

3.3 The Plastic Zone at the Tip of a Crack:

The preceding sections in this chapter discuss the elastic stress analysis of cracks. In Eqs. 3.6 it may be noticed that the elastic analysis predicts that the stresses at the crack tip (i.e. $r \rightarrow 0$) tend toward infinity. In reality infinite stresses cannot occur and metals respond to this tendency toward high stress by yielding in the immediate vicinity of the crack tip. It is of interest to discuss the extent of this zone of plastic deformation in order to evaluate its consequences in disturbing the elastic prediction of the crack tip stress field.

A first approximation of the location of the elastic-plastic interface may be obtained by using the elastic stress field equations, Eqs. 3.6. Presuming in addition that the stress state will be that of plane stress, i.e. $\sigma_z = \tau_{xz} = \tau_{yz} = 0$ everywhere, Eqs. 3.6 may be substituted into a suitable yield criterion which will result in an equation for the contour of the interface. Choosing von Mise's criterion that result is:

$$\frac{r_o}{w} = \cos^2 \frac{\theta}{2} \left[1 + 3 \sin^2 \frac{\theta}{2} \right] \qquad 3.11$$

where

$$w = \frac{K_1^{\,2}}{2\,\sigma_{y.p.}^{\,2}} \qquad 3.12$$

Now it may be noted that the parameter, w, is the width of the predicted plastic zone directly ahead of the crack. Thus, w, is not only an estimate of the extent of plasticity along the line of propagation of the crack, but is as well representative of the relative size of the whole plastic zone.

For the particular configuration shown in Figure 2.1 the stress intensity factor was given in Eq. 3.10. Substituting Eq. 3.10 into 3.12 results in

$$w = \frac{\sigma^2 \; a}{2\,\sigma_{y.p.}^{\,2}} \qquad 3.13$$

An analogous result was stated previously in Eq. 2.14 which is now clarified by the derivation of Eq. 3.13.

Stimson and Eaton (Ref. 29) have numerically solved a similar problem for the contour of the elastic-plastic interface using the methods of theory of plasticity. Both the size and shape of their contours agree within a few percent with Eqs. 3.11 and 3.13 for stress levels, σ, of 2/3 the yield stress, $\sigma_{y.p.}$, or less. Hence, Eqs. 3.11, 3.12 and 3.13 may be regarded as a suitable estimate of the plastic zone size for the stress levels ordinarily of interest.

Moreover, in fatigue, work hardening at the crack tip will tend to improve the assumption of using the elastic stress distribution to locate the contour of the interface. Hence, the preceding results, Eqs. 3.11 and 3.12 will be employed in analyzing fatigue crack propagation without further hesitation.

Finally, Eq. 3.13 implies that the width of the plastic zone ahead of a crack is small compared to the crack length for ordinary stress levels. Hence, it may be concluded for the configuration shown in Figure 2.1 that the plastic zone at the crack tip is ordinarily so small that it will hardly disturb the distribution of stresses in a plate with a crack. Again, the cyclic nature of the loading in fatigue will cause work hardening which improves the accuracy of employing the elastic distribution of stress given by Eqs. 3.6 in the region surrounding the tip of the crack but outside the plastic zone, since work hardening subsequently in-creases the yield stress of the material, etc.

For the above reasons it seems justifiable to retain the stress-intensity-factor concept even though a small yield zone exists near the crack tip. For such a case the stress-intensity-factor may be regarded as a local stress field parameter which describes the intensity of stresses in the region surrounding the plastic zone at a crack tip.

3.4 An Interpretation of the Griffith-Irwin Theory of Static Failure:

The validity of making use of an elastic parameter, the stress-intensity-factor, in situations where a small amount of yielding exists near the crack tip has been argued on semi-intuitive grounds. If these arguments do not seem acceptable on that basis, their results may be subject to experimental verification in a direct fashion.

The Griffith-Irwin theory of static failure of bodies containing cracks has experimentally proven to be successful (Ref. 2). Since this theory may be interpreted in terms of direct application of stress intensity factor concepts, its success may be regarded as at least partial validation of the arguments in the preceding sections of this chapter. Therefore a brief discussion of the Griffith-Irwin theory will be presented, in which only those points which add to the purpose of this dissertation will be considered.

The Griffith-Irwin theory may be stated in the following way:

When the magnitude of the stress-intensity-factor for a crack tip in a given material reaches a certain critical value, rapid crack extension commences.

Though some attention must be paid to the effect of plate thickness on the crack tip stress state, i.e. plane stress vs. plane strain, the above statement has been extensively verified experimentally (Ref. 27). Moreover, it has also been shown to apply to the type of stress intensity factors associated with bending of plates (Refs. 26 and 30).

The verification of the usefulness of stress-intensity-factors in predicting static failure, including the failure in the last cycle of fatigue, not only adds to the expectatation of their applicability to fatigue crack propagation phenomena, but makes it plausible to treat the second and third phase of fatigue, as outlined in section 1.1, by a single method. However, in applying these concepts to fatigue crack growth caused by fluctuating loads, final acceptance of the justification of that application should rest on direct experimental evidence.

3.5. A Summary of Introductory Discussions:

The first three chapters have cleared the way for meeting the primary objective of this work. The importance of the role of an analysis of crack growth in understanding fatigue phenomena has been pointed out. The need for a method which may be applied to various configurations and types of load-time histories has been discussed. In addition, the shortcomings of previous investigations have been reviewed and the advantages of introducing a fundamental crack tip stress parameter clarified.

Finally, the concept of the crack tip stress-intensity-factor was introduced. The use of an elastic parameter of this type in cases where some limited plasticity occurs near a crack tip was considered. It was concluded that the application of stress-intensity-factors to the analysis of cracking phenomena has enjoyed some success in the past and it appears that further application to fatigue crack growth may be fruitful.

Therefore, this discussion shall proceed at once to introduce stress-intensity-factor concepts in an analysis of crack growth phenomena. The objective of this analysis will be to attempt to develop useful quantitative methods for determining the rate of crack growth in structures.

CHAPTER IV - CRACK PROPAGATION CAUSED BY A STATIONARY LOADING

4.1 The Time History of the Stress-Intensity-Factor at a Crack Tip:

The stress-intensity-factor for a particular configuration will depend upon the points of application and magnetude of the loads and the length of the crack as noted in Chapter III. Moreover, referring to Eqs. 3.6, stress-intensity-factors are linear factors in the equations for the linear elastic stress distribution near a crack tip. Since the elastic stresses must be directly proportional to the applied load, the stress-intensity-factors must contain the load as a linear factor, i.e.

$$K = P \cdot f(a) \qquad 4.1$$

where P denotes load and f(a) represents the dependency on crack length.

Now, even if the load-time history is stationary, the stress-intensity-factor time history will not be stationary in general, since the crack length will change with time. A question arises as to how the non-stationary quality of K will affect rates of crack growth.

Let $\{P\}$ be a parameter which describes the magnitude of the load-time history applied to a body; for example, it could be regarded as the amplitude of a sinusoidal loading or the root mean square value of a random loading or a similar general property describing the magnitude of a load-time history.

Moreover, let $\{K\}$ denote a corresponding parameter describing the magnitude of the time history of the stress-intensity-factor. Provided that f(a) is a "slowly changing" function (which will be shown later to be the case), Eq. 4.1 may be applied to relate these parameters by regarding the crack length or f(a) as for practical purposes constant for an interval of time during which the relationship is:

$$\{K\} = \{P\} \quad f(a) \qquad 4.2$$

Then the time rate of change of the parameter of K is:

$$\frac{d \{K\}}{dt} = \frac{\partial \{K\}}{\partial \{P\}} \cdot \frac{d \{P\}}{dt} + \frac{\partial \{K\}}{\partial a} \cdot \frac{da}{dt} \qquad 4.3$$

If the loading also happens to be periodic* then cycle number, N, may be substituted for time, t, or:

---

* A periodic loading is denoted here as one which may be slowly amplified in time so that $\frac{d \{P\}}{dN}$ is not necessarily zero.

$$\frac{d\{K\}}{dN} = \frac{\partial\{K\}}{\partial\{P\}} \cdot \frac{d\{P\}}{dN} + \frac{\partial\{K\}}{\partial a} \cdot \frac{da}{dN} \qquad 4.4$$

But it is not sufficient to merely compute the rate of change of the parameter $\{K\}$. Its importance relative to other aspects of crack propagation must be included.

The effects of the load-time history on the points in a body through which the crack will pass are of primary importance. Moreover, only those points which are within the plastic zone ahead of the crack will be accumulating the effects of the load-time history. Therefore it is of interest to evaluate the change of the parameter $\{K\}$ during the time interval in which a particular material point suffers damage, i.e. the time it takes for the crack to traverse one plastic zone width, w. Moreover, it is significant to compute the ratio, R, of the change in the parameter to the size of the parameter itself in order to obtain a measure of the relative rate of change of the properties stress-intensity-factor time-history as it affects crack growth. Therefore it is of interest to compute:

$$R = \frac{\{K\}\big|_{a+w} - \{K\}\big|_{a}}{\{K\}\big|_{a}} = \frac{1}{\{K\}}\left[\frac{d\{K\}}{d a} \cdot w + \ldots\right] \qquad 4.5$$

where the second order terms in the expansion shall be ignored. Note that:

$$\frac{d\{K\}}{da} = \frac{\frac{d\{K\}}{dt}}{\frac{da}{dt}} = \frac{\frac{d\{K\}}{dN}}{\frac{da}{dN}} \qquad 4.6$$

Substituting Eqs. 4.6, 4.3 and 4.4 into 4.5 results in:

$$R = \frac{w}{\{K\}} \cdot \frac{1}{\frac{da}{dt}} \left[\frac{\partial\{K\}}{\partial\{P\}} \cdot \frac{d\{P\}}{dt} + \frac{\partial\{K\}}{\partial a} \frac{da}{dt} \right]$$

or

$$= \frac{w}{\{K\}} \cdot \frac{1}{\frac{da}{dN}} \left[\frac{\partial\{K\}}{\partial\{P\}} \cdot \frac{d\{P\}}{dN} + \frac{\partial\{K\}}{\partial a} \cdot \frac{da}{dN} \right] \qquad 4.7$$

Rearranging and substituting derivatives of Eq. 4.2 leads to:

$$R = \frac{\frac{1}{\{P\}} \cdot \frac{d\{P\}}{dt}}{\frac{1}{w} \cdot \frac{da}{dt}} + \frac{w}{\{K\}} \cdot \frac{\partial\{K\}}{\partial a} \qquad 4.8$$

or for periodic loading:

$$R = \frac{\frac{1}{\{P\}} \frac{d\{P\}}{dN}}{\frac{1}{w} \frac{da}{dN}} + \frac{w}{\{K\}} \cdot \frac{\partial\{K\}}{\partial a} \qquad 4.9$$

Eqs. 4.8 and 4.9 allow evaluation of the relative rate at which parameters of the time-histories of stress-intensity-factors are changing. It remains to determine the magnitude of R which causes the non-stationary character of the stress-intensity-factor to affect the rate of crack propagation. Experimental evidence is required for that determination.

4.2 Evaluation of the Effect of Non-Stationary Stress-Intensity-Factor Time Histories:

Since experimental data are available only for stationary sinusoidal loading, it will be used in an attempt to evaluate the effect of the non-stationary quality of stress-intensity-factors as measured by R. Moreover, data taken in experiments incorporating the configuration shown in Figure 2.1 will be used.

Let the maximum value of the stress intensity factor, K_M, during a cycle of loading, be regarded as the parameter $\{K\}$, of interest. For the configuration chosen, see Eq. 3.10,

$$\{K\} = K_M = \sigma_M \sqrt{a} \qquad 4.10$$

The plastic zone size also depends on the maximum value of the applied stress, therefore from Eq. 3.12

$$w = \frac{\sigma_M^2 \; a}{2 \, \sigma_{y.p.}^2} \qquad 4.11$$

Since the loading itself is stationary

$$\frac{d \{P\}}{dN} = 0 \qquad 4.12$$

Substituting these results, Eqs. 4.10, 4.11 and 4.12 into Eq. 4.9 gives:

$$R = \frac{1}{4}\left[\frac{\sigma_M}{\sigma_{y.p.}}\right]^2 \qquad 4.13$$

Previous investigators have ignored the non-stationary character of the time history of the crack tip stress field in analyzing their experimental data. For example the data shown on Fig. 2.3 correlate nicely even though σ_M covers a range of values from 0.13 $\sigma_{y.p.}$ to 0.64 $\sigma_{y.p.}$. Other data on various materials have also been correlated in a similar way for stress levels, σ_M, up to the yield point, $\sigma_{y.p.}$, itself. Therefore, this positive correlation of data with no apparent effect of the non-stationary character of the stress-intensity-factors shows that up to R = 1/4 (and perhaps higher) non-stationary effects may be neglected.

The preceding result implies that for stationary loading the time history of the stress-intensity-factor at a crack tip may be regarded as stationary, i.e. characterized by its quasi-stationary properties, in analyzing crack propagation. That is to say that the properties of the stress-intensity-factor time history which are significant are those computed from the load time history regarding the crack length as momentarily constant. Furthermore, non-stationary loadings whose parameters are slowly and continuously changing with time may be regarded as quasi-stationary provided that the resulting R, as computed by Eqs. 4.8 or 4.9, is not greater than 1/4. These results simplify the analysis of crack propagation.

4.3 A Hypothesis for Analyzing Crack Propagation Caused by any Particular Stationary Loading:

In the preceding section it was shown that the time history of the stress-intensity-factor at a crack tip may be regarded as quasi-stationary if the loading itself is stationary. This result implies that the significant properties of the K-time history as they effect crack growth may be regarded as those which occur if the crack length is momentarily held constant. Adopting a quasi-stationary view point from Eq. 4.2 it may be noted that the parameters of the K-time history are the same as those of the load time history except for a modification in magnitude. (It is assumed that the rate of load application is slow enough so that dynamic effects, i.e. stress waves, may be ignored. That is to say that the stress analysis problem is quasi-static).

Now if two bodies are subjected to identical load-time histories except for a difference in magnitude, then the stress-intensity-factors for cracks within those bodies will have identical quasi-stationary time histories except for another difference in magnitude which depends upon f(a) in each. As the crack lengths in these bodies change, the quasi-stationary time histories of K change in magnitude. Therefore, there may be comparable situations existing for these cracks where for a given crack length in one of the bodies, there will be some crack length in the other for which the quasi-stationary time histories of K will be the same in magnitude as well as other characteristics. The following hypothesis is presented as a means of comparing crack extension rates for such situations (Ref. 31).

> If the quasi-stationary time histories of the stress-intensity-factors at two crack tips are the same in a given material, then the crack propagation rates will be the same.

This hypothesis will form the basis for treating crack propagation caused by any particular stationary load-time history (Ref. 32). But first it should be investigated and justified (Ref. 33) in the light of existing experimental evidence.

4.4 Verification of the Hypothesis for the Case of Sinusoidal Loading:

If a sinusoidal loading is imposed on a body, then the quasi-stationary (crack length considered constant) K-time history is also sinusoidal. The character of the sinusoidal K-time curve may be described by the frequency and two other parameters. For the present, frequency shall be ignored and the maximum value of the stress-intensity-factor, K_M, and the ratio, β, of the maximum to minimum values of K shall be employed as the other parameters.

Referring to Eq. 4.1, it is observed that the ratio, β, is also the load ratio or

$$\beta = \frac{K_M}{K_{min.}} = \frac{P_M}{P_{min.}} \qquad 4.14$$

During a test with stationary sinusoidal loading, the load ratio remains constant, hence β is constant. However as the test proceeds the crack grows and K_M changes its value.

Therefore the hypothesis stated in the preceding section implies that in a given material subjected to sinusoidal loading, crack tips which experience the same values of K_M and β are experiencing the same quasi-stationary K-time history and as a consequence should be growing at the same rate. In other words data from tests on a given material which employ the same β are expected to form a single curve (or scatter band) on a K_M vs $\frac{da}{dN}$ diagram. Obtaining a single curve amounts to verification of the hypothesis provided that the data cover an adequate range of extension rates, $\frac{da}{dN}$ to establish a broad trend.

The data of McEvily and Illg (Ref. 7) on 7075-T6 aluminum alloy are sufficient to establish such a trend, (as previously noted in sections 1.3 and 2.8). Figure 4.1 shows the type of configuration they used in their tests. The stress-intensity-factor for this configuration, including a correction factor for finite width is (see Ref. 22, 27, or 28):

$$K_1 = \sigma\sqrt{a} \cdot \sqrt{\frac{2b}{\pi a} \tan\frac{\pi a}{2b}} \qquad 4.15$$

Figure 4.2 shows their data for both 2 inch and 12 inch wide panels plotted as suggested in the preceding paragraph. It is noted that the correlation into a single curve is quite definite, but further and more striking evidence is available to verify the aforementioned hypothesis.

It is conceivable that the observed correlation on Figure 4.2 may be due to the special configuration employed (Fig. 4.1). However, some additional test results are available (see Ref. 1) for a configuration with widely different properties. This alternative configuration is shown in Figure 4.3 and its stress-intensity-factor including a finite width correction is (see Ref. 2, 22, or 28):

$$K_1 = \frac{P}{\pi\sqrt{a}} \cdot \frac{1}{\sqrt{\frac{b}{\pi a} \sin\frac{\pi a}{b}}} \qquad 4.16$$

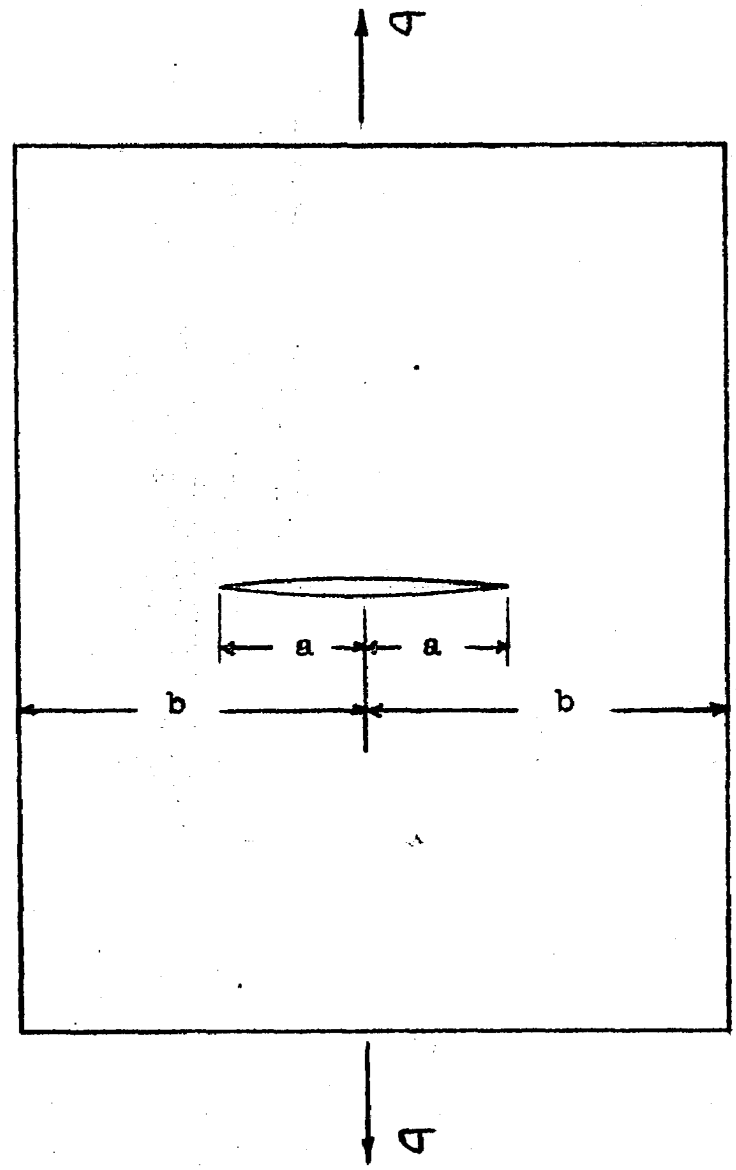

Fig. 4.1

A Finite Width Sheet
containing a Crack

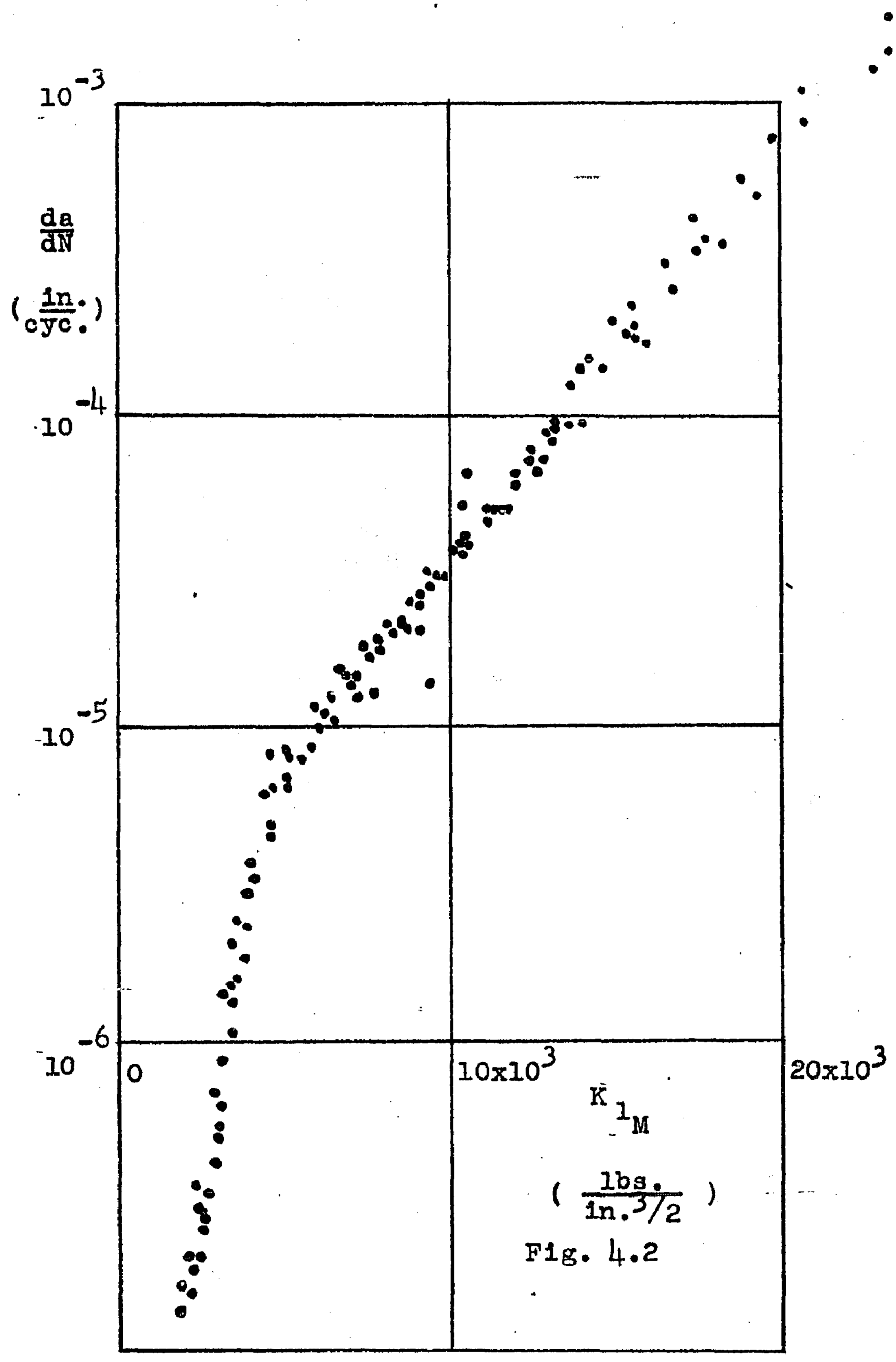

Fig. 4.2

Correlation of McEvilly and Illg's Data on 7075 T6 Aluminum Alloy using Stress-Intensity-Factors

From Eq. 4.16 it may be noted that the stress-intensity-factor tends to decrease as the crack length increases, contrary to the behavior of the previous configuration (Fig. 4.1 and Eq. 4.15).

Figure 4.4 shows the data on 7075-T6 aluminum alloy from this alternative type of test configuration with the scatter band from Figure 4.2 indicated by the solid lines. Again, the correlation into a single curve of data from tests using stationary sinusoidal loading of a given, β , is observed. The wide difference in the characteristics of the configurations and the broad range of data employed shows that without a doubt the correlation implies that the hypothesis stated in the previous section is a reasonable description of the role of stress-intensity-factors in crack propagation phenomena.

References 1, 21, 32, 33 and 34 contain some further discussion of the above technique of data correlation and include similar plots (using the configuration of Fig. 4.1) for some other metal alloys. However, Figures 4.2 and 4.4 have served the immediate purpose; they have verified the hypothesis.

4.5 A Further Note on the Discontinuous Nature of Crack Growth:

In section 1.2 it was noted that crack growth is a

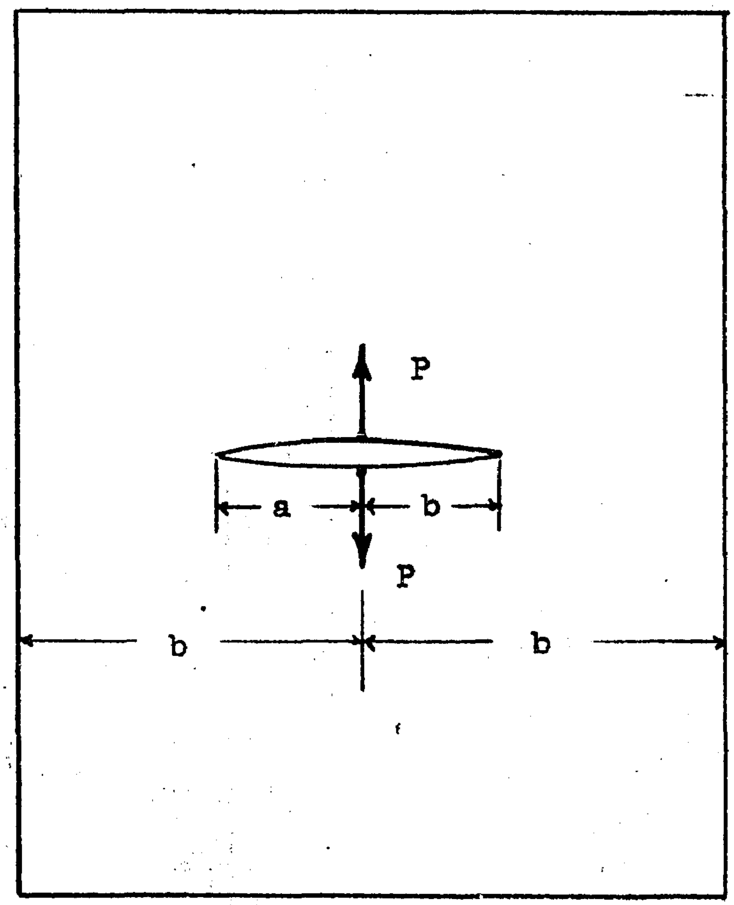

Fig. 4.3

A Wedge Force Test Configuration
for Finite Width Sheets

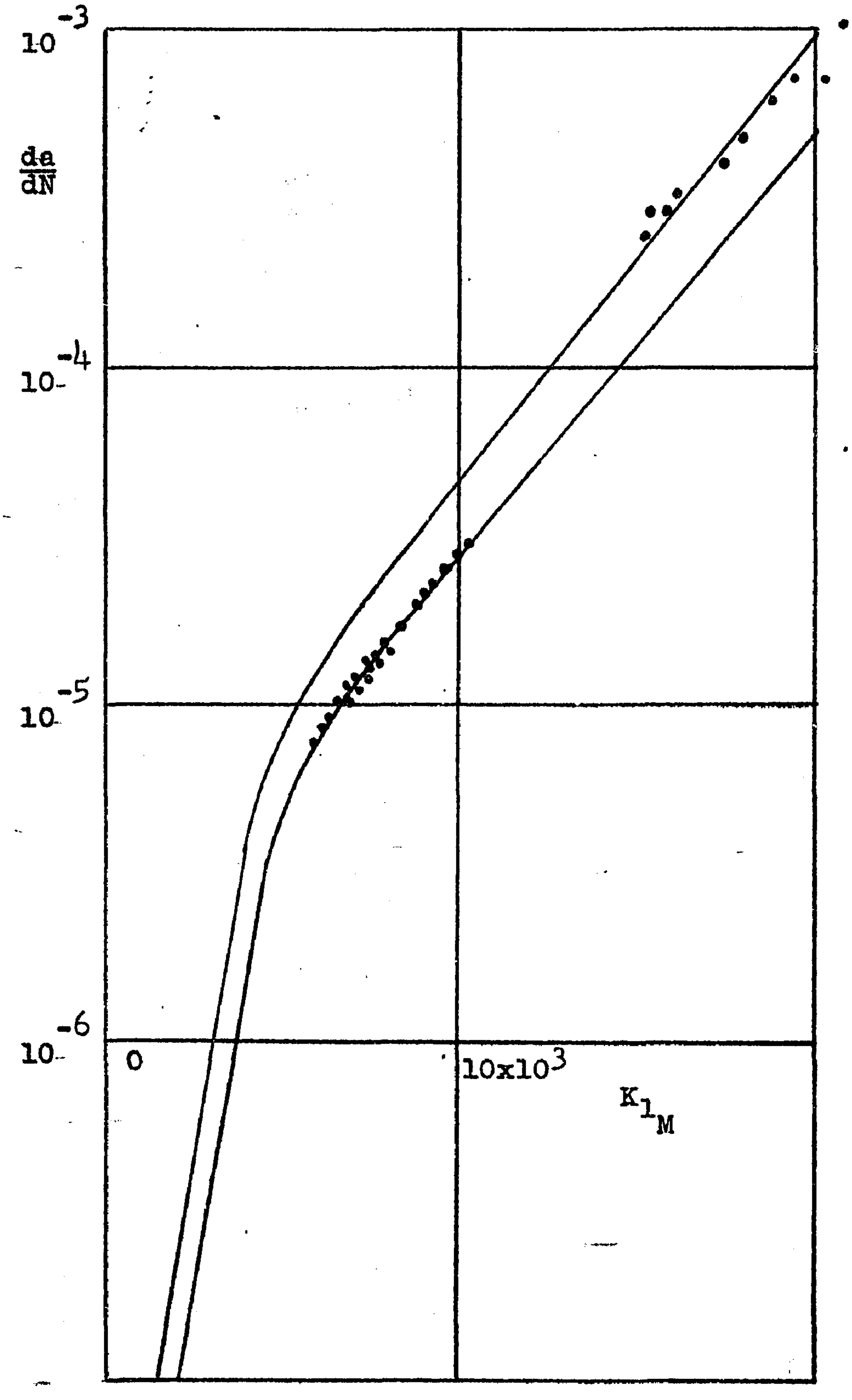

Fig. 4.4

A Comparison of Wedge Force Test Data with McEvilly and Illg's Scatter Band for 7075 T6 Aluminum Alloy discontinuous phenomenon on a micro-level of observation, which is of no practical interest provided that a useful description of crack propagation can be written in terms of average growth rates. This description involves the assumption that the intervals covered by the averaging process may be regarded as sufficiently long so that the crack growth is effectively represented by the average rate of growth in the intervals. Yet, simultaneously the intervals must be sufficiently short so that the predicted life of a body based on a summation involving the growth rates over the intervals of averaging may be accurately represented by an integration process upon viewing the phenomenon as continuous.

This assumption is tacitly involved in any attempt to write a crack propagation law of the form of Eq. 2.1, yet it has not been either recognized or investigated directly in previous work (Refs. 3, 4, 5, 6, 8, 9, 11, 12, 13, 14, 15, 17, 18, 32, 33, 34, etc). However, Bhandari (Ref. 35) has at least observed the high sensitivity of the integration process to interval size in numerical computations of life. And some investigators (Refs. 3 and 9) have made some limited experimental observations of the level of discontinuity in the process.

Valluri (Ref. 3) has suggested that the level of discontinuity in the crack growth process is related to the plastic zone size. On that basis he also argues that the crack growth rate, as a consequence, should be proportional to the plastic zone size. Since there is no extensive enough data available on the level of discontinuity itself, his suggestions must be evaluated by probing the result of his implied consequence, which may be stated:

$$\frac{1}{w} \cdot \frac{da}{dN} = \text{const} \qquad 4.17$$

Eq. 4.17 supposedly should apply to any given material subjected to stationary sinusoidal loading with a particular load ratio, β. McEvily and Illg's data is again plotted on Figure 4.5 using the coordinates, K_M and $\frac{1}{w}\frac{da}{dN}$. The data correlates nicely into a single curve which shows that the data is not insensitive to these variables. However, Valluri's observation that $\frac{1}{w}\frac{da}{dN}$ should be a constant is obviously erroneous. Moreover, this error reemphasizes the fact that the dimensional arguments of several investigators (see sections 2.3, 2.4, 2.5, and 2.8) do not agree with the data on crack growth. For example Liu (section 2.4) had also argued that Eq. 4.17 can be derived from dimensional considerations. Therefore an approach other than dimensional analysis must be taken to investigate the effect of the discontinuous nature of crack growth.

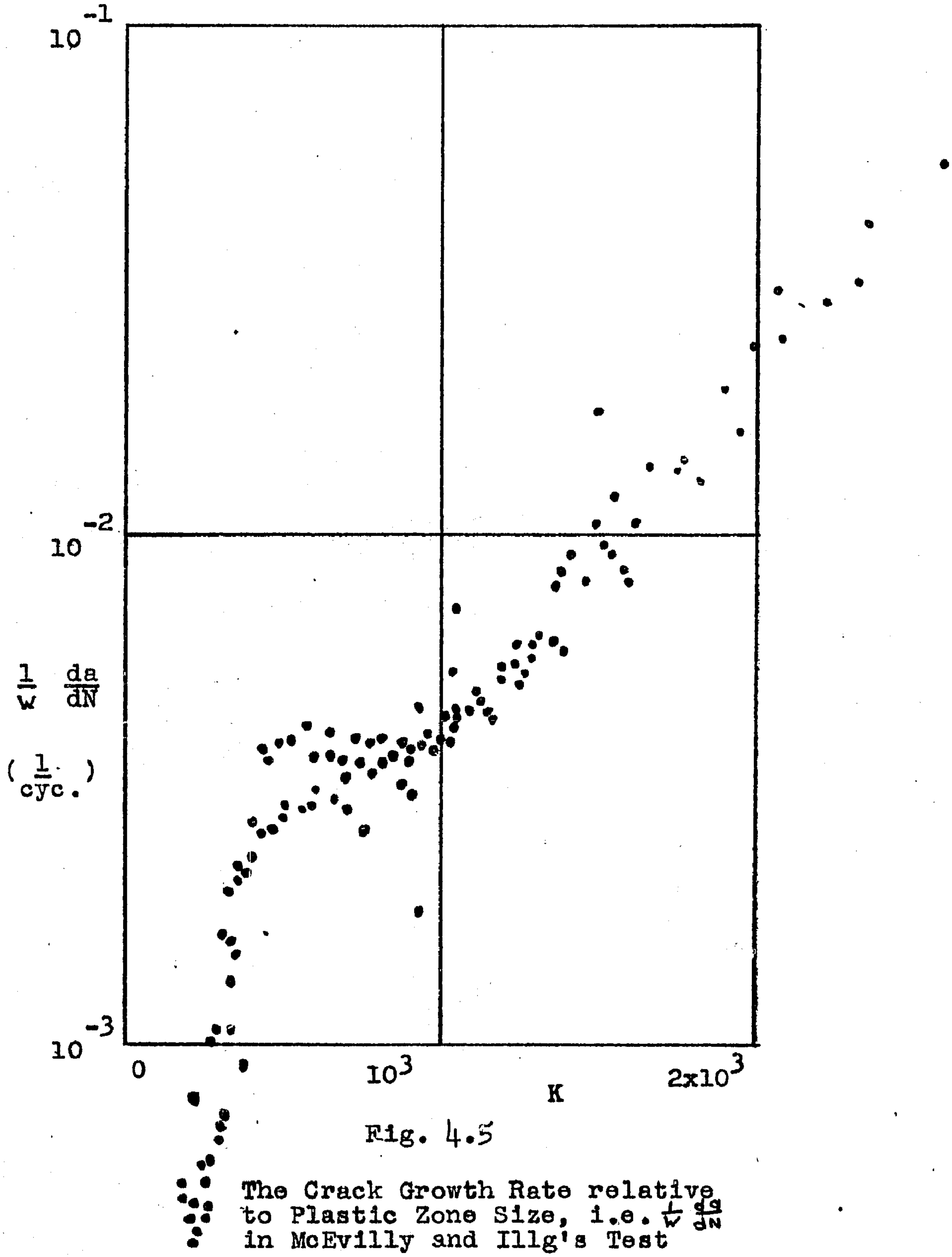

Fig. 4.5

The Crack Growth Rate relative to Plastic Zone Size, i.e. $\frac{1}{W}\frac{da}{dN}$ in McEvilly and Illg's Test

But before continuing, one final point should be made with respect to Figure 4.5. The values of $\frac{1}{w}\frac{da}{dN}$ are shown to be a function of K for a material. This is of interest in evaluating the effect of mildly non-stationary loading using Eq. 4.9. The factor $\frac{1}{w}\frac{da}{dN}$ in that equation may be evaluated in terms of properties of K, i.e. $\{K\}$, from data representations such as Figure 4.5, which aids in computing R.

Returning to the discussion of continuity in crack growth, a semi-experimental approach may be suggested. For the moment let the discussion be restricted to the case of stationary sinusoidal loading, specifically the experiments of McEvily and Illg. Since their data correlates very well on graphs such as Figures 4.2 and 4.5 it may be assumed that their intervals in taking data are sufficiently large to avoid difficulty in continuous representation of a discontinuous process. Thus Figure 4.2 suggests that:

$$\frac{\Delta a}{\Delta N} = F_2 (K_M, \beta , c_i) \qquad 4.18$$

where Δa and ΔN correspond to their data intervals. The question remains whether a continuous process representation of Eq. 4.18, i.e.

$$\frac{da}{dN} = F_2 (K_M, \beta , c_i) \qquad 4.19$$

will lead to the same result in life calculations. That is equivalent to inquiring whether summing Eq. 4.18,

$$N-N_o = \sum_{a_o}^{a} \frac{\Delta a}{F_2(K_M, \beta, C_i)} \qquad 4.20$$

leads to the same result as integrating Eq. 4.19,

$$N-N_o = \int_{a_o}^{a} \frac{da}{F_2(K_M, \beta, C_i)} \qquad 4.21$$

In their own work, McEvily and Illg (Ref. 4) note that experimental curves of a vs. N which are equivalent to the result of summing, as in Eq. 4.20, are almost identical in shape to computed curves which are equivalent to the results of integrating, as in Eq. 4.21. However, they do not find that the curves are completely coincidental to each other.

This lack of coincidence may be attributed to either a difficulty in establishing the initial value, N_o, or a difficulty in accuracy of the early portion of the integration process where the values of F_2 are very small compared to the latter portion. Hence the beginning and early portion of the integration will receive attention here.

When cracks in bodies are small, that is early in the life, they may almost always be represented by the configuration shown in Figure 2.1 (or its equivalent).

For that configuration, Fig. 2.3 and Eq. 2.22 suggest that the crack extension rate is approximately proportional to the crack length squared, i.e.

$$\frac{da}{dN} = F_2 = D_4 \, a^2 \qquad 4.22$$

where D_4 depends upon the properties of the sinusoidal loading. Substituting this result, Eq. 4.22 into Eq. 4.21 gives:

$$N - N_o = \frac{1}{D_4} \int_{a_o}^{a} \frac{da}{a^2} = \frac{1}{D_4} \left[\frac{1}{a_o} - \frac{1}{a} \right] \qquad 4.23$$

Simply by inspecting Eq. 4.23, it is noted that integrating over many orders of magnitude of crack length from a_o to a will always be an inherently inaccurate process if a_o is difficult to establish. However, integrating over one or perhaps two orders of magnitude of crack length can usually be farily accurately accomplished, but not without some difficulty.

In conclusion, it is clear that crack propagation laws which apply to a limited range of data are useless for meeting the objective of predicting structural life. Moreover, the process of computing lives, which involve several orders of magnitude of change in crack length, from the rate of crack extension is very sensitive to inaccuracies in the initial values and early portion of the integration. In fact, this also explains the wide scatter in lives of fatigue specimens with initial crack-like imperfections of different sizes!

However, no evidence has been found which leads to the conclusion that it is not possible to find a crack propagation law, which upon integration will take on proper connotations even if a view that crack growth is a continuous process is adopted. A result of this discussion is that that assumption has been shown to be justified from tests employing sinusoidal loading. The justification arises from McEvily and Illg's observation that integration as in Eq. 4.21 leads to an a vs. N curve prediction which agrees with the trends of observed a vs. N data.

On the other hand it was shown that other aspects of crack growth, and not the continuous process assumption, cause some difficulties in life prediction methods. Hence, if a general crack propagation law may be formulated, which predicts the broad data trend, it will at least not be invalid as a consequence of being conceived in terms of continuous process variables. And even though it may not always be possible to predict structural lives accurately using such a law it may at least be possible to establish the full trend caused in total fatigue life due to changing some variable or to predict portions of lives with sufficient accuracy. That will be a matter open to investigation in specific applications rather than for further concern herein.

4.6 The Effect to Frequency and Load Ratio on Crack Propagation Under Stationary Sinusoidal Loading:

In a previous discussion (section 4.4) it was shown that data from crack propagation tests employing stationary loading can be correlated using stress-intensity-factor concepts. Specifically, the data for a given load ratio, β, form a single curve on Figures 4.2 and 4.4.
It is appropriate to consider the effect of changes in frequency and load ratio on data plotted on similar diagrams.

First, the data plotted on Figure 4.2 covered a range of frequencies from 20 to 1800 c.p.m. Upon inspecting that figure the conclusion can be drawn that crack growth in 7075-T6 aluminum alloy is not noticably affected by frequency. Data on 7075-T6 from other sources concur with this conclusion. However, 2024-T3 aluminum alloy (see Ref. 34) does have a noticable, but small frequency effect. Specifically, a change in frequency from 20 to 1200 c.p.m. causes a reduction in crack growth rate by a factor of about 2. This is said to be small since a change in load level of less than 20% will produce the same effect on the rate of crack growth. Henceforth, frequency will be regarded as a variable of secondary importance compared to magnitude of load.

The effect of load ratio is another matter. Figure 4.6 shows some data (from Ref. 1) on 7075-T6 aluminum alloy for a variety of load ratios, β , from 1.22 to 20. It may be observed that changes in β have a considerable effect on the crack growth rate, i.e. β is as important a variable as magnitude of loading. However, it is possible to describe a sinusoidal loading interms of other variables which reduce the number of primary parameters to one.

The quasi-stationary sinusoidal variation of stress-intensity-factors may be alternately represented in terms of the double-amplitude (or range), ΔK and the mean value, K mean. Thus the variables, ΔK and γ , are chosen, where:

$$\gamma = \frac{K_{mean}}{\Delta K} = \frac{P_{mean}}{\Delta P} \qquad 4.24$$

Figure 4.7 shows the data on Figure 4.6 replotted in terms of these new variables. It is observed that the stress-intensity-factor range, ΔK, is far more influential than the ratio of the mean to the range, γ . This result is also applicable for 2024-T3 aluminum alloy (Ref. 32).

Therefore, the stress-intensity-factor range, ΔK, will be regarded as A fundamental local stress parameter

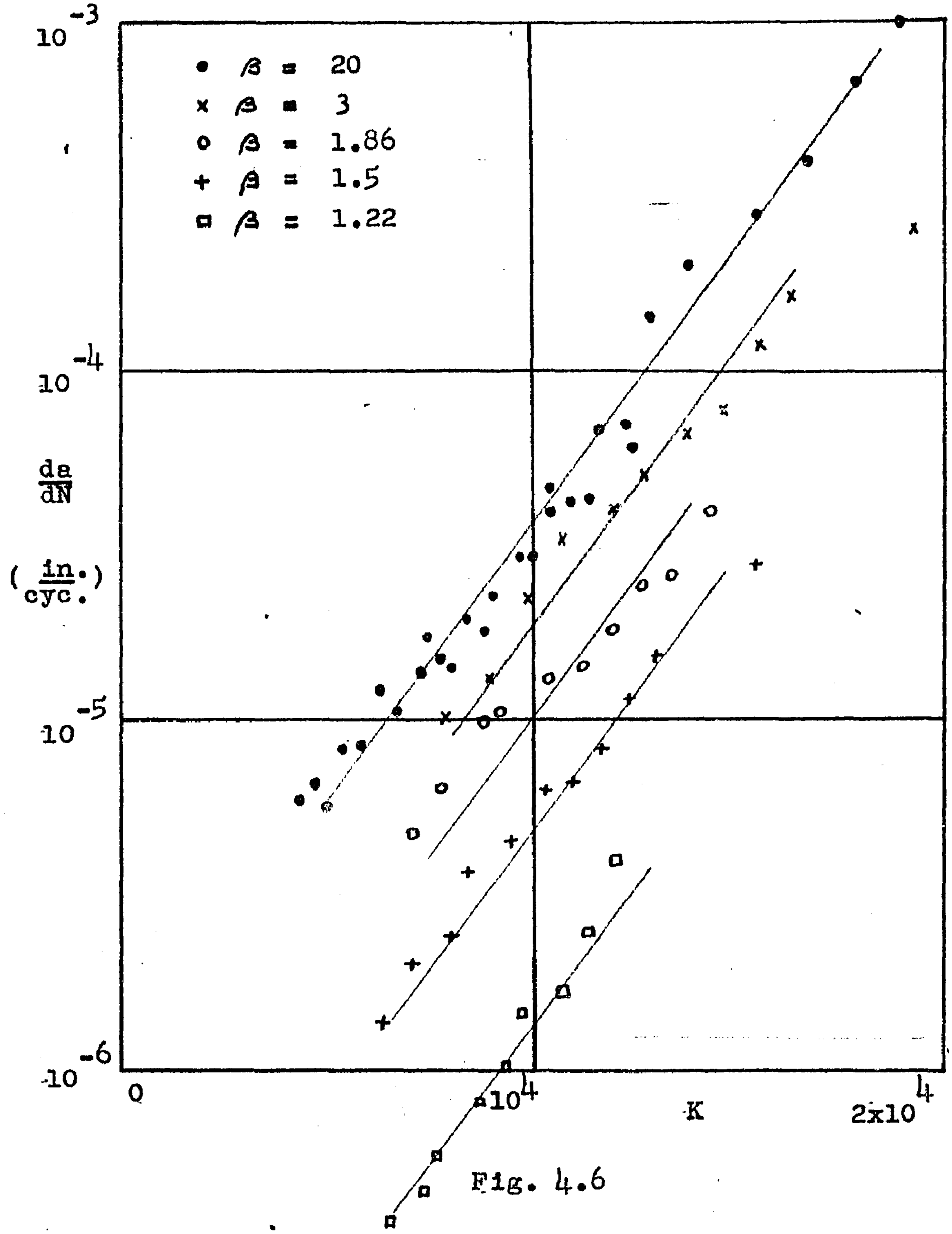

Fig. 4.6

The Effect of Load Ratio, in Sinusoidal Loading Test on 7075 T6 Aluminum Alloy from Donaldson and Anderson

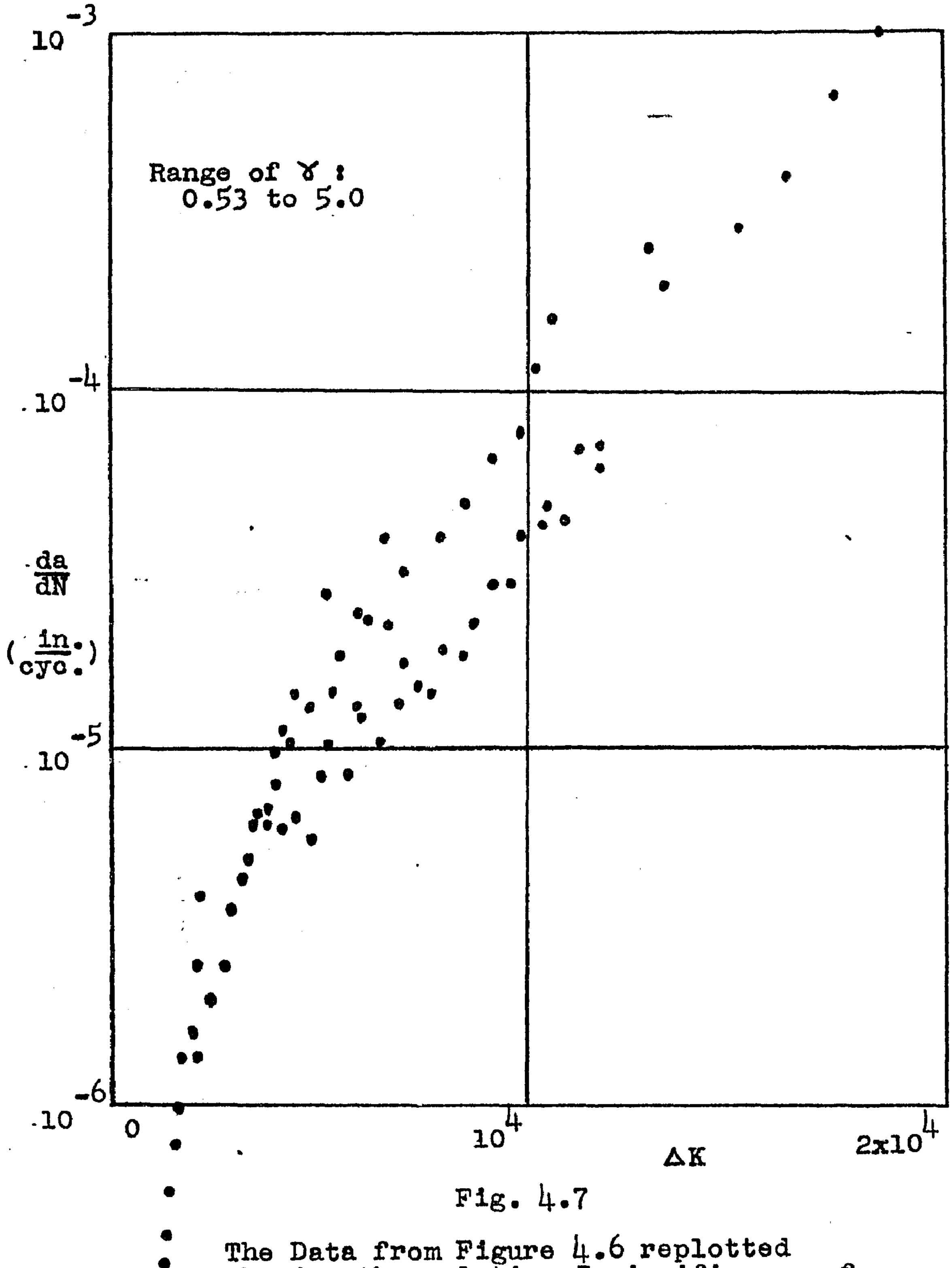

Fig. 4.7

The Data from Figure 4.6 replotted showing the relative Insignificance of Mean Load, γ compared to Load Range, ΔK in subsequent discussions. The mean value of the stress-intensity-factor will be regarded as of relatively secondary importance. And frequency effects for the most part will be neglected.

4.7 Interference Between Crack Surfaces:

Stress solutions to crack problems, where interference between the crack surfaces occurs, are not available. Moreover, the roughness of natural fatigue crack surfaces leaves considerable doubt as to the accuracy of application of such solution to conditions where interference occurs.

Therefore, all of the data discussed previously in this work were restricted to tests employing sinusoidal loads where the whole load cycle was in the "tension range", i.e. β positive (or γ greater than 0.5).

On the other hand Illg and McEvily, (Ref. 37), have provided some data on crack propagation under fully reversed loading, $\beta = -1$, for 2024-T3 aluminum alloy. Reference 1 shows this data, plotted in the manner of Fig. 4.2 and the following conclusions may be drawn. The crack propagation rate under fully reversed loading is approximately (only slightly larger than) the rate obtained in tests where only the tension portion of the cycle is applied. It is evident that a crack should be fully closed during the compression portion of the load cycle on an ideally elastic body. Therefore, during that portion of the cycle there would be no fluctuation of the stress-intensity-factor. However, in a real body the yielded material at the crack tip tends to "prop" the crack open slightly upon unloading from the tension range. Consequently, a slight effect of further closing occurs upon going into compression.

These conclusions are based on limited amount of data but the implications are clear. Loadings which cause the stress-intensity-factor to take on negative values may be treated by neglecting the variation of the stress-intensity-factor during its negative excursions without large errors. A complete and accurate treatment of the effects of closing of cracks will require additional data and further analysis not available at this time.

4.8 Stationary Periodic Loading, Other Than Sinusoidal:

Sinusoidal loading is perhaps the simplest periodic loading to analyze and the easiest to produce in the laboratory. That explains the emphasis which investigators have placed on it in their analyses and experimentation. However, it is hardly the only type of periodic loading sustained by structural elements in which fatigue and/or crack propagation must be considered in design. Thus, it is appropriate to discuss other types of stationary periodic loading.

Apparently the hypothesis as stated in section 4.3 may be directly applied to all periodic loadings provided that:

(1) The period is short enough so that the assumption that the stress-intensity-factor time history may be represented by its quasi-stationary characteristics (see sections 4.1 through 4.5) is justifiable.

(2) The period is short enough so that the assumption that crack growth may be represented as a continuous process (see sections 1.2 and 4.5) is justifiable. It is self-evident that periodic loadings with very long periods, i.e. a great many peaks per cycle, may cause violations of the above restrictions, (1) or (2) or both. For example a periodic loading with so many peaks per cycle that the life of a specimen is less than one cycle can be visualized. For such an example the above restrictions are surely violated; the loading must alternatively be considered to be a non-periodic deterministic loading. Non-periodic loadings are outside the scope of this section and shall be left for later discussion.

Returning to the clarification of "short period loadings", it is of interest to consider the question of how short is "short enough" (as stated in (1) and (2) above) to make the application of the hypothesis justifiable. This question can only be answered via the same experimental techniques which were cited in the verificating the hypothesis for sinuosidal loading. Rigorously, for each new type of periodic wave form encountered, data on crack growth must be examined on a graph of the form of Figure 4.3 or 4.7 or their equivalent. Correlation of the data into a single curve is required. Moreover, restriction (1) must be evaluated by establishing an interval of R, as defined by Eq. 4.9, within which the correlation of data is acceptable. And finally, the plausibility of the continuous process restriction, (2), must be evaluated, proceeding as in section 4.5, in order to assure the usefulness of the results in making life predictions.

Of course the repetition involved in fully establishing these justifications for every conceivable periodic loading is an infinite task. It seems more plausible to suggest that the "length" of a periodic loading may be characterized by some parameter, such as the total excursion of the load per cycle, (reasons for this choice of a particular parameter will become apparent later). Then these restrictions might be examined for periodic loadings of various lengths until a general criterion of a "short enough" period is established.

In the end a more satisfactory method of attack of periodic loadings as well as other types may be an attempt to develop a crack propagation theory for arbitrary deterministic loadings. But in discussions to follow it will be observed that such an attack may not be fruitful for sometime to come. In the meantime the hypothesis proposed here may be used to treat a great variety of periodic loadings adequately.

4.9 Stationary Random Loadings:

A stationary random process is one which is neither periodic nor deterministic in any sense, yet has some statistical properties which are constant, i.e. stationary, with time. In this discussion of stationary random loadings it will be assumed that the load-time history is continuous and that the statistical distribution of the magnitudes of the deviation of the load from its mean is Gaussian. The assumption of a Gaussian distribution serves to facilitate analysis and is typical of a type of random loading often encountered in structures. In addition many of the remarks to be made here apply to other types of distributions as well. Thus the assumption of a Gaussian nature is not a severe restriction.

A random loading, which is continuous, stationary, and Gaussian, can be completely characterized by its power spectrum, $S(\omega)$, see Bendat (Ref. 36). For the purpose of discussing crack propagation it is convenient to define some special characteristics of random loadings in terms of the power spectrum. The power spectrum will account only for deviation of the load from its mean.

First, the moments, M_r, of the area under the power spectrum are:

$$M_r = \int_{-\infty}^{\infty} \omega^r \; S(\omega) \, d\omega \qquad 4.25$$

Then the average number of times the load passes through a certain magnitude, α , per unit time for a Gaussian process is (Ref. 36):

$$\bar{N}_\alpha = \frac{1}{\pi} \sqrt{\frac{M_2}{M_o}} \; e^{-\left(\frac{\pi \alpha^2}{M_o}\right)} \qquad 4.26$$

where α is defined as the difference between the magnitude of the load at any instant and its mean value over a long period of time (tending toward infinity).

The average number of peaks or troughs (maxima or minima) in the loading per unit time is (Ref. 36):

$$\bar{Q} = \frac{1}{2\pi} \sqrt{\frac{M_4}{M_2}} \qquad 4.27$$

Moreover, the average total excursion, $\bar{V}$, of the load per unit time is:

$$\bar{V} = \int_{-\infty}^{\infty} \bar{N}_\alpha \; d\alpha = \frac{\sqrt{M_2}}{\pi} \qquad 4.28$$

which is the sum of all the individual rises and falls, h, between successive maxima and minima in the load-time history. Therefore the average excursion without a reversal in direction, $\bar{h}$, is (Ref. 37):

$$\bar{h} = \frac{\bar{V}}{2\bar{Q}} = \frac{M_2}{\sqrt{M_4}} \qquad 4.29$$

Finally, it can be observed directly from Eq. 4.26, that the mean square deviation in the load from its mean value is:

$$\overline{\sigma_p^{\,2}} = \frac{M_o}{2\pi} \qquad 4.30$$

and the average number of crossings of the mean load level per unit time is:

$$\bar{N}_o = \frac{1}{\pi}\sqrt{\frac{M_2}{M_o}} \qquad 4.31$$

It is seen above that the averages of characteristics of a random load of this type may be written in terms of moments of the power spectrum. The determination of statistical distributions of these characteristic quantities is more difficult. The problem of the distribution of the intervals between crossings of the mean load level has been treated by Rice (see Ref. 36) and others. The distributions of other quantities such as rises and falls, etc., have been analyzed by Beer et. al. (Refs. 37 and 38).

Some of these quantities and their distributions obviously have an influence on the rate of crack propagation. Therefore, treating crack propagation in general under random loading will necessitate the determination of the influence of each of the quantities.

However, without much difficulty some conclusions may be drawn and a method of analyzing crack propagation for any particular random loading can be stated from the direct application of the hypothesis in section 4.3. A discussion of this application will follow immediately. The problem of treating random loading crack extension in a more general way, (i.e. by direct determination of the influence of various statistical quantities describing a random load), will be brought up in a subsequent chapter.

4.10 Application of the Hypothesis to Crack Propagation Under Random Loading:

In order to analyze crack propagation under a specific random load, an investigation of the time histories of the stress-intensity-factors at crack tips in bodies subjected to this loading is required. In particular, consider the quasi-stationary K-time histories (i.e. with crack length considered constant).

Eq. 4.1 may be applied which leads to the conclusion that the quasi-stationary K-time history is simply the load-time history multiplied by a constant factor (which depends upon the crack length). That is to say that since the power spectrum of the load, $s_p(\omega)$, is defined from the frequency spectrum, $F^T(\omega)$, by

$$S_p(\omega) = \lim_{T\to\infty} \frac{1}{2T} \left| F^T(\omega) \right|^2 \qquad 4.32$$

and the frequency spectrum is the Fourier transform of the load-time history, P(t) or

$$F^T(\omega) = \int_{-T}^{T} P(t)\, e^{-i\omega t}\, dt \qquad 4.33$$

In order to find the quasi-stationary power spectrum of the stress-intensity-factor, $S_K(\omega)$, it is sufficient to multiply the load P(t) by the factor f(a) according to Eq. 4.1 and perform the indicated operations in Eqs. 4.32 and 4.33 which leads to:

$$S_K(\omega) = f(a)^2\, S_p(\omega) \qquad 4.34$$

Thus it has been shown that the quasi-stationary power spectrum of the stress-intensity-factor is identical to the power spectrum of the load except for a change in magnitude. Moreover, as the crack length, a, changes the magnitude of $S_K(\omega)$ changes but its functional form does not. Hence, the K-time history, characterized by $S_K(\omega)$, is the same as the load-time history or $S_p(\omega)$ except for the magnetude factor which changes but only slightly from one instant, (crack length), to the next.

Assuming that the rate of change of the magnitude of $S_K(\omega)$ does not appreciably effect the rate of crack propagation, the hypothesis in section 4.3 may be interpreted as:

> The rate of crack propagation caused by a particular random loading, $S_p(\omega)$, will be a function of the magnitude of the quasi-stationary power spectrum of the stress-intensity-factor, $S_K(\omega)$, in a given material.

The magnitude of a power spectrum of K may be alternately represented by its area, M_{o_K}, by the mean square deviation in K, $\overline{\sigma_K^2}$, by $\bar{h}_K$, the mean excursion in K, or other quantities. The above interpretation of the hypothesis suggests that crack growth rate data for a given material caused by a particular random loading, can be correlated into a single curve on a plot such as Figure 4.2 where K_{IM} is replaced by one of the aforementioned quantities. That is to say:

$$\frac{da}{dt} = F\,(\bar{h}_K\ ,\ C_i) \qquad 4.35$$

for a particular random loading. The choice of $\bar{h}_K$ as the variable to represent the magnitude of $S_K(\omega)$ is made for convenience in comparing this result, Eq. 4.35, with those to follow later.

The question of equivalence of the load time histories of random loads may be opened with regard to more general applicability of Eq. 4.35. Consider a particular load time history P(t). Since the rate of application of load (frequency) has no effect on crack growth rate (provided that the rate is based on an equivalent time) and the magnetude of applied load is "scaled" in its appearence in stress-intensity-factor, then P'(t) is equivalent to P(t) provided,

$$P'(t) = A_1 \; P(A_2 t) \qquad 4.36$$

where A_1 and A_2 are arbitrary constants. Recalling Eq. 4.1, it is observed that Eq. 4.36 may be rewritten in terms of stress-intensity-factors,

$$K'(t) = A_1 \; K(A_2 t) \qquad 4.37$$

It remains to explore the implications of these equivalent stress-intensity-factor time-histories.

The correlation function, $\varphi_K(\tau)$, of the original time history is:

$$\varphi_K(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} K(t)\; K(t+\tau)\; dt \qquad 4.38$$

For the equivalent loading (primed) the correlation function is:

$$\varphi_{K'}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} K'(t)\; K'(t+\tau)\; dt \qquad 4.39$$

which upon substituting Eq. 4.37 and changing the variable of integration to A_2 t results in

$$\varphi_{K'}(\tau) = A_1^{\,2}\,\varphi_K\,(A_2\tau) \qquad 4.40$$

The power spectrum of the stress-intensity-factor for original loading is defined by

$$S_K\,(\omega) = \int_{-\infty}^{\infty} \varphi_K\,(\tau)\;e^{-i\omega\tau}\,d\tau \qquad 4.41$$

Similarly for the equivalent loading

$$S_{K'}(\omega) = \int_{-\infty}^{\infty} \varphi_{K'}(\tau)\;e^{-i\omega\tau}\,d\tau \qquad 4.42$$

which upon substitution of Eq. 4.40 and a change of the variable of integration to $A_2\tau$ gives

$$S_{K'}(\omega) = \frac{A_1^{\,2}}{A_2}\;S_K\,\left(\frac{\omega}{A_2}\right) \qquad 4.43$$

This result implies that equivalent power spectra are those which can be formed by any arbitrary linear modification of the amplitude and/or frequency scales.

Refering to Eq. 4.25 for a definition of moment of a power spectrum, it is found that

$$M_{r_{K'}} = \int_{-\infty}^{\infty} \omega^r S_{K'}\,(\omega)\;d\omega \qquad 4.44$$

Substituting Eq. 4.43 and changing the variable of integration to $\frac{\omega}{A_2}$ leads to

$$M_{r_{K'}} = A_1^{2} \, A_2^{r} \, M_{r_K} \qquad 4.45$$

The effect of the time scale change may be noted in the zero crossings of the equivalent loading. From Eq. 4.31 the number of zero crossings per unit time may be found and introducing Eq. 4.45 results in:

$$\bar{N}_{o_{K'}} = \frac{1}{\pi} \sqrt{\frac{M_{2_{K'}}}{M_{o_{K'}}}} = \frac{A_2}{\pi} \sqrt{\frac{M_{2_K}}{M_{o_K}}} = A_2 \, \bar{N}_{o_K} \qquad 4.46$$

Hence the process of the equivalent loading is speeded by a factor, A_2.

In order to account for the increased speed of the process, Eq. 4.35 must be modified in order for it to apply to all equivalent loadings. Let the function, F, be redefined by

$$F = N_{o_K} \, F^{*} \qquad 4.47$$

Substitution this result into Eq. 4.35 gives

$$\frac{da}{dt} = N_{o_K} \, F^{*} \quad (\bar{h}_K, \, c_i) \qquad 4.48$$

Defining a non-dimensional time by

$$t^{*} = N_{o_K} \cdot t \qquad 4.49$$

Eq. 4.48 may be rewritten,

$$\frac{da}{dt*} = F* \quad (\bar{h}_K, Ci) \qquad 4.50$$

Eq. 4.50 is a single expression for the crack growth rates caused by all equivalent random loads. Therefore the application of the hypothesis to random loading in a more general way has been defined.

It seems self-evident that such an application is justifiable, though no experimental evidence is available to support this conclusion.

4.11 Concluding Remarks on the Crack Propagation Hypothesis:

The hypothesis stated in this chapter allows comparison of crack propagation rates for a given material and stationary loading of any particular wave form, including random (or statistically described) wave forms. That is to say that it allows prediction of the crack growth rates for a configuration, such as a complicated structural member, from data on crack growth rates using another configuration, such as the uniformly stressed plate with a central crack, which is convenient in the laboratory. This ability to compare growth rates for different configurations represents a considerable economy if the crack growth resistance of a structure is to be investigated.

Curves such as Fig. 4.2, 4.6, and/or 4.7 may be drawn which give a material's whole crack growth characteristics for a given wave form of loading on a single diagram. Data from this type of diagram may be incorporated in numerical integration procedures to estimate the remaining lives of structures in which cracks are discovered or in a similar fashion to estimate the inspection intervals appropriate for discovery of cracks prior to complete failure. Moreover, the relative desirability of one material compared to another, as far as their crack growth characteristics are concerned, can be visually assessed from this type of representation.

The use of this representation, employing the implications of the hypothesis, has allowed rather comprehensive evaluations of the relative roles of the primary and secondary variables in the crack growth process. It has been illustrated that mean load plays a minor role compared to variation or range in load in crack propagation. Moreover, frequency or rate of loading effects, barely observable in 2024-T3 and not visible at all in 7075-T6 aluminum alloy crack growth data, can readily be noted as nearly negligible compared to the range of excursion of the load. Hence, the hypothesis has also aided in understanding the importance of the variety of variables associated with crack propagation.

But the preceding analysis is lacking in two ways which demand further discussion. First, it would be advantageous to be able to establish a crack propagation law in functional form which predicts broad trends of crack growth behavior. And second it remains to discover ways to relate the crack propagation rate under one type of wave form of loading to another. In the end it will be desirable to attempt to develop theoretical approaches to the problem of crack propagation analyses interrelating growth rates for arbitrary types of load-time histories. In view of the fact that experimental data is only available for sinusoidal loadings such attempts might seem a bit ambitious. But since the foregoing hypothesis has allowed the establishment of some fundamental trends in crack growth behavior, further theories may be developed which, if nothing more, suggest some critical experimentation in the future.

CHAPTER V - AN EMPIRICAL CRACK PROPAGATION THEORY AND ITS IMPLICATIONS

5.1 Introductory Observations:

In the earlier portions of this work a great deal of emphasis was placed on finding the appropriate parameters to describe crack propagation. It was argued that stress-intensity-factors are a convenient fundamental parameter. Moreover, Figure 4.7 shows that the range of variation in stress-intensity-factors is of primary importance in crack growth processes, compared to the mean level of stress-intensity-factors. A further observation from experimental data is that frequency is at most a relatively minor variable.

That is to say that for sinusoidal loading one may write:

$$\frac{da}{dN} = F_4(\Delta K, \gamma, f, C_i) \qquad 5.1$$

on the basis of the observed facts from experimentation and the crack propagation hypothesis presented earlier. Since γ, and f are minor variables in Eq. 5.1, the general trend of behavior may be established in the form:

$$\frac{da}{dN} = F_5(\Delta K, C_i) \qquad 5.2$$

A corresponding trend is undoubtedly followed for all types of stationary loading, provided that the "period" of the loading is short enough so that an interval of R exists in which the hypothesis applies. This provision was clarified previously.

Therefore, for all types of stationary loading, it is expected that Eq. 5.2 can be generalized to read:

$$\frac{da}{dt^*} = F_6\ (\overline{var.\ K},\ C_i) \qquad 5.3$$

where t^* is a characteristic time for the loading and $\overline{var.\ K}$ is the average quasi-stationary variation or excursion in K per unit time, t^*. It is of interest now to attempt to find an approximate form of the function F_5 (and the function F_6 if possible) which indicates the broad trend of crack growth behavior for the primary load variable only.

An analytic attempt to find these functional forms would require the adoption of a "model" of the mechanism of crack propagation involving a variety of additional assumptions. Rather than adopt such a "model" at this time, it seems more reasonable to avoid new assumptions and if possible, determine the function F_5 and perhaps F_6 empirically from the available data.

5.2 An Empirical Crack Propagation Theory for Stationary Sinusoidal Loading:

Figure 2.3 and Eq. 2.22 almost suggest the form of an empirical crack propagation theory by themselves. Noting that Eq. 2.20 and Eq. 3.10 indicate that the parameter, p, is in reality ΔK for the configuration used in the tests whose results are plotted on Fig. 2.3, leads one to try data representations of the form $\ln \frac{da}{dN}$ vs. $\ln \Delta K$.

Figure 5.1 shows all of McEvily and Illg's data for 7075-T6 aluminum alloy using the suggested representation. Figure 5.2 shows the scatter band of Figure 5.1 with selected 7075-T6 data from other investigations, as well as the "wedge force" test data from Figure 4.4, superimposed. Figure 5.3 shows typical data for 2024-T3 aluminum alloy (Ref. 1) on a similar representation. And finally, Figure 5.4 shows typical data from various materials on a single diagram.

Figures 5.1 through 5.4 display all the results of the data available on various materials at this time. Moreover, the trend of each of these figures clearly indicates a straight line of a slope of 4 on the $\ln \frac{da}{dN}$ vs. $\ln \Delta K$ diagrams. The implied empirical relationship is therefore:

$$\frac{da}{dN} = \frac{(\Delta K)^4}{M} \qquad 5.4$$

where M is thought of as a material parameter rather than a constant. Changes in relative mean load, γ, frequency, f, and other minor variables in crack propagation, as well as environmental conditions, may be thought of as causing changes in M for a given material. Therefore, Eq. 5.4 may be regarded as an approximate empirical theory of crack propagation giving the main trend of behavior under sinusoidal loading.

Table 5.1 lists some values for M (computed from the data in Reference 1) for various materials under a variety of conditions (i.e. a variety of frequencies, relative mean load ratios, and environments).

5.3 A Generalization of the Empirical Theory:

As was the case in proceeding from Eq. 5.2 to 5.3, the empirical theory for sinusoidal loading, Eq. 5.4, may be extended and generalized. In accordance with Eq. 5.3 the result is:

$$\frac{da}{dt^*} = \frac{(\overline{\text{var. } K})^4}{M_P} \qquad 5.5$$

where M_P is the material parameter which depends on the wave form of the loading, as well as the other minor variables mentioned earlier, in addition to environment. Therefore, Eq. 5.5 may be regarded as a generalization of the theory and it applies to any particular stationary loading. Of course, the limitations placed on R and other assumptions associated with the hypothesis in the last

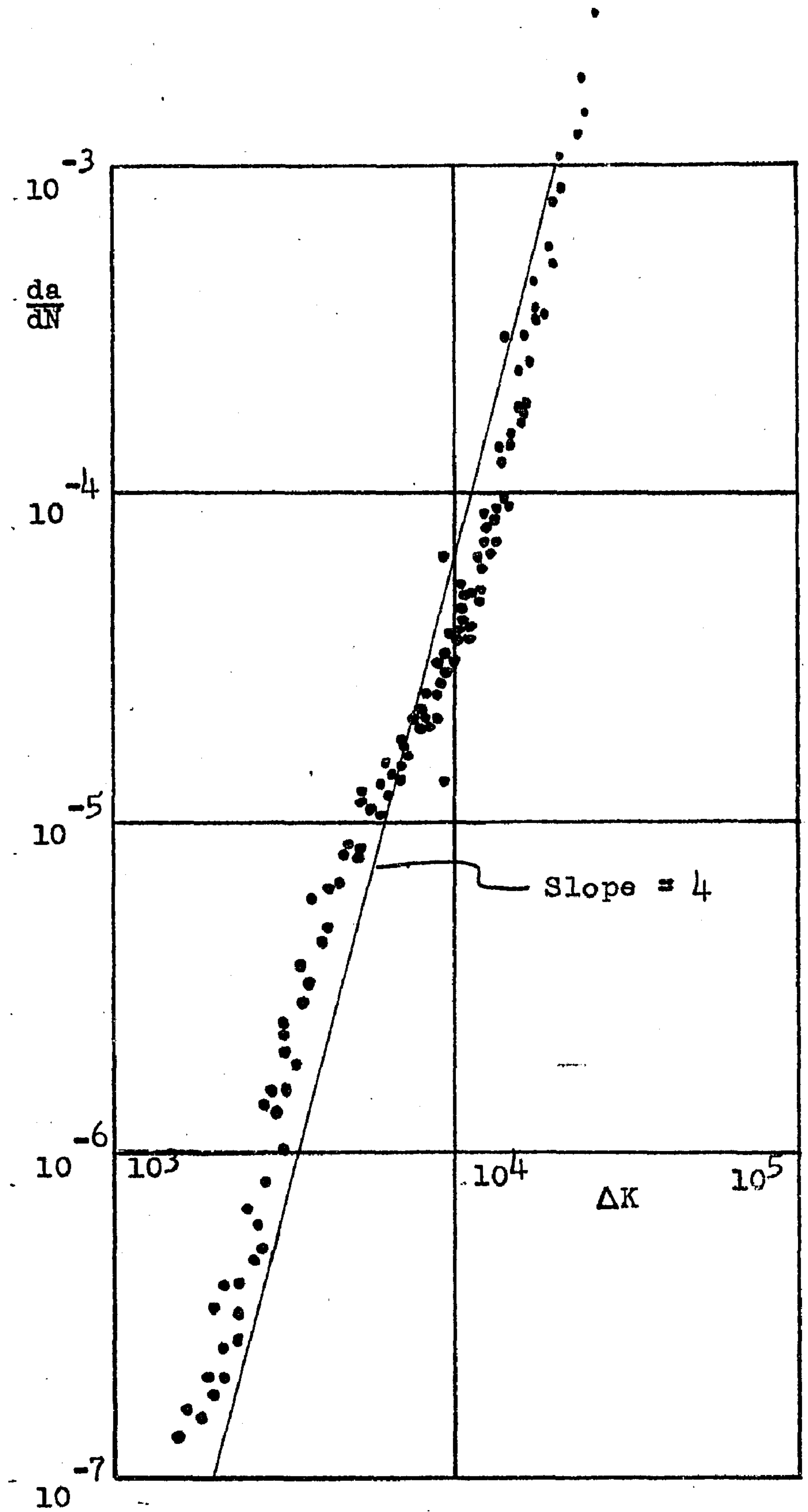

Fig. 5.1

A Log-Log Representation of McEvilly and Illg's Data on 7075 T6 Aluminum Alloy

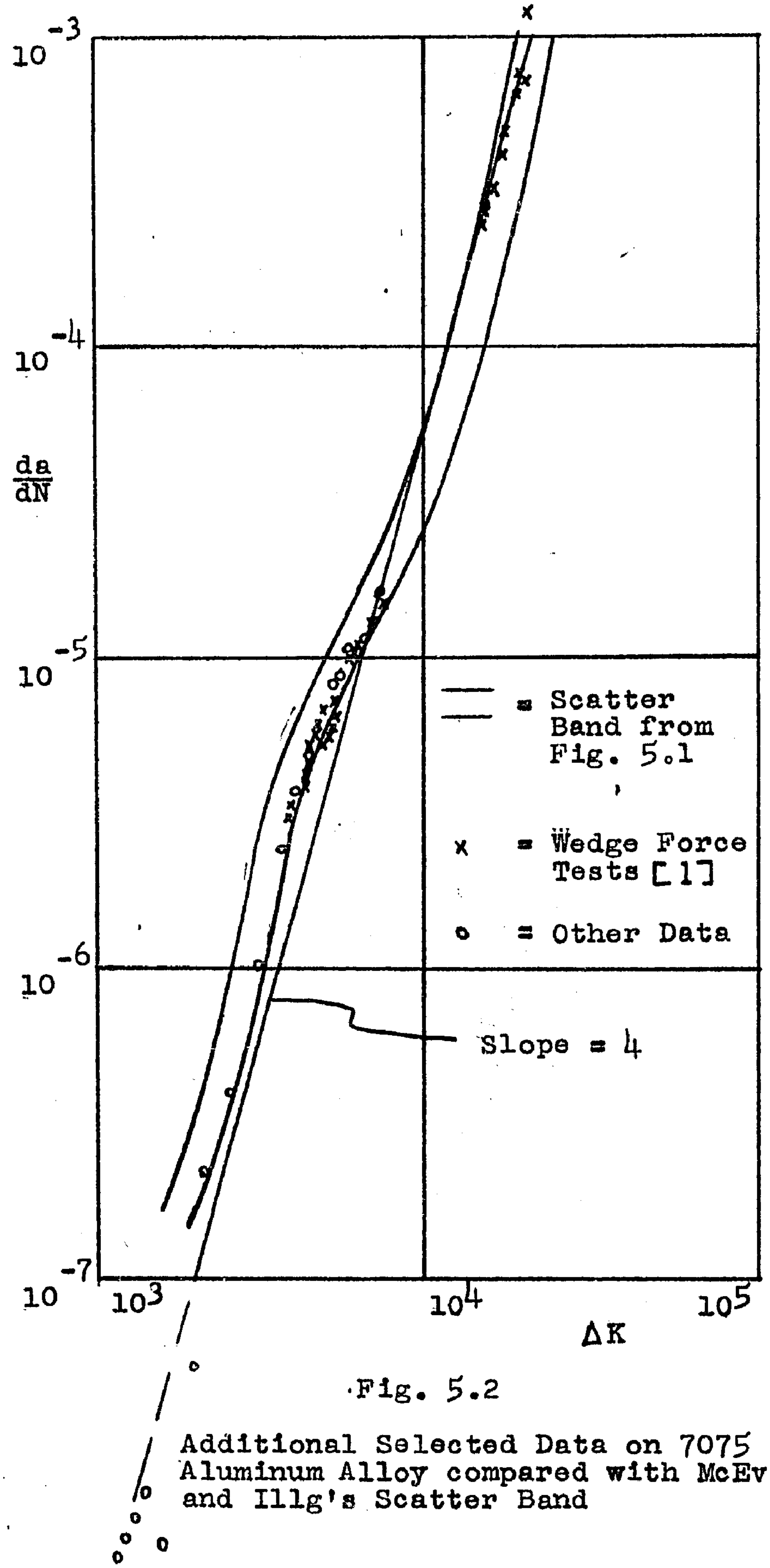

Fig. 5.2

Additional Selected Data on 7075 T6 Aluminum Alloy compared with McEvilly and Illg's Scatter Band

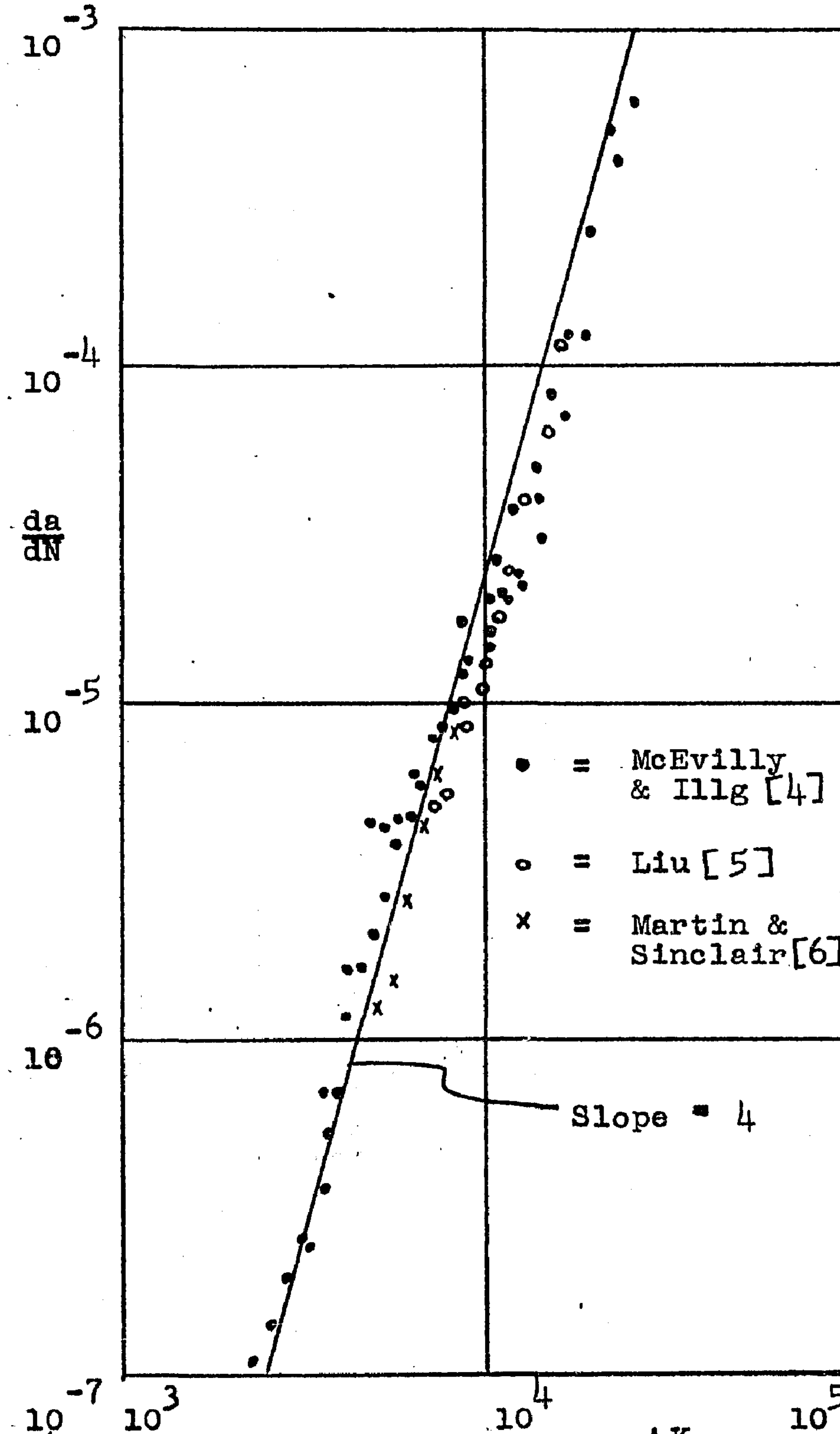

Fig. 5.3

Test Data on 2024 T3 Aluminum Alloy

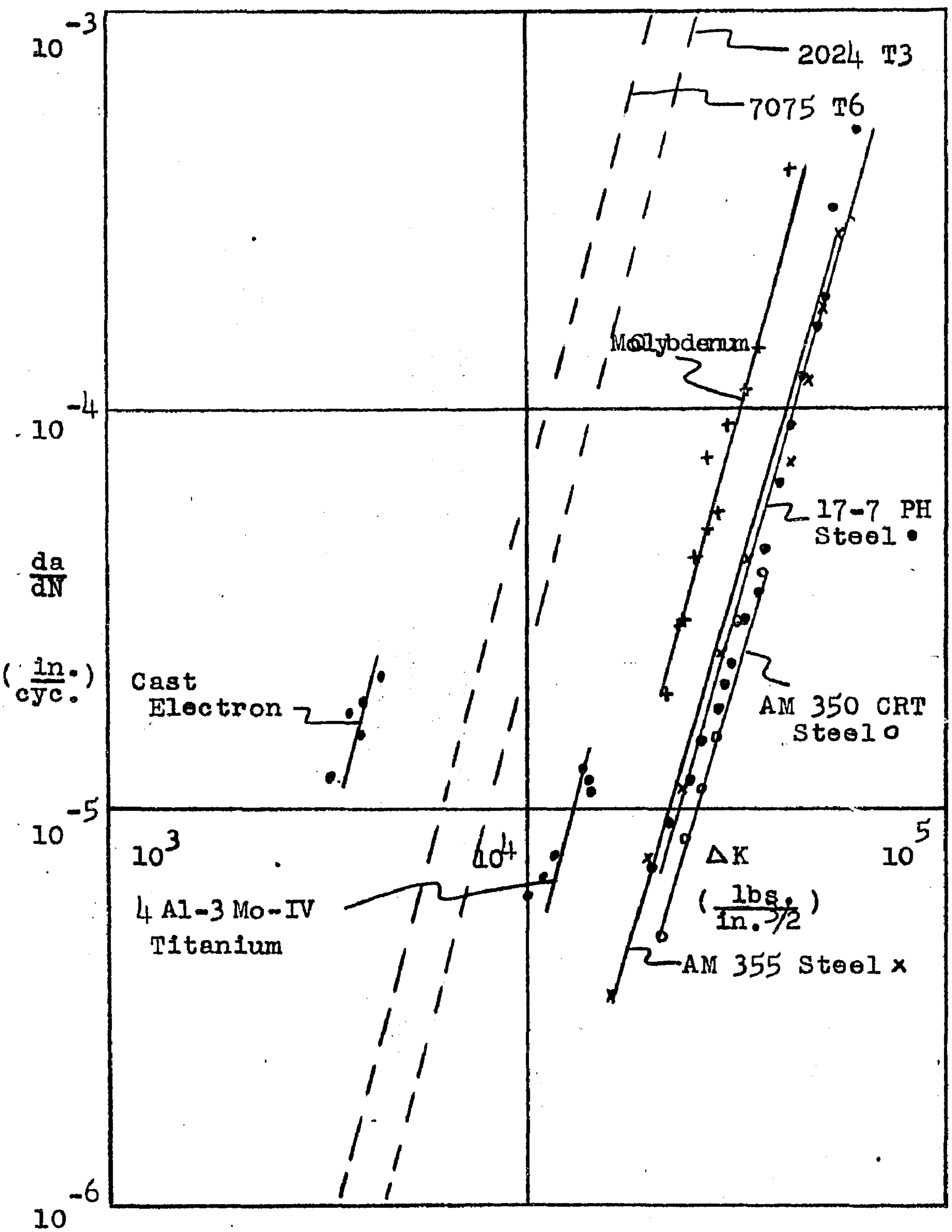

Fig. 5.4

Data on Various Materials compared to earlier Results for 2024 T3 and 7075 T6 Aluminum Alloys

TABLE 5.1

Material	Condition	f	γ	Temperature	Number of Data Points	M	Origin of Data	Remarks
2024 (Alum. Alloy)	T-3	(Various)	(About 0.5)	Room	64	4.1×10^{20}	Figure 5.3	This is the data shown on figures in this work (see Ref. 1 for further details on the data).
7075 (Alum. Alloy)	T-6	(Various)	(About 0.5)	Room	116	1.5×10^{20}	Figure 5.1	
7075 (Alum. Alloy)	T-6	(Various)	(About 0.5)	Room	47	1.5×10^{20}	Figure 5.2	
AM 350 (Steel)	CRT	120 CPM	(About 0.5)	Room	7	3.9×10^{22}	Figure 5.4	
17-7 PH (Steel)	RH 1095°F		(About 0.5)	Room	18+	3.0×10^{22}	Figure 5.4	
AM 355 (Steel)	SCT	500 CPM	(About 0.75)	Room	9	2.3×10^{22}	Figure 5.4	
Molybdenum		500 CPM	(About 0.70)	Room	11	1.0×10^{22}	Figure 5.4	
4 Al-3 Mo-1V (Titanium)	(At 1075°F 4 Hours)		(About .7 0)	Room	6+	2.9×10^{21}	Figure 5.4	
Cast Electrom (Magnesium)		1000 CPM	(About 0.75)	Room	5+	1.5×10^{19}	Figure 5.4	
7075 (Alum.)	T-6	1800 CPM	0.5	Room	29	1.8×10^{20}	Figure 4.6	(See also Ref. 1 for details.) (Boeing tests.)
		1800 CPM	1.0	Room	11	8.5×10^{19}		
		1800 CPM	1.76	Room	10	4.5×10^{19}		
		1800 CPM	2.5	Room	11	2.6×10^{19}		
		1800 CPM	5.0	Room	9	9.2×10^{18}		
2024 (Alum.)	T-3	2000 CPM	0.5	Room	27+	5.0×10^{20}	Reference 1	(Boeing Tests)
		1800 CPM	1.0	Room	10	2.0×10^{20}		
		1800 CPM	1.76	Room	11	7.6×10^{19}		
		1800 CPM	2.5	Room	10	5.9×10^{19}		
		1800 CPM	5.0	Room	10	3.5×10^{19}		
2024 (Alum.)	T-3	1200 CPM	0.55	Room	23	10×10^{20}	Reference 4	(See also Ref. 34)
		50 CPM	0.55	Room	6	4.1×10^{20}		
		20 CPM	0.55	Room	5	4.1×10^{20}		
4330 Mod. (Steel)	UTS 180-200	82.2 CPM	0.55	70°F	7+	1.3×10^{22}	Reference 1	
		90 CPM	0.55	-30°F	7+	2.4×10^{22}		

chapter also apply here.

Finally, on comparing Eqs. 5.4 and 5.5 it appears to be desirable to discuss the possibility of defining the variation in K in such a way that the material parameter in Eq. 5.5 is the same as that in Eq. 5.4. That is to say that it is self-evident that the influence of wave form on M_p is that of a factor which might be associated with the variation in K instead of M_p. But any attempt to determine the particular way of defining $\overline{\text{var. K}}$ which reduces M_p to M requires further assumptions, i.e. a new "model". A discussion of "models" will be included in the chapter to follow.

5.4 Implications of the Empirical Theory:

A major implication of the empirical theory is that the discussion of discontinuous crack growth in section 4.5 may be seen to apply to all types of stationary loading. Eq. 4.23 may be written in a more general form via the following approach.

Again, attention to accuracy of the integration process is turned to the early portion of the integration (see section 4.5). In that case Figure 2.1 is representative of the usual configuration and its stress-intensity-factor is, referring to Eq. 3.10, is:

$$K = \sigma \sqrt{a} \qquad 5.6$$

Then, for sinusoidal loading:

$$\Delta K = \Delta\sigma \sqrt{a} \qquad 5.7$$

or for any stationary loading:

$$\overline{\text{var. } K} = \overline{(\text{var. } \sigma)} \sqrt{a} \qquad 5.8$$

Substituting these results, Eqs. 5.7 and 5.8 into Eqs. 5.4 and 5.5 leads to:

$$\frac{da}{dN} = \frac{(\Delta\sigma)^4 a^2}{M} \qquad 5.9$$

and

$$\frac{da}{dt^*} = \frac{(\overline{\text{var.}\sigma})^4 a^2}{M_p} \qquad 5.10$$

Notice that Eq. 5.9 is identical to a previous result, Eq. 2.22. Thus upon integrating Eqs. 5.9 and 5.10, it is observed that:

$$N - N_o = \frac{M}{(\Delta\sigma)^4} \left[\frac{1}{a_o} - \frac{1}{a} \right] \qquad 5.11$$

and

$$\bar{t}^* - \bar{t}^*_o = \frac{M_p}{(\overline{\text{var.}\sigma})^4} \left[\frac{1}{a_o} - \frac{1}{a} \right] \qquad 5.12$$

Thus the character of the early portion of the integration process is the same for all stationary loadings within the restriction of agreement with the empirical theory, i.e. Eqs. 5.4 and 5.5.

It can also be noted that upper and lower bounds on M can be easily found on data representation such as Figures 5.1 through 5.4 by drawing two straight lines with slopes of 4, which bound the data for a given material. From these bounds the limits of error in the integration process may be deduced, since from Eqs. 5.11 and 5.12 it can be seen that the resulting errors are linearly related to M or M_p.

A further ramification of the empirical theory presented here is its general disagreement with laws proposed by previous investigators. Specifically, the data represented on Figures 5.1 through 5.4 and the resulting functional form of Eqs. 5.4 and 5.5 for the broad trend of behavior suggest that many previous laws as mentioned in Chapter II, are in error. However, the use of a local stress parameter here in obtaining correlation of data in the form of Figures 5.1 through 5.4 is in agreement with McEvily and Illg's adoption of a local stress parameter (Ref. 4). The law they proposed on empirical grounds, Eq. 2.18, has been replaced by a much simpler expression, Eq. 5.4, which is only slightly less accurate but decidely more versatile.

A comment on the engineering significance of the preceding hypothesis in contrast to the empirical theory seems appropriate. Since Eqs. 5.11 and 5.12 estimate the crack propagation life of the configuration of greatest interest in structures, Fig. 2.1, and since errors in their results may be easily bounded, perhaps they will find wide engineering application due in part to their simplicity. However, direct integration from the results of the hypothesis will remain the more accurate and more versatile approach regardless of the application.

CHAPTER VI - MECHANICAL MODELS IN CRACK PROPAGATION ANALYSES

6.1 The Continuum Concept of Model Analysis.

In the preceding chapters it was shown that crack growth rates in bodies with different configurations may be compared through the application of local crack tip stress parameters, specifically stress-intensity-factors. These comparisons are only possible for situations where the wave form of the loading imposed on the bodies was the same. The wedge forces tests, Fig. 4.3, with sinusoidal loading, whose results are compared with other sinusoidal loading experiments on Fig. 4.4, illustrate the method of treating different configurations clearly. It remains to attempt to also develop means of comparing crack growth rates in bodies with different wave forms of loading imposed.

The preceding analyses embody a "model" of crack propagation as a result of the hypothesis stated in section 4.3. The model regards the crack tip as a system which responds in a particular way to a given time history of local stresses. That is to say that crack tips in a given material, which experience the same time history of stress-intensity-factors, will grow at the same rate. The success of that hypothesis in itself implies that continuum models (or a systems concept) are at least partially successful in describing crack growth. There seems to be no reason to expect that the continuum concept cannot be applied to determine the effect of the wave form of the loading on crack growth rate as well.

Moreover, the empirical law developed in Chapter V appears to be appropriate for all materials, (i.e. a slope of 4 fits the data for all of the various materials shown on Figures 5.1 through 5.4). Since the metallurgical structure of these materials is quite different, their general agreement with the same empirical law based on continuum concepts implies that a description of crack growth of a rather universal nature may be possible using the continuum approach. If micro-processes variables (such as dislocation movement stress within crystals, etc.), beneath the continuum level are required in describing crack growth, then the success of the preceding analyses is ambiguous with that requirement. For that reason it seems feasible to proceed to attempt an analysis of mechanical models of the crack propagation process using an entirely continuum approach in the discussion.

6.2 Significant Variables in the Wave Form of the Load.

The objective of the discussion of "models" in this chapter is to develop that which is more commonly termed rules of "accumulation of damage" in the fatigue field in general. The word damage is appropriate here to describe the cumulative effects of fluctuations in load at a crack tip, and ahead of the crack tip with its plastic zone. Outside of the plastic zone, in the elastic regions of a body, the processes of deformation will be regarded as reversible and therefore no accumulative effects will be considered to be present (compared to the damage accruing in the plastic zone).

From the above view of damage and from the experimental results cited in previous sections, some conclusions may be drawn regarding the significant variables describing the wave form of a loading covering crack growth. First, since the damage is occurring within the plastic zone at the crack tip, the fluctuation of the local stress surrounding the plastic zone may be regard as the immediate cause of damage. The wave form of the time history of stress-intensity-factors is therefore of primary importance in the analysis. If the crack length may be regarded as constant for the sake of analysis, as was justified earlier, the wave form of the stress-intensity-factor and the load are effectively the same.

In section 4.6 the effect of frequency or rate of load application was observed to be negligible compared to the effect of range of variation in load. Moreover, the relative mean load (as described by γ for sinusoidal loading) was shown to be of secondary importance compared to range of variation of load. Hence, it may be concluded that the primary cause of crack growth is the extent of rises and falls of the load, exclusive of the rate at which the rises and falls occur and the mean load present. In addition the observation of delays in crack growth following an overload (Ref. 10) indicates that the crack growth process is sensitive to the sequence in which the rises and falls in the load occur.

Finally, the results of Chapter 5 indicate that the crack growth rate depends on range of variation of load to approximately the fourth power for all the materials for which data are available.

The above observations may be regarded as restrictions which aid in the development of a suitable "model" of crack propagation. In summary they are:

(1) A model of crack propagation should regard damage as occuring in the plastic zone at the crack tip.

(2) A model should show sensitivity to rises and falls in the load (but sensitivity to the rate of load application or the mean load is not required here).

(3) A model should be sensitive to the sequence in which rises and falls in load occur.

(4) A model should show a dependency of the rate of crack growth on the magnitude or range of the rises and falls to about the fourth power.

(5) A model should be applicable to any configuration via employing stress-intensity-factor concepts.

A model which meets the above five requirements may be regarded as one which meets the minimum acceptable restrictions. In addition, a suitable model should be capable of incorporating the minor effects (rate of load application, mean load, and environmental effects) within its "material constants". Finally, the ultimate criterion of the acceptability of any model will be its ability to predict the broad trends of changes in crack growth rates caused by differences or changes in the wave form of loads.

6.3 The Damage Accruing in the Plastic Zone at a Crack Tip.

Previous investigators have considered a variety of models in attempts to analyze crack growth. Often complicated models were used where their adoption now appears to be quite unnecessary. That is to say that in this dissertation it has been shown that crack growth under sinusoidal loading, (or any other particular stationary loading), can be completely analyzed via stress-intensity-factor concepts without requiring a model synthesizing the accumulation of damage. Whereas, for example, the crack propagation analyses of Head (Ref. 11) and Liu (Ref. 15), discussed in sections 2.2 and 2.4, embody models to formulate laws of crack growth for particular stationary loadings. Even though their models were unnecessary for the purpose for which they were employed and often led to questionable results (see section 2.8), they are at least informative in developing new models here.

In the majority of previous analyses damage was regarded as being directly related to the extent of plastic straining near the crack tip (Refs. 3, 4, 11, 15, 17, and 18). (It should be noted that linearly work hardening to a given stress level is equivalent to attaining a given plastic strain in interpreting the preceding statement). Essentially two ingredients are present in each analysis. They are: first, an assumption required to permit an estimation of the amount of plastic strain present and second, an assumption required to determine a criterion of failure due to the plastic straining. In general two or more similar assumptions will be required in each of the new models to follow.

The above pair of required assumptions will be reviewed in more specific detail for a few of the previous investigations before proceeding further. McEvily and Illg (Ref. 4) in determining the proper form of a crack propagation law based their analysis on examining only the point of material directly ahead of the crack tip. They presumed that that point of material would exhibit linear strain hardening to a given fracture stress and that the point is subjected to strains proportional to σ_∞.

On the other hand McClintock (Ref. 17) directed attention to the strains occurring within a small circular sector ahead of the crack. Furthermore he assumed that (as in the case of torsion) the strain distribution in the plastic zone varies with 1/r, where r is the distance from the crack tip. He then argued that, when the average plastic strain in his circular sector approached Coffin's (Ref. 34) critical plastic strain, the crack traverses the sector. Since McClintock's circular sector was quite arbitrarily chosen to facilitate an integration, he himself raised some doubt about its choice.

Head (Ref. 11) analyzed a line of plastic elements directly ahead of the crack, as discussed in section 2.2, and presumed that these elements work harden to a fracture stress whereupon the crack traverses them one by one.

Liu (Ref. 15) devoted attention to all of the elements throughout the plastic zone. For all of these elements he assumed that the elastic distribution of strains is a suitable approximation and that geometrically similar points absorb a characteristic amount of hysteresis energy for failure. (Hysteresis energy for an ideally plastic material is equivalent to plastic strain). Valluri (Ref. 3) used arguments equivalent to those of Liu.

Therefore, the essential difference between previously proposed models of crack growth lies in the assumptions of the distribution of strains in the plastic zone and the choice of elements in which plastic strains are significant. All the previous investigations cited above adopt a failure criteria which is equivalent to an ideal elasto-plastic material for which Coffin's criteria of a critical accumulated plastic strain to failure is appropriate.

Moreover, the crack propagation laws resulting from previous investigations are quite similar (see Chapter II), i.e. the laws of Head, Liu and Valluri all imply that the rate of crack propagation is proportional to $(\sigma\sqrt{a})^2$ (for the configuration shown in Figure 2.1). McClintock's model is the only one which differs greatly. It results in a prediction that the rate of crack growth is proportional to either $(\sigma\sqrt{a})^4$ of $(\sigma\sqrt{a})^{10}$, the choice of which depends on micro-process variables rather than continuum concepts. The similarities in these laws bring up an interesting point. Apparently there is a wide variety of assumptions which lead to identical laws when applied to the configuration of Figure 2.1 and sinusoidal loading. That is to say that agreement of a law derived from some model with test results from that configuration and sinusoidal loading is no guarantee that the assumptions of the model are "correct" at all. Such a model may give entirely erroneous results when adapted to other configurations or wave forms of loading. Hence any model proposed is speculative and unproved until it has been shown to agree with a variety of configurations and wave forms of loading through direct agreement with the broad trends of experimental results.

Nevertheless, it seems appropriate to construct some theoretical models here in order to draw some general conclusions about model analysis and in order to point out specifically the type of experimentation required to permit advances in crack propagation analyses through model analysis. Upon reviewing previous investigations and the physical phenomena associated with crack growth, the following point of view of damage seems appropriate.

First, Coffin's critical accumulated plastic strain to failure criteria (or its equivalent) is not only adopted in all significant previous investigations, but no other reasonable choice based on continuum concepts seems to be available. Willner and McClintock, (Ref. 40) have made it clear that Coffin's Criteria is a most reasonable choice on the basis of experimental evidence indirectly related to crack growth. Consequently it will be adopted as the failure criteria in the analyses to follow.

Second, it remains to assertain which elements within the plastic zone are those for which plastic straining actually contributes to final failure. Previous investigations invariably disagree on the choice of the elements which are of interest. It seems plausible that at least the plastic strains in all infinitesimal elements through which the crack will pass must be embodied in a complete analysis. That is to say that the smallest rationally acceptable region of interest is the line extension of the crack projected across the plastic zone (that is more correctly the plane of extension of the crack of an infinitesimal thickness, upon considering the problem in three spatial dimensions). However, Coffin's criteria may be applicable only to average strains throughout finite regions in which case an area must be considered rather than a line (again, an area is equivalent to a volume upon adopting a third dimension). The only characteristic area which does not involve an arbitrary (human) choice is the whole plastic zone. Therefore it is plausible that models which draw attention to plastic strains along either the line of crack extension or the whole plastic zone are the only acceptable ones if they are to employ Coffin's criteria.

Third and finally it is necessary to make an assumption in order to determine the magnitude plastic strains occurring within the elements of interest. A reasonable stress-strain behavior must be assumed in addition to an acceptable distribution of strains in order to develop a model in which accumulated plastic strain is employed as the failure criteria. Two choice of strain distribution prevale in previous investigations. Either the elastic strain distribution is considered acceptable into the plastic range (i.e. strains vary with $1/\sqrt{r}$) or as McClintock (Ref. 17) noted the plastic strain distribution may be assumed to be analogous to that in torsion (i.e. strains vary with $1/r$ within the plastic zone and $1/\sqrt{r}$ in the elastic region near the crack tip). The question of the strain distribution which should be adopted in a model cannot be fully resolved short of solving the elasto-plastic crack problem with fluctuating load. Current theories of plasticity are not adequate to solve this problem in general. Hence it will be left unresolved in the model analysis to follow.

It is appropriate at this point to attempt to formulate some models within the spirit of the above discussion of plausible assumptions of: a failure criterion, the elements of interest in accumulation of damage and the plastic strain distribution. Moreover, the results of the analysis of the models should be reviewed in light of the five requirements listed in section 6.2.

6.4 A Model Devoting Attention to the Line of Crack Propagation and Assuming a Linear (Elastic) Strain Distribution.

In the three preliminary sections of this Chapter the proper motivations of a model analysis and assumptions of previous investigations were discussed. At this time it seems appropriate to develope a model which is close to the assumptions of previous investigations, yet of a general enough nature to attempt to satisfy the requirements listed in section 6.2. With that purpose in mind the following assumptions shall be made:

(1) The material within the damage zone (the plastic zone) behaves in an ideal elasto-plastic manner. Figure 6.1 shows a diagram of this type of behavior.

(2) The strains within the damage zone are distributed according to those resulting from an entirely elastic analysis.

(3) The significant damage is accumulated only in those elements which do in fact fail, i.e. the line of elements within the plastic zone directly ahead of the crack.

(4) Coffin's critical plastic strain theory is an appropriate failure criteria. That is to say that the crack will have propagated through any element which has sus-

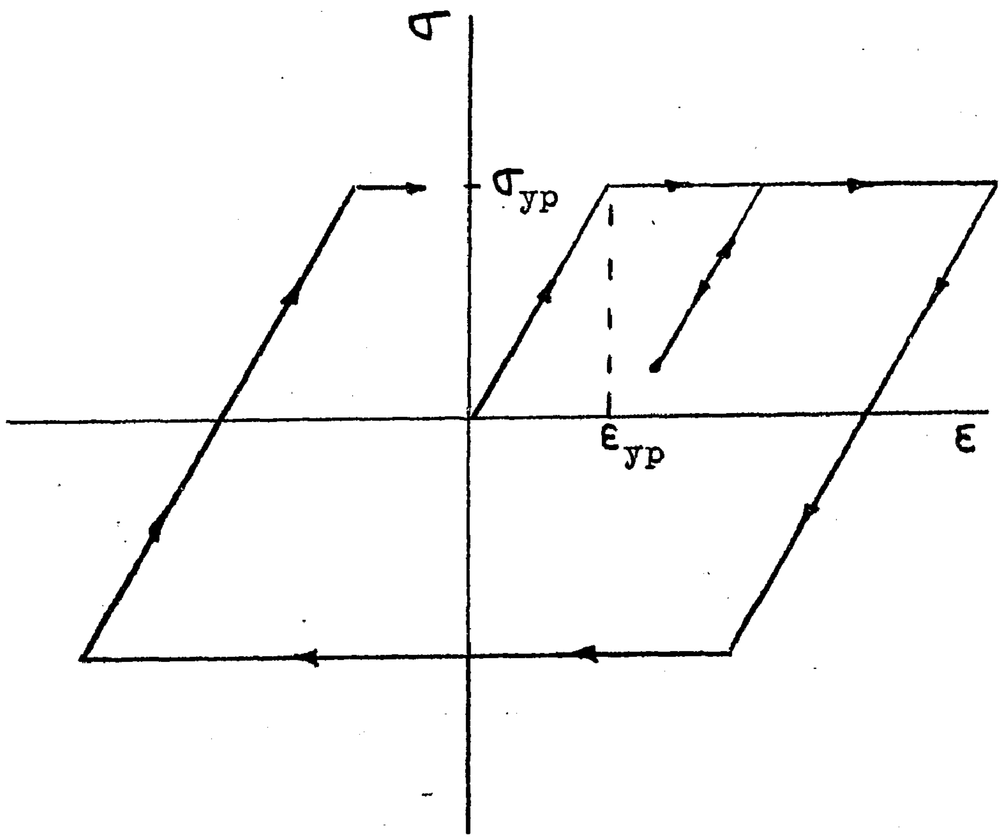

Fig. 6.1

The Stress-Strain Behavior of an Ideal
Elasto-Plastic Material

tained a total accumulated plastic strain which characterizes failure in the material.

Let the element on the line of crack extension a distance r ahead of the crack at a time t be considered; see Figure 6.2. The rate at which such an element is receiving plastic deformation is of interest. According to the above assumption of a strain distribution equivalent to the elastic case, the strain, ε , in the element can be computed from Eqs. 3.6 and Hooke's Law which results in

$$\varepsilon = \frac{K}{E \sqrt{2r}} \qquad 6.1$$

where E is an elastic constant related to Young's modulus. (ε may be regarded as octahedral shear strain by adjusting the interpretation of E). The stress-intensity-factor is related to the load and crack length according to Eq. 4.1 or Eq. 6.1 becomes

$$\varepsilon = \frac{P \cdot f(a)}{E \sqrt{2r}} \qquad 6.2$$

Let the rate of crack growth or crack velocity, v, be written in terms of the rate of change of r as well as a. That is:

$$v = \frac{-dr}{dt} = \frac{da}{dt} \qquad 6.3$$

Thus it is noted that dr and da are equivalent.

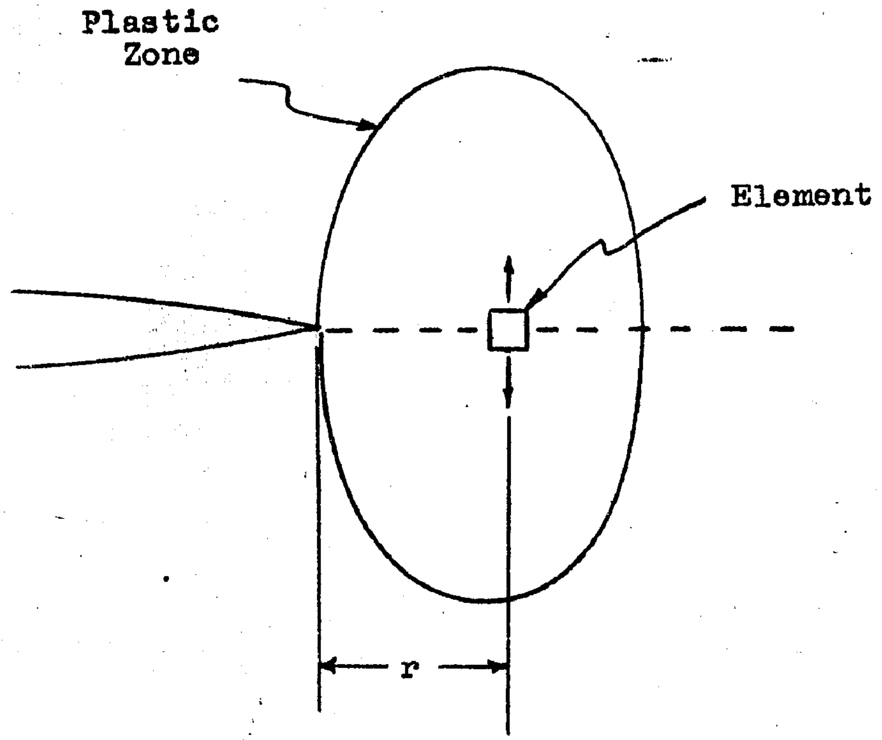

Fig. 6.2

An Element accumulating Damage within
the Plastic Zone

Differentiating Eq. 6.2 to obtain the rate at which total strain is occurring in the element gives:

$$\dot{\varepsilon} = \frac{\dot{P}}{E}\frac{f(a)}{\sqrt{2r}} + \frac{P}{E}\left[\frac{f'(a)}{\sqrt{2r}} + \frac{f(a)}{(2r)^{3/2}}\right]\frac{da}{dt} \qquad 6.4$$

But it was shown in sections 4.1 through 4.4 that the crack growth rate, and consequently the rate of accumulation of damage, depends only on the quasi-stationary variation of the stress-intensity-factor; i.e. with the crack length considered constant. Therefore, it is justifiable to ignore the second term on the right hand side of Eq. 6.4 in computing the rate of strain for damage analyses. Eq. 6.4 is reduced to

$$\dot{\varepsilon} = \frac{\dot{P}\, f(a)}{E\sqrt{2r}} \qquad 6.5$$

However, the rate of total strain, $\dot{\varepsilon}$, must now be reinterpreted to obtain the rate of plastic strain, $\dot{\varepsilon}_p$. Referring to Figure 6.1, increments of strain are either wholly elastic or wholly plastic for an ideal elasto-plastic material.

Therefore, during the time in which plastic increments of strain occur, Eq. 6.5 gives the desired rate of plastic straining. As a consequence attention must be given to the determination of the particular increments of strain which are plastic.

Eq. 6.5 states that an increment of strain corresponds to an incremental change in the magnitude of the external load, P. Moreover, from Figure 6.1 it may be noted that elastic strain occurs following any reversal in sense of strain increments, until either the strain proceeds in the reversed direction an amount, $2\varepsilon_{y.p.}$, or returns to the point of reversal and proceeds in the original sense. All other increments of strain are plastic. Since strain and load are analogous according to Eq. 6.2, the preceding statement may be reinterpreted. Wholly plastic strains are sustained in the element considered from changes in load as expressed by Eq. 6.5, except for the changes in load following a reversal in sense of the increments of loading and until the load has proceeded in the reversed sense an amount,

$$H = \frac{2\varepsilon_{y.p.} E \sqrt{2r}}{f(a)} \qquad 6.6$$

or until it returns to the point of reversal and proceeds in the original sense. Eq. 6.6 was computed directly from Eq. 6.2 to obtain the extent of reversal in load, H, from the point of reversal corresponding to a reversal in strain of $2\varepsilon_{y.p.}$.

Therefore, Eq. 6.5 may be used to compute increments of plastic strain if only those incremental changes in load which cause plastic strain are included in $\dot{P}$. With that in mind, the average or probable rate of accumulation of plastic strain, $\dot{\varepsilon}_p$, in the element of interest is

$$\dot{\varepsilon}_p = \frac{T(H)\ f(a)}{E\ \sqrt{2r}} \qquad 6.7$$

where T(H) is the average rate of rise and fall of the load excluding those rises or falls following a reversal as explained above. That is to say that, referring to Figure 6.3, T(H) may be computed from a load time history by determining its total average rise and fall per unit time excluding any rises or falls within a load interval, H, following a reversal. That means that on Figure 6.3, the rises and falls for the heavied portion of the curve should be counted and divided by the total time to compute the average rate of rise and fall to be counted, T(H). Now as time proceeds the total plastic strain in the element of interest will reach the critical plastic strain, $\varepsilon_{p_{cr}}$, i.e.

$$\varepsilon_{p_{cr}} = \int_0^{t_{cr}} \dot{\varepsilon}_p \, dt \qquad 6.8$$

During that time the crack grows and according to Eq. 6.3 increments of time correspond to increments of r by:

$$dt = -\frac{dr}{v} \qquad 6.9$$

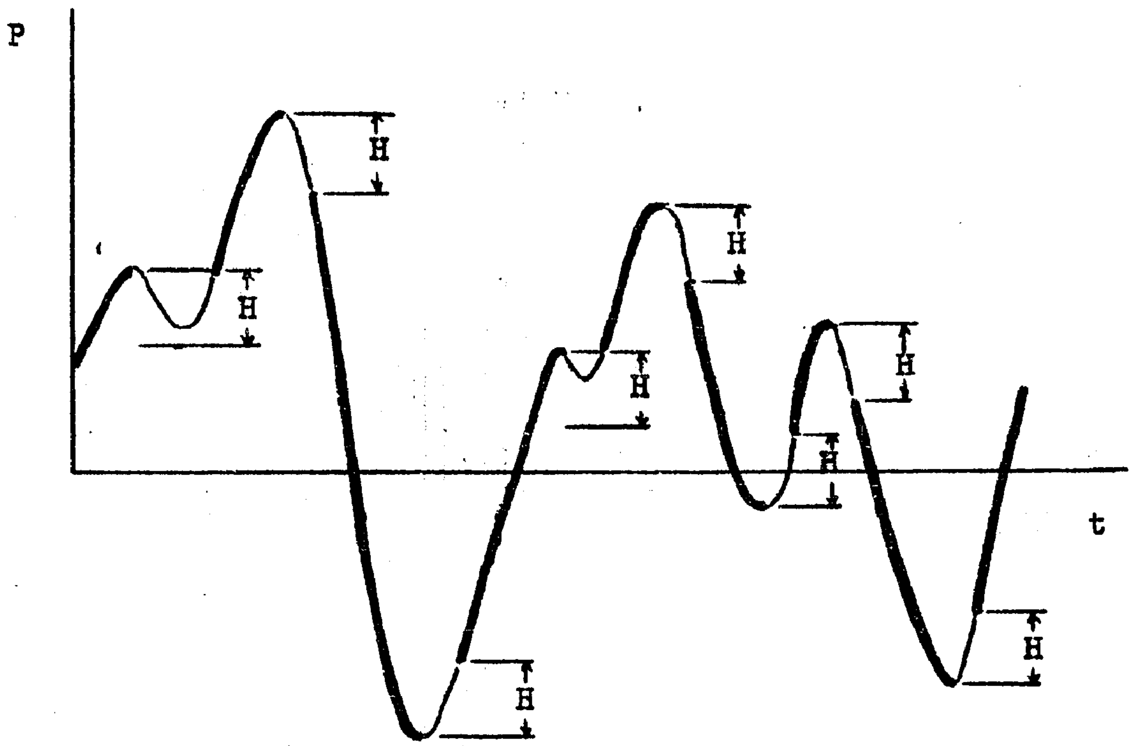

Fig. 6.3

The counted (heavied) Rise and Fall in an Arbitrary Load Time History

Substituting Eqs. 6.7 and 6.9 into 6.8 gives:

$$\varepsilon_{p_{cr}} = \int_0^\infty \frac{T(H)\ f(a)}{E\ \sqrt{2r}} \cdot \frac{dr}{v} \qquad 6.10$$

During the time that the element of interest is within the zone of plastic deformation, the crack length changes only a small amount, i.e. the width of the plastic zone, which via Eq. 3.13 and analogous expressions is small compared to the crack length itself. Thus f(a) may be considered as a constant with respect to the integration in Eq. 6.10. Moreover, the crack growth rate, v, is in a like manner constant during a change of crack length of one plastic zone width. The integral in Eq. 6.10 is simplified by these observations and becomes

$$\varepsilon_{p_{cr}} = \frac{f(a)}{E\ v} \int_0^\infty T(H)\ \frac{dr}{\sqrt{2r}} \qquad 6.11$$

Reusing the argument preceding Eq. 6.11, Eq. 6.6 may be differentiated to give:

$$\frac{dr}{\sqrt{2r}} = \frac{f(a)}{2\,\varepsilon_{y.p.}E}\ dH \qquad 6.12$$

which on substitution into Eq. 6.11 results in:

$$v = \frac{da}{dt} = \frac{\left[f(a)\right]^2}{2\,\varepsilon_{p_{cr}}\,\varepsilon_{yp}\ E} \int_0^\infty T(H)\ dH \qquad 6.13$$

This result, Eq. 6.13, is the desired relationship expressing the crack growth rate, $\frac{da}{dt}$; in terms of a property the load time history, $\int_0^\infty T(H)\, dH$; a configuration factor, f(a), which accounts for variation of the stress intensity factor with crack length; and a single material constant, $2\varepsilon_{p_{cr}} \varepsilon_{y.p.} E$, (upon lumping these factors together).

It is interesting to note that this expression for crack growth rate, Eq. 6.13, quite obviously meets all of the listed requirements in section 6.2 with the possible exception of number (4). Its agreement or disagreement with (4) is yet to be established. That question can be settled most easily by specializing Eq. 6.13 to the case of sinusoidal loading.

For periodic loading Eq. 6.13 may be rewritten as:

$$\frac{da}{dN} = \frac{[f(a)]^2}{m_o} \int_0^\infty T_N(H) \quad dH \qquad 6.14$$

where m_o is a lumped material constant and $T_N(H)$ is the counted rise and fall in the load per cycle. It remains to compute the integral of $T_N(H)$ in Eq. 6.14 for sinusoidal loading.

Figure 6.4 shows a sinusoidal loading of range Δ P with one cycle indicated where $T_N(H)$ is the rise and fall of the heavied portion of the curve. It can be seen that:

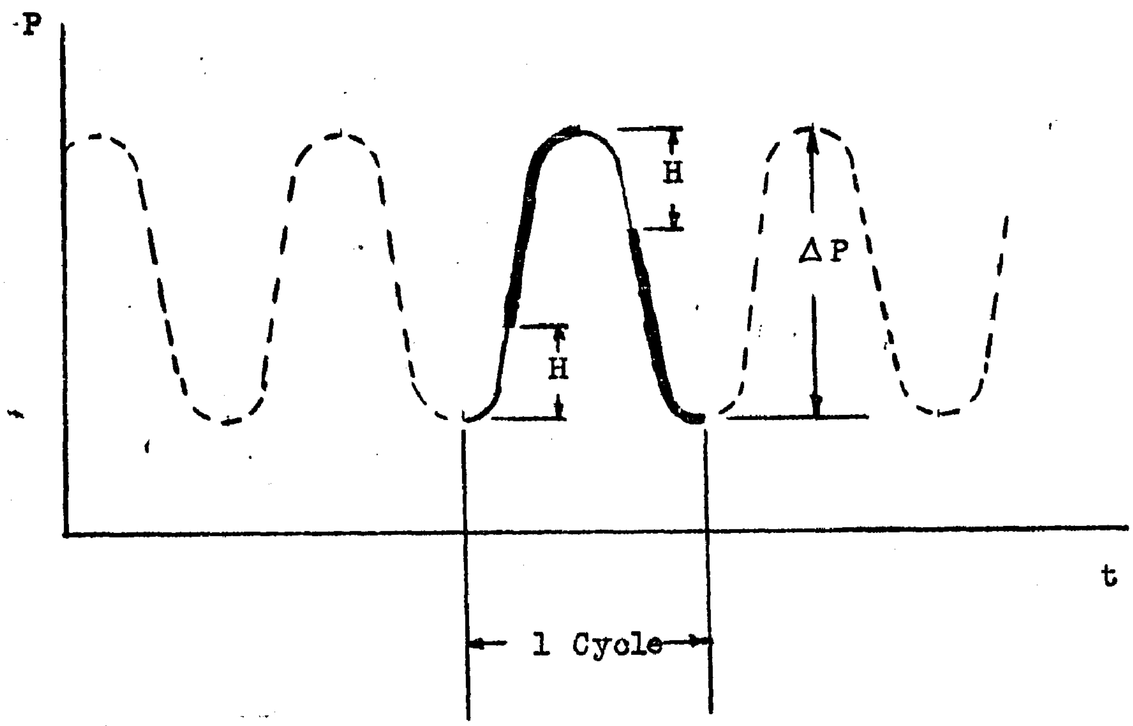

Fig. 6.4

The counted (heavied) Rise and Fall in a Cycle of Sinusoidal Loading

$$T_N(H) = 2\left[\Delta P - H\right] \quad (H \leq \Delta P)$$

$$= 0 \quad (H \geq \Delta P) \qquad 6.15$$

Therefore the integral becomes

$$\int_0^{\infty} T_N(H)\, dH = \int_0^{\Delta P} 2\left[\Delta P - H\right] dH + 0 \qquad 6.16$$

$$= \Delta P^2$$

Substituting Eq. 6.16 into Eq. 6.14 leads to

$$\frac{da}{dN} = \frac{\left[\Delta P \cdot f(a)\right]^2}{m_o} \qquad 6.17$$

Referring to Eq. 4.1 it can be readily seen that

$$\Delta K = \Delta P \cdot f(a) \qquad 6.18$$

Whereupon Eq. 6.17 reads

$$\frac{da}{dN} = \frac{\Delta K^2}{m_o} \qquad 6.19$$

Eqs. 6.17 and 6.19 are the special forms for sinusoidal loadings of the more general relationship Eq. 6.14. It can now be seen that this model leads to a dependency of the crack growth rate on the square of the range of rises and falls of the load (or stress intensity factor). This result is in opposition to requirement number (4) of section 6.2, and is further clarified by comparing Eq. 6.19 to Eq. 5.4 and the data on the Figures in Chapter V. Therefore, the model does not meet the requirement of agreeing with the broad trend of data from sinusoidal loading experiments.

However, when Eq. 6.19 is further specialized for the configuration of Figure 2.1 by utilizing Eq. 5.7, it results in:

$$\frac{da}{dN} = \frac{(\Delta\sigma)^2 a}{m_o} \qquad 6.20$$

which is identical to Liu's results, Eq. 2.15 and is in general agreement with the proposals of Head (section 2.2), Frost (section 2.3) and Valluri (section 2.7). Therefore, though the writer disagrees with a model exhibiting such a behavior, it is in fact in agreement with much of the recent work done by others in the field. It not only consolidates their work but in its general form, Eq. 6.13 or 6.14, it is much more versatile than their analyses. Therefore, it may be regarded as a generalization of previous theories and as such might be subjected to a some further investigation to determine its broader implications.

For example, let Eq. 6.14 be considered for specialization to the case of narrow band Gaussian random loadings. Bendat (Ref. 36) notes that narrow band random loading is characterized by a sine-wave-like load time curve with a slowly varying range or amplitude. He observes that the probability density function q(R) for the amplitude, R, about the mean load is:

$$q(R) = \frac{R}{\sigma_P^2} e^{-\left(\frac{1}{2}\frac{R^2}{\sigma_P^2}\right)} \qquad 6.21$$

From Eq. 6.16 it is observed that a single cycle of the sine-like curve of amplitude, R, results in a contribution of $(2R)^2$ to the integral of $T_N(H)$. But the integral must be computed as an average for the distribution of peaks given in Eq. 6.21 or

$$\int_0^\infty T_N(H)\, dH = \int_0^\infty (2R)^2\, q(R)\, dR \qquad 6.22$$

which can be integrated and (in agreement with Bendat) is

$$\int_0^\infty T_N(H)\, dH = 8\overline{\sigma_P^2} \qquad 6.23$$

where $\overline{\sigma_P^2}$ is the mean square (or standard) deviation of the load about its mean as defined in Eq. 4.30. Substituting Eq. 6.23 into Eq. 6.14 results in

$$\frac{da}{dN} = \frac{8 \overline{\sigma_P^2} \left[f(a)\right]^2}{m_o} \qquad 6.24$$

which is the expression for crack growth rate under a narrow band random loading for the model under discussion. Further, it can be noted from Eq. 4.1 that the mean square deviation in K about its mean is

$$\overline{\sigma_K^2} = \overline{\sigma_P^2} \left[f(a) \right]^2 \qquad 6.25$$

which upon substitution into Eq. 6.24 gives:

$$\frac{da}{dN} = \frac{8 \overline{\sigma_K^2}}{m_o} \qquad 6.26$$

This result for narrow band random load, Eq. 6.26, can be compared directly to the result for sinusoidal loading, Eq. 6.14. Consequently, the model predicts that the crack growth rate in a material for a narrow band random loading and a sinusoidal loading should be equal if 8 $\overline{\sigma_K^2}$ for the random loading is equal to ΔK^2 for the sinusoidal loading. Such a comparison would be a simple matter to examine if some narrow band random loading crack growth data were available; they are not, but this should provide some additional motivation for experimenters.
Specifically, the experiment suggested here is to compare the rate of crack growth in identical specimens in the

configuration of Figure 4.1 under the same mean load and where the R.M.S. deviation of the sinusoidal load is the same as the R.M.S. deviation of the narrow band random load and where both loadings are of the same characteristic frequency. If the crack growth rates are also the same, that result indicates agreement with the model. This is merely a first suggestion of a test to investigate models in the manner in which they suit their ultimate objective, their ability to predict differences in crack growth rates for various wave forms of loading. Suggestions can easily be added for other types of loading by computing the integral of T(H) for new loadings for incorporation in Eq. 6.13 or 6.14. The methods employed above are adequate for deterministic loadings and Beer et. al. (Refs. 37, 38 and 41) have discussed means of computing like integrals for rangdom loadings in general.

In conclusion the model presented in this section is a versatile one whose special case results are in agreement with the majority of the previous investigations of consequence. However, this model does not agree with the broad trend of crack growth data noted in Chapter V. Moreover, it may be noted here that the common element of this model and previous investigations which leads to disagreement with the trend of data in Chapter V is the assumption of the equivalent of the elastic strain distribution within the plastic zone at the tip of the crack. Therefore, that assumption will be altered in the model to be discussed subsequently.

6.5 A Model Resulting in 4th Power Dependency on Load Range.

In the preceding section a model based on assuming a strain distribution equivalent to the elastic strain distribution, resulted in a prediction of the dependency of crack growth rate on the 2nd power of load range. Moreover, it was noted that all the previous investigations which make this or an equivalent assumption also come to the same result. Since the broad trend of crack growth rate data appears to depend on the 4th power of the load range, it seems reasonable to attempt to modify or avoid that assumption. The model to be discussed now is one which avoids the assumption of a specific strain distribution completely.

Consider two or more crack tips in a given material which are surrounded by stress fields of different intensities. The distribution of the stresses outside the plastic zone will be the same for each crack as noted in Chapter III. Hence it can be seen that the plasticity problem of determining the stresses and strains within the plastic zone is the same except for a scale factor of absolute size which depends on the magnitude of the stress-intensity-factor. The result of this view is equivalent to assuming that geometrically similar plasticity problems produce geometrically similar distributions of stresses and strains.

The size of the plastic zone for a single load application may be estimated by Eq. 3.12. But in analyzing crack propagation the interest lies in examining the resulting size of the plastic zone caused by the change (rise or fall) in the load following a reversal in sense of the increments of loading and until the sense again reverses. Moreover, upon reversal of the sense of increments of load the stress at any point which has yielded (or nearly yielded) must change an amount $2\sigma_{y.p.}$ before subsequent yielding occurs (see Fig. 6.1). Therefore, Eq. 3.12 must be modified by introducing the rise or fall in the stress-intensity-factor, h_K, for K and $2\sigma_{y.p.}$ for $\sigma_{y.p.}$, in order to estimate the size of the yield zone w_h which results from a single rise or fall in load. The result is

$$w_h = \frac{(h_K)^2}{8\,\sigma_{y.p.}^2} \qquad 6.27$$

In a fluctuating load a sequence of rises and falls occurs and Eq. 6.27 is proposed as an estimate plastic zone size which occurs for each reversal.

In addition a new point of view of Coffin's critical plastic strain criteria (Ref. 39) will be employed. For a material behaving as in Figure 6.1, i.e. as an ideal elastoplastic material, the critical accumulated plastic strain is equivalent to a critical amount of energy absorption or dissipation. Moreover, the absorption measured in tests employed by Coffin to obtain this criteria was for whole specimens and not single infinitesimal elements near crack tips. It seems feasible to conclude that perhaps Coffin's criteria should only be applied over whole specimens or regions of specimens. Let the view be taken that it should be applied to the whole region surrounding a crack tip in analyzing crack growth. This view is consistent with assuming that the energy dissipation per unit crack extension is a constant, which results in:

$$\frac{da}{dt} \approx \frac{dW}{dt} \qquad 6.28$$

where W is plastic work or energy absorbed near the crack tip.

Now, if as noted earlier in this section the crack tip plasticity problems are geometrically similar, then the work absorbed, W_h, for any particular rise or fall will be proportional to the volume of the plastic zone. Since the crack tip strain field is a plane (2-dimensional) field, this statement results in

$$W_h = c_7 \; w_h^2 \qquad 6.29$$

Incorporating Eq. 6.27 in 6.29 gives

$$W_h = c_7 \left[\frac{(h_K)^2}{8 \; \sigma_{y.p.}^2} \right]^2 \qquad 6.30$$

If $\bar{Q}$ is the average number of peaks or troughs in load time curve per ~~unit~~ time, then $2\bar{Q}$ is the average number of rises and falls. Therefore, the average rate at which work is absorbed is:

$$\overline{\frac{dW}{dt}} = \frac{\overline{W_h}}{2\bar{Q}} \qquad 6.31$$

interpreting $\frac{da}{dt}$ as the average rate of crack growth in Eq. 6.28, Eqs. 6.30 and 6.31 may be substituted intc it, which gives:

$$\frac{da}{dt} = \frac{c_8}{128 \; \sigma_{y.p.}^4} \cdot \frac{\overline{h_K^4}}{\bar{Q}} \qquad 6.32$$

Eq. 6.32 may be simplified by lumping the material parameters or

$$\frac{da}{dt} = \frac{\overline{h_K^{\;4}}}{m_1 \; \bar{Q}} \qquad 6.33$$

The desired result of a dependency of the crack growth rate on the 4th power of the fluctuations in the stress-intensity-factor, h_K, (or, as a consequence, the load) has been obtained. However, reviewing the list of restrictions for the results of model analysis in section 6.2, it is noted that number (3) is not satisfied. This model is not sensitive to the sequence of rises and falls in loading, which has been observed experimentally to have an effect on the growth rate of cracks.

The reason for the lack of sequence dependency in this model can be traced to the simplifying assumptions associated with elastic unloading following a reversal in sense of increments of loading. That is to say that the residual stress and strains of prior yielded zones affect the formation of new overlapping plastic zones at a crack tip. It is improbable to propose a detailed analysis of these effects for this model, since an essential part of the model's development was the avoiding of an assumption of a specific strain distribution.

However, before dismissing the analysis of this model, its advantages should also be clarified. First, the model does allow a reasonable estimate of the total work dissipated in crack extension (short of sequence dependency). It shows that such an estimate depends on the distribution of rises and falls in any particular loading.

In addition all other authors (Refs. 3, 4, 5, 12, 14, 16 and 17) proposed models and compared results with data from sinusoidal loading tests alone. If, as they implied, such a comparison is by itself significant (without considering the restrictions of section 6.2) then it is appropriate to test Eq. 6.33 by specializing it for sinusoidal loading. Let a unit of time be one cycle whereupon dt is dN and $\overline{Q}$ is 1. Moreover, for a sinusoidal loading $h_K = \Delta K$ for each rise and fall. Introducing these developments into Eq. 6.33 results in

$$\frac{da}{dN} = \frac{(\Delta K)^4}{m_1} \qquad 6.34$$

This result is identical to the empirical law, Eq. 5.4, followed by the broad trend of the data on Figures 5.1 through 5.4. Therefore, though some shortcomings of this model have already been discussed here, this model is in fact the only entirely continuum based model of crack propagation to date which agrees with the broad trend of data from sinusoidal loading tests. Moreover, if one should disagree with validity of this trend, then the previous model in section 6.4 is one which combines and generalizes all the other models which have proposed to date. Consequently, the two models proposed in this dissertation occupy a rather advantageous position compared to previous proposals.

Anticipating the likelihood that narrow band random loading test data may become available, it is appropriate to specialize Eq. 6.33 for that particular type of loading. First, regard one cycle (or positive zero crossing) as a unit of time, then dt is dN and $\overline{Q}$ is one or

$$\frac{da}{dN} = \frac{\overline{(h_K^4)}}{m_1} \qquad 6.35$$

Referring to Eq. 4.1 (or better yet, Eq. 4.2) it is observed that

$$h_K = h_P \cdot f(a) \qquad 6.36$$

Substituting Eq. 6.36 into 6.35 gives

$$\frac{da}{dN} = \frac{\overline{(h_P^4)}\left[f(a)\right]^4}{m_1} \qquad 6.37$$

where h_P is the rise or fall between peaks or

$$h_P = 2R \qquad 6.38$$

The R's are the peak heights of the narrow band loading which are distributed according to Eq. 6.21. Therefore,

$$\overline{(h_P^4)} = 16 \int_0^{\infty} R^4 \cdot q(R) \quad dR \qquad 6.39$$

which upon integration results in

$$(\overline{h_P^4}) = 128 \left[(\overline{\sigma_P^2}) \right]^2 \qquad 6.40$$

Substituting this result into Eq. 6.37 gives

$$\frac{da}{dN} = \frac{128 \left[(\overline{\sigma_P^2}) \right]^2 \left[f(a) \right]^4}{m_1} \qquad 6.41$$

Making use of Eq. 6.25 leads to

$$\frac{da}{dN} = \frac{128 \left[\overline{\sigma_K^2} \right]^2}{m_1} \qquad 6.42$$

Upon comparison with Eq. 6.34, Eq. 6.42 predicts that the crack growth rate caused by a narrow band random load will be twice that of a sinusoidal load of the same R.M.S. intensity if all other factors are the same. (For comparison, the model in section 6.4 predicted a factor of one). It will be an easy matter to examine this result when narrow band random loading test data become available.

6.6 Remarks on Model Analysis

In the preceding sections it was shown that the two new models discussed have decided advantages over those proposed by other authors. In summary the advantages of the new models are that:

(1) They are developed in a manner so that they may be

applied to a loading of arbitrary wave form.

(2) They are based entirely on continuum concepts and the stress-intensity-factor concept is incorporated which allows more general application to bodies of various materials and various configurations.

(3) They are based on assumptions which result in predictions of the broad trends of crack growth characteristics without being encumbered by the minor variables involved (such as mean load, frequency, or microprocess variables).

Therefore the two models which have been presented are a first step in the development of a complete analysis of the effect of the wave form of the loading on crack growth. These models should be examined by comparing their predictions with the rate of crack growth determined experimentally under loads with various wave forms. As has been suggested earlier, narrow band random loading and then broad band random loads with various shaped power spectra seem to be the most advantageous loadings to suggest for future tests. These random loads provide a variety of sequences (wave forms) of rises and falls of load within a single type of loading which will also test applicability of these theories to loading of a statistical or random nature. Only after such test results are available will it be possible to reassess the underlying assumptions of the models presented in order to improve them. Further analyses of new models would be rather speculative at this point.

At least the openings for improvements have been made clear in the analysis of the two models presented. Perhaps most important is the incorporation of a more realistic strain distribution within the plastic zone at the crack tip in subsequent models. Though these openings for improvement have been discussed at length, they should not be allowed to overshadow the advances made here in developing models. Most important is the fact that it has been shown that entirely continuum based models are both generally acceptable for a description of crack growth and do in fact allow prediction of the broad trend of crack growth rates observed in the laboratory to date.

CHAPTER VII - CONCLUSIONS

The following general results and conclusions have been established in this work.

(1) It has been found that the crack propagation laws presented by Head, Frost and Dugdale, and Liu do not agree with the broad trend of test results.

(2) The method of analysis of crack growth of McEvilly and Illg (NASA) was found to be correct though open to improvement in generality.

(3) The use of crack-tip stress-intensity-factors as the local stress parameter in analyzing crack growth was shown to be appropriate.

(4) Crack growth rate data from sinusoidal loading tests show a positive correlation when compared on the basis of equivalent stress-intensity-factor time-histories in a given material.

(5) It was found that the non-stationary character of stress-intensity-factor time-histories, caused by change in crack length, does not affect the rate of crack growth in the test data available to date. That is to say that the properties of the time variations of stress-intensity-factors may be computed considering the crack length constant in analyzing fatigue crack growth.

(6) It was shown that the discontinuous nature of fatigue crack growth does not exclude an analysis of crack growth as a continuous process for the purpose of making life predictions.

(7) It was observed that stress-intensity-factor methods may be directly applied to random loading in which case the conditions for equivalence of stress-intensity-factor time-histories were established.

(8) An empirical crack propagation theory was established which describes the broad trend of crack growth behavior. In essence this theory is the observation that the rate of crack growth is proportional to the fourth power of the amplitude of variation of the stress-intensity-factor for a given material and type of load time-history.

(9) The observation that a continuum based empirical law with a single material constant is in general agreement with test data for all of the variety of materials tested to date leads to the conclusion that:

It is plausable to consider continuum based model analyses of crack propagation in order to predict differences in the rates of crack growth under various types of load time-histories.

(10) Two models of crack growth were presented which are adaptable to any wave form of loading including random loads. The first of the models resulted in a generalization

and unification of the models proposed by previous investigators, but it was in disagreement with the broad trend of available data. The second model showed that it is possible to devise continuum based models in complete agreement with the broad trend of data observed by the empirical law given earlier.

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