Power Measurements And Optimization Of Mixing In Solids Blending.

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POWER MEASUREMENTS AND OPTIMIZATION OF MIXING IN SOLIDS BLENDING

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DEDICATION

I would like to dedicate this thesis to my wife Rosalie, as a token of my love and appreciation for the many sacrifices she has borne and the generous understanding she has shown during the course of this work.

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The author wishes to thank Professor Curtis W. Clump for his friendly encouragement and guidance of this study. Special thanks are also extended to Professor Sutton Monro as well as to other members of the Committee for their helpful advice and continued interest.

I would also like to express my sincere appreciation to Ivan J. Taylor for useful discussions and for loan of valuable equipment.

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1. ABSTRACT

The quantitative relationships between four of the important variables involved in the mixing of dry solids and the Degree of Mixing were investigated. Once the functional relationship became known an optimization procedure was followed to arrive at the best levels of the variables which would yield a good mix. The experimental plan involved employing a one-third replicate of a three-level factorial design. The four variables studied were, the rate of rotation, the volume per cent of the mixer filled, the angle of inclination of the mixer, and the number of mixer rotations. All these factors were varied simultaneously.

In addition the power requirements were also determined for every combination of levels used in the experimentation. Here again, a quantitative relationship between the power consumed and the variables was evolved.

A cylindrical mixer constructed from "Plexiglas", capable of being adjusted to a given set of experimental needs was utilized to mix undyed and dyed sand. All runs were conducted using a one-to-one ratio of white to colored sand. Random samples were drawn after completion of every run to determine the percentage of the dyed sand in the blended product. The standard deviation of the composition of the colored sand was calculated and was used as a measure of the Degree of Mixing.

Power consumption was measured by employing a surface-strain-gage device bonded on to the drive-shaft of the mixer. When the shaft is loaded in torsion, the torque pick-up translates the torsion into changes in electrical output; this output was amplified and recorded on an oscillograph.

The data obtained were analyzed by the Analysis of Variance techniques and numerous mathematical models were constructed and examined with the help of a high-speed digital computer.

For the range of conditions that were investigated, it was found that with regard to the Degree of Mixing, the factor of prime significance is the volume per cent of the mixer charged. Next in order of importance comes the number of mixer rotations. In addition, the rate of rotation-number of rotations interaction as well as the rate of rotation-volume per cent interaction was found to be of some significance. However, for the power consumption, the speed of rotation is most important, followed by the volume per cent of the mixer filled, which in turn is followed by the angle of inclination. Furthermore the interactions that appeared to be significant were the speed-volume interaction, the speed-angle interaction and the volume-angle interaction.

Quantitative expressions incorporating these effects were obtained to correlate the Degree of Mixing with the variables on the one hand, and power required and the input variables on the other. The relationship between the Degree of Mixing measured by the standard deviation S* and the variables is given by

$$S^* = 93.027 - 4.324B + 0.0456B^2 + 0.708G - 0.009G^2 - 0.0179AG + 0.00997AB .$$

The following equation represents the relationship between Power consumed and the input variables

$$P^* \doteq 0.00454 + 0.000507A + 0.000109B - 0.00111F + 0.000033AB.$$

In the above equations S* represents the Degree of Mixing or the standard deviation and P* is the Power required measured in terms of horsepower. As for the variables, A stands for the rate of rotation, B is the per cent of the mixer charged, F denotes the angle of inclination and G is equal to the number of mixer rotations.

The predicted optimum conditions were successfully checked out by subsequent experimentation.

The optimum conditions for obtaining the maximum Degree of Mixing defined by the minimum value of the standard deviation were found to be a rate of fifteen rotations per minute, with the mixer loaded to about forty-six per cent of its total volume, a zero degree angle of inclination and a value of thirty for the number of mixer rotations.

2. INTRODUCTION

The terms "mixing" and "blending" are used interchangeably and may be looked upon as representing a process in which two or more materials are interspersed in space with respect to one another. In spite of the fact that the mixing of solids is a part of many industrial operations, it has received little attention in the literature, relative to other unit operations. Among others, the industries involving the manufacture of food, drugs, glass, pigments, soap powders, paints and plastics, fertilizers and explosives, all employ the blending operation in various processing steps. However, a lot more needs to be known about the fundamental aspects of solids mixing.

Mixing has been considered to be a statistical process for a long time and the majority of the existing literature deals with finding a means by which the pattern of mixing can be followed. These methods involve an index commonly called the "Degree of Mixing". This index therefore, is essentially the result of analyses of a number of samples, taken either at intervals during or at the end of the mixing process, for the composition or other property of one component; this permits us to gauge the quality of the mixture.

The various statistical degrees of mixing being employed in this area are primarily the mean, standard deviation or variance of the composition of one component throughout the samples from the mean composition of that component in the mixture.

The measure for the "Degree of Mixing" employed in this study is the standard deviation, since it represents a variation of the uniformity within the mixture. In a mixture of colored and uncolored sands, for example, by counting the number of dyed and undyed particles, the percentage of dyed particles in each spot sample can be calculated.

Hence,

$$S^* = \sqrt{\frac{\sum_{i=1}^{n} (p_i - P)^2}{n}}$$

where p_i = the percentage of dyed component in sample i

P = the population mean (the actual percentage of the dyed component in the mixture as known from charging the cylinder)

n = the number of spot samples

and S^* = the estimate of the population standard deviation or the Degree of Mixing.

A low value of S* indicates that the product is a better mix than one that has a higher value of S*.

Mechanism of Mixing

Lacey (19) has suggested that the mixing of solids occurs by three principal mechanisms:

1. the transfer of groups of adjacent particles from one location to another - "convective mixing"
2. the distribution of particles over a freshly developed area or surface - "diffusive mixing"
3. the setting up of slip planes within the mass - "shear mixing".

The extent to which the mechanisms occur in any given mixer is controlled by the mode of operation of the mixer. For example, in a horizontal drum mixer, mixing is primarily of a "diffusional nature". However, this analogy is broad and does not consider all possible ways in which mixing is effected.

Though one index may not completely describe all the mixing action occurring in any one mixer, an index such as the standard deviation allows the mixing of solids to be followed with reasonable accuracy.

Among the important contributions to the literature in the area of solids mixing, may be mentioned the detailed account by Weidenbaum (28,29). When perusing the literature, one finds that there is no consistent criterion that has been used by various workers in the field, to express the state of the final product. However, when one attempts any kind of interpretation of the data, the serious handicap is not so much the variety of ways in which the mixedness is measured, as much as the disadvantages that go along with the classical forms of experimental procedures. Thus the long-standing and classic idea of experimentation is to vary one of the many variables at a time, keeping all the others constant at a set of arbitrarily chosen conditions. But what is not often recognized is that in order to be able to obtain a more realistic picture of the effect of any given variable, it is necessary for the other variables to be simultaneously varied. This is because the results obtained from experimentation keeping all but one variable constant need not be the same as when two or more variables are varied simultaneously. Herein lies the advantage in the utilization of statistically designed experiments. They help elucidate not only the effect of each factor but also the possible ways in which each factor may be modified by the variation of others. To cite an example in a recent paper Donald and Roseman (12) clearly state that an arbitrary standard operating condition was chosen and one variable investigated at a time and further add that the results do not show whether the indicated trends apply at other than the single condition investigated. Obviously the utility of such studies is quite limited.

Hence, an urgent need exists for evolving quantitative data based upon a well designed and planned series of experiments wherein such interactions can be examined.

Because of the classical methods of experimentation, it has not been possible to amalgamate the different pockets of information available in the literature into a unified whole along strictly scientific lines.

Besides, the literature on power requirements in solids blending is very sparse indeed. Weidenbaum (28) notes that Oyama measured the power consumption for certain systems under different mixing conditions. But, unfortunately no further details are given and the results are presented only in a graphical manner with the speed of rotation on the abscissa and the power consumed in kg. - m./sec. on the ordinate. Thus one would have difficulty in quantitatively predicting the power requirements under a given set of conditions. Therefore one can easily recognize the importance of gaining knowledge with respect to the power consumption in solids mixing.

More recently, Hager (16, 17) has explored the effect of certain variables upon blending utilizing a statistically designed experiment. It consisted of experiments which are components of a two-level factorial design, wherein the effect of seven variables was investigated. However from any two-level series of experiments, one can only qualitatively indicate the relative importance of the variables.

Hence, the objectives of this work are two-fold. Firstly to quantitatively relate the Degree of Mixing to the significant variables of interest and to their interactions thereby obtaining a functional relationship which upon subsequent treatment will yield the best or optimum conditions for producing a good random mixture. Secondly, the aim is to determine the dependence of power consumption on the operating variables. Once such a quantitative expression for the power requirements becomes available, then it will be possible to predict the power needs and also to discuss its relationship with S*, the Degree of Mixing, for any given set of operating conditions.

3. EXPERIMENTAL PROGRAM

3.1 Plan of Experiments

Utilizing a two-level factorial design without replication, Hager (16,17) has investigated the effects of seven variables on dry solids blending. The seven variables examined were:

A. The rate of rotation

B. The per cent of mixer charged, by volume

C. The number of baffles

D. The volume per cent of each component

E. The height of baffles

F. The angle of inclination of the cylinder

G. The number of mixer rotations

The experiments were conducted using a right cylindrical mixer.

The standard deviation of the composition of one of the components from the mean composition of that component, was calculated and used as a measure of the Degree of Mixing. The data was analyzed by the analysis of variance with half-normal plots constructed to aid in the analysis.

The conclusions from this two-level series of runs were that the main effects of A,B,D,F, and G, as listed above, are significant; whereas the effect of C and E, were not significant.

Even though the effect of Cand E were not significant relative to the influence of the other variables on the Degree of Mixing, it had been concluded that increasing the number of baffles improves the mixing for a 30 per cent charged cylinder, but hinders mixing when the cylinder is 50 per cent charged. Similarly the baffle height was also found to be unimportant. Since the influence of variables C and E had not been found to be significant, it was felt that no useful purpose would be served by retaining them in the present study.

Thus only five of the original seven variables are worthy of further consideration and serious investigation.

For the significant factors, the favorable levels (out of the 2 levels for each factor) are as indicated in Table 1.

The data from a two-level factorial experiment if fitted to a model will yield only a linear function. Optimization of a linear function is not possible. At least three levels for each of the variables, are essential for any optimization study. Because of the fact that many variables are involved, this would result in an unreasonably large number of runs. Therefore, a complete factorial design would have been impractical. Hence a plan involving a three-level factorial design was adopted for this study.

TABLE 1

FAVORABLE LEVELS OF SIGNIFICANT FACTORS

Factor	Level	
	High	Low
A. Rate of Rotation	58 (only at B = 30 F = 14 G = 30)	21 (everywhere else)
B. Percent Mixer Charged	50 (only at a. A=21, F=0, G=9 b. A=21, F=14, G=30 c. A=58, F=0, G=30 B has no effect at d. A=21, F=0, G=30)	30
D. Volume Percent of Dyed Component		20
F. Angle of Inclination		0°
G. Number of Rotations	30	

Having decided on a three-level design, the next question that needs to be answered pertains to the inclusion of the right variables viz. those that had been found to be significant. Upon closer scrutiny of the variables A, B, D, F and G, it is seen that the factor D, representing the percentage of the dyed component is not an inherent variable. This is due to the fact that, when the value of D is less than 50 per cent, there is a greater probability for the resulting mixture to have a lower standard deviation (indicating that it is a better mix) than otherwise. The fact that the standard deviation is dependent on the percentage of the dyed component in the mixture is amply borne out by the experimental data published by Chudzikiewicz (6). Besides Donald and Roseman (12) have shown that the final degree of mixing is independent of the Proportion of the components. Thus the variable D was held at a value equal to 50 per cent for all the experiments in the present study. The remaining significant variables, therefore are A, B, F and G.

Initially it was thought that the variable G denoting the number of mixer rotations should also be fixed at a value of 30. This was due to the belief that mixing was essentially complete by the time this many rotations took place. However on re-examination of this earlier stand it was found that there might be danger in pre-fixing G at a value of 30. Were G to be fixed and not varied it would be impossible to talk about the quantitative main effect of G especially in the early stages of the mixing process when it is quite critical. In addition it would have masked the influence of interplay between G and the other variables. Hence it was decided to retain G as a variable.

In the case of the factor F which stands for the angle of inclination, one naturally tends to assume that increasing the angle could not possibly aid the process of mixing. This is also borne out by the results obtained from the earlier study. But since these results were obtained from just a two-level study, a skeptical question that arises has to deal with not knowing whether there is an angle somewhere inbetween these levels that might be better. In order to clearly answer this question it was decided to retain the variable F for further investigation. Another important reason for this retention of F was the desire to learn about the variation of power with angle of inclination.

As for A which is the speed of rotation, there was little doubt that it should be examined further.

Thus there are four factors that need to be incorporated into a three-level factorial design. Rather than use the complete design, since it was felt that a fractional replicate of the complete design would be just as good in obtaining the desired information, a one-third replicate of a factorial experiment composed of the above-mentioned four significant factors each at three levels, was employed.

The form of the design for a complete factorial experiment from which to draw the particular combinations of the levels of the variables at which to conduct the experiments in the current series, is shown in Table 2.

In order to facilitate the selection of a representative fractional design from the parent arrangement, a completely randomized design was adopted as given in a National Bureau of Standards publication (9).

Information on similar designs as well as on pertinent mathematical discussions are available in standard texts on statistics (11, 8, 4, 18).

The pattern of the $1/3 \times 3^4$ factorial design is detailed in Table 3 which indicates tha various combination of the levels for the four factors A, B, F and G, at which the different experiments were carried out. The nomenclature is in keeping with those found in the literature and the numbers 0, 1 and 2 refer to the three levels of each variable.

TABLE 2

COMPLETE 3^4 FACTORIAL DESIGN IN A, B, F AND G

			G_0	G_1	G_2
A_0	B_0	F_0	0000*	0001	0002
		F_1	0010	0011	0012*
		F_2	0020	0021*	0022
	B_1	F_0	0100	0101	0102*
		F_1	0110	0111*	0112
		F_2	0120*	0121	0122
	B_2	F_0	0200	0201*	0202
		F_1	0210*	0211	0212
		F_2	0220	0221	0222*
A_1	B_0	F_0	1000	1001	1002*
		F_1	1010	1011*	1012
		F_2	1020*	1021	1022
	B_1	F_0	1100	1101*	1102
		F_1	1110*	1111	1112
		F_2	1120	1121	1122*
	B_2	F_0	1200*	1201	1202
		F_1	1210	1211	1212*
		F_2	1220	1221*	1222
A_2	B_0	F_0	2000	2001*	2002
		F_1	2010*	2011	2012
		F_2	2020	2021	2022*
	B_1	F_0	2100*	2101	2102
		F_1	2110	2111	2112*
		F_2	2120	2121*	2122
	B_2	F_0	2200	2201	2202*
		F_1	2210	2211*	2212
		F_2	2220*	2221	2222

TABLE 3

PLAN FOR 1/3 x 3^4 FACTORIAL DESIGN IN A, B, F AND G

EXPT. NO:	a b f g
1	0 0 0 0
2	1 2 1 2
3	2 1 2 1
4	0 0 2 1
5	1 2 0 0
6	2 1 1 2
7	0 0 1 2
8	1 2 2 1
9	2 1 0 0
10	0 1 0 2
11	1 0 1 1
12	2 2 2 0
13	0 1 2 0
14	1 0 0 2
15	2 2 1 1
16	0 1 1 1
17	1 0 2 0
18	2 2 0 2
19	0 2 0 1
20	1 1 1 0
21	2 0 2 2
22	0 2 2 2
23	1 1 0 1
24	2 0 1 0
25	0 2 1 0
26	1 1 2 2
27	2 0 0 1

On comparing Tables 2 and 3, it can be seen that those combinations marked with an asterisk in Table 2 have gone into the making of Table 3. This gives one an idea of the layout of the current design.

3.2 Selection of Levels

The two existing levels for each of the variables that had been used in the previous study, had been selected on the basis of published literature wherever possible. Considering, first of all the variable A which represents the speed of rotation of the mixer in revolutions per minute, one of the conclusions that had been reached was that a speed of 21 rather than 58 RPM gave a better mix. However since this does not enable us to determine the overall trend of the effect of A, a speed of 35 RPM was picked as the new third level of A. Moreover because the purpose on hand is to be able to predict and confirm the optimum value of the variables, it was felt that this value should enable one to determine in a quantitative manner the dependence of both the Degree of Mixing as well as the power on this variable A.

Next, examining the existing levels of B, it is seen that mixing appears to be better at a 30 per cent load than when the mixer carried a 50 per cent load. Logically speaking, one would not attempt, to load the cylinder beyond its 50 per cent capacity, because, the greater the proportion of the mixer filled, smaller is the "available" space over which the particles could maneuver and mix freely. This naturally reduces the likelihood of obtaining a better mix. Therefore a value lower than 30 per cent seemed to be a more likely choice. The new third level for the variable B was finally fixed at 15 per cent. Besides, this covers a sufficiently wide range for this variable, to enable us to investigate fully the effect of variable B on the Degree of Mixing as well as on the Power consumption.

Coming now to F which represents the angle of inclination, one of the earlier levels of F viz. 14 degrees (the other being zero degrees) had itself been selected based on Coulson and Maitra's (10) paper, wherein a range of 0 to 33 degrees had been employed. The results from these earlier studies clearly indicate the inadvisability of experiments with angles of inclination exceeding 14 degrees. The value that was picked for the third level of the angle of inclination was 7 degrees. The results from experiments conducted using these three levels for the angle should dispel any doubts concerning the possible occurrence of an angle at an intermediate value where mixing might be improved.

As for the variable G which is the number of mixer rotations, it had been found that for a particular treatment combination when the Degree of Mixing was plotted against the variable G, the value of the standard deviation which is a measure of the Degree of Mixing, did not change beyond a value of G equal to 30, indicatinrg that mixing is essentially complete. This graph is reproduced here from (16) and is shown in Figure 1. The reasons for deciding to keep G a variable have been given in an earlier section. Since one of the objectives was to explore and ascertain how the Degree of Mixing is related to G, and because G is critical in the initial period immediately following the commencement of the mixing process, the third level of G was chosen to be 20 rotations between the old levels of 9 and 30 rotations. This should enable us to obtain a true picture of exactly how G and its interactions with the other variables affect S*.

As mentioned earlier, the variable D representing the percentage of dyed component in the mixture was kept at a value of 50 per cent so that the charge consisted of a one to one ratio of dyed to undyed sand.

The levels of the various factors as well as the notation used to indicate what combination of levels were used in a particular experiment, are shown in Table 4. For example, taking Experiment Number 8 in Table 3, the treatment combination is given by 1 2 2 1, which from Table 4 would mean that this

FIGURE 1

STANDARD DEVIATION AS A FUNCTION
OF THE NUMBER OF ROTATIONS

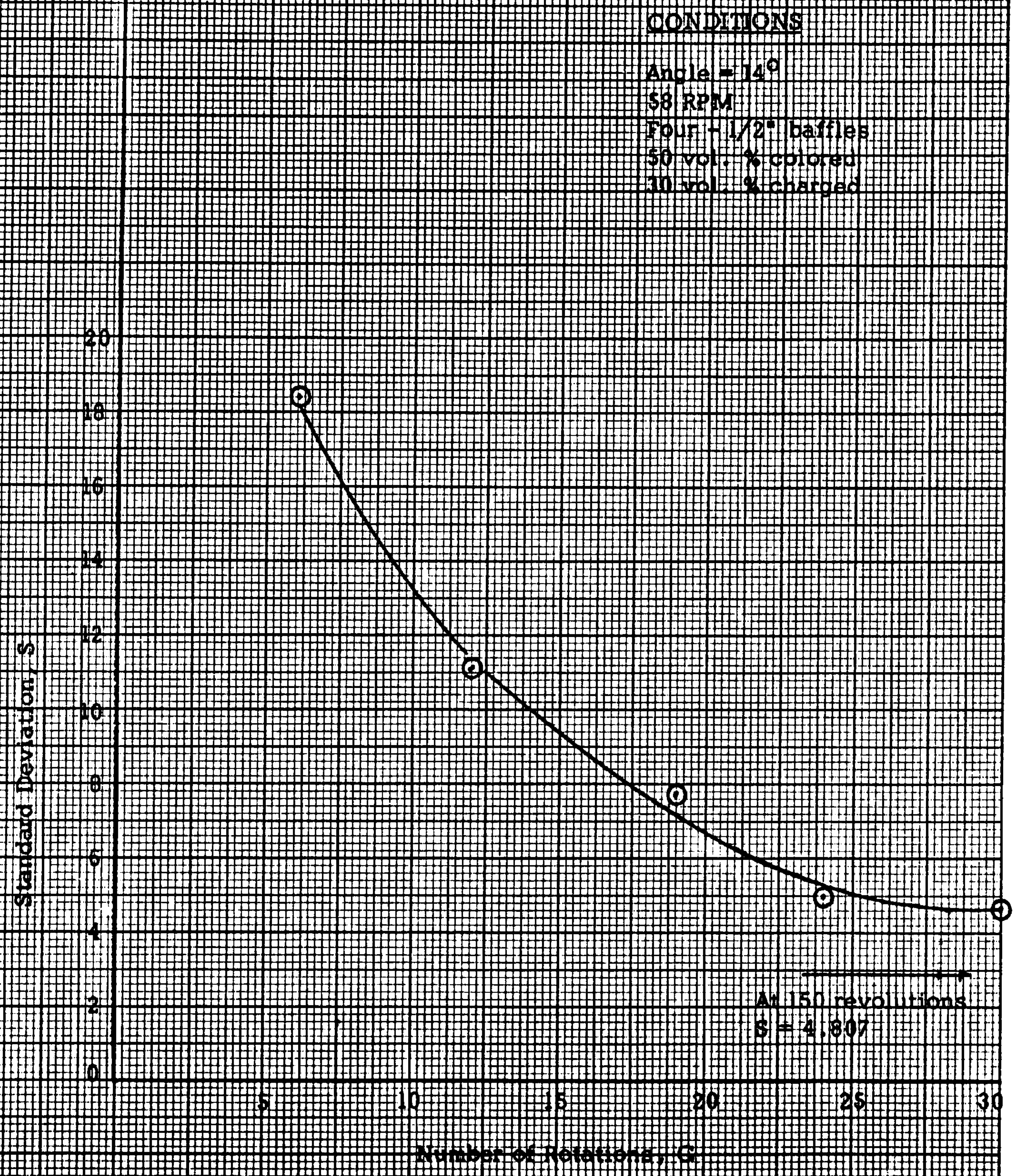


TABLE 4

LEVELS OF EACH FACTOR USED IN THIS STUDY

FACTORS	LEVELS		
	0	1	2
A	21	35	58
B	15	30	50
F	0°	7°	14°
G	9	20	30

experiment was conducted under the conditions where the value of A was 35 RPM, the value of B was 50 per cent, that of F was 14 degrees while G had a value of 20 rotations. Obviously, for conducting any experiment, Table 4 would have to be used in conjunction with Table 3 to determine the exact conditions corresponding to a given experiment.

3.3 Equipment

3.3.1 Mixer A right circular cylinder constructed out of "Plexiglas" was employed in this study to mix dry particulate solids. Its internal dimensions are such that it measures 17 inches in length and 11.5 inches across the diameter, with a wall thickness of 1/4 inch. The length to diameter ratio is therefore very close to 1.5 which is the same ratio that has been quite commonly used by workers in this field such as Coulson and Maitra (10), Donald and Roseman (12), Fisher (14), Blumberg and Maritz (3) and others.

Extensive remodelling of the existing equipment was necessitated to adapt it to the objectives and conditions of the present study. The entire set-up is shown in Figure 2.

The cylinder is mounted horizontally in a support assembly specially constructed for the purpose, out of suitable "A"- frames. The two ends of the cylinder are removable and to facilitate sampling

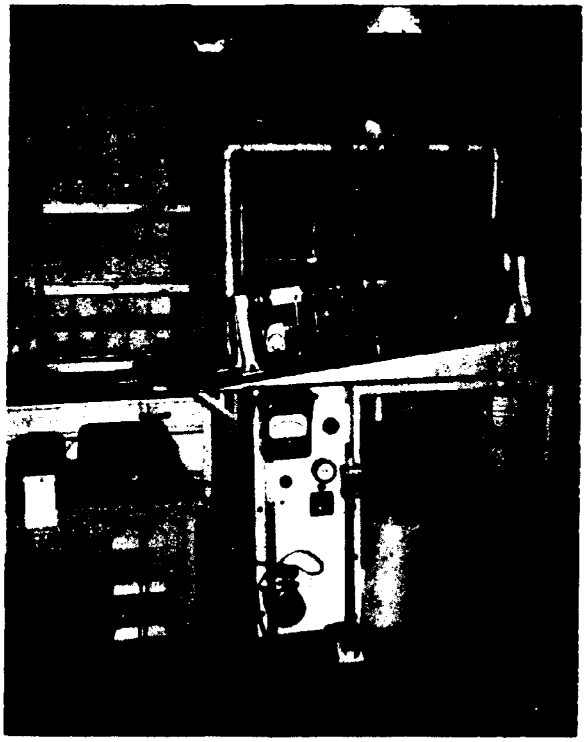


FIGURE 2
EXPERIMENTAL SET-UP

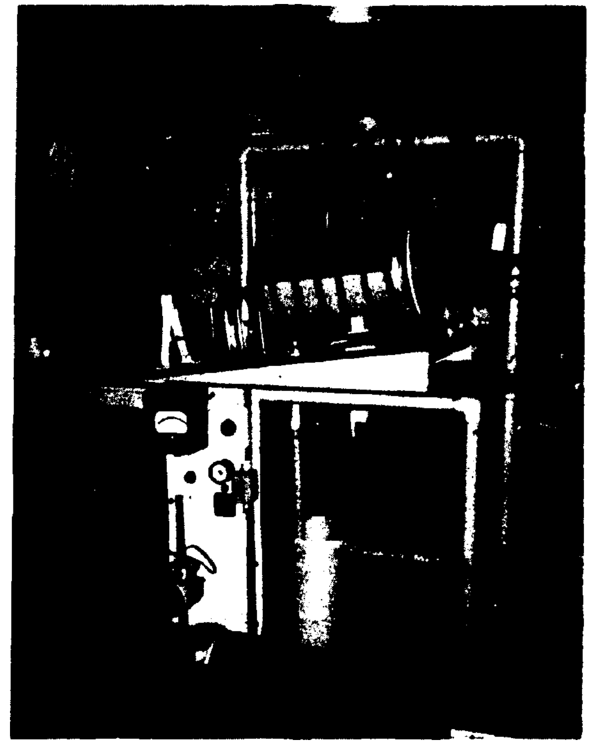


FIGURE 7
SAMPLING GUIDE IN POSITION

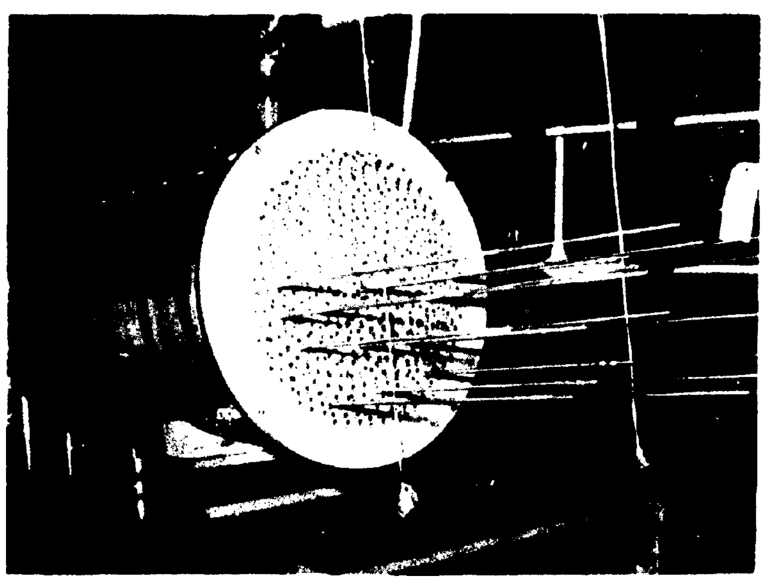


FIGURE 6
SAMPLING GUIDE

during the course of this study, it was necessary to remove the end opposite to that on which the driving mechanism was located. During the sampling procedure, in order to prevent spillage of sand, a plastic sheet mounted on a hoop and positioned just inside this end of the mixer, acts as a retainer to confine the sand. Even though the cylinder is built so that upto four baffles spaced ninety degrees apart can be mounted on the inside, this facility was not used because the present study did not call for the use of any baffles.

In order to facilitate loading and unloading of the mixer, six holes were drilled through its wall, along a line parallel to the axis of rotation. These holes are 13/16 of an inch in diameter and are spaced two inches apart. This enables the mixer to be charged uniformly. A tank mounted above the cylinder is used in loading a portion of the charge into the mixer . During the rotation of the cylinder these holes are plugged by tight fitting "Plexiglas" stoppers which in turn are held in place by wide rubber bands cut out from an innertube.

The "A"-frames which support the cylinder as well as the other frames used for accomodating the Power measuring device, are each provided with a ball bearing assembly to carry the shaft of the mixer. This reduces frictional losses.

The driving mechanism consists of a 1/4 horsepower, 1725 RPM, direct current, compound motor. It was wired to function as a shunt type motor so as to reduce the dependence of speed on load.

The D.C. electrical circuit for this set-up is shown in Figure 3. A voltage of about 115 volts is generated by a direct current generator and is fed as input to the motor. This input voltage is regulated by the variable resistance R-1. Hence the input voltage may be read off from V-1. Introduction of a variable resistance R-2, into the armature circuit of the motor, permits a limited control of the speed of the motor. The voltmeter V-2 is used to read the voltage drop across this resistance R-2. A spring type toggle switch which is connected in series with the voltmeters permits these instruments to be disconnected from the system upon starting the motor. This facility is provided so as to prevent starting surges from damaging the instruments. There was also a provision for introduction of an ammeter in series with the circuit without interruption of the current flow. This may be done by making use of an ammeter block placed in a position just preceding the on-off switch. However, in this study there was no necessity for using this added facility. Here again, if one were to use the ammeter, it would not be introduced until after the motor is running in order to prevent

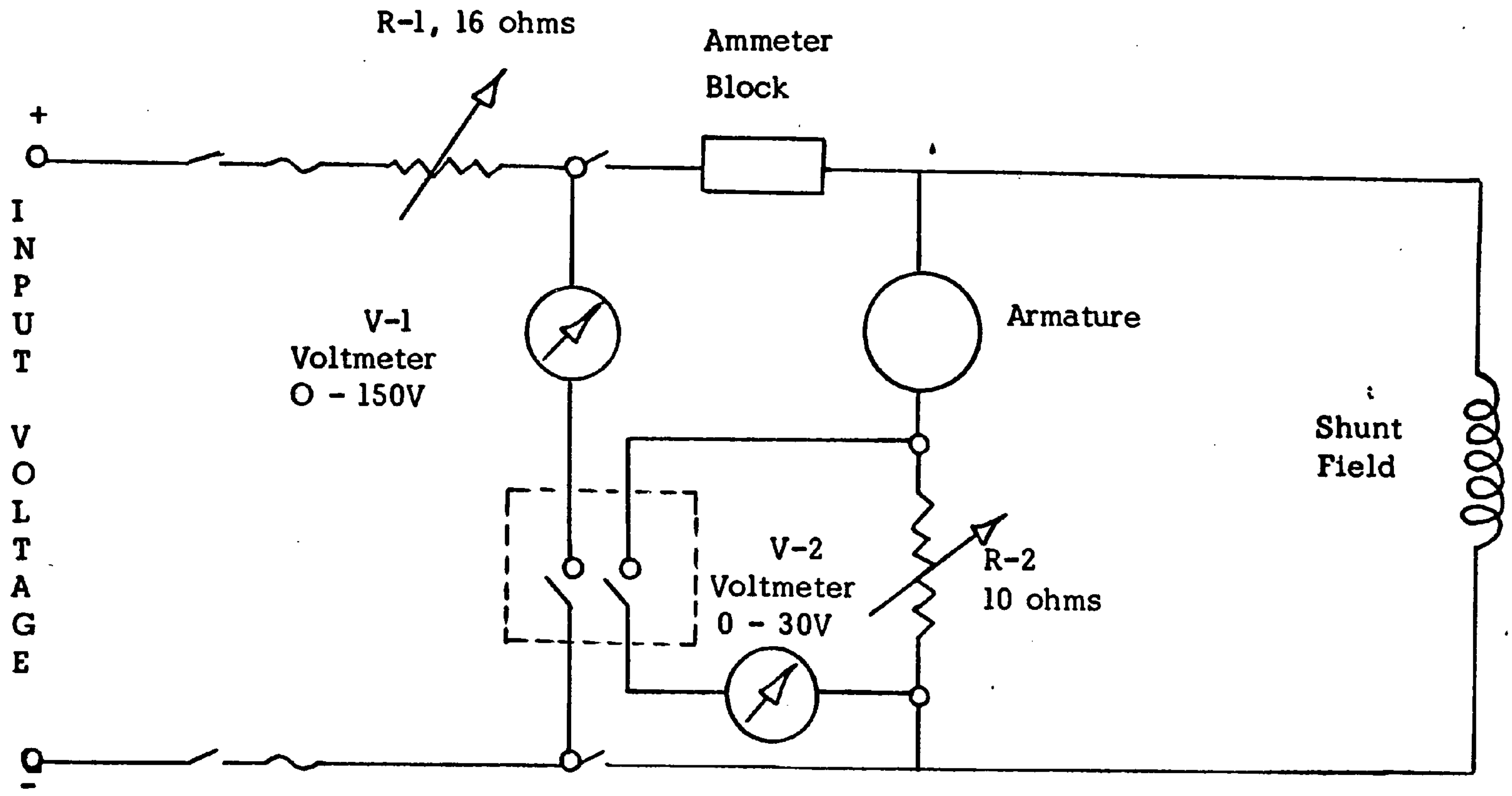


FIGURE 3. D.C. ELECTRICAL CIRCUIT

any damage to the instrument.

As for the transmission system, it consists of a V-belt step pulley arrangement along with a Boston Gear Reducer positioned in between the motor and the mixer. The gear reducer has a reduction ratio of 50 to 1. In order to prevent slippage of the belt which occurs at high loads with corresponding high speeds, there is an adjustable belt tightener located on the mixer side of the reducer; the required tension is obtained by adjusting the screws on this belt tightener.

The pulley arrangement used in the set-up is shown in Figure 4. The details of the different rotational speeds possible for various belt combinations of the pulleys, is shown in Table 5. This table shows the ideal speeds at which the pulley-reducer drive assembly is capable of rotating an empty cylinder. The values in Table 5, have been calculated on the basis of a motor speed of 1725 RPM coupled with a reduction ratio of 50 to 1 between the motor and the mixer. Thus, Figure 4 and Table 5, would be used in conjunction with each other, to determine what particular pulley combination must be used in order to attain the desired speed. As previously noted, some additional variation in speed could also be attained. Since the capacity of the motor used was only 1/4 horsepower, it was found that the motor did not have sufficient power to reach the

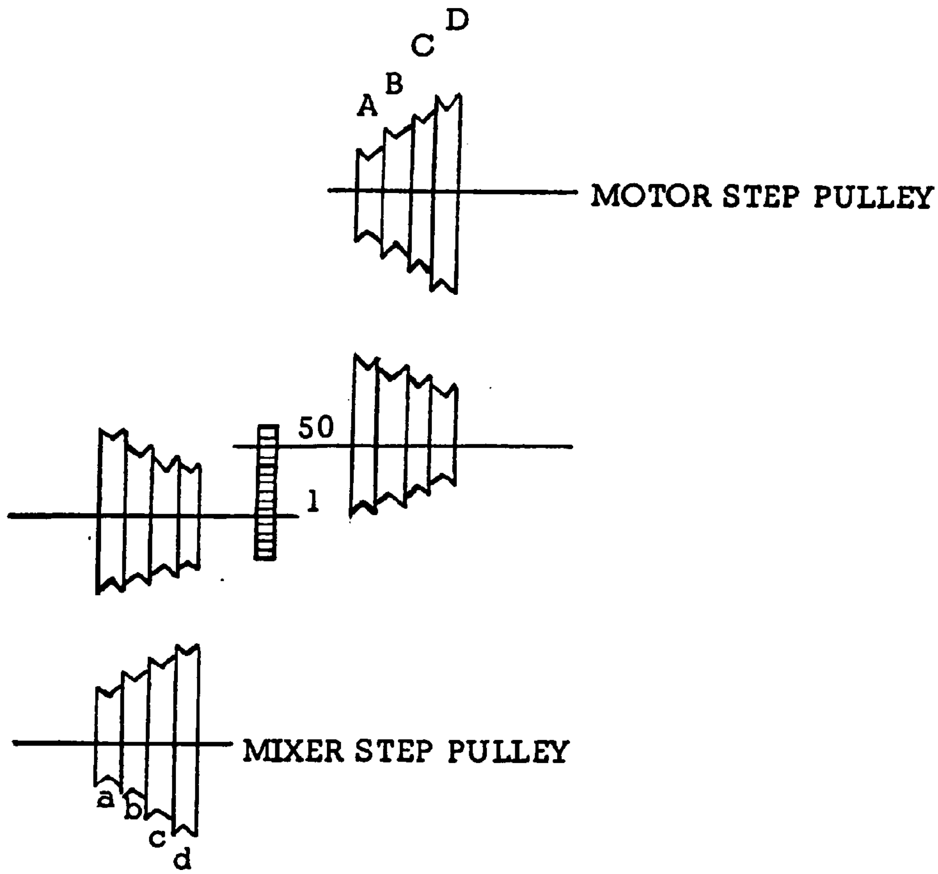


FIGURE 4. PULLEY ARRANGEMENT

TABLE 5

IDEAL SPEEDS OF THE PULLEY-REDUCER DRIVE

Motor Step Pulley		Mixer Step Pulley		Speed RPM
Step	Size inches	Step	Size inches	
A	2	a	1-3/4	39.6
		b	2-1/2	22.4
		c	3-1/4	13.3
		d	4	7.6
B	2-5/8	a	1-3/4	61.4
		b	2-1/2	34.8
		c	3-1/4	20.6
		d	4	11.7
C	3-3/8	a	1-3/4	102.0
		b	2-1/2	57.7
		c	3-1/4	34.1
		d	4	19.5
D	4	a	1-3/4	158.5
		b	2-1/2	90.0
		c	3-1/4	52.3
		d	4	30.3

higher speeds at high loads. It can thus be seen, that rotational speeds can be accurately set and maintained over a wide range of speeds.

At this point it might be mentioned that the speed is measured with the help of a Veeder Root revolution counter along with a precision timer. The revolution counter is mounted below the cylinder and registers the number of rotations completed. The electric timer which is capable of accurate measurement upto 1/10 of a second indicates the time, enabling the speed to be easily calculated.

All of the component parts of the equipment viz. the motor, transmission arrangement, mixer, power-measuring device are assembled and mounted together on a plywood board hinged on one end. This is to allow the entire setup to be inclined at any desired angle with the horizontal. In keeping with the levels of the angle of inclination called for in this study, two wooden wedges were constructed, one of which was a 7 degree wedge and the other a 14 degree one. They can be introduced under the mounting board which supports the mixer, whenever a treatment combination calls for the use of one. There are screws with which one can securely fasten the apparatus to the bench for horizontal runs. In addition for runs to be conducted at an angle, the supporting board of the mixer, the wedge and the bench are all held securely by means of nuts and bolts. The equipment is raised and lowered by means of a windlass which facilitates the insertion and removal of the wedge.

3.3.2 Solids The type of sand used for this mixing study was Ottawa Sand. The sand was screened to give a close particle size distribution and thus establish the desired spot sample size for a specified hole volume. The Sieve Analysis as well as the physical properties of the sand are given in Tables 6A and 6B. Details on the techniques of sieve analyses may be obtained in the literature (25).

A dye selling under the name of "Buffalo Black" was used to obtain blue sand from the white sand. This dye is soluble in water. The blue and the white sands used in the blending therefore, differed only in color.

TABLE 6 A

SIEVE ANALYSIS OF SOLIDS
U.S. STANDARD SIEVES

Mesh	Cumulative Wt. %
+ 45	0.9
- 45 + 50	58.7
- 50 + 60	98.7
- 60	100

TABLE 6 B

PROPERTIES OF SOLIDS

Material	Ottawa Sand
Bulk Density, gm/cc	1.5
Natural Color-white Angle of Repose - 33^o	Dyed Color - Blue Angle of Repose - 33^o

3.3.3 Sampling Apparatus and Guide The equipment used for removing the samples for analyses, consisted of twelve sampling devices, each one similar to that shown in Figure 5. This is similar to the sampling probe used by Weidenbaum (29) and Blumberg and Maritz (3). The probes are constructed of brass, except for the steel tip. The purpose of having a sharp steel tip is two-fold. On the one hand, it affords easy penetration of the plastic retaining barrier and on the other it reduces the disturbance caused in pushing the probe through the sand. Each of the six holes on a given sampling device is carefully drilled to within ± 0.001 inches in depth in order to insure that any difference in sample size would not be attributed to differences in the sampling devices.

A sampling guide as depicted in Figure 6, is used to aid random sampling. It is constructed out of "Plexiglas" and is 3/4 inch in thickness. It has 572 guide holes each of which is 3/16 inches in diameter. The holes are numbered and are located on concentric bands 3/16 inches wide. The holes in a particular band are spaced equidistant apart and their number is a function of the area in that band. The bands are so located that they overlap a half thickness . In order to facilitate rotational movement of this guide along the rim of the mixer at the sampling end, it is provided with four fasteners. Each of these fasteners has a ball-bearing tip that rides in an aluminum groove. This groove

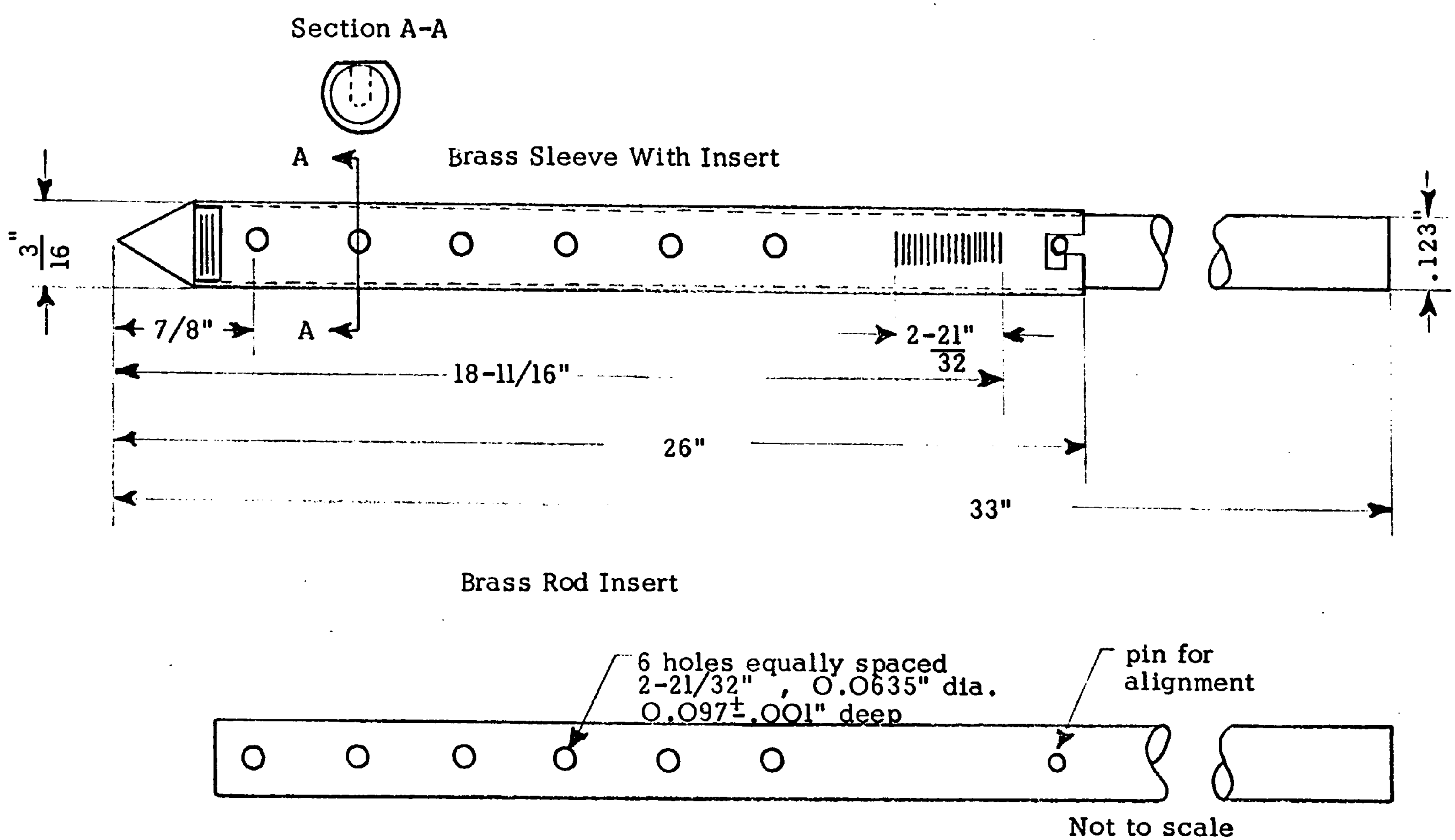


FIGURE 5. SAMPLING DEVICE

is attached on to the outside circumference of the cylinder. Figure 7 illustrates the equipment with the guide in its proper position.

3.3.4 Counting Apparatus The method selected for the purpose of analysing the samples collected, was that of counting the blue particles as well as the white particles in a given spot sample. Hence, a suitable apparatus was made and is shown in Figure 8.

The equipment is made up of a 5 power magnifying lens suitably mounted in a "Plexiglas" holder. At the base of this holder is a wooden block which is provided with a carved out section capable of receiving the receptacle with the sample to be analysed. This receptacle consists of a Syracuse Watchglass to the inner base of which is affixed a piece of 1/4 inch grid paper. The lens holder itself is capable of free movement both laterally as well as vertically. This facilitates transfer of the sample to and from the receptacle and also permits manual adjustments and focussing.

A microscope lamp mounted on a wooden base provides sufficient light for illuminating the sample. It is portable and it was found convenient to locate the lamp to the left of the sample in the receptacle. A dual counter was used for accurate and

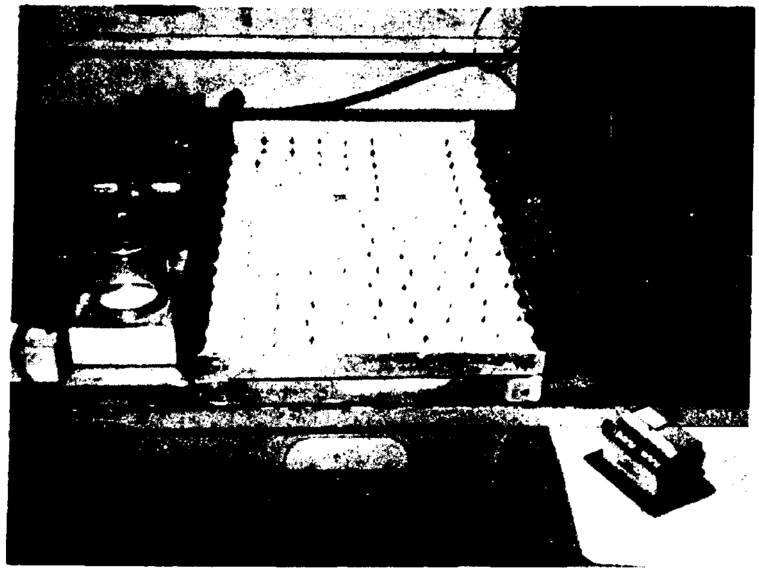


FIGURE 8
COUNTING APPARATUS

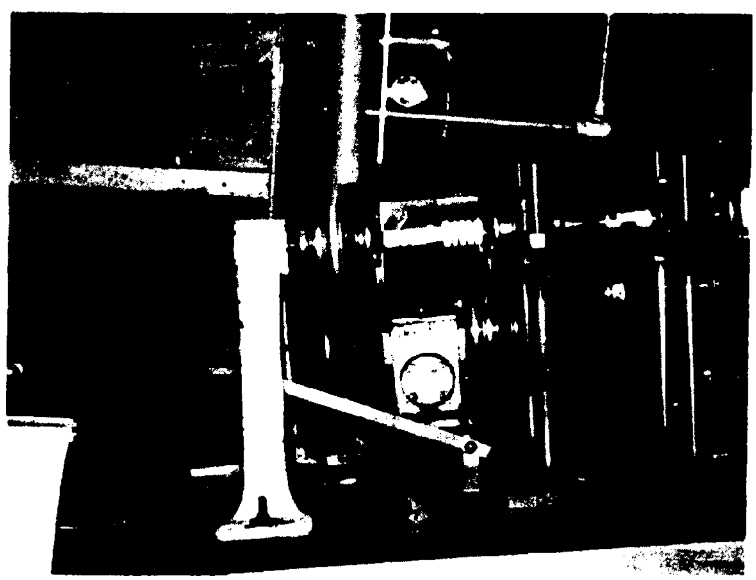


FIGURE 10
TORQUE MEASURING ASSEMBLY

efficient counting of the particles viewed through the magnifying lens.

Several cardboard pill boxes numbered in serial order were used to hold the samples to be counted.

3.3.5 Equipment for Power Measurement. A surface-strain-gage type transmission dynamometer was built to measure torque transmitted to the rotating shaft. This torque pick-up is essentially a short length of shaft which when loaded in torsion translates the torsion into changes in electrical output. Thus, when coupled into the line of shafting, the pick-up assembly will measure the torque being transmitted while in no way influencing or changing its magnitude.

There are several literature references available on the fundamentals of strain-gage work. Among these may be mentioned publications by Perry and Lissner (22), Aronson and Nelson (1) and Buckelew (5), the last one being a recent article of general interest. For a detailed discussion on the different techniques available for the measurement of shaft horsepower, one should consult the feature article on "Torque Instrumentation" (21). Other pertinent information on such devices may also be obtained from manufacturers such as Baldwin-Lima-Hamilton (20).

The torquemeter used in this study, however was built from scratch (24). Since the existing shaft was found to be too short, a new drive shaft of sufficient length was procured. Next, the torsion sensitive element in the torque pick-up consisting of four strain gages were bonded on to the shaft and placed at a 45 degree angle to the shaft axis. The arrangement is shown in Figure 9. A,B,C and D represent the four strain gages. They are exactly equi-spaced circumferentially and are interconnected into a Wheatstone bridge circuit, so as to respond only to principal strains caused by torsion and to cancel out the effects of bending and thrust loads, as well as temperature changes. However, bending stresses (which might be caused by misalignment of the parts or bearings), when added to the high torsional stress in the measuring section of the shaft, could approach the fatigue limit of the steel. Therefore, a flexible coupling was used for mounting the torque-pickup. Silver slip rings rotating with the shaft and stationary brushes in the casing provide electrical connection between the gages, and allow the strain gage bridge to be energized and its resistive unbalance to be measured by a suitable indicating or recording instrument. This bridge unbalance caused by the gage resistance change bears a linear relationship to the torsional strain. Figure 9 schematically depicts the manner in which the strain gages are affixed to the

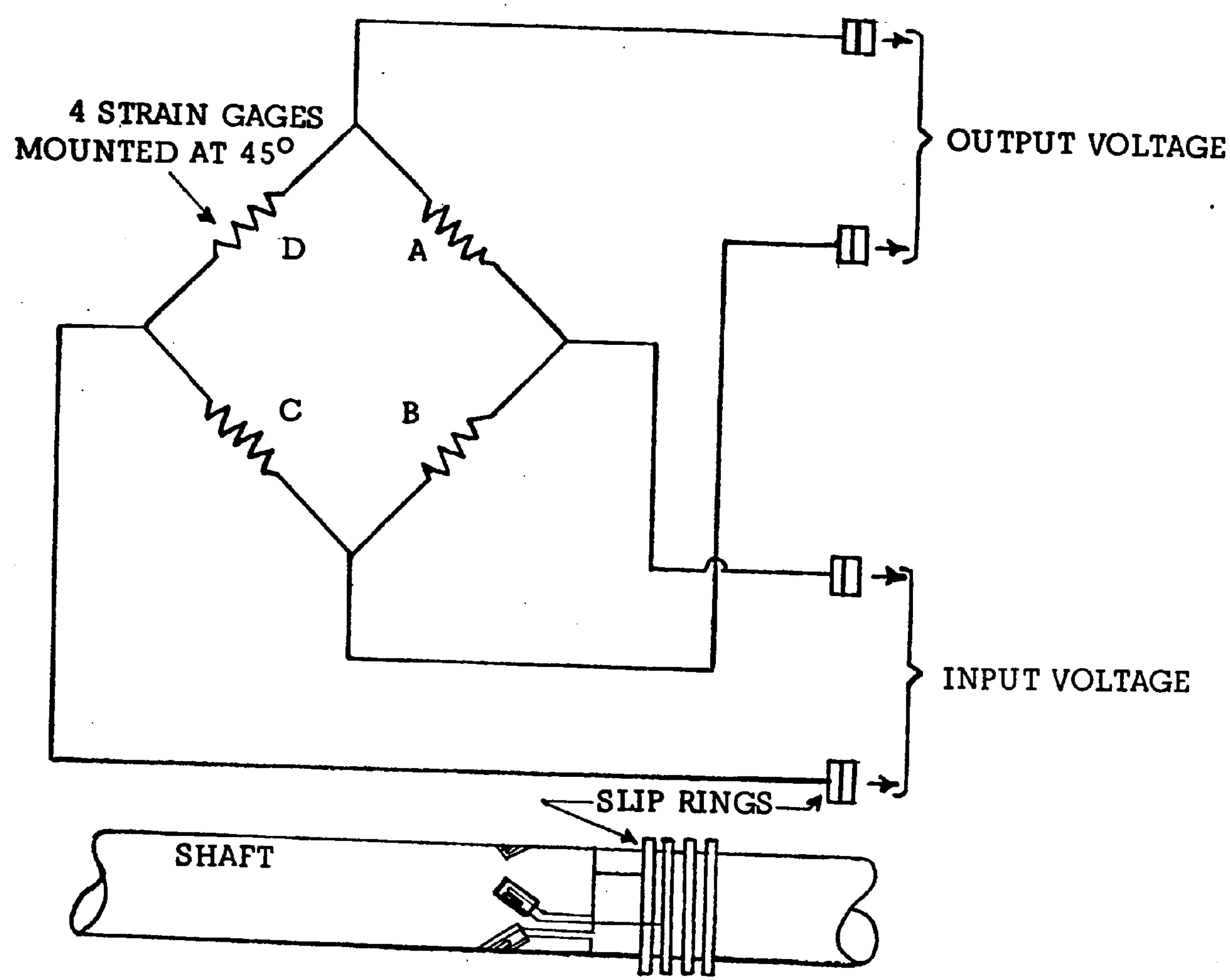


FIGURE 9

STRAIN GAGE AND SLIP RING ASSEMBLY

torque shaft and also shows how they are wired to the slip rings for input and output connections. The torque measuring assembly used in this study is illustrated in Figure 10.

Facilities were also provided for lifting the brushes, in order to eliminate brush wear when torque readings are not required.

Since the bridge output is frequently insufficient to drive the measuring instruments, amplification becomes necessary. Both D.C. and A C. amplifiers are available for strain-gage work.

The amplifier used in this work was BL-320 Universal Amplifier manufactured by The Brush Development Company.

An oscillograph or recorder is required to produce a record showing the variation of the phenomenon under study (in this case, the strain or torque) against time. The Model BL-202 double-channel Direct Inking Oscillograph was employed for this purpose.

Calibrating equipment also is required to provide convenient and accurate overall calibration of the system so that the deflection on the oscillograph can be calibrated in terms of the strain. In addition to the above mentioned oscillograph, a Strain Indicator put out by the Baldwin-Lima-Hamilton Corporation was used for initial calibration of the torque measuring device.

The above mentioned Universal Amplifier, when used with the Brush Magnetic Direct Inking Oscillograph, results in a unit called the "Universal Strain Analyzer". It provides a complete unit for the measurement of strain or other phenomenon where a resistance sensitive pick-up is employed. This combination of equipment records static and, or dynamic strains. Most studies of dynamic strain require that the instrument be capable of accurately following and recording the instantaneous values of strain. The above combination is therefore useful because dynamic strain contains static or low-frequency components.

The amplifier used contains two legs of the Wheatstone bridge network. The third and the fourth legs were connected externally, one being the active gage, the other for sensitivity doubling. Resistance and phase balance controls are provided on the panel of the amplifier. Provision is also made for connecting an internal calibrating resistor in the bridge circuit and adjusting the overall gain. The amplifier can be calibrated for individual gages, so that the strain can be read-off directly in microinches per inch from the chart deflection. This amplifier is so constructed that it is also applicable for use with any resistance sensitive pick-up.

The heart of the oscillograph equipment is the magnetic pen motor. The oscillograph provides a chart drive mechanism for pulling the radially ruled paper at constant speed under the point of the pen, resulting in a record of the variation of torque against time. Three chart speeds are available - 5, 25, and 125 mm/sec., corresponding roughly to 1/5, 1, and 5 inches/sec. A hinged door permits access to the inkwell, pen, and chart paper for minor adjustments and notations without removing the cover, although the cover can be easily removed for reloading chart paper.

For further details, one should refer to the available literature (27). The relationship between the chart reading and torque was obtained by calibration as detailed in a later section.

3.3.6 Accessories for Analysis Several pieces of additional equipment were employed in this work. Even though their roles were minor, they were essential for the purposes of processing the sand.

The initial screening of the Ottawa Sand to obtain to obtain a close size distribution was done on a style No: 3, Rotex Sieve Shaker. This machine is capable of holding only one sieve at a time, and therefore divides the initial material into an oversize and an underize. An additional pass over another screen is necessary to isolate a product of a size distribution intermediate between the size limits of the original material. The sieve cloth utilized conforms to U.S. Standard specifications and is constructed of bronze wire.

Because of the fact that there was only a limited supply of carefully screened sand, it was necessary to re-use the sand repeatedly after mixing. In order to facilitate washing of the sand mixture to obtain white sand, a fifty-five gallon drum cut in half and provided with a canvas separator at about eight inches from the open end, was used. Two circular wooden bands held the canvas separator in place. On the inner side of the drum there are welded flanges to which these bands are bolted. So as to prevent sand from escaping down the sides of the drum, a coating of expoxy resin was applied all around the junction of the canvas and wood. The water for washing the sand, was provided through a shower head. Affixed to the bottom of the drum is an outlet with an attached hose leading to a lower level in the laboratory, thus providing sufficient head. This permits an improved washing procedure because of faster outflow of wash water.

The washed sand was then dried in a tunnel dryer. The same dryer was also used to dry the dyed sand. It contains two racks, each capable of holding four trays filled with sand to be dried. When available, steam was used to provide the heat input. More frequently however, a gas conversion burner mounted at the inlet of the dryer, was employed as the source of heat supply.

A Rotap Sieve Shaker with U.S. Standard Sieves was used to perform sieve analyses.

3.4 Preliminary Experiments

Under this title will be included, primarily two types of experiments that were necessary. The first series of experiments were concerned with the Calibration of the Power measuring unit as well as with obtaining results from preliminary power measurements. The other series of operations pertained to the preparation of solids for performing the actual experiments called for in the Design.

3.4.1. Calibration of the Power Measuring Unit. For purposes of calibration, a twelve inch steel bar with a circular ring welded on to one of its ends was fabricated. This end was attached on to the drive shaft of the mixer and was held securely in place by a screw. The bar was also provided with holes 2 inches apart, to permit suspension of suitable weights at a known distance from the center. Knowing the magnitude of the weight and the radial distance at which it is suspended, the torsional force or torque can be calculated. For purposes of testing the device, initially an SR-4 Strain Indicator was used to determine the relationship between torque and the indicator reading. This relationship is linear in nature and is shown in Figure 11.

Since the torque measuring device had successfully passed the test, steps were then undertaken to install the Amplifier and Oscillograph, in order to obtain, the correlation between the torque and the chart reading. Hence, it was necessary to calibrate the new recording set-up. As mentioned earlier the "Universal Analyzer" unit which was used in this study, consists essentially of an A.C.- Wheatstone bridge, voltage amplifier, discriminator, and D.C. power amplifier. The four arms of the Wheatstone bridge terminate at the four binding posts located at the left front corner of the Amplifier. It is necessary that the bridge be completed before the unit can be operated. This was done by properly connecting the four leads from the torque pick-up to the input terminals on the Amplifier. Calibration can then be done by first centering the penmotor pen on the chart, prior to connecting the Oscillograph (recorder) to the Amplifier, by making use of the centering lever at the side of the penmotor. This is obviously done with the recorder running. The Amplifier and the Oscillograph are then connected by the interconnecting cable and the twin outlet A.C. cord and then the A.C. power is turned on. The Amplifier switch is then thrown to the "on" position and about fifteen to twenty

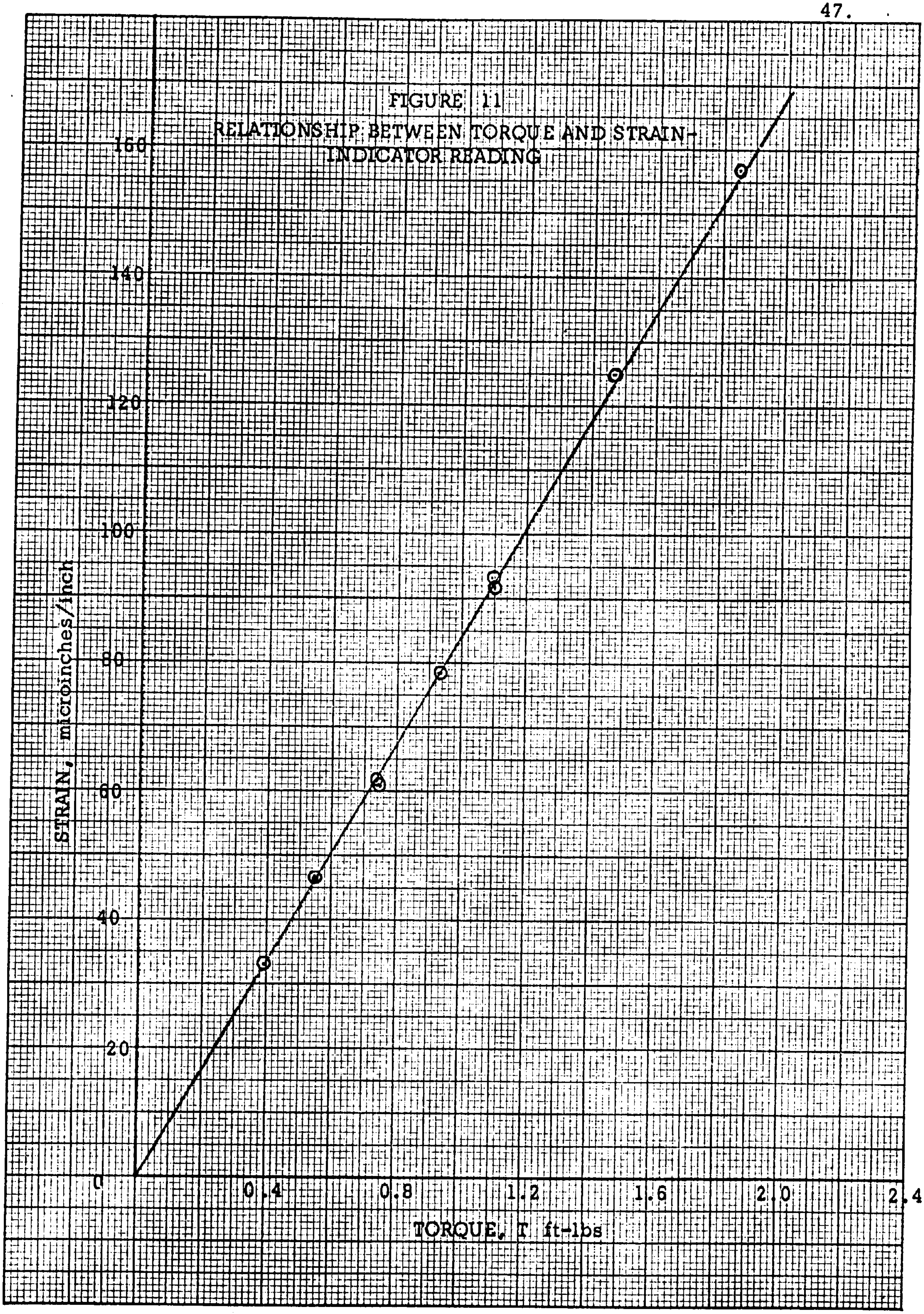


FIGURE 11
RELATIONSHIP BETWEEN TORQUE AND STRAIN-INDICATOR READING
STRAIN, microinches/inch
TORQUE, T ft-lbs
0
20
40
60
80
100
120
140
160
0.4
0.8
1.2
1.6
2.0
2.4

minutes are allowed for the Amplifier to be "warmed up" and stabilized. The oscillograph is also turned on. The Amplifier attenuator is turned to the "off" position and the "D.C. Gain" control to the "D.C. Amplifier Only" position. The pen is then centered by adjusting the "D.C. Center" control. This control electrically centers the pen motor pen when the "D.C. Gain" control is in either the "D.C. Amplifier Only" position with zero input, or in the "Bridge On" position with the "Attenuator" in the "Off" position.

Then the "D.C. Gain" control is turned to the "Bridge On" position. The "A.C. Gain" is turned to a position of about half its full capacity. Next the "Attenuator" knob is turned to the "1" position which provides maximum amplification of the output. Balancing is accomplished by depressing the "Balance" switch and operating the "Bridge Balance (R) and (C)" controls, adjusting the "(R)" control first. Balance is indicated by a minimum deflection of the pen from the center of the chart.

The "A.C. Gain" is then re-adjusted, while depressing the "Calibrate" switch so that the maximum displacement of the pen will be 15 lines from the center. This prevents the pen from accidentally running amok. The set-up is then ready to record any strain. In a manner similar to the one employed for test purposes with the Strain Indicator, weights were suspended at known distances from the axis and the magnitude of the recorder output from the oscillograph was read-off and correlated with the corresponding torque which produced the strain in the first place. Numerous experiments were conducted in order to determine this relationship. Larger than the normal number of runs were necessitated, because of the failure of the welded joint at the end of the cylinder next to the driving mechanism. This consequently called for recalibration of the assembly and had to be repeated on a couple of occasions as a result of the failure of the welded joint. Hence, in order to avoid additional risk of possible future ruptures, it was decided to start with a single piece of 3 inch diameter shaft and machine it down to a 1/2 inch diameter size everywhere except at one end where a composite 3 inch diameter disc-like piece was retained. The torque pick-up had therefore to be recalibrated with this new arrangement. The results obtained for Calibration, have all been condensed into a representative form as shown in Figure 12. The abscissa represents the Torque T, in ft.-lbs. whereas the ordinate shows the chart reading R. One unit of chart reading (which measures 1 cm. in width) actually corresponds to 5 lines on the chart paper.

From Figure 12, it is seen that the slope m, of the line is given by

$$m = \frac{3.805 \text{ units}}{4.000 \text{ ft.-lbs.}}$$

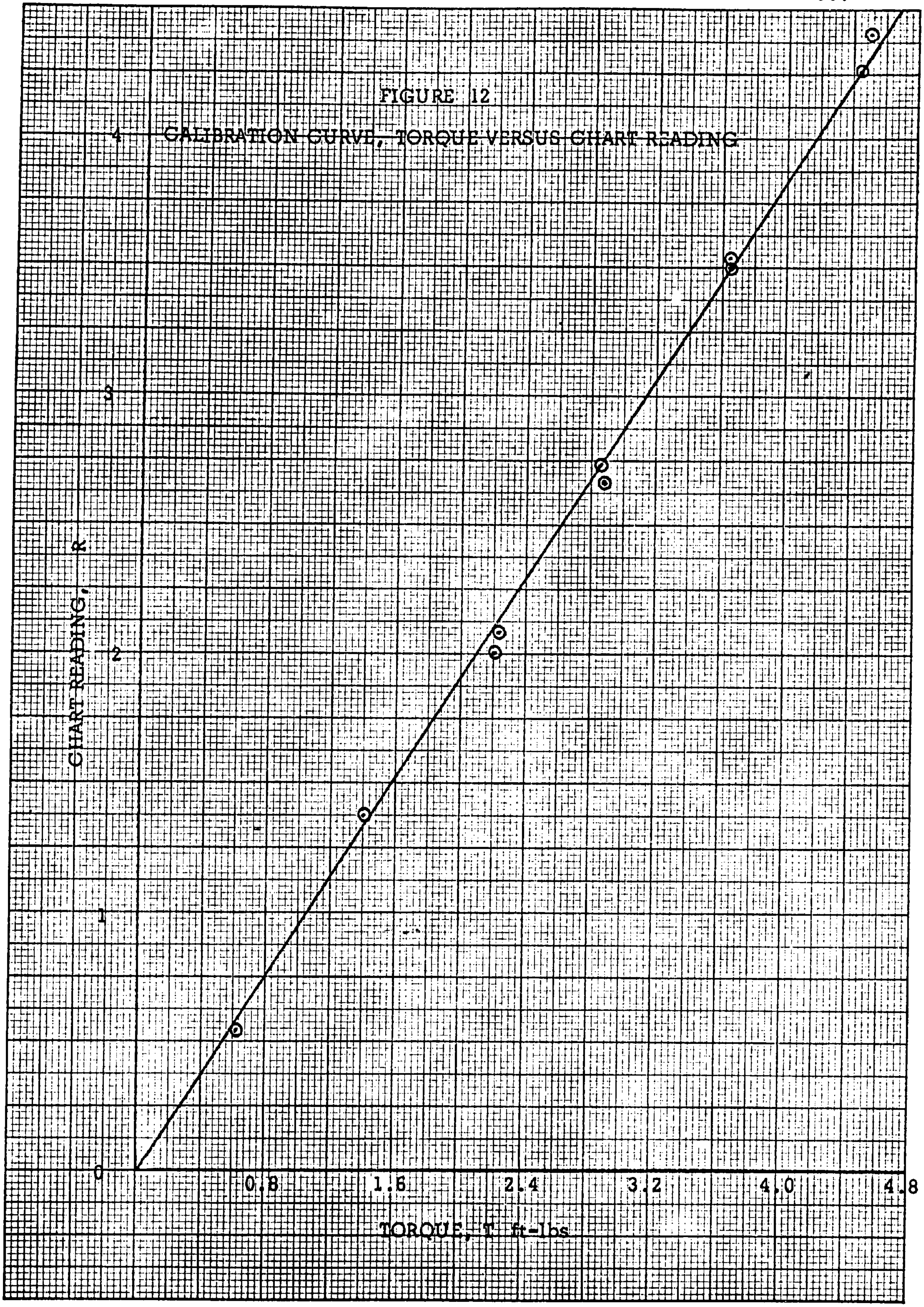


FIGURE 12

CALIBRATION CURVE, TORQUE VERSUS CHART READING

Hence

$$T = \frac{4.000}{3.805}$$

which yields

$$T = 1.05\,R$$

This equation therefore represents the correlation between the torque transmitted by the shaft and the corresponding chart reading produced at an Attenuator setting of 1.

Next a series of runs were conducted with the value of the rate of rotation fixed at 58 RPM, at varying angles to determine power consumption under the no-load conditions. The horsepower required for rotating the empty cylinder was found to be the same for any angular position of the assembly, within the range of zero to fourteen degrees.

When the cylinder was rotated at a speed of 58 RPM, the torque as measured from the oscillograph recording was found to be equal to 0.4 ft.-lbs.

In general from the knowledge of the torque required T, and the rotational speed N, the power consumption P* measured in units of horsepower can be calculated by using the following equation:

$$P^* = \frac{2\pi\, T\, N}{33{,}000}$$

where $P^* =$ power, HP.

$T =$ torque, ft.-lb.

$N =$ rotational speed, RPM.

In the above-mentioned case therefore, the power would be given by

$$P^* = \frac{2\pi T N}{33,000}$$

$$P^* = \frac{2\pi \times 0.4 \times 58}{33,000} = 4.4 \times 10^{-3} \text{ HP.}$$

Hence it is seen that for a rotational speed of 58 RPM, the conversion factor from measured Torque T, to Power required P*, is given by

$$P^* = 11 \times 10^{-3} T$$

where again P is in units of horsepower. Similarly for the other levels of the rotational speeds, viz. 35 RPM and 21 RPM, the corresponding conversion factors may be calculated and are shown below.

For a speed of 35 RPM

$$P^* = 6.666 \times 10^{-3} T.$$

For 21 RPM the conversion factor is

$$P^* = 3.99 \times 10^{-3} T.$$

3.4.2 Preliminary Power Measurements. Yet another series of experiments was conducted, with the level for the variable A viz. the rate of rotation kept constant at 58 RPM. The objectives of this series were, to obtain general performance data on power as well as to gain an insight into the nature of variation of power with varying loads and angles of inclination. Even though the speed was held constant at 58 RPM, the load B, was varied over the range from the no-load condition to a load of 50 per cent; the experiments were conducted at four levels, namely 0, 15, 30 and 50 per cent of the variable B.

The angle of inclination F, was also varied from 0° to 14° with each of the runs being conducted at 0°, 7° and 14°. As for the number of rotations G, for the sake of uniformity in each case the cylinder was allowed to rotate for 58 revolutions.

The results obtained from all these runs are shown in Table 7. The letters A, B, F and G denote the variables as explained earlier. Just prior to conducting each run, it was necessary to calibrate the recording equipment so as to make it ready for that particular run.

It is seen from the trend of results in Table 7, that at a constant rate of rotation and for a given angle of inclination, the power consumption increases with the volume per cent of the cylinder charged which is as expected. It is also seen that with

TABLE 7

POWER CONSUMPTION AT VARYING LOADS
AND DIFFERENT ANGLES

A RPM	B Volume Percent	F Degrees	G Rotations	P* Horsepower
58	0	0	58	0.004
58	15	0	58	0.075
58	30	0	58	0.130
58	50	0	58	0.147
58	0	7	58	0.004
58	15	7	58	0.072
58	30	7	58	0.107
58	50	7	58	0.121
58	0	14	58	0.004
58	15	14	58	0.056
58	30	14	58	0.066
58	50	14	58	0.069

a speed of 58 RPM and a given load, the power requirements tend to decrease as the angle of inclination increases.

3.4.3 Preparation of Solids Under this section of the preliminary experiments may be classified the operations of screening, washing, dyeing and drying.

It was through the screening operation, that a close particle size distribution of the sand was obtained. Once the size distribution is known, the desired spot sample size for a specified hole volume can be found. This spot sample size refers to the number of particles of sand per sample and was ascertained to have an approximate value of 105 particles.

The size distribution had been determined from an earlier study which dealt with the relationships between the number of particles, the probe hole size and the particle size.

About a ton of Federal Fine Grade Ottawa Sand had been screened on a Rotex Sieve Shaker employing a 50 mesh sieve. (U.S.Standard), in order to eliminate all the fines. The product had then been screened over a 45 mesh sieve to eliminate the oversize. This procedure was repeated until only a very small portion of the sand was lost per sieving pass. The remaining product consisting of about three hundred pounds which had been used in the previous study, was utilized in the present work.

Since the charge consists of colored sand besides the white sand, it was necessary to obtain a suitable dye for purposes of coloring the white sand. As mentioned earlier a dye selling under the name "Buffalo Black" marketed by the National Aniline Division of the Allied Chemical Corporation, was selected; it has desirable qualities such as being water-soluble and is easily washed free of the sand when necessary. Neither does it flake or wear off.

Since the sand mixture used for dyeing, is composed of 50 per cent colored sand, it was found that roughly five grams of dye are necessary to color eighteen pounds of this sand mixture. Also, during the dyeing process, it is essential that the water for dispersing the dye into the sand be used only sparingly. Otherwise, there is a tendency for pockets of poorly dyed material to remain in the final product. Also, it is important to keep the sand constantly stirred while it is drying. This is to prevent the surface layers from being darker than those beneath the surface and to assure that the dyed sand is as uniform in color as possible. It was found that periodic stirring and raking at least once every half hour during the drying process, minimises the chance of obtaining a nonuniform product. The colored sand thus obtained is then stored in a separate container for later use.

When it was necessary to obtain white sand from the mixed product for further experimentation, it was washed in the equipment described in an earlier section. Repeated washings are necessary to ensure that all the dye has been completely removed. The effluent wash water may be tested with a piece of white filter paper, to detect if any traces of dye are still present. Washings are continued until there is no trace of the dye in the washed sand. This wet sand is then dried in the oven, to obtain the white sand for charging into the mixer. The white sand is also stored separately to await later use.

Since it had been concluded in the previous study that the attrition of the sand particles is very negligible, frequent sieve analyses were not performed. However, preliminary tests were conducted to ascertain the size distribution. The results from these tests have been incorporated in Table 6A presented earlier.

Prior to conducting each of the various experiments, it was necessary to determine data on speed as a function of both the voltage drop V_2 across the resistance in the armature circuit, as well as the input voltage V_1. The speed was then measured and the necessary adjustments made, until the desired rate of rotation corresponding to any given particular treatment combination was achieved.

3.5 Procedure

When a particular experiment is to be performed, the operations detailed below are carried out.

The experimental conditions for any given experiment is determined from Tables 3 and 4. Thus, a treatment combination is selected from Table 3 and the corresponding level (for each of the variables) at which the experiment is to be conducted, is found from Table 4.

The cylinder is filled with a dummy load consisting of a used sand mixture upto the extent dictated by the level of variable B for that particular run, and the apparatus is inclined to whatever angle is called for by the level of variable F. Adjustments were made on the belt tightener when necessary. Then the input voltage V_1 and the voltage drop V_2 across the armature of the motor, were adjusted to obtain a speed close to that required by the treatment combination. This procedure is repeated with fine adjustments of the resistances in the circuit, until the exact speed called for in the experiment is attained. To attain this speed may require trials, numbering any where from five to a dozen, each one with slightly different adjustments. However, this procedure ensures the fact that a given experiment will truly be conducted at the speed called for. Once this speed is achieved, the voltage and resistance settings are left undisturbed until the start of the next run.

Simultaneously, the power measuring gear is activated and allowed to warm up. After the desired speed has been obtained, the mixer is brought to rest in its normal position. The calibration procedures, as detailed earlier, are repeated and the oscillograph made ready to record the torque. When the mixer is at rest, the pen traverses the center line of the chart paper showing no strain. The motor is turned on and the mixer is allowed to rotate for one minute and a continuous recording of the torque is made. This enables one to discover whether or not the system is functioning properly and affords an opportunity to correct any malfunction, prior to the actual run. Such initial torque-recordings are repeated where necessary, in order to assist in selecting the best Attenuator factor to be used in the actual mixing experiment. Besides, they also provide a double-check on the power output obtained at the end of the experiment. The calibration adjustments are also left undisturbed; until the conclusion of the specified run.

Now that the conditions of the variables have been fixed as per the requirements, the initially charged used sand mixture is unloaded. The end of the cylinder away from the driving mechanism is removed. The inside surface of the cylinder is then coated with an antistatic agent marketed under the name Merix Compound No: 79 Concentrate. At the same time a piece of polyethylene about

4 mils thick, is stretched over a hoop provided with steel wires that criss-cross diametrically. The inside surface of this assembly which acts as a barrier preventing spillage of sand during sampling, is also coated with the antistatic agent. Both the inner surfaces of the mixer and the retainer are then allowed to dry. Obviously, the purpose of the application of such an agent is to prevent the build-up of static electricity. Otherwise, there might be a tendency for particles of sand to adhere to the walls and thereby prevent one from obtaining an unobstructed view of what actually takes place within the mixer.

When completely dry, the retainer assembly is pushed into position at the end of the cylinder and held in place by small bolts. These bolts are such that they pass through the circumference of the cylinder into suitably tapped holes on the metallic hoop. Next, the end of the cylinder is mounted and held firmly in place by several screws. After the end has been fixed in position, the "A"-frame is slid over the bearing assembly on the shaft. Meanwhile the jack-type supporting arrangement underneath the mixer is lowered. The "A" - frame is then held in place by means of bolts projecting upwards from the mounting board. Also, the bearing assembly over the shaft is held within the "A" - frame support by tightening the allen screw provided for this purpose.

Depending on the value of the variable B applicable to that particular treatment combination, the appropriate quantity of white sand is first weighed on a scale and then charged into the mixer via the overhead tank and a funnel. If the level of B happens to be 50 per cent, the quantity of undyed sand charged into the mixer amounts to 25 per cent or half the total charge; if B has a value of 30 percent the charge of white sand would be 15 per cent of the total volume and so on for every other treatment combination. Once the cylinder has been loaded with the white sand, the charging ports are plugged shut and held in place by means of the wide rubber bands. The mixer is then rotated for about 100 revolutions so as to warm up the driving mechanism. At the same time the power measuring equipment is also allowed to function.

Whenever the power is turned on to rotate the mixer, it is advisable to disconnect the voltmeters by using the spring type switch.

At the end of this initial warmup run, the cylinder with its contents is stopped; so is the oscillograph. The mixer is then returned to its normal position which is indicated by an arrow on the side being vertical. In this position the surface of sand in the mixer is horizontal.

The other half of the requisite charge composed of the dyed sand is then weighed. This dyed sand is divided into six equal portions and charged onto the surface of the existing white sand, through the six charging holes.

So as to create a minimum of disturbance at the dyed-undyed sand interface, a funnel with an extension attached to its end is used. This also makes possible a more uniform distribution of sand.

Depending on the angle of inclination called for in the selected treatment combination, the entire assembly is either positioned horizontally or at the desired angle. If the run calls for a zero angle of inclination, the mounting board is fastened to the bench. But if the experiment is to be run at an angle, the entire assembly is drawn up by raising the mounting board by means of the windlass. When raised to a reasonable angle, the appropriate wedge is introduced under the board and the windlass is carefully released, until the mounting board sits squarely on the wedge which has already been bolted to the bench. Bolts are used to lock the mounting board onto the wedge. This entire operation is carefully done so as not to disturb the charge of sand inside the mixer.

After the cylinder and its contents have been thus positioned, the recorder is turned on and re-calibrated if necessary. Once the final calibration is done, the amplifier and oscillograph are left running until the completion of the run. At this point, it is interesting to note that even though three chart speeds are available, the one most convenient was found to be a chart speed of 5 mm/sec. or 30 cm/minute.

Just prior to starting the motor, the timer as well as the rotation counter are reset to zero.

The exact starting procedure consists of switching the voltmeters out of the circuit by means of the toggle switch. Because the switch is spring loaded, it must be held in place in the disconnected position until the motor switch is thrown to the "on" position. Then the motor and the timer are activated simultaneously. Immediately the value of the voltmeter readings are noted down. Normally, no further adjustments of the resistances beyond those made for speed, during the initial dummy run, will be required. The mixing process is allowed to continue until the desired number of rotations, as prescribed by the treatment combination, is reached. The input power to the motor is cut off just prior to the point where the required number of rotations is completed. This is to enable the cylinder to come to a halt in its normal position, with the arrow on its side pointing upward. At the same time the clock is also stopped; the actual speed at which the experiment was conducted can therefore be calculated. The torque-recorder is also switched off simultaneously.

The cylinder is backed off slowly until the surface of sand in the mixer is in a nearly horizontal plane. The jack-type support is raised and placed in contact with the bottom of the cylinder, where it is locked in place. The "A"-frame and the cylinder end on the sampling side are dismantled from the assembly. The sampling guide is mounted onto the groove and is revolved. When it stops it is locked in place by adjusting screws.

A random number table (26) is consulted in order to randomly choose the holes through which each of the twelve sampling probes are to be inserted. After the position of a numbered hole in the guide is determined, the probe is carefully introduced into the hole upto a certain depth. This depth of entry is also selected in a random manner. The depth can be varied because each of the probes are provided with seventeen notches. This facility allows one to obtain samples at different axial positions. The probe which is inserted with its holes closed is manipulated by opening and closing the holes in order to trap the spot samples.

The number of captured samples is found by counting the number of exposed holes in the probe to ensure that this number is more than 35. When necessary, some of the probes had to be withdrawn and reinserted in order to obtain this many samples.

The sampling probes are then carefully withdrawn and each of the spot samples is transferred to a separate pill box. These boxes are numbered to correspond with the numbers assigned to the sample holes on each probe.

Next, in order to ascertain the number of samples that will be representative of the whole, a preliminary calculation was made. All the experiments were conducted with a one to one ratio of colored sand to white sand. Hence, the probability of finding a colored particle at any point in the mixture is constant and is equal to the proportion of the colored particle in the whole mixture, which in this case is 1/2. The calculation therefore yields a value of nine, for the requisite number of samples. But to be on the sure side, it was decided to select fifteen samples randomly. Each of these samples were counted to determine the number of dyed and undyed particles. In a few cases thirty samples were randomly selected and each one counted to find out the number of dyed and undyed particles. As will be seen later the results obtained with thirty samples are not different from those found by using fifteen, indicating that the latter figure represents quite a sufficient number of samples.

After the sampling probes are withdrawn, the holes that were pierced into the polyethylene barrier were sealed with Scotch tape. The end assembly is then placed in its correct position and reassembled.

The strip chart with the recorded value of power consumed, is then labelled and stored for analysis.

Depending on the conditions under which the next run is to be conducted, the appropriate amount of sand is either added to or removed from the mixer and the angle of inclination is also adjusted to that required by the experiment. As before, the speed at which the run is to be conducted is attained by repeated adjustments of the resistances in the circuit. Once the required speed is reached, these adjustments are left undisturbed. Similarly, the power measuring system is turned on, the calibration procedures repeated and initial torque recordings obtained. These adjustments are also left intact for the next run.

The mixer is then emptied of its contents of mixed sand through the charging hole on the driven end of the cylinder. An adaptor is attached to this hole and is closed by means of a plug. The cylinder is rotated so that the adaptor and plug are in line with a brass funnel, to the bottom of which is attached a flexible hose. The plug is removed and the mixed sand is allowed to discharge into a suitable drum placed beneath the cylinder. This sand mixture is either washed free of the dye or is colored with the dye depending on the need. The details of these two procedures have been given in an earlier section.

4. TREATMENT OF DATA

4.1 Calculation of Sample Mean and Population Mean

Subsequent to the counting of each sample in order to determine the number of white and blue sand, the proportion of dyed sand in each spot sample was calculated. From this, the percentage p_i of the dyed sand was then evaluated for each spot sample. The population mean $\bar{p}$ was obtained by dividing the sum of the values of p_i by the number of spot samples.

4.2 Calculation of Degree of Mixing

The measure employed for the Degree of Mixing was the standard deviation of the composition of the dyed sand. It is given by

$$S^* = \sqrt{\frac{\sum_{i=1}^{n} (p_i - P)^2}{n}} \qquad \text{----(A)}$$

where p_i = the percentage of the dyed component in sample i

P = the population mean which is equal to the percentage of the dyed component in the mixture as known from charging the cylinder

n = number of spot samples

and S^* = the estimate of the population standard deviation.

It was because of the fact that the standard deviation brings out the variation of the uniformity within the mixture, that it was chosen to represent measure of the Degree of Mixing. Thus, a low standard deviation which corresponds to a high uniformity of composition within the mixture would indicate a good mix. By the same token, mixtures depicting a high value for the standard deviation are undesirable as they are poorly mixed.

At this point it is appropriate to talk about the nature of the sample variance with respect to any question that may arise about the unbiasedness property. In general, the sample variance S^2, is an unbiased estimate of population variance, σ^2, only when n-1 is used in its denominator (4, 2, 15). Thus, when defined as

$$S = \sqrt{\frac{\sum_{i=1}^{n} (p_i - \bar{p})^2}{n - 1}} \qquad \text{----(B)}$$

where $\bar{p}$ ($= \sum p_i / n$) represents the estimate of the population mean, it would denote the unbiased estimate. In special cases if the population mean P is known, (as is very seldom true in practice) such as in the present study where it has a value of 50 per cent, then the actual mean square of the deviation from P is an unbiased estimate of the population variance σ^2, and

$$(S*)^2 = \frac{\sum_{i=1}^{n}(p_i - P)^2}{n} \qquad \text{-------}(A^1)$$

Strictly speaking, even though generally the sample variance s^2 is an unbiased estimate of the population variance, its square root s however, is not an unbiased estimate of the population standard deviation; s^2 is mathematically tractable whereas its square root s is not (15). This also applies to S* which is the square root of $(S*)^2$ given in Equation (A^1) above.

A question may then arise as to why S* is used as a measure for the Degree of Mixing rather than $(S*)^2$. Primarily, one is attempting to measure the Degree of Mixing in the Physical world and a convenient statistic is necessary. The standard deviation S* is more useful for discussing the mixing process because it comes out linear and is in the same units as the unit of measurement. Even though S* is intractable, it is physically more meaningful a measure. Whatever bias there may be, is very small. In fact, to show that the bias is indeed small one can calculate $(S_1{}^*)^2$ and then $S_1{}^*$ as follows

$$S_1{}^* = \sqrt{\frac{\sum_{i=1}^{n}(p_i - P)^2}{n - 1}} \qquad \text{----} (C)$$

and compare it with S*. From the results it will be seen later, that there is no difference between the two.

All three measures of the Degree of Mixing viz. S*, S and S_1^* were calculated using Equations (A), (B) and (C) respectively. The value of S has been included in the results for purposes of comparison with the previous study and for reference.

4.3 Calculation of Power Consumption

The oscillograph recording of the value of the torque was read off in chart units, wherein one chart unit corresponds to five lines on the chart paper. It was then multiplied by the Attenuator setting and the conversion factor (as detailed earlier) in order to obtain the average value of the torque required for that particular experiment. Depending upon the level for the variable A viz. rate of rotation at which the experiment was conducted, the horsepower expended P*, was evaluated from the equation

$$P^* = \frac{2 \pi T N}{33{,}000}$$

The nomenclature and the conversion factors are the same as those given earlier.

All the above computations were performed on a desk calculator capable of printing out the results, primarily to obtain the results of a run in a detailed fashion, immediately after the experiment is completed.

A full discussion and a detailed account of the manner in which these results were handled with aid of a digital computer will be given in a subsequent chapter.

5. RESULTS

5.1 Mean and Standard Deviation S.

The calculated values of the population mean $\bar{p}$ and the estimate of the population standard deviation S around $\bar{p}$, calculated by using Equation (B) given earlier, along with the data from which they were generated are presented in Appendix I.

5.2 Degree of Mixing

The values of S^* amd S_1^* were calculated from Equations (A) and (C) respectively and are reported in Table 8. It may be recalled that both S^* as well as S_1^* represent the Degree of Mixing.

For the sake of comparison, the values of S are also included in Table 8. For Experiments 10, 12 and 13, when thirty instead of fifteen spot samples were taken, the results are not very different as shown, indicating that fifteen samples are quite sufficient for this study.

When one examines the results and treatment combinations given in Table 8, it will be seen that six of the experiments used in the present study are under conditions similar to those used by Hager (16). It must be remembered however, that even though the conditions for the present study are close to the ones

TABLE 8

DEGREE OF MIXING AT VARIOUS TREATMENT COMBINATIONS

Expt. No.	Treatment Combination	Degree of Mixing		
	a b f g	S*	S_1*	S
1	0 0 0 0	39.992	41.396	41.355
2	1 2 1 2	4.718	4.884	4.706
3	2 1 2 1	19.040	19.708	19.642
4	0 0 2 1	44.943	46.520	44.994
5	1 2 0 0	6.300	6.521	6.362
6	2 1 1 2	4.032	4.173	4.010
7	0 0 1 2	44.211	45.762	44.207
8	1 2 2 1	7.999	7.729	7.680
9	2 1 0 0	22.037	22.810	21.638
10●	0 1 0 2	9.163	9.485	9.807
11	1 0 1 1	46.693	48.331	45.514
12●	2 2 2 0	11.673	12.083	12.080
13●	0 1 2 0	6.315	6.536	5.413
14	1 0 0 2	38.913	40.279	40.110
15	2 2 1 1	3.377	3.495	3.491
16	0 1 1 1	7.501	7.764	7.764
17	1 0 2 0	48.327	50.023	47.213
18	2 2 0 2	4.285	4.435	4.080
19	0 2 0 1	6.077	6.289	6.029
20	1 1 1 0	20.917	21.651	21.639
21	2 0 2 2	19.934	20.634	20.487
22	0 2 2 2	4.084	4.228	4.225
23	1 1 0 1	6.196	6.414	6.405
24	2 0 1 0	42.271	43.755	43.822
25	0 2 1 0	6.386	6.609	6.410
26	1 1 2 2	13.592	14.069	13.606
27	2 0 0 1	39.208	40.584	40.200

● When 30 instead of 15 samples were taken the following values were obtained.

	S*	S_1*	S
Expt. No: 10	7.914	8.049	8.274
Expt. No: 12	14.153	14.345	14.242
Expt. No: 13	6.337	6.446	5.903

used earlier, they are not exactly the same. This is because baffles (whose effects upon mixing were found to be relatively less pronounced compared to the other variables) were not used in the current series. The difference lies in the fact that Hager (16) had always incorporated the lower level for the variable C, representing the number of baffles, in the runs that had similar conditions. The results from these six experiments under discussion are listed in Table 9, along with those from Hager's study. It can be seen from Table 9, that wherever mixing is good i.e. whenever S* has a value of approximately 12 or less, the results obtained are quite compatible. However, when one inspects the results for Experiments 9 and 12 of the current series, it is seen that they tend to oppose the results obtained earlier. One would therefore suspect the cause of this discrepancy to be the abence of baffles; this would mean that under one condition (Experiment No: 9) the absence of baffles hinders mixing while under some other condition (Experiment No: 12) this absence aids the mixing process. Indeed, an examination of the conclusions from Hager's study clearly indicates this to be the case and shows how the presence of baffles improves mixing for a cylinder that is 30 per cent charged, as is the case in Experiment No: 9, while hindering mixing when the mixer is filled to the extent of 50 per cent, which is the case in Experiment No: 12. Therefore, wherever

TABLE 9

RESULTS FROM EXPERIMENTS
WITH AND WITHOUT BAFFLES

PREVIOUS STUDY (with baffles)				CURRENT STUDY (without baffles)			
Run No.	Treatment Combn.	Degree of Mixing		Expt. No.	Treatment Combn.	Degree of Mixing	
		S	S*			S	S*
63	ad	11.718	13.557	9	2 1 0 0	21.638	22.037
18	dg	6.006	6.282	10	0 1 0 2	9.807	9.163
49	abdf	23.441	23.364	12	2 2 2 0	12.080	11.673
119	df	11.118	11.015	13	0 1 2 0	5.413	6.315
2	abdg	5.381	5.750	18	2 2 0 2	4.080	4.285
34	bdfg	5.983	6.531	22	0 2 2 2	4.225	4.084

differences occur between results from related experiments in the previous study and the current one, these may be easily explained on the presence of baffles in the earlier study and their interactions if any, with the other variables.

These experiments may be considered to be control runs in a sense, because they did provide a guideline to determine whether or not the mixing process was under control. The results clearly depict that the process was well under control as indicated by the absence of any untoward behavior in the experiments.

5.3 Power Consumption

The value of P* which represents the amount of power required measured in horsepower, has been tabulated in Table 10 along with the conditions at which the experiments were carried out.

5.4 Correlation of the Results

First of all, the results were examined graphically to determine trends in variation of the Degree of Mixing S* and the Power consumed P* with varying values of the variables. In order to do this, the results were separated into 9 sets consisting of 3 values each and a series of graphs were plotted of S* and P* versus the variable under examination, with two other variables held constant in each set, while the fourth variable was different for every point in each set.

TABLE 10

POWER CONSUMPTION AT DIFFERENT TREATMENT COMBINATIONS

Experiment No.	Treatment Combination	Power Required P*
	a b f g	Horsepower
1	0 0 0 0	0.018
2	1 2 1 2	0.079
3	2 1 2 1	0.078
4	0 0 2 1	0.015
5	1 2 0 0	0.083
6	2 1 1 2	0.105
7	0 0 1 2	0.016
8	1 2 2 1	0.063
9	2 1 0 0	0.110
10	0 1 0 2	0.045
11	1 0 1 1	0.027
12	2 2 2 0	0.092
13	0 1 2 0	0.038
14	1 0 0 2	0.027
15	2 2 1 1	0.133
16	0 1 1 1	0.042
17	1 0 2 0	0.026
18	2 2 0 2	0.144
19	0 2 0 1	0.047
20	1 1 1 0	0.058
21	2 0 2 2	0.046
22	0 2 2 2	0.039
23	1 1 0 1	0.059
24	2 0 1 0	0.053
25	0 2 1 0	0.045
26	1 1 2 2	0.053
27	2 0 0 1	0.057

In order to illustrate graphically, the variation of S* and P* with respect to the variable A, which represents the rate of rotation, the results were arranged into the nine sets shown in Table 11. In this arrangement, the values of variables B and F are held constant in each of the nine sets; that of G varies with every point within each set.

These S* values were plotted against A as depicted in Figure 13. It must be noted that even though A is plotted on the abscissa, the variation in S* on the ordinate scale is not caused by A alone, but also by simultaneous variation of the other variables involved in the experiments. One can therefore appreciate the difficulty in attempting to represent the variation of more than two variables on the plane of the graph paper. Each of the nine sets in Figure 13, is identified by numbering the graph corresponding to that particular set. As indicated, in addition to the value of A, that of variable G at each individual point in a set has also been identified. As for the levels of variables B and F which as already mentioned are held constant in each set, they are easily determined by referring to Table 11. For example, in the case of Set 1, the treatment combinations reveal that B and F are both held at the 0 level, i.e. at a 15 per cent load, and 0^{o} angle of inclination. Notice that in each set, the value of A increases in an orderly

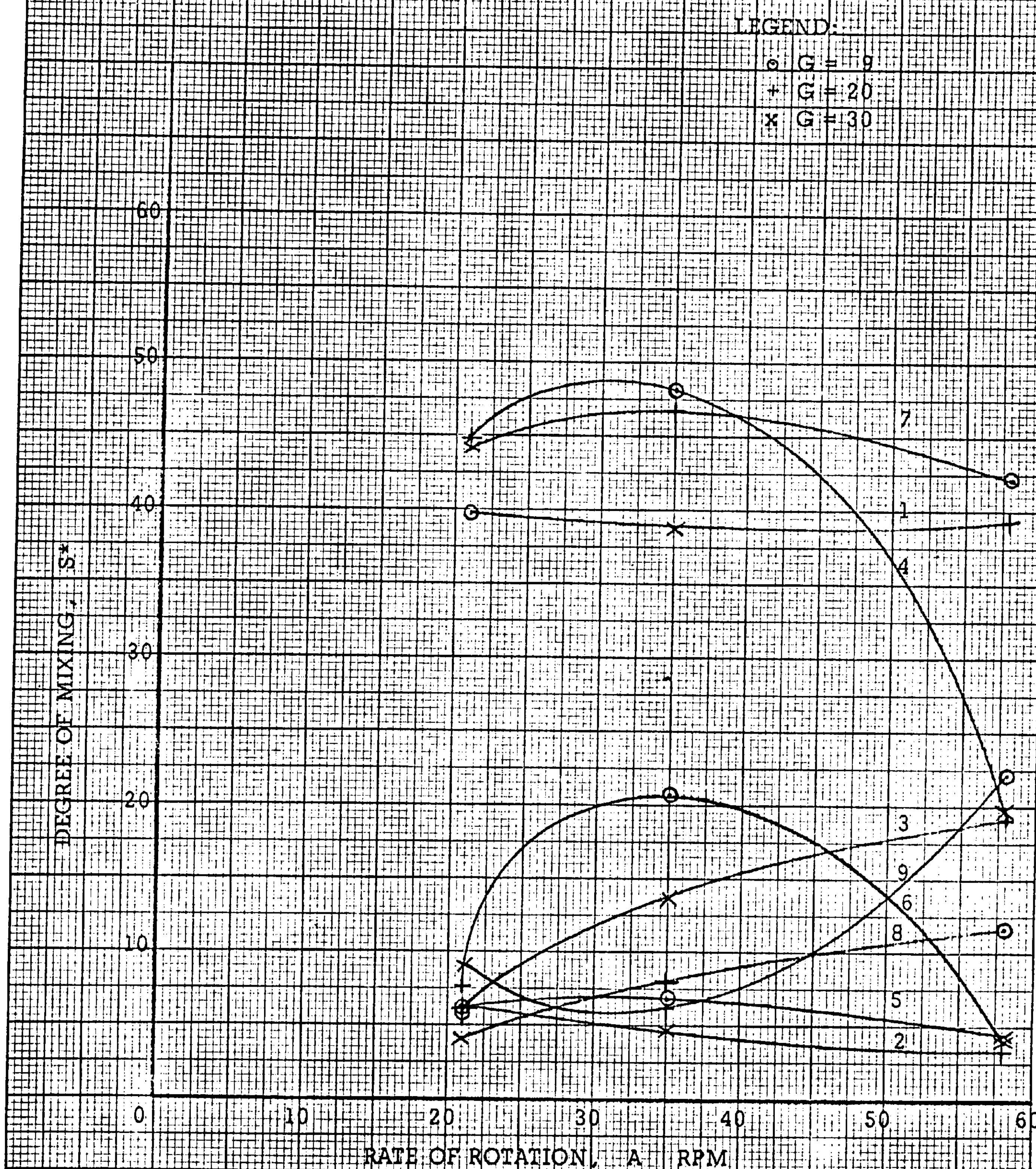
TABLE 11

RESULTS ARRANGED FOR VARIATION IN A

Expt. No.	Treatment Combn. a b f g	S*	P* hp.
	Set 1		
1	0 0 0 0	39.992	0.018
14	1 0 0 2	38.913	0.027
27	2 0 0 1	39.208	0.057
	Set 2		
25	0 2 1 0	6.386	0.045
2	1 2 1 2	4.718	0.079
15	2 2 1 1	3.377	0.133
	Set 3		
13	0 1 2 0	6.315	0.038
26	1 1 2 2	13.592	0.053
3	2 1 2 1	19.040	0.078
	Set 4		
4	0 0 2 1	44.943	0.015
17	1 0 2 0	48.327	0.026
21	2 0 2 2	19.934	0.046
	Set 5		
19	0 2 0 1	6.077	0.047
5	1 2 0 0	6.300	0.083
18	2 2 0 2	4.285	0.144
	Set 6		
16	0 1 1 1	7.501	0.042
20	1 1 1 0	20.917	0.058
6	2 1 1 2	4.032	0.105
	Set 7		
7	0 0 1 2	44.211	0.016
11	1 0 1 1	46.693	0.027
24	2 0 1 0	42.271	0.053
	Set 8		
22	0 2 2 2	4.084	0.039
8	1 2 2 1	7.999	0.063
12	2 2 2 0	11.673	0.092
	Set 9		
10	0 1 0 2	9.163	0.045
23	1 1 0 1	6.196	0.059
9	2 1 0 0	22.037	0.110

FIGURE 13

DEGREE OF MIXING S* VERSUS RATE OF ROTATION A



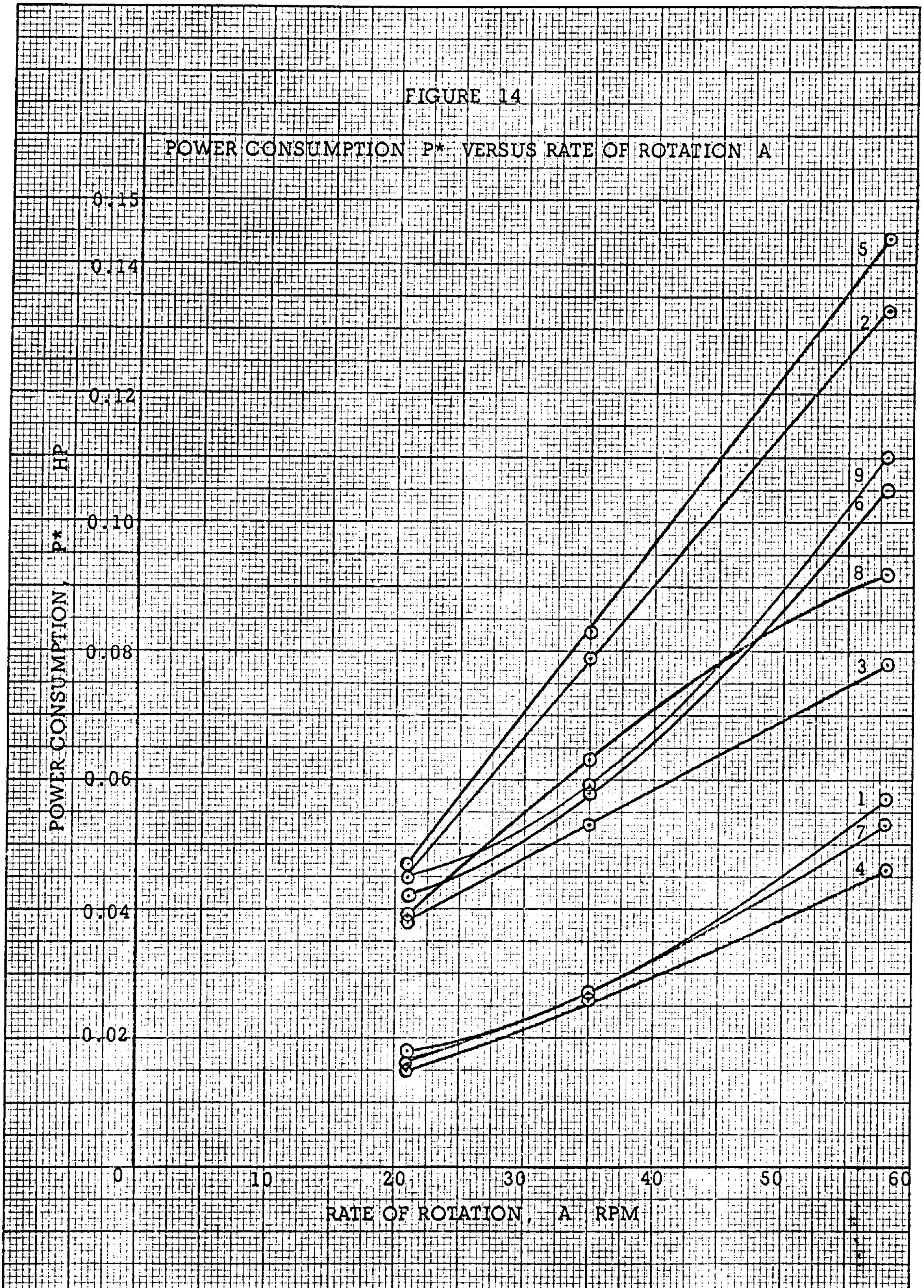


FIGURE 14

POWER CONSUMPTION P* VERSUS RATE OF ROTATION A fashion through its levels viz. 0 , 1 and 2, while G takes on a value at the level dictated by the original Design.

Examination of Figure 13, reveals that sets 7, 1 and 4 form a family of curves that show poor mixing as indicated by the large values for S*. A search for a common trait in these three sets shows that all three have the same value for variable B (see Table 11) viz. 15 per cent. This would indicate that a load of this magnitude should not be used because of the poor mixing that occurs.

Another tentative conclusion from Figure 13 would be that there appears to be no appreciable effect of variable A upon S*, as evidenced by lack of a marked change in S* with increasing A, in most cases. If this is not immediately evident, in cases like sets 4,6, and 9, then a closer look at the conditions of the experiments in these sets is in order. Thus, in Set 4 for instance, the third point has a value of S* = 19.934, because the cylinder was allowed to rotate for 30 rotations which is greater than for the other two experiments in Set 4. As for Set 6, the point corresponding to Experiment No: 20, has a higher S* value equal to 20.917, primarily because the cylinder was allowed to rotate for only 9 revolutions. A similar situation exists in Set 9, where S* for experiment No: 9 increases to a value of 22.037 due to the fact that variable G was kept at its 0 level of 9 rotations.

It may also be noticed that at any value of A, as long as there is a load of 50 per cent, the Degree of Mixing has a low value indicating good mixing.

A speed of about 30 RPM or less is preferable as this does not entail a loss in the quality of the mixture.

The values of P* the power consumption have also been shown in Table 11 for the nine sets. Figure 14 shows a plot of these P* values against the rate of rotation A. In this case it may be observed that, power which represents a rate of work function is essentially independent of the number of rotations G.

From Figure 14, it is evident that Power Consumption increases almost linearly with increase in A. Moreover, P* also increases with increase in the level of variable B. This is evidenced by sets 1, 7 and 4 all of which have B at its 0 level, consuming the least power, with progressive increase upto sets 2, 5 and 8 where the level of variable B is 2. There also seems to be a tendency for decrease in P* with increasing angles of inclination F.

In Table 12, the results have been arranged for variation in B; so that in every one of the nine sets, the value of variable B increases from its 0 level upto its 2 level. The values of A and F are held constant in any given set for every run in that set. Hereagain, G varies simultaneously with B. The Degree of Mixing S* is plotted against B in Figure 15. Only six of the nine sets of Table 12, have

TABLE 12

RESULTS ARRANGED FOR VARIATION IN B

Expt. No.	Treatment Combn. a b f g	S*	P* hp.
	Set 1		
1	0 0 0 0	39.992	0.018
10	0 1 0 2	9.163	0.045
19	0 2 0 1	6.077	0.047
	Set 2		
11	1 0 1 1	46.693	0.027
20	1 1 1 0	20.917	0.058
2	1 2 1 2	4.718	0.079
	Set 3		
21	2 0 2 2	19.934	0.046
3	2 1 2 1	19.040	0.078
12	2 2 2 0	11.673	0.092
	Set 4		
4	0 0 2 1	44.943	0.015
13	0 1 2 0	6.315	0.038
22	0 2 2 2	4.084	0.039
	Set 5		
14	1 0 0 2	38.913	0.027
23	1 1 0 1	6.196	0.059
5	1 2 0 0	6.300	0.083
	Set 6		
24	2 0 1 0	42.271	0.053
6	2 1 1 2	4.032	0.105
15	2 2 1 1	3.377	0.133
	Set 7		
7	0 0 1 2	44.211	0.016
16	0 1 1 1	7.501	0.042
25	0 2 1 0	6.386	0.045
	Set 8		
17	1 0 2 0	48.327	0.026
26	1 1 2 2	13.592	0.053
8	1 2 2 1	7.999	0.063
	Set 9		
27	2 0 0 1	39.208	0.057
9	2 1 0 0	22.037	0.110
18	2 2 0 2	4.285	0.144

FIGURE 15

DEGREE OF MIXING S* VERSUS PERCENTAGE LOAD B

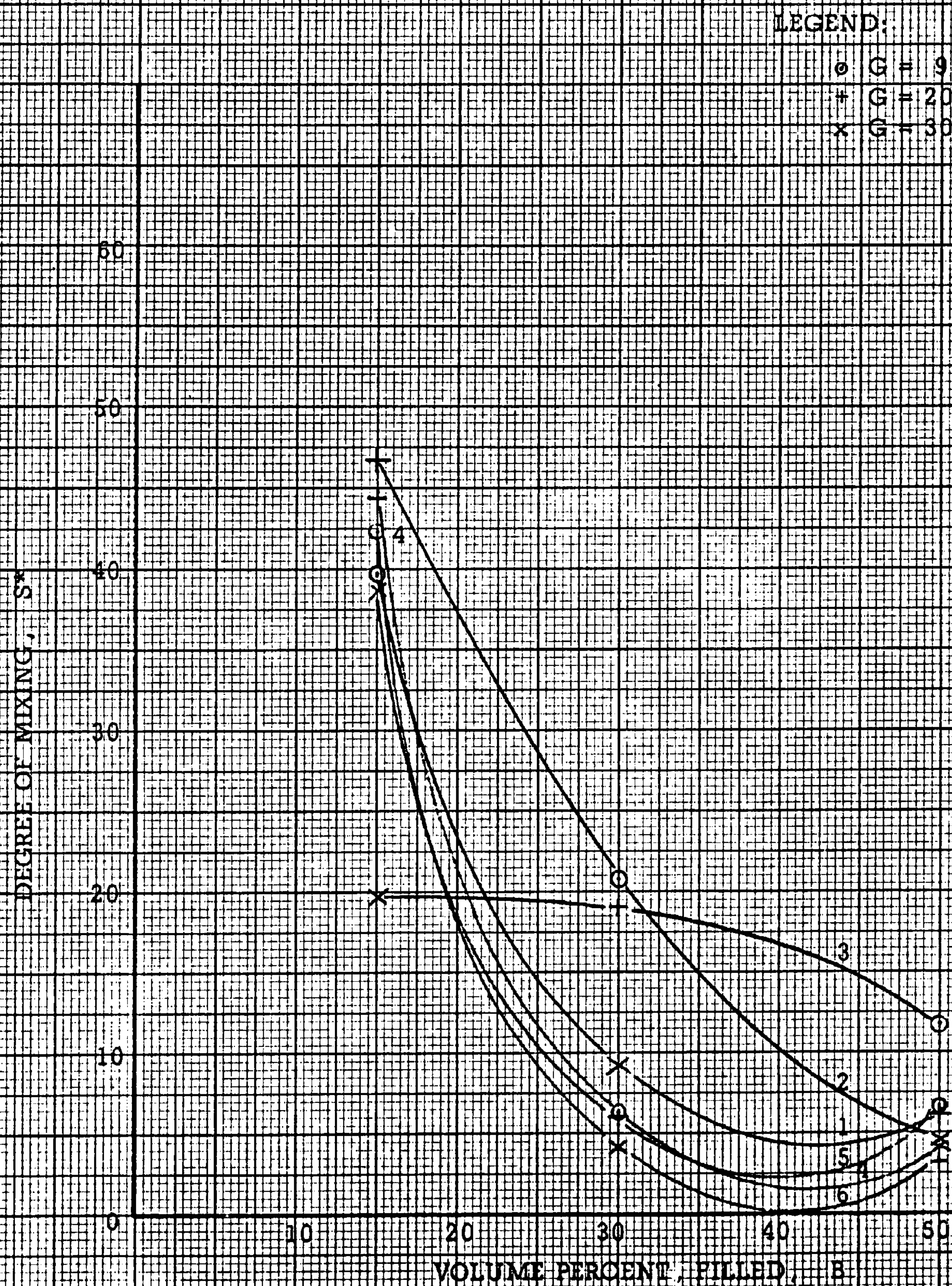
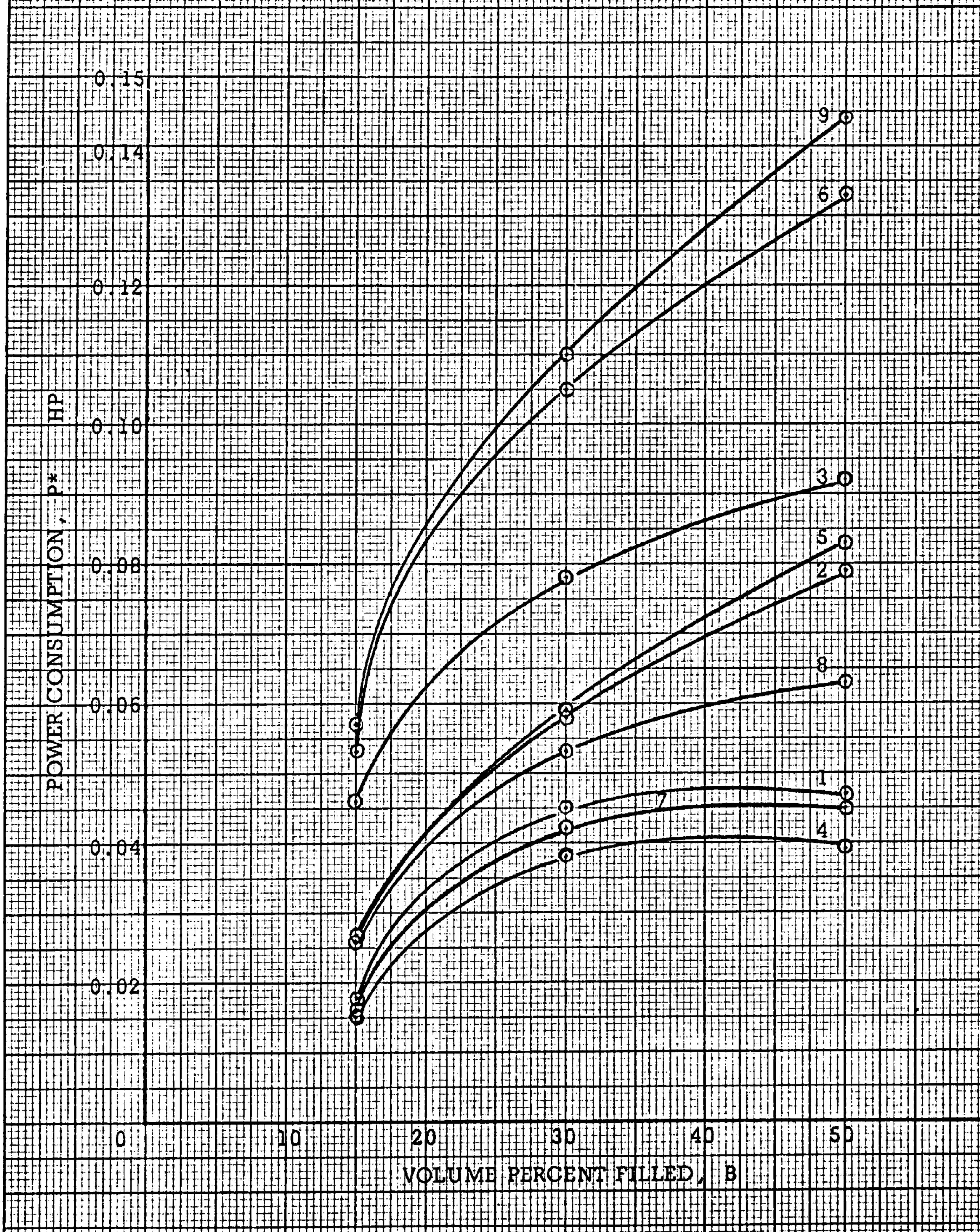


FIGURE 16

POWER CONSUMPTION P* VERSUS VOLUME PERCENT FILLED B



been plotted in Figure 15, because the remaining three also show a similar pattern of behavior. The main feature in this relationship between S* and B, is that there is an optimum value of B somewhere between 30 and 50 per cent load. Specifically from the nature of the graphs in Figure 15 as well as from the results in Table 12, this value seems to be close to a value of B equal to 40 per cent. Even more so than indicated by Figure 13, it is apparent from Figure 15, that a value of B of 15 per cent will almost invariably yield a poorly blended product, and should therefore be avoided. If Experiment No: 21 of Set 3 with an S* value of 19.934, seems to be different from the crowd of the other similarly loaded ones, it is because of the fact that in this particular run the angle of inclination F was at its 2 level of 14 degrees.

The P* values given in Table 12, were plotted against the Volume Per Cent filled B in Figure 16. These graphs which have been numbered according to their sets in Table 12, could be divided roughly into three groups. The first would be formed by sets 1, 7 and 4. Their common feature of course, is that they all have variable A at its 0 level viz. 21 RPM. The second group would consist of sets 5,2 and 8, while the last grouping would include sets 9, 6 and 3. The second group has a common speed of 35 RPM and the last one a rate of rotation of 58 RPM.

Here also, as from Figure 14, it appears that P* increases directly with speed A, load B, and inversely with angle of inclination F.

In order to determine the effect of the angle of inclination F, upon both S* and P*, the results were arranged as in Table 13. While F increases from its 0 level to its 2 level in every set, A and B are held constant in a given set and G is also allowed to vary simultaneously. The values of S* taken from Table 13 have been plotted against F for the various sets, as shown in Figure 17.

A clearly defined influence of F upon S* if any, is difficult to detect from this Figure. This is because of the simultaneous variation of G with F. Thus in Set 8, Experiment No: 20 has an S* value of 20.917 which is larger than the other two, not so much because it was conducted at an angle of 7 degrees (equivalent to level 1 for F) but because it was permitted only 9 revolutions.

The inference drawn earlier viz. that a load of 15 per cent produces a poor mix, is also borne out by Figure 17. Perhaps the only area of operations where increasing the angle of inclination might improve mixing, is this very region of small loads, provided sufficient time (G is a function of time) is allowed for mixing. For example, Experiment No: 21 in Set 9 shows a relatively lower value

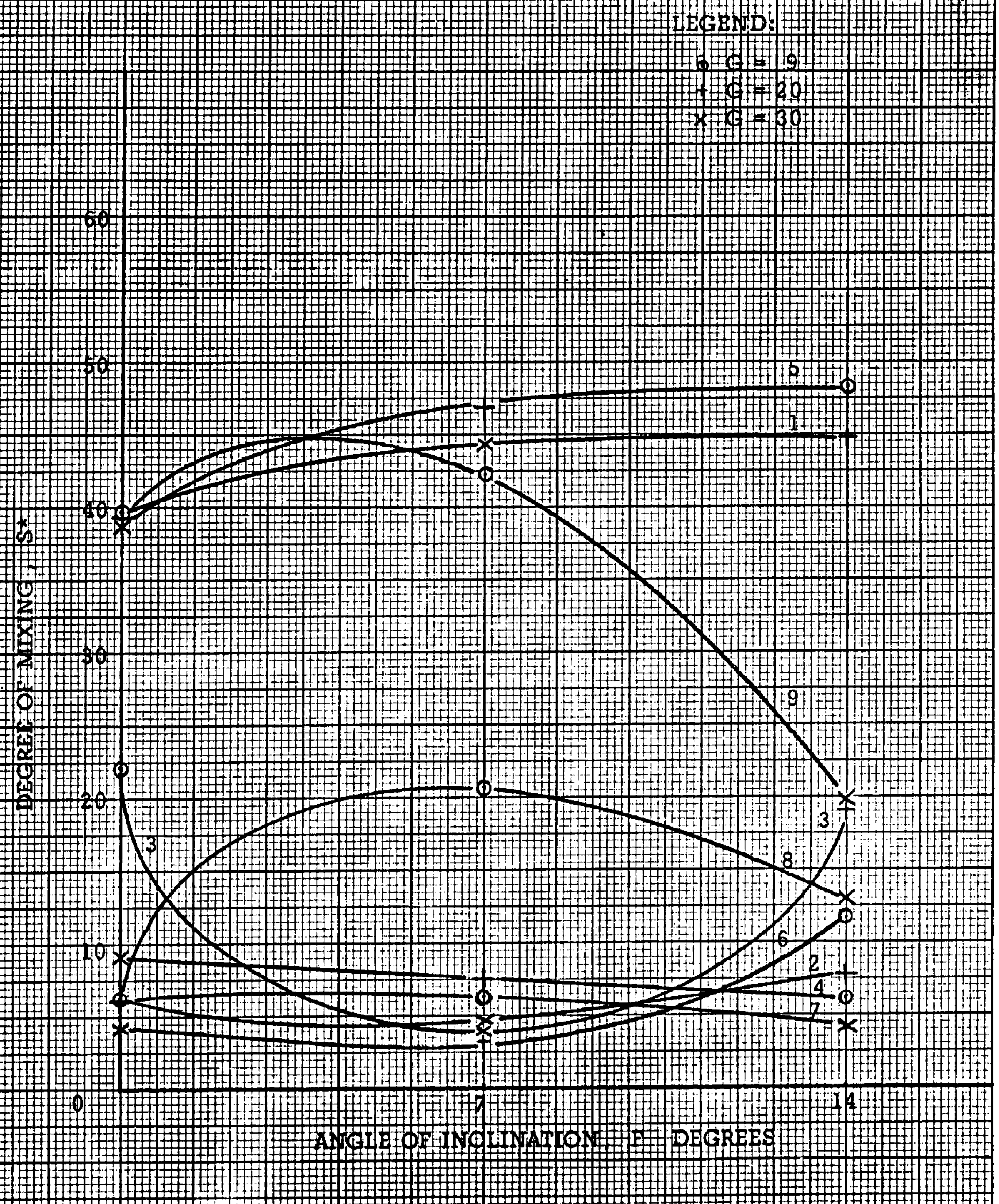
TABLE 13

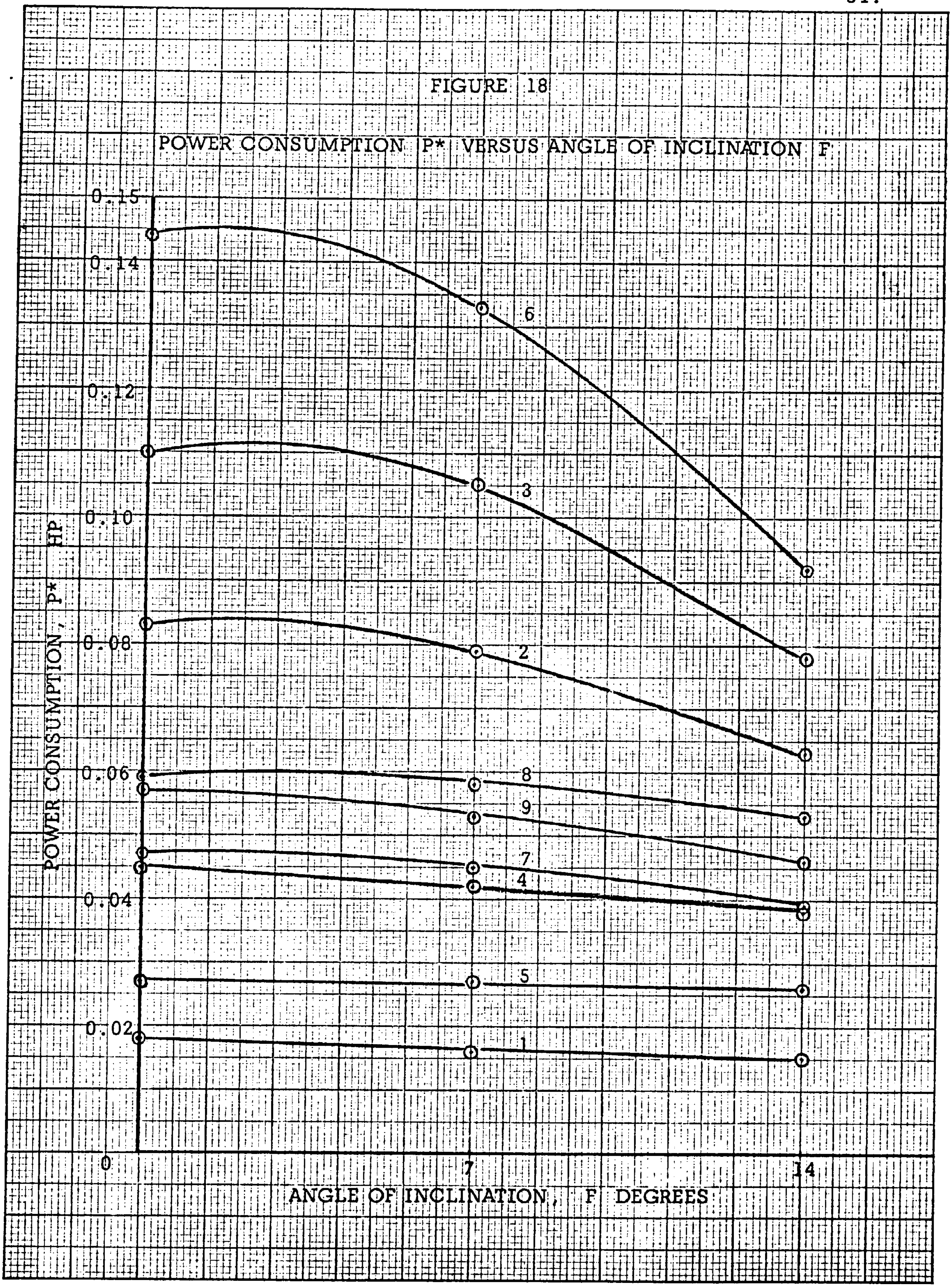
RESULTS ARRANGED FOR VARIATION IN F

Expt. No.	Treatment Combn. a b f g	S*	P* hp.
	Set 1		
1	0 0 0 0	39.992	0.018
7	0 0 1 2	44.211	0.016
4	0 0 2 1	44.943	0.015
	Set 2		
5	1 2 0 0	6.300	0.083
2	1 2 1 2	4.718	0.079
8	1 2 2 1	7.999	0.063
	Set 3		
9	2 1 0 0	22.037	0.110
6	2 1 1 2	4.032	0.105
3	2 1 2 1	19.040	0.078
	Set 4		
10	0 1 0 2	9.163	0.045
16	0 1 1 1	7.501	0.042
13	0 1 2 0	6.315	0.038
	Set 5		
14	1 0 0 2	38.913	0.027
11	1 0 1 1	46.693	0.027
17	1 0 2 0	48.327	0.026
	Set 6		
18	2 2 0 2	4.285	0.144
15	2 2 1 1	3.377	0.133
12	2 2 2 0	11.673	0.092
	Set 7		
19	0 2 0 1	6.077	0.047
25	0 2 1 0	6.386	0.045
22	0 2 2 2	4.084	0.039
	Set 8		
23	1 1 0 1	6.196	0.059
20	1 1 1 0	20.917	0.058
26	1 1 2 2	13.592	0.053
	Set 9		
27	2 0 0 1	39.208	0.057
24	2 0 1 0	42.271	0.053
21	2 0 2 2	19.934	0.046

FIGURE 17

DEGREE OF MIXING S* VERSUS ANGLE OF INCLINATION F





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of 19.934 for S* than other comparable values. In spite of this, the final product as indicated by such an S* value, cannot be considered to be good, let alone optimum.

Next, in order to determine the influence of the angle of inclination F, on power consumption P*, Figure 18 was drawn. The values are those given in Table 13. There appears to be a decrease in the power requirements with an increase in the angle of inclination. This effect is slightly more pronounced at higher loads combined with higher rates of rotation.

The results when arranged to find out the effect of the variable G upon the Degree of Mixing S*, yield Table 14. These values are illustrated in graphical form in Figure 19. In practically every case, it can be seen that an increase in the number of rotations is beneficial to mixing. In each set, A and B have constant values whereas the value of F varies from experiment to experiment within a set. In Table 14, Set 8 has within it Experiment No: 26; when plotted on Figure 19, it appears as though increasing the number of rotations actually hinders mixing, which certainly is unreasonable. However, upon examination of the other conditions, it is seen that F has a value of 14 degrees corresponding to its level of 2; this is the reason why Experiment No: 26 has a slightly higher value of S*.

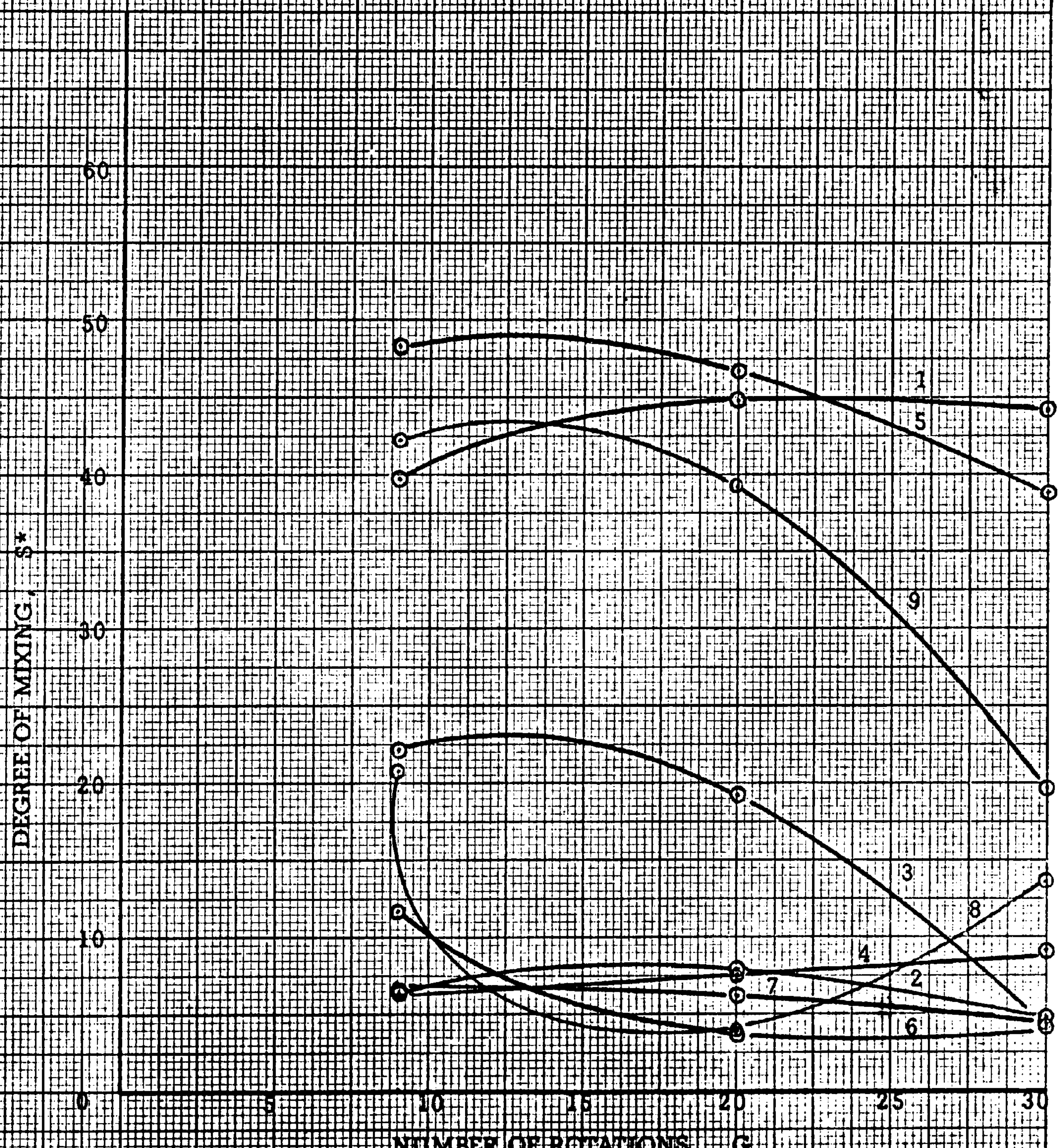
TABLE 14

RESULTS ARRANGED FOR VARIATION IN G

Expt. No.	Treatment Combn. a b f g	S*	P* hp.
	Set 1		
1	0 0 0 0	39.992	0.018
4	0 0 2 1	44.943	0.015
7	0 0 1 2	44.211	0.016
	Set 2		
5	1 2 0 0	6.300	0.083
8	1 2 2 1	7.999	0.063
2	1 2 1 2	4.718	0.079
	Set 3		
9	2 1 0 0	22.037	0.110
3	2 1 2 1	19.040	0.078
6	2 1 1 2	4.032	0.105
	Set 4		
13	0 1 2 0	6.315	0.038
16	0 1 1 1	7.501	0.042
10	0 1 0 2	9.163	0.045
	Set 5		
17	1 0 2 0	48.327	0.026
11	1 0 1 1	46.693	0.027
14	1 0 0 2	38.913	0.027
	Set 6		
12	2 2 2 0	11.673	0.092
15	2 2 1 1	3.377	0.133
18	2 2 0 2	4.285	0.144
	Set 7		
25	0 2 1 0	6.386	0.045
19	0 2 0 1	6.077	0.047
22	0 2 2 2	4.084	0.039
	Set 8		
20	1 1 1 0	20.917	0.058
23	1 1 0 1	6.196	0.059
26	1 1 2 2	13.592	0.053
	Set 9		
24	2 0 1 0	42.271	0.053
27	2 0 0 1	39.208	0.057
21	2 0 2 2	19.934	0.046

FIGURE 19

DEGREE OF MIXING S* VERSUS NUMBER OF ROTATIONS G



Some other graphical representations were also constructed; but they did not reveal indications substantially different from the ones already given and are therefore not included here.

One of the chief conclusions that can be drawn from the examination of the results and graphs, is that wherever the factor B is held at its lowest level viz. 15 per cent of the mixer charged, the value of S* was very high and therefore the extent of mixing very poor. This tendency to give very high values for S* is observed at different combinations of the levels of the other factors A, F and G; but in all these cases B has a value of 15 per cent.

With regard to the Power consumption P* measured in units of horespower, a general trend is observed viz. that P* varies directly as A, directly as B and inversely as F. Power being the rate of work is independent of G.

Even though this much can be inferred from the results, there remains quite a lot that needs to be known before one can attempt to evolve a quantitative functional relationship between S* and P* on the one hand and the variables on the other. For instance, from the graphical treatment, we have no idea of the manner in which interactions between the variables affect the Degree of Mixing and the Power consumption. Another aspect of the problem that remains unknown, is the relative importance of each of variables and their interactions. Moreover, virtually no quantitative information can be gathered from graphical representations; at best only qualitative pointers are available. Since quantitative functional relationships must first be generated before any optimization procedure can be adopted, new avenues of interpreting the results had to be developed.

Initially it was thought that it might be advantageous to separate the results into two groups viz. one where the level of variable B which is the percent volume charged was 15 per cent, and the other group where it had a value other than 15 per cent. The reason for this was the fact that in cases belonging to the former group, very little mixing actually took place. Further in the interest of keeping the analysis simple, it was thought that it would be preferable to concentrate the analytical efforts on the group consisting of runs where the value of B was not 15 per cent. Also, it is quite certain that the optimum which we are seeking is not in the region of B equal to 15 per cent. Therefore the results from all the experiments where B had a value other than 15 per cent, were further analysed by means of planar plots and more graphs. However, these efforts though somewhat productive did not yield definitive pointers as to what variables in terms of their main effects and interactions should be included in the model that would quantitatively describe both the Degree of Mixing as well as Power as a function of the variables under study.

Upon re-examination of the previous stand, it was felt that neglecting those readings where the variable B did have a value of 15 per cent (undersirable though this may be from the standpoint of efficient mixing) would probably entail a loss of other information, which loss would not be very desirable. Hence, it was decided that it would be better to retain the whole body of the results as a unit and attempt analysis of the same.

This called for a new strategy, different from the one adopted earlier, whereby the relative importance of the main effects and interactions of the variables involved, could be gauged. The technique for the analysis was as follows. It was evident that the effect of B was very significant both from observations and the graphs, and in addition since the effect of B appeared to also bear a quadratic relationship to the Degree of Mixing, it was decided to fit a quadratic equation in B to the Degree of Mixing measured by S* and to examine the resulting residuals.

The quadratic equation in B expressing this relationship is

$$S^* = 89.413 - 3.944B + 0.0455B^2 \qquad \text{--------(D)}$$

This equation was obtained by feeding in the results into a GE-225 digital computer and asking for a polynomial fitting by sequential least squares. Details on computer techniques connected with this as well as other subsequent methods of handling the results with the aid of this computer are given in Appendix II.

An analysis of variance on the effect of this regression showed that both the linear as well as the quadratic effect of B were significant. In fact attempts to suit higher order equations in B, to the results, did not yield a better fitting model.

As the effect of B, both linear and quadratic were extracted by the above procedure, the variation in the residuals would then be caused by the other three variables viz. A, F and G. The residuals were calculated; and the method adopted for analysing them was the Analysis of Variance (ANOVA for short) technique. By definition (23), the analysis of variance is a statistical technique for analysing measurements depending on several kinds of effects operating simultaneously, to decide which kinds of effects are important and to estimate the effects. Naturally, a theory of analysing measurements has implications about how the experiments should be planned and the observations taken; this means adherance to an experimental design, such as in the present study. The analysis of variance is probably the most powerful procedure in the field of experimental statistics. For an interesting discussion on the assumptions underlying the analysis of variance, the papers by Eisenhart (13) and Cochran(7) should be consulted.

It may be mentioned that Yates Method (31) for the analysis of variance which was used in the previous study, can be used only in the case of two-level factorial experiments and is therefore not applicable in the present case.

The above-mentioned residuals were analysed by the technique of analysis of variance (See Appendix II) to determine the relative importance of the main effects and interactions of A, F and G. The results of this ANOVA is shown in Table 15.

It can be seen that the main effect of G is most significant. Also that the main effect of F is not significant at all. This is indicated by the extremely small value of the F-ratio in the last column, corresponding to the source F. Similarly, the relative effect of each of the other factors and interactions can be estimated from the magnitude of their mean squares and F-ratio values given in Table 15.

Since Table 15 shows that the effect of variable F is not significant, it might be useful to recall at this point, that qualitatively this is what was initially proposed. But at that time, the question arose as to whether there might not be some angle between 0 and 14 degrees at which mixing might be significantly improved. Hence, here there is evidence to show that the main effect as well as interactions of F are minor indeed.

Since the effect of variable F is minor, an analysis on the S* data with respect to A, B and G was attempted and the results

TABLE 15

ANALYSIS OF VARIANCE ON RESIDUALS FROM B

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	40.939	20.470	0.81
F	2	4.275	2.138	0.08
G	2	235.602	117.800	4.68
AF	4	128.694	32.173	1.28
AG	4	249.968	62.492	2.48
FG	4	143.955	35.989	1.43
Residual	8	201.435	25.179	
TOTAL	26	1004.869		

** Tabulated Values for F: $F_{0.05}$ (2, 8) = 4.46

$F_{0.05}$ (4, 8) = 3.84

from that ANOVA is shown in Table 16. It is interesting to note that the results in Table 16 are quite in agreement with those found earlier from graphs and Table 15. Thus it is seen in Table 16 that B is the variable that has prime influence on the mixing process. This is in agreement with what was suspected and deduced from the graphs. Next comes the variable G representing the number of rotations, which is as expected because a certain number of rotations must elapse before a good mixture results.

It may be added that on comparing Tables 15 and 16, it is seen that the Residual Mean Square values are 25.179 and 21.071 respectively; the former is caused by random error and interactions of B with A, F and G; the latter is due to random error and F and the interactions of F with A, B and G. Their small magnitudes reflect the fact that the accuracy in these experiments is very good.

In order to check the consistency of these results they were treated in several different ways. Since the main effect of A upon S* seems to be rather small both from Tables 15 and 16, the S* values were subjected to the analysis of variance under the assumption that only B, F and G were instrumental in producing the variation in S*. The results are shown in Table 17. Since the effect of A is higher than that of F, the residual mean square should be larger than in Table 16. This residual mean square from Table 17 is 63.130 which is clearly larger than 21.071 in Table 16.

TABLE 16

ANALYSIS OF VARIANCE ON THE S* VALUES (A, B AND G)

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	52.029	26.015	1.23
B	2	6079.050	3039.525	144.24
G	2	212.786	106.393	5.05
AB	4	246.515	61.628	2.92
AG	4	256.076	64.019	3.04
BG	4	69.224	17.306	0.82
Residual	8	168.572	21.071	
TOTAL	26	7084.255		

** Tabulated Values for F: $F_{0.05}\ (2,8) = 4.46$

$F_{0.05}\ (4,8) = 3.84$

TABLE 17

ANALYSIS OF VARIANCE ON S* (B,F AND G)

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
B	2	6079.050	3039.525	48.14
F	2	3.502	1.751	0.03
G	2	212.787	106.393	1.68
BF	4	92.052	23.013	0.36
BG	4	69.224	17.306	0.27
FG	4	122.597	30.649	0.48
Residual	8	505.042	63.130	
TOTAL	26	7084.255		

**Tabulated Value for F: $F_{0.05}(2,8) = 4.46$

$F_{0.05}(4,8) = 3.84$

Attempts were made to fit a polynomial in F to the S* values. The purpose was to obtain residuals from such a fit and anlyse them for the effects of A, B and G. The results showed that even a linear effect F on S* is negligible, not to mention the futility of attempting fits with quadratic or higher order functions. This is quite in order because in this case one would expect a poor fit in F for S* (because of the insignificance of F) and consequently a large residual mean square should result and this is exactly what happens; and the analysis of the residuals for A,B and G shown in Table 18, exhibits remarkable agreement with the results obtained in Table 16. Whereas the residual mean square in Table 16 is 21.071, the corresponding figure when the analysis is performed on the residuals from F, is 20.634 indicating that the main effect and interactions of F are truly negligible as indicated earlier. Therefore the optimum and incidentally the most convenient value for the variable F is at a level of 0°. It may be added that had the results been otherwise, they would have been surprising.

To complete the series of tests pertaining to the analysis of variance on S*, three more programs were run and the results from these tests are shown in Part 1 of Appendix III. Suffice it to say, that all these results are in complete agreement with those

TABLE 18

ANALYSIS OF VARIANCE ON RESIDUALS FROM F

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	52.029	26.015	1.26
B	2	6079.050	3039.525	147.31
G	2	212.786	106.393	5.16
AB	4	246.515	61.628	2.99
AG	4	256.078	64.019	3.10
BG	4	69.224	17.306	0.84
Residual	8	165.070	20.634	
TOTAL	26	7080.753		

** Tabulated Values for F: $F_{0.05}(2,8) = 4.46$

$F_{0.05}(4,8) = 3.84$

obtained in Tables 15 through 18.

In order to show that the difference between S^* and S_1^* due to any bias is indeed very small, these above calculations were repeated for S_1^*; the results of analyses on S_1^* are included in Part 2 of Appendix III.

As a result of all these analyses of variance, one can obtain a quantitative rating on how the various effects stand with regard to their influence on the Degree of Mixing S^*. One therefore has a firm and legitimate footing from which to launch the process of trying out various models and arriving at the best mathematical model.

As far as the Degree of Mixing is concerned, the variables and interactions in order of their importance are B, G, AG and AB.

Next, considering the question of Power consumption P^*, it too was analysed in a manner similar to that for the Degree of Mixing. However in the case of P^*, since it is independent of G, an analysis of variance with respect to A, B and F was sufficient to give the necessary information. The results are as shown in Table 19.

With regard to Power P^* therefore, the variables of major significance are A, B and F in that order; as for interactions, the important ones are AB and AF with BF also being of slight importance.

TABLE 19

ANALYSIS OF VARIANCE ON POWER P* (A, B AND F')

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F**
A	2	0.0152	0.00759	308.18
B	2	0.0113	0.00563	228.79
F	2	0.00119	0.000598	24.28
AB	4	0.00147	0.000367	14.90
AF	4	0.000669	0.000167	6.79
BF	4	0.000404	0.000101	4.11
Residual	8	0.000197	0.0000246	
TOTAL	26	0.030374		

** Tabulated Values for F: $F_{0.05}(2,8) = 4.46$

$F_{0.05}(4,8) = 3.84$

It must be realized that these results for S* and P* do not distinguish between the linear and quadratic main effects of the variables, nor can one directly classify the effects from the linear-linear, linear-quadratic, quadratic-linear or quadratic-quadratic combination of factors teaming up to give an interaction effect. It is not possible to use the method of orthogonal polynomials to decipher these individual effects, because that method is applicable only to the case of equispaced levels.

Hence, the avenue open was to attempt to directly fit regression models involving the different significant variables by the technique of Multiple Linear Regression. (See Appendix II) The procedure that was undertaken to achieve this goal, consisted in selecting various models based upon the earlier analyses of variance and adding one variable at a time to determine its influence on the fit by examining among other items, the residual mean square as well as a quantity R^2 called the Coefficient of Determination (also called "Maximum Likelihood") which in the ordinary case reduces to the square of the multiple correlation coefficient. The magnitude of the residuals also indicate the goodness of fit, small values being naturally preferred.

Numerous models for the Degree of Mixing S* , with different combinations of the significant terms were tried out in sequence and the results are shown in Table 20. The corresponding equations of the various models of interest are shown in Table 21.

Testing with such varied models was also conducted for S_1*; these results are shown in Table IV.1 of Appendix IV. The equations corresponding to these models are given in Table IV.2 of the same Appendix. As can be observed, both S* and S_1* behave in the same manner.

The best representation of the data with the minimum number of significant terms but with sufficient accuracy is given by Equation 7 in Tables 20 and 21. The criterion was to strike a balance between a complicated model on the one hand and still obtaining a more than adequate representation of the data on the other, without adding too many terms to close the gap of the last few percentage points from a perfect fit.

TABLE 20

RESULTS FROM DIFFERENT MATHEMATICAL MODELS FOR S*

Variables No.	Variables Name	Residual Mean Square	Change in RMS	Source	Change as % Original RMS	Cumulative Per Cent***	Equation No.	R^2
0	0	272.471	----	----	----	----	1	----
1	B	84.744	187.627	B	68.898	68.898	2	0.70094
2	B, B^2	41.883	42.861	B^2	15.730	84.628	3	0.85811
3	B, B^2, G	34.709	7.174	G	2.633	87.261	4	0.88731
4	Above & G^2	36.019	- 1.310	G^2	0.481	86.780	5	0.88814
5	Above & AG	35.024	0.995	AG	0.366	87.146	6	0.89618
6	Above & AB	25.344	10.680	AB	3.552	90.698	7	0.92845
7	Above & A^2G	24.807	0.537	A^2G	0.197	90.895	8	0.93348
8	Above & AG^2	24.703	0.104	AG^2	0.038	90.933	9	0.93723
9	Above & A^2G^2	25.943	- 1.240	A^2G^2	-0.455	90.478	10	0.93774
10	Above & A^2B	27.156	- 1.213	A^2B	-0.445	90.033	11	0.93867
11	Above & AB^2	23.934	3.222	AB^2	1.179	91.212	12	0.94930
12	Above & A^2B^2	24.202	- 0.268	A^2B^2	-0.095	91.117	13	0.95217

*** Cumulative Per Cent = (R^2 based on unbiased variances) x 100

TABLE 21

EQUATIONS CORRESPONDING TO DIFFERENT MODELS FOR S^*

Equation No.	
1	$S^* = 19.562$
2	$S^* = 49.515 - 0.946B$
3	$S^* = 89.413 - 3.944B + 0.0455B^2$
4	$S^* = 95.761 - 3.944B + 0.0455B^2 - 0.323G$
5	$S^* = 93.027 - 3.944B + 0.0455B^2 + 0.0271G - 0.00901G^2$
6	$S^* = 93.027 - 3.944B + 0.0455B^2 + 0.195G - 0.00901G^2 - 0.00444AG$
7	$S^* = 93.027 - 4.324B + 0.0456B^2 + 0.708G - 0.009G^2 - 0.0179AG + 0.00997AB$
8	$S^* = 93.027 - 4.324B + 0.0456B^2 + 0.218G - 0.009G^2 + 0.0107AG + 0.00997AB - 0.000356A^2G$
9	$S^* = 93.027 - 4.224B + 0.0456B^2 - 0.528G + 0.0146G^2 + 0.0303AG + 0.00736AB - 0.000356A^2G - 0.000622AG^2$
10	$S^* = 93.027 - 4.224B + 0.0456B^2 - 1.187G + 0.0401G^2 + 0.0688AG + 0.00736AB - 0.000834A^2G - 0.00211AG^2 + 0.0000185A^2G^2$
11	$S^* = 93.027 - 3.927B + 0.0456B^2 - 2.268G + 0.0663G^2 + 0.132AG - 0.00998AB - 0.00162A^2G - 0.00364AG^2 + 0.0000375A^2G^2 + 0.000216A^2B$
12	$S^* = 93.027 - 5.291B + 0.0673B^2 - 0.322G + 0.0191G^2 + 0.0805AG + 0.0259AB - 0.00162A^2G - 0.00239AG^2 + 0.0000375A^2G^2 + 0.000216A^2B - 0.000571AB^2$
13	$S^* = 93.027 - 7.897B + 0.109B^2 + 3.402G - 0.0712G^2 - 0.136AG + 0.178AB + 0.00108A^2G + 0.00287AG^2 - 0.000028A^2G^2 - 0.00167A^2B - 0.00299AB^2 + 0.0000301A^2B^2$

A number of other models with a variety of combinations of the variables were also tried, but none of them were found to be significantly better than Equation 7.

So, with regard to the Degree of Mixing, this was considered to be the best model and the optimization technique was applied to Equation 7.

$$S^* = 93.027 - 4.324B + 0.0456B^2 + 0.708G - 0.009G^2 - 0.0179AG + 0.00997AB \qquad \text{------------(7)}$$

A similar method of approach utilizing the various techniques adopted in the treatment of the Degree of Mixing was also applied to the correlation of data on Power Consumption. Here again, numerous mathematical models were constructed and tested. The important results from these tests for P*, are given in Table 22. The corresponding equations equivalent to their assigned numbers as per the last-but-one column of Table 22, are shown in Table 23. The mathematical model that gives the best representation of the data on P* is given by Equation 66 of Table 23, reproduced below.

$$P^* = 0.00454 + 0.000507A + 0.000109B - 0.00111F + 0.000033AB \qquad \text{-------(66)}$$

It is seen from Table 22 that the residual mean square corresponding to this equation has a value of 11.111×10^{-5}. Referring

TABLE 22

RESULTS FROM DIFFERENT MATHEMATICAL MODELS FOR POWER P*

Variables No.	Variables Name	Residual Mean Square x 10^5	Change in RMS x 10^5	Source	Change as % Original RMS	Cumulative Per Cent***	Equation No.	R^2
0	0	116.824	----	----	----	----	48	----
1	A	60.966	55.858	A	47.813	47.813	49	0.49820
2	A,B	20.579	40.387	B	34.571	82.384	50	0.83739
3	A,B & F	16.740	3.839	F	3.287	85.671	51	0.87324
4	A,B, F & F^2	17.014	-0.274	F^2	-0.236	85.435	65	0.87676
4	A,B, F&AB	11.111	5.629	AB	4.817	90.488	66	0.91952
5	Above &AF	9.037	2.074	AF	1.776	92.264	67	0.93752
6	Above & BF	7.728	1.309	BF	1.120	93.384	68	0.94911
7	Above & B^2F	5.205	2.523	B^2F	2.160	95.544	69	0.96743
8	Above & BF^2	4.647	0.558	BF^2	0.478	96.022	70	0.97246
9	Above & B^2F^2	4.905	-0.258	B^2F^2	-0.222	95.800	71	0.97254

TABLE 23

EQUATIONS CORRESPONDING TO DIFFERENT MODELS FOR POWER P*

Equation No.	
48	$P^* = 0.0592$
49	$P^* = 0.000207 + 0.00155A$
50	$P^* = -0.0429 + 0.00155A + 0.00136B$
51	$P^* = -0.0351 + 0.00155A + 0.00136B - 0.00111F$
65	$P^* = -0.0365 + 0.00155A + 0.00136B + 0.0000952F - 0.0000862F^2$
66	$P^* = 0.00454 + 0.000507A + 0.000109B - 0.00111F + 0.000033AB$
67	$P^* = -0.00918 + 0.000868A + 0.000109B + 0.00085F + 0.000033AB - 0.0000516AF$
68	$P^* = -0.0189 + 0.000868A + 0.000417B + 0.00224F + 0.000033AB - 0.0000516AF - 0.0000441BF$
69	$P^* = -0.0189 + 0.000868A + 0.000417B - 0.000875F + 0.000033AB - 0.0000514AF + 0.00019BF - 0.00000356B^2F$
70	$P^* = -0.0189 + 0.000868A + 0.000369B - 0.000875F + 0.000033AB - 0.0000516AF + 0.000232BF - 0.00000356B^2F - 0.00000296BF^2$
71	$P^* = -0.0194 + 0.000868A + 0.000381B - 0.000834F + 0.000033AB - 0.0000516AF + 0.000249BF - 0.00000399B^2F - 0.00000441BF^2 + 0.0000000338B^2F^2$

back to Table 19, one can see that this quantity had a value of 2.46×10^{-5}. This is the residual mean square value that would result with a perfect fit; and as can be seen the Coefficient of of Determination R^2, has a very high value of about 0.92. This indicates that Equation 66 is an excellent mathematical model representing the variation of P* in terms of the variables causing this variation. It may be noted that addition of the AF and BF terms give a slight improvement in the fit; but in view of the fact that a 0^o level for the variable F is the optimum, these terms do not in fact contribute significantly to improving the model for P*.

Thus, if one is interested in determining the power consumption under any given set of conditions, we have available at our disposal a quantitative relationship which would enable us to obtain the desired information and reach an intelligent decision.

5.5 Optimization

The optimization procedure (11, 30) involved differentiating the best response-surface equation in S* obtained earlier with respect to one variable at a time, and equating the resulting partial derivative relationships to zero. These were then solved simultaneously in order to determine the optimum values of each of the variables that satisfy these equations.

This treatment on Equation 7 resulted in the following values for the main variables:

$A = 13.8$ RPM

$B = 45.9$ Per cent

$F = 0^{o}$

and $G = 25.5$ rotations.

These then are the optimum values predicted as a result of the correlation and analysis of the data.

It is indeed satisfying to note that these conditions arrived at after mathematical analysis of the results are quite in agreement with what would be expected both from logical reasoning as well as experience.

It was felt it would be worthwhile to confirm these predicted optimum conditions, by subsequent experimental observations.

Naturally, the conditions under which the next experiment was conducted was dictated by the predicted optimum values. This experiment designated as Experiment No: 28 was actually conducted at the following conditions:

$A = 13$ RPM

$B = 45$ Per cent

$F = 0^{o}$

$G = 30$ rotations.

The speed is quite close to the predicted value and could also be physically maintained. With regard to variable G it was felt that a level of 30 rotations would ensure complete mixing, even though the predicted value of G is a little over 25 rotations; besides, this slight increase does not call for a greatly increased time of mixing.

The resulting mean and standard deviation S obtained from the data of Experiment No: 28 are listed in Table 24. The values of S^* and S_1^*, are also included therein.

Power measurements were not repeated for this experiment because it was felt that not much would be gained by it, especially in view of the re-calibration that would be required.

Now from Equation 7, the predicted value for S^* is 2.778 under the conditions of this experiment. The experimentally observed value was found to be 3.982. The variance corresponding to the calculated value of 2.778 is 7.719 and that corresponding to the observed value of 3,982 is 15,856. Since the latter was generated from 15 observations, a chi-square test (χ^2 - test) with 15 degrees of freedom gives a value of

$$\chi^2 = \frac{15 \times 15.856}{7.719} = 30.8$$

The tabulated value of χ^2 at the 5 per cent level of significance, with 15 degrees of freedom is 24.996 while that at the 1 per cent level is 32.801. On comparing the test-statistic of 30.8 with these values it is seen that perhaps there is a marginal distinction between the observed and the calculated values. One of

TABLE 24

RESULTS FROM EXPERIMENT NO: 28

a	b	p_i	$\bar{p}$	S
64	122	52.459		
67	114	58.771		
64	115	55.652		
55	107	46.728		
64	120	53.333		
51	108	47.222		
62	117	52.991		
61	112	54.464		
57	118	48.305		
54	118	45.762		
49	108	45.370		
53	116	45.689		
58	118	49.152		
51	105	48.571		
52	106	49.056	50.235	4.114

$S^* = 3.982$

and $S_1^* = 4.122$

the reasons for this may be the presence of a single value of the means p_i tabulated in Table 24, different from the rest. An examination of this Table 24 reveals exactly what is suspected. Thus the values of p_i are so distributed that nine of them are under 50, five of them all below 55.6 and there is one value close to 59 which is different from the rest. If one recalculates the observed value with 14 degrees of freedom leaving the odd value out, it is seen that S* is 3.390, while the predicted value is still 2.778. If the chi-square test is repeated it is seen that the test statistic

$$\chi^2 = \frac{14 \times 11.493}{7.719} = 20.843$$

The tabulated value with 14 degrees of freedom and at a 5 per cent significance level, is 23.685; and for a 1 per cent level the value is 29.141. It can therefore be clearly seen that the tabulated values are greater than the test-statistic and consequently the hypothesis that the observed value $S^* > \sigma_o$ where $\sigma_o =$ 2.778 is rejected.

We are now in the region where the order of magnitude of S* is quite low. In fact, physically it will not be possible to reach the value of 2.778 which is the predicted low. Besides, a minimum of experimental error does not permit a closer approach than the one already available. This would indicate that we are in a flat region and that it will be difficult to distinguish between the results obtainable with levels of 44, 45 and 46 per cent for variable B. Philosophically, the true difference in the S* values between experiments with 45 and 46 per cent loads is very very small. In order to distinguish between them, it will be necessary to conduct a very large number of operations; and it becomes unprofitable to carry them out. This means that the conditions of Experiment No: 28 indeed represent the optimum values.

Another important justification for the fact that we are indeed at the optimum, is that different equations predict the same optimum conditions. This is the case, even with the addition of the most recent results. For example, the results from Experiment No: 28 were added to the bulk of the results obtained earlier and examined exhaustively in a manner similar to that used earlier for S* and P*. The results are shown in Table 25. The equations corresponding to the different models are shown in Table 26. In these Tables, it is Equation 93 which corresponds to Equation 7 determined earlier.

$$S^* = 92.953 - 4.312B + 0.0455B^2 + 0.701G - 0.00852G^2 - 0.0181AG + 0.00993AB \qquad (93)$$

The optimization procedure was applied to Equation 93 and the following are the newly predicted optimum conditions:

A = 15 RPM   F = 0°

B = 45.8 Per Cent   G = 25 rotations.

TABLE 25

RESULTS FROM DIFFERENT MATHEMATICAL MODELS FOR S*
(INCLUDING THE OPTIMUM)

Variables No.	Variables Name	Residual Mean Square	Change in RMS	Source	Change as % Original RMS	Cumulative Per Cent***	Equation No.	R^2
0	0	271.049	----	----	-----	-----	87	----
1	B	81.801	189.248	B	69.820	69.820	88	0.70938
2	B, B^2	40.210	41.591	B^2	15.345	85.165	89	0.86264
3	B, B^2, G	33.626	6.584	G	2.429	87.594	90	0.88973
4	B, B^2 G, G^2	34.912	- 1.286	G^2	-0.526	87.120	91	0.89028
5	Above & AG	33.433	1.479	AG	0.545	87.665	92	0.89949
6	Above & AB	24.193	9.240	AB	3.409	91.074	93	0.93058

*** Cumulative Per Cent = (R^2 based on unbiased variances) x 100

TABLE 26

EQUATIONS CORRESPONDING TO DIFFERENT MODELS FOR S*
(INCLUDING THE OPTIMUM)

Equation Number	
87	$S^* = 19.006$
88	$S^* = 49.624 - 0.952B$
89	$S^* = 89.438 - 3.946B + 0.0456B^2$
90	$S^* = 95.099 - 3.917B + 0.0452B^2 - 0.308G$
91	$S^* = 92.785 - 3.914B + 0.0452B^2 - 0.0194G - 0.0074G^2$
92	$S^* = 93.018 - 3.943B + 0.0455B^2 + 0.195G - 0.00895G^2 - 0.00447AG$
93	$S^* = 92.953 - 4.312B + 0.0455B^2 + 0.701G - 0.00852G^2 - 0.0181AG + 0.00993AB$

It can easily be seen that these values are the same as those obtained earlier from Equation 7.

Even though optimum conditions for each of the variables have been firmly established, it was decided to conduct yet another experiment to absolutely reconfirm the conclusions. Therefore Experiment No: 29 was carried out under following conditions:

A = 17 RPM

B = 55 Per Cent

F = 0°

G = 30 rotations.

It was felt that a level of 17 RPM for variable A, would be just right to get information at an intermediate level between the 0 level of 21 RPM used for the entire series and the low value of 13 RPM used for Experiment No: 28. It may be recalled that variable A has virtually no influence on the Degree of Mixing, within reasonable limits, of course. This obviously includes the range of 13 through 58 RPM.

Since B is the variable having a prime influence on the Degree of Mixing, a load of greater than half the capacity of the mixer should give a somewhat higher value for S* as poorer mixing would be expected. The greater the loading over the 50 per cent capacity, the lesser will be the quality of the resulting mix.

As for variables F and G, they were left at their best values of 0^{o} and 30 rotations respectively.

The predicted value of S* for Experiment No: 29 from Equation 7 is 6.479; that predicted from Equation 93 is 6.846.

One would definitely expect the results from Experiment No: 29 to be larger than both the predicted optimum of 2.778 as well as the results from Experiment No: 28. The results from Experiment No: 29 are shown in Table 27. Clearly the value of 5.351 for S* is greater than 2.778 as well as 3.390, the latter value corresponding to 14 degrees of freedom.

One can show by means of a χ^2-test that this value of S* is significantly different from the optimum value predicted by Equation 7 viz. 2.778. Thus, the test-statistic

$$\chi^2 = \frac{15 \times (5.351)^2}{(2.778)^2}$$

$$= \frac{15 \times 28.633}{7.717}$$

$$= 55.654 .$$

The tabulated value at the 1 per cent level of significance is 30.578 (that at the 5 per cent level is 24.996) for 15 degrees of freedom. Since the test-statistic of 55.654 is much larger than the tabulated value, the hypothesis that they are equal is rejected. Hence the results show that the conditions of

TABLE 27

RESULTS FROM EXPERIMENT NO: 29

a	b	p_i	$\overline{p}$	S
45	104	43.269		
60	109	60.550		
51	116	43.965		
56	98	57.142		
60	112	53.571		
62	116	53.448		
60	116	51.724		
54	116	46.551		
52	109	47.706		
58	115	50.434		
68	117	58.119		
49	106	46.226		
57	117	48.717		
52	116	44.827		
64	115	55.652	50.793	5.478

$S^* = 5.351$

and $S_1^* = 5.539$

Experiment No: 29 are quite different from the optimum ones, which is as it should be.

One can also make a comparison between the observed value of S* for Experiment No: 29 with those predicted by Equations 7 and 93. As indicated earlier Equation 7 predicted a value of 6.479 for S* under conditions of Experiment No: 29. The observed value is 5.351. A chi-square test gives the following value for the test-statistic:

$$\chi^2 = \frac{15 \times 28.633}{41.989}$$

$$= 10.228$$

The tabulated value for a 5 per cent level of significance and corresponding to 15 degrees of freedom is 24.996; and as the test-statistic is much lower than the tabulated value, the hypothesis that the observed value is from a different population than the calculated value, is rejected. The observed and predicted values are therefore in agreement.

A similar calculation may be performed to compare the observed value with that which Equation 93 predicts viz. 6.846. Hence,

$$\chi^2 = \frac{15 \times 28.633}{48.867}$$

$$= 9.164$$

Here too, just as in the last case, the conclusion that there is no difference between the observed and the predicted values holds good.

Having established that Experiment No: 28 does indeed give the optimum conditions for obtaining the best mix, it was felt it would be advantageous to incorporate the data from the additional results of Experiment No: 29 into the existing bulk of information and to see what happens. Hence, all the results available from experimental observations were pooled together and analysed by techniques quite similar to those used for obtaining the values shown in Tables 20 and 25. The new results with information from conditions outside the original range for variable B, are shown in Table 28 . Equations equivalent to the numbers given in Table 28 are shown in Table 29. In these tables the equation of interest corresponding to Equations 7 and 93 obtained earlier, is Equation 104. It is reproduced below:

$$S^* = 92.552 - 4.287B + 0.0449B^2 + 0.712G - 0.00889G^2 - 0.0181AG + 0.01AB \qquad \text{---------(104)}$$

This equation when subjected to the optimization treatment, produced the following set of conditions:

A = 15 RPM

B = 46 Per Cent

F = 0^{o}

G = 25 rotations.

TABLE 28

RESULTS FROM DIFFERENT MATHEMATICAL MODELS FOR S*
(INCLUDING EXPERIMENT NO: 29)

Variables No.	Variables Name	Residual Mean Square	Change in RMS	Source	Change as % Original RMS	Cumulative Per Cent***	Equation No.	R^2
0	0	267.799	----	----	-----	-----	98	-----
1	B	80.937	186.862	B	69.777	69.777	99	0.70856
2	B, B^2	39.396	41.541	B^2	15.512	85.289	100	0.86340
3	B, B^2 G	32.393	7.003	G	2.615	87.904	101	0.89200
4	Above & G^2	33.544	- 1.151	G^2	-0.530	87.474	102	0.89263
5	Above & AG	32.490	1.054	AG	0.394	87.868	103	0.90034
6	Above & AB	23.149	9.341	AB	3.488	91.356	104	0.93208

*** Cumulative Per Cent = (R^2 based on unbiased variances) x 100

TABLE 29

EQUATIONS CORRESPONDING TO DIFFERENT MODELS FOR S*
(INCLUDING EXPERIMENT NO: 29)

Equation No.	
98	$S^* = 18.535$
99	$S^* = 48.956 - 0.924B$
100	$S^* = 88.008 - 3.827B + 0.0435B^2$
101	$S^* = 94.677 - 3.871B + 0.0445B^2 - 0.315G$
102	$S^* = 92.247 - 3.875B + 0.0445B^2 - 0.00249G - 0.00799G^2$
103	$S^* = 91.759 - 3.85B + 0.044B^2 + 0.212G - 0.0101G^2 - 0.004AG$
104	$S^* = 92.552 - 4.287B + 0.0449B^2 + 0.712G - 0.00889G^2 - 0.0181AG + 0.01AB$

It is interesting to note that these values are exactly the same as those predicted and experimentally verified earlier.

Hence, there are available three equations viz. Equations 7, 93 and 104 which give an adequate quantitative representation of the Degree of Mixing S^* in terms of the variables influencing its variation. Any one of these equations may be used whenever necessary.

Furthermore, there is also available a quantitative functional relationship for Power Consumption P^*. Equation 66, obtained earlier, represents the dependence of P^* upon the variables affecting the power requirements.

6. DISCUSSION OF RESULTS

The contents of this chapter are intended to supplement the discussions already presented in the preceding chapter, as and when the individual problems were considered.

From the data presented in Table 8 and from the nature of the graphs given in Figures 13 through 19, one of the striking features apparent is the fact that very little mixing takes place when the load in the cylinder is 15 per cent of its full capacity. This is so because, the sand is raised only a small vertical distance before pockets of sand start slipping on the inside surface of the cylinder. This slipping of sand on the inside surface of the mixer prevents slip-planes from being generated within the body of sand and therefore hinders mixing. There is no bodily transfer of groups of particles which also retards mixing. An increase in the value of variable A representing the speed, is of almost no beneficial effect because the mixer simply rotates faster, past the sand which continually slips on the inside. Allowing the mixer rotations to increase is of no avail either, because the chief obstacle is not the time factor or the number of rotations G. A possible way of improving the mixing appears to be to raise the level of variable F or the angle of inclination. For example, Experiment No: 21 with a

14^{o} angle, shows a marked improvement over experiments with the same load but a smaller angle. This improvement is possible because the entire load is now in the lower corner L, of the mixer as in Figure 20(a). When the cylinder rotates, the material entrapped within its lowest section, because it has now accumulated to a sufficient quantity, is lifted up by the cylinder and dumped back onto a freshly formed surface, aiding mixing. However, since in the majority of cases where the charge amounts to 15 per cent of full capacity, the mixing is very poor, it is best to stay away from small loads. Moreover, it would be more practical as well as economical to work with higher loads.

When small loads are unavoidable, mixing may be improved by inclining the mixer at an angle, while at the same time allowing an adequate number of rotations to take place.

Next, examining the analysis of variance given in Table 15, it is seen that after B, G is the variable having the greatest influence upon the Degree of Mixing. All the other main effects and interactions are truly not very significant; also, that the interaction which may have some influence on S* is AG. The fact that this rate of rotation-number of rotations interaction is the one to be considered after the main effects of B and G, is clearly brought out in Tables 15, 16 and 18; the magnitude of this interaction can increase in two ways. At a constant value

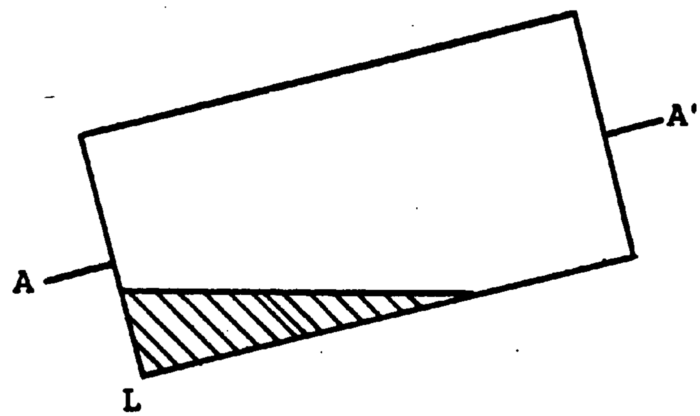


FIGURE 20(a). POSITION OF SAND IN THE MIXER

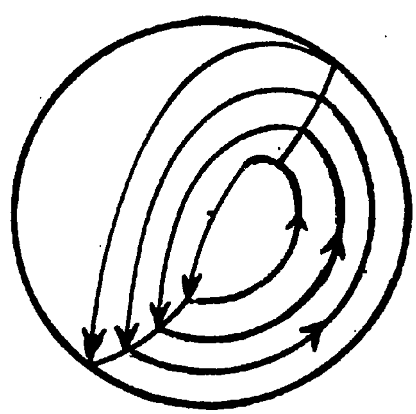


FIGURE 20(b). MOVEMENT OF SAND IN THE MIXER of A, increasing the number of rotations G would, in general, tend to aid the mixing process. But increasing G much beyond a certain value (30 in this case) will not be of much help. On the other hand, if A is increased either with or without simultaneous increase in G, a situation is reached where the sand is thrown in a parabolic trajectory as shown in Figure 20(b). This slows down the mixing process as the formation of a central tunnel of unmixed material is encouraged. If one increases the speed still further, a stage will be approached where the sand swings with the cylinder wall with practically no relative motion between the two. It can thus be seen that different kinds of actions occur in the mixer at different conditions of speed.

Next in line, comes the AB or the rate of rotation-volume per cent interaction. Here again, the statements made above in connection with the A component in an interaction involving A holds good. As for the contribution of variable B, there seems to be little doubt that a higher load in the range of 45 to 50 per cent is definitely preferable. It may be noted that while intermediate loads of around 30 per cent give a better mix than smaller loads, they are not necessarily the best. An increase in this interaction either due to increase in A (which increase is not of major consequence) or in B (an increase in which, is very desirable) in essence produces a better mixture.

From Table 19, it is seen that as far as the Power consumption P* is concerned, the main effects of A and B are of prime significance. One would expect this behavior, in that, with increasing rates of rotation, increasing amounts of power will have to be expended in order to maintain the required speeds. Similarly, one would have to spend more power to mix larger and larger loads. Rather than merely state the dependence of P* upon A and B qualitatively, Table 19 permits one to determine this dependence in a quantitative manner.

It may be remarked that the rather high degree of accuracy in the experimentation is reflected by the small values of the Residual Mean Square in Tables 15 through 19.

As mentioned earlier, increasing the angle of inclination is, in general, detrimental to mixing. The explanation for this effect lies in the fact that with reasonable loads, an increase in the angle causes material to be trapped in the lowest part of the mixer. When there is sufficient material in the mixer, an inclined position does not allow the entire bulk of the sand to mix along slip-planes or over a freshly developed surface to the same extent as when the experimental set-up is horizontal. However, increasing the angle of inclination has a tendency to cause a decrease in P*. Even though at first sight this appears to be advantageous, a re-examination is in order. Since the primary goal is achievement of a good mix, and as mixing deteriorates with increasing angles, one would not desire to keep the mixer at any level other than horizontal. The slight advantage in the form of reduced power consumption with an inclined load is far outweighed by the disadvantage of poor mixing. Since it would defeat the main purpose if operations were to be continued at positions other than the horizontal, there is no doubt that the optimum level for variabie F is indeed zero degrees.

As far as variable A is concerned, its optimum is dictated in the following manner. Since the Degree of Mixing is not especially influenced by A, virtually any value of A within the range employed may be used. Naturally, one would prefer a low rate of rotation as it would then mean correspondingly reduced power requirements. Applying these two criteria for S* and P* with respect to A, it is seen that a speed of 15 RPM is more than adequate to give good mixing, while at the same time not calling for large amounts of power.

Now, the optimum value for the Degree of Mixing S*, predicted by Equation 7 is 2.778. Experiment No: 28 which was then conducted with B kept at a 45 per cent load, gave an experimental S* value of 3.390 with 14 degrees of freedom. These two values have already been shown to be compatible. It is extremely unlikely that if one went to a load of say 46 per cent, (which in reality would mean a difference of a little over 15 ozs. in a total charge of 43 lbs. 13 ozs.) one would then obtain the lowest value for S*; this is why many more experiments in that region were not conducted. Here is a case wherein one strikes a balance between experience and practicability on the one hand and extreme mathematical precision on the other.

The mathematical models given by Equations 93 and 104 both predict the same optimum conditions for the different variables. This is in spite of the fact that the conditions of the final experiments were outside the original range of the variables. This consequently should increase the confidence in the model especially for the range investigated and small extensions thereof.

7. CONCLUSIONS

In the field of solids blending, the dependence of the Degree of Mixing S* and the Power requirements P* upon four variables A, B, F and G was investigated; these variables represent the rate of rotation, per cent load, angle of inclination and the number of mixer rotations, respectively.

Quantitative relationships between S* and the variables as well as between P* and the variables have been evolved, using a three-level factorially designed series of experiments. For the range of conditions explored and for small extrapolations thereof, the variable B was found to be critical with respect to the Degree of Mixing. The variable G was also found to be quite significant. The rate of rotation-number of rotations interaction and the rate of rotation-volume per cent load interaction were also found to have an influence on the Degree of Mixing measured by S*.

It was observed that when the mixer was charged with loads in the vicinity of 15 per cent of its full capacity, virtually no mixing actually took place.

As this behavior occurs even with different levels of the other variables, if the objective is to achieve good mixing, it may be concluded that small loads should be avoided.

From observations as well as analyses, it has been shown that the angle of inclination F is not significant.

The quantitative relationship between the Degree of Mixing S* and the variables is given by

$$S^* = 93.027 - 4.324B + 0.0456B^2 + 0.708G - 0.009G^2 - 0.0179AG + 0.00997AB.$$

The corresponding correlation for Power consumed P* is

$$P^* = 0.00454 + 0.000507A + 0.000109B - 0.00111F + 0.000033AB$$

where P* is measured in units of horsepower.

The former function involving S*, was optimized and the predicted optimum conditions for each of variables were obtained. These predicted values were confirmed by subsequent experimental observations. The optimum conditions for blending two solids in a cylinder were found to be as follows:

A = 15 RPM

B = 46 per cent

F = 0 degrees

and G = 30 rotations.

The quantitative functional relationships developed above will enable one, not only to ascertain the Degree of Mixing under any given set of conditions, but also to calculate the Power requirements under these or other conditions.

8. NOMENCLATURE

a	number of colored (blue) particles in a spot sample
A	rate of rotation, RPM
b	total number of particles in a spot sample
B	volume per cent of the mixer filled
F	angle of inclination, degrees
F-ratio	ratio of variances
G	number of rotations of the mixer
m	slope of the torque vs. chart-reading graph
n	number of spot samples
N	rotational speed in equation for power, RPM
p_i	the percentage of dyed component in sample "i"
$\overline{p}$	estimate of the population mean, $\overline{p} = \Sigma p_i/n$
P	the population mean or the actual percentage of the dyed component as known from the initial charge
P*	Power consumption, hp.
R	chart reading for torque
RMS	Residual Mean Square

R^2 — Coefficient of Determination

s^2 — sample variance, in general

S — estimate of the population standard deviation,

$$S = \sqrt{\frac{\sum_{i=1}^{n} (p_i - \overline{p})^2}{n - 1}}$$

S^* — estimate of the population standard deviation (biased)

$$S^* = \sqrt{\frac{\sum_{i=1}^{n} (p_i - P)^2}{n}}$$

S_1^* — estimate of the population standard deviation (unbiased)

$$S_1^* = \sqrt{\frac{\sum_{i=1}^{n} (p_i - P)^2}{n - 1}}$$

T — torque, ft. lbs.

σ_o — population standard deviation

σ^2 — population variance, in general

abfg — when used as a unit refers to the treatment combination

9. LITERATURE CITED

1. Aronson, M., and R. C. Nelson, Editors, "Strain Gage Instrumentation", Instruments Publishing Company, Inc., Pittsburgh, Pennsylvania, 1958.

2. Bennett, C. A., and N. L. Franklin, "Statistical Analysis in Chemistry and the Chemical Industry", John Wiley and Sons, Inc., New York, 1954.

3. Blumberg, R., and J.S. Maritz, Chem. Eng. Science, Vol. 2, 240, 1953.

4. Bowker, A. H., and G. J. Lieberman, "Engineering Statistics", Prentice-Hall, Inc., Englewood Cliffs, N. J., 1961.

5. Buckelew, J., Bell Laboratories Record, Vol. 40, No:11, 398, 1962.

6. Chudzikiewicz, R., International Chem. Eng., Vol. 1, 124, 1961.

7. Cochran, W. G., Biometrics, Vol. 3, No: 1, 22, 1947.

8. Cochran, W. G., and G. M. Cox, "Experimental Designs", 2nd Ed., John Wiley and Sons, Inc., New York, 1957.

9. Connor, W. S., and M. Zelen, "Fractional Factorial Experiment Designs for Factors at Three Levels", U.S. Dept. of Commerce, National Bureau of Standards, Applied Mathematics Series, 54, 1959.

10. Coulson, J. M.,and N.K., Maitra, Ind. Chemist, Vol. 26, 55, 1950.

11 Davies, O. L., Editor, "Design and Analysis of Industrial Experiments", 2nd Ed., Hafner Publishing Co., New York, 1960.

12. Donald, M. B.,and B. Roseman, Brit. Chem. Eng., Vol.7, 749, 823, 922, 1962.

13. Eisenhart, C., Biometrics, Vol. 3, No: 1, 1, 1947.

14. Fisher, R. K., Ph.D. Thesis, Princeton University, 1963.

15. Freeman, Harold, "Introduction to Statistical Inference", Addison-Wesley Publishing Company, Inc., Reading, Mass., 1963.

16. Hager, T. C., Ph.D. Thesis, Lehigh University, 1963.

17. Hager, T. C., and C. W. Clump, Paper presented at the A.I.Ch.E. Meeting, San Juan, Puerto Rico, 1963.

18. Kempthorne, O., "The Design and Analysis of Experiments", John Wiley and Sons, Inc., New York, 1952.

19. Lacey, P.M.C., J. Appl. Chem., Vol. 4, 257, 1954.

20. Literature on SR-4 Products supplied by Instrumentation Division, Baldwin-Lima-Hamilton, Waltham, Mass., 1963.

21. "Measurement of Shaft Horespower", Feature Article, Instruments and Control Systems, Vol. 35, No: 9, 111, 1962.

22. Perry, C. C., and H. R. Lissner, "The Strain-Gage Primer", McGraw-Hill Book Company, Inc., New York, 1962.

23. Scheffé, H., "The Analysis of Variance", John Wiley and Sons, Inc., New York, 1959.

24. Taylor, Ivan, J., Lehigh University, Private Communications, 1963.

25. "Testing Sieves and Their Uses", Handbook 53, Published by the W. S. Tyler Company, Cleveland, Ohio, 1963.

26. The Rand Corporation, "A Million Random Digits With 1000,000 Normal Deviates", The Free Press, Glencoe, Illinois, 1955.

27. "Universal Analyzer", Model BL-320 and "Amplifiers", Publications by The Brush Development Company, Cleveland, Ohio, 1950.

28. Weidenbaum, S. S., "Advances in Chemical Engineering", Vol. II, Edited by Drew, T. B., and J. W. Hoopes, Academic Press, New York, 1958.

29. Weidenbaum, S. S., Ph.D. Thèsis, Columbia University, 1953.

30. Wilde, D. J., "Optimum Seeking Methods", Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964.

31. Yates, F., Imp. Bur. Soil Sci. Tech. Comm., Vol. 35, 1937.

APPENDIX I

MEAN AND STANDARD DEVIATION S

Experiment No: 1

Treatment Combination 0000

a	b	p_i	$\bar{p}$	S
60	94	72.340		
55	113	48.672		
114	116	98.275		
72	116	62.068		
64	116	55.172		
1	113	0.884		
1	116	0.862		
45	94	47.872		
103	116	88.793		
109	110	99.090		
111	111	100.000		
0	112	0.000		
1	118	0.847		
2	112	1.785		
117	117	100.000	51.777	41.355

Experiment No: 2

Treatment Combination 1212

a	b	p_i	$\bar{p}$	S
49	104	47.115		
55	102	53.921		
65	112	58.035		
49	102	48.039		
58	107	54.205		
48	102	47.058		
51	106	48.113		
57	116	49.137		
54	104	51.923		
46	109	42.201		
60	121	49.586		
48	106	45.283		
40	102	39.215		
58	111	52.252		
57	118	48.305	48.959	4.706

Experiment No: 3

Treatment Combination 2121

a	b	p_i	$\overline{p}$	S
89	114	78.070		
101	114	88.596		
27	114	23.684		
44	105	41.904		
7	120	5.833		
56	112	50.000		
63	106	59.433		
55	112	49.107		
63	111	56.756		
45	96	46.875		
52	109	47.706		
68	106	64.150		
53	111	47.747		
64	110	58.181		
62	112	55.357	51.559	19.642

Experiment No: 4

Treatment Combination 0021

a	b	p_i	$\bar{p}$	S
12	120	10.000		
100	100	100.000		
46	109	42.201		
111	111	100.000		
2	107	1.869		
99	99	100.000		
91	104	87.500		
4	114	3.508		
102	102	100.000		
0	112	0.000		
2	101	1.980		
96	97	98.969		
105	105	100.000		
78	101	77.227		
100	102	98.039	61.419	44.994

Experiment No: 5

Treatment Combination 1200

a	b	p_i	$\bar{p}$	S
66	106	62.264		
44	111	39.639		
45	108	41.666		
56	105	53.333		
52	110	47.272		
50	104	48.076		
46	96	47.916		
39	93	41.935		
49	100	49.000		
43	93	46.236		
60	111	54.054		
52	103	50.485		
44	111	39.639		
51	98	52.040		
63	113	55.752	48.620	6.362

Experiment No: 6

Treatment Combination 2112

a	b	p_i	$\bar{p}$	S
53	109	48.623		
61	111	54.954		
50	109	45.871		
47	114	41.228		
52	111	46.846		
65	118	55.084		
55	108	50.925		
44	102	43.137		
54	113	47.787		
55	105	52.380		
50	102	49.019		
56	109	51.376		
56	111	50.450		
48	107	44.859		
58	115	50.434	48.864	4.010

Experiment No: 7

Treatment Combination 0012

a	b	p_i	$\overline{p}$	S
93	111	83.783		
78	97	80.412		
76	96	79.166		
105	116	90.517		
79	94	84.042		
108	108	100.000		
0	111	0.000		
112	112	100.000		
108	108	100.000		
2	114	1.754		
97	99	97.979		
113	117	96.581		
4	111	3.603		
2	109	1.834		
2	115	1.739	61.427	44.207

Experiment No: 8

Treatment Combination 1221

a	b	p_i	$\overline{p}$	S
64	113	56.637		
47	106	44.339		
62	111	55.855		
55	110	50.000		
53	107	49.532		
59	103	57.281		
56	100	56.000		
60	112	53.571		
60	106	56.603		
43	114	37.719		
59	112	52.678		
50	108	46.296		
44	100	44.000		
34	109	31.192		
47	103	45.631	49.155	7.680

Experiment No: 9

Treatment Combination 2100

a	b	p_i	$\bar{p}$	S
74	101	73.267		
67	116	57.758		
83	118	70.338		
72	113	63.716		
58	113	51.327		
53	97	54.639		
55	105	52.380		
24	116	20.689		
60	112	53.571		
56	111	50.450		
92	103	89.320		
100	107	93.457		
46	115	40.000		
72	104	69.230		
16	112	14.285	56.961	21.638

Experiment No: 10

Treatment Combination 0102

a	b	p_i	$\bar{p}$	S
65	98	66.326		
64	114	56.140		
73	114	64.035		
64	116	55.172		
60	110	54.545		
44	105	41.904		
54	118	45.762		
50	105	47.619		
50	106	47.169		
51	106	48.113		
71	103	68.932		
64	102	62.745		
53	110	48.181		
49	110	44.545		
51	120	42.500	56.692	9.807
61	115	53.043		
62	105	59.047		
58	118	49.152		
52	112	46.428		
62	108	57.407		
61	116	52.586		
54	118	45.762		
52	109	47.706		
61	111	54.954		
57	106	53.773		
42	110	38.181		
47	125	37.600		
60	114	52.631		
46	113	40.707		
56	117	47.863	52.907	8.274

Experiment No: 11

Treatment Combination 1011

a	b	p_i	$\overline{p}$	S
88	101	87.128		
89	97	91.752		
95	95	100.000		
0	121	0.000		
2	111	1.801		
1	110	0.909		
110	111	99.099		
106	110	96.363		
96	96	100.000		
108	109	99.082		
19	118	16.101		
101	101	100.000		
109	116	93.965		
1	109	0.917		
101	102	99.019	65.742	45.514

Experiment No: 12

Treatment Combination 2220

a	b	p_i	$\overline{p}$	S
57	107	53.271		
52	116	44.827		
61	116	52.586		
58	122	47.540		
48	97	49.484		
56	114	49.122		
54	99	54.545		
66	108	61.111		
54	113	47.787		
42	105	40.000		
45	110	40.909		
54	106	50.943		
59	115	51.304		
29	108	26.851		
81	97	83.505	50.252	12.080
58	109	53.211		
55	120	45.833		
47	114	41.228		
52	104	50.000		
54	117	46.153		
33	95	34.736		
64	111	57.657		
47	100	47.000		
49	115	42.608		
57	123	46.341		
62	104	59.615		
70	99	70.707		
0	118	0.000		
33	108	30.555		
60	102	58.823	47.941	14.242

Experiment No: 13

Treatment Combination 0120

a	b	p_i	$\bar{p}$	S
60	115	52.173		
53	109	48.623		
48	113	42.477		
53	107	49.532		
50	112	44.642		
45	112	40.178		
43	113	38.053		
48	111	43.243		
45	105	42.857		
57	116	49.137		
54	117	46.153		
47	101	46.534		
50	118	42.372		
69	117	58.974		
53	102	51.960	46.460	5.413
65	117	55.555		
49	115	42.608		
57	117	48.717		
43	109	39.449		
49	111	44.144		
64	111	57.657		
53	116	45.689		
41	110	37.272		
48	109	44.036		
57	111	51.351		
57	112	50.892		
69	117	58.974		
51	107	47.663		
52	105	49.523		
58	109	53.211	48.449	5.903

Experiment No: 14

Treatment Combination 1002

a	b	p_i	$\overline{p}$	S
61	110	55.454		
64	111	57.657		
24	119	20.168		
71	118	60.169		
74	112	66.071		
1	114	0.877		
102	104	98.076		
104	104	100.000		
17	117	14.529		
108	108	100.000		
109	109	100.000		
0	115	0.000		
36	119	30.252		
123	123	100.000		
0	115	0.000	53.550	40.110

Experiment No: 15

Treatment Combination 2211

a	b	p_i	$\overline{p}$	S
54	105	51.428		
57	109	52.293		
55	111	49.549		
51	107	47.663		
64	117	54.700		
53	109	48.623		
60	110	54.545		
50	100	50.000		
54	105	51.428		
53	102	51.960		
50	104	48.076		
45	111	40.540		
52	107	48.598		
50	107	46.728		
59	115	51.304	49.829	3.491

Experiment No: 16

Treatment Combination 0111

a	b	p_i	$\overline{p}$	S
68	104	65.384		
68	106	64.150		
56	120	46.666		
55	119	46.218		
49	98	50.000		
67	115	58.260		
64	114	56.140		
48	105	45.714		
49	110	44.545		
47	117	40.170		
45	109	41.284		
55	110	50.000		
51	118	43.220		
50	106	47.169		
51	100	51.000	49.994	7.764

Experiment No: 17

Treatment Combination 1020

a	b	p_i	$\overline{p}$	S
99	99	100.000		
102	104	98.076		
103	104	99.038		
3	115	2.608		
109	110	99.090		
91	102	89.215		
1	113	0.884		
102	102	100.000		
2	117	1.709		
93	93	100.000		
3	114	2.631		
0	113	0.000		
93	94	98.936		
113	115	98.260		
110	111	99.099	65.969	47.213

Experiment No: 18

Treatment Combination 2202

a	b	p_i	$\overline{p}$	S
57	115	49.565		
57	104	54.807		
56	106	52.830		
56	100	56.000		
48	105	45.714		
57	104	54.807		
59	109	54.128		
54	98	55.102		
58	106	54.716		
49	104	47.115		
48	114	42.105		
56	100	56.000		
52	105	49.523		
54	106	50.943		
51	100	51.000	51.623	4.080

Experiment No: 19

Treatment Combination 0201

a	b	p_i	$\overline{p}$	S
52	115	45.217		
80	129	62.015		
55	112	49.107		
49	105	46.666		
54	118	45.762		
46	104	44.230		
55	103	53.398		
50	112	44.642		
66	119	55.462		
56	117	47.863		
65	119	54.621		
60	115	52.173		
61	109	55.963		
70	119	58.823		
48	115	41.739	50.512	6.029

Experiment No: 20

Treatment Combination 1110

a	b	p_i	$\overline{p}$	S
79	118	66.949		
57	108	52.777		
31	102	30.392		
31	117	26.495		
18	117	15.384		
16	105	15.238		
40	110	36.363		
65	113	57.522		
63	100	63.000		
38	104	36.538		
66	107	61.682		
69	99	69.696		
100	112	89.285		
54	103	52.427		
72	109	66.055	49.320	21.639

Experiment No: 21

Treatment Combination 2022

a	b	p_i	$\overline{p}$	S
54	93	58.064		
0	106	0.000		
47	101	46.534		
69	98	70.408		
0	108	0.000		
62	103	60.194		
59	96	61.458		
60	97	61.855		
47	95	49.473		
50	100	50.000		
47	99	47.474		
57	103	55.339		
53	114	46.491		
56	111	50.450		
60	106	56.603	47.622	20.487

Experiment No: 22

Treatment Combination 0222

a	b	p_i	$\overline{p}$	S
53	95	55.789		
53	91	58.241		
53	114	46.491		
50	109	45.871		
54	106	50.943		
43	94	45.744		
48	109	44.036		
56	117	47.863		
48	101	47.524		
57	109	52.293		
57	102	55.882		
52	106	49.056		
54	109	49.541		
56	113	49.557		
52	97	53.608	50.162	4.225

Experiment No: 23

Treatment Combination 1101

a	b	p_i	$\overline{p}$	S
61	102	59.803		
59	105	56.190		
58	99	58.585		
64	109	58.715		
56	120	46.666		
54	116	46.551		
54	113	47.787		
54	121	44.628		
44	92	47.826		
48	102	47.058		
48	117	41.025		
49	116	42.241		
60	117	51.282		
57	119	47.899		
61	104	58.653	50.327	6.405

Experiment No: 24

Treatment Combination 2010

a	b	p_i	$\bar{p}$	S
68	103	66.019		
0	117	0.000		
107	115	93.043		
2	123	1.626		
0	109	0.000		
3	111	2.702		
92	92	100.000		
111	112	99.107		
3	116	2.586		
60	108	55.555		
4	111	3.603		
80	115	69.565		
86	107	80.373		
104	105	99.047		
104	109	95.412	51.242	43.822

Experiment No: 25

Treatment Combination 0210

a	b	p_i	$\bar{p}$	S
59	110	53.636		
75	117	64.102		
61	100	61.000		
74	121	61.157		
47	104	45.192		
55	106	51.886		
53	104	50.961		
48	100	48.000		
55	115	47.826		
47	103	45.631		
48	114	42.105		
52	94	55.319		
58	121	47.933		
55	109	50.458		
51	106	48.113	51.554	6.410

Experiment No: 26

Treatment Combination 1122

a	b	p_i	$\bar{p}$	S
42	103	40.776		
1	112	0.892		
49	105	46.666		
58	109	53.211		
64	115	55.652		
54	113	47.787		
58	109	53.211		
47	108	43.518		
51	97	52.577		
60	104	57.692		
44	92	47.826		
56	100	56.000		
52	100	52.000		
46	98	46.938		
49	113	43.362	46.540	13.606

Experiment No: 27

Treatment Combination 2001

a	b	p_i	$\bar{p}$	S
61	96	63.541		
66	105	62.857		
84	98	85.714		
97	102	95.098		
50	105	47.619		
0	117	0.000		
73	97	75.257		
94	114	82.456		
102	106	96.226		
25	118	21.186		
0	109	0.000		
2	115	1.739		
107	108	99.074		
96	96	100.000		
0	114	0.000	55.384	40.200

APPENDIX II

COMPUTER TECHNIQUES

A GE-225 digital computer was used for performing the analysis and correlation of the results. A concise account of each of the programs used and pertinent details are given below. Complete details on the programs and corresponding instructions for their use are available in the Library of the Computer Laboratory at Lehigh University.

1. POLYNOMIAL FITTING BY SEQUENTIAL LEAST SQUARES

This program catalogued under the number D 6-504, was used to obtain both S* and P* as polynomial functions of the different variables, taken one at a time.

The purpose is to obtain the best coefficients in the least squares sense to the model

$$\hat{Y} = C_1 x^{k_1} + C_2 x^{k_2} + \ldots + C_m x^{k_m}$$

where x is the input variable under study while $\hat{Y}$ is the output variable. $C_1, C_2, \ldots C_m$ are the coefficients to be determined and $k_1, k_2 \ldots k_m$ denote the various powers of the component terms of a polynomial equation of the required degree.

Among the quantities included in the output are an Analysis of Variance table, the coefficients, the variance of the coefficients, the x_i and Y_i values, and the expected $\hat{Y}_i$ values.

Under option 1, a sequence of models was also obtained with all of the statistical parameters listed above. These models are of the form

$$\hat{Y} = C_1x^{k_1} + C_2x^{k_2} + \ldots + C_{m-2}x^{k_{m-2}} + C_{m-1}x^{k_{m-1}}$$

$$\hat{Y} = C_1x^{k_1} + C_2x^{k_2} + \ldots + C_{m-2}x^{k_{m-2}}$$

$$\hat{Y} = C_1x^{k_1}.$$

The coefficients in each of the reduced models are, of course, entirely different from the preceding model in the sequence despite the same lettering. This added information was obtained with a slight increase in computer running time. For example, x_i and Y_i are printed out with each of the $\hat{Y}_i$'s . Thus in a model with 6 terms (or order 6), $\hat{Y}_i$ was printed out under option, for models of order 5, 4, 3, 2 and 1 in addition to the model of order 6. This enables quick comparison of the efficiency of successive powers of a polynomial to be made, so as to determine the best fit.

Employing option 2, additional information partly of interest in debugging was printed out. This includes for each order the coefficients and the diagonal of the inverse product moment matrix.

Although the input forms specify one pair of x_i, Y_i values per card, any number of such pairs may be punched per card.

2. ANALYSIS OF VARIANCE

The purpose of using the analysis of variance program (No: D 3.005) was to compute the information necessary for interpretation of the results. Even though this program is primarily designed for handling a complete factorial experiment, the results from the present study can be suitably arranged to obtain the desired information. In some cases, pertinent and justifiable assumptions were made when necessary.

The program can also be used for other types of experimental designs, such as nested classifications, randomized blocks etc.

The input values of the observations were punched in a prescribed order, depending upon their nature and on the conditions from which they originated.

For instance, for obtaining the results shown in Table 15, the residuals obtained from the fitting of a quadratic equation in B, were used as input.

In order to get the results shown in Table 16, the data on S* had to be re-arranged as if they were produced by the action of A, B and G alone (without F) and then fed into the computer as the input values. Similarly for obtaining the results in Table 19, P* values had to be arranged such that only A, B and F caused its variation.

3. MULTIPLE LINEAR REGRESSION

Multiple Regression is a subject of such broad and varied usage that no single computer program is likely to be economical, convenient and adaptable for every use. No single arithmetic treatment, in fact, is likely to suffice.

Certain of the quantities calculated by the multiple linear regression program have a very definite and useful meaning, if the correct mathematical model has been selected and the variables are properly distributed. These same quantities may, on the other hand have no such meaning and actually may be misleading to the user if the model and distribution of variables do not apply. Now, in the present case, it was precisely with the idea of being able to pick the correct variables (main effects and interactions) which are important, that the previously mentioned analyses of variance programs were run. So that, once the significant variables have been isolated and relatively classified, they may then be used within the various models, in the proper fashion.

Let the vectors X_t where

$$X_t = (X_{1t}, X_{2t}, ---- X_{mt}) \quad (t = 1, 2, ---- n)$$

represent sets of observations on the same group of variables X_{it}.

The basic program obtains a linear representation of these variables denoted as dependent in terms of others denoted as independent. One can take X_{mt} to be the dependent variable, leaving the other X_{it} as independent variables and denote it as X_{mt} or $\bar{Y}_t$, whichever is more convenient. In this model the coefficients B_i (i = 0, 1, 2 ----m-1) as shown in

$$\bar{Y}_t = \sum_{i=0}^{m-1} B_i X_{it} + e_t \quad \text{----------(E)}$$

are determined to minimise

$$E = \sum_{t=1}^{n} e_t^{\,2} . \quad \text{---------------(F)}$$

Minimum E is obtained by use of coefficients in Equation (E) which satisfy the linear system of equations

$$\sum_{j=0}^{m-1} \left(\sum_{t=1}^{n} X_{it} X_{jt} \right) B_j = \sum_{t=1}^{n} X_{it} \bar{Y}_t \quad (i=0,\ 1,2,\text{---}m-1). \quad \text{----------(G)}$$

The output from a basic regression program (No: D 3.503) will include, among others, the following listed quantities.

Symbol	Meaning
R^2	Coefficient of Determination
CONSTANT TERM	B_o or zero according to the representation printed.
COEFFICIENT	Coefficient B_i of X_i in the regression.
VAR. NO.	The index i of the transformed variable being analysed on this line.

The quantity R^2 is so called because in the ordinary case it reduces to the square of the multiple correlation coefficient.

In addition, a quantity called "CONF. LVL", is also printed on the output. On assumptions that the model is correct, the subsequent values apply: a level of zero means less than 90 per cent confidence that a variable is significant; a level of 1 means 90 to 95 per cent confidence; and a level of 3 means at least 99 per cent confidence. These numbers are intended only as a qualitative guide. The program takes no action based on this level.

Residuals and Cumulative Residuals are also printed out. Naturally, the residual refers to the difference between the observed and calculated values of the dependent variables. Cumulative residuals are also provided.

APPENDIX III

1. RESULTS FROM ANALYSES OF VARIANCE ON S*

TABLE III.1

ANALYSIS OF VARIANCE ON S* (A, B AND F)

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	52.029	26.015	0.44
B	2	6079.050	3039.525	51.42
F	2	3.502	1.751	0.03
AB	4	246.515	61.629	1.04
AF	4	138.217	34.554	0.58
BF	4	92.052	23.013	0.39
Residual	8	472.887	59.110	
TOTAL	26	7084.255		

** Tabulated Values for F: $F_{0.05}$ (2,8) = 4.46

$F_{0.05}$ (4,8) = 3.84

TABLE III.2

ANALYSIS OF VARIANCE ON S* (A, F, AND G)

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	52.029	26.015	0.033
F	2	3.502	1.751	0.002
G	2	212.786	106.393	0.135
AF	4	138.217	34.554	0.044
AG	4	256.076	64.019	0.081
BG	4	122.597	30.649	0.039
Residual	8	6299.046	787.381	
TOTAL	26	7084.255		

** Tabulated Values for F: $F_{0.05}$ (2,8) = 4.46

$F_{0.05}$ (4,8) = 3.84

TABLE III.3

ANALYSIS OF VARIANCE ON RESIDUALS FROM G

Source	Degree of Greedom	Sum of Squares	Mean Square	F**
A	2	52.029	26.015	0.80
B	2	6079.050	3039.525	93.48
F	2	3.502	1.751	0.05
AB	4	246.515	61.628	1.89
AF	4	138.217	34.554	1.06
BF	4	92.052	23.013	0.71
Residual	8	260.100	32.512	
TOTAL	26	6871.468		

** Tabulated Values for F: $F_{0.05}$ (2,8) = 4.46

$F_{0.05}$ (4,8) = 3.84

2. RESULTS FROM ANALYSES OF VARIANCE ON S_1*

TABLE III.4

ANALYSIS OF VARIANCE ON RESIDUALS FROM B

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	53.492	26.746	0.88
F	2	3.786	1.893	0.06
G	2	227.569	113.784	3.78
AF	4	146.062	36.515	1.21
AG	4	276.202	60.050	2.29
FG	4	126.921	31.730	1.05
Residual	8	240.715	30.089	
TOTAL	26	1074.751		

** Tabulated Values for F: $F_{0.05}(2,8) = 4.46$

$F_{0.05}(4,8) = 3.84$

TABLE III.5

ANALYSIS OF VARIANCE ON S_1* (A, B AND G)

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	53.547	26.773	1.21
B	2	6528.675	3264.337	148.01
G	2	227.360	113.680	5.15
AB	4	266.097	66.524	3.02
AG	4	276.455	69.113	3.13
BG	4	75.164	18.791	0.85
Residual	8	176.442	22.055	
TOTAL	26	7603.743		

** Tabulated Values for F: $F_{0.05}$ (2,8) = 4.46

$F_{0.05}$ (4,8) = 3.84

TABLE III.6

ANALYSIS OF VARIANCE ON S_1* (B, F AND G)

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
B	2	6528.675	3264.337	47.90
F	2	3.792	1.896	0.03
G	2	227.360	113.680	1.67
BF	4	96.582	24.145	0.35
BG	4	75.164	18.791	0.27
FG	4	126.952	31.738	0.46
Residual	8	545.217	68.152	
TOTAL	26	7603.743		

** Tabulated Values for F: $F_{0.05}\ (2,8) = 4.46$

$F_{0.05}\ (4,8) = 3.84$

TABLE III.7

ANALYSIS OF VARIANCE ON RESIDUALS FROM F

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	53.545	26.772	1.24
B	2	6528.387	3264.194	151.80
G	2	227.347	113.674	5.28
AB	4	266.100	66.525	3.10
AG	4	276.451	69.113	3.21
BG	4	75.154	18.788	0.87
Residual	8	172.651	21.581	
TOTAL	26	7599.638		

** Tabulated Values for F: $F_{0.05}\ (2,8) = 4.46$

$F_{0.05}\ (4,8) = 3.84$

TABLE III.8

ANALYSIS OF VARIANCE ON S_1* (A, B AND F)

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	53.547	26.773	0.42
B	2	6528.675	3264.337	51.40
F	2	3.798	1.896	0.03
AB	4	266.097	66.524	1.05
AF	4	146.061	36.515	0.57
BF	4	96.582	24.145	0.38
Residual	8	508.988	63.623	
TOTAL	26	7603.743		

** Tabulated Values for F: $F_{0.05}$ (2,8) = 4.46

$F_{0.05}$ (4,8) = 3.84

TABLE III.9

ANALYSIS OF VARIANCE ON S_1* (A, F AND G)

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	53.547	26.773	0.032
F	2	3.792	1.896	0.002
G	2	227.360	113.680	0.134
AF	4	146.061	36.515	0.043
AG	4	276.455	69.113	0.082
FG	4	126.952	31.738	0.037
Residual	8	6769.574	846.197	
TOTAL	26	7603.743		

** Tabulated Values for F: $F_{0.05}(2,8) = 4.46$

$F_{0.05}(4,8) = 3.84$

TABLE III.10

ANALYSIS OF VARIANCE ON RESIDUALS FROM G

Source	Degrees of Freedom	Sum of Squares	Mean Square	F**
A	2	53.547	26.773	0.76
B	2	6528.285	3264.143	92.72
F	2	3.791	1.895	0.05
AB	4	266.097	66.524	1.89
AF	4	146.061	36.515	1.04
BF	4	96.582	24.145	0.69
Residual	8	281.627	35.203	
	26	7375.992		

** Tabulated Values for F: $F_{0.05}(2,8) = 4.46$

$F_{0.05}(4,8) = 3.84$

APPENDIX IV

Each of the model indicated in the following tables were obtained by an individual and separate running of the Multiple Linear Regression Program.

1. RESULTS FROM DIFFERENT MATHEMATICAL MODELS FOR S_1*

TABLE IV.1

RESULTS FROM DIFFERENT MATHEMATICAL MODELS FOR S_1*

Variables No.	Variables Name	Residual Mean Square	Change in RMS	Source	Change as % Original RMS	Cumulative Per Cent***	Equation No.	R^2
0	0	292.452	----	----	----	-----	72	----
1	B	90.543	201.909	B	69.040	69.040	73	0.70231
2	B, B^2	44.794	45.749	B^2	15.643	84.683	74	0.85861
3	B, B^2 G	37.099	7.695	G	2.631	87.314	75	0.88781
4	Above & G^2	38.532	- 1.433	G^2	-0.590	86.824	76	0.88851
5	Above & AG	37.478	1.054	AG	0.360	87.184	77	0.89649
6	Above & AB	27.064	10.414	AB	3.562	90.746	78	0.92881
7	Above & A^2G	26.572	0.492	A^2G	0.168	90.914	79	0.93360
8	Above & AG^2	26.458	0.114	AG^2	0.038	90.952	80	0.93737
9	Above & A^2G^2	27.815	- 1.357	A^2G^2	-0.463	90.489	81	0.93781
10	Above & A^2B	29.075	- 1.260	A^2B	-0.431	90.058	82	0.93882
11	Above & AB^2	25.668	3.407	AB^2	1.165	91.223	83	0.94936
12	Above & A^2B^2	25.798	- 0.130	A^2B^2	-0.045	91.178	84	0.95249

*** Cumulative Per Cent = (R^2 based on unbiased variances) x 100

TABLE IV.2

EQUATIONS CORRESPONDING TO DIFFERENT MODELS FOR S_1^*

Equation No.	
72	$S_1^* = 20.228$
73	$S_1^* = 51.291 - 0.981B$
74	$S_1^* = 92.512 - 4.079B + 0.0471B^2$
75	$S_1^* = 99.084 - 4.079B + 0.0471B^2 - 0.334G$
76	$S_1^* = 96.424 - 4.07B + 0.0471B^2 + 0.00622G - 0.00876G^2$
77	$S_1^* = 96.424 - 4.07B + 0.0471B^2 + 0.180G - 0.00876G^2 - 0.00458AG$
78	$S_1^* = 96.424 - 4.471B + 0.0471B^2 + 0.712G - 0.00876G^2 - 0.0186AG + 0.0103AB$
79	$S_1^* = 96.424 - 4.471B + 0.0471B^2 + 0.215G - 0.00876G^2 + 0.0104AG + 0.0103AB - 0.00036A^2G$
80	$S_1^* = 96.424 - 4.369B + 0.0471B^2 - 0.557G + 0.0157G^2 + 0.0307AG + 0.00763AB - 0.00036A^2G - 0.000645AG^2$
81	$S_1^* = 96.424 - 4.369B + 0.0471B^2 - 1.194G + 0.0403G^2 + 0.0678AG + 0.00763AB - 0.000822A^2G - 0.00208AG^2 + 0.0000179A^2G^2$
82	$S_1^* = 96.424 - 4.047B + 0.0471B^2 - 2.365G + 0.0688G^2 + 0.136AG - 0.0111AB - 0.00167A^2G - 0.00374AG^2 + 0.0000385A^2G^2 + 0.00234A^2B$
83	$S_1^* = 96.424 - 5.454B + 0.0695B^2 - 0.355G + 0.02G^2 + 0.0833AG + 0.259AB - 0.00167A^2G - 0.00245AG^2 + 0.0000385A^2G^2 + 0.000234A^2B - 0.000589AB^2$
84	$S_1^* = 96.424 - 8.278B + 0.114B^2 - 3.677G - 0.0779G^2 - 0.152AG + 0.191AB + 0.00125A^2G + 0.00325AG^2 - 0.0000325A^2G^2 - 0.00182A^2B - 0.00321AB^2 + 0.0000326A^2B^2$

2. OPTIMIZATION

The optimum values predicted by using Equation 77 in S_1* (this Equation is equivalent to Equation 7 for S*) are

A = 14.3 RPM

B = 45.7 Per Cent

F = 0°

G = 25.4 rotations.

All the calculations performed with S* were repeated using the S_1* values. These values also behave in exactly the same manner as S* and the results predicted the same optimum conditions as in the case of S*.

APPENDIX V

VITA

The author, Venkatachalam Seshan, is the son of C. S. Venkatachalam and Lakshmi (Ammal) Venkatachalam. He was born on December 25, 1938 at Kottayam, Kerala, India.

He obtained his elementary education at the Little Flower School in Bombay and completed his secondary education at the St. Xavier's High School, Vile Parle, Bombay, graduating in 1954.

In June 1954, he entered the Ramnarain Ruia College and passed the Intermediate Examination in Science from the University of Bombay, in June 1956. On the basis of his first class at this examination, he was admitted to the Department of Chemical Technology in June 1956. During his undergraduate training, he worked at the National Rayon Corporation as well as with Polychem Ltd. He received the degree of Bachelor of Chemical Engineering in 1960 and was placed in the first class. While attending the University of Bombay, he was elected Member of the Managing Committee of the Technological Association.

After working as a Chemical Engineer with J. K. Chemicals, Ltd., Bombay, India, he joined the University of Bombay as Graduate Assistant in Chemical Technology, in June 1960, and was in charge of the Ore-Dressing and Fuel Technology Laboratories, until August 1961. During this time he conducted both research and teaching.

In September 1961, he was awarded a Research Fellowship by the University of Louisville Institute of Industrial Research. He was initiated into the Sigma Xi Honor Society in May 1962. In September 1962, he received the degree of Master of Chemical Engineering from the University of Louisville, Louisville, Kentucky.

The author joined Lehigh University in September 1962, in order to continue his doctoral program in Chemical Engineering. From 1962 to 1965 he held the William C. Gotshall Scholarship.

He spent the summer of 1963 with E.I. du Pont de Nemours and Company at their Experimental Station in Wilmington, Delaware. He will be joining the Du Pont organization in October, 1965.