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TO THE STRENGTH OF STEEL COLUMNS

by

Reidar Bjørhovde

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Erik, Anne and Knut

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ABSTRACT

The study presented in this dissertation represents an attempt to arrive at a more rational and adequate means of assessing the strength of centrally loaded, pinned-end, initially curved, prismatic steel columns. The investigation has been based on the concept of the maximum strength of a column, whereby all pertinent column strength factors have been taken into account, including the initial out-of-straightness.

Two conceptually different methods of solution have been utilized. The first part of the dissertation describes the deterministic investigation, for which the maximum strength column curves for a large number of different structural shapes have been developed. The column types included represent rolled and welded wide-flange and box shapes of a variety of steel grades, sizes, manufacturing methods, and so on. The theoretically determined maximum strengths have been found to compare well with a number of column test results (accuracy of ± 5 percent). The deterministic column strength data have been analyzed, in order to detect the influence of the most important column strength factors, with the ultimate goal of developing a set of multiple column curves. Each of the

curves in the set represents the average of the strength of the columns that are to be designed according to it, and the set of curves therefore illustrates an improvement over the presently used column design rules. The set of multiple column curves that has been developed is based on an initial out-of-straightness of 1/1000, which constitutes the specification maximum allowable.

The second method of solution is based on a probabilistic computation of the column maximum strength. An extensive evaluation of the statistical characteristics of the cross-sectional dimensions and the other geometric properties of the column shape, of the yield stress, the residual stresses in the shape, and of the initial out-of-straightness, has been carried out. The probabilistic characteristics have been substantiated by the results of actual measurements. These data have been utilized towards the development of a quasi-steady probabilistic solution for the maximum column strength, whereby the deterministic, incremental relationships governing the behavior of the column have been formulated so as to account for the random nature of the parameters involved. This represents the first time that a non-linear structural engineering problem has been solved on the basis of probabilistic concepts, including the effects of material and geometric non-linearities.

The results of the probabilistic study indicate that the initial out-of-straightness is by far the factor contributing the most to the random variation of the strength of a particular column. The yield stress only has a very small effect, and the influence of the random variability of the residual stresses and of the cross-sectional properties is small within the limitations of the study. The randomness of the residual stresses studied are indicative only of the $\pm$-variations that may occur at any point throughout the cross section. The random nature of the entire residual stress pattern in a shape was not included in the investigations. The gross residual stress distribution therefore remains one of the most important factors influencing the strength of columns in general.

Further analyses of the data have provided material for the development of a set of probabilistic multiple column curves, which have been based on an assumed mean value 1/1470 of the out-of-straightness. The probabilistic set of curves is believed to represent a scientifically sound solution, and is regarded as more adequate than the deterministic multiple column curves.

1. INTRODUCTION

The column is one of the basic elements of any structure, and its strength and behavior have long constituted some of the most important and crucial problems to face the structural engineering researcher and designer. Numerous theories and attempts at arriving at the most rational and representative solution of the problem have been formulated, but most of them have been hampered by deficiencies with more or less far-reaching consequences.

Of major importance in all of these solutions is the fact that the strength and behavior have been investigated on the basis of deterministic concepts. The parameters of influence have been assumed to take on predetermined values, not subjected to the natural laws of variation that are inherent in almost any physical phenomenon, and thus beyond the reach of human control. Whereas conceptually obvious, it is inconceivably complex to incorporate all factors and their variability into a practical solution, and some simplification and assumptions therefore have to be made in order to arrive at a practicable column model. This does not imply that the study and its results will be of lesser value, but rather that it represents a step in the direction of improving the assessment of column strength.

The study presented in this dissertation deals with two basically different methods of arriving at a better representation of the strength of centrally loaded, initially curved, prismatic steel columns. The first approach is based on a strictly deterministic computation of the column's maximum strength, and the variation of the strength is attempted to be accounted for by finding the strength for columns with different shapes, sizes, materials, and so on, and analyzing these data. The second approach is founded on probabilistic concepts, whereby the variation of the relevant strength parameters is considered explicitly in the calculation of the maximum strengths. This method therefore circumvents several of the steps necessary in the other approach and allows a direct analysis of the separate and joint effects of the variables. It will be shown that many of the results obtained by the latter solution could not possibly be deduced from the data obtained in the first part of the investigation. This represents the first time that the concepts of probability theory have been applied towards the solution of the problem of an inelastic, initially curved column, and hence provides a theory that may be used in many other areas of similar nature.

1.1 A Historical Review of the Column Strength Problem

The first to realize the nature of the column strength problem, and thus the first to present a rigorous solution, was Leonard Euler[1], who published his classical work in 1759. The controversy related to the column problem did not emerge until more than one hundred years later, however, when the non-linear column strength theories were conceived and developed. Having presented the original tangent modulus theory in 1889[2], Engesser later modified it to account for the remarks by Considère[3] and the criticism given by Jasinsky[4], and thus in 1895 set forth the reduced modulus (double modulus) theory of column strength[5]. This theory thereby came to be regarded as the correct solution of the column problem, particularly after von Kármán[6] conducted a number of tests which showed a good correlation between his test results and the theory.

Considerable confusion began to arise when, in the years that followed, the results of most column tests that were performed exhibited strengths closer to the tangent modulus loads than to the reduced modulus loads. It is a remarkable fact that a clarification did not come until fifty years later, when Shanley published his now famous treatise[7,8], whereby the tangent modulus concept was reinstated as the correct representation of the strength of an initially perfectly straight column. It was then realized that the tangent modulus load is a lower bound, and the reduced modulus load an upper bound to the column strength[9,10].

All of the above mentioned theories dealt with the strength of an initially perfectly straight column, and its behavior up to the point of bifurcation of the equilibrium (buckling). A major step forward towards the understanding of the column's behavior when subjected to loads above the tangent modulus load was made by Duberg and Wilder[11], and their results were later confirmed by Johnston[12], Fujita[13], and Tall[14].

It was found fairly early that an initial curvature of the column would influence its behavior strongly[10]. The results arrived at by Lin[15] enunciated the phenomenon well for inelastic columns, and other investigators confirmed and expanded the theory[16]. Dutheil[17,18] proposed a mathematically much simpler approach, which led to very satisfactory results.

Residual stresses were not considered in the before-mentioned investigations by Lin and Dutheil, although their existence had been known since the end of the nineteenth century. Osgood[19] and Yang et al.[20] presented a well-developed theory of column strength where this was taken into account, and the results had a far-reaching impact on the studies of the column problem. A large number of investigations on this topic were conducted in the following years, of which the works by Tall[14], Huber and Beedle[21], and Huber and Ketter[22] give but a brief survey.

The strength of an inelastic column with residual stresses and initial curvature therefore was regarded as an exceedingly complex problem, although attempts had been made to solve it[22]. A most practicable approach was developed by Tall[23], but it was not until a few years later, when the full impact of the electronic computer era came into effect, that the problem could be studied systematically. Detailed analyses were presented by Fujita[13], Tall[14,23], and Batterman and Johnston[24], and other investigators generalized and expanded on the methods involved[25].

All of these studies were conducted primarily in order to arrive at a better assessment of the column and its behavior, but often also with a view towards developing rules for practical design purposes. These rules usually were expressed in the form of a column curve, relating strength and column length in some fashion. It is an amazing fact that some of the earliest column design curves, notably the ones developed by Tredgold (1822)[10], Rankine-Gordon[10], Tetmajer[26], and Johnson[27], compare very well with considerably more recent curves. Some of the presently employed specifications in various countries are still based on the early developments, but most of them have utilized more modern approaches[28]. In the United States an equation proposed by Bleich[9] later was modified by the Column Research Council[28], and has since formed the basis for the present and previous versions of the column design rules of the AISC Specification[29].

1.2 Structural Safety and the Variation of Column Strength

The uncontrollable, random variation of the parameters influencing the strength of a column, and hence the variation of the strength itself, is inseparably tried to the question of the safety of the structure of which the column is a part. The importance of the variability was recognized fairly early, but the manner in which it was accounted for may only be classified as somewhat arbitrary. Similar statements may be made about the loadings incorporated in the analyses of structures. Together, load and strength form a structural entity, and the method of handling of the two components has a major influence upon the final outcome, of which economy plays an important role.

A rational treatment of a problem involving random variables can only be accomplished by the employment of the principles of probability theory and mathematical statistics. This was realized long ago in engineering sciences such as electronics and applied physics, and thus the concepts of reliability and a reliability theory evolved. The essential aspect of all of these disciplines lies in the recognition of the fact that a failure of the system considered, be it a structure or an assembly of electronic components, never can be ruled out as an impossible event. It may, however, be asserted that the probability of this event become as small as desired, and there is no reason why this probability should not be assigned different values for different types of structures or structural elements. The final result will be uniform and consistent factors of safety that are based on sound scientific principles.

It was not until well into the twentieth century that the questions of structural safety, loading, and strength were tied together with the concepts of probability theory and mathematical statistics. A brief note by Prot[30] in 1936 seems to have provided the starting point, and in the following years a number of studies on the general nature of the problem were made by Freudenthal[31,32], Pugsley[33], Streletskij[34], and Basler[35]. A crucial investigation on the statistical nature of the strength of materials was conducted by Weibull[36], and his concepts later were expanded by several others, notably Volkov[37].

Many of the above mentioned and of the more recent studies were aimed at implementing the probabilistic concepts of risk and safety into the formats of codes and specifications. Particularly noteworthy are the works of Cornell[38,39], Ang[40,41], Benjamin[42], and Lind et al.[43] Haugen[44] and Benjamin and Cornell[45] developed practical methods for the design of various structural systems and components, which thus represented another step towards the implementation of the probabilistic concepts.

A major obstacle in this process of evolution was and remains the lack of actual data on the variation of the many relevant characteristics, such as the material and geometric properties of structural steel shapes. Although such data are not essential to the analyses, and only serve to substantiate the assumptions and the results; this shortcoming is rapidly being removed through extensive investigations. The comprehensive series of tests of centrally loaded columns[46,47], carried out in Europe over the past few years, is a good example of such studies. These tests were conducted primarily to establish the statistical characteristics of the strength of centrally loaded columns, and were done in conjunction with detailed theoretical analyses.

2. ON THE THEORIES OF COLUMN STRENGTH

The controversy about the column problem has always been rooted in the question of the proper representation of the strength of all columns. Assuming that the tangent modulus and the reduced modulus theories are well known, a detailed description of the concepts involved will not be presented here. One of the basic issues of both of these approaches lies in the assumption that initially a column is perfectly straight, which thus provides for its treatment as an eigenvalue problem (buckling, bifurcation of the equilibrium).

Following the recognition that the reduced modulus load is a purely theoretical quantity, which in practice never can be attained (upper bound), the lower bound characteristic of the tangent modulus load has been one of the reasons for its forming the basis for the column strength criteria in a number of specifications. Being a safe, yet a conservative estimate of the strength of a column, this approach has afforded the concepts on which a rational inelastic column theory can be based.

The principal shortcoming of the tangent modulus theory lies in the assumption of an initially perfectly straight member. It does take into account the non-linearity of the column properties, as expressed by the presence of residual stresses, and for some materials also a non-linear stress-strain relationship, but it cannot treat imperfections in the form of out-of-straightness, eccentric application of the load, and so on. Imperfections like these are always present in real columns, as real columns come from the manufacturer and are installed in the structure; and the imperfections must be incorporated in a fully realistic approach. The tangent modulus load is a fair representation of the strength of a column, as long as the imperfections are small, but its validity ceases to exist when they attain any appreciable magnitudes. This may be realized by considering the curves shown in Fig. 1, which gives a schematic illustration of the various inelastic column strength concepts.

The shortcomings of the tangent modulus theory can be alleviated by replacing it with the maximum strength theory, which essentially means that the bifurcation phenomenon will be replaced by a load-deflection approach. The column thus starts to bend at the onset of loading, and gradually approaches a limiting state, beyond which the load must decrease with increasing deflection, in order that equilibrium between the external and the internal forces is maintained. The differences between the two methods of approach are schematically illustrated in Fig. 1.

A large number of factors influence the maximum strength of a column, and each of them must be duly taken into account in any analysis that aims at an improvement over the present conditions. Briefly stated, the important factors can be summarized as follows:

1. Grade of steel
   (a) Stress-strain relationship
   (b) Yield stress

2. Manufacturing method (i.e. residual stresses)
   (a) Rolled columns
   (b) Welded columns
      (α) Welded from flame-cut plates
      (β) Welded from universal mill plates with as-rolled edges

3. Size of shape

4. Cross section of column (wide-flange, box, etc.)

5. Bending axis

6. Magnitude and shape of out-of-straightness

The first five of these items are the same for all column strength theories, whether the tangent modulus, the reduced modulus, or the maximum strength theory is considered. The out-of-straightness is unique for the maximum strength approach, however, and plays a very important role for a certain range of slenderness ratios[13,14,24].

The yield stress is the most significant factor as far as the grade of steel is concerned, and a wide variety of grades is commercially available. The form of the stress-strain relationship changes as the yield stress increases, with a resulting loss of the typical yield plateau. This has an effect that is outlined in more detail in Section 3.3.2. The most important factors for the strength of the very short columns are the strain-hardening properties of the steel, but this is considered a topic beyond the scope of this study.

Besides the out-of-straightness, the residual stresses produced by cooling after rolling, by cold-straightening and by welding, constitute the single most important factor for the column strength. Being heavily dependent on the method of manufacture, distinction has to be made between rolled and welded shapes; and for welded shapes whether the component plates are universal mill plates with as-rolled edges, or flame-cut plates. The size of the shape and its component plates also influence the magnitude and distribution of the residual stresses, in particular such that the stresses exhibit a pronounced variation through the plate thickness in the heaviest shapes[48,49].

Shape size and method of manufacture have a certain bearing on the actual yield stress of the full cross section. Most significant are the effects of welding and flame-cutting, which produce small areas of very high yield stress close to welds and to flame-cut edges[50,51]. The yield stress also decreases as the thickness of the steel increases[52,53]. In particular the effects of welding and flame-cutting are important to consider in the evaluation of column strength.

A large number of structural shapes are available, but only a limited number of these are used as column sections. The rolled and welded wide-flange shape and the welded box-shape constitute by far the ones most widely used [54]. The choice of major and minor bending axes usually follows the type of shape.

The real configuration of the initial out-of-straightness of a column may be very complicated, often expressed as a crookedness about both axes simultaneously.

The magnitude of the out-of-straightness is limited by the specifications, which also give the requirements regarding the allowable variations of dimensions, weight, and so on, for structural shapes. Certain assumptions and simplifications therefore must be made in order to implement the effects of these imperfections, and the choices made on this point are discussed further in Section 3.3.2. A detailed analysis of the effects of all possible structural imperfections may be regarded as exceedingly difficult.

Recently a few studies of column strength based on probabilistic concepts have been performed. Common to all of these is that already known deterministic theories of column strength have been employed, and a few of the variables concerned have been treated as random variables. A more complete discussion and evaluation of these investigations is given in the part of this dissertation that deals with a probabilistic evaluation of the maximum column strength (Chapter 4).

3. DETERMINISTIC SOLUTIONS OF THE COLUMN STRENGTH PROBLEM

3.1 Methods of Implementing or Reducing the Variation of Column Strength

A variety of deterministic methods aimed at the implementation of more rational and accurate methods of column design can be employed. Some of these are related to the manufacture of the columns, while others are based on theoretical developments[55,56]. The main problem connected with the manufacture-related approaches lies in the formulation of requirements that will duly consider all pertinent factors, such as the rate of cooling after rolling, which is a task so complex as to be impossible. Most important among the theoretically based methods is the one that utilizes several column strength curves, to each of which related column categories are assigned. This is denoted the concept of multiple column curves, and was used in a very simple form for the first time by the German specification for design of columns DIN 4114 (1959). It now has been employed to a much wider extent in methods proposed in American[25] and European[57] developments.

The following chapters outline and discuss the relative merits of each of the basic approaches mentioned above. Specific attention is paid to the multiple column curve solution, which is considerably simpler and easier to put to use than any of the other methods. Strictly speaking, from the point of view of strength, the best solution is the one where every type of column could be represented by its own design curve; that is, for example, one curve for annealed rolled wide-flange shapes, one for hot-rolled box-shapes, one for rolled wide-flange sections of high-strength steel, and so on. This would, however, complicate the approach so much that the practical advantages might be lost. The final number of column curves therefore should be such that an optimum of practicality and rationale can be attained.

3.2 Solutions Related to the Manufacture of Columns

Under appropriate control, a number of manufacturing and post-fabrication operations may be used to enhance the strength of a column. The most important of these methods and their effects are discussed below.

1. Flame-cutting of plates for welded built-up shapes

Numerous tests and theoretical investigations have shown that built-up columns welded from oxygen-cut component plates exhibit a greater strength than similar columns made from universal mill plates with as-rolled edges[25,50,58]. This is due to the more favorable residual stress distribution is such sections, whereby flame-cut plates have high tensile residual stresses at the edges, and the edges of universal mill plates are in compression[48,50,51].

2. <u>Use of minimum weld size</u>

It has been shown[25,48,51] that the influence of welding on the residual stresses in the shape decreases as the size of the shape increases; that is, the percentage of the final residual stresses which can be attributed to the welding process becomes less as the shape becomes larger. Although it is clearly advantageous to keep the weld size to a minimum, both for this reason, and the fact that larger welds may cause significant distortions of the shape, the effects as far as an increase of the column strength is concerned will be almost insignificant the heavier the shape[25,48,51]. It will, however, play a role for the small and medium size shapes.

3. Control of cooling after rolling

The usual process of cooling after rolling of shapes and plates takes place on a cooling bed, which allows the heat stored in the specimens to escape more or less freely, depending on whether other members are located in the vicinity. Under no circumstances are planned controls imposed during this stage of the manufacture, and the rate of cooling will be fairly high. A slower cooling rate will provide for a more even escape of the heat, with reduced residual stresses as a result[59].

There are various means of achieving slower cooling rates, some of which resemble postheating and annealing (see below). It is thus possible to place the rolled specimens in a temperature-controlled chamber, which forcibly will reduce the speed of heat emission; or local heat input may be provided, for instance, along the flange tips of a rolled wide-flange shape after it has been rolled and cooled regularly.

4. <u>Use of preheating or postheating</u>

Both of these methods will alter the residual stress distribution in plates and shapes, and the result will be an increased strength of the section[50]. The extent of alteration depends greatly on the amount and location of the heat input used. The heating of the flange tips of a rolled wide-flange shape, mentioned above, is one method of postheating which seems to possess promising characteristics.

Annealing is a limiting case of postheating, and is basically a furnace heating and cooling operation that reduces the residual stresses to near-zero values. The strength of annealed columns is very significantly higher than that of regularly cooled columns[21,25,50].

5. <u>Use of cold-straightening</u>

Cold-working of the material will also have an effect on the distribution of the residual stresses. A roller-straightened (rotorized) shape is continuously yielded along its length, whereas gagging is a purely localized process. Both methods alter the initial residual stress pattern in the shape to one that is more favorable insofar as buckling is concerned, resulting in a higher column strength[52]. Much work remains, however, to determine all the effects and implications of the various straightening procedures.

Some of the above mentioned operations, like oxygen-cutting, result in improved column performance, but cannot be controlled sufficiently to guarantee a particular strength. Cold-straightening, in all likelihood, can be so controlled that a predetermined strength can be achieved, but this is a topic open for additional research.

Tests of actual columns may be considered as a possible basis for the establishment of design criteria. Economy and the strength of the heavier columns limit the usefulness of tests, however, since it must be regarded as impossible to be able to cover all variables in a proper fashion.

3.3 The Multiple Column Curve Approach

The manufacture-related techniques of enhancing the strength of columns, described above, have been shown to provide a much too complex approach to the solution of the problem at hand. The theoretically based methods, in particular the multiple column curve procedure, thus provide more practicable means of studying the variation of the column strength. The following sections outline the basic ideas of the concept of multiple column curves, and describe and analyze the results of an extensive investigation of the maximum strength of steel columns. The outcome of the study is used to develop a set of column curves that will represent the strength of all columns in a rational fashion, previously not attainable.

3.3.1 The Concept of Multiple Column Curves

A striking illustration of the variation of the strength of a number of different column types is given by Fig. 2, which shows the test results for approximately 100 columns. The differences in column strengths are caused by differences in column shape, steel grade, size, manufacturing method, and so on, but each test point can be predicted within an accuracy of ± 5 percent. It is clear that the use of a single column curve would significantly over- or underestimate the strength of many columns.

The essence of the multiple column curve concept therefore lies in the fact that no one column curve can represent the strength of all types of columns rationally and adequately. By introducing several curves, to each of which columns of related behavior and strength are assigned, the difference between the assessed and the actual column strength will not be completely eliminated, but rather reduced to an acceptable level. This idea is illustrated by Fig. 3, from which it may be seen that the variation of the strength of the column types assigned to, for instance, the lower of the three curves, is substantially smaller than the variation of the strength of all columns together. Whereas an increase of complexity will be the result of utilizing several column curves, significant gains may be expected in terms of accuracy and economy. The best solution will be the one where an optimum of complexity and gains has been achieved.

The implementation of the multiple column curve concept in a deterministic image presents numerous problems, of which the evaluation of a sufficient number of data by far outweighs all the others. This is mainly due to the large number of factors that influence the strength of a column, which require that in order to have a representative sample of results, all possible combinations of the variables should be considered. Although highly desirable, the use of actual (measured) values of the column strength parameters may severely limit the amount of strength data that can be obtained. Theoretical values, for example, of the residual stresses in a shape, can be used, but this may impair the quality of the results. In the deterministic investigation that is described in the subsequent parts of this chapter, real magnitudes of the column strength parameters have been employed, and it is believed that a fairly good representation of practical conditions has been achieved. In the European study of the same problem[57], for example, purely theoretical data form the basis for the computations; and an extensive series of tests was undertaken in order to substantiate the findings[46,47].

The study that will be described here has been based on the maximum column strength concept. Prior to this, a similar investigation based on tangent modulus loads had been conducted, in order to study the rationale and possibilities of utilizing multiple column curves in the design of columns[25,60,61]. This aim was basically fulfilled, and Fig. 4 shows the possible multiple column curves that constituted the essence of the work. This investigation is not described here, but it may be noted that one of the three curves shown in Fig. 4 is the same as the present CRC Curve.

3.3.2 An Investigation Based on Maximum Column Strengths

The study of tangent modulus loads, mentioned above, was based on data already available in various reports and publications. A literature search revealed that similar information on the maximum strength of columns was practically non-existent, except for a few, basically idealized, investigations[13,14,24,58,62]. In addition, some of these studies only dealt with the maximum strength of initially perfectly straight columns, whereby the very important influence of the out-of-straightness was omitted from consideration.

The main problem connected with the initiation of the present investigation consisted of obtaining experimental results for the residual stress distributions, yield stresses and cross-sectional properties of a representative variety of shapes in different steel grades. An extensive search for such data was carried out, and the information thus obtained was used as input data for a maximum strength computer program that had been developed.

The following factors were included in the study:

1. Grade of steel (yield stress): ASTM A7, A36, A242, A441, A572(50), A572(65), and A514[60].

2. Manufacturing method: Rolled wide-flange columns (W), welded wide-flange (H) and box-columns, manufactured from flame-cut and universal mill plates.

3. Size of shape: Light and heavy. (A shape is defined as light if the thickness of all component plates is less than one inch; otherwise it is heavy)[54].

4. Cross section of column: Wide-flange and box (cf. item 2).

5. Bending axis: Two, as appropriate (major and minor).

6. Out-of-straightness: Four values were chosen, namely, L/500, L/1000, L/1500, and L/2000, where L is the length of the column. For simplicity the shape of the out-of-straightness was assumed to be that of half sine-wave, with the maximum value occurring at the mid-height of the column.

By comparing the above with the list of parameters given in Chapter 2 (page 15), it will be seen that some factors have been omitted, and that for others, specific but representative choices have been made. The reasons for the omissions and the choices are outlined later in this chapter. In particular, the magnitude L/1000 chosen as one of the values of the initial out-of-straightness, conforms with the straightness-requirements of the specifications for the delivery of structural steel shapes[63].

Tables 1 through 3 give the data for all the columns included in the investigation. Fifty-six different combinations of shape, steel grade, and so on, are represented; and with two column curves for each shape (one for the major and another for the minor axis bending), a total of 112 maximum strength column curves has been generated. It may be noted that this number by no means exhausts all the possible combinations of shape and steel grade. Shape/steel grade combinations for which column curves are not available are limitations that have been imposed due to the lack of data for residual stress distributions, since actually measured residual stresses were used. The 112 maximum strength curves do, however, constitute a representative variety of column shapes and steel grades, and provide material sufficiently reliable for the establishment of preliminary multiple column curves.

The complexity of the maximum strength problem leaves no formula by which to predict the magnitude of this load, as opposed to the somewhat simpler tangent modulus and reduced modulus approaches. The most practicable solution makes use of a digital electronic computer, and a computer program named MAXLD2 was developed for this purpose. Essentially the program is based on the same theoretical approach as that used in earlier studies[13,14,23,24,58], and an outline of the method is given below. A listing of the program MAXLD, which formed the basis for MAXLD2, but is of a less general and versatile nature, may be found in Ref. 64.

The deterministic principle for the computation of the maximum strength of a centrally loaded column subjected to in-plane bending basically consists of finding the axial load P, which for the given deflection δ at the column mid-height provides for a state of equilibrium between the external and the internal forces and moments. The equilibrium equations are thus:

$$P = P_{int} \tag{1}$$

and

$$P \cdot \delta = M_{int} \tag{2}$$

where P_{int} and M_{int} are the internal force and moment, respectively.

Considering Fig. 5, it may be seen that the total deflection δ consists of three parts, namely:

$$\delta = e + v_p + \Delta \tag{3}$$

where e is the initial out-of-straightness, v_p the deflection due to the applied load (P), and Δ the eccentricity of the applied load.

The total stress in an element in the cross section at any stage of loading may be expressed non-dimensionally as:

$$\frac{\sigma_i}{\sigma_y} = \frac{\varepsilon_i}{\varepsilon_y} = \frac{\varepsilon_{ri}}{\varepsilon_y} + \frac{\varepsilon_p}{\varepsilon_y} + \theta \cdot \frac{\xi_i}{\varepsilon_y} \quad \text{for } |\sigma_i| < \sigma_{yi} \qquad (4a)$$

where σ_i, ε_i = stress and strain in element i in the cross section

σ_y, ε_y = overall (average) yield stress and strain for the cross section, as obtained in a stub column test.

ε_{ri} = residual strain in element i, given by $\varepsilon_{ri} = \sigma_{ri}/E$, where σ_{ri} is the residual stress in the element, and E the modulus of elasticity

ε_p = axial strain in element i due to load P

θ = curvature of column due to deflection v_p

ξ_i = distance from the centroid of element i to the bending axis considered (major or minor)

Equation (4a) is valid for the case when the element stress, σ_i, is less than the yield stress of the element, σ_{yi}. If the element yield stress is exceeded, Eq. (4a) must be replaced by the relations

$$\frac{\sigma_i}{\sigma_y} = \frac{\sigma_{yi}}{\sigma_y} \quad \text{if } \sigma_i \geq \sigma_{yi} \tag{4b}$$

and

$$\frac{\sigma_i}{\sigma_y} = -\frac{\sigma_{yi}}{\sigma_y} \quad \text{if } \sigma_i \leq -\sigma_{yi} \tag{4c}$$

Equations (4b) and (4c) reflect the yielding in tension and compression, respectively, of element i.

Assuming that the initial and all subsequent deflected shapes of the column may be described by a half sine-wave, thus:

$$v = e \cdot \sin\frac{\pi x}{L} \tag{5}$$

where the coordinate x is measured along the length of the column; the column curvature at mid-height is found from

$$\theta = \frac{1}{R} \approx \frac{d^2v}{dx^2} \tag{6}$$

Differentiating (5) accordingly, the curvature at mid-height ($x = L/2$) becomes:

$$\theta = \frac{\pi^2}{L^2} \cdot v_p \tag{7}$$

since only the deflection created by the applied load is relevant.

The column's load-deflection curve is obtained by incrementing the deflection v, and finding the corresponding equilibrium load P. For every value of v, the magnitude of P that satisfies the equilibrium condition is found through an iteration procedure. This is accomplished by assuming values of the axial strain ε_p, and computing the corresponding axial loads such that internal and external force and moment equilibrium is reached. Hence, with an assumed value of ε_p, all factors in Eq. (4a) are known, and the internal force and moment in the cross section can be calculated from the relationships:

(a) <u>Force</u>:

$$\frac{P_{int}}{P_y} = \frac{1}{A \cdot \sigma_y} \sum_{i=1}^{n} \sigma_i A_i \tag{8}$$

where A is the total area of the cross section, A_i the area of element i, n the number of elements in the cross section, and $P_y = A \cdot \sigma_y$ is the yield load of the cross section.

(b) <u>Moment:</u>
$$\frac{M_{int}}{P_y} = \frac{1}{A \cdot \sigma_y} \sum_{i=1}^{n} \sigma_i \xi_i A_i \tag{9}$$

Having found the internal force and moment according to Eqs. (8) and (9), the state of equilibrium can be investigated by using (from (1) and (2)) the expression

$$P_{int} = \frac{M_{int}}{\delta} \tag{10}$$

from which is established:

$$\frac{\Delta P}{P_y} = \frac{P_{int}}{P_y} - \frac{M_{int}}{P_y \cdot \delta} \gtrless 0 \tag{11}$$

ΔP represents the amount of out-of-equilibrium of the axial force P that corresponds to the assumed axial strain. Absolute equality in Eq. (11) is practically impossible to attain, and therefore an accuracy requirement is imposed by the inequality

$$\frac{\Delta P}{P_y} \leq \left(\frac{\Delta P}{P_y}\right)_{min} \tag{12}$$

If (12) is not satisfied, a new value of the axial strain ε_p is chosen, and the iteration process carried out until the sufficient accuracy is obtained.

The procedure described leads to the determination of the load-deflection curve for the column, and the highest load obtained constitutes the maximum strength of the column.

The following assumptions were made in the analysis:

1. The material exhibits an idealized linearly elastic-perfectly plastic stress-strain relationship in every fiber of the cross section.

2. The initial and all subsequent deflected shapes of the column can be described by a half sine-wave.

3. The residual stresses are uniform through the thickness, and constant along the length of the column.

4. Sections that are originally plane remain so for the range of deflections considered.

5. Yielded fibers unload elastically.

6. The yield stress may vary across the width of the column, but is assumed to be constant through the thickness of the component plates, and along the length of the column.

7. Only stresses and strains at the mid-height cross section of the column are considered.

The effects of using an elastic-plastic stress-strain relationship depend on the type of steel considered, and may be schematically illustrated as shown in Fig. 6. The discrepancy between the real and the assumed stress-strain curve for the mild structural steel shown in Fig. 6a, is very small for strains less than ε_{st}, the strain-hardening value. The principal implication of the assumption therefore will be the neglecting of any strain-hardening, which probably will result in a slightly lower maximum strength for very short columns. For medium-length and long columns it is believed that the strain-hardening will have no significant effect.

For a column made of high-strength steel, the effects outlined above will be more pronounced. This may be conceived by regarding Fig. 6b, which compares the real and the assumed stress-strain relationships for a steel of this type. The theoretical maximum strength will be underestimated to an extent that depends on how the yield strength for the material is defined.

The assumption of a column that maintains the configuration of its deflected shape presents several advantages to the numerical solution; although strictly speaking, it is not correct. The proper approach is to determine the actual shape iteratively at each increment of load, but previous investigations[24] have indicated that this method of solution gives maximum strengths only very slightly higher than the simplified approach. The method utilized is therefore somewhat conservative, although the difference is so small as to be considered negligible. Its main computational advantage lies in the ease with which the curvature of the column is found (cf. Eq. (7)), which is needed when stresses and strains at a section are computed.

A sinusoidal initial out-of-straightness presents a fair assumption, but any second- or higher-order curve with continuous derivatives, being symmetric about the middle, and with the maximum deflection appearing there, may be used for the present method of analysis. The real shape of the initial crookedness, however, may be very different, as found in numerous investigations[25]. A detailed analysis of columns exhibiting arbitrary initial deflections is not possible with the approach utilized for the computer program MAXLD2, since stresses and strains are considered only at the mid-height of the column. It could be accomplished by making use of a finite element technique, whereby the column is divided into a number of elements along the length and throughout the cross section. The initial boundary conditions then easily can be taken into account. It is not possible to give a full outline of the effects of the assumed versus the real out-of-straightness at this stage.

It is a good approximation to assume that the residual stresses are constant along the length of the column, although some variation has been detected in a few studies[21,65]. The variation appears to be completely random, and thus cannot be accounted for in a deterministic study such as the present one. It therefore would serve no purpose to make the extensive and costly measurements that would be needed. Thus, the input data for the computer program are the residual stresses measured in one cross section of one specimen.

More important is the effect of assuming constant residual stress through the thickness of the component plates of a column. The assumption is satisfactory for thicknesses below approximately one inch, but the variation becomes increasingly pronounced as the thickness increases[48,49,50,51]. It has been shown, however, that for shapes where the plates are about 2 inches thick, the difference in the two tangent modulus loads thus obtained is less than

2 percent[50], with the load based on uniform residual stresses the higher of the two. The difference may be larger in other cases, and for certain slenderness ratios. Other investigations[24] have indicated that the residual stress does not play as important a part for the maximum strength of a column, as it does for the tangent modulus load. This is substantiated by the results of the present and previous studies[25,56,61], indicating significantly smaller differences between maximum strengths than between tangent modulus loads. The reason for this is to be found in the presence of the initial out-of-straightness. It therefore is expected that only if the thickness increases to values in the range of 5 to 6 inches or larger may the discrepancy be of some significance.

The validity of the Navier-Bernoulli hypothesis, namely, that plane sections remain plane, may be questionable for the very short columns, but will hold true for the medium-long and the long columns. Due to this assumption, for very short columns the predicted maximum strength probably will be lower than the correct one. A comparison between experiment and theory, to be given later in this chapter, shows that this may be true.

It is of great importance that the computer program used is able to handle shapes with a yield stress that varies throughout the cross section, particularly for welded, built-up shapes. It is a well-known fact that the material in and adjacent to the welds exhibit significantly higher yield stress than the base metal; and if the flanges of a wide-flange shape are made from flame-cut plates, the flame-cut edges also possess this property[50,51,58]. The strength of hybrid columns, that is, columns where the component plates are of different steel grades, can therefore also be found with this program. The results will be more accurate than those based on an average yield stress, unless this quantity has been derived from stub-column test results.

Most studies of the inelastic response of structures and structural elements make use of the assumption that already yielded fibers unload elastically. The concept holds well for most structural metals. The variation of the yield strength along the length and through the thickness of the component plates of a column is of a random nature, similar to that described above for the residual stresses. It cannot be taken into account in the deterministic study that is conducted here, but it also presents a problem of almost insurmountable difficulty to a probabilistic method of solution.

The present version of the computer program can only accept major and minor axis bending of wide-flange and box-shapes. Sections with different geometric configuration can only be treated after relatively extensive alterations and additions have been made to the program. However, a survey on the utilization and manufacture of heavy columns has indeed indicated that box- and wide-flange shapes are used much more than any other types[54].

3.3.3 A Discussion of the Results of the Maximum Strength Investigation

A relatively brief description of the overall results of the maximum strength investigation is presented in this section, together with several comparisons of the maximum strengths predicted by the computer program and those obtained in column tests. A detailed analysis of the results, with special reference to the development of deterministic multiple column curves, is given in Section 3.3.4.

A good indication of the quality of the theoretical results, and thus of the validity of the assumptions and simplifications made, can be obtained by comparing them with some available column test data. Previous studies at Lehigh University have provided an abundance of such information, and a representative sample of test results was extracted from some of the papers listed in the bibliography of Ref. 25.

Tables 4 through 6 give the specific details for the columns that were used for the comparisons. The columns have been separated with regard to manufacturing method (rolled and welded), cross section (wide-flange and box), size (light and heavy), and steel grade, in order to analyze the performance of the computer program for a variety of column strength parameters. It is believed that the choices made provide such an opportunity. Included in the tables are also the values of the ratio

$$\alpha = \frac{(P_{max}/P_y)\ \text{theory}}{(P_{max}/P_y)\ \text{experiment}} \tag{13}$$

where P_{max} is the maximum strength, and P_y the yield load; which illustrates the deviation between theory and experiment.

It may be seen that, except for two or three cases, where the reliability of the test result is somewhat questionable, the value of α lies primarily between 0.95 and 1.05. This indicates a difference between test and theory of $\pm$ 5 percent, which is in agreement with previous findings [58]. The weighted average of α, based on the number of data in each of the Tables 4 through 6, is equal to 0.97. Similarly, denoting the absolute value of the difference between the theoretical and the experimental maximum strength by β,

$$\beta = \left| (P_{max}/P_y)_{theory} - (P_{max}/P_y)_{experiment} \right| \qquad (14)$$

the weighted average of this quantity is equal to $\pm$ 0.05. This confirms the previous statement, namely, that the predicted strength usually compares with the experimental one to an accuracy of $\pm$ 5 percent.

The data available do not provide sufficient information to allow a separation of the effects of the various assumptions made. There are indications, however, that, for example, for short columns the theory seems to give loads slightly lower than do the tests. The differences are not more pronounced for columns of the higher strength steels, than they are for columns of mild steel; nor does the method of manufacture seem to have any bearing on the results. These statements are true for all column lengths.

The reason that some tests results do not give the impression of being fully reliable, may be attributed to the influence of factors such as errors in alignment, small amounts of end restraint, cold-straightening of the column, and factors of this kind. For example, only a minor form of end restraint will lead to a significantly higher column strength than what would have been obtained for a real pinned-end column.

Figure 7 shows the band of all the 112 maximum strength column curves that have been developed, using an initial out-of-straightness of L/1000. The diagram is of the usual non-dimensional form for column curves, with P_{max}/P_y - values plotted as the ordinate, and the non-dimensionalized slenderness ratio λ as the abscissa. λ is given by the expression

$$\lambda = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}}\,\frac{L}{r} \qquad (15)$$

where r is the radius of gyration for the bending axis in question, and all the other quantities have been defined previously. The non-dimensional representation is used to allow a direct comparison of the performance of the different shapes and steel grades.

Only the upper and lower envelope curves for the band are indicated in Fig. 7, since the number and the density of the curves between these two limits prevent a meaningful illustration of each separate curve. The width of the band is largest for the intermediate slenderness ratios, and tapers off towards the ends. For low slenderness ratios the variation of the maximum strength is influenced more by the variation of the yield stress than any other factor; that is, the differences in yield stress obtained in tension tests from the same material account for the variation of the maximum strength. An analysis of the frequency distribution of the maximum strength for various slenderness ratios shows that the maximum strength exhibits less kurtosis (= peakedness of frequency distribution) for $\lambda = 0.3$ than do the tension test results[25,66], but the general tendency prevails. The frequency distribution exhibits increasing kurtosis and skewness (= departure from a symmetic distribution) with an increasing slenderness ratio. Figures 8a through 8c show the frequency distribution histograms for the maximum strength for three typical λ-values.

The increasing kurtosis and skewness are indications to the effect that for high slenderness ratios, factors such as the residual stress and the yield stress have a decreasing influence. In fact, the maximum strength of a very long column will approach the Euler, or elastic, buckling load. This is also due to the diminishing effect of the initial out-of-straightness on such columns. It is substantiated by the curves in Fig. 7, which approach the Euler curve as λ increases. Other investigations have given similar results[24].

The band of column curves shown in Fig. 7 was analyzed statistically throughout the range of slenderness ratios, and Fig. 9 illustrates the results of the statistical computations. The most important information in Fig. 9 is given by the arithmetic mean curve, which shows the gradual shifting of the mean; from being closer to the lower envelope curve at low λ-values, to being located in the vicinity of the upper envelope curve at high λ-values.

Included in the figure are also the curves for the 2 1/2- and the 97 1/2-percentiles, between which 95 percent of the 112 column curves are located. These percentiles have been used instead of the more commonly known quantities

$$M - 2 \cdot s \quad \text{and} \quad M + 2 \cdot s$$

where M is the arithmetic mean and s the standard deviation. The two expressions above give similar bounds, but are based on a normal (Gaussian) frequency distribution. The distribution of the maximum strength does not possess the Gaussian characteristics, as evidenced by the histograms in Fig. 8, and analyses of the coefficients of skewness and kurtosis show that they are never equal to 0.0 and 3.0, respectively. These are the magnitudes associated with a normal distribution.

For comparative purposes, column curve bands, similar to the one shown in Fig. 7, were developed by using initial out-of-straightness L/500 and L/2000. The three bands are compared in Fig. 10, and it may be seen that there is a significantly larger distance between the curves pertaining to the out-of-straightness L/500 and L/1000 than there is between the curves corresponding to the values L/1000 and L/2000. The column curves for different e/L-values, where e is the initial crookedness, for each of the columns included in the study, exhibit the same tendencies. Only a comparatively small amount of strength therefore is gained by reducing the initial crookedness from L/1000 to L/2000.

A further comparison of the three column curve bands is given by Figs. 11 and 12. Figure 11 illustrates the frequency distribution curves of the maximum strength for the slenderness ratio $\lambda = 1.5$ (as an example), and Fig. 12 shows the relationship between the coefficient of variation and λ. It is found that the maximum strength clusters more and more around certain values as the slenderness ratio increases, and also that this tendency is more pronounced, the smaller the magnitude of the initial out-of-straightness[25].

The standard deviation and the coefficient of variation exhibit similar patterns of variation, namely, pronounced maxima for the intermediate slenderness ratios. The maximum standard deviation occurs in the range of $\lambda = 0.8$ to 0.9[25], whereas the coefficient of variation reaches its highest level for $\lambda = 1.0$ to 1.1. The reason for this difference is that the coefficient of variation, V, is a function of the arithmetic mean, thus:

$$V = \frac{s}{M} \cdot 100 \quad \text{(in percent)} \tag{16}$$

Figure 12 also shows that the dispersion of the maximum column strength is largest for columns with a large initial crookedness, for the lowest and highest slenderness ratios. The converse is true for the intermediate λ-values. These results emphasize the previous findings with regard to the column strength band widths.

3.3.4 Development of Deterministic Multiple Column Curves

A general description of the overall results of the investigation was given in the preceding section, but a detailed discussion of the influence of the various parameters and the separate and joint effects on the maximum strength was not presented. The aim of this section is to provide an analysis of the available data with the specific purpose of preparing the multiple column strength curves, and to present the resulting set of curves.

The results are analyzed with respect to all of the column strength parameters considered in the study, with the exception of the bending axis and the initial out-of-straightness. The bending axis has been excluded as a factor, due to the desire to keep the major and minor axis column curves within the same column strength category; that is, that they belong to one and the same of the multiple column curves. It will be seen that this has been achieved for almost all the column types included. The omission of the out-of-straightness is a result of the choice of a particular value as the basis for the multiple column curves, namely, the value $e/L = 1/1000$.

Figure 13 shows the column curve band for the 22 curves (11 shapes) for rolled wide-flange columns in steel grades ASTM A7 and A36. The full band occupies a somewhat wide range of P_{max}/P_y-values in the middle and lower portions of the 112-curve band (see Fig. 7). A detailed analysis[25] shows that heavy rolled wide-flange columns tend to be relatively weaker than their light counterparts, except when cold-straightening has been applied. This also has been found in other investigations[49]. The width of the band for the light rolled A7/A36-shapes, together with the other data, indicates that the light shapes may be located in a middle (of three) column curve category, whereas heavy shapes seem to belong to the lowest located group. Figure 14 similarly shows that light rolled A242-columns may be assigned either to the upper or to the middle category. The width and the location of the band of curves for the shapes of A514-steel, shown in Fig. 15, are a clear indication that columns of the highest strength steels belong to the upper category. There is no significant difference between the curves for heavy and light A514-columns. This shows, for example, that the residual stresses have a relatively small influence on the load-carrying capacity of columns of high-strength steel, and this is further evidenced by the location of the band of curves.

The analysis of the data for welded wide-flange columns is considerably more complicated than that for the rolled shapes, since several additional column strength parameters must be taken into account. The results of the analyses of the data for these columns are given in Figs. 16 through 26.

Figure 16 illustrates the band of 34 column curves, pertinent to welded wide-flange columns of steel grades A7 and A36. A relatively large variation of the strength prevails, and the lower envelope curve closely follows the corresponding curve for the band of 112 column curves (Fig. 7).

The effect of the size of the shape is analyzed in Figs. 17 and 18. Figure 17 compares the strengths of heavy and light columns made of flame-cut plates, and Fig. 18 gives the same information for columns of universal mill plates. It is evident that the size of the shape is not important, since the column curve bands for the heavy and the light shapes practically overlap each other completely. Only for the universal mill columns does a small difference occur, such that the heavy shapes have a slightly lower relative strength.

Figures 19 and 20 separate the column curves of Fig. 16 with respect to the method of manufacture. It will be noted that both heavy and light universal mill columns are clearly weaker than their flame-cut counterparts. This concurs with the results of other investigations[14,50,58]. It has been found[25] that the type of weld that is used at the flange-web junction has no influence on the strength of flame-cut shapes, except that groove-welded columns are very slightly weaker than fillet-welded columns for low slenderness ratios. The difference is more pronounced for universal mill columns, for which the groove welded sections are weaker for all slenderness ratios.

These analyses indicate that both heavy and light welded flame-cut columns of A7 and A36 steel may be assigned to the middle column strength category. Universal mill columns, on the other hand, may preferably be located in the lowest of the three categories. There is no reason to distinguish between sections welded with fillet-welds and sections welded with groove-welds at the flange-web junction.

The band of column strength curves for welded wide-flange shapes of steel grade A441 is shown in Fig. 21, and may be of limited usefulness, since only heavy shapes are represented in the band of 8 curves. However, the trend exhibited by the curves seems to indicate that columns of this type may be assigned to the middle column strength category. A study of the effect of different types flange-web welds gives results analogous to those for columns of A7 and A36 steel, although the effect is less pronounced for the universal mill shapes of A441 steel[25]. The type of flange-web weld therefore does not require any special considerations.

The 4 available column curves for welded wide-flange shapes of steel grade A572 (50), illustrated in Fig. 22, also yield only a limited amount of information. The characteristics of this steel are somewhat similar to those of A441, however, and the location of the column curve band is very much the same. It therefore may be deduced that welded wide-flange columns of A572 (50) steel naturally fall into the middle column strength category.

Figure 23 shows the column curve band for welded wide-flange columns of A514-steel. The band is located relatively close to the upper envelope curve for the 112-curve band (see Fig. 7), but the width is somewhat larger than that for the rolled and heat-treated A514-shapes (see Fig. 15), and the band runs somewhat lower. It is found that some welded A514-columns may be assigned to the upper column strength category, and others to the middle one. The larger variation of the strength as compared to the rolled shapes, is probably caused by factors such as the welding and the flame-cutting.

The results obtained for the available hybrid welded wide-flange shapes are presented in Fig. 24. Note that two column curve bands are shown, rather than the usual one, in order to distinguish between hybrid shapes with A514-flanges and with A441-flanges. Although a total of only 8 column curves are available, the differences between the two major types of columns are highly pronounced. Welded hybrid H-shapes with flame-cut A514-flanges therefore may be assigned to the upper column strength category, and those with flame-cut and universal mill A441-flanges to the middle category.

Figures 25 and 26 provide the analysis of the column strength curves for the welded box-shapes included in the study. The column curve bands for heavy and light box-shapes of A7 and A36 steel, shown in Fig. 25, illustrate that the heavy shapes are relatively stronger than the light ones for a large range of slenderness ratios. This result is in agreement with previous findings[64]. The general trend of the curves shows that welded box-shapes of A7 and A36 steel may be placed in the middle column strength category.

The data for welded A514 box-shapes are limited, but nevertheless do illustrate a tendency similar to that of the rolled and welded wide-flange sections of the same material. Figure 26 thus indicates that box columns of A514 steel may be assigned to the upper column strength category.

For each of the set of data on the various types of columns, a decision thus has been made as to which column strength category the column best fits into. The final grouping of the maximum strength column curves therefore can be performed, and Tables 7 through 9 contain the data for the column types that have been assigned to each of the three column strength categories.

It may be seen that category 1 (Table 7) contains columns of high-strength steels, in addition to hybrid wide-flange shapes with A514-flanges. Also included in this category are stress-relieved sections of all steel grades, regardless of manufacturing method, shape, size, and bending axis. The commonly used methods of stress-relief (annealing) remove practically all cooling and welding residual stresses(50,52,66), which thus account for the increased strength. It is believed that further investigations on the influence of cold-straightening will show that many columns that are straightened this way also may be assigned to category 1. This is substantiated by the results of previous studies(52), but may not hold true for the heaviest shapes.

Column strength category 2 (Table 8) contains the largest number and the greatest variety of shapes, steel grades, manufacturing methods, and so on. This is as expected, since category 2 occupies a band of column curves that is centrally located in the 112-curve band (Fig. 7). It may be noted that the minor (weak) axis bending cases for some of the columns of high-strength steels are located in this category, whereas the major (strong) axis bending cases belong to the category above. Otherwise, all of the other columns are represented by both bending axes.

At this stage, relatively few column types have been assigned to column strength category 3. Only two steel grades are represented, namely, ASTM A7 and A36; and heavy rolled, heavy welded, and light welded wide-flange universal mill shapes are included. The small number of curves, and their somewhat large scatter, is the reason for some of the adjustments that are made for the initially evaluated curve 3.

The column curve bands for the three categories, together with some statistically evaluated curves, are shown in Fig. 27 through 29. Numerical values of the most important statistical properties of the three bands are given in Tables 10 through 12. The most notable feature for all categories is the fact that the arithmetic mean and the median are practically equal for all slenderness ratios. The coefficients of variation for categories 1 and 2 are significantly smaller than those of the 112-curve band [25], whereas category 3 exhibits a comparable scatter. The distribution of column strength within each category is not normal, and the use of mean plus or minus two standard deviations to define the 95 percent confidence intervals therefore will not be appropriate.

The arithmetic mean curves for the three bands form the initial choice of the possible multiple column curves. These curves are also shown in Figs. 27 through 29, and Fig. 30 illustrates the three curves drawn in the same diagram. This is denoted Alternative 1 of the multiple column curves.

The locations of the curves of Alternative 1 with respect to each other are somewhat unreasonable; in particular, Curves 2 and 3 are placed too closely together. Based on a presumed lowering of Curve 3 by the introduction of more column curves into this category, which is an assumption partially substantiated by the statistics presented by Fig. 29, Alternative 2 of the multiple column curves has been developed. The resulting curves are shown in Fig. 31, and by a comparison with those of Alternative 1, it will be seen that Curve 3 has been lowered slightly for all slenderness ratios, and that Curve 2 is slightly higher for low λ-values, and slightly lower for high λ-values. The location of Curve 1 has not been altered. The imposed changes are not severe, however, such that the curves of Alternative 2 do not deviate much from the arithmetic mean curves of the three bands.

The mathematical expressions representing the curves of Alternative 2 are not developed here. Such relationships are given for the set of multiple column curves presented in Fig. 32, which provide a simplified solution, and from a practical standpoint, one that is easier to use. These curves, by originating at the point where $P_{max}/P_y = 1.0$ and $\lambda = 0.15$, do take into account the strength-raising effects of strain-hardening for short columns, where no overall column buckling occurs.

The simplified Curves 1 and 2 may, for a large range of slenderness ratios be represented by parabolas, and Curve 3 by a straight line. From certain λ-values on, all three curves have the shape of hyperbolas. In general, the relationships can all be expressed by the formula

$$(P_{max}/P_y) = a + b\cdot\lambda^{m_1} + c\cdot\lambda^{m_2} \qquad (17)$$

where a, b, and c are constant coefficients that may or may not be zero; and m_1 and m_2 are exponents, whose values determine the shape of the curve.

The calculations leading to the determination of the column curve equations are not shown here. The expressions arrived at are as follows:

Curve 1

$$(P_{max}/P_y)=0.99+0.122\cdot\lambda-0.38\cdot\lambda^2 \qquad \text{for } 0.15\leq \lambda\leq 1.2 \quad (18a)$$

$$(P_{max}/P_y)=0.05 + 0.778\cdot\lambda^{-2} \qquad \text{for } 1.2\leq\lambda \leq 1.8 \quad (18b)$$

$$(P_{max}/P_y)=0.013+0.895\cdot\lambda^{-2} \qquad \text{for } 1.8\leq\lambda \leq 2.8 \quad (18c)$$

$$(P_{max}/P_y)=\lambda^{-2}\text{(=Euler Curve)} \qquad \text{for } \lambda \geq 2.8 \qquad (18d)$$

Curve 2

$$(P_{max}/P_y)=1.035-0.204\cdot\lambda-0.232\cdot\lambda^2 \qquad \text{for } 0.15\leq\lambda\leq 1.0 \qquad (19a)$$

$$(P_{max}/P_y)=-0.111+0.62\cdot\lambda^{-1}+0.091\cdot\lambda^{-2} \qquad \text{for } 1.0\leq\lambda\leq 2.0 \qquad (19b)$$

$$(P_{max}/P_y)=0.012+0.838\cdot\lambda^{-2} \qquad \text{for } 2.0\leq\lambda\leq 3.6 \qquad (19c)$$

$$(P_{max}/P_y)=\lambda^{-2}\text{(=Euler Curve)} \qquad \text{for } \lambda\geq 3.6 \qquad (19d)$$

Curve 3

$$(P_{max}/P_y)=1.095-0.63\cdot\lambda \qquad \text{for } 0.15\leq\lambda\leq 0.8 \qquad (20a)$$

$$(P_{max}/P_y)=-0.128+0.695\cdot\lambda^{-1}-0.097\cdot\lambda^{-2} \qquad \text{for } 0.8\leq\lambda\leq 2.2 \qquad (20b)$$

$$(P_{max}/P_y)=0.009+0.767\cdot\lambda^{-2} \qquad \text{for } 2.2\leq\lambda\leq 5.0 \qquad (20c)$$

$$(P_{max}/P_y)=\lambda^{-2}\text{(=Euler Curve)} \qquad \text{for } \lambda\geq 5.0 \qquad (20d)$$

For all three curves:

$$(P_{max}/P_y)=1.0 \qquad \text{for } 0\leq\lambda\leq 0.15 \qquad (21)$$

Note that all three curves coincide with the Euler-curve for elastic buckling from different λ-values on (Eqs. (18d), (19d), and (20d)). This has been done to account for the fact that the out-of-straightness plays a negligible role for the strength of the very long columns[24].

Equations (18) through (21) may now be used to find the maximum strength of a column, given the non-dimensional slenderness ratio, λ, and the yield load, P_y. The equations also could have been written with (σ_{max}/σ_y) replacing (P_{max}/P_y), where σ_{max} is the maximum (critical) stress of the column. Conversion from non-dimensional to dimensional slenderness ratio, L/r, is easily accomplished.

A brief discussion of a comparison between the multiple column curves developed here, and the ones prepared in Europe[57], may be of interest. Figure 33 shows the proposed European multiple column curves and the data for the column types that belong to each of them. The original number of curves is three (curves a, b, and c in Fig. 33)[57], but recent developments indicate that two more (curves a° and d in Fig. 33) may be added[67]. These are shown as dashed lines in Fig. 33.

Figure 34 illustrates the comparison of the European curves and the simplified multiple column curves of

Fig. 32. The correlation between the two sets of curves is very good, provided all five European curves are taken into account. The original three of the latter curves do not cover more than approximately 50 percent of the band of all column strength curves (cf. Fig. 7), and hence do not provide a suitable alternative to the curves developed in this study. The reasons for the insufficient coverage are the types of structural steel and shape sizes commonly used in Europe. High-strength steels, such as ASTM A514, are not available, and neither are the types of heavy shapes that frequently are utilized in American construction practice[54]. The tentative curves, a° and d, have been developed for possible future use[67].

It should be noted that the European investigation has been based entirely on theoretical data for residual stresses, yield stresses, and so on. The theoretical results have been compared with the outcome of the extensive series of column tests performed in various European countries[46,47], based on the assumption that approximately 97 1/2 percent of all column tests should lie above the pertinent column strength curve. The comparison yields a fairly favorable result.

The three deterministically developed multiple column curves of Fig. 32 are meant to be used together with a column curve selection table. The table is used to decide which of the multiple column curves is applicable to a particular column type. Although the table is indispensable for the selection process because of the large number of steel grades, shapes, manufacturing methods, and so on, that are available to the designer, it will not be presented here. A detailed discussion of the table and how to use it is provided in Ref. 25.

3.4 Summary of Deterministic Solutions

A number of deterministic methods of implementing more rational and accurate solutions of the column strength problem have been presented in this part of the dissertation. It has been shown that the methods related to the manufacture of columns present numerous possibilities, but also that these methods may be difficult to utilize for practical solutions. The theoretically based approach, using the concept of multiple column curves, thereby appears to be the method most viable and practical.

The results of the investigation illustrate beyond doubt that by using the multiple column curve concept, that is, by utilizing several column strength curves, to each of which columns of related behavior and strength are assigned, the deviation between the actual and the design strength of the columns will be reduced significantly. Measured values of residual stresses, yield stresses, and cross-sectional dimensions imply a realistic basis for the theoretical computations, and by explicitly taking into account the initial crookedness that is found in all real columns, the solution appears to assume even further closeness to reality.

It has been found that the incremental, iterative computation of the column maximum strength gives results that correlate well with the available experimental data. The main problem associated with the deterministic multiple column curve approach therefore has been the gathering of a representative amount of data on all the various type of columns that may be used in practice. Lacking the specifics for some column types, the multiple column curves presented may be regarded as preliminary only; but, nevertheless, do provide essential information on the practical use of the concept.

The most important disadvantage of the deterministic solution lies in the fact that the column strength parameters have been treated as fixed values. It has long been known and accepted that not only does the column strength exhibit significant variation, but also that the variation in part is caused by the random variability of the column strength parameters. With some factors assuming extreme values, for example, the yield stress of a steel may be 32 ksi, as opposed to a specified magnitude of 36 ksi, the interaction of extreme values may cause the column strength to fall outside the band of variation. Such cases can only be accounted for in a study where the randomness is taken directly into account.

4. PROBABILISTIC ANALYSIS OF COLUMN STRENGTH

The deterministic modeling of any problem implies the use of the fundamental concept of a one-to-one correspondence between the dependent and the independent variables*, and any variation of the pertinent parameters is omitted from consideration. A probabilistic model takes the variability explicitly into account, and the resulting solution thereby becomes expressed as a number of values, of which some are more likely to occur than others. The concept of the probability of the occurrence of an event thus is naturally introduced, whereby the multi-valued solution of the problem is expressed as a function - either a probability density function or a distribution function[45,68,69]. Both of these are real-valued (single-valued) and continuous for the range of the variables concerned, and the density function is a direct representation of the derivative of the distribution function, assuming that this is such that continuous derivatives do exist.

*The terms "dependent" and "independent" are used here in a deterministic sense, to designate the variables of an equation, say, $z = x^2 + 3y - 5xy$, where z is the dependent, and x and y the independent variables. In probabilistic usage the words imply certain very important characteristics, which is explained below.

The assumption of continuity of the density and the distribution function is a simplification, since no real phenomena possess strictly continuous characteristics in the mathematical sense. It does, however, provide several advantages, mainly by allowing easier handling of the very often complicated mathematics that arise in problems of this nature, but may not be necessary where only a numerical (not closed form) solution is being sought.

The simplest possible problem is encountered when only one variable, for instance, y, is involved. The probability density function then may be denoted f(y), and the distribution function of y is given by F(y), and the two are related by the relationship

$$f(y) = \frac{dF(y)}{dy} = F'(y) \tag{22}$$

Most real phenomena involve several variables, however, and the density and distribution function may be expressed as

$$f(y_1 y_2 y_3 \cdots y_n) = \frac{\partial^n F(y_1 y_2 \cdots y_n)}{\partial y_1 \partial y_2 \cdots \partial y_n} \tag{23}$$

where y_1, y_2, ...y_n are the n variables. This equation holds true whether the n variables are statistically independent* or not[68]. The function $f(y_1 y_2 \ldots y_n)$ is commonly denoted the joint probability density function of the variables y_1 through y_n, and may be thought of as representing a surface in an n-dimensional space.

Any development or illustration of the mathematical concepts of the theory of probability, other than what is essential to the understanding of the problem being studied here, is assumed known, and consequently is not presented here. Detailed developments of the mathematical methods of probability theory and of mathematical statistics may be found in Refs. 45, 68, 69, 70, and 71.

*Statistical independence implies that the multi-variable probability density function may be written as the product of the density functions of each of the variables, namely: $f(y_1 y_2 \ldots y_n) = f(y_1) f(y_2) \ldots f(y_n)$. The problem encountered is considerably simplified if the variables are independent.

4.1 The Variation of Column Strength and Its Probabilistic Treatment

The probabilistic treatment of the column strength problem represents a fairly novel development, although its relationship to the limit state methods of design and to the reliability analyses of strength and loads was conceived some time ago. The major obstacle of a successful solution of the problem was inherent in the lack of closed-form deterministic relationships of sufficiently general nature, which are essential to the implementation of probabilistic concepts in most structural engineering problems[34]. Approaches of this kind are termed quasi-steady statistical methods, and evolve from deterministic relationships between the random variables[34].

Possibly the first to realize the probabilistic nature of the column strength problem, Dutheil[17] suggested that the results of column tests be evaluated statistically, and the outcome compared with an appropriate theoretical analysis. This implied the solution of the inelastic column problem, but the absence of high-speed computers prevented any advancement for several years. The strength of columns in the elastic range was investigated, however, particularly with regard to the influence of random initial deflections[72,73,74,75], but this did not contribute significantly to the understanding of the much more complicated problem of the inelastic column. One of the first attempts in this direction was provided by Chung[76,77], who presented a tangent modulus based solution, where the modulus of elasticity, the tangent modulus of elasticity, and the yield stress were treated as random variables. This involved, among other things, the evaluation of very complex multiple integrals, and Ravindra and Galambos[78] therefore suggested a simple first-order probabilistic solution procedure. Both of these approaches seemed to give satisfactory results.

The tangent modulus approach also was utilized by Rokach[79], who developed a first-order probabilistic method, and compared his results with the data provided by the experimental investigation in Europe[46,47]. A procedure of somewhat similar character was presented by Carpena[80], wherein the arithmetic mean and the variance of the column strength were developed on the basis of the partial derivatives[45] of the probability density function of the strength. Augusti and Baratta[81] made an analysis based on the column strength approach suggested by Dutheil[17,18], using as random variables the initial out-of-straightness, the yield stress, and the slenderness ratio, but neglecting the residual stresses in the column.

Common to all of the above mentioned solution procedures is their treatment of the random column strength parameters as independent, which is an assumption that is not correct in general, but will not have very significant effects. Most of the approaches have utilized a simplified tangent modulus method (Duberg-Wilder idealized column model[11]), thereby analyzing the strength of an initially perfectly straight column. It has previously (see Chapter 2) been shown that the tangent modulus load is a good approximation to the strength of a column, provided the out-of-straightness is small, but that it may give results that differ substantially from the strength of other columns. The only method that has taken the initial crookedness into account ignores the residual stresses, and therefore cannot be considered satisfactory, either.

An improvement over the above mentioned studies can be arrived at, by conducting an analysis based on the maximum strength concept. The necessary deterministic relationships for this approach are available in the form of incremental, iterative equations (see Section 3.3.2), whereby the expressions relating the random variables may be established. All of the important column strength parameters therefore will be explicitly accounted for, including the initial out-of-straightness. This has been the basis for the investigation that is presented in the following sections, and it is believed that the solution procedure and the results illustrate the generality and advantages of such an approach.

4.2 The Method of Analysis of Column Strength

The subsequent sections of the dissertation outline in detail the column strength parameters that are to be considered, their assumed or measured statistical properties, and the derived statistical properties of factors such as the cross-sectional area and the yield load. These data are utilized in an extensive presentation of the method of evaluation of the maximum column strength. The final section of this chapter is used to analyze the variation of the maximum strength, and the factors that contribute significantly to this variation.

4.2.1 The Variation of the Column Strength Parameters

The probabilistic treatment of the maximum strength of a column is essentially a study of a structure which exhibits a random non-linear behavior. It therefore is necessary to establish the mathematical laws that reflect the random nature of the pertinent factors, prior to the formulation of the equations that govern the maximum strength.

The probabilistic nature of the column strength factors will be expressed in terms of probability density functions or distribution functions and their characteristic quantities, such as the mathematical expectation (arithmetic mean of a sample) and the variance. The form of the functions will be assumed, and as far as possible and feasible, substantiated by the results from measurements. It may be mentioned, however, that very few investigations have been conducted that have dealt systematically with the statistical properties of the parameters relevant to this study; and of those that actually are available, only some present data that may be regarded as statistically significant.

With the mathematical relationships thus being established, the primary problem involves the estimation of the functional parameters. Several methods may be used for this purpose, of which the method of moments and the method of maximum likelihood seem to possess the most valuable characteristics[44,45]. Whereas the method of moments has many advantages, the method of maximum likelihood is generally considered to be unsurpassed as a statistical approach to the majority of the measurement problems that are encountered in the physical sciences. This method utilized all of the experimental information in the most direct and efficient fashion possible, to yield an unambiguous estimate of the parameter sought for. In addition to prescribing the statistic which should be used, it will provide an approximation of the distribution of the statistic, such that approximate confidence intervals can be established. The method is therefore essentially based on the assumption that the sample of numerical values is representative of the population of all possible values. The main disadvantage of the method of maximum likelihood lies in the fact that the functional relationship must be known or assumed.

The method of maximum likelihood is based on the use of the sample likelihood function $L(\phi)$, where ϕ is a known parameter of the distribution of the random variable y. The joint probability density function of a random sample of values $y_1, y_2, \ldots, y_n$ may then be expressed as

$$\begin{aligned} f(y_1, y_2, \ldots y_n | \phi) &= f(y_1) f(y_2) f(y_3) \ldots f(y_n) \\ &= \prod_{i=1}^{n} f(y_i | \phi) \end{aligned} \tag{24}$$

where the form $f(\cdot|\phi)$ indicates that ϕ is known[45].

Usually, however, specific values of y are known, and ϕ is the only unknown factor. The expression given by

Eq. (24) therefore may be regarded as a function of ϕ only, which gives the relative likelihood of having observed the particular set of values y_1, y_2, ..., y_n as a function of ϕ. The likelihood function of the sample is thereby

$$L(\phi|y_1,y_2,\ldots y_n) = \prod_{i=1}^{n} f(y_i|\phi) \tag{25}$$

The maximum-likelihood estimator $\hat{\phi}$ is the value of ϕ which makes the likelihood function attain a maximum value. Thus, the solution of

$$\left(\frac{dL(\phi)}{d\phi}\right)_{\phi=\hat{\phi}} = 0 \tag{26}$$

yields the value of $\hat{\phi}$. The equation will be easier to handle if a logarithmic transformation of $L(\phi)$ is used[44].

Most problems involve several parameters ϕ, which may be expressed as a vector $\{\phi\}$. The likelihood function is then

$$L(\{\phi\}|y_1,y_2,\ldots,y_n) = \prod_{i=1}^{n} f(y_i|\{\phi\}) \tag{27}$$

The vector of estimators that maximize the likelihood function is

$$\{\hat{\phi}\} = \{\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_r\} \quad (28)$$

which indicates that r estimators are involved. A logarithmic transformation of Eq. (27) gives

$$\log[L(\{\phi\}|y_1, y_2, \ldots, y_n)] = \log[\prod_{i=1}^{n} f(y_i|\{\phi\})]$$

or

$$\log[L(\{\phi\}|y_1, y_2, \ldots, y_n)] = \sum_{i=1}^{n} \log[f(y_i|\{\phi\})] \quad (29)$$

Equation (29) requires the solution of a set of r simultaneous equations, since the likelihood function is maximized with respect to each $\hat{\phi}_i$. Hence:

$$\sum_{i=1}^{n} \frac{\partial}{\partial \phi_j}\{\log[f(y_i|\{\phi\})] = 0 \quad \text{for } j = 1, 2, \ldots, r \quad (30)$$

As an example, the mean value M and the standard deviation s of a normally distributed variate y are to be estimated from a sample of size r, and the estimators of M and s will be designated by $\hat{M}$ and $\hat{s}$, respectively. The probability density function of y is

$$f(y) = \frac{1}{s\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y-M}{s}\right)^2\right] \tag{31}$$

and the likelihood function becomes

$$L(\{M,s\}) = \prod_{i=1}^{r} \frac{1}{s\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y-M}{s}\right)^2\right] \tag{32}$$

Differentiating $L(\{M,s\})$ with respect to $M = \hat{M}$ gives:

$$\left(\frac{\partial \log L}{\partial M}\right)_{M=\hat{M}} = \sum_{i=1}^{r} \frac{y_i - \hat{M}}{s^2} = 0$$

and therefore

$$r \cdot \hat{M} - \sum_{i=1}^{r} y_i = 0$$

The maximum-likelihood estimator of the mean is therefore

$$\hat{M} = \frac{1}{r} \sum_{i=1}^{r} y_i \tag{33}$$

A similar evaluation with respect to the standard deviation leads to:

$$\left(\frac{\partial \log L}{\partial s}\right)_{s=\hat{s}} = \sum_{i=1}^{r} \left(-\frac{1}{\hat{s}} + \frac{(y_i - \hat{M})^2}{\hat{s}^3}\right) = 0$$

and thus:

$$\hat{s} = \sqrt{\frac{1}{r} \sum_{i=1}^{r} (y_i - \hat{M})^2} \tag{34}$$

Equations (33) and (34) express the common definitions of the mean and the standard deviation of a sample.

The above discussion of the method of maximum likelihood provides a necessary background for the development of the characteristics of the column strength parameters. The method has been used in the computation of the magnitudes of the relevant column strength statistics, for which the set of mathematical formulas needed will be evaluated on the following pages. The column strength parameters to be considered are:

1. Cross-sectional properties of wide-flange and box shapes.
2. Mechanical properties of the steel.
3. Residual stress variation in a particular column category.
4. Out-of-straightness of the column.

Included among the cross-sectional properties are thus the size and cross section of the shape, and whether the section is rolled or welded. The consideration of the residual stresses account for the other parameters related to the manufacture of the columns, such that the list of four items covers the most important column strength factors. It should be noted that the random variation of the residual stress distribution in shapes as a whole is not to be treated; only the $\pm$-variations of the residual stresses measured in a shape of a particular manufacturing method.

1. Statistical Characteristics of the Cross-Sectional Properties

Figure 35 illustrates a wide-flange and a box-shape, and the designation of the various cross-sectional dimensions pertinent to each type. Rolled and welded sections are treated separately, due to reasons that are outlined further below.

(a) Rolled wide-flange shape dimensions

The dimensions b_h, h_h, t_h, and w_h are given by the steel manufacturer, and must satisfy certain tolerances. The American specifications[63] set no requirements for the flange thickness, t_h, and the web thickness, w_h; but specify the tolerances foı the width b_h and the height h_h. Other specifications[82,83] provide very detailed requirements for all cross-sectional measurements, and common to most of these is that the tolerances are given in the form of equal $\pm$-values.

The two flanges are assumed to be of equal thickness, although they may exhibit different t_h, but this assumption presents negligible implications. The shape as a whole is assumed doubly symmetric, such that the center of the web is located at the center of the two, equally wide, flanges. The entire shape thereby may be enclosed in a rectangle of size $b_h \times h_h$. These assumptions neglect the possibility of having a shape with sloping flanges (I), with flanges being displaced relative to each other (I), and with an off-center web (I). It is believed, however, that the assumptions are reasonable and that the factors thereby neglected assume relatively insignificant values.

Each of the quantities b_h, h_h, t_h, and w_h are assumed statistically independent and normally distributed. The form of the tolerance requirements, together with the results from some investigations(53,83), substantiate the use of normally distributed variates. The probability density functions for the four quantities therefore may be given by:

(1) Total height, h_h:

$$f(h_h) = \frac{1}{\sigma_h^h\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2} \cdot \left(\frac{h_h - H_h}{\sigma_h^h}\right)^2\right] \tag{35}$$

where H_h and σ_h^h denote the arithmetic mean and the standard deviation of h_h, respectively. In the following, capital letters are used to designate the means of the variates considered. Similarly:

(2) Flange width, b_h:

$$f(b_h) = \frac{1}{\sigma_b^h\sqrt{2\pi}} \cdot \exp[-\frac{1}{2}(\frac{b_h - B_h}{\sigma_b^h})^2] \quad (36)$$

(3) Flange thickness, t_h:

$$f(t_h) = \frac{1}{\sigma_t^h\sqrt{2\pi}} \cdot \exp[-\frac{1}{2}(\frac{t_h - T_h}{\sigma_t^h})^2] \quad (37)$$

(4) Web thickness, w_h:

$$f(w_h) = \frac{1}{\sigma_w^h\sqrt{2\pi}} \cdot \exp[-\frac{1}{2}(\frac{w_h - W_h}{\sigma_w^h})^2] \quad (38)$$

where B_h, T_h, and W_h are the means, and σ_b^h, σ_t^h, and σ_w^h the standard deviations of the flange width, flange thickness, and web thickness, respectively. The magnitude of the tolerance is set equal to 2 standard deviations(44), such that, for example, a flange thickness less than $(T_h - 2 \cdot \sigma_t^h)$ will occur with a probability of approximately 0.025 (2.5 percent). For consistent assessment of the numerical values of the tolerances, and because the ASTM Specifications(63) do not contain data for the thicknesses, the tolerance requirements given by Ref. 82 will be used. Table 13 gives a summary of these values.

With the total height of the shape being specified, the net depth of the web, d_h, becomes a derived quantity given by

$$d_h = h_h - 2t_h \tag{39}$$

d_h will be distributed normally, since it is a linear function of two independent random variables[45,68]. The mean, D_h, and the standard deviation, σ_d^h, therefore become:

Mean of d_h: $$D_h = H_h - 2T_h \tag{40}$$

Standard deviation: $$\sigma_d^h = \sqrt{(\sigma_h^h)^2 + 4(\sigma_t^h)^2}\,^* \tag{41}$$

and the probability density function of d_h is given by

$$f(d_h) = \frac{1}{\sigma_d^h\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2}\left(\frac{d_h - D_h}{\sigma_d^h}\right)^2\right] \tag{42}$$

(b) Welded wide-flange shape dimensions

Being manufactured from plates, the dimensions d_h, b_h, t_h, and w_h are supplied by the manufacturer, and it is

*The standard deviation of the sum of two identically distributed independent random variables x and y is $\sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2}$ (44). The standard deviation of the sum of two absolutely correlated variables, such as x+x, is $\sigma_{x+x} = 2\sigma_x$.

assumed that the tolerances applied for the dimensions of rolled wide-flange shapes (Table 13) also apply for welded wide-flange shapes. The assumptions stated above for the rolled shape are used for the welded section also, and the probability density functions given by Eqs. (36)-(38) and (42) therefore are directly applicable. It should be noted, however, that the net web depth (d_h) is given directly for the welded shape, and will no longer be a derived quantity (see Eq. (39)). Consequently, equations (40) and (41) do not apply in this case. It is assumed further that the flanges and the web are completely joined, with no gap appearing between the two component plates.

(c) Welded box-shape dimensions

The dimensions d_b, b_b, t_b, and w_b exhibit characteristics similar to those of the corresponding dimensions of welded wide-flange shapes, and the same assumptions therefore are made. It is further assumed that the cross section represents a perfectly rectangular shape, with the implication that the two "flange" thicknesses vary simultaneously and equally, and so also the "web" thicknesses, the "flange" widths and the "webs" depths. The probability density functions for the cross-sectional dimensions of the box shape are found by substituting the subscript h, used for the random variables and their means in Eqs. (36), (38), and (42), by the subscript b. Equations (39)-(41) are not applicable for the depth d_b, since it is an independent quantity.

(d) Cross-sectional areas of wide-flange and box shapes

The deterministic expressions for the cross-sectional areas of wide-flange and box shapes are:

$$\text{Wide-flange shape:} \quad A_h = 2\cdot b_h\cdot t_h+d_h\cdot w_h$$
$$\text{Box shape:} \quad A_b = 2(b_b\cdot t_b+d_b\cdot w_b) \tag{43}$$

where each of the factors involved is independent of the others. The areas will be normally distributed, because each of the contributing parameters are. Hence:

(1) For wide-flange shapes:
The formula for the area is written as

$$A_h = 2\cdot b_h\cdot t_h+d_h\cdot w_h = 2\cdot u_h+z_h \tag{44}$$

where u_h and z_h become statistically independent random variables. Their means and standard deviations are[44]:

Means: $$U_h = B_h \cdot T_h$$
$$Z_h = D_h \cdot W_h \tag{45}$$

Standard deviations:

$$\sigma_u^h = \sqrt{B_h^2(\sigma_t^h)^2 + T_h^2(\sigma_b^h)^2 + (\sigma_b^h\sigma_t^h)^2}$$
$$\sigma_z^h = \sqrt{D_h^2(\sigma_w^h)^2 + W_h^2(\sigma_d^h)^2 + (\sigma_d^h\sigma_w^h)^2} \tag{46}$$

The mean and the standard deviation of the area A_h are therefore

$$\overline{A}_h = 2 \cdot U_h + Z_h = 2 \cdot B_h \cdot T_h + D_h \cdot W_h \tag{47}$$

and

$$\sigma_A^h = \sqrt{4(\sigma_u^h)^2 + (\sigma_z^h)^2} \tag{48}$$

The probability density function for the area assumes the the form of the expression given by Eq. (35). It should be noted that when a rolled wide-flange shape is considered, the factor D_h in Eqs. (45)-(47), and the factor σ_d^h in Eq. (46), are given by Eqs. (40) and (41). For a welded wide-flange shape the statistical characteristics of each of the dimensions are directly applicable.

(2) For box shapes:

The formula for the area is written as

$$A_b = 2(b_b \cdot t_b + d_b \cdot w_b) = 2(u_b + z_b) \tag{49}$$

with the characteristics of u_b and z_b given by Eqs. (45) and (46), replacing subscripts and superscripts h with b; thus:

$$\text{Means:} \qquad \begin{aligned} U_b &= B_b \cdot T_b \\ Z_b &= D_b \cdot W_b \end{aligned} \tag{50}$$

Standard deviations:

$$\begin{aligned} \sigma_u^b &= \sqrt{B_b^2(\sigma_t^b)^2 + T_b^2(\sigma_b^b)^2 + (\sigma_b^b \sigma_t^b)^2} \\ \sigma_z^b &= \sqrt{D_b^2(\sigma_w^b)^2 + W_b^2(\sigma_d^b)^2 + (\sigma_d^b \sigma_w^b)^2} \end{aligned} \tag{51}$$

The properties of the cross-sectional area of a box shape are therefore:

$$\text{Mean:} \qquad \bar{A}_b = 2(U_b + Z_b) = 2(B_b T_b + D_b W_b) \tag{52}$$

Standard deviation:

$$\sigma_A^b = 2\cdot\sqrt{(\sigma_u^b)^2 + (\sigma_z^b)^2} \tag{53}$$

(3) Moments of inertia of wide-flange and box shapes

The usual formulas for the moments of inertia of the two principal types of sections dealt with are as follows:

(1) Wide-flange shape:

About the major (x-) axis: $$I_x^h = \frac{w_h d_h^3}{12} + \frac{b_h t_h (d_h + t_h)^2}{2} \tag{54a}$$

About the minor (y-) axis: $$I_y^h = \frac{t_h b_h^3}{6} \tag{54b}$$

The contribution of the web has been neglected in the expression for I_y^h. It should be noted that the two terms on the right-hand side of Eq. (54a) are not statistically independent, since the factor d_h occurs in both.

(2) Box shape:

Assuming a general, rectangular shape of the cross section, the principal moments of inertia are:

About the major (x-) axis: $$I_x^b = \frac{w_b d_b^{\,3}}{6} + \frac{b_b t_b (d_b - t_b)^2}{2} \tag{55a}$$

About the minor (y-) axis: $$I_y^b = \frac{t_b b_b^{\,3}}{6} + \frac{d_b w_b (b_b + w_b)^2}{2} \tag{55b}$$

The two right-hand side terms in both of these equations are not statistically independent.

Based on these deterministic relationships, the probability density functions for the various moments of inertia now may be developed, since the probability density functions of all the cross-sectional dimensions are known. However, since in the computer program utilized, the moments of inertia are computed on an incremental basis, with the specific purpose of checking and redistributing the residual stresses to achieve equilibrium; it is not necessary to find the probability density functions from Eqs. (54) and (55). Instead, the following approach is used:

(a) Number of firite area elements in each flange (in a box shape this refers to the component plates with thickness t_b): n_f

(b) Number of finite area elements in each web (in a box shape this refers to the component plates with thickness w_b): n_w

(c) Width of flange: b_h (wide-flange) or b_b (box)

The widths of the finite area elements are assumed equal, thus:

$$dt_f^h = \frac{b_h}{n_f} \quad \text{(wide-flange)} \tag{56a}$$

$$dt_f^b = \frac{b_b}{n_f} \quad \text{(box)} \tag{56b}$$

With b_h (b_b) normally distributed, dt_f^h and dt_f^b also become distributed as such, namely:

Means: $$DT_f^h = \frac{B_h}{n_f} \quad \text{and} \quad DT_f^b = \frac{B_b}{n_f} \tag{57}$$

Standard deviations:

$$\sigma_{dt}^h = \frac{\sigma_b^h}{n_f} \quad \text{and} \quad \sigma_{dt}^b = \frac{\sigma_b^b}{n_f} \tag{58}$$

In all of these equations the superscripts h and b refer to wide-flange and box shapes, respectively.

The area of an element of the flange therefore becomes:

$$dA_f^h = dt_f^h \cdot t_h = \frac{1}{n_f} \cdot b_h \cdot t_h \quad \text{(wide-flange)} \tag{59a}$$

and

$$dA_f^b = dt_f^b \cdot t_b = \frac{1}{n_f} \cdot b_b \cdot t_b \quad \text{(box)} \tag{59b}$$

Since b_h (b_b) and t_h (t_b) both are normally distributed, the elemental area $dA_f^h(dA_f^b)$ also follows this probability law. Consequently:

Mean elemental areas:

$$DA_f^h = DT_f^h \cdot T_h = \frac{1}{n_f} \cdot B_h \cdot T_h \tag{60a}$$

and
$$DA_f^b = DT_f^b \cdot T_b = \frac{1}{n_f} \cdot B_b \cdot T_b \tag{60b}$$

where Eq. (60a) is valid for the wide-flange, and Eq. (60b) for the box shape. The standard deviation of the elemental area distribution is

$$\sigma_{dA_f}^h = \frac{1}{n_f} \sqrt{B_h^2 (\sigma_t^h)^2 + T_h^2 (\sigma_b^h)^2 + (\sigma_b^h \sigma_t^h)^2} \tag{61}$$

which concerns a wide-flange shape. A similar expression can be found for the box shape, by replacing the superscripts and subscripts h with b.

Equations similar to Eqs. (56)-(61) can be developed for the web, as follows:

Depth of web: d_h (wide-flange) or d_b (box)

The width of the finite area elements in the web is constant, thus:

$$dw_w^h = \frac{d_h}{n_w} \quad \text{(wide-flange)} \tag{62a}$$

$$dw_w^b = \frac{d_b}{n_w} \quad \text{(box)} \tag{62b}$$

The distribution parameters of dw_w^h (dw_w^b) are

$$\text{Means:} \qquad DW_w^h = \frac{D_h}{n_w} \quad \text{and} \quad DW_w^b = \frac{D_b}{n_w} \tag{63}$$

Standard deviations:

$$\sigma_{dw}^h = \frac{1}{n_w}\sigma_d^h \quad \text{and} \quad \sigma_{dw}^b = \frac{1}{n_w}\sigma_d^b \tag{64}$$

where, for a rolled wide-flange shape, $D_h = H_h - 2T_h$ and $\sigma_d^h = \sqrt{(\sigma_h^h)^2 + 4(\sigma_t^h)^2}$ (cf. Eq. (40) and (41)). The area of the element therefore becomes

$$dA_w^h = dw_w^h \cdot w_h = \frac{1}{n_w} \cdot d_h \cdot w_h \quad \text{(wide-flange)} \tag{65a}$$

and

$$dA_w^b = dw_w^b \cdot 2w_b = \frac{2}{n_w} \cdot d_b \cdot w_b \quad \text{(box)} \tag{65b}$$

where it will be noticed that $2w_b$ has been used in Eq. (65b), to account for the two webs of the box shape.

The distribution parameters of dA_w^h and dA_w^b are

Means: $$DA_w^h = DW_w^h \cdot W_h = \frac{1}{n_w} \cdot D_h \cdot W_h \tag{66a}$$

and $$DA_w^b = DW_w^b \cdot 2W_b = \frac{2}{n_w} \cdot D_b \cdot W_b \tag{66b}$$

where Eq. (66a) is for the wide-flange and Eq. (66b) for the box shapes. The standard deviation of the elemental web area distribution for a wide-flange shape is given by

$$\sigma_{dA_w}^h = \frac{1}{n_w} \sqrt{D_h^2 \cdot (\sigma_w^h)^2 + W_h^2 (\sigma_d^h)^2 + (\sigma_d^h \sigma_w^h)^2} \tag{67a}$$

and for the box shape

$$\sigma_{dA_w}^b = \frac{2}{n_w} \sqrt{D_b^2 (\sigma_w^b)^2 + W_b^2 (\sigma_d^b)^2 + (\sigma_d^b \sigma_w^b)^2} \tag{67b}$$

Having established the distribution characteristics of the elemental areas of the flanges and the webs, the general expression for any moment of inertia now may be written as

$$I = d_c^2 \cdot w \cdot b + \sum_{i=1}^{k} db_i \cdot x_i^2 \cdot t \qquad (68)$$

and the two terms on the right-hand side of Eq. (68) are statistically independent. The factors used in Eq. (68) designate the following:

k = n_f or n_w, depending on whether the minor or the major axis moment of inertia is being computed, respectively.

db_i = the width of element i, equal to dt_f^h (dt_f^b) if the minor axis is considered, and equal to dw_w^h (dw_w^b) if the major axis is considered.

x_i = the coordinate of the centroid of element i, measured in the coordinate system for the cross section (see Fig. 35). x_i corresponds to x_i if the minor axis is considered, and to y_i if the major axis moment of inertia is to be computed.

t = the thickness of element i; equal to $2t_h$ ($2t_b$) for minor axis bending, and equal to w_h ($2w_b$) for major axis bending.

d_c = the distance from the centerline of the flange to the centroid of the shape. $d_c = \frac{1}{2}(d_h+t_h)$ for major axis bending of a wide-flange shape; $d_c = 0$ for minor axis bending of a wide-flange shape (no contribution from web); $d_c = \frac{1}{2}(d_b-t_b)$ for major axis bending of a box shape; and $d_c = \frac{1}{2}(b_b+w_b)$ for minor axis bending of a box shape.

w = $2t_h$ ($2t_b$) for major axis bending of a wide-flange shape (box shape), and equal to $2w_b$ for minor axis bending of a box shape. w = 0 for minor axis moment of inertia computation for a wide-flange shape.

b = b_h (b_b) for major axis bending of a wide-flange shape (box shape); equal to d_b for minor axis bending of a box shape; it does not enter into the picture for minor axis bending of a wide-flange shape, since there is no contribution from the web in this case.

Each of the factors given in Eq. (68) are random variables, and the moment of inertia therefore also becomes a random variable. Its distribution characteristics can be evaluated, but are not essential to the understanding of the problem. For example, the mean moment of inertia is given by

$$\bar{I} = D_c^2 \bar{W}\ \bar{B} + \sum_{i=1}^{k} DB_i \cdot X_i^2 \cdot \bar{T} \qquad (69)$$

and the standard deviation may be found by using, for instance, a first-order form of the partial derivatives of $\bar{I}$[45]. This implies that if the standard deviation of a function $\Psi = f(y_1, y_2)$ is to be found, a good approximation is given by

$$\sigma_\Psi \approx \left[\left(\frac{\partial \Psi}{\partial y_1}\right)^2 \sigma_{y_1}^2 + \left(\frac{\partial \Psi}{\partial y_2}\right)^2 \sigma_{y_2}^2 \right]^{1/2} \tag{70a}$$

or, if the function is $\Psi = f(y_1, y_2, \ldots, y_j)$:

$$\sigma_\Psi \approx \left[\sum_{i=1}^{j} \left(\frac{\partial \Psi}{\partial y_i}\right)^2 \sigma_{y_i}^2 \right]^{1/2} \tag{70b}$$

2. Statistical Characteristics of the Mechanical Properties of Steel

The assumed linearly elastic-perfectly plastic stress-strain relationship for the steel basically involves two factors; namely, the yield stress, σ_y, and the modulus of elasticity, E. It has been found that the modulus of elasticity indeed does vary to a certain degree, but the fluctuations are so small as to be considered negligible[84, 85]. The coefficient of variation for E is as low as 0.1 to 0.5 percent, which is very small when compared to the variability of the other strength parameters. The modulus of elasticity therefore will be regarded as a constant.

The only mechanical property of concern which must be considered as a random variable therefore is the yield stress, σ_y. This causes the stress-strain relationship to assume a form as illustrated by Fig. 36, where the mean yield stress is represented by the solid line, and the upper and lower bounds by the dashed lines.

With the specification yield stress being a specified minimum value[63], it is reasonable to assume that the distribution of yield stresses will assume a positively skewed form. This assumption has been substantiated by several investigations[36,53,65,66,85], involving a large number of tests with tension specimens. It naturally follows that it is not appropriate to assume that the distribution of σ_y follows a normal probability law. By analyzing the data from approximately 60000 tension tests[53], it has been found that a truncated normal distribution will be a fair representation of the variation of σ_y, but a considerably better correlation is attainable if a Type I asymptotic extreme value distribution (Gumbel distribution) is used. It is believed that other types of extreme value distributions, possibly in particular the Type III (Weibull) distribution, may be applied with equally successful outcome.

The mathematical representation of the Type I distribution assumes a variate that may vary between $+\infty$ and $-\infty$, which in reality is not the case for σ_y. However, the parameters of the equation may be so formulated that the probability of obtaining σ_y-values outside specified bounds will be negligible[45]. The distribution function for σ_y, or for the largest of many independent random variables with a common exponential type of upper-tail distribution, may be expressed as:

$$F_{\sigma_y}(\sigma_y) = \exp[-e^{-\kappa(\sigma_y - p)}] \quad \text{for } -\infty \leq \sigma_y \leq \infty \tag{71}$$

and the probability density function of σ_y is derived from above as

$$f_{\sigma_y}(\sigma_y) = \kappa \cdot \exp[-\kappa(\sigma_y - p) - e^{-\kappa(\sigma_y - p)}] \tag{72}$$

The mode (=most frequently occurring value) of the distribution is given by the parameter p, and κ is a measure of the dispersion. The mean and the standard deviation of σ_y thereby becomes[45]

$$\text{Mean:} \quad \bar{\sigma}_y = p + \frac{\zeta}{\kappa} \approx p + \frac{0.577}{\kappa} \tag{73}$$

where $\zeta \approx 0.577$ is known as Euler's constant.

Standard deviation:

$$\sigma_{\sigma_y} = \frac{\pi}{\kappa \cdot \sqrt{6}} \approx \frac{1.282}{\kappa} \tag{74}$$

It is possible to arrive at an expression that is mathematically more tractable, by introducing a transformed yield stress, σ_{ya}. By setting

$$\sigma_{ya} = \kappa(\sigma_y - p) \tag{75}$$

a variable is obtained which is distributed according to a Type I probability law, with mode p = 0, and $\kappa = 1$. The distribution function of σ_{ya} becomes

$$F_{\sigma_{ya}}(\sigma_{ya}) = \exp[-e^{-\sigma_{ya}}] \tag{76}$$

and the probability density function is

$$f_{\sigma_{ya}}(\sigma_{ya}) = \exp[-\sigma_{ya} - e^{-\sigma_{ya}}] \tag{77}$$

Tables for the Type I distribution are given in terms of the transformed variate[45]. A transformation from σ_{ya}- to σ_y-values is easily accomplished, since it is clear that

$$\begin{aligned} & f_{\sigma_y}(\sigma_y) = \kappa \cdot f_{\sigma_{ya}}[(\sigma_y - p)\kappa] \\ \text{and} \quad & F_{\sigma_y}(\sigma_y) = F_{\sigma_{ya}}[(\sigma_y - p)\kappa] \end{aligned} \tag{78}$$

Assuming that a consistent probability level of 2.5 percent is used for all of the column strength parameters, this corresponds for the yield stress to a value of σ_{ya} of approximately -1.3[45] (see Fig. 37, which illustrates the probability density functions of σ_y and σ_{ya}. The value of σ_y which corresponds to this value of σ_{ya} will be assumed to be the specified yield stress, σ_{ym}; for example, σ_{ym} = 36 ksi for ASTM A36 steel, for a specified range of material thicknesses[63]. Consequently, with $\sigma_{ya} = -1.3$, σ_{ym} is given by

$$\sigma_{ym} = p + \frac{\sigma_{ya}}{\kappa} = p - \frac{1.3}{\kappa} \tag{79}$$

The mode corresponds to $\sigma_{ya} = 0$, and the most frequently occurring value of σ_y thus becomes equal to p. The upper limit of σ_y is set equal to the value beyond which magnitudes of the yield stress appear with a probability of 2.5 percent or less. The corresponding value of σ_{ya} is approximately 3.7[45], such that the maximum value of σ_y is

$$\sigma_{y,max} = p + \frac{3.7}{\kappa} \tag{80}$$

3. The Yield Load of a Cross Section

The yield load of a cross section is given by the deterministic expression

$$P_y = \sigma_y \cdot A \tag{81}$$

where both σ_y and A are random variables, thus making P_y a random variable. The cross-sectional area is a function of the dimensions of the shape, and the necessary probabilistic relationships for all of these quantities are given by Eqs. (35) through (69). The yield stress is a function of the thickness of the material, whereby σ_y decreases with increasing thickness[48,49,50,53,59]. A deterministic functional relationship between the thickness and the yield stress is assumed, based on the data from some experimental investigations[53]. Figure 38 illustrates the principles involved in this development. The equation is given as

$$\sigma_y(t)_{min} = \sigma_{ym} \cdot (1-\eta \cdot \frac{t}{t_{min}}) \tag{82}$$

where t represents the material thickness; t_{min} the thickness below which σ_{ym} is constant (cf. Fig. 38 for values of $t/t_{min} \leq 1$); and η is a reduction factor. The results from some investigations[53] have indicated that the factor η may be assigned an approximate value of 0.04. Equation (82) thereby becomes

$$\sigma_y(t)_{min} = \sigma_{ym} \cdot (1-0.04 \cdot \frac{t}{t_{min}}) \tag{83}$$

and for most practical purposes t_{min} may be set equal to about 3/4"; thus giving the expression

$$\sigma_y(t)_{min} = \sigma_{ym}(1-0.04\cdot t_a) \tag{84}$$

where t_a is a dimensionless thickness.

The relationship given by Eq. (84) is assumed to be valid, regardless of the magnitude of σ_{ym}. It also will be assumed that the Type I asymptotic extreme value distribution of the yield stress is maintained, and thus is independent of the material thickness; such that for any given thickness, σ_y follows the probability law given by Eqs. (71) and (72).

With the cross-sectional area being a random variable, whose characteristics are expressed by a normal distribution, the following is known:

(a) <u>For wide-flange shape</u>:

$$\left.\begin{array}{ll} \text{Mean area:} & \bar{A}_h = 2B_hT_h+D_hW_h \\ \text{Standard deviation:} & \sigma_A^h = \sqrt{4(\sigma_u^h)^2+(\sigma_z^h)^2} \end{array}\right\} \qquad (85) \text{ (cf. Eqs. (47)-(48))}$$

(b) <u>For box shape:</u>

$$\left.\begin{array}{ll}\text{Mean area:} & \bar{A}_b = 2\cdot(B_bT_b+D_bW_b) \\ \text{Standard deviation:} & \sigma_A^b = 2\cdot\sqrt{(\sigma_u^b)^2+(\sigma_z^b)^2}\end{array}\right\} \quad (86)$$

(cf. Eqs. (52)-(53))

The general form of the probability density function for the area is therefore

$$f_A(A) = \frac{1}{\sigma_A\sqrt{2\pi}}\cdot\exp[-\frac{1}{2}(\frac{A-\bar{A}}{\sigma_A})^2] \qquad (87)$$

where A may be A_h or A_b; $\bar{A}$ may be $\bar{A}_h$ or $\bar{A}_b$; and σ_A represents either σ_A^h or σ_A^b.

It has been shown above that the yield stress is a function of the thickness, and σ_y and A therefore are not strictly statistically uncorrelated (independent). The probability density function for the yield load, P_y, may be given as

$$f_{P_y}(P_y) = f(\sigma_y\cdot A) \qquad (88)$$

and this joint density function should be evaluated on the basis of the dependence between σ_y and A. However, for a specific shape, where a particular set of dimensions and their dispersion characteristics are given; σ_y may be treated as a factor statistically independent of the area. This may be realized by considering the deterministic relationship between σ_y and t (cf. Eqs. (82)-(84)). A good approximation of Eq. (88) is thereby provided by the expression

$$f_{P_y}(P_y) = f_{\sigma_y}(\sigma_y) \cdot f_A(A) \qquad (89)$$

such that the probability density function for P_y becomes

$$f_{P_y}(P_y) = \frac{1}{\sigma_A\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2}\left(\frac{A-\bar{A}}{\sigma_A}\right)^2 - \kappa(\sigma_y - p) - e^{-\kappa(\sigma_y - p)}\right] \qquad (90)$$

where Eqs. (72) and (87) have been substituted for the proper terms in Eq. (89). Introducing the transformed yield stress, σ_{ya}, the above becomes the density function for the transformed yield load, P_{ya}, thus:

$$f_{P_{ya}}(P_{ya}) = \frac{1}{\sigma_A\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2}\left(\frac{A-\bar{A}}{\sigma_A}\right)^2 - \sigma_{ya} - e^{-\sigma_{ya}}\right] \qquad (91)$$

The mean and the variance of the yield load may be found by the commonly used methods, since in general the mean (expectation), for instance, is given by[68]

$$E(\Psi) = \int_{\text{Range of } \Psi} \Psi \cdot f(\Psi) \cdot d\Psi \qquad (92)$$

where Ψ represents a function of one or more variables. For the particular case studied here, $f(\Psi)$ is given by Eq. (91), and Ψ is equal to $\sigma_{ya} \cdot A$. Equation (92) therefore becomes a double integral, the range of which is given by the ranges of σ_{ya} and A; and the two functions for these may be integrated separately (due to their independence). A similar expression may be given for the evaluation of the variance, and hence the standard deviation, of P_{ya} and P_y.

The evaluation of the necessary integrals will be quite complicated, but with the pertinent variables being independent, the mean and standard deviation may be found from:

Mean: $$\bar{P}_y = \bar{\sigma}_y \cdot \bar{A} \tag{93}$$

Substituting for $\bar{\sigma}_y$ from Eq. (73) and for $\bar{A}$ from Eqs. (47) and (52):

Mean yield load for wide-flange shape:

$$\bar{P}_y^h \simeq (p+\frac{0.577}{\kappa})(2B_hT_h+D_hW_h) \tag{94}$$

and mean yield load for a box shape:

$$\bar{P}_y^b \simeq 2\left(p+\frac{0.577}{\kappa}\right)(B_b T_b + D_b W_b)$$

The approximate magnitude of the standard deviation is

$$\sigma_{P_y} \cong \sqrt{\bar{A}^2 \sigma_{\sigma_y}^{\ 2} + \bar{\sigma}_y^2 \sigma_A^2 + \sigma_A^2 \sigma_{\sigma_y}^{\ 2}} \tag{96}$$

and the specific expressions pertinent to the types of shapes studied may be found by substituting for $\bar{A}$ and σ_A from Eqs. (85) and (86), and for $\bar{\sigma}_y$ and σ_{σ_y} from Eqs. (73) and (74), respectively.

4. Statistical Characteristics of the Residual Stresses

In the previous studies of the probabilistic nature of the column strength where the residual stresses have been taken into account[76,77,78,79,80], the basic concepts involved necessitated the use of deterministic relationships between the residual stresses and the various cross-sectional properties[76,77,79]. The maximum strength solution that is presented here, is based on an incremental, iterative approach; the closed form expression, relating the residual stresses to the particular shape, is therefore not necessary.

The magnitude of the residual stress in any element in the cross section, be it compressive or tensile, will be assumed to vary randomly such that it follows a normal probability law. However, it is also necessary to establish the total residual stress distribution in the shape, such that (1) force equilibrium and (2) moment equilibrium about the two principal bending axes are maintained. Thus, although the residual stress within any element will be assumed to vary randomly, and independently of the residual stresses in the neighboring or any other elements in the cross section; the stresses in all elements at any time together must satisfy the overall equilibrium of the cross section. The assumption of normality is substantiated by the results from measurements of the residual stresses in a number of identical cross sections of the same shape[21,65,86,87].

The residual stresses are, however, highly influenced by a number of factors, such as the width and the thickness of the material, the geometry of the cross section, the heat inputs created by welding and flame-cutting, and so on. It is assumed that the results of measurements made on the various rolled and welded shapes, for which data are available, are representative of the mean residual stresses in the shapes. This assumption cannot be fully substantiated, but is made to provide a necessary starting point for the analysis. It is known, however, that the studies that have been made on the statistical nature of the residual stresses in particular shapes, indicate deviations about the mean of approximately ±3 to 5 ksi[65,87]. This corresponds to coefficients of variation of about 5 to 10 percent. These measurements have been made on identical shapes, thereby omitting the random nature of the overall residual stress magnitudes and patterns, as between rolled, welded, universal mill, and flame-cut shape parameters. This distribution is greatly influenced by the manufacturing method, and therefore has a most significant effect on the column strength.

Consequently, the following expression represents the probability density function for the residual stress in element i of the cross section:

$$f_{\sigma_r}^{\,i}(\sigma_{ri}) = \frac{1}{\sigma_{\sigma_{ri}}\sqrt{2\pi}}\cdot\exp\left[-\frac{1}{2}\left(\frac{\sigma_{ri}-\bar{\sigma}_{ri}}{\sigma_{\sigma_{ri}}}\right)^2\right] \tag{97}$$

where $\bar{\sigma}_{ri}$ is the mean residual stress in element i, and $\sigma_{\sigma_{ri}}$ is the standard deviation of the residual stress in element i. Furthermore, it is known that

$$|\sigma_{ri}| \le \sigma_{yi} \tag{98}$$

which states that the residual stress in any element can only be as large as the yield stress of the element. The force equilibrium of the cross section requires that

$$\sum_{i=1}^{m} \sigma_{ri} \cdot db_i \cdot t = 0 \tag{99}$$

where the algebraic summation is extended over the entire cross section. Equation (99) may also be written as

$$2 \cdot \sum_{i=1}^{n_f} \sigma^f_{ri} \cdot dt^h_f \cdot t_h + \sum_{i=1}^{n_w} \sigma^w_{ri} \cdot dw^h_w \cdot w_h = 0 \tag{100a}$$

for a wide-flange shape,

where the first term in Eq. (100a) represents the summation over all elements in the two flanges, and the second term expresses the summation over the web. σ^f_{ri} and σ^w_{ri} denote the residual stress in element i in the flange and in the web, respectively. All of the other factors have been defined previously. Similarly, the force equilibrium requirement for a box shape may be expressed as

$$2 \cdot \{ \sum_{i=1}^{n_f} \sigma^f_{ri} \cdot dt^b_f \cdot t_b + \sum_{i=1}^{n_w} \sigma^w_{ri} \cdot dw^b_w \cdot w_b \} = 0 \tag{100b}$$

It should be noted that the numeral 2 that appears in Eqs. (100a) and (100b), does not imply that the terms following are to be multiplied by two, but rather that the summations must be extended over both flanges and both webs (in the case of a box shape) separately.

Figure 39 gives a qualitative illustration of the randomly varying residual stresses. It follows that the mean residual stress distribution must satisfy

$$\sum_{i=1}^{m} \bar{\sigma}_{ri} \cdot db_i \cdot t = 0 \tag{101}$$

with $m = 2n_f + n_w$ for a wide-flange shape, and $m = 2(n_f+n_w)$ for a box shape. The factors db_i and t have been defined previously (cf. Eq. (69)). The standard deviation must satisfy the equation

$$\sum_{i=1}^{m} \sigma_{\sigma_{ri}} \geq 0 \tag{102}$$

Moment equilibrium about each of the principal bending axes requires that

$$\sum_{i=1}^{\ell} \sigma_{ri} \cdot db_i \cdot t \cdot x_i = 0 \tag{103}$$

where $\ell = 2n_f$ for the case of a wide-flange shape bending about the minor axis; $\ell = 2n_f+n_w$ for major axis bending of a wide-flange shape, and all flange elements have the same $x_i (=\frac{1}{2}(h_h-t_h))$; and $\ell = 2(n_f+n_w)$ for box shapes. In the case of the box shape, all elements of the "flanges" considered

(="webs" for minor axis, = "flanges" for major axis bending) have equal x_i's $(=\frac{1}{2}(b_b+w_b)$ for minor axis, $= \frac{1}{2}(d_b-t_b)$ for major axis bending).

The factors in the moment equilibrium equation are all random variables, and it follows that

$$\sum_{i=1}^{\ell} \overline{\sigma}_{ri} \cdot DB_i \cdot \overline{T} \cdot X_i = 0 \tag{104}$$

when the computations are based on the mean values. A requirement to the standard deviation of the moment equation, similar to Eq. (102), can be formulated, but will not be given here.

The importance of the form of the overall residual stress distribution in the shape cannot be sufficiently emphasized. This is the reason why, for example, flame-cut and universal mill welded wide-flange shapes exhibit such different column behavior and strength. It is a topic open for future research to apply the concepts of probability theory towards the determination of the probability for obtaining a particular residual stress pattern in a given shape. However, such a study is considered beyond the scope of the investigation presented in this dissertation.

4. Statistical Characteristics of the Initial Out-of-Straightness

Most specifications for the delivery of structural steel shapes and plates contain straightness requirements for the members (see, for example, Ref. 63), whereby the initial crookedness is required to be less than a certain value. This indicates that the magnitude of the out-of-straightness will follow a negatively skewed distribution, and actual measurements have proved this to be the case[93].

Several probability density functions can be used that will fit this type of variation, of which the most practicable ones may be the Rayleigh, the Type I (Gumbel), and the Type III (Weibull) asymptotic extreme value distributions. The Type I distribution for smallest values[45] is utilized here, in order to arrive at a similarity with the distribution of the yield stress. The distribution function and the probability density function assume the following form[45]:

$$F_{e_L}(e_L) = 1-\exp[-e^{\mu(e_L-q)}] \quad \text{for } -\infty \le e_L \le \infty \tag{105}$$

and

$$f_{e_L}(e_L) = \mu\cdot\exp[\mu(e_L-q)-e^{\mu(e_L-q)}] \tag{106}$$

where e_L has been used to denote the initial out-of-straightness, instead of e; in order to avoid confusion with e as the base for the natural logarithms. Equations (105) and (106) indicate a variate that may assume values between $-\infty$ and $+\infty$, which in reality is not the case. This discrepancy can be circumvented in a fashion similar to what is used for the yield stress, which also follows a Type I distribution.

The mean value of the out-of-straightness becomes[45]:

$$\bar{e}_L = q-\frac{\zeta}{\mu}\approx q-\frac{0.577}{\mu} \tag{107}$$

and the standard deviation is

$$\sigma_{e_L} = \frac{\pi}{\sqrt{6}\mu}\approx\frac{1.282}{\mu} \tag{108}$$

The factor q represents the mode of the distribution, and μ is a measure of the dispersion.

Similar to the case of the yield strength, a transformed out-of-straightness may be introduced by

$$e_{La} = (e_L-q)\mu \tag{109}$$

whereby the distribution and the density functions become

$$F_{e_{La}}(e_{La}) = 1-\exp[-e^{e_{La}}] \tag{110a}$$

and

$$f_{e_{La}}(e_{La}) = \exp[e_{La}-e^{e_{La}}] \tag{110b}$$

Equation (110b) illustrates a distribution with mode $q = 0$, and $\mu = 1$.

The maximum allowable out-of-straightness is assumed as L/1000, and it is further assumed that values larger than this may occur with a probability of 2.5 percent. This corresponds to a value of e_{La} of 1.3 (antisymmetry with the transformed, Type I largest value distribution). Hence:

$$e_{L,max} = q+\frac{e_{La}}{\mu} = q+\frac{1.3}{\mu} = \frac{L}{1000} \tag{111}$$

The smallest possible out-of-straightness is zero, corresponding to a perfectly straight column. It is arbitrarily assumed that this value occurs with a probability of 1 percent, and the value of e_{La} thus becomes -4.6[45]. Hence:

$$e_{L,min} = q+\frac{e_{La}}{\mu} = q-\frac{4.6}{\mu} = 0 \tag{112}$$

Equations (111) and (112) can be used to determine the magnitudes of q and μ, thus:

From Eq. (112): $q\mu = 4.6$

From Eq. (111): $q\mu = \mu\cdot(\frac{L}{1000})-1.3$

and the solution of this set of equations yields for q and μ:

$$\mu = \frac{5.9}{(L/1000)} \tag{113a}$$

and

$$q = 0.78 \cdot \left(\frac{L}{1000}\right) = \frac{L}{1280} \tag{113b}$$

It is therefore seen that the most frequently occurring value of the out-of-straightness is L/1280. The mean and the standard deviation are (using Eqs. (107) and (108)):

$$\text{Mean:} \quad \bar{e}_L \approx q - \frac{0.577}{\mu} = 0.78 \cdot \left(\frac{L}{1000}\right) - \frac{0.577}{5.9} \cdot \left(\frac{L}{1000}\right) \tag{114}$$

from which is given: $\underline{\bar{e}_L} \approx 0.68 \cdot \left(\frac{L}{1000}\right) = \underline{\frac{L}{1470}}$

The standard deviation is:

$$\underline{\sigma_{e_L}} = \frac{\pi}{\mu\sqrt{6}} \approx \frac{1.282}{5.9} \cdot \left(\frac{L}{1000}\right) = 0.22\left(\frac{L}{1000}\right) = \underline{\frac{L}{4600}} \tag{115}$$

Figure 40 shows the probability density functions for the initial out-of-straightness and its transformed counterpart, e_{La}.

Figures 41 through 44 present the most important results from a few numerical examples, whereby the theoretical concepts developed have been put to practical use. The hypothetical variation of the cross-sectional area of two typical rolled wide-flange shapes, namely, W8x31 and W14x26, is illustrated by the curves in Fig. 41. It is particularly interesting to note that the coefficient of variation for the area of the smaller shape is approximately 5 times larger than that for the larger shape. This implies that variations of the area have more far-reaching consequences, the smaller the shape.

The variation of the cross-sectional area of welded wide-flange shapes and of box shapes also was investigated, although the results are not shown here. Among the most interesting results is the finding that welded and rolled wide-flange shapes of equal dimensions have almost exactly the same distribution characteristics. This illustrates that the use of the net depth of the web as a derived quantity for rolled wide-flange shapes (cf. Eqs. (39)-(41)), is not of any practical importance.

Figure 42 illustrates the hypothetical variation of the yield stress for a steel of grade ASTM A36, with thickness 3/4" and 3". Equation (84) has been used to derive σ_{ym} for the steel with thickness 3". Figure 43 shows a three-dimensional representation of the probability density function for the yield load of a shape W8x31 of steel grade ASTM A36. The density "surface" has been developed by finding the marginal probability density functions for P_y, given various values of the yield stress, σ_y, and the cross-sectional area, A. Figure 44 depicts the residual stress distribution in a welded wide-flange shape H12x79 of steel grade ASTM A572 (50)[87]. The diagram shows the distribution of the mean residual stresses, and the 95 percent confidence interval for the distribution of these. The confidence intervals have been calculated on the basis of sets of many measurements made at the same location in identical shapes, and for several locations throughout the shape.

A tabulation of the characteristic values of some of the most frequently utilized ASTM steel grades[54] is presented in Table 14. These values have been computed on the basis of the probability density function for the yield stress, given in Eqs. (71) through (80); with σ_{ym} as specified in Ref. 63, and the magnitudes of $\sigma_{y,max}$ estimated from the information provided by Refs. 63 and 88.

Table 14A gives the statistical data for some of the typical column types that have been studied. Included are the number of measurements utilized to assess the statistical characteristics of the residual stresses and the yield stresses, together with the computed variations of the maximum column strength for one particular out-of-straightness. The column strength variation is explained in Section 4.2.3.

The sets of equations and other criteria that have been established in this part of the dissertation, form the extensive and necessary background for the development of the probabilistic, incremental equations, that illustrate the behavior and the maximum strength of the column.

4.2.2 Probabilistic Evaluation of Maximum Column Strength

The maximum strength of a centrally loaded, initially curved, pinned-end, prismatic column in principle may be expressed by the following equation, where P_{max} denotes the maximum strength:

$$P_{max} = f(\sigma_y, \sigma_r, E, b, t, d, w, e_L, L) \qquad (116)$$

The function given in Eq. (116) represents a multidimensional probability density function, or a response surface, since the parameters involved may be treated as random variables.

The probabilistic characteristics of the factors σ_y, σ_r, b, t, d, w, and e_L, in addition to the characteristics of several derived variables that are functions of one or more of these basic column strength parameters, already have been established. The modulus of elasticity, E, exhibits so small a variation, that it may be treated as a constant. The length of the column, L, is a deterministic quantity, and the slenderness ratio (λ or L/r) therefore also will be incorporated as such in the analysis of the maximum strength. This is not strictly correct, since the slenderness ratio, L/r, is a function of the length and the cross-sectional properties;

the non-dimensionalized slenderness ratio, λ, also is a function of the yield stress, σ_y. Both σ_y and the cross-sectional properties have been established as random variables. However, with the maximum strength being computed from a load-deflection analysis, as opposed to being the solution of an eigenvalue problem, the length or the slenderness ratio may be regarded as fixed input-values.

The column is assumed to be subjected to a deterministic load, P, which remains as such from the onset of loading and until the maximum strength is reached. The load-deflection analysis of the column therefore will result in the determination of a semi-probabilistic load-deflection curve with the deflection as a random variable if P is the input-value,and with P as a random variable if the deflection is the deterministic starting-point for the computation. The semi-probabilistic P-δ-relationships are schematically illustrated in Fig. 45.

The deterministic Eqs. (1) through (12), depicting the behavior of the column, are basically applicable also when a probabilistic approach is utilized. This is a direct illustration of the application of a quasi-steady statistical approach[34], whereby deterministic equations express the relationships between random variables. The column maximum strength, found from the solution of the incremental, iterative expressions, therefore becomes a fixed value for every given set of values of the strength parameters for a given column. The variation of the column strength factors, as illustrated in Section 4.2.1, provides for a varying strength of the column, thus leading to the determination of the probabilistic characteristics of the strength (cf. Fig. 45b) for a given column and length. Introducing other values of the length (or the slenderness ratio), and finding the distribution of the strength for each of these, will eventually lead to a set of column curves expressing the total variation of the strength. This set of column curves will be denoted the column curve spectrum, in order to avoid confusion with the term column curve band, which has been used in a different context in Chapter 3.

The following equations represent a rewriting of Eqs. (1) through (12), whereby the probabilistic nature of the variables concerned has been accounted for. The factors that have been considered as random variables are designated by a tilde (~), for instance, the yield stress of the steel is indicated by $\tilde{\sigma}_Y$.

The equilibrium of external and internal axial forces requires that

$$\tilde{P} = \tilde{P}_{int} \qquad (117)$$

and the standard deviation of the load is

$$\sigma_P = \sigma_{P,int} \geq 0 \qquad (118)$$

Equation (118) implies that the forces do not assume fixed (deterministic) values. σ_P is derived from the variation of the load, which is found below.

Moment equilibrium is provided by

$$\tilde{P} \cdot \tilde{\delta} = \tilde{M}_{int} \qquad (119)$$

and the standard deviation of the moment may be approximated by the partial derivatives[45], thus:

$$\sigma_M \approx \left[\left(\frac{\partial M}{\partial P}\right)^2 \cdot \sigma_P^2 + \left(\frac{\partial M}{\partial \delta}\right)^2 \cdot \sigma_\delta^2 \right]^{1/2} \qquad (120a)$$

Substituting for $\tilde{M} = \tilde{M}_{int}$ from Eq. (119), this becomes

$$\sigma_M \approx [\bar{\delta}^2 \cdot \sigma_P^2 + \bar{P}^2 \cdot \sigma_\delta^2]^{1/2} \qquad (120b)$$

where $\bar{\delta}$ and $\bar{P}$ are the mean values of the deflection $\tilde{\delta}$ and the force $\tilde{P}$, respectively, and σ_P and σ_δ are the corresponding standard deviations.

Neglecting any amount of eccentric application of the axial load, the total deflection of the column at any instant is expressed as

$$\tilde{\delta} = \tilde{e}+\tilde{v}_P \tag{121}$$

where the terms e and v_P have been defined previously. The standard deviation of the deflection $\tilde{\delta}$ is

$$\sigma_\delta = \sqrt{\sigma_e^2+\sigma_v^2} \tag{122}$$

and the mean value

$$\bar{\delta} = \bar{e}+\bar{v}_P \tag{123}$$

The magnitudes of $\bar{e}$ and σ_e are given by Eqs. (114) and (115); and $\bar{v}_P$ and σ_v may be derived from the distribution characteristics of the load, $\tilde{P}$. (A number of loads satisfy the equilibrium equations, and the distribution of these loads may be used to find $\bar{v}_P$ and σ_v).

The total stress in an element i of the cross section is given by

$$\frac{\tilde{\sigma}_i}{\tilde{\sigma}_y} = \frac{\tilde{\varepsilon}_i}{\tilde{\varepsilon}_y} = \frac{1}{\tilde{\varepsilon}_y}[\tilde{\varepsilon}_{ri}+\tilde{\varepsilon}_p+\tilde{\theta}\cdot\tilde{\xi}_i] = \Phi(\varepsilon) \tag{124}$$

The distribution of $\tilde{\sigma}_y$ is depicted by Eqs. (71) through (74), and the properties of the distribution of $\tilde{\varepsilon}_y$ are easily derived therefrom, since

$$\tilde{\sigma}_y = \tilde{\varepsilon}_y\cdot E \tag{125}$$

where E is a deterministic quantity. The probability density function and the distribution function for $\tilde{\varepsilon}_y$ are found by substituting σ_y/E for σ_y in Eqs. (71) and (72). The mean and the standard deviation of $\tilde{\varepsilon}_y$ become

Mean: $$\bar{\varepsilon}_y = \frac{1}{E}\cdot\bar{\sigma}_y \tag{126a}$$

and standard deviation:

$$\sigma_{\varepsilon_y} = \frac{1}{E}\cdot\sigma_{\sigma_y} \tag{126b}$$

In a similar fashion for the residual strain, $\tilde{\varepsilon}_{ri}$:

$$\tilde{\varepsilon}_{ri} = \frac{1}{E}\cdot\tilde{\sigma}_{ri} \tag{127}$$

and the distribution of $\tilde{\sigma}_{ri}$ is illustrated by Eq. (97). The parameters of the distribution of $\tilde{\varepsilon}_{ri}$ are therefore:

Mean: $$\bar{\varepsilon}_{ri} = \frac{1}{E}\cdot\bar{\sigma}_{ri} \quad (128a)$$

Standard deviation: $$\sigma_{\varepsilon_{ri}} = \frac{1}{E}\cdot\sigma_{\sigma_{ri}} \quad (128b)$$

In the deterministic solution procedure, specific values of the strain ε_p are chosen, and Eq. (4a) then is used to find the stresses and strains in the elements throughout the cross section. This allows the establishment of the axial load P, which satisfies the equilibrium conditions. For the probabilistic approach, however, a probability density function is chosen as the representation of $\tilde{\varepsilon}_p$; a set of axial loads which satisfy the equilibrium conditions therefore can be found. The type of density function that represents $\tilde{\varepsilon}_p$ is not important, since basically only the mean value of $\tilde{\varepsilon}_p$ is used, and a normal probability law therefore will be assumed. Thus:

$$f_{\varepsilon_p}(\varepsilon_p) = \frac{1}{\sigma_{\varepsilon_p}\sqrt{2\pi}}\cdot\exp\left[-\frac{1}{2}\left(\frac{\varepsilon_p-\bar{\varepsilon}_p}{\sigma_{\varepsilon_p}}\right)^2\right] \quad (129)$$

Approximate values of the mean, $\bar{\varepsilon}_p$, and the standard deviation, σ_{ε_p}, may be used in the computations.

The centroidal distance, $\tilde{\xi}_i$, follows the distribution laws that have been developed for the cross-sectional dimensions (cf. section 4.2.1), and it therefore becomes a normally distributed variate. The curvature of the column, $\tilde{\theta}$, attains a value at midheight that is given by

$$\tilde{\theta}_m = \left(\frac{\pi}{L}\right)^2 \cdot \tilde{v}_p \tag{130}$$

For an infinitesimally small axial load, the curvature will be distributed as the term $(\pi/L)^2 \cdot \tilde{e}^2$, since for this small load, $v_p \cong e$.

The distribution characteristics of all of the factors in Eq. (124) thus have been established, and the parameters of the distribution of the strain in element i are found as:

Mean: $$\overline{\left(\frac{\varepsilon_i}{\varepsilon_y}\right)} = \frac{1}{\bar{\varepsilon}_y}[\bar{\varepsilon}_{ri} + \bar{\varepsilon}_p + \bar{\theta} \cdot \bar{\xi}_i] \tag{131}$$

and the standard deviation is

$$\sigma(\varepsilon_i/\varepsilon_y) \cong \Big[\left(\frac{\partial \Phi}{\partial \varepsilon_{ri}}\right)^2 \sigma^2_{\varepsilon_{ri}} + \left(\frac{\partial \Phi}{\partial \varepsilon_p}\right)^2 \sigma^2_{\varepsilon_p} + \left(\frac{\partial \Phi}{\partial \theta}\right)^2 \sigma^2_\theta + \left(\frac{\partial \Phi}{\partial \xi_i}\right)^2 \sigma^2_{\xi_i} + \left(\frac{\partial \Phi}{\partial \varepsilon_y}\right)^2 \sigma^2_{\varepsilon_y}\Big]^{1/2} \tag{132}$$

Substituting for $\Phi(\varepsilon)$ from Eq. (124) leads to:

$$\sigma(\varepsilon_i/\varepsilon_y) \simeq \frac{1}{\bar{\varepsilon}_y}[\sigma^2_{\varepsilon_{ri}}+\sigma^2_{\varepsilon_p}+\bar{\xi}_i^2\cdot\sigma_\theta^2+\bar{\theta}^2\cdot\sigma^2_{\xi_i} - \frac{(\bar{\varepsilon}_{ri}+\bar{\varepsilon}_p+\bar{\theta}\cdot\bar{\xi}_i)^2}{\bar{\varepsilon}_y^2}\cdot\sigma^2_{\varepsilon_y}]^{1/2} \tag{133}$$

Introducing the coefficient of variation of $\tilde{\varepsilon}_y$ as

$$V_{\varepsilon_y} = \frac{\sigma_{\varepsilon_y}}{\bar{\varepsilon}_y} \tag{134}$$

the last term in Eq. (133) may be expressed by

$$-(\bar{\varepsilon}_{ri}+\bar{\varepsilon}_p+\bar{\theta}\cdot\bar{\xi}_i)^2\cdot V^2_{\varepsilon_y}$$

The coefficients of variation of the yield stress are identical to those of the yield strain, and a computation of such values from the data provided by Table 14, shows that all the coefficients of variation are close to 0.04. $V^2_{\varepsilon_y}$ therefore becomes a small number, and the standard deviation of the strain in element i is approximated well by

$$\sigma(\varepsilon_i/\varepsilon_y) = \frac{1}{\bar{\varepsilon}_y}[\sigma^2_{\varepsilon_{ri}}+\sigma^2_{\varepsilon_p}+\bar{\xi}_i^2\sigma_\theta^2+\bar{\theta}^2\cdot\sigma^2_{\xi_i}]^{1/2} \tag{135}$$

With the mean and the standard deviation of the elemental strain thus determined, the stresses in all of the elements in the cross section can be found. It should also be noted that stress limitations, similar to those given by Eqs. (4a) and (4b), must be imposed.

The internal force and moment are given by the expressions:

Force:
$$\frac{\tilde{P}_{int}}{\tilde{P}_y} = \frac{1}{\bar{A}\cdot\bar{\sigma}_y} \sum_{i=1}^{n} \tilde{\sigma}_i \cdot \tilde{A}_i \tag{136a}$$

Moment:
$$\frac{\tilde{M}_{int}}{\tilde{P}_y} = \frac{1}{\bar{A}\cdot\tilde{\sigma}_y} \sum_{i=1}^{n} \tilde{\sigma}_i \cdot \tilde{\xi}_i \cdot \tilde{A}_i \tag{136b}$$

These equations make use of the distribution characteristics of $\tilde{P}_y$, given by Eqs. (90) and (93)-(96), the characteristics of $\tilde{A}$, given by Eqs. (47), (48), (52), and (53); of $\tilde{\sigma}_y$, given by Eqs. (71) through (74); and of the elemental areas $\tilde{A}_i$, together with the centroidal distances $\tilde{\xi}_i$, given by Eqs. (56) through (67). The properties of $\tilde{\sigma}_i(\tilde{\varepsilon}_i)$ are given above. The summations illustrated by Eqs. (136a) and (136b) are easily carried out, and the distribution characteristics of $\tilde{P}_{int}$ and $\tilde{M}_{int}$ are determined simultaneously.

An accuracy requirement of a form similar to that given by Eqs. (11) and (12) may be introduced, thus:

$$\left(\frac{\tilde{\Delta P}}{\bar{P}_y}\right) = \frac{\tilde{P}_{int}}{\bar{P}_y} - \frac{\tilde{M}_{int}}{\bar{P}_y \cdot \tilde{\delta}} \leq \left(\frac{\tilde{\Delta P}}{\bar{P}_y}\right)_{min} \tag{137}$$

The distribution of this out-of-equilibrium force can be developed, since the properties of the density functions of the other factors in Eq. (137) are known.

One of the most significant computational advantages of this approach is inherent in the fact that a range of ε_p-values is taken into account at the same time. This eliminates the time-consuming and error-prone repetition of the calculations, that previously often resulted from incorrectly assumed ε_p-values. In this probabilistic method, an adjustment of the distribution of ε_p is carried out, as soon as the computation indicates the magnitude of the mean value of ε_p.

Various numerical methods may be applied, in order to determine the distribution of the maximum strength. The use of a Monte Carlo approach[44] was contemplated, but was discarded as an inefficient and expensive solution procedure. It may prove advantageous if purely theoretical values for the column strength parameters are used. For all practical purposes, however, the complete distribution of the maximum strength is not needed, since basically the upper and lower bounds, and a central distribution parameter, such as the mean, will provide the information necessary. This basically is what has been done in the study presented here, and the following section gives an outline of the most important results.

4.2.3 The Variation of Maximum Column Strength

The large amounts of data that have been produced prevents a presentation of every conceivable result, and a selection of the findings for a few typical columns therefore has been made. The information and the discussion of the results that are given in this section of the dissertation are thus but a part of what is available, but nevertheless illustrate and emphasize all of the important aspects of the investigation.

Figure 46 shows the column curve spectra for the major and minor axis bending of a typical light rolled wide-flange shape, namely, W8x31. The steel grade is given as ASTM A36. Each column curve spectrum reflects the variation of the maximum column strength of this shape, when all of the column strength parameters vary between their respective extreme values. The spectra therefore illustrate the 95 percent confidence intervals for the maximum strength of the shape, such that there is only a probability of 5 percent that the strength of a randomly chosen W8x31 (A36) column will fall outside the interval. For reasons that are given below, it may be stated that the upper limit of each spectrum is indicative of columns with an initial out-of-straightness of 1/10,000; and the lower limit of columns with $e/L = 1/1000$. These two values correspond to the limits of the 95 percent confidence interval for the distribution of the out-of-straightness.

It may be seen that the largest variation of the major axis strength occurs for $\lambda \approx 0.9-1.0$, with upper and lower limits as approximately (for $\lambda = 0.9$) 0.823 and 0.712, respectively. The higher of the two values represents an increase of the strength of about 15 percent, when compared to the lower limit. Similarly, the largest deviation of the minor axis strength appears for λ-values of 0.9 to 1.1, indicating changes in the strength of about 30 percent. Both of these may be considered as very significant.

In order to detect and analyze the effects of the variability of the other column strength parameters, major and minor axis column curve spectra were prepared, for which the initial out-of-straightness was kept constant. Figure 47 illustrates the two spectra that were produced for the W8x31 shape, maintaining e/L at its mean value of 1/1470. Other values of the out-of-straightness also were studied, but the findings are identical to those for e/L = 1/1470.

Within the limitations and assumptions imposed by the study, the data presented in Fig. 47 show that the influence of the variability of the yield stress and the cross-sectional properties, and of the ±-variations of the residual stresses in any particular shape with a specific manufacturing method, is relatively small for the variation of the maximum strength. The two column curve spectra both indicate maximum strengths that lie within a range of 3 to 7 percent (from the upper to the lower limit); depending on the magnitude of the slenderness ratio. Extended analyses of the data, furthermore, show that this variation - however small it may be - almost in its entirety may be attributed to the variation of the yield stress. This statement is substantiated by the information provided by Table 15, which gives the most important statistics for the two spectra in Fig. 47. Means and coefficients of variation for the major and minor axis strength are given for three typical slenderness ratios, using the maximum, mean, and minimum values of the yield stress of this steel. For comparison, a similar set of data has been included in Table 15 for the column curve spectra that are based on an out-of-straightness of 1/10,000.

For each value of the yield stress, the corresponding mean values of the maximum strength are clearly different, although the deviations are very small. The magnitudes of the coefficients of variation, which are indicative of the influence of the variations of the residual stresses and the cross-sectional properties, are extremely small. Thus, it is seen that the coefficients of variation attain values between 0 and 0.6 percent, and a further analysis of the data shows that there is no systematic influence of the varying residual stresses and cross-sectional dimensions. These statements are true for all slenderness ratios, and the numbers in the last column of Table 15 illustrate that the conclusions also hold for other values of the out-of-straightness.

The reason for the lack of influence of the $\pm$ residual stress variation about the mean of the overall distribution in the shape, is due to the overriding influence of the initial out-of-straightness, which strongly governs the behavior and strength of the column.

It is more difficult to establish the cause of the absence of any influence of the variability of the cross-sectional properties, particularly for such a small shape as the W8x31, where the coefficient of variation for the area is 5.6 percent (see Section 4.2.1). It is believed, however, that the effects of the geometric properties are almost completely overridden by the variation of the yield stress.

Investigations identical to the one described above have been carried out for a number of rolled and welded wide-flange and box shapes of various steel grades. The conclusions arrived at are the same, although it must be stated that the influence of the yield stress increases with increasing range ($\sigma_{y,max}-\sigma_{ym}$). Although still negligible when compared to the yield stress and the out-of-straightness, for heavy rolled shapes the variability of the cross-sectional dimensions has a certain bearing on the variation of the maximum strength. This has been evidenced by the results from the study of such shapes.

Figure 48 shows the major axis column curve spectrum for the W8x31 shape, together with the curves depicting its dispersion characteristics. Due to the fact that the two most important factors, namely, the initial out-of-straightness and the yield stress, both are distributed according to an extreme value distribution, it might have been expected that the maximum column strength also would exhibit distribution properties as such. The anti-symmetry of the σ_y- and e-distributions intuitively might lead to the false conclusion that P_{max} will follow some symmetric probability density function, for example, a normal distribution. The much greater importance of the out-of-straightness overrides the effect of the yield stress, however, and the result is a significantly skewed distribution. This is indicated in Fig. 48, and Fig. 49 illustrates the probability density function for the maximum strength of the W8x31 (A36) column with a slenderness ratio $\lambda = 0.9$, bent about the major axis. It was found that a Type I (Gumbel, largest value) asymptotic extreme value distribution fits the data very well, with mode $P_{max}/P_y = 0.741$, and dispersion factor equal to 45. (The large dispersion factor is necessary, in order to make the area under the curve in Fig. 49 equal to 1 (one). This is one of the fundamental requirements of any probability density function).

The data presented in Fig. 50 are analogous to those of Fig. 48, but represent the column curve spectrum for the minor axis bending of the W8x31 shape. The skew distribution of the maximum strength prevails, although it may be noted that it is significantly more pronounced for the intermediate and high slenderness ratios, when compared to the data in Fig. 48. This is a property common to many column curve spectra for minor axis bending, as is seen in the following.

Probability density functions, such as the one shown in Fig. 49, can be developed for each slenderness ratio for any column. This is not essential, however, since the main interest is focused either on the extremes or some centrally located characteristic of the spectrum. The following data therefore are presented in this simplified, but sufficiently informative fashion.

Figures 51 and 52 show the column curve spectra for minor and major axis bending, respectively, of a typical heavy rolled wide-flange shape W14x426 (ASTM A36). The distribution of the major axis maximum strength is highly skewed for λ-values between 0.7 and 1.1, and for minor axis strength for slenderness ratios larger than approximately 0.9. It is interesting to note that the skewness for the major axis strength distribution is reversed for $\lambda>1.3$, such that the mean is located closer to the upper limit curves. This is a very uncommon feature, and it has not been detected for any other columns studied. It is also known that the strength of very long, initially curved columns, approaches the elastic buckling load for a straight member.

The variation of the maximum strength of a rolled wide-flange column of high-strength steel is illustrated in Fig. 53, which shows the column curve spectrum for a shape W8x31 (ASTM A514). The amount of variation is significantly smaller than that of the previously discussed columns, and it is partly caused by a diminishing influence of the out-of-straightness for increasing material strength. The main reason for the smaller variation is due to the fact that the magnitude of the residual stresses is small, compared to the yield stress of the steel.

Figure 54 gives the column curve spectrum for a light welded flame-cut wide-flange shape H12x79 of steel grade ASTM A572 (50), bent about the minor axis. The smaller width of the spectrum is partly indicative of a column of higher strength steel, but its other characteristics are similar to those of rolled wide-flange shape spectra.

The column curve spectrum for the heavy welded universal mill wide-flange shape H15x290 (ASTM A36), shown in Fig. 55, exhibits properties very similar to those of the heavy rolled wide-flange shapes. This also has been substantiated by other findings in the study. The excessive skewness that prevails for slenderness ratios larger than 0.6 is again an example typical of minor axis bending of wide-flange shapes.

The column curve spectra for two typical welded box shapes □24x774 (ASTM A36, heavy) and □6x20 (ASTM A514, light) are shown in Figs. 56 and 57, respectively. The amount of maximum strength variation is fairly small for both shapes, compared to the findings for wide-flange shapes. The results for the other types of box shapes, included in the investigation, exhibit the same tendencies. It therefore may be deduced that box shapes in general are less influenced by the variation of the column strength parameters, regardless of the steel grade used, within the limitations imposed by the use of single profiles and measurements made on these.

The results presented above give a detailed illustration of the variation of the maximum column strength for a variety of shapes and steel grades. The factors contributing most significantly to this variation have been clearly established; and the next, and final, step in the probabilistic analysis is therefore the development of the multiple column curves.

4.3 Development of Multiple Column Curves

The results presented in the foregoing section (Section 4.2.3) of the dissertation have shown, that given the assumed variations in yield stress, cross section geometry, and residual stress, that the random variation of the maximum strength of one particular column type is mainly due to the variation of the initial out-of-straightness, and to a very small extent also to the variation of the yield stress and the residual stress variation.

Based on this knowledge, it therefore stands to reason that the upper limit of the strength of a column (2.5 percent probability level), is well described by a column curve based on an initial out-of-straightness of 1/10,000; and the lower limit by a column curve based on a value of e/L of 1/1000. The probability density function of the out-of-straightness, shown in Fig. 40, indicated a probability of 2.5 percent that e_L will become less than L/10,000; and a probability of 2.5 percent that e_L will become larger than L/1000. The development of a set of column curves that adequately and rationally may represent the strength of all conceivable types of columns therefore presents the following basic problems:

1. The choice of strength criterion for a particular column; the magnitude of the out-of-straightness; the use of the lower limit or some central strength characteristic as representative of the real or typical capacity of the member.

2. The classification of the various types of columns; which columns exhibit similar strength and dispersion properties.

3. The number of column curves that are necessary to use, in order to arrive at an optimum of complexity and gains.

All of these problems have been discussed at some length in the course of development of the set of deterministic multiple column curves (cf. Chapter 3), but could only be partly solved, due to the lack of certain indispensable data. This information now has been furnished, in the form of the knowledge of the causes and implications of the variation of the strength of a specific column.

The number of curves in the set of multiple column curves has been set at three. It should be noted, however, that one of the very significant advantages of the probabilistic approach is the ease with which practically any number of such curves may be developed. It does, however, require data on the probability density functions for the maximum strength of all the column types that are to be included. Such an investigation is considered beyond the scope of this study

It has been decided to base the set of probabilistic multiple column curves on mean values for all of the column strength parameters; in particular such that $e/L = 1/1470$ is the basic out-of-straightness. The use of the mean values of the column strength parameters is arbitrary, since any set of data with similar probabilistic bases may be used. It therefore will not be difficult to implement the use of a lower limit of the strength as a basis for the curves[89].

The only remaining problem therefore is the classification of the various types of columns into three separate categories. Having decided on a particular out-of-straightness, the only column strength parameter of importance will be the yield stress of the steel, together with the following column types:

1. Light and heavy rolled wide-flange shapes.

2. Light and heavy welded wide-flange shapes (flame-cut, universal mill).

3. Light and heavy box shapes.

At this point, the investigation ceases to be of a strictly probabilistic nature, since the arrival at a set of appropriate multiple column curves only involves the grouping of a large number of probabilistically determined maximum strength curves. The method of classification is the same as that used in Section 3.3.4, and it is not considered to be necessary to provide all the background material that was utilized previously. Of overall importance, however, is the fact that the significance of each of the column strength parameters has been established on a sound, probabilistic basis; and that the analysis may be performed only by choosing the value of the out-of-straightness.

The results of the classification of the column types are summarized in Tables 16 through 18, and Fig. 58 shows the final set of possible multiple column curves. The equations describing the three curves are as follows:

Curve 1P:

$$(P_{max}/P_y)=0.979+0.205\cdot\lambda-0.423\cdot\lambda^2 \quad \text{for } 0.15\leq \lambda\leq 1.2 \quad (138a)$$

$$(P_{max}/P_y)=0.03+0.842\cdot\lambda^{-2} \quad \text{for } 1.2\leq \lambda\leq 1.8 \quad (138b)$$

$$(P_{max}/P_y)=0.018+0.881\cdot\lambda^{-2} \quad \text{for } 1.8\leq \lambda\leq 2.6 \quad (138c)$$

$$(P_{max}/P_y)=\lambda^{-2}\text{(=Euler Curve)} \quad \text{for } \lambda\geq 2.6 \quad (138d)$$

Curve 2P:

$$(P_{max}/P_y)=1.03-0.158\cdot\lambda-0.23\cdot\lambda^2 \quad \text{for } 0.15\leq \lambda\leq 1.0 \quad (139a)$$

$$(P_{max}/P_y)=-0.163+0.803\cdot\lambda^{-1}+0.056\cdot\lambda^{-2} \quad \text{for } 1.0\leq \lambda\leq 1.8 \quad (139b)$$

$$(P_{max}/P_y)=0.018+0.815\cdot\lambda^{-2} \quad \text{for } 1.8\leq \lambda\leq 3.2 \quad (139c)$$

$$(P_{max}/P_y)=\lambda^{-2}\text{(=Euler Curve)} \quad \text{for } \lambda\geq 3.2 \quad (139d)$$

Curve 3P:

$$(P_{max}/P_y)=1.091-0.608\cdot\lambda \quad \text{for } 0.15\leq \lambda\leq 0.8 \quad (140a)$$

$$(P_{max}/P_y)=0.021+0.385\cdot\lambda^{-1}+0.066\cdot\lambda^{-2} \quad \text{for } 0.8\leq \lambda\leq 2.0 \quad (140b)$$

$$(P_{max}/P_y)=0.005+0.9\cdot\lambda^{-2} \qquad \text{for } 2.0\leq \lambda\leq 4.5 \qquad (140c)$$

$$(P_{max}/P_y)=\lambda^{-2}\,(=\text{Euler Curve}) \qquad \text{for } \lambda\geq 4.5 \qquad (140d)$$

For all three curves:

$$(P_{max}/P_y)=1.0 \qquad \text{for } 0\leq \lambda\leq 0.15 \qquad (141)$$

The appearance of these equations and of the curves is similar to those developed in the deterministic part of the investigation, and a comparison of the two sets of data will be presented in Chapter 5. The reason for the similarity between the deterministic and the probabilistic multiple column curves is the fact that both sets of curves are based on mean values for the residual stresses, yield stresses, and cross-sectional properties. The basic difference appears in the value of the initial out-of-straightness. A conversion of the equations into expressions relating critical stresses and dimensional slenderness ratios is easily accomplished, once the yield stress of the material is known.

A topic of future research is provided by a study aimed at the probabilistic classification of column types into the various column strength categories.

4.4 Summary of Probabilistic Solutions

A probabilistic approach for the solution of the column maximum strength problem has been devised in this part of the dissertation, whereby the random variability of the column strength parameters has been taken into account. An extensive section has been devoted to the analysis of the random nature of the cross-sectional properties, the yield stress, the residual stresses, and the initial out-of-straightness. Data from several experimental investigations on the statistical properties of the column strength parameters were studied, and the choice of probability density functions for the strength factors have been made to correlate well with these data. A few numerical examples have been worked out, illustrating the use of the theory, and it has been found that the theory fits well with actual results.

The computation of the maximum strength has been evaluated on the basis of a quasi-steady statistical approach, whereby the previously used deterministic relationships have been rewritten to account for the stochastic characteristics of the factors involved. This represents the first time that the concepts of probability theory have been applied towards the solution of a non-linear structural problem, where the basic relationships are given in the form of incremental, iterative equations.

The analyses of the data produced indicate that the random variation of the strength of a particular column, given its manufacturing method, yield stress, and so on, almost in its entirety may be attributed to the variation of the initial out-of-straightness. The random variability of the yield stress accounts for a small amount of the column strength variation, and its influence increases slightly as the magnitude of σ_y increases. Larger range of the yield stress variation, namely, $(\sigma_{y,max}-\sigma_{ym})$, also influences the column strength to a certain small extent. In the presence of significant out-of-straightness, it has been shown that the random nature of the cross-sectional properties and of the residual stresses, that is, the $\pm$ variation of the residual stresses about their mean pattern in the shape, do not contribute as much variation in the column strength as does that of the out-of-straightness. The lack of influence of the residual stresses may be attributed to the fact that the strength and behavior of the column is more influenced by the out-of-straightness than of any other factor, which therefore overrides the influence of the other parameters. It must be stated, however, that the random nature of the overall residual stress distribution in a shape has not been studied, and the effects of differences in the residual stress pattern as influenced by the various manufacturing methods are of profound importance for the column behavior and strength.

It has been found that the variation of the strength of a particular column with a given slenderness ratio, is well represented by a Type I asymptotic extreme value probability density function. This is a result of the overall influence of the initial out-of-straightness. The maximum strength of an initially curved column therefore will not be distributed according to a normal (Gaussian) probability law.

The analysis of the available maximum strength data has been used to develop a set of multiple column curves, with a method of classification identical to that utilized in the deterministic part of the study. The multiple column curves have been based on the assumed mean value of the initial out-of-straightness, namely, $e/L = 1/1470$. The choice of the mean value of the initial crookedness has been based on the need for a consistent set of column strength parameters as the basis for the curves, since the mean values of all of the other factors (residual stresses, yield stress, cross-sectional dimensions) have been utilized.

5. COMPARISON OF DETERMINISTIC AND PROBABILISTIC METHODS OF SOLUTION

The essence of the deterministic study is provided by the column category data given in Tables 7 through 9, and by the set of multiple column curves shown in Fig. 32. Similarly, the final outcome of the probabilistic investigation is given by the data in Tables 16 through 18, and by the multiple column curves illustrated in Fig. 58. The comparison of the results from the two parts of the study therefore is based on a comparison of the tables and the two figures.

Figure 59 shows the two sets of multiple column curves drawn in the same diagram. It is evident that the curves are closely related, which may be attributed to the fact that the out-of-straightness is the single most important column strength parameter, and that the methods of column classification are the same. The probabilistic curves, labeled 1P, 2P, and 3P, are located above the three deterministic curves for all slenderness ratios. The largest differences between the curves occur for curves 1P and 1, for λ-values in the range of 0.7 to 1.1; and for curves 2P and 2, for λ-values between approximately 0.9 and 1.4. The increases in strength thus provided, when utilizing the probabilistic curves instead of the deterministic ones, amount to 5 to 8 percent, depending on the slenderness ratio. This is partly a result of using the mean value of the out-of-straightness as the basis for the curves, rather than the maximum value of 1/1000. The differences between curves 3P and 3 are fairly small, amounting to maximum gains in strength of approximately 1 to 4 percent. The reasons for the smaller increase in strength for the lowest curve are partly to be found in the fact that the out-of-straightness has a smaller influence on the maximum strength of the heaviest types of columns. These are the types of columns that belong to the lowest column strength category.

The most significant cause of the closeness of the curves 3P and 3, however, is the difference in the types of columns that belong to the two categories. This may be realized by considering the data provided by Tables 9 and 18. Thus, category 3P contains only the minor axis bending cases of the heavy rolled and heavy and light welded wide-flange universal mill columns of A36 steel, whereas category 3 (Table 9) also contains the major axis bending cases. The latter type did provide for an increase in the curve 3 strengths, but these cases now have been assigned to category 2 (cf. Table 8). Hence the resulting smaller increases in strength when utilizing curve 3P instead of curve 3.

The column strength categories included in groups 1P and 2P are different from those of groups 1 and 2, as may be seen by comparing the data in Tables 7 and 16, and 8 and 17, respectively. Category 1P thereby contains both bending axis cases for the light rolled wide-flange A242-shapes, as well as both cases for the light welded flame-cut wide-flange shapes of A514 steel (Table 16). Category 1 only contains the cases of major axis bending for these two column types (Table 7). Similarly, category 2P contains the major axis bending cases of A36 heavy rolled wide-flange shapes; as well as the major axis cases of light and heavy welded wide-flange universal mill columns of A36 steel. Both of these previously were located in category 3, as mentioned above.

It is believed that the probabilistically based set of multiple column curves presents a good representation of the strength of the various types of columns included. The belief is supported and substantiated by the detailed analyses of the column strength, that were made possible only by the use of probabilistic considerations of the column strength factors.

6. SUMMARY AND CONCLUSIONS

The following summarizes the most significant findings of the study presented in this dissertation:

1. A probabilistic method for the solution of the problem of the maximum strength of centrally loaded, initially curved, pinned-end, prismatic steel columns has been developed. This represents the first time that a structure exhibiting a random non-linear behavior, for which the basic relationships are given in the form of incremental, iterative equations, has been treated within the context of probability theory. The method is basically founded on a quasi-steady probabilistic approach, whereby deterministic expressions govern the relationships between random variables. The concepts involved in the development may be used in future studies of problems of similar nature, and the method therefore may have a significant influence on the advancement of the use of probabilistic principles in topics pertinent to civil engineering.

2. It is known that the strength of all types of columns exhibits a significant variation, due to differences in column cross section, steel grade, manufacturing

method, and so on. In an attempt to develop more rational and adequate means of assessing the strength of real columns, a number of methods of implementing the column strength variation in design have been explored. A set of column curves - multiple column curves - have been developed to provide strength classification for selected categories of structural shapes used as columns. The variety of column types available are thereby classified on the basis of similar behavior and strength, and column curves that are representative of the strength of each of the categories are developed. Two sets of multiple column curves, each containing three curves, have been provided by the investigations based on deterministic and probabilistic concepts.

3. The maximum strength of an initially curved, centrally loaded, pinned-end, prismatic steel column provides the most realistic representation of the strength of actual columns of this kind. This is attributed to the ever-present out-of-straightness of real columns.

In addition to the above conclusions, the following provides a summary of the other results of the investigation:

4. A computer program was developed for the deterministic part of the investigation. The program is based on an incremental, iterative approach, and takes into account the residual stresses, the yield stress and its variation throughout the shape, and the initial out-of-straightness. So far only wide-flange and box shapes have been included, but modifications in the program will make it possible to handle other shapes as well.

5. A large number of column curves, representing a variety of wide-flange and box shapes in several steel grades, sizes, manufacturing methods, and so on, have been developed. The curves have been analyzed with regard to a classification of the columns into three column strength categories. Based on these analyses, a set of multiple column curves have been evaluated, which are based on an initial out-of-straightness of 1/1000. This value represents the maximum allowable crookedness, according to the specifications for the delivery of structural steel shapes.

6. The statistical characteristics for the cross-sectional dimensions and other geometric properties, for the yield stress of the steel, for the $\pm$ variation of the resi-

dual stresses about their mean, and finally also for the initial out-of-straightness, have been studied extensively. Comparisons with actually measured values indicate that the choices made for the probability density functions of the factors are reasonable.

7. The results from the probabilistic study indicate that the variation of the strength of a particular column, given its manufacturing method, almost in its entirety may be attributed to the variation of the initial out-of-straightness. The variability of the yield stress has a very small effect, but this increases with increasing yield stress and yield stress range of variation. The random variation of the residual stresses about their mean, and of the cross-sectional properties, do not contribute significantly to the random variation of the maximum column strength. The probabilistic nature of the overall residual stress distribution in the shape has not been studied. The pattern of residual stress in the shape therefore remains one of the most significant column strength parameters.

8. Due to the overall importance of the initial out-of-straightness, the maximum strength of a specific column will be distributed in a skew fashion. It was found

that a Type I asymptotic extreme value distribution will represent the random column strength variation well.

9. Column curve spectra have been developed for a large number of wide-flange and box columns, in various steel grades, sizes, and so on. A column curve spectrum is defined as the set of column curves, that exhibit the variation of the strength of a specific column for all slenderness ratios and values of the column strength parameters.

10. Based on the column curve spectra available, a set of multiple column curves has been developed. The mean value of the initial out-of-straightness, 1/1470, was chosen as the basis for this development, based on the desire to use mean values for all column strength parameters.

11. A comparison between the deterministic and the probabilistic set of multiple column curves reveals that the latter provide for somewhat higher column strengths. This is partly due to the smaller value of the crookedness, which has created changes in the classification of the column types.

7. NOMENCLATURE

7.1 Symbols

The following is a list of the symbols that have been used throughout this dissertation. It should be noted that in the cases when the same symbol is given twice, and one of them appears with a tilde (~) above; the latter indicates the same quantity, that is being treated as a random variable. (For instance, σ_y and $\tilde{\sigma}_y$, with $\tilde{\sigma}_y$ being the random variable).

Δ	= eccentricity of applied axial load at column ends
$\Delta P, \tilde{\Delta P}$	= the difference between the applied axial load, and the axial load that corresponds to equilibrium between internal and external forces
$\Phi(\varepsilon)$	= a function that expresses the total strain in an element in the cross section of the column
α	= a ratio given by $\alpha = \frac{(P_{max}/P_y)_{theory}}{(P_{max}/P_y)_{experiment}}$, used to compare theoretical and experimental maximum column strengths
β	= a factor given by $\beta = \lvert (P_{max}/P_y)_{theory} - (P_{max}/P_y)_{experiment} \rvert$, used to compare theoretical and experimental maximum column strengths

$\delta, \tilde{\delta}$	=	general symbol for the total deflection of the column
$\bar{\delta}$	=	mean value of the total deflection of the column
$\varepsilon_i, \tilde{\varepsilon}_i$	=	the total strain in element i in the cross section of the column
$(\overline{\varepsilon_i/\varepsilon_y})$	=	mean value of total strain/yield strain in element i in the cross section
$\varepsilon_p, \tilde{\varepsilon}_p$	=	axial strain in the cross section, due to the column load P
$\bar{\varepsilon}_p$	=	mean value of the axial strain in the cross section, due to the column load P
$\varepsilon_{ri}, \tilde{\varepsilon}_{ri}$	=	residual strain in element i in the column cross section
$\bar{\varepsilon}_{ri}$	=	mean value of the residual strain in element i in the column cross section
ε_{st}	=	strain at the onset of strain-hardening of the steel
$\varepsilon_y, \tilde{\varepsilon}_y$	=	yield strain of steel
$\bar{\varepsilon}_y$	=	mean value of the yield strain of steel
ζ	=	Euler's constant, approximately equal to 0.577
η	=	constant
$\theta, \tilde{\theta}$	=	curvature of the column, due to the applied axial load
$\bar{\theta}$	=	mean value of the curvature of the column, due to the applied axial load

$\theta_m, \tilde{\theta}_m$	=	curvature at the mid-height of the column, due to the applied axial load
κ	=	dispersion factor for the Type I (Gumbel, largest value) asymptotic extreme value distribution of the yield stress
λ	=	non-dimensionalized slenderness ratio
μ	=	dispersion factor for the Type I (Gumbel, smallest value) asymptotic extreme value distribution of the initial out-of-straightness
$\xi_i, \tilde{\xi}_i$	=	distance from the centroid of element i in the cross section to the bending axis considered
$\bar{\xi}_i$	=	mean value of the distance from the centroid of element i in the cross section to the bending axis considered

<u>Symbols designating stresses:</u>

$\sigma_i, \tilde{\sigma}_i$	=	total stress in element i in the cross section of the column
$\sigma_{ri}, \tilde{\sigma}_{ri}$	=	residual stress in element i in the cross section of the column
$\bar{\sigma}_{ri}$	=	mean value of the residual stress in element i
$\sigma^f_{ri}, \sigma^w_{ri}$	=	residual stress in element i of the flange and the web, respectively

$\sigma_y, \tilde{\sigma}_y$ = yield stress of steel

$\bar{\sigma}_y$ = mean value of the yield stress of steel

σ_{ya} = transformed yield stress of steel

σ_{yi} = yield stress of the material in element i in the cross section

$\sigma_{y,max}$ = maximum value of the yield stress (probability of occurrence $\leq 2.5\%$)

σ_{ym} = specified yield stress

$\sigma_{y,p}$ = most frequently occurring value of the yield stress (mode)

Symbols designating standard deviations of random variables:

σ_δ = standard deviation (st. dev.) of the deflection of a column

σ_{ε_p} = st. dev. of the axial strain in the cross section, due to the load P

$\sigma_{\varepsilon_{ri}}$ = st. dev. of the residual strain in element i in the cross section

σ_{ε_y} = st. dev. of the yield strain of steel

σ_θ = st. dev. of the curvature of the column

σ_{ξ_i} = st. dev. of the distance from the centroid of element i to the bending axis considered

$\sigma_{\sigma_{ri}}$ = st. dev. of the residual stress in element i

σ_{σ_y} = st. dev. of the yield stress of the steel

σ_Ψ = st. dev. of a function of random variables, Ψ

$\sigma(\varepsilon_i/\varepsilon_y)$ = st. dev. of (total strain in element i/yield strain)

σ_A = st. dev. of the area of the cross section of a column

σ_M = st. dev. of the moment acting on the cross section of a column

σ_P = st. dev. of the external axial load

σ_{P_y} = st. dev. of the yield load for a column

$\sigma_{P,int}$ = st. dev. of the internal axial load, acting on the cross section

σ_b^b, σ_b^h = st. dev. of the flange width of a box shape and a wide-flange shape, respectively

σ_d^b, σ_d^h = st. dev. of the web depth of a box shape and a wide-flange shape, respectively

$\sigma_{dA_f}^h$ = st. dev. of the area of a flange-element in a wide-flange shape

$\sigma_{dA_w}^b, \sigma_{dA_w}^h$ = st. dev. of the area of a web-element in a box shape and a wide-flange shape, respectively

$\sigma_{dt}^b, \sigma_{dt}^h$ = st. dev. of the width of a flange-element in a box shape and a wide-flange shape, respectively

$\sigma_{dw}^b, \sigma_{dw}^h$ = st. dev. of the width of a web-element in a box shape and a wide-flange shape, respectively

σ_{e_L} = st. dev. of the initial out-of-straightness

σ_h^h = st. dev. of the total (overall) depth of a wide-flange shape

σ_t^b, σ_t^h = st. dev. of the flange thickness of a box shape and a wide-flange shape, respectively

σ_u^b, σ_u^h = st. dev. of the random variable u, pertaining to a box shape and a wide-flange shape, respectively

σ_v = st. dev. of the column deflection, due to the axial load P

σ_w^b, σ_w^h = st. dev. of the web thickness of a box shape and a wide-flange shape, respectively

σ_z^b, σ_z^h = st. dev. of the random variable z, pertaining to a box shape and a wide-flange shape, respectively

ϕ = a parameter (e.g., the mean) of the distribution of a random variable

$\hat{\phi}$ = maximum likelihood estimator of the parameter ϕ

Ψ = general symbol for a function of random variable(s)

$A, \tilde{A}$ = area of the cross section

A_b, A_h = cross-sectional area of a box shape and a wide-flange shape, respectively

$\bar{A}_b, \bar{A}_h$ = mean cross-sectional area of a box shape and a wide-flange shape, respectively

$A_i, \tilde{A}_i$ = area of element i in the cross section of the column

$\bar{B}$ = mean value of a width, b

B_b, B_h = mean value of the flange width of a box shape and a wide-flange shape, respectively

D_c = mean value of the distance from the center-line of the component plate regarded as flange, to the centroid of the shape

D_b, D_h = mean value of the web depth of a box shape and a wide-flange shape, respectively

DA_f^b, DA_f^h = mean value of the area of a flange-element of a box shape and a wide-flange shape, respectively

DA_w^b, DA_w^h = mean value of the area of a web-element of a box shape and a wide-flange shape, respectively

DB_i = mean value of the width of element i in the cross section

DT_f^b, DT_f^h = mean value of the width of a flange-element in a box shape and a wide-flange shape, respectively

DW_w^b, DW_w^h = mean value of the width of a web-element in a box shape and a wide-flange shape, respectively

E = modulus of elasticity

$E(\Psi)$ = mathematical expectation (= mean) of a function of random variables

$F(y)$ = the distribution function of a random variable y

H_h = mean value of the total (overall) depth of a wide-flange shape

I = general symbol for moment of inertia

$\bar{I}$ = mean value of the moment of inertia

I_x^b, I_y^b = moment of inertia for a box shape about the major and minor axes, respectively

I_x^h, I_y^h = moment of inertia for a wide-flange shape about the major and minor axes, respectively

L = length of the column

$L(\phi)$ = sample-likelihood function with parameter ϕ

M = general designation of the arithmetic mean

$\hat{M}$ = maximum likelihood estimator of the arithmetic mean

$M_{int}, \tilde{M}int$ = internal moment in a cross section of the column

$P, \tilde{P}$ = external axial load of the column

$\bar{P}$ = mean value of the external axial load

$P_{int}, \tilde{P}_{int}$ = internal axial load in a cross section of the column

P_{max} = maximum strength (load) of the column

$P_y, \tilde{P}_y$ = yield load of the column (no overall column buckling occurring)

$\overline{P}_y$ = mean value of the yield load

$\overline{P}^b_y, \overline{P}^h_y$ = mean value of the yield load of a box and a wide-flange column, respectively

P_{ya} = transformed yield load

R = radius of curvature of the center-line of the column

$\overline{T}$ = mean value of the thickness of an element in the cross section

T_b, T_h = mean value of the flange thickness of a box shape and a wide-flange shape, respectively

U_b, U_h = mean values of the random variables u_b and u_h, pertaining to box and wide-flange shapes, respectively

V = coefficient of variation (general)

V_{ε_y} = coefficient of variation of the yield strain

$\overline{W}$ = mean value of the web thickness of the cross section (general)

W_b, W_h = mean value of the web thickness of a box shape and a wide-flange shape, respectively

X_i = mean value of the coordinate of the centroid of element i in the cross section, depending on which of the principal bending axes is considered

Z_b, Z_h = mean values of the random variables z_b and z_h, pertaining to box and wide-flange shapes, respectively

a	=	constant
b	=	constant; in some expressions denoting the width of a component plate
b_b, b_h	=	flange width of a box shape and a wide-flange shape, respectively
c	=	constant
d_b, d_h	=	web depth of a box shape and a wide-flange shape, respectively
d_c	=	distance from the center-line of the component plate regarded as flange, to the centroid of the shape
dA_f^b, dA_f^h	=	area of a flange-element of a box shape and a wide-flange shape, respectively
dA_w^b, dA_w^h	=	area of a web-element of a box shape and a wide-flange shape, respectively
db_i	=	width of element i (general)
dt_f^b, dt_f^h	=	width of a flange-element of a box shape and a wide-flange shape, respectively
dw_w^b, dw_w^h	=	width of a web-element of a box shape and a wide-flange shape, respectively
$e, \tilde{e}, e_L, \tilde{e}_L$	=	initial out-of-straightness of the column at mid-height
$\bar{e}_L$	=	mean value of the initial out-of-straightness
e_{La}	=	transformed initial out-of-straightness

$e_{L,max}$	=	maximum initial out-of-straightness
$e_{L,min}$	=	minimum initial out-of-straightness
$f(y)$	=	the probability density function of a random variable y
h_h	=	total (overall) height of a wide-flange shape
k	=	number of elements in flange or web
m,n	=	total number of elements in the cross section
m_1,m_2	=	polynomial exponents
n_f	=	number of elements in the flange
n_w	=	number of elements in the web
p	=	mode of the Type I (Gumbel, largest value) asymptotic extreme value distribution of the yield stress (= $\sigma_{y,p}$)
q	=	mode of the Type I (Gumbel, smallest value) asymptotic extreme value distribution of the initial out-of-straightness
r	=	radius of gyration
s	=	standard deviation of a sample of data
$\hat{s}$	=	maximum likelihood estimator of the standard deviation
t	=	thickness of an element in the cross section (general)
t_a	=	non-dimensional thickness (actual thickness divided by 3/4")

t_b, t_h = flange thickness of a box shape and a wide-flange shape, respectively

t_{min} = minimum thickness, below thich $\sigma_{y,min}$ is constant ($t_{min} \approx 3/4"$)

u_b, u_h = random variables, pertaining to box and wide-flange shapes, respectively

v = deflection (general)

$v_p, \tilde{v}_p$ = deflection of the column, due to the applied axial load

$\bar{v}_p$ = mean value of the deflection caused by the applied axial load

w = thickness (general)

w_b, w_h = web thickness of a box shape and a wide-flange shape, respectively

x = coordinate, measured along the length of the column

x_i = coordinate of the centroid of element i in the cross section, given by the bending axis considered

$y_1, y_2, \ldots y_n$ = random variables

z_b, z_h = random variables, pertaining to box and wide-flange shapes, respectively

7.2 Glossary and Abbreviations

The following terms and expressions have been used frequently in the dissertation, and this glossary is intended to provide brief, but exhaustive definitions of the most significant items. The listing is given alphabetically, according to the first letter in the first word of each term.

Band of Column Curves:

A number of column curves for a variety of columns, drawn in the same diagram. For instance, drawing the column curves for all rolled wide-flange columns of A36 steel in the same figure, provides a column curve band.

Column Curve Spectrum:

The set of column curves that illustrates the variation of the strength of one particular column type, as caused by the variation of the column strength parameters.

Deterministic:

A deterministic analysis is one where the factors of concern do not exhibit random (unpredictable) variation; that is, there is a one-to-one correspondence between the functionally dependent and independent parameters.

Distribution Function:

A function that gives the probability of the event that the random variable is less than or equal to a certain value. For instance, $F(y_1) = P(y \leq y_1)$ gives the probability that y is less than or equal to the value y_1.

First-Order Probabilistic Approach:

A solution procedure that is based on probabilistic concepts and the randomness of the variables, but only to the extent of considering the first-order dispersion quantities (i.e. mean and variance).

Heavy and Light Columns:

A column is heavy if any one of its component plates has a thickness greater than or equal to one inch; otherwise it is light.

Initial Out-of-Straightness:

Deviation from perfect straightness about any axis of the column; exhibited by the column after the manufacture is completed, but prior to the application of any external load.

Maximum Strength of a Column:

The maximum load-carrying capacity of a column. The load is specifically given by the point on the load-deflection curve which has a horizontal tangent ($dP/d\delta = 0$). Beyond this point, an increase of the deflection is accompanied by a decrease of the load.

Multiple Column Curves:

A set of several column curves, where each curve is a fair representation of the strength of the column types assigned to it. This involves the categorizing of all columns in terms of behavior and strength, and those with related properties are assigned to one and the same of the multiple column curves.

Probabilistic:

A probabilistic analysis is one where the factors of concern do exhibit random variation, and the probabilities of their assuming specific values are taken into account.

Probability Density Function:

A function specifying the behavior of a continuous random variable over the range of variation. For an infinitesimally small interval dy, the probability density function gives the probability that the random variable assumes a value between y and (y+dy) by the expression $f(y) \cdot dy$.

Quasi-Steady Statistical (Probabilistic) Approach:

A solution procedure that is based on (known) deterministic relationships between the parameters considered, but where the parameters are treated as random variables. The variables do not vary with time; hence the use of the word steady.

Random Variable:

A numerical variable, whose specific value cannot be predicted with certainty before an experiment.

Semi-Probabilistic:

A semi-probabilistic analysis is one where some of the factors involved are treated as random variables, whereas the other factors are considered as deterministic quantities.

Statistical Independence:

Two (or more) random variables are said to be statistically independent if the probability of the occurrence of one of them does not influence the probability of the occurrence of the others.

Tangent Modulus Load:

The load at which a centrally loaded, initially perfectly straight, prismatic column; unrestrained between the pinned ends, may assume a deflected position. An infinitesimally higher load requires that the column deflects, in order to maintain structural equilibrium.

8. <u>TABLES AND FIGURES</u>

TABLE 1

DATA FOR ROLLED COLUMNS INCLUDED IN INVESTIGATION OF MAXIMUM STRENGTHS

Material*	Light or Heavy**	No. of Column Curves†	Column Sections
A7	L	18	W4x13,8x24,8x31,8x67, 12x50,12x65,5x18.5, 14x43,8x31(Annealed)
A7	H	2	W14x426
A36	H	2	W12x161
A242	L	6	W8x31,12x50,12x65
A514	L	8	W8x17,8x31,10x33,12x45
A514	H	2	W12x120
"A514"††	H	2	W10x112
		Sum of No. of Curves = 40	

*Designation according to ASTM Specifications(60).

**L = Light; H = Heavy.

†Includes both major and minor bending axes column curves.

††This is steel type USS 5Ni-Cr-Mo-V, with a nominal yield strength of 130 ksi (measured 140 ksi).

TABLE 2

DATA FOR WELDED H-COLUMNS INCLUDED IN INVESTIGATION OF MAXIMUM STRENGTHS (Notation as in Table 1)

Material	Light or Heavy	No. of Column Curves	Column Sections*
A7	L	8	H7x28(UM,FC),H10x62(UM,FC)
A36	L	2	H12x79(FC)
A36	H	24	H12x210(FC),H20x354(FC), H14x202(FC),H15x290(FCF and FCG,UMF and UMG),H24x428(UM, 2xFC)**,H24x428(Stress-relieved), H23x681(FC)
A572(50)	L	2	H12x79(FC)
A572(50)	H	2	H14x202(FC)
A441	H	8	H15x290(FCF and FCG,UMF and UMG)
A514	L	6	H7x28(FC,SH),H10x62(FC)
Hybrid	L	8	H7x28(FC;Fl A514,Web A441) H7x28(FC;Fl A514,Web A36) H7x28(FC;Fl A441,Web A36) H7x28(UM;Fl A441,Web A36)
		Sum of No. of curves = 60	

*The following notation is used for manufacturing method designation: UM = Universal mill; FC = Flame-cut; SH = Sheared; FCF = Flame-cut plates and fillet welds; FCG = Flame-cut plates and groove welds; UMF = Universal mill plates and fillet welds; UMG = Universal mill plates and groove welds.

**Two shapes with flame-cut plates, but different preheating techniques.

TABLE 3

DATA FOR WELDED BOX-COLUMNS INCLUDED IN INVESTIGATION OF MAXIMUM STRENGTHS (Notation as in Tables 1 and 2)

Material	Light or Heavy	No. of Column Curves	Column Sections
A7	L	4	□ 6x20,10x65
A36	L	2	□ 10x65
A36	H	2	□ 24x774
A514	L	4	□ 6x20,10x65
		Sum of No. of Curves = 12	

TABLE 4

COMPARISON OF THEORETICAL AND EXPERIMENTAL MAXIMUM COLUMN STRENGTHS FOR ROLLED WIDE-FLANGE COLUMNS
(Notation as in Tables 1 through 3)

Shape	Steel Grade	Light or Heavy	Axis	Experiment				Theory	α ++
				e/L	λ	L/r	P_{max}/P_y	P_{max}/P_y	
W12x161	A36	H	W	0.002	0.49	50	0.78	0.83	1.06
W 8x31	A242	L	W	0.0009 0.0013	0.75 1.00	54 72	0.82 0.78+	0.77 0.62	0.94 (0.80)
W12x120	A514	H	W	? 0.0002	0.55 0.92	30 50	0.89 0.82	-** 0.85	- 1.04
W10x112	"A514"*	H	W	0.0001	1.07	49	0.73	0.75	1.03

*This is steel type USS 5Ni-Cr-Mo-V, with a nominal yield strength of 130 ksi (measured 140 ksi).

**Since the out-of-straightness for the tested column is unknown, no comparison with the theory can be made.

+The reliability of this test result is highly questionalbe.

++The factor α is given by the ratio $\alpha = \dfrac{(P_{max}/P_y)\,\text{theory}}{(P_{max}/P_y)\,\text{experiment}}$

TABLE 5

COMPARISON OF THEORETICAL AND EXPERIMENTAL MAXIMUM COLUMN STRENGTHS FOR WELDED WIDE-FLANGE COLUMNS
(Notation as in Tables 1 through 4)

Shape	Steel Grade	Light or Heavy	Axis	Experiment				Theory	α^*
				e/L	λ	L/r	P_{max}/P_y	P_{max}/P_y	
H12x79	A36	L	W	0.0002	0.35	30	0.97	0.93	0.96
				0.0003	0.70	60	0.76	0.75	0.99
				0.0002	1.05	90	0.68	0.64	0.94
H14x202	A36	H	W	0.0009	0.34	30	0.97	0.94	0.97
				0.0006	0.68	60	0.84	0.78	0.93
				0.0002	1.02	90	0.64	0.60	0.94
H12x79	A572(50)	L	W	0.0003	0.40	30	0.90	0.91	1.01
				0.0011	0.81	60	0.76	0.70	0.92
				0.0002	1.22	90	0.60	0.58	0.97
H14x202	A572(50)	H	W	0.0006	0.84	60	0.80	0.70	0.88
				0.0008	1.26	90	0.61	0.53	0.87
H10x62	A514	L	W	0.0004	0.68	35	0.90	0.90	1.00
				0.0003	1.07	55	0.79	0.83	1.05

*See Table 4

TABLE 6

COMPARISON OF THEORETICAL AND EXPERIMENTAL MAXIMUM COLUMN STRENGTHS FOR WELDED BOX COLUMNS
(Notation as in Tables 1 through 5)

Shape	Steel Grade	Light or Heavy	Axis*	Experiment				Theory	α^+
				e/L	λ	L/r	P_{max}/P_y	P_{max}/P_y	
□ 6x20	A7**	L	P	0.0006 0.0002	0.44 0.70	32 51	0.93 0.75	0.90 0.73	0.97 0.97
□ 10x65	A7	L	P	0.0003 0.0006 0.0007	0.34 0.56 0.90	30 50 80	0.94 0.82 0.64	0.96 0.85 0.62	1.02 1.04 0.97
□ 6x20	A514	L	P	0.0007 0.001	0.76 1.15	40 60	0.91 0.69	0.85 0.63	0.94 0.91
□ 10x65	A514	L	P	0.0001 0.0005	0.56 0.94	30 50	0.94 0.87	0.91 0.82	0.97 0.94

*P = one of the principal axes (these box-shapes are square).

**Measured yield strength = 55.6 ksi ($>> \sigma_y$ of nominal A7).

+See Tables 4 and 5.

TABLE 7

COLUMN TYPES BELONGING TO COLUMN STRENGTH CATEGORY 1 (MAXIMUM STRENGTH STUDY)
(Notation as in Tables 1 through 3)

Material+	Manufacturing Method	Shape*	Light or Heavy	Axes
A242	Rl	W	L	S
A514 A514 A514 A514	Rℓ We/FC We/UM We	W H H B	LH L L L	SW S S SW
Hybrid: Fl A514 &Web A441 Fl A514 &Web A36	 We/FC We/FC	 H H	 L L	 SW SW
All stress-relieved columns (regardless of steel grade, manufacturing method, shape, size, and axes).				

*B = Box-shape.

+Included among the A514-steels are the special extra-high strength steels, such as USS 5Ni-Cr-Mo-V (σ_y = 130 ksi).

TABLE 8

COLUMN TYPES BELONGING TO COLUMN STRENGTH CATEGORY 2 (MAXIMUM STRENGTH STUDY)
(Notation as in Tables 1 through 3, and 7)

Material	Manufacturing Method	Shape	Light or Heavy	Axes
A7/A36	Rl	W	L	SW
A7/A36	We/FC	H	LH	SW
A7/A36	We	B	LH	SW
A242	Rl	W	L	W
A572(50)	We/FC	H	LH	SW
A441	We/FC	H	H	SW
A441	We/UM	H	H	SW
A514	We/FC	H	L	W
A514	We/UM	H	L	W
Hybrid:				
Fl A441 &Web A36	We/FC	H	L	SW
Fl 441 &Web A36	We/UM	H	L	SW

TABLE 9

COLUMN TYPES BELONGING TO COLUMN STRENGTH CATEGORY 3 (MAXIMUM STRENGTH STUDY)
(Notation as in Tables 1 through 3, and 7 and 8)

Material	Manufacturing Method	Shape	Light or Heavy	Axes
A7/A36	R1	W	H	SW
A7/A36	We/UM	H	LH	SW

TABLE 10

STATISTICAL PROPERTIES OF THE BAND OF CURVES CONTAINING POSSIBLE COLUMN STRENGTH CURVE 1
(30 curves, e/L = 1/1000)

λ	Arithmetic Mean*	Median	Percentiles $2\frac{1}{2}$	Percentiles $97\frac{1}{2}$	Standard Deviation	Coeff. of Variation
0.3	0.974	0.979	0.951	0.990	0.012	1.2
0.5	0.937	0.936	0.895	0.967	0.027	2.8
0.7	0.878	0.878	0.830	0.935	0.037	4.2
0.9	0.787	0.782	0.741	0.851	0.037	4.7
1.1	0.655	0.651	0.600	0.705	0.029	4.4
1.3	0.513	0.513	0.478	0.543	0.017	3.2
1.5	0.396	0.399	0.362	0.415	0.012	3.0
1.7	0.311	0.311	0.277	0.324	0.010	3.1
1.9	0.252	0.252	0.230	0.261	0.006	2.4

*Column Strength Curve 1 follows the arithmetic mean curve.

TABLE 11

STATISTICAL PROPERTIES OF THE BAND OF CURVES CONTAINING POSSIBLE COLUMN STRENGTH CURVE 2
(70 curves, e/L = 1/1000)

λ	Arithmetic Mean*	Median	Percentiles $2\frac{1}{2}$	Percentiles $97\frac{1}{2}$	Standard Deviation	Coeff. of Variation
0.3	0.936	0.937	0.889	0.970	0.020	2.2
0.5	0.849	0.845	0.780	0.921	0.036	4.3
0.7	0.749	0.743	0.679	0.849	0.046	6.2
0.9	0.646	0.637	0.567	0.760	0.050	7.7
1.1	0.539	0.541	0.458	0.633	0.045	8.2
1.3	0.439	0.442	0.373	0.493	0.032	7.3
1.5	0.355	0.359	0.305	0.390	0.023	6.3
1.7	0.290	0.292	0.252	0.311	0.016	5.6
1.9	0.239	0.241	0.211	0.255	0.012	4.9

*Column Strength Curve 2 follows the arithmetic mean.

TABLE 12

STATISTICAL PROPERTIES OF THE BAND OF CURVES CONTAINING POSSIBLE COLUMN STRENGTH CURVE 3
(12 curves, e/L = 1/1000)

λ	Arithmetic Mean*	Median	Percentiles $2\frac{1}{2}$	$97\frac{1}{2}$	Standard Deviation	Coeff. of Variation
0.3	0.914	0.920	0.894	0.937	0.020	2.2
0.5	0.795	0.802	0.744	0.854	0.044	5.6
0.7	0.674	0.684	0.601	0.760	0.063	9.4
0.9	0.567	0.576	0.490	0.654	0.067	11.7
1.1	0.470	0.477	0.402	0.544	0.058	12.4
1.3	0.383	0.392	0.332	0.444	0.043	11.1
1.5	0.314	0.319	0.274	0.358	0.032	10.1
1.7	0.260	0.260	0.223	0.290	0.024	9.3
1.9	0.218	0.219	0.190	0.243	0.019	8.7

*Column Strength Curve 3 follows the arithmetic mean.

TABLE 13

TOLERANCE REQUIREMENTS FOR CROSS-SECTIONAL PROPERTIES OF ROLLED WIDE-FLANGE SHAPES (According to Euronorm 34-62 (Ref. 82).)

<table>
<tr><th rowspan="2">Wide-Flange Shapes with Specified Total Depths of</th><th colspan="5">Tolerances</th></tr>
<tr><th>Flange Width</th><th>Total Depth</th><th>Flange Thickness</th><th>Web Thickness</th><th>Area</th></tr>
<tr><td>-160mm(6.3")</td><td rowspan="8">±3mm (=±0.12")</td><td>+4mm=+0.16"
-2mm=-0.08"</td><td rowspan="2">±1.5mm=±0.06"</td><td rowspan="2">±1mm =±0.04"</td><td rowspan="8">±6%</td></tr>
<tr><td>160-220mm(8.7")</td><td rowspan="4">±3mm=±0.12"</td></tr>
<tr><td>220-260mm(10.2")</td><td rowspan="6">±2mm =±0.08"</td><td rowspan="5">±1.5mm =±0.06"</td></tr>
<tr><td>260-300mm(11.8")</td></tr>
<tr><td>300-400mm(15.7")</td></tr>
<tr><td>400-500mm(19.7")</td><td>±4mm=±0.16"</td></tr>
<tr><td>500-700mm(27.6")</td><td rowspan="2">±5mm=±0.20"</td></tr>
<tr><td>700-1000mm(39.4")</td><td>±2mm=±0.08"</td></tr>
</table>

TABLE 14

STATISTICAL CHARACTERISTICS OF SOME IMPORTANT ASTM STEEL GRADES
(For material with thickness < 3/4"; value in ksi)
(Hypothetical Data)

Steel Grade*	σ_{ym}+	Mode $\sigma_{y,p}$	Mean $\bar{\sigma}_y$	$\sigma_{y,max}$**	St. deviation σ_{σ_y}	Dispersion Factor κ
A36	36	37.6	38.3	42	1.54	0.83
A242	50	52.1	53.0	58	2.05	0.63
A441	50	52.1	53.0	58	2.05	0.63
A572(50)	50	52.1	53.0	58	2.05	0.63
A572(65)	65	67.6	68.8	75	2.56	0.50
A514	100	103.9	105.6	115	3.85	0.33

*Designation according to Ref. 63

**Estimated value, based on data in Refs. 63 and 88

+Specified yield stress (63)

TABLE 14A

BASES FOR THE ASSESSMENT OF THE STATISTICAL CHARACTERISTICS OF RESIDUAL STRESSES AND YIELD STRESSES IN SOME SHAPES. (Number of measurements of each quantity)

Column Shape	Number of measurements for		Coefficient of Var.of maximum Col.strength (computed)
	Residual str. σ_r	Yield stress σ_y	
W8x31 (A36)	20	20	3-7%
W14x426 (A36)	1	1	2-8%
H12x79 (A572(50))	5	5	3.5-7%
H14x202 (A572(50))	5	5	3-7%
H15x290 (A36)	1	1	2-8.5%
□6x20 (A514)	1	3	3-7%
□10x65 (A36)	12	12	3-7%

TABLE 15

COLUMN CURVE SPECTRUM DATA FOR A LIGHT ROLLED WIDE-FLANGE SHAPE W8x31 (ASTM A36) (Various values of the column strength factors)*

σ_y (ksi)	Axis	e/L	1/1470 (Mean)			1/10,000 (Min)
		λ	0.3	0.9	1.5	0.9
42 (Max)	y	Mean C.ofV.	0.959 0.17	0.659 0.11	0.363 0.61	0.729 0.08
	x	Mean C.ofV.	0.976 0.04	0.759 0.07	0.393 0.20	0.827 0.08
38.3 (Mean)	y	Mean C.ofV.	0.956 0.11	0.644 0.09	0.355 0.62	0.711 0.13
	x	Mean C.ofV.	0.974 0.05	0.748 0.19	0.389 0.28	0.818 0.08
36 (Min)	y	Mean C.ofV.	0.955 0.07	0.633 0.39	0.351 0.23	0.699 0.09
	x	Mean C.ofV.	0.974 0	0.742 0.20	0.385 0.21	0.811 0.12

*Column strength means given in terms of the ratio P_{max}/P_y, and coefficients of variation are given in percent.

TABLE 16

COLUMN TYPES ASSIGNED TO CATEGORY 1P
(Probabilistic Study)
(Notation as in Tables 1 through 3, and 7)

Material	Manufacturing Method	Shape	Light or Heavy	Axes
A242	Rolled	W	L	SW
A514	Rolled	W	LH	SW
A514	Welded and Flame-Cut	H	L	SW
A514	Welded and Universal Mill	H	L	S
A514	Welded	B	L	SW
Hybrid:				
FL A514 &Web A441	Welded and Flame-Cut	H	L	SW
FL A514 &Web A36	Welded and Flame-Cut	H	L	SW
All stress-relieved shapes				

TABLE 17

COLUMN TYPES ASSIGNED TO CATEGORY 2P
(Probabilistic Study)
(Notation as in Tables 1 through 3)

Material	Manufacturing Method	Shape	Light or Heavy	Axes
A36	Rolled	W	L	SW
A36	Rolled	W	H	S
A36	Welded, Flame -Cut	H	LH	SW
A36	Welded, Universal Mill	H	LH	S
A36	Welded	B	LH	SW
A572(50)	Welded, Flame -Cut	H	LH	SW
A441	Welded, Flame -Cut	H	H	SW
A441	Welded, Universal Mill	H	H	SW
A514	Welded, Universal Mill	H	L	W
Hybrid:				
FL A441 &Web A36	Welded, Flame -Cut	H	L	SW
FL A441 &Web A36	Welded, Universal Mill	H	L	SW

TABLE 18

COLUMN TYPES ASSIGNED TO CATEGORY 3P
(Probabilistic Study)
(Notation as in Tables 1 through 3)

Material	Manufacturing Method	Shape	Light or Heavy	Axes
A36	Rolled	W	H	W
A36	Welded, Universal Mill	H	LH	W

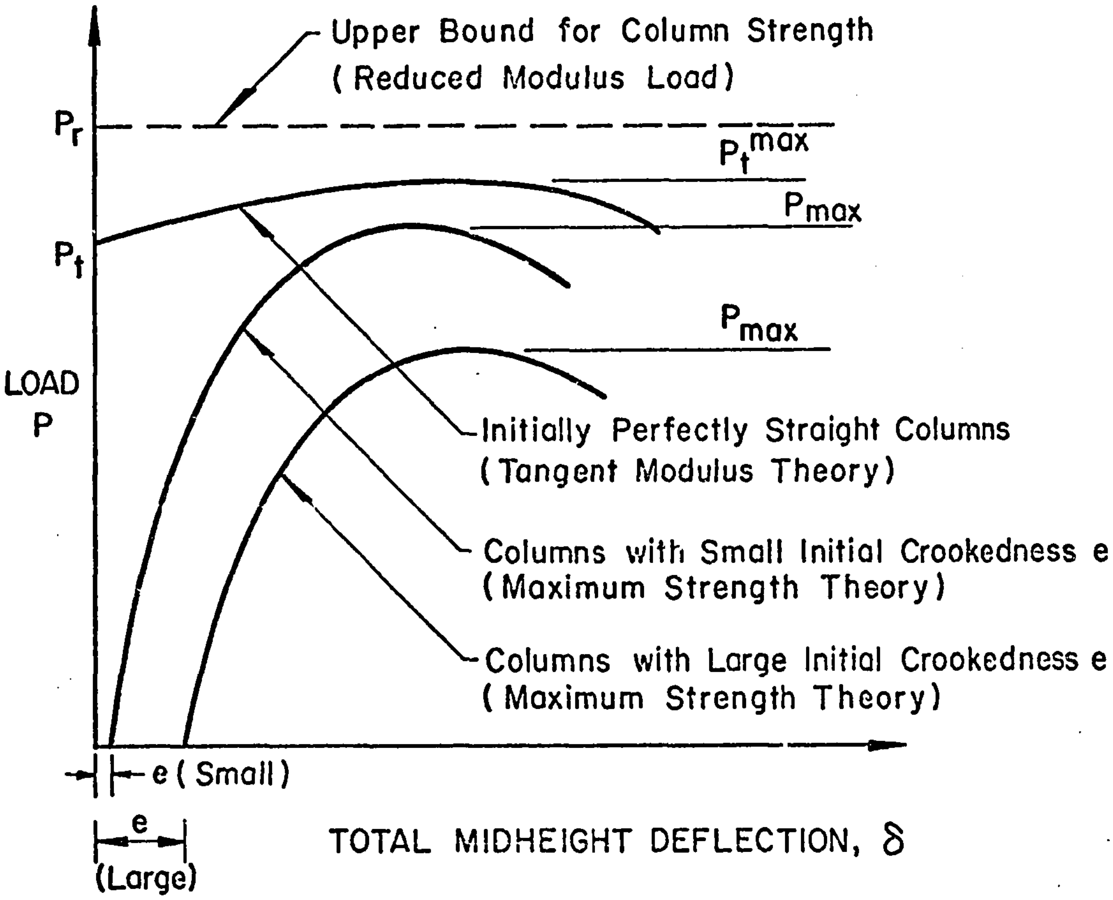


Fig. 1 A Schematic Illustration of the Various Inelastic Column Strength Concepts

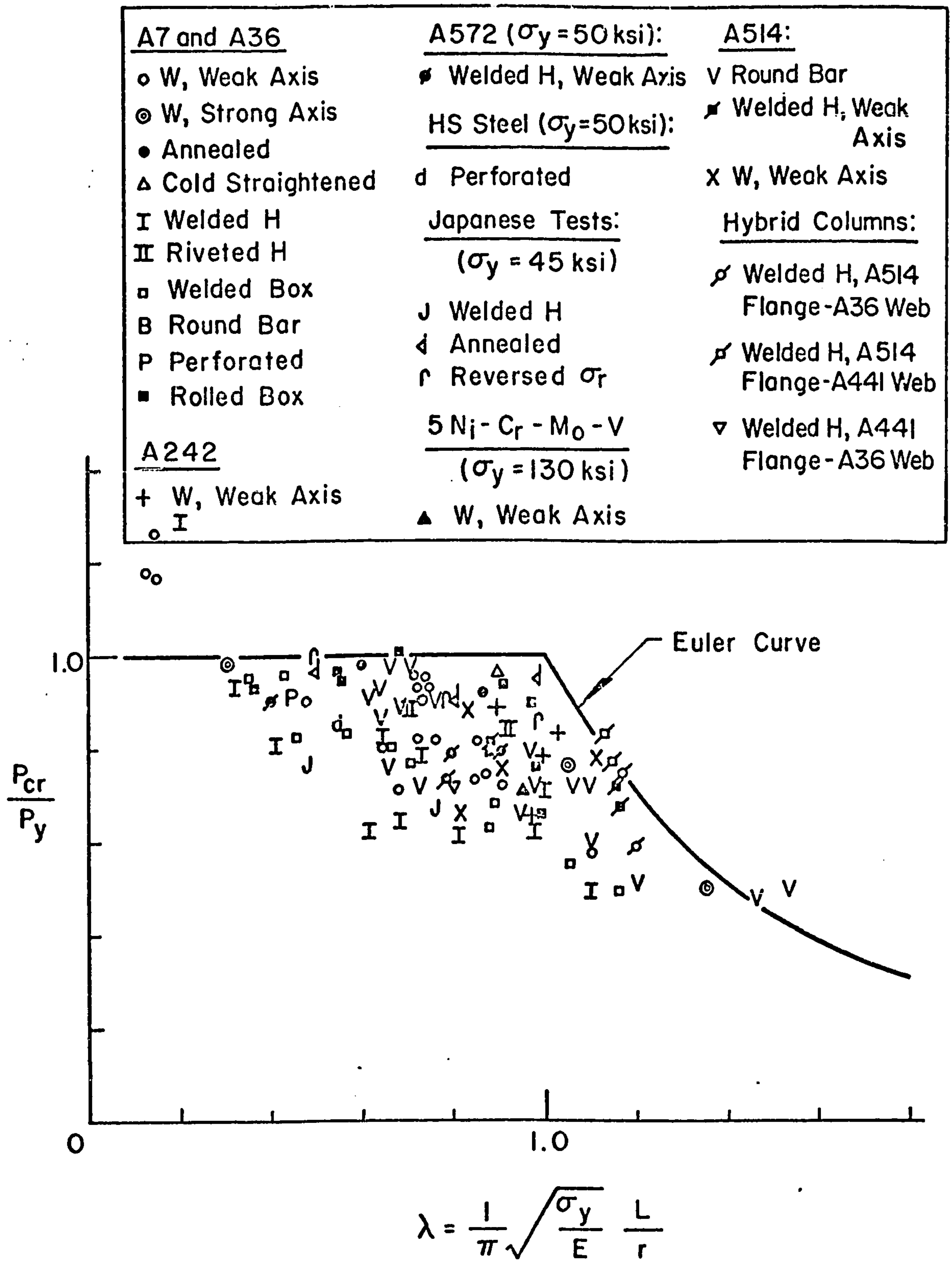
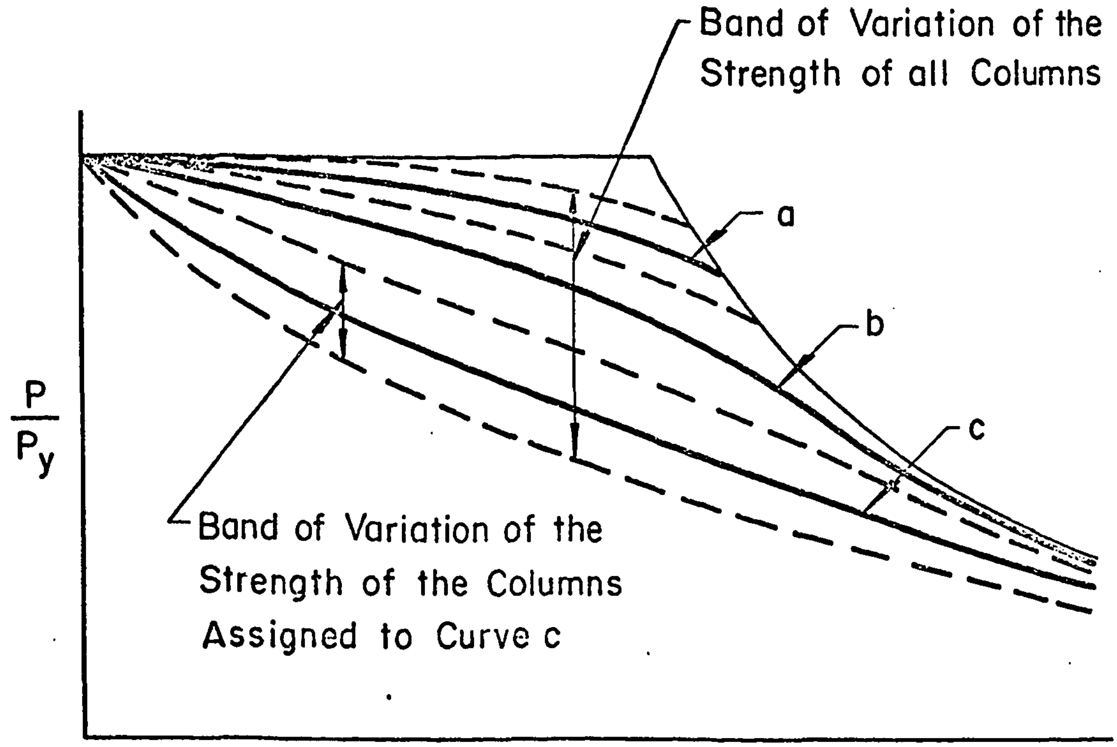


Fig. 2 Test Results for Approximately 100 Centrally Loaded Columns



$$\lambda = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}}\frac{L}{r}$$

Fig. 3 The Variation of Column Strength for Each Multiple Column Curve Category, Compared to the Variation of the Strength of All Columns

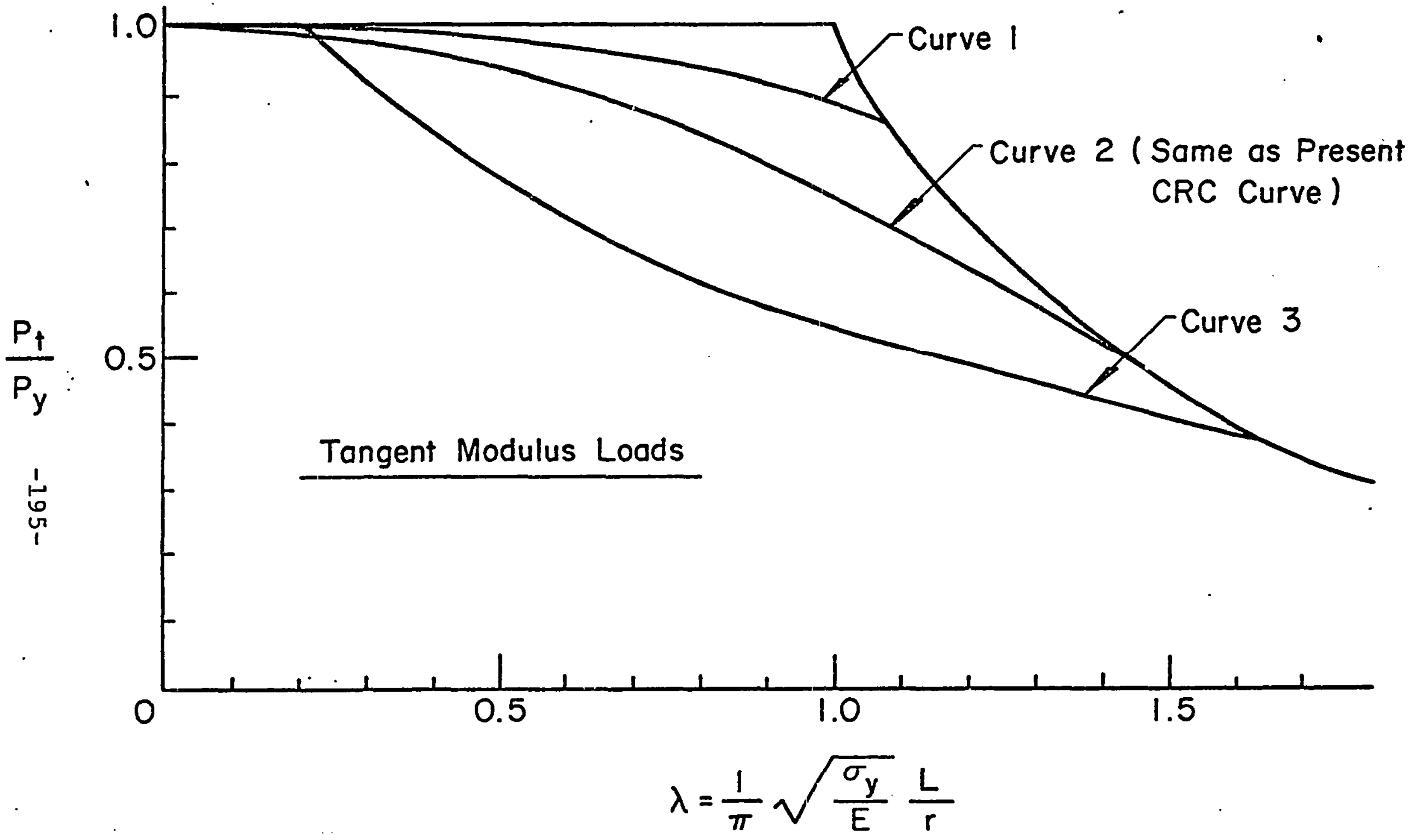


Fig. 4 Possible Multiple Column Curves, Based on the Tangent Modulus Concept

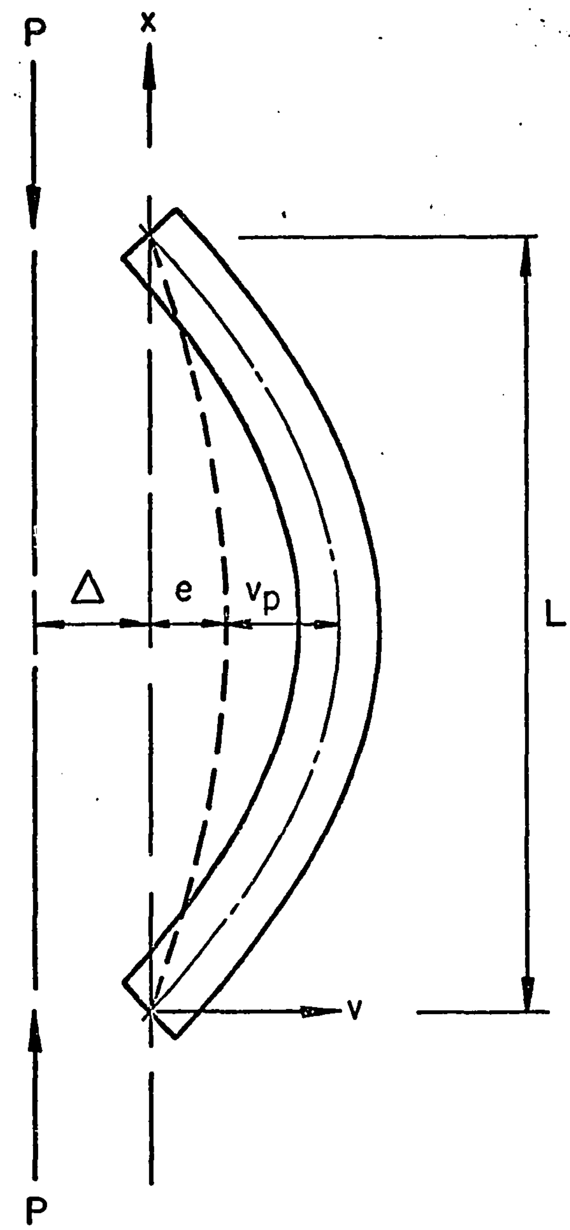
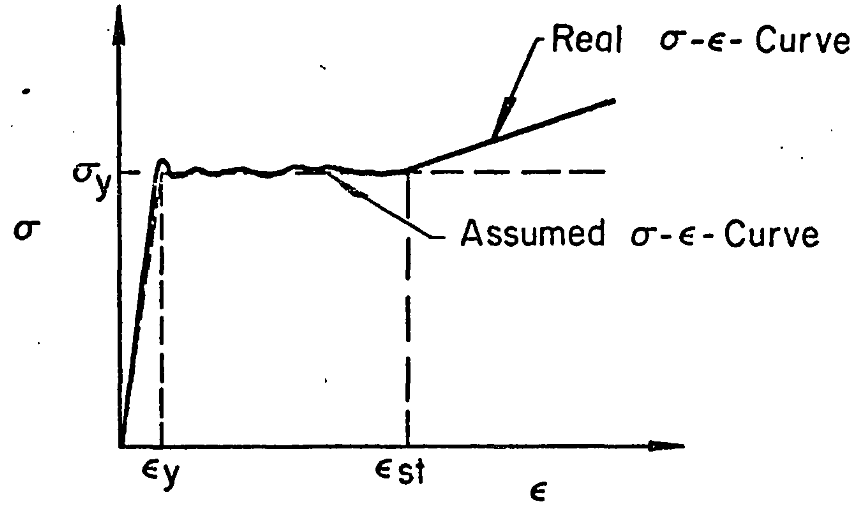
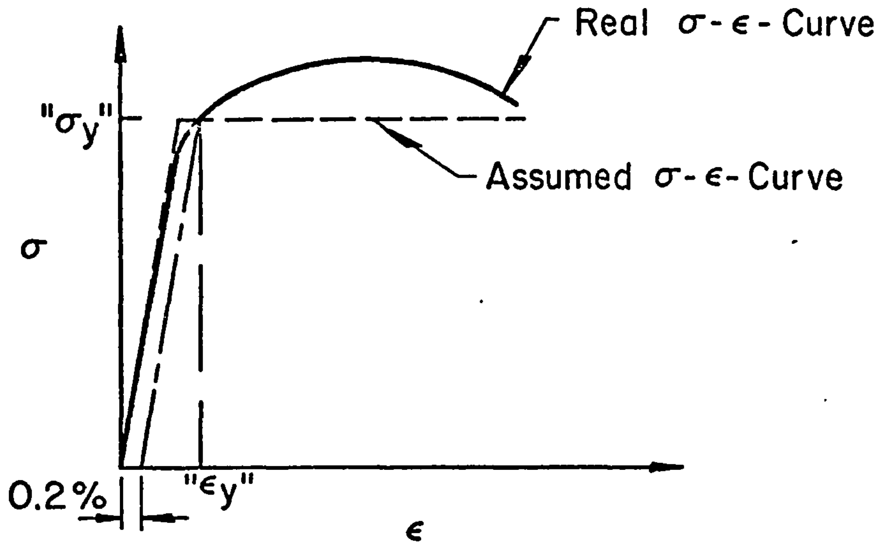


Fig. 5 A Centrally Loaded Column and Its Deflected Shape Characteristics



(a) Mild Structural Steel



(b) High Strength Steel

Fig. 6 Real and Assumed Stress-Strain Curves for Mild and High-Strength Structural Steels

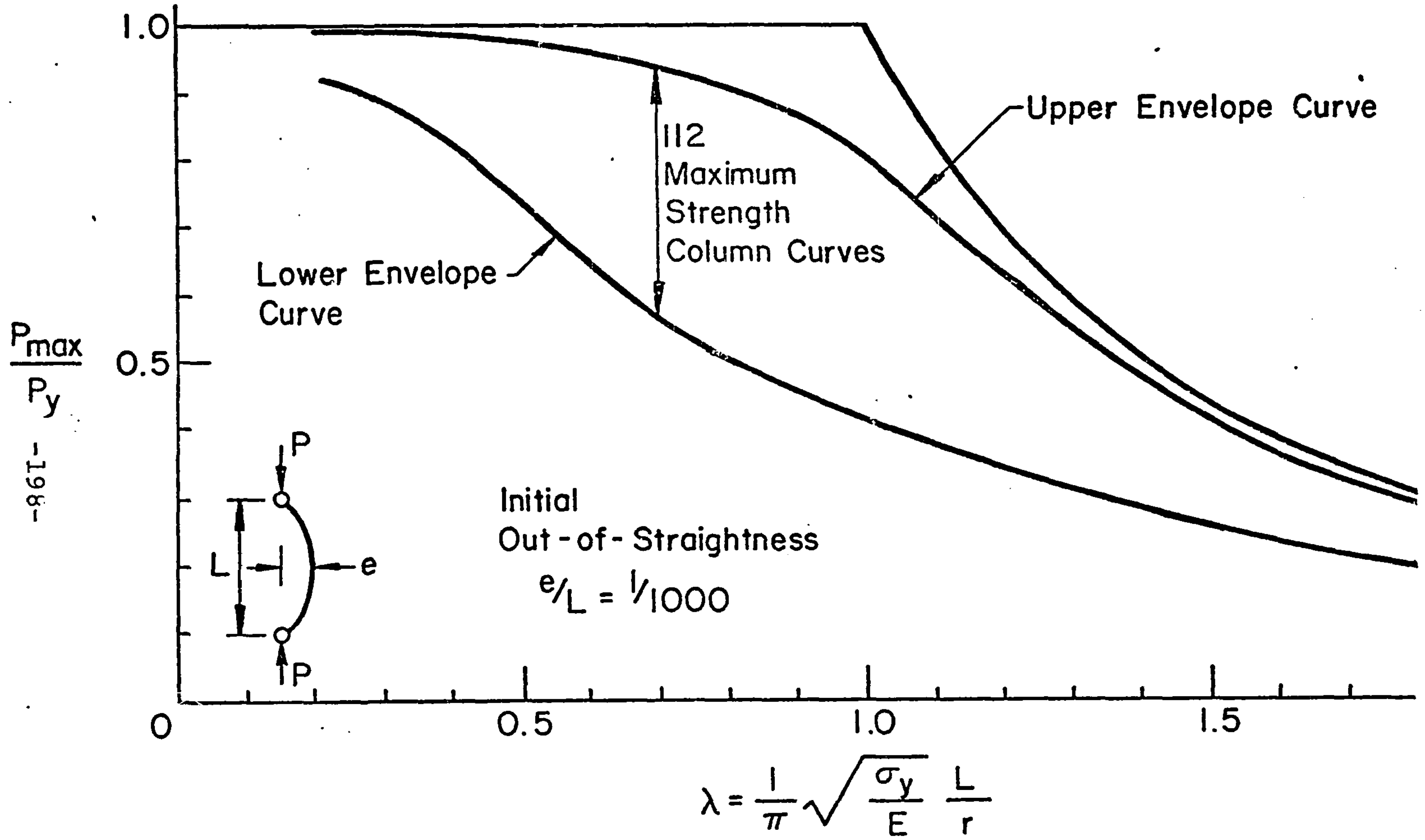


Fig. 7 The Band of all 112 Maximum Strength Column Curves, Based on an Initial Out-of-Straightness e/L = 1/1000

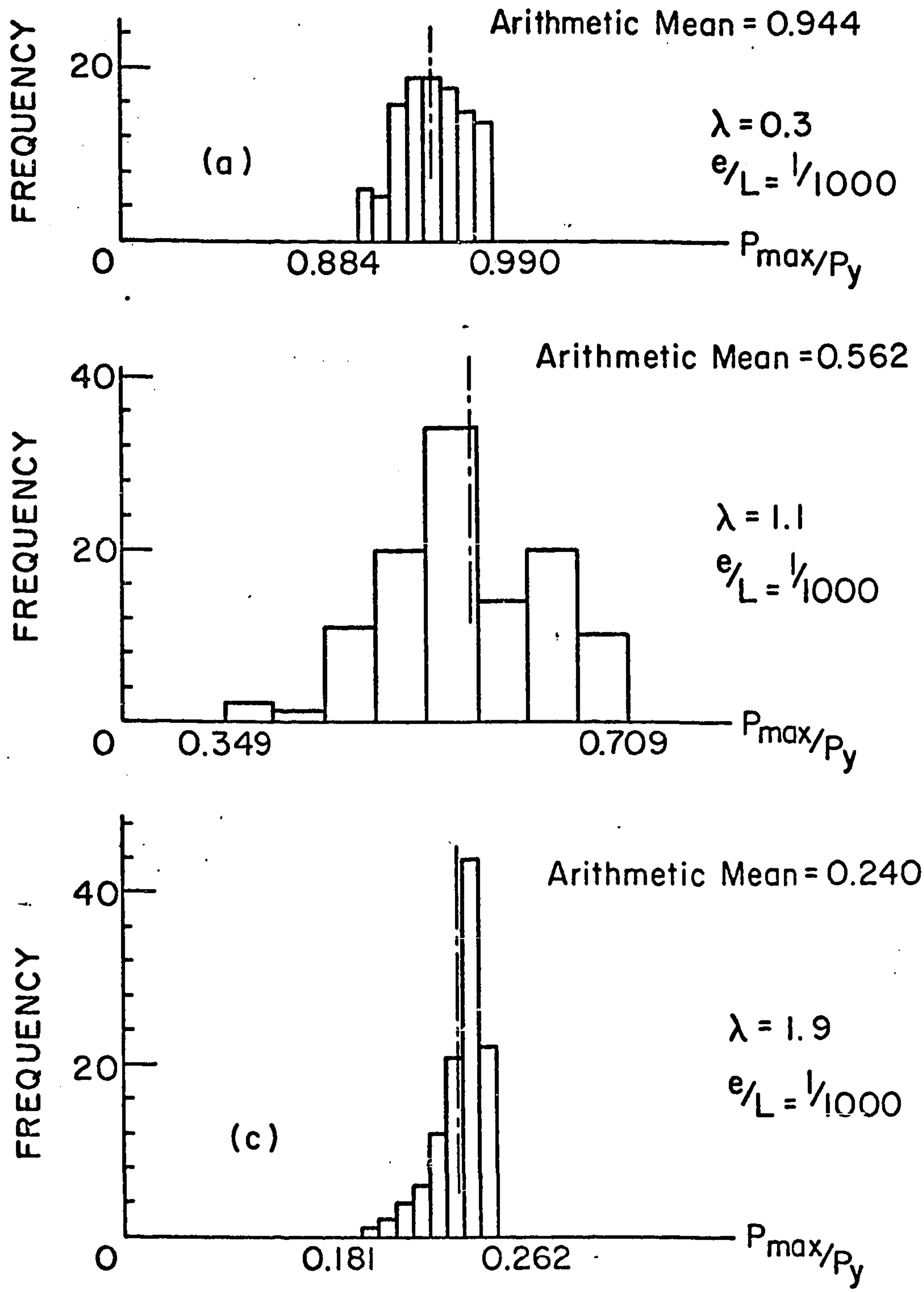


Fig. 8 Typical Frequency Distribution Histograms for the Maximum Strength of All 112 Column Curves with Initial Out-of-Straightness e/L = 1/1000

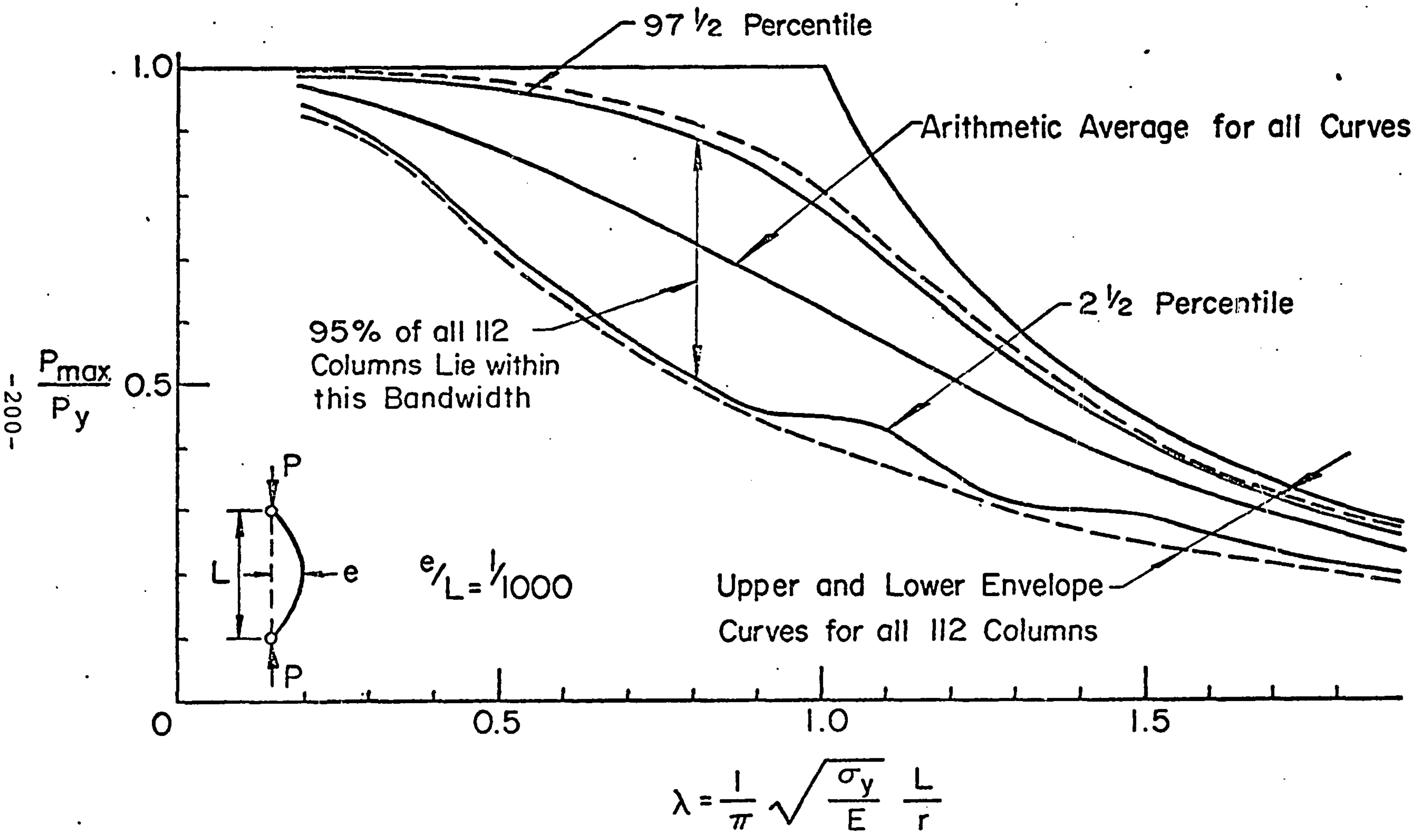


Fig. 9 Central Tendency and Dispersion Characteristics for the Band of 112 Column Curves (Initial Out-of-Straightness $e/L = 1/1000$)

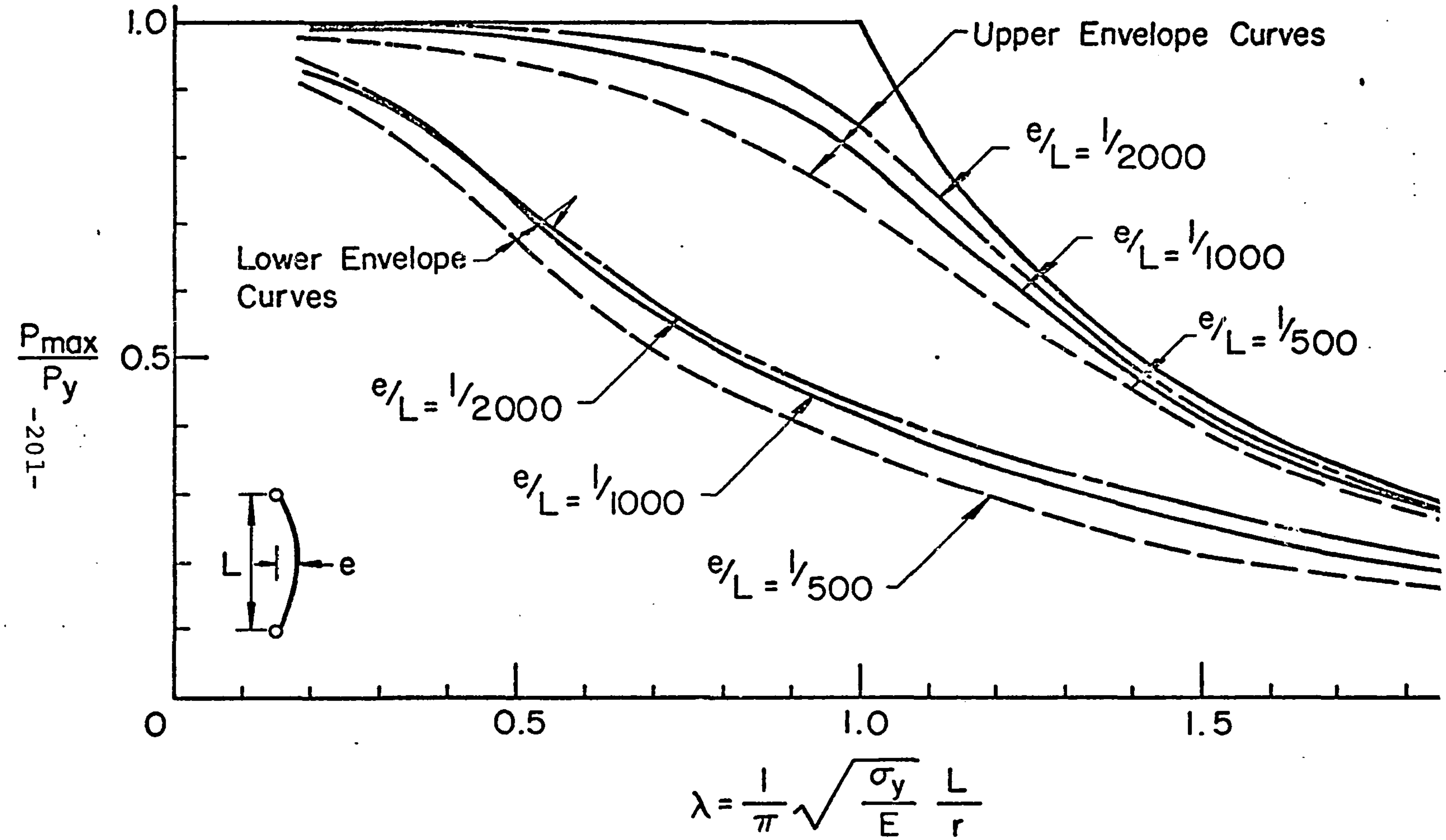


Fig. 10 The Effect of Initial Out-Straightness, Illustrated by a Comparison of the Three Bands of 112 Column Curves, (Based on Initial Out-of-Straightness e/L = L/500, L/1000, and L/2000)

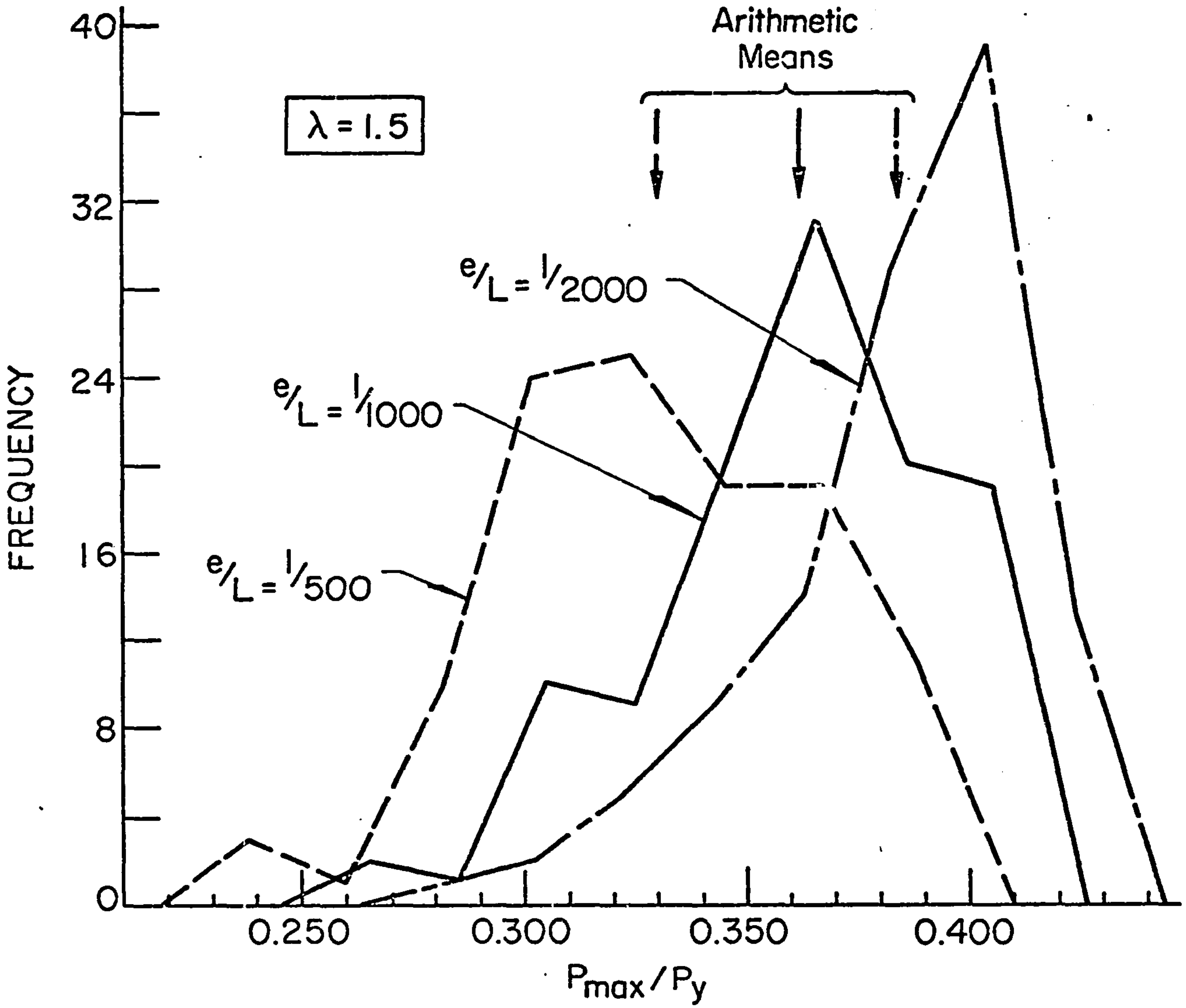


Fig. 11 Comparison of the Frequency Distribution Curves of Maximum Strength for $\lambda = 1.5$, with Three Initial Out-of-Straightnesses

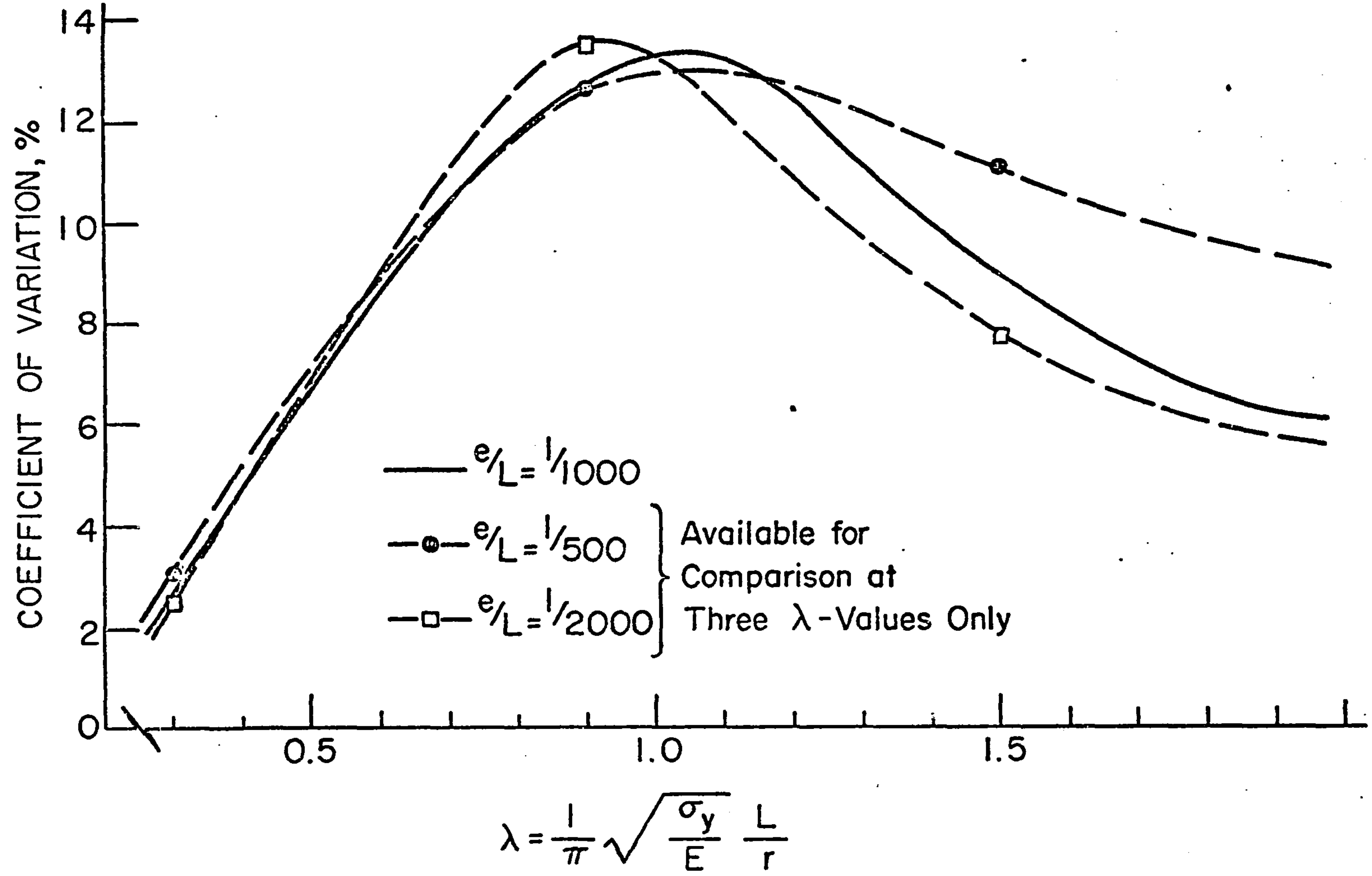


Fig. 12 The Relationship Between the Coefficient of Variation and the Non-Dimensional Slenderness Ratio for all 112 Maximum Strength Column Curves

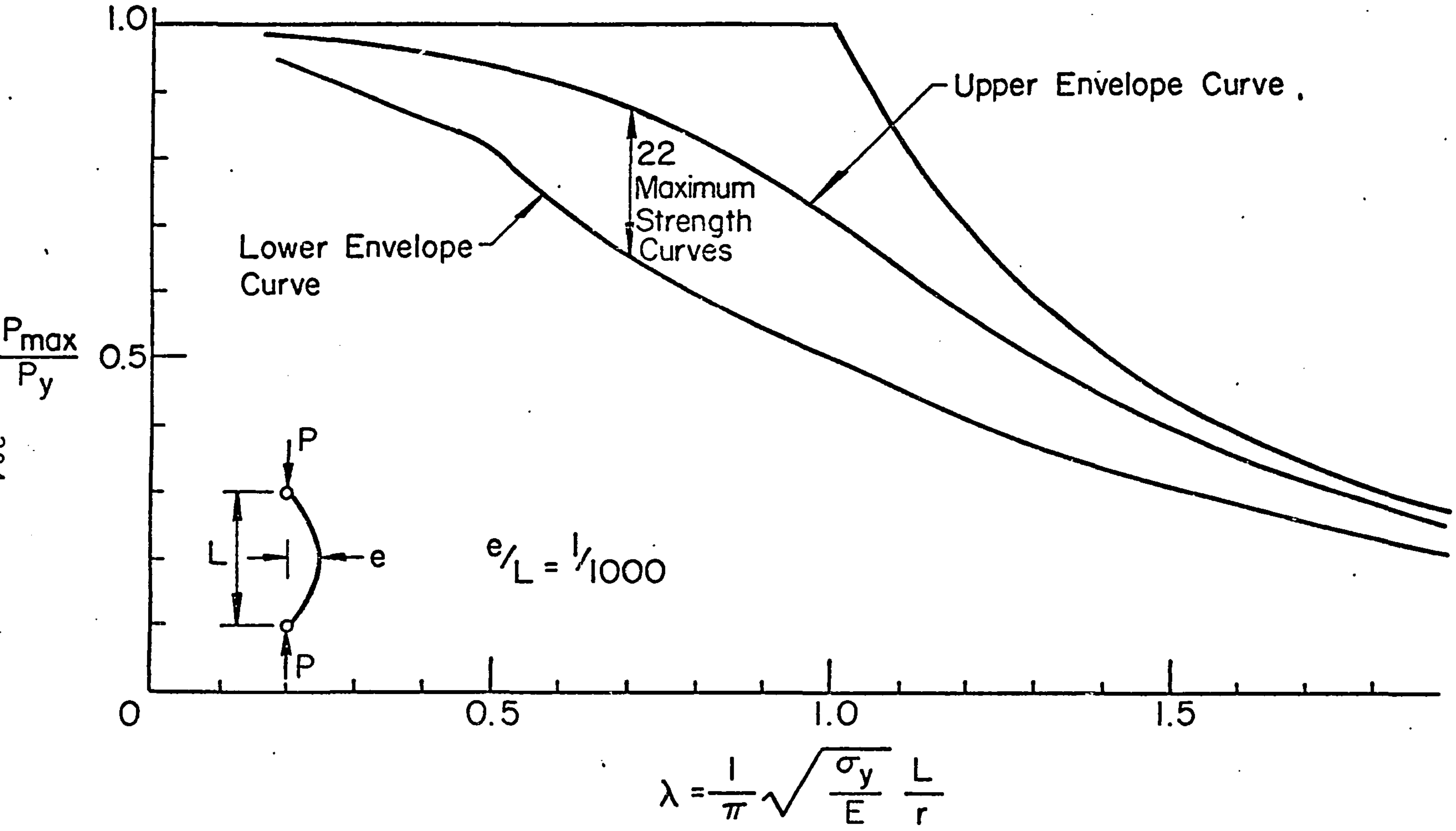


Fig. 13 The Band of Maximum Strength Column Curves for Rolled Wide-Flange Shapes of Steel Grades ASTM A7 and A36 (Heavy and Light Shapes, 22 curves, e/L = 1/1000)

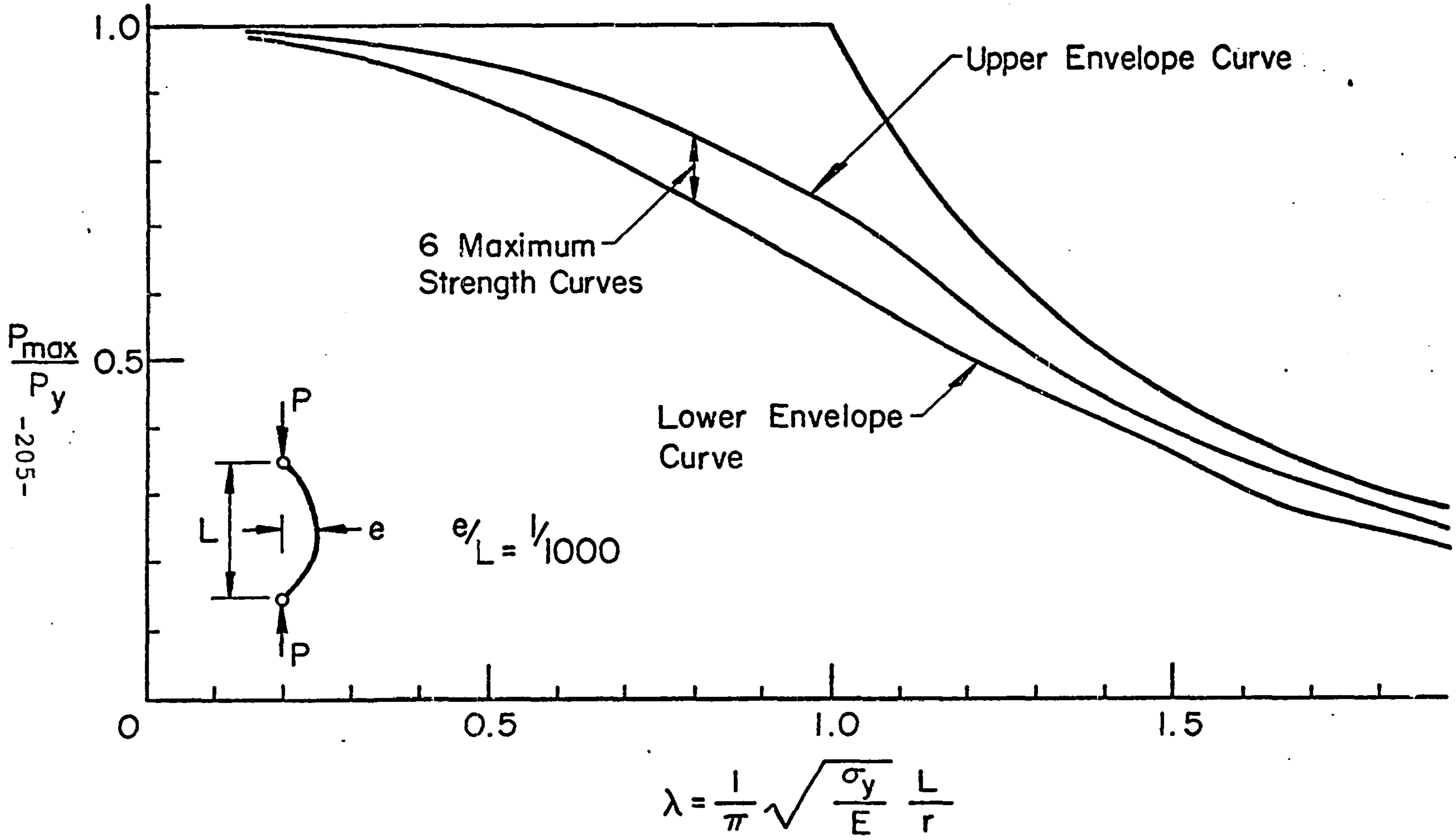


Fig. 14 The Band of Maximum Strength Column Curves for Rolled Wide-Flange Light Shapes of Steel Grade ASTM A242 (6 curves, e/L = 1/1000)

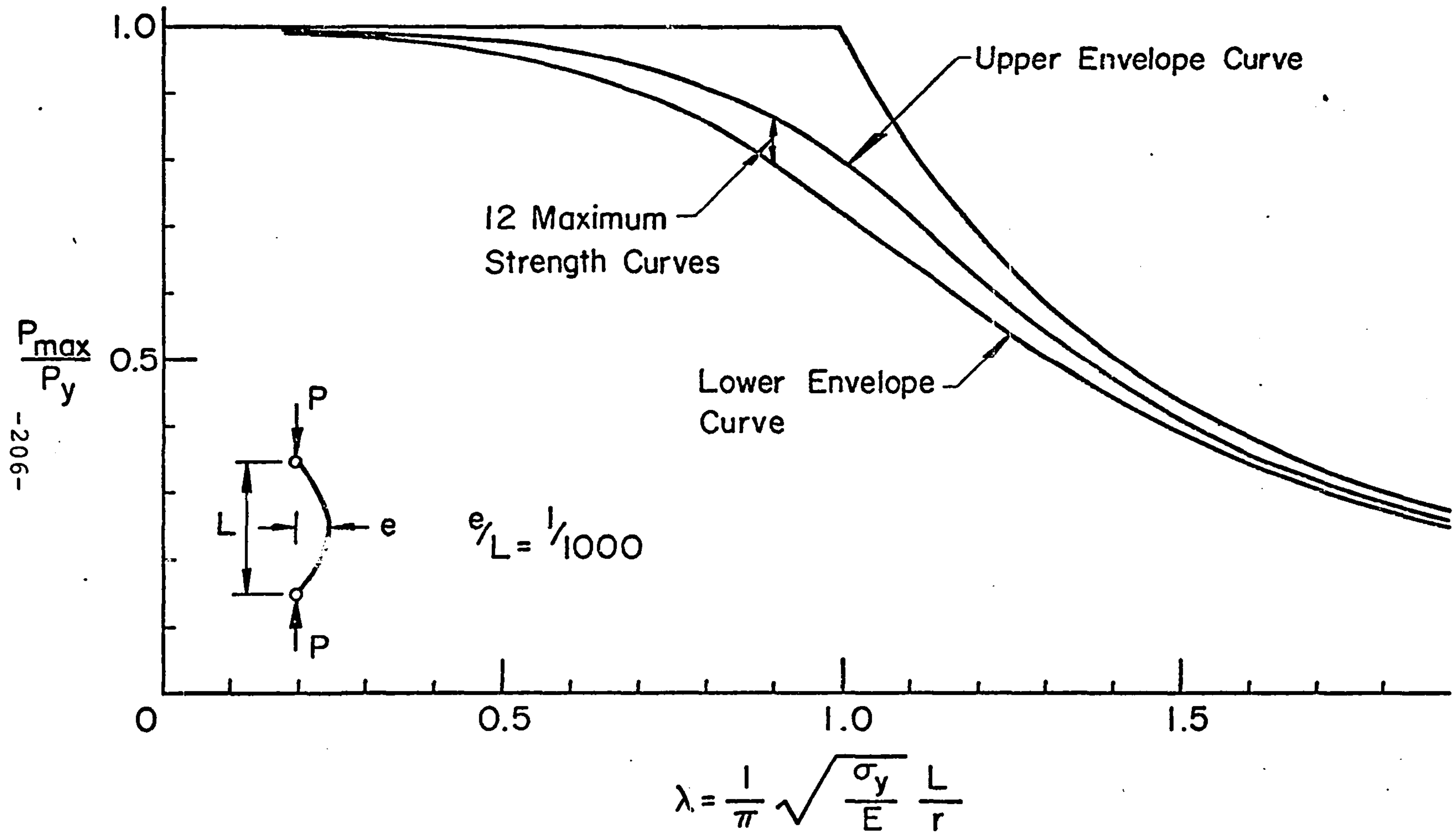


Fig. 15 The Band of Maximum Strength Column Curves for Rolled Wide-Flange Shapes of Steel Grade ASTM A514 (Heavy and Light Shapes, 12 curves, e/L = 1/1000)

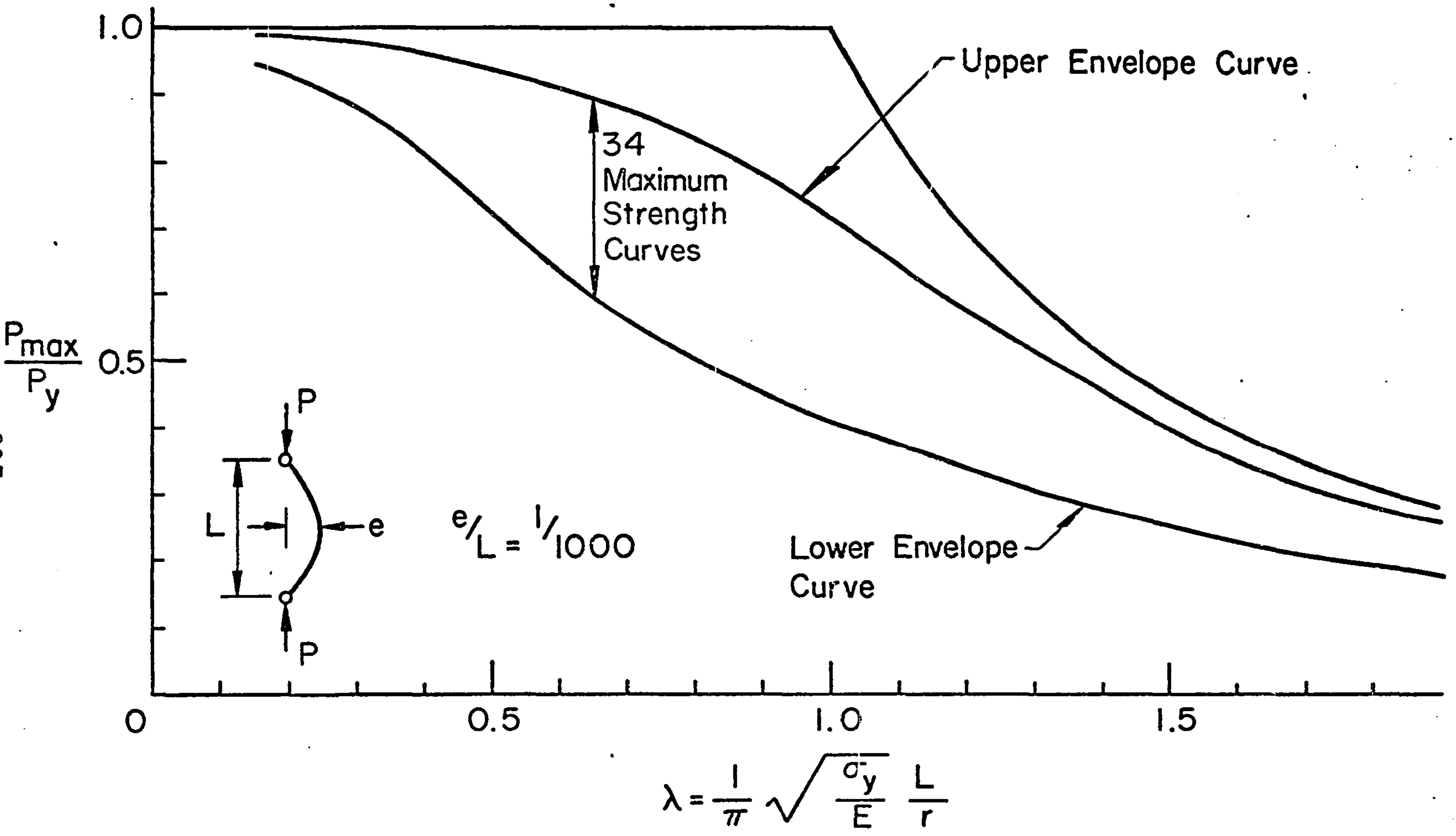


Fig. 16 The Band of Maximum Strength Column Curves for Welded Wide-Flange Shapes of Steel Grades ASTM A7 and A36 (Heavy and Light Shapes, 34 curves, e/L = 1000)

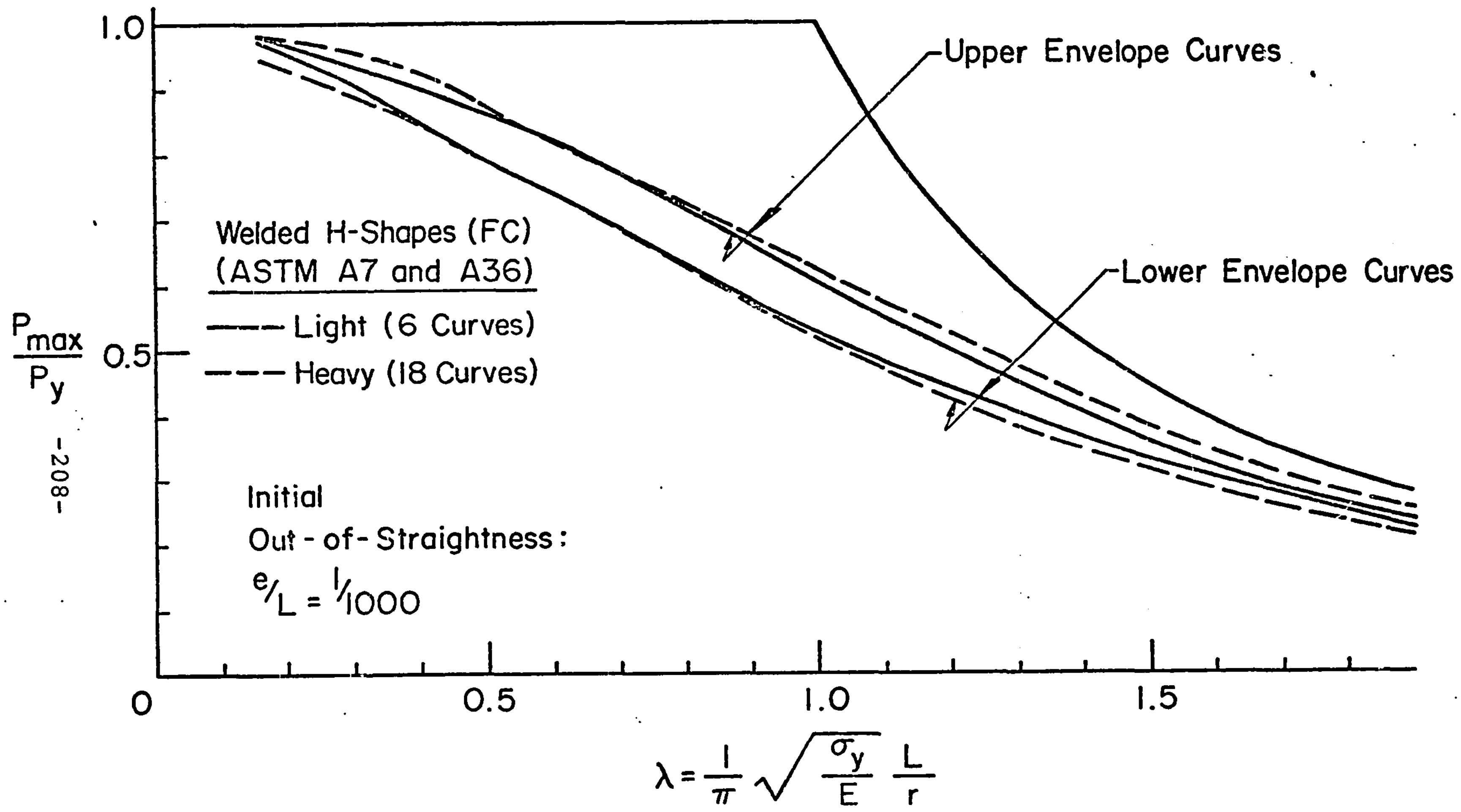


Fig. 17 Comparison of the Maximum Strength Column Curves for Heavy and Light Welded Flame-Cut H-Shapes in Steel Grades ASTM A7 and A36 (e/L = 1/1000)

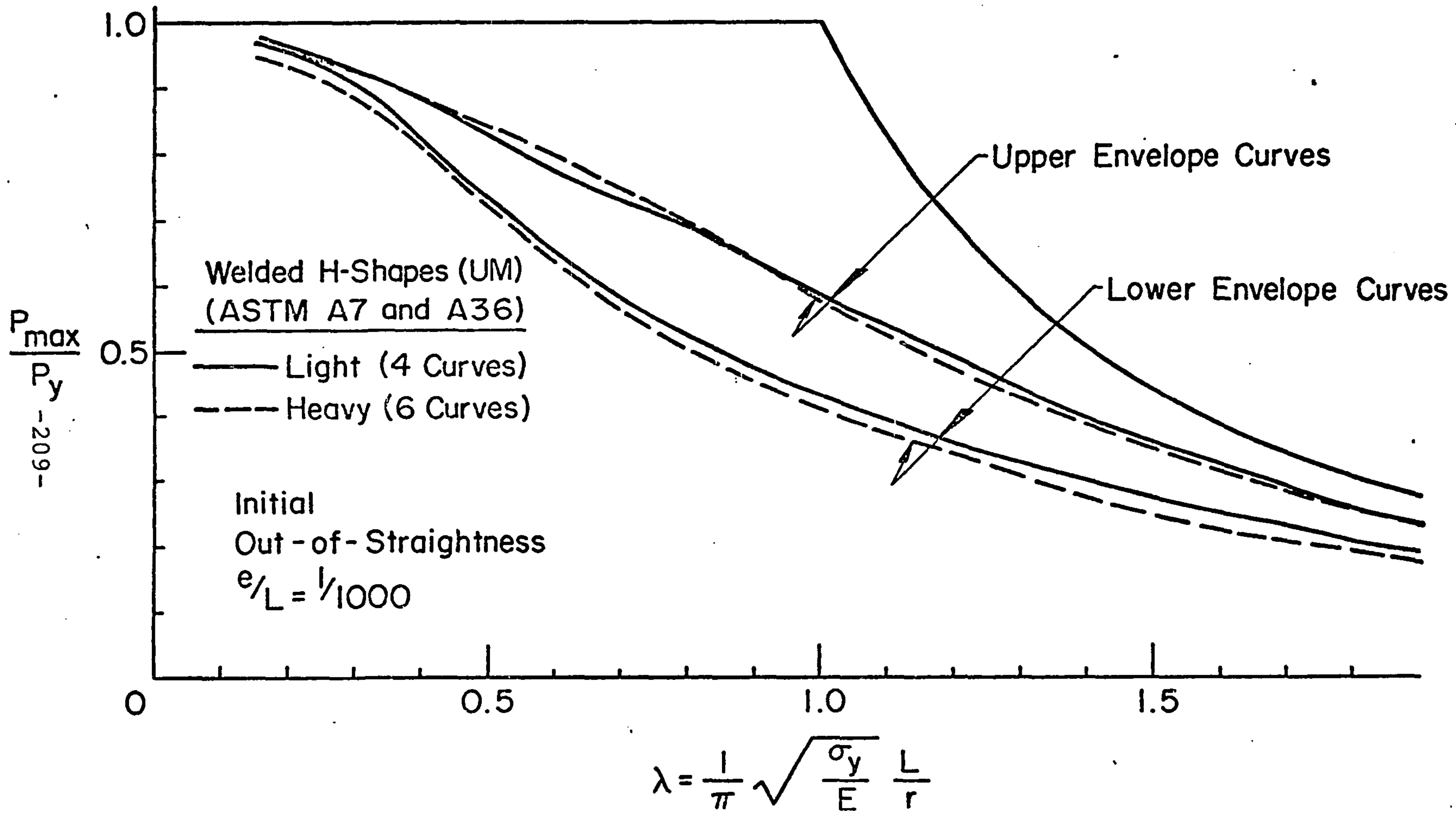
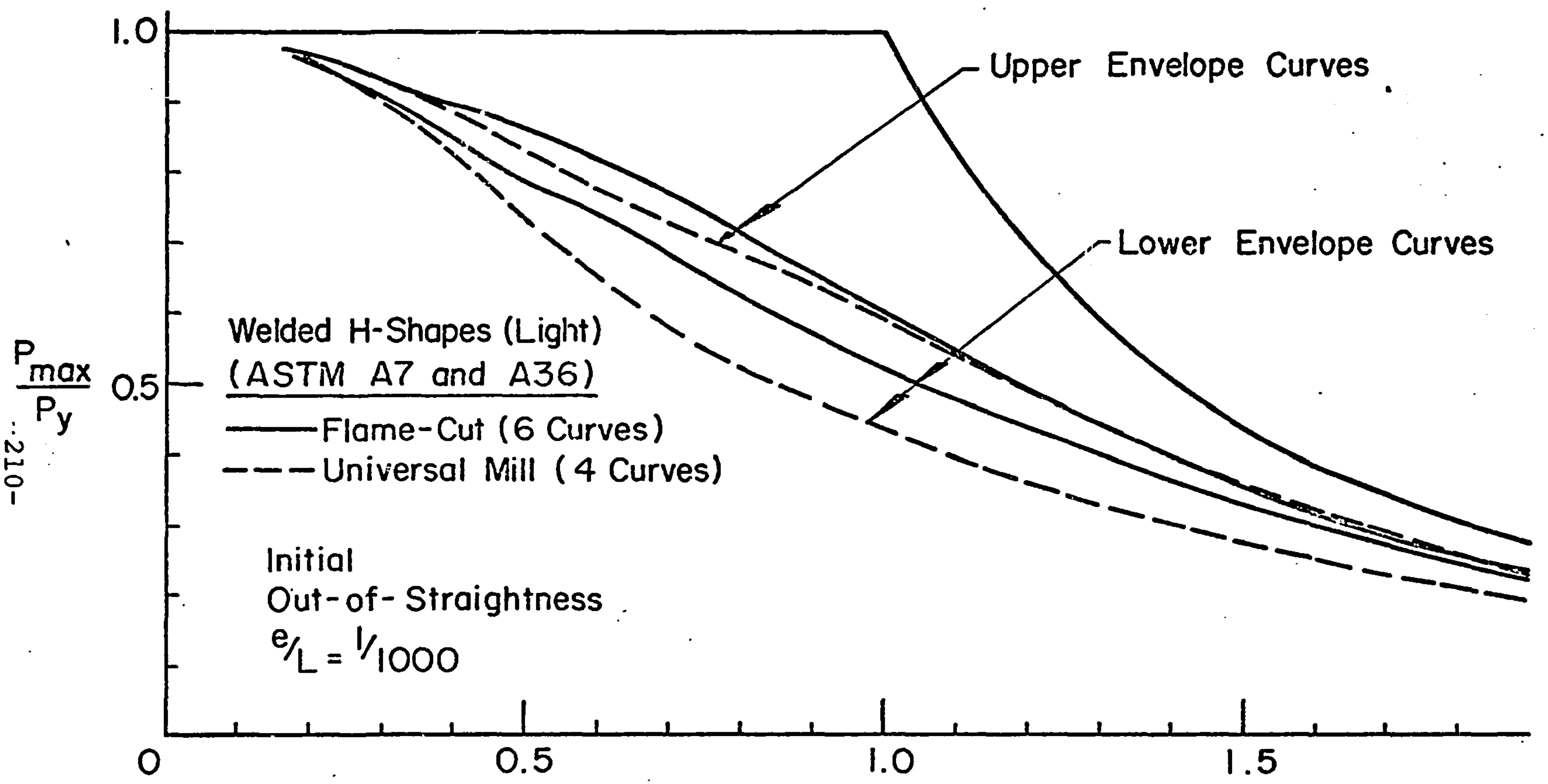


Fig. 18 Comparison of the Maximum Strength Column Curves for Heavy and Light Welded Universal Mill H-Shapes in Steel Grades ASTM A7 and A36 (e/L = 1/1000)



$$\lambda = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}}\,\frac{L}{r}$$

Fig. 19 Comparison of the Maximum Strength Column Curves for Light Welded Flame-Cut and Universal Mill H-Shapes in Steel Grades ASTM A7 and A36 (e/L = 1/1000)

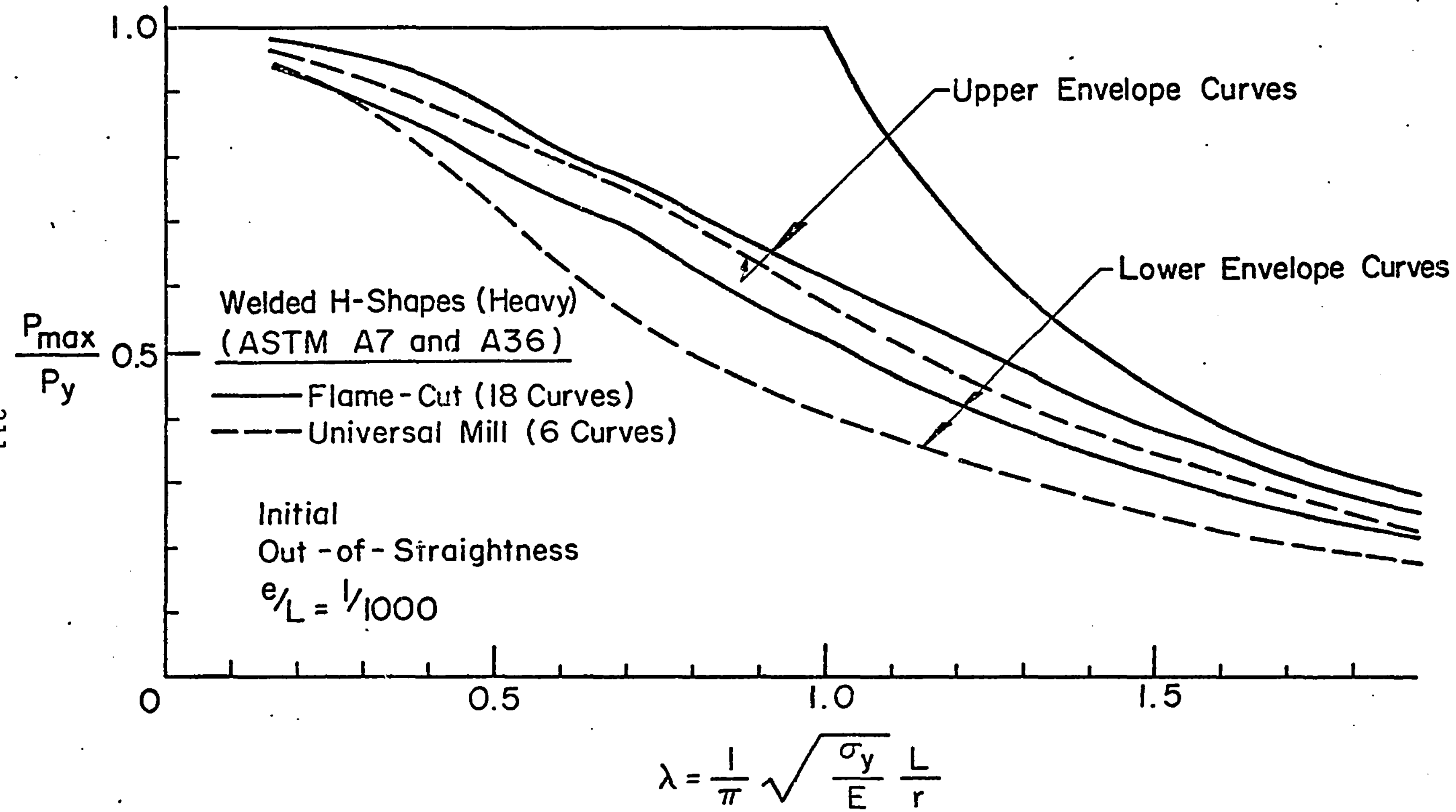


Fig. 20 Comparison of the Maximum Strength Column Curves for Heavy Welded Flame-Cut and Universal Mill H-Shapes in Steel Grades ASTM A7 and A36 (e/L = 1/1000)

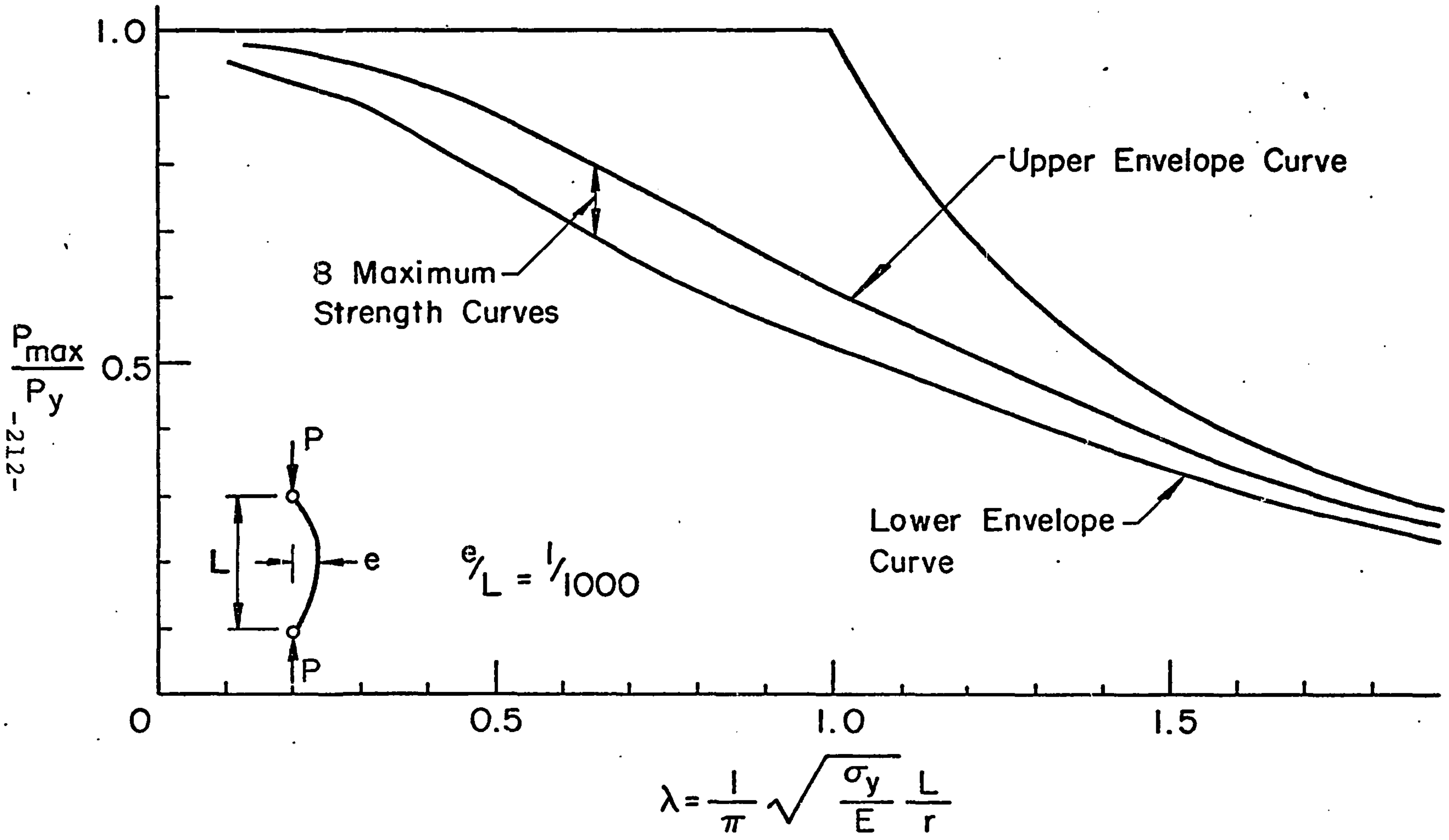


Fig. 21 The Band of Maximum Strength Column Curves for Welded Wide-Flange Heavy Shapes of Steel Grade ASTM A441 (8 curves, e/L = 1/1000)

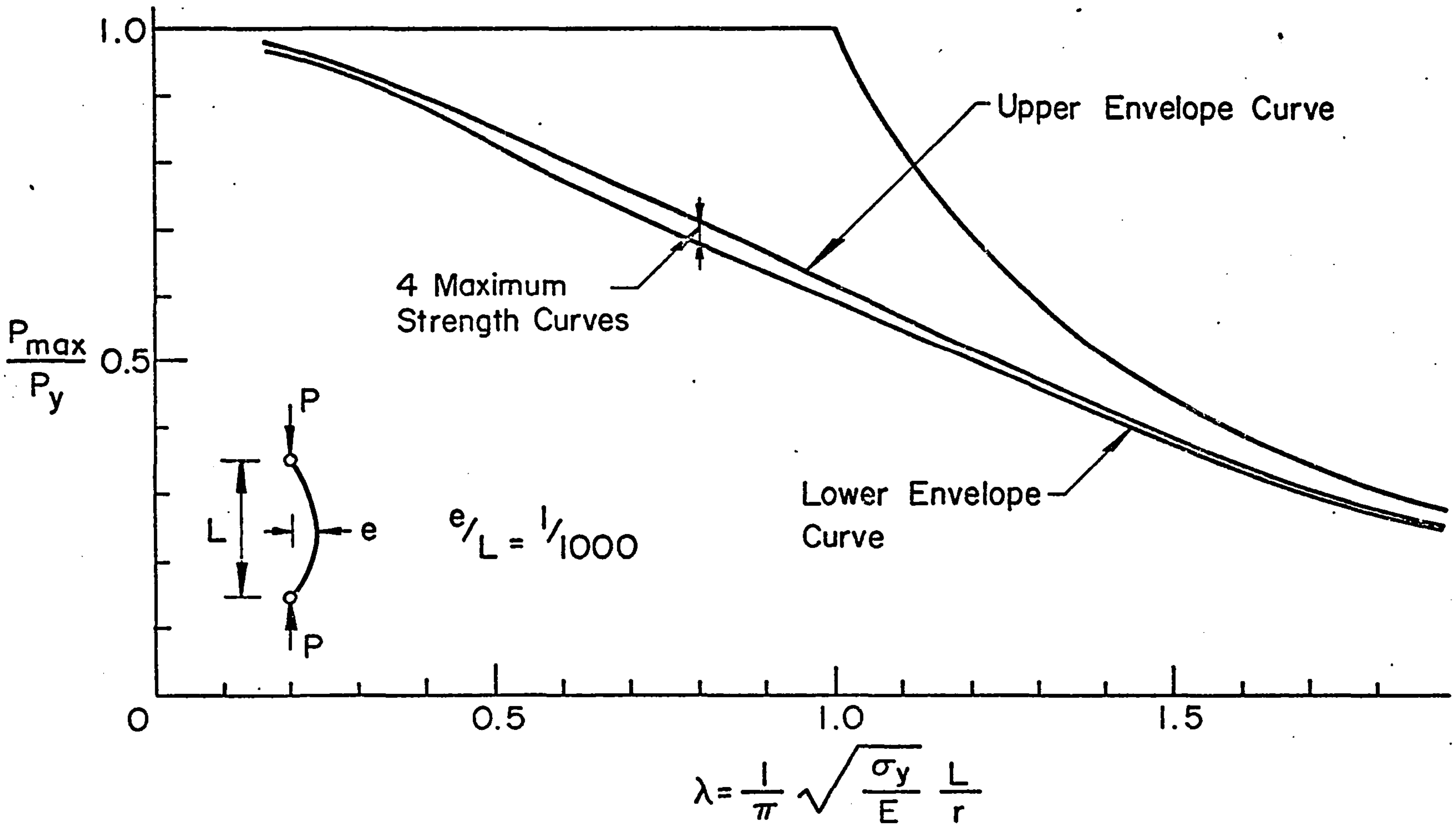


Fig. 22 The Band of Maximum Strength Column Curves for Welded Wide-Flange Shapes of Steel Grade ASTM A572 (50) (Heavy and Light Shapes, Flame-Cut, 4 curves, e/l = 1/1000)

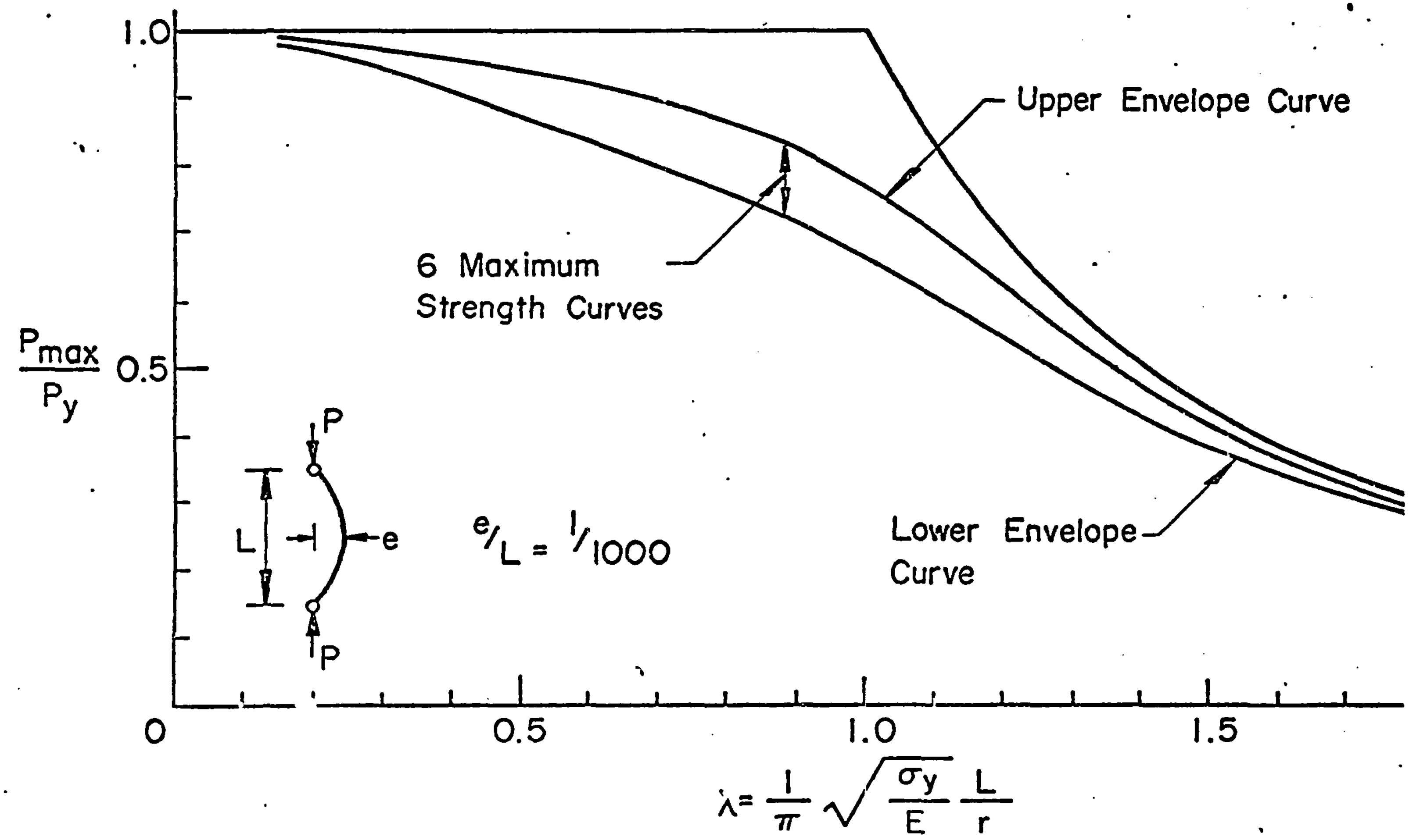


Fig. 23 The Band of Maximum Strength Column Curves for Welded Wide-Flange Light Shapes of Steel Grade ASTM A514 (6 curves, e/L = 1/1000)

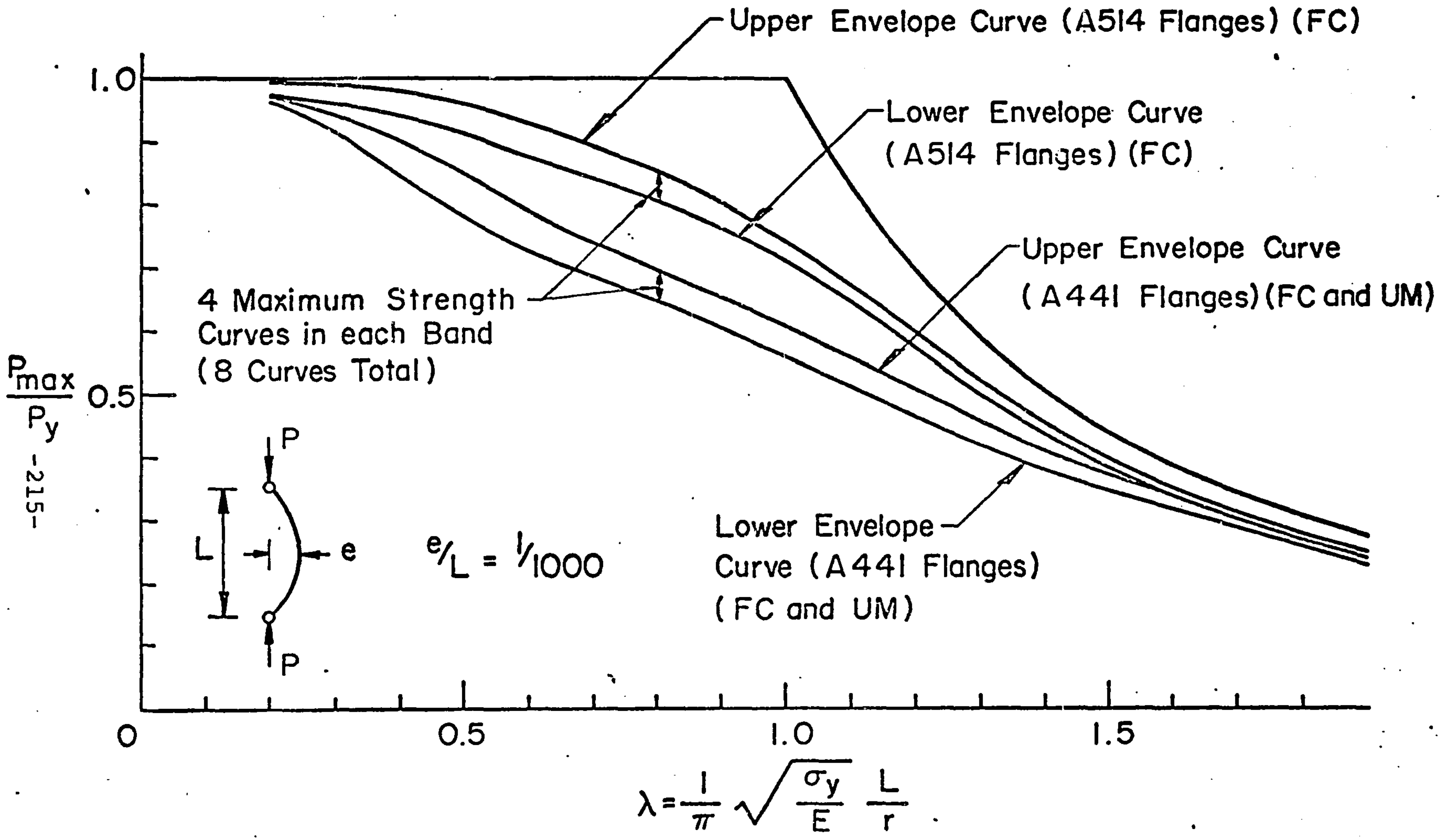


Fig. 24 The Bands of Maximum Strength Column Curves for Welded Wide-Flange Light Hybrid Shapes (Flanges: A514, A441; Web: A441, A36) (8 curves, e/L = 1/1000)

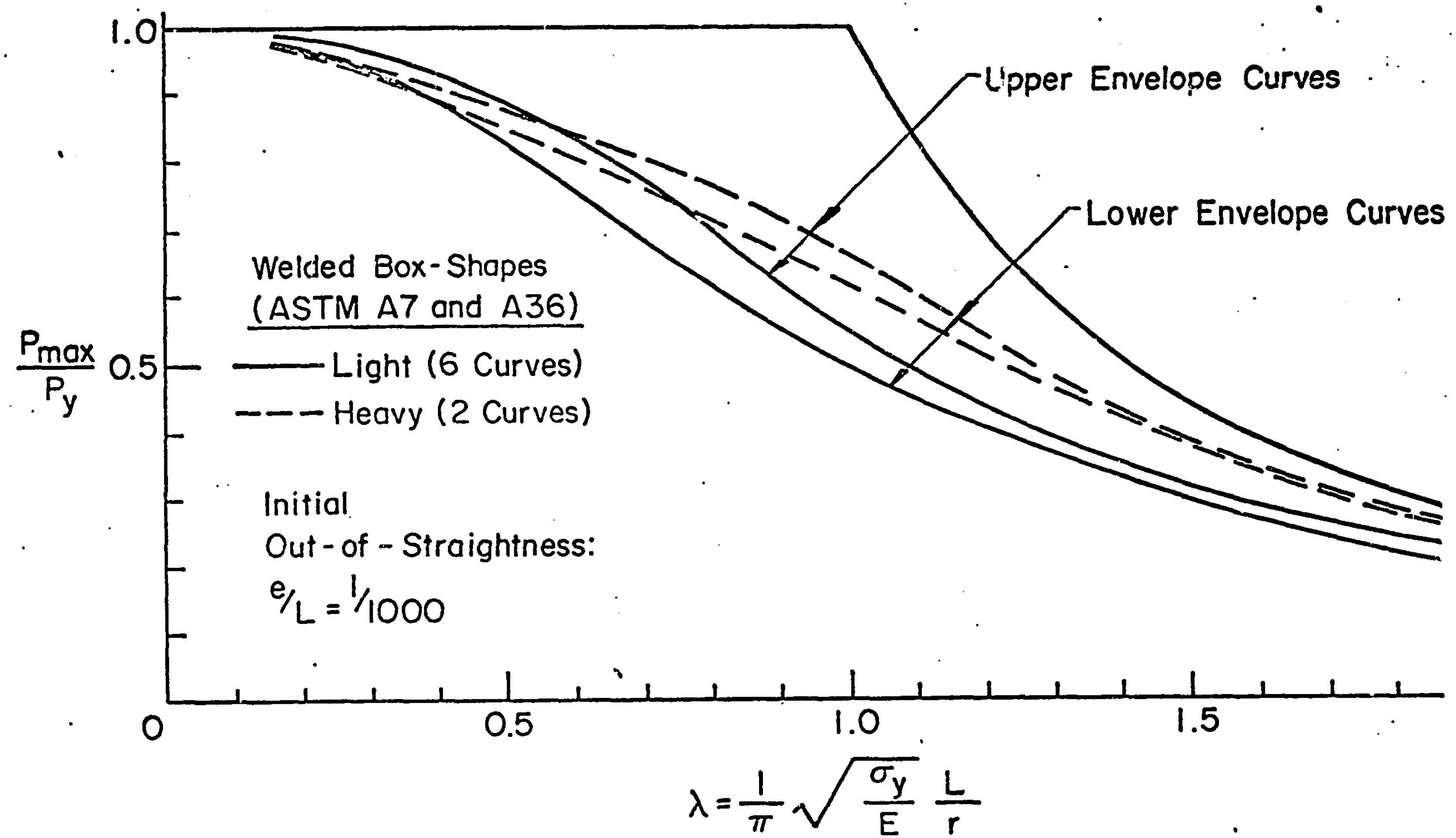


Fig. 25 Comparison of the Maximum Strength Column Curves for Heavy and Light Welded Box-Shapes in Steel Grades ASTM A7 and A36 ($e/L = 1/1000$)

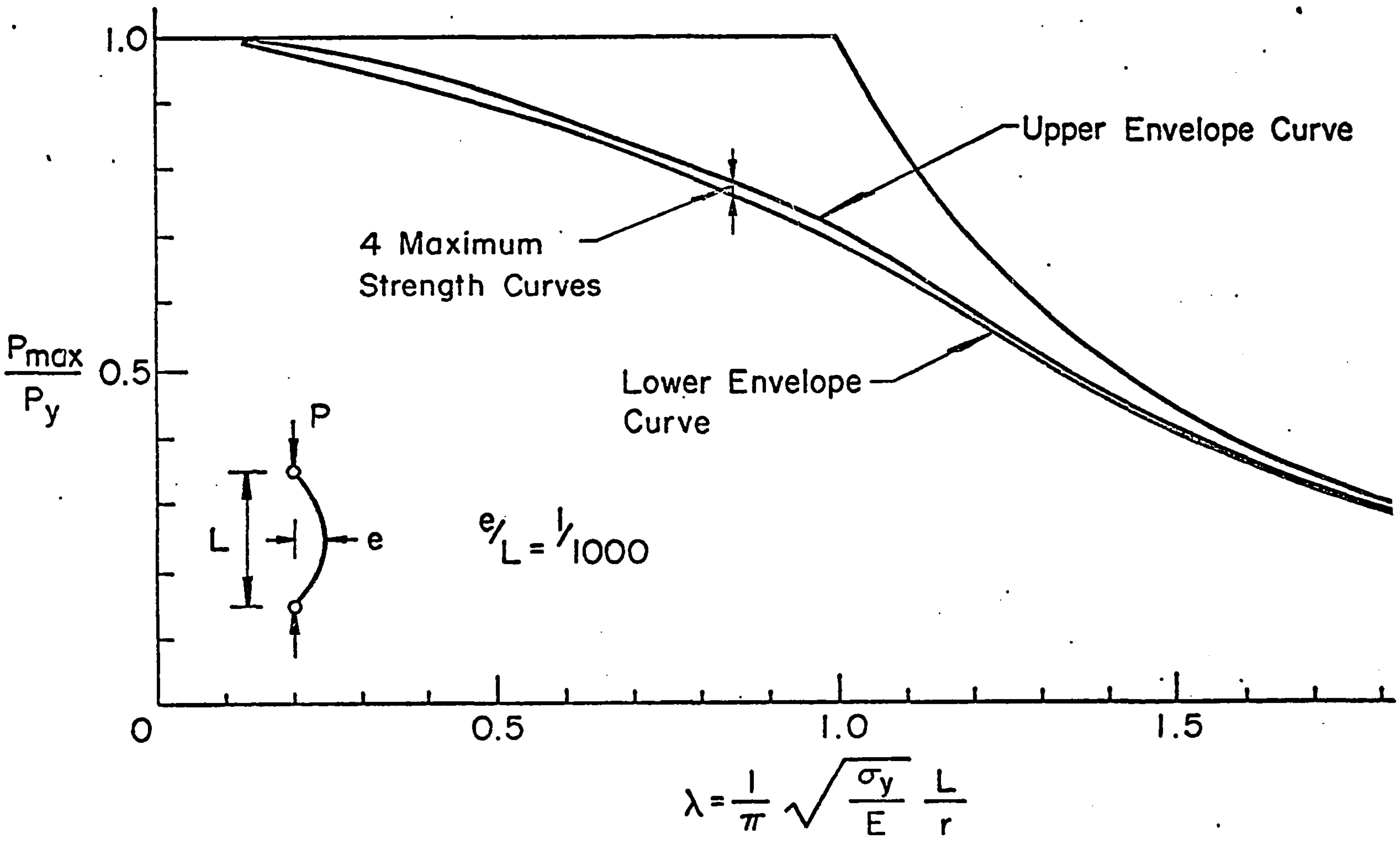


Fig. 26 The Band of Maximum Strength Column Curves for Welded Light Box-Shapes of Steel Grade ASTM A514 (4 curves, e/L = 1/1000)

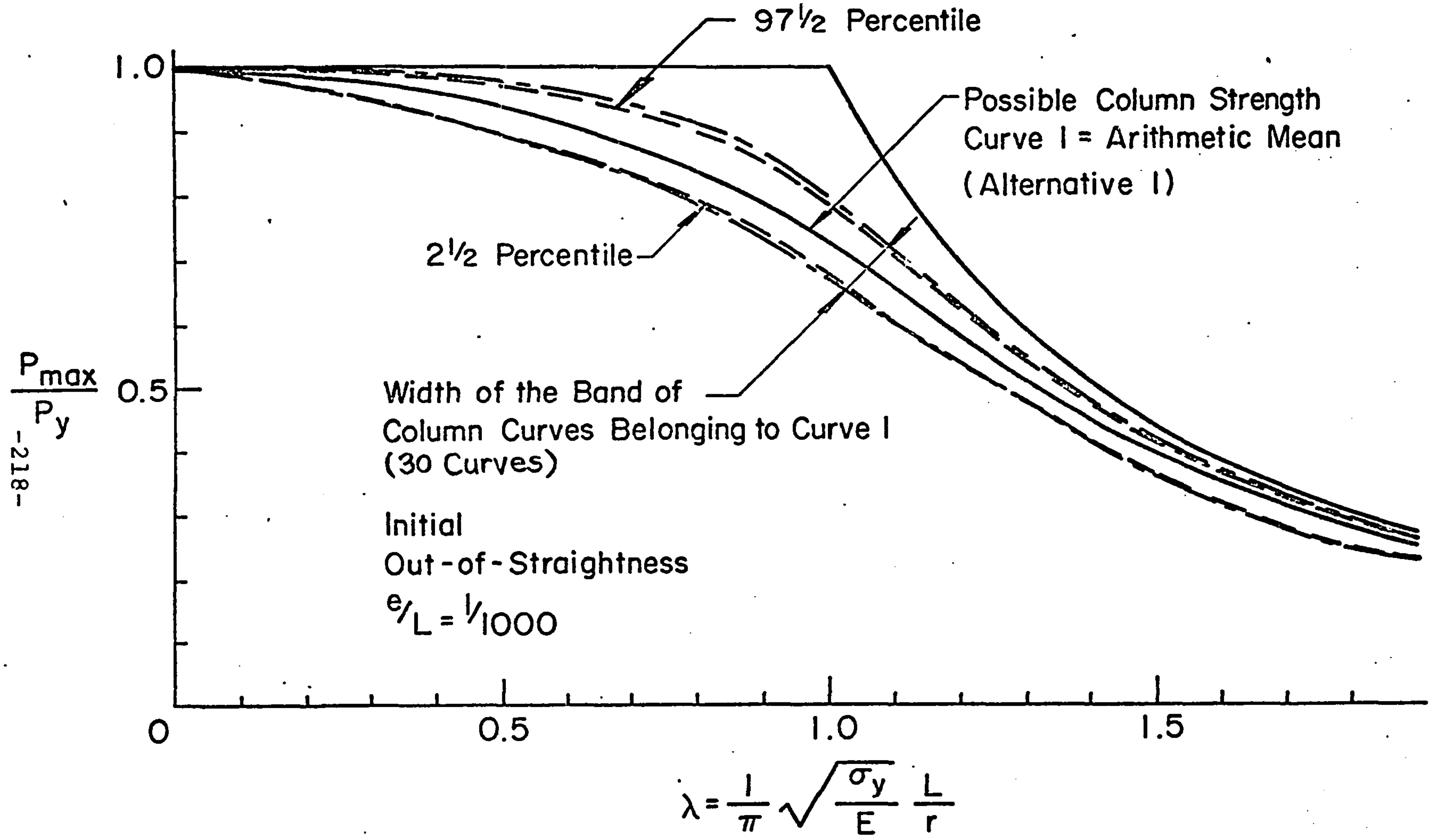


Fig. 27 Possible Column Strength Curve 1, and the Statistical Properties of the Band of Column Curves that Belong to It

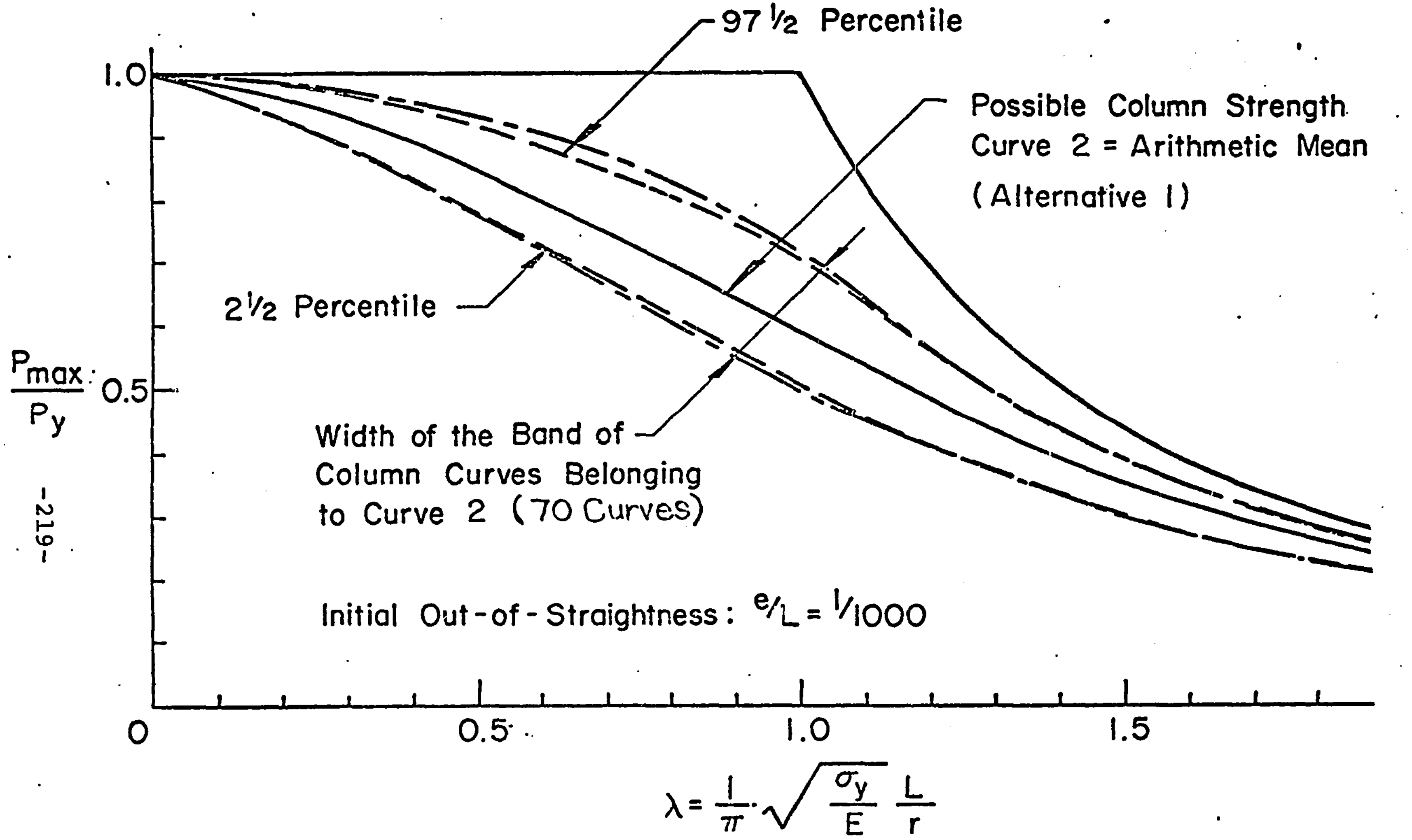


Fig. 28 Possible Column Strength Curve 2, and the Statistical Properties of the Band of Column Curves that Belong to It

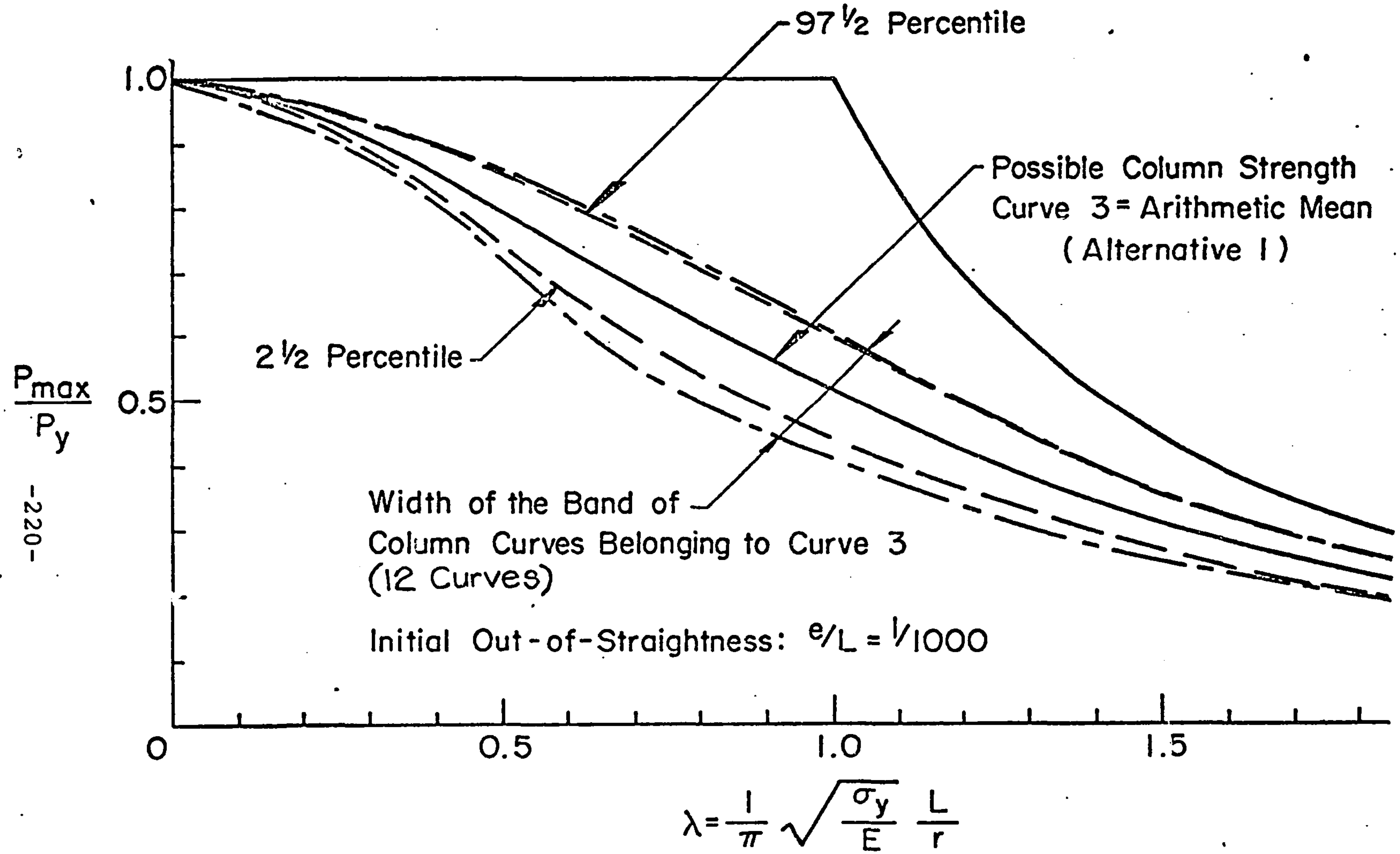


Fig. 29 Possible Column Strength Curve 3 and the Statistical Properties of the Band of Column Curves that Belong to It

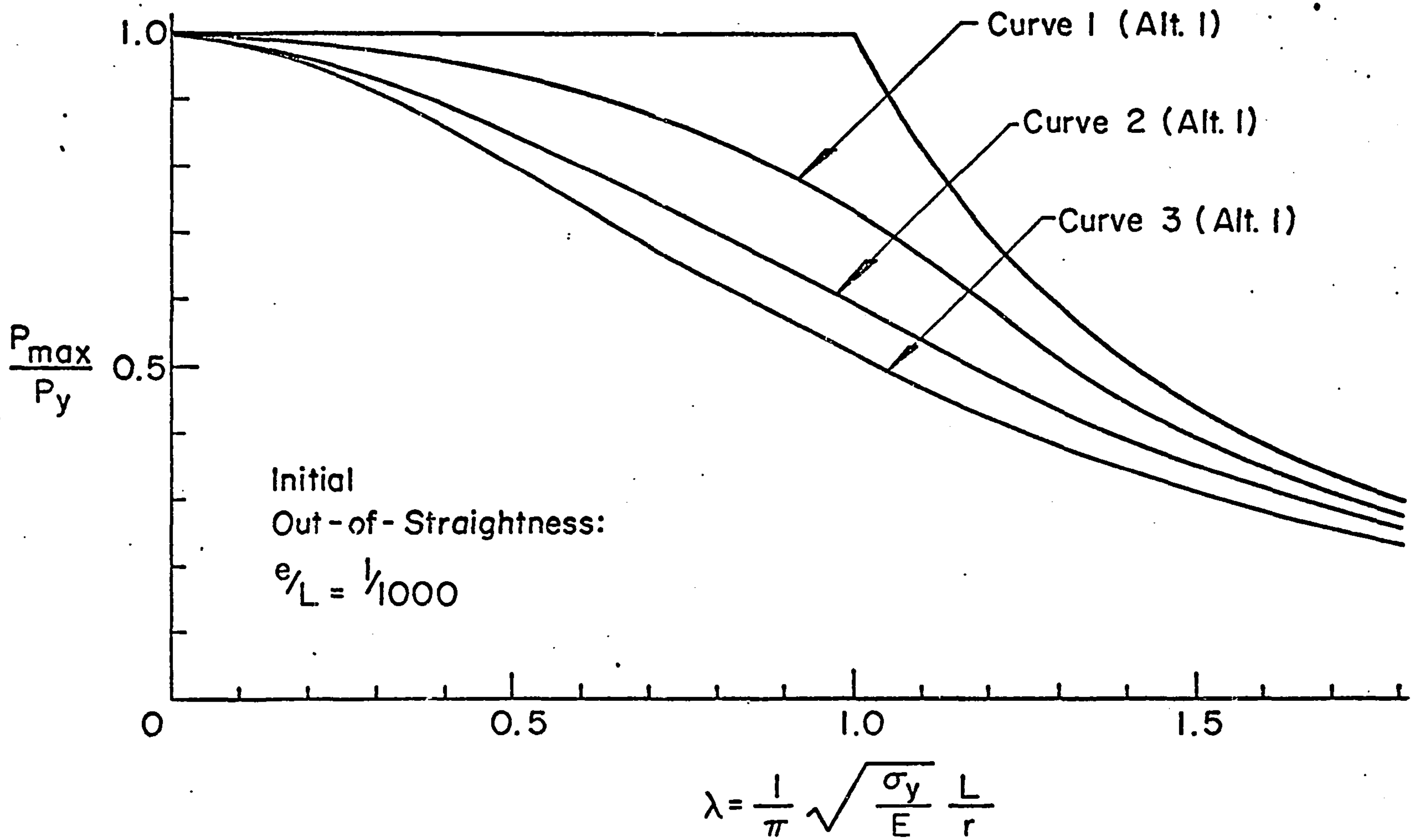


Fig. 30 Possible Maximum Strength Multiple Column Curves, Based on Initial Out-of-Straightness $e/L = 1/1000$ (Alternative 1)

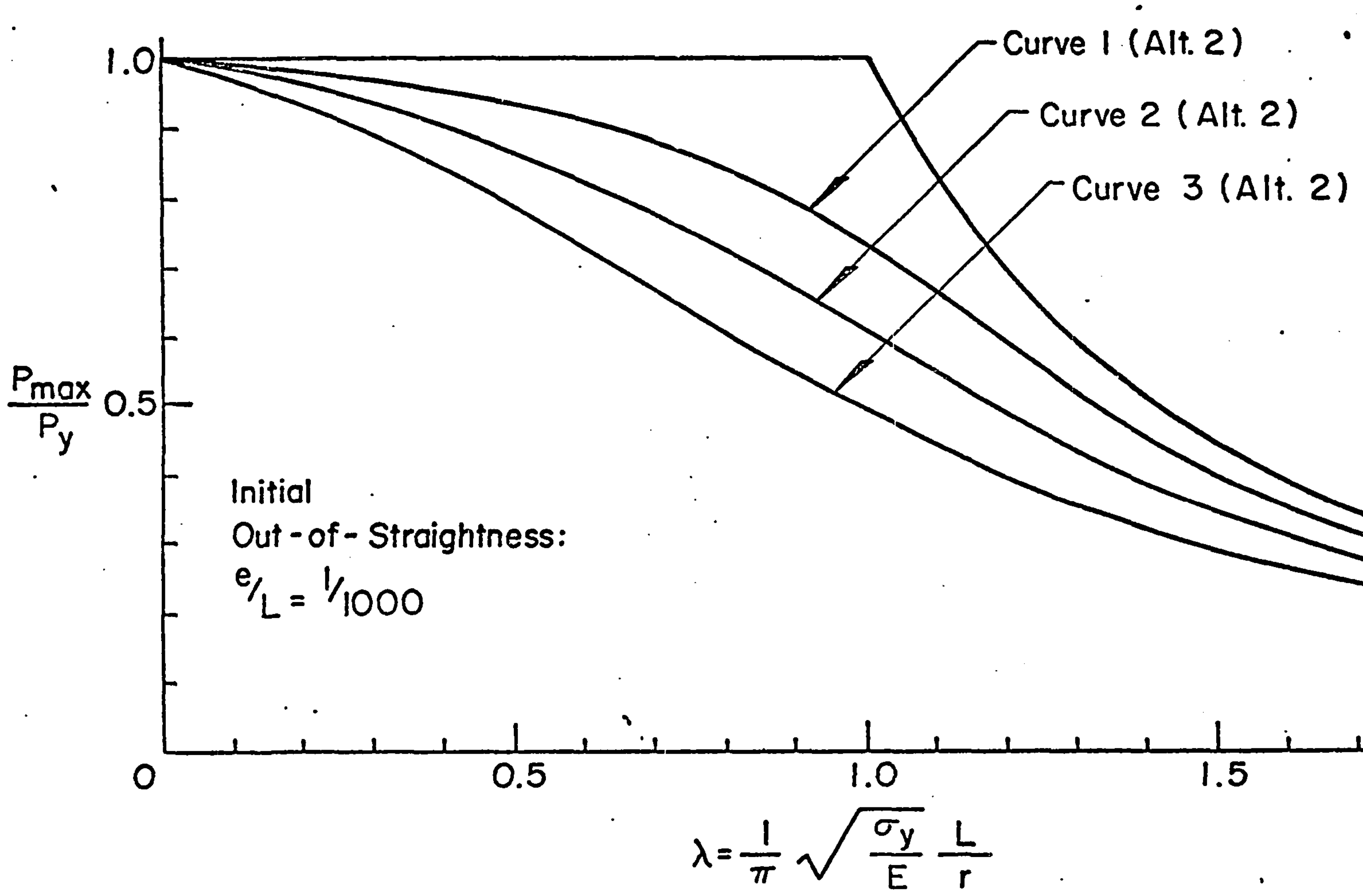


Fig. 31 Modified (Alternative 2) Maximum Strength Multiple Column Curves, Based on Initial Out-of-Straightness e/L = 1/1000

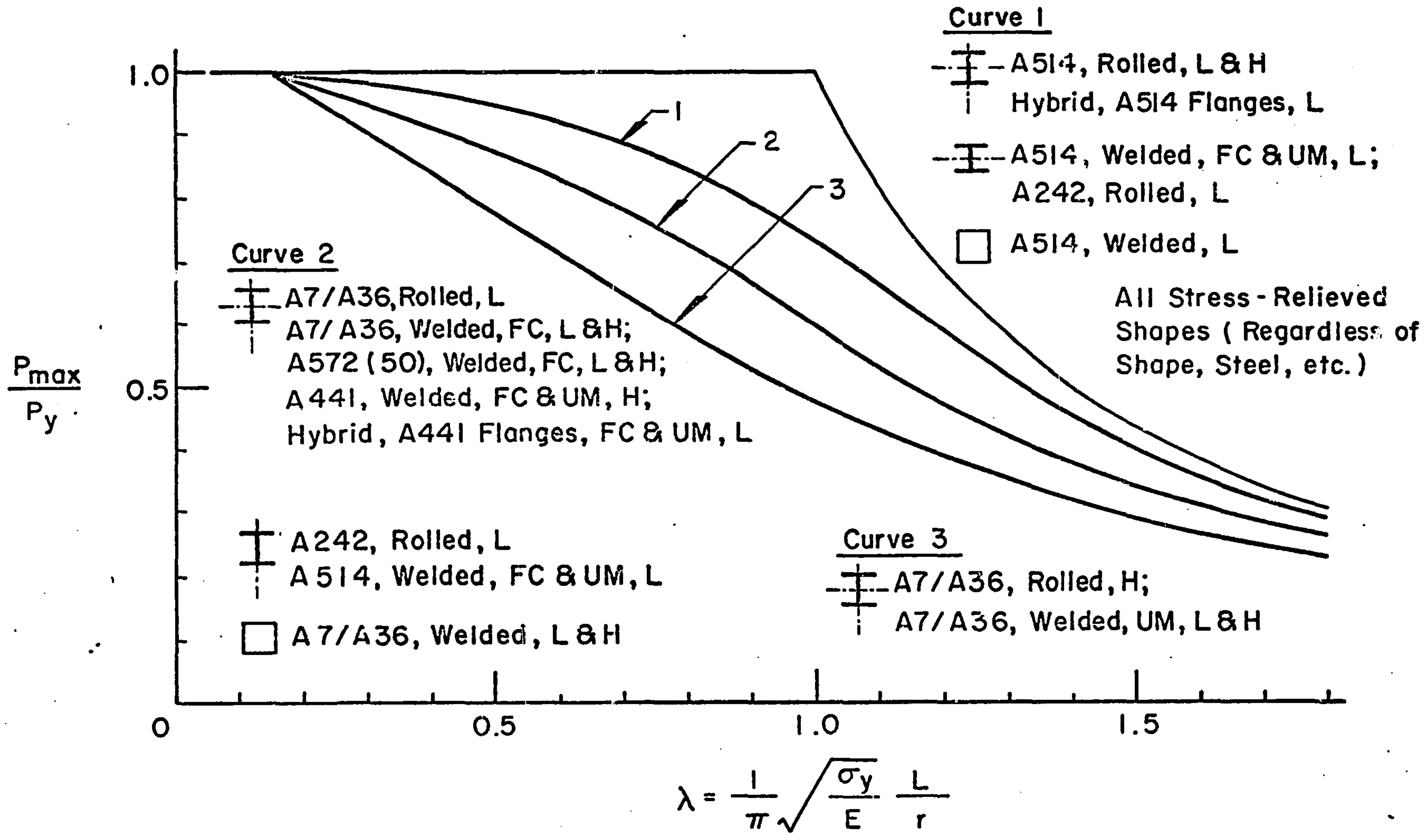


Fig. 32 Simplified Maximum Strength Multiple Column Curves
(Based on Initial Out-of-Straightness e/L = 1/1000)

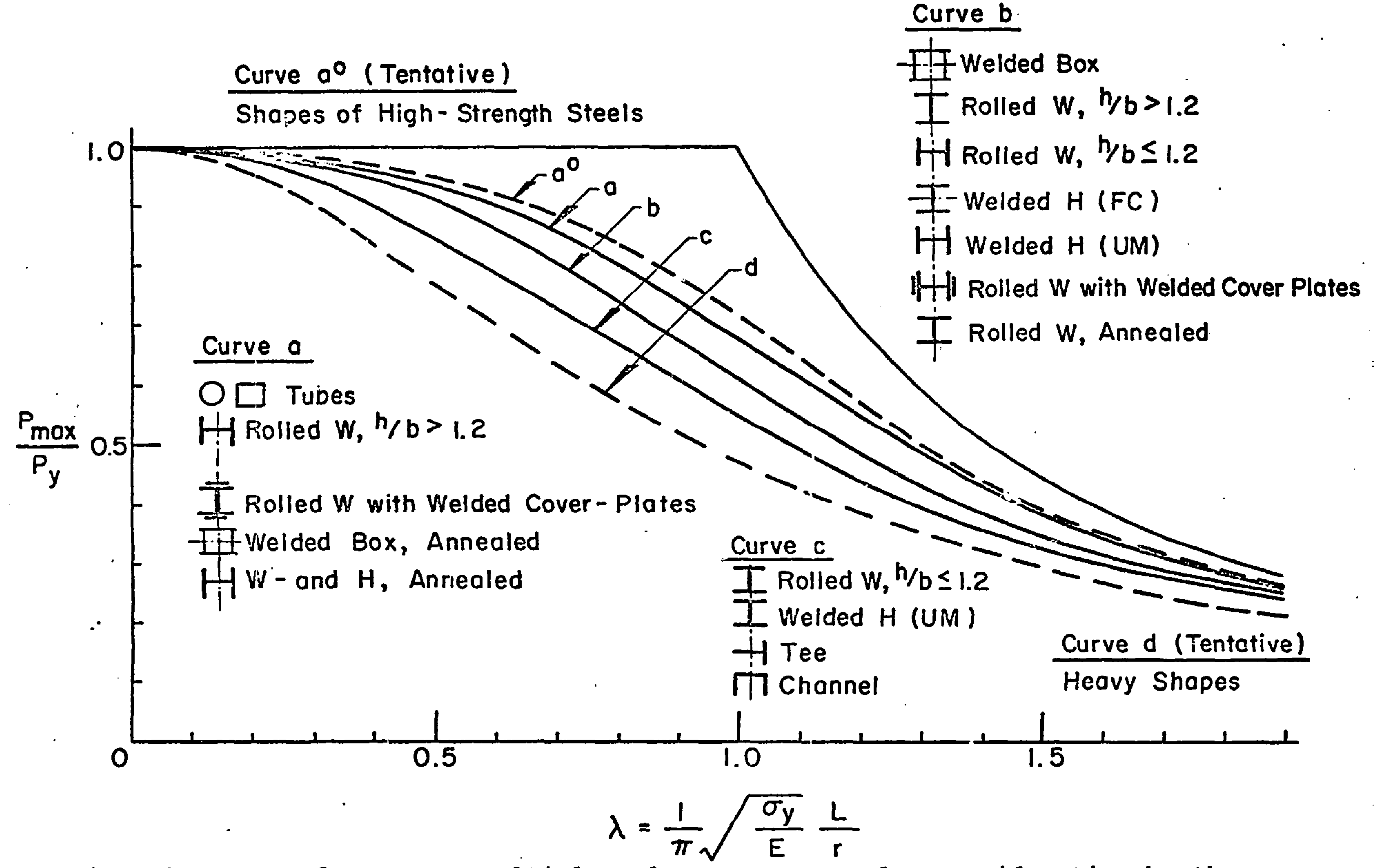


Fig. 33 Proposed European Multiple Column Curves, under Consideration by the European Convention of Constructional Steelworks (ECCS). (Based on Initial Out-of-Straightness e/L = 1/1000)

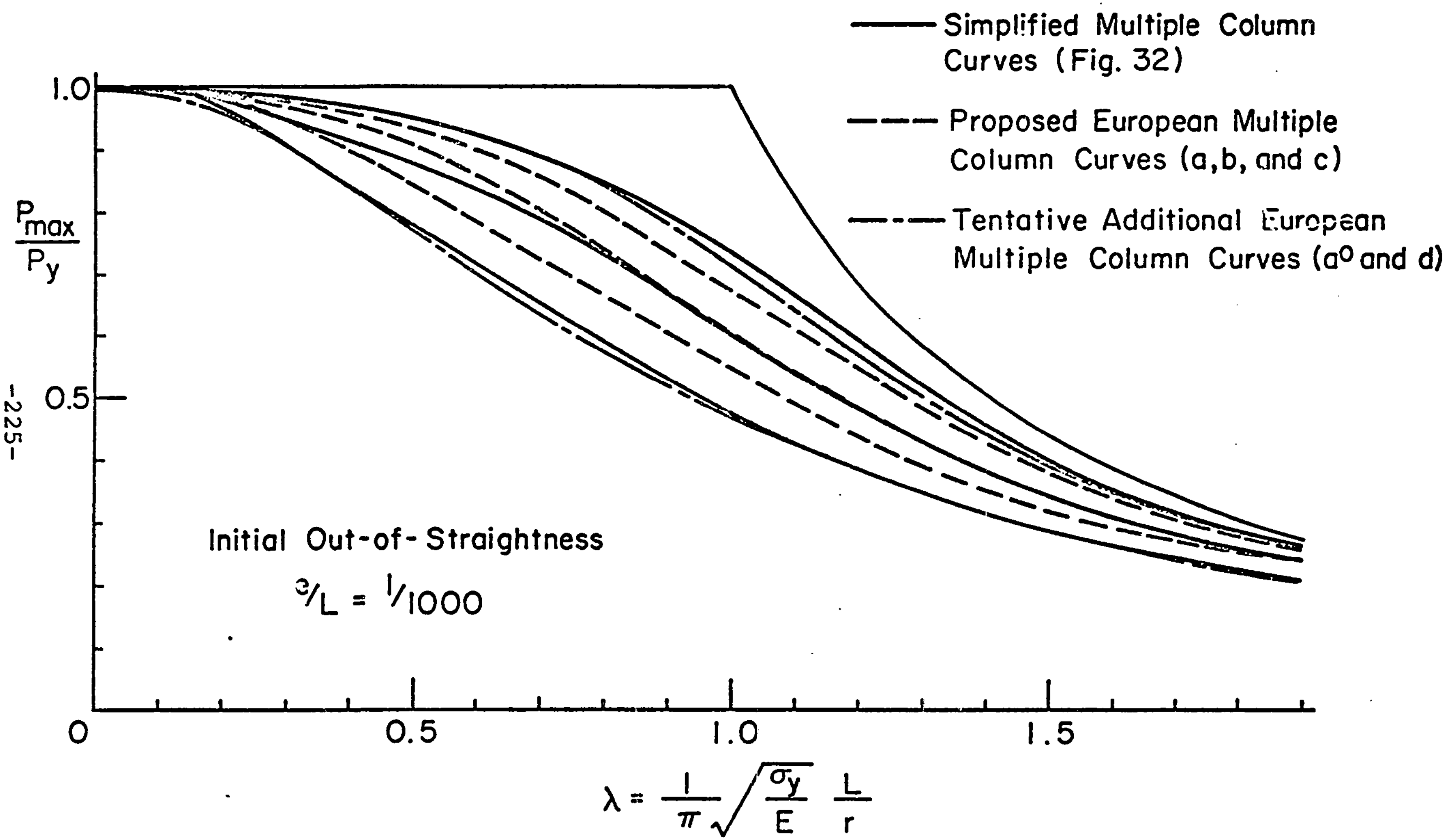


Fig. 34 Comparison of the Simplified Multiple Column Curves (Fig. 32), and the Proposed and Tentative Additional European Multiple Column Curves (Initial Out-of-Straightness e/L = 1/1000)

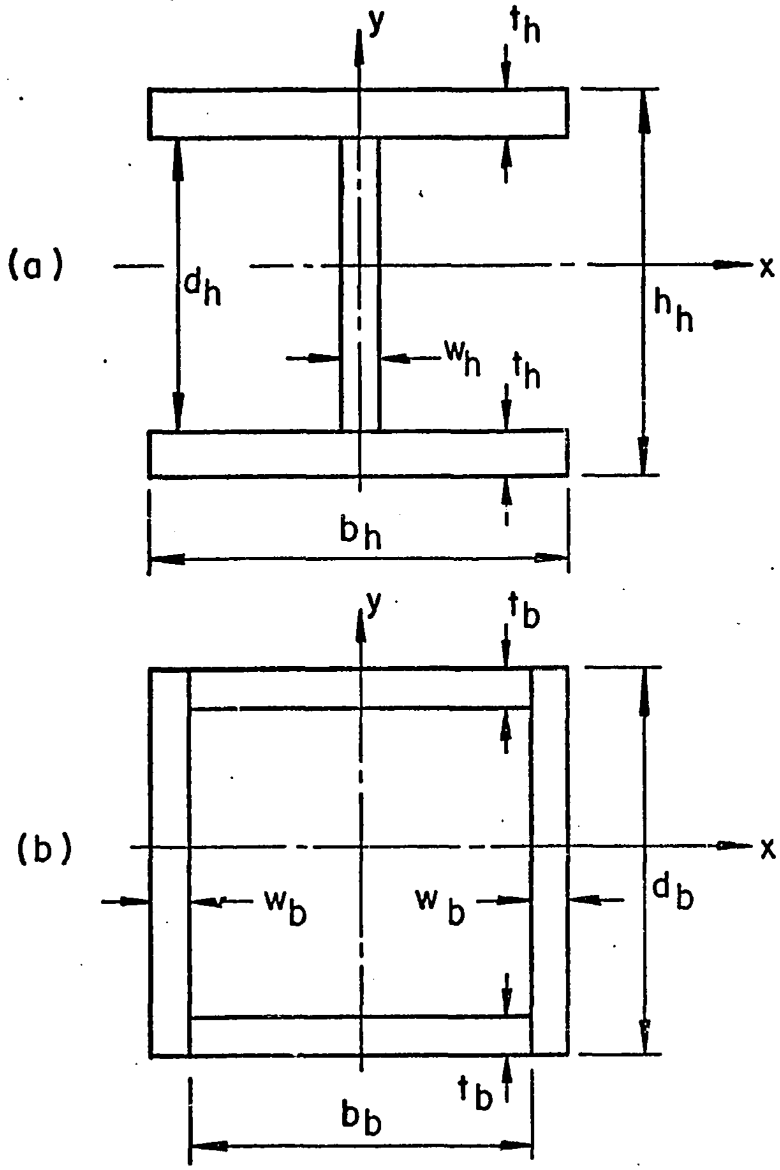


Fig. 35 Cross-Sectional Dimensions of Wide-Flange and Box Shapes

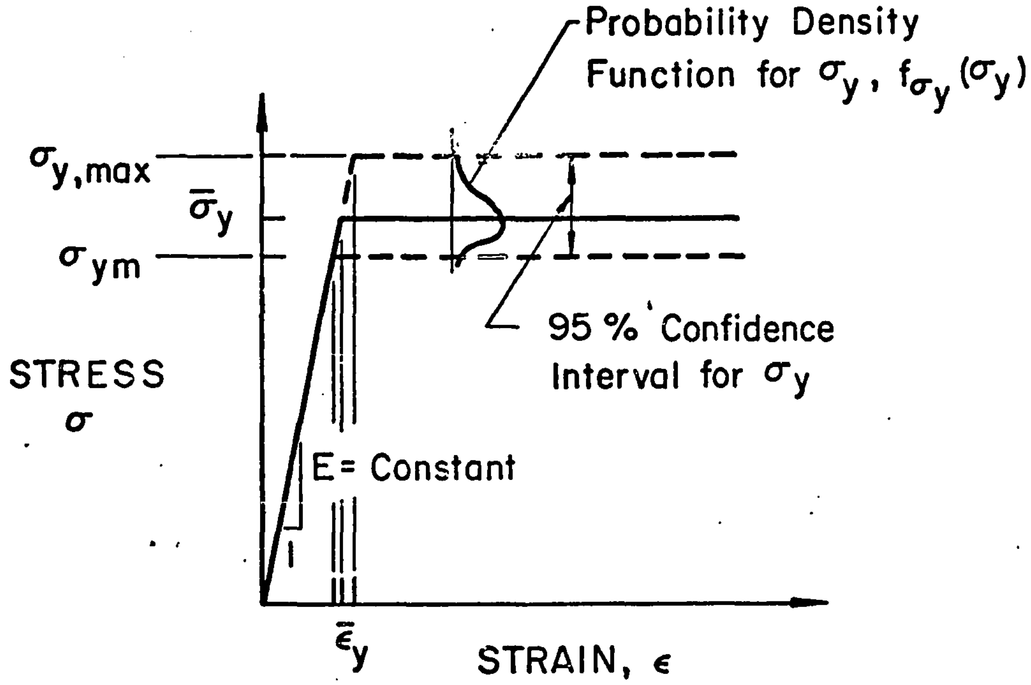
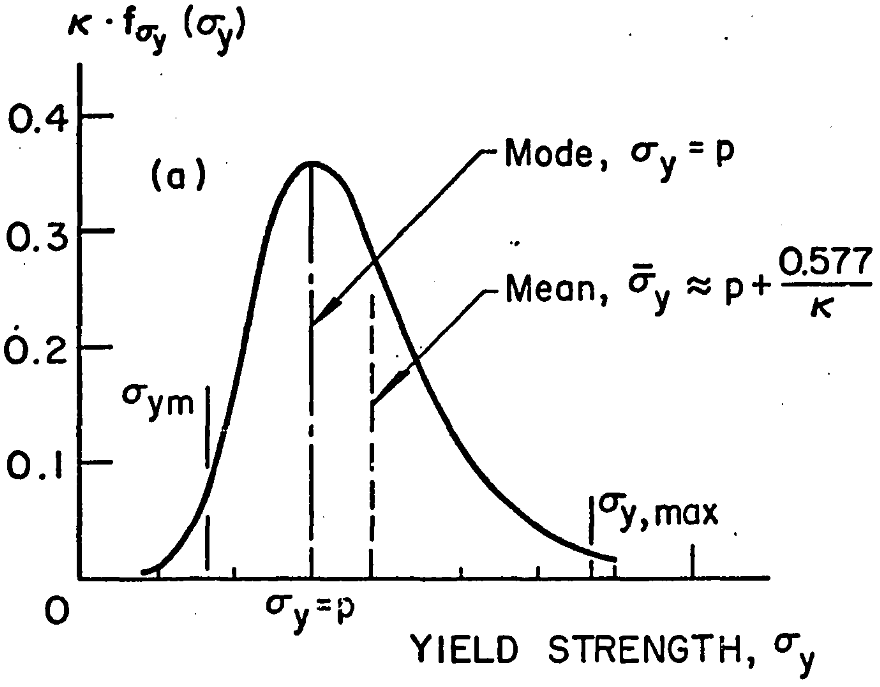


Fig. 36 Schematic Illustration of the Statistical Characteristics of the Mechanical Properties of Steel



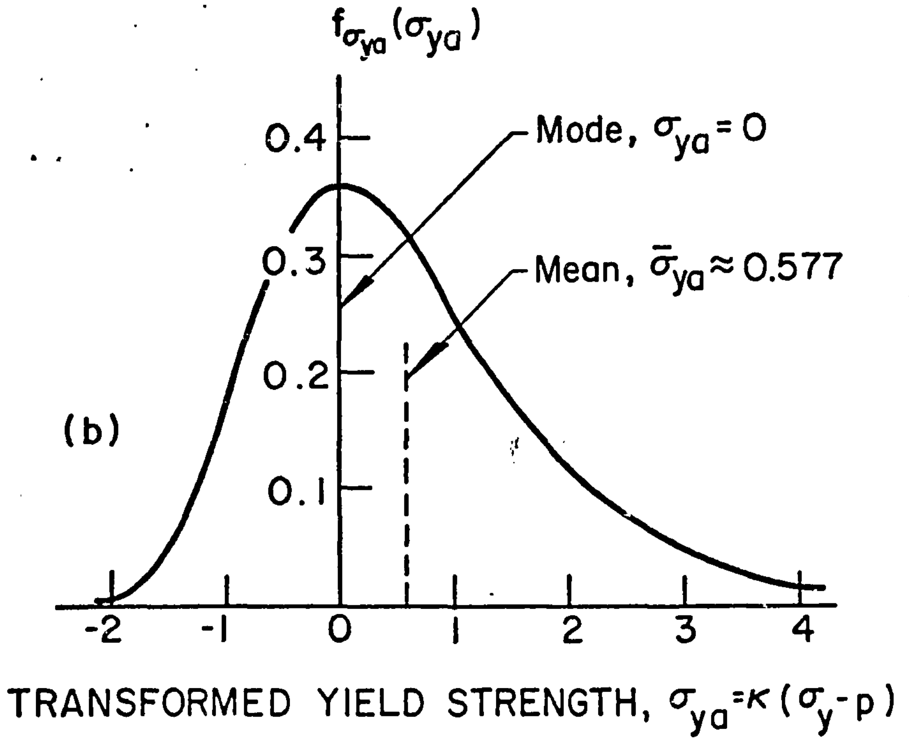


Fig. 37 The Type I (Gumbel) Asymptotic Extreme Value Probability Density Functions for the Yield Stress, σ_y, and Its Transformed Counterpart, σ_{ya}

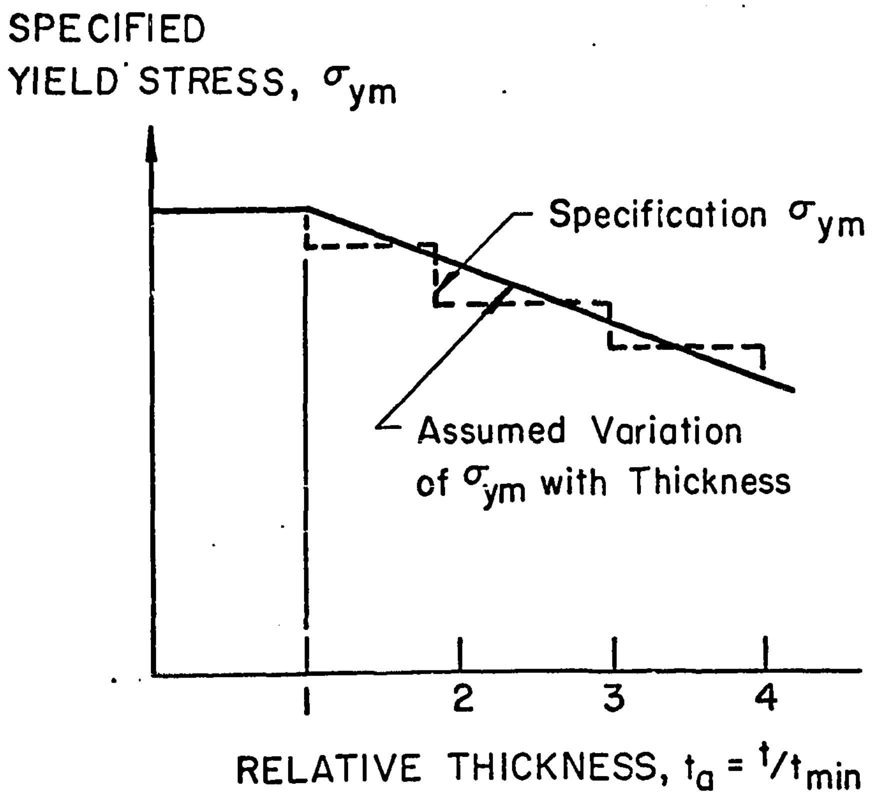


Fig. 38 The Relationship Between the Yield Stress and the Thickness of the Material

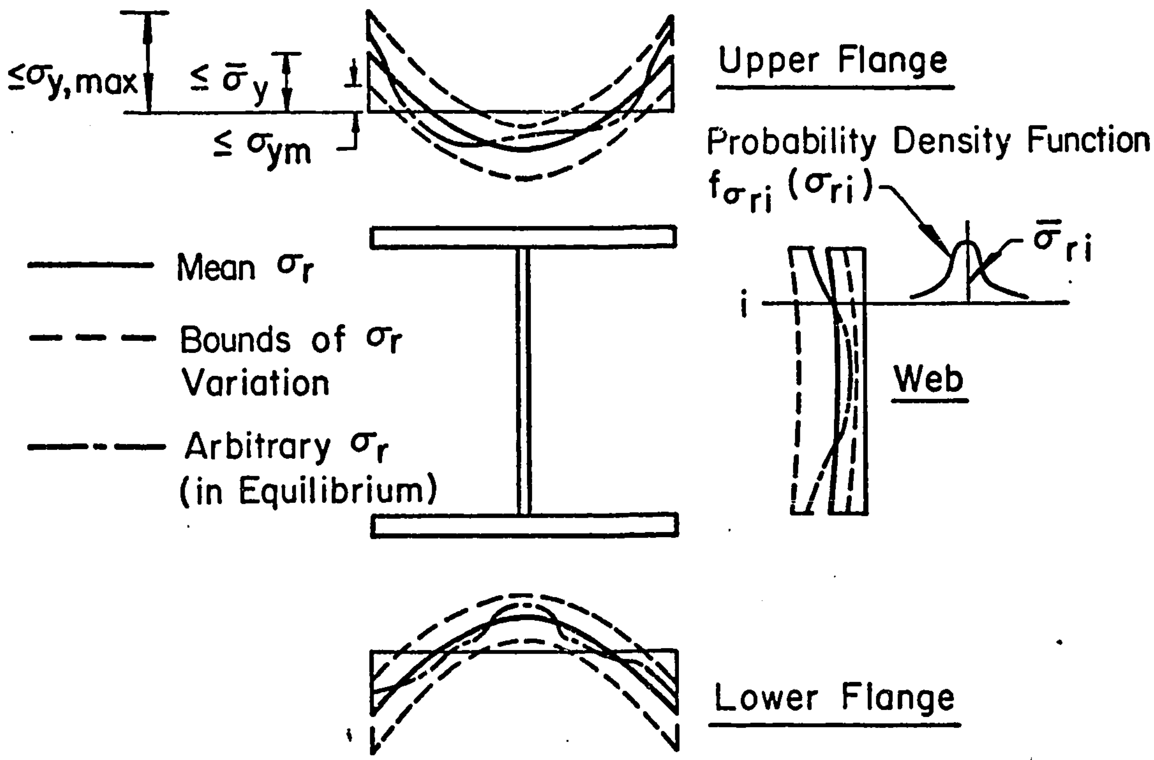
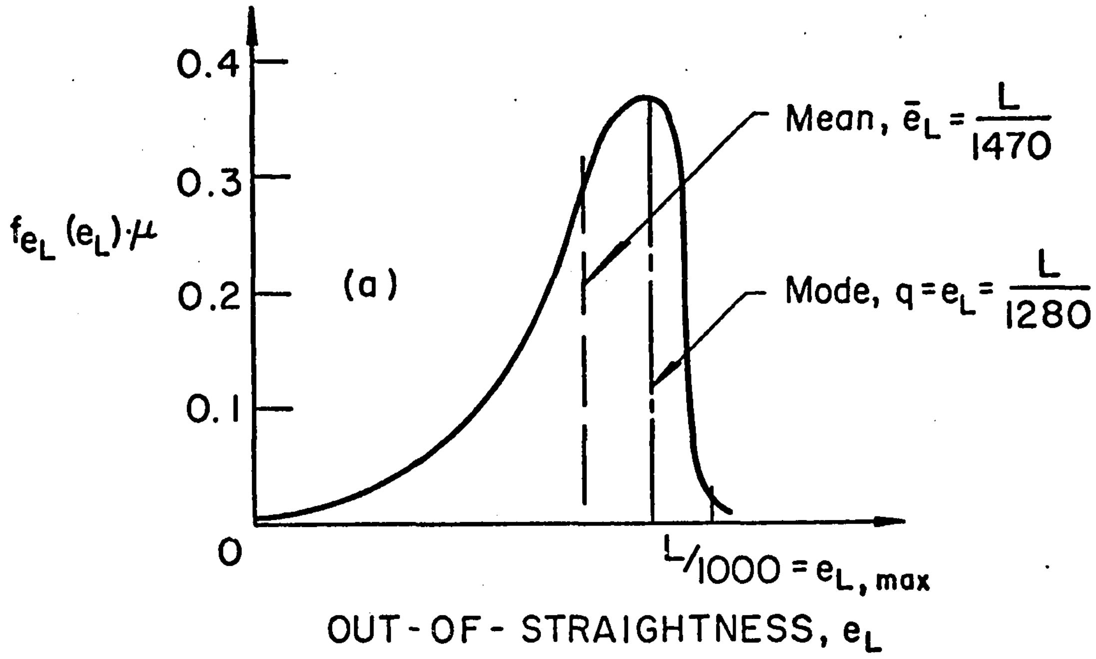


Fig. 39 Qualitative Illustration of the Mean Residual Stress Distribution in a Shape, and the Bounds Between Which the Residual Stresses May Vary



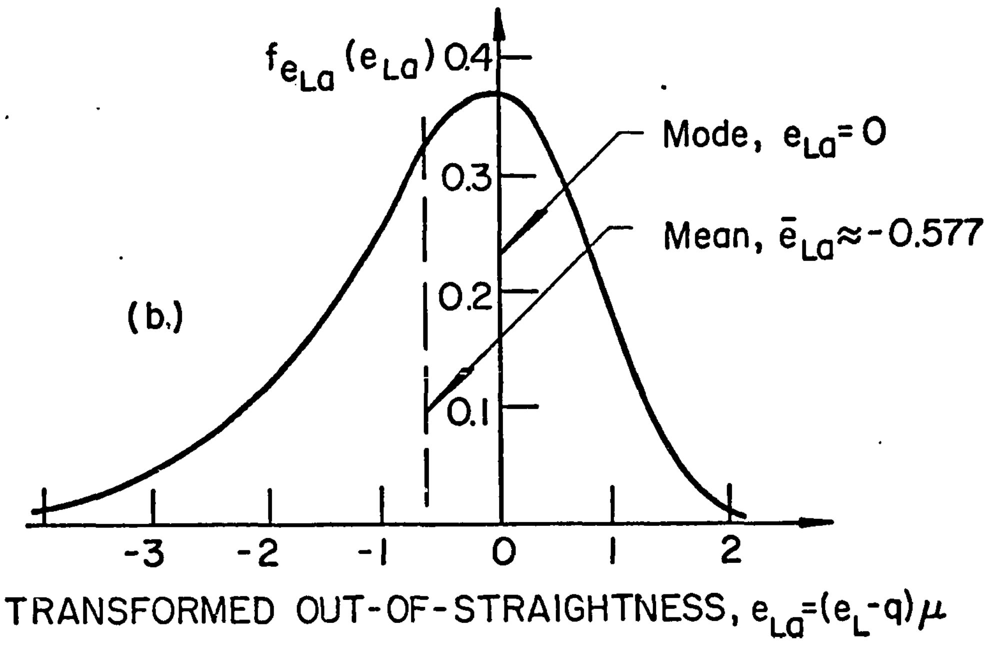


Fig. 40 The Type I (smallest value) Asymptotic Extreme Value Distributions for the Initial Out-of Straightness, e_L, and Its Transformed Counterpart, e_{La}

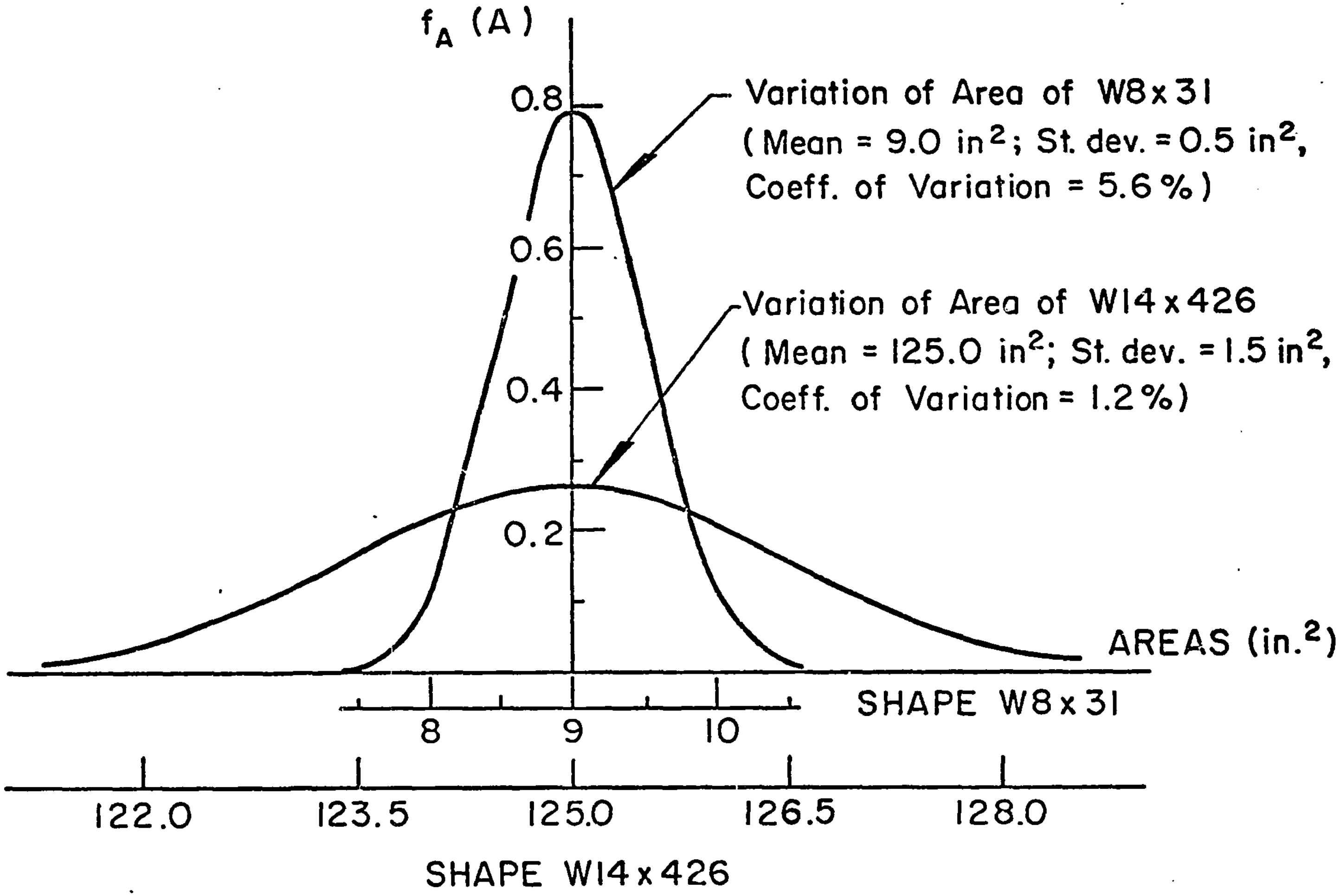


Fig. 41 The Hypothetical Distributions of the Cross-Sectional Areas of the Rolled Wide-Flange Shapes W8x31 and W14x426

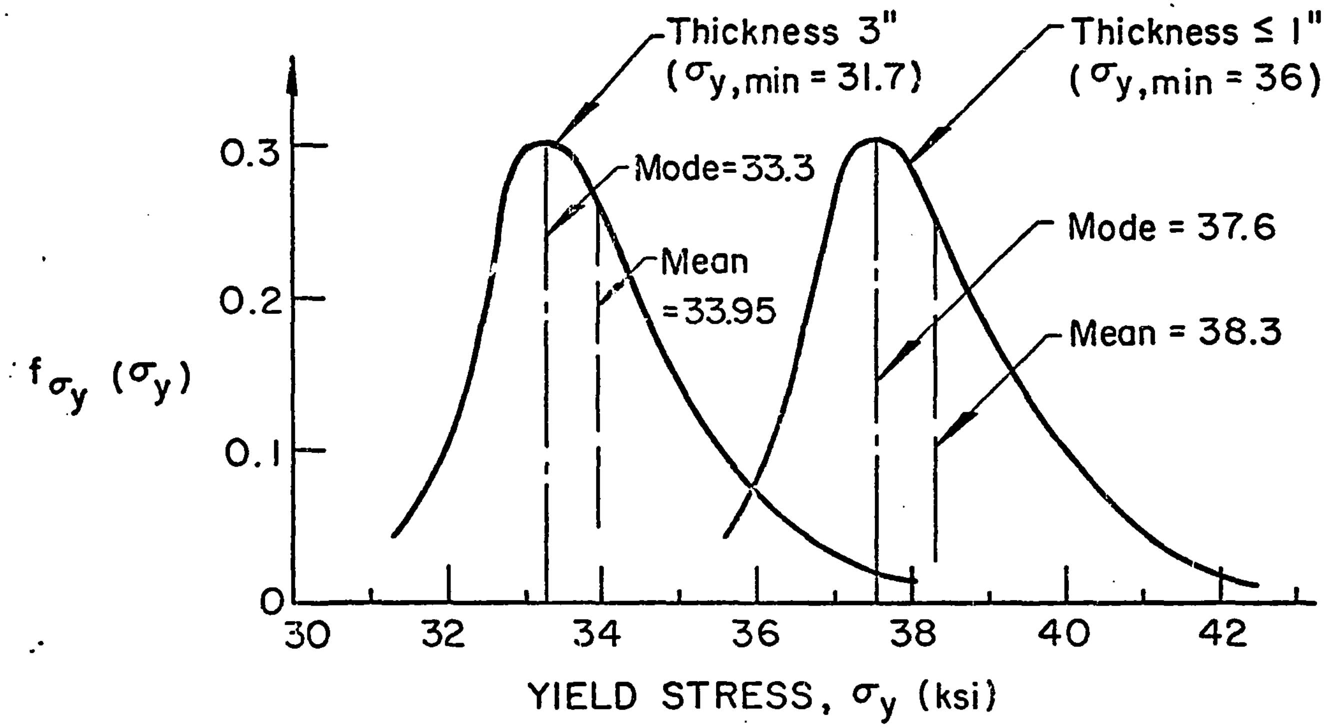


Fig. 42 The Hypothetical Distribution of the Yield Stress for Steel ASTM A36 With Thicknesses 3/4" and 3"

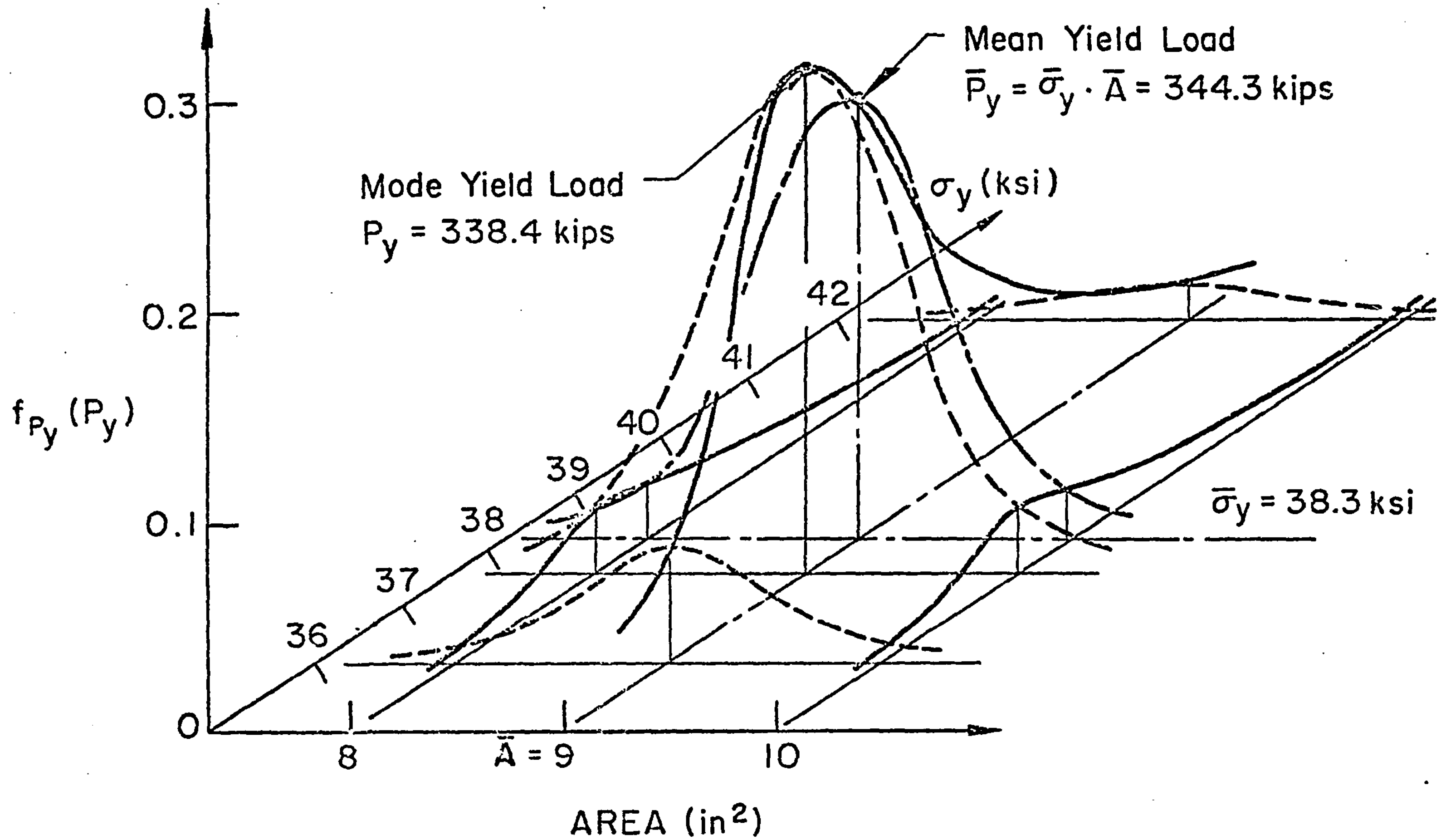


Fig. 43 Three-Dimensional Illustration of the Hypothetical Probability Density Function for the Yield Load of a Rolled Wide-Flange Shape W8x31 of Steel Grade ASTM A36

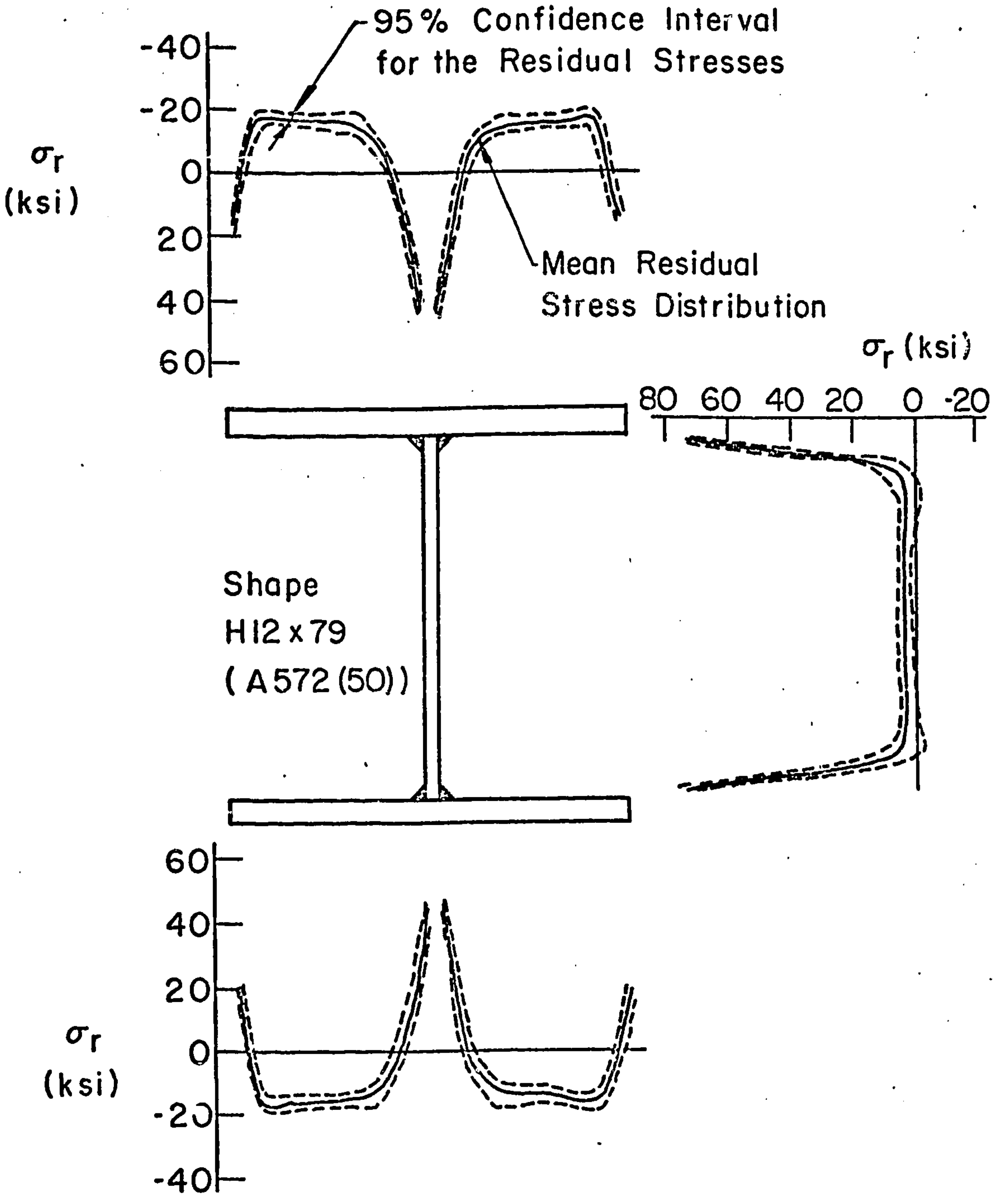
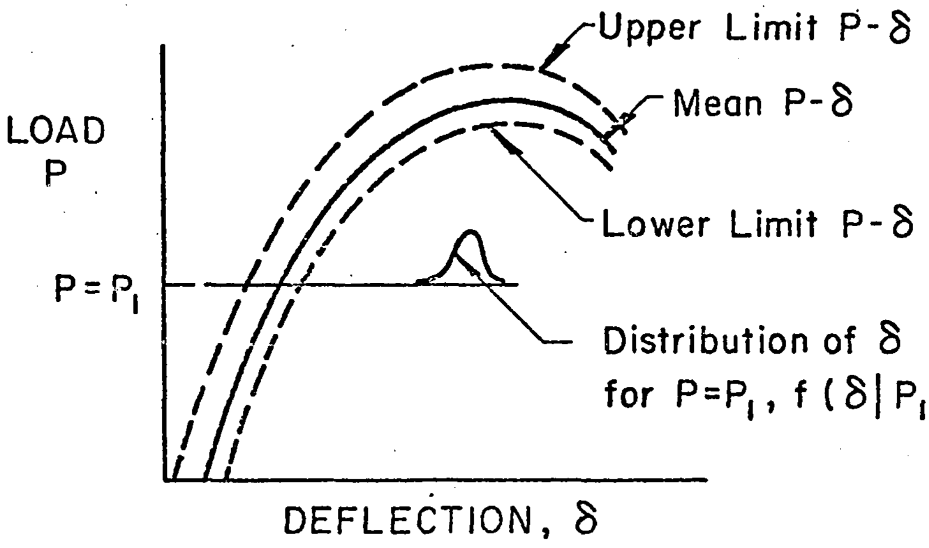
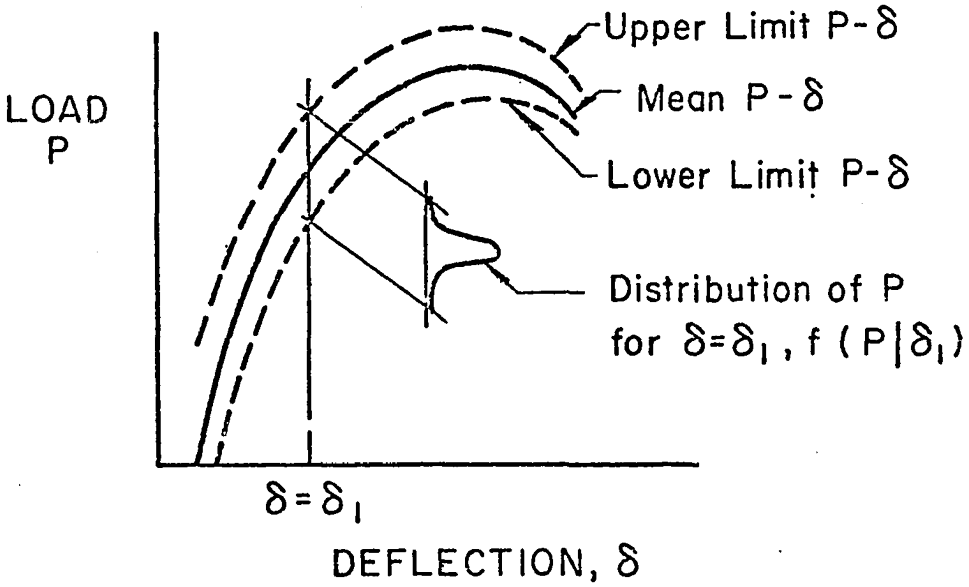


Fig. 44 The Measured Variation of the Residual Stress Distribution in a Welded Wide-Flange Shape H12x79 of Steel Grade ASTM A572(50), Illustrated by Mean Residual Stresses and Their 95 Percent Confidence Intervals(87)



(a) Deterministic Load, Probabilistic Deflection



(b) Deterministic Deflection, Probabilistic Load

Fig. 45 Schematic Illustration of Semi-Probabilistic Load-Deflection Curves

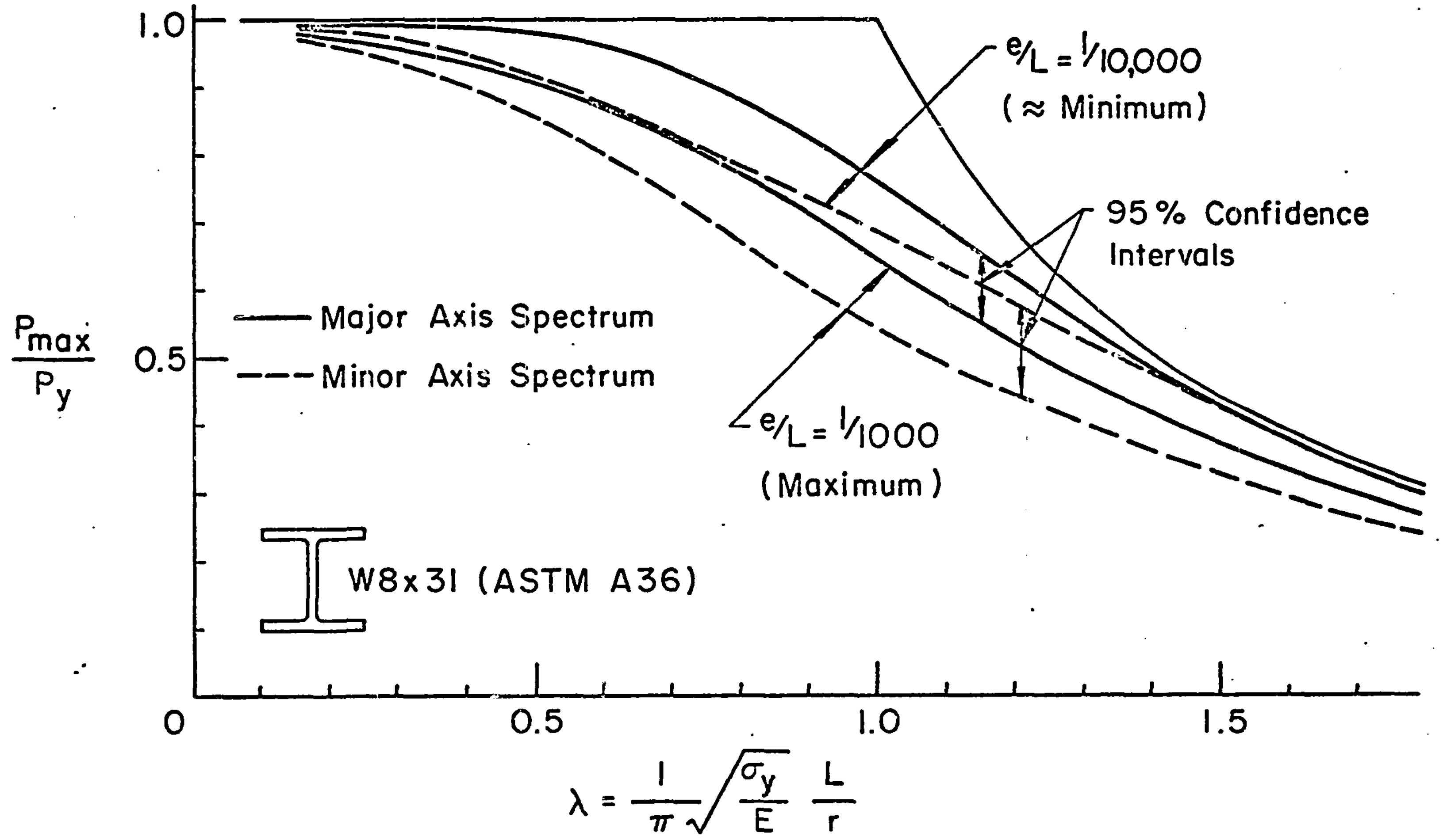


Fig. 46 The Column Curve Spectra for Major and Minor Axis Bending of a Light Rolled Wide-Flange Shape W8x31 (ASTM A36)

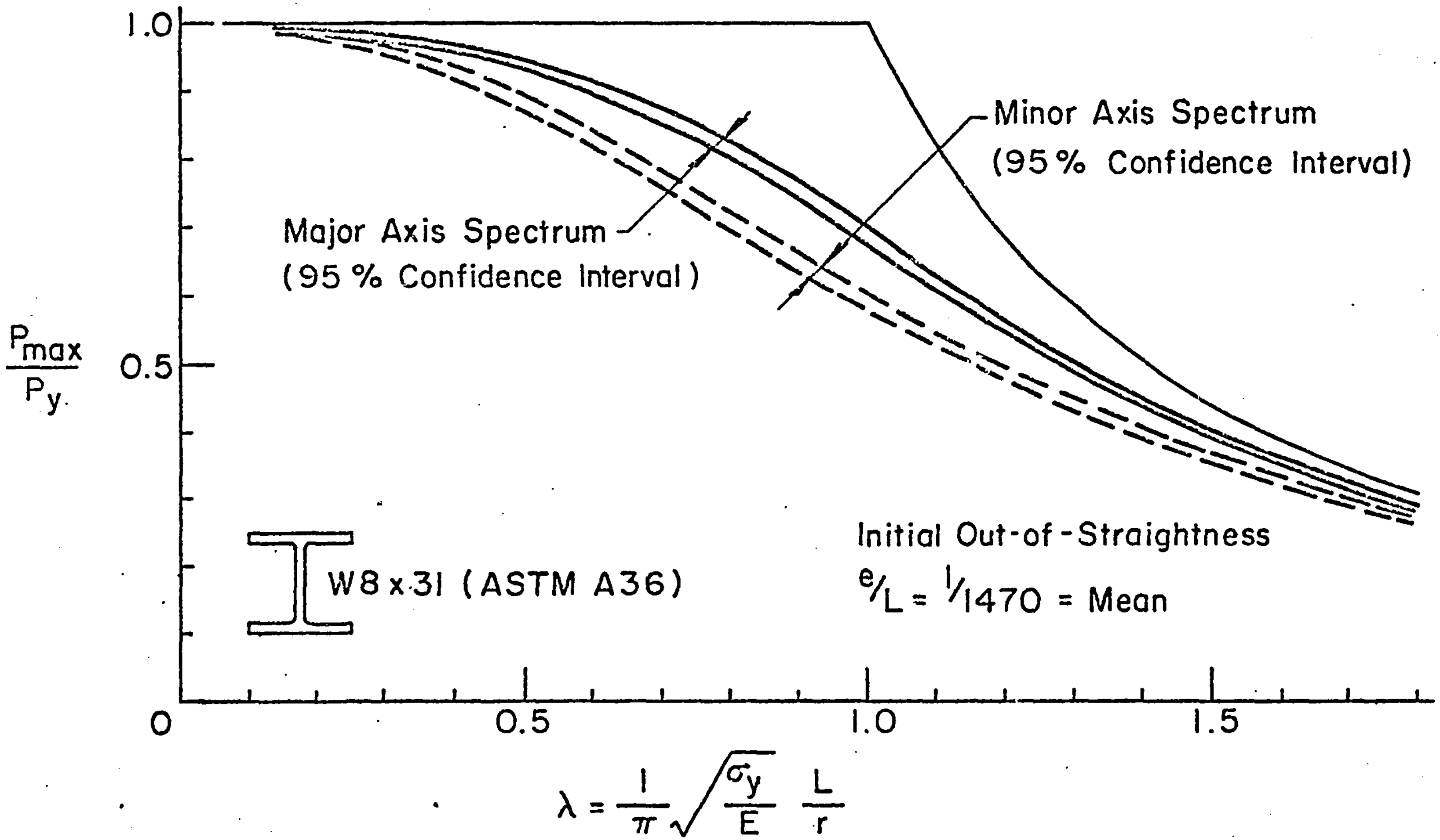


Fig. 47 The Column Curve Spectra for Major and Minor Axis Bending of a Light Rolled Wide-Flange Shape W8x31 (ASTM A36) for One Specific Out-of-Straightness (e/L = 1/1470 = Mean)

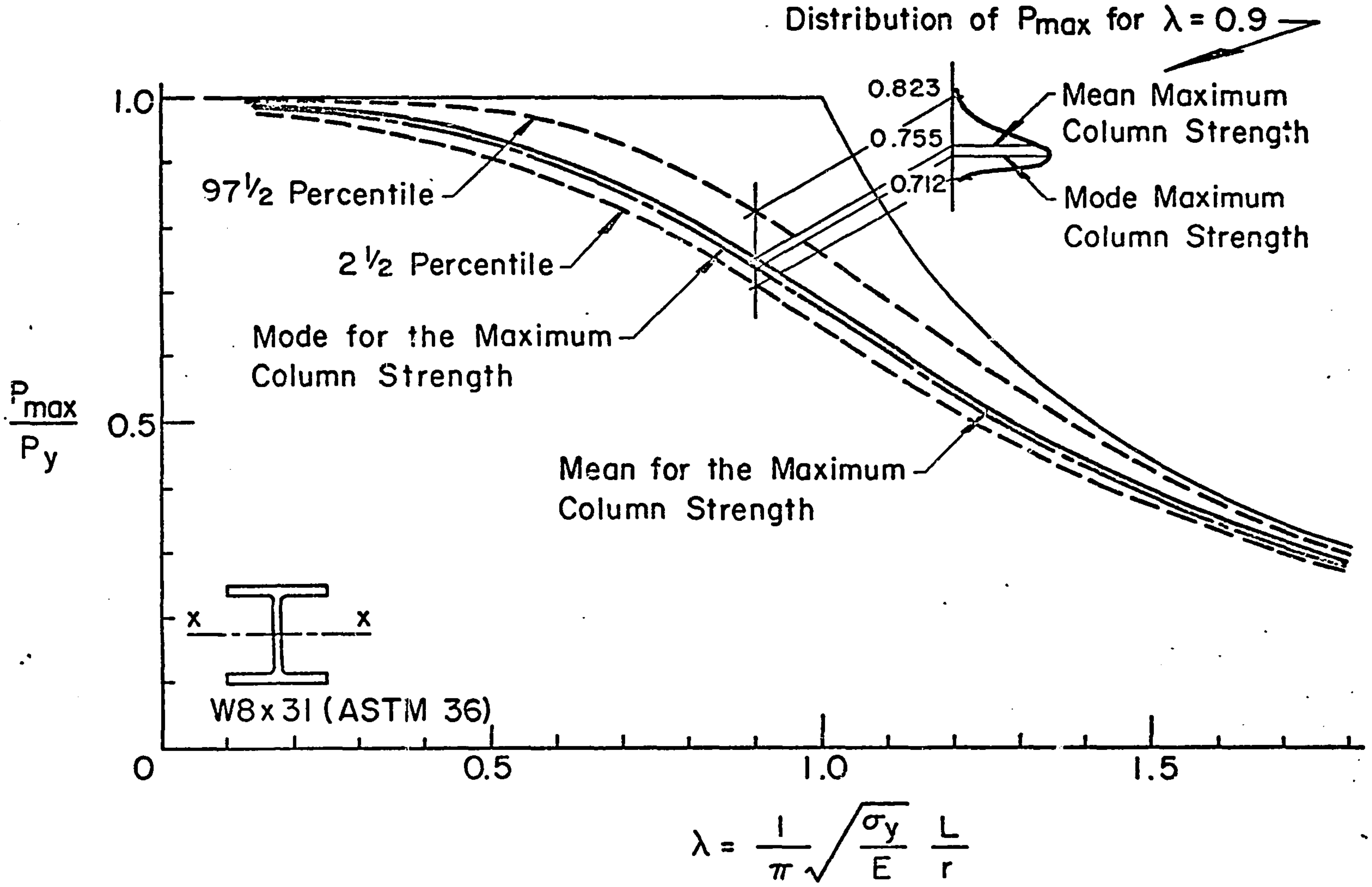


Fig. 48 The Column Curve Spectrum and Its Characteristics, for a Light Rolled Wide-Flange Shape W8x31 (ASTM A36), Bent About the Major Axis

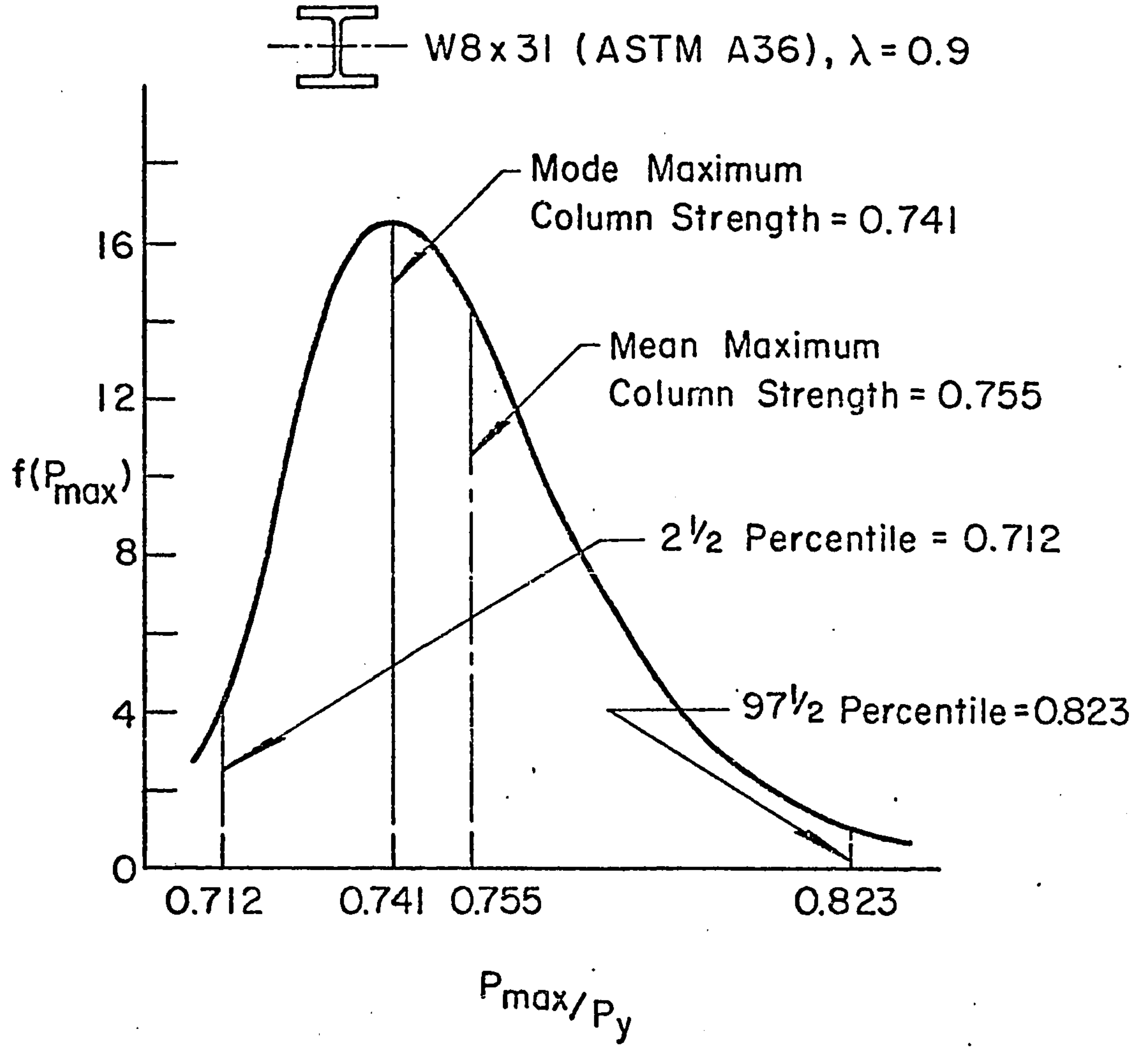


Fig. 49 The Probability Density Function for the Maximum Column Strength of a Rolled Wide-Flange Shape W8x31 (ASTM A36), With Slenderness Ratio $\lambda = 0.9$, Bent About the Major Axis

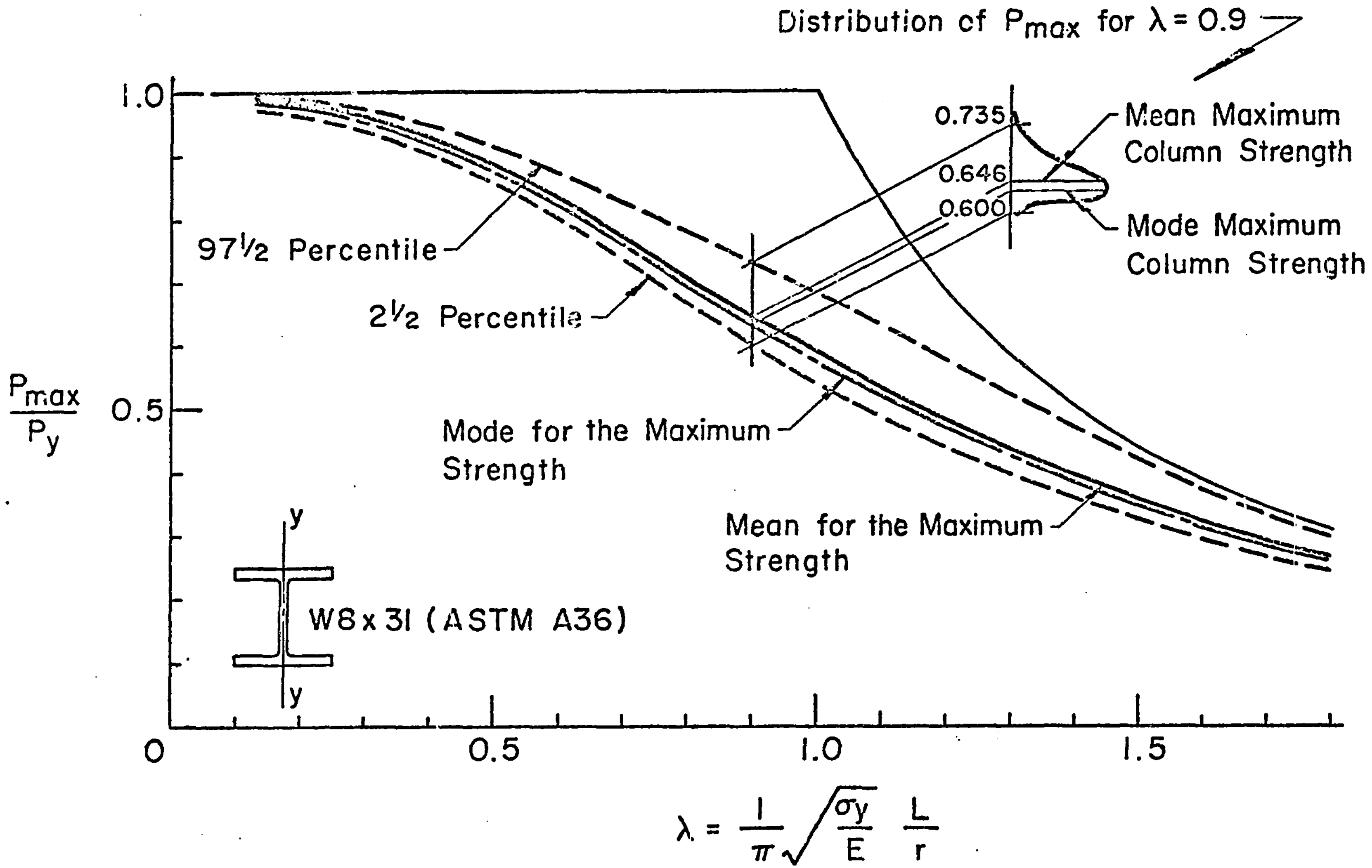


Fig. 50 The Column Curve Spectrum and Its Characteristics, for a Light Rolled Wide-Flange Shape W8x31 (ASTM A36), Bent About the Minor Axis

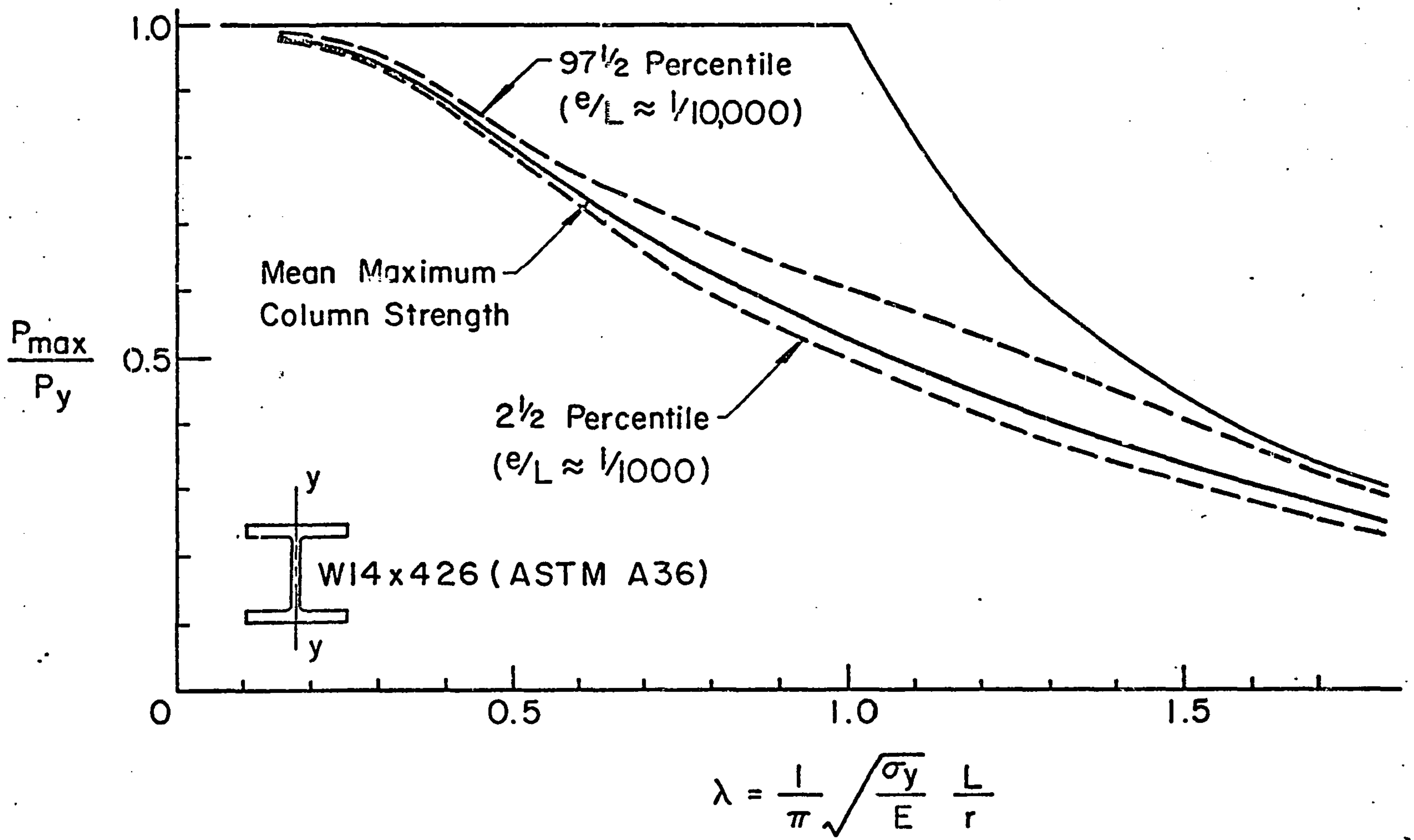


Fig. 51 The Column Curve Spectrum for a Heavy Rolled Wide-Flange Shape W14x426 (ASTM A36), Bent About the Minor Axis

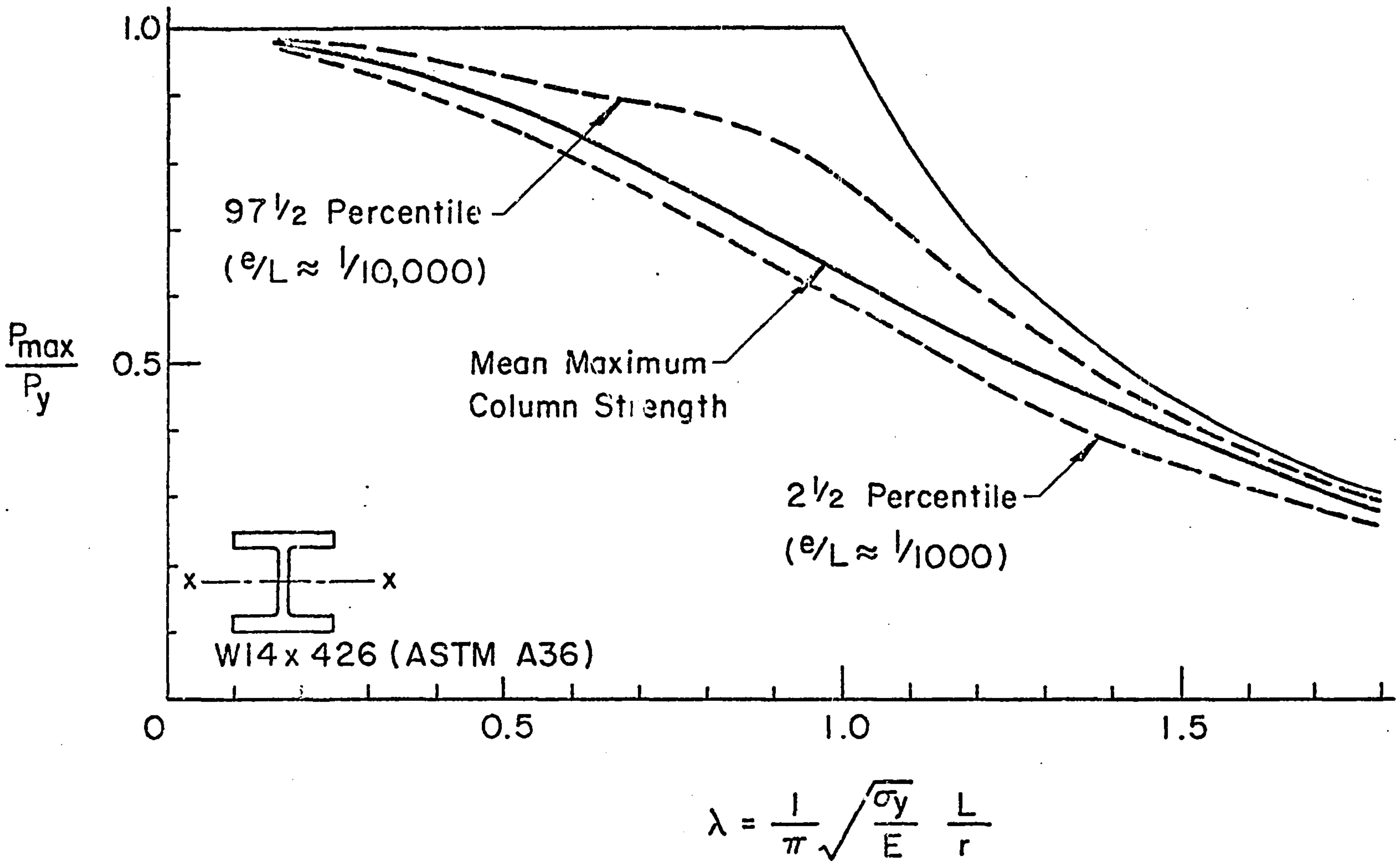


Fig. 52 The Column Curve Spectrum for a Heavy Rolled Wide-Flange Shape W14x426 (ASTM A36), Bent About the Major Axis

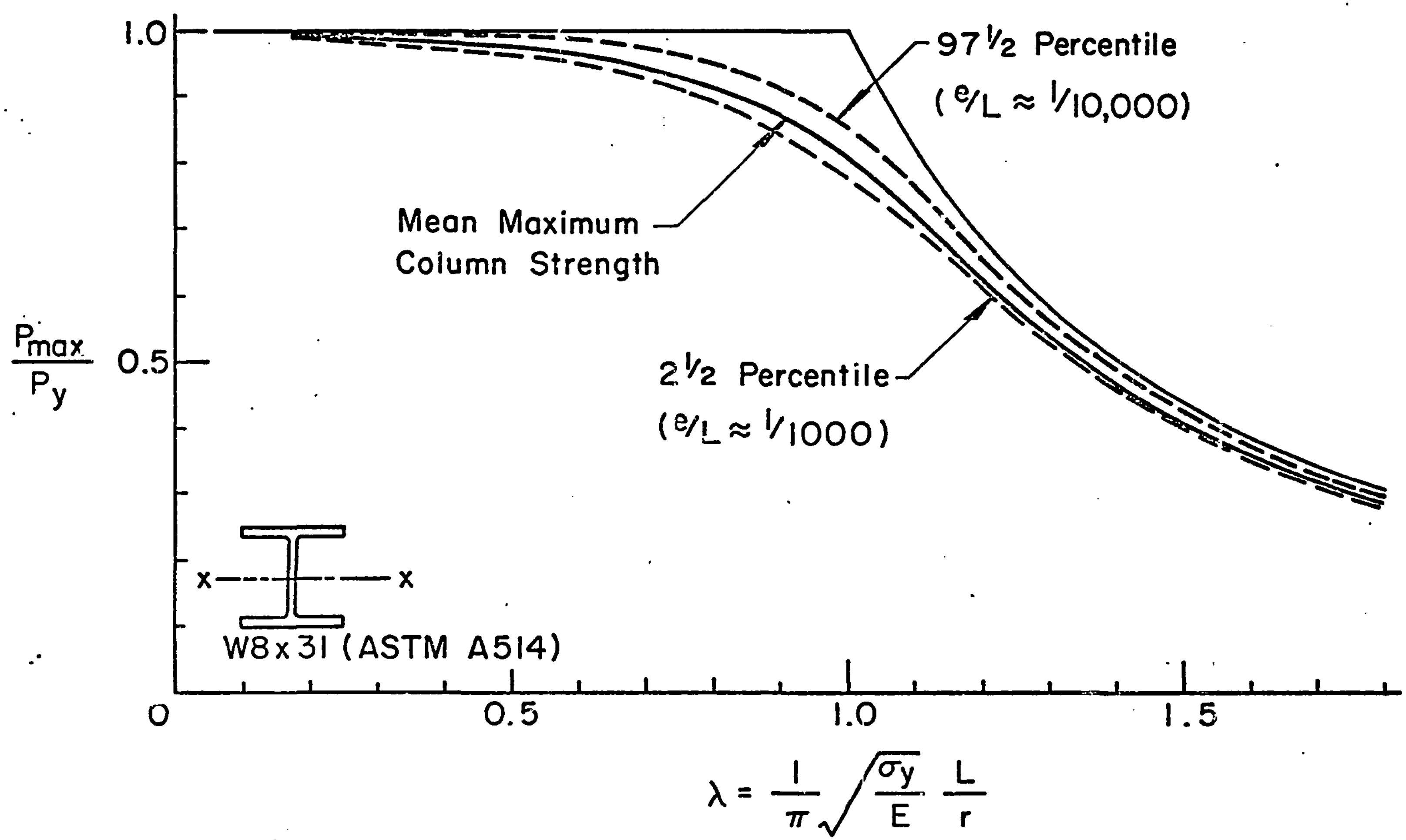


Fig. 53 The Column Curve Spectrum for a Light Rolled Wide-Flange Shape W8x31 (ASTM A514), Bent About the Major Axis

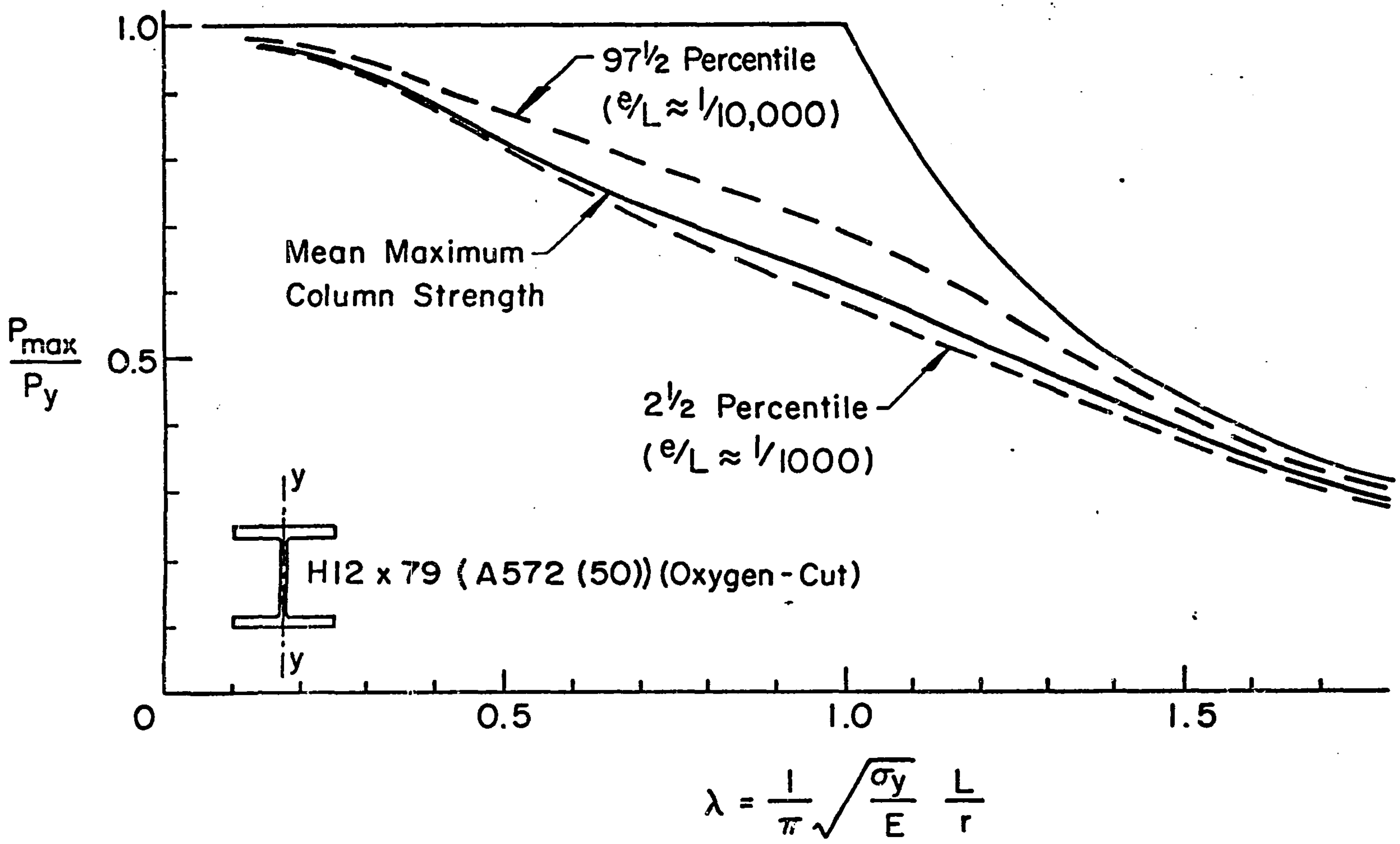


Fig. 54 The Column Curve Spectrum for a Light Welded Flame-Cut Wide-Flange Shape H12x79 (ASTM A572(50)), Bent About the Minor Axis

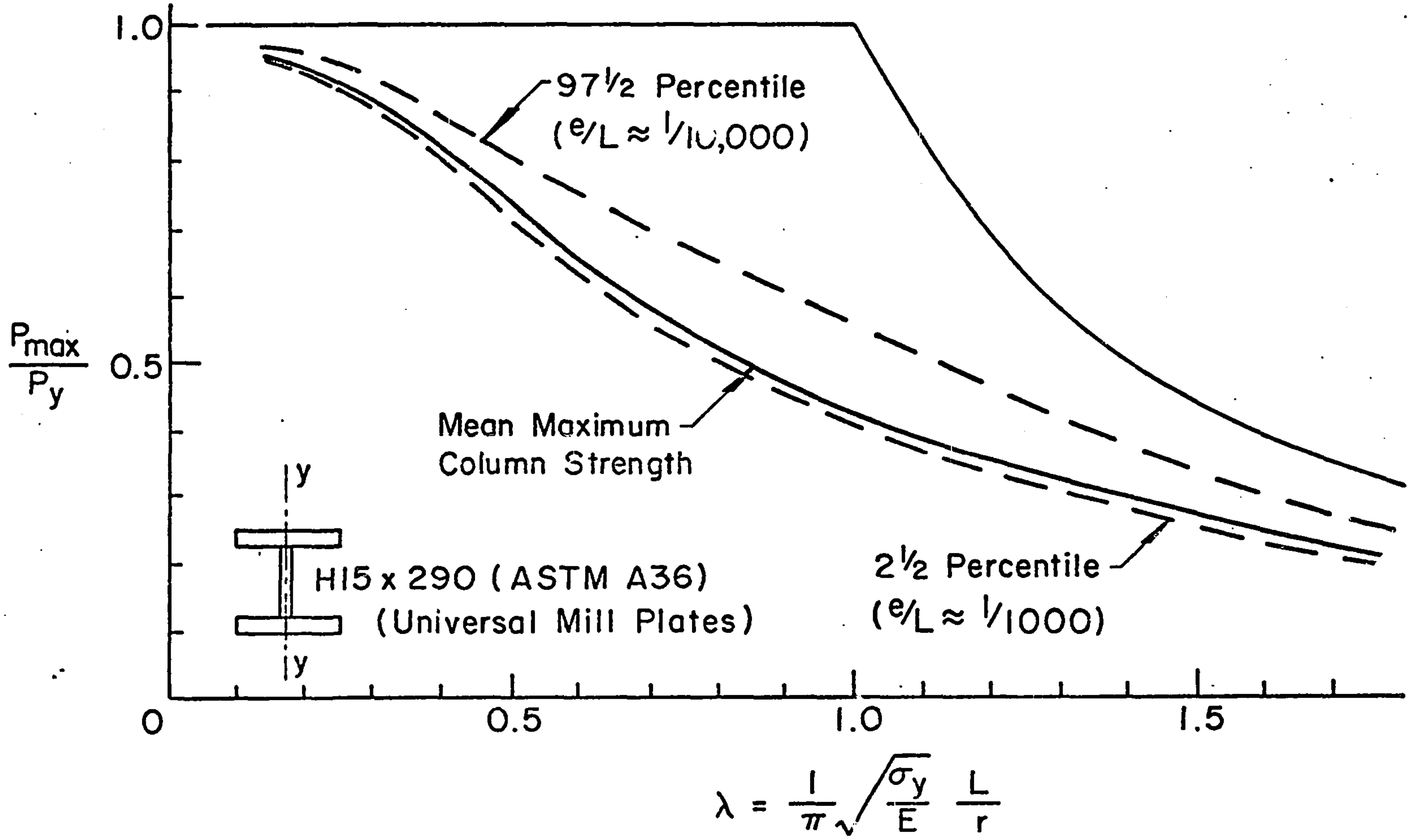


Fig. 55 The Column Curve Spectrum for a Heavy Welded Universal Mill Wide-Flange Shape H15x290 (ASTM A36), Bent About the Minor Axis

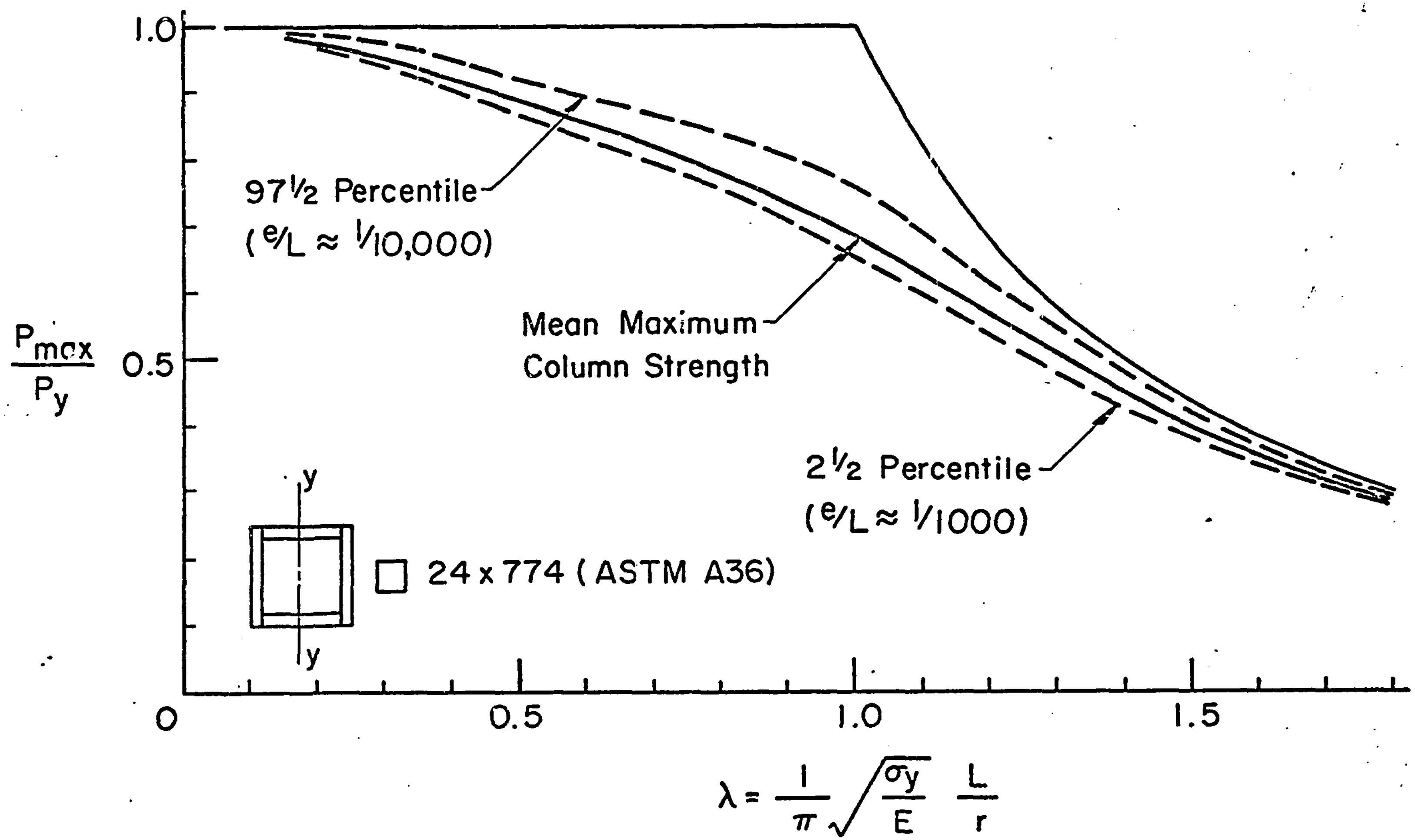


Fig. 56 The Column Curve Spectrum for a Heavy Welded Box Shape □ 24x774 (ASTM A36), Bent About a Principal Axis

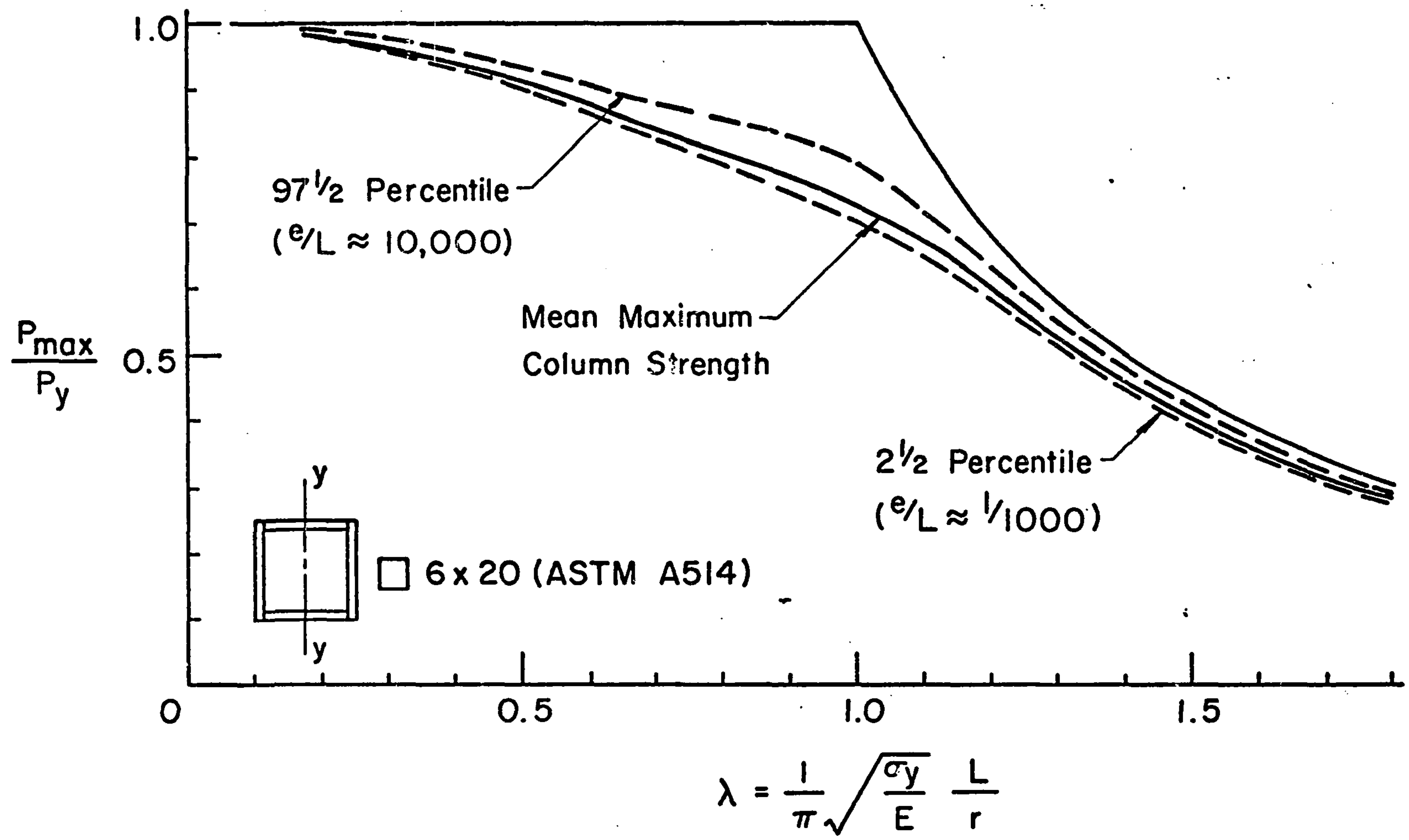


Fig. 57 The Column Curve Spectrum for a Light Welded Box Shape □ 6x20 (ASTM A514), Bent About a Principal Axis

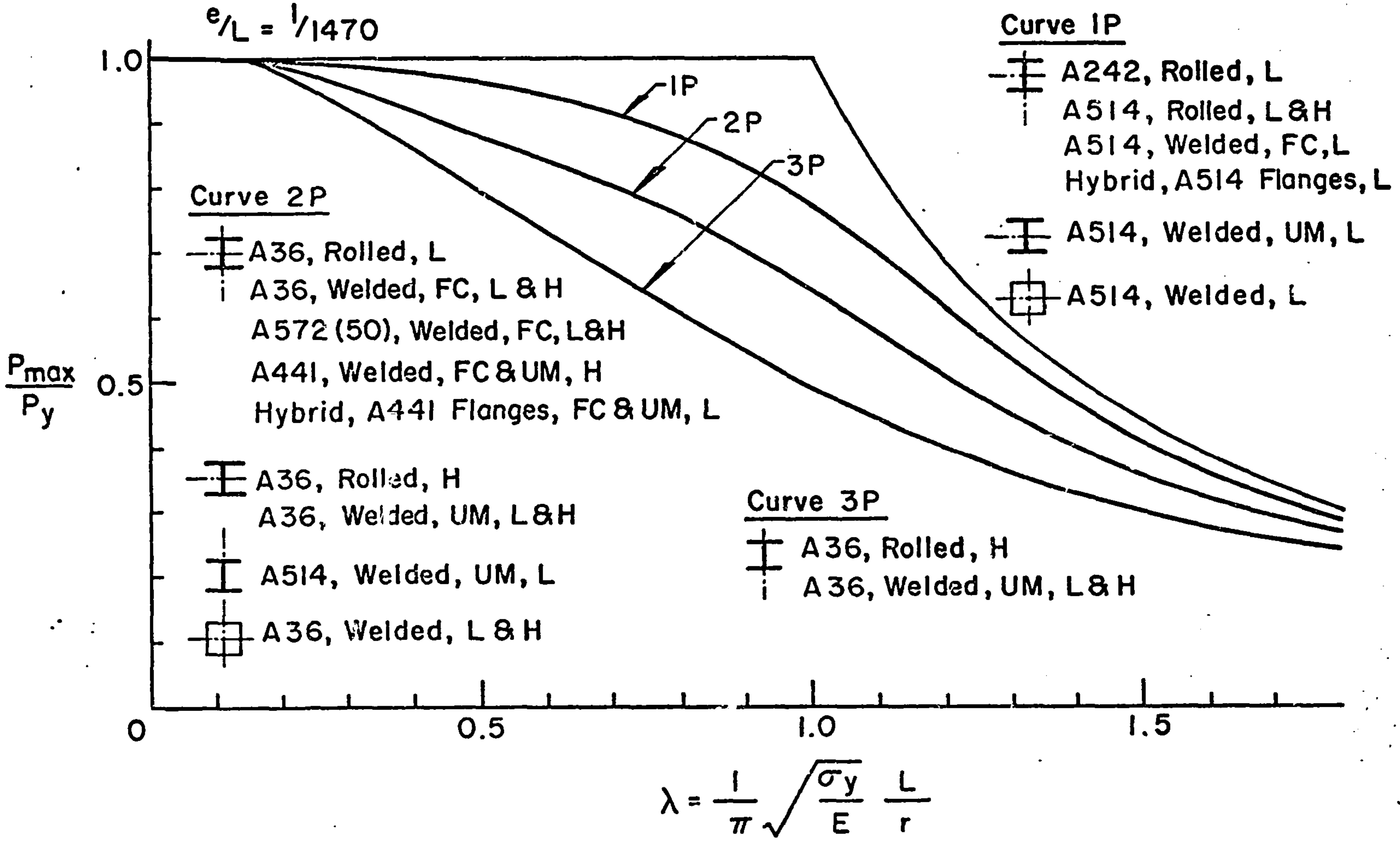


Fig. 58 Possible Multiple Column Curves, Based on an Initial Out-of-Straightness e/L = 1/1470 (Probabilistic Study)

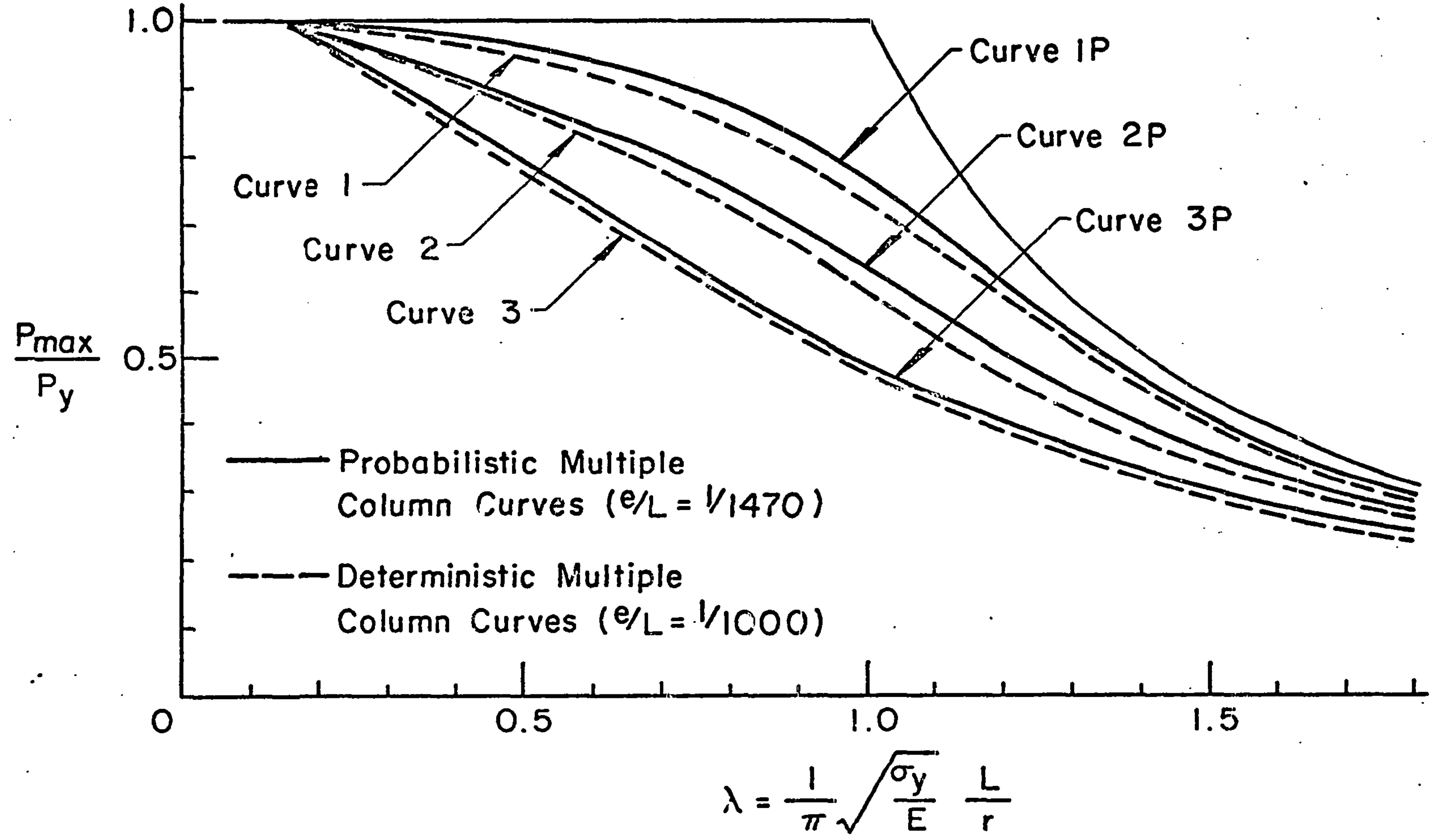


Fig. 59 A Comparison of the Sets of Deterministic and Probabilistic Multiple Column Curves

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