Automated Color-matching Of Printed Ink Films.

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AUTOMATED COLOR-MATCHING OF PRINTED INK FILMS

by
Paul Henry Hoffenberg

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ABSTRACT

The prime objective of an automated procedure for the color-matching of printing inks is to provide the color-matcher with the optimum combination of basic inks to use in the formulation and to supply a good first-shot formula. Such a procedure requires a mathematical model representative of a printed substrate through which the reflectance and ultimately the visual appearance of the printed substrate can be related to the optical properties of the various materials in the system.

The procedure described here employs the Allen three-layer model of a printed substrate in which Kubelka-Munk Theory is applied to the three layers in series to yield the reflectance at the top of the print. Experimental procedures have been developed for the determination of the Kubelka-Munk absorption and scattering coefficients of printing inks and paper substrates and for the determination of additional model parameters.

The experimental techniques and computer programs developed in this work have been applied to a system of 13 commercial lithographic printing inks, with letterpress printing on two uncoated offset papers. The effects of fluorescence have not been considered and have been excluded wherever possible.

Match calculations have been carried out for standards

of known and unknown composition. For additional comparisons, six commercial matches to the same printed standard have been procured. All the matches have been evaluated with respect to color difference under both daylight and artificial (tungsten) light.

For this limited system, the automated procedure has been shown to be superior to purely visual techniques in its ability to select the best combination of basic inks. Also, the first-shot computer matches would seem to be approaching the routine performance of commercial colormatchers in terms of match color difference.

I. INTRODUCTION

For color-matching in any medium there can be said to be five requirements for an ideal commercial color-match:

1. The colorants used should have properties suitable for the end use of the product (e.g., light-fastness, chemical resistance, Federal Drug Administration restrictions, etc.).
2. The match should be within a specified tolerance.
3. A minimum number of colorants should be used in the formulation.
4. The match should be satisfactory for a variety of light sources and for different observers.
5. The cost of raw materials should be a minimum.

For color-matching done solely by visual means, only rarely are these five requirements satisfied simultaneously.

With present-day technology an experienced color-matcher working by visual means can easily satisfy the first requirement by restricting his palette to colorants with the requisite properties. He then selects a combination of colorants from the palette and mixes them in a trial-and-error fashion, constantly comparing the trial mixture to the match standard, until he obtains a match within the required tolerance. With further trials he can bring the match as close to the standard as he desires.

In this manner the color-matcher can satisfy the first two requirements, but in so doing, he often leaves the remaining requirements to chance. By continually altering the shade of the trial mixture he may have incorporated more colorants in the formulation than are really necessary. The match may fail when it is viewed under a different light source. In fact, it may even be disputed under the first light source by a second observer with slightly different color perception. Finally, in attempting to comply with the first four requirements, he may have increased the cost of the match beyond the minimum.

The reason for these difficulties is that there are many possible combinations of colorants which may be used to match a shade for a given light source and a given observer. For example, in the case of textiles three dyes would be the maximum number of colorants needed to achieve a visual match. From a palette of 10 dyes there might be 30-40 possible 3-dye matching combinations. The match from each of these combinations would differ in its ability to stand up under different viewing conditions and would differ in cost. There would be, however, one optimum combination with regard to the above requirements, and it is this combination the color-matcher would like to find.

The determination of the optimum combination by using the manner described above for each colorant combination is clearly impractical in terms of time and effort expended. However, if suitable theory can be found to relate the appearance of a colored object to the optical properties of its constituent colorants, the problem becomes one for which a computer is ideally suited.

This study is part of an attempt to supply such a procedure for the printing ink industry. The work is limited to a palette of 13 commercial printing inks, two offset paper substrates differing only in surface finish, and letterpress printing techniques. The primary objective is to develop a technique capable of selecting the optimum combination of basic inks from the palette, and, secondly, to supply an approximate "first-shot" formula.

II. THE NATURE OF COLOR PERCEPTION (1, 2)

A. Tristimulus Value Matching

For colored objects seen by reflected light (as opposed to self-luminous objects) the perceived color depends on three factors: a light source emitting a characteristic distribution of radiant energy, a colored object which modifies this energy distribution in reflecting it, and a human eye which detects and analyzes the modified energy distribution and gives rise to stimuli for the color to the brain. These stimuli to the brain are three in number, and their magnitudes are referred to as the tristimulus values of the color. Whereas the reflectance curve of the object is its ultimate physical description, it is the tristimulus values that characterize the perceived color of the object.

The fact that the massive amount of information impinging on the eye is reduced to only three numbers is a result of human color vision being mediated by three independent mechanisms. It is precisely for this reason that many colorant combinations can be used to make a visual match under one set of viewing conditions. To match a standard it is necessary to match only three numbers, the tristimulus values. Mathematically, this requires three degrees of freedom. Thus, for example, a dyed textile can be matched with a mixture of three dyes.

This matter will perhaps be made more clear by referring to Fig. 1. Color concerns itself with the so-called visible region of the electromagnetic spectrum extending from roughly 380 to 750 nm. For some purposes, including match calculations, the spectral portions below 400 nm and above 700 nm can be neglected due to the eye's sharply reduced sensitivity in these regions. In Fig. 1a, then, is seen the energy distribution (E) of a typical light source. This energy distribution is modified by the spectral reflectance (R) of the object, seen in Fig. 1b. Since reflectance is defined as the ratio at each wavelength of reflected light intensity to incident light intensity, the modified energy distribution leaving the object is determined by a point by point multiplication of E by R across the spectrum. When this light distribution enters the eye the visual system acts as if it were three separate colored filters through which the light passes simultaneously (rather than consecutively)*. These three hypothetical filters have transmission curves labeled $\bar{x}$, $\bar{y}$, and $\bar{z}$ in Fig. 1c. The energy transmitted through each of the filters is determined by continuing the point by point multiplication, i.e., $E \cdot R \cdot \bar{x}$, $E \cdot R \cdot \bar{y}$, and $E \cdot R \cdot \bar{z}$,

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* This is, of course, an inaccurate anatomical or physical description of the eye, but the analogy will serve for the present purpose. For details see Reference (3).

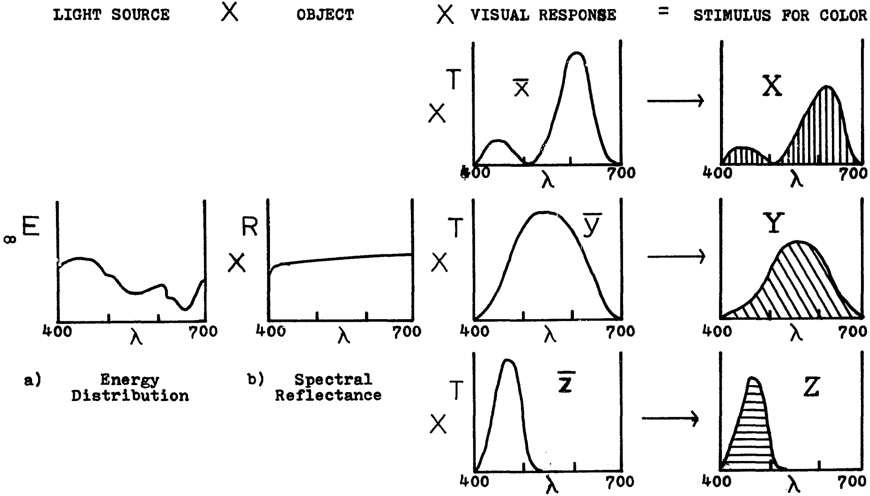


Figure 1. Tristimulus Values of Object 1

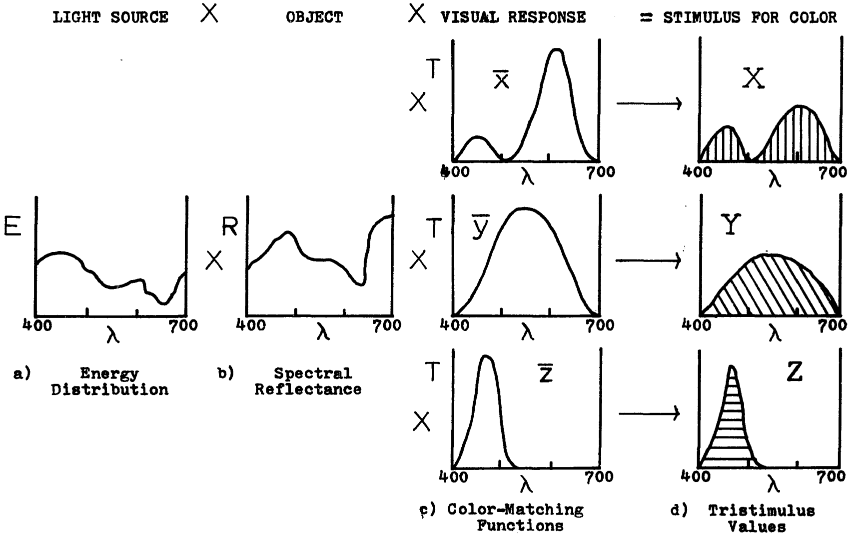
a) Energy Distribution

b) Spectral Reflectance

c) Color-Matching Functions

d) Tristimulus Values

Figure 2. Tristimulus Values of Object 2



a) Energy Distribution

b) Spectral Reflectance

c) Color-Matching Functions

d) Tristimulus Values

to get the final curves in Fig. 1d. It is the integrated areas X, Y, and Z under the three respective final curves that represent the outputs of each of the three receptor systems. X, Y, and Z are the tristimulus values of the color. In mathematical terms,

$$\left.\begin{aligned} X &= k\int_{\lambda=700}^{\lambda=400} ER\bar{x}d\lambda \\ Y &= k\int_{\lambda=700}^{\lambda=400} ER\bar{y}d\lambda \\ Z &= k\int_{\lambda=700}^{\lambda=400} ER\bar{z}d\lambda \end{aligned}\right\} \qquad \text{(II.1)}$$

where k is the normalizing factor

$$k = 100/\int_{\lambda=700}^{\lambda=400} E\bar{y}d\lambda$$

Fig. 2 shows the same calculation for a second object with a reflectance curve entirely different from that of the first object. The light source and visual functions are the same as before, and for this case the second object is a visual match to the first. The final sets of curves do not match in shape, but their respective area or tristimulus values are equal.

However, if the light source or visual functions are changed for the two objects there will be two new sets of tristimulus values, and these sets will probably fail to match. That is, in general, each of the colored objects will change in appearance under a new light source or for a new observer. The respective changes are likely to be in different directions so that the objects fail to match under the new viewing conditions.

This phenomenon is referred to as metamerism: When two colored samples have different spectrophotometric curves (due to their having different constituent colorants) and match under one set of viewing conditions, but fail to match under different viewing conditions, the match is said to be metameric.

The foregoing considerations shed some light on the problems confronting the color-matcher working by purely visual means. He need not have any knowledge of energy distributions, reflectance curves, visual color-matching functions, or tristimulus values. He merely mixes colorants, compares the trial match with the standard, and mentally assesses any difference. Since color vision is tridimensional in nature, the perceived difference can be described with from one to three adjectives representing differences in hue (redder, greener, or bluer), differences in lightness (lighter or darker), and differences in saturation (grayer or more saturated). The color-matcher corrects accordingly and continues this process until he achieves a match. What has in fact occurred is that he has matched the tristimulus values of the standard and trial match.

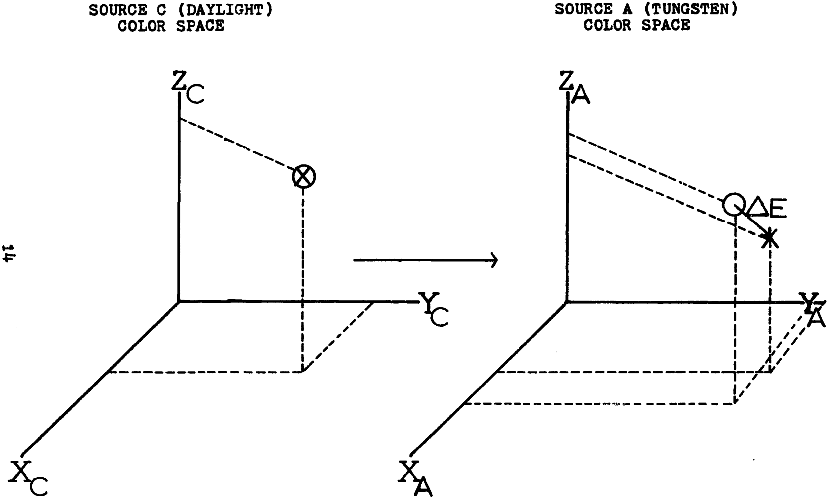
If the color-matcher next compares the sample and standard under a new light source, the match will likely be metameric; that is, it will fail. If the failure is pronounced, the color-matcher must start anew with a different combination of colorants and hope he has made a better guess at a good combination.

B. Metamerism Indices

The key to the solution of this problem with instrumental and computer techniques is finding suitable theory to relate the optical properties of the colorants to the reflectance curve of the trial match sample. If such theory is available one can proceed as follows: The reflectance curve of the standard is measured and tristimulus values are calculated for a standard light source such as CIE Source C (3) and a standard observer such as the 1931 CIE 2° Standard Observer (3). Tristimulus values are also found for a markedly different light source such as CIE Source A (3). Then, for each combination of colorants in the palette, mathematical techniques are employed to find a match formula which has a reflectance curve that will generate tristimulus values identical to those of the standard under Source C illumination. Source A tristimulus values are then generated from the reflectance curve of the match of the moment. The differences between the Source A tristimulus values for the match and standard are used to calculate a "metamerism index" as will be described below. This metamerism index gives a measure of metamerism, or the degree of mismatch under Source A for that particular formulation and the standard. Match formulas and metamerism indices are similarly calculated for all possible colorant combinations in the palette. The different formulations can then be ranked in terms of metamerism, and, if desired, cost of raw materials.

The concept of metamerism can be better understood by reference to Fig. 3. Since color is tridimensional one can think in terms of plotting a color in a three-dimensional Cartesian coordinate space whose axes are the tristimulus values X, Y, and Z. The Source C and Source A tristimulus values for a standard are indicated by the circles in Figs. 3a and 3b, respectively. Note that the coordinates are not the same for the two points; hence, one considers a Source C and a Source A space as indicated by the subscripts. The calculated Source C match for a particular colorant combination naturally plots coincidently with the standard in Source C space, as represented by the cross in Fig. 3a. However, in Source A space, the sample and standard are no longer coincident. The difference vector in three dimensions indicated by the arrow has a length which represents the color difference under Source A for an exact match under Source C. This is used as the definition of the metamerism index.

In general practice, the three-dimensional rectangular coordinate space of Fig. 3 is not used. Instead, the basic



SOURCE C (DAYLIGHT)
COLOR SPACE

SOURCE A (TUNGSTEN)
COLOR SPACE

Figure 3. Standard (○) and Match (X) in Source C and Source A Color Space

tristimulus information is transformed to a different three-dimensional space, and the color difference is calculated by an often complicated set of equations. In this work the metamerism indices are calculated using the Adams Chromatic Value system (4) and Adams color-difference equations (5):

$$\left.\begin{aligned} V_X &= 2.529(100X/X_{pd})^{1/3} - 1.838 \\ V_Y &= 2.529(100Y/Y_{pd})^{1/3} - 1.838 \\ V_Z &= 2.529(100Z/Z_{pd})^{1/3} - 1.838 \end{aligned}\right\} \qquad (II.2)$$

where "pd" refers to the value of a perfect diffuser and V_i has a maximum value of 10.000, and

$$\left.\begin{aligned} \alpha &= 50(V_X - V_Y) \\ \beta &= 20(V_Z - V_Y) \\ \gamma &= 11.5(V_Y) \end{aligned}\right\} \qquad (II.3)$$

$$\text{Metamerism index} = [(\Delta\alpha)^2+(\Delta\beta)^2+(\Delta\gamma)^2]^{\frac{1}{2}} \qquad (II.4)$$

where the Δ's are the differences between the values for the sample and standard.

Although a spectral match would be a perfect non-metameric match, a spectral match might be impossible if the colorants used in the standard were not available in the matching palette. This is especially likely to be the case, for example, if the standard is a dyed textile or paper, and the match must be made in a paint medium. If the demand for a spectral match is quite rigid, such a match may be made by using more than the minimum number of colorants. There is, however, a danger in this approach.

Color space is not uniform, and slight spectral differences in one region (e.g., a green) would show a larger visual difference than the same order of spectral differences for a different region (e.g., off-white). Thus the closest spectral match may not be a visual match.

III. THEORETICAL BASIS FOR COLOR-MATCHING

A. Kubelka-Munk Theory

For all media in which light is modified by both absorption and scattering, Kubelka-Munk Theory (6) is the common choice of theoretical framework by which reflectance curves are related to optical properties of colorants and substrates. This theory treats a layer of dispersed pigment or other scattering material as if the layer were homogeneous. It is assumed that there is diffuse illumination directed downward on the layer and scattering of the light is in either the forward (downward) or backward (upward) direction. Conceptually then, there are two beams passing through the layer, one in the downward direction and one in the upward direction. As each beam passes through an elementary thickness dx of material (where x is measured from the bottom of the film) there is both absorption and scattering proportional to the beam intensity at that point(intensity I for the downward beam and intensity J for the upward beam). The proportionality constants are the absorption (K) and scattering (S) coefficients. This is summarized by the differential equations

$$\begin{aligned} dJ &= -(K+S)Jdx + SIdx \\ -dI &= -(K+S)Idx + SJdx. \end{aligned} \qquad \text{(III.1)}$$

It should be noted first, that these equations are to be applied at each wavelength, with K and S varying with wavelength. Second, the minus sign in front of the second equation is explained by the downward direction being that of decreasing x.

These equations can be simultaneously integrated to relate the reflectance R at the top of the layer to K, S, the total layer thickness X, and the background reflectance R_g behind the layer

$$R = f(K,S,X,R_g) = \frac{1 - R_g(a - b\cdot\coth bSX)}{a - R_g + b\cdot\coth bSX}, \qquad \text{(III.2)}$$

where

$$a = (K+S)/S \text{ and} \qquad \text{(III.3)}$$

$$b = (a^2 - 1)^{\frac{1}{2}}. \qquad \text{(III.4)}$$

All the quantities except X are wavelength dependent.

B. Single-Constant Theory

For the case of a completely opaque layer, neither the background nor the layer thickness have any effect. This is apparent from allowing X to approach infinity in eq. (III.2) —an opaque layer is an "infinitely thick" layer— to yield the much more simple relationship

$$K/S = (1 - R_\infty)/2R_\infty, \qquad \text{(III.5)}$$

where the subscript "∞" refers to the opaque layer.

This equation is quite useful in the case of dyed textiles where it is assumed that all the scattering is due to the textile fiber itself. The uptake of dye by the fiber changes only the absorption coefficient except for very high concentrations of dye. Furthermore, it is assumed that the dye permeates the entire fiber to yield a homogeneous structure, and several folded layers of the material constitute a single opaque layer.

This situation leads to the overall K/S ratio for a dyed textile being a linear combination of K/S ratios for the respective dyes and undyed fibers. This is illustrated as follows:

$$(K/S)_{total} = \frac{K_{fiber} + c_1K_1 + c_2K_2 + \cdots}{S_{fiber}}, \qquad \text{(III.6)}$$

where the c's are concentration factors and the subscripts refer to dye number 1, 2, etc. This can be rewritten as

$$(K/S)_{total} = (K/S)_{fiber} + c_1(K/S)_1 + c_2(K/S)_2 + \cdots \qquad \text{(III.7)}$$

where $(K/S)_i$ represents the K/S ratio for a unit concentration of the "ith" dye on the particular substrate.

Besides the relatively simple mathematics, textile color-matching has the advantage of a rather small number of substrates; e.g., nylon, wool, cotton, etc. For each substrate the dyes-substrate system can be calibrated from reflectance measurements on materials dyed at unit concentrations. This information is obtained once and for all and is retained for all match calculation work.

An analog computer called COMIC I (7) has been built and is in use in industrial textile color-matching. Digital computer programs have also been written to accomplish this purpose (8).

The same mathematics and computer programs apply to opaque layers of pastel-shade paints, where virtually all the scattering is due to a white pigment such as titanium dioxide (TiO_2) or zinc oxide (ZnO). In this case, the K/S ratios for the colored pigments (or basic colored paints) are determined from mixtures of small amounts of the color with white.

C. Two-Constant Theory

The next step in complexity occurs when the scattering contribution of the colored pigments in paints or plastics can no longer be neglected in relationship to the white pigment present. The basic equation that applies here is

$$(K/S)_{total} = \frac{c_1K_1 + c_2K_2 + c_3K_3 + \cdots}{c_1S_1 + c_2S_2 + c_3S_3 + \cdots} . \quad (III.8)$$

This creates the need to know both K and S separately for each pigment or basic paint in the mixture.

When the colorant layer ceases to be fully opaque, one must revert to the general Kubelka-Munk equation (III.2). It is additionally necessary to measure the thickness of the layer, and the background reflectance. K_{total} and S_{total} are given by the numerator and denominator, respectively, of eq. (III.8).

It is appropriate at this point to note that Kubelka-Munk Theory serves to predict the reflectance at the top of the colorant layer. It does not account for the events at the refractive index discontinuity between air and the colorant medium. It is necessary to account for these effects by applying the so-called Saunderson correction (9):

$$R' = k_1 + (1-k_1)(1-k_2)R/(1-k_2R), \qquad \text{(III.9)}$$

where R' is the corrected reflectance that would be seen by the eye or measured with a spectrophotometer; R is the theoretical reflectance; k_1 is the value of the front surface reflectance; and k_2 is the fraction of the light intensity diffusely incident on the front surface from within which is reflected back into the layer. In this work k_1 is assigned the value 0.04 for a continuous medium of refractive index 1.5; this is given by the Fresnel equation at normal incidence

$$R = \frac{(n-1)^2}{(n+1)^2}. \qquad \text{(III.10)}$$

The theoretical value of 0.6 suggested by Judd (10) is taken as the value of k_2.

Kubelka-Munk Theory also does not take into account the effects of fluorescent materials. Therefore, this work has been restricted to non-fluorescent materials, leaving fluorescence considerations for future investigation.

IV. THE ALLEN THREE-LAYER MODEL OF A PRINTED SUBSTRATE (11)

A. Mathematical Model*

The argument up to this point has been confined to a single homogeneous layer, but a printed substrate is anything but homogeneous. There are at least three layers present: 1) ink on top of the paper substrate, 2) ink which has penetrated into the paper, and 3) plain paper beneath the region of penetration. There is most certainly a fourth layer of vehicle alone interposed between the penetrated ink and plain paper layers, but the model proposes to incorporate this within the intermediate layer. Fig. 4 is a sketch of the model.

Each layer is, in general, non-opaque. Therefore, if K, S, and X are known for each of the three layers, the reflectance at the top of the print may be calculated by applying eq. (III.2) three times in series, starting with the bottom layer. That is, the background reflectance behind the composite is assumed to be $R_g = 0$, and the reflectance R_I is calculated for the top of layer I. This is used as a background for layer II, and so on, to the top of layer III. The Saunderson correction,

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* Although Dr. Eugene M. Allen's mathematical model has been published (11), this writer feels that it is such a central part of this thesis, that it must be recapitulated here.

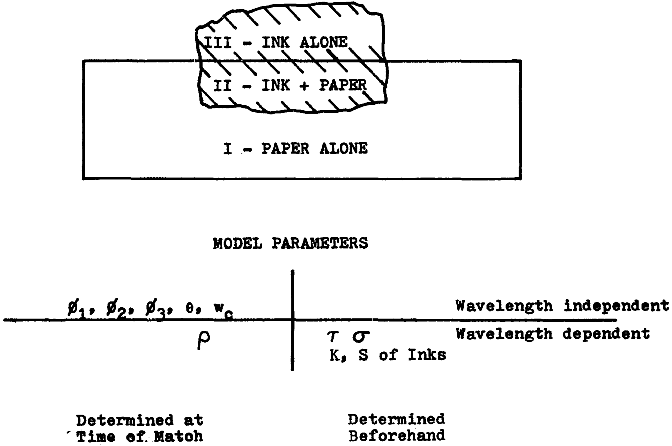


Figure 4. The Allen Three-Layer Model of a Printed Substrate

eq. (III.9), is then applied to the theoretical reflectance R_{III} to yield the final value R (without a subscript).

The major obstacle in color-matching of printing inks is the influence of the substrate. Each substrate contributes its own color properties, but also, by virtue of its physical structure, affects the distribution of ink through the paper. Thus, the thicknesses of each of the three layers are generally not known for a new paper substrate. These difficulties are multiplied by the number of available substrates, which is virtually infinite. Each new job may call for a new substrate.

Two adjustable model parameters ϕ_1 and ϕ_2 account for the film thicknesses. Of the total applied ink film thickness w_c (in units of g/m^2 rather than units of length) it is assumed that some fraction ϕ_1 remains on top; the remaining ink, $(1-\phi_1)w_c$, penetrates the paper. The ink that penetrates the paper does so to some fractional depth ϕ_2 of the total paper thickness. Thus layer II contains $(1-\phi_1)w_c$ g/m^2 of ink and $\phi_2 w_p$ g/m^2 of paper. Layer III contains $(1-\phi_2)w_p$ g/m^2 of plain paper. The term w_p is the basis weight of the paper.

As indicated in the above paragraph, the thickness terms used in the model, which correspond to the term X in the Kubelka-Munk notation, are in units of weight per unit area rather than length. This is more convenient from an experimental point of view and is consistent with the standard manner in which paper substance is reported.

The parameters ϕ_1 and ϕ_2, along with a third parameter ϕ_3 which characterizes the ink-paper-proofing-press system, are dimensionless and wavelength-independent. As seen in the mathematical definition below in eq. (IV.10), ϕ_3 describes how much of the paper fibers in layer II are partially wet by vehicle. The assignment of "ϕ-values" or magnitudes of these parameters is done by a calibration procedure that will be described later.

B. Determination of K and S of Printing Inks

Since, at each wavelength and for each ink, two unknowns K and S must be determined, two equations are required. The procedure is to start with the white ink. If a sufficiently thin film of known thickness can be applied to a black and white substrate, where the black and white background reflectances are known, and where the <u>film reflectances differ for measurement over the two backgrounds</u>, eq. (III.2) can be used twice to supply the two necessary equations. The substrate must be receptive to the ink so that uniform films are obtained, but also impermeable to the ink so that only a single layer is present. Although the two simultaneous equations (III.2), one for each background and corresponding film reflectance, cannot be solved directly due to the hyperbolic cotangent function, the solutions K and S are obtainable by numerical analysis techniques.

The same procedure can be attempted for each of the colored inks. Here, however, there are certain to be some wavelengths where the pigment absorption is so strong that the thin films are virtually opaque. The two equations for the two different backgrounds then degenerate into one, and an additional equation is required. This is provided by a film over a known white background, where the film is composed of a mixture of the colored ink with the above-mentioned white. These procedures are in accordance with the suggestion of Davidson and Hemmendinger (12).

C. Characterization of the Substrate

Since the substrate may change with every job, an economically practical automated matching procedure must provide a rapid, reliable means of determining the optical and physical properties of paper substrates. For white papers, the most important factor is the "scattering power" Ψ_p of the paper, a dimensionless quantity which is the product of the paper scattering coefficient S_p (in $[g/m^2]^{-1}$) and the basis weight w_p (paper thickness in $[g/m^2]$). This quantity may be calculated by an equation given by Kubelka (13):

$$\Psi_p = S_p w_p = \left[\operatorname{Arcoth}\left(\frac{a-R}{b}\right) - \operatorname{Arcoth}\left(\frac{a-R_g}{b}\right)\right]/b \qquad \text{(IV.1)}$$

where

$$a = 1+K_p/S_p = (1+R_\infty^2)/2R_\infty, \qquad \text{(IV.2)}$$

$$b = (a^2-1)^{\frac{1}{2}} = (1-R_\infty^2)/2R_\infty, \qquad \text{(IV.3)}$$

K_p is the absorption coefficient, R is the reflectance of a single sheet over a black background, R_g is the reflectance of the black background, and R_∞ is the reflectance of an "infinite thickness" pile of sheets. All the variables in eqs. (IV.1, IV.2, and IV.3) are wavelength-dependent.

A simplifying assumption made by Dr. Allen (11) is that paper scattering powers for different substrates will not differ significantly on a relative basis. By relative basis is meant the ratio of the scattering power at any wavelength to the scattering power at a reference wavelength, the latter being chosen as 700 nm, the wavelength of least scattering. This defines the "tau-function" $\tau(\lambda)$:

$$\tau(\lambda) = \Psi_p(\lambda)/\Psi_p(700) = \Psi_p(\lambda)/\theta, \qquad \text{(IV.4)}$$

where θ is the absolute scattering power at 700 nm, and may be considered a wavelength-independent quantity.

At the present time, the validity of the assumption of invariance of tau with paper substrates is not known. This thesis confines itself to two papers (Finch Offset-Smooth Finish and -Vellum Finish). These two papers do show slight differences in their tau functions, but in this work matches have been calculated using the tau function corresponding to the particular match substrate. Future work is necessary to determine if one tau function will serve for all papers, or if papers can be sorted into classes, each with its own tau function. The criterion for judging the suitability of a particular tau function is not how well it is reproduced for different papers, but rather, how good the calculated matches are if the particular tau function is used for different papers.

The advantage that the tau function offers is that the reflectance of the single sheet over a black background need be measured only at 700 nm and not over the entire spectrum. The <u>absolute</u> scattering power θ is calculated at 700 nm, and the product of θ and the <u>relative</u> scattering power $\tau(\lambda)$ generates the absolute scattering power $\Psi_p(\lambda)$. It may be noted that in eqs. (III.2, III.3, and III.4) the absorption and scattering coefficients never appear separately; it is the absorption and scattering <u>powers</u> or the ratios of the coefficients that occur. This obviates the need to measure the total paper thickness w_p.

The absorption power $\Upsilon_p(\lambda)$ of the paper is determined by the following scheme:

$$\rho(\lambda) = (1-R_{\infty p})^2/2R_{\infty p} = K_p/S_p = K_p w_p/S_p w_p = \Upsilon_p/\Psi_p, \quad \text{(IV.5)}$$

$$\Upsilon_p = \rho\Psi_p = \rho\tau\theta. \quad \text{(IV.6)}$$

At this point all the variables concerned with layers I and III have been expressed in measureable or calculable quantities. It is the intermediate layer II that presents the most difficulties. The ink contained in this layer will contribute its normal absorption and scattering properties. The absorption power of the paper will also be unchanged. However, even if the vehicle present in that layer is virtually transparent, it will change the scattering properties of the paper due to partial wetting of the fibers. This becomes readily apparent if one observes the increased translucency of paper printed with vehicle, or, for that matter, paper wet by water or solvent.

For a sheet printed with vehicle alone, the scattering power Ψ_v can be determined in the same manner as the determination of Ψ_p, the scattering power of plain paper. However, it would be impractical to do this for each new job. Consequently, Dr. Allen has devised (11) the "sigma" (σ) function to give a measure of the decrease in scattering power due to partial immersion of the paper fibers in vehicle. This function will now be explained.

In Fig. 5 four relative scattering curves are displayed. The τ curve (curve 1) has been previously described. If it were possible to immerse a paper sheet completely in vehicle, the immersed sheet would have a scattering coefficient $S_p''(\lambda)$ in units of $[g/m^2$ of unimmersed paper$]^{-1}$. This appears on a relative basis $(S_p''(\lambda)/S_p(700))$ as curve 3. Unfortunately, it would be difficult or impossible to determine curve 3 experimentally since complete fiber wetting could not be guaranteed. However, the assumption may be made that such a relative scattering curve exists, and is again invariant with respect to changing paper-vehicle systems. What can be determined experimentally is curve 2, the relative scattering curve $S_p'(\lambda)/S(700)$ of a sheet printed with vehicle. This is available since $\Psi_v = S_p'w_p$, where S_p' is also in units of $[g/m^2$ of plain

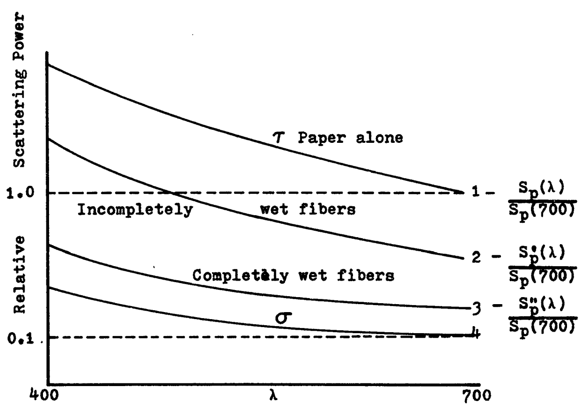


Figure 5. Relative Scattering Curves for Paper and Paper-Vehicle

paper$]^{-1}$. The further assumption is made that curve 2 is a linear combination of curves 1 and 3; that is,

$$S'_p(\lambda)/S_p(700) = k\tau(\lambda) + (1-k)S''_p(\lambda)/S_p(700). \quad \text{(IV.7)}$$

If a single data point is known on curve 3, eq. (IV.7) can be solved for k, and the rest of curve 3 can be calculated. But, since no points on curve 3 are available, an artifice is used. Curve 4 is the sigma function defined by

$$\sigma(\lambda) \equiv k'\tau(\lambda) + (1-k')S''_p(\lambda)/S_p(700), \text{ and, arbitrarily,} \quad \text{(IV.8)}$$
$$\sigma(700) \equiv 0.1.$$

Thus, $\sigma(\lambda)$ is also a linear combination of curve 1 and curve 3, but the constant k' is fixed by the assignment of the value 0.1 at 700 nm. It can be shown further (11) that

$$\sigma(\lambda) = \frac{0.9\Psi_v(\lambda) - [\Psi_v(700) - 0.1\Psi_p(700)]\tau(\lambda)}{\Psi_p(700) - \Psi_v(700)}. \quad \text{(IV.9)}$$

Eliminating $S''_p(\lambda)/S_p(700)$ from eq. (IV.7) by means of eq. (IV.8) gives

$$\left.\begin{aligned} S'_p(\lambda)/S_p(700) &= \frac{(1-k)}{(1-k')}\sigma + \frac{(k-k')}{(1-k')}\tau \\ &= \left(\frac{1-k}{1-k'}\right)\sigma + \left[1 - \left(\frac{1-k}{1-k'}\right)\right]\tau \\ &= \phi_3\sigma + (1-\phi_3)\tau \end{aligned}\right\} \quad \text{(IV.10)}$$

Thus, the vehicle print is described as a linear combination of scattering contributions from partially wet fibers, as represented by σ, and unwet fibers, as represented by τ.

D. Summary of Model Equations

The absorption and scattering <u>powers</u> of each layer are as follows:

Layer III -

$$\mathcal{T}_{III}(\lambda) = \phi_1 w_c K_c(\lambda)$$

$$\Psi_{III}(\lambda) = \phi_1 w_c S_c(\lambda)$$

Layer II -

$$\mathcal{T}_{II}(\lambda) = (1-\phi_1) w_c K_c(\lambda) + \phi_2 \theta \rho(\lambda) \tau(\lambda) \qquad \text{(IV.11)}$$

$$\Psi_{II}(\lambda) = (1-\phi_1) w_c S_c(\lambda) + \phi_2 \theta [\phi_3 \sigma(\lambda) + (1-\phi_3) \pi(\lambda)]$$

Layer I -

$$\mathcal{T}_{I}(\lambda) = (1-\phi_2) \theta \rho(\lambda) \tau(\lambda)$$

$$\Psi_{I}(\lambda) = (1-\phi_2) \theta \tau(\lambda)$$

The series solution then applies the following equation three times:

$$R_J = \frac{1-R_{J-1}(a_J - b_J \cdot \coth b_J \Psi_J)}{a_J - R_{J-1} + b_J \cdot \coth b_J \Psi_J}, \qquad \text{(IV.12)}$$

for J = I, II, III and $R_0 = 0.0$.

The print reflectance R is given by

$$R = 0.04 + 0.384 R_{III}(1 - 0.6 R_{III}) \qquad \text{(IV.13)}$$

E. Calibration - Assignment of Phi-Values

In order to apply the above system of equations (IV.11, IV.12, and IV.13) it is necessary to calibrate the entire system of inks-paper-proofing-press by assigning values to the three parameters ϕ_1, ϕ_2, and ϕ_3. This is done by means of a "calibrating print." One of the basic inks from the palette is printed on the job substrate under the identical conditions by which the match print will be made; i.e., the same impression, press speed, and applied ink film thickness. This calibrating print is then measured for reflectance when dry. By computer calculation the set of ϕ-values is then found which gives the best forced-fit between the measured print reflectance and the print reflectance calculated from the mathematical model. This set of ϕ-values is retained for calculating matches for that substrate-basic-ink system.

F. Match Calculation by the Color-Matcher

Reference to Fig. 4 will help to explain the procedure used by the color-matcher. The K and S values of the inks are determined beforehand and are stored in the memory of the computer. The τ and σ functions are also stored in the computer memory. At the time of the match the color-matcher must measure the reflectance across the spectrum of an infinite thickness pile of substrate sheets and the reflectance at 700 nm of a single sheet over a known black background. He also prepares the calibrating print, records the applied film weight, and measures the reflectance of the dry calibrating print. Finally, he measures the reflectance of the standard.

All the reflectance data, the film weight, and the designation of the calibrating ink are input data for the match calculation computer program. The program uses the reflectance data on the substrate to compute the functions ρ and θ. The ϕ-values are then calculated to complete the system calibration, and the match calculation is begun. The color-matcher chooses a first-shot formula from the computer output, prepares the mixture, and finishes the match.

V. EXPERIMENTAL WORK

A. Substrate Characterization

The τ-functions for Finch Offset-Smooth Finish and Finch Offset-Vellum Finish have been previously determined by this writer (14); the values appear in Table 1. Also, the σ-function has been determined (14) for Finch Offset-Vellum Finish and #3003 Safflower Alkyd Varnish (Superior Varnish & Drier Co.). Although the values of the σ-function can be shown to be independent of the film thickness of the vehicle print used in the determination, consideration of eq. (IV.9) shows that more reliable and reproducible values are obtained at higher film thicknesses. Conseqently, the values of σ used in this work were the average of five sets of data from prints having 3.7 to 6.4 microns of vehicle (14). (see Table 1). Since the two substrates differ only in surface finish and not fiber structure, σ was not redetermined for the Smooth Finish stock. It was also assumed that differences in vehicle refractive index were small enough to avoid redetermining σ for different vehicles.

It must be noted that these two offset papers were a somewhat unfortunate choice for letterpress printing from a practical point of view, because their surface roughness

Table 1. Paper Optical Properties

a) Tau-Functions of Finch Offset Papers

	Vellum				Smooth			
λ(nm)	avg.	min.	max.	std. dev.	avg.	min.	max.	std. dev.
400	1.204	1.115	1.312	.050	1.319	1.221	1.540	.093
420	1.186				1.294			
440	1.188				1.269			
460	1.170				1.238			
480	1.154				1.220			
500	1.141	1.128	1.160	.009	1.188	1.164	1.227	.018
520	1.117				1.164			
540	1.091				1.138			
560	1.070				1.117			
580	1.060				1.085			
600	1.045	1.032	1.055	.007	1.069	1.056	1.084	.010
620	1.034				1.054			
640	1.024				1.057			
660	1.019				1.040			
680	1.001				1.030			
700	1.000				1.000			

b) Variations in Sigma-Function with Film Thickness

	All 10 Sheets				5 Higher Thicknesses			
λ(nm)	avg.	min.	max.	std. dev.	avg.	min.	max.	std. dev.
400	-.259	-1.414	.378	.470	-.137	-.203	-.061	.052
420	-.117				-.097			
440	-.067				-.040			
460	-.007				-.020			
480	.018				-.013			
500	.048	- .090	.347	.118	.006	-.009	.018	.010
520	.061				.033			
540	.092				.038			
560	.106				.053			
580	.091				.071			
600	.103	.024	.258	.064	.077	.043	.128	.029
620	.120				.089			
640	.126				.093			
660	.100				.077			
680	.099				.089			
700	.100				.100			

requires inordinately high ink film thicknesses to assure full coverage. This is not a theoretical weakness, but rather is disturbing to the professional ink chemist. The papers were chosen primarily as the only uncoated stocks on hand which did not display inclusion of fluorescent brighteners.

B. Characterization of Basic Inks

The inks used in this work were a series of thirteen singly-pigmented, lithographic, commercial ("off-the-shelf") inks furnished by the Lewis Roberts Co. Ink identification numbers and pigment names appear in Table 2.

A previously developed experimental procedure and associated computer programs (14) for K and S determinations involved making letterpress prints of known area on black-and-white checkerboard Morest Charts (Morest Co., Form 05). The prints were made on a hand-driven cylinder-flat Vandercook No. 4 Proof Press from a 50 Duro rubber plate (2½ x 4½ x .085") glued to a piece of metal (3½ x 5½ x .055"); the plate was held in a wooden base with a dovetail clip arrangement. The area density (g/m^2) of the prints was determined from the known area of the plate and the weight of ink transferred, the latter being obtained by weighing the printing plate to the nearest 0.1 mg before and after impression.

This method was not entirely satisfactory for inks specially formulated for this work (14), and not at all

Table 2. Ink Identification No.'s and Pigment Names

Ink No.	Pigment Type
White CL1	Rutile TiO_2
Black CL1	Carbon black (untoned)
Yellow CL1BM	Benzidine
Yellow CL1BO*	Benzidine
Red CL1LC*	Red lake C
Red CL1LR*	Lithol Rubine
Red CL1W*	Yellow Permanent Red 2B
Red CL1RH	Rhodamine Y/S
Purple CL1RH*	Rhodamine B/S
Blue CL1A	Alkali G/S
Blue CL1PC	Phthalocyanine (peacock)
Blue CL1V	Victoria
Green CL1PC	Phthalocyanine

* Found to be significantly fluorescent

satisfactory for the CL1 series of inks. The primary problem was one of obtaining uniform ink films on an impenetrable black-and-white substrate (often a low energy surface). Consequently, a new method was developed which involved printing from a NPIRI wedge plate onto 0.005" Di-Sta (polyethylene terephthalate) film. A wedge plate is a type-high plate of steel in which rectangular channels of uniform depth have been milled. In the plate used there were three channels of dimensions 9.65 x 3.80 cm and 5, 10, and 15 microns deep, respectively (see Fig. 6). The plate was inked by a drawdown with an Adco straight-edge blade so that one entire channel was filled*. Excess ink was then wiped from the shoulders of the plate so that only the ink in the channel would be printed. Prints on the plastic film were made on the Vandercook press at 10 mils impression. Since it was impossible to weigh accurately the 15-25 mg transferred by weighing the heavy plate, the plastic film had to be weighed before and after impression.

A major difficulty was encountered in gaining accurate weights of the plastic film. Due to buildup of electrostatic charge, the film weight changed as rapidly as 1.0 mg/minute. Ionization of the air in the balance chamber with a radioactive source did slow the weight change somewhat, but the most stable weights were obtained with an

---

* Studies have indicated that this process fills the channels to the 80% level. The respective channels are therefore labeled 12μ, 8μ, and 4μ.

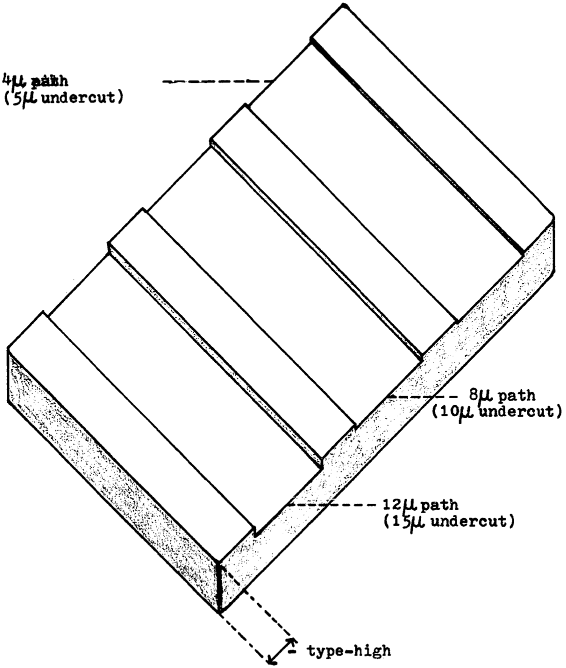


Figure 6. NPIRI Wedge Plate

anti-electrostatic aerosol spray (Anti-Staticum, Ciba Photochemical). An important characteristic of this spray was the fact that it did not significantly change the ink receptivity of the film for the inks studied. However, upon drying, the spray left a mottled residue on the the film which had to be carefully and gently removed before printing.

When the prints had dried for twenty-four hours, black and white **backgrounds** were provided by 6 mil drawdowns of black and white inks on the reverse sides of the printed plastic films. After two additional days of drying, the prints were measured for reflectance.

Four areas of each print were measured: the white background measured through the plastic film; the black background similarly measured; and the printed areas over the black and white backgrounds. To insure opaque backgrounds and plane geometry at the spectrophotometer measurement port, the plastic films were backed by squares of black or white glass.

Three prints were made of White CL1 from the 12μ path of the wedge plate and three from the 8μ path. The same was done for Red CL1W in order to observe any effects due to different film thicknesses. For all the remaining inks except Black CL1 **three prints were made** from the 12μ path; for Black CL1 only two prints were made from the 12μ path.

For each of the colored inks a bleach or tint was made by mixing a small amount of colored ink with White CL1. One print of each bleach was made on the Di-Sta film from the 12μ path. A white background was provided for each bleach print, and, when dry, each print was measured for the reflectance of the white background (through the plastic film) and the bleach over the white background. An example of a Di-Sta print appears in Fig. 7.

The reflectance data and print film thicknesses were the input data for computer programs to calculate the K- and S-values. In the case of the White CL1 the calculations were done by the so-called "black-white" method. It was assumed as a first approximation that the Di-Sta film and all ink vehicles had the same refractive indices. Therefore, at each ink/air or Di-Sta/air interface it was necessary to apply the Saunderson correction, eq. (IV.13), in reverse to get the theoretical reflectance, i.e., the reflectance before the events at the refractive index discontinuity:

$$R_{TH} = (R_M - .04)/(.6R_M + .36) \qquad (V.1)$$

where R_{TH} = the theoretical reflectance and

R_M = the measured reflectance.

With the corrected reflectances of the black and white backgrounds and the corrected ink film reflectances over black and over white, K and S were found using eq. (III.2) with a Newton-Raphson iteration technique; that is, the K and S were found which would give the correct ink film

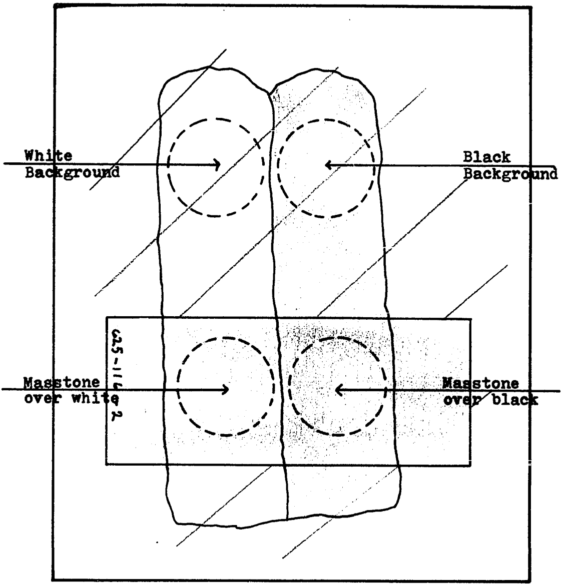


Figure 7. Wedge Plate Print on Di-Sta Film

reflectances over the black and white backgrounds.

For each colored ink the calculations were carried out by the black-white method, and additionally by using the relationship between the K and S of the white (W), K and S of the bleach (B), and the K and S of the colored ink (C):

$$K_B = (PAW \cdot K_W + K_C)/(PAW + 1)$$
$$S_B = (PAW \cdot S_W + S_C)/(PAW + 1) \qquad (V.2)$$

where PAW = parts by weight of white per part colored ink. Both methods were applied at each wavelength, and the computer program calculated an error index for each method. The error index for the black-white method was the absolute value of the difference between the measured tint reflectance and the tint reflectance calculated from the black-white K and S values. The error index for the "mass-tint" method was the absolute value of the difference between the measured reflectance of the masstone over black and the value calculated using the mass-tint K and S values. At each wavelength the more accurate set of values (that with the lower error index) was indicated in the printout.

Ink densities were measured by weighing volumes of the inks delivered from IPI or IGT ink pipettes.

C. Reflectance Measurements

All reflectance measurements in this work were carried out on a Kollmorgen KCS-40 color measurement system which incorporates an abridged spectrophotometer and a Digital

Equipment Corporation PDP-8L mini-computer and teletype unit complete with paper tape capabilities. This spectrophotometer has diffuse illumination of the sample and standard with a choice of CIE Source A or CIE Source D_{6500}. Sample and standard are presented at ports of an integrating sphere, and the measuring beams are at 8° from the normal. Monochromatization is achieved by passage of the beams (after reflection has occurred) through a series of interference filters. For the characterization of the inks and papers, the measurements were taken at 16 points in the spectrum, from 400 to 700 nm at 20 nm intervals, using Source A illumination. All measurements were made with the specular component included, and the reflectance values were simultaneously printed out on the teletype and punched on paper tape. For the various computer programs used, the paper tapes were read into a second teletype linked with a General Electric time-sharing computer service.

In the measurement of the prints on Di-Sta film, the area illuminated was 0.75" in diameter, and the measuring beam was 0.50" in diameter. All other measurements used 1.125" and 0.625" as the diameters of the areas illuminated and viewed, respectively.

The "working" reflectance standards were matched squares of Vitrolite glass which had been calibrated on a Trilac spectrophotometer against freshly prepared plaques of Eastman White Reflectance Standard ($BaSO_4$) assumed to have

100% reflectivity. By means of a calibration procedure the KCS-40 automatically calculated absolute reflectivities of the samples (or, at least their reflectivities vs. $BaSO_4$ as 100%).

D. Calibration Procedure

Four calibrating prints of Blue CL1PC and four calibrating prints of Red CL1LR all on Finch Offset-Smooth Finish were prepared on the Vandercook No. 4 Proof Press by printing from the rubber plate previously described at a printing speed of 220±5 ft/min. (15). Exact measurement of the impression was unavailable due to the deformability of the rubber plate, but the impression cylinder (undercut by 0.040") was packed with 6 sheets of 0.006" tympan, and the 0.0035" Finch Offset-Smooth Finish sheet was backed by a second sheet. Area densities (g/m^2) of all prints were determined by plate weighing. Reflectance measurements were made on the dry prints with a backing of Black Nashua Cover Velour having a reflectivity of from 1.9 to 4.1%. Sets of ϕ-values were calculated for all the calibrating prints. The computer program employed in the calculation used the basic mathematical model with a combination of Newton-Raphson and Steepest Descent iteration to find the best forced fit. However, unlike the previous program to calculate ϕ-values (14), this version used as the fit criterion the mean square of the tristimulus value differences between the measured and calculated curves of the print. Although tristimulus value differences were minimized, there can be no metamerism problems in the calculation (both the measured and calculated reflectances are based on identical colorants). The use of tristimulus values amounts to a fit based on visual rather than spectrophotometric agreement.

It should be noted at this point that the integration of tristimulus values on a digital computer requires digital rather than anolog data. The reflectance data, and of course the optical characteristics of the model parameters, consisted of sets of 16 points. This transforms the integrals of eqs. (II.1) into summations of the form

$$
\begin{aligned}
X &= k\sum_{\lambda=400}^{\lambda=700} E_\lambda R_\lambda \bar{x}_\lambda \Delta\lambda \\
Y &= k\sum_{\lambda=400}^{\lambda=700} E_\lambda R_\lambda \bar{y}_\lambda \Delta\lambda \qquad\qquad (\text{V.3}) \\
Z &= k\sum_{\lambda=400}^{\lambda=700} E_\lambda R_\lambda \bar{z}_\lambda \Delta\lambda
\end{aligned}
$$

and $k = 100/\sum_{\lambda=400}^{\lambda=700} E_\lambda \bar{y}_\lambda \Delta\lambda$, with $\Delta\lambda = 20$ nm.

The actual scheme for the calculation of tristimulus values in this and the other computer programs used in this work may be more readily understood in the notation of matrix algebra. In this form eqs. (V.3) become

$$
\begin{bmatrix} \bar{x}_{400} & \cdots\cdots & \bar{x}_{700} \\ \bar{y}_{400} & \cdots\cdots & \bar{y}_{700} \\ \bar{z}_{400} & \cdots\cdots & \bar{z}_{700} \end{bmatrix}
\begin{bmatrix} E_{400} & 0 & \cdots\cdots & 0 \\ 0 & E_{420} & 0 \cdots & 0 \\ \vdots & & & \vdots \\ \cdots & \cdots & \cdots & \cdots \\ \vdots & & & \vdots \\ 0 & \cdots\cdots & 0 & E_{700} \end{bmatrix}
\begin{bmatrix} R_{400} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ R_{700} \end{bmatrix}
=
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
$$

or, more compactly,

$$[\tilde{\bar{x}}][\tilde{E}][\tilde{R}] = [\tilde{X}] \text{ .} \qquad (V.4)$$

Since the matrix $[\tilde{\bar{x}}]$ of the color-matching functions are always those of the 2° Standard Observer, and since the matrix $[\tilde{E}]$ is limited to that for Sources A, C, and D_{6500}, the matrices $[\tilde{\bar{x}}]$ and $[\tilde{E}]$ may be combined to form the (3 x 16) matrix denoted by $[\widetilde{\bar{x}E_S}]$. This avoids repetative calculations. Then

$$[\widetilde{\bar{x}E_S}][\tilde{R}] = [\tilde{X}_S] \qquad (V.5)$$

for S = Source A, C, or D_{6500}.

Values of the matrix $[\widetilde{\bar{x}E_S}]$ have been tabulated for the CIE Sources, but the original data are at 10 nm intervals, from 380 to 770 nm. The elements of the matrix for 20 nm intervals was calculated by the relationship

$$(\bar{x}E_S)^{20}_{\lambda} = (\bar{x}E_S)^{10}_{\lambda} + \tfrac{1}{2}\left[(\bar{x}E_S)^{10}_{\lambda-10} + (\bar{x}E_S)^{10}_{\lambda+10}\right] \qquad (V.6)$$

where $(\bar{x}E_S)^{20}_{\lambda}$ represents the matrix element at wavelength λ for the 20 nm factors, and $(\bar{x}E_S)^{10}_{\lambda}$, $(\bar{x}E_S)^{10}_{\lambda-10}$, and $(\bar{x}E_S)^{10}_{\lambda+10}$ represent the matrix elements of the 10 nm factors at wavelengths λ, λ-10, and λ+10, respectively.

In subsequent work calibrating prints of Blue CL1PC were made on Finch Offset-Vellum Finish. The press packing and speed were the same, but two sheets of the Vellum paper (caliper 0.0045") were used instead of two sheets of the Smooth. The ∅-values were calculated by an even more streamlined computer program which employed a Newton-Raphson iteration alone.

E. "First-Shot" Match Calculations

1. Matches to Standards of Known Composition

Six arbitrary mixtures were prepared, each containing four of the palette inks. Prints of the mixtures were made on the appropriate papers. The mixture formulas, print identification numbers, substrate names and applied film thicknesses appear in Table 3. The prints of the mixtures served as standards to be matched.

When the printed standards had dried for a day they were measured for reflectance on the KCS-40 spectrophotometer. The reflectance data served as input data for a computer program called MATCH1 which calculated matches from four palette inks specified by the operator. Additional input data were the θ-value of the substrate, the infinite thickness reflectance of the substrate, and the ϕ-values and film thickness of the calibrating print. Already stored in the computer were the K and S values of all the inks, the tristimulus weighting factors $[\widetilde{\bar{x}E_S}]$ for Sources A and C, and the τ and σ curves of the substrate.

Four inks, rather than three, are required for printing ink color-matching to provide three degrees of freedom under the constraint of maintaining a specific film thickness. If only three inks were used and the concentrations found for the first two, the third concentration would be fixed by the film thickness constraint. Thus, the MATCH1 program used three colored inks with a fourth neutral ink

Table 3. Standards of Known Composition

Ink Mixture No. and Color	Formula		Sub-strate*	Print No.	Film Thick-ness#
625-147-A Dark Gray	White CL1	79.98%	FOS	625-147-A-2	4.8
	Black CL1	9.98			
	Yellow CL1BM	5.04			
	Blue CL1PC	5.00			
		100.00			
625-147-B Salmon	White CL1	79.97	FOS	625-147-B-5	4.9
	Yellow CL1BO	10.01			
	Red CL1W	5.00			
	Red CL1LC	5.02			
		100.00			
625-147-C Pink	White CL1	90.02	FOS	625-147-C-2	4.9
	Red CL1RH	4.98			
	Yellow CL1BM	3.99			
	Purple CL1RH	1.01			
		100.00			
625-140-1 Olive Green	White CL1	80.00	FOS	625-140-4	7.9
	Blue CL1V	3.00			
	Red CL1RH	1.99			
	Yellow CL1BM	15.01			
		100.00			
625-141-1 Light Blue	White CL1	92.00	FOV	625-141-3	6.3
	Blue CL1PC	3.00			
	Blue CL1A	2.00			
	Green CL1PC	3.00			
		100.00			
685-17-1 Dark Blue-Green	White CL1	68.00	FOV	685-19-2	8.3
	Black CL1	2.00			
	Blue CL1PC	10.00			
	Green CL1PC	20.00			
		100.00			

* FOS - Finch Offset-Smooth Finish
FOV - Finch Offset-Vellum Finish

\# microns

(in this case opaque white) to allow for the fixed film thickness.

The matching program calculated matches by the procedure described earlier. A Newton-Raphson iteration was used with the initial formula consisting almost entirely of white ink, with small amounts of each of the three colored inks. Since it was previously demonstrated (14) that a matching print must duplicate the film thickness of the calibrating print, the density of the formula was calculated for each iteration, and the absolute amounts of each ink were adjusted to maintain the proper film thickness. A typical example of the computer output appears in Fig. 8. Note that the area density (g/m^2) at which the match print is to be made is indicated on the output.

For each standard the match formula and metamerism index were evaluated with respect to the formula of the standard. Since the inks to be used in the calculation were specified by the operator as the inks used in the standard, each calculated formula was expected to approximate the formula of the corresponding standard, and the calculated metamerism index was expected to be low. If these criteria were satisfied the match was then recalculated by a program called MATCH22 which tried all combinations in the palette of white plus three colored inks, and ranked the various formulas in terms of increasing metamerism (lowest metamerism first). If the system was working

```
                    PRINTING INK COLOR MATCH


          SAMPLE: PRINT NO. 625-147-A-2
          PAPER STOCK: FINCH OFFSET SMOOTH
          SCATTERING POWER AT 700 NM: 5.540

          AREA DENSITY OF PRINT: 7.398 G/SQ. M

          PHI VALUES:
              PHI(1) = 0.054
              PHI(2) = 0.456
              PHI(3) = 0.520

          MATCHING FORMULA:
              WHITE CL1             75.25 PER CENT
              BLACK CL1             12.74 PER CENT
              YELLOW CL1BM           4.60 PER CENT
              BLUE CL1PC             7.41 PER CENT

          COLOR DIFFERENCE BETWEEN SAMPLE AND MATCH, ACU
              SOURCE C (RESIDUAL MISMATCH):  0.0
              SOURCE A (METAMERISM INDEX):   0.0


     WAVE-                        R E F L E C T A N C E
    LENGTH,                      SAMPLE          MATCH
     NM

     400                         0.105           0.109

     420                         0.110           0.110

     440                         0.109           0.109

     460                         0.110           0.111

     480                         0.113           0.113

     500                         0.117           0.117

     520                         0.117           0.117

     540                         0.114           0.115
```

Figure 8. MATCH1 Output
(cont. next page)

560	0.108	0.108
580	0.101	0.101
600	0.097	0.096
620	0.094	0.094
640	0.094	0.095
660	0.097	0.099
680	0.097	0.100
700	0.095	0.096

(Fig. 8. cont.)

properly, the actual combination of inks used in the standard would be indicated as the least metameric.

The computed results for standards Salmon 625-147-B-5 and Pink 625-147-C-2 affected substantially the subsequent course of the work. Consequently, these results will be described briefly here.

For the Salmon Print 625-147-B-5 the MATCH1 program reported "no match possible with this set of inks," even when given the proper set of inks to use. For the Pink Print 625-147-C-2 the MATCH1 program calculated a match formula in which one ink concentration was in error by nearly 60%. As a result the calculated metamerism index had the relatively high value of 1.4 (Adams Chromatic Value Units). By way of comparison, in one of the commercial textile matching programs a metamerism index of 0.5 is the upper limit of acceptability. Further problems with the Pink standard were apparent in the results of the MATCH22 program, in which the correct ink combination was rated eighth in metamerism rather than the least metameric.

a. Detection of Fluorescence

These problems prompted a study of the basic inks for possible fluorescent character. The studies were carried out first by observation of the printed standards and the Di-Sta prints of the basic inks under an ultra-violet lamp in a dark room. Some of the prints, both on paper and on Di-Sta, showed a visible glow indicative of fluorescence.

Further study of the basic inks was done by reflectance measurements on the KCS-40 spectrophotometer of the Di-Sta prints over the black background both with Source A and Source D_{6500} sample illumination. In each case the same sample area was measured with each illuminant.

The so-called "reverse optics" of the KCS-40, diffuse sample illumination with a full energy spectrum and monochromatization after reflection, combined with the availability of two different sources of illumination, allows the detection of fluorescence. For non-fluorescent samples, since a spectrophotometer is a ratio measuring device (i.e., it measures the ratio of the light intensity reflected by the sample compared to that of the white standard), the reflectance curve is independent of the source of illumination. However, the "reflectance" curves of fluorescent samples are dependent on the source of sample illumination. For example, suppose a particular sample absorbs energy at 450 nm and re-emits energy (fluoresces) at a longer (lower energy) wavelength such as 600 nm. The total "reflected" intensity at 600 nm is really the sum of the reflected and fluorescent components. If one sample illuminant has relatively more energy at 450 nm than a second illuminant, the sample will fluoresce more at 600 nm with the first source than with the second. Consequently, the two "reflectance," or more properly spectral radiance factor, curves will differ.

The criterion used for assessing the presence of "significant" fluorescence in any of the inks was a difference at any wavelength of 1.0% R or greater between the Source A and Source D_{6500} curves (e.g., $R_A = 80.00\%$ and $R_D = 81.00\%$). This resulted in the elimination of the five inks from the palette indicated in Table 2 by the asterisks.

Dark Gray Standard 625-147-A-2 did not contain fluorescent inks. The last three standards in Table 3 were formulated from the reduced palette of eight inks, and all subsequent match calculations in this work were calculated with the reduced palette.

For all the standards of known composition, the least metameric match formula indicated by the MATCH22 computer program output was mixed and printed at the specified film thickness (area density). If the correct ink combination was not chosen as the least metameric, that formula was mixed and printed also.

2. Matches to Standards of Unknown Composition

Color swatches of printed solids (not half-tones or so-called 4-color process prints) were cut from magazines to serve as standards of unknown composition. All the standards were measured on the KCS-40 under both Sources A and D_{6500} to avoid fluorescent standards. Matches were then computed for the non-fluorescent standards by the MATCH22 program.

Due to the limited palette of inks, many of the match calculations yielded formulations for which even the least metameric ink combinations had relatively high metamerism indices. This was an indication that suitable inks to match these standards were simply not present in the palette. Since a highly metameric match is always a poor one, these standards were discarded.

Suitably low metamerism indices were obtained from the MATCH22 calculations for five standards (including a neutral gray painted standard). For each of these, the least metameric formulation was mixed and printed at the indicated print area density.

F. Commercial Matches to a Pink Standard

A pink printed standard of formula 625-147-C (Table 3) was cut into six pieces. A piece of the standard and a supply of Finch Offset-Smooth Finish paper was given to representatives of six member ink companies of the National Printing Ink Research Institute (NPIRI). Each representative was asked to have finished commercial matches made to the standard and to return the match prints and formulations to the NPIRI Color Laboratory for evaluation.

This experiment was initiated before the detection of fluorescence in the print. However, for visual color-matchers, such slight fluorescence would not constitute a problem since visual signals would be matched, not calculated tristimulus values.

G. Evaluation of Commercial and "First-Shot" Computer Matches

1. Color Differences

Each sample and standard match-pair was measured on the KCS-40 spectrophotometer. This spectrophotometer has the facility to calculate color differences between a standard and a sample in MacAdam units using the Friele-MacAdam-Chickering (FMC-2) mathematics (16, 17, 18). Since the matches were all made (either by calculation or by visual means) for daylight conditions, and to avoid errors in the calculations due to fluorescence, the reflectance measurments upon which the calculations were based used Source D_{6500} illumination, Source D_{6500} Tristimulus Integrating Factors (19), and a 38 point curve from 380 to 750 nm at 10 nm intervals.

Because direct comparisons between color differences in MacAdam units and Adams Chromatic Value Units (ACV) are not entirely meaningful and possibly misleading, all the color differences were also calculated in ACV using a Wang 700 Model programmable desk calculator.

2. Metamerism Indices

Metamerism index calculations were also carried out on all the sample and standard match-pairs from reflectance measurements of the prints. The metamerism index was defined earlier as the color difference under Source A for an exact match under Source C. In this evaluation Sources D_{6500} and A were used. The tristimulus values of all the prints were calculated from the actual Source D_{6500} and

Source A reflectance curves. These tristimulus values were then transformed to the Adams Chromatic Value space by eqs. (II.2 and II.3). Since the match-pairs were not exact matches under Source D_{6500}, for each match-pair the vector representing the residual Source D_{6500} color difference was subtracted from the Source A color difference vector. The length of the resultant vector was then taken as the metamerism index in ACV units. With the use of this approach, the error in the original daylight match would have a lessened affect on the metamerism evaluation. A poor match could still be low in metamerism, i.e., it might have the right colorants but the wrong proportions.

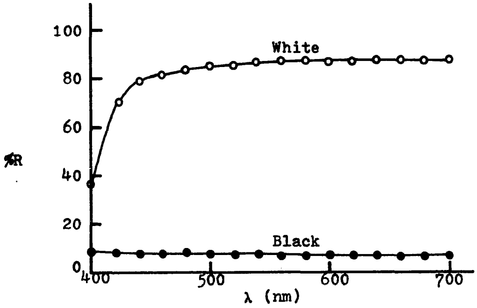
VI. RESULTS

A. Characterization of Basic Inks

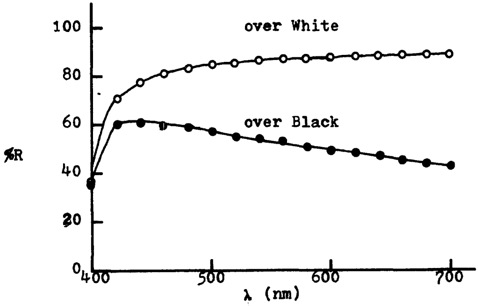
1. Absorption (K) and Scattering (S) Coefficients

Typical reflectance data on the Di-Sta prints of White CL1 are illustrated in Fig. 9. It should be kept in mind that the points on the graphs represent the actual data used; the curves are drawn through the points so the data can be more easily visualized. The data were used for the K and S calculation by the black-white method. Similar curves representing the reflectance data for the K and S calculations of one of the colored inks (Red CL1W) appear in Fig. 10. For the colored inks the calculations were carried out by both the black-white and mass-tint methods. Typical results of the computer programs appear in the form of the printed output in Figs. 11 and 12 for inks White CL1 and Red CL1W, respectively.

As mentioned above, for each of the inks White CL1 and Red CL1W three Di-Sta prints were made from the 12μ path of the wedge plate and three from the 8μ path to study the effects of different film thicknesses. The resultant K and S data appear in Tables 4 and 5. For each set of three prints of the same ink from the same channel, separate averages of K and S were calculated. The averages

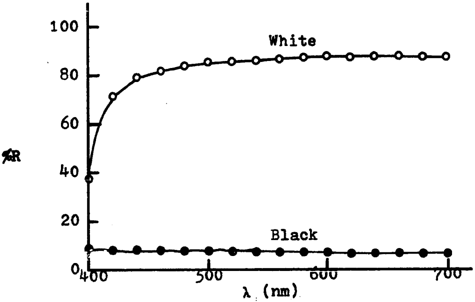


a) Background Reflectances

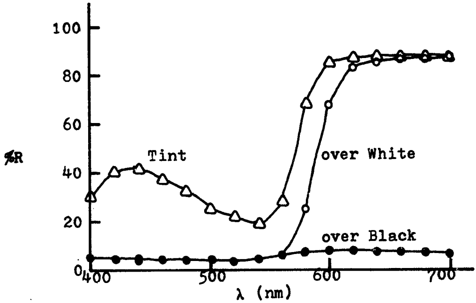


b) Masstone Reflectances

Figure 9. Reflectance Curves for White CL1 on Di-Sta



a) Background Reflectances



b) Masstone and Tint Reflectances

Figure 10. Reflectance Curves for Red CL1W on Di-Sta

Figure 11. Sample Computer Output for K and S Determination of White CL1

```
      DETERMINATION OF ABSORPTION AND SCATTERING
                COEFFICIENTS OF STANDARD WHITE

  STANDARD WHITE CL1 - SAMPLE NO. 625-116-1
  FILM WEIGHT, 7.227000 GMS/M↑2


   I N P U T  R E F L E C T A N C E   V A L U E S
   WAVE-     WHITE      DRAWDOWN       BLACK      DRAWDOWN
  LENGTH,  BACKGROUND     OVER      BACKGROUND      OVER
    NM                   WHITE                      BLACK
                       BACKGROUND                 BACKGROUND
  -------  ----------  ----------   ----------   ----------
    400      0.3621      0.3685       0.0821       0.3605
    420      0.7036      0.7129       0.0791       0.6073
    440      0.7826      0.7783       0.0766       0.6102
    460      0.8129      0.8097       0.0744       0.5981
    480      0.8329      0.8314       0.0728       0.5849
    500      0.8450      0.8455       0.0716       0.5701
    520      0.8537      0.8555       0.0703       0.5558
    540      0.8603      0.8643       0.0694       0.5398
    560      0.8651      0.8715       0.0687       0.5252
    580      0.8693      0.8768       0.0681       0.5110
    600      0.8710      0.8801       0.0675       0.4973
    620      0.8718      0.8812       0.0670       0.4825
    640      0.8715      0.8812       0.0666       0.4684
    660      0.8724      0.8828       0.0662       0.4562
    680      0.8727      0.8833       0.0660       0.4444
    700      0.8731      0.8834       0.0661       0.4329
```

(cont. next page)

(Fig. 11, cont.)

WAVE-LENGTH, NM	TWO-CONSTANT THEORY		ONE-CONSTANT THEORY
	ABSORPTION COEFFICIENT	SCATTERING COEFFICIENT	K/S VALUE
400	0.068716	0.411689	0.167128481
420	0.007059	0.584892	0.012440073
440	0.003601	0.541430	0.006440113
460	0.002293	0.495573	0.004450084
480	0.001581	0.459318	0.003345776
500	0.001150	0.426086	0.002733456
520	0.000879	0.398203	0.002345515
540	0.000626	0.369782	0.002034150
560	0.000423	0.345877	0.001799379
580	0.000295	0.324695	0.001637634
600	0.000181	0.305578	0.001541541
620	0.000133	0.286530	0.001510284
640	0.000096	0.269380	0.001510284
660	0.000038	0.255233	0.001465504
680	0.000005	0.242225	0.001451675
700	-0.000006	0.229927	0.001448918

Figure 12. Sample Computer Output for K and S Determination of Red CL1W

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DETERMINATION OF ABSORPTION AND SCATTERING COEFFICIENTS
                   OF COLORED PIGMENTS


PIGMENT                RED CL1W 625-117-1
STANDARD WHITE         WHITE CL1


FILM THICKNESSES
   MASSTONE OVER WHITE:  4.091 GRAMS PER SQ. METER
   MASSTONE OVER BLACK:  4.091 GRAMS PER SQ. METER
   TINT:                 6.818 GRAMS PER SQ. METER



 WAVE-    WHITE    MASSTONE  WHITE     TINT    BLACK  MASSTONE
LENGTH,   BACKG.     OVER    BACKG.    OVER    BACKG.   OVER
  NM       FOR      WHITE     FOR     WHITE            BLACK
         MASSTONE   BACKG.    TINT    BACKG.           BACKG.
------   --------  --------  ------  ------  ------  ------

 400     0.3712    0.0479    0.3439  0.3019  0.0874  0.0435

 420     0.7107    0.0470    0.6582  0.4134  0.0848  0.0424

 440     0.7885    0.0455    0.7337  0.4183  0.0825  0.0413

 460     0.8178    0.0418    0.7701  0.3844  0.0805  0.0404

 480     0.8376    0.0411    0.7937  0.3231  0.0791  0.0404

 500     0.8486    0.0404    0.8065  0.2507  0.0778  0.0404

 520     0.8569    0.0404    0.8156  0.2255  0.0766  0.0404

 540     0.8635    0.0430    0.8271  0.1889  0.0758  0.0418
```

(cont. next page)

(Fig. 12, cont.)

560	0.8686	0.0610	0.8401	0.2757	0.0751	0.0583
580	0.8729	0.2459	0.8517	0.6811	0.0745	0.0683
600	0.8741	0.6793	0.8595	0.8437	0.0738	0.0750
620	0.8747	0.8294	0.8648	0.8696	0.0732	0.0733
640	0.8742	0.8542	0.8695	0.8756	0.0727	0.0714
660	0.8755	0.8632	0.8711	0.8789	0.0723	0.0699
680	0.8751	0.8665	0.8734	0.8815	0.0719	0.0688
700	0.8761	0.8693	0.8763	0.8839	0.0720	0.0682

* NOTE: TINT HAS 19.949 PARTS OF WHITE PER PART OF PIGMENT.

DETAILS OF COEFFICIENTS AND ERROR INDICES

WAVE-LENGTH, NM	ABSORPTION COEFF. BLACK-WHITE METHOD	ABSORPTION COEFF. MASS-TINT METHOD	SCATTERING COEFF. BLACK-WHITE METHOD	SCATTERING COEFF. MASS-TINT METHOD	ERROR INDEX BLACK-WHITE METHOD	ERROR INDEX MASS-TINT METHOD
400	0.4465	0.8838	0.0058	0.0369	0.0271	0.0043
420	0.5003	1.2887	0.0046	0.0482	0.1190	0.0046
440	0.5232	1.2524	0.0021	0.0365	0.1251	0.0048
460	0.6545	1.5000	0.0008	0.0115	0.1285	0.0014
480	0.7441	2.1648	0.0012	0.0125	0.1679	0.0007
500		3.5215		0.0074		0.0000

NO BLACK-WHITE VALUES --
R OVER WHITE NOT GREATER THAN R OVER BLACK

(cont. next page)

(Fig. 12, cont.)

```
520             4.1110              0.0086             0.0000
  NO BLACK-WHITE VALUES --
         R OVER WHITE NOT GREATER THAN R OVER BLACK

540        0.6801   5.4993     0.0060   0.0869     0.3067 0.0012

560        0.5457   2.5319     0.0552   0.2990     0.2584 0.0027

580        0.1053   0.2278     0.0148   0.4939     0.0834 0.1684

600        0.0152   0.0282     0.0039   1.3294     0.0101 0.5339

620        0.0030   0.0086     0.0007   2.0643     0.0018 0.6725

640        0.0013   0.0114    -0.0006   4.6632     0.0022 0.7640

660        0.0008   0.0066    -0.0015   3.0231     0.0008 0.7384

680        0.0006   0.0041    -0.0020   1.9219     0.0005 0.6834

700        0.0004   0.0043    -0.0025   2.1594     0.0006 0.7023



WAVE-      TWO-CONSTANT THEORY                            ONE-
LENGTH,                                                 CONSTANT
  NM      ABSORPTION   SCATTERING    METHOD OF        THEORY
          COEFFICIENT  COEFFICIENT  CALCULATION    K/S VALUE
------    -----------  -----------  -----------    ----------
 400        0.8838       0.0369      MASSTINT        2.3718
 420        1.2887       0.0482      MASSTINT        2.2886
 440        1.2524       0.0365      MASSTINT        2.3142
 460        1.5000       0.0141      MASSTINT        3.0252
 480        2.1648       0.0025      MASSTINT        4.8268
 500        3.5215       0.0074      MASSTINT        8.6941
 520        4.1110       0.0086      MASSTINT       10.8977
 540        5.4993       0.0869      MASSTINT       15.6098
 560        2.5319       0.2990      MASSTINT        7.0754
 580        0.1053       0.0148     BLACKWHITE       0.3282
 600        0.0152       0.0039     BLACKWHITE       0.0475
 620        0.0030       0.0007     BLACKWHITE       0.0303
 640        0.0013      -0.0006     BLACKWHITE       0.0305
 660        0.0008      -0.0015     BLACKWHITE       0.0300
 680        0.0006      -0.0020     BLACKWHITE       0.0305
 700        0.0004      -0.0025     BLACKWHITE       0.0301
```

Table 4. Variation of K and S Values with Film Thickness for White CL1

a) Absorption Coefficients at 20 nm Intervals from 400 nm to 700 nm

Twelve-Micron Path				Eight-Micron Path				Grand Avg.
1	2	3	Avg.	4	5	6	Avg.	
.0687	.0657	.0623	.0656	.0755	.0760	.0716	.0744	.0700
.0071	.0072	.0066	.0069	.0073	.0079	.0068	.0073	.0071
.0036	.0039	.0034	.0036	.0037	.0042	.0034	.0038	.0037
.0023	.0025	.0021	.0023	.0024	.0027	.0021	.0024	.0024
.0016	.0018	.0014	.0016	.0016	.0019	.0014	.0016	.0016
.0011	.0013	.0010	.0012	.0011	.0014	.0010	.0012	.0012
.0009	.0010	.0008	.0009	.0008	.0010	.0007	.0008	.0009
.0006	.0007	.0005	.0006	.0005	.0008	.0005	.0006	.0006
.0004	.0006	.0003	.0004	.0004	.0006	.0003	.0004	.0004
.0003	.0004	.0002	.0003	.0002	.0004	.0002	.0003	.0003
.0002	.0003	.0001	.0002	.0001	.0003	.0001	.0001	.0002
.0001	.0002	.0001	.0001	.0000	.0002	.0001	.0001	.0001
.0001	.0002	.0000	.0001	.0000	.0001	.0000	.0000	.0001
.0000	.0001	.0000	.0001	.0000	.0001	.0000	.0000	.0000
.0000	.0001	.0000	.0000	.0000	.0001	.0000	.0000	.0000
.0000	.0001	.0000	.0000	.0001	.0000	.0001	.0000	.0000

b) Scattering Coefficients at 20 nm Intervals from 400 nm to 700 nm

Twelve-Micron Path				Eight-Micron Path				Grand
1	2	3	Avg.	4	5	6	Avg.	Avg.
.4117	.3936	.3706	.3920	.4402	.4373	.4164	.4313	.4116
.5849	.5873	.5375	.5699	.5703	.5659	.5544	.5636	.5667
.5414	.5501	.5060	.5325	.5320	.5262	.5216	.5266	.5295
.4956	.5039	.4651	.4882	.4865	.4839	.4775	.4836	.4854
.4593	.4677	.4326	.4532	.4509	.4499	.4440	.4483	.4507
.4261	.4346	.4025	.4211	.4195	.4181	.4109	.4162	.4186
.3982	.4058	.3764	.3935	.3911	.3912	.3848	.3891	.3913
.3698	.3766	.3506	.3657	.3637	.3645	.3579	.3620	.3638
.3459	.3529	.3276	.3421	.3403	.3417	.3347	.3347	.3405
.3247	.3315	.3086	.3216	.3199	.3215	.3150	.3188	.3202
.3056	.3111	.2896	.3021	.3004	.3030	.2944	.2992	.3007
.2865	.2923	.2723	.2837	.2818	.2850	.2775	.2815	.2826
.2694	.2750	.2561	.2669	.2649	.2683	.2608	.2647	.2658
.2552	.2608	.2434	.2531	.2511	.2540	.2473	.3508	.2520
.2422	.2479	.2311	.2404	.2385	.2415	.2343	.2381	.2392
.2299	.2353	.2194	.2282	.2257	.2299	.2224	.2260	.2271

Table 5. Variation of K and S Values with Film Thickness for Red CL1W

a) Absorption Coefficients at 20 nm Intervals from 400 nm to 700 nm

Twelve-Micron Path				Eight-Micron Path				Grand
1	2	3	Avg.	4	5	6	Avg.	Avg.
0.884	0.883	0.883	0.883	0.890	0.890	0.891	0.891	0.887
1.289	1.288	1.288	1.288	1.294	1.293	1.294	1.294	1.291
1.252	1.252	1.252	1.252	1.257	1.256	1.257	1.256	1.254
1.500	1.500	1.500	1.500	1.503	1.503	1.503	1.503	1.501
2.165	2.164	2.165	2.165	2.166	2.167	2.167	2.167	2.166
3.522	3.522	3.522	3.522	3.522	3.522	3.522	3.522	3.522
4.111	4.111	4.111	4.111	4.111	4.111	4.111	4.111	4.111
5.499	5.480	5.486	5.489	5.482	5.501	5.491	5.491	5.490
2.532	2.528	2.528	2.530	2.531	2.534	2.533	2.533	2.531
0.105	0.107	0.109	0.107	0.117	0.113	0.111	0.113	0.110
0.015	0.015	0.016	0.016	0.017	0.016	0.016	0.016	0.016
0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
0.001	0.001	0.002	0.001	0.002	0.002	0.002	0.002	0.001
0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

b) Scattering Coefficients at 20 nm Intervals from 400 nm to 700 nm

Twelve-Micron Path				Eight-Micron Path				Grand
1	2	3	Avg.	4	5	6	Avg.	Avg.
0.037	0.034	0.034	0.035	0.061	0.061	0.063	0.062	0.048
0.048	0.045	0.044	0.046	0.094	0.091	0.096	0.094	0.070
0.037	0.032	0.032	0.034	0.078	0.076	0.078	0.077	0.056
0.014	0.010	0.012	0.012	0.038	0.037	0.039	0.038	0.025
0.012	0.009	0.011	0.011	0.020	0.023	0.024	0.022	0.017
0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
0.084	0.060	0.069	0.072	0.063	0.090	0.075	0.075	0.074
0.299	0.286	0.286	0.291	0.297	0.305	0.302	0.302	0.296
0.015	0.015	0.017	0.016	0.019	0.016	0.016	0.017	0.016
0.004	0.003	0.007	0.005	0.007	0.004	0.004	0.005	0.005
0.001	0.000	0.003	0.001	0.003	0.000	0.001	0.001	0.001
0.000	0.000	0.002	0.001	0.001	0.000	0.000	0.000	0.000
0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

for the two different channels did not show any marked trends or serious discrepancies with film thickness, so the K and S values for each ink were taken as the grand averages indicated in Tables 4 and 5.

The average K and S values for all the CL1 series inks appear in graphical form in Fig. 13. Again, the plotted points represent the calculated data, but the curves are drawn through the points for easier visualization.

Two further observations should be made concerning the K and S results. First, at some wavelengths for some inks the computer iterated to small negative values of K or S. Such values, of course, have no physical meaning but most likely are due to the limits of precision of the spectrophotometer. If the average K or S at these wavelengths for the series of prints of the particular ink were also negative, the values were set equal to zero. This was the case for White CL1 K(700); Yellow CL1BM S(640), S(660), S(680), S(700); Yellow CL1BO K(700); Red CL1RH S(680), S(700); Red CL1W S(640), S(660), S(680), S(700); and Blue CL1V S(460).

Second, at some wavelengths for some inks, the error indices for the black-white and mass-tint methods were close in magnitude. In some of these cases the method selected by the computer resulted in K and S values which seemed incongruous with respect to the values at the remaining wavelengths; i.e., the K and S curves had sharp peaks

Figure 13 a) Absorption (K) and Scattering (S) Coefficients of White CL1

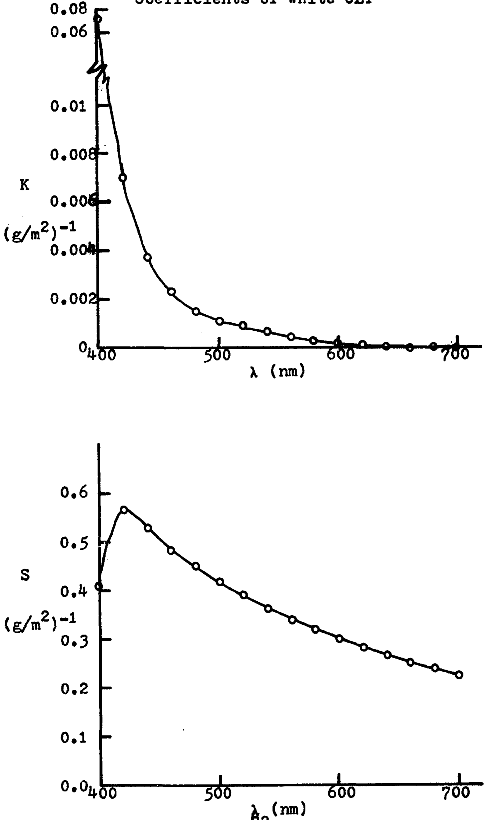
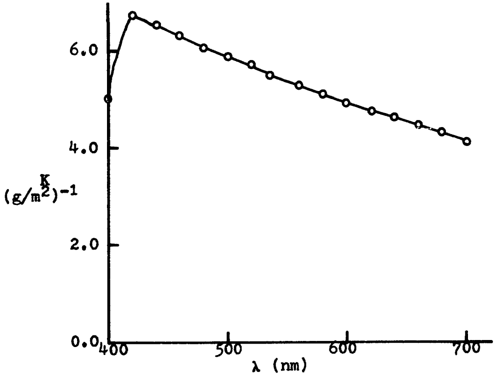


Figure 13 b) Absorption (K) and Scattering (S) Coefficients of Black CL1



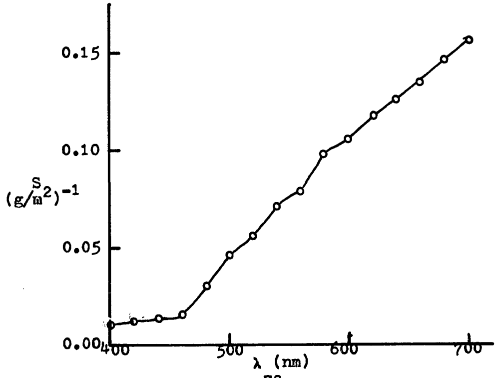


Figure 13 c) Absorption (K) and Scattering (S) Coefficients of Yellow CL1BM

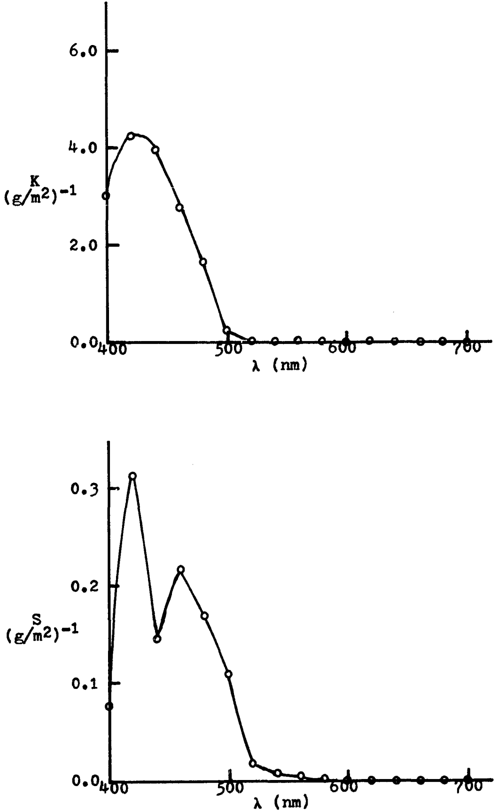


Figure 13 d) Absorption (K) and Scattering (S) Coefficients of Yellow CL1BO

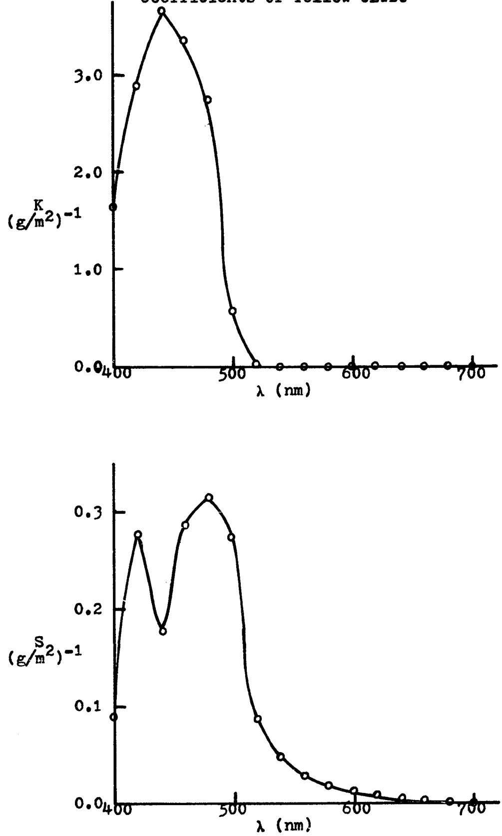


Figure 13 e) Absorption (K) and Scattering (S) Coefficients of Red CL1LC

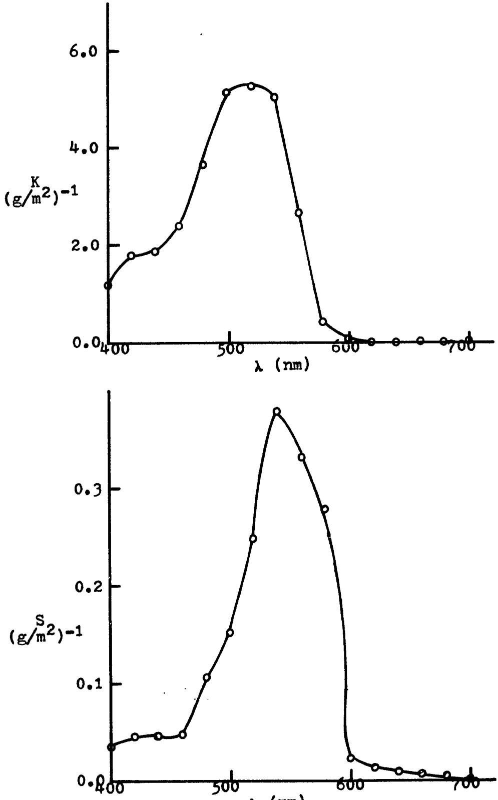


Figure 13 f) Absorption (K) and Scattering (S) Coefficients of Red CL1LR

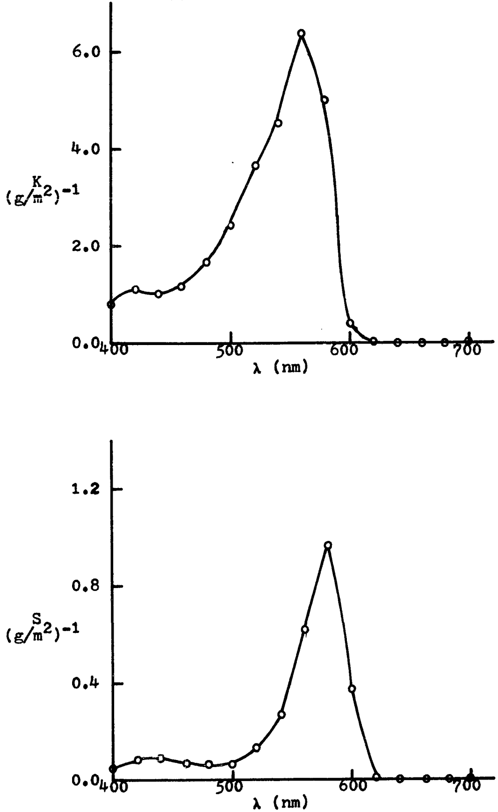


Figure 13 g) Absorption (K) and Scattering (S) Coefficients of Red CL1W

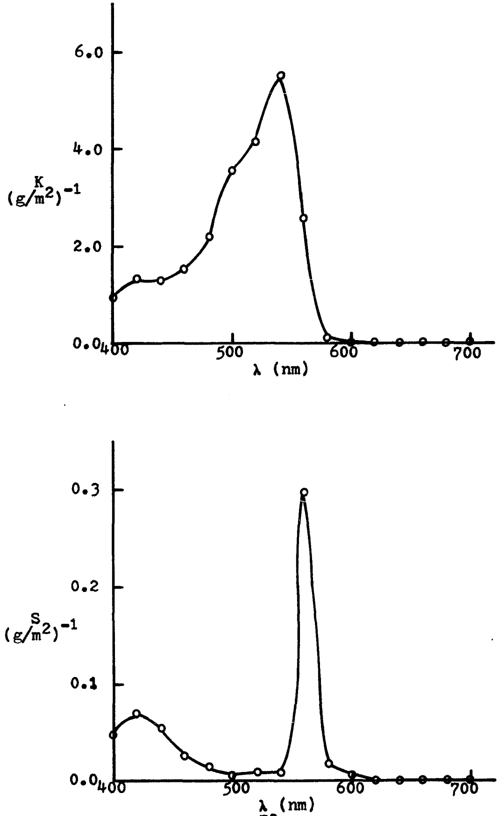


Figure 13 h) Absorption (K) and Scattering (S) Coefficients of Red CL1RH

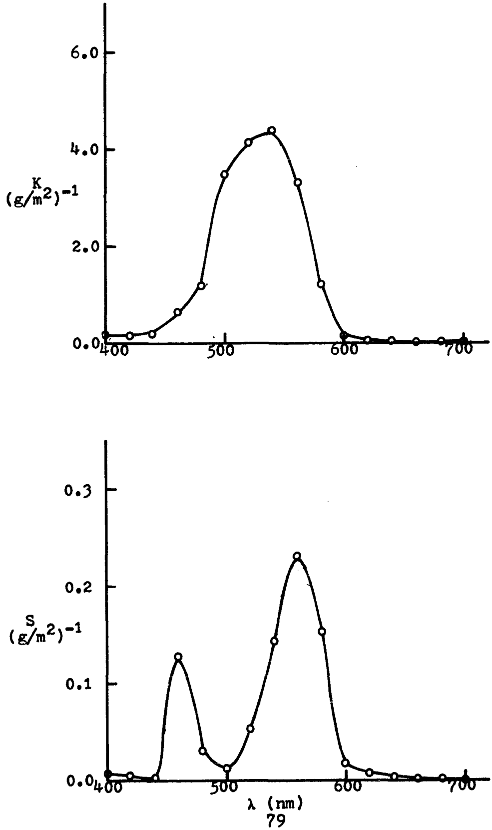


Figure 13 i) Absorption (K) and Scattering (S) Coefficients of Purple CL1RH

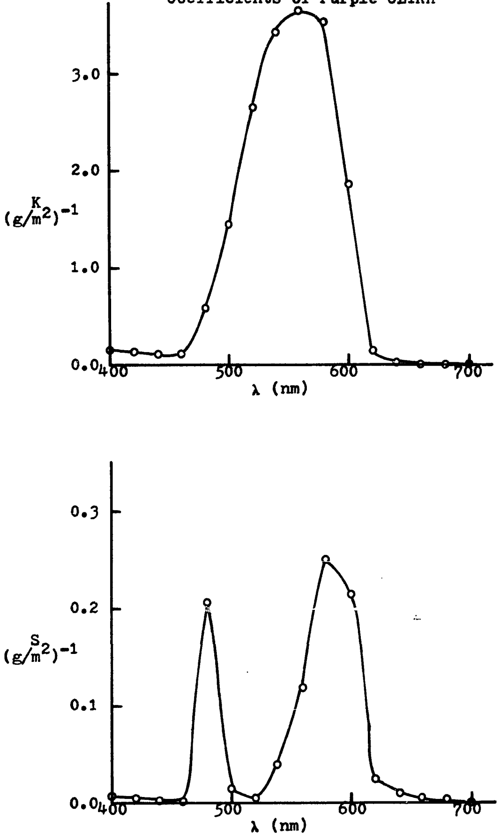


Figure 13 j) Absorption (K) and Scattering (S) Coefficients of Blue CL1A

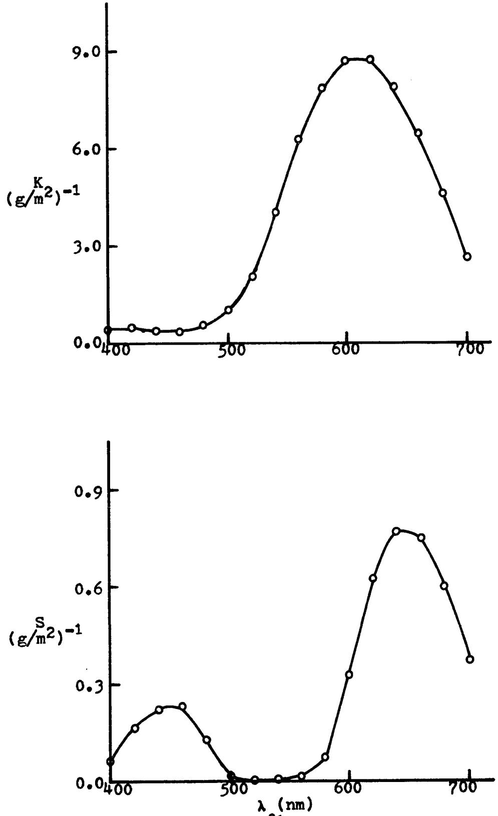


Figure 13 k) Absorption (K) and Scattering (S) Coefficients of Blue CL1PC

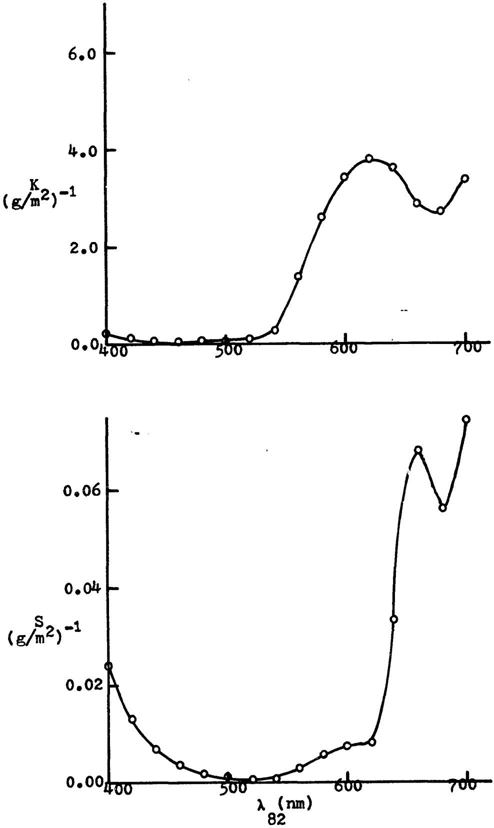


Figure 13 1) Absorption (K) and Scattering (S) Coefficients of Blue CL1V

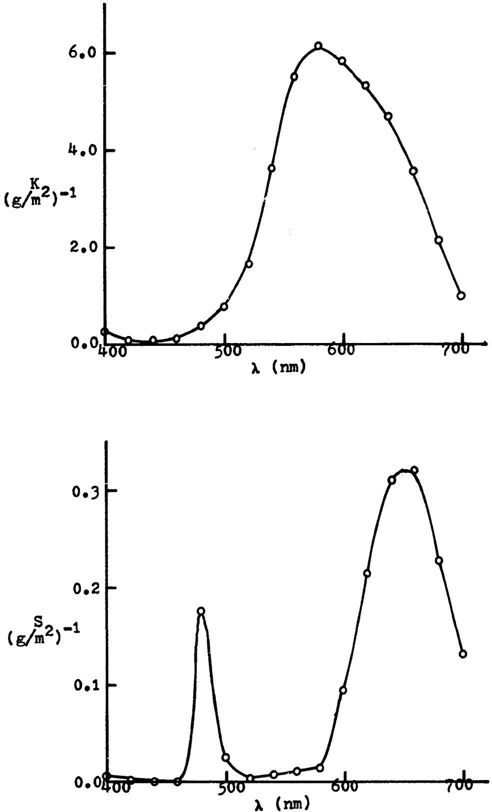
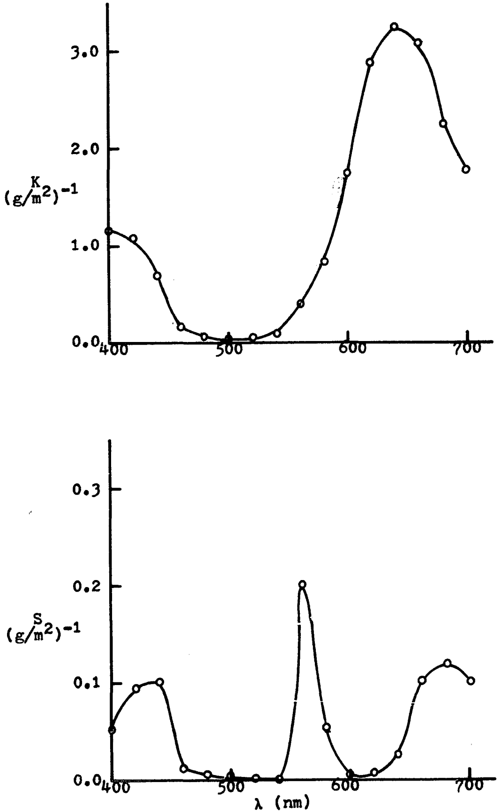


Figure 13 m) Absorption (K) and Scattering (S) Coefficients of Green CL1PC



or discontinuities. For these cases the values for the alternative method were preferred if they appeared to form more reasonable curves. This was done for the following cases: Green CL1PC at 460 nm, mass-tint values replaced with black-white values; Blue CL1PC at 540 nm, mass-tint values replaced with black-white values; Blue CL1A at 400, 440 nm, black-white values replaced with mass-tint values.

2. Ink Densities

The ink densities at 72° F appear in Table 6.

B. System Calibration - ϕ-Values

Fig. 14 shows a typical printed output of the calibration program using a combination of Newton-Raphson and Steepest Descent iteration. The sets of ϕ-values found for calibrating prints on Finch Offset-Smooth Finish appear in Table 7. The wide diversity of the different sets of ϕ-values is immediatèly apparent as are the strange physical implications of some of the values.

In six of the eight cases, ϕ_1 ranges between 0.020 and 0.085. These are reasonable values for the fraction of the applied ink that remains on top of the substrate. The other two values, 0.279 and 0.393, are unrealistic fractions for this uncoated paper.

If an ink film of from 5 to 7μ thickness is printed on a substrate which is 88.9μ (3.5 mils) thick, one might expect maximum ink penetration to a depth of 20 or 30μ by capillary action. Thus the largest reasonable value of ϕ_2

Table 6. Densities of the CL1 Series of Inks at 72° F

Ink	Density, g/cm^3	
	Individual Det'ns	Avg.
Yellow CL1BM	1.012*	
	1.008*	
	1.012	
	1.002	1.008
Yellow CL1BO	1.003*	
	0.998*	
	0.986	
	0.985	0.993
Red CL1LC	1.146	
	1.134	
	1.154	1.144
Red CL1W	1.123	
	1.108	1.116
Red CL1RH	1.071	
	1.064	1.068
Red CL1LR	1.043	
	1.046	1.044
Purple CL1RH	1.088	
	1.103	1.095
Blue CL1A	1.046	
	1.043	1.044
Blue CL1V	1.046	
	1.045	1.046
Blue CL1PC	1.149	
	1.142	1.146
Green CL1PC	1.066	
	1.082	1.074
White CL1	1.668	
	1.681	1.675
Black CL1	1.094	
	1.079	1.087

* using 2 cm^3 of ink, all other values using 3.2 cm^3

Figure 14. Sample Computer Output of Calibration Program

```
          PHI VALUES FROM CALIBRATING PRINT 625-144-5

          PAPER STOCK: FINCH OFFSET SMOOTH
          SCATTERING POWER AT 700 NM: 5.540

          COMPOSITION OF CALIBRATING INK:
              RED CL1LR        100.0 PER CENT
              RED CL1RH          0.  PER CENT
              RED CL1W           0.  PER CENT
              PURPLE CL1RH       0.  PER CENT

          AREA DENSITY OF PRINT:  5.873 G/SQ. M


         PHI(1)     PHI(2)     PHI(3)        MEAN SQ.
                                          TRIST. DIFF.

 I.  NEWTON-RAPHSON ITERATION

         0.000      1.000      0.000         14191.0
        -0.001      0.366      0.015          5355.8
         0.002      0.152     -0.185          1764.6
         0.011      0.085     -0.563           511.3
         0.031      0.123     -0.181           187.2

 II.  STEEPEST DESCENT PROGRAM

         0.011      0.085     -0.563           511.3
         0.019      0.079     -0.563           224.7
         0.020      0.079     -0.563           219.8
         0.037      0.099     -0.564           139.2
         0.037      0.099     -0.564           139.2


                    FINAL PHI VALUES:
                        PHI(1) = 0.037
                        PHI(2) = 0.099
                        PHI(3) =-0.564
```

(cont. next page)

(Fig. 14, cont.)

REFLECTANCE VALUES CORRESPONDING TO FINAL SET OF PHI'S

WAVE-LENGTH, NM	REFLECTANCE FOUND	REFLECTANCE CALCD.
400	0.072	0.078
420	0.071	0.069
440	0.073	0.074
460	0.067	0.066
480	0.059	0.057
500	0.050	0.050
520	0.047	0.049
540	0.050	0.052
560	0.057	0.058
580	0.079	0.073
600	0.143	0.180
620	0.452	0.460
640	0.688	0.629
660	0.755	0.674
680	0.779	0.689
700	0.792	0.692

Table 7. ϕ-Values from Calibrating Prints

Ink	Substrate	Print No.	Film Thickness		ϕ_1	ϕ_2	ϕ_3
			(g/m^2)	(μ)			
Blue CL1PC	FOS	625-144-1	6.616	5.8	0.020	0.050	-0.147
		-2	7.330*	6.4*	0.279	0.140	0.138
		-3	7.908*	6.9*	0.060	0.059	-0.098
		-4	7.221	6.3	0.393	0.162	0.087
Red CL1LR	FOS	625-144-5	5.873	5.6	0.037	0.099	-0.564
		-6	7.207	6.9	0.054	0.456	0.520
		-7	7.826	7.5	0.085	0.508	0.593
		-8	7.922	7.6	0.074	0.587	0.604
Blue CL1PC	FOV	625-142-1	9.490	8.3	0.101	-0.030	4.043
		-2	10.067	8.8	0.031	-0.133	1.473

* Subtraction error found in weight/area determination;

values should be 7.468 6.5
8.046 7.0

should be about 0.337. Three out of the eight values of ϕ_2 are larger than this value. The remaining five values fall within the realm of credulity.

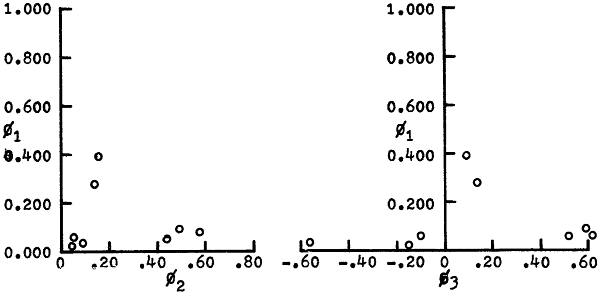
The value of ϕ_3 represents how much scattering in layer II is due to paper fibers partially wet by vehicle, and $(1-\phi_3)$ represents how much scattering is due to unwet fibers. Negative values of ϕ_3, of which there are three out of eight, would seem to indicate that the paper in layer II scatters more than plain paper. Yet, the remaining five values indicate that a substantial amount of scattering is due to partially wet fibers.

However, one should not demand that the ϕ-values be realistic. They should rather be thought of as adjustable parameters which force a fit between calculated and measured reflectance curves (actually tristimulus values). As such they may compensate for all the experimental uncertainties, including the arbitrary definition of $\sigma(700)$ as 0.100, and including the semi-empirical nature of Kubelka-Munk Theory. In previous work (14) it was seen that vastly different sets of ϕ-values could be used to generate virtually identical reflectance curves. This and the above results suggest that there may be more than one unique and "accurate" set of ϕ-values, and raise the possibility that the ϕ-values are not mutually independent. Indeed there may well be some relationship among the ϕ-values. For example, if $\phi_1 = 1.000$ it would follow in the sense of their original definitions that $\phi_2 = \phi_3 = 0.000$. Also, if $\phi_1 = 0.000$, most probably, $\phi_2 \neq 0.000$ and $\phi_3 \neq 0.000$.

A brief attempt was made to investigate the possible relationship. The hypothesis was put forth that if the ϕ-parameters formed a three-dimensional "ϕ-space," the various solutions for calibrating the system would all lie on the same curve through this ϕ-space. To test this hypothesis the eight ests of values, which constituted eight points in the ϕ-space, were plotted in two dimensions at a time. That is, plots were made of ϕ_1 vs. ϕ_2, ϕ_1 vs. ϕ_3, and ϕ_2 vs. ϕ_3; the plots appear in Fig. 15. The plots in fact represent the projections of the eight points on each of the three coordinate planes.

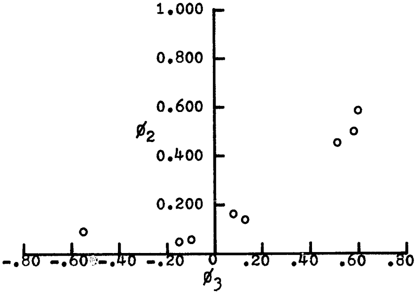
The original hope was that the various solutions would lie along an easily characterized curve, preferably a straight line, in which case extrapolations might be made to solutions which had realistic physical implications and could be applied for all calibrating prints. A careful study of the data in Figs. 15a, b, and c indicated that defining such a curve, if it indeed did exist, would require the acquisition of much additional data with no assurance of any substantial gain in understanding of the system.

The ϕ-values for two prints of Blue CL1PC on Finch Offset-Vellum Finish also appear in Table 7. These values were calculated using a program involving Newton-Raphson iteration alone. While the program was more efficient in



a) ϕ_1 vs. ϕ_2

b) ϕ_1 vs. ϕ_3



c) ϕ_2 vs. ϕ_3

Figure 15. Plots in "ϕ-Space" of Eight Sets of ϕ-Values

converging to solutions, the solutions still could not be interpreted in physical terms.

C. "First-Shot" Computer Matches

1. Matches to Standards of Known Composition

The first match computed using the MATCH1 program was for the Pink Standard 625-147-C-2. The match was computed three times; first with the data from calibrating print 625-144-6 of Red CL1LR, second with the data from calibrating print 625-144-5 of Red CL1LR, and third with the data from calibrating print 625-144-2 of Blue CL1PC. Each of the three runs gave slightly different formulations of the specified four basic inks, and metamerism indices of 1.9, 1.4, and 1.5 ACV, respectively. Although the value of 1.4 for the metamerism index was rather high, the corresponding formula was chosen to be mixed and printed at the specified film thickness.

In Fig. 16 appear the results of the MATCH1 computer output including the description of the standard, the Ø-values used, and plots of the measured reflectance curves of the standard and match print, and the calculated reflectance curve of the match print. If the <u>measured</u> curve for the <u>match print</u> was coincident with the <u>calculated</u> curve of the match print, the <u>standard</u> and <u>match print</u> would be a visual match under Source C (at least for the Standard Observer). Any differences between these two curves represent theoretical and experimental uncertainties. Naturally,

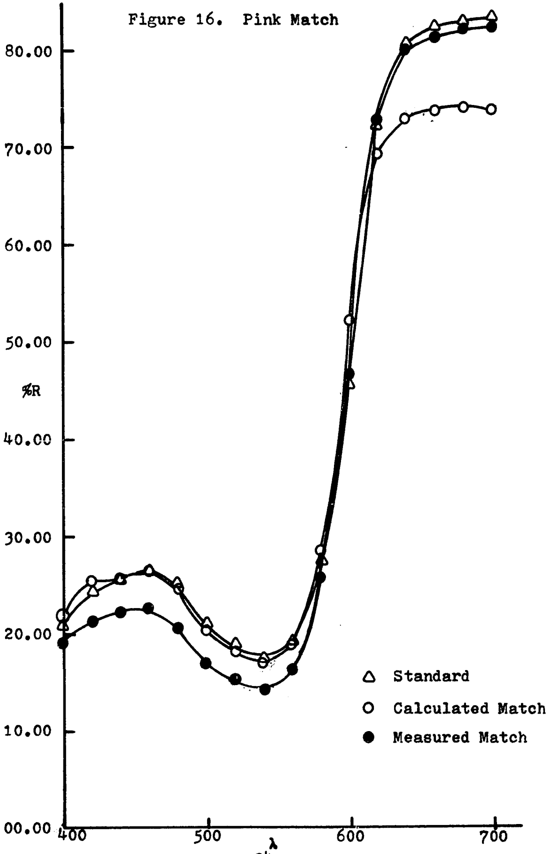


Figure 16. Pink Match
80.00
70.00
60.00
50.00
%R
40.00
30.00
20.00
10.00
00.00
400
500
600
700
λ
△ Standard
○ Calculated Match
● Measured Match

(Fig. 16, cont.)

	Calculated Match	Match Print
Standard:	Print 625-147-C-2	Print 625-150-D-3
Paper Stock:	Finch Offset-Smooth	
Scattering Power at 700 nm:	5.540	
Film Thickness:	4.9 microns	5.0 microns
Phi Values:		
Phi(1) =	0.037	
Phi(2) =	0.099	
Phi(3) =	-0.564	
Matching Formula:		
White CL1	85.94 %	85.95 %
Yellow CL1BM	5.32	5.30
Red CL1RH	7.94	7.95
Purple CL1RH	0.80	0.80
Metamerism Index:	1.4 ACV	

the closer the match print curve was to the standard curve, the less metameric the match would be.

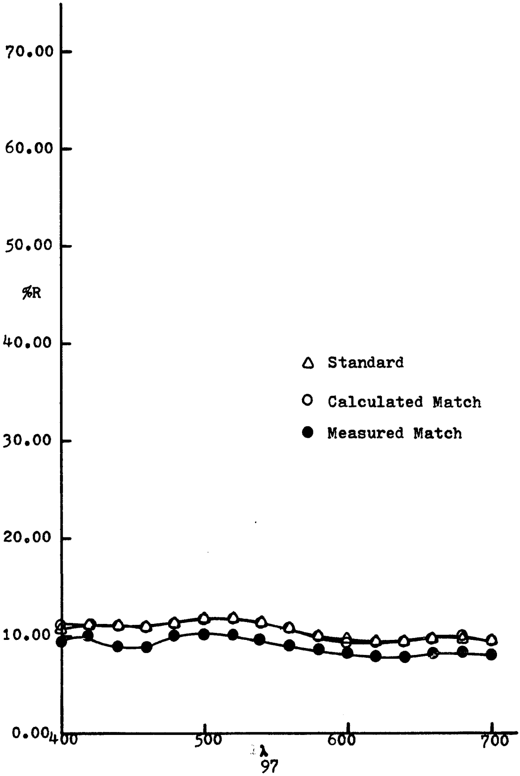
The metamerism indices computed by MATCH1 or MATCH22 based on the calculated curves will be referred to as the theoretical metamerism indices. The metamerism indices calculated using the Wang Calculator and based on the reflectance measurements of the standards and match prints will be referred to as the experimental metamerism indices. The experimental metamerism indices for all the matches will be discussed later.

Match calculations for Dark Gray Standard 625-147-A-2 were run with the MATCH1 program, and the results appear in Fig. 17. The ø-values were from calibrating print 625-144-6 of Red CL1LR. Since the theoretical metamerism index was 0.0, no need was seen to try other ø-values or to run the MATCH22 program for this standard. As in the previous figure the data for the match print also appear in Fig. 17.

As mentioned above, the MATCH1 program failed to find a match for Salmon Standard 625-147-B-5. The reason for this is that, during the calculation, at least one of the components was called for at a negative concentration algebraically less than 1.00 % of the total ink area density. A branch in the program tested for this condition and ended the calculation if the criterion was satisfied.

The need for such a branch is evident from the follow-

Figure 17. Dark Gray Match



(Fig. 17, cont.)

	Calculated Match	Match Print
Standard:	Print 625-147-A-2	Print 625-151-E-3
Paper Stock:	Finch Offset-Smooth	
Scattering Power at 700 nm:	5.540	
Film Thickness:	4.8 microns	4.8 microns
Phi Values:		
Phi(1) =	0.054	
Phi(2) =	0.456	
Phi(3) =	0.520	
Matching Formula:		
White CL1	75.25 %	75.25 %
Black CL1	12.74	12.75
Yellow CL1BM	4.60	4.60
Blue CL1PC	7.41	7.40
Metamerism Index:	0.0 ACV	

ing considerations. Since a tristimulus match requires matching three numbers, it is always possible to compute a mathematical match, given the necessary three degrees of freedom. However, for an extreme example, in calculating the match to an orange print from a combination of three blue inks and a white ink, it is a virtual certaintly that the match formula will contain at least one negative concentration. Since such formulations are physically impossible to make, the computer program must have a criterion by which to reject them without the dependence on any preselection of basic inks by the operator. Yet, very small negative concentrations should be permitted, to allow for experimental uncertainty. In addition, the waste of costly computer time must be avoided. For this latter reason a reasonable rejection criterion must be presumed and applied sometime during the iteration process, rather than at the completion of the iteration to a final formula. Unfortunately, without extensive knowledge of the mathematical behavior of the model and iteration process, one can only guess at the cutoff criterion and when to apply it. It is conceivable that iteration to positive concentrations may proceed through the region of negative concentrations.

The MATCH22 program* was written with the same logic

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* Although the MATCH22 program gave correct answers, it was never finally debugged to give a presentable and explicitly labeled output; no sample output is given here

as the MATCH1 program but used the combinatorial approach; it calculated all possible match combinations of white plus three colored inks in the 13 ink palette (220 possible combinations). The MATCH22 program was first run with the Pink Standard 625-147-C-2 to try to ascertain the reason for the high theoretical metamerism index of 1.4. With the rejection criterion set at -1.0 % of the total area density, 69 formulations were found, of which 36 had no negative concentrations. The actual ink combination used in the standard was selected as only the eighth lowest in metamerism. With the rejection criterion changed to -0.1 %, 38 formulas were found, 30 had no negative concentrations, and the actual combination was still eighth lowest in metamerism. These results led to the study of the inks for possible fluorescence and to the reduction of the palette to eight inks. As previously stated, the three subsequent standards of known composition and all the subsequent matches were made from this palette. In this limited palette there were only 35 possible combinations of white plus three colored inks.

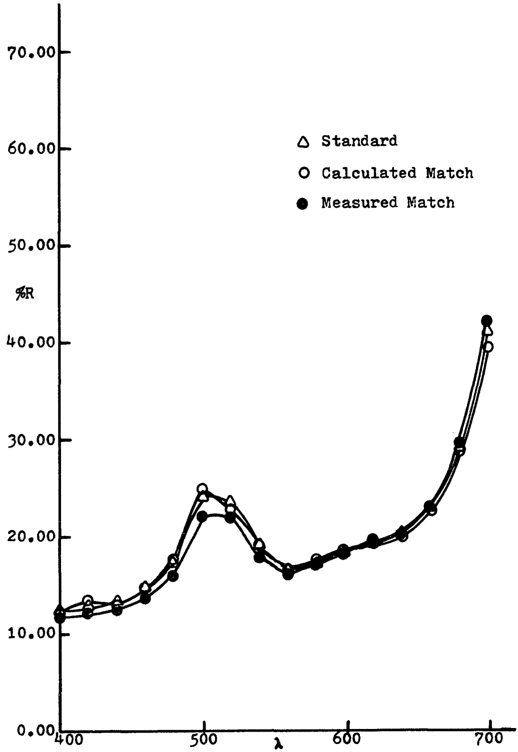
For the Olive Green Standard 625-140-4 and the Light Blue Standard 625-141-3 matches were successfully calculated by the MATCH1 program. Also, in both of these cases the MATCH22 program, using a rejection criterion of -1.0 % of the total area density, selected the actual combination of inks as being the least metameric. For the Olive Green, 9 formulations were found, 8 of which had only positive concentrations. For the Light Blue, 20 formulation were found, 9 of which had only positive concentrations. The results for the match prints appear in Figs. 18 and 19, respectively.

In the case of the Dark Blue-Green Standard 685-19-2 the MATCH1 program computed the slightly high metamerism index of 0.6. When run with the MATCH22 program, 11 formulations were found, all of which had only positive concentrations. However, the correct combination was selected as only the third lowest in metamerism. Of the two formulas selected as being lower in metamerism, both contained the white, black, and green, but one substituted Blue CL1A for Blue CL1PC, and the other substituted Blue CL1V for Blue CL1PC. Both of these two formulas had metamerism indices of 0.4. In addition to the formula with the correct ink combination, the formula containing the Blue CL1A was mixed and printed. The results appear in Fig. 20.

a. Fluorescence in Basic Inks

The determination of fluorescence was carried out by the measurement of the Di-Sta prints on the KCS-40 spectrophotometer. In each case the masstone printed over the white background was measured without using the white vitrolite glass backing; the print was backed only by the black rubber sponge of the sample holder. Each print was measured for "reflectance" under Source D_{6500}, and then,

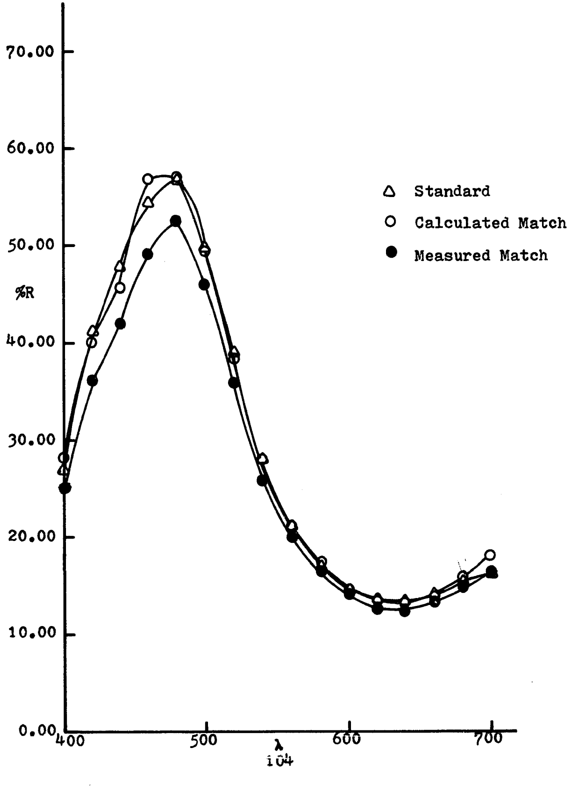
Figure 18. Olive Green Match



(Fig. 18, cont.)

	Calculated Match	Match Print
Standard:	Print 625-140-4	Print 685-15-3
Paper Stock:	Finch Offset-Vellum	
Scattering Power at 700 nm:	4.982	
Film Thickness:	7.9 microns	6.6 microns
Phi Values:		
Phi(1) =	0.101	
Phi(2) =	-0.030	
Phi(3) =	4.043	
Matching Formula:		
White CL1	77.33 %	77.33 %
Yellow CL1BM	16.85	16.85
Red CL1RH	2.55	2.55
Blue CL1V	3.27	3.27
Metamerism Index:	0.2 ACV	

Figure 19. Light Blue Match

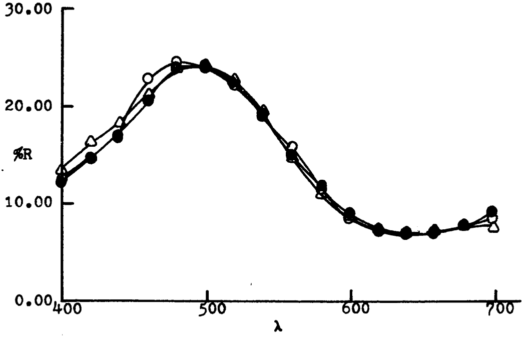


(Fig. 19, cont.)

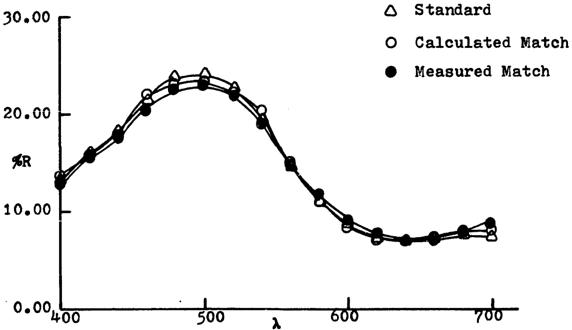
	Calculated Match	Match Print
Standard:	Print 625-141-3	Print 685-14-3
Paper Stock:	Finch Offset-Vellum	
Scattering Power at 700 nm:	4.982	
Film Thickness:	6.3 microns	6.4 microns
Phi Values:		
Phi(1) =	0.101	
Phi(2) =	-0.030	
Phi(3) =	4.043	
Matching Formula:		
White CL1	91.24 %	91.24 %
Blue CL1A	2.69	2.69
Blue CL1PC	1.19	1.19
Green CL1PC	4.88	4.88
Metamerism Index:	0.2 ACV	

Figure 20. Dark Blue-Green Matches

a) Lowest Metamerism Formula



b) Correct Combination Formula



(Fig. 20, cont.)

	Calculated Match (Lowest Metamerism Formula)	Match Print
Standard:	Print 685-19-2	Print 685-26-2
Paper Stock:	Finch Offset-Vellum	
Scattering Power at 700 nm:	4.982	
Film Thickness:	8.3 microns	8.1 microns
Phi Values:		
Phi(1) =	0.101	
Phi(2) =	-0.030	
Phi(3) =	4.043	
Matching Formula:		
White CL1	64.84 %	64.84 %
Black CL1	1.82	1.82
Blue CL1A	1.56	1.56
Green CL1PC	31.78	31.78
Metamerism Index:	0.4 ACV	

	Calculated Match (Correct Combination Formula)	Match Print
Standard:	Print 685-19-2	Print 685-25-3
Paper Stock:	Finch Offset-Vellum	
Scattering Power at 700 nm:	4.982	
Film Thickness:	8.3 microns	8.8 microns
Phi Values:		
Phi(1) =	0.101	
Phi(2) =	-0.030	
Phi(3) =	4.043	
Matching Formula:		
White CL1	67.20 %	67.20 %
Black CL1	2.33	2.33
Blue CL1PC	8.81	8.81
Green CL1PC	21.66	21.66
Metamerism Index:	0.6 ACV	

without moving the print, the spectrophotometer was set up for Source A and a second "reflectance" measurement made. The Source D and Source A curves were examined wavelength by wavelength to see what differences existed. Table 8 shows for each ink the wavelength at which the maximum Source D - Source A reflectance difference occurred and the magnitude of the difference. Although this difference is not a measure of the true fluorescence it gives an indication as to which inks would be likely to cause problems. Although the Benzidine Yellow CL1BM and the Rhodamine Red CL1RH were not perfectly free of fluorescence, they were retained in the palette so as to have some representation of the long wavelength portion of the spectrum.

2. Matches to Standards of Unknown Composition

The MATCH22 program was run for nine standards of unknown composition; eight of these were printed standards, and the ninth was a painted standard. Table 9 identifies the standards and shows for each the total number of matches found, the number of matches with no negative concentrations and the lowest theoretical metamerism index.

Due to the limited palette of eight inks, four of the standards chosen had unacceptably high values for even the lowest metamerism index. No prints were made of these matches, but even these negative results are valuable in that they indicate the absence of suitable inks in the palette.

Table 8. Fluorescence Indications in Basic Inks

Ink	Wavelength (nm)	Difference in Source D-Source A Reflectance (%R)
White CL1	-	-*
Black CL1	-	-*
Yellow CL1BM	520	1.57
Yellow CL1BO	520	3.13
Red CL1LC	600	3.09
Red CL1LR	620	3.34
Red CL1W	600	3.09
Red CL1RH	620	1.21
Purple CL1RH	620	2.22
Blue CL1A	-	-*
Blue CL1PC	-	-*
Blue CL1V	-	-*
Green CL1PC	-	-*

* The precision of the spectrophotometer in this test is estimated as 0.3-0.4 %R. Only differences greater than this limit of precision appear in the table.

Table 9. MATCH22 Calculations for Standards of Unknown Composition

Standard	No. Formulas Found total	all positive conc.'s	Lowest Metamerism Index (ACV)
Blue 685-31-1	7	0	5.8
Green -31-2	8	0	1.4
Aqua -31-3	16	8	0.8
Blue -31-4	9	8	0.3
Black -34-1	7	5	0.5
Black -34-2	5	5	0.4
Blue -34-3	13	12	1.7
Blue -34-5	9	5	3.4
Gray #264	24	5	0.3

Of the remaining five standards and match calculations, four had minimum metamerism indices of 0.5 or below. These match formulations were mixed and printed. The fifth match had the relatively high metamerism index of 0.8, but this match was mixed and printed anyway. The results appear in Figs. 21 through 25. Since the matches were calculated with the MATCH22 program only, these figures show only the measured curves of the standard and match print.

D. Commercial Matches to a Pink Standard

The six commercial matches to the Pink Standard are described in Fig. 26 and Table 10. Fig. 26 displays the six measured reflectance curves of the match prints (the standard curve and one match print curve on each plot). Since there are no curves available for the theoretical tristimulus matches of the six respective formulations, the only judgement that can be made based on the inspections of the curves is one concerning metamerism. Table 10 shows the pigment constitution of each match formulation and of the standard. It is readily seen that six experienced color-matchers took widely different approaches towards achieving a match.

E. Evaluation of Commercial and "First-Shot" Computer Matches

Table 11 shows the color difference and experimental metamerism index data for the finished commercial matches to the Pink Standard. The color differences in MacAdam units were calculated by the software of the KCS-40

Figure 21. Aqua Match

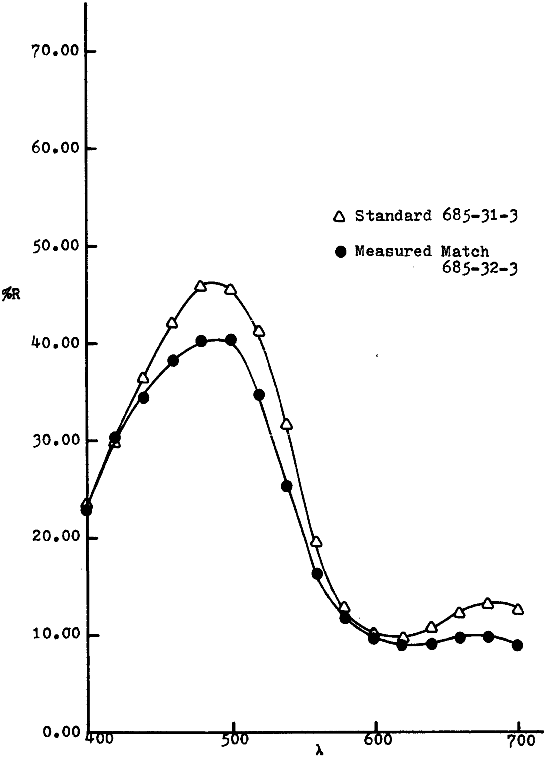


Figure 22. Blue Match

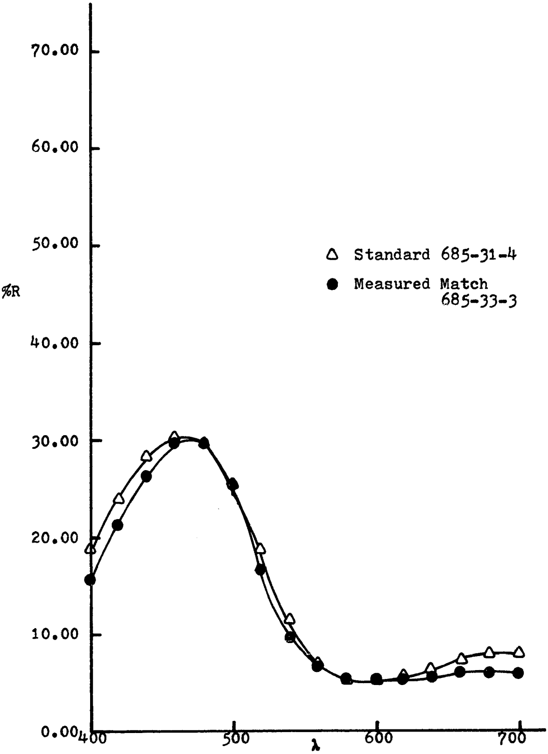


Figure 23. Black 1 Match

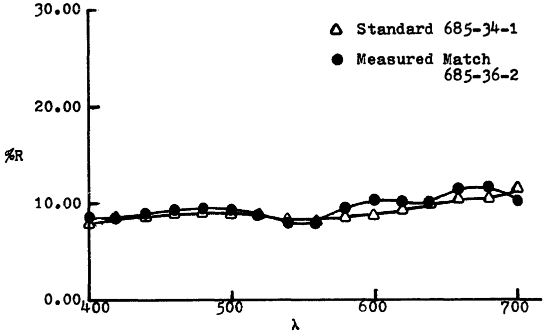


Figure 24. Black 2 Match

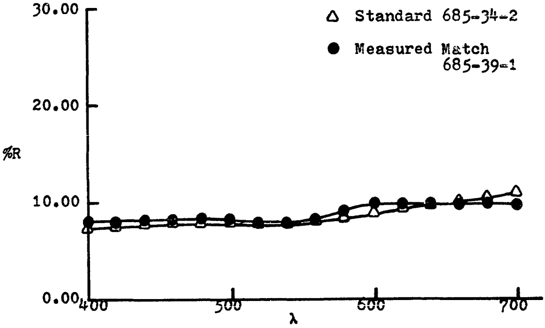


Figure 25. Gray Match

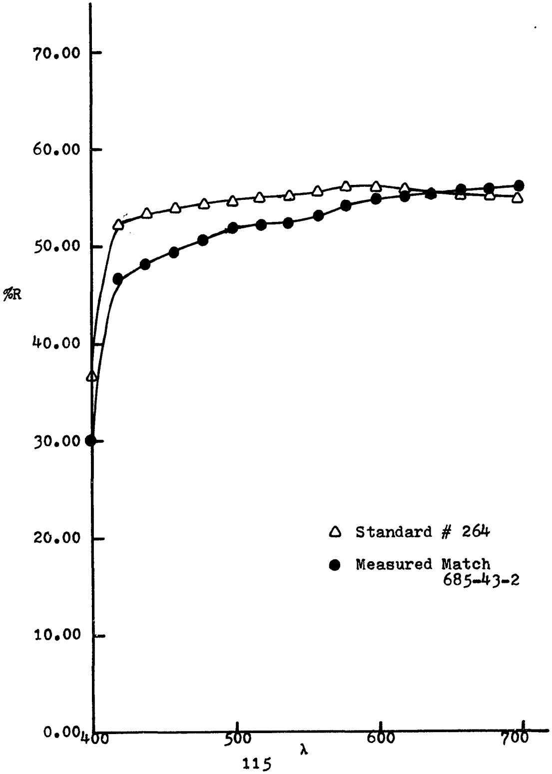


Figure 26a. Pink Commercial Match 1

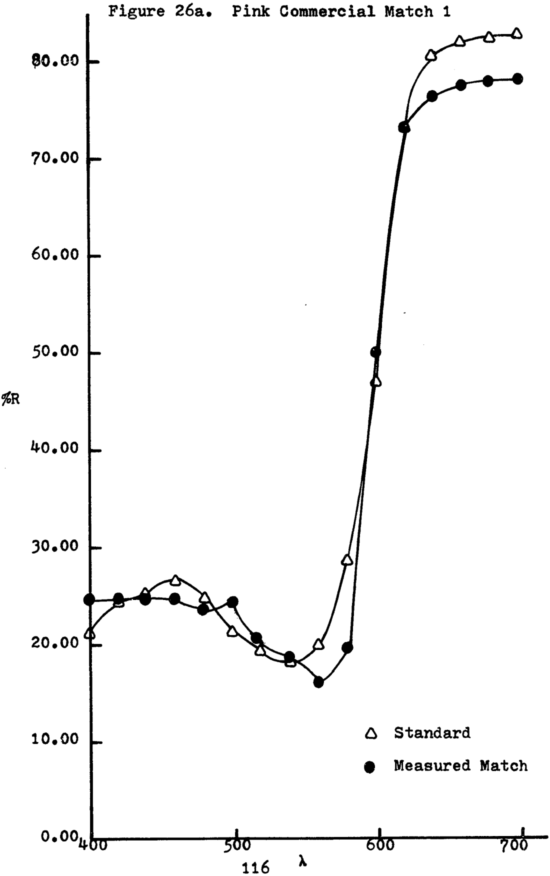


Figure 26b. Pink Commercial Match 2

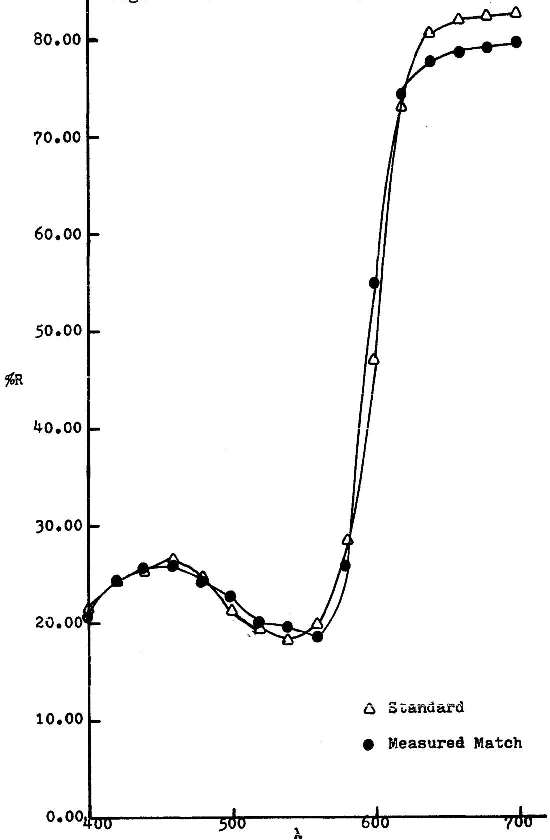


Figure 26c. Pink Commercial Match 3

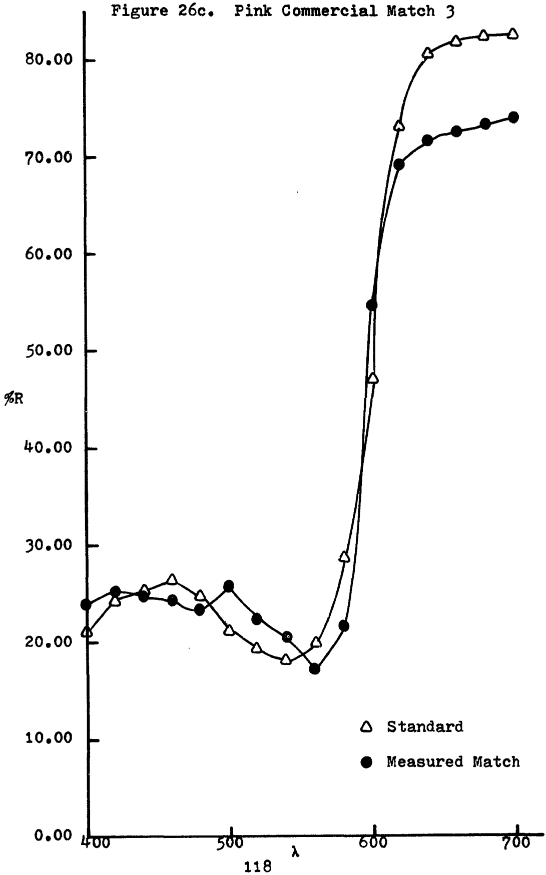
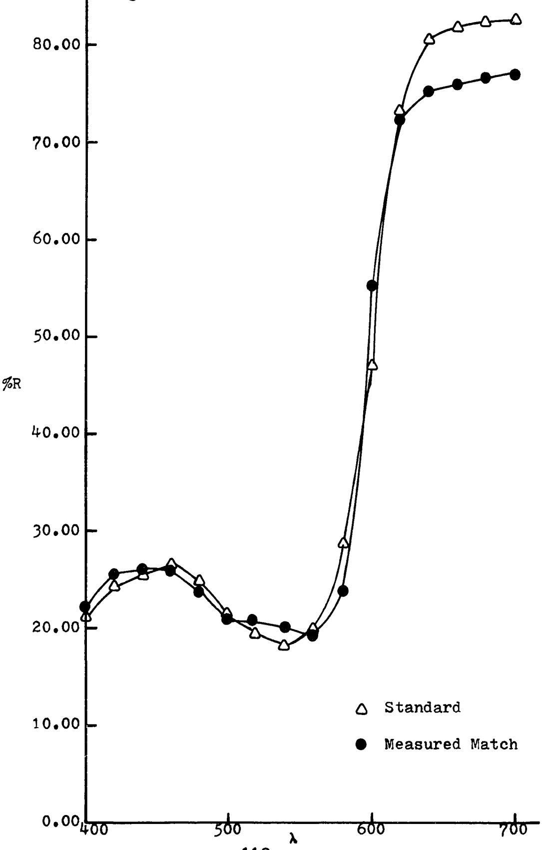


Figure 26d. Pink Commercial Match 4



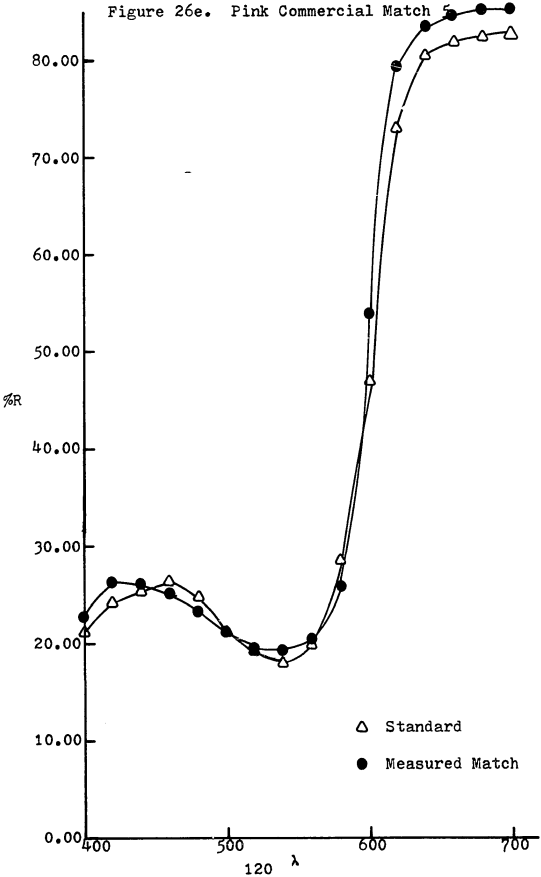


Figure 26e. Pink Commercial Match 5

Figure 26f. Pink Commercial Match 6

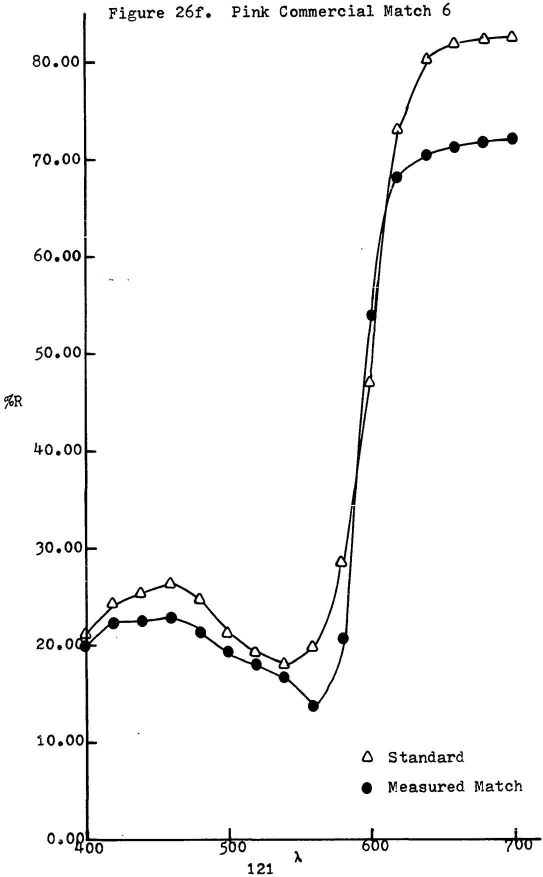


Table 10. Pigments Used in Commercial Matches

Color-Matcher	Pigments Used in Match
Color Lab (Standard)	Rutile Benzidine Yellow Rhodamine (Yellow-shade) Rhodamine (Blue-shade)
Ink Company 1	Benzidine Yellow Calcium Lithol Rubine Barium Red Lake C Furnace Carbon Black Rhodamine (Blue-shade) Rutile Calcium Carbonate
Ink Company 2	Calcium Lithol Rubine Chrome Yellow (Lemon-shade) Chrome Yellow (Med. shade) Burnt Umber Rutile
Ink Company 3	Lithol Rubine Benzidine Yellow Calcium Carbonate Rutile Channel Carbon Black
Ink Company 4	Rutile Chrome Yellow (Med. shade) Macatawa Red Tungstated Rhodamine Rhodamine (Yellow-shade) Furnace Carbon Black
Ink Company 5	Rutile Calcium Lithol Red Molybdate Orange
Ink Company 6	Rutile Calcium Carbonate Watchung Red Chrome Yellow (Med. shade) Channel Carbon Black

Table 11. Color Differences of Finished Industry Matches

Ink Company	Color Difference		Metamerism Index
	MacAdam Units	ACV Units	(Expt'l) ACV Units
4	1.8	1.0	0.5
2	2.3	1.1	0.2
1	2.7	1.5	1.4
5	8.8	3.6	2.6
3	9.6	5.1	0.5
6	11.0	4.2	0.5
Avg.	6.0	2.8	1.3

spectrophotometer from reflectance measurements of the standard and match prints. Since one Mac Adam unit is defined as a just noticeable difference (jnd), it is seen that some of the matches are quite good, some are rather poor, and the average color difference represents a match of perhaps borderline acceptability. The same observations may be made concerning the experimental metamerism indices in ACV. The ACV formulas used were adjusted to conform closely to the NBS unit, where 1.0 NBS was the original definition of a commercial tolerance. Since the metamerism information was in ACV, The color difference data were recalculated in ACV. It may be seen that, although there is no simple relationship between FMC and ACV mathematics or units, the results are comparable.

Table 12 shows the same data for the computer calculated "first-shot" matches. The pink match is listed, but not included in the averages due to the fluorescence problem. As indicated by the averages, the color differences of the "first-shot" computer matches compare well with the commercial finished matches. More important, however, the average of the metamerism indices is significantly lower than the average for the commercial matches. In addition, the computer performed just as well on the unknown standards, where it had no guarantee of the presence of the correct inks in the palette, as it did on the known standards, where there was such a guarantee.

Table 12. Color Differences of Computer-Calculated "First-Shot" Matches

	Color	Color Difference				Metamerism Index	
		MacAdam Units		ACV Units		(Expt'l) ACV Units	
	Pink (Fluorescent)	(18.3)		(6.4)		(5.4)	
Known Stds.	Dark Gray	6.7	Avg. 4.9	3.6	Avg. 2.8	0.4	Avg. 0.8
	Olive Green	5.8		2.7		0.6	
	Light Blue	4.5		3.0		1.3	
	Dark Blue-Green	4.1		3.1		1.0	
	Dark Blue-Green	3.3		1.6		0.5	
Unknown Stds.	Aqua	9.4	Avg. 6.0	8.8	Avg. 3.7	1.9	Avg. 0.8
	Blue	2.9		2.0		0.2	
	Black 1	7.1		2.9		0.5	
	Black 2	3.3		1.3		0.3	
	Gray	7.1		3.4		1.1	
	Average	5.4		3.2		0.8	

It may be argued, quite validly, that the commercial results are represented for only one region of color space, and that the comparisons are really only meaningful to the extent that the color difference formulas approximate a transformation to a uniform color space. However, an important point may be raised in the defense of the automated system. In the practice envisioned, a color-matcher would build on the first-shot computer results and quickly achieve a finished match. The color-difference and metamerism results for such finished matches could hardly fail to be superior to the first-shot results.

VII. DISCUSSION

This thesis could not be considered complete without an explanation of the overriding philosophy and motivation that guided this work. This project was conceived and financed in large part by the printing ink industry as a study of the technological feasibility of automated color-matching of printing inks. As such it was dominated by the rather pragmatic objective of developing a system of color-matching over a relatively short period of time. For this reason, some of the areas of theoretical or practical interest were not as fully investigated as other circumstances might have allowed. The desire was to get a working system, and it is believed that this was achieved. The following discussion is devoted primarily to those areas in which more exacting investigation might have lead to an improved system. Possible sources of experimental or theoretical uncertainty will be described, but it must be emphasized that any changes in the theoretical or experimental approach can only be considerd improvements by observing the end product, the ability of the system to select the optimum ink combination and to provide a good first-shot formulation.

The first considerations concern the substrate and the mathematical assumptions and parameters that deal with the substrate. The τ function and the assumption of invariance of τ from paper to paper (or at least invariance for a given class of papers) are used in an attempt to rapidly characterize the substrate optical properties. Although the assumption of invariance of τ was not tested here, it has been demonstrated (14) that there are variations among sheets of the same paper and between the τ functions of two papers that presumably differ only in surface finish (Finch Offset-Vellum Finish and -Smooth Finish). However, these differences are relatively unimportant. A color-matcher could easily measure the reflectance of the single sheet of the job substrate across the whole spectrum, rather than at 700 nm alone. In addition, the overall matching procedure may be (relatively) insensitive to small variations in τ or θ.

The values of the σ function (14) used in this work were the averages from five sets of data, and the five sets showed a fair degree of scatter. Also, the data were based on a system of FOV paper, #3003 Safflower Alkyd, and letterpress printing from a magnesium plate. The assumption was that σ would be invariant from system to system. It might have been desirable to recalculate σ using the average of perhaps 10 or more vehicle prints. Also, different vehicle-paper-plate combinations might have been studied. However, the fact that successful match calculations were obtained with the current σ values for systems using vehicles other than #3003 Safflower Alkyd, paper other than FOV, and a rubber rather than a magnesium plate, suggests that the overall matching procedure is not very sensitive to the values of σ.

Concerning the optical characteristics of the basic inks, the discrepancies due to fluorescence have already been mentioned, but further details are offered here. The K and S determinations were based on reflectance measurements with Source A sample illumination. Since Source A has a relatively small amount of fluorescence-exciting radiation, the K and S values were probably not very much in error. However, Kubelka-Munk Theory does not deal with the fluorescent flux, so when matches were computed for Source C (which has much mor fluorescence-exciting energy) the results naturally erred in relation to the standards with fluorescent inks.

It is possible to separate a spectral radiance factor curve into the true reflectance and fluorescent protions (20). Extensions to Kubelka-Munk Theory have been made (21, 22) which take into account fluorescence, re-absorption, secondary emission, and true and apparent quantum efficiencies of fluorescence. Such modifications may allow the calculation of accurate spectral radiance factor curves and tristimulus values for fluorescent samples. The major uncertainty lies in the affects of "quantum mechanical quenching" of fluorescence, i.e., the affects of molecular interactions. For the many complex pigment and vehicle molecules present in printing inks this would be a formidable theoretical problem.

In view of the relatively small degree of fluorescence present in normal printing inks, the most practical approach to the problem may be to determine the K and S values of the inks with measurements using Source D_{6500} illumination. Then, the tristimulus matches would be calculated for Source D_{6500} also. This will not eliminate errors in the Source A metamerism index, but it may reduce them and should lead to more accurate formulations for the Source D_{6500} tristimulus matches.

Two additional factors concern both the K and S values and many of the reflectance measurements. First, there is the matter of the reflectance standards. As mentioned above, the working vitrolite standards were calibrated against freshly prepared $BaSO_4$ plaques. But the calibration was done on the Trilac spectrophotometer and the results used on the KCS-40 spectrophotometer. Some time after all the experimental work was completed for this thesis, a new fresh $BaSO_4$ plaque was made. This was measured against the vitrolite standards. If the Trilac and KCS-40 spectrophotometers were comparable, it was expected that the reflectance of the $BaSO_4$ would be 100.0 %. As seen in the results of Table 13, the results were somewhat higher than

Table 13. Check on Vitrolite Calibration

A fresh $BaSO_4$ plaque was measured on the KCS-40 spectrophotometer against vitrolites calibrated on the Trilac spectrophotometer. For correct calibration the reflectance of the $BaSO_4$ should be 100.0 %.

Wavelength, nm	Reflectance, %R
380	107.78
390	103.35
400	101.08
410	101.51
420	101.39
430	101.50
440	101.66
450	101.64
460	101.63
470	101.70
480	101.90
490	101.93
500	102.03
510	102.00
520	101.95
530	102.01
540	101.98
550	102.11
560	102.10
570	102.14
580	102.33
590	102.33
600	102.31
610	102.34
620	102.32
630	102.21
640	102.27
650	102.28
660	102.35
670	102.42
680	102.49
690	102.57
700	102.69
710	102.82
720	102.97
730	103.13
740	103.32
750	103.58

expected, indicating that the reflectance values used in this work were also some 2-3% high.

Second, many of the reflectance measurements were made with the so-called automatic range of the KCS-40 spectrophotometer. With this mechanism, whenever the sample reflectance dropped below 10 %, a 10 % reduction screen was placed in front of the white standard measuring beam to increase the precision. When the sample reflectance went above 20 % the screen was automatically removed. However, due to improper hardware design in the spectrophotometer or software error in the computer program of the mini-computer that drives the instrument, the screen was not automatically set in the out position at the beginning of a run. This meant that the presence or absence of the screen at the start of a run depended on the previous run. For example, if a blue sample was run immediately following a calibration check of the white standards, the screen would be in the out position until the blue reflectance peak was passed and a reflectance lower than 10 % was encountered. At the end of the first measurement of the blue sample, 700 nm, the screen would likely remain in place. A repeat run on the blue sample would now start with the screen in place, and, if the blue reflectance peak never exceeded 20 %, the screen would remain in place. All of this would cause no problems if the 10 % reduction screen had a transmission value of 10 % across the spectrum. Unfortunately, this was not the case. By adjusting the software in the minicomputer it was possible to measure the screen transmission; the results appear in Table 14. This problem, of which the instrument manufacturer was not fully aware, was not discovered until the match for Dark Blue-Green Standard 685-19-2 was being calculated. From that point on, all reflectance measurements were made with the 100 % range, so the screen would not be used.

The reflectance data for the K and S determinations were corrected for the screen and vitrolite calibration errors, and the K and S values have been recalculated. However, this also was done after the completion of the data gathering for this thesis. It would have been desirable to repeat the work with the corrected K and S values, but the computer costs were prohibitive.

The latter statement gives rise to a final consideration. An analytic error analysis using exact partial derivatives, etc. would have been extremely difficult in view of the complexity of the mathematical model and match computations. A more practical approach would have been a numerical analysis procedure, i.e., perturbing the system variables one or more at a time and observing the affects on the match calculations. But again, in view of the computer cost of $30-$50 per match calculation and the number of possible permutations of the perturbations of system variables, this possiblity was prohibitively expensive.

Table 14. Transmission of 10 % Reduction Screen

Wavelength, nm	Transmission, %T
380	10.49
390	10.49
400	10.49
410	10.50
420	10.57
430	10.50
440	10.50
450	10.44
460	10.44
470	10.45
480	10.44
490	10.44
500	10.38
510	10.40
520	10.30
530	10.31
540	10.39
550	10.31
560	10.32
570	10.31
580	10.30
590	10.31
600	10.29
610	10.30
620	10.32
630	10.27
640	10.30
650	10.30
660	10.30
670	10.24
680	10.31
690	10.32
700	10.29
710	10.29
720	10.41
730	10.29
740	10.41
750	10.39

Further work is now in progress to develop a commercially practical system. This addtional work deals with a system including a new set of basic inks, dry offset printing, and three new substrates, an uncoated paper, a coated paper and a paper board. The mathematical model and computer programs are being revised to avoid the need for making weighed prints and to achieve a faster system with lower costs. The approach also uses a transparent base as the fourth ink need to deal with the film thickness constraint. Since transparent pigments (fillers) are less expensive than opaque white and are ordinarily used commercially to build up the solids content of an ink for viscosity considerations, this also conforms more closely to commercial practice.

VIII. CONCLUSIONS

1. Experimental procedures and computer programs have been developed for determining the Kubelka-Munk absorption (K) and scattering (S) coefficients of letterpress and lithographic printing inks.

2. A process for the automated color-matching of printed ink films has been developed for a limited system of non-fluorescent basic inks, two non-fluorescent uncoated offset papers, and letterpress printing.

3. The system successfully achieves the objectives of selecting the least metameric match combination and providing a sound "first-shot" formulation.

4. The "first-shot" computer matches approached the results of commercial color-matchers in terms of color difference and exceeded them in terms of metamerism.

5. The technological feasibility of the automated color-matching of printed ink films has been demonstrated.

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VITA

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