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THE DESIGN OF SOLID WASTE SYSTEMS: AN APPLICATION OF GEOMETRIC PROGRAMMING TO PROBLEMS IN MUNICIPAL SOLID WASTE MANAGEMENT

by

Wilson W. Yale

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ABSTRACT

The purpose of this research is the development of nonlinear solid waste management models that more accurately approximate reality than previous attempts. Since the literature reviewed indicates that the operating of solid waste management systems do not pay for themselves, only cost minimization models are investigated. The nonlinear phenomena of economies of scale and diseconomies of scale pervade the cost structure of municipal solid waste models. Therefore, each mathematical model is a nonlinear programming problem which is solved by geometric programming. The nonlinear objective function models the fixed and variable costs of the solid waste system while the constraints impose technological, legal, and social restrictions upon the system. In general, the resulting geometric program can have reversed constraints, inactive constraints, and large degrees of difficulty. A solution procedure with a hueristic rule is developed and then utilized to obtain solutions to the geometric programs.

A case study of a solid waste system for

Lehigh and Northampton Counties is modeled and solved. For certain structures of municipal solid waste systems, the accompanying geometric program has degree of difficulty zero for which a global minimal solution is uniquely determined. One of the most advanced and efficient municipal solid waste systems in the United States is shown to have this structure.

CHAPTER 1

OVERVIEW

1.1 Purpose and Contributions of the Research

The purpose of this research is the development of nonlinear solid waste management models that more accurately approximate reality than previous attempts. Since the literature reviewed indicates that the operation of solid waste management systems does not generate profits, only cost minimization models will be investigated. Each mathematical model is a nonlinear programming problem whose objective function is to be minimized subject to several constraints. The objective function reflects the fixed and variable costs of the solid waste system while the constraints mirror the technological, legal, and social restrictions imposed upon the system.

Previous attempts at solving the solid waste problem include forming a linear programming model of the system. Although the linear program can be efficiently solved by various software codes, the well documented nonlinear phenomenon of economies of scale in these solid waste management models is ignored.

Another attempt attacks the nonlinearity of economies of scale by piecewise linear approximation (96). This approximation technique can appreciably augment the size and complexity of the linear program to be solved. The proposed research will employ posynomial geometric programming which can more naturally accommodate the nonlinearities found in solid waste management models.

The contributions of this research are:

1) Modeling solid waste systems in a natural way with multiplicative relationships and economies of scale in order to obtain more accurate solutions to solid waste systems than LP (linear programming) approximations.
2) Solving these models in a more computationally efficient manner with geometric programming.
3) Obtaining a global optimum to the mathematical models corresponding to certain structures of solid waste management models.
4) Developing and computerizing an algorithm to solve the nonlinear models.
5) Modeling and solving a case study of a solid waste system for a region in the Lehigh Valley of Pennsylvania.

1.2 General Background

Solid waste management is an increasingly complex problem confronting intermediate and large cities throughout the world. Some of the topics that come under the study of solid waste management are refuse collection, recycling systems, disposal system analysis, mathematical programming, mathematical modeling, law, economics, and politics. Because of the extensive variety of components in solid waste management systems, these systems are frequently developed on an *ad hoc* basis (1, 2). A brief overview of the solid waste management problem will be presented here.

Municipal solid waste is composed of residential and commercial solid waste generated within a community where:

a) residential solid waste comprises all solid waste that normally originates in a residential environment.

b) commercial solid waste comprises all solid waste generated by stores, offices, and other activities that do not actually produce a product.

Two general classes of solid waste materials are combustibles and noncombustibles. Thermal energy is

stored in solid waste combustibles such as paper, leaves, plastics, and tires. Noncombustible resources from solid waste include ferrous and nonferrous metals, carbon black from tires, and glass. The table below illustrates the valuable energy and material components derived from solid waste in Tampa and New Orleans (3).

	Tampa	New Orleans
Combustibles	76.0	85.1
Ferrous Metals	7.0	4.9
Aluminum	1.0	.8
Other Nonferrous Metals	<1.0	.3
Glass	3.0	6.0
Plastic	12.0	6.1
Ash and Dirt	0	0

(The above values in the table are percentages by weight.)

Hereafter, references to waste or solid waste should be interpreted as municipal solid waste unless specifically stated otherwise.

When designing solid waste management systems, collection, recycling, and landfilling define the major subsystems. First, the waste must be collected.

The most common method of collection is by compactor trucks. However, in Sweden there is a trial system consisting of a network of pneumatic tubes leading from drop-off points directly into the disposal plant (4). Second, the solid waste may be processed for recycling and/or disposal at a landfill site. Some systems merely mass burn the solid waste for reduction through incineration with a stoker. Although the stoker may be gas fired, the solid waste generates additional thermal energy, all of which could be used for firing a boiler for an electricity generating turbine (3). About 10.9 per cent of the electrical energy produced in the United States by steam could be produced by solid waste which burns with one-tenth of the sulphur content of coal (5). St. Louis' Union Electric Company has been obtaining 20 per cent of its power from solid waste (3). The noncombustible wastes such as ferrous metals, nonferrous metals (especially aluminum), carbon black from tires, and glass can be extracted and recycled (3, 6, 7, 8, 9). In fact, in Connecticut a statewide resource recovery system including collection, recycling, and landfilling subsystems is currently being implemented (7).

By utilizing a process called pyrolysis (heating in the absence of air) to recycle tires, an

organic fuel is one of the products (3). Goodyear Tire and Rubber Corporation and the Oil Shale Corporation are utilizing the pyrolysis technique with scrap tires at a test facility. Goodyear proposes a plant that will produce 10 million pounds of steel, 14 million gallons of oil, 63 million pounds of carbon black (a key ingredient in the production of tires), and enough energy to run itself from 11 million tires (9). Recycling of the tires' components results in a volume reduction of between 95 and 98 per cent. Furthermore, the remaining slag residue (2 to 5%) could be used for road surfacing (3).

The sanitary landfill is not only where most of the residue from recycling solid waste is dumped, but in many cases where the raw refuse is deposited. Pulverizers and shredders are used to reduce the volume of solid waste for landfill dumping (3). Some of the problems with landfill dumping are ground water pollution and gas generation.

System analysis and solid waste management direct the flow of solid waste with control and feedback systems which involve close interactions with people. Here are eight constraints about which solid waste management systems are developed:

1) The physical state of the area
2) Laws and ordinances
3) Control and monitoring agencies
4) Processes of disposal
5) Composition of refuse
6) Collection services
7) Economics
8) Societal preferences

Notice that some of the constraints can be easily quantified while others cannot. Therefore, measures of effectiveness not only include an analysis of economic benefits, but they must also involve intangible benefits. The proposed research will investigate intangible and tangible economic consequences of solid waste systems through the solution of mathematical models.

Solid waste recovery can be economically desirable. Recycled aluminum needs less than 5 per cent of the energy required to make it from virgin material (8). As a consequence, reclaimed aluminum was selling for over 200 dollars per ton in 1974 (8). However, the net benefits of reclaiming the aluminum must be greater than the next best alternative for processing aluminum in order to be economically attractive. In a Connecticut study, resource recovery

has economies of scale that favor reclamation over dumping in landfill sites, but that depends upon stable market conditions. Projections in this study indicate that for a 2,000-ton-per-day facility, fuel recovery costs are less than a nearby landfill facility and material recovery costs are less than a remote landfill facility (7). Another study points out that 6 million dollars are spent to dispose of 220 million tons of urban waste which contains 990 million dollars of metals, glass, and paper (3). For example, the solid waste plant at Lowell, Massachusetts, which was erected by Raytheon, processed a ton of refuse worth $15.76 for $4.06 where the capital investment for this 250-ton-per-day plant is estimated at 1.4 million dollars (3).

The legal environment encourages resource recovery. Some recently passed acts are:

a) Resource Conservation and Recovery Act of 1976, which includes provisions to eliminate substandard land disposal sites and to promote resource recovery.

b) Clean Air Acts of 1965 and 1970, which prohibit substandard incinerators from operating.

c) Ocean Dumping Act of 1972, which prohibits ocean dumping to a large degree.

d) Federal Water Pollution Act Amendments of 1972, which reduced the amount of wastes that can be deposited into streams.

The effectiveness of this legislation has been recently challenged. The United States sued West Chester County, New York, for the restoration of the streams and tidal marshes surrounding the county's Croton Point landfill site. In addition, five municipal incinerators in Nassau County, New York, were closed because of pollution control violations.

Presently, many areas cannot support a large enough facility to economically warrant refuse reclamation over landfilling. Yet as natural resources become increasingly scarce and pollution laws become more stringent, reclamation sites will become economically attractive as they will become legal necessities. Even if solid waste recovery is not economical, the refuse must be disposed by some solid waste system. In this research, cost minimization models of solid waste management systems are investigated to determine the minimum total cost of these systems regardless of their economic attractiveness.

CHAPTER 2

THE SOLID WASTE MANAGEMENT PROBLEM

Part 1 - Resource Recovery

2.1 Introduction

"Resource recovery is a general term used to describe the extraction of economically usable materials or energy from waste materials" (10). The historic trend of resource recovery indicates that man will be recycling the lowest proportion of domestic solid waste--less than one per cent in the 1970's--than ever before in mankind's history. With the advent of the industrial revolution and the rise of the standard of living, the proportion of domestic solid waste recycled has plummeted (See Figure 2.1). However, the percentages of certain materials being recovered from all solid wastes are not as low as for domestic solid wastes (See Figure 2.2).

Using secondary materials instead of virgin materials often results in lower energy use, less air and water pollution, and less solid waste to dispose (11). For a fair comparison of secondary versus virgin materials, one must look at the mining,

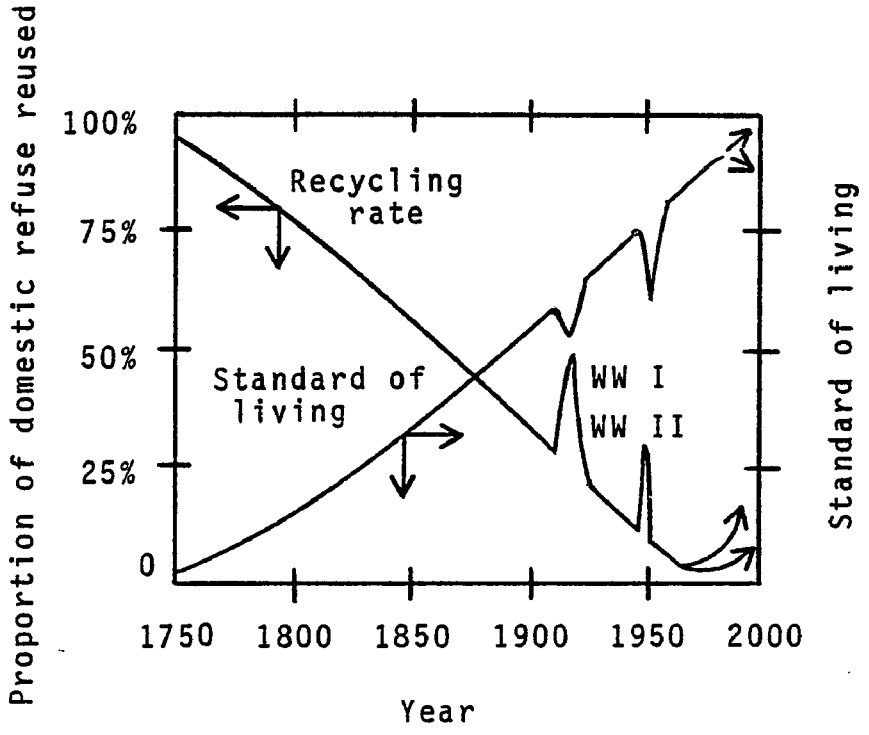


FIGURE 2.1. Estimate of Domestic Waste Recycled.

Source: Wilson, David Gordon, Handbook of Solid Waste Management. Van Nostrand Reinhold Company, (1977), p. 275.

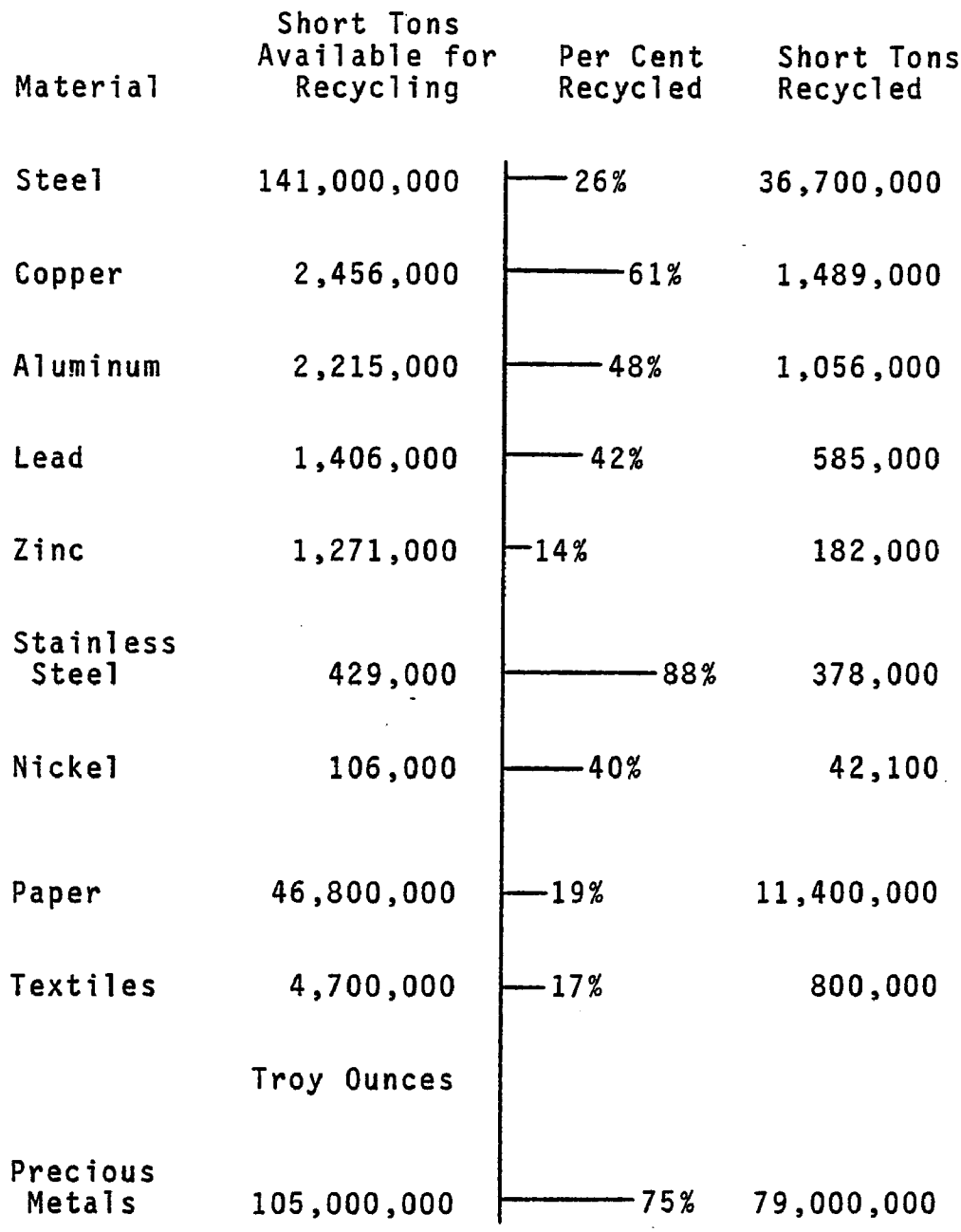


FIGURE 2.2. The Percentage of Materials Being Recycled (1971 Estimates). Source: Glyson, Eugene H., et al., The Problem of Solid Waste Disposal. College of Engineering, University of Michigan, (1972), p. 10.

processing, fabrication, manufacturing, and disposal demands for each type of material. The collection and recovery of waste material is equivalent to the mining and processing of virgin materials, respectively. If environmental and societal costs could be associated with virgin materials, resource recovery may prove to be far superior economically.

Only 25 per cent of the United States consumption of some of the most heavily recycled materials such as metals, glass, paper, and rubber are being recovered from solid waste (19). Most of these recycled materials come from manufacturing processes which generate homogeneous wastes. The industrial community finds that virgin materials from distant lands are economically more desirable than processing municipal solid waste (19).

2.2 Resource Recovery Incentives

The most prevalent resource recovery incentives stem from the current economic and legal scenarios. The scarcity and rising costs of virgin materials and the diminishing cost of resource recovery processes are two economic incentives. In the United States, higher proportions of virgin materials with higher transportation costs are imported (12). Furthermore,

major technological advances in recovery processes are lowering the recovery costs and increasing the recovery yields. Even though resource recovery technology is in its primitive stages as compared to virgin material technology, current studies (166, 172) indicate that resource recovery can be economically desirable.

Increasing amounts of solid waste, as shown in Table 2.1, are being produced each year, while conventional disposal capacity is being radically reduced. The amount produced per person and the population are growing. New legislation prohibits open dumping and may force closure of many landfills (14). Moreover, dumping at sea is prohibited, and many incinerators must cease operations (14). Nevertheless, over 90 per cent of the solid waste generated in the United States is handled by landfilling, ocean dumping, and incineration.

2.3 Availability and Costs of Materials

The potential demand for several nonrenewable minerals cannot be met with virgin materials (10). Coal as a principal fuel resource will be available for at least 2,300 years at 1970 rates of utilization, or 111 years if the consumption rate increases by the

Year	Per Capita Production of Municipal and Industrial Solid Waste
1920	2.0 pounds/day
1969	5.3 pounds/day

TABLE 2.1. Per Capita Production of Municipal and Industrial Solid Waste for Two Cities in the United States.

Source: Government Publication PB 233178, April, 1974.

expected rate of 4.1 per cent. Conservative consumption growth estimates listed in Table 2.2 predict that most major resources will be depleted in less than 200 years. In terms of mineral value, the United States is producing substantially less than what it consumes, and the gap is widening.[1]

---

[1]For detailed accounts of the status of the world's natural resources, see the following tables:

a) Abundance, Mass, Reserves, and Resources of Some Metals in the Earth's and United States Crust (15)

b) World Production and Consumption of Selected Minerals (16)

c) Net Supply of Principal Minerals by Components (17)

d) Comparisons of the World and United States Production and United States Imports of Principal Minerals and Mineral Fuels in 1976 (17)

e) Value of Selected Minerals and Minerals Products Imported and Exported by the United States in 1970, by Commodity Groups and Commodities (17)

f) United States Exports of Principal Minerals and Products (17).

Resource	Known Reserves	Projected Rate of Growth (%/year)	Exponential Index (Years)
Aluminum	1.17×10^9 tons	6.4	31
Chromium	7.75×10^8 tons	2.6	95
Coal	5×10^{12} tons	4.1	111
Cobalt	4.8×10^9 lb	1.5	60
Copper	308×10^6 tons	4.6	21
Gold	353×10^6 troy oz	4.1	9
Iron	1×10^{11} tons	1.8	93
Lead	91×10^6 tons	2.0	21
Manganese	8×10^8 tons	2.9	46
Mercury	3.34×10^6 flasks	2.6	13
Molybdenum	10.8×10^9 lb	4.5	34
Natural gas	1.14×10^{15} cu ft	4.7	22
Nickel	147×10^9 lb	3.4	53
Petroleum	455×10^9 bbl	3.9	20
Platinum group	429×10^6 troy oz	3.8	47
Silver	5.5×10^9 troy oz	2.7	13
Tin	4.3×10^6 long tons	1.1	15
Tungsten	2.9×10^9 lb	2.5	28
Zinc	123×10^6 tons	2.9	18

This table summarizes the known reserves of some important resources and shows some measures of how long they are likely to last. The exponential index allows for growth in consumption at the annual rate shown. (Data: Mineral Facts and Problems, 1970, U.S. Bureau of Mines.)

TABLE 2.2. Resource Lifetimes.

Source: Randers, J. and Meadows, D., "The Dynamics of Solid Waste," Technology Review, pp. 20-32 (Mar./Apr. 1972).

2.4 Reclamation Industry and Waste Systems

The complex interactions of the different components in the reclamation industry are shown in Figure 2.3. The alternate paths for wastes to pass from consumers (sources) to scrap-consuming industries can make the development of reclamation systems complex. For illustrative purposes, the movement of ferrous scrap in the steel industry will be analyzed (See Figure 2.4). There are two types of scrap--"home" scrap which is scrap produced from production runs and milling operations, and "obsolescent" scrap which is scrap from consumers of steel (10). The home scrap tends to be of high purity, since segregation of different grades of steel in mass production is profitable. However, obsolescent scrap is mixed with other materials when discarded; thus the value of the ferrous scrap plummets. A proportion of .02 per cent copper contamination in the ferrous scrap can render the scrap worthless (10) (See Figure 2.5). Therefore, separating the ferrous metal from the heterogeneous waste mixture is the ferrous-solid waste problem.

The current solid waste industry outlook is encouraging. The Environmental Protection Agency (EPA) estimates that the volume of residential and commercial solid wastes generated in the United States is increasing

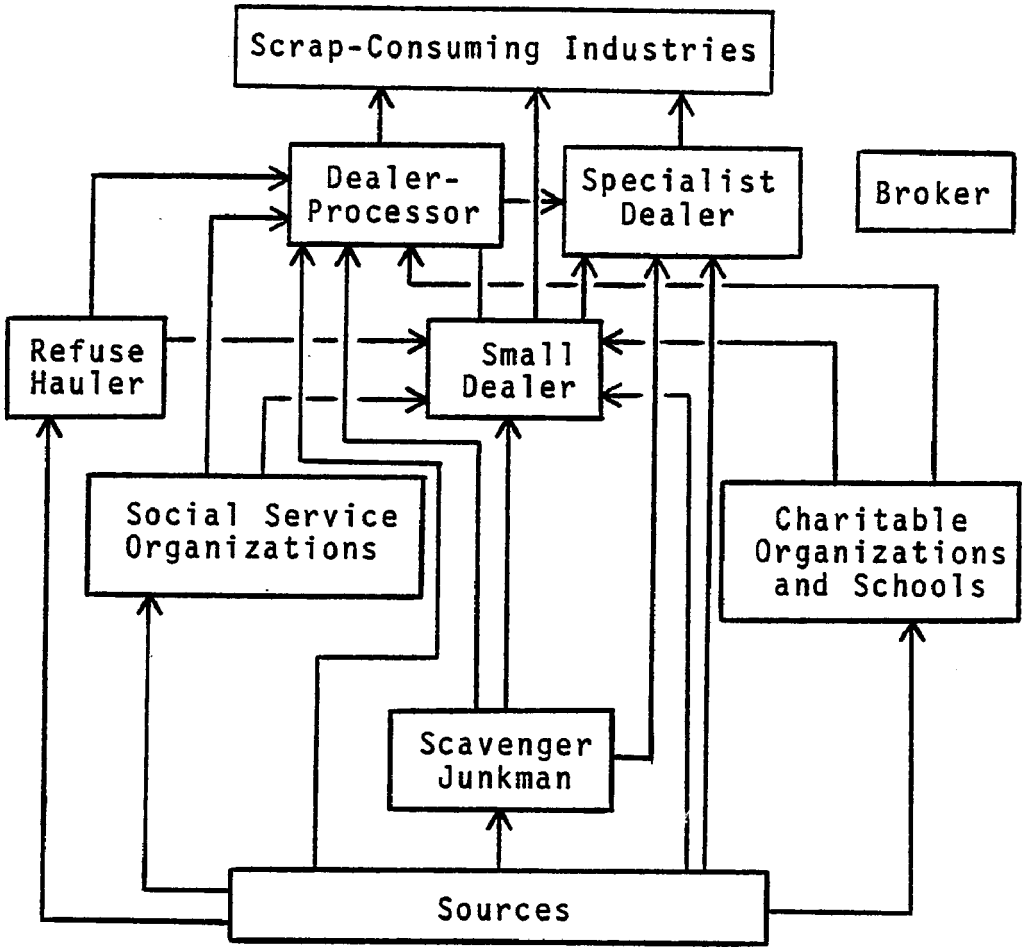


FIGURE 2.3. Structure of the Salvage Industry and Flow of Commodities.

Source: Arsen Darnay and William E. Franklin, "Salvage Markets--For Materials in Solid Wastes," U.S. Environmental Protection Agency, Report (SW-29c), 1972.

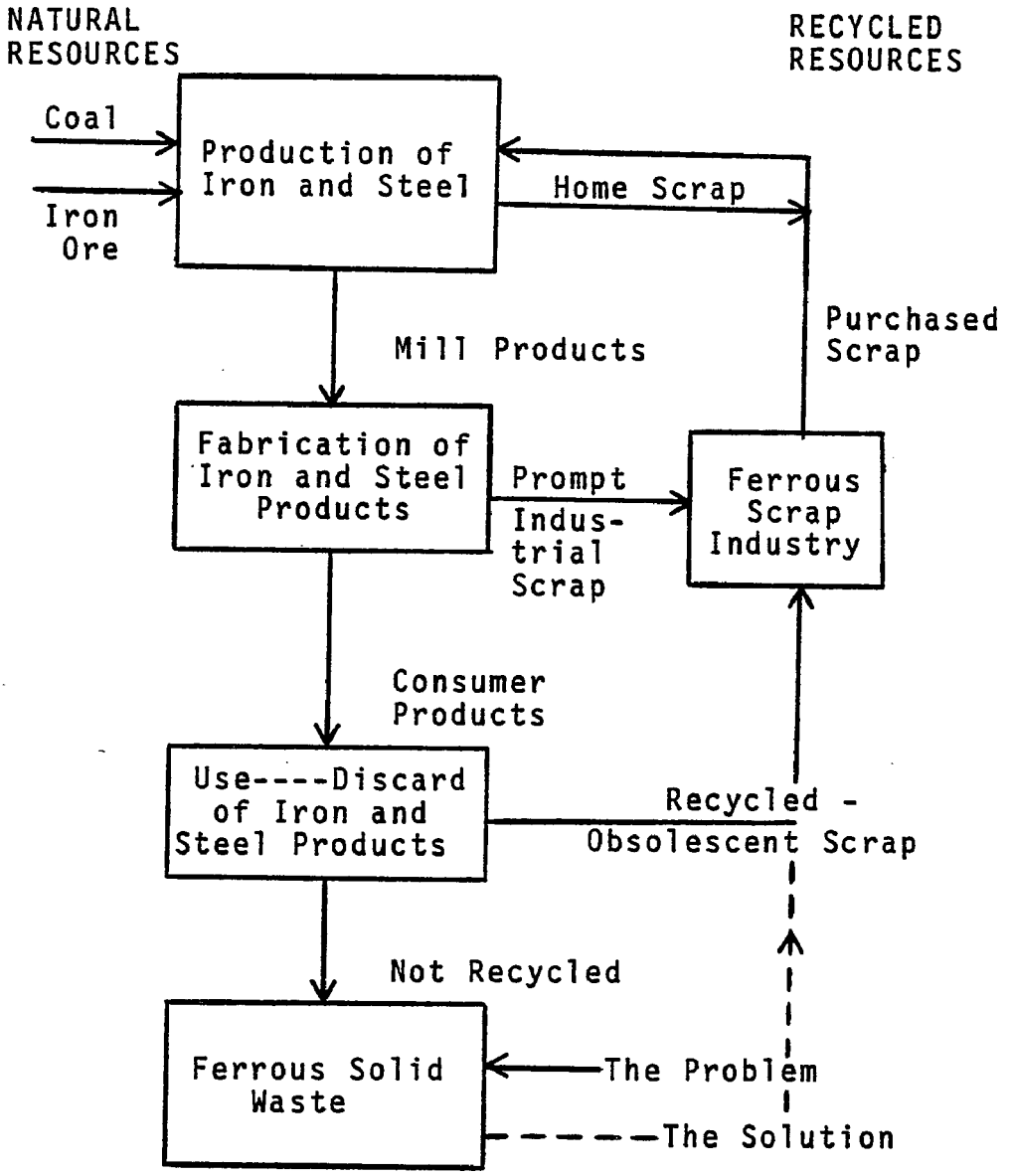


FIGURE 2.4. The Scrap Cycle and Ferrous Solid Waste.

Source: W. J. Regan, R. W. James, and T. J. McLeer, "Identification of Opportunities for Increased Recycling of Ferrous Solid Waste," U.S. Environmental Protection Agency, Report (SW-45-d.1), 1972.

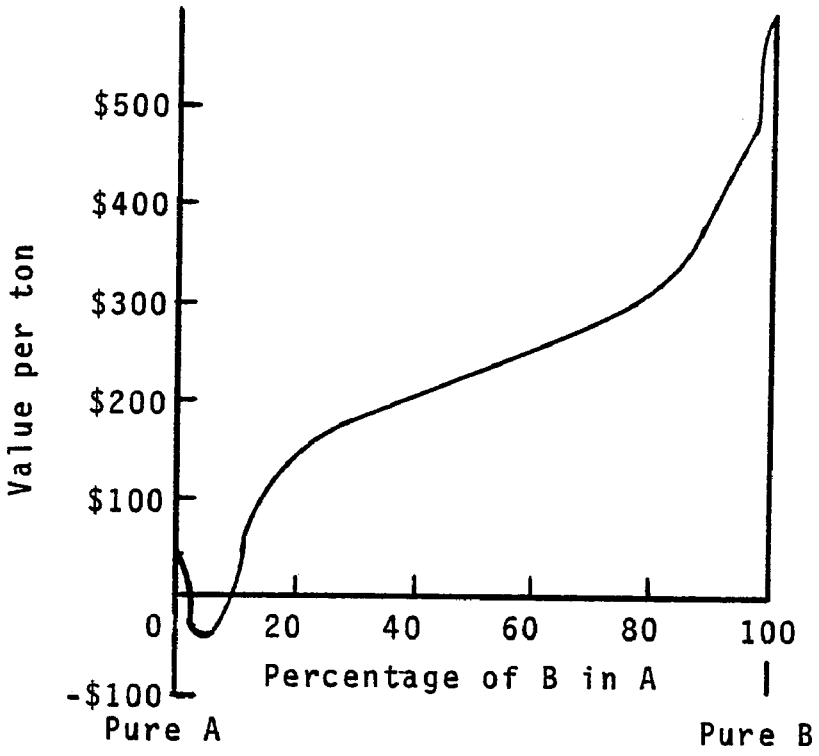


FIGURE 2.5. Effects of Contamination on Value.

Source: Wilson, David G., Handbook of Solid Waste Management. Van Nostrand and Reinhold Company, (1977), p. 288.

at an annual rate of 3 per cent. Currently, the United States is generating 300 million tons of solid waste annually. Browning-Ferris Industries, the largest publicly held company engaged in providing solid waste collection, processing/recovery, and disposal services to commercial, industrial, residential, and governmental customers throughout the United States and Canada, expects the waste systems to maintain a growth rate of 8 per cent annually (20). Furthermore, a recent Columbia University survey for the National Science Foundation concluded that private firms provide lower cost collection services than municipalities (10).

2.5 Specifications of Recycled Materials

Quality standards for recycled materials are by agreement between buyer and seller in the United States, whereas in Europe the standards are determined by legislation. The principal associates for recycling standards are the National Association of Recycling Industries and the Institute of Scrap Iron and Steel. The National Association of Recycling Industries for paper, cotton and synthetics and nonferrous scrap can be found in Tables 10.9, 10.10, and 10.11 of reference 10. The Institute of Scrap Iron and Steel recycling

standards for iron and steel scrap can be found in Tables 10.12 and 10.13 of reference 10. For European standards, see Tables 10.14 through 10.21 of reference 10. These standards explain in detail the different qualities and the conditions of sale for each material. Some outdated purity specifications should be updated to reflect the current technology of recycling.

2.6 Reclaimed and Virgin Material Use and Availability

A large proportion of aluminum is supplied by imports making recycling more attractive (16). The United States imports nearly all of the virgin ores needed to manufacture aluminum (24, 25). Since 1946, the United States has been importing more bauxite than it produces; in fact, 89 per cent of it (21, 22). Furthermore, the United States is a net exporter of scrap aluminum to West Germany, Italy, and Japan (23). Aluminum is the most valuable commodity on a per unit per weight basis in municipal waste (24, 25), even though only 1/4 to 1-1/2 per cent of municipal waste is composed of nonferrous metals such as aluminum, copper, zinc, and lead (26). United States production of beryllium, cobalt, chromium, and copper falls far short of domestic demand; there is no domestic production for some minerals, for example, columbium (16).

One-half of the United States' steel and copper production is from scrap. It has been estimated that every lead molecule is recycled 100 times before ultimate disposal, or equivalently, twice as much lead is recycled as is mined (24). It also has been estimated that 90 per cent of ferrous metals could be recovered from industrial and municipal wastes (24, 26). Industrial scrap is one major source, but reclamation of municipal solid waste can be another (27).

Soda ash which is needed to manufacture glass from virgin materials is in short supply, thereby creating a demand for recycled glass (28). Furthermore, more energy is used to manufacture glass from virgin materials than from reclaimed glass. However, manufacturers want color sorted glass, and the recovery of such glass is not cost effective with current technology.

Waste glass has a multitude of uses in addition to being a substitute for raw materials used in the production of glass. Waste glass can be utilized to make road paving material which is more resilient to studded tires (29). Building materials manufactured from glass have desirable properties such as high strength and low water absorption (11, 30). Furthermore, glass rubble structural panels yield handsome

and inexpensive finishes with decorative marble and granite facings (31).

The paper industry is deriving 20 per cent of its raw materials, which is equivalent to 200,000,000 trees, from waste paper (24). Most of this recycled paper is used in the production of boxboard (32). Nevertheless, recycled waste paper constitutes a meager 0.3 per cent contribution to the production inputs over the 1956 to 1974 period, while virgin materials use claims the balance (33). Even though the growth rate of virgin material use is 6 per cent annually, recycled paperboard capacity is expected to be 3.6 per cent for the years 1975 through 1977 (33). Recycling of newsprint is a "potentially explosive area for waste paper use in the future" (32). The United States Forest Service released a report in December of 1972 stating that the demand for timber could exceed the supply in the 1980's (34).

In lieu of using expensive animal feeds, spent grains utilized in the production of beer can be dried and dewatered for feed. Hog feed can be obtained from waste food of restaurants, institutions, residences, and markets (24). Cement dust filtered out of the air during the production of portland cement can be fed to cattle that not only gain weight faster, but also

produce tastier meat (35).

For detailed statistics on important resources such as carbon black, paper products and paper, plastics, peat, rubber, sand, gravel ash, slag, and glass, see Chapter 10 of reference 10 for reproductions of charts, many of which are from the Minerals Yearbook of the United States Bureau of Mines, or consult the most current yearbook. Other information discussed in Chapter 10 of reference 10 is the environmental impact, the change and trend in America's demand, and the uses of various materials.

2.7 Separation of Mixed Waste for Reclamation

After mechanical separation of residential solid waste into combustibles and noncombustibles, the combustibles can be incinerated to produce thermal energy, pyrolyzed (thermal decomposition in the absence of oxygen) to produce organic fuels, and fermented to produce alcohol. Further separation of the noncombustibles such as the metals and glass can be effected by manipulating existing or induced differences in the physical properties of the materials being separated. A well developed mining technology includes the separation techniques that key on such properties as color,

luster, size, shape, tenacity, brittleness or friability, structure, texture, surface characteristics, specific gravity, magnetic susceptability, electrical conductivity, radioactivity, and decrepitation (11). Differences in the properties of materials can be chemically induced by drying to improve electrical conductivity, by oxidation to affect magnetic properties, and by the application of a vacuum to porous materials to change their specific gravity (11). Now, separation techniques that key off the induced properties can be applied to the altered materials.

Eight unit processes utilized in solid waste separation are:

1) processes incidental to separation
2) mechanical and hand separation
3) washing and scrubbing
4) gravity concentration
5) magnetic separation
6) electrical (electrostatic) separation
7) flotation.

Processes incidental to separation of municipal solid waste include alteration of the waste's chemical properties, size reduction, sizing, and fluid-solid separation. Chemical alteration techniques were

previously mentioned. Size reduction of the waste occurs through the application of three mechanical forces: tension forces, compression forces, and shear forces (36). Some of the equipment designs employed in size reduction, sizing, and fluid-solid separation are crushers, cage disintegrators, shears, shredders, cutters and clippers, rasp mills, drum pulverizers, disk mills, wet pulpers, and hammermills. Of the four types of crushers used in mining--jaw, roll, impact, and gyrating--the impact crusher is found to be better adapted to municipal solid waste (36). Cage disintegrators apply a compression force to the input material, but as for most crushers, cage disintegrators are restricted to brittle materials. Shears have direct applicability to solid waste by reducing the input to one-foot cubes. Shredders, cutters, and clippers apply both tension and shear forces to the input waste. The cutting type is vulnerable to damage by unsorted solid waste. The pierce-and-tear type is best suited for fibrous and ductile wastes.

Rasp mills apply all three mechanical forces to the solid waste through the movement of large arms which move the waste over rasping pins to obtain a two-inch product; the irreducible input items are rejected. Rasp mills are primarily used for composting. Drum

pulverizers may have contrarotating beaters or baffles that force the solid waste through holes inside of the drum. The effect is similar to the rasp mill. A disk mill is a precise machine consisting of a contrarotating disk especially designed for pulpable materials whose material input size is limited to two-inch diameters. These machines literally tear the solid waste "to bits" (36). Wet pulpers are especially adaptable to fibrous wastes where unpulpables such as cans and tires are rejected. Hammermills are used in the largest percentage of municipal waste size reduction systems since they can accommodate the widest range of feed with respect to material composition. All three mechanical forces are applied to the acceptable materials; materials too bulky are rejected.

Preliminary separation and sizing can be performed by vibrating screens even though most size reduction designs produce a relatively homogeneous size output (11, 36). Fluid-solid separation can be accomplished by centrifuges (11). Hand and mechanical sorting are the two general classes of sorting. For hand sorting to be successful, the material to be removed must have a readily distinguishable quality (11). Darnay et al discovered that no class of materials can repay the cost of hand sorting with respect

to the economic conditions of the United States in 1972 (37).

Gravity separation methods sort materials according to their specific gravities. Heavy media or sink/float separation employs a heavy liquid medium that separates materials whose specific gravities are greater than the liquid medium from those that are not (11, 38).

Jigging consists of shaking a basket in water that is filled with materials of different specific gravities. The materials will separate according to their specific gravities--the heaviest at the bottom. Table separation is similar to jigging; however, a table is shaken in lieu of a basket (11).

Magnetic separators remove ferrous metals from the influent waste. Hence, after the ferrous metals have been removed, a charged roll places an electrostatic charge on the nonconductors and lifts them away from the nonferrous conducting metals (11).

Flotation methods make use of a pulp for separation purposes by changing the surface chemistry. The pulp is aerated with air drawing the hydrophobic particles to the top in a froth. Nevertheless, the particle size must be between 20 mesh down to 5 microns.

The previously mentioned unit processes are the most common in solid waste systems, but other unit processes do exist.

Material identification is necessary to determine the market price of the reclaimed material. Information on pages 406 through 414 in reference 10 illustrate the methods of identification for metal alloys, plastic materials, and natural and synthetic fibers. Tables 10.156 through 10.157 on pages 406 through 410 of reference 10 yield the identification procedures for metal alloys; pages 410 through 414 contain tests for plastics and fibers.

2.8 Composting

Composting is the "biochemical degredation of the organic fraction of solid waste material, having as its end product a humus-substance that is used, primarily, for soil conditioning" (12). In addition to soil conditioning, compost can be used for clean landfill and cattle feed (11). A properly run compost system has a useful by-product that is produced naturally in the absence of obnoxious by-products (12). Seventy to eighty per cent of municipal solid waste can be composted; however, it is one of the most expensive alternatives to solid waste disposal (39, 12)

in the United States. Composting has operated quite successfully in Europe because the market for compost is stronger in Europe than in America (12).

2.9 Energy Aspects of Reclamation: Energy Recovery

"It seems apparent that the implementation of energy recovery systems in the near future is inevitable when the energy demands of this country and the world are considered" (page 440, 12). The Union Electric Company in St. Louis is not the only private corporation that views refuse fuel as an economically feasible alternative to fossil fuels (40). Furthermore, legislation may force more communities to recycle solid waste (10, 41).

Incineration reduces the volume of municipal solid waste from 80 to 85 per cent, diminishes the weight by 60 to 65 per cent, and can produce thermal energy to generate steam to power turbines (10, 26). The technology utilized consists of pipes with fins to collect heat for the water. This system, called a waterwall, needs much less air than the non-energy-collecting refractory incineration technology; therefore, less heat and fewer particles are expelled into the atmosphere (26). However, markets for the steam

must be close, even at 1975 fuel prices. Some disadvantages of incinerating solid waste are: high capital equipment costs including pollution abatement equipment, operating costs which are higher than landfills, and generation of residue that must be landfilled (12).

Solid waste can be chemically converted by pyrolysis, hydrogenation, wet oxidation, and hydrolysis. Biochemical energy recovery processes include anaerobic digestion and biological fractionation (41). Hydrogenation converts cellulose waste into a low-sulfur fuel oil by heating the material under pressure in the presence of water and carbon monoxide. At the Bureau of Mines Research Center in Pittsburgh, two barrels of low-sulfur fuel oil have been derived from one ton of dry municipal refuse at 380°C and 5,000 psi. Wet oxidation converts cellulose at 160-320°C and 1,000-2,000 psi into carbon monoxide, carbon dioxide, a mixture of organic acids and a fibrous sludge; these gases are used to drive a turbine (11). In a University of California at Los Angeles study, Golueke et al reports that wet oxidation is not a very good method (42). Hydrolysis converts cellulose waste into fermentable sugars by heating the material with acid at elevated temperatures. Ethanol, citric acid and

animal fodder can be produced from the fermented sugars (10, 11, 43). Anaerobic digestion enlists anaerobic bacteria to convert cellulose waste at an elevated temperature in an anaerobic environment to methane (natural gas), carbon dioxide, hydrogen sulfide, and solid residue (10, 11). Golueke et al reports that hydrolysis is not a desirable method either (42).

Pyrolysis of solid waste yields a low BTU gas containing hydrogen, methane, carbon monoxide and carbon dioxide; a liquid fuel including tars and gums; and a solid fuel from the char residue (10, 11, 14). The major benefits of the process are:

a) a drastic reduction in the volume of the solid waste in a way that the residue is less hazardous than incineration.

b) the production of an energy product.

In fact, Union Carbide utilizes a high temperature gas pyrolysis upon municipal solid waste to produce a gas so similar to natural gas that the only modification necessary for burner conversion is a larger nozzle due to its lower BTU content (44). Goodyear Tire and Rubber Corporation and the Oil Shale Corporation are employing the pyrolysis technique with scrap tires at

a test facility. As a result, Goodyear has proposed a plant that will produce 10 million pounds of steel, 14 million gallons of oil, 63 million pounds of carbon black (a principal ingredient in the production of tires), and enough energy to run itself from 11 million tires (45).

Solid dry fuel called refuse derived fuel (RDF) appears to be an "...immediate potential for full-scale application...of an energy recovery system" (12). The Union Electric Company of St. Louis obtains 10 per cent of the energy requirements for a generating plant from RDF (46). A planned regional RDF system for St. Louis is expected to be competitive with landfilling (47). A 7,000-10,000 tpd (tons per day) regional RDF system in Connecticut and a 7,400 tpd RDF system for the Tennessee Valley Authority (TVA) are planned (48, 47); a 200 tpd RDF plant is operating today at Ames, Iowa (47). RDF recovery process appears to be the preferred technology in northern New Jersey (49). In fact, in northern New Jersey, 11 per cent of the electric power could be derived from RDF, saving 23 per cent of that region's fuel oil consumption (50).

An RDF recovery system usually includes:

1) receiving and primary shredding operations

2) separation of light and heavy organic materials
3) secondary shredding of the organic portion for fuel
4) magnetic separation of ferrous metals
5) further classification.

Such a recovery system in Illinois produces:

Material	Per Cent by Weight
Boiler Fuels	60
Ferrous Metals	7
Aluminum	5
Paper (newsprint and corrugated)	8
Moisture (loss)	10
Residue	15

The boiler fuel, a low sulfur content RDF, has an energy value of between 40 and 50 per cent of that of coal.

There are indeed some disadvantages of utilizing RDF systems. Converting conventional boilers to RDF burning boilers can be very costly (49). Even though the RDF has a significantly lower sulfur content than coal, the particulate emissions slightly rise and the

gaseous emissions are only negligibly reduced when burning the RDF-coal mixture (46). Moreover, a possible nuclear energy preference in the future could reduce the market for refuse fuel.

One advantage of RDF use is that the conversion for municipal solid waste to RDF could approach 80 per cent. Also, the capital cost of RDF systems is lower than incinerators and in northern New Jersey, they seem to be competitive with landfilling (26). Additionally, RDF displaces fossil fuels for alternate uses such as the production of plastics.

2.10 Energy Aspects of Reclamation: Energy Consumption in Production of Virgin and Secondary Materials

"In general, the production of a given quantity of metals, in particular, and to a lesser extent, of glass and paper, requires more energy for the production of virgin materials than for the same grade of material recovered from wastes" (10). This energy gap between production requirements of virgin materials and secondary materials seems to be widening. For example, since 1950, the use of mechanical power in United States mines has increased by several multiples

(21) due to increased mechanization and productivity and the extraction of poorer grades of ore (10). Energy requirement comparisons between the production of some important metals from virgin materials and secondary materials is shown in Table 2.3 The numbers in the table represent the equivalent coal energy in kilowatt hours (kwh) per ton of the metal produced from either virgin or secondary materials. These numbers reflect assumed transmission and electrical generation efficiency of the conversion from coal to electricity for the most efficient generating station at 40 per cent and 29.8 per cent for today's average. A detailed breakdown of energy requirements for each process in the production of a ton of the five metals above can be found in reference 51.

"The energy cost of retrieving scrap glass from waste is far higher than the energy cost of mining raw materials, and that therefore, from an energy standpoint, it makes little sense to recycle, in the sense of remelt, either returnable or throwaway bottles. The only energy savings are realized in reuse of the container: retrieving it as raw material is wasteful of energy. Substantially more (60 times) energy is used to obtain waste glass as a replacement for crushed stone in making road-pavement materials"(52).

Metal	Present Source	Equivalent coal energy in kwh/ton of metal	Reprocessing
Magnesium	Sea water	90,821(103,739)	1,395(1,875)
Aluminum	50% bauxite	51,379(63,892)	Al scrap 1,300-2,000
Iron	High grade hematite	4,270(4,289)	Iron and steel scrap 1,240(1,666)
	Magnetic taconite	4,656(4,727)	
Copper	1% sulfide ore	13,532(15,193)	98% Cu scrap 635(853)
			Impure Cu scrap 1,555(1,727)
Titanium	High grade rutile ore	126,115(154,779)	
	Ilmenite rocks	149,440(186,090)	Ti scrap 39,000 (52,416)
	Ilmentite beach sands	153,055(190,948)	
	Ferruginous rocks	152,813(190,622)	

Numbers not in parentheses represent an electrical generation and transmission efficiency of 40%; numbers in parentheses represent an efficiency of 29.8%.

TABLE 2.3. Summary of the Energy Requirements for the Production and Recycle of Metals.

Source: J. C. Bravard, H. B. Blora, II and Charles Portal, "Energy expenditure associated with the production and recycle of metals," Oak Ridge National Laboratory-National Science Foundation RANN program, report ORNL-NSF-LP-24, 11/72.

These observations concerning the reuse of glass for paving materials were taken from the Black-Clawson municipal solid waste plant. Comparatively speaking, the Black-Clawson plant is a high-energy-use system (10). Throwaway glass bottles and aluminum cans consume three times the energy necessary for returnables assuming that 30 per cent of them are filled eight times. A bimetal can only consumes about two times as much energy (53).

Less energy demanding processes may be more attractive, especially if the separation processes require only a small incremental energy use. Nevertheless, general statements cannot be made; each situation must be analyzed as a specific case.

2.11 Economics of Resource Recovery

The cost versus capacity relationship of owning and operating a comprehensive resource recovery operation such as the Black-Clawson possesses the property of economies of scale (pages 11-13 of 11; page 139 of 54). Without exception, the unit processes reviewed in this paper also exhibited this economic property with respect to cost versus capacity. However, this author discovered that "RELIABLE and MEANINGFUL cost and performance data on unit processes for the

recovery and utilization of mixed municipal solid waste are not generally available from existing operations owing to: the limited application of many equipment designs and processing techniques, the failure to collect meaningful data on the applications that do exist, and the proprietary nature of much of the available cost and performance data" (page 945 of 36). Furthermore, additional "...data that concern the relationships between particle size, capacity, and efficiency of separation for processing solid waste and various component and byproduct streams..." are desperately needed (page 33 of 36). Similar laments have been voiced by other authors (page 328 of 12; page 232 of 55). Hence, the cost relationships to be discussed are from the scant few sources that this author discovered after arduous researching.

2.12 Size Reduction

Performance and cost data on 27 solid waste size reduction machines manufactured by 21 companies can be found in Figure 12 on page 15 of reference 36. The purchase prices of these machines exhibit economies of scale with respect to their capacities (pages 15 and 19 of 36). (See Figures 2.6 and 2.7.)

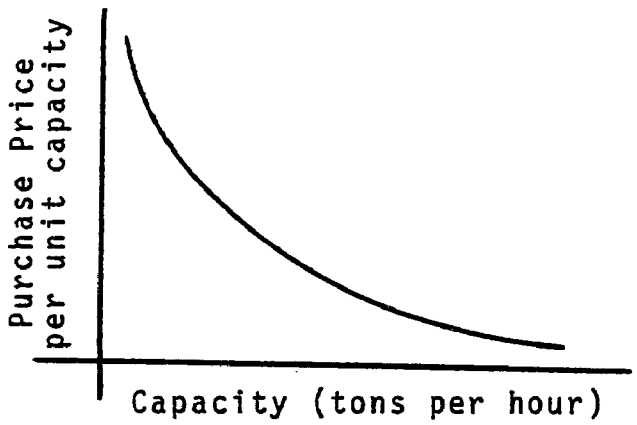


FIGURE 2.6. Average Price Curve for Size Reduction Machinery.

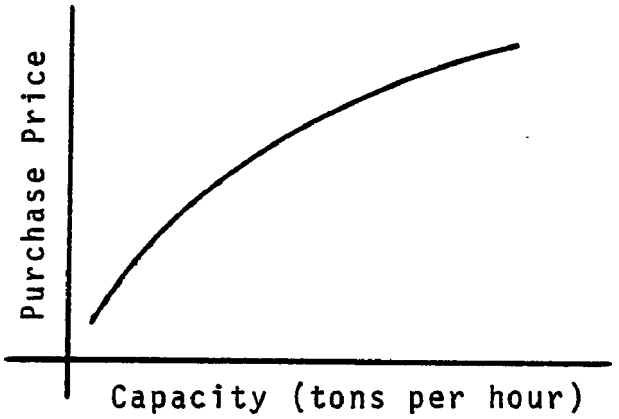


FIGURE 2.7. Total Price Curve for Size Reduction Machinery.

Source: (Figures 2.6 and 2.7) Drobny, N. L., Hull, H. E., and Testin, R. F., "Recovery and Utilization of Municipal Solid Waste," Report SW-100, Battelle Memorial Institute, Columbus Laboratories, under Contract No. PH 86-67-265, USEPA.

The machines consist of various combinations and configurations of crushers, cage disintegrators, shears, shredders, cutters and clippers, rasp mills, drum pulverizers, disk mills, wet pulpers, and hammermills (36). "Although the data are...scattered, the trend indicates that the capital cost of size reduction equipment in general does not increase linearly with capacity, but follows the classical 0.6 exponent relationship" (page 18 of 36). The "six-tenths rule for capital costs is:

$$\frac{\text{Purchase price}_2}{\text{Purchase price}_1} = \left(\frac{\text{Capacity}_2}{\text{Capacity}_1} \right)^{0.6}$$

where Purchase price$_i$ is the cost to purchase a machine with a processing capacity of Capacity$_i$, i=1,2.

The shape of this relationship is illustrated in Figure 2.7. The operation costs, including maintenance, power, and labor costs also illustrate economies of scale. (See Figure 2.8.)

The total product cost versus product size curve for size reduction machinery from twelve manufacturers exhibits the property of increasing costs as the product size diminishes. The data corroborating

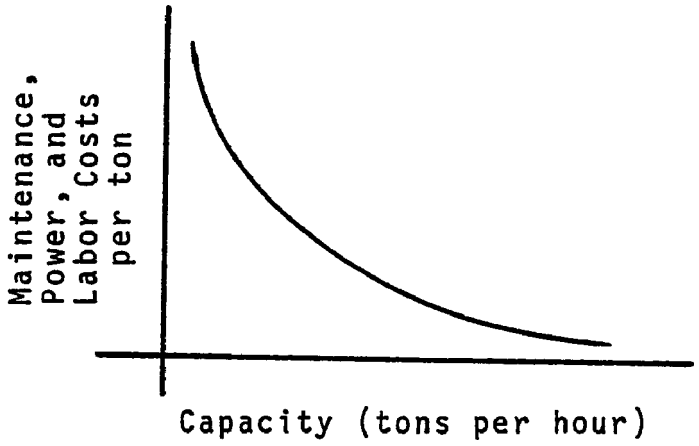
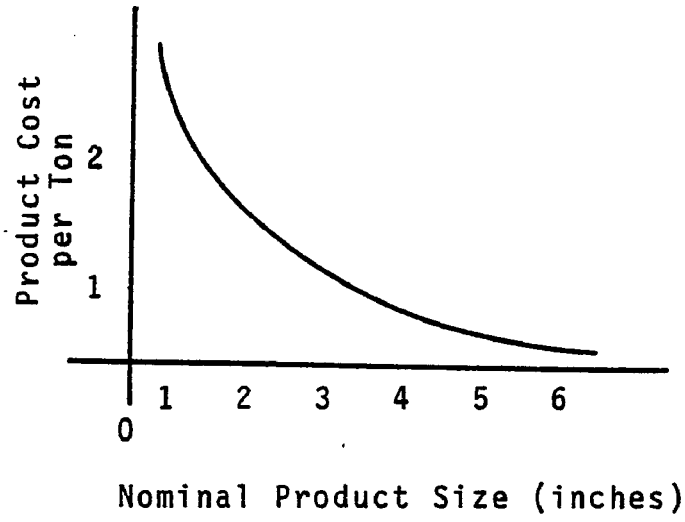


FIGURE 2.8. Average Cost Curve for Operations.

the above property was obtained from either the manufacturers or demonstration projects (pages 12-17 of 36). "Here again, the increasing cost trend with smaller product size roughly follows the 0.6 exponent relationship" (page 17 of 36). (See Figure 2.9.) However, the Madison, Wisconsin demonstration project data indicates an exponential relationship of -0.45 for the unit process. Unadjusted cost data versus product size (Table 2.4) for the Gondard hammermill in Holland exemplifies diseconomies of scale primarily because the hammermill's "...power requirement increases sharply in inverse proportion to the size of the final product" (page 150 of 10). The diseconomies of scale phenomenon is described almost exactly by a -0.19 exponential relationship in Table 2.4. This exponential factor reflects the effects of product size on running the entire mill at Madison, Wisconsin.

2.13 Separation

Separation equipment such as magnetic separators, vibrating screens, flotation and wet separators, and optical separators demonstrate economies of scale with respect to purchase price versus capacity (pages 26,



where

$$\frac{\text{Product Cost}_2/\text{ton}}{\text{Product Cost}_1/\text{ton}} = \frac{\text{Product Cost}_2}{\text{Product Cost}_1}$$

$$= \left(\frac{\text{Product Size}_2}{\text{Product Size}_1} \right)^{-0.6}$$

FIGURE 2.9. Six-Tenths Rule of Increasing Costs.

Cost Item	Annual Cost	Grate Size 3-1/2 Inch Annual Tonnage 10,750	5 Inch 11,500	6-1/4 inch 12,050
Labor	$39,800	$3.70	$3.46	$3.30
Amortization	32,200	2.99	2.80	2.67
Power	variable	.34	.30	.30
Lighting	2,300	.21	.20	.19
Water	200	.02	.02	.02
Gas-heat	1,200	.11	.10	.10
Hammer Wear	1,600-1,710	.16	.15	.14
Mill Maintenance	850-950	.08	.08	.08
Small Equipment	800	.07	.07	.07
General Supplies	1,100	.10	.10	.09
Front End Loader Operation	500	.05	.04	.04
Transportation to Landfill*	3,250	.30	.28	.27
Other	1,700	.16	.15	.14
TOTAL		$8.29 ton	$7.75 ton	$7.41 ton

*Based on round trip of less than 1/2 mile.

TABLE 2.4. Unadjusted Cost Data for Gondard Mill (June 1968 through May 1969).

Source: Reinhardt, John J. and Robert K. Ham, "Final report on a milling project at Madison, Wisconsin," U.S. Environmental Protection Agency, 1973.

34, 41, 50 of 36; page 327 of 12; page 81 of 57). (See Figures 2.10 and 2.11).

The literature reviewed indicates that product cost versus the efficiency of separation exhibits diseconomies of scale. First, the efficiency of many of the separation processes depends upon the degree of size reduction as a preliminary step to separation. For example, "...efficient magnetic separation is affected by the degree of size reduction provided and the extent to which ferrous materials are physically liberated from other types of materials" (page 26 of 36). Efficient separation by flotation requires finely divided material between 20 and 200 mesh (page 36 of 36). Efficient gravity separation by dense media, by size stoners, and by mineral jigs demands material sizes from 10 mesh to 3 inches, from 1/16 to 2 inches, and from fines to 1-1/2 inches, respectively (36). A second cause for increasing costs associated with the efficiency of separation equipment is the unit processes of separation themselves. For example, a nonferrous metals separation technology based upon eddy current techniques is such a process. "The efficiency of operation is basically related to force magnitude and size effects. Efficiency is higher for

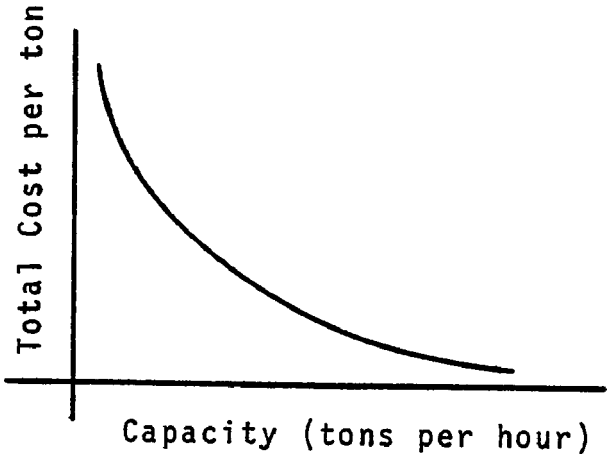


FIGURE 2.10. Average Cost Curve for Separation Equipment.

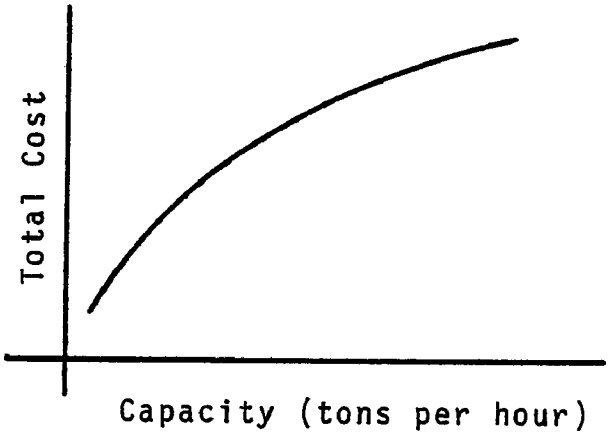


FIGURE 2.11. Total Cost Curve for Separation Equipment.

those techniques exhibiting greater forces and less restrictive size limitations" (page 80 of 54).

An eddy current system called PULSORT (See Figure 2.12) removes nonferrous metals from municipal solid waste at a greater efficiency when operated at a lower mode setting. (See Figure 2.13.) However, the power requirements and hence the costs rise at an increasing rate with respect to mode number or efficiency. (See Figure 2.14.)

"It is seen that recovery falls with increasing mode number, especially for the smaller aluminum." (See Figure 2.15.) "Reference to Figure 2.16 shows that recovery of mixed nonferrous metals also falls with increasing mode number. This relationship has been found to be true for all cases studied" (page 84 of 54).

The efficiency of an eddy current separator also depends upon the feed rate of a nonmetallic separator such as a trommel and the belt speed through the electrical field of the eddy current separator. Assuming that the output of trommel feeds directly into eddy current separator, the efficiency of removing aluminum from the municipal solid waste decreases as the load to trommel and the eddy current

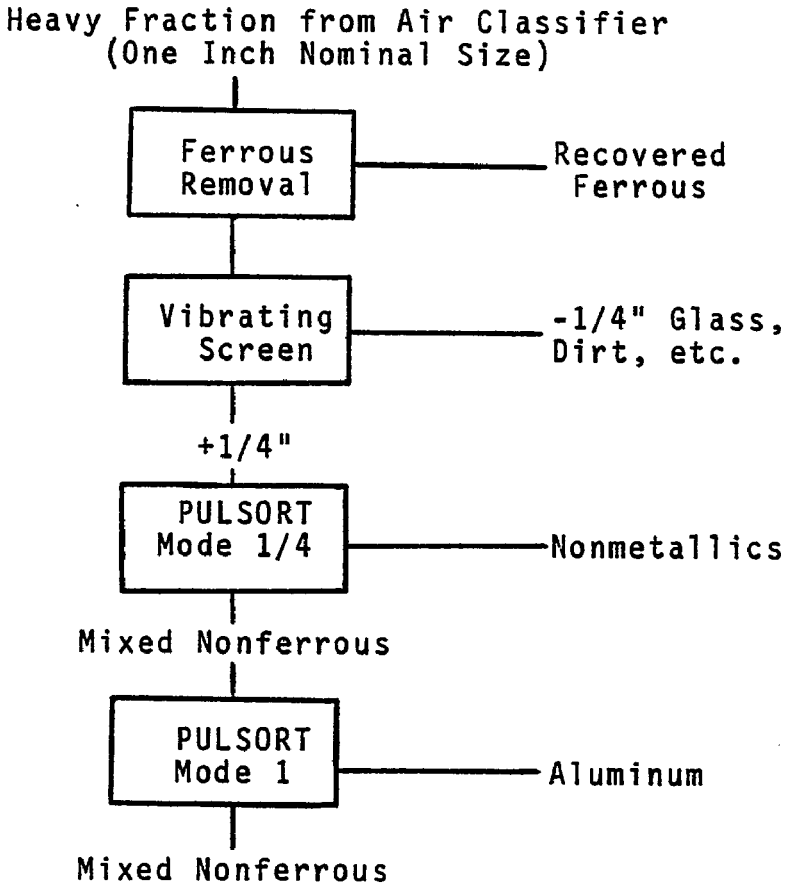


FIGURE 2.12. Flow Diagram for System Designed to Recover Separate Aluminum and Mixed Nonferrous Fractions.

Source: Figures 2.12 to 2.16: Aleshin, Eugene, ed., Fourth Mineral Waste Utilization Symposium, U.S. Bureau of Mines, (1974), pp. 77-84.

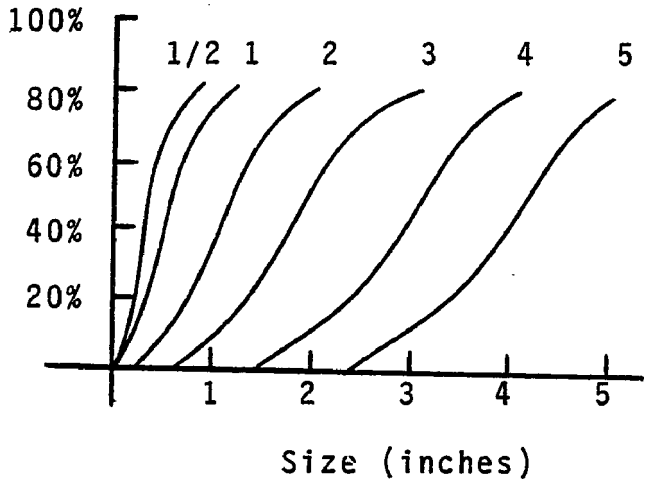


FIGURE 2.13. Aluminum Recovery Dependence upon Size and Operating Mode Number. Source: See Figure 2.12.

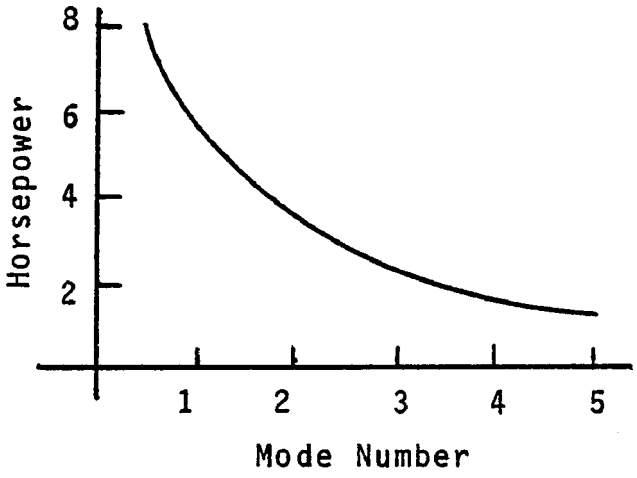


FIGURE 2.14. Typical Input Power Requirements for Various PULSORT Operating Modes. Feed throughput is 1 ton/hour of heavy fraction (30 lb/ft^3) of shredded and air classified municipal refuse. Source: See Figure 2.12.

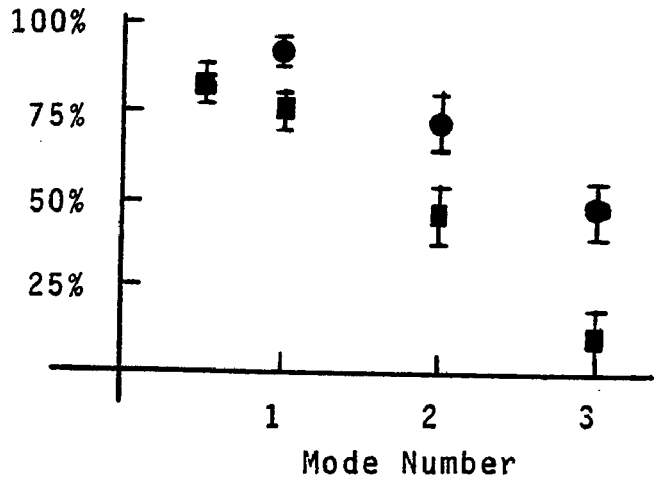


FIGURE 2.15. Aluminum Recovery as a Function of Mode Number. Nominal Size of Shredded Aluminum Can Pieces is 1-1/2" for Data Represented by Circles and 1/2" for Data Represented by Squares. Source: See Figure 2.12.

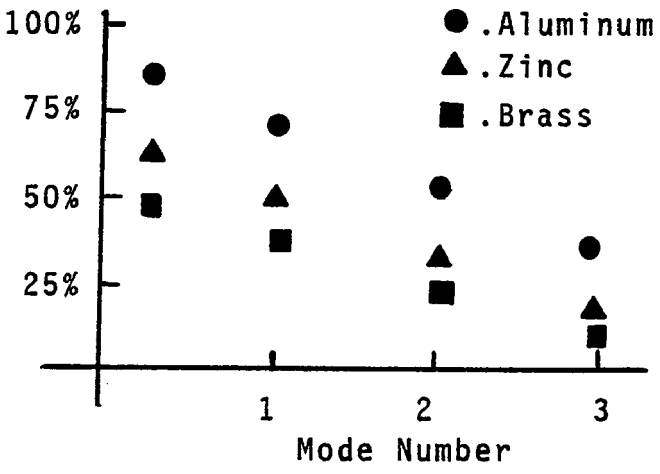


FIGURE 2.16. Mixed Nonferrous Metals Recovery for Various Values of Mode Numbers. Source: See Figure 2.12.

separator increases.

In fact, Blayden (56) reports that when feed rates to the trommel were increased from 0.1 to 0.2 tons-per-hour, the recovery efficiency decreased from 99 to 80 per cent of the weight of aluminum in the waste stream. (See Figure 2.17) Since the feed rate is doubled, the cost per ton processed for this unit process must decrease by 50 per cent. (See Figure 2.18.) Notice that a 50 per cent decrease in speed only yields a 25 per cent increase in recovered aluminum which, depending upon the price of aluminum, may prove to be cost effective.

2.14 Composting

In addition to separation equipment, the total cost of composting systems possesses economies of scale with respect to total costs and plant capacity (page 64 of 36).

2.15 Heat Recovery Systems

The conventional refractories with or without heat recovery and the water wall with waste heat recovery follow the "six-tenths" rule; consequently, they adhere to the economies of scale property (pages 162-

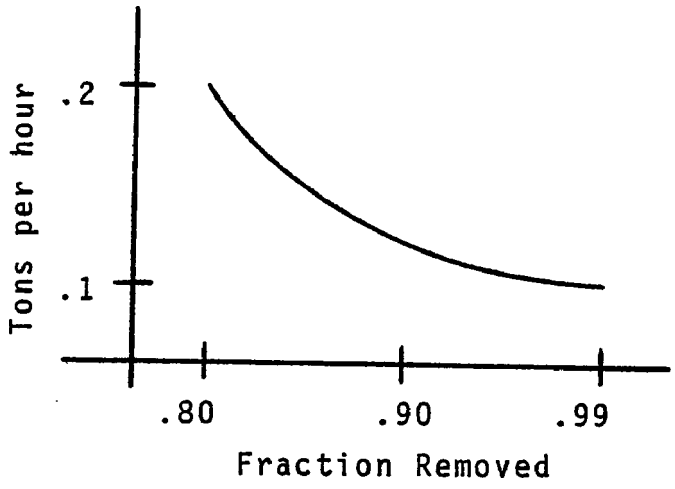


FIGURE 2.17. Trommel Feed Rates Versus Efficiency.

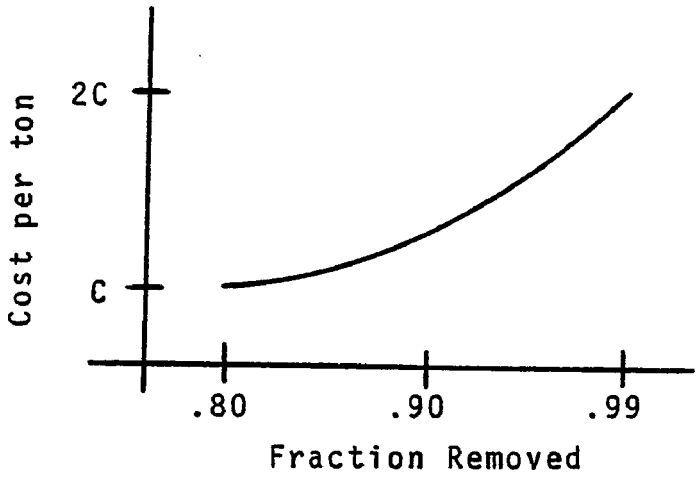
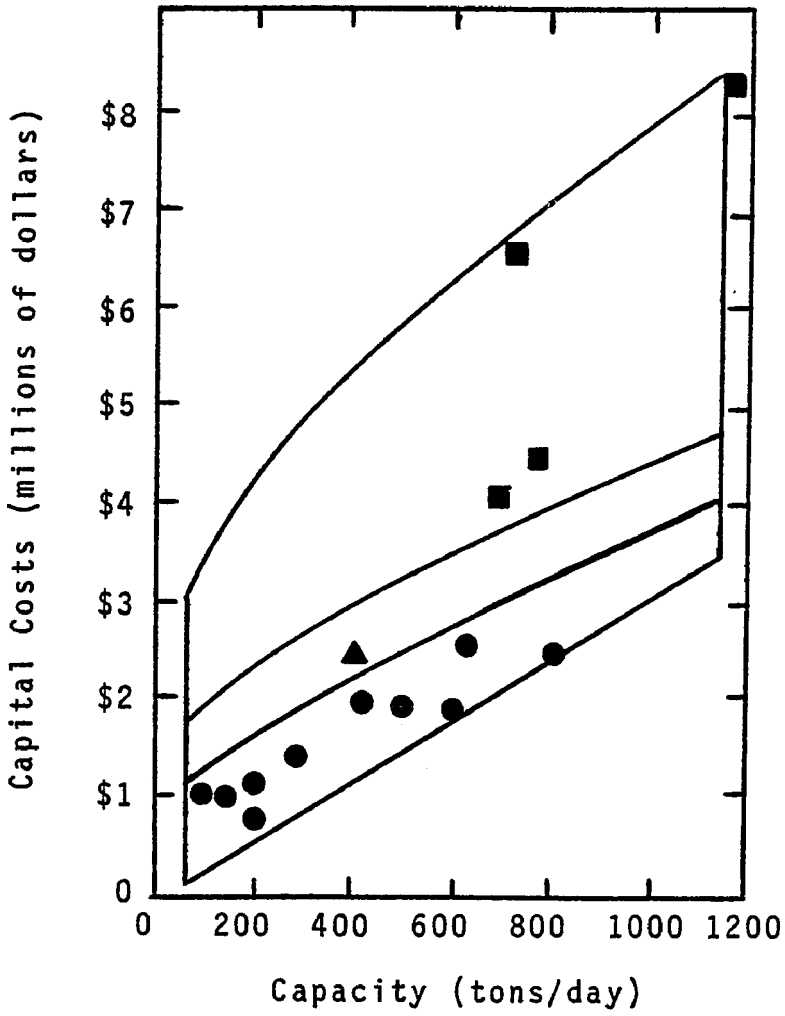


FIGURE 2.18. Trommel Average Costs.



■ Conventional Refractory with Waste-Heat Recovery

● Conventional Refractory without Waste-Heat Recovery

▲ Water Wall with Waste-Heat Recovery

FIGURE 2.19. Municipal Incinerators: Capital Costs.

Source: Glyson, Eugene A., et al., The Problem of Solid Waste Disposal. College of Engineering, University of Michigan, 1972.

163 of 39; page 83 of 18, 36, 50, 55, 56, 71). (See Figure 2.19.) "The most completely developed device for pyrolyzing municipal waste is the Lantz Converter" (page 78 of 36). It also exhibits the "six-tenths rule for the capital costs. The net disposal cost for this pyrolysis system has been found to have the economies of scale property (56).

As previously stated, an RDF system includes shredding and separation operations. Hence, the efficiency and quality of the RDF is a direct relationship of the fraction and quality of the organic materials removed from the waste stream by the interrelation of these unit operations. Both separation and shredding operations were shown to exhibit economies of scale with regard to input capacity and diseconomies of scale with regard to efficiency.

2.16 Resource Recovery Projects in the United States

Various resource recovery plants across the United States are being tested for technical and economical feasibility. Even though these projects differ according to location, technology, and stage of development, their success or failure aids in defining the state of the art for feasible resource recovery

systems. Brief statements will be made concerning the more prominent recovery systems.

Franklin, Ohio Recovery Project: A 150 tpd (tons per day) test facility has recovered low-grade paper fiber, ferrous metals, glass, and aluminum, while operating at a 35 tpd rate. Of the incoming solid waste, 50 per cent of the paper, 95 per cent of the metals, and 60 per cent of the glass were recovered. The low-grade paper has been sold to a roofing manufacturer, while the rejected paper was incinerated with sewage. The EPA (Environmental Protection Agency) estimates that the gross cost of a similarly sized and designed facility operating at 150 tpd without aluminum and glass recovery is $24.60 per ton while generating $12.20 in revenues from the sale of recoverables. Therefore, a net cost in 1975 dollars is $12.40 per input ton of solid waste (40).

Houston, Texas Recovery Plant: Browning-Ferris and ERDA (Energy Research and Development Administration) are currently operating a 500 tpd plant where metals and a refuse derived fuel with the trademark of REF-FUEL are converted. The plant is supposed to demonstrate that REF-FUEL is a low cost alternative to fossil fuels (20, 16).

Washington, D.C. Recovery Project: A proposed 650 tpd plant could not get funding (40).

Lowell, Massachusetts Incinerator Residue Separation Project: Raytheon Service Company and the United States Bureau of Mines have proposed a 250 tpd plant that is expected to recover 75-85 per cent (39) of the incinerator residue and possibly recover up to 95 per cent at a net cost of $0.55 per ton in 1975 dollars (40). The implementation never materialized because Lowell withdrew support in July 1975 (40).

St. Louis, Missouri/Union Electric Company Project: A government funded 650 tpd project terminated in June 1975 was deemed a success by Union Electric Company. A proposed 8,000 tpd recovery system at a cost of $70 million would recover ferrous metals, refuse derived fuel to burn in Union's electric generating system, and nonferrous metals (40, 58). Union's RDF plant has been eulogized as the most cost effective plant of its kind (59). After investing $14 million in the project, political problems have thwarted it:

a) Local zoning ordinances have killed the building of a necessary transfer station.

b) A recently approved Missouri referendum

precludes an electric company from including carrying costs on funds invested in new facilities not fully operational.

The Missouri/Illinois Bi-State Department Agency may resurrect the project through monetary support and condemnation of transfer station sites. Since the landfills could be exhausted by 1980 and the incinerators are rather decrepit, the Union project may be the only alternative (40).

Delaware Material and Energy Recovery Project: The state of Delaware and the EPA are planning to design and build a resource recovery facility that will process 485 tpd of municipal solid waste, 15 tpd of industrial solid waste, 230 tpd of digested sewage sludge (39, 40). The plant is expected to recover ferrous and nonferrous metals, glass, gas from pyrolysis, and RDF at a net cost of $13.00 per ton, which includes revenues and gross costs of the recovery and the disposal of residue. The plant is anticipated to be completed three years after the full service contract has been signed by a contractor. Since the contract is still under negotiation, the net cost of resource recovery may be highly competitive with current landfill costs (40).

Saugus, Massachusetts Energy Recovery Project: Wheelabrator-Frye, Incorporated and DeMatteo Construction Company have formed a partnership called RESCO (Refuse Energy Systems Company) to build a 1,200 tpd waterwall system which was financed with tax-exempt pollution control bonds. General Electric is under a long-term contract with RESCO to buy the steam produced by the waterwall system; 16 municipalities in the Boston area have long-term disposal contracts with RESCO at $13 to $14 per ton in 1976, with increases adjusted by an economic indicator (40). The system focuses upon energy recovery; however, material recovery is planned. The waterwall incinerator possesses pollution abatement equipment which removes over 99 per cent of the particles in the exhaust smoke. The President's Council on Environmental Quality, the EPA and the Environmental Industrial Council awarded Wheelabrator-Frye with the "Premier Award for Excellence" for its role in this plant (40).

Baltimore, Maryland Pyrolysis Project: In mid-1975, Monsanto Enviro-Chem systems, Incorporated, designed, built, and initially operated the 1,000 tpd plant. Monsanto has abandoned the project and recommended that Baltimore follow suit. Nevertheless, the

city has continued to operate the plant and is currently violating air pollution laws. Maryland is currently applying pressure to Baltimore to add the needed $6 million pollution abatement equipment (40).

Ames, Iowa Resource Recovery Project: This currently operating 200 tpd facility has 25-year disposal contracts with 14 communities. Eighty-four per cent of the solid waste is converted to RDF, 9 per cent is recovered materials such as ferrous and aluminum, and 7 per cent is landfilled. The net cost of disposal is $14.30 per ton (40).

San Diego, California Pyrolysis Project: A 200 tpd plant is jointly sponsored by the EPA and Occidental Research Corporation, who is the plant designer. The plant is being designed to recover materials and oil with a BTU rating that is 75 per cent of number 6 fuel oil. This recovered oil will be sold to San Diego Gas and Electric Company. Although the economics of this plant are not competitive with landfill costs (in 1975 dollars), a 1-1/2 year observation period should determine whether there exists potential economics competitive enough to favor recovery (40).

Hempstead, New York Resource Recovery Project: Construction of the 1,500 to 1,800 tpd resource recovery plant commenced in June 1976. According to a long-term agreement, the Long Island Power and Light Company has assented to purchase steam generated by the plant to drive electricity generating turbines. Similar agreements with material buyers have been consummated for the purchase of recovered ferrous and nonferrous metals and color sorted glass (40).

Baltimore County, Maryland Resource Recovery Project: Teledyne National is the prime contractor for the 1,500 tpd facility. The facility is currently operating at 625 tpd and recovering ferrous metals, nonferrous metals, and RDF (27, 40).

New Orleans, Louisiana Materials Recovery Project: The 650 tpd facility designed by the National Center for Resource Recovery reclaims ferrous metals, aluminum, and mixed glass, and processes organic residue for open landfilling. For New Orleans, resource recovery is the least cost disposal method (40).

Milwaukee, Wisconsin Resource Recovery Project: "With the 1977 start-up of its Americology Plant, Milwaukee becomes the largest city in the United

States to begin a comprehensive resource recovery program...[and]...to process all of its municipal solid waste for resource recovery" (60). The contract to process all of Milwaukee's waste terminates in 15 years. "Contracts for the sale of newspapers, ferrous metals, glass, aluminum and air classified refuse have been negotiated and shipments should begin in January 1977" (61). Ninety per cent of the incoming solid waste is being recovered (60). The city is currently paying only $10.00 per ton for disposal, which in the future will be adjusted according to the consumer price index (40).

Summary:

Many of the projects have encountered technical, marketing, institutional, and political problems which have negatively affected the expected economies of scale. The successful projects have been in areas where the alternatives to resource recovery possess higher costs, usually where land is scarce or non-existent, for instance, in New Orleans. Political co-operation appears to be the last major obstacle for successful resource recovery systems, for example, Milwaukee and St. Louis. Three reasons why the

momentum for the development of comprehensive resource recovery systems such as Milwaukee's may be gaining strength are:

1) The legal environment is restricting and contracting the capacity of non-recovery disposal systems throughout the United States (14, 62, 63).
2) The disposal cost of nonrecovery disposal systems is rising, for instance, the cost of land for landfilling.
3) The prices of the recovered materials are rising to greater offset the recovery costs (61).

Part 2 - Sanitary Landfilling

2.17 Introduction

"The process of sanitary landfilling is an engineering construction project for the burial of solid waste at the optimum compaction in sealed cells of earth, designed to produce a minimum impact on the environment" (64). The Committee on Sanitary Landfill Practice of the Sanitary Engineering Division of ASCE (American Society of Civil Engineers) defines landfilling as: "...a method of disposal of refuse on

land without creating nuisances or hazards to public health and safety, utilizing the principles of engineering to confine the refuse to the smallest practical volume, and to cover it with a layer of earth at the conclusion of each day's operation, or at such more frequent intervals as may be necessary" (65).

Site selection requires the adherence to planning goals and zoning laws of the municipality; an analysis of the climate, topography, natural features, and cost considerations; and the acceptance of the public. The master plan for a proposed landfill site should be consulted for future zoning of adjacent land. For example, a landfill site should not be constructed near a present or future airport since accidents can be caused by the birds attracted to the airport (10, 26). An insight to some landfill characteristics such as odor generation, water table pollution, blowing litter, erosion, and frost-penetration depth could be obtained by studying the climate. The topography is also closely related to the water table pollution, and other characteristics such as landfill capacity, drainage, and ultimate site use. The pertinent cost considerations are the developmental costs and the cost of land per ton of waste

disposed. The suitability and volume of soil cover must satisfy government regulations to avoid costly importation of cover soil. A recommended soil cover is called sandy loam, which is 45 to 80 per cent sand and the remainder clay (66). A table for suitability of soil types can be found in reference 67.

2.18 Site Planning and Design

A sanitary landfill operation incorporates the interaction of many facilities. Therefore, comprehensive site data should be collected corroborating the planning of:

1) detailed drainage facilities, all-weather access roads, fencing, buildings, sanitation, water, lighting, telephone, scales, equipment-maintenance yard, and all other site improvements;
2) a series of cut-and-fill plans presenting the landfill at various stages of completion;
3) a detailed description of daily operating procedures;
4) a phased analysis of equipment and manpower requirements for the life of the site;

5) an estimate of projected capital and operating costs including final finished landfill landscaping improvements; and

6) an ultimate land-use plan for the site (10).

This planning procedure is constrained by technological, legal, and economic restraints.

The equipment and manpower requirements depend upon the solid-waste flow. For example, Table 2.5 illustrates the landfill site personnel used in Los Angeles County. Equipment requirements not only depend upon the daily volume, but also the characteristics of the landfill. The most common landfill site equipment includes crawler or rubber tired tractors with ancillary attachments such as dozer blades and buckets. At large landfill site operations, more specialized vehicles and equipment are employed: draglines, scrapers, and steel-wheel compactors (10). For data concerning landfill equipment needs as a function of solid waste handled, consult references 67 and 173.

2.19 Environmental Factors

Leachate, gas generation, odors, fires, vectors, noise, aesthetics, dust, and air pollution compose the

Solid-waste flow, tons/day	200	500	1000	4000
Equipment operator	1	2	4	12
Laborer	0	0	1	2
Weighmaster	1	1	1	3
Foreman	0	0	1	1

TABLE 2.5. Landfill Site Personnel Used in Los Angeles County.

Source: Wilson, David G., Handbook of Solid Waste Management, Van Nostrand and Reinhold Company, (1977), p. 229.

critical environmental factors in landfill design.

Leachate is a "liquid emanation from a land disposal cell that contains dissolved, suspended, and/or microbial contaminants from the solid waste" (page 729 of 10). The primary cause of leachate is the inflow of water, although organic decomposition and the compaction of the solid waste also contributes (39, 10). Excessive saturation of the landfill by rainfall or a high water table can cause water pollution. Table 2.6 presents the range of pollutant concentrations in leachate from typical domestic refuse (page 234 of 10). A comparison of leachate and sewage is shown by Table 2.7. A treatment plant constructed at the landfill site can treat the diverted leachate (39, 26). In any event, the following measures can reduce the water inflow to the landfill:

1) Locate the landfill so as to avoid direct groundwater or surface-water contact.
2) Use proper surface-cover grading to increase rainwater runoff.
3) Plant final fill site-cover with vegetation which will avoid erosion while enhancing runoff and evapo-transportation.

Ion	Concentration-mg/liter Range
Iron	200 - 1700
Zinc	1 - 135
Phosphate	5 - 130
Sulfate	25 - 500
Chloride	100 - 2400
Sodium	100 - 3800
Nitrogen	20 - 500
Hardness (as $CaCO_3$)	200 - 5250
Chemical oxygen demand	100 - 51,000
Total residue	1000 - 45,000
Nickel	0.01 - 0.8
Copper	0.10 - 9.0
pH	4.00 - 8.5

TABLE 2.6. Range of Pollutant Concentrations in Leachate from Typical Domestic Refuse.

Source: Wilson, D. G., Handbook of Solid Waste Management. Van Nostrand and Reinhold Company, (1977), p. 234.

Component	Leachate	Sewage	Potable Water
Iron	1,600[a]	-	0.3
Hardness[b]	2,500	-	150
Suspended Solids	750	300	1
Total Solids	10,000 to 28,000	500	-
Biochemical Oxygen Demand (BOD)	25,000	200	-

[a]All units are milligrams per liter (mg/l).

[b]As measured in calcium carbonate.

TABLE 2.7. Water Quality of Leachate.

Source: F. Flower, "The Refuse Landfill," Executive Housekeeper, Vol. 22, No. 2, February 1974, p. 24.

4) Use a nonpermeable cover material with maximum lateral-flow characteristics.

5) Delay leachate generation by placing solid waste and soil cover with low moisture contents and high compaction densities.

6) Install drainage facilities to divert off-site surface and subsurface waters. It is possible to determine the local rainfall and solid-waste parameters in order to calculate the estimated field capacity which will cause leachate generation.[1]

Many of these measures are regulatory rules in New Jersey (66).

The decomposition process of solid waste generates a series of gases such as carbon dioxide, hydrogen sulfide, methane, and nitrogen. The major gaseous products for landfills in Los Angeles comprise 55 to 60 per cent methane and 40 to 45 per cent carbon dioxide (pages 15-25 of 39). This methane

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[1]Taken from Government Publication PB 233178, April, 1974.

buildup can cause fires. Another study in Southern California illustrates the different proportions of gases generated with different methods of operation. (See Table 8.9 of 10). Better absorption of odor causing gases in landfills has been shown in more recent investigations (page 236 of 10).

Other environmental problems such as noise can be alleviated by careful routing of the vehicles. Aesthetics of the site can be enhanced with a handsome facade and natural fences of trees and shrubbery to absorb noise and block the landfill site from public view. Air pollution can be avoided by spreading the solid waste with proper soils.

2.20 Public Acceptance

Public opposition of a sanitary landfill operation could come from one or more groups:

1) nearby property owners
2) citizens against the additional costs
3) citizens who desire the status quo
4) citizens in favor of resource reclamation
5) neighbors concerned about environmental pollution
6) politically opposed citizens (pages 239-240 of 10).

Some points to be stressed in a public relations program to help thwart opposition are:

1) accurate information about successful existing landfills
2) comprehensive plans for vehicular regulation
3) utilization of public-opinion survey to pinpoint opposition
4) citizen involvement in the development
5) marketing the site through well trained personnel and media (page 240 of 10).

The major obstacle is usually the public hearing where a poorly run presentation could cause termination of the project. A good presentation should explain both positive and negative aspects of a proposed landfill site by:

1) defining a landfill
2) presenting all reasonable alternatives
3) recommending a prototype landfill
4) proposing a proforma analysis, and
5) complying with statutory regulations.

2.21 Operating Procedures

The area method and the trench method comprise

the two basic methods of landfilling. The area method is best suited for concave topology and the trench method for relatively flat topology. For the area method, cover soil is obtained from the walls of the concave area, whereas in the trench method, the cover soil is obtained from the soil excavated to create the trench. In both methods, the open working face should be minimized, the collection vehicles should not have to travel over uncovered waste, and the cover soil should be close to the working face. Furthermore, waste receiving, spreading and compaction, type of covering, and site maintenance are similar in both methods.

The waste is accepted at a receiving station where truck scales weigh the amount of incoming waste for the billing computation and the gathering of statistics. Receiving time should be kept at a minimum to avoid losing haulers to other sites.

After dumping the waste near the working face, a dozer spreads the waste on a slope of 20 to 30 degrees in layers no thicker than two feet. To reduce fires and potential harborage for rodents, the dozer passes over each two-foot layer from two to five times, preferably five times.

At the end of each day, the previously mentioned sandy loam soil cover of not less than six inches must be applied to the active surface. A minimum of one foot of cover should be placed on areas not to be used for 30 days, and two feet of cover for areas that are inactive for more than six months. These guidelines for a sanitary landfill are becoming laws. For example, in New Jersey, the active area is limited to 1/3 of an acre, daily cover is required, and two feet of final cover is compulsory for indefinitely inactive areas (68).

2.22 Special or Processed Wastes

Crushable bulky items such as automobiles, demolition debris, and tires tend to work their way to the surface and therefore should be disposed at the toe of the working face. Pipe, rolls of cable, reinforcing bars, and coils of wire should be immediately covered with soil to avoid damaging landfill equipment. Dead animals should be buried immediately.

Shredded wastes accelerate the decomposition process and increase the efficiency of the landfill. The increased landfill density causes fewer fires

because of fewer voids for methane pockets, reduces the disease vector such as rats and flies, requires less cover soil per ton of waste, and lengthens the life of the landfill (69).

High density bailing of solid waste possesses more benefits than landfilling shredded waste. The higher density bails are stacked to yield a virtually settlement free landfill mass (64). Furthermore, bailing results in reduced haul and transfer costs (10).

2.23 Monitoring

Leachate and gas monitoring can be effected by utilizing wells drilled into the landfill. For groundwater monitoring, one well should be drilled upstream and two downstream. At a properly run landfill site, leachate is treated and does not pollute the groundwater. Gas monitoring exposes the direction of gas flow and the rate of decomposition.

2.24 Final Land Use

Reclamation of the landfill should be a part of the master plan. Reclamation projects include parks, golf courses, and parking lots. Construction

of buildings is discouraged since the load bearing capacity is relatively low, the landfill is composed of corrosive soil for subsurface utilities, and the landfill generated gases could enter the buildings.

2.25 Economics of Sanitary Landfilling

For many areas, landfilling is the least cost method of disposal. In Ohio and New York, the sum of the landfill operating costs and the direct costs such as land and equipment have been relatively inexpensive as compared to the available alternatives. In addition, the landfilling method of disposal is a proven technique which is probably a safer bet than the new recovery technologies. The common measures of landfilling costs tend to be underestimated; intangible costs such as the economic impact upon rental rates and land values are omitted from the cost analysis since they cannot be easily quantified (page 212 of 12). Because of shortages of virgin materials and more stringent environmental laws, the opportunity cost of burying recoverable materials is becoming positive.

As illustrated in Figures 2.20 and 2.21, the cost of owning and operating equipment for landfill

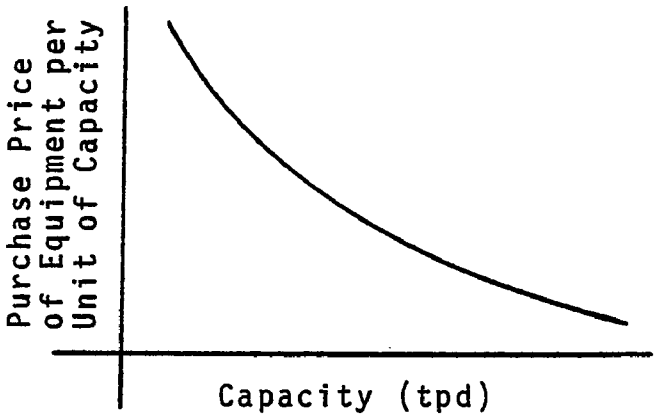


FIGURE 2.20. Average Landfill Site Cost: Capital Equipment.

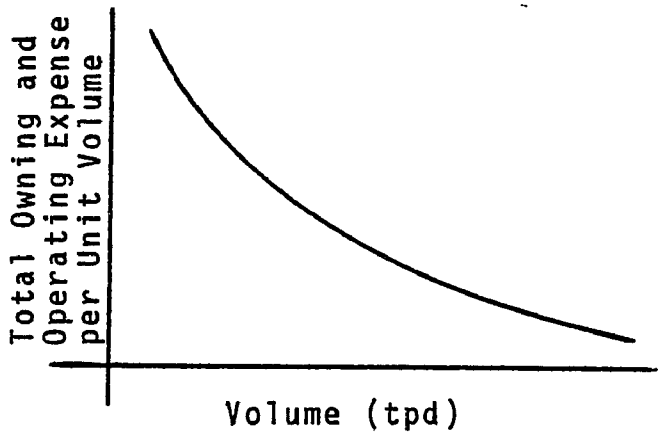


FIGURE 2.21. Average Landfill Site Cost: Owning and Operating Expense.

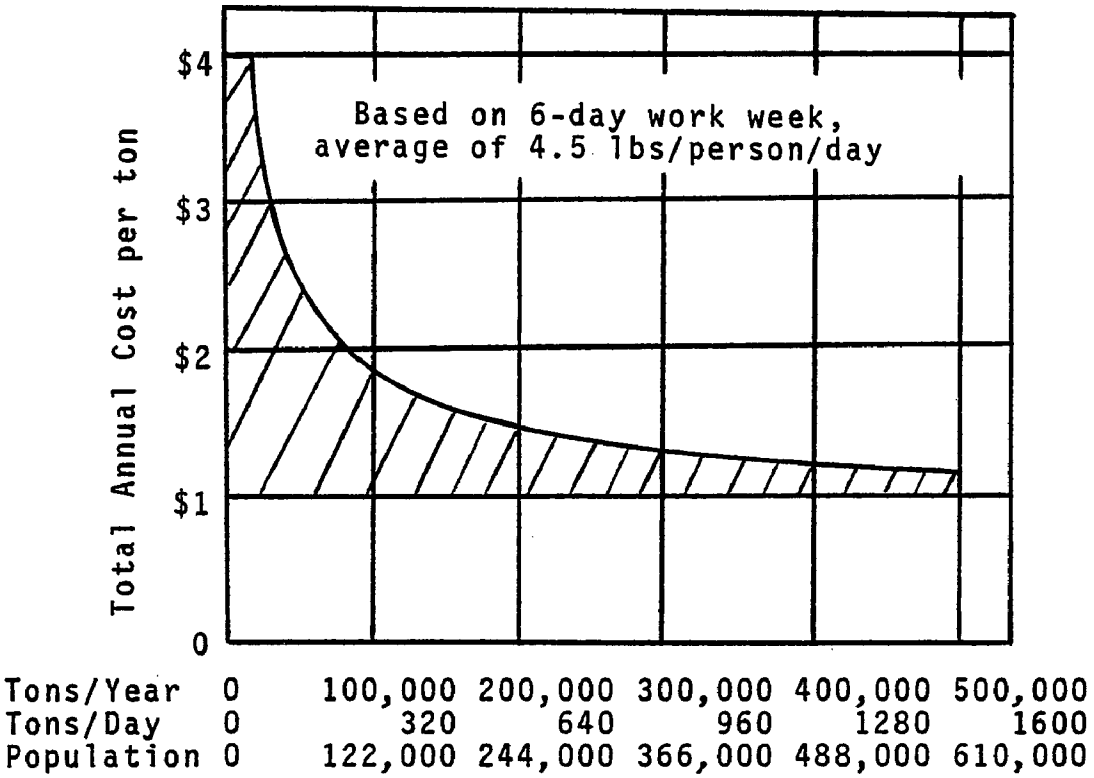


FIGURE 2.22. Six-tenths Rule of Average Annual Disposal Costs at Landfill Sites

Source: Wilson, D. G., Handbook of Solid Waste Management, van Nostrand and Reinhold Company, (1977), p. 233.

sites exhibits economies of scale (pages 154-157 of 39, 56; Tables 4-6 and 4-10 of 70). Moreover, the annual disposal cost at a landfill has the property of economies of scale following the "six-tenths" rule (page 46 of 18; page 214 of 12; page 222 of 55; page 111 of 70, 71; page 105 of 24). (See Figure 2.22.)

Part 3 - Collection and Transportation

2.26 Introduction: Types of Waste

About four billion tons of solid waste have been generated in 1972. Of that amount, 3.7 billion tons are agricultural and mining wastes which are easily disposed in rural areas. The remainder is composed of urban solid waste, of which 110 million tons are industrial and 250 million tons are municipal solid waste. "Municipal waste is a collective term referring to residential, commercial and demolition solid wastes" (Chapter 4 of reference 10).

Residential wastes are comprised of household wastes whose composition might be: 35-45 per cent paper, 10-20 per cent food wastes, 5-10 per cent glass and ceramics, 5-10 per cent metals, 5-15 per cent

grass and leaves, 2-6 per cent textiles, and less than 4 per cent wood, plastics and rubber. Commercial wastes are similar to residential wastes, with possibly more food from restaurants and more textile wastes from hotels. Demolition wastes are created from the razing and the construction of buildings. Demolition and industrial wastes are usually handled by private contractors and will not be analyzed in this work.

2.27 Collection System Parameters

Crew size can be minimized while still offering excellent service and high cost savings. No more than a two-man crew is necessary for curb or alley pickup. Similarly, no more than a three-man crew is necessary for the backyard pickup (72). New developments include a mechanical pickup apparatus for single crew operation.

Equipment needed for collection should be watertight for all weather service. The optimal size of the vehicle can be determined by charts in reference 73. The equipment should be maintained by the system and should be discarded when beyond its useful life. For instance, Hartford, Connecticut reported savings in equipment cost by reducing the replacement period (10).

Compactors are the preferred solid waste collection equipment. Side-loading vehicles are preferred for driver participation. Manual front loaders are dangerous and manual rear loaders make communication between loaders and driver difficult. Mechanical handling of the solid-waste containers is the most efficient where uniform containers are utilized and vehicle access is unburdensome (Figures 4.3 and 4.9 of reference 10).

For transporting solid waste over long distances, rail and barge hauling is the most economical. It is not recommended unless: at least 1,000 tpd are transported over round-trip distances greater than 100 miles; baling occurs at the transfer station; and the railroad cars are exclusively for solid waste. Tractor-trailer hauling is the most economical for round trips under 100 miles and if barge hauling is available under 60 miles (74).

A pneumatic pipeline system in Sweden has been operating for more than ten years without clogging (page 115 of 10). Refuse is transported through underground pipes to the resource recovery plant at a rate of 27.4 tpd (75). The waste must be less than 20 inches in size in order to pass through the pipeline.

The system is claimed to be more economical in the long run, since it is totally automatic and requires no vehicular collection fleet with crewmen (75). However, the pneumatic system becomes impractical and uneconomical for distances greater than a couple of miles or for excessive volumes (page 115 of 10, 76).

A slurry (a fluid mixture of water and fine, insoluable particles) pipeline becomes less costly per ton-mile as the haulage and volume increase; however, size reduction of the solid waste is required before entering the system. Since the cost of purchasing and maintaining size reduction equipment at each generation point is economically prohibitive, a centrally located reduction site is necessary. If a pipeline system is most desirable, a pneumo-slurry system could be developed. A pneumatic pipeline system from the generating sources to a central size reduction operation which is less than a couple miles away, a size reduction operation, and a slurry pipeline system from the reduction site to the ultimate disposal site comprise this long distance pipeline transportation system. A semi-automatic slurry system consisting of trucks collecting and removing the waste from the sources to the size reduction sites may be

more economical than using the pneumatic system for collection and removal. The pneumo-slurry transport system appears to be technologically feasible with present transport technology (pages 116-120 of 10, 76, 77). Furthermore, for Philadelphia, a pneumo-slurry transport system is expected to be cheaper than truck collection when the distance to the disposal site is greater than 50 miles.

2.28 Economics: Collection and Transportation

For a specific area and fixed distance, as the population density increases, the cost of collecting solid waste per person decreases (page 39 of 71; page 59 of 42). This implies that as the population density increases, the collection cost per ton decreases, or the cost per person decreases, assuming that each person generates the same quantity of solid waste on the average. This economies of scale relationship, depicted in Figures 2.23 and 2.24, is corroborated by the fact that as the population density increases, fewer collection stops are made. Furthermore, larger and more efficient collection vehicles can be employed (page 82 of 26, 64). Some actual data taken from reference 55 on collection costs for the "Best-Known

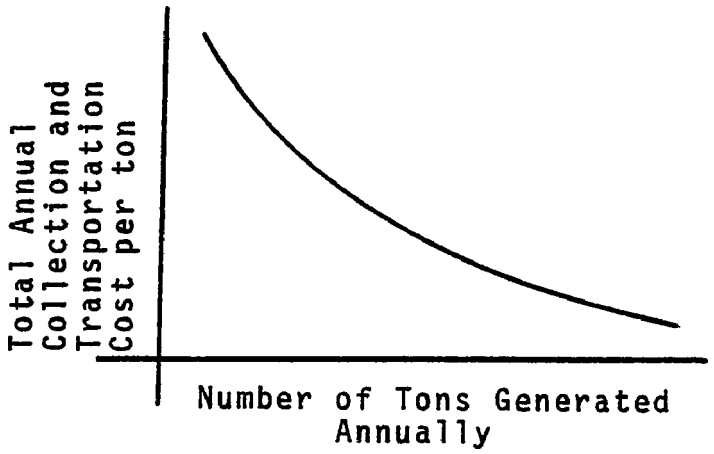


FIGURE 2.23. Total Average Cost Curve for Collection and Transportation.

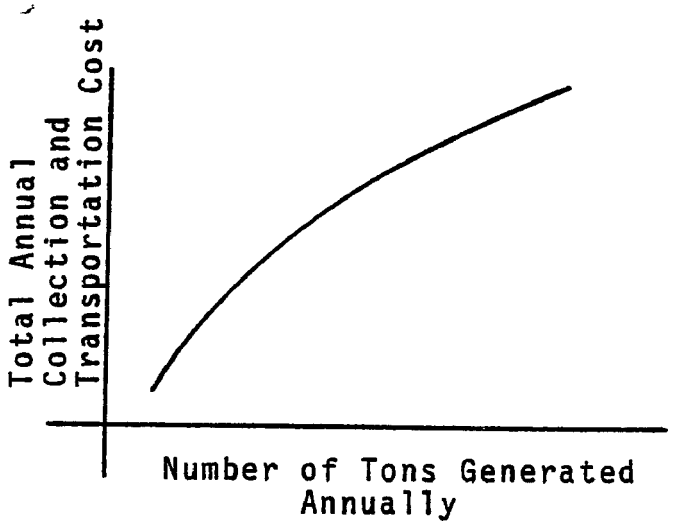


FIGURE 2.24. Total Cost Curve for Collection and Transportation.

Collection Productivities" is shown in Table 2.8. Assuming a nonlinear exponential relationship such as

$$\frac{(\text{Cost/Ton})_2}{(\text{Cost/Ton})_1} = \left[\frac{\text{Tons}_2/\text{Crew Hour}}{\text{Tons}_1/\text{Crew Hour}} \right]^{\alpha} ,$$

where $\alpha < 0$, to reflect decreasing average costs, the values of α for crew sizes of 1, 2, and 3 are -0.78, -0.75, and -1.17, respectively.

Economies of scale is also observed for long distance transport of solid waste from transfer stations (page 45 of 57; page 61 of 42, 78). Table 2.9 illustrates the decreasing average costs associated with railroad hauling for 50 miles. The decreasing average costs follow a negative exponential relationship between average cost and tons hauled.

An analysis of the direct haul costs to the disposal site versus transfer costs and haulage costs to disposal site determines whether a transfer station is needed. Transfer stations tend to be favored when environmental characteristics include traffic congestion, large quantities of solid waste, and a distant disposal site. A breakeven analysis comparable to the one illustrated in Figure 2.25 is recommended before deciding upon transfer stations. Furthermore, total

Crew Size	Tons Collected per Crew-Hour	No. of People Served/Truck/Wk	Cost/ Ton Col- lected
1	2.2	7763	8.44
1	1.3	3636	12.71
2	2.1	7517	11.82
2	1.2	5130	18.00
3	3.8	7060	10.34
3	3.0	6316	13.65

TABLE 2.8. Best-Known Collection Productivities.

Source: Page 242 of Reference 55.

	Tons Shipped by Rail for 50 Miles	Cost/Ton
	50	$5.00
	100	2.65
	1000	1.90
	2500	1.30
Variables:	T	U

Remarks:

Let $\frac{U_2}{U_1} = \left(\frac{T_2}{T_1} \right)^{\alpha}$ be the assumed relationship between U and T. Then $\alpha \varepsilon [-.34, -.22]$.

TABLE 2.9. Decreasing Average Costs Associated with Railroad Hauling.

Source: Page 45 of Reference 57.

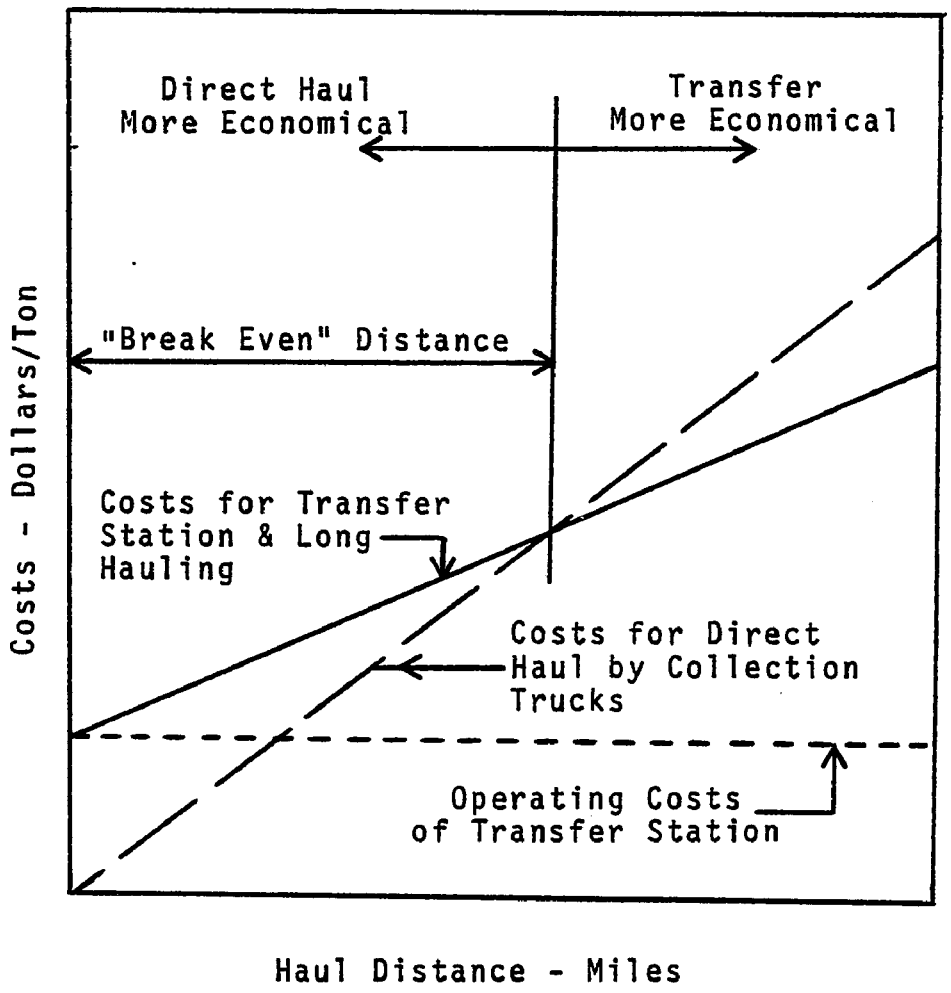


FIGURE 2.25. Representative Economic Analysis of Transfer Stations. Source: Hagerty, D. J., et al., Solid-Waste Management, Van Nostrand Reinhold, New York, 1973.

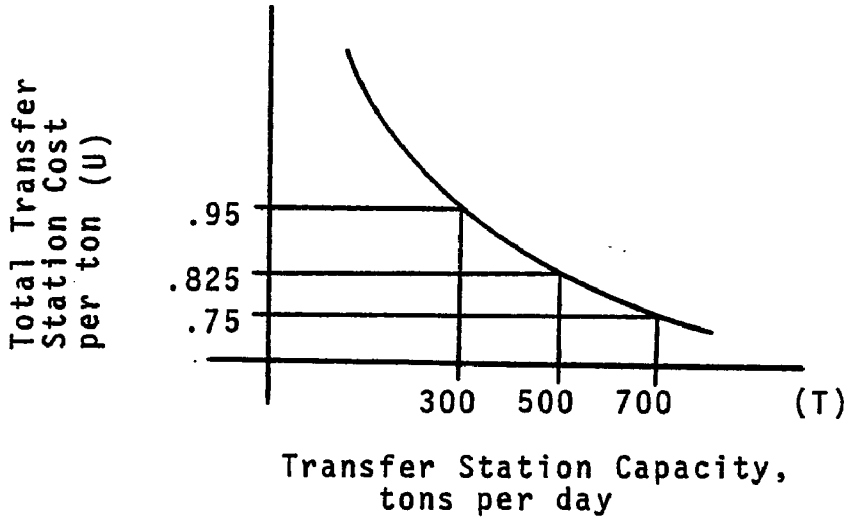


FIGURE 2.26. Average Cost of Transfer Stations. Source: p. 44 of reference 57.

Remarks:

Assuming $\frac{U_2}{U_1} = \left(\frac{T_2}{T_1} \right)^{\alpha}$, then $\alpha = -0.28$.

transfer station costs exhibit economies of scale with respect to capacity (pages 158-161 of 39; page 44 of 57). (See Figure 2.26.)

Part 4 - Legal Environment

Although the Resource Conservation and Recovery Act of 1976 (PL 94-580) is a group of amendments to the Solid Waste Disposal Act of 1965, the 1976 Act, hereafter referred to as the Act, basically replaces the 1965 Act. Included in the lengthy and complicated Act are provisions to eliminate substandard land disposal sites such as open dumps and to encourage resource recovery (20, 79). A timetable for compliance of the Act is shown in Tables 2.10 through 2.13.

Some earlier federal laws also encourage resource recovery systems. The Clean Air Acts of 1965 and 1970 caused many incinerators to be closed (20). The Federal Ocean Dumping Act of 1972 prohibits ocean dumping to a large degree (20). The Federal Water Pollution Act Amendments of 1972 reduced the amount of waste that can be dumped into streams (20).

The effectiveness of the new legislation has been recently challenged. The United States government sued Westchester County to preserve streams and

October 1976 Act signed by Gerald Ford.

April 1978 EPA promulgates criteria to assist states in developing a state hazardous waste program. If state does not develop program or does not meet criteria, EPA will take over task.

EPA promulgates regulations establishing performance standards for owners and operators of facilities for treatment, storage, and disposal of hazardous wastes.

EPA also publishes regulations requiring these owners and operators to have a permit for treatment, storage, and disposal of hazardous wastes.

EPA promulgates regulations establishing standards for generators and transporters of hazardous wastes.

EPA promulgates criteria for identifying characteristics of and for listing hazardous wastes, promulgates regulations identifying the characteristics of, and lists, hazardous wastes.

July 1978 Generators, transporters, and those who treat, store, or dispose of hazardous waste must file statement with the EPA.

October 1978 Regulations under this subtitle (including permit requirement) take effect.

Note: An Interim Permit status is available for existing facilities that have filed a statement with EPA until EPA makes final decision on Permit application. Final permit may include timetable for necessary modifications.

TABLE 2.10. Deadline Dates for Actions Under Resource Conservation and Recovery Act of 1976: Hazardous Waste Management.

Source: The interpretation of the Act by the Joint Planning Commission, Lehigh-Northampton Counties.

October 1976	Act signed by Gerald Ford.
April 1977	EPA develops guidelines for planning areas.
October 1977	Governor identifies planning area boundaries.
	EPA develops regulations containing criteria for defining "open dumps".
April 1978	EPA develops State Plan guidelines, state and local officials decide who will develop and implement the Plan.
	Now or, if later, when State Plan approved, disposal in open dumps and development of new open dumps prohibited.
July 1978	If no planning agency chosen, Governor designates such State agency.
October 1978	EPA publishes inventory on all U.S. "open dumps".
1983	Maximum time limit to allow open dumps to come into compliance with EPA minimum criteria.

TABLE 2.11. Deadline Dates for Actions Under Resource Conservation and Recovery Act of 1976: State or Regional Solid Waste Plans.

Source: See Table 2.10.

October 1976	Act signed by Gerald Ford.
October 1978	EPA to complete and submit research and studies on Composition of Waste Stream, a Priorities Study for solid waste recovery strategies and methods, a Small-Scale and Low Technology Study, Front-End Source Separation, Mining Waste, Sludge, and Airport Landfills.
October 1979	EPA completes and submits research and studies on Glass and Plastic, Tires, Functioning of Resource Recovery Facilities, and a Study by the Resource Conservation Committee on resource conservation and its effects on the public, markets, and solid waste.

Note: Once these reports are completed, EPA will prepare a plan for research, development, and demonstration based on these study findings.

TABLE 2.12. Deadline Dates for Actions Under Resource Conservation and Recovery Act of 1976: Special Studies Under Subtitle H.

Source: See Table 2.10.

October 1976	Act signed by Gerald Ford.
October 1978	Publish guidelines for the development of specifications for the classification of recovered materials from waste. (Physical and chemical properties important in deciding their usefulness in replacing virgin materials in various industrial, commercial, and governmental uses. Identify geographical location of existing or potential markets for recovered materials, identify economic and technical barriers to their use, and encourage development of new uses for these materials.

TABLE 2.13. Deadline Dates for Actions Under Resource Conservation and Recovery Act of 1976: Duties of the Secretary of Commerce in Resource and Recovery.

Source: See Table 2.10.

tidal marshes at its landfill site at Croton Point, and to separate reusable materials (12). Five municipal incinerators in Nassau County were closed in June 1975 because of state and federal law violations. These closings are causing overloading of nearby landfills and encouraging resource recovery. For example, in North Hempstead, New York, the local incinerator was closed because of air quality violations forcing an existing landfill to be filled in four years instead of lasting a decade (22). In 1971, the Hackensack Meadows Development Commission of northern New Jersey forbade filling any virgin wetlands (80).

Since a case study of a municipal solid waste system in Pennsylvania is presented in Chapter 6, the prominent solid waste acts of that state are briefly reviewed. The two principal solid waste acts of Pennsylvania are PA ACT 241, entitled Pennsylvania Solid Waste Management Act, and PA ACT 198, entitled Pennsylvania Solid Waste Resource Recovery Act (62, 63). Enacted in 1968 and revised in 1970 and 1972, ACT 241 contains provisions for environmental planning and for regulating solid waste disposal (62, 79). ACT 198 basically recognizes solid waste as a resource (63, 79).

CHAPTER 3

COST MINIMIZATION MODELS FOR SOLID WASTE MANAGEMENT

3.1 Introduction

The solution to solid waste management problems includes the assessment of many complex interactions in terms of transportation systems, land use patterns, urban growth and development, political and social concerns, and public health considerations. "A necessary step in the process of developing a systems analysis approach to solid waste management is the establishment of a mathematical modeling structure" (page 137 of 39). The three major areas of a solid waste management system are collection including storage, transfer and transportation of solid waste, and disposal including any accompanying treatment such as resource recovery and/or incineration. The three alternatives normally considered in such a system are:

1) Direct shipment from municipalities to an ultimate disposal site.
2) Direct shipment to a transfer station where solid waste is transferred to large vehicles and then shipped for ultimate disposal.

3) Direct shipment from municipalities to a processor (incinerator or recovery system) where waste is processed and the residue is shipped for ultimate disposal.

"The political and social factions of a region may be hostile to the least costing system and more interested in the equity of the system. The cheapest system often involves one giant incinerator or landfill with the nearby residents receiving all the adverse impacts from the region's waste" (page 156 of 81). A pure equity model might be one where everyone disposes their own waste; however, the cost and overall environmental impact would be prohibitive. In Milwaukee, Americology, Inc., a subsidiary of American Can Company, processes all of the city's waste at one resource recovery plant which does not adversely impose upon nearby Milwaukee residents and which has promising economics (60). Therefore, the desired design should be equitable when: no single area receives an unreasonable share of the system's impacts; it is efficient to keep costs low; and no changes could be made that would improve the design in favor of one community without hurting another (81).

The objective of the solid-waste mathematical model is to obtain the most economically efficient

solution subject to equity and technical constraints. A mathematical model that minimizes the aggregate cost subject to equity and technical constraints can give an indication of the best system. A preliminary screening of as many possible choices of facilities to ones technically feasible and potentially equitable can appreciably reduce the dimension of the problem; only the most economically efficient model needs to be determined. These simpler models will be the only ones that will be analyzed in detail by this work. However, a more general model with equity variables which was adapted from two recent articles will be presented.

The choice of a suitable variable for measuring equity is difficult, since such nebulous forces as utility preferences come into play. For illustrative purposes, assume that a more equitable system disposes its waste near its source, leading to a larger number of small facilities (81). The two measures of equity used are the total ton-miles of waste shipped during the planning period and total tonnage shipped over political boundaries.

There are many solid waste facility models illustrated in the literature (See references 82 through 98) that may stress certain points, but in

general, they are similar. The dynamic facility location and waste haul model partially taken from page 540 of reference 10 and page 538 of reference 81 illustrates such a model.

The objective function, equation 3.1, is the total cost to construct and to operate over the entire planning period. λ_t is a discount factor in order that the analysis is performed with respect to net present value. A term could be added to reflect the estimated net present value of the salvage value of the system. Any revenues generated by a resource recovery process are assumed to be embedded in the cost function as negative costs. The cost functions $F_{it}(Y_{it})$, $\gamma_{it}(P_{it})$, $C_{ijt}(X_{ijt})$ and $C'_{ght}(T_{ght})$ are usually concave functions that represent the prevalent economies of scale nature of all the costs. $F_{it}(Y_{it})$ often displays an initial charge which is independent of capacity.

For any feasible system, all the waste generated must be handled by the system. Equation 3.2 forces the system to have the correct level of capacity throughout the planning period.

Equation 3.3 states that all waste shipped to a processing plant must be treated.

DYNAMIC FACILITY LOCATION AND WASTE HAUL MODEL

Minimize:

$$\sum_{i=1}^{m}\sum_{t=1}^{r} \lambda_t \gamma_{it}(P_{it}) + \sum_{t=1}^{r}\sum_{j=1}^{n}\sum_{i=1}^{m} \lambda_t C_{ijt}(X_{ijt}) +$$

$$\sum_{t=1}^{r}\sum_{i=1}^{m} \lambda_t F_{it}(Y_{it}) + \sum_{t=1}^{r}\sum_{h=1}^{m-a}\sum_{g=1}^{a} \lambda_t C'_{ght}(T_{ght}) \qquad (3.1)$$

Subject to:

$$\sum_{i=1}^{n} X_{ijt} \geq G_{jt}, \text{ for all } j,t \qquad (3.2)$$

$$\sum_{j=1}^{m} X_{ijt} - P_{it} \leq 0, \text{ for all } i,t \qquad (3.3)$$

$$P_{g\tau} - \sum_{t=1}^{\tau} Y_{gt} - q_g \leq 0, \text{ for all } \tau\in\{1,\ldots,r\},$$

$$g\in\{1,2,\ldots,a\} \qquad (3.4a)$$

$$\sum_{t=1}^{\tau} (P_{ht} - Y_{ht}) - q'_h \leq 0, \text{ for all } \tau\in\{1,\ldots,r\},$$

$$h\in\{1,\ldots,m-a\} \qquad (3.4b)$$

$$\alpha_g P_{gt} - \sum_{h=1}^{m-a} T_{ght} \leq 0, \text{ for all } g,t \qquad (3.5)$$

$$\sum_{t=1}^{r} \sum_{j=1}^{n} \sum_{i=1}^{m} d_{ij} X_{ijt} + \sum_{t=1}^{r} \sum_{h=1}^{m-a} \sum_{g=1}^{a} d'_{gh} T_{ght} \leq A \qquad (3.6)$$

$$\sum_{t=1}^{r} \sum_{j=1}^{n} \sum_{i=1}^{m} e_{ij} X_{ijt} \leq B \qquad (3.7)$$

$$\sum_{t=1}^{r} Y_{it} \leq S_i, \text{ for all } i \qquad (3.8)$$

$$X_{ijt} \geq 0, \text{ for all } i,j,t \qquad (3.9)$$

$$T_{ght} \geq 0, \text{ for all } g,h,t \qquad (3.10)$$

$$Y_{it} \geq 0, \text{ for all } i,t \qquad (3.11)$$

$$P_{it} \geq 0, \text{ for all } i,t \qquad (3.12)$$

where:

X_{ijt} = units of waste shipped from generating area j to site i in time period t.

Y_{it} = capacity added to facility i at the beginning of period t.

P_{it} = units of waste processed at site i in period t.

T_{ght} = units of waste shipped from treatment site g to final disposal site h in period t.

$F_{it}(Y_{it})$ = cost of expanding capacity of site i by Y_{it} units in period t.

$\gamma_{it}(P_{it})$ = cost of processing P_{it} units at site i in period t.

$C_{ij}(X_{ijt})$ = cost of shipping X_{ijt} units from generating area j to i in period t.

$C'_{ght}(T_{ght})$ = cost of shipping T_{ght} units from treatment site f to final disposal site h in period t.

λ_t = financial factor for period t.

G_{jt} = units of waste generated in generating area j in period t.

q_g = initial capacity of treatment site g.

q'_h = initial capacity of final disposal site h.

α_g = weight reduction factor of treatment site g.

e_{ij} = equity coefficient: 0, if site i is in area j, and 1, otherwise.

d_{ij} = distance between site i and generating area j.

d'_{gh} = distance between treatment site g and disposal site h.

A = maximum allowable ton-miles of waste shipment.

B = maximum allowable tons of waste shipped over district boundaries.

S_i = total possible expansion capacity of site i.

m = number of feasible facility sites i.

n = number of generating areas (districts) j.

a = number of intermediate processing sites g.

m-a = number of final disposal sites h.

r = number of time periods t.

Equations 3.4a and 3.4b are the processing capacities of the intermediate sites and ultimate sites, respectively. Constraint 3.4a states that the amount of waste processed at intermediate site g during period t cannot exceed the initial period processing capacity of the facility plus all of the capacity expansions; analogously, for the ultimate sites represented in constraint 3.4b.

Constraint 3.5 is the balance constraint between the treatment site and the ultimate disposal site.

The equity constraints are represented by constraints 3.6 and 3.7 where equation 3.6 limits the total ton-miles of waste shipments and equation 3.7 limits the number of tons of waste shipped across political boundaries.

The constraint that limits the level of site expansion is 3.8.

3.2 Computer Programming Models and Algorithms

Skelly (99) and Walker (100) view the solid waste management problem as primarily a capacitated warehouse location problem with the variable costs illustrating economies of scale. Stated verbally,

the problem is:

MINIMIZE:

(TRANSPORTATION COSTS + VARIABLE WASTE DISPOSAL COSTS + FIXED FACILITY OVERHEAD AND CAPITAL COSTS - REVENUES FROM RECLAIMED MATERIALS)

SUBJECT TO:

1) ALL REFUSE GENERATED AT A SOURCE IS ASSIGNED TO SOME SITE
2) THE REFUSE SHIPPED TO A DISPOSAL SITE DOES NOT EXCEED THE SITE'S CAPACITY
3) ALL REFUSE SHIPPED INTO A TRANSFER STATION OR AN INTERMEDIATE SITE IS SHIPPED TO A FINAL DISPOSAL OR ANOTHER INTERMEDIATE SITE (WITH A POSSIBLE REDUCTION IN WEIGHT)

Generally speaking, the cost minimization models of solid waste systems fall within the framework of the verbal model above (1, 2, 81, 101, 102, 103, 104).

The complexities involved in attempting to obtain an optimal solution to the problem are the nonlinear variable cost curves coupled with the

fixed charges. Mixed integer programming algorithms yield exact solutions to the fixed charge problem with linear variable cost curves for small problems, but become computationally prohibitive for large problems (100, 105-109). Of the available hueristic algorithms, none can guarantee a global optimum (100, 106, 110, 111, 112).

The fixed charge problem is encountered when there is a fixed cost associated with an activity. The variable cost can be proportional to the level of the activity, or it can be non-linearly related. Let x_j, $j = 1, 2, \ldots n$, denote the level of activity, say, the number of units processed per unit of time. Assume that the variable cost is proprotional to x_j with proportionality constant c_j. Let k_j be the fixed cost of activity j which is only incurred when x_j is greater than 0. Assuming that all of the constraints upon the x_j's are linear, the mathematical formulation[1] of the fixed charge problem is:

---

[1]Taken from references 100 and 113.

Minimize:

$$Z = \sum_{j=1}^{n} f_j(x_j) = \sum_{j=1}^{n} (c_j x_j + k_j \delta_j)$$

Subject to:

$$\underline{A}\underline{x} \leq \underline{b}, \text{ and}$$

$$\underline{x} \geq 0$$

$$\text{Where } \delta_j = \begin{cases} 0, \text{ when } x_j = 0 \\ 1, \text{ when } x_j \neq 0 \end{cases}$$

$$j = 1, 2, \ldots, n.$$

Figure 3.1 illustrates the type of concave objective function in a linear fixed charge problem.

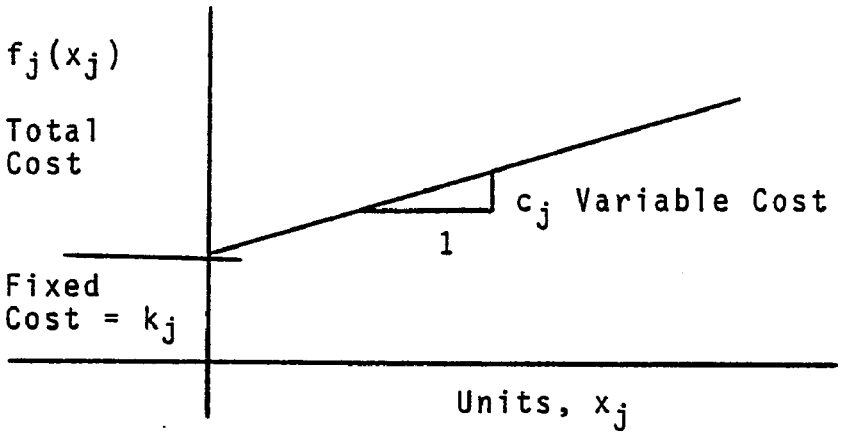


FIGURE 3.1. Cost Function With Fixed Charge

Some solution techniques to the linear fixed charge problem are briefly described below. Efroymson and Ray (114) have developed a branch and bound technique where a large number of LP's must be solved and where there are no limits on the facilities' capacity. This method can only handle simple plant location problems (103). Spielberg developed an exact algorithm similar to Efroymson and Ray's which can handle some additional constraints (103, 114). Gray's exact algorithm which is a mixed integer algorithm required more than 10 minutes on the average and as much as 22 minutes on a Burroughs B5500 system to solve a 5x7 problem (100, 103, 107). Kuehn and Hanberger (115) Kuhn and Baumol (108), Manne (116), and Balinski (109) developed hueristics and approximations that they say produce good solutions to the fixed charge problem (103).

This author did not find any hueristic algorithms or references to any that surpassed Walker's hueristic algorithm in reference 100. When tested against Spielberg's and Gray's exact algorithms, Walker's procedure yielded the optimal solution in almost all test cases attempted (100, 103). In fact, all solutions were achieved in from 0.50 to 0.0012 of the time needed by Gray's algorithm; only two

inconsistent solutions were off by less than two per cent (100). Furthermore, in reference 100, Walker tested his algorithm against hueristic algorithms developed by Cooper and Drebes (110), Steinberg (106), Denzler (111), and Cooper (112). For the test cases exhibited in reference 100, only Walker's hueristic converged upon the global solution. Furthermore, barring differences in programming skills, Walker believes his algorithm appears to be faster and to obtain the optimal solution a higher percentage of the time (100). A major drawback of Walker's algorithm is that it can only guarantee achieving a local optimum which he surmises could be the optimal solution or at least a good solution (1, 2, 103).

Many of the previously mentioned solution procedures require large sophisticated computers and long running times which are costly. A rather simple computer model written by McNamara (117) requires only 60000_8 words of memory on a CDC6400 computer for a problem whose waste sources and landfill sites sum to 30. The FORTRAN coded computer model executes in less than a minute and comprises a deck with fewer than 300 cards. The program utilizing Walker's algorithm required 8.9 minutes to solve a 25-city, 9-site problem on an IBM360/65 computer (page 594

of 100). McNamara's solution procedure involves some manual interventions to circumvent some of the complex programming necessary to model the fixed charge aspect of the problem. Since the algorithm assumes the facilities are operating at full capacity, facilities with low utilizations are removed from the problem and a new problem is solved. The solution procedure ends when no more facilities can be removed.

In the literature reviewed, only Walker's algorithm attacks the problem of the fixed charge type with nonlinear costs. In his treatment of concave disposal costs to reflect economies of scale, the nonlinear cost functions are estimated by piecewise linear approximation. This approximation procedure vastly increases the number of variables in the LP. Hence, the size and complexity of the LP problem is immensely augmented. The FORTRAN program, WRAP (101), which is designed to aid in solid waste planning, utilizes Walker's algorithm (100). It is currently being distributed by the Systems Management Programs, United States Environmental Protection Agency under the direction of Donna Krabbe. In WRAP, nonlinear cost curves are approximated from data input of the intercepts and slopes of the approximating linear segments (See Figure 3.2).

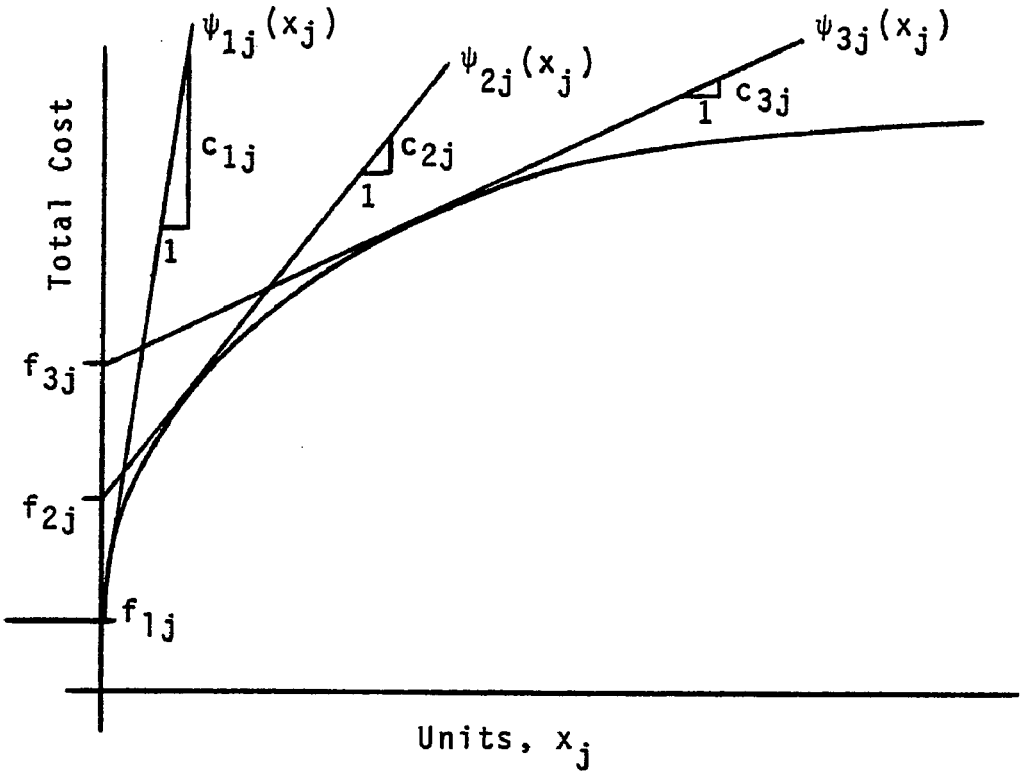


FIGURE 3.2. Piecewise Linear Approximation.

(See following page for Legend)

LEGEND - FIGURE 3.2.

The curve $\phi_j(x_j)$ is approximated with 3 linear segments, where

$c_{ij} \equiv$ the slope of the i^{th} segment of $\psi_j'(x_j)$,

$f_{ij} \equiv$ y- intercept of the i^{th} segment of $\psi_j'(x_j)$.

By concavity, $c_{1j} > c_{2j} > c_{3j}$ and

$$f_{3j} > f_{2j} > f_{1j}.$$

Let $x_j = \sum_{i=1}^{3} \Delta_{ij}$. Then the approximating function, ψ_j', of $\phi_j(x_j)$ can be written

$$\psi_j'(x_j) = \text{MIN} \sum_{i=1}^{3} \psi_{ij}(x_j) = \sum_{i=1}^{3} (c_{ij}\Delta_{ij} + f_{ij}\delta_{ij}),$$

$j = 1, 2, \ldots, n$ (= number of processes),

Subject to $\sum_{i=1}^{3} \Delta_{ij} = x_j$,

Where $\delta_{ij} = \begin{cases} 0, & \text{if } \Delta_{ij} = 0 \\ 1, & \text{if } \Delta_{ij} > 0 \end{cases}$, and

$$\Delta_{ij} \geq 0, \; j = 1, 2, \ldots, n.$$

3.3 Summary

The WRAP program can only guarantee approximate local optimal solutions. The only nonlinear cost curves analyzed by WRAP are the processing costs; all others are assumed to be linear or fixed. However, variable and fixed costs exhibit economies of scale for haulage, for processing at resource recovery plants and landfill sites, and for capital equipment (36, 118). Moreover, resource recovery plant processing costs also have the diseconomies of scale property.

Geometric programming can handle posynomial nonlinear cost functions and constraints in a natural way. Therefore, geometric programming should be a computationally more efficient and more accurate solution technique for solid waste management problems. The proposed research will investigate the utilization of geometric programming to aid in solving the nonlinear concave disposal costs directly without linear approximations, and the development of more efficient or better methods of handling the fixed costs which exhibit economies of scale in solid waste management problems.

CHAPTER 4

GEOMETRIC PROGRAMMING

Part 1 - An Overview of Geometric Programming

4.1 Introduction

Since the middle Eighteenth Century, differential calculus and calculus of variations have been applied to certain types of optimization problems in geometry and physics. Since the late 1940's, a new class of optimization problems called mathematical programming problems has attracted increasing interest. Mathematical programming problems are classified as optimization problems in Economics; chiefly, problems in the allocation of scarce resources. With his development of the simplex algorithm for solving linear programs, G. B. Dantzig has spurred the great impetus for the growth of interest in mathematical programming. The simplex algorithm is an efficient computation scheme that can be readily applied to many real world problems. Furthermore, the duality theory of linear programming has provided a vast store of economic information from the dual variables

whose values represent shadow prices. A global optimum is guaranteed by the simplex procedure for consistent programs. Only excessively large problems cannot be solved with this algorithm on modern computers.

Unlike linear programming, few nonlinear programs can be solved for any solution, let alone global solutions. The small subset of the class of nonlinear programs for which solutions or approximate solutions seems obtainable is studied most extensively. These programs are usually comprised of objective functions that are separable or quadratic with linear constraints. The solution technique utilized most often is a transformation of the objective function into a linear function, followed by the application of linear programming to the transformed program. Solutions to nonlinear programs with nonseparable objective functions and/or constraints can sometimes be attacked with calculus. However, many computational problems usually arise when calculating solutions with calculus. If the rare incidence of the nonlinear program satisfying the Kuhn-Tucker conditions occurs, a global optimal solution can be obtained for reasonably sized problems.

4.2 Historical Development of Geometric Programming

C. Zener wrote the first paper (119) where a degree of difficulty zero and an unconstrained posynomial geometric program was solved. R. J. Duffin utilized theory concerning the geometric inequality (120) and duality theory to develop the theoretical basis for Zener's technique. C. Zener removed the zero degree of difficulty constraint by first proposing a perturbation procedure (121) followed by a further extension employing overlapping subsystems (122). Constrained geometric programs were introduced by A. Charnes and W. W. Cooper in 1962, and by R. J. Duffin and E. L. Peterson in 1964 (123). In 1966, Duffin and Peterson constructed a comprehensive duality theory for posynomial geometric programming (124).

Some extensions of posynomial geometric programming included stochastic geometric programming developed by M. Avriel (125). U. Passy first analyzed geometric programs with negative coefficients called signomials (126, 127).

4.3 Solving Signomial Programs

There are three basic approaches toward obtaining a solution to signomial programs. First,

the primal program could be directly attacked with an algorithm such as the one developed by Reklaitis (128) or Staats (129). Second, several authors (130 through 135) have developed algorithms to take advantage of the dual structure of a primal program. Third, R. J. Duffin suggests transforming the signomial program into a posynomial program (136, 137). Several algorithms that solve signomial programs have appeared in the literature (133, 138 through 141). Since a geometric program with reversed constraints is equivalent to a signomial program, it can be solved by the above methods.

A geometric program with degrees of difficulty and loose constraints can be approached by the surrogate constraint (129) and augmented procedures (140, 142).

Since its inception, geometric programming has been applied to problems which cut across many science and engineering disciplines. Mechanical engineering problems such as the optimal design of journal bearings (143), gas transmission compressors (144), welded beams (144), and pressure vessels (145), were solved by geometric programming. The Williams-Otto process (146) and alkylation process

optimization (141) were analyzed with geometric programming. Concerning the subject of biochemistry, geometric programming was applied to a model of the chemistry of the human respiratory system (144). In the business arena, the mathematical programming technique was applied to the marketing mix problem (147) and the minimization of capital investment for batch processes (148). In civil engineering, such problems as the optimal design of cofferdams (149) and industrial waste treatment plants (150) were attacked by geometric programming. Furthermore, problems in tool engineering (151, 152), ship design (153, 154), nuclear engineering (155), modular design (156), gas production facilities design (159), reliability engineering (157), and resource allocation engineering (158) were analyzed by geometric programming.

4.4 Basic Geometric Programming Theory

The notation and theorems used to present the basic geometric programming theory in this section are from reference 160. First, the primal program and its respective dual are presented with a statement of the relationships between the primal and dual variables. Using the notation presented on pages 78 and

79 of reference 160, the primal and dual programs are now described.

Primal Program A

Minimize:

$$g_0(\underline{t}), \tag{4.1}$$

Subject to:

$$\left.\begin{aligned} & g_1(\underline{t}) \leq 1, \\ & g_2(\underline{t}) \leq 1, \\ & \vdots \\ & g_p(\underline{t}) \leq 1, \end{aligned}\right\} \tag{4.2}$$

$$\underline{t} > 0, \tag{4.3}$$

Where:

$$g_k(\underline{t}) = \sum_{i \epsilon J[k]} c_i t_1^{a_{i1}} t_2^{a_{i2}} \ldots t_m^{a_{im}},$$

$$k = 0,1,2,\ldots,p, \tag{4.4}$$

$$J[k] = \{m_k, m_{k+1}, m_{k+2}, \ldots, n_k\},$$

$$k = 0,1,2,\ldots,p, \tag{4.5}$$

$$m_0 = 1, \; m_1 = n_0+1, \; m_2 = n_1+1, \ldots, m_p = n_{p-1}+1,$$

$$n_p = n \tag{4.6}$$

$$a_{ij} \in \{\text{real numbers}\}, \text{ for all } i \in \{1,2,\ldots,n\}$$

$$\text{and for all } j \in \{1,2,\ldots,m\}, \qquad (4.7)$$

$$c_i > 0, \text{ for all } i \in \{1,2,\ldots,n\}. \qquad (4.8)$$

Equation 4.1 is called the primal function. The vector $\underline{t}$ is comprised of the primal variables. The primal constraints are composed of two classes: forced, which are illustrated by 4.2 and natural, which are illustrated by 4.3. The restrictions of 4.7 and 4.8 force the functions in 4.4 to be posynomials. The matrix (a_{ij}) is the exponent matrix.

Dual Program B

Maximize:

$$v(\underline{\delta}) = \left[\prod_{i=1}^{n} \left(\frac{c_i}{\delta_i}\right)^{\delta_i}\right] \prod_{k=1}^{p} \lambda_k(\underline{\delta})^{\lambda_k(\underline{\delta})}, \qquad (4.9)$$

Subject to:

$$\sum_{i \in J[0]} \delta_i = 1, \qquad (4.10)$$

$$\sum_{i=1}^{n} a_{ij}\ \delta_i = 0,\ j = 1,2,\ldots,m, \tag{4.11}$$

$$\underline{\delta} \geq 0, \tag{4.12}$$

Where:

$$\lambda_k(\underline{\delta}) = \sum_{i \in J[k]} \delta_i,\ k = 1,2,\ldots,p, \tag{4.13}$$

and

(4.5), (4.6), (4.7), (4.8) hold.

Equation 4.9 is called the dual function. The vector $\underline{\delta}$ is comprised of the dual variables. Each dual variable corresponds to a term in the primal program. The factors c_i in the dual are coefficients in the primal program. There is one $\lambda_k(\underline{\delta})$ factor in the dual objective function for each constraint in the primal. The normality condition, 4.10, the orthogomality condition, 4.11, and the positivity conditions, 4.12, comprise the dual constraints.

The following theorem is the main theorem of geometric programming which is called the first

duality theorem of geometric programming. It states the relationship between the primal and the dual variables. However, the theorem requires that the primal program be superconsistent: there exists a $\underline{t}^*$ such that $g_k(t^*) < 1$ for all $k \in \{1,2,\ldots,p\}$.

Theorem 4.1

Suppose that Primal Program A is superconsistent and that the primal function $g_0(\underline{t})$ attains its constrained minimum value at a point that satisfies the primal constraints. Then

(i) The corresponding Dual Program B is consistent and the dual function $v(\underline{\delta})$ attains its constrained maximum value at a point which satisfies the dual constraints.

(ii) The constrained maximum value of the dual function is equal to the constrained minimum value of the primal function.

(iii) If $\underline{t}'$ is a minimizing point for Primal Program A, there are non-negative Lagrange multipliers such that the Lagrange function

$$L(\underline{t},\underline{\mu}) = g_0(\underline{t}) + \sum_{k=1}^{p} \mu_k\left[g_k(\underline{t}) - 1\right] \qquad (4.14)$$

has the property

$$L(\underline{t}', \underline{\mu}) \leq g_0(\underline{t}') = L(\underline{t}', \underline{\mu}') \leq L(\underline{t}, \underline{\mu}') \qquad (4.15)$$

for arbitrary $t_j > 0$, $j = 1,2,\ldots,m$, and arbitrary $\mu_k \geq 0$, $k = 1,2,\ldots,p$. Moreover, there is a maximizing vector $\underline{\delta}'$ for Dual Program B whose components are

$$\delta_i' = \begin{cases} \dfrac{c_i t_1^{a_{i1}} t_2^{a_{i2}} \ldots t_m^{a_{im}}}{g_0(\underline{t})}, & i \in J[0], \\ \dfrac{\mu_k c_i t_1^{a_{i1}} t_2^{a_{i2}} \ldots t_m^{a_{im}}}{g_0(\underline{t})}, & i \in J[k],\ k = 1,2,\ldots,p, \end{cases} \qquad (4.16)$$

where $\underline{t} = \underline{t}'$ and $\underline{\mu} = \underline{\mu}'$. Furthermore,

$$\lambda_k(\underline{\delta}') = \frac{\mu_k'}{g_0(\underline{t})}, \quad k = 1,2,\ldots,p. \qquad (4.17)$$

(iv) If $\underline{\delta}'$ is a maximizing point for Dual Program B, each minimizing point $\underline{t}'$ for Primal Program A satisfies the system of equations

$$c_i t_1^{a_{i1}} \ldots t_m^{a_{im}} = \begin{cases} \delta_i' \; v \; (\underline{\delta}'), & i \in J[0], \\ \delta_i' \; / \; \lambda_k \; (\underline{\delta}'), & i \in J[k], \end{cases} \tag{4.18}$$

where k ranges over all positive integers for which $\lambda_k(\underline{\delta}') > 0$.

Notice that when the degree of difficulty n - (m+1) equals zero, a unique solution to the dual can be easily obtained by solving the m+1 equations, 4.10 and 4.11, in n(=m+1) unknowns. To obtain the primal solution vector, first take the logarithm of 4.18 obtaining

$$\sum_{j=1}^{m} a_{ij} \ell n \; t_j = \ell n \; \delta_i' + \ell n \; v \; (\underline{\delta}') - \ell n \; c_i, \; i \in J[0], \tag{4.19}$$

and

$$\sum_{j=1}^{m} a_{ij} \ell n \; t_j = \ell n \; \delta_i' - \ell n \; \lambda_k(\underline{\delta}') - \ell n \; c_i, \; i \in J[k], \tag{4.20}$$

then solve the linear system of equations for $\ell n\ t_j$, $j = 1,2,\ldots,m$. More often than not, the degree of difficulty is greater than zero; hence, an application of the above procedure yields an underdetermined linear system in variables $\ell n\ t_j$, $j = 1,2,\ldots,m$. The factors $\lambda_k(\underline{\delta}')$, $k = 1,2,\ldots,p$, in the dual objective function are Lagrange multipliers multiplied by the constant factor $[g_0(\underline{t}')]^{-1}$.

The second duality theorem of geometric programming states sufficient conditions for the hypothesis of the first theorem to be true.

Theorem 4.2

If Primal Program A is consistent and there is a point $\underline{\delta}^*$ with positive components which satisfies the constraints of Dual Program B, the primal function $g_0(\underline{t})$ attains its constrained minimum value at a point $\underline{t}'$ which satisfies the constraints of Primal Program A.

Part 2 - Some Properties and Limitations of Geometric Programming

4.5 Introduction

Several properties and limitations of geometric programming are now illuminated. Global optimality and geometric programming, geometric programs with reversed constraints, geometric programs with loose constraints, degenerate geometric programs and canonical geometric programs are the primary topics to be discussed. Since the investigation of global optimality depends upon the notion of convexity, a brief review of some definitions and properties concerning convex functions and convex sets can be found in the Appendix.

4.6 Global Optimality and Geometric Programming

The underlying structure of Primal Program A is revealed by forming an equivalent transformed program (160). For the posynomials

$$g_k(\underline{t}) = \sum_{i \in J[k]} c_i \, t_1^{a_{i1}} \, t_2^{a_{i2}} \ldots t_m^{a_{im}},$$

$$k = 0,1,2,\ldots,p, \qquad (4.21)$$

of Primal Program A, let

$$t_j = e^{z_j}, \quad j = 1,2,\ldots,m. \tag{4.22}$$

Then the transformed functions

$$g_k(\underline{z}) = \sum_{i \in J[k]} c_i e^{\sum_{j=1}^{m} a_{ij} z_j},$$

$$k = 0,1,2,\ldots,p, \tag{4.23}$$

are positive exponential functions where

$$z_j \in \{\text{real numbers}\} \text{ and } t_j > 0, \quad j = 1,2,\ldots,m.$$

The transformed primal program is now presented:

Transformed Primal Program A_z

Minimize:

$$g_0(\underline{z}) \tag{4.24}$$

Subject to:

$$\left.\begin{aligned} & g_1(\underline{z}) \leq 1, \\ & g_2(\underline{z}) \leq 1, \\ & \vdots \\ & g_p(\underline{z}) \leq 1, \end{aligned}\right\} \tag{4.25}$$

where:

$$g_k(\underline{z}) = \sum_{i \in J[k]} e^{\sum_{j=1}^{m} a_{ij} z_j} ,$$

$$k = 0,1,2,\ldots,p, \tag{4.26}$$

$$J[k] = \{m_k, m_k+1, m_k+2,\ldots,n_k\},$$

$$k = 0,1,2,\ldots,p, \tag{4.27}$$

$$m_0 = 1,\ m_1 = n_0+1,\ m_2 = n_1+1,\ldots,m_p = n_{p-1}+1$$

$$n_p = n, \tag{4.28}$$

$$a_{ij} \in \{\text{real numbers}\}, \text{ for all } i \in \{1,2,\ldots,n\}$$

$$\text{and for all } j \in \{1,\ldots,m\}, \tag{4.29}$$

$$c_i > 0, \text{ for all } i \in \{1,2,\ldots,n\}, \tag{4.30}$$

$$z_j \in \{\text{real numbers}\}, \text{ for all } j \in \{1,2,\ldots,m\}. \tag{4.31}$$

Duffin, Peterson, and Zener show that any non-trivial primal program can always be formulated such that its exponent matrix (a_{ij}) has rank equal to the

number of primal variables, m, which is strictly less than the number of terms, n (page 84 of reference 160). All geometric programs discussed in this research are assumed to be nontrivial.

The following theorem states the most important property of Transformed Primal Program A_z.

Theorem 4.3 (160)

Transformed Primal Program A_z is a convex program. Each positive exponential function $g_k(\underline{z})$, $k = 0,1,2,\ldots,p$ is convex. Even though the posynomials in Primal Program A may not all be convex, all functions 4.24 through 4.25 of Program A_z are convex. Therefore, Transformed Primal Program A_z is a convex program where every local minimum of $g_0(\underline{z})$ is also a global minimum of $g_0(\underline{z})$.

4.7 Geometric Programs with Reversed Constraints

The minimizing stationary point for Primal Program A is also the global minimizing solution; this cannot be said for posynomial programs with reversed constraints. In general, a geometric program with reversed constraints is no longer a convex program even when transformed, as illustrated

in Section 4.6. There is no guarantee of optimality at the saddle point of the Lagrangian, since it is no longer composed of a positive linear combination of convex functions.

A posynomial program with reversed constraints, Primal Program A^R, is now defined:

Primal Program A^R

Minimize:

$$g_0(\underline{t}) \tag{4.32}$$

Subject to:

$$\left.\begin{aligned} & g_1(\underline{t}) \leq 1, \\ & g_2(\underline{t}) \leq 1, \\ & \vdots \\ & g_p(\underline{t}) \leq 1, \end{aligned}\right\} \tag{4.33}$$

$$\left.\begin{aligned} & g_{p+1}(\underline{t}) \geq 1, \\ & g_{p+2}(\underline{t}) \geq 1, \\ & \vdots \\ & g_{p+q}(\underline{t}) \geq 1, \end{aligned}\right\} \tag{4.34}$$

where:

$$g_k(\underline{t}) = \sum_{i \in J[k]} c_i t_1^{a_{i1}} t_2^{a_{i2}} \ldots t_m^{a_{im}}, \tag{4.35}$$

$$k = 0,1,2\ldots,p,\ p+1,\ p+2,\ldots,p+q \tag{4.36}$$

$$J[k] = \{m_k,\ m_k+1,\ m_k+2,\ldots,\ n_k\},$$

$$k = 0,1,2,\ldots,p,\ p+1,\ p+2,\ldots,p+q \tag{4.37}$$

$$m_0 = 1,\ m_1 = n_0+1,\ m_2 = n_1+1,\ldots,\ m_{p+q} =$$

$$n_{p+q-1}+1,\ n_{p+q} = n \tag{4.38}$$

$$a_{ij} \in \{\text{real numbers}\},\ \text{for all } i \in \{1,2,\ldots,n\}$$

$$\text{and for all } j \in \{1,2,\ldots,m\}, \tag{4.39}$$

$$c_i > 0,\ \text{for all } i \in \{1,2,\ldots,n\}, \tag{4.40}$$

$$\underline{t} > \underline{0}\ . \tag{4.41}$$

To form the transformed Primal Program A_Z^R, let

$$t_j = e^{z_j},\ j = 1,2,\ldots,m, \tag{4.42}$$

then all the posynomials in A_Z^R have the form:

$$g(\underline{z}) = \sum_i c_i e^{\sum_{j=1}^{m} a_{ij} z_j}\ . \tag{4.43}$$

Hence, the Transformed Program A_Z^R is:

Transformed Primal Program A_Z^R

Minimize:

$$g_0(\underline{z}) \tag{4.44}$$

Subject to:

$$\left.\begin{aligned} & g_1(\underline{z}) \leq 1, \\ & g_2(\underline{z}) \leq 1, \\ & \vdots \\ & g_p(\underline{z}) \leq 1, \end{aligned}\right\} \tag{4.45}$$

$$\left.\begin{aligned} & g_{p+1}(\underline{z}) \geq 1, \\ & g_{p+2}(\underline{z}) \geq 1, \\ & \vdots \\ & g_{p+q}(\underline{z}) \geq 1, \end{aligned}\right\} \tag{4.46}$$

where:

$$g_k(\underline{z}) = \sum_{i \in J[k]} e^{\sum_{j=1}^{m} a_{ij} z_j}, \qquad k = 0,1,2,\ldots,p,\ p+1,\ldots,\ p+q, \tag{4.47}$$

$$J[k] = \{m_k, m_k+1, m_k+2, \ldots, n_k\},$$

$$k = 0,1,2,\ldots,p, p+1,\ldots, p+q \qquad (4.48)$$

$$m_0 = 1, m_2 = n_0+1, m_2 = n_1+1,\ldots,$$

$$m_{p+q} = n_{p+q-1}+1, n_{p+q} = n, \qquad (4.49)$$

$$a_{ij} \varepsilon \{\text{real numbers}\}, \text{ for all } i\varepsilon\{1,2,\ldots,n\}$$

$$\text{and for all } j\varepsilon\{1,2,\ldots,m\} \qquad (4.50)$$

$$c_i > 0, \text{ for all } i\varepsilon\{1,2,\ldots,n\}, \qquad (4.51)$$

$$z_j \varepsilon \{\text{real numbers}\}, \; j = 1,2,\ldots,m. \qquad (4.52)$$

Assume that $\underline{z}$ has real valued components. Then, $g_k(\underline{z})$, $k = 0,1,2,\ldots,p, p+1,\ldots, q+p$, can be shown to be convex. Now the respective complements of the sets

$$\{\underline{z}: g_k(\underline{z}) < 1\}, \; k\varepsilon \{p+1, p+2,\ldots, p+q\},$$

which are convex are the sets

$$\{\underline{z}: g_k(\underline{z}) \geq 1\}, \; k\varepsilon \{p+1, p+2,\ldots,p+q\},$$

called reverse convex sets which are not necessarily convex. Program A_Z^R consists of minimizing a convex object function $g_0(\underline{z})$ over the feasible region which is the intersection of the convex sets

$$\{\underline{z}: g_k(\underline{z}) \leq 1\}, k \in \{1,2,\ldots,p\}, \tag{4.53}$$

and the reverse convex sets

$$\{\underline{z}: g_k(\underline{z}) \geq 1\}, k \in \{p+1, p+2,\ldots, p+q\}. \tag{4.54}$$

Hence, the convexity of the feasible region of $\underline{z}$ may have been destroyed by the presence of reverse constraints. Since A_Z^R may not be a convex program, a local minimizing solution to A_Z^R need not be a global minimizing solution.

The Lagrangian for Program A_Z^R is not composed of a positive linear combination of convex functions. Let

$$G_k(\underline{z}) = \begin{cases} g_k(\underline{z}) & , k = 0, \\ g_k(\underline{z}) - 1, & k = 1,2,\ldots,p, \\ 1 - g_k(\underline{z}) & , k = p+1,\ldots, p+q, \end{cases} \tag{4.55}$$

Then the corresponding Lagrangian for Program A_Z^R is

$$L(\underline{z}, \underline{\mu}) = g_0(\underline{z}) + \sum_{k=1}^{p} \mu_k \left[g_k(\underline{z}) - 1 \right]$$

$$+ \sum_{k=p+1}^{p+q} \mu_k \left[1 - g_k(\underline{z}) \right]. \qquad (4.56)$$

Notice that for $\underline{\mu} \geq 0$, the Lagrangian is a convex function in terms of $g_k(\underline{z})$, $k = 0,1,2,\ldots,p$, but concave in terms of $g_k(\underline{z})$, $k = p+1,\ldots,p+q$. Thus the saddle point solution to the Lagrangian need not be the global optimum.

4.8 Degenerate and Canonical Programs

A geometric program is said to be degenerate if any of the primal terms can approach zero without causing at least one other primal term to approach positive infinity; otherwise, the program is said to be canonical (160). Degenerate programs cannot be solved by the method of geometric programming; the associated dual (equation 4.9) is undefined because the dual variable associated with the vanishing primal term is also zero. The degenerate

terms should be removed such that a canonical program can be formulated. Peterson (161) indicates that naturally occurring design problems will be canonical. A properly formulated geometric programming model should always be canonical because of the underlying opposing cost relationship that pervades economic allocation problems.

4.9 Loosely Constrained Programs

Loosely constrained geometric programs present formidable computational problems because all of the dual variables associated with a loose (inactive) constraint at optimality are zero. The Kuhn-Tucker complementary slackness property of optimal solutions requires that the Lagrangian multipliers accompanying a loose constraint equal zero. Since the dual variables corresponding to terms in loose constraints are directly proportional to the Lagrangian multiplier of that constraint (equation 4.16), all of the dual variables associated with that constraint are zero. Hence, the dual objective function is undefined.

Recall that the Lagrangian function for Primal Program A is:

$$L(\underline{t}, \underline{\mu}) = g_0(\underline{t}) + \sum_{k=1}^{p} \mu_k \left[g_k(\underline{t}) - 1 \right]. \qquad (4.57)$$

If $\underline{t}^*$ is a solution to Primal Program A, then the Kuhn-Tucker theorem necessitates that there exists a vector $\underline{\mu}^*$ that satisfies the following relations:

a) $g_k(\underline{t}) - 1 \leq 0$, for all $k \in \{1,2,\ldots,p\}$. (4.58)

b) $\mu_k \left[g_k(\underline{t}) - 1 \right] = 0$, for all $k \in \{1,2,\ldots,p\}$. (4.59)

c) $\mu_k \geq 0$, for all $k \in \{1,2,\ldots,p\}$. (4.60)

d) $$\frac{\partial g_0(\underline{t})}{\partial t_j} + \sum_{k=1}^{p} \mu_k \frac{\partial g_k(\underline{t})}{\partial t_j} = 0 ,$$

for all $j \in \{1,2,\ldots,m\}$. (4.61)

Therefore, if $g_k(\underline{t}) < 1$ for some $k \in \{1,2,\ldots,p\}$, then by the complementary slackness relation 4.59, $\mu_k = 0$. But

$\mu_k = 0 \rightarrow$

$\delta_i = 0$ for all $i \in J[k]$ by relation 4.16 $\rightarrow$

$v(\underline{\delta})$ is undefined.

The loose constraints at optimality should be removed from the Primal Program A. Since it is often not obvious whether a constraint will be loose at optimality, some authors have developed solution procedures that add slack variables to all constraints to assure that the constraints remain tight (142, 130). Such a procedure by McNamara is subsequently discussed.

Part 3 - Solution Procedure

4.10 Introduction

This part reviews two of the current solution procedures for solving geometric programs with nonzero degrees of difficulty, reversed constraints, and negative coefficients. The solution procedures developed by J. R. McNamara and R. J. Duffin are explained in some detail, since they are incorporated in the hybrid solution procedure utilized to solve nonlinear solid waste models developed in Chapters 5 and 6.

4.11 Geometric Programs with Degrees of Difficulty: J. R. McNamara's Solution Procedure

J. R. McNamara (142) developed an algorithm to

solve geometric programs with degrees of difficulty. If the geometric program satisfies certain conditions, an augmented geometric program with degree of difficulty zero can be constructed by adding slack variables to the constraint equations and by constraining these variables in a particular fashion. Furthermore, these slack variables prevent any constraints from becoming inactive, which can lead to computational difficulties.

Suppose that Primal Program A comprised of equations 4.1 through 4.8 has a degree of difficulty greater than zero, i.e., n-m-1 > 0, where n and m are the number of terms and the number of primal variables, respectively. Without loss of generality, assume that:

a) the objective function, $g_0(\underline{t})$ contains m terms, and its exponent matrix $(a_{ij})_{i,j\in\{1,2,\ldots,m\}}$ has rank m, and

b) there exists an active forced constraint in Program A at optimality (page 18 of 142).

Assumption a) guarantees that the augmented program constructed has the degree of difficulty equal to

zero, while assumption b) forces all constraints in the augmented program to be active (equalities) at the optimal solution.

To form the augmented program from Primal Program A, each term in the system of p forced constraints is multiplied by a slack variable t_i, $i\varepsilon\{m+1,m+2,\ldots,n\}$ followed by appending a constraint imposed upon the product of the slack variables. The resultant program is illustrated below:

Primal Program A^0

Minimize:

$$g_0(\underline{t}) = \sum_{i\varepsilon J[0]} c_i t_1^{a_{i1}} t_2^{a_{i2}} \ldots t_m^{a_{im}}, \quad (4.62)$$

Subject to:

$$g_k(\underline{t},\underline{t}_s) = \sum_{i\varepsilon J[k]} c_i t_1^{a_{i1}} t_2^{a_{i2}} \ldots t_m^{a_{im}} t_i \leq 1,$$

$$k = 1,2,\ldots,p \quad (4.63)$$

$$g_{p+1}(\underline{t}_s) = t_{m+1}^{b_{m+1}} t_{m+2}^{b_{m+2}} \ldots t^{b_n} \leq 1,$$

$$t_1 > 0, \ldots, t_n > 0, \quad (4.64)$$

Where:

$$b_i \leq -1, \text{ for all } i \in \{m+1, m+2, \ldots, n\}, \qquad (4.65)$$

and $\underline{t}_s$ is a vector of primal slack variables. The degree of difficulty is $(n+1)-m-1=(n+1)-n-1=0$, since $m=n$.

McNamara proves that if $(\underline{t}', \underline{t}_s')$ solves Program A^o and $\underline{t}_s'$ satisfies

$$t_i' \geq 1, \text{ for all } i \in \{m+1, m+2, \ldots, n\}, \qquad (4.66)$$

then $\underline{t}'$ solves Program A. Condition 4.66 guarantees that no constraints are violated, and represents the stopping mechanism for the solution procedure.

The Solution Procedure

Given the augmented Primal Program A^o, form its respective Dual Program B^o, as shown by equations 4.9 through 4.13. To satisfy condition 4.11, the transpose of the exponent matrix of Program A^o, A^T, is right multiplied by the dual vector $\underline{\delta}$, and equated to zero, i.e., $A^T \underline{\delta} = 0$. However, the central matrix

of this solution procedure, $\underline{\gamma}$, is computed by applying several elementary row and column operations to A^T to obtain the form below.

$$\underline{\gamma} = (\gamma_{ji}) = \begin{matrix} \text{term} \\ \downarrow \\ \\ \uparrow \\ \text{variable} \end{matrix} \begin{bmatrix} 1 & \cdots & 0 & \gamma_{1,m+1} & \cdots & \gamma_{1,n} & 0 \\ \vdots & & \vdots & & & & \vdots \\ \cdot & 1 & \cdot & & \gamma_{ji} & & \cdot \\ \vdots & & \vdots & & & & \vdots \\ 0 & \cdots & 1 & \gamma_{m,m+1} & \cdots & \gamma_{m,n} & 0 \\ 0 & \cdots & 0 & 1 & & 0 & b_{m+1} \\ \vdots & & \vdots & & & & \vdots \\ \cdot & 0 & \cdot & & 1 & & \cdot \\ \vdots & & \vdots & & & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 & b_n \end{bmatrix} \tag{4.67}$$

Certainly,

$$\underline{\gamma}\,\underline{\delta}' = 0 \rightarrow$$

$$A^T\underline{\delta}' = 0. \tag{4.68}$$

Furthermore, condition 4.10,

$$\sum_{i \in J[0]} \delta_i = 1, \text{ and } 4.68$$

$$\rightarrow$$

$$\delta'_{n+1} = 1/\Sigma_{j=1}^{m} \Sigma_{i=m+1}^{n} \gamma_{ji} b'_i \,, \tag{4.69}$$

$$\delta'_j = \delta'_{n+1} \left(\sum_{i=m+1}^{n} \gamma_{ji} b'_i \right), \; j \in \{1,2,\ldots,m\}, \tag{4.70}$$

and

$$\delta'_i = - \delta'_{n+1} \, b'_i \,, \; i \in \{m+1,\ldots,n\}. \tag{4.71}$$

Notice that the program remains canonical if the uniquely determined vector, $\underline{\delta}'$, is strictly positive. The restriction of

$$\sum_{i=m+1}^{n} \gamma_{ji} \, b'_i > 0, \; j \in \{1,2,\ldots,m\} \tag{4.72}$$

along with

$$b'_i \leq -1, \; i \in \{m+1, \; m+2,\ldots,n\}. \tag{4.73}$$

ensures that the problem remains canonical. However, if there exists $j \in \{1,\ldots m\}$ such that $\gamma_{ji} \geq 0$ for all

$i \in \{m+1,\ldots,n\}$, then there does not exist a $\underline{b}$ which satisfies 4.72. Thus the problem is not canonical, and cannot be solved by this method.

The solution procedure is:

a) Determine a preliminary exponent vector $\underline{b}'$ which satisfies 4.72 and 4.73.

b) Compute $\underline{\delta}'$ by utilizing 4.69, 4.70, and 4.71.

c) Calculate $v(\underline{\delta}')$ with formula 4.9.

d) Solve the linear system of 4.19 and 4.20 for $(\underline{t}, \underline{t}_s)$.

e) If $\underline{t}'_s \geq 1 - \varepsilon \cdot \underline{1}$ for ε sufficiently small, stop; the solution vector $\underline{t}'$ solves Program A within an ε neighborhood of a forced constraint. Otherwise, continue.

f) Find $t'_r = \min \{t_i \mid t_i < 1, i \in \{m+1,\ldots,n\}\}$.

Let $b''_r = b'_r / t'_r$, $r \in \{m+1,\ldots,n\}$. (4.74)

g) Go to b).

The solution obtained by this procedure, say $\underline{t}^o$, is approximately the true optimal solution, $\underline{t}^*$,

depending upon the size of the stopping criteria, ε, i.e., $g_0(\underline{t}^0) \simeq g_0(t^*)$. However, the procedure may fail to converge while utilizing rule 4.74. Furthermore, condition 4.72 may not be satisfied. McNamara states that this procedure has worked well in the past and that "it has always been possible to modify rule 4.74 and achieve convergence"(142). One modification that he mentions is

$$b_r'' = b_r' + (1-t_r') b_r' \tag{4.75}$$

If b_r fails to satisfy 4.72, then the following rule is invoked:

$$b_r'' = b_r' + \tfrac{1}{2}(\bar{b}_r'' - b_r') \tag{4.76}$$

where

$$\left|\bar{b}_r''\right| = \min_{\{\gamma_{ji}>0,\ j=1,\ldots,m\}} \left| \sum_{\substack{i=m+1 \\ i\neq r}}^{i=n} \gamma_{ji}\ b_i'/\gamma_{jr} \right| \tag{4.77}$$

Whenever an infeasible $\underline{b}$ vector is generated by either rule 4.74 or rule 4.75, the procedure must continuously employ rule 4.76.

4.12 Geometric Programming with Reversed Constraints and Negative Coefficients: R. J. Duffin's Condensation Procedure

Many nonlinear engineering problems cannot be modeled without negative coefficients and reversed constraints. In reference 162, Duffin reviews his calculation technique to transform a signed geometric program with possibly reversed constraints to an approximating posynomial geometric program in standard form. The calculation procedure consists of solving a converging sequence of posynomial programs in accord with a rule to adjust certain weights at each interaction. The condensation method is now exemplified.

Let $g_k(\underline{t})$, $k = 0,1,2$ be posynomials. Then define the signed program:

Program A_s

Minimize:

$$g_0(\underline{t}) \tag{4.78}$$

Subject to:

$$g_1(\underline{t}) - g_2(\underline{t}) \leq 1. \tag{4.79}$$

Since $g_k(\underline{t}) > 0$ for k = 1,2, and 4.79 holds, there exists $t_{m+1} > 0$ such that

$$g_1(\underline{t}) \leq t_{m+1} \leq g_2(\underline{t}) + 1$$

$$\rightarrow$$

$$t_{m+1}^{-1}\, g_1\,(\underline{t}) \leq 1 \leq t_{m+1}^{-1}\, g_2\,(\underline{t}) + t_{m+1}^{-1}$$

or

$$t_{m+1}^{-1}\, g_1\,(\underline{t}) \leq 1 \quad \text{and} \quad t_{m+1}^{-1}\, g_2\,(\underline{t}) + t_{m+1}^{-1} \geq 1.$$

Define a new posynomial

$$G_2(t_1,t_2,\ldots,t_m,\, t_{m+1})$$

$$= t_{m+1}^{-1}\, g_2\,(\underline{t}) + t_{m+1}^{-1} \geq 1 \qquad (4.80)$$

which has the form

$$G_2(t_1,t_2,\ldots,\, t_{m+1}) = \sum_{i=1}^{N} u_i\ .$$

Since

$$G_2(\underline{t},\ t_{m+1}) \geq \prod_{i=1}^{N} \left(\frac{u_i}{\xi_i}\right)^{\xi_i} = \overline{G}_2(\underline{t},\ t_{m+1})$$

for

$$\sum_{i=1}^{N} \xi_i = 1 \text{ and } \xi_i > 0, \text{ for all } i\epsilon\{1,2,\ldots,N\},$$

constraint 4.79 can be approximated with the posynomial constraint

$$\left[\overline{G}_2\ (\underline{t},\ t_{m+1})\right]^{-1} \leq 1. \qquad (4.81)$$

The point $\underline{t}^0$ satisfies 4.81, then

$$G_2(\underline{t}) \geq \overline{G}_2(\underline{t}) \geq 1.$$

The new posynomial program is:

Program A_c

Minimize:

$$g_0(\underline{t}) \qquad (4.78)$$

Subject to:

$$t_{m+1}^{-1}\, g_1(\underline{t}) \leq 1 \tag{4.82}$$

$$\left[\overline{G}_2\ (\underline{t},\ t_{m+1})\right]^{-1} \leq 1 \tag{4.83}$$

Given a feasible point $\underline{t}^1$ that is consistent with 4.82 and 4.83 defined by

$$\xi_i^1 \ = \ u_i(\underline{t}^1)/G_2(\underline{t}^1),$$

$$i\varepsilon\{1,2,\ldots,N\}, \tag{4.84}$$

solve A_c with parameters, ξ_i^1 , $i\varepsilon\{1,2,\ldots,N\}$ for a minimum $\underline{t}^2$. Recompute the ξ_i^j , $i\varepsilon\{1,2,\ldots,N\}$, parameters until for j=n-1

$$g_0(\underline{t}^n) - g_0(\underline{t}^{n-1}) \leq \varepsilon \tag{4.85}$$

for a given tolerance ε. When condition 4.85 is satisfied, the procedure terminates, since, if $\underline{t}^*$ is the true optimum, then

$$g_0(\underline{t}^*) \leq g_0(\underline{t}^n) \leq g(\underline{t}^{n-1}) \leq \ldots \leq g_0(\underline{t}^1).$$

4.13 A Solution Procedure for Geometric Programs with Degrees of Difficulty, Reversed Constraints, and Loose Constraints

The nonlinear solid waste models solved in Chapter 6 are formulated as geometric programs which are characterized by large degrees of difficulty, reversed constraints, and loose constraints. The solution procedure utilized for solving these programs is a hybrid composed of the solution procedures developed by McNamara and Duffin (142, 162). The procedure involves condensing the reversed constraints followed by augmenting the program with slack variables to create a geometric programming problem possessing degree of difficulty zero. The solution to the original program, Program P, requires the iterative estimation of certain parameters associated with the condensed and augmented program, Program CA.

Assume that Program P is in standard geometric programming form except for reversed constraints such as 4.80. Condense the reversed constraints in

Program P as illustrated by equations 4.80 through 4.81 to form Program C. Since the condensed program should be in standard form, alter the condensed program as necessary to satisfy conditions a and b of McNamara's procedure. Now construct the augmented program, Program CA, as illustrated by relations 4.62 through 4.64. Given a feasible primal vector $\underline{t}$, compute the initial components of the weight vector $\underline{\xi}$ with formula 4.84. The iterative solution procedure is:

Phase 1

Solve Program CA with McNamara's procedure.

Phase 2

If 4.85 is satisfied, stop; otherwise, revise the weight vector with formula 4.84 and go to Phase 1.

A geometric program in standard form is solved in Phase 1 for each vector of weights calculated in Phase 2. Even though the solutions to the geometric programs computed in Phase 1 are global minimums,

there is no guarantee that the sequence of geometric programs solved by the solution procedure will converge upon the global minimum. (Refer to Section 4.7.)

CHAPTER 5

MODELING NONLINEAR SOLID WASTE MANAGEMENT MODELS WITH GEOMETRIC PROGRAMMING

5.1 Introduction

In this chapter, the components of a general nonlinear model for municipal solid waste management are developed. This model is intended to determine the preliminary design of municipal solid waste systems from socially and politically acceptable alternatives. The nonlinear model consists of an objective function and several constraints which transform the verbal model in Chapter 3 into a quantitative format. The solution to the model describes the routes through the solid waste system for which a specified number of tons per day of municipal solid waste should be handled in an optimal fashion.

5.2 Description of the Proposed Nonlinear Solid Waste Management Model

The following model illustrates process selection at the resource recovery plant and material handling of the waste through a transportation network. The capital and operating costs represented

in the objective function are generated by the resource recovery plant, the landfill sites, and the hauling of the waste throughout the transportation network. Landfill capacity, technological, legal, and balance constraints comprise the four classes of restrictions imposed upon the model.

The nodes of the transportation network consist of the resource recovery plants, the landfill sites, and the generating centroids. The landfills and resource recovery plants are restricted to areas which are politically and socially acceptable as nuisance sites while the generating centroids are determined by the municipal solid waste analyst. Computation of the generating centroids requires the analyst to partition the study area into several regions which include towns, cities, and boroughs each assigned a geographical location or map coordinates, and a solid waste factor. The quantity of waste is assumed to be concentrated at coordinates representing the town, city, or borough geographically, or at the center of population. The calculation of the location of the generating centroid is now shown:

$$\sigma_{EW} = \frac{\sum_{i=1}^{r} \omega_i d_i^{EW}}{\sum_{i=1}^{r} \omega_i} \qquad (5.1)$$

$$\sigma_{NS} \quad \frac{\sum_{i=1}^{r} \omega_i d_i^{NS}}{\sum_{i=1}^{r} \omega_i} \qquad (5.2)$$

where (σ_{EW}, σ_{NS}) are the waste centroidal coordinates, and where:

d_i^{EW} ≡ the distance from the origin in the east-west direction for town i.

d_i^{NS} ≡ the distance from the origin in the north-south direction from town i.

ω_i ≡ the tons per day of solid waste generated at town i.

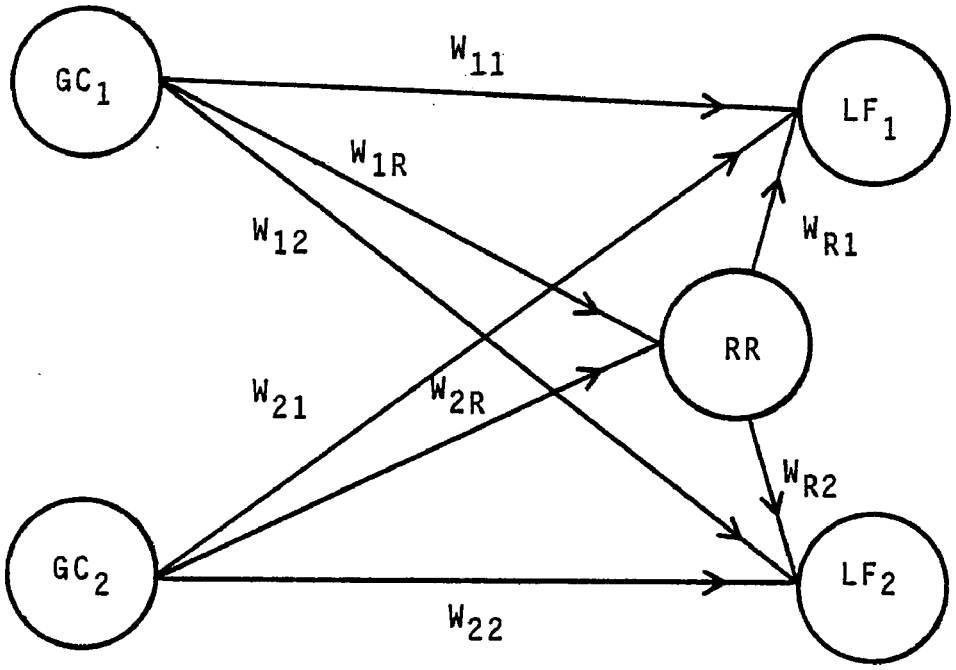
r ≡ the number of towns in the region.

σ_{EN} ≡ the map coordinate for the east-west centroid location.

$\sigma_{NS} \equiv$ the map coordinate for the north-south centroid location.

The solid waste that flows from the generating centroids to the resource recovery plant is reduced by a series of processes such as magnetic separation and/or pyrolysis and/or pulverization at the plant. The nonrecycled residue from the resource recovery plant along with the remaining unprocessed waste at the generating centroids is shipped to the landfill site(s). For illustrative purposes, Figure 5.1 depicts the overall structure of a solid waste management model composed of one resource recovery plant, two generating centroids and two landfill sites; while Figure 5.2 portrays the sequential nature of the modeling for the resource recovery processes.

The mathematical expressions for the nonlinear solid waste model will be illuminated by deriving an example model for a system composed of two sources, one resource recovery plant, and two landfills (Figure 5.1). The underlying structure of this model is comprehensive enough to expose the mathematical relationships present in larger systems without burdening the reader with the more



Legend

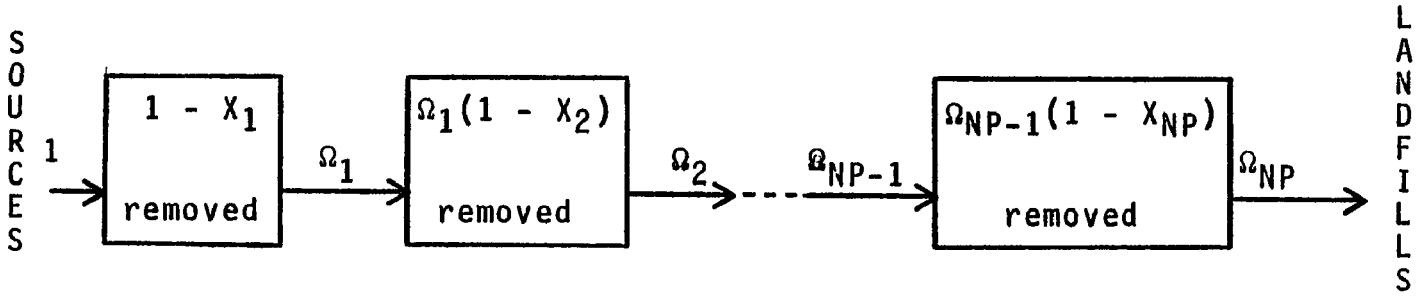
GC_i ≡ generating centroid i, i=1,2.

LF_i ≡ landfill site i, i=1,2.

RR ≡ resource recovery plant.

W_{ij} ≡ tpd of solid waste at i shipped to j.

FIGURE 5.1. Solid Waste Management Model: Two Generating Centroids, One Resource Recovery Plant, Two Landfill Sites.



Legend

X_j ≡ fraction of solid waste entering process j that is remaining after process j, j = 1,2,...,NP, where NP is the number of processes.

Ω_k ≡ fraction of solid waste entering the resource recovery plant that is remaining after process k, i.e.,

$$\Omega_k = \prod_{j=1}^{k} X_j, \quad k = 1,2,\ldots,NP.$$

FIGURE 5.2. Sequential Process Model at Resource Recovery Plant.

cumbersome notation present in the general model. After the derivation of the small example, the general model will be stated.

5.3 A Solid Waste Model Comprised of One Resource Recovery Plant, Two Generating Centroids (Sources), and Two Landfill Sites

Landfill Capacity Constraint

Let

$D_{LF_i} \equiv$ capacity of the i^{th} landfill site in tons per day, i = 1,2, and

$W_{ij} \equiv$ the tons per day of solid waste at i shipped to j.

The amount to be sent to the i^{th} landfill site must be less than or equal to its capacity. Therefore, the constraints are

$$\sum_{j=1}^{2} W_{ji} + W_{Ri} \leq D_{LF_i} ,$$

$$i = 1,2. \qquad (5.3)$$

Technological and Legal Constraints

Let

$L_j \equiv$ the implicit lower bound on X_j which is due to the current technology only being capable of feasibly removing the fraction $1 - L_j$ by the j^{th} process, or is due to the fraction of the material in municipal solid waste that is treated by process j being equal to $1 - L_j$; L_j must be between 0 and 1,

and

$U_j \equiv$ the upper bound on X_j which is due to a legal maximum remaining of the particular solid waste that process j does not remove and that can be dumped at the landfill site; it also must be between 0 and 1, and greater than L_j, where $j = 1,2,\ldots,NP$.

Recall

$X_j \equiv$ fraction of solid waste entering process j that is remaining after process j, $j = 1,2,\ldots,NP$.

Then for the lower bound constraints, we have

$$(1 - X_j) \prod_{i=1}^{j-1} X_i \leq 1 - L_j$$

or

$$\prod_{i=1}^{j-1} X_i \leq (1 - L_j) + \prod_{i=1}^{j} X_i ,$$

$$\text{for all } j \in \{1,2,\ldots,NP\}. \qquad (5.4)$$

The upper bound constraints are simply written

$$U_j^{-1} X_j \leq 1,$$

$$\text{for all } j \in \{1,2,\ldots,NP\}. \qquad (5.5)$$

Balance Constraints

Let

$S_i \equiv$ tons per day of municipal solid waste generated at the i^{th} source, i=1,2.

To insure that 100% of the solid waste is removed from the generating centroids and the recycling plant,

$$\sum_{j\in\{1,2,R\}} W_{ij} \geq S_i \text{ , for } i = 1,2, \qquad (5.6)$$

and

$$(W_{1R} + W_{2R}) \prod_{j=1}^{NP} X_j \leq W_{R1} + W_{R2}, \qquad (5.7)$$

respectively.

Daily Processing and Capital Equipment Costs at the Resource Recovery Plant

At a given level of technology and for fixed X_j, $j = 1,2,\ldots,NP$, the capital equipment costs and operating costs illustrate economies of scale with respect to increases in capacity. This is true for separators, grinders, shredders, and a host of other equipment (36). (See Figures 5.3 and 5.4.)

On the contrary, the processing costs exhibit diseconomies of scale when increasing fractions of the waste stream are removed by some process j. (See Figure 5.5.) The total daily processing and capital equipment cost relationship is now shown.

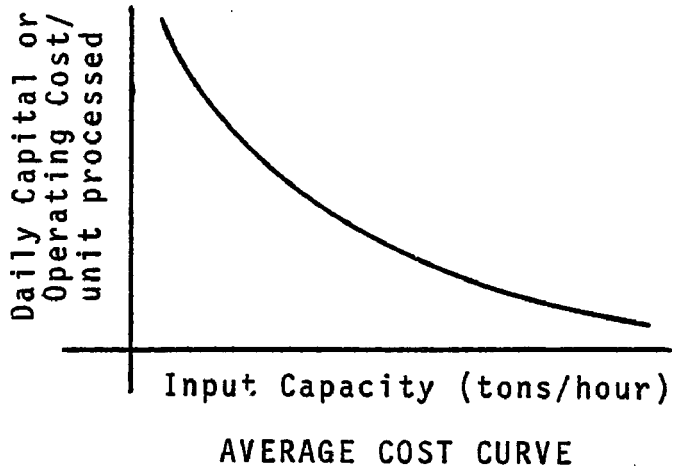


FIGURE 5.3. Average Capital and Operating Costs versus Capacity at the Resource Recovery Plant.

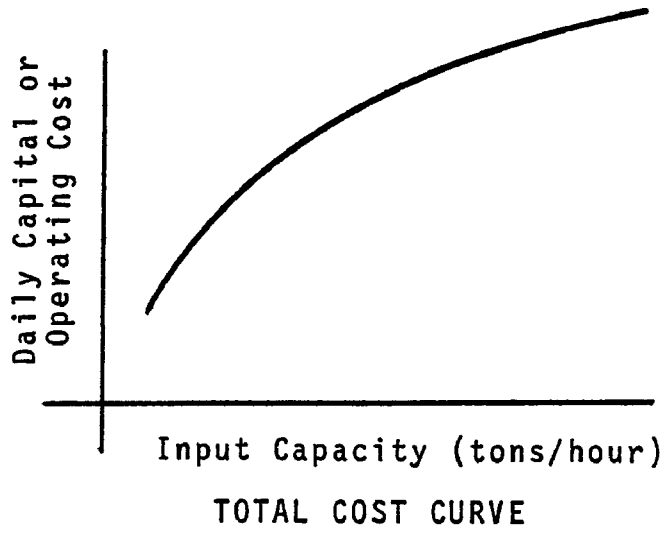


FIGURE 5.4. Total Capital and Operating Costs versus Capacity at the Resource Recovery Plant.

Let

K_R ≡ total daily processing and capital equipment cost,

d_R, d'_{Ri} ≡ fixed constants greater than zero,

e_R, e'_{Ri} ≡ fixed constants where e_{Ri}, $e'_{Ri} \in (0,1)$; their range guarantees the economies of scale property, and

f_{Ri} ≡ fixed constants less than zero for diseconomies of scale with respect to the fraction removed and the cost of equipment, i = 1,2,...,NP.

Define the cost relationship as:

$$K_R = d_R \left[W_{1R} + W_{2R} \right]^{e_R}$$

$$+ \sum_{k=1}^{NP} d'_{Rk} \left[(W_{1R} + W_{2R}) \prod_{j=1}^{k-1} X_j \right]^{e'_{Rk}} X_k^{f_{Rk}} \qquad (5.8)$$

The first cost term models the building and land costs associated with the plant; it, therefore, follows the economies of scale property with respect

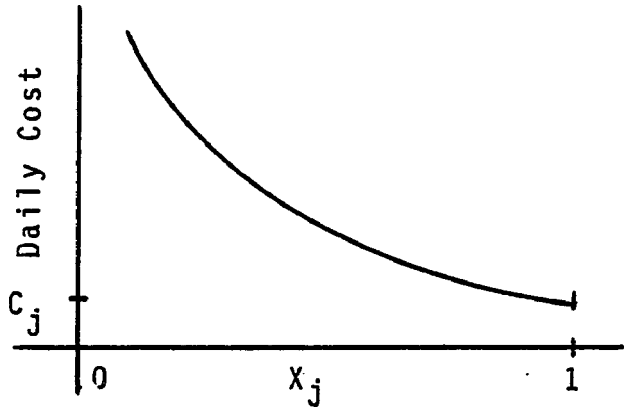


FIGURE 5.5. Processing Costs versus Level of Efficiency at Resource Recovery Plant.

Source: Liebman, Jon, "Mathematical Models of Solid Waste Collection and Disposal," presented at the 38th National Meeting of the Operations Research Society of America (1970), p. 17.

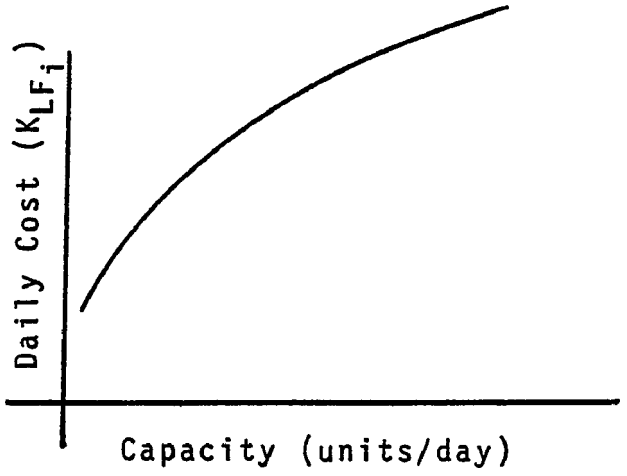


FIGURE 5.6. Total Landfill Costs versus Capacity.

to the plant size. The second term includes the economies of scale property for the capital equipment, amortized purchasing costs, and equipment operating costs. It also contains the diseconomies of scale factor for the increasing cost phenomenon associated with reclaiming higher fractions of the municipal solid waste.

Total Daily Landfill Site Costs

Let

$K_{LF_i} \equiv$ daily operating and owning costs of the i^{th} sanitary landfill, $i = 1,2$.

The cost function has the property of economies of scale which is illustrated in Figure 5.6 (page 86 of 71; pages 68-69 of 36).

Let

$d_{LF_i} \equiv$ fixed constant strictly greater than 0, $i = 1,2$, and

$e_{LF_i} \equiv$ fixed constant where $0 < e_{LF_i} < 1$, $i = 1,2$.

Now,

$$K_{LF_i} = d_{LF_i} \left[W_{1i} + W_{2i} + W_{Ri} \right]^{e_{LF_i}}, \quad i = 1,2. \tag{5.9}$$

Daily Transportation Costs

For a specific area, as the population density increases, the cost per person decreases (page 39 of 71). This implies that the collection cost per ton decreases as the population density increases, assuming that each person generates the same quantity of solid waste on the average. The long distance transport of solid waste from transfer stations exhibits economies of scale (page 45 of 57; page 61 of 42, 78). Little is said concerning the direct hauling of municipal solid waste to the resource recovery plants of landfills from their respective collection areas, except that larger capacity trucks are more efficient. Given that the compactor truck sizes are fixed, the total daily transportation costs may exhibit increasing costs to a landfill when experiencing heavy traffic conditions at the landfill because of the longer

time spent in a queue at the landfill. Since the resource recovery plant is being specifically designed for a specific waste load, a slight economies of scale could be experienced both at the unloading platforms for unprocessed waste and at the loading platforms of the unreclaimed residue waste.

The total daily cost relationship for transporting municipal solid waste is now illustrated where for economies of scale $\tau_{ij} \in (0,1)$ (See Figure 5.7), while for diseconomies of scale, $\tau_{ij} > 1$ (See Figure 5.8).

Let T_{ij} be the daily cost relationship for transporting solid waste from i to j.

Let $\alpha_{ij} \equiv$ fixed constants greater than 0, and

$\tau_{ij} \equiv$ fixed constants where $0 < \tau_{ij} < 1.5$, all i, j.

Then

$$T_{ij} = \alpha_{ij} W_{ij}^{\tau_{ij}} \qquad (5.10)$$

for all $i \in \{1,2\}$, for all $j \in \{1,2,R\}$, and

$$T_{Ri} = \alpha_{Ri} W_{Ri}^{\tau_{Ri}} \text{ , for all } i \in \{1,2\}. \qquad (5.11)$$

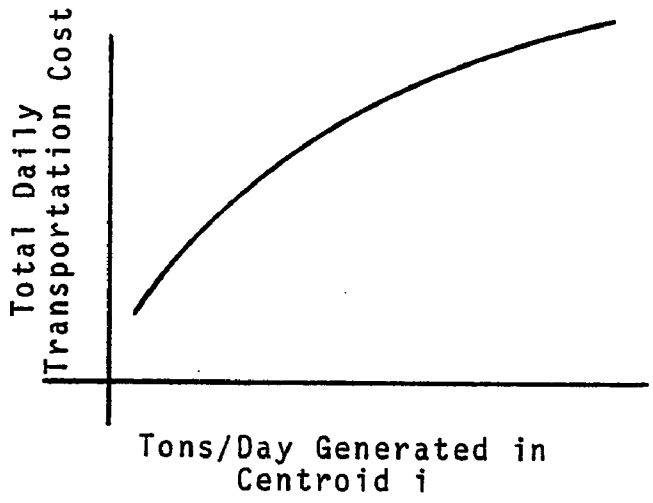


FIGURE 5.7. Total Daily Transportation Cost versus Tons Hauled, Exhibiting Economies of Scale.

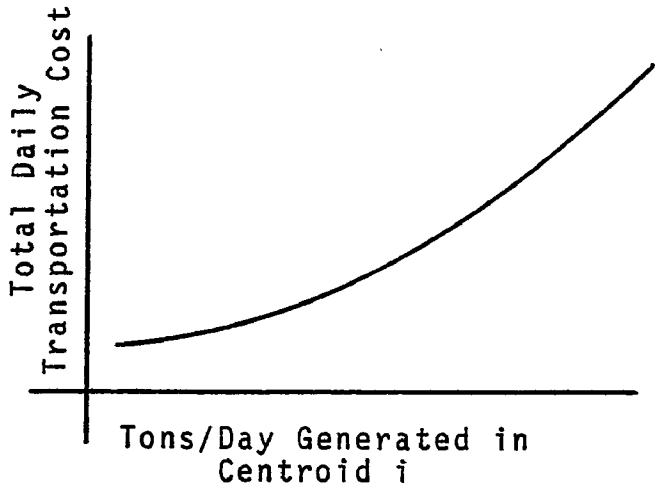


FIGURE 5.8. Total Daily Transportation Cost versus Tons Hauled, Exhibiting Diseconomies of Scale.

The mathematical model is:

Minimize:

$$K_R + \sum_{i=1}^{2} \left[K_{LF_i} + T_{Ri} + \sum_{j\in\{1,2,R\}} T_{ij} \right] \tag{5.12}$$

Subject to:

$$\sum_{k=1}^{2} W_{ki} + W_{Ri} \leq D_{LF_i}, \quad i=1,2, \tag{5.3}$$

$$\prod_{i=1}^{j-1} X_i \leq (1 - L_j) + \prod_{i=1}^{j} X_i,$$

$$\text{for all } j\in\{1,2,\ldots,NP\}, \tag{5.4}$$

$$U_j^{-1} X_j \leq 1, \text{ for all } j\in\{1,2,\ldots,NP\}, \tag{5.5}$$

$$\sum_{j\in\{1,2,R\}} W_{ij} \geq S_i \ , \ i = 1,2, \text{ and} \tag{5.6}$$

$$\left[\sum_{i=1}^{2} W_{iR} \right] \prod_{j=1}^{NP} X_j \leq \sum_{j=1}^{2} W_{Rj} \tag{5.7}$$

The objective function (5.12) and the constraints (5.4), (5.6), and (5.7) are not in standard geometric programming form. Therefore, the following transformations and condensations are necessary.

Transform (5.9) to:

$$\kappa_{LF_i} = d_{LF_i} Z_i^{e_{LF_i}}, \quad i = 1,2 \tag{5.13}$$

subject to

$$Z_i^{-1} \left(\sum_{j \in \{1,2,R\}} W_{ji} \right) \leq 1, \quad i = 1,2. \tag{5.14}$$

Transform (5.8) to

$$\kappa_R = d_R Z_3^{e_R} + \sum_{k=1}^{NP} d'_{Rk} \left[Z_3 \prod_{j=1}^{k-1} X_j \right]^{e'_{Rk}} X_k^{f_{Rk}} \tag{5.15}$$

subject to

$$Z_3^{-1} \left[\sum_{j=1}^{2} W_{jR} \right] \leq 1 \tag{5.16}$$

Constraint (5.4) needs to be condensed by Duffin's procedure (176). Therefore, by condensation, we obtain

$$\prod_{i=1}^{j-1} X_i \left(\frac{1-L_j}{\xi_{2j-1}} \right)^{-\xi_{2j-1}} \left(\frac{\prod_{i=1}^{j} X_i}{\xi_{2j}} \right)^{-\xi_{2j}} \leq 1,$$

$$\xi_{2j-1} + \xi_{2j} = 1, \text{ for all } j \in \{1,2,\ldots,NP\}. \qquad (5.17)$$

Condensation is also necessary for constraints (5.6) and (5.7). Therefore, for (5.6), we have

$$S_1 \left(\frac{W_{11}}{\xi_1'} \right)^{-\xi_1'} \left(\frac{W_{12}}{\xi_2'} \right)^{-\xi_2'} \left(\frac{W_{1R}}{\xi_3'} \right)^{-\xi_3'} \leq 1, \qquad (5.18)$$

$$S_2 \left(\frac{W_{21}}{\xi_4'} \right)^{-\xi_4'} \left(\frac{W_{22}}{\xi_5'} \right)^{-\xi_5'} \left(\frac{W_{2R}}{\xi_6'} \right)^{-\xi_6'} \leq 1, \qquad (5.19)$$

and for (5.7) we obtain

$$Z_3 \prod_{j=1}^{NP} X_j \left[\frac{W_{R1}}{\xi'_7}\right]^{-\xi'_7} \left[\frac{W_{R2}}{\xi'_8}\right]^{-\xi'_8} \leq 1, \qquad (5.20)$$

where

$$\sum_{i=1}^{3} \xi'_i = \sum_{i=4}^{6} \xi'_i = \sum_{i=7}^{8} \xi'_i = 1.$$

The model in standard geometric programming form is

Minimize:

$$\kappa_R + \sum_{i=1}^{2} \left[\kappa_{LF_i} + T_{Ri} + \sum_{j\in\{1,2,R\}} T_{ij}\right] \qquad (5.21)$$

Subject to:

$$\sum_{k=1}^{2} W_{ki} + W_{Ri} \leq D_{LF_i}, \quad i = 1,2, \qquad (5.3)$$

$$\prod_{i=1}^{j-1} X_i \left[\frac{1-L_j}{\xi_{2j-1}}\right]^{-\xi_{2j-1}} \left[\frac{\prod_{i=1}^{j} X_i}{\xi_{2j}}\right]^{-\xi_{2j}} \leq 1,$$

for all $j\in\{1,2,\ldots,NP\}$, (5.17)

$$U_j^{-1} X_j \leq 1,$$

for all $j \in \{1,2,\ldots,NP\}$, (5.5)

$$S_i \left(\frac{W_{i1}}{\xi_{3i-2}}\right)^{-\xi_{3i-2}} \left(\frac{W_{i2}}{\xi_{3i-1}}\right)^{-\xi_{3i-1}} \left(\frac{W_{iR}}{\xi_{3i}}\right)^{-\xi_{3i}} \leq 1,$$

$$i = 1,2, \tag{5.22}$$

$$Z_3 \prod_{j=1}^{NP} X_j \left(\frac{W_{R1}}{\xi'_7}\right)^{-\xi'_7} \left(\frac{W_{R2}}{\xi'_8}\right)^{-\xi'_8} \leq 1, \tag{5.23}$$

where

$$\sum_{i=1}^{3} \xi'_i = \sum_{i=4}^{6} \xi'_i = \sum_{i=7}^{8} \xi'_i = 1,$$

$$Z_i^{-1} \left(\sum_{j \in \{1,2,R\}} W_{ji}\right) \leq 1,$$

$$i = 1,2, \tag{5.24}$$

$$Z_3^{-1} \left(\sum_{j=1}^{2} W_{jR}\right) \leq 1. \tag{5.25}$$

5.4 The General Model

The development of the general nonlinear municipal solid waste model is identical to the previous example, except for limits of summation and some appended subscripts. The presentation of the general case exposes the nature of a municipal solid waste system comprised of NS sources; NRR resource recovery plants with NP_ℓ processes in the ℓ^{th} plant, $\ell = 1,2,\ldots,NRR$; and NLF landfill sites. When applying the general model to actual municipal solid waste management problems, some terms and/or constraints may be inapplicable and should be omitted. The general model is now stated:

Landfill Capacity Constraints

Let

$X_j^\ell \equiv$ fraction of solid waste entering process j at plant ℓ that is remaining after process j, $j=1,\ldots,NP_\ell$ where NP_ℓ is the number of processes at plant ℓ.

$D_{LF_i} \equiv$ capacity of the i^{th} landfill site in tons per day (tpd), $i = 1,2,\ldots,NLF$, where NLF is the number of landfills.

$W_{ij} \equiv$ tons per day of municipal solid waste shipped from source i to landfill j, where $i \in \{1,2,\ldots,NS\}$ and $j \in \{1,2,\ldots,NLF\}$.

$W_{R_\ell j} \equiv$ tons per day of municipal solid waste shipped from resource recovery plant R_ℓ to landfill j, where $\ell \in \{1,2,\ldots,NRR\}$, NRR is the number of resource recovery plants, and $j \in \{1,2,\ldots,NLF\}$.

$W_{iR_\ell} \equiv$ tons per day of municipal solid waste shipped from source i to resource recovery plant R_ℓ where $i \in \{1,2,\ldots,NS\}$ and $\ell \in \{1,2,\ldots,NRR\}$.

The set of constraints to prevent violating the landfills' daily capacities is:

$$\sum_{i=1}^{NS} W_{ij} + \sum_{\ell=1}^{NRR} W_{R_\ell j} \leq D_{LF_j}$$

$$j \in \{1,2,\ldots,NLF\}. \qquad (5.26)$$

Technological and Legal Constraints

Let

L_j^{ℓ} ≡ the lower bound on X_j^{ℓ} which is due to the current technology only being capable of feasibly removing $1 - L_j^{\ell}$ by the j^{th} process, or is due to the fraction of material in municipal solid waste that is treated by process j being equal to $1 - L_j^{\ell}$; it must be between 0 and 1,

and

U_j^{ℓ} ≡ the upper bound on X_j^{ℓ} which is due to a legal maximum remaining of the particular solid waste that process j does not remove and that can be dumped at the landfill site; it must also be between 0 and 1 and strictly greater than L_j^{ℓ} for all $j \in \{1,\ldots,NP_{\ell}\}$ and $\ell \in \{1,\ldots,NRR\}$.

Then the lower bounded constraints are:

$$(1 - X_j^{\ell}) \prod_{i=1}^{j-1} X_i^{\ell} \leq 1 - L_j^{\ell} \rightarrow$$

$$\prod_{i=1}^{j-1} X_i^{\ell} \leq (1 - L_j^{\ell}) + \prod_{i=1}^{j} X_i^{\ell} ,$$

for all $j \in \{1,2,\ldots,NP_{\ell}\}$ and

for all $\ell \in \{1,2,\ldots,NRR\}$. (5.27)

The upper bound constraints are simply written

$$\left[U_j^{\ell}\right]^{-1} X_j^{\ell} \leq 1,$$

for all $j \in \{1,2,\ldots,NP_{\ell}\}$ and

for all $\ell \in \{1,2,\ldots,NRR\}$. (5.28)

Balance Constraints

Let

$S_i \equiv$ amount of solid waste generated at the i^{th} source in tpd, $i = 1,2,\ldots,NS$, where NS is the number of generating centroids.

To insure that 100 per cent of the solid waste is removed from the generating centroids and the recycling plant,

$$\sum_{j=1}^{NLF} W_{ij} + \sum_{\ell=1}^{NRR} W_{iR_\ell} \geq S_i \ ,$$

for all $i \in \{1,2,\ldots,NS\}$, and (5.29)

$$\left[\left(\sum_{i=1}^{NS} W_{iR_\ell}\right) \prod_{j=1}^{NP_\ell} X_j^\ell\right] \leq \sum_{j=1}^{NLF} W_{R_\ell j}$$

for all $\ell \in \{1,2,\ldots,NRR\}$. (5.30)

Daily Processing and Capital Equipment Costs at the Resource Recovery Plant

Let

K_{R_ℓ} ≡ total daily processing and capital equipment cost,

d_{R_ℓ}, $d'_{R_\ell i}$ ≡ fixed constants > 0,

e_{R_ℓ}, $e'_{R_\ell i}$ ≡ fixed constants in the open interval (0,1); their range guarantees the economies of scale property,

and

$f_{R_\ell i}$ ≡ fixed constants < 0 for diseconomies of scale with respect to the fraction removed and the cost of equipment, $i \in \{1,2,\ldots,NP_\ell\}$, $\ell \in \{1,2,\ldots,NRR\}$.

Define the cost relationships as:

$$K_{R_\ell} = d_{R_\ell}\left[\sum_{i=1}^{NS} W_{iR_\ell}\right]^{e_{R_\ell}} + \sum_{k=1}^{NP_\ell} d'_{R_\ell k}\left[\left(\sum_{i=1}^{NS} W_{iR_\ell}\right)\prod_{j=1}^{k-1} X_j^\ell\right]^{e'_{R_\ell k}} X_k^{f_{R_\ell k}},$$

for all $\ell \in \{1,2,\ldots,NRR\}$. (5.31)

Landfill Site Costs

Let

$K_{LF_i} \equiv$ daily operating and owning costs of the i^{th} sanitary landfill,

$d_{LF_i} \equiv$ fixed constant strictly greater than 0,

and

$e_{LF_i} \equiv$ fixed constant where $0 < e_{LF_i} < 1$, $i = 1,2,\ldots,NLF$.

Now,

$$K_{LF_i} = d_{LF_i} \left[\sum_{k=1}^{NS} W_{ki} + \sum_{\ell=1}^{NRR} W_{R_\ell i} \right]^{e_{LF_i}}$$

for all $i \in \{1,2,\ldots,NLF\}$. (5.32)

Total Daily Transportation and Collection Costs

Let T_{ij}, $T_{R_\ell j}$, and T_{iR_ℓ} be the daily cost relationships for transporting solid waste from i to j, from R_ℓ to j, and from i to R_ℓ, respectively.

Let α_{ij}, $\alpha_{R_\ell j}$, and α_{iR_ℓ} be fixed constants greater than 0,

and τ_{ij}, $\tau_{R_\ell j}$, and τ_{iR_ℓ} be fixed constants where $0 < \tau_{ij} < 1.5$, $i\varepsilon\{1,2,\ldots,NS\}$, $j\varepsilon\{1,2,\ldots,NLF\}$, $\ell\varepsilon\{1,2,\ldots,NRR\}$.

Then

$$T_{ij} = \alpha_{ij} W_{ij}^{\tau_{ij}}, \quad i\varepsilon\{1,2,\ldots,NS\},$$

$$j\varepsilon\{1,\ldots,NLF\}, \tag{5.33}$$

$$T_{R_\ell j} = \alpha_{R_\ell i} W_{R_\ell i}^{\tau_{R_\ell i}}, j\varepsilon\{1,2,\ldots,NLF\},$$

$$\ell\varepsilon\{1,2,\ldots,NRR\} \tag{5.34}$$

$$T_{iR_\ell} = \alpha_{iR_\ell} W_{iR_\ell}^{\tau_{iR_\ell}}, \quad i\varepsilon\{1,2,\ldots,NS\},$$

$$\ell\varepsilon\{1,2,\ldots,NRR\}. \tag{5.35}$$

Therefore, the general nonlinear model is

Minimize:

$$\sum_{\ell=1}^{NRR} K_{R_\ell} + \sum_{i=1}^{NLF} K_{LF_i} + \sum_{i=1}^{NS} \sum_{j=1}^{NLF} T_{ij} +$$

$$\sum_{\ell=1}^{NRR} \sum_{j=1}^{NLF} T_{R_\ell j} + \sum_{i=1}^{NS} \sum_{\ell=1}^{NRR} T_{iR_\ell} \qquad (5.36)$$

Subject to:

$$\sum_{i=1}^{NS} W_{ij} + \sum_{\ell=1}^{NRR} W_{R_\ell j} \leq D_{LF_j},$$

for all $j \in \{1,2,\ldots,NLF\}$, (5.26)

$$\sum_{j=1}^{NLF} W_{ij} + \sum_{\ell=1}^{NRR} W_{iR_\ell} \geq S_i ,$$

for all $i \in \{1,2,\ldots,NS\}$, (5.29)

$$\left(\sum_{i=1}^{NS} W_{iR_\ell}\right) \prod_{j=1}^{NP_\ell} X_j^\ell \leq \sum_{j=1}^{NLF} W_{R_\ell j},$$

for all $\ell \in \{1,2,3,\ldots,NRR\}$, (5.30)

$$\prod_{i=1}^{j-1} X_i^\ell \leq (1 - L_j^\ell) + \prod_{i=1}^{j} X_i^\ell ,$$

for all $j \in \{1,2,\ldots,NP_\ell\}$,

for all $\ell \in \{1,2,\ldots,NRR\}$, (5.27)

$$\left[U_j^\ell\right]^{-1} X_j^\ell \leq 1,$$

for all $j \in \{1,2,\ldots,NP_\ell\}$,

for all $\ell \in \{1,2,\ldots,NRR\}$. (5.28)

Analogous to the small sample model of one resource recovery plant, two sources and two landfills, the landfill costs (5.32) and the resource recovery costs (5.31) need to be transformed into standard geometric programming form by a change of variable, while the constraints (5.27), (5.29), and (5.30) require the application of R. J. Duffin's condensation procedure.

The landfill costs, K_{LF_i}, will be transformed into proper geometric programming form by a change of variable, as shown:

Let

$$\kappa_{LF_i} = d_{LF_i} Z_i^{e_{LF_i}},$$

$$\text{for all } i \in \{1,2,\ldots,NLF\}, \tag{5.37}$$

subject to

$$Z_i^{-1} \left[\sum_{k=1}^{NS} W_{ki} + \sum_{\ell=1}^{NRR} W_{R_\ell i} \right]^{e_{LF_i}} \leq 1$$

$$\text{for all } i \in \{1,2,\ldots,NLF\}. \tag{5.38}$$

The resource recovery plant costs will also be transformed into proper geometric programming form by a change of variable.

Let

$$\kappa_{R_\ell} = d_{R_\ell} Z_{R_\ell}^{e_{R_\ell}} +$$

$$\sum_{k=1}^{NP_\ell} d'_{R_\ell k} \left[Z_{R_\ell} \prod_{j=1}^{k-1} X_j^\ell \right]^{e'_{R_\ell k}} X_k^{\ell\, f_{R_\ell k}} \qquad (5.39)$$

subject to

$$Z_{R_\ell}^{-1} \left(\sum_{i=1}^{NS} W_{iR_\ell} \right) \leq 1,$$

$$\text{for all } \ell \in \{1,2,\ldots,NRR\}. \qquad (5.40)$$

Constraints (5.27), (5.29), and (5.30) will now be condensed. For the constraints of (5.27), we have:

$$\prod_{i=1}^{j-1} X_i^{\ell} \left(\frac{1 - L_j^{\ell}}{\xi_{2j-1}^{\ell}} \right)^{-\xi_{2j-1}^{\ell}} \left(\frac{\prod_{i=1}^{j} X_i^{\ell}}{\xi_{2j}^{\ell}} \right)^{-\xi_{2j}^{\ell}} \leq 1, \quad (5.41)$$

where

$$\xi_{2j-1}^{\ell} + \xi_{2j}^{\ell} = 1,$$

for all $j \in \{1,2,\ldots,NP_{\ell}\}$,

and for all $\ell \in \{1,2,\ldots,NRR\}$. (5.42)

For constraints (5.29), the associated condensed constraint is

$$S_i \prod_{j=1}^{NLF} \left(\frac{W_{ij}}{\xi_{v_{ij}}} \right)^{-\xi_{v_{ij}}} \prod_{\ell=1}^{NRR} \left(\frac{W_{iR_{\ell}}}{\xi_{v_{iR_{\ell}}}} \right)^{-\xi_{v_{iR_{\ell}}}}$$

$$\leq 1, \quad (5.43)$$

where

$$\sum_{j=1}^{NLF} \xi_{v_{ij}} + \sum_{\ell=1}^{NRR} \xi_{v_{iR_\ell}} = 1,$$

for all $i \in \{1,2,\ldots,NS\}$. (5.44)

For constraint (5.30), the proper transformed equation is

$$\sum_{\ell=1}^{NRR} Z_{R_\ell} \prod_{j=1}^{NP_\ell} X_j^{\ell} \prod_{j=1}^{NLF} \left(\frac{W_{R_\ell j}}{\xi_{v_{R_\ell j}}} \right)^{-\xi_{v_{R_\ell j}}}$$

$$\leq 1, \qquad (5.45)$$

$$\sum_{j=1}^{NLF} \xi_{v_{R_\ell j}} = 1,$$

for all $\ell \in \{1,2,\ldots,NRR\}$. (5.46)

Therefore, the general model in standard geometric programming form is

Minimize:

$$\sum_{\ell=1}^{NRR} {}^{K}R_{\ell} + \sum_{i=1}^{NLF} {}^{K}LF_{i} + \sum_{i=1}^{NS} \sum_{j=1}^{NLF} T_{ij} +$$

$$\sum_{\ell=1}^{NRR} \sum_{j=1}^{NLF} T_{R_{\ell}j} + \sum_{i=1}^{NS} \sum_{\ell=1}^{NRR} T_{iR_{\ell}} \qquad (5.47)$$

Subject to:

5.26, 5.28, 5.38, 5.40, 5.41, 5.42, 5.43, 5.44, 5.45, and 5.46.

5.5 A Small Example Illustrating the Impact of Different Starting Parameters Upon the Final Solution

A small numerical example, Program 5.1, will be analyzed for the sensitivity of different choices

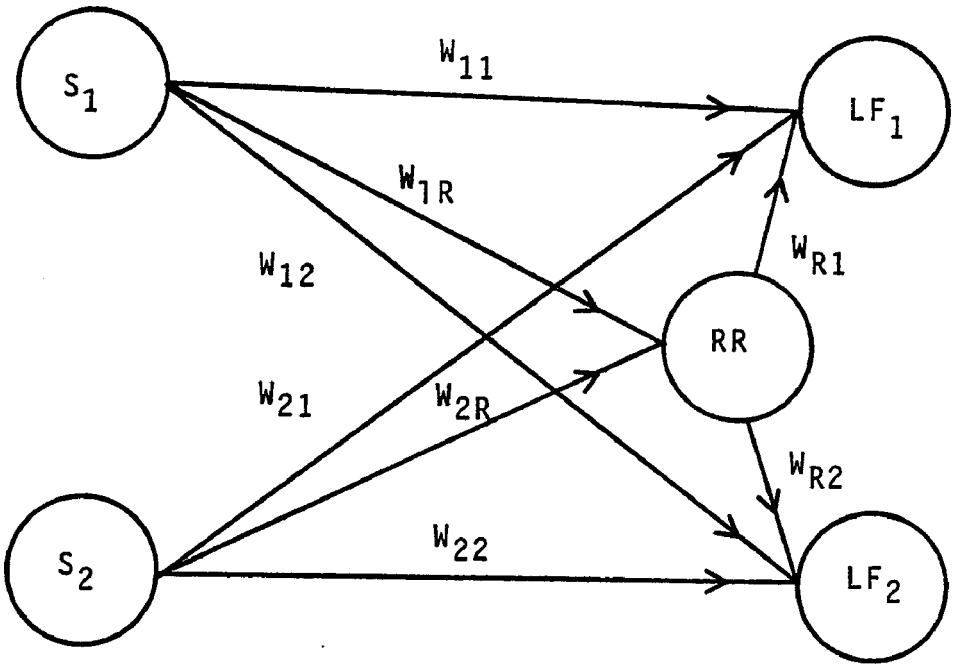
of the condensation weights upon the final solution to the problem. Basically, the effects of choosing condensation weights for balance constraints 5.6 and 5.7 are examined. This example problem models a solid waste system composed of two sources, two landfill sites, and one resource recovery with two processes. (See Figures 5.9 and 5.10.) The cost expression of the resource recovery plant for Program 5.1 is identical to that of the Lehigh Valley Case Study, namely Problem 6.1 in Chapter 6.

The transportation costs for the three types of routes which are source-to-recovery plant, source-to-landfill, and recovery-plant-to-landfill are randomly selected from the cost matrix of Table 6.3. The exponents for these costs are similarly chosen from the transportation cost equation 6.9. The two sources are assumed to generate 1000 tpd jointly. Program 5.1 is now stated:

Program 5.1

Minimize:

$$16\ W_{11}^{1.05} + 14\ W_{12}^{1.1} + 11\ W_{1R}^{.9} + 15\ W_{21}^{1.1}$$



Legend

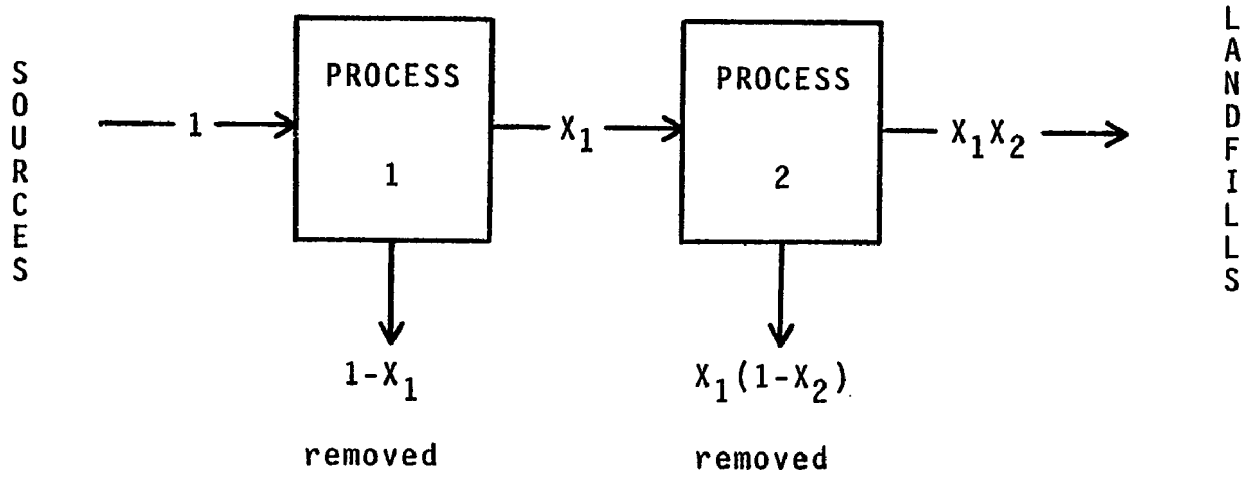
S_i ≡ generating source i, i=1,2.

LF_i ≡ landfill site i, i = 1,2.

RR ≡ resource recovery plant

W_{ij} ≡ tons per day of municipal solid waste shipped from i to j, $i \in \{1,2,R\}$, $j \in \{1,2,R\}$, $i = R \rightarrow j \neq R$, $j = R \rightarrow i \neq R$.

FIGURE 5.9. Solid Waste Management Model for Problem 5.1: Two Generating Centroids, One Resource Plant, Two Landfill Sites.



Legend

X_j ≡ fraction of solid waste entering j that is remaining after process j, j = 1,2,...,NP, where NP is the number of processes.

FIGURE 5.10. Resource Recovery Plant with Two Processes.

$$16\ W_{22}^{1.06} + 6\ W_{2R}^{.95} + 40\ Z^{.65} +$$

$$Z^{.62}X_1^{-1.62} + 15Z^{.68}X_1^{.68}X_2^{-1.58} +$$

$$14\ W_{R1}^{.78} + 13\ W_{R2}^{.85}. \qquad (5.48)$$

Subject to:

$$\sum_{j\in\{1,2,R\}} W_{1j} \geq 600 \qquad (5.49)$$

$$\sum_{j\in\{1,2,R\}} W_{2j} \geq 400 \qquad (5.50)$$

$$\sum_{i=1}^{2} W_{iR} \leq Z \qquad (5.51)$$

$$ZX_1X_2 \leq W_{R1} + W_{R2} \qquad (5.52)$$

Since constraints 5.49, 5.50, and 5.52 are not in standard geometric form, condensation of the

constraints is necessary for the application of the solution procedure. After condensing 5.49, 5.50, and 5.52, Program 5.2 is formed:

Program 5.2

Minimize:

$$16\ W_{11}^{1.05} + 14\ W_{12}^{1.1} + 11\ W_{1R}^{.9} +$$

$$15\ W_{21}^{1.1} + 16\ W_{22}^{1.06} + 6\ W_{2R}^{.95} +$$

$$40Z^{.65} + Z^{.62}X_1^{-1.62} + 15\ Z^{.68}X_1^{.68}X_2^{-1.58} +$$

$$14\ W_{R1}^{.78} + 13\ W_{R2}^{.85} \qquad (5.53)$$

Subject to:

$$600 \prod_{i=1}^{3} \xi_i^{\xi_i}\ W_{11}^{-\xi_1}\ W_{12}^{-\xi_2}\ W_{1R}^{-\xi_3} \leq 1 \qquad (5.54)$$

$$400 \prod_{i=4}^{6} \xi_i^{\xi_i}\ W_{21}^{-\xi_4}\ W_{22}^{-\xi_5}\ W_{1R}^{-\xi_6} \leq 1 \qquad (5.55)$$

$$W_{1R}\ Z^{-1} + W_{2R}\ Z^{-1} \leq 1 \qquad (5.56)$$

$$\prod_{i=10}^{11} \xi_i^{\xi_i} \; Z \; X_1 X_2 \; W_{R1}^{-\xi_{10}} \; W_{R2}^{-\xi_{11}} \leq 1 \qquad (5.57)$$

where

$$\sum_{i=1}^{3} \xi_i = \sum_{i=4}^{6} \xi_i = \sum_{i=10}^{11} \xi_i = 1. \qquad (5.58)$$

In general, geometric programs with negative terms and/or reversed constraints lose the property that every local minimum is also a global minimum (144, 138, 163, 164). Notice that the system costs generated from different initial condensation weights are not identical; yet they are within 8.5 per cent of one another. (See Exhibits 5.1, 5.2, and 5.3.) This author does not infer that these disparities are due to the numerical approximation error of the solution procedure, but presents a possible method of checking a local minimum with several other local minimal solutions. An exhaustive search over all the weights, ξ_i, $i \in \{1,2,\ldots,11\}$, would reveal the global optimal solution; however, this could be expensive and time consuming. A suggested procedure

EXHIBIT 5.1

Program 5.1 Solved With Non-Polarized Initial Weights

a) Solution

		Destinations		
		LF_1	LF_2	LF_3
Sources	S_1	28	21	551
	S_2	8	13	379
	RR	167	16	-

Tons per day shipped to resource recovery plant: Z=930

i	Fraction of Waste Input to Process i Remaining After Processing
1	$X_1 = 0.21$
2	$X_2 = 0.94$

System Cost = $12,534 per day

b) Initial Condensation Weights

$\xi_1 = 0.33$ $\xi_2 = 0.33$ $\xi_3 = 0.34$

$\xi_4 = 0.33$ $\xi_5 = 0.33$ $\xi_6 = 0.34$

$\xi_{10} = 0.50$ $\xi_{11} = 0.50$

c) Final Condensation Weights

$\xi_1 = 0.05$ $\xi_2 = 0.03$ $\xi_3 = 0.92$

$\xi_4 = 0.02$ $\xi_5 = 0.03$ $\xi_6 = 0.95$

$\xi_{10} = 0.91$ $\xi_{11} = 0.09$

EXHIBIT 5.2

Program 5.1 Solved With Partially Polarized Initial Weights

a) Solution

		Destinations		
		LF_1	LF_2	LF_3
Sources	S_1	56	7	537
	S_2	2	3	394
	RR	233	5	-

Tons per day shipped to resource recovery plant: Z=931

i	Fraction of Waste Input to Process i Remaining After Processing
1	$X_1 = 0.24$
2	$X_2 = 1.08$

System Cost = $11,969 per day

b) Initial Condensation Weights

$\xi_1 = 0.80$ $\xi_2 = 0.10$ $\xi_3 = 0.10$

$\xi_4 = 0.33$ $\xi_5 = 0.33$ $\xi_6 = 0.34$

$\xi_{10} = 0.50$ $\xi_{11} = 0.50$

c) Final Condensation Weights

$\xi_1 = 0.09$ $\xi_2 = 0.01$ $\xi_3 = 0.90$

$\xi_4 = 0.01$ $\xi_5 = 0.01$ $\xi_6 = 0.98$

$\xi_{10} = 0.98$ $\xi_{11} = 0.02$

EXHIBIT 5.3

Program 5.1 Solved With Polarized Initial Weights

a) Solution

	Destinations		
	LF_1	LF_2	LF_3
S_1	74	11	515
S_2	16	7	376
RR	2	170	-

Tons per day shipped to resource recovery plant: Z=928

i	Fraction of Waste Input to Process i Remaining After Processing
1	$X_1 = 0.21$
2	$X_2 = 0.92$

System Cost = $13,089 per day

b) Initial Condensation Weights

$\xi_1 = 0.80$ $\xi_2 = 0.10$ $\xi_3 = 0.10$

$\xi_4 = 0.80$ $\xi_5 = 0.10$ $\xi_6 = 0.10$

$\xi_{10} = 0.10$ $\xi_{11} = 0.90$

c) Final Condensation Weights

$\xi_1 = 0.12$ $\xi_2 = 0.02$ $\xi_3 = 0.86$

$\xi_4 = 0.04$ $\xi_5 = 0.02$ $\xi_6 = 0.94$

$\xi_{10} = 0.01$ $\xi_{11} = 0.99$

for comparing the solution would be to initially solve the problem when the elements in each set of the weights corresponding to a condensed constraint have identical values. Then resolve the problem with the initial weights having opposite polarization to the final weights computed in the previous solution. In practice, this method appears to expose the widest range of local optimal solutions.

This problem of not having every solution computed being a global optimum is shared by the linear programming models WRAP and RECOVR.

CHAPTER 6

TWO APPLICATIONS OF GEOMETRIC PROGRAMMING TO MUNICIPAL SOLID WASTE SYSTEMS

Part 1 - Lehigh Valley Case Study

6.1 Introduction

Regional solid waste management plans are not only important social concerns, but are becoming legal necessities as well. In fact, one provision of the recently passed Resource Conservation and Recovery Act (1976) requires that every state develop a solid waste plan. Since the passage of Pennsylvania Act 241, the Department of Environmental Resources (DER) has acquired the responsibility of developing solid waste disposal plans for all areas of the state except for very sparsely populated regions.

The DER subsequently has delegated this responsibility to local governments or local regional planning agencies such as the Joint Planning Commission of Lehigh and Northampton Counties (JPC). Over the last few years, the JPC has responded to this responsibility by compiling and publishing a

solid waste plan in 1971 (165), followed by several supplemental updates in the years 1974, 1975, 1976, and 1977 (166). This plan comprises a collection of existing data and descriptions of existing facilities, accompanied by future extrapolations of waste generation rates and disposal requirements.

This author knows of only one study attempting to determine an optimal solid waste management plan for the Lehigh and Northampton County region. Vincent Viscomi, who is a professor at Lafayette College, and Aurel Arndt, who is the business manager of the Lehigh County Authority (LCA) and also a resident expert in waste disposal systems for the region, have utilized the linear programming model RECOVR written by J. R. McNamara (117) of Lehigh University to obtain the optimally designed solid waste system. In this study, the optimal solution is determined by minimizing a linear function representing the aggregate costs of the waste system subject to several linear constraints.

To better approximate reality, this case study will present a nonlinear mathematical programming model that includes the diseconomies of scale and the economies of scale properties represented

by the solid waste system costs. Furthermore, specific recovery processes and their levels of operation are described by this model, but are not included as design variables in Arndt's linear model.

The goal of this case study is to examine the economics of a regional approach to solid waste management for the Lehigh and Northampton Counties for the year 1990. A transportation network model is constructed of nodes representing generating centroids, landfills, and a resource recovery plant, and of arcs representing the flow of municipal solid waste. Since only politically and socially acceptable nuisance sites are considered in the model for landfill sites and the resource recovery plant, the measures of effectiveness will be aggregate costs which include collection and transportation costs, resource recovery plant costs, and landfill costs. Therefore, the minimization of a nonlinear cost function subject to several constraints determines the optimal solid waste plan for the region.

The Lehigh and Northampton County region is located in the eastern portion of Pennsylvania,

approximately 50 miles north of Philadelphia. Sixty-two municipalities, including the three large population centers of Allentown, Bethlehem, and Easton, are situated within the region's borders. The population growth of Allentown, Bethlehem, and Easton is relatively stable, while the adjacent suburban areas have experienced growth.

The case study consists of three separate phases:

1. Collection of the data.
2. Development of the nonlinear computer model.
3. Analysis of the solution.

Much of the data has been obtained from Aural Arndt of the LCA and from a consulting study completed for the LCA by Malcolm Pirnie (174). The computer model of this two-county region exemplifies the application of geometric programming to a solid waste management system composed of eight generating centroids, four landfill sites, and one resource recovery plant.

6.2 Collection of Data for Waste Generation Rates

The data for solid waste generation rates in tons per operating day (tpd) have been obtained

from the JPC Solid Waste Plan (165). Although the report contains data for residential, institutional/commercial, agricultural, and industrial solid wastes, only residential and institutional wastes are included in this research. The generation estimates have been taken from Tables 29 and 37 of the report. Table 29 contains the projected yearly tons of household solid waste, while Table 37 contains data on commercial/institutional solid waste. To obtain the daily tonnage of municipal solid waste, the values of Tables 29 and 37 are summed, divided by 300 operating days per year, and then reduced by 12 per cent, as recommended by the JPC, to reflect source separation and source recovery efforts.

For the case study model, eight generating sources are formed by partitioning the study region into eight areas; the generating rates are computed by summing all the waste generated by the municipalities in those areas. For generation rates beyond 1975, a growth rate of 8.5 per cent per five-year period, which is the geometric mean for the figures that the JPC Plan reports (165), is applied to the 1975 base levels. (See Table 6.1.)

No.[2]	Source	Municipalities	Generation Rate (tpd)[1] 1975	1980	1985	1990
1	Western Lehigh County	Slatington, Washington, North Whitehall, Lynn, Lowhill, Heidelberg, Weisenberg Townships. Macungie, Albertis; Lower Macungie, Upper Macungie Townships.	65	71	77	83
2	Eastern Lehigh County	Emmaus, Coopersburg, Fountain Hill, Salisbury, Upper Milford, Lower Milford, Upper Saucon Townships. Coplay, Catasauqua; Hanover, Whitehall, South Whitehall Townships.	171	186	201	218
3	Allentown	Allentown	314	341	370	401

[1]A geometric mean of 8.5 per cent per five-year period is assumed. This is roughly what the actual figures in the JPC suggest for the period 1970-1990.

[2]This is the subscript for S_i, i.e., S_4 is the fourth source.

TABLE 6.1. Generation Rates of Lehigh and Northampton Counties. (See next page.)

	No.*	Source	Municipalities	Generation Rate (tpd)* 1975	1980	1985	1990
NORTHAMPTON COUNTY	4	Northwestern Northampton County	Bethlehem, Northampton, North Catasauqua, Walnutport, Chapman Quarries; Moore, Lehigh, East Allen, Allen Townships.	51	55	60	65
	5	Northeastern Northampton County	Portland, E. Bangor, Bangor, Roseto, Pen Argyl, Wind Gap; Upper Mt. Bethel, Washington, Plainfield, Bushkill Townships.	53	58	62	68
	6	Southcentral Northampton County	Nazareth, Tatamy, Stockerton, Wilson, W. Easton, Glendon; Forks, Upper Nazareth, Lower Nazareth, Palmer, Williams, Lower Mt. Bethel Townships. Hellertown; Bethlehem, Lower Saucon, Hanover Townships.	136	148	160	174
	7	Bethlehem	Bethlehem	167	181	197	213
	8	Easton	Easton	80	87	94	102

* See footnotes for Lehigh County.

TABLE 6.1 (cont'd.) Generation Rates of Lehigh and Northampton Counties.

To insure that all the municipal solid waste generated at each of the eight sources is removed, the following constraints are constructed:

Let

$J = \{1,2,3,4,R\}$; then

$$\sum_{j \in J} W_{ij} \geq 83, \tag{6.1}$$

$$\sum_{j \in J} W_{2j} \geq 218, \tag{6.2}$$

$$\sum_{j \in J} W_{3j} \geq 401, \tag{6.3}$$

$$\sum_{j \in J} W_{4j} \geq 65, \tag{6.4}$$

$$\sum_{j \in J} W_{5j} \geq 68, \tag{6.5}$$

$$\sum_{j \in J} W_{6j} \geq 74, \tag{6.6}$$

$$\sum_{j \in J} W_{7j} \geq 213, \tag{6.7}$$

$$\sum_{j \in J} W_{8j} \geq 102. \qquad (6.8)$$

6.3 Landfill Capacity and Costs, Collection and Transportation Costs

The daily maximum capacities for various operating landfills are difficult to determine. The JPC 1977 Supplement (166) and Arndt of the LCA report widely differing figures for the daily maximum capacities of regional landfills. This author has telephoned a few of the landfills and discovered that they do not refuse anyone except when there are severe weather conditions, or when the waste has been generated outside of the region. However, if there is an increase in the inflow of municipal solid waste to a properly run landfill, customers will spend longer times in the landfill service queue, which will drive up their cost per ton. Therefore, the combined collection and transportation costs are modeled with a nonlinear function adhering to the property of diseconomies of scale with respect to the tons per day. This increasing cost relationship acts somewhat as a constraint upon a congested landfill by inducing

haulers to go elsewhere. This author feels that the above approach better estimates reality than utilizing poor estimates of landfill site capacities.

Conversely, the haulage costs to and from the resource recovery plant are assumed to have slight economies of scale. Since the resource recovery system has not been constructed, it is assumed that the costs of this system will exhibit economies of scale with respect to capacity. Hence, the exponents of all variables, W_{iR} and W_{Ri}, representing the tons per day of waste entering and leaving the recovery plant, are slightly less than one, while all other transportation cost terms have exponents that are slightly greater than one.

The transportation and landfill cost data are taken from the work papers associated with the Malcolm Pirnie report (174). Collection costs are not included; transportation and landfill costs are combined. The transportation costs are calculated under the assumptions that:

1. Compactor trucks are fully loaded at the geographical center of a partitioned

section of the region.

2. Distance from the geographical center to the destination is measured by the crow-fly method.
3. The standard speed of 35 miles per hour and a turnaround time of ten minutes at the landfill sites are used.

Although there are disparities in the landfill disposal costs between Arndt and Viscomi's paper (167) and the JPC report (165), the former paper's figures are used because they are combined with the only transportation costs that could be found. The cost values are assumed to be in 1976 dollars; therefore, the costs are inflated at a rate of 7.5 per cent for 13 years. This rate is obtained by computing the geometric mean of the most current annual inflation rates for the consumer price index of all commodities (168). (See Tables 6.2 and 6.3.)

Therefore, the transportation and disposal cost relationships for the entire system are:

$T_1 + T_2$, where

Year	Consumer Price Index (All Items)	Growth Rate
1974	147.7	
		1.09140/year
1975	161.2	
		1.05769/year
1976	170.5	
		1.07097/½ year
1977 July	182.6	

Since the Geometric Mean = $(1.09140)^{.4}$ $(1.05769)^{.4}$ $(1.07097)^{.2}$

$= 1.07372,$

a 7.5% Inflation Rate from the end of 1976 to the beginning of 1990 is assumed.

TABLE 6.2. Inflation Rate Calculation.

Generating Centroid	Lehigh County		Northampton County		Resource Recovery Plant
	Helena LF_1	Oswald LF_2	Bethlehem LF_3	Chrin LF_4	
S_1	16.44 [1]	14.39	23.89	26.91	10.91
S_2	15.34	15.80	18.82	20.92	6.04
S_3	15.52	15.98	17.49	20.46	4.46
S_4	14.82	19.82	18.82	20.51	5.89
S_5	25.17	31.08	23.74	22.05	12.42
S_6	20.25	23.38	15.39	15.21	4.28
S_7	17.90	19.28	15.72	16.87	1.46
S_8	22.07	24.94	16.90	13.90	6.25
RR	14.31	13.98	12.70	14.03	-

[1] All figures have units of dollars per ton.

TABLE 6.3. Estimated Transportation and Landfill Costs Measured in 1990 Dollars.

$$16.44\, W_{11}^{1.1} + 14.39\, W_{12}^{1.11} + 23.89\, W_{13}^{1.05} +$$
$$26.91\, W_{14}^{1.05} + 10.91\, W_{1R}^{.95} + 15.34\, W_{21}^{1.1} +$$
$$15.80\, W_{22}^{1.1} + 18.82\, W_{23}^{1.06} + 20.92\, W_{24}^{1.04} +$$
$$6.04\, W_{2R}^{.98} + 15.52\, W_{31}^{1.1} + 15.98\, W_{32}^{1.1} +$$
$$17.49\, W_{33}^{1.09} + 20.46\, W_{34}^{1.04} + 4.46\, W_{3R}^{.97} +$$
$$14.82\, W_{41}^{1.1} + 19.82\, W_{42}^{1.09} + 18.82\, W_{43}^{1.09} +$$
$$20.51\, W_{44}^{1.05} + 5.89\, W_{4R}^{.99} + 25.17\, W_{51}^{1.05} +$$
$$31.08\, W_{52}^{1.05} + 23.74\, W_{53}^{1.05} + 22.05\, W_{54}^{1.06} +$$
$$12.42\, W_{5R}^{.96} + 20.25\, W_{61}^{1.07} + 23.38\, W_{62}^{1.05} +$$
$$15.39\, W_{63}^{1.08} + 15.21\, W_{64}^{1.09} + 4.28\, W_{6R}^{.95} +$$
$$17.9\, W_{71}^{1.08} + 19.28\, W_{72}^{1.07} + 15.72\, W_{73}^{1.09} +$$
$$16.87\, W_{74}^{1.09} + 1.46\, W_{7R}^{.96} + 22.07\, W_{81}^{1.05} +$$
$$24.94\, W_{82}^{1.05} + 16.90\, W_{83}^{1.09} + 13.90\, W_{84}^{1.1} +$$
$$6.25\, W_{8R}^{.97}$$

$$= T_1 \,, \qquad (6.9)$$

and where

$$14.31\ W_{R1}^{.78} + 13.98\ W_{R2}^{.90} + 12.70\ W_{R3}^{.85} + 14.03\ W_{R4}^{.79} = T_2. \qquad (6.10)$$

6.4 Economic Factors in Materials Recovery

"At present, ferrous metals and paper are the only materials recycled from municipal solid waste in substantial amounts" (page 64 of reference 169). The consulting study (174) completed in May 1976 for the LCA suggests a resource recovery system with refuse derived fuel (RDF), ferrous metals, and glass recovery. Another recent study (169) reports that the recovery of aluminum and other nonferrous metals is technically feasible. Since readily available markets exist in the region for ferrous and combustible materials, only the recovery of these two materials is analyzed (See Figure 6.1.) An important caveat in considering a materials recovery system is that no system has yet been fully implemented. "Recovery efficiencies and product specifications will not be

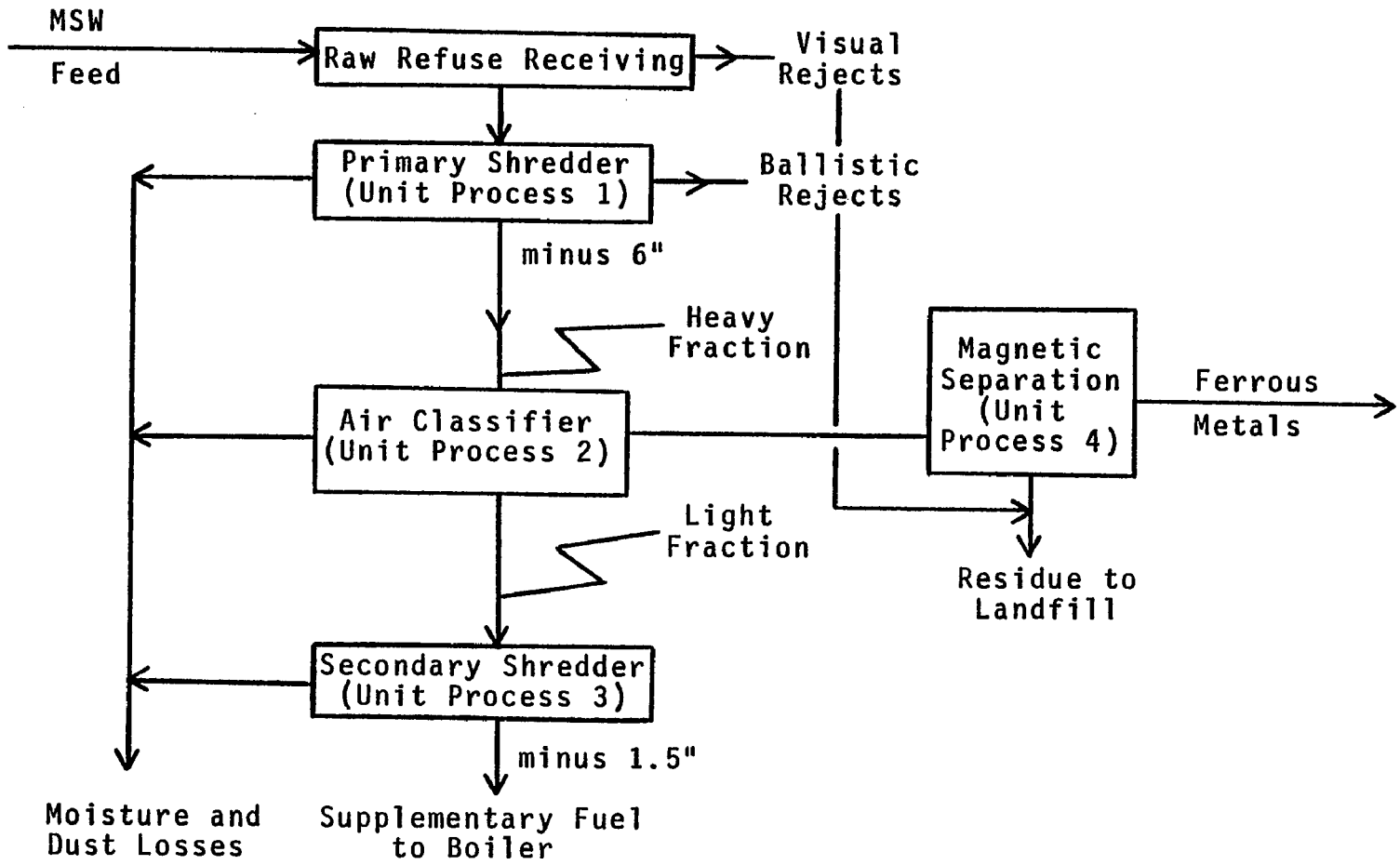


FIGURE 6.1. Resource Recovery Plant Schematic for the Lehigh Valley Case Study.

reliably known until a system has a background of operational experience" (169). Therefore, the costs and credits for these operations are estimated from extrapolations of test results.

The resource recovery model in this research includes the recovery of RDF and ferrous metals, where approximately 80 per cent of the waste could be used to produce RDF (pages 2-12 of references 165, 174; page 7 of reference 170), while only about 7 per cent of the waste is ferrous scrap (page 75 of 170, 171). The mathematical model consists of a RDF recovery process, process 1, and a ferrous metal recovery process, process 2, each of which can consist of several unit processes. The first two unit processes of the proposed resource recovery plant in Figure 6.1 are shared by recovery processes 1 and 2 at the assumed rates of 80 per cent for the RDF process 1 and 20 per cent for the ferrous metal process 2 for cost assignment purposes. The former percentage estimates the fraction of combustible materials in the waste stream, while the latter percentage

approximates the fraction of noncombustible materials processed by the shared unit processes.

The unit operations costs for the primary and secondary shredders, air classifier, and magnetic separator of this recovery plant are obtained from reference (169), but need to be inflated from the end of 1976 to the beginning of 1990. An inflation rate of 10 per cent per annum is assumed for waste processing equipment which has been the inflation rate experienced by the LCA. The 10 per cent per annum inflation rate is also assumed for land and building costs. Operating costs are assumed to escalate at the rate of the consumer price index of 7.5 per cent per annum. (See Table 6.4.) The unit processing costs which are based upon a 15-year amortization at 7 per cent interest rate include all equipment and ancillaries for each unit operation not including the building and land costs (170). Operating costs include purchased power, supplies, and maintenance costs predicted on a plant processing 1,000 tpd of municipal solid waste for 330 days per year, while removing 70 per cent of the input stream as RDF and 5 per cent as

	Unit Operation	Capital Cost[1] ($/Ton msw)	Operating Cost[2] ($/Ton msw)
1.	Primary Shredding (to -4 in.)	1.69	5.56
2.	Air Classification	1.07	3.64
3.	Secondary Shredding (to -1 in.)	0.55	2.66
4.	Magnetic Materials Recovery	0.28	1.13
	TOTALS	3.59	12.98

The costs above do not include the cost of land and buildings. The costs for the land and buildings are assumed to be equal to the capital costs shown above. This was the experience at the LCA Lower Saucon Site (171). Hence, the land and buildings costs are assumed to be $3.59/Ton msw.

[1]Capital equipment costs which are amortized at 7 per cent per annum are escalated at 10 per cent per year, from the end of 1976 to the beginning of 1990.

[2]Operating costs are escalated at 7.5 per cent per year from the end of 1976 to the beginning of 1990.

TABLE 6.4. Estimated Unit Operations Costs for 1990 in a 1000 tpd Plant.

ferrous scrap (167, 170). The building and land costs are assumed to equal the capital equipment cost because this was the experience of the now defunct LCA recovery plant (171).

Based upon LCA experience at its recovery plant, ferrous scrap generates a revenue of $22 per ton FOB recovery plant. Therefore, if inflated to 1990 prices at a rate of 7.5 per cent per annum for 13 years, the revenue is $56.33 per ton, FOB recovery plant. The RDF produced by process 1 is assumed to be salable for $9.50 per ton delivered to the Martins Creek Generating Plant of Pennsylvania Power and Light Company. From work sheets that have accompanied the LCA consulting report (174), the cost to transport the RDF to the generating plant is approximately $2.66 per ton. Hence, the net revenue is $6.84 per ton recovered. (See Exhibit 6.1.) After inflating at 7.5 per cent per annum for 13 years, the net revenue is $17.51 per ton RDF received at the generating plant in 1990 dollars. Assuming that the recovery plant is recovering 70 per cent of the input stream in RDF and 5 per cent of the input stream in ferrous

EXHIBIT 6.1

Calculation: Net Revenue from RDF Recovery

20 minutes each way at 25 mph	=	96 minutes
Turn Around Time	=	30 minutes
Total Time		126 minutes

126 minutes at $0.019 per ton-min hauled = $2.39/ton RDF received.

Assume the cost of the driver is $8/hour or $0.16/minute, and the large trucks used to haul the RDF carry approximately 75 tons of RDF; then the labor cost per ton of RDF received is:

$$\$0.16 \text{ min} \times \frac{1 \text{ trip}}{75 \text{ tons}} \times \frac{126 \text{ min}}{\text{trip}} = \$0.27/\text{ton RDF received.}$$

Therefore, the total cost is $2.66/ton RDF received.

→

The Net Revenue per ton RDF received is:

$9.50/ton - $2.66/ton = $6.84/ton.

EXHIBIT 6.2

Calculation: Net Revenue per ton of Municipal Solid Waste

Assumptions:

1000 tpd plant operating 330 days per year with 70 per cent recovery of msw as RDF and 5% recovery of msw as ferrous scrap.

Ferrous Metals:

	$56.33/ton ferrous metals
x	5% recovered per ton msw
	$ 2.82/ton of msw

RDF:

	$17.51/ton RDF delivered
x	70% recovered per ton msw
	$12.26/ton of msw

metals, the net revenues can then be computed on a dollars-per-ton-municipal-solid-waste basis, yielding $12.26 RDF revenues and $2.82 in ferrous material revenues per ton of Municipal Solid Waste (msw). (See Exhibit 6.2.)

Let

$C_i \equiv$ a fixed positive coefficient for the i^{th} term, i=1,2,3,

$\alpha_i \equiv$ a fixed positive parameter for the i^{th} term, i=1,2,3,

$X_j \equiv$ fraction of msw remaining after passing through process j, j=1,2,

$\beta_j \equiv$ a fixed negative parameter for x_j, j=1,2, and

$Z \equiv$ tons per day of municipal solid waste that enters the resource recovery plant.

Then,

$$C_1 Z^{\alpha 1} + C_2 Z^{\alpha 2} X_1^{\beta 1} + C_3 (ZX_1)^{\alpha 3} X_2^{\beta 2}$$

will be assumed to be the cost relationship with

$\alpha_1, \alpha_2 \in (0,1)$ and $\beta_1, \beta_2 \in (-2,-1)$ and where

$$Z \geq \sum_{i=1}^{8} W_{iR}. \tag{6.11}$$

This mathematical expression is the assumed relationship that describes the resource recovery plant costs. The exponents α_i, i=1,2, model the economies of scale property for building, land, and capital equipment costs with respect to capacity. The exponents β_i, i=1,2, model the diseconomies of scale property for processing costs versus level of efficiency. The fixed cost for process i, i=1,2, with a capacity of $Z \prod_{j=1}^{i-1} X_j$ tons per day can be defined by letting $X_i=1$.

Let $C_1 Z^{\alpha_1}$ be the net cost for the building and land.

Let $C_2 Z^{\alpha_2} X_1^{\beta_1}$ be the net cost for process 1 while letting

$C_3 (ZX_1)^{\alpha_3} X_2^{\beta_2}$ be the net cost for process 2. If we assume that all costs are computed for a 1000 tpd plant removing 70 per cent of the input stream

as RDF (process 1) and 5 per cent of the input stream as ferrous metals (process 2), then, if X_i is defined as the fraction of input municipal solid waste remaining from process i,

$$1 - X_1 = .70 \rightarrow X_1 = .30$$

and

$$X_1(1 - X_2) = .05 \rightarrow X_2 = .83\overline{3}.$$

Furthermore, we will assume that the cost relationships above approximately follow the six-tenths rule of decreasing average costs associated with increasing capacity. Hence, $\alpha_1 = .65$, $\alpha_2 = .62$, and $\alpha_3 = .68$.

The property of increasing costs with respect to removing higher fractions of materials from municipal solid waste will be assumed to be approximately -1.60. An illustration of aluminum separation in Chapter 2 illustrates that these exponents could be at most -1.00, and are probably less. Hence, let $\beta_1 = -1.62$ and $\beta_2 = -1.58$.

The coefficients for the terms in the resource recovery cost relation are computed by

EXHIBIT 6.3

Calculation: Net Costs for the Resource Recovery Plant

I. Apportioning the Total Costs

[1] T_{msw} ≡ tons of municipal solid waste.	Cost per T_{msw}[1]	Proportion allocated to Process	Total Process Cost per T_{msw}
A. Process 1 (RDF)			
a) Primary Shredding (to 4 in.)	7.25	80%	5.80
b) Air-Classification	4.71	80%	3.77
c) Secondary Shredding (to 1 in.)	3.21	100%	3.21
			12.78
B. Process 2			
a) Primary Shredding	7.25	20%	1.45
b) Air-Classification	4.71	20%	0.94
c) Magnetic Material Recovery	1.41	100%	1.41
			3.80
C. Land and Building	$3.59/$T_{msw}$		

(See next page.)

EXHIBIT 6.3 (continued)

Calculation: Net Costs for the Resource Recovery Plant

II. Computation of Net Costs

Suppose we apportion the revenues as computed in Exhibit 6.2 directly to the processes that generated them. Then the Net Costs are as follows:

A. Process 1

$$\text{Net Cost} = \$12.78/T_{msw}^{1} - \$12.26/T_{msw} = \$.52/T_{msw}$$

B. Process 2

$$\text{Net Cost} = \$3.80/T_{msw} - \$2.82/T_{msw} = \$.98/T_{msw}$$

C. Land and Building $\$3.59/T_{msw}$

[1] $T_{msw} \equiv$ tons of municipal solid waste.

equating each term to the net-cost-per-ton value computed in Exhibit 6.3. Since the net costs in Exhibit 6.3 are computed with respect to a 1000 tpd plant, let Z = 1000. For the land and building cost term,

$$C_1 Z^{.65} = C_1\ 1000^{.65} = \$3.59/\text{ton} \times 1000\ \text{tpd}$$

$$\rightarrow \quad C_1 = \$40.30/\text{day--ton}.$$

For process 1,

$$C_2 Z^{\alpha_2} X_1^{\beta_1} = C_2\ 1000^{.62}\ .3^{-1.62}$$

$$= \$0.52/\text{ton} \times 1000\ \text{tpd}$$

$$\rightarrow \quad C_2 = \$1.02/\text{day--ton}.$$

For process 2,

$$C_3 (ZX_1)^{\alpha_3} X_2^{\beta_2} = C_3\ 300^{.68}\ .83^{-1.58}$$

$$= \$.98/\text{ton} \times 1000\ \text{tpd}$$

$$\rightarrow \quad C_3 = \$15.08/\text{day--ton}.$$

Therefore, the resource recovery plant cost

relationship is

$$40.3\ Z^{.65} + 1.02\ Z^{.62}\ X_1^{-1.62} + 15.08\ (ZX_1)^{.68}\ X_2^{-1.58} = T_{RR} \cdot \qquad (6.12)$$

6.5 The Mathematical Model of the Case Study Problem

In this section, the mathematical model for the Lehigh and Northampton study is formulated and solved. First, the technological and balance constraints will be presented, followed by the mathematical program for this model consisting of eight sources, four landfills, and one resource recovery plant with two processes.

For two processes at the recovery plant, the technological constraints are:

$$1 - X_1 \leq 1 - .20, \text{ or}$$

$$.2\ X_1^{-1} \leq 1 \qquad (6.13)$$

for process 1, and

$$(1 - X_2)\, X_1 \leq (1-.93), \text{ or}$$

$$X_1 \leq .07 + X_1X_2 \tag{6.14}$$

for process 2.

To insure that all of the residue at the resource recovery plant is removed, let

$$\left(\sum_{i=1}^{8} W_{iR} \right) X_1X_2 \leq \sum_{j=1}^{4} W_{Rj}. \tag{6.15}$$

The mathematical program for the Lehigh Valley regional solid waste management system is:

Minimize:

$$T_1 + T_{RR} + T_2 \tag{6.16}$$

Subject to:

(6.1) through (6.8), (6.11) and (6.13) through (6.15).

As described in Chapter 5, the above program is not in standard geometric programming form, and

therefore, transformations by change of variable and by condensation must be performed upon constraints 6.1 through 6.8, 6.14, and 6.15. Hence, from constraints 6.1 through 6.8, we obtain

$$83 \prod_{i=1}^{5} \xi_i^{\xi_i} \; W_{11}^{-\xi_1} \; W_{12}^{-\xi_2} \; W_{13}^{-\xi_3} \; W_{14}^{-\xi_4} \; W_{1R}^{-\xi_5} \leq 1, \tag{6.17}$$

$$218 \prod_{i=6}^{10} \xi_i^{\xi_i} \; W_{21}^{-\xi_6} \; W_{22}^{-\xi_7} \; W_{23}^{-\xi_8} \; W_{24}^{-\xi_9} \; W_{2R}^{-\xi_{10}} \leq 1, \tag{6.18}$$

$$401 \prod_{i=11}^{15} \xi_i^{\xi_i} \; W_{31}^{-\xi_{11}} \; W_{32}^{-\xi_{12}} \; W_{33}^{-\xi_{13}} \; W_{34}^{-\xi_{14}} \; W_{3R}^{-\xi_{15}} \leq 1, \tag{6.19}$$

$$65 \prod_{i=16}^{20} \xi_i^{\xi_i} \; W_{41}^{-\xi_{16}} \; W_{42}^{-\xi_{17}} \; W_{43}^{-\xi_{18}} \; W_{44}^{-\xi_{19}} \; W_{4R}^{-\xi_{20}} \leq 1 \; , \tag{6.20}$$

$$68 \quad \prod_{i=21}^{25} \xi_i^{\xi_i} \; W_{51}^{-\xi_{21}} \; W_{52}^{-\xi_{22}} \; W_{53}^{-\xi_{23}} \; W_{54}^{-\xi_{24}} \; W_{5R}^{-\xi_{25}} \leq 1, \tag{6.21}$$

$$74 \quad \prod_{i=26}^{30} \xi_i^{\xi_i} \; W_{61}^{-\xi_{26}} \; W_{62}^{-\xi_{27}} \; W_{63}^{-\xi_{28}} \; W_{64}^{-\xi_{29}} \; W_{6R}^{-\xi_{30}} \leq 1, \tag{6.22}$$

$$213 \quad \prod_{i=31}^{35} \xi_i^{\xi_i} \; W_{71}^{-\xi_{31}} \; W_{72}^{-\xi_{32}} \; W_{73}^{-\xi_{33}} \; W_{74}^{-\xi_{34}} \; W_{7R}^{-\xi_{35}} \leq 1, \tag{6.23}$$

and

$$102 \quad \prod_{i=36}^{40} \xi_i^{\xi_i} \; W_{81}^{-\xi_{36}} \; W_{82}^{-\xi_{37}} \; W_{83}^{-\xi_{38}} \; W_{84}^{-\xi_{39}} \; W_{8R}^{-\xi_{40}} \leq 1. \tag{6.24}$$

Constraint (6.15) is transformed to

$$\prod_{i=44}^{47} \xi_i^{\xi_i} \; ZX_1X_2 \; W_{R1}^{-\xi_{44}} \; W_{R2}^{-\xi_{45}} \; W_{R3}^{-\xi_{46}} \; W_{R4}^{-\xi_{47}} \leq 1. \tag{6.25}$$

where

$$Z \geq \sum_{i=1}^{8} W_{iR}. \tag{6.11}$$

The proper form of (6.14) is

$$.07^{-\xi_{49}} \prod_{i=48}^{49} \xi_i^{\xi_i} \; X_1^{1-\xi_{48}} \; X_2^{-\xi_{48}} \leq 1. \tag{6.26}$$

Finally,

$$\sum_{j=1}^{5} \xi_{5(i-1)+j} = 1, \tag{6.27}$$

for all $i \in \{1,2,\ldots,8\}$,

$$\sum_{i=44}^{47} \xi_i = 1, \qquad (6.28)$$

and

$$\sum_{i=48}^{49} \xi_i = 1. \qquad (6.29)$$

The mathematical program in standard geometric programming form is:

Problem 6.1

Minimize:

$$T_1 + T_{RR} + T_2 \qquad (6.16)$$

Subject to:

(6.17) through (6.29), (6.11), and (6.13).

Problem 6.1 has 18 degrees of difficulty, and satisfies conditions a and b of McNamara's procedure in Section 4.11 of Chapter 4. Hence, the program can be solved with the hybrid solution procedure

developed in Chapter 4. The solution to Problem 6.1 is shown in Table 6.5, where a run is defined as one iteration of the solution procedure. The computer resources necessary to solve the problem are illustrated in Table 6.6.

A second problem, Problem 6.2, with 50 per cent higher resource recovery costs is solved to reflect an economic scenario that is more hostile to material reclamation. Problem 6.2 is formed by replacing equation (6.16) in Problem 6.1 with

$$T_1 + T_{RR}' + T_2 \tag{6.30}$$

where

$$TT_{RR}' = 60.5\, Z^{.65} + 1.53 Z^{.65} X_1^{-1.62} + 22.6 Z^{.68} X_1^{.68} X_2^{-1.58} \tag{6.31}$$

The solution shown in Table 6.7 indicates that resource recovery is still favored, but not to the degree of the previous solution. Furthermore, the fact that $X_2 > 1$ indicates that recovery of ferrous

Run 1

Sources	Destinations				
	LF_1	LF_2	LF_3	LF_4	RR
S_1	16	17	13	12	27
S_2	37	36	36	36	81
S_3	65	63	61	67	182
S_4	14	11	12	12	21
S_5	12	10	13	13	23
S_6	12	11	15	14	27
S_7	31	30	34	31	123
S_8	16	14	18	21	40
RR	34	31	26	33	-

$X_1 = 0.22$, $X_2 = .94$, $Z = 561$

System Cost = $25,755/day

Run 2

Sources	Destinations				
	LF_1	LF_2	LF_3	LF_4	RR
S_1	14	15	9	8	41
S_2	26	25	25	24	133
S_3	39	38	37	39	316
S_4	11	7	8	8	35
S_5	10	7	11	11	35
S_6	7	7	11	10	50
S_7	14	14	15	14	203
S_8	10	7	12	16	61
RR	57	46	31	54	-

$X_1 = 0.21$, $X_2 = .93$, $Z = 908$

System Cost = $21,329/day

TABLE 6.5. Solution to Problem 6.1. (See next page.)

Run 3

Sources	Destinations				
	LF_1	LF_2	LF_3	LF_4	RR
S_1	10	11	5	5	54
S_2	15	15	14	14	178
S_3	16	15	15	15	375
S_4	8	4	5	5	48
S_5	8	4	9	7	52
S_6	3	3	6	6	64
S_7	3	3	4	4	219
S_8	5	3	7	11	85
RR	77	52	27	74	-

$X_1 = .21$, $X_2 = .93$, $Z = 1113$

System Cost = \$17,159/day

Run 4

Sources	Destinations				
	LF_1	LF_2	LF_3	LF_4	RR
S_1	7	8	3	2	61
S_2	7	7	7	6	196
S_3	5	5	5	5	393
S_4	5	2	3	2	57
S_5	4	2	5	5	56
S_6	1	1	3	3	70
S_7	1	1	1	1	212
S_8	2	1	5	6	78
RR	90	51	20	83	-

$X_1 = .21$, $X_2 = .92$, $Z = 1134$

System Cost = \$14,437/day

TABLE 6.5. Solution to Problem 6.1. (See next page.)

Run 5

Sources	Destinations				
	LF_1	LF_2	LF_3	LF_4	RR
S_1	5	6	2	1	72
S_2	4	3	3	3	212
S_3	2	2	2	1	386
S_4	2	2	1	1	52
S_5	2	1	3	3	60
S_6	0	0	1	1	78
S_7	1	1	1	1	212
S_8	1	0	3	3	95
RR	103	45	12	88	-

There is no reason to perform another iteration, since the solution obviously chooses the solution that nearly all (about 96 per cent) waste first passes through the resource recovery plant, where 79 per cent (1-.21) of the municipal solid waste input stream is reclaimed RDF and 1.3 per cent [(1-.94)(.21)] of the waste input stream is reclaimed ferrous metals.

$X_1 = .21$, $X_2 = .94$, $Z = 1169$

System Cost = \$13,501/day or \$11.03/ton

TABLE 6.5. Solution to Problem 6.1. (End)

Run	Number of Iterations (Phase I: McNamara's Procedure)	System Seconds (CDC6400)
1	102	33.8
2	116	36.4
3	113	36.3
4	146	41.2
5	186	47.2

Maximum number of words of memory = 26,624; Compilation time = 1.8 seconds.

TABLE 6.6. Computer Resources Needed to Compute the Solution to Problem 6.1.

		Destinations				
		LF_1	LF_2	LF_3	LF_4	RR
Sources	S_1	12	17	8	5	44
	S_2	21	21	20	20	152
	S_3	25	22	21	26	314
	S_4	12	6	7	7	35
	S_5	8	5	10	10	41
	S_6	6	6	12	12	46
	S_7	8	7	8	8	205
	S_8	8	9	14	8	68
	RR	86	58	32	79	-

$X_1 = 0.24$, $X_2 = 1.09$, $Z = 911$

System Cost = \$20,955/day or \$17.12/ton

TABLE 6.7. Solution to Problem 6.2.

metals should not be considered. To avoid having X_i, i=1,2, driven to values above unity, the constraints

$$X_i \leq 1, \quad i=1,2 \tag{6.32}$$

may be appended to the mathematical program.

Since the solution of Problem 6.2 indicates that the ferrous recovery process should be dropped from the plant, the appropriate cost terms are removed from the model and a new problem, Problem 6.3, is solved. Problem 6.3 is formed by replacing expression 6.30 in Problem 6.2 with

$$T_1 + T_{RR}^{[2]} \tag{6.33}$$

where

$$T_{RR}^{[2]} = 60.5\, Z^{.65} + 1.53\, Z^{.62} X_1^{-1.62}$$

and by deleting constraint 6.26. The substantial reduction in the total system cost from \$17.12 per ton for Problem 6.2 (Table 6.7) to \$12.45 per ton

for Problem 6.3 (Table 6.8) reflects the costs savings of not owning and operating the capital equipment that comprises the ferrous metal recovery process.

Reflecting upon Problem 6.1, we should realize that process 2 is operating at a relatively low level of efficiency, that is, recovering only 6 per cent of the waste input to the process. Therefore, it may be more economical to operate the resource recovery plant without process 2. To determine whether process 2 should be eliminated from the resource recovery plant, Problem 6.4, which is equivalent to Problem 6.1 sans process 2, will be formed and solved. Problem 6.4 is constructed from Problem 6.1 by replacing 6.16 with

$$T_1 + T_{RR}^{[3]} + T_2 \qquad (6.34)$$

where

$$T_{RR}^{[3]} = 40.3\, Z^{.65} + 1.02\, Z^{.62} X_1^{-1.62}$$

and by deleting constraint 6.26. The total system

		Destinations				
		LF_1	LF_2	LF_3	LF_4	RR
Sources	S_1	5	7	1	1	69
	S_2	2	2	2	2	217
	S_3	2	1	1	1	409
	S_4	2	0	0	1	65
	S_5	1	1	3	3	58
	S_6	0	0	1	1	73
	S_7	0	0	0	0	214
	S_8	0	0	1	3	99
	RR	178	54	7	117	-

$X_1 = .297$, $Z = 1204$

System Cost = \$15,235.56/day or \$12.45/ton

TABLE 6.8. Solution to Problem 6.3.

cost of Problem 6.4 (Table 6.9) is slightly less than that of Problem 6.1 (Table 6.5). Since the system costs vary by less than 6 per cent, one could infer that the system costs for Problems 6.1 and 6.4 are approximately identical, and that the ferrous recovery process is essentially breaking even at an efficiency rate of about 6 per cent when the process' capacity is 245 tpd.[1]

In conclusion, for the cost data utilized in Problems 6.1 and 6.4, the recovery of refuse derived fuel by process 1 appears to be economically desirable, while the recovery of ferrous metals by process 2 breaks even. Assuming the level of efficiency does not diminish, a small increase in the waste input to this process would decrease the average cost per ton below that of the alternative of not including the process. This is because process 2 exhibits economies of scale with respect to capacity. As for Problems 6.2 and 6.3, the

---

[1] From Table 6.5, note that the input capacity to the plant Z is 1169 tpd and that the fraction of municipal solid waste remaining from process 1, X_1, is 0.21. Therefore, the capacity of process 2 should be: ZX_1 = 1169 tpd x 0.21 = 245 tpd.

		Destinations				
		LF_1	LF_2	LF_3	LF_4	RR
Sources	S_1	1	1	1	1	78
	S_2	2	2	1	1	215
	S_3	3	3	2	2	400
	S_4	1	1	1	1	62
	S_5	1	1	1	1	64
	S_6	0	0	0	0	72
	S_7	0	0	1	1	212
	S_8	1	1	1	1	99
	RR	187	37	4	84	-

$X_1 = .250$, $Z = 1209$

System Cost = \$12,697.15/day or \$10.37/ton

TABLE 6.9. Solution to Problem 6.4.

system cost disparities are substantially separated to conclude that the ferrous recovery process should not be included in the resource recovery plant.

6.6 Optimality Analysis of the Solution Model

Since Problem 6.1 has reversed constraints, the solution procedure computes a solution that may not be a global minimum. Except for constraint 6.26, the elements in each set of condensation weights corresponding to a reversed constraint initially had identical values. (See Table 6.10.) However, after several iterations of the solution procedure, the weights diverged substantially from their initial settings. (See Table 6.11.) As suggested in Section 5.5 of Chapter 5, the values of the starting weights for constraints 6.17 through 6.25 are polarized away from the dominant column of weights, ξ_{i+j}, $j\varepsilon\{0,5,10,15,\ldots,35\}$, $i\varepsilon\{1,2,\ldots,5\}$, for run 5, Problem 6.1. (Compare Table 6.11 with Table 6.12.) Now the solution procedure is applied to Problem 6.1A, which is Problem 6.1 with the starting values for the condensation weights taken from Table 6.12. Notice that the final condensation weights for both problems depicted in

Constraint	ξ_{i+1}[1]	ξ_{i+2}	ξ_{i+3}	ξ_{i+4}	ξ_{i+5}	SET
6.17	.2	.2	.2	.2	.2	1
6.18	.2	.2	.2	.2	.2	2
6.19	.2	.2	.2	.2	.2	3
6.20	.2	.2	.2	.2	.2	4
6.21	.2	.2	.2	.2	.2	5
6.22	.2	.2	.2	.2	.2	6
6.23	.2	.2	.2	.2	.2	7
6.24	.2	.2	.2	.2	.2	8
6.25	$\frac{\xi_{44}}{.25}$	$\frac{\xi_{45}}{.25}$	$\frac{\xi_{46}}{.25}$	$\frac{\xi_{47}}{.25}$		
6.26	$\frac{\xi_{48}}{.75}$	$\frac{\xi_{49}}{.25}$				

[1] i = (SET-1)5

TABLE 6.10. Initial Condensation Weights for Problem 6.1.

Constraint	ξ_{i+1}[1]	ξ_{i+2}	ξ_{i+3}	ξ_{i+4}	ξ_{i+5}	SET
6.17	.06	.07	.02	.01	.84	1
6.18	.02	.02	.02	.02	.92	2
6.19	.01	.01	.01	0.0	.97	3
6.20	.03	.03	.02	.02	.90	4
6.21	.03	.01	.04	.04	.88	5
6.22	0.0	0.0	.01	.01	.98	6
6.23	0.0	0.0	0.0	0.0	1.00	7
6.24	.01	0.0	.03	.03	.93	8
6.25	ξ_{44} .42	ξ_{45} .18	ξ_{46} .05	ξ_{47} .35		
6.26	ξ_{48} .74	ξ_{49} .26				

[1] $i = (SET-1)5$

TABLE 6.11. Final Condensation Weights for Problem 6.1.

Constraint	ξ_{i+1}[1]	ξ_{i+2}	ξ_{i+3}	ξ_{i+4}	ξ_{i+5}	SET
6.17	.20	.20	.20	.35	.05	1
6.18	.20	.20	.35	.20	.05	2
6.19	.20	.35	.20	.20	.05	3
6.20	.20	.20	.35	.20	.05	4
6.21	.20	.35	.20	.20	.05	5
6.22	.35	.20	.20	.20	.05	6
6.23	.20	.20	.20	.35	.05	7
6.24	.20	.35	.20	.20	.05	8
6.25	ξ_{44} .05	ξ_{45} .30	ξ_{46} .60	ξ_{47} .05		
6.26	ξ_{48} .75	ξ_{49} .25				

[1] $i = (SET-1)5$

TABLE 6.12. Initial Condensation Weights for Problem 6.1A.

Tables 6.11 and 6.13, and the solutions for both problems illustrated in Tables 6.5 and 6.14 are nearly identical. Thus, it would appear that their respective solutions may represent multiple global minimums or solutions close to the global minimum. The only significant disparities in the solutions of the two problems resulted from the level of the ferrous recovery process operation and from the distribution of resource recovery residue to the four landfills. The level of ferrous recovery for Problem 6.1A is about twice that recommended by the solution of Problem 6.1, while the landfill sites 2 and 3 receive the bulk of the residue from the resource recovery plant in Problem 6.1A, but receive the least of the residue in Problem 6.1.

Notice that for Problems 6.1 and 6.1A, both sets of initial starting weights corresponding to constraint 6.26 are identical. (Compare Table 6.10 with Table 6.12.) Unlike the sets of condensation weights associated with constraints 6.17 through 6.25, the condensation weights for constraint 6.26 cannot range freely over the interval (0,1) when subject to its normality condition 6.29. For example, visualize the condensation of constraint 6.14

Constraint	ξ_{i+1}[1]	ξ_{i+2}	ξ_{i+3}	ξ_{i+4}	ξ_{i+5}	SET
6.17	.05	.07	.01	.01	.86	1
6.18	.01	.01	0.0	0.0	.98	2
6.19	.01	.01	.01	0.0	.97	3
6.20	.05	0.0	.02	.02	.91	4
6.21	.01	.01	.03	.04	.91	5
6.22	.01	0.0	.03	.03	.93	6
6.23	0.0	0.0	0.0	0.0	1.00	7
6.24	0.0	0.0	.01	.03	.96	8
6.25	ξ_{44} .01	ξ_{45} .46	ξ_{46} .52	ξ_{47} .01		
6.26	ξ_{48} .73	ξ_{49} .27				

[1] $i = (SET-1)5$

TABLE 6.13. Final Condensation Weights for Problem 6.1A.

Sources	Destinations				
		LF_1	LF_2	LF_3	LF_4	RR
	S_1	4	6	1	1	70
	S_2	2	2	1	1	213
	S_3	2	2	2	1	389
	S_4	3	0	1	1	60
	S_5	1	1	2	3	61
	S_6	1	0	2	2	75
	S_7	1	1	1	1	211
	S_8	0	0	1	3	93
	RR	3	113	126	3	-

$X_1 = .21$, $X_2 = .89$, $Z = 1178$

System Cost = \$13,517/day or \$11.04/ton

TABLE 6.14. Solution for Problem 6.1A.

which is approximated with

$$X_1 \leq \left(\frac{X_1X_2}{\xi_{48}}\right)^{\xi_{48}} \left(\frac{.07}{\xi_{49}}\right)^{\xi_{49}} \rightarrow$$

$$X_1 \left(\frac{X_1X_2}{\xi_{48}}\right)^{-\xi_{48}} \left(\frac{.07}{\xi_{49}}\right)^{-\xi_{49}} \leq 1 \rightarrow$$

$$\xi_{48}^{\xi_{48}} \; \xi_{49}^{\xi_{49}} \; .07^{-\xi_{49}} \; X_1^{1-\xi_{48}} \; X_2^{-\xi_{48}} \leq 1. \tag{6.26}$$

where

$$\xi_{48} = \frac{X_1X_2}{.07+X_1X_2} \tag{6.35}$$

and

$$\xi_{49} = \frac{.07}{.07+X_1X_2} \cdot \tag{6.36}$$

From 6.35, 6.13, and 6.14, we obtain

$$.2 \leq X_1 \leq .07 + \frac{.07\ \xi_{48}}{1-\xi_{48}} \quad \rightarrow$$

$$.2 \leq .07 + \frac{.07\ \xi_{48}}{1-\xi_{48}} \quad \rightarrow$$

$$\xi_{48} \geq 0.65. \tag{6.37}$$

Since $\xi_{48} = .90$ is feasible and represents a greater polarization than $\xi_{48} = .65$. Problem 6.1B, composed of Problem 6.1A with $\xi_{48} = .90$ will be solved. In this case, the final solution value of the objective function for Problem 6.1B (Table 6.15) is considerably higher than the solutions of Problems 6.1 (Table 6.5) and 6.1A (Table 6.14). The previous fact corroborates the belief that Problems 6.1 and 6.1A represent good solutions.

		Destinations				
		LF_1	LF_2	LF_3	LF_4	RR
Sources	S_1	12	17	3	2	51
	S_2	5	11	13	8	193
	S_3	4	4	4	4	382
	S_4	15	3	7	7	37
	S_5	4	1	7	8	49
	S_6	6	2	16	15	36
	S_7	1	1	1	1	209
	S_8	2	2	6	14	78
	RR	1	135	140	1	-

$X_1 = .28$, $X_2 = .95$, $Z = 1035$

System Cost = \$17,841/day or \$14.48/ton

TABLE 6.15. Solution for Problem 6.1B.

6.7 Reasonableness of the Solution to the Model

Based upon initial cost estimates utilized in Problem 6.1, the total system cost computed from Problem 6.1 is $13,501 per day, or $11.03 per ton processed. The solution values of the design variables indicate that 96 per cent of the municipal solid waste generated in the Lehigh Valley should be processed at the resource recovery plant. Furthermore, 79 per cent of the incoming waste should be recovered as RDF (refuse derived fuel), while 1.3 per cent of the waste should be recovered as ferrous metals. However, the total system cost is slightly less when ferrous metal recovery is omitted from the system. The total system cost of Problem 6.4 is $12,697 per day, or $10.37 per ton. In this case, all of the total waste generated should be processed at the resource recovery plant, where 75 per cent of the waste is reclaimed as RDF.

When the net costs of Problem 6.1 for processing municipal solid waste at the resource recovery plant are unilaterally raised by 50 per cent to form Problem 6.2., the total system cost for Problem 6.2 is $20,955 per day, or $17.12 per ton.

In this case, less than 75 per cent should be processed at the recovery plant, where 76 per cent of the input is recovered as RDF and where no ferrous recovery is recommended. The total system cost when ferrous metal recovery is omitted declines appreciably to $15,235 per day, or $12.45 per ton. Furthermore, it is recommended by this solution of Problem 6.3 that all of the Lehigh Valley's municipal solid waste should be processed at the resource recovery plant, where 70 per cent is recycled as RDF.

Since the resource recovery is favored in Problems 6.1 through 6.4, then the alternative of a municipal solid waste system where all waste is deposited at landfills should have a higher total system cost. Therefore, an estimate of the cost of such a landfill system will now be computed. Since all paths in the landfill system lead directly from a waste generating centroid to a landfill site, the cost expression for a particular path is

$$\alpha_{S_iLF_j} W_{S_iLF_j}^{\tau_{S_iLF_j}}, \quad i\in\{1,2,\ldots,8\}, \quad j\in\{1,2,3,4\}, \tag{6.38}$$

where the values for $\alpha_{S_iLF_j}$ can be found in Table 6.16, and the values for $\tau_{S_iLF_j}$ can be found in Table 6.17.

A lower bound for the landfill system cost can be computed by assuming the least cost combination of parameter values from Tables 6.16 and 6.17 for the parameters of each source. Therefore, define the parameters associated with each route emanating from source S_i to a landfill LF_j:

$$\alpha_{S_iLF} = \text{MIN}\{\alpha_{S_iLF_j} \mid j = 1,2,3,4\} \quad (6.39)$$

and

$$\tau_{S_iLF} = \text{MIN}\{\tau_{S_iLF_j} \mid j = 1,2,3,4\}. \quad (6.40)$$

Because of the increasing cost trend for each route, that is $\tau_{S_iLF} > 1$, for all $i\varepsilon\{1,2,\ldots,8\}$, the minimum system cost can be obtained by equally apportioning the waste from each source S_i to each landfill LF_j. Obviously, the total system cost under these assumptions is strictly less than the cost computed with the parameter values of Tables 6.16

		Destinations			
		LF_1	LF_2	LF_3	LF_4
Sources	S_1	16.44	14.39	23.89	26.91
	S_2	15.34	15.80	18.82	20.92
	S_3	15.52	15.98	17.49	20.46
	S_4	14.82	19.82	18.82	20.51
	S_5	25.17	31.08	23.74	22.05
	S_6	20.25	23.38	15.39	15.21
	S_7	17.90	19.28	15.72	16.87
	S_8	22.07	24.94	16.90	13.90

TABLE 6.16. Estimated Transportation and Landfill Costs Measured in 1990 Dollars.

		Destinations			
		LF_1	LF_2	LF_3	LF_4
Sources	S_1	1.10	1.11	1.05	1.05
	S_2	1.10	1.10	1.06	1.04
	S_3	1.10	1.10	1.09	1.04
	S_4	1.10	1.09	1.09	1.05
	S_5	1.05	1.05	1.05	1.06
	S_6	1.07	1.05	1.08	1.09
	S_7	1.08	1.07	1.09	1.09
	S_8	1.05	1.05	1.09	1.10

TABLE 6.17. Transportation Cost Term Exponents.

EXHIBIT 6.4

Lower Bound Computation for the Landfill System

i	$\alpha_{S_i LF}$	$\tau_{S_i LF}$	Quantity of msw at Generating Centroid S_i	Total Cost[1] to dispose of S_i
1	14.39	1.05	83	1389.92
2	15.32	1.04	218	3918.96
3	15.52	1.04	401	7483.06
4	14.82	1.05	65	1107.40
5	22.05	1.05	68	1727.59
6	15.21	1.05	74	1302.33
7	15.72	1.07	213	4422.57
8	13.90	1.05	102	1667.02
		TOTALS	1224 tpd	$23,018.85/day or $18.81/ton

[1]Total Cost to dispose of $S_i = 4\ \alpha_{S_i LF}(S_i/4)^{\tau_{S_i LF}}$, $i=1,2,\ldots,8$.

and 6.17. Thus, as illustrated in Exhibit 6.4, a lower bound for the cost of the landfill system is $23,018.85 per day, or $18.81 per ton processed. One should conclude from this analysis that a resource recovery facility should be included in the solid waste system for the Lehigh Valley.

6.8 Dimensionality of the Computer Model

The size of the nonlinear mathematical programming problem depends upon the number of components and their respective relationship with one another in the system being studied. Using the notation of Chapter 5, a formula for computing the size of the problem by the measures of the total number of terms and the total number of decision variables will be constructed. Assuming that the transportation and landfill disposal costs are combined as in the case study, the formula for calculating the number of terms, N_T, in the problem is:

$$N_T = NS(2NRR+NLF+1) + \sum_{\ell=1}^{NRR} (2+2NP_\ell)+(NRR)(NLF) \qquad (6.41)$$

where

NS = number of sources,

NRR = number of resource recovery plants,

NLF = number of landfill sites, and

NP_{ℓ} = number of processes at resource recovery plant ℓ, ℓ=1,2,...,NRR.

For the case study,

$$NS=8,\ NRR=1,\ NLF=4,\ NP_1=2 \rightarrow$$

$$N_T=8(2+4+1) + (2+2(2))+4 = 66.$$

The number of variables in this computer model is equal to the number of terms in the objective function, which is

$$N_V = NS(NRR+NLF) + \sum_{\ell=1}^{NRR} (1+NP) + NRR(NLF) \tag{6.42}$$

where

$$N_V = 8(1+4) + (1+2) + 1(4) = 47$$

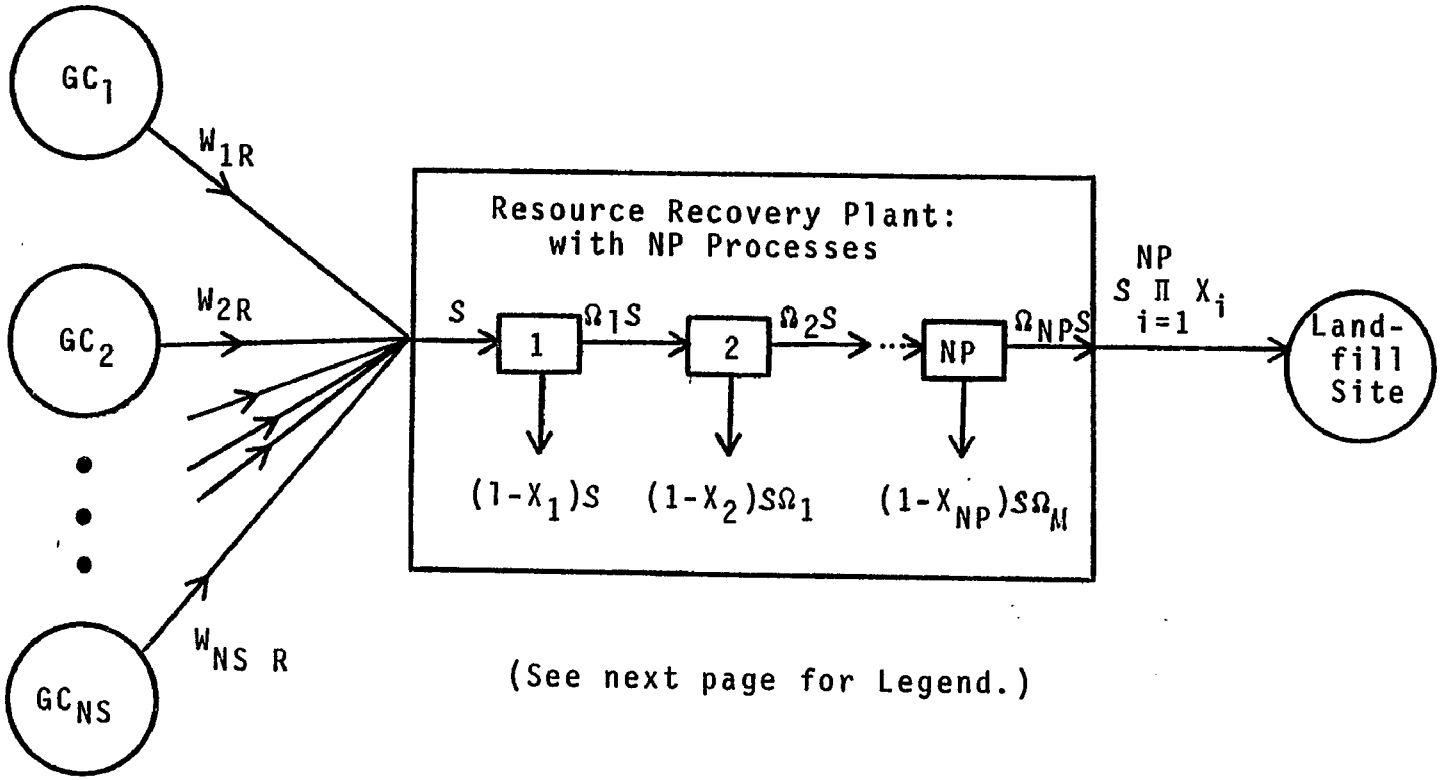
for the case study problem.

The size of the FORTRAN program used to solve the case study problem is directly related to the number of terms, N_T. A forty-term problem needs 16,354 words of memory, while a seventy-term problem needs about 26,624 words of memory on the CDC6400 computer at Lehigh University. Hence, approximately 341 additional words of memory are needed for each new term as computed by the formula represented by N_T. Furthermore, this program, which is composed of about 300 lines of FROTRAN code, required approximately 1.5 seconds of central processor time for compilation on the CDC6400 computer.

Part 2 - A Nonlinear Municipal Solid Waste Model with Global Solutions

6.9 Introduction

A solid waste management system that processes all of a municipality's solid waste will now be investigated. This system consists of multiple sources, one resource recovery plant and one landfill site (See Figure 6.2.) The locations of the resource recovery plant and the landfill site are assumed to have been determined by a preliminary study of



(See next page for Legend.)

FIGURE 6.2. Solid Waste Management Model: Multiple Sources, one Resource Recovery Plant, one Landfill Site.

LEGEND FOR FIGURE 6.2

$GC_i \equiv$ waste generating centroid i, i=1,2,...,NS, where NS is the number of waste generating centroids.

$W_{i\,R} \equiv$ tons per day of municipal solid waste transported to the resource recovery plant from waste generating centroid i, i=1,2,...,NS.

$X_j \equiv$ fraction of solid waste entering process j that is remaining after process j, j=1,2,...,NP, where NP is the number of processes.

$S \equiv \sum_{i=1}^{NS} W_{i\,R}$, the total amount of municipal solid waste generated less source separation.

$\Omega_k \equiv$ fraction of solid waste entering the resource recovery plant that is remaining after process k, i.e.,

$$\Omega_k = \prod_{j=1}^{k} X_j, \quad k = 1,2,\ldots,NP.$$

$M \equiv NP-1.$

available nuisance sites surrounding the municipality. The proper capacity of the recovery plant should be computed from an analysis of the expected volume of waste generated by the municipality. The purpose of this investigation is to determine the minimal cost design of the described municipal solid waste system.

6.10 The Mathematical Model

The costs associated with this system are the capital and operating costs associated with the hauling, recovery processing, and landfilling operations. Since the capacity, Z, of the recovery plant has been predetermined and the waste generating centroids are assumed to be generating a fixed amount of municipal solid waste per day, namely GC_i, $i=1,2,\ldots,NS$, their respective costs are fixed and can therefore be eliminated from the objective cost function. The method of modeling the landfill constraints is very similar to the Lehigh Valley case. However, in this case the increasing cost relationship is assumed to represent the escalating costs of overburdening the chosen landfill with the municipality's solid waste. This relationship discourages

filling up the landfill too rapidly, where only more expensive alternatives exist. Neglecting inflation, the rising costs associated with disposing waste at different sites over a fixed horizon are illustrated in Figure 6.3. These rising costs are attributed to greater transporting distances and/or less efficient landfill sites.

The cost equation is now developed.

Let

$$C_i \left(S \prod_{j=1}^{i-1} X_j \right)^{\alpha_i} X_i^{\beta_i} , \qquad (6.43)$$

be the net cost per day for processes i=1,2,...,NP. For the NP processes, $\alpha_i \in (0,1)$, i=1,2,...,NP, represents the decreasing net processing costs that accompany higher input capacities for process i, i=1,2,...,NP. Concerning the landfill site, α_{NP+1} could be slightly less than one to reflect the economies of scale associated with hauling and landfilling of larger quantities of municipal solid waste. The parameter β_i, i=1,2,...,NP represents the increasing processing costs related to removing

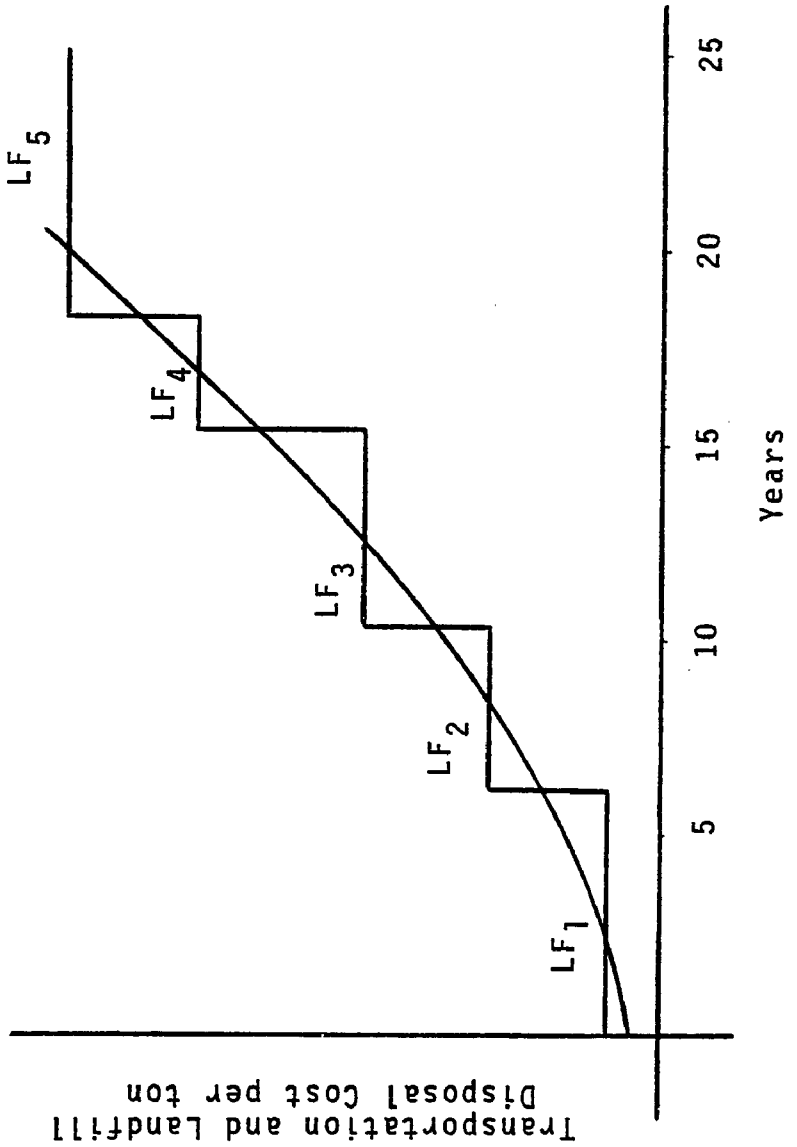


FIGURE 6.3. Increasing Average Costs of a Landfill Over Time.

higher fractions of recoverable materials from the waste stream; hence $\beta_i < 0$, for all $i \in \{1,2,\ldots,NP\}$. The parameter β_{NP+1} for the landfill cost relationship is also negative to reflect the costs of burdening the single landfill with the municipality's solid waste. The parameter C_i, $i=1,2,\ldots,NP+1$ is a positive fixed design coefficient, while S is the total amount of solid waste input to the processing plant in tons per day. Therefore, for a system with NP processes, the total system cost relation is

$$\sum_{i=1}^{NP+1} C_i \left(S \prod_{j=1}^{i-1} X_j \right)^{\alpha_i} X_i^{\beta_i} . \tag{6.44}$$

To assure that all of the municipality's solid waste is reclaimed at the recovery plant or buried at the landfill site, let

$$\prod_{j=1}^{NP+1} X_j \leq \varepsilon \tag{6.45}$$

where ε is a positive and negligibly small real number.

Therefore, the mathematical program is

Minimize:

$$\sum_{i=1}^{NP+1} C_i \left(S \prod_{j=1}^{i-1} X_j \right)^{\alpha_i} X_i^{\beta_i} \tag{6.44}$$

Subject to:

$$\prod_{j=1}^{NP+1} X_j \leq \varepsilon. \tag{6.45}$$

Since the program comprised of minimizing 6.44 subject to 6.45 is a standard geometric program with zero degrees of difficulty, the solution obtained by geometric programming is a unique global optimum. Let

$$X_j^* \text{ , } j=1,2,\ldots,NP+1,$$

be the optimal solution to 6.44 while satisfying 6.45. If for some j, $X_j^* \geq 1$, then the process is too costly and should be deleted from the processing system. If for a process j, the fraction removed

of a material in the municipal solid waste input stream, namely $1-X_j^*$, exceeds the actual content of that material in the municipal solid waste, simply set X_j at its maximum feasible level and resolve the problem.

6.11 Example Problem With Two Processes

A small example problem composed of a resource recovery facility with two processes will exemplify the applicability of the general model. Suppose the quantity of municipal solid waste entering the recovery plant, S, is 1000 tpd, and that the correct exponential parameters are:

$$\underline{\alpha} = (.8, .8, 1.0)$$

and

$$\underline{\beta} = (-2, -3, -.1).$$

Furthermore, suppose the net cost per ton for process 1 is \$3.00 per ton and for process 2 is \$4.00 per ton when the processes are respectively reclaiming 45 per cent and 11 per cent of the 1000 tpd waste stream. The transportation cost from the resource recovery plant to the

landfill is combined with the landfill disposal cost. It is assumed that when 440 tpd are shipped from the recovery facility to the landfill, the cost per ton is $10.00. Hence, the vector of the objective function coefficients is:

$$\underline{C} = (3.61, 7.23, 5.01). \quad \text{(See Exhibit 6.5.)}$$

Finally, the constraint tolerance parameter, ε, is $0.001 \rightarrow C_4=1000.0$. The example, Problem 6.5, is now stated:

Minimize:

$$907\ X_1^{-2} + 1816\ X_1^{-.8} X_2^{-3} + 5010\ X_1X_2X_3^{-.1} \tag{6.46}$$

Subject to:

$$1000\ X_1X_2X_3 \leq 1 \tag{6.47}$$

The dual program of Problem 6.5 is:

EXHIBIT 6.5

Calculation: Objective Function Coefficients

Process 1 Cost Term

$$\frac{\text{cost/day for Process 1}}{\text{tons/day input to Process 1}} = \frac{C_1 S^{\alpha_1} X_1^{\beta_1}}{S}$$

$$= \frac{C_1\ 1000^{.8}\ (1 - .45)^{-2}}{1000} = \$3/\text{ton}$$

→

$C_1 = 3.61$, since $X_1 = 1 - .45 = .55$.

Process 2 Cost Term

$$\frac{\text{cost/day for Process 2}}{\text{tons/day input to Process 2}} = \frac{C_2(SX_1)^{\alpha_2} X_2^{\beta_2}}{SX_1}$$

$$= \frac{C_2\left[1000(1-.45)\right]^{.8}\ (.8)^{-3}}{1000(1 - .45)} = \$4/\text{ton}$$

→

$C_2 = 7.23$, since $.11 = X_1(1-X_2) \rightarrow X_2 = .80$.

(See next page.)

EXHIBIT 6.5 (continued)

Transportation and Landfill Cost Term

$$\frac{\text{cost/day for transportation and disposal}}{\text{tons/day disposed}}$$

$$\frac{C_3(SX_1X_2)^{\alpha_3} X_3^{\beta_3}}{(SX_1X_2)} = C_3(0.001)^{-.1}$$

$$= \$10/\text{ton}$$

$$\rightarrow$$

$$C_3 = 5.01.$$

Maximize:

$$v(\underline{\delta}) = \left[\frac{907}{\delta_1}\right]^{\delta_1}\left[\frac{1816}{\delta_2}\right]^{\delta_2}\left[\frac{5010}{\delta_3}\right]^{\delta_3}\left[\frac{1000}{\delta_4}\right]^{\delta_4}\ \delta_4^{\delta_4} \tag{6.48}$$

Subject to:

$$\begin{bmatrix} -2 & .8 & 1 & 1 \\ 0 & -3 & 1 & 1 \\ 0 & 0 & -.1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{6.49}$$

and

$$\underline{\delta} > 0. \tag{6.50}$$

Notice that this geometric programming problem has degree of difficulty equal to zero. Hence, a unique global minimum can be obtained by solving the linear system 6.49 (page 12 of reference 160). The solution to the dual program is:

$$\underline{\delta}^* = (0.33,\ 0.18,\ 0.49,\ 0.049).$$

Therefore, $v(\underline{\delta}) = 9275$.

By utilizing the primal-dual variable relation 4.3.16, we obtain

$$X_1 = \left[\delta_1 \quad v(\underline{\delta})/907\right]^{-1/2} = .54$$

$$X_2 = \left[\delta_2 \quad v(\underline{\delta})/1816X_1^{.8}\right]^{-1/3} = .87$$

$$X_3 = \left[\delta_3 \quad v(\underline{\delta})/15010\ X_1X_2\right]^{-10} = 0.00139$$

Hence, process 1 and process 2 should respectfully reclaim 460 tpd and 70 tpd of the input waste, while the remaining 480 tpd should be disposed at the landfill site. The total system cost per day for this optimally designed system is $9,275. The corresponding cost contributions of each process and of the landfill to the total system can be directly obtained from the dual variables, namely, 33 per cent for process 1, 18 per cent for process 2, and 49 per cent for the landfill disposal. These relative contributions are invarient regardless of the values of the coefficients of any of terms.

6.12 Case Study: Milwaukee-Americology

"Americology Division of American Can Company has designed, engineered, and constructed a new $18 million plant, and has a 15-year contract with the city to dispose of 100 per cent of Milwaukee's city collected solid waste" (page 1 of reference 60). "Milwaukee is the largest American city to process all of its municipal solid waste for resource recovery" (page 12 of reference 60). "The clean, non-polluting operation of Milwaukee's Americology resource recovery plant permits the facility to be located near the center of the city" (page 1 of reference 60). The process utilized by Americology is depicted in Figure 6.4. The net cost of $10 per ton for processing the municipal solid waste at the Americology plant approximately equals the cost of landfill disposal in the area (172). The plant is currently processing about 1000 tpd of municipal solid waste, but it is designed for a maximum future load of 1500 tpd. About "...90 per cent of the city's waste is being recovered for new uses..." (60). Based upon the current input to the Americology facility, the following percentages of

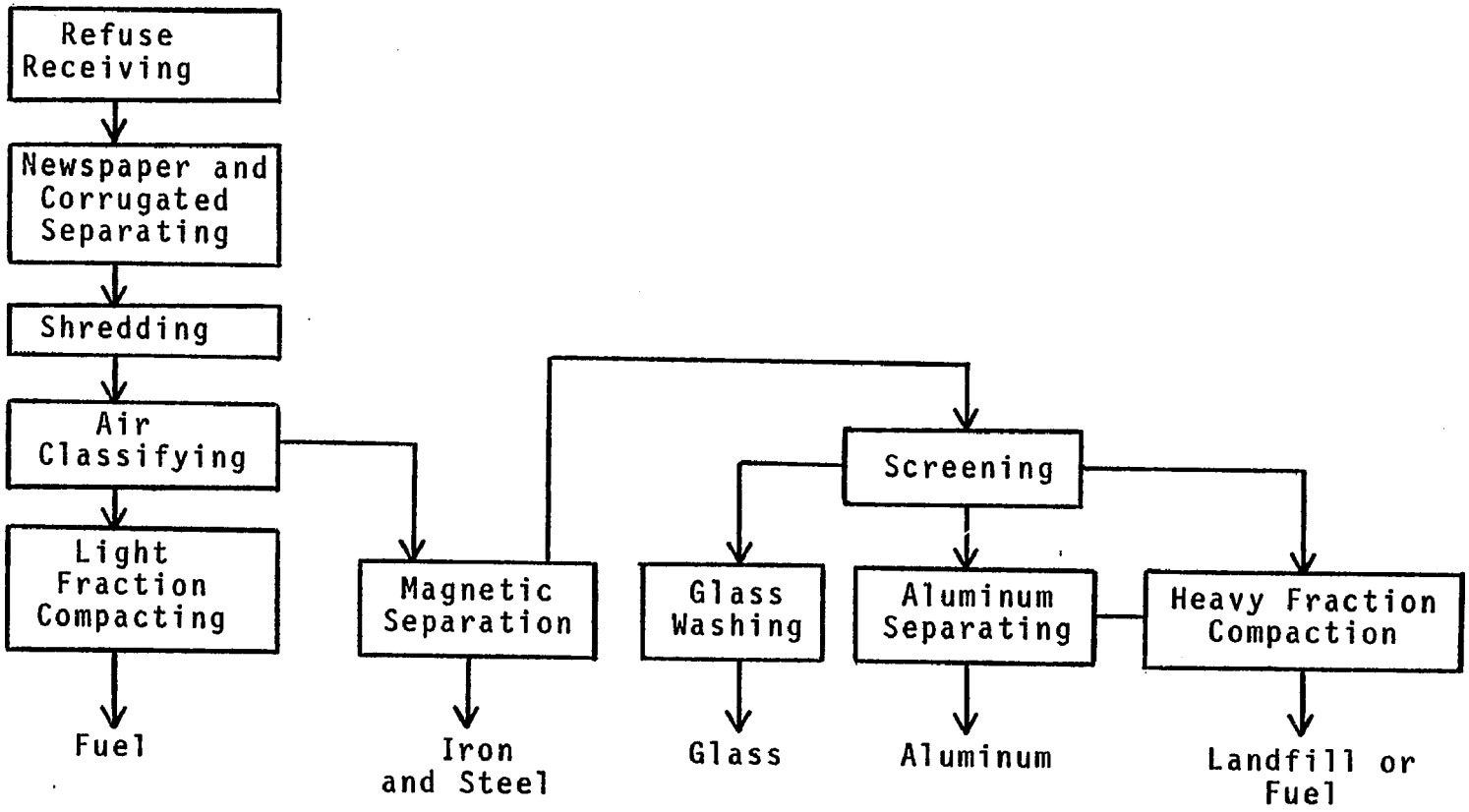


FIGURE 6.4. Resource Recovery Plant Schematic of Americology.

Milwaukee's municipal solid waste can be recovered each year:

a) 65 per cent in fiber fuel for electric power generation by the Wisconsin Electric Power Company,
b) 7 per cent in ferrous metals,
c) 0.5 per cent in aluminum,
d) 5 per cent in newspaper and corrugated paper, and
e) 10 per cent glassy aggregate.

The remaining unrecovered solid waste, which amounts to 12.5 per cent, or 125 tpd, can be hauled to the nearest landfill in about five round trips per day when utilizing a compactor tractor trailor (pages 2-23 of reference 165).

Notice that the Americology disposal system naturally fits the model composed of 6.44 and 6.45. All flows of municipal solid waste from the waste generating sources converge at the resource recovery plant, where 100 per cent of the solid waste is processed. Although not stated, it can be reasonably assumed that the 125 tpd residue from the recovery plant is shipped to the least cost landfill

site. For example, all but two of the nine landfills in the Lehigh Valley can easily handle a 125 tpd load (Table 3 of reference 165). An analysis analogous to the Lehigh Valley case study in Part 1 could be effected to apportion the several unit operations depicted in Figure 6.4 into possibly four material recovery processes: a fuel recovery process, an iron and steel recovery process, a glass recovery process, and an aluminum recovery process.

There are indications that municipal solid waste systems composed of either one resource recovery plant and one landfill, or simply one landfill, are not only recommended for Milwaukee, but recommended as the best alternative in other areas of the United States. A solid waste system where all of the waste is deposited at a single landfill is the least cost county solid waste system for six counties in Southwestern Indiana and Northern Kentucky (page 80 of reference 70). However, only landfill disposal was studied. A single resource recovery plant coupled with these single landfill solid waste systems may prove to be economical if

the amount of solid waste generated in the area is great enough to achieve the benefits of economies of scale.

CHAPTER 7

SUMMARY, CONCLUSIONS, AND FUTURE STUDY AREAS

From existing data, transportation, landfilling, and resource recovery costs of municipal solid waste systems are shown to exhibit the nonlinear phenomenon of economies of scale and/or diseconomies of scale. Little work has been done on the nonlinear solid waste management problem. Only one paper could be found that indirectly models the economies of scale property with piecewise linear approximation. No papers could be discovered that model the nonlinear phenomenon of diseconomies of scale or that directly solve the nonlinear solid waste models with a nonlinear programming technique. In this research a nonlinear solid waste management model is developed which includes the nonlinear phenomena of economies of scale and diseconomies of scale to reflect reality.

The general nonlinear solid waste model in this dissertation quantitatively illuminates the economic relationships from both macro and micro viewpoints with respect to municipal solid waste management systems. From a macrosystem viewpoint, the municipal solid

waste system can be modeled with an interconnecting network of waste generating centroids, resource recovery plants, and landfills. The economic interactions of these entities are represented with nonlinear cost functions. From a microsystem viewpoint, the internal multistage process structure of a resource recovery plant is effectively modeled with multiplicative relationships between certain plant design variables. The nonlinear cost relation for each process contains economic information describing the interacting costs for operating at different levels of efficiency and capacity. Therefore, the solution to the model exposes the design characteristics for both the underlying macrosystem and microsystem comprising the nonlinear solid waste model. For certain types of nonlinear solid waste models, this solution can be shown to be a global optimum.

This author believes that resource recovery and/or volume reduction of municipal solid waste at a processing plant will become overwhelmingly favorable in the near future. The results of the Lehigh Valley case study indicate that resource recovery is favored even when the estimated net costs of recovery are increased by 50 per cent. This finding appears to be valid, since the volume of municipal solid waste

processed at the plant is commonly regarded to be high enough for resource recovery to be competitive with other alternatives such as landfilling. Moreover, the system cost of the Lehigh Valley system with resource recovery is substantially lower than that of a pure landfill system. Since collection and transportation costs constitute the largest component of the solid waste management system cost (page 34 of 56), they will be a major determining factor towards favoring resource recovery. The transportation costs will be rising rapidly due to the escalating costs of labor and the tendency of landfills to be located greater distances from the waste generating centers. Available land for landfilling operations is becoming scarce and costly to purchase. For example, the cost for the Croton Point landfill in Westchester County, New York, should include the costs of land, the costs of daily operations, and the costs associated with refurbishing the environment which was mandated by a recent United States government suit against the county. Resource recovery is in its infancy of technological development; therefore, advances in cost saving recovery processes can be expected in the near future. Furthermore, as daily generation rates of municipal solid waste increase in future years, the

average cost to recover waste will diminish for the resource recovery plants possessing the property of economies of scale with respect to capacity. Recently passed legislation is requiring both landfill and incinerator owners to adopt new expensive operating procedures and to purchase expensive pollution abatement equipment. Since the public is becoming aware of the external costs of disposal practices which pollute the environment, social costs are being levied in the form of pollution abatement equipment and procedure costs. All these factors contribute toward encouraging resource recovery.

The nonlinear mathematical model developed in this paper only investigates the minimization of positive net costs, even though some studies including Arndt's study of the Lehigh Valley region suggest that the solid waste system with resource recovery can possess negative net costs (profits). A current review of the well known and operating resource recovery systems unveils the reality that they are operating at positive net cost levels which, in many cases, are much higher than expected (183). If the scrap values of certain recoverable materials in municipal solid waste are profitable to extract, then the material concentration of these valuable materials may be

reduced by source separation. Higher scrap values for certain materials such as aluminum and newspaper have already induced source separation and reclamation. Moreover, legislation that causes source recovery can also diminish the amount of valuable recoverable materials in municipal solid waste. For example, Vermont has instituted a return deposit fee on all beverage containers sold in the state. Furthermore, the future markets for the recovered materials are volatile. For the Lehigh Valley, reclamation of refuse derived fuel (RDF) generates substantially more revenue per ton of solid waste than the ferrous metal recovery, but the existing market could weaken if Pennsylvania Power and Light converts to an alternate energy source such as nuclear, solar, or geothermal energy. Although negative net costs can be modeled with geometric programming by utilizing condensation techniques (138, 144, 163, 164), the merits of including the negative costs do not outweigh the disadvantages of greater computational difficulties resulting from negative terms in the objective function.

This author believes that the nonlinear programming technique geometric programming is a natural mathematical tool for solving municipal solid waste

management models because of their inherent nonlinearities. The nonlinear phenomenon of economies of scale can be accommodated directly with geometric programming sans linear estimations. Piecewise linear approximation procedures for estimating this nonlinear phenomenon result in very large and complicated linear programs. However, when the economies of scale, the diseconomies of scale, and the multistage processing properties are included in a municipal solid waste management model, the resulting mathematical relations are nonseparable functions which do not lend themselves to be solved by piecewise linear approximation techniques. Furthermore, a global solution can be obtained with geometric programming for certain types of solid waste problems which are not required to be convex mathematical programs. In fact, one of the most successful municipal solid waste systems with resource recovery, Americology, Inc., falls within the constructs of this model. Geometric programming yields useful economic information: the dual variables corresponding to the primal terms in the objective function equal the fractions of the total cost that each term contributes. This research has demonstrated that geometric programming is an appropriate technique for solving these nonlinear programs.

In Chapter 4 a hybrid solution procedure composed of two previously developed algorithms is constructed to calculate solutions to mathematical models of solid waste systems. The FORTRAN program of the solution procedure comprises about 300 lines of code and compiles in less than two seconds on a CDC6400 computer. The time and memory requirements for the FORTRAN program are related to the size of the solid waste management model; the Lehigh Valley case study problem could have been solved on a small computer with only 27,000 words of memory. Furthermore, with the addition of an unsophisticated data preparation routine, the resulting FORTRAN program could be employed by a solid waste management practitioner who is unfamiliar with geometric programming. Therefore, after the practitioner has modeled a reasonably sized solid waste management system and has completed the data analysis for the system, only a small office computer and the FORTRAN coded algorithm are needed to solve the model. This practicality of the solution procedure suggests a new outlook when designing municipal solid waste systems.

Some applications of the general nonlinear programming model derived in Chapter 5 may require reversed constraints in the mathematical model. Since

global optimality to geometric programs with reversed constraints cannot be guaranteed, this paper presents a hueristic procedure to aid in determining the optimal solution or a solution believed to be near the optimal solution. The hueristic rule determines a method of revising initial values of condensation weights to expose what is believed to be the widest range of local optimums. In practice, only a few problems need to be solved where the minimum of the solutions to the problems appears to be the optimum or at least very close to the optimum.

Although techniques for solving geometric programs with reversed constraints and negative terms exist, to the author's knowledge, none can guarantee global optimality. This research analyzes the effect of different choices of certain parameters upon the solution to these programs. It is suggested that these types of geometric programs be investigated to determine if there exist special structures for which global solutions can be easily obtained.

It has been the experience of this author and others that good cost data for unit operations of municipal solid waste management systems are either not available to the public or do not exist. It is

suggested that the current government supported projects provide extensive and detailed data to the public to encourage further research of municipal solid waste management systems.

One possible extension of this research is to investigate a dynamic version of the static model presented in this paper. The solution to a dynamic model could produce information such as:

a) When and how much to expand a facility or process;

b) When to delete and add certain facilities, processes, and capital equipment; and

c) When to refinance an existing system if interest rates change or if some debt instruments are coming due.

The data utilized to compute the input parameters is stochastic in nature. Hence, the general model could be extended to reflect the stochastic nature of the data. Stochastic geometric programming has been developed and could possibly be embedded in the hybrid solution procedure developed in this research to solve the stochastic model. However, an extensive statistical analysis of the study area is required to generate probability distributions for the input parameters.

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APPENDIX

CONVEXITY

The purpose of this appendix is to present those properties of convex sets and convex functions that are necessary to expose the underlying theory and structure of geometric programming. The theorems and definitions of this appendix have been adapted from reference 160.

Let $\underline{x}$, $\underline{y}$ ϵE_N, the N dimensional Euclidean vector space. Define the line segment, xy, joining $\underline{x}$ and $\underline{y}$ as all points $\delta_1\underline{x} + \delta_2\underline{y}$, where $\delta_1 + \delta_2 = 1$, and $\delta_1, \delta_2 \geq 0$.

Definition A.1

A set S of points in E_N is said to be convex if the line segment xy joining any two points $\underline{x}$ and $\underline{y}$ of S is contained in S.

Theorem A.1

If A and B are convex subsets of E_N, their intersection $A \cap B$ is also a convex subset of E_N.

Definition A.2

A function f defined on a convex subset S of E_N is said to be convex if f satisfies the inequality

$$f(\delta_1 \underline{x} + \delta_2 \underline{y}) \leq \delta_1 f(\underline{x}) + \delta_2 f(\underline{y})$$

for each point $\delta_1 \underline{x} + \delta_2 \underline{y}$ belonging to an arbitrary line segment xy contained in S. A convex function is said to be strictly convex if the inequality

$$f(\delta_1 \underline{x} + \delta_2 \underline{y}) < \delta_1 f(\underline{x}) + \delta_2 f(\underline{y})$$

is satisfied when $\underline{x} \neq \underline{y}$ and $\delta_1, \delta_2 \neq 0$.

Theorem A.2

A function f which has continuous partial derivatives of the second order on a convex subset S of E_N is convex on S if the related function

$$\phi(S) = f([1-s]\ \underline{x} + s\underline{y})$$

satisfies the inequality

$$\frac{d^2\phi}{ds^2} \geq 0,\ 0 \leq s \leq 1,$$

for arbitrary points $\underline{x}$ and $\underline{y}$ in S. Furthermore, f is strictly convex if the preceding inequality is strict when $\underline{x}$ and $\underline{y}$ are distinct points of S.

Corollary A.1

A function f which has continuous partial derivatives of the second order on a convex subset S of E_N is convex on S if the quadratic form

$$Q(\underline{q},\underline{z}) = \sum_{i,j=1}^{N} \left.\frac{\partial^2 f}{\partial z_i \, \partial z_j}\right|_{\underline{z}} q_i q_j$$

is positive semidefinite for each point $\underline{z}$ belonging to S. Furthermore, f is strictly convex if this quadratic form is positive definite for each $\underline{z}$ in S. Notice that

$$\frac{d^2\phi}{ds^2} = \sum_{i,j=1}^{N} \left.\frac{\partial^2 f}{\partial z_i \, \partial z_j}\right|_{\underline{z}} (y_i - x_i)(y_j - x_j),$$

where

$$\underline{z} = [1-s]\underline{x} + s\underline{y} \; .$$

Let

$\underline{q} = \underline{y} - \underline{x}$, then

$$Q(\underline{q};\underline{z}) = \frac{d^2\phi}{ds^2} \geq 0 \rightarrow f \quad \text{is convex.}$$

The hyperplane tangent to the hypersurface representing the convex function f lies below the hypersurface. This is the property stated in the next theorem.

Theorem A.3

If f is a convex function on a convex set S and if f is differentiable at a point $\underline{y}$ belonging to S, then

$$f(\underline{x}) + \nabla f(\underline{x}) \cdot [\underline{y}-\underline{x}] \leq f(\underline{y})$$

for each point $\underline{y}$ in S.

Theorems A.4 and A.5 indicate how convex functions can be generated from other convex functions. Theorem A.4 states that the family of convex functions is closed under composition, while Theorem A.5 states the closure relationship for linear combinations of convex functions.

Theorem A.4

If h is a monotone nondecreasing convex function defined on E_1 and if f is a convex function defined on a convex subset S of E_N, the composite function h(f) is a convex function with domain S.

Theorem A.5

If the functions f_i, i=1,2,...,M, are convex on the same convex domain S of E_N, the linear combination

$$\sum_{i=1}^{M} c_i f_i$$

is also a convex function on S when the coefficients c_i are non-negative constants.

Convex sets can be formed by convex functions constrained in a certain fashion. This property is stated by Theorem A.6.

Theorem A.6

If f is a convex function defined on a convex subset S of E_N and c is an arbitrary number, then all points $\underline{z} \in S$ which satisfy the inequality $f(\underline{z}) \leq c$ form a convex subset of S.

The definition of concave functions can be obtained from the definition of convex functions simply by reversing the sign of the function f, i.e. a function f is concave if, and only if, -f is convex. Therefore, the concave functions possess theorems analogous to the previously mentioned properties of convex functions.

Definition A.3

A function f defined on a convex subset S of E_N is said to be concave if f satisfies the inequality

$$f(\delta_1 \underline{x} + \delta_2 \underline{y}) \geq \delta_1 f(\underline{x}) + \delta_2 f(\underline{y})$$

for each point $\delta_1 \underline{x} + \delta_2 \underline{y}$ belonging to an arbitrary line segment xy contained in S where $\underline{x}$ and $\underline{y}$ are in S, $\delta_1 + \delta_1 = 1$, and $\delta_1, \delta_2 \geq 0$. A concave function f is said to be strictly concave if the strict inequality

$$f(\delta_1 \underline{x} + \delta_2 \underline{y}) > \delta_1 f(\underline{x}) + \delta_2 f(\underline{y})$$

is satisfied when $\underline{x} = \underline{y}$ and $\delta_1, \delta_2 = 0$.

Let f be a convex function. Then for each convex set S in the domain of f, a convex program can be defined:

minimize f over S.

If $\underline{x}' \in S$ and $f(\underline{x}') \leq f(\underline{x})$ for each $\underline{x} \in S \cap N_\varepsilon$, where N_ε is some sufficiently small neighborhood, then $\underline{x}'$ is said to be a local minimizing point for f over S; $\underline{x}' \in S$ is a global minimizing point for f over S if $f(\underline{x}') \leq f(\underline{y})$ for all $\underline{y} \in S$. Clearly every local

minimizing point $\underline{x}'$ is also a global minimizing point for $\underline{x}'$ over S. Since if $\underline{x}'$ is a local minimizing point and $\underline{y} \varepsilon S$, then for $0 < s < \varepsilon$,

$$f(\underline{x}') \leq f([1-s]\underline{x}' + s\underline{y}) \leq [1-s]\, f(\underline{x}') + sf(\underline{y})$$

$$\rightarrow \quad f(\underline{x}') \leq f(\underline{y}) \quad \rightarrow$$

$\underline{x}'$ is a global minimizing point for f over S. Moreover, if $\underline{x}_1$ and $\underline{x}_2$ are global minimizing points for f over S, then $f(x_1) = f(x_2)$, since $f(x_1) \leq f(x_2)$.

Theorem A.7

The set of minimizing points for a convex program is convex.

If M is the minimizing value, let $\underline{z} \varepsilon S_M$, then S_M is the set of minimizing points for f over S if $f(z) \leq M$. The result follows directly from Theorem A.5. Furthermore, if $\underline{x}, \underline{y} \varepsilon S_M$, then $\delta_1 \underline{x} + \delta_2 \underline{y} \varepsilon S_M$ and

$$f(\delta_1 \underline{x} + \delta_2 \underline{y}) = \delta_1 f(\underline{x}) + \delta_2 f(\underline{y})$$

for f convex over S and S_M the set of minimizing points for f over S.

Let f be a convex function on a convex set S. Then $\underline{x}' \in S$ is a stationary point for f if f is differentiable at $\underline{x}'$ and $\nabla f(\underline{x}') = \underline{0}$.

Theorem A.8

Each stationary point for a convex function on a set S is also a minimizing point for f over S. This theorem follows immediately from Theorem A.3. However, the converse is not true.

Let $G_k(\underline{z})$, $k=0,1,2,\ldots,p$ be convex functions on $E_m \rightarrow G_k(\underline{z}) \leq 0$ for $k=1,2,\ldots,p$. The set of points $\underline{z}$ that satisfy some constraint $G_k(\underline{z}) \leq 0$, $k \in \{1,2,\ldots,p\}$ is convex by Theorem A.6. Therefore, the set of points $\underline{z}$ that satisfy all $G_k(z) \leq 0$, $k \in \{1,2,\ldots,p\}$ is convex since an intersection of convex sets is convex by Theorem A.1. The resulting convex program comprises minimizing $G_0(\underline{z})$ over a convex subset of E_m. The program is said to be consistent if there exists a point that satisfies all its constraints. It is said to be superconsistent if there exists a point $\underline{z}^*$ satisfying $G_1(\underline{z}^*) < 0$, $G_2(z^*) < 0, \ldots, G_p(z^*) < 0$, i.e., there is a point that can simultaneously satisfy all constraints as strict inequalities.

One method of solving the convex program is by Lagrange. Observe that the Lagrange function

$$L(z,\underline{\mu}) = G_0(\underline{z}) + \sum_{k=1}^{p} \mu_k \, G_k(\underline{z})$$

which is convex, since it is composed of a linear combination of convex functions. The μ_k's are called Lagrange multipliers.

Theorem A.9 (Kuhn-Tucker Theorem)

Suppose that the convex program of minimizing $G_0(\underline{z})$ subject to $G_k(\underline{z}) \leq 0$, $k=1,2,\ldots,p$, is super-consistent where $G_k(\underline{z})$, $k=0,1,2,\ldots,p$, are convex functions. Let $G_k(\underline{z})$, $k=0,1,2,\ldots,p$, have continuous first partial derivatives on E_m. Then z' is a solution to the program if and only if there is a vector $(\underline{z}', \underline{\mu}')$ satisfying

(i) $\mu_k' \geq 0$, $k=1,\ldots,p$,

(ii) $\mu_k' \, G_k(\underline{z}')=0$, $k=1,2,\ldots,p$, and

(iii) $L(\underline{z}', \underline{\mu}) \leq L(\underline{z}',\underline{\mu}') \leq L(\underline{z},\underline{\mu}')$

for all $\underline{z} \in E_m$ and for all $\mu_k \geq 0$, where

$$L(\underline{z},\underline{\mu}) = G_0(z) + \sum_{k=1}^{p} \mu_k \, G_k(\underline{z}).$$

The superconsistency assumption in Theorem A.9 guarantees the existence of the Lagrange multipliers. Suppose the convex program minimizes $G_0(z)$ subject $G_1(z) \leq 0$ is not superconsistent. If z' is the only point in the domain of this program and $\frac{d\, G_0(z')}{dz} \neq 0$, then there is no $\mu' \geq 0$ such that $L(z,\mu')$ is stationary at z'.

Condition (iii) of Theorem A.9 unveils the duality nature of the convex program with the aid of the Lagrangian function.

VITA

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