Volume, number of trades, and price adjustment process: Theory and empirical properties.

Volume, Number of Trades, and Price Adjustment Process: Theory and Empirical Properties

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Abstract

We develop a variant of the standard sequential trade model to discover the roles of both number of trades and trade size simultaneously. We demonstrate that the significance of the number of trades for the market maker's pricing depends on his prior probabilistic belief on the value of the underlying asset, the informedness of the market, and the elasticity of the informed traders' demand and supply schedules. We also show the economic situations where the number of trades rather than the trading volume sends a better signal to the market maker.

We implement empirical tests to see the implications of our theoretical model. We examine the relationships among the number of trades, the bid-ask spread, the past-price variability, the market informedness, and the firm size while the trading volume is held constant. The results indicate that the market maker generally raises the bid-ask spread when he experiences a relatively large number of transactions. There appears to be a persistent, positive effect of the number of trades on the market maker's setting of the bid-ask spread. We also find that past-price variability, market informedness, and firm size can effect the number of trades, but have a statistically insignificant effect on the bid-ask spread.

We further test whether the stock return has a positive relationship with the number of trades after considering the stochastic characteristics of its volatility. We examine whether the established time-series property of observed returns, AR(1) process, comes from the traders' strategic behavior rather than from simple measurement error in calculating returns. Our results show it to be plausible that this truly is the case.

Chapter I. Introduction

"Knowledge is power" -- a much overused cliché. In the finance world there are seemingly endless examples supporting this simple statement. As a result, a large amount of research concentrates on the power of financial knowledge and its influence on the ebb and flow of the big board. Recent studies of security prices and information demonstrate that a security's price (in particular, its bid-ask spread) does in fact depend on the number of trades (transactions) and their volume.[1] The presumption is that the number and volume of trades somehow reflect the information possessed by the market participants. Thus, on Wall Street, the bid-ask spread can depend indirectly (through volume and transaction data) on the information of the traders.

For example, on average, the bid-ask spread is larger for a large-size transaction than for a small-size transaction. If a large-size transaction suddenly occurs, then the market maker assumes that information is circulating to which he is not privy, and moves to protect himself by setting up a larger bid-ask spread.[2] For infrequently traded securities, a transaction in itself signals new information, again to which he is not privy. Therefore, following a trade the market maker sets a higher bid-ask spread. But, for frequently traded securities, both the presence and the absence of trade provides information. In particular, with an extended absence of trade, the market maker assumes that an information event has

[1] The number of trades is synonymous with the frequency of trades in the literature. But since the latter can confuse us with the terminology of the cyclical movement, the former is used in this dissertation unless the author explicitly uses the frequency of trades.
[2] For this dissertation, when we refer to the term, market maker, it implies a dealer who buys and sells stock to provide liquidity to the market place. Chapter III will describe this in more detail.

not occurred. He accordingly lowers the bid-ask spread. Moreover, the bid-ask spread is decreasing in the time between trades.

Easley and O'Hara (1987), in a model where investors are allowed to trade different sizes, investigate the time path of the bid-ask spread. They focus on an adverse-selection problem with both informed and uninformed traders. The informed traders have perfect information about a security's value and proceed on that information, while the uninformed participants trade for liquidity reasons. The informed participants, on average, trade larger quantities than do the uninformed traders. Thus, a market maker uses trade size as an imperfect indicator of a security's true value and sets the bid-ask spread according to trade size data. In particular, the bid-ask spread is increasing in trade size.

Easley, Kiefer, O'Hara, and Paperman (1996) examine the bid-ask spread of infrequently traded securities. With these securities, when a trade actually occurs, the market maker presumes that an information event (to which he is not privy) lies behind it, and he accordingly raises the bid-ask spread. Easley and O'Hara (1992) pursue the idea that the timing of trades is related to the frequency with which a security is traded.[3] They show that the lack of trade provides the market maker with a signal of the low possibility of his facing an informed trader. Spreads will then depend on the time between trades, with spreads decreasing as this time increases.

[3] Diamond and Verrecchia (1987) also deal with the role of time focused on the speed with which the stochastic process of prices impounds information. They use *the first passage time approach* with the assumption of the existence of short sale constraints. They conclude that bad news is related with no trade since a trader informed of bad news may be unable to trade if short sales are prohibited. This empirically implies that both bid-ask prices should fall following a period of no trade.

From observing the applications of this conventional wisdom and reading the supporting research, one might simply conclude that, for frequently traded securities, a larger volume or a large number of trades in a given trading period does in fact lead the market maker to set up a larger bid-ask spread. On average this might be true, but there are economic situations that the research models fail to address.

This dissertation examines those situations. For example, the aforementioned models give no explanation about differences in the bid-ask spreads between a small number of large-size trades and a large number of small-size trades (both having similar gross volumes) within a given trading period.[4] If investors have information about the asset's true value, that information can be revealed in their trading behavior, which can be reflected in either trade size or number of trades. In this situation both the number of trades and trade size are the key variables in determining differences in the bid-ask spreads.

In chapter III, we develop a model in which the number of trades and trade size for frequently traded securities are both considered. By fixing the gross volume, we can separate the effects of both the number of trades and trade size on the bid-ask spread. This model helps us to focus on economic situations which the research misses and to see under what conditions, more frequent trading (or larger-size trading) reveals the existence of better information, making the bid-ask spread larger.

There are two general scenarios, each illustrating the significance of the number of trades and the trade size. Case A identifies a scenario in which a greater number of trades (again holding the gross daily volume constant) provides greater information to the

[4] The typical or implicit assumption about the relationship between volume and the number of trades in the literature is that the more frequently the stock is traded, given a trading period, the greater the volume.

market maker. Suppose, for example, that informed trader's demand and supply schedules are perfectly inelastic, and that, with a positive information event, some traders always place large orders while others always place small orders. The size of an individual's demand is independent of the amount of the trader's information, but dependent on its existence. In this case, the trade size provides the market maker with no information, while the number of trades does.

Case B presents an alternative scenario in which greater trade size (and thus a lower number of trades) provides greater information than does the number of trades to the market maker. In this scenario, there is a small variation in the past value of a security and the market maker's prior probabilistic belief reflects this small past variation. Also, the market maker has a strong belief that at least some of the traders are informed and that informed participants trade large amounts of the security. In this case, if a market participant places a large-size demand, the market maker decides that the security's value is high, and thus sets a high ask price. If the market maker is presented with a large-size supply, the market maker assumes that the security's value is low, and sets a low bid price. Thus, a larger-size transaction results in a higher bid-ask spread.

To investigate the effect of both the number of trades and the trade size of a security, this dissertation must extend the models of information-based trading in the literature. In the adverse-selection models that examine the effect of the number of trades on the bid-ask spread, only one unit per trade given a trading period is allowed [Easley and O'Hara (1992), Easley, Kiefer, O'Hara and Paperman (1996)]. In the models that deal with the different trade size, only one trade is allowed in a given trading period [Easley and

O'Hara (1987)]. Thus, these models do not permit the consideration of the effects of both the number of trades and trade size on the price. In addition, even though we analyze the overall bid-ask spread behavior for the frequently traded security by using the results from both Easley and O'Hara (1987) and Easley and O'Hara (1992), there exists an ambiguity in the effects of the number of trades and the trade size on the bid-ask spread. Easley and O'Hara (1987) show that a large-size transaction has a higher bid-ask spread than a small-size transaction. Easley and O'Hara (1992) indicate that the larger the number of trades of a security, the higher the bid-ask spread. Thus, when holding gross volume constant, the effects of changes in the trade size (which implies a change in the number of trades) on the bid-ask spread is ambiguous. Our dissertation builds a model that clarifies these changes.

Since the purpose of this dissertation is to discover the roles of both the number of trades and the trade size simultaneously, we add one important features to the standard adverse-selection model [Glosten and Milgrom (1985)]. The investors can vary the size of their own trades with holding the gross volume per trading period constant. Thus, a change in gross volume may be due to, perhaps, one large order or it may be due to many small orders.

Numerical results shown in our model confirm those of Easley and O'Hara (1987) and Easley and O'Hara (1992): given a trading period, the bid-ask spread is increasing in either the trade size or the number of transactions. These results are consistent with those of the standard incomplete information models. The number of trades and the trade size provide information to the market maker that statistical informed participants are trading, and that the market maker adjusts the bid-ask spread accordingly. However, our results dealing with number of trades are more specific.

In addition to confirmation of these established results, our model reveals interesting effects of the number of trades upon the market maker's price setting in a competitive market. Since our model considers both the number of trades and trade size in one model, these effects are different in some aspects from those of previous studies. It demonstrates that the significance of the number of trades for the market maker's pricing depends, not only on the market maker's prior probabilistic belief about the value of the underlying asset, but also on the informedness of the market and the elasticity of the informed traders' demand and supply schedules. Since the market maker's belief about the value of the underlying asset is conditional on the past and present information about the security price, we use the past-price variability as a proxy variable for his prior belief.

We also show that the number of trades sends a better signal to the market maker in the following situations: (i) given small past-price variability, when a small fraction of informed traders exists in the market and the informed trader's demand schedule is perfectly inelastic, the relationship between the bid-ask spread and the number of trades is positive. (ii) Given large-past price variability, when a large fraction of informed traders exists in the market and the order of the informed trader increases with the quantity of information, the relationship between the bid-ask spread and the number of trades (or trade size) is negative (or positive). These results mirror those in cases A and B of our general scenarios.

The results of a sequential trade model in chapter III, related to the number of trades, gives empirical insights for chapter IV. The first concern is the relationship between the bid-ask spread and the number of trades when gross volume is held constant for a given trading period. This testing possibly reveals how the number of trades, relative to the trade size, influences the market maker in setting his bid-ask spread. Another concern is whether the variables -- past-price variability, market informedness, and firm size (firm's market value) -- can affect the number of trades. Testing this second hypothesis shows the degree to which the three variables influence the number of trades of individual investors. Then, considering the results of both together, we naturally come to a third hypothesis. How do those variables affect the market maker's bid-ask spread? We can see the relationship between those variables and the market maker's bid-ask spread. In addition, we can get an idea of how much the impacts of the three variables on the number of trades can be represented in the market maker's bid-ask spread.

When daily volume is held constant, we find a statistically significant, positive relationship between the number of daily transactions and its lagged variables with the bid-ask spread. This indicates that the market maker generally raises the bid-ask spread when he experiences a relatively large number of (daily) transactions. When he observes several days over a particular period having similar volumes, he considers more transactions as an indication of the existence of a higher possibility that there is something happening which he does not know. The statistically positive coefficients for the lagged variables of the number of trades reveal that the positive effect of the number of trades on the market maker's bid-ask spread is persistent. We also discover the relationships between number of trades and the three variables -- past-price variability, market informedness, and firm size (firm's market value) -- when holding gross volume constant for a given trading period. Our results find that past-price variability, market informedness, and firm size can affect both number of trades and bid-ask spread. However, from a statistical viewpoint, their effects on bid-ask spread are less clear.

The effect of the number of trades on the security price implicitly implies the time dependence in the rate of information arrival. Chapters III and IV also show how the investor's strategic behavior reflects the number of trades, which in turn affects the market maker's bid-ask spread. In chapter V, we raise two questions based on this observation. (i) Do stock returns have a positive relationship with the number of trades after considering the stochastic characteristic of its volatility? (ii) Can the established time-series property of observed returns, [AR(1) process], come from the traders' strategic behavior rather than from simple measurement error in calculating the returns? By using *the Exponential Generalized Autoregressive Conditional Heteroskedasticity* (E-GARCH) model, we find that there is a statistically positive relationship between the number of trades and stock returns. When we compare the stock return volatility between an AR(1) stock return and one with the number of trades variable, we find that the specification for the relationship between stock returns and number of trades is as good as one for an AR(1) stock return. Thus, it is plausible that the established AR(1) stock returns process come from the traders' strategic behavior rather than from measurement errors.

Chapter II. Information-Based Models and Related Empirical Results in the Study of Market Microstructure

Market microstructure is the study of the process and outcomes of exchanging assets under explicit trading rules [Jarrow *et al.* (1996)]. Since prices act as signals for allocations of scarce social resources in the market mechanism, understanding the positive and normative implications of the relationship between trading structures and price behavior is important. For a positive study, we analyze specific phenomena of pricing in stock markets by looking into the interaction between the mechanics of the trading process and its outcomes. A normative analysis tells which is the most efficient market mechanism.

This dissertation is interested in building a theoretical model for the role of number (frequency) of trades in the price adjustment process and in testing its related empirical hypotheses. In chapter II, we describe the previous research related with this specific objective. We first examine the two most common information-based approaches in market microstructure research, sequential trade models and batch models (strategic trader models). We then discuss the concept of the rational expectations equilibrium (REE) and its evolution because the noisy rational expectations equilibrium (NREE) batch-style model is a primary research tool for the role of volume in the price adjustment process. In the next section, we review theoretical models of the relationship between the volume and the stock price adjustment process. Reviewing all of this literature will provide insights into our theoretical and empirical modeling for the role of number of trades in the stock market.

Finally, we survey the empirical studies dealing with the effects of volume and number of trades on stock prices.

II. 1. Sequential Trade Models

We can simplify the trading process in these models as follows: the market maker sets up the bid and ask prices based on his information, which is better than the information of the uninformed but worse than that of the informed. He then opens his trading window. There is always one trader at the window and the market maker does not know whether the trader is uninformed or informed. The trader sells or buys one unit of stock at the market maker's bid or ask price.

This is different from the batch model surveyed in the next section. A batch is defined as a number or quantity of things produced at one time or to be taken together as a set. Since a batch model takes all transactions as one set during the same period to clear the market, it cannot examine transactions on *a trade-by-trade basis* like in sequential trade models. While the batch models provide a number of important insights into the behavior of market price, that approach cannot provide information about the intricate details of the stock price process.

Thus, sequential trade models (quote-driven models) have the advantage of investigating the evolution of the stock price process across time, and also of identifying the determinants of the bid-ask spread. Glosten and Milgrom (1985) is a seminal paper for this analysis. In this section we review the Glosten and Milgrom (1985) model and other various sequential trade models which examine the effect of trade size or number of trades on the bid-ask spread.

II. 1. 1. Glosten and Milgrom (1985)

Why is there bid-ask spread in the stock market? Bagehot's (1971) simple, but insightful, paper provides market microstructure researchers with very useful approaches for this question. He insists that if informed traders buy when the stock is priced too low, given their information, and sell when it is too high, then the market maker loses to them. To offset these losses by gains from trading with uninformed traders, a bid-ask spread develops. This is called as a market maker's *adverse-selection.*

Copeland and Galai (1983) first formalize this concept in a one-period model with a risk neutral competitive dealer, showing that a bid-ask spread emerges from the market maker's adverse selection. Even though this approach provides a convincing explanation about the existence of a bid-ask spread, the assumption that the value of the stock is known at the end of the period makes it inappropriate for a study of the evolution of asset prices.

This is the starting point by Glosten and Milgrom (1985). They do not model private information as being revealed to all market participants immediately after each trade. Instead, they allow further trading until such a time as information is revealed; only then resolving the informational differences between the informed and the uninformed.

Thus, in a present period the market maker and traders can learn from previous trades and adjust their behavior.

In the Glosten and Milgrom (1985) model, all traders are risk neutral and act competitively. Each trade involves one unit of the asset. Trades take place sequentially, and at either the market maker's bid or ask price. A fraction of these trades comes from uninformed traders who trade for exogenous liquidity reasons. The remainder of the trades comes from informed traders. The market maker is in a competitive market and thus sets bid and ask prices in such a way that his expected profit on any trade is zero.

They derive five propositions. (i) At all times the ask exceeds the bid, and if insider trading can occur, then the expectation of true value lies strictly between the bid and ask prices. (ii) The sequence of transaction prices forms a *martingale* relative to the market maker's information and the public information.[1] (iii) If trade is reasonably balanced (i.e., the probability of a purchase, given that a trade occurred, is bounded away from zero and one), then the expectation of the number of trades times the average spread squared is bounded by a number that is independent of the pattern of trade. This suggests that the average spread is proportional to one over the square root of average volume, and spreads decline with the number of trades.[2] (iv) If trade is reasonably balanced in the sense of the

[1] A martingale implies that tomorrow's price is expected to be the same as today's price. Mathematically, a process X is called a martingale [relative to $(\{F_n\}, P)$] if (i) X is adapted, (ii) $E(|X_n|) < \infty, \forall n$, and (iii) $E(X_n | F_{n-1}) = X_{n-1}$, a.s. $(n \geq 1)$ [see Williams (1991)] In Glosten and Milgrom (1985), the martingale property implies that observed prices represent fair assessments of future value conditional on the market maker's information set, and that the market maker never *ex post* regrets a trade.

[2] They explain the negative relationship between volume and spreads using the market maker's fixed costs rather than the adverse selection costs in their model. Also, they implicitly assume that the more trades occur in a given period, the more volume we have.

(iii), then the expectations of the market maker and the traders converge as the number of traders increases. (v) There are some results of comparative statistics: the ask price increases and the bid price decreases when, other things being equal, the insiders' information at time *t* becomes better, or the ratio of informed to uninformed arrival rates at *t* is increased, or the elasticity of uninformed supply and demand at time *t* increases.

II. 1. 2. Trade size: Easley and O'Hara (1987)

The Glosten and Milgrom (1985) sequential trade approach prompts authors to address questions other than the existence of a bid-ask spread in market microstructure. Two of the most prolific authors, Easley and O'Hara (1987), investigate the role of trade size on a bid-ask spread. They consider a sequential trade model similar to Glosten and Milgrom (1985), but assume two different aspects. First, individuals can trade various amounts of the security. For the sake of simplification, they are assumed to submit orders for either large or small trades. The informed can choose an order size across trades to maximize their expected profits. The uninformed are assumed to randomly use both order sizes.[3] Second, the market makers and the uninformed traders do not know whether new information exists and, if any, what it is.

In their model, there are two types of equilibria, separating or pooling, depending on the market parameters. The informed traders can separate themselves from the uninformed

[3] As Glosten and Milgrom (1985) do, Easley and O'Hara (1987) also assume that only one trade occurs in a given trading period.

traders by choosing the large quantity of trading, or they can pool themselves with the uninformed traders, trading either a large or a small quantity of trades. A market maker sets up the bid-ask spread after observing the order sizes. If his spread is constant across trade sizes, the informed make more money from large trades than from small trades. This leads them to submit only large orders. Considering, however, if the market maker's spread varies across trade sizes, it must be greater for larger trades since the market maker would know that large trades come from informed traders. Therefore, whether the informed traders pool or separate, the spread must increase with trade size. This result induces the following conclusion: prices in their model do not satisfy *a Markov property*, since the conditional distribution of the next period's price depends on the entire history of past prices, i.e., on order flow.

II. 1. 3. Time between trades: Diamond and Verrecchia (1987) and Easley and O'Hara (1992)

Other research has used the sequential trade approach to analyze the role of time in the trading process.[4] Both the Glosten and Milgrom (1985) sequential trade approach and the Kyle (1985) batch approach cannot show the meaningful role of time. The Glosten and

[4] Hausman, Lo, and MacKinlay (1992) do very significant empirical research in this context. They estimate the conditional distribution of trade-to-trade price changes using *ordered probit*, a statistical model for discrete random variables [see Maddala (1993)]. This approach recognizes that transaction price changes occur in discrete increments, typically eighth of a dollar, and occur at irregularly-spaced time intervals. Unlike existing models of discrete transactions price, ordered probit can quantify the effects of other economic variables, like volume, past price changes, and the time between trades, on price changes. They estimate the ordered probit model *via* a maximum likelihood method and use the parameter estimates to measure several transaction-related quantities, such as the price impact of trades of a given size, the tendency towards price reversals from one transaction to the next, and the empirical significance of price discreteness.

Milgrom model treats the order arrivals by using an exogenous probability of arrival that is independent of any time parameter. In the Kyle model, a market maker does not know the individual order arrivals since all trades during a given period are batched as a set. Diamond and Verrecchia (1987) and Easley and O'Hara (1992) try to deal with the role of the time in price adjustment by using the variants of the Kyle model and the Glosten and Milgrom model respectively.

Diamond and Verrecchia (1987) use *the first passage time* approach to know the speed of price adjustment with the assumption of the existence of short sale constraints.[5] They conclude that bad news is related with no trade since a trader informed of bad news may be unable to trade if short sales are prohibited. This would provide an empirical implication that both bid and ask prices should fall following an arrival of no trade to incorporate this bad news potential.

In another analysis of this research type, Easley and O'Hara (1992) consider a variant of a typical sequential trade model for frequently traded securities. In their model, potential buyers and sellers trade one unit of an asset, given a trading period, with a market maker. As a major distinction of their model from the typical Glosten-Milgrom model, they consider 'event uncertainty', which implies that information events need not occur. This shows, in particular, that the lack of trade provides a signal of the existence (or non-existence) of any new information while trades provide signals of the direction of any new information. Thus, traders learn from both trade and the lack of trade because each may be correlated with properties of the underlying information structure.

They demonstrate that spread will depend on the time between trades, with spreads decreasing as this time increases.[6] Because the absence of trades is correlated with volume, their model also predicts the relationship between spreads and both normal and unexpected volume.[7]

II. 1. 4. Frequency (number) of trades: Easley, Kiefer, O'Hara, and Paperman (1996)

The research by Easley, Kiefer, O'Hara, and Paperman (1996) is also noteworthy because of the way it deals with a trade model and its effect of the number of trades on the bid-ask spread. They use a model very close to that of Easley and O'Hara (1992). They turn their attention to the possibility that if non-synchronicity is purposeful and informationally motivated, then the serial dependence it induces in asset returns should be considered genuine, since it is the result of economic forces rather than measurement error. Thus, they think that purely statistical models of non-synchronous trading are clearly inappropriate and that an economic model of strategic interactions is needed for this issue.

They examine the bid-ask spread behavior for the infrequently traded security. They focus on the possibility of informedness when infrequently traded securities are suddenly

[5] Here, first passage times imply the length of time before the price process is approximately efficient.
[6] If we interpret this result within a framework of a given trading period, bid-ask spreads increase as do the number of trades. This result is similar to the result of our model. However, there is an important caveat to keep in mind. The implications could differ since the number of trades can be the same even though the time intervals between trades are not even.
[7] They define normal volume as the amount of liquidity (uninformed) trades and unexpected volume as the amount of informed trades. They show that the initial spread is decreasing in normal volume. However, their predictions about the relationship between spread and both normal and unexpected volume are only a general statement that 'it takes volume to move price'.

traded.[8] For this analysis, they add the assumption of *a Poisson process* arrival rate of a transaction to the Easley and O'Hara (1992) model. They demonstrate that less actively traded securities are considered riskier by the market maker because he assumes that there is information out there which he does not know precipitating the trade when they are traded. Thus he increases the bid-ask spread.

All the sequential trade models the we have described assume that traders and market makers behave competitively. However, what if only one or a few traders monopolize the new information? As Kyle (1985) points out, a strategy of trading to disguise the true information might lead to higher trading profits. In sequential trade models, traders are not sure when their turn to trade will arrive since this is determined by a exogenous probability. Thus, in these model, informed participants cannot change the timing of their trades in order to affect the transmission of their information. Thus, researchers have turned to batch models in order to examine this issue.

II. 2. Batch models (strategic trader models)

One aspect of the batch model (the strategic trader model) that differs from the earlier microstructure models is found in its explicit link to the REE (the rational expectations equilibrium). They all use the rational expectations concept.[9] In the complete market with

[8] They theoretically model the bid-ask spread behavior in a thin market. However, their empirical tests are actually based on a thick market. They also assume one-unit per transaction as in Easley and O'Hara (1992).
[9] This is the reason why a good understanding about the REE in market microstructure is required. Chapter II. 3 will show further investigation of the REE.

the differential information, traders may try to draw inferences of others' information from prices or other market statistics, and then update their prior beliefs about the value of securities. It is the informational role of prices and market statistics that the REE concept was contrived to solve. If we relate the REE concepts to batch models in the market microstructure context, the interaction between an informed trader's conjecture about market maker's pricing policy and the market maker's inference about the informed trader's information is an important aspect. It is the essence of the REE.

Kyle's two papers (1985 and 1989) represent an analysis of market microstructure with these (Walrasian) batch models. Kyle basically considers the stock market mechanism as order-driven. His two seminal papers, however, are different from each other even though they both use batch models. The earlier paper (1985) deals with the market orders, while the later paper (1989) uses limit orders (price-contingent orders).

Kyle focuses only on the strategic behavior by an informed trader in his two papers. As a natural extension of Kyle's framework, Admati and Pfleiderer (1988) and Foster and Viswanathan (1990) look into the case whereby the uninformed traders also pursue the strategic behavior. This section will describe these seminal batch model papers. They show the paradigms that trigger tremendous further research.

II. 2. 1. Market orders: Kyle (1985)

Kyle (1985) examines a batch model of market interactions. In his batch model, each trader submits *a market order*, which states a buying or selling order as a function of the market price. A market maker then observes the net order flow for each price, and sets the market-clearing price. Even though these kinds of batch models do not provide us with understanding of a characterization of bid-ask spread or transaction prices on a trade-by-trade basis, they shed light on the effect of the informed traders' strategies (not the uninformed traders' strategies) on prices.

Kyle tries to answer the following questions in his paper: (i) how quickly is new private information about the value of an underlying asset incorporated into market prices? (ii) How valuable is private information to an inside trader? (iii) How does noise trading affect the volatility of prices? (iv) What determines the liquidity of a speculative market?

In his model, a market maker does not observe the individual quantities traded by the insider or noise traders separately, nor does the market maker consider them as having any other kind of special information. As a result, price fluctuations are always a consequence of order flow innovations. An informed trader, who is risk neutral, is assumed to maximize expected profits. He has rational expectations, and knows the pricing model and the distribution of noise trades. He acts as an intertemporal monopolist in the asset market, taking into account explicitly the effects that his trading at one auction has on the price determined by market makers. These prices are assumed to equal the expectation of the liquidation value of the asset, conditional in the market makers' information sets, at the dates the prices are determined. Thus, market makers earn, on average, zero profits.[10] The arrival of the noise traders is assumed to follow *a Brownian motion.*[11] One implication of this assumption is that the quantity traded by the uninformed participants in one auction is independent of that quantity traded in each other auction.

In Kyle's model each auction trading takes place in two steps. In step one, the insider and the noise traders simultaneously place market orders. When making a choice, the insider's information consists of his private observation of the liquidation value of the asset, as well as past prices and past quantities traded by himself. He does not observe current or future prices, and quantities traded by noise traders. The random quantities traded by noise traders is distributed independently from both present or past quantities traded by the insider and noise traders. In step two, the market maker sets a market-clearing price and trades the associated quantity. When doing so, their information consist of observations of the current and past aggregate quantities traded by the insider and noise traders combined. We call these aggregate quantities the *order flow*.

With this framework, Kyle derives a single auction equilibrium and extends this idea to the sequential game style by using the concept of *the sequential equilibrium.*[12]

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[10] Thus, the Kyle model (1985) cannot explain the bid and ask prices in the stock market. This is different from the Glosten and Milgrom model (1985).

[11] A continuous, adapted process ω is called *a standard Brownian motion* (*a Wiener process*) on a filtered probability space $(\Omega, F, \mathrm{F}, P)$ If (i) $\omega_0 = 0$ a.s., (ii) for any $0 \leq t \leq s \leq T$, the increment $\omega_s - \omega_t$ is independent of F_t and is normally distributed with mean zero and variance $s - t$ [see Malliaris and Brock (1982)].

[12] A strategy profile and system of beliefs (σ, μ) is a sequential equilibrium of the extensive form game if it has the following properties: (i) strategy profile σ is sequentially rational given belief system μ, (ii)

Finally it is developed and applied to the continuous auction equilibrium by making the discrete auctions into the continuous auctions by the convergence concept.

Whether in the single period auction or in the multiple period auction, both equilibrium solutions to his model have the similar, general linear form in which prices are linearly related to the order flow, and the order strategy by the informed is linearly related to the true value of the asset. The market maker updates his beliefs by using Bayes' rule and sets prices equal to the mean of his posterior beliefs. This price clears the market.

His result on *the market depth* is one of the most interesting characteristics of his paper to the empiricist. The market depth is the order flow which the market can accommodate without affecting prices, i.e., how the informed trader selects a strategy to take advantage of his information across time. Kyle states that the informed trader smoothly or gradually exploits his information across time so that it leads to the constant volatility as the time periods of the model are shortened to approach a continuous auction.[13] Kyle also investigates the price path. His model shows that prices follow a martingale. He further demonstrates that prices will eventually reflect the informed trader's new information. One consequence of this equilibrium is that in order to disguise new information the informed trader profits more spreading trades over time rather than trading in one block.

There exists a sequence of completely mixed strategies $\{\sigma^k\}_{k=1}^{\infty}$, with $\lim_{k\to\infty} \sigma^k = \sigma$, such that $\mu = \lim_{k\to\infty} \mu^k$, where μ^k denotes the beliefs derived from strategy profile σ^k using Bayes' rule. The sequential equilibrium notion requires that beliefs be justifiable as coming from some set of totally mixed strategies that are close to the equilibrium strategies σ (i.e., a small perturbation of the equilibrium strategies), [see Mas-Colell, Winston, and Green (1995)].

II. 2. 2. Limit orders (price-contingent orders): Kyle (1989)

In Kyle's earlier paper (1985), the single informed trader is not permitted to submit limit orders (price-contingent orders), so it does not represent an actual trading mechanism. In his later paper, Kyle (1989), informed players can submit limit orders. These informed traders are imperfect competitors who explicitly consider the effect of their trades on market prices. In this model, a Baysian Nash equilibrium is a profile of trading strategies, where each trading strategy is a demand function that is submitted to a centralized market.

There are two important reasons for modeling informed traders as imperfect competitors. First, the stylized facts about speculative markets suggest that the better-informed traders are large. For example, in a stock market, arbitrageurs with private information about merger prospects buy and sell significant percentages of the outstanding equity of publicly held companies. Therefore, it is implausible to assume that the large traders involved do not affect prices when they trade and that these traders do not take into account their effect on prices in choosing the quantities traded. The second reason for modeling informed traders as imperfect competitors is that models based on perfect competition do not have reasonable properties for stock prices. The *schizophrenia problem* by Hellwig (1980) is a good example.[14]

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[13] This is a *random walk*! While this behavior is theoretically consistent with efficient markets, it does not appear to be constant with the empirical behavior of prices [see II. 5.].

[14] Grossman (1976) proposes that the equilibrium price aggregates the available information perfectly in the perfect competition world. Thus, the equilibrium price *fully reveals* the private information as a *sufficient statistic*. Any unrevealed information in the price is not worth communicating because it would only be treated as noise. This is why Hellwig (1980) states that the traders are slightly schizophrenic. If traders consider the covariance between noise in their own information and noise in the price, this covariance

Kyle's (1989) analysis is based on the one period, single risky asset market in which a Walrasian auctioneer clears all orders at a single price. This market includes this fictional auctioneer, noise traders, informed speculators, and uninformed speculators. Noise traders purchase a random, exogenous, inelastic quantity. After private observations are realized, each speculator chooses a demand function. Since the market-clearing price is determined after the demand functions are chosen, each speculator realizes that his choice of a demand function influences the market-clearing price.

Kyle assumes that speculators have CARA (constant absolute risk aversion) utility functions and that the random variables are normally distributed. With imperfect competition, this makes the speculator's objective function quadratic [see Ingersoll (1987)]. Hence, a symmetric equilibrium exists in which strategies are linear.

Equilibrium prices in this imperfectly competitive REE are less informative than they are in a competitive REE since traders specifically take account of their actions on the market price. Thus, they will not completely trade away their informational advantage. This provides a very important property. Informed traders earn a positive return to information, which, in turn, solves the Hellwig's schizophrenia problem in the competitive models.[15]

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makes them neglect their own information when they pay attention to information contained in the price [see chapter II. 3].

[15] This conclusion seems to differ from Grossman's (1981). Grossman insists that a Nash equilibrium with limit order strategy is equal to a perfect competitive equilibrium in the case of an industry with a large fixed cost.

II. 2. 3. Strategic uninformed traders: Admati and Pfleiderer (1988) and Foster and Viswanathan (1990)

In Kyle's papers (1985 and 1989), uninformed traders are restricted not to act strategically. In the equilibrium of Kyle's models, the uninformed traders always earn a negative expected profit. This does not properly reflect reality since they can behave strategically. Once the uninformed behave in that way, the informed traders will also optimally react to their changes. Then the stock pricing will change. A lot of interesting aspects in the study of market microstructure come from allowing the uninformed to behave strategically. Both Admati and Pfleiderer (1988) and Foster and Viswanathan (1990) are significant seminal papers to show that patterns can occur in trading behavior by focusing on this uninformed traders' ability to trade strategically.

The first aspect that several researchers [Jain and Joh (1988) and others] report is *the U-shaped pattern* of the average volume traded across hours of the day.[16] They point out the heavy trading in the beginning and the end of the trading day and the relatively light trading in the middle. Admati and Pfleiderer (1988) note this phenomenon and then attempt to answer the following three questions in reference to the uninformed traders having strategic behavior: (i) why does trading tend to be concentrated in particular time periods within the trading day? (ii) Why are returns (or price changes) more variable in some periods and less variable in others? (iii) Why do the periods of higher trading volume also tend to be the periods of higher return variability?

[16] The average volume is defined as follows: (the total hourly trading volume for the period of the analysis)/(the number of each specific hour for the period of the analysis).

Even though the market structure is similar with the discrete time version of the Kyle (1985) model, Admati and Pfleiderer (1988) endow the traders with different characteristics: informed traders repeatedly receive information one period before it becomes public. This short life of the information leads the informed to choose their optimal order size in each period rather than deciding when to trade, as in Kyle's (1985) issue of market depth. Uninformed liquidity traders are assumed to be one of two types. There are discretionary uninformed traders having exogenous liquidity needs with trades that cannot be split and whose only decision is when to trade. There are also non-discretionary liquidity traders who are identical to noise traders in the market and also cannot split their orders between periods.

Admati and Pfleiderer (1988) demonstrate that discretionary liquidity traders prefer to concentrate their buying in one period and selling in another under some conditions. This comes from the following reasons: the market maker's commission with more uninformed trading can be lower since they feel less danger of the adverse selection by gaining more from the relatively large uninformed to offset the losses to the informed. Thus, if all the uninformed traders clump together, they can improve their terms of trades and thus minimize their expected losses, and also increase the liquidity of the market in the period which they trade. In their model, since the optimal informed traders' quantity depends on the interaction of the strategic behavior between the informed and the discretionary uninformed, informed traders also trade more actively in periods when liquidity trading is a concern. Each informed trader only trades one unit. Since the total number of informed traders is exogenous, the increased trading by the informed fails to nullify the positive effects of the uninformed traders' clumping. This does not necessarily preclude the simultaneity of liquidity selling and buying.

Something more is needed to show the patterns in prices. We need an equilibrium in which liquidity buyers and sellers choose different times to transact. They argue that if information acquisition is endogenous, then this concern can be resolved. They derive the existence of the equilibrium with the trader's endogenous acquisition of information. In this equilibrium, they show that more traders become privately informed in periods of concentrated liquidity trading and prices become more informative. Thus, all the patterns they have identified in volume and price variability emerge as consequences of the interacting strategic decisions of the informed and liquidity traders.

Their key assumptions for these results are the independence of trade between periods, short life of the informed traders' information, and the incapability of uninformed traders' splitting orders across the periods. However, it is the very same assumptions that lead volume not to play any role as a market statistic since trade in one period provides no information about trade in subsequent periods. Hence, volume provides no information and there is no endogenous learning problem by either the uninformed or the market makers. Besides this, the model is not robust in that changes in assumptions regarding traders' behavior could bring about equilibria with different qualitative features or no equilibrium at all.

Another example of a study dealing with strategic uninformed traders is the work by Foster and Viswanathan (1990). They also use the basic structure of Kyle's (1985) continuous auction model. They assume that trade occurs only once a day, there exists a single risk neutral informed trader, and there is a noisy public signal. They demonstrate that a trading pattern arises because the informational advantage of the informed trader is reduced as time passes. If a public signal is not informative, then the informed trader simply chooses trading volume to release the same amount of information each day in order to offset only discretionary effects. There is no pattern to stock prices and variances. If a public signal is informative, however, the value of the informed traders' information deteriorates across time so the informed trader can accelerate his trades. Patterns may emerge. They argue that this result may possibly explain the empirical regularities in volumes and stock returns.[17]

II. 3. A Rational Expectations Equilibrium (REE) and Its Evolution

Section II.1 and II.2 described two representative modeling approaches in market microstructure research -- the sequential trade framework and the batch models. In these two models, the trading behavior of the informed and uninformed traders play a significant role in the process by which new information is transmitted into price changes and prices converge to their true values. One of the main issues in this dissertation is to understand

[17] French and Roll (1986), Jain and Joh (1988), and Chan, Chung, and Johnson (1995) show the results that have the empirical interest about this issue.

how prices adjust to new information related to a market statistic, such as trade size and number of trades. For this, one needs to know more about the process by which price adjustments occur. This price adjustment process has been researched by the use of noisy rational expectations equilibrium (NREE) models. Thus, it is important to first review the rational expectations equilibrium (REE) models.

II. 3. 1. A standard Walrasian economy

Market microstructure deals with the strategic behavior among traders who have asymmetric information. Thus, trader's expectations about other traders' behavior play a very significant role for the price process in the study of the market microstructure. As aforementioned, there are two approaches for the market microstructure research, the sequential trade model and the batch model. The batch model, which helps to understand how prices adjust to new information related to a market statistic, uses explicitly the rational expectations concept. This is the reason why we need to understand the evolution of the REE from the Walrasian equilibrium concepts.

A Walrasian economy is a fairly unrealistic mechanism. Nonetheless, it pervasively populates the literature on general equilibrium [see Mas-Colell, Winston, and Green (1995)]. It concerns an individual known as the *Walrasian auctioneer* who stands up in front of the entire population of an economy and calls out a price vector. Each consumer and producer consults preferences and determines what he desires to buy or sell and produce at the prescribed prices. Each consumer and producer reports back to the auctioneer the net trade and production desired and the Walrasian auctioneer tries to find a price vector that clears the market.

Suppose that we have the following simple two-asset single period economy. In the Grossman and Stiglitz (1980) model, traders can have different information. In a simplified version of their paper, as analyzed in Huang and Litzenberger (1988), individuals have the same information. In their analysis, the economy has two assets -- risky and risk-free -- which are independently and normally distributed random variables. There are N agents in the economy and they have the negative exponential utility function, $U(W_i) = -e^{-\theta \tilde{W}_i}$.[18] Here $\tilde{W}_i$ indicates the individual agent's wealth and θ denotes the agent's coefficient of absolute risk aversion, which is assumed to be the same across among all individuals. The price of the risk-free asset is normalized to one, and the price of the risky asset is denoted by p. The true value of the risky asset, v, is unknown and normally distributed with mean μ_v and variance σ_v^2. Without loss of generality, the total number of shares of the risky asset is one. Thus, traders have the following expected utility maximization problems:

$$\text{Maximize } EU(\tilde{W}_i)$$

$$\text{subject to } px + B \leq I_i, \tag{2.1}$$

[18] Using the negative exponential utility function has two advantages. One is that there is no *wealth effect*. The second is that the individual's expected utility maximization problem can be boiled down to one tractable expected mean-variance optimization problem and the solution has a linear form.

where I_i denotes the agent's endowment of bonds, x denotes the fraction (or number) of shares of the risky asset, and B denotes the number of shares of the risk-free asset that the agent buys. Since the agent's wealth at the end of the period is $\tilde{W}_i = x\tilde{V} + B$ and it is also a random variable, its distribution is $N\left(x\mu_v + B, x^2\sigma_v^2\right)$.

By using the assumptions of the negative exponential utility functions and of the normal random variables, the individual's maximization problem can be converted to

$$\text{Maximize } \{E\left(\tilde{W}_i\right) - \frac{1}{2}\theta \operatorname{var}\left(\tilde{W}_i\right)\}$$

$$\text{subject to } px + B \le I_i \, . \tag{2.2}$$

Solving this equation, the individual agent's optimal demand and the market clearing price are

$$x_i^* = \frac{\mu_v - p}{\theta\sigma_v^2} \tag{2.3}$$

$$p^* = \mu_v - \frac{\theta\sigma_v^2}{N} \; . \tag{2.4}$$

This shows that price is not in the indirect expected utility function. Therefore, agents cannot learn anything from prices. This result stems from the assumption that agents have identical information.

II. 3. 2. A fully revealing rational expectations equilibrium

A very natural question is whether we can still use the concept of the Walrasian equilibrium even when one assumption of the Walrasian economy is loosened, allowing agents to have diverse information sets. For differential information among the agents, we have to consider that individual agents make their decisions given their own information. If we take this into account by using the conditional expected mean and variance, then we can directly use the trader's above optimal demand function. In order to look into the individuals' differential information, an individual's private signal on the value of the risky asset is denoted as $\tilde{y}_i = \tilde{v} + \tilde{\varepsilon}_i$, where $\tilde{v}$ is a true value of the risky asset and $\tilde{\varepsilon}_i$ is a noise term. The noise term, $\tilde{\varepsilon}_i$, follows the normal distribution with zero mean and variance, σ_ε^2.

Now, the individual's optimal demand in the differential information can be given by

$$x_i{}^* = \frac{E(\tilde{v}|\Omega_i) - p}{\theta \operatorname{var}(\tilde{v}|\Omega_i)} , \tag{2.5}$$

where Ω_i denotes the agent's information set. Since we are interested in the precision of trader's information rather than variance, the inverse of variance, $\frac{1}{\operatorname{var}(\cdot)}$, is defined as the precision of trader's information, $\tau_{(\cdot)}$. Then the trader's optimal demand and the market-clearing price are

$$x_i^* = \frac{\dfrac{\tau_v \mu_v + \tau_\varepsilon \tilde{y}_i}{\tau_v + \tau_\varepsilon} - p}{\dfrac{\theta}{\tau_v + \tau_\varepsilon}} , \qquad (2.6)$$

$$p = \frac{\tau_v \mu_v}{\tau_v + \tau_\varepsilon} - \frac{\theta}{N(\tau_v + \tau_\varepsilon)} - \frac{\tau_\varepsilon}{N(\tau_v + \tau_\varepsilon)} \sum \tilde{y}_i \ . \qquad (2.7)$$

As we can see from (2.6) and (2.7), the equilibrium price, with the differential information among the individuals, includes the average of each individual's private signal. Thus, as Grossman (1976) states, traders might be able to learn the true value (or future prices) of the risky asset from observing the equilibrium price.

How can agents learn something from prices? In other words, how does one form expectations from prices? In *the theory of rational expectations* one finds the answer to these questions [see Blume, Bray, and Easley (1982) etc.].[19]

Grossman (1976), who is the first to indicate the need to consider the existence of noise traders, argues that price aggregates the private information in the economy. Therefore, uninformed traders form their beliefs about a future price from observing current

[19] *An explicit dynamic learning model*, which is the forefather of the recent market microstructure models in many ways, has a different framework from the rational expectations theory. It is a general equilibrium in which each agent considers a finite collection of models, one of which is a correct description of the rational expectations equilibrium. This model is interesting because it shows that under a Bayesian type of

prices. Current prices are based on the information of the informed traders. He states that the price reveals the mean of the private signals and is ***a sufficient statistic*** for the true value. Hence, given a price, the private information becomes redundant. The rational expectation equilibrium is the identical to the previous Walrasian equilibrium where agents share all their information before trading. In general, if a parameter is known only to some agents in the economy, the equilibrium price in this economy would be the function of this parameter. Since the price is common knowledge, all agents would get to learn the parameter by inverting this function. Price becomes the great information equalizer. That is the essence of the ***fully revealing rational expectations equilibrium***.

This result can be more concretely explained in the context of the above-mentioned economic setting. By using the concept of REE, traders have the conjectures that the price has the linear relationship of the mean of all the private information,

$$p^* = \alpha_0 + \alpha_1 \sum_{i=1}^{N} \tilde{y}_i \quad , \tag{2.8}$$

or rearranging the terms as

$$\left(\frac{p^* - \alpha_0}{\alpha_1 N} \right) = v + \frac{\sum \varepsilon_i}{N} = \bar{y} \quad . \tag{2.9}$$

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learning process the rational expectations equilibrium is locally stable, but that non-rational equilibria may also be locally stable.

Since the traders believe that the equilibrium price reveals the sufficient statistic for the true value, the price, a public signal, dominates the private signal. Hence, their information sets based on $\left(\tilde{v} \middle| v+\varepsilon_i, v+\frac{\sum \varepsilon_i}{N}\right)$ become equal to $\left(\tilde{v} \middle| v+\frac{\sum \varepsilon_i}{N}\right)$. Therefore, the conditional expected mean and precision on the information set, $\left(v+\frac{\sum \varepsilon_i}{N}\right)$, are

$$E\left(\tilde{v} \middle| v+\frac{\sum \varepsilon_i}{N}\right) = \frac{\tau_v \mu_v + N\tau_\varepsilon y_i}{\tau_v + N\tau_\varepsilon} , \tag{2.10}$$

$$precision\left(\tilde{v} \middle| v+\frac{\sum \varepsilon_i}{N}\right) = \tau_v + N\tau_\varepsilon . \tag{2.11}$$

After substituting these conditional expected mean and variance for the individual's optimal demand function and clearing the market, we can then derive the equilibrium price, $p = \frac{\tau_v \mu_v}{\tau_v + \tau_\varepsilon} - \frac{\theta}{N(\tau_v + \tau_\varepsilon)} - \frac{\tau_\varepsilon}{N(\tau_v + \tau_\varepsilon)} \sum \tilde{y}_i$, which is consistent with the prior conjectured parameters:

$$\alpha_0 = \frac{\tau_v \mu_v}{\tau_v + N\tau_\varepsilon} - \frac{\theta}{N(\tau_v + N\tau_\varepsilon)} , \tag{2.12}$$

$$\alpha_1 = \frac{\tau_\varepsilon}{\tau_v + N\tau_\varepsilon} . \tag{2.13}$$

Hence, agents' conjectures on the prices become true, price fully reveals the average of private information, and the traders with less precise information could learn the true value of the risky asset from a public signal -- the price.

The fully revealing rational expectations equilibrium concept, however, has problems. According to Grossman and Stiglitz (1980), there is a paradox in this fully revealing REE. If the equilibrium price is a sufficient statistic for all the information in the market, then the agents will not collect costly private information. Thus the price cannot be a sufficient statistic.

II. 3. 3. A noisy rational expectations equilibrium

How can we solve or eliminate this possibility of fully revealing prices? It can be resolved if the price system aggregates information only *partially*. Then, the price information is no longer a sufficient statistic for an investor's private signal. In such a case, an individual's optimal demand for the risky asset will depend, not only upon the price information, but also upon his own private signal.

For the fully revealing REE in the above single period economy, the only source of uncertainty is about the end-of-period payoff of the risky asset. To make the REE partially revealing, Grossman (1977) introduces one more source of noise into the economy -- the uncertainty of the (aggregate) supply of the risky asset.[20] That noise should be uncorrelated

[20] How one handles the aggregate supply, is very important. There are three approaches: One approach, presented by Brown and Jennings (1989), assumes that the supply uncertainty is an exogenously given

with the return of the risky asset. The equilibrium price becomes the functions of a parameter and a noise term. Hence it is not possible to back out the parameter by knowing the price. This price is called a *noisy rational expectations equilibrium* (NREE) *price*. Within the NREE price framework, the trader conjectures that the equilibrium price system is a following linear function of private information, $\sum \tilde{y}_i$ and supply uncertainty, $\tilde{S}$:

$$p^* = \beta_0 + \beta_1 \sum \tilde{y}_i - \beta_2 \tilde{S} \ , \tag{2.14}$$

This can be rearrange as

$$\left(\frac{p^* - \beta_0}{\beta_1 N}\right) = \left(v + \frac{\sum \varepsilon_i}{N}\right) - \frac{\beta_2}{\beta_1 N}\tilde{S} \ , \tag{2.15}$$

which equals

$$\bar{y} - \frac{\beta_2}{\beta_1 N}\tilde{S} \ , \tag{2.16}$$

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random supply of a risky asset. Grundy and McNichols (1989) assume that the aggregate supply depends on individuals' endowments. In this case, we have to be very careful of any correlation between the individuals' endowments and any information about per capita supply. The final approach, found in Blume, Easley, and O'Hara (1994), assumes that the aggregate supply is fixed [see Chapters II. 4].

where β_0, β_1, and β_2 are positive parameters. Given this conjectured price functional, we can obtain the individual's optimal demand function conditional on a realized price, his private signal, and his supply of the risky asset:

$$x_i^* = \frac{E\left(\tilde{v} \middle| v + \frac{\sum \varepsilon_i}{N}, \frac{\beta_2}{\beta_1 N}\tilde{S}\right) - p}{\theta Var\left(\tilde{v} \middle| v + \frac{\sum \varepsilon_i}{N}, \frac{\beta_2}{\beta_1 N}\tilde{S}\right)}. \tag{2.17}$$

The derivation of x_i^* is identical to the case of the fully revealing REE with one exception. A trader builds his expected value about the true value of the risky asset conditional on his own signal, the realized price, and his supply.

After getting the conditional expected mean and variance, $E\left(\tilde{v} \middle| v + \frac{\sum \varepsilon_i}{N}, \frac{\beta_2}{\beta_1 N}\tilde{S}\right)$ and $precision\left(\tilde{v} \middle| v + \frac{\sum \varepsilon_i}{N}, \frac{\beta_2}{\beta_1 N}\tilde{S}\right)$, and then substituting them for the individual's optimal demand function, and finally clearing the market, we can get the same equilibrium price, $p^* = \beta_0 + \beta_1 \sum \tilde{y}_i - \beta_2 \tilde{S}$, that the traders conjecture under the uniquely solved values for β_0, β_1, and β_2. Therefore, we have a NREE where there is no one-to-one relationship between the price of the risky asset and the sufficient statistic $\bar{y}$. Thus, price just partially reveals the private signals.

II. 4. A Noisy Rational Expectations Equilibrium Model for the Relationship between Volume and Stock Prices Adjustments

Prices play two roles -- market clearing and information aggregation -- in complete markets with asymmetric information. However, the price's role in information aggregation is not complete. Thus, by observing market data, uninformed traders may be able to learn something about the remaining information on an asset's value that the price cannot aggregate. From this market data, researchers believe volume to be the most representative variable since it includes information from aggregate trading demands (supplies). For the study of volume's role, the NREE provides us with the most influential tool.

II. 4. 1. Differences in opinion (belief) as an approach

This approach is based on the assumption that there exist differences in opinion among traders. The traders receive a common (public) information but differ in the way in which they interpret it. Also, each trader believes absolutely in the validity of his interpretation.[21]

***Harris and Raviv* (1993).** Harris and Raviv (1993) try to characterize the evolution of prices and trading volume. They focus on speculative trading as the major factor to both pay attention to a speculative motive rather than a hedging motive and also to explain surges in market activity following public announcements. They hypothesize that the speculative trading on a particular asset comes from disagreement among traders over the relationship between the announcement and the true value of a risky asset. Furthermore, this disagreement arises from the different interpretations rather than from different information among traders.

Harris and Raviv (1993) assume that speculative traders have common, prior beliefs about the final payoffs of a particular asset. As information about this asset becomes publicly available, they revise their prior beliefs regarding the payoffs using Bayes' rule and their own models (likelihood functions) of the relationship between signals and the asset's returns. This assumption causes different interpretations of the public information since their models have the same functional forms but with varying parameters. There are two distinct groups of speculative traders differentiated by the extent of the disagreement about the significance of information. Speculative traders either increase or decrease their probabilities for high payoffs depending on whether the information received is favorable or unfavorable.

Two more essential assumptions are used in their model.[22] They assume that the speculator's preferences are risk neutral rather than risk averse. This assumption implies that the demand functions are infinitely elastic. They therefore conclude that trade will occur only when the cumulative information between two groups changes from favorable to

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[21] This approach is fairly well accepted in accounting literature where the role of public information is important. However, Holthausen and Verrecchia (1990) state that it is also appropriate to interpret the informational signal as private information rather than public information.

[22] By using the risk neutrality of the trader's utility and the concept of *price taking group's reservation price*, Harris and Raviv (1993) avoid using the typical mean-variance equilibrium approach. Nevertheless, this becomes a shortcoming of their model since they fail to derive analytical conclusions with the risk averseness added.

unfavorable, or vice versa. Another important assumption in their model is related with intertemporal price changes. They assume that one group can exercise sufficient market power in each period to offer a price on a take-it-or-leave-it basis to another group. This price will equal the *price taking group's reservation price.*

Under this setting, they reach the following empirical and theoretical conclusions: (i) there is a positive correlation between absolute price changes and speculative volume, as well as one between absolute changes in the mean forecast of the final payoff and speculative volume,[23] (ii) There exists a negative serial correlation of price changes, (iii) Volume has a positive autocorrelation, (iv) There exist patterns in volume, such as overnight, weekends, and holidays.

II. 4. 2. The differential information approach with its variations [24]

The previous approach using differences in opinion describes the story about the price-volume relationship when it assumes that there are varying interpretations among traders who have the same public information. There is another widely used approach, based on different private information, dealing with this relationship between the price movement and the trading volume.

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[23] They argue that this result is closer to the relationship between volatility of returns and the number of trades since, in their model, the size of each trade is normalized to unity, but the number of trades is endogenous.

[24] Researchers use differential information as general terminology when they set up the different informational structure of the informed and uninformed traders. Usually, the differential information implies that the informed have better and exclusive information. However, it is beneficial to clarify the

***Wang* (1994) *and He and Wang* (1995).** The conventional *representative agent model* of asset prices in the complete market does not explain well the role of trading volume and its empirical stylized facts. Both Wang (1994) and He and Wang (1995) focus on the weakness of this representative agent model paradigm. While both examine the dynamic relationship between volume and prices in the infinite period model, Wang (1994) and He and Wang (1995) use different assumptions about traders' investment opportunities and about the information structure. This requires one to use different mathematical tools.

Wang (1994) extends the Grossman and Stiglitz (1980) standard competitive model in three different ways. He first allows the traders to use the dynamic trading strategies to maximize their life time expected utilities. Second, he explicitly adopts the non-informational (liquidity) and informational trading as the traders' optimizing behavior. Third, he considers the public information. He and Wang (1995) also share all these same aspects.

For the traders' investment opportunity sets, Wang (1994) assumes that each share of a risky asset pays a dividend, and there exists a constant returns to scale of the risky production technology. He also assumes that the true dividend process and the true excess rate of return of the technology follow AR(1) process. Informed traders are assumed to have perfect private information about a risky asset's dividend process and the returns on private investment opportunities, while uninformed traders possess only a noisy signal of

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distinction between better and exclusive, since He and Wang (1995) use only the latter case, but still call it differential information.

the dividend process. Basically, he assumes the existence of the superior information for the informed relative to the uninformed.

By using the typical recursive procedure, he derives an individual trader's optimal portfolio and a REE. In the REE, the equilibrium price of the risky asset has a linear relationship with the conditional expected dividend process, the true dividend process, and return on private investment opportunities. Given these results, he reaches the following conclusions: (i) there is a positive correlation between the trading volume and the absolute value of contemporary price changes. (ii) There is a positive relationship between the trading volume and the absolute dividend changes. (iii) There is a surge in trading volume around the public information announcement. And (iv) there is a negative dynamic relationship between return and volume.

When compared with Wang (1994), He and Wang (1995) examine scenarios with different agent investment opportunities and information structure. They assume that a random supply of the stock follow an AR(1) process. This allows them to use fractions for the demands of the informed and the uninformed. They also assume that each investor receives different information about the true underlying asset value rather than superior or inferior information. Under a continuous time setting, they derive the linear equilibrium price in which it is summarized in a single variable, a conditional expectation of the underlying asset value. In addition to Wang's (1994) results, (i) and (iii), they demonstrate that trading volume generated by new (exogenous) information, private or public, is accompanied by significant price changes, while volume generated by existing (private and endogenous) information is not.

Even though the trading volume emerges as the result of the trader's optimal demands and it is correlated with variables like traders' heterogeneity, it still only plays the role for market-clearing in Wang (1994) and He and Wang (1995). Thus, traders do not extract any information from the trading volume, nor can they use any of the correlation implied by volume to form their demands.

***Foster and Viswanathan* (1993a and 1995).** Both papers by Foster and Viswanathan (1993a and 1995) use the same theoretical model. They assume the Grossman and Stiglitz (1980) standard economy and *the elliptically contoured class* (ECC) *distributions* of the variables to study the volume and volatility in a speculative market. Since their 1995 model can comprehensively describe their theoretical and empirical results on this issue, it will be examined in this section.

Their models consist of a single market maker, a single asset, the informed, and the uninformed. The informed traders receive a signal about the true value (i.e., final liquidation value) of a risky asset. The market maker knows the net order flow and sets up the market-clearing price without identifying which orders come from which traders. In their multi-period setting, the assumptions unique to these papers are the traders' endogenous information acquisitions and the elliptically contoured class (ECC) distributions for variables. For the traders to acquire the (private) information, they assume that some traders decide whether to pay a fixed cost to acquire private information after observing the public information (signal). Traders buy the information simultaneously, and the number of informed traders is announced before trading starts.[25] They also use distribution functions from the ECC which are needed to maintain the linear equilibrium and to derive some results that the typical normal distribution cannot provide.[26]

They set up the theoretical model with endogenous informed agent speculative trading to show that (i) there is a unique linear Nash equilibrium; (ii) trading volume and the variance of price changes are conditionally heteroskedastic; (iii) volume is positively autocorrelated,[27] and (iv) when prices, volumes and other information are available to all traders, then trading volume becomes redundant and is not required. When only prices and volumes are available, both of them are useful in predicting functions of future prices and volumes.

By using half-hourly price changes and volume data for IBM during the year 1988, as reported by the Institute for the Study of Security Markets (ISSM), they want to test their theoretical model using moment restrictions based on the price change and trading volume series predicted by the model. When they apply these restrictions to their observed data,

[25] Foster and Viswanathan (1993a and 1995) assume that the number of informed traders finally become public information.
[26] The elliptical contour lines of density functions in this class name the distribution. This ECC consists of broad lines of distributions: the normal distribution, the multivariate *t*, mixture of normals, and multivariate double exponential and so on. For the definition and the properties of the ECC distribution, see Foster and Viswanathan (1993) and Ingersoll (1987).
[27] Their earlier paper (1993a) only pays attention to building the theoretical model by using an ECC distribution, implicitly assuming the multivariate *t* distribution. However, their later paper (1995) shows not only a theoretical model of ECC distribution focused on a mixture of normals, but also empirical results. For the result of (ii), the 1995 model assumes that the unobservable latent variable which drives the liquidity trading governs the conditional heteroskedasticity in volatility and has a stochastic process.

they can know whether the model is accepted or rejected. To implement this test, they use *the simulated method of moments* (SMM) procedure.[28]

The parameter estimation results reject their model, but it still has value. This rejection appears to be from the inability of the model to fit the higher order lagged dependence in the volume process. Thus, the trading volume seems to come from the informed traders, but it appears that these traders have imprecise information.

***Blume, Easley, and O'Hara* (1994).** To examine the price-volume relationship, Blume, Easley, and O'Hara (1994) focus on the market information that is inherent in the volume statistic and what investors can learn from observing trading volume. This might play a significant role in a technical analysis, like observing sequences of prices themselves. By solving this learning problem, they demonstrate how the volume statistic itself affects the adjustment of prices to new information.

Besides a given typical Grossman and Stiglitz's (1980) standard economy, Blume, Easley, and O'Hara (1994) classify the investors into two groups according to the difference in information: the informed group has the better precision (quality) on the true value of the risky asset. Their assumption about the informational difference between the informed and the uninformed belongs to the superior approach as mentioned earlier. They also use *the*

[28] The authors provide an example for this. They say that with endogenous informed trading, the decision to acquire information depends, in a nonlinear fashion, on the conditional second moment of the latent variable, given the trading history. Rather than ignoring some moments or not testing the model, the SMM allows the use of moments computed from a simulation of the model, rather than analytic moments, making tests feasible.

common error term for both investors' information structures. Thus, both types of investors have uncertainty about the price, even as time goes to infinity.

The most important characteristic among their assumptions is related with *the contemporaneous linkages problem* in rational expectations analyses. O'Hara (1995) clearly states this problem as follows: in rational expectations models, traders use the information contained in their current trade to determine that trade. Thus, traders use the price (and volume) at which the trader executes to determine the trade they make. If traders use this contemporaneous information to form demands, however, then the simultaneity of the process can vitiate any information the trade may have.

This is what Blume, Easley, and O'Hara (1994) investigate. They follow Hellwig's suggestion to avoid this simultaneity problem. Thus, they allow investors to condition on all information up to, but not including, the market statistic resulting from their desired trade. Hence, they assume that the aggregate supply is fixed.

To attain the typical linear NREE, they demonstrate that, with precision fixed, volume is strictly convex (V-shape) in price. Moreover, the steepness and dispersion of the V-shape depends on the quality and dispersion of the underlying information. These theoretical results match well with the stylized empirical relationship between price and volume. They suggest that the price-volume link may be explained in the context of the quality of traders' information.

They conclude that the volume as a market statistic equalizes the informational difference between traders. Thus, volume provides traders with the ability to discriminate the effects of the quality of information from the direction of information impounded in price. They argue that a trader observing only a high price cannot determine whether price is high because of a high average quality signal or perhaps an average signal with high quality. Volume picks up signal quality in a way independent from price because volume is not normally distributed. In the technical analysis, the trading volume may be valuable by revealing some information, but not all.

II. 5. Empirical Studies: Number of Trades, Volume, and Stock Return Volatility

There is very little theoretical and empirical research relating to the number of trades while theoretical and empirical research on the relationship between volume and volatility has been well documented. Since theoretical achievements about volume (or number of trades) and their roles in stock price movements are reviewed in previous sections in this chapter, their empirical results will be briefly described first, formulating ideas about the relationship of stock return with the number of trades after considering the stochastic characteristics of its volatility for chapter V. The empirical research related to the number of trades will then be studied.

II. 5. 1. Empirical studies on the relationship between volume and volatility

One Wall Street adage says that it takes volume to move prices.[29] This is a very concise explanation of the relationship between volume and price volatility. Interest in this relationship has become part of the mainstream studies about market microstructure. It is assumed in this research that the volume, (or trade size), contains information about true stock values. The following empirical stylized facts about the relationship between volume and the stock price movements are noteworthy since they give insight into empirical research related to the number of trades in the stock price:[30] (i) the correlation between volume and the absolute value of price change is positive in both equity and futures markets, (ii) The correlation between volume and the price changes is positive in equity markets, (iii) Expected trading volume is conditionally heteroskedasitic, (iv) The autocorrelation of volume is positive, and (v) Volume is relatively heavy in bull markets and light in bear markets.[31]

[29] Here, volume is defined as the total number of shares (or dollar amounts) traded in a day. Some of the research in this chapter uses volume as a trade size of individual transaction. However, the implications of the results in both cases can be considered as similar once we restrict our concerns to a daily period.

[30] For these stylized facts, see Tauchen and Pitts (1983); Hasbrouck (1988); Jain and Joh (1988); Gallant, Rossi, and Tauchen (1992); Stickel and Verrecchia (1994); Chan, Chung, and Johnson (1995); Bessembinder, Chan, and Seguin (1996). A thorough summary is found in Karpoff (1987). Lamoureux and Lastrapes (1990) argue that autocorrelation in volume accounts for the autocorrelation in volatility. Campbell, Grossman, and Wang (1993) examine the relationship between volume and the autocorrelation of the stock return.

[31] The distribution of volume, skewed positively, is often included in the stylized facts.

II. 5. 2. Empirical studies related to number of trades

Jones, Kaul, and Lipson (1994) are the first researchers who draw our attention to the potential significance of the role of the number of trades. By using the daily data of NASDAQ-NMS over the 1986-1991 period, they investigate the relationship between volatility, volume, and the frequency (number) of trades. For the measurement of volatility and estimation, they use a two-step procedure: they first estimate an OLS regression with 12 lagged variables of the stock return and then use the absolute residuals from this regression as the dependent variables for the following OLS regressions. They do this in order to find the relationship among volume, frequency of trades, and volatility. They argue that it is the occurrence of transactions *per se*, and not their size, that generates volatility in the stock market. Volume (or trade size) has no information beyond that contained in the frequency of transactions.

Keim and Madhavan (1995) examine the behavior of institutional traders. By using data on the equity transactions of 21 institutions of differing investment styles, they analyze the motivations for trade, the determinants of trade duration, and the choice of order type. Standard theoretical models state that the price impact per share, and hence the benefit from order fragmentation, is inversely related to overall trading activity. This is based on two rationales. One is that the market maker's inventory control costs decrease with trading frequency (number of trades). The other is that asymmetric information costs are less severe for widely-followed stocks.

While Keim and Madhavan's (1995) analysis mainly provides some support for the predictions made by theoretical models, there is one of exceptional finding worthy of our attention. They find that trade duration increases with liquidity, by holding the order quantity constant, which is opposite to the predictions by those established theoretical models. Their result paves the way for other direction of interpretation between the asymmetric information and the liquidity, and provides insight into the role of number of trades in stock prices.

Chapter III. The Effects of Number of Trades and Trade Size on Bid-Ask Spread in a Security Market

In this chapter, we first present a sequential trade model of security price formation based on the Glosten and Milgrom (1985) model. Since the purpose of this dissertation is to discover the roles of both number of trades and the trade size simultaneously, there are some important additions and changes. For our interpretation of the results, we view our model from a daily point of reference rather than from an intraday reference because we have a special interest in the role of the number of trades in the stock market. This is useful. The arrival rate of informed traders may depend on their private signals, the preceding trade history, the bid-ask spread, and other various market factors. Thus, as Andersen (1996) points out, it is too strong to solve the dynamic complications related with both the information occurrence and the price adjustment into the equilibrium by using only transaction (intraday) data.

Afterward, we derive the bid-ask spreads for each analyzed case. Finally, we characterize the roles of the number of trades and the trade size by numerical analysis.

III. 1. The Model

We consider a variant of the established sequential trade model of market making. In this model, potentially informed and uninformed traders buy or sell an asset with a market maker who sets bid-ask prices.

Information Structure, Investors and The Market Maker. As we see in the tree diagram, Figure 3.1, we consider an asset with the eventual dollar value represented by a random variable V, V to be in $[0, \infty]$. However, without loss of generality, we restrict V to be in $[0, 1]$. We define an informational event as the occurrence of a signal about V. The five signal values, 1, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, and 0, occur with probabilities, s_1, s_2, $1 - 2s_1 - 2s_2$, s_2, and s_1 respectively.

Investors (informed and uninformed) and a market maker figure in this model. The fraction of the traders who are informed is α, where $0 < \alpha < 1$. Informed traders receive a signal about the value of the underlying asset (V). The support of the distribution of V represents the actual value of the underlying asset. Thus, the distribution of V is symmetric.[1]

Figure 3.1 also shows that market participants can trade either one or two units of the security at the market maker's posted bid-ask prices during the trading period. Let γ_1 (γ_2) denote the fraction of informed traders who trade two units when the asset's true value is either 0 or 1 ($\frac{1}{4}$ or $\frac{3}{4}$). The usual assumption that net demand is decreasing in price brings us to $\gamma_1 \geq \gamma_2$. From the parameters γ_1, and γ_2, we can determine the elasticity of the informed traders' demand and supply schedules with respect to the received information about the value of the underlying asset. For example, when $\gamma_1 = \gamma_2 = 0$, the demand (supply) of the informed traders is perfectly inelastic. When $\gamma_1 = \frac{1}{4}$, $\gamma_2 = 0$, the

[1] The assumption about the symmetric distribution can be broadened for generalization. This will be our further research.

amount of order increases with the quality of information. The former is considered as group 1, the latter as group 2.

Uninformed traders are purely liquidity traders who trade the stocks for their consumption or portfolio considerations. Since they do not have specific information about the asset's true value, we assume that the probabilities of their buying and selling one unit per trade (a small-size trade; $x = 1$) and two unit per trade (a large-size trade; $x = 2$) are both equal to $\frac{1}{4}$.

The market makers know the probabilistic structure of the investor and the probabilistic distribution of the value on the underlying asset. They also are competitive with each other and make correct statistical inference from the observed data.[2] Thus, a market maker, in setting the bid and ask prices, earns zero expected profit. The market maker, informed traders, and uninformed traders are risk-neutral. As Easley and O'Hara (1992) put it, for the market maker, the assumption of the risk neutrality excludes any direct inventory effects on the market maker's prices, but still retains any information effects of inventory. Moreover, for the informed traders, this assumption rules out any strategic behavior by the informed traders and makes the analysis possible in the potential existence of multiple informed traders.

[2] In a dealership market, the dealer (as a market maker) buys and sells stock not only for others but also for his own account. So, his responsibility is two-fold: He must maintain his inventory as well as deal with information. For the analysis, both inventory and information effects could be considered together. However, most agree that the information effect overwhelms the inventory effect [See Madhavan and Smidt (1993)]. Thus, we only deal with the information effect.

Trade Process. As in a standard Glosten Milgrom (1985) model, we also use a pure dealership market. The market maker performs no brokerage services, and in effect all orders are market orders. Both informed traders and uninformed participants trade. In a competitive market, the informed traders' transactions will reflect their information, either selling if they know bad news or buying if they know good news. Therefore, if someone wants to sell to the market maker, it could signal that the trader received bad news; and if someone wants to buy from the market maker, this could signal that the trader received good news. It could also mean, however, that the trader is uninformed and simply needs to trade for liquidity reasons. Selling (buying) one unit or two units per transaction could reveal the investors' accuracy of their information signals. The market maker is free to change the bid and ask prices at any time after one investor has placed an order and before the next trader arrives.[3] The trading period is divided into discrete intervals of time, $t = 1, 2, \ldots\ldots, T$, with two units traded each period.

Figure 3.1 summarizes the information structure of the model and the trading process. At the first node, nature determines what kind of information will occur out of the five possible economic situations with its affect on the value of the underlying asset being either extremely positive, positive, neutral, negative, or extremely negative. The five nodes before the dotted line in Figure 3.1 are reached only at the beginning of the trading period. Then traders arrive. Once the information event occurs, an informed trader is selected with probability α. He then may choose to buy or sell based on the probabilities in Figure 3.1. These probabilities are varied according to whether he

[3] This means that the price per unit is constant in the order size since it is set before order size is stated.

belongs to group 1 or group 2. Similarly, with the probability $(1 - \alpha)$, an uninformed trader is selected and he may choose to buy or sell. If the information belongs to the neutral value, i.e. $V = \frac{1}{2}$, then only the uninformed traders will trade.

III. 2. Bid-Ask Spread

The market maker, using Bayes rule, utilizes the arrival of trades and the size of the trade to update his belief about the value of an underlying asset. Since the market maker cannot tell which kind of information actually occurs, he protects himself by adjusting his belief about the value of the stock, conditional on the type of the trade that occurs. As the market maker receives a trade order, his expectation of the asset's value changes, and this, in turn, causes his prices to change.

In our numerical analysis, we fix the volume per trading period at two units.[4] Fixing volume allows us to compare the informational roles of both number of trades and trade size in the price adjustment process. In particular, we compare the market maker's price of one two-unit trade (a smaller number of large-size trades) with that of two consecutive one-unit trades (a larger number of a small-size trade). In our numerical analysis the two consecutive trades should have the same trading direction, that is if the first trade is a selling order, then the next should also be a selling order, and vice versa. If the trades are not consecutively traded in the same direction, then price changes become null.

[4] We also consider the exceptional case of the one unit volume per trading period in order to compare the previous research.

For the case of one two-unit trade per period, the ask price, $p_a(x = 2)$, is the expected value of the asset conditional on the history of the process prior to the arrival of orders at that time. By Bayes rule, the market maker's posterior probability on five possible values of the underlying asset is[5]

$$pr_{a1} \equiv prob(V = 1|x = 2) = \frac{1}{A}\left[prob(V = 1)\, prob(x = 2|V = 1)\right], \tag{3.1}$$

$$pr_{a2} \equiv prob(V = \frac{3}{4}|x = 2) = \frac{1}{A}\left[prob(V = \frac{3}{4})\, prob(x = 2\Big|V = \frac{3}{4})\right], \tag{3.2}$$

$$pr_{a3} \equiv prob(V = \frac{1}{2}|x = 2) = \frac{1}{A}\left[prob(V = \frac{1}{2})\, prob(x = 2\Big|V = \frac{1}{2})\right], \tag{3.3}$$

$$pr_{a4} \equiv prob(V = \frac{1}{4}|x = 2) = \frac{1}{A}\left[prob(V = \frac{1}{4})\, prob(x = 2\Big|V = \frac{1}{4})\right], \tag{3.4}$$

$$pr_{a5} \equiv prob(V = 0|x = 2) = \frac{1}{A}\left[prob(V = 0)\, prob(x = 2|V = 0)\right], \tag{3.5}$$

where

$$A \equiv prob(V = 1)\, prob(x = 2|V = 1) + prob(V = \frac{3}{4})\, prob(x = 2|V = \frac{3}{4})$$

$$+\ prob(V = \frac{1}{2})\, prob(x = 2|V = \frac{1}{2}) + prob(V = \frac{1}{4})\, prob(x = 2|V = \frac{1}{4})$$

[5] We denote p_a (p_b) as an ask (bid) price and pr_{ai} as a probability of specific economic situations where we denote a as an ask price and i denotes each informational signal of the value of the underlying asset; 1 denotes extremely positive information, 2 positive information, 3 neutral information, 4 negative information, and 5 extremely negative information. We also denote pr_{Ai} as a probability of specific economic situations as in pr_{ai}. However, the only difference between pr_{ai} and pr_{Ai} is that the former is used

$$+\ prob(V=0)\,prob(x=2|V=0)$$

$$=\frac{1}{4}(1-\alpha)+\alpha\left(\frac{1}{2}+\gamma_1\right)s_1+\alpha\left(\frac{1}{2}+\gamma_2\right)s_2\ . \qquad (3.6)$$

Thus, the subsequent ask price becomes

$$p_a(x=2)=E(V|x=2)=Pr_{a1}*(1)+Pr_{a2}*(\frac{3}{4})+Pr_{a3}*(\frac{1}{2})+Pr_{a4}*(\frac{1}{4})+Pr_{a5}*(0)$$

$$=\frac{1}{8A}\left[(1-\alpha)+4\alpha(1+2\gamma_1)s_1+3\alpha(1+2\gamma_2)s_2\right]\ . \qquad (3.7)$$

Similar calculations show the bid price, $p_b(x=-2)$, as

$$p_b(x=-2)=E(V|x=-2)=\frac{1}{8A}\left[(1-\alpha)+\alpha(1+2\gamma_2)s_2\right]\ . \qquad (3.8)$$

Since the bid and ask prices in equations 3.7 and 3.8 are the prices which the market maker will put up when he faces the investor who wants to sell (buy), therefore, the bid-ask spread for the case of one transaction with two units per trading period is

$$[p_a(x=2)-p_b(x=-2)]\equiv p(2,1)=\frac{1}{4A}\alpha\left[2(1+2\gamma_1)s_1+(1+2\gamma_2)s_2\right]\ . \qquad (3.9)$$

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for two unit volume per trading period while the latter is used for two consecutive trades of one unit per trading period.

We denote $p(i,j)$ as a bid-ask spread, where i indicates the trade size and j indicates the number of trades for the trading period. Since α, γ_1, γ_2, s_1, and s_2 are in both denominator and numerator, we can not immediately tell how they influence the bid-ask spread.

For the case of two consecutive one-unit transactions per trading period, the market maker's posterior probability on five possible values of the underlying asset is:[6]

$$pr_{A1} \equiv prob(V=1|x=1,x=1)=\frac{1}{B}\left[prob(V=1)\left(prob(x=1|V=1)\right)^2\right], \qquad (3.10)$$

$$pr_{A2} \equiv prob(V=\frac{3}{4}|x=1,x=1)=\frac{1}{B}\left[prob(V=\frac{3}{4})\left(prob\left(x=1\middle|V=\frac{3}{4}\right)\right)^2\right], \qquad (3.11)$$

$$pr_{A3} \equiv prob(V=\frac{1}{2}|x=1,x=1)=\frac{1}{B}\left[prob(V=\frac{1}{2})\left(prob\left(x=1\middle|V=\frac{1}{2}\right)\right)^2\right], \qquad (3.12)$$

$$pr_{A4} \equiv prob(V=\frac{1}{4}|x=1,x=1)=\frac{1}{B}\left[prob(V=\frac{1}{4})\left(prob\left(x=1\middle|V=\frac{1}{4}\right)\right)^2\right], \qquad (3.13)$$

$$pr_{A5} \equiv prob(V=0|x=1,x=1)=\frac{1}{B}\left[prob(V=0)\left(prob(x=1|V=0)\right)^2\right], \qquad (3.14)$$

where

$$B \equiv prob(V=1)\left(prob(x=1|V=1)\right)^2 + prob(V=\frac{3}{4})\left(prob\left(x=1\middle|V=\frac{3}{4}\right)\right)^2$$

[6] For the case of two consecutive transactions with one unit per trading period, we assume that when the market maker receives one unit for the present transaction, his belief on having one unit for the next transaction is independent from the present transaction.

$$+ \ prob(V = \frac{1}{2})\left(prob\left(x = 1 \middle| V = \frac{1}{2} \right) \right)^2 + prob(V = \frac{1}{4})\left(prob\left(x = 1 \middle| V = \frac{1}{4} \right) \right)^2$$

$$+ \ prob(V = 0)\left(prob(x = 1 | V = 0) \right)^2$$

$$= s_1 \left[\alpha^2 \left(\frac{1}{2} - \gamma_1 \right)^2 + \frac{1}{2}\alpha(1-\alpha)\left(\frac{1}{2} - \gamma_1 \right) \right]$$

$$+ \ s_2 \left[\alpha^2 \left(\frac{1}{2} - \gamma_2 \right)^2 + \frac{1}{2}\alpha(1-\alpha)\left(\frac{1}{2} - \gamma_2 \right) \right] + \frac{1}{16}(1-\alpha)^2 \ . \qquad (3.15)$$

Thus, the ask price is then

$$p_a(x = 1, x = 1) = E(V | x = 1, x = 1)$$

$$= Pr_{A1} * (1) + Pr_{A2} * (\frac{3}{4}) + Pr_{A3} * (\frac{1}{2}) + Pr_{A4} * (\frac{1}{4}) + Pr_{A5} * (0)$$

$$= \frac{1}{32B}\left[(1-\alpha)^2 + 8\alpha(1 - 2\alpha\gamma_1)(1 - 2\gamma_1)s_1 + 6\alpha(1 - 2\alpha\gamma_2)(1 - 2\gamma_2)s_2 \right]. \qquad (3.16)$$

Similar calculations show that the bid price, $p_b(x = -1, x = -1)$ is

$$p_b(x = -1, x = -1) = E(V | x = -1, x = -1)$$

$$= \frac{1}{32B}\left[(1-\alpha)^2 + 2\alpha(1 - 2\alpha\gamma_2)(1 - 2\gamma_2)s_2 \right] \ . \qquad (3.17)$$

Therefore, the bid-ask spread for the case of two consecutive one-unit transactions per trading period is

$$[p_a(x=1,x=-1)-p_b(x=1,x=-1)] \equiv p(1,2)$$

$$=\frac{1}{8B}\alpha\left[2(1-2\alpha\gamma_1)(1-2\gamma_1)s_1+(1-2\alpha\gamma_2)(1-2\gamma_2)s_2\right] \quad . \qquad (3.18)$$

Equation (3.18) shows that α, γ_1, γ_2, s_1, and s_2 are in both a denominator and a numerator as in the previous case of the one two-unit transaction. Thus, we also cannot immediately tell how they influence the bid-ask spread.

In summary, equations (3.9) and (3.18) show the bid-ask spreads for one two-unit transaction and two consecutive one-unit transactions per trading period respectively. We need to analyze the difference between the bid-ask spreads in these two cases to investigate the roles of both the number of trades and the trade size in the stock price adjustment. The difference between the bid-ask spreads associated with two one-unit trades and one two-unit trade is

$$\Delta p \equiv p(2,1) - p(1,2) = \frac{1}{4A}\alpha\left[2(1+2\gamma_1)s_1+(1+2\gamma_2)s_2\right]$$

$$- \frac{1}{8B}\alpha\left[2(1-2\alpha\gamma_1)(1-2\gamma_1)s_1+(1-2\alpha\gamma_2)(1-2\gamma_2)s_2\right] \quad . \qquad (3.19)$$

Recall that $p(2,1)$ denotes the bid-ask spread for one two-unit transaction while $p(1,2)$ denotes the bid-ask spread for the two consecutive one-unit transactions within that same trading period. Thus, the difference between those two spreads comes from what is learned from observing the different numbers of the trades, holding total volume constant. If the value of the difference between those spreads is positively (negatively) related with the change in the value of a certain parameter, such as α, γ_1, γ_2, s_1, and s_2, the market maker infers that the parameter is negatively (positively) related to the number of trades.

In next section we compare our results with the previous models. For these comparisons, we need the bid-ask spread of the case in which there exists only one transaction with one unit. By the similar calculation with the above cases, the bid-ask spread for the case of one transaction with one unit per trading period is

$$p(1,1) = \frac{1}{4C}\alpha\left[2(1-2\gamma_1)s_1 + (1-2\gamma_2)s_2\right], \tag{3.20}$$

where

$$C \equiv \frac{1}{4}(1-\alpha) + \alpha\left(\frac{1}{2}-\gamma_1\right)s_1 + \alpha\left(\frac{1}{2}-\gamma_2\right)s_2 . \tag{3.21}$$

III. 3. Numerical Results

We compute the values of the effects of the number of trades on bid-ask spread for a variety of settings. Equation (3.19) shows the effect of the number of trades and the trade size on the bid-ask spread and consists of the following parameters: s_1, s_2, α, γ_1, and γ_2. We will use these parameters in the numerical analysis. Recall that the market maker's prior belief about the given economic situations is denoted as s_1 and s_2. The fraction of the informed traders is represented by α. The variables, γ_1, and γ_2, show the extent of the demand by the informed traders. Table 3.1 summarizes how we change the values of the parameters for the analyses.

In this section, we show that our model confirms the results of previous research, which includes the following: (i) the bid-ask spread increases as trade size increases [Easley and O'Hara (1987)]; (ii) For the frequently traded securities, bid-ask spreads increase as do the number of trades [Easley and O'Hara (1992)];[7] (iii) The bid-ask spread increases as the market becomes more informed [Glosten and Milgrom (1985) amongst many others]. In addition, our model identifies the economic conditions for which the number of trades sends a more influential signal to the market maker than does the trade size.

Easley, Kiefer, O'Hara, and Paperman (1996) examine the relationship between frequency (number) of trades and the bid-ask spread for infrequently traded securities. They demonstrate that the bid-ask spread decreases in trading frequency. In these less actively traded securities, a transaction in itself signals new information. So, they argue that the risk of information-based trading for individual stocks becomes large. The market maker sets a higher bid-ask spread for these securities. Their investigation is different from ours. Their model is dynamic and examines the timing of trades, while our model is static and examines the number of trades and trade size within a trading period (also holding the gross volume per trading period constant).[8]

III. 3. 1. A relationship between trade size and bid-ask spread

We compare our result with those of the Easley and O'Hara (1987) model. They show that the bid-ask spread is increasing in the trade size. To confirm this result, we need two equations. Equation (3.9), $p(2,1)$, shows the bid-ask spread when there exists a two-unit trade during the trading period. Equation (3.20), $p(1,1)$, shows the bid-ask spread when there exists only one unit traded during the trading period. The difference between these two spreads is as follows:

$$p(2,1) - p(1,1) = \frac{1}{4A}\alpha\left[2(1+2\gamma_1)s_1 + (1+2\gamma_2)s_2\right] - \frac{1}{4C}\alpha\left[2(1-2\gamma_1)s_1 + (1-2\gamma_2)s_2\right], \qquad (3.22)$$

[7] Interpreting within a framework of a given trading period, we can annotate the results of Easley and O'Hara (1992) like this. See the footnote 6 in section II. 1. 3.

[8] As we point out in chapter II, they theoretically model the bid-ask spread behavior in a thin market. However, their empirical tests are actually based on a thick market.

where

$$\text{A} = \frac{1}{4}(1-\alpha) + \alpha\left(\frac{1}{2}+\gamma_1\right)s_1 + \alpha\left(\frac{1}{2}+\gamma_2\right)s_2,$$

$$\text{C} = \frac{1}{4}(1-\alpha) + \alpha\left(\frac{1}{2}-\gamma_1\right)s_1 + \alpha\left(\frac{1}{2}-\gamma_2\right)s_2. \qquad (3.23)$$

Table 3.2 shows that all the signs are positive regardless of the values of other variables, which indicates that the bid-ask spread is increasing in the trade size.[9] This confirms the Easley and O'Hara (1987) result: if a large-size transaction suddenly occurs, then the market maker assumes that there is information to which he is not privy, and moves to protect himself by setting up a larger bid-ask spread.

III. 3. 2. A relationship between number of trades and bid-ask spread

We compare our results with those of the Easley and O'Hara (1992). Recall that we interpret their results within a framework of a given trading period. They argue that for the frequently traded securities, both the presence and the absence of trade provide information. In particular, as the period without trade lengthens, the market maker raises his probabilistic belief that an information event has not occurred. He accordingly lowers the bid-ask spread. So, the bid-ask spread decreases as time between trades increases. As in the previous case, we need two equations. Equation (3.18), $p(1,2)$, shows bid-ask spread when there exists

[9] We perform the numerical analyses with all parameter values as described in Table 3.1. For tables in this chapter, we show a sample of results that have larger intervals than in Table 3.1 when we increment the parameter values. These results are congruent with those using all parameter values.

two consecutive trades with one unit during the trading period. Equation (3.20), $p(1,1)$, shows the bid-ask spread when there exists only one unit trade during the trading period. Then we examine the signs which indicate the difference between these two spreads. They are as follows:

$$p(1,2) - p(1,1) = \frac{1}{8B}\alpha\left[2(1-2\alpha\gamma_1)(1-2\gamma_1)s_1 + (1-2\alpha\gamma_2)(1-2\gamma_2)s_2\right]$$

$$- \frac{1}{4C}\alpha\left[2(1-2\gamma_1)s_1 + (1-2\gamma_2)s_2\right], \qquad (3.24)$$

where

$$\mathrm{B} = s_1\left[\alpha^2\left(\frac{1}{2}-\gamma_1\right)^2 + \frac{1}{2}\alpha(1-\alpha)\left(\frac{1}{2}-\gamma_1\right)\right]$$

$$+ s_2\left[\alpha^2\left(\frac{1}{2}-\gamma_2\right)^2 + \frac{1}{2}\alpha(1-\alpha)\left(\frac{1}{2}-\gamma_2\right)\right]$$

$$+ \frac{1}{16}(1-\alpha)^2,$$

$$\mathrm{C} = \frac{1}{4}(1-\alpha) + \alpha\left(\frac{1}{2}-\gamma_1\right)s_1 + \alpha\left(\frac{1}{2}-\gamma_2\right)s_2 . \qquad (3.25)$$

Table 3.3 shows that all the signs are positive regardless of the values of other variables. This implies that the bid-ask spread becomes larger as the number of trades increases and confirms the Easley and O'Hara (1992) result when we restrict our concern to the given trading period.

III. 3. 3. Our contribution: bid-ask spread differences when both number of trades and trade size are included

Now we examine the effect of both the number of trades and the trade size on bid-ask spread. This analysis shows the economic conditions in which the number of trades has a greater effect on the market maker's pricing than does the trade size. Our method is similar to those cases previously mentioned. We need two equations. Equation (3.9), $p(2,1)$, shows bid-ask spread when there exists two consecutive trades with one unit during the trading period. Equation (3.18), $p(1,2)$, shows bid-ask spread when there exists only one unit trade during the trading period. Then we examine the signs which indicate the difference between these two spreads. They are as follows:

$$
\begin{aligned}
&p(1,2) - p(2,1) \\
&\quad = \frac{1}{4A}\alpha\left[2(1+2\gamma_1)s_1 + (1+2\gamma_2)s_2\right] \\
&\quad - \frac{1}{8B}\alpha\left[2(1-2\alpha\gamma_1)(1-2\gamma_1)s_1 + (1-2\alpha\gamma_2)(1-2\gamma_2)s_2\right], \qquad (3.26)
\end{aligned}
$$

where

$$A = \frac{1}{4}(1-\alpha) + \alpha\left(\frac{1}{2}+\gamma_1\right)s_1 + \alpha\left(\frac{1}{2}+\gamma_2\right)s_2 ,$$

$$B = s_1\left[\alpha^2\left(\frac{1}{2}-\gamma_1\right)^2 + \frac{1}{2}\alpha(1-\alpha)\left(\frac{1}{2}-\gamma_1\right)\right]$$

$$+ s_2\left[\alpha^2\left(\frac{1}{2}-\gamma_2\right)^2 + \frac{1}{2}\alpha(1-\alpha)\left(\frac{1}{2}-\gamma_2\right)\right] + \frac{1}{16}(1-\alpha)^2 . \qquad (3.27)$$

Table 3.4 shows that signs are positive as well as negative depending on the values of other variables. The positive sign implies that the trade size is a more influential signal for the market maker's pricing than is the number of trades. The negative sign shows the opposite -- the number of trades is a more influential signal for the market maker's pricing.

In subsequent sections, we will examine this result in more detail to see which economic situations leads to which outcome. We will find the results of the numerical analyses showing the relationships among number of trades, trade size, and bid-ask spread by concentrating on our model's three key variables. Those variables are market maker's prior belief, market informedness, and, elasticity of the informed traders' demand and supply schedules on the new information.

III. 3. 3. 1. Two general scenarios

There are two general scenarios, each illustrating the significance of the number of trades and the trade size related in the stock price adjustment process.

Case A identifies a scenario in which a greater number of trades (again fixing the gross daily volume) provides greater information to the market maker. Suppose, for example, that informed traders' demand and supply schedules are perfectly inelastic, and that, with a positive information event, some informed traders always place large orders while others always place small orders. The size of an individual's demand is independent of the amount of the trader's information, but dependent on its existence. In this case, the trade size provides the market maker with no information, while the number of trades does.[10]

Case B presents an alternative scenario. Greater trade size (and thus a lower number of trades) provides greater information than does the number of trades to the market maker. In this scenario, there is a small variation in the past value of a security and the market maker's prior probabilistic belief reflects this small past variation. Also, the market maker has a strong belief that at least some of the traders are informed and that informed participants trade large amounts of the security. In this case, if a market participant places a large-size demand, the market maker decides that the security's value is high, and thus sets a high ask price. If the market maker is presented with a large-size supply, the market maker assumes that the security's value is low, and sets a low bid price. Regardless of demand or supply, higher volume transactions result in higher bid-ask spreads.

Our numerical analyses examine whether our model explains these two general scenarios. For case A, we assume $\gamma_1 = \gamma_2 = 0$ in equation (3.26). This implies that the

[10] According to Brock and Kleidon (1992), the elasticity of informed traders' demand and supply schedules generally are less elastic at the open and close than at other times of the trading day.

informed traders' demand and supply schedules are perfectly inelastic. For case B, we assume that the elasticity of the informed traders' demand and supply schedules with respect to information about the value of the underlying asset is large ($\gamma_1 = 0.25$, or $\gamma_1 = 0.45$, $\gamma_2 = 0$) and that the market is well informed ($\alpha = 0.9$). Our model demonstrates these scenarios. Table 3.5 shows these results. The next sections will present more detail examination of this.

III. 3. 3. 2. The positive relationship between market informedness and bid-ask spread

In a perfectly competitive market, informed traders tend to use to their advantage new information as quickly as possible. The market maker, fully aware of this tendency, thus reacts predictably to certain actions. Hence, there exists a positive relationship between market informedness and bid-ask spread. Our model confirms this market microstructure generalization which is related with the adverse-selection component of the bid-ask spread. However, our results dealing with number of trades are more specific. They show that as the market becomes more informed, the trade size has more influence on bid-ask spread than the number of trades does. They also reveal how the informed trader's elasticity of information is related with this.

As he encounters more informed traders in the market and knows that informed traders use new information quickly, one large-size trade is to him indicative that traders now possess better information. Inversely, when he encounters two consecutive small-size trades, he considers these trades to also be based on better information, but without the signal to the market maker that a rapid rise in price is imminent. So as a protective measure, the market maker increases the bid-ask spread more readily on one large-size trade than he would for two consecutive small-size trades.

Our model shows that this happens when γ_1 is large γ_2 is small, i.e., the large elasticity of the informed traders' demand and supply schedules with respect to information about the value of the underlying asset ($\gamma_1 = 0.45$, $\gamma_2 = 0$, or 0.2). Table 3.6 shows this result.

III. 3. 3. 3. The relationship between number of trades and bid-ask spread

It is timely to note here that the market maker's reaction to the informedness of traders is a reaction, not only to the quality of the information, based on his prior belief, but also to the effect that information will have on the price. For our purposes in this dissertation, we assume that the market-maker reacts to the information and anticipates the effect that it will have on the price. So both the market informedness and the market maker's prior belief (the past-price variability) are key factors in the setting up of the bid-ask prices.

In this dissertation we are concerned more with the informational role of the number of trades on the bid and ask prices than with the trade size. These two variables are directly related to the bid and ask prices. Thus, we use them as indicators of situations where the number of trades plays a significant role in pricing. Just as volume and number of trades relate to market informedness, so does past-price variability relate to the market maker's prior belief. The market maker's belief on the underlying asset value are conditional on the past and present information about the security price. Therefore past-price variability is a proxy variable for his prior belief. To understand the role of past-price variability on the bid-ask spread, we can analyze both small and large past-price variability.

Small past-price variability. Consider a situation where the past-price variability of the security is relatively stable. In such a case the market maker's prior belief function has a small variance. In our model, this occurs when the market maker's prior belief is centered in the middle range of the security's value.

Numerical results in Table 3.7.1 show two economic situations in which more numbers of transaction (two consecutive small-size trades) or larger trade size (one large-size trade) represent the existence of better information in the market to which the market maker is not privy. In our results, number of trades is more valuable for the market maker when the elasticity of the informed traders' demand and supply schedules, with respect to information about the value of the underlying asset, is either very small or perfectly inelastic ($\gamma_1 = 0$, 0.05, or 0.25, $\gamma_2 = 0$), with low market informedness ($\alpha = 0.1$). The trade size in our model is more valuable for the market maker when the elasticity of the informed traders' demand and supply schedules is very large ($\gamma_1 = 0.45$, $\gamma_2 = 0$), with high market informedness ($\alpha = 0.9$).

We can interpret these results as follows: let us look into the case when the number of trades is more valuable for the market maker. In a market with a small fraction of informed traders, if the market maker receives any trades, he considers that those trades come mainly from uniformed traders. Under this market condition, however, if he receives two consecutive small-size trades rather than a large-size trade, he conjectures that they are more likely to come from traders with relatively better information than if one large-size trade occurs. By assigning his prior belief to the middle range of the security's value, he anticipates that the value of this security will neither increase nor decrease rapidly. So, if he encounters two consecutive small-size trades, he thinks that this type of a trading pattern is as expected. That is, the market maker interprets two consecutive trades to be based on neutral information without any particular signal and thus no imminent rapid rise or fall in price occurs.

However, if he observes one large-size trade, he considers the trade to arise from purely uninformed traders. Therefore, two consecutive small-size trades are more convincing to the market maker that they arise from better informed traders than if one large-size trade occurs. He sets up a larger bid-ask spread in two consecutive small-size trades to protect himself from the investors' informational edge.

Then next interpretation is for the case when the trade volume is more valuable for the market maker. In a market with a large fraction of informed traders, if the market maker receives any trades, he considers that they mainly come from the informed. In this case, if two consecutive small-size trades occur, he conjectures quite well that two informed traders have neutral information. This is possible since the market maker has already assigned his prior belief to the middle range of the security's value. If he receives a large-size trade and he thinks it mainly comes from an informed trader, then he considers it to contain some new information which he does not know. This is unexpected and too risky for the market maker to ignore. Thus, one large-size trade is more convincing that it comes from a better informed trader than two consecutive small-size trades are. Therefore, he sets up a larger bid-ask spread in one large-size trade for the purpose previously mentioned.

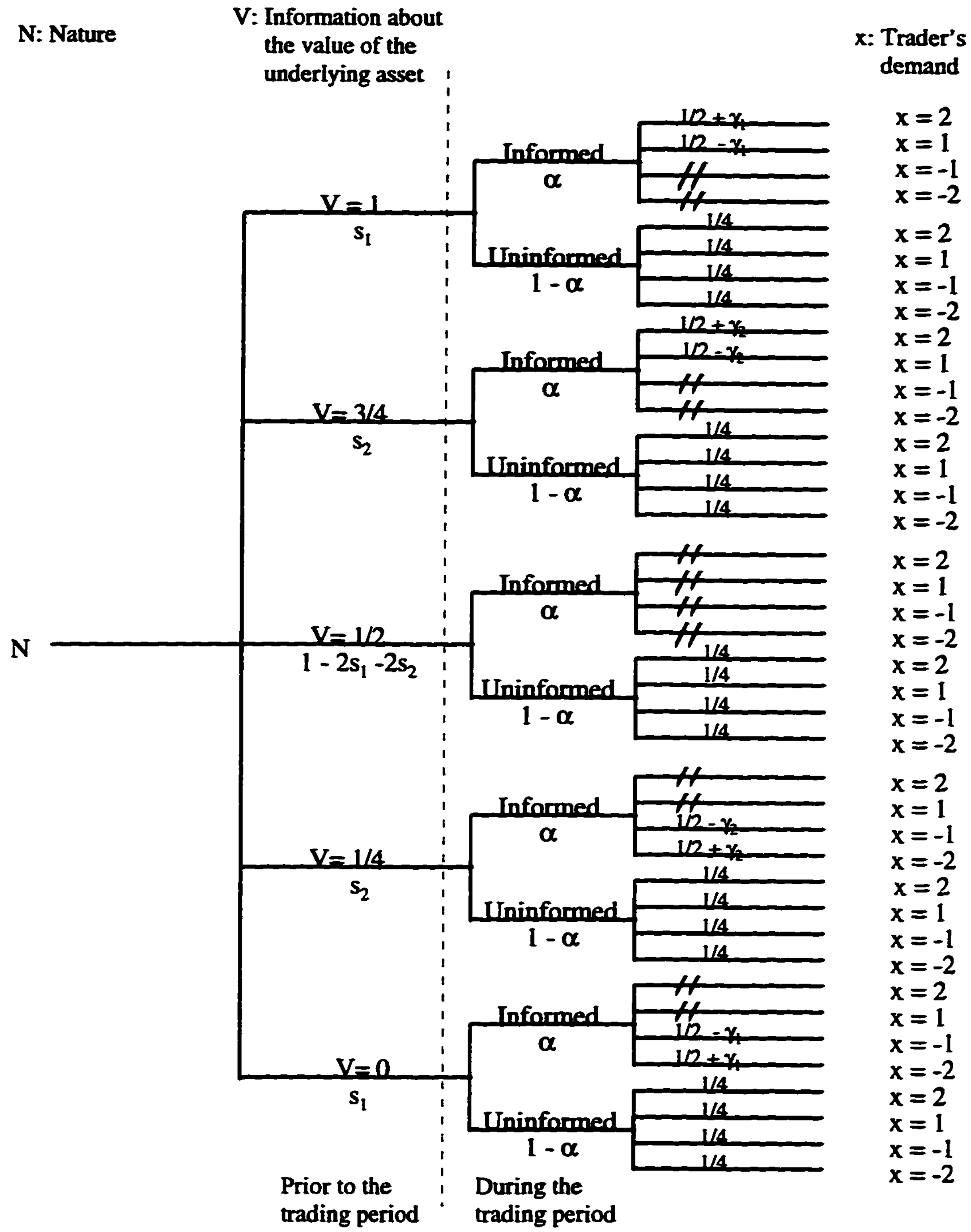
Large past-price variability. Now consider a situation where the past-price variability of the security is relatively large. It is difficult in such a case for the market maker to conjecture the security's real value. If the market maker is not sure of the underlying asset's true value at the beginning of the trading period, his prior belief is not helpful in his decision-making process. In our model, therefore, he evenly assigns a probability on the given probable values.

Numerical results in Table 3.7.2 show also two economic situations in which more numbers of transaction (two consecutive small-size trades) or larger trade size (one large-size trade) represent the existence of better information in the market to which the market maker is not privy. As in our results on the small past-price variability, number of trades is more valuable for the market maker when the elasticity of the informed traders' demand and supply schedules, with respect to information about the value of the underlying asset, is either very small or perfectly inelastic (γ_1 = 0, 0.05, or 0.25, γ_2 =0), with low market informedness ($\alpha = 0.1$). As in the small past-price variability, the trade size in our model is more valuable for the market maker when the elasticity of the informed traders' demand and supply schedules is very large (γ_1 = 0.45, γ_2 =0), with high market informedness (α = 0.9).

To interpret these results, let us look into the case when the number of trades is more valuable for the market maker. In a market with a small fraction of informed traders, new trades are considered to come mainly from the uninformed. However, two consecutive small-size trades rather than one large-size trade indicate to the market maker a likelihood that two consecutive trades still come from uninformed traders, but ones who have relatively better information. Based on the aforementioned reactions, the market maker takes action accordingly. He has little to go on in this case. He only focuses on what he does know, the repetition in trading. Therefore, he sets up a larger bid-ask spread in two consecutive trades to protect himself.

Our next interpretation concerns the case when the trade volume is more valuable for the market maker. The reaction is different if the same two consecutive small-size trades occur in a market with a large, rather than a small, fraction of the informed. Even though he is not sure what the security's true value is, he is pretty sure that there exists a large number of the informed. Thus, the market maker conjectures that trades mainly come from informed traders. His interpretation of the signal is thus based on the size of the trade. One large-size trade comes from better informed traders. The bid-ask spread becomes larger.

We need to note the result that the informational roles of both number of trades and trade size have regularity regardless of small or large past-price variability. Also, the small (large) fraction of the informed traders in the market is related with the informed traders' inelastic (elastic) demand and supply schedules. For the next chapter, these results give meaningful implications: we may expect that the market maker's prior belief (past-price variability) does not play a significant role for either traders' number of trades or the market maker's bid-ask spread. In the situation where we cannot get the data for the elasticity of the informed traders' demand and supply schedules, the positive relationship between the market informedness and the informed traders' elasticity can facilitate us to perform empirical analyses.



In this diagram V is a signal about the value of the underlying asset. α is the probability that a trader is informed, 1 - α the probability that he is uninformed; γ_1 is the fraction of informed traders who trade 2 units when the asset's value is either 0 or 1, γ_2 the fraction of informed traders who trade 2 units when the asset's value is either 3/4 or 1/4; s_1 denotes the probability the signal is either extremely good or bad, s_2 the probability it is either good or bad, and $1 - s_1 - s_2$ the probability it is neutral.

Figure 3. 1 Tree diagram of the market process

Parameter	Starting point	Increment	Ending point
α	0.10	0.10	0.90
γ_1	0.05	0.05	0.45
γ_2	0.00	0.05	0.20
s_1	0.05	0.05	0.35
s_2	0.05	0.05	0.35

* Summation of the market maker's prior probabilities about the value of the underlying asset is 1.
We restrict $\gamma_1 > \gamma_2$, except in the case of $\gamma_1 = \gamma_2 = 0$.

Table 3.1 Numerical examples - parameter values

α	γ1	γ2	s1	s2	P(2,1)-P(1,1)
0.1	0.05	0	0.05	0.05	0.00214
0.1	0.05	0	0.05	0.2	0.00204
0.1	0.05	0	0.05	0.35	0.00195
0.1	0.05	0	0.2	0.05	0.00802
0.1	0.05	0	0.2	0.2	0.00766
0.1	0.05	0	0.35	0.05	0.01319
0.1	0.25	0	0.05	0.05	0.01069
0.1	0.25	0	0.05	0.2	0.01019
0.1	0.25	0	0.05	0.35	0.00974
0.1	0.25	0	0.2	0.05	0.04013
0.1	0.25	0	0.2	0.2	0.03833
0.1	0.25	0	0.35	0.05	0.06605
0.1	0.25	0.2	0.05	0.05	0.0149
0.1	0.25	0.2	0.05	0.2	0.02598
0.1	0.25	0.2	0.05	0.35	0.03572
0.1	0.25	0.2	0.2	0.05	0.04395
0.1	0.25	0.2	0.2	0.2	0.05272
0.1	0.25	0.2	0.35	0.05	0.06953
0.1	0.45	0	0.05	0.05	0.01925
0.1	0.45	0	0.05	0.2	0.01835
0.1	0.45	0	0.05	0.35	0.01753
0.1	0.45	0	0.2	0.05	0.0723
0.1	0.45	0	0.2	0.2	0.06906
0.1	0.45	0	0.35	0.05	0.11922
0.1	0.45	0.2	0.05	0.05	0.02346
0.1	0.45	0.2	0.05	0.2	0.03415
0.1	0.45	0.2	0.05	0.35	0.04353
0.1	0.45	0.2	0.2	0.05	0.07615
0.1	0.45	0.2	0.2	0.2	0.08353
0.1	0.45	0.2	0.35	0.05	0.12276
0.5	0.05	0	0.05	0.05	0.01458
0.5	0.05	0	0.05	0.2	0.01067
0.5	0.05	0	0.05	0.35	0.00833
0.5	0.05	0	0.2	0.05	0.03736
0.5	0.05	0	0.2	0.2	0.02964
0.5	0.05	0	0.35	0.05	0.04544
0.5	0.25	0	0.05	0.05	0.07304
0.5	0.25	0	0.05	0.2	0.05339
0.5	0.25	0	0.05	0.35	0.0417
0.5	0.25	0	0.2	0.05	0.19005
0.5	0.25	0	0.2	0.2	0.15
0.5	0.25	0	0.35	0.05	0.23577
0.5	0.25	0.2	0.05	0.05	0.09847

Table 3.2 A comparison with the result of Easley and O'Hara (1987):trade size

α	γ1	γ2	s1	s2	P(2,1)-P(1,1)
0.5	0.25	0.2	0.05	0.2	0.11968
0.5	0.25	0.2	0.05	0.35	0.1236
0.5	0.25	0.2	0.2	0.05	0.20252
0.5	0.25	0.2	0.2	0.2	0.18519
0.5	0.25	0.2	0.35	0.05	0.24191
0.5	0.45	0	0.05	0.05	0.13199
0.5	0.45	0	0.05	0.2	0.09635
0.5	0.45	0	0.05	0.35	0.07519
0.5	0.45	0	0.2	0.05	0.35654
0.5	0.45	0	0.2	0.2	0.27778
0.5	0.45	0	0.35	0.05	0.46534
0.5	0.45	0.2	0.05	0.05	0.15811
0.5	0.45	0.2	0.05	0.2	0.16457
0.5	0.45	0.2	0.05	0.35	0.15952
0.5	0.45	0.2	0.2	0.05	0.37321
0.5	0.45	0.2	0.2	0.2	0.32328
0.5	0.45	0.2	0.35	0.05	0.47831
0.9	0.05	0	0.05	0.05	0.03333
0.9	0.05	0	0.05	0.2	0.01667
0.9	0.05	0	0.05	0.35	0.01111
0.9	0.05	0	0.2	0.05	0.03466
0.9	0.05	0	0.2	0.2	0.03004
0.9	0.05	0	0.35	0.05	0.02733
0.9	0.25	0	0.05	0.05	0.17087
0.9	0.25	0	0.05	0.2	0.08387
0.9	0.25	0	0.05	0.35	0.05572
0.9	0.25	0	0.2	0.05	0.19326
0.9	0.25	0	0.2	0.2	0.1575
0.9	0.25	0	0.35	0.05	0.15938
0.9	0.25	0.2	0.05	0.05	0.18667
0.9	0.25	0.2	0.05	0.2	0.09986
0.9	0.25	0.2	0.05	0.35	0.06825
0.9	0.25	0.2	0.2	0.05	0.16745
0.9	0.25	0.2	0.2	0.2	0.11166
0.9	0.25	0.2	0.35	0.05	0.13159
0.9	0.45	0	0.05	0.05	0.32698
0.9	0.45	0	0.05	0.2	0.15327
0.9	0.45	0	0.05	0.35	0.10097
0.9	0.45	0	0.2	0.05	0.47569
0.9	0.45	0	0.2	0.2	0.31976
0.9	0.45	0	0.35	0.05	0.46858
0.9	0.45	0.2	0.05	0.05	0.36856
0.9	0.45	0.2	0.05	0.2	0.18581
0.9	0.45	0.2	0.05	0.35	0.12422
0.9	0.45	0.2	0.2	0.05	0.4893
0.9	0.45	0.2	0.2	0.2	0.31762
0.9	0.45	0.2	0.35	0.05	0.47071

Table 3.2 - *Continued*

α	$\gamma 1$	$\gamma 2$	s1	s2	P(1,2) - P(1,1)
0.1	0.05	0	0.05	0.05	0.01758
0.1	0.05	0	0.05	0.2	0.03313
0.1	0.05	0	0.05	0.35	0.0458
0.1	0.05	0	0.2	0.05	0.04684
0.1	0.05	0	0.2	0.2	0.05839
0.1	0.05	0	0.35	0.05	0.07124
0.1	0.25	0	0.05	0.05	0.0123
0.1	0.25	0	0.05	0.2	0.02858
0.1	0.25	0	0.05	0.35	0.04183
0.1	0.25	0	0.2	0.05	0.02842
0.1	0.25	0	0.2	0.2	0.04239
0.1	0.25	0	0.35	0.05	0.043
0.1	0.25	0.2	0.05	0.05	0.00958
0.1	0.25	0.2	0.05	0.2	0.01929
0.1	0.25	0.2	0.05	0.35	0.02789
0.1	0.25	0.2	0.2	0.05	0.02607
0.1	0.25	0.2	0.2	0.2	0.03437
0.1	0.25	0.2	0.35	0.05	0.04099
0.1	0.45	0	0.05	0.05	0.00762
0.1	0.45	0	0.05	0.2	0.02458
0.1	0.45	0	0.05	0.35	0.03838
0.1	0.45	0	0.2	0.05	0.01079
0.1	0.45	0	0.2	0.2	0.02727
0.1	0.45	0	0.35	0.05	0.0139
0.1	0.45	0.2	0.05	0.05	0.0048
0.1	0.45	0.2	0.05	0.2	0.01492
0.1	0.45	0.2	0.05	0.35	0.02389
0.1	0.45	0.2	0.2	0.05	0.00804
0.1	0.45	0.2	0.2	0.2	0.01788
0.1	0.45	0.2	0.35	0.05	0.01122
0.5	0.05	0	0.05	0.05	0.19349
0.5	0.05	0	0.05	0.2	0.19354
0.5	0.05	0	0.05	0.35	0.17476
0.5	0.05	0	0.2	0.05	0.28565
0.5	0.05	0	0.2	0.2	0.22819
0.5	0.05	0	0.35	0.05	0.29065
0.5	0.25	0	0.05	0.05	0.13885
0.5	0.25	0	0.05	0.2	0.17304
0.5	0.25	0	0.05	0.35	0.16383
0.5	0.25	0	0.2	0.05	0.20769
0.5	0.25	0	0.2	0.2	0.1875
0.5	0.25	0	0.35	0.05	0.23434
0.5	0.25	0.2	0.05	0.05	0.11124

Table 3.3 A comparison with the result of Easley and O'Hara (1992): number of trades

α	γ1	γ2	s1	s2	P(2,1)-P(1,1)
0.5	0.25	0.2	0.05	0.2	0.14663
0.5	0.25	0.2	0.05	0.35	0.15272
0.5	0.25	0.2	0.2	0.05	0.20585
0.5	0.25	0.2	0.2	0.2	0.19332
0.5	0.25	0.2	0.35	0.05	0.24165
0.5	0.45	0	0.05	0.05	0.10206
0.5	0.45	0	0.05	0.2	0.16456
0.5	0.45	0	0.05	0.35	0.16153
0.5	0.45	0	0.2	0.05	0.1146
0.5	0.45	0	0.2	0.2	0.16369
0.5	0.45	0	0.35	0.05	0.12524
0.5	0.45	0.2	0.05	0.05	0.05982
0.5	0.45	0.2	0.05	0.2	0.12282
0.5	0.45	0.2	0.05	0.35	0.13948
0.5	0.45	0.2	0.2	0.05	0.08011
0.5	0.45	0.2	0.2	0.2	0.12931
0.5	0.45	0.2	0.35	0.05	0.09724
0.9	0.05	0	0.05	0.05	0.23869
0.9	0.05	0	0.05	0.2	0.09587
0.9	0.05	0	0.05	0.35	0.06019
0.9	0.05	0	0.2	0.05	0.15384
0.9	0.05	0	0.2	0.2	0.07677
0.9	0.05	0	0.35	0.05	0.10971
0.9	0.25	0	0.05	0.05	0.19948
0.9	0.25	0	0.05	0.2	0.07979
0.9	0.25	0	0.05	0.35	0.05045
0.9	0.25	0	0.2	0.05	0.13416
0.9	0.25	0	0.2	0.2	0.03879
0.9	0.25	0	0.35	0.05	0.10074
0.9	0.25	0.2	0.05	0.05	0.29174
0.9	0.25	0.2	0.05	0.2	0.13496
0.9	0.25	0.2	0.05	0.35	0.08893
0.9	0.25	0.2	0.2	0.05	0.21944
0.9	0.25	0.2	0.2	0.2	0.11338
0.9	0.25	0.2	0.35	0.05	0.16609
0.9	0.45	0	0.05	0.05	0.21163
0.9	0.45	0	0.05	0.2	0.09253
0.9	0.45	0	0.05	0.35	0.05899
0.9	0.45	0	0.2	0.05	0.15063
0.9	0.45	0	0.2	0.2	0.06699
0.9	0.45	0	0.35	0.05	0.1057
0.9	0.45	0.2	0.05	0.05	0.23928
0.9	0.45	0.2	0.05	0.2	0.12865
0.9	0.45	0.2	0.05	0.35	0.08681
0.9	0.45	0.2	0.2	0.05	0.18825
0.9	0.45	0.2	0.2	0.2	0.09704
0.9	0.45	0.2	0.35	0.05	0.15288

Table 3.3 - *Continued*

α	γ1	γ2	s1	s2	P(2,1) - P(1,2)
0.1	0.05	0	0.05	0.05	-0.01544
0.1	0.05	0	0.05	0.2	-0.03109
0.1	0.05	0	0.05	0.35	-0.04385
0.1	0.05	0	0.2	0.05	-0.03882
0.1	0.05	0	0.2	0.2	-0.05072
0.1	0.05	0	0.35	0.05	-0.05805
0.1	0.25	0	0.05	0.05	-0.00161
0.1	0.25	0	0.05	0.2	-0.01838
0.1	0.25	0	0.05	0.35	-0.03209
0.1	0.25	0	0.2	0.05	0.01171
0.1	0.25	0	0.2	0.2	-0.00405
0.1	0.25	0	0.35	0.05	0.02304
0.1	0.25	0.2	0.05	0.05	0.00532
0.1	0.25	0.2	0.05	0.2	0.00669
0.1	0.25	0.2	0.05	0.35	0.00783
0.1	0.25	0.2	0.2	0.05	0.01788
0.1	0.25	0.2	0.2	0.2	0.01834
0.1	0.25	0.2	0.35	0.05	0.02854
0.1	0.45	0	0.05	0.05	0.01162
0.1	0.45	0	0.05	0.2	-0.00623
0.1	0.45	0	0.05	0.35	-0.02085
0.1	0.45	0	0.2	0.05	0.06151
0.1	0.45	0	0.2	0.2	0.0418
0.1	0.45	0	0.35	0.05	0.10533
0.1	0.45	0.2	0.05	0.05	0.01866
0.1	0.45	0.2	0.05	0.2	0.01923
0.1	0.45	0.2	0.05	0.35	0.01965
0.1	0.45	0.2	0.2	0.05	0.0681
0.1	0.45	0.2	0.2	0.2	0.06566
0.1	0.45	0.2	0.35	0.05	0.11154
0.5	0.05	0	0.05	0.05	-0.17891
0.5	0.05	0	0.05	0.2	-0.18287
0.5	0.05	0	0.05	0.35	-0.16643
0.5	0.05	0	0.2	0.05	-0.24829
0.5	0.05	0	0.2	0.2	-0.19854
0.5	0.05	0	0.35	0.05	-0.24521
0.5	0.25	0	0.05	0.05	-0.06581
0.5	0.25	0	0.05	0.2	-0.11965
0.5	0.25	0	0.05	0.35	-0.12213
0.5	0.25	0	0.2	0.05	-0.01765
0.5	0.25	0	0.2	0.2	-0.0375
0.5	0.25	0	0.35	0.05	0.00142

Table 3.4 The bid-ask spread differences when both number of trades and trade size are included

α	γ1	γ2	s1	s2	P(2,1) - P(1,2)
0.5	0.25	0.2	0.05	0.05	-0.01277
0.5	0.25	0.2	0.05	0.2	-0.02695
0.5	0.25	0.2	0.05	0.35	-0.02912
0.5	0.25	0.2	0.2	0.05	-0.00334
0.5	0.25	0.2	0.2	0.2	-0.00813
0.5	0.25	0.2	0.35	0.05	0.00026
0.5	0.45	0	0.05	0.05	0.02993
0.5	0.45	0	0.05	0.2	-0.06822
0.5	0.45	0	0.05	0.35	-0.08634
0.5	0.45	0	0.2	0.05	0.24194
0.5	0.45	0	0.2	0.2	0.11409
0.5	0.45	0	0.35	0.05	0.3401
0.5	0.45	0.2	0.05	0.05	0.09829
0.5	0.45	0.2	0.05	0.2	0.04176
0.5	0.45	0.2	0.05	0.35	0.02004
0.5	0.45	0.2	0.2	0.05	0.29309
0.5	0.45	0.2	0.2	0.2	0.19397
0.5	0.45	0.2	0.35	0.05	0.38107
0.9	0.05	0	0.05	0.05	-0.20536
0.9	0.05	0	0.05	0.2	-0.0792
0.9	0.05	0	0.05	0.35	-0.04908
0.9	0.05	0	0.2	0.05	-0.11918
0.9	0.05	0	0.2	0.2	-0.04673
0.9	0.05	0	0.35	0.05	-0.08238
0.9	0.25	0	0.05	0.05	-0.02862
0.9	0.25	0	0.05	0.2	0.00407
0.9	0.25	0	0.05	0.35	0.00526
0.9	0.25	0	0.2	0.05	0.0591
0.9	0.25	0	0.2	0.2	0.11871
0.9	0.25	0	0.35	0.05	0.05864
0.9	0.25	0.2	0.05	0.05	-0.10508
0.9	0.25	0.2	0.05	0.2	-0.0351
0.9	0.25	0.2	0.05	0.35	-0.02068
0.9	0.25	0.2	0.2	0.05	-0.052
0.9	0.25	0.2	0.2	0.2	-0.00171
0.9	0.25	0.2	0.35	0.05	-0.0345
0.9	0.45	0	0.05	0.05	0.11535
0.9	0.45	0	0.05	0.2	0.06075
0.9	0.45	0	0.05	0.35	0.04198
0.9	0.45	0	0.2	0.05	0.32506
0.9	0.45	0	0.2	0.2	0.25277
0.9	0.45	0	0.35	0.05	0.36288
0.9	0.45	0.2	0.05	0.05	0.12928
0.9	0.45	0.2	0.05	0.2	0.05716
0.9	0.45	0.2	0.05	0.35	0.03741
0.9	0.45	0.2	0.2	0.05	0.30105
0.9	0.45	0.2	0.2	0.2	0.22058
0.9	0.45	0.2	0.35	0.05	0.31782

Table 3.4 - ***Continued***

Case A: P(2,1) - P(1,2) is negative if $\gamma_1 = \gamma_2 = 0$

α	γ1	γ2	s1	s2	P(2,1) - P(1,2)
0.1	0	0	0.05	0.05	-0.01899
0.1	0	0	0.05	0.2	-0.03436
0.1	0	0	0.05	0.35	-0.04687
0.1	0	0	0.2	0.05	-0.05153
0.1	0	0	0.2	0.2	-0.06249
0.1	0	0	0.35	0.05	-0.07811
0.5	0	0	0.05	0.05	-0.20833
0.5	0	0	0.05	0.2	-0.20000
0.5	0	0	0.05	0.35	-0.17857
0.5	0	0	0.2	0.05	-0.30000
0.5	0	0	0.2	0.2	-0.23810
0.5	0	0	0.35	0.05	-0.29762
0.9	0	0	0.05	0.05	-0.24759
0.9	0	0	0.05	0.2	-0.10250
0.9	0	0	0.05	0.35	-0.06472
0.9	0	0	0.2	0.05	-0.15375
0.9	0	0	0.2	0.2	-0.08629
0.9	0	0	0.35	0.05	-0.10786

Case B: P(2,1) - P(1,1) becomes larger if α is large, γ_1 is large, & $\gamma_2 = 0$

α	γ1	γ2	s1	s2	P(2,1) - P(1,2)
0.9	0.25	0	0.05	0.2	0.00407
0.9	0.25	0	0.05	0.35	0.00526
0.9	0.25	0	0.2	0.05	0.05910
0.9	0.25	0	0.2	0.2	0.11871
0.9	0.25	0	0.35	0.05	0.05864
0.9	0.45	0	0.05	0.05	0.11535
0.9	0.45	0	0.05	0.2	0.06075
0.9	0.45	0	0.05	0.35	0.04198
0.9	0.45	0	0.2	0.05	0.32506
0.9	0.45	0	0.2	0.2	0.25277
0.9	0.45	0	0.35	0.05	0.36288

Table 3.5 Two general scenarios

α	$\gamma 1$	$\gamma 2$	s1	s2	P(2,1) - P(1,2)
0.1	0.45	0	0.05	0.05	0.01162
0.5	0.45	0	0.05	0.05	0.02993
0.9	0.45	0	0.05	0.05	0.11535
0.1	0.45	0	0.2	0.2	0.0418
0.5	0.45	0	0.2	0.2	0.11409
0.9	0.45	0	0.2	0.2	0.25277
0.1	0.45	0.2	0.05	0.05	0.01866
0.5	0.45	0.2	0.05	0.05	0.09829
0.9	0.45	0.2	0.05	0.05	0.12928
0.1	0.45	0.2	0.2	0.2	0.06566
0.5	0.45	0.2	0.2	0.2	0.19397
0.9	0.45	0.2	0.2	0.2	0.22058

Table 3.6 The positive relationship between market informedness and bid-ask spread

1. Small past-price variability

s1	s2	α	γ1	γ2	P(2,1) - P(1,2)
0.05	0.35	0.1	**0**	**0**	**-0.04687**
0.05	0.35	0.1	**0.05**	**0**	**-0.04385**
0.05	0.35	0.1	**0.25**	**0**	**-0.03209**
0.05	0.35	0.1	0.25	0.2	0.00783
0.05	0.35	0.1	0.45	0	-0.02085
0.05	0.35	0.1	0.45	0.2	0.01965
0.05	0.35	0.9	0	0	-0.06472
0.05	0.35	0.9	0.05	0	-0.04908
0.05	0.35	0.9	0.25	0	0.00526
0.05	0.35	0.9	0.25	0.2	-0.02068
0.05	0.35	0.9	**0.45**	**0**	**0.04198**
0.05	0.35	0.9	0.45	0.2	0.03741

2. Large past-price variability

s1	s2	α	γ1	γ2	P(2,1) - P(1,2)
0.2	0.2	0.1	**0**	**0**	**-0.06249**
0.2	0.2	0.1	**0.05**	**0**	**-0.05072**
0.2	0.2	0.1	**0.25**	**0**	**-0.00405**
0.2	0.2	0.1	0.25	0.2	0.01834
0.2	0.2	0.1	0.45	0	0.04180
0.2	0.2	0.1	0.45	0.2	0.06566
0.2	0.2	0.9	0	0	-0.08629
0.2	0.2	0.9	0.05	0	-0.04673
0.2	0.2	0.9	0.25	0	0.11871
0.2	0.2	0.9	0.25	0.2	-0.00171
0.2	0.2	0.9	**0.45**	**0**	**0.25277**
0.2	0.2	0.9	0.45	0.2	0.22058

Table 3.7 The relationship between number of trades and bid-ask spread

Chapter IV. An Empirical Examination of the Relationship between Bid-Ask Spreads and Number of Trades

In chapter III, we presented a sequential trade model that predicts the effects of the number of trades and trade size (volume per transaction) on the market maker's pricing for the frequently traded securities. The next step in our analysis is to test the structural model.

Recall the results from the model in the previous chapter. Our model reveals that the significance of the number of trades for the market maker's pricing depends, not only on the market maker's prior probabilistic belief about the value of the underlying asset, but also on the market informedness and the elasticity of the informed traders' demand and supply schedules. We also show that the number of trades sends a better signal to the market maker in the following situations: given small past-price variability, when a small fraction of the informed traders exists in the market and the informed traders' demand and supply schedules are perfectly inelastic, the relationship between the bid-ask spread and the number of trades is positive. Given large past-price variability, when a large fraction of informed traders exists in the market and the amount of their order increases with the quantity of information, the relationship between the bid-ask spread and the number of trades is negative.

The following empirical hypotheses are based on the model in chapter III. Among others, that model uses a variable, the elasticity of the informed traders' demand and supply schedules, for which there is no available data. Nevertheless, when examining the whole picture, considering all variables, we were able to formulate workable hypotheses using the ample supply of data from other variables.[1] The first concern is the relationship between the bid-ask spread and the number of trades when volume is held constant for a given trading period. This testing possibly reveals how the number of trades, relative to the trade size, impacts the market maker on setting his bid-ask spread. Another concern is whether the variables -- past-price variability, market informedness, and firm size (firm's market value) -- can affect the number of trades. Testing this second hypothesis shows the degree to which the three variables influence the number of trades of individual investors. Then, considering the results of both the first and the second hypotheses together, we naturally come to a third hypothesis. How do those variables affect the market maker's bid-ask spread? We can see the relationship between those variables and the market maker's bid-ask spread. In addition, we can get an idea of how much the impacts of the three variables on the number of trades can be represented in the market maker's bid-ask spread.

In this chapter we test these empirical implications. First, we describe data, proxy variables, and some testable hypotheses. Then we show the results of the regression analyses.

[1] The results in chapter III show that the positive relationship between market informedness and the informed traders' elasticity. In chapter IV, one can use this result as a frame of reference. Since we analyze the market situation for the frequently traded securities, we add the firm size variable to proxy the market liquidity. In chapters IV and V, volume is defined as the total shares (or dollar amounts) of stocks traded in a day. Chapter III uses a simple model in which volume represents a trade size for an individual transaction. However, the implications of the model in chapter III can be extended into both this and following chapters once we restrict the period of our concerns to a daily basis.

IV. 1. Data Description and Proxy Variables

The files of the NASDAQ-NMS (the National Association of Securities Dealers Automated Quotation - National Market System) of the CRSP (the Center for Research in Security Prices) provide daily data for the bid-ask spread and the number of trades. These data are proper since the NASDAQ-NMS actually has competitive market makers, as we assumed in our model.

Since our study focuses on the price impacts of the information in relatively thick markets, the stocks with no trades are removed from the sample.[2] In order to obtain the data with this criterion, we restrict it to having consecutive trading for the whole sample period.[3] The period of analysis is from January 1990 to December 1992. In total, our final daily data set for bid-ask spread and number of trades consists of 759 time-series and cross-sectional observations of 317 firms.

We also use three additional variables: the market maker's prior belief about the value of an underlying asset, the market informedness, and the market depth (market liquidity). Since we cannot directly obtain measures for these variables, we need to use proxies for them. We have only monthly data for the market informedness variable (the number of institutions holding a firm's shares). Thus, we end up having 36 time-series data

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[2] The market depth is classified into two categories: thin and thick. The thick (thin) market implies that liquidity is high (low) and price impacts are low (high).

[3] Based on this criterion, table 4.1 shows that the average daily bid-ask spread of our sample is 0.35. This can be comparable to the average spread of NYSE and AMEX firms, 0.42 [see Kaul and Nimalendran (1990)]. Empirical insight may be found regarding similar issues in NYSE and AMEX. However, the data selection criterion used here may also cause the problem called *incidental truncation*. We will discuss this in section IV.2.1.3.

for the three proxy variables to analyze. The rationales for these proxies and the data sources are as follows:

Past-Price (or Return) Variability. As in chapter III, the market maker's belief on the value of an underlying asset is conditional on the past and present information about the security price. The market maker does not have special information about the underlying asset's value. Therefore past-price variations can be instrumental in the formation of his prior belief. We use this as a proxy for the market maker's prior belief. The following basic relationships between the past-price variability and the market maker's prior belief work depending on given economic situations: when the price of a security is relatively stable, the market maker can assign a prior belief to specific values. When the past-price variability of a security is relatively large, it is difficult for the market maker to conjecture the security's real value.

This variable is also related to individual traders. For uninformed traders, the securities which have a small past-price variability are attractive. The label 'uninformed trader' is somewhat misleading. They do come into a trade with some public information. They do not simply invest randomly. Common logic would steer even the uninformed traders away from securities with relatively large past-price variability because they are riskier.

Informed traders react in an opposite way. For informed traders, the securities which have a large past-price variability are attractive. Assume that private information is useful, i.e., it can be used to make more precise predictions of future return than an expected return conditional just on public information. Private information then becomes more valuable for firms with higher return (or past-price) variability. Ceteris paribus, as return variability increases, so does the probability of getting large deviations between the expected return conditional on both private and public information and the expected return conditional just on public information. The expected trading profit is an increasing function of this probability and it follows that the expected trading profit based on private information will be higher for a firm with higher return variability. We use the standard deviations of the daily bid (or ask) prices within the month as the past-price variability variable from NASDAQ-NMS of CRSP.

Ownership Structure. We use the number of institutions holding a firm's shares as a proxy for the market informedness (the fraction of the informed traders) in a given trading period. In his series of papers (1989a, 1989b), Bhushan (1989a) develops a model of information collection about publicly traded firms in a economy by using a noisy rational expectations equilibrium. Bhushan (1989b) also examines the major determinants of the number of analysts following a firm, which represents the informed traders. In his paper (1989b), he investigates the following five firm characteristics which influence both aggregate demand and supply of analyst services: (i) ownership structure, (ii) firm size, (iii) return variability, (iv) number of lines of business, and (v) correlation between firm return and market return. He concludes that almost all of these firm's characteristics significantly affect the extent of the analyst following a firm. The empirical results generally accord well with economic intuition.

Among the factors that lead to differences in the number of analysts following a firm, the ownership structure of a firm is of special interest. Bhushan considers the following three variables relating to the ownership structure of a firm: (i) the number of institutions holding a firm's shares, (ii) the percentage of its shares held by these institutions, and (iii) the degree to which the firm is closely held by insiders. The results of his regression show that the aggregate demand for analyst services increases as more institutions hold shares in a firm, or as the percentage held by them increases. It also shows that the demand for analyst services comes from non-insiders. In other words, this demand increases as the fraction of the firm held by insiders decreases.

We give attention to the positive relationship between the number of analysts following a firm and the number of institutions holding a firm's shares. While the number of analysts following a firm might be a better proxy for the market informedness, due to data limitations, we use the number of institutions holding a firm's shares as a proxy. We cannot get data for the number of analysts following a firm in NASDAQ-NMS.

Despite this restriction, we still can use the number of institutions holding a firm's shares as a proxy for the market informedness. As Bhushan (1989b) puts it, an increase in the number of institutions holding a firm's shares suggests an increase in the concentration of ownership by institutional investors. If the acquisition of analyst services is not cost-effective for a small investor, more concentration of ownership by institutional investors would imply increased demand for analyst services. Also, if institutions provide the majority of transactions business to the analyst, then the supply of his services is likely to increase as the number of institutions holding a firm's shares increases. For the number of institutions holding a firm's shares, the corresponding monthly data from Standard and Poor's *Security Owner Stock Guide* will be used.

Firm Size (Firm's Market Value). The chapter III model focuses on the informational role of the number of trades in frequently traded securities, which implies a relatively thick market. Now we explicitly take this market liquidity to test empirically its impact on the relationship between the number of trades and the bid-ask spread.

Liquid markets are generally viewed as those which accommodate trading with the least effect on price. In this context, firm size can be a proxy for the market liquidity. The following example shows the rationale for using the firm size as a proxy for this: suppose that we take two different transactions, each with the same amount. One trade is related to a firm with a huge capitalization, and the other is related to one with a small capitalization. A trade in a firm with a huge market value is more likely to be executed without substantially disturbing the price than the same trade in a firm with a small market value. We calculate each firm's size by multiplying its number of shares outstanding with its price per share. For each security, we take the number of shares outstanding on December 31 in 1989 and the closing average bid and ask prices at the end of each month for the three-year period from January 1990 to December 1992.

IV. 2. Empirical Hypothesis

Our model is concerned with price movements across the number of trades while holding the volume (trade size) constant for a given trading period. From all of our model's predictions, we pay attention to three trade variables which are related to the testable implications: market maker's prior belief, market informedness, and firm size. For the empirical tests, we interpret a given trading period as either a day or a month. Thus, we hold the daily (or monthly) volume constant.

In this section, we want to first determine the relationship between the bid-ask spread and the number of trades, while holding daily volume constant. This will tell us how the number of trades, relative to the trading volume, impacts the market maker on setting his bid-ask spread. Second, we examine whether the three trade variables can affect the number of trades. This will show how much those three variables influence the number of trades of the individual investors. Then, considering the results of both the first and the second hypotheses together, the third seems like a natural course of investigation: how do those variables affect the market maker's bid-ask spread? We can see the relationship between those variables and the market maker's bid-ask spread. In addition, we can get an idea of how much the impacts of the three variables on the number of trades can be represented in the market maker's bid-ask spread.

IV. 2. 1. The relationship between number of trades and bid-ask spread

A prediction from our theoretical model about the relationship between number of trades and bid-ask spread shows two possible relationships. (i) Given small past-price variability, when a small fraction of the informed trades exists in the market and the informed traders' demand and supply schedules are perfectly inelastic, the relationship between the bid-ask spread and the number of trades is positive. (ii) Given large past-price variability, when a large fraction of informed traders exists in the market and the amount of their order increases with the quantity of information, the relationship between the bid-ask spread and the number of trades is negative.

For empirical tests, these theoretical predictions can be summarized as follows: we will have an ambiguous relationship between the number of trades and the bid-ask spread. This relationship depends on the effects of influential variables like past-price variability, market informedness, and firm size, on both number of trades and bid-ask spread.

Based upon these considerations, we employ the following structural model for the empirical representation of the relationship between the number of trades and the bid-ask spread:

$$(\text{Bid-Ask Spread})_{i,t} = \alpha_0 + \alpha_1(\text{Number of Trades})_{i,t} + \alpha_2(\text{Volume})_{i,t} + \sum_{j=3}^{7}\alpha_j(\text{Number of Trades})_{i,t+2-j} + \sum_{j=8}^{12}\alpha_j(\text{Volume})_{i,t+7-j} + \varepsilon_{i,t} \quad (4.1)$$

where i and t denote a security and day, respectively. We use the number of trades and the volume as regressors.[4] This will separate the effect of a daily trading volume on bid-ask spread from that of a daily number of trades. We also consider the dynamic effects of these two variables on the bid-ask spreads using the lagged variables for each. To select the order of lagged variables, we use *Akaike's Information Criterion* (AIC), BIC (*Baysian Information Criterion*), and *adjusted* R^2. Then we pool all sample firms across the sample period for the regression analyses.[5]

IV. 2. 1. 1. Descriptive statistics

Table 4.1 presents some descriptive statistics for the variables used in the regression equation (4.1). The variables are bid-ask spreads, number of trades, and volume. The table contains their average, standard deviation, quintile, skewness, and kurtosis. All reported

[4] In stead of the multiple regression, we can use a simple one in which bid-ask spread is a dependent variable and number of trades is an independent variable. Then, we can construct decile portfolios for these two variables by sorting according to the size of a daily volume in order to consider its fixed size. However, this modeling may cause the *self-selection problem*.

[5] As you will see in Table 4.2, the effects of lagged variables for both number of trades and volume on bid-ask spreads are very persistent. This makes it very difficult to select the order of the lagged variables. After considering *Akaike's Information Criterion* (AIC), BIC (*Baysian Information Criterion*), and *adjusted* R^2, we

numbers are obtained using daily observations. The second column reports the total number of observations.

The numbers of Table 4.1 reflect the diverse sample of securities used in this study. For example, the daily volume ranges between 100 shares and 26.8 million shares. The trading frequency exhibits also a substantial cross-sectional variation; the number of transactions varies between 1 and 9,454 per day. We need to look closely at the statistics for this variable. The distribution for the number of trades is skewed to the left, but it does show that this variable's daily median is 43. This frequency is relatively high compared to the securities that we removed from our sample. Recall that we eliminated from the sample the securities with no trades during the sample periods. If the median (or mean) daily number of trades is very close to 0 rather than our 43 (or 98), then removing these would lose much information for our analysis, which is called *incidental truncation.*

IV. 2. 1. 2. Empirical results

We estimate regression equation (4.1) using Hansen's (1982) *Generalized Method of Moments* (GMM) procedure between January 1990 and December 1992. Heteroskedasticity and autocorrelation are adjusted by using a Bartlett kernel with an optimal bandwidth as in Andrews (1991). Because the set of instruments we use is identical to the set of regressors, coefficient estimates are identical to those obtained by *Ordinary Least Squares* (OLS)

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end up with five lagged variables for each. We also use BIC to complement the AIC's tendency toward taking higher orders of lagged variables.

estimation. However, standard errors, and therefore inferences, differ from those obtained using OLS.

Part A in Table 4.2 presents estimates of regression equation (4.1) to test directly whether, when volume is held constant for a given trading period, the daily number of trades is related to the daily bid-ask spread. It contains the coefficient estimates of number of daily transactions, volume, their lagged variables, and adjusted R^2, with their heteroskedasticity and autocorrelation consistent t-statistics using a Bartlett kernel with an optimal bandwidth as in Andrews (1991).

The most notable aspect of the evidence in Part A in Table 4.2 is the signs of the coefficients for the number of trades and its lagged variables. When daily volume is held constant, we see a statistically significant, positive relationship between the number of daily transactions with the bid-ask spread. This result indicates that the market maker generally raises the bid-ask spread when he experiences a relatively large number of (daily) transactions. When he observes several days over a particular period having similar daily volumes, he considers more transactions as an indication of the existence of a higher possibility that there is something happening which he does not know. Thus, he wants to protect himself from this asymmetric information. He quotes a higher bid-ask spread to compensate for his possible loss to the informed traders. The statistically positive coefficients for the lagged variables of the number of trades reveal that the positive effect of the number of trades on the market maker's bid-ask spread is persistent.

Besides this main purpose of study, Part A in Table 4.2 reveals that the volume and its lagged variables have statistically negative coefficients. This tells that the bid-ask spread decreases as the daily volume increases because the market price approaches a better equilibrium price.[6] As in the number of trades, the negative coefficients of the volume's lagged variables show that the volume has a persistent effect on the bid-ask spread.

To see more clearly these conventional descriptions of the relationships among bid-ask spread, number of trades, and volume, we can consider the following approach: first, we estimate regression (4.1) without volume and its lagged variables (see Part B). It suggests that the relationship between bid-ask spread and number of trades is negative with a statistical significance. We then estimate regression equation (4.1) (see Part A). It reveals that the bid-ask spread has a positive relationship with number of trades and a negative relationship with volume. Both are statistically significant. This raises a question.

Why is the relationship between bid-ask spread and number of trades changing? How can we get a negative relationship between bid-ask spread and number of trades in Part B even though we have a positive relationship in Part A? This might be answered by looking at the empirical results of a very highly positive correlation between number of trades and volume. Part A shows that the correlation between number of trades and volume is 0.70. Thus, when we just run a regression of bid-ask spread on the number of trades and its lagged variables, we might have a missing variable effect. The potential effect of volume can be possibly compounded with the number of trades to produce differing results.

[6] If we consider a trade size for an individual transaction, however, the bid-ask spread has a positive relationship with it due to the market maker's protection from asymmetric information.

The negative coefficient tells us that the effect of the volume on the bid-ask spread overwhelms the effect of the number of trades.[7]

IV. 2. 1. 3. An incidental truncation corrected regression

Our study focuses on the price impacts of the information in relatively thick markets. Thus, we remove the stocks with no trades during the period of the analysis from the sample. This may cause *the selection problem - incidental truncation.* In this section, we briefly consider the possibility of this problem in our analyses. However, the difference in data (daily vs. monthly data) between this analysis and the previous one in regression equation (4.1) can provide us with a simple indication about the extent of a possible selection problem in the regression equation (4.1).

Suppose that two random variables, y and z have a bivariate distribution with correlation ρ, and that we are interested in the distribution of y, given that z exceeds a particular value. For our case, the variable, y denotes bid-ask spread and number of trades, z is positive. Intuition suggests that if y and z are positively correlated, the truncation of z could push the distribution of y to the right [see Greene (1993)].

Heckman (1979) suggests a two-step estimation procedure to correct the selection problem. This procedure is similar to *the two-stage least squares* (2SLS) *method*, but is

[7] We can consider the effect from multicollinearity of volume and number of trades. But *heteroskedasticity and autocorrelation t-values* using a Bartlett kernel with an optimal bandwidth as in Andrews (1991) show that coefficient estimates are statistically significant even though the possible effect of multicollinearity will cause higher standard errors. This result is contrasting with Jones, Kaul, and Lipson (1994b). They suggest

different from 2SLS method by using the *probit* model for the regression of the first stage.

Even though our sample may have a selection problem like Heckman's, it is somewhat different since the number of trades variable, z is used in both the first and the second equation. Thus, we use the following recursive system for a path analysis:[8]

$$(\text{Bid - Ask Spreads})_{i,t} = \alpha_0 + \alpha_1(\text{Number of Trades})_{i,t} + \alpha_2(\text{Volume})_{i,t} + \varepsilon_{i,t}\,, \qquad (4.2a)$$

$$(\text{Number of Trades})_{i,t} = \beta_0 + \beta_1(\ln(\text{Past - Price Variability}))_{i,t-1}$$
$$+\ \beta_2(\ln(\text{Market Informedness}))_{i,t-1} + \beta_3(\ln(\text{Firm Size}))_{i,t-1}$$
$$+\ \beta_4(\ln(\text{Volume}))_{i,t} + \varepsilon_{i,t} \qquad (4.2b)$$

In regression equation (4.2a,b), we use 6 variables.[9] Note that we use monthly data to run this recursive system because that is the only data available for the three regressors in the first stage regression, (4.2b). Table 4.3 presents estimates of the regression equation (4.2a,b). It contains the coefficient estimates of number of daily transactions, volume, past-price variability, market informedness, firm size, volume, and adjusted R^2, with their *t-statistics*. Table 4.3 shows that the coefficients of number of trades and volume in equation

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that the number of trades might be the primary determinant in the price adjustment process of a complete market with differential information, or differences in belief.

[8] We do not use lagged variables for each independent variables for the same reasons as in footnote 10 from section IV.2.2.

[9] Here and in regression equations (4.3) and (4.4), we use the natural logarithm for independent variables creating a better model specification.

(4.2a) are positive and negative respectively. These are the same as in regression equation (4.1).

In section IV.2.1.1, we noted that the median (or mean) daily number of trades for our sample is 43 (or 98). This is far from 0, probably indicating that the selection bias may not be large. Considering these two results together, a selection problem may be minimal.

IV. 2. 2. Effects of past-price variability, market informedness, and firm size on number of trades

We investigate the effects of past-price variability, market informedness, and firm size on the numbers of transactions. This will show whether these three variables can affect the number of trades.

The number of trades is affected by investors rather than the market maker. The portion of the informed traders in the market is significantly influential on the number of transactions. Public information, like past-price variability, will influence the investor's decision when to trade. This decision is also related to the number of trades in a given trading period. The firm size as a proxy for the market liquidity impacts the number of transactions; they will trade more frequently in that market situation if the market provides investors with better liquidity.

We control the daily volume as we do in the regression equation (4.1). The structural model for the empirical representation of the relationship among the number of trades, past-price variability, market informedness, and firm size is as follows:[10]

$$\begin{aligned}(\text{Number of Trades})_{i,t} = \beta_0 &+ \beta_1(\ln(\text{Past - Price variability}))_{i,t-1} \\ &+ \beta_2(\ln(\text{Market Informedness}))_{i,t-1} \\ &+ \beta_3(\ln(\text{Firm Size}))_{i,t-1} + \beta_4(\ln(\text{Volume}))_{i,t} + \varepsilon_{i,t}. \qquad (4.3)\end{aligned}$$

Our model predicts that β_1 is either negative or positive depending upon investors' informedness. For uninformed traders, the securities with a small past-price variability are attractive because they are less risky. The informed traders with private information are attracted to securities with a higher past-price variability since the use of that information can increase their return. Based on the results of chapter III, we predict that β_1 may not be statistically significant.

The coefficient β_2 on the market informedness is predicted to be positive. Under a perfect competition among traders, they want to use new information quickly, which leads them to trade a large-size transaction rather than more trades. Thus, the individual informed trader wants to immediately execute a large-size trade. However, when the

[10] Since we use proxy variables, we take the Hausman (1978) specification test for measurement error to build this regression equation. We will mention this later (see section IV.2.2.2). Here, we do not take lagged variables for each independent variables for the following reasons: (i) for the analyses of equations (4.3) and (4.4), we use monthly data while, for the equation (4.1), we use daily data. (ii) When we use monthly data, the effects of the lagged variables on the market maker's pricing appear to be unrealistic. (iii)

individual informed traders become part of a like-minded group, the relationship between number of trades and market informedness turns positive. The mere number of transactions drive the relationship.

We expect the greater is the firm size, the greater is the number of trades, so that $\beta_3 \geq 0$. If the market can have a great capability for any kind of transactions with the least effect on price, investors can trade whatever they want. From the positive daily correlation between number of trades and volume in Table 4.2, we can also expect a positive relationship between two variables.

IV. 2. 2. 1. Descriptive statistics

Table 4.4 reports summary descriptive statistics for the variables, bid-ask spreads, number of trades, standard deviation of bid (or ask) prices (past-price variability), number of institutions holding a firm's shares (market informedness), firm size (firm's market value), and volume. The table contains their average, standard deviation, quintile, skewness, and kurtosis. All are from monthly observations. The second column shows the total number of observations.

The numbers of Table 4.4 indicate similar data properties with Table 4.1. Like the other table, they are also a diverse sampling. For example, the number of transactions varies between 1 and 1,412 per day on a monthly average. The range of average volume is

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We have also run regressions with the lagged variables. But their results are not different form the ones without the lagged variables form a statistical viewpoint.

between 2.3 thousand shares and 4.3 million shares. The firm's market value is from 21.4 thousand dollars to 16.0 million dollars on a monthly average.

IV. 2. 2. 2. The Hausman specification test

We use proxies for three variables -- market maker's prior belief about the true value of an underlying asset, market informedness, and market liquidity. Since a proxy variable by definition contains measurement error, the use of proxy variables leads the estimators to be biased and inconsistent. Thus, we will consider the two-stage least squares (2SLS) estimation using the instrument variables (IV) rather than OLS estimation. However, before using the 2SLS by the IV variables, we must determine which estimation method provides a better specification. For the preliminary estimates, we use the Hausman (1978) specification test.

Hausman has devised a test for the presence of errors of measurement. Under the hypothesis of no measurement error, both $\mathbf{b}$, the least squares estimators, and $\mathbf{b}_{IV}$, the instrumental variables estimator, are consistent estimators of β, although least squares is efficient, while the IV estimator is inefficient. But if the hypothesis is false, only $\mathbf{b}_{IV}$ is consistent. The test, then, examines the difference between $\mathbf{b}$ and $\mathbf{b}_{IV}$. Under a hypothesis of no measurement error, $plim(\mathbf{b} - \mathbf{b}_{IV}) = 0$. If measurement error exists, this *plim* will be non-zero. For the test statistic, he uses *the Hausman m-statistic*, which follows *a chi-square distribution* [see Greene (1993)].

To use the 2SLS by the IV estimation we must find an "instrument" for each regressor that is contemporaneously correlated with the error term. To be this new independent variable, it should have a high correlation with its original regressor variable and also be contemporaneously uncorrelated with the error term. Possible instrumental variables are as follows: (i) any variable in the model that is independent of the errors, (ii) lags of variables in the system, (iii) low degree polynomials in the exogenous variables. For our instrumental variables in this chapter, we use the first-order lagged variables of the regressors: past-price variability, market informedness, and firm size. The *p-value* of the Hausman *m*-statistic, 31%, in Table 4.5, rejects the null hypothesis that the 2SLS is better than the OLS. Therefore, we use the OLS estimation.

IV. 2. 2. 3. Empirical results

Table 4.5 presents estimates of regression equation (4.3), testing whether the three variables -- past-price variability, market informedness, and firm size -- can affect the number of trades using monthly-based data. It shows estimation results of both the OLS and the 2SLS methods. It consists of the coefficient estimates of past-price variability, market informedness, firm size, volume, and adjusted R^2. It also presents the heteroskedasticity and autocorrelation consistent t-statistics using a Bartlett kernel with an optimal bandwidth as in Andrews (1991).

The evidence in Table 4.5 shows that the relationships among variables are generally consistent with our expectations. It also reveals that both the OLS and the 2SLS methods have very similar results. The relationship (β_1) between number of trades and the past-price variability is negative, but statistically insignificant. However, the sign of the coefficient may imply that the effect by uninformed traders is greater than that of informed traders during the sample periods. For the uninformed traders, the securities with a small past-price variability are attractive because they are less risky.

The relationship (β_2) between number of trades and market informedness is positive. Firm size and number of trades show also a statistically significant, positive relationship (β_3). The relationship (β_4) between number of trades and volume is statistically and significantly positive. These results are as expected.

IV. 2. 3. Effects of past-price variability, market informedness, and firm size on bid-ask spread

Regression equation (4.1) shows the relationship between number of trades and bid-ask spread, while the regression equation (4.3) reveals the effects of past-price variability, market informedness, and firm size on the investors' numbers of trades when holding volume constant. From both regression equations (4.1) and (4.3), we can naturally deduce another relationship among bid-ask spread, past-price variability, market informedness, and firm size. This shows the market maker's behavior. The impact of the three variables on the number of trades can be represented in the market maker's bid-ask spread.

We also control average monthly volume as we do in regression equation (4.3). The reduced-form model for the empirical representation of the relationship among the bid-ask spread, past-price variability, market informedness, and firm size are as follows:

$$\begin{aligned}(\text{Bid - Ask Spreads})_{i,t} = \gamma_0 + \gamma_1(\ln(\text{Past - Price Variability}))_{i,t-1} \\ + \gamma_2(\ln(\text{Market Informedness}))_{i,t-1} \\ + \gamma_3(\ln(\text{Firm Size}))_{i,t-1} + \gamma_4(\ln(\text{Volume}))_{i,t} + \varepsilon_{i,t}\,. \qquad (4.4)\end{aligned}$$

Our model predicts that γ_1 is positive for the following reasons: when the price of a security is relatively stable, the market maker can assign a higher probabilistic prior belief to specific values. This leads the market maker to set a relatively smaller spread. However, when the past-price variability of a security is relatively large, it is difficult for the market maker to conjecture the security's real value. This leads the market maker to set a relatively larger spread. However, based on the results of chapter III, we predict that γ_1 may not be statistically significant.

The coefficient γ_2 on the market informedness is predicted to be positive since the market maker sets up a larger bid-ask spread against *adverse-selection* and its price concession.[11]

[11] Price concessions imply the expected price impact of the trades incurred by the trader in exchange for immediate order execution. Price concessions consist of a temporary component as compensation for the specialist or other intermediary for providing liquidity, and a permanent component reflecting any new information revealed by the trade.

We expect that the sign of γ_3 is either negative or positive. Since firm size is used as a proxy for market liquidity, a larger firm size represents a more liquid market for that security. Thus, the bid-ask spread becomes smaller as the market becomes more liquid. However, it does not always happen in this way. As O'Hara (1995) puts it, the market maker's viewing aggregated order flow, rather than facing individual orders, may provide a very different view of liquidity. If price varies with the trade size, the spread for large traders may be significantly larger than the small trade spread. The larger trade size may happen more frequently in the larger-size company's securities. In this case, the larger firm size may bring about larger bid-ask spreads. How does one compare the liquidity of markets?

The coefficient of γ_4 is predicted to be negative since the market price approaches a better equilibrium price as the daily volume increases.

IV. 2. 3. 1. Empirical results

Before estimating regression equation (4.4), we do the Hausman specification test for measurement error as we do for regression equation (4.3). The *p-value* of the Hausman *m*-statistic, 30%, in Table 4.6, rejects the null hypothesis that the 2SLS is better than the OLS. Therefore, we use the OLS estimation.

Table 4.6 presents estimates of regression equation (4.4), testing whether the three variables -- past-price variability, market informedness, and firm size -- can affect the bid-ask spread using monthly-based data. It contains estimates of the coefficient of past-price variability, market informedness, firm size, volume, and adjusted R^2. It also presents the heteroskedasticity and autocorrelation consistent t-statistics using a Bartlett kernel with an optimal bandwidth as in Andrews (1991).

The evidence in Table 4.6 is relatively less clear for the relationships among the past-price variability, the market informedness, firm size with the bid-ask spread. The relationship (γ_2) between bid-ask spread and market informedness is positive, but statistically insignificant. However, the sign of the coefficient may confirm our expectation that the market maker sets up a larger bid-ask spread against adverse-selection and its price concession. Firm size and bid-ask spread show a statistically significant negative relationship (γ_3). The negative sign may show that the bid-ask spread becomes smaller in a more liquid market. We expected that the relationship between bid-ask spread and past-price variability (γ_1) is positive, but not statistically significant. The negative coefficient for γ_1, though it is insignificant, is different from our expectation. This may imply that the past-price variability does not significantly impact the market maker's prior belief. Since the interpretation of its sign is so confusing, further investigation is needed.[12]

[12] We can possibly consider a structural shift for the coefficient on the past-price variability.

	No. of Obs.	Bid-Ask Spread (in dollars)	Number of Trades	Volume
	240,890			
Average		0.3475	97.8	186,922.9
Std. Dev.		0.2144	192.8	401,538.4
Maximum		13.0000	9,454.0	26,841,073.0
Q3		0.5000	95.0	185,325.0
Median		0.2500	43.0	72,075.0
Q1		0.2500	21.0	26,951.0
Minimum		0.0313	1.0	100.0
Skewness		2.6448	8.6	10.9
Kurtosis		57.7132	159.0	293.1

This table contains the average, standard deviation, quintile, skewness, and kurtosis for the variables: bid-ask spreads, number of trades, and volume. The descriptive statistics are based on daily data for the entire sample period and are reported for all 240,890 securities that have a consecutive trading between January 1990 and December 1992. The individual-firm statistics are averaged across firms. The second column reports the total number of observations.

Table 4. 1 Daily descriptive statistics of the data for NASDAQ-NMS securities used in the regression of bid-ask spread on number of trades, volume, and their lagged variables, 1990-1992

Part A: Regression of daily bid-ask spread on daily number of trades, volume, and their lagged variables together

	Parameter Estimates		
	Estimates	*t-values*	$\overline{R}^2$
			0.032
α_0	0.3646	146.63*	
α_1	0.00004075	5.69*	
α_2	-0.00000005	-13.87*	
α_3	0.00003947	4.82*	
α_4	0.00004099	8.76*	
α_5	0.00005065	10.01*	
α_6	0.00004513	8.34*	
α_7	0.00003829	5.97*	
α_8	-0.00000003	-7.20*	
α_9	-0.00000003	-12.47*	
α_{10}	-0.00000003	-14.17*	
α_{11}	-0.00000003	-13.94*	
α_{12}	-0.00000003	-13.56*	

Chi-square statistic: 4,763.99 (0.0000)
Correlation between number of trades and volume: 0.70

This table contains estimates of the following regression:

$$(\text{Bid - Ask Spread})_{i,t} = \alpha_0 + \alpha_1(\text{Number of Trades})_{i,t} + \alpha_2(\text{Volume})_{i,t} + \sum_{j=3}^{7}\alpha_j(\text{Number of Trades})_{i,t+2-j} + \sum_{j=8}^{12}\alpha_j(\text{Volume})_{i,t+7-j} + \varepsilon_{i,t}$$

where i and t denote a security and day, respectively. It contains the coefficient estimates of number of daily transactions (α_1), volume (α_2), five lagged variables of number of trades (α_3-α_7), five lagged variables of volume (α_8-α_{12}), and adjusted R^2, with their *t-statistics*. The *t-statistics* are heteroskedasticity and autocorrelation consistent estimators using a Bartlett kernel with an optimal bandwidth as in Andrews (1991). * denotes significance at the 0.05 level. We estimate the regression using GMM. *Chi-square statistic* comes from the model specification test with the *p-value* in parenthesis.

Table 4. 2 Regression results of daily bid-ask spread on daily number of trades, volume, and their lagged variables for NASDAQ-NMS securities, 1990-1992

Part B: Regression of daily bid-ask spread on daily number of trades and its lagged variables alone

	Parameter Estimates		
	Estimates	*t-values*	$\overline{R}^2$
			0.014
α_0	0.3617	146.17*	
α_1	-0.0000682	-15.25*	
α_2	-0.00001032	-3.69*	
α_3	-0.00001491	-5.28*	
α_4	-0.00001052	-3.73*	
α_5	-0.00001634	-6.10*	
α_6	-0.00002443	-7.23*	
Chi-square statistic: 3,643.25 (0.0000)			

This table contains estimates of the following regression:

$$(\text{Bid - Ask Spread})_{i,t} = \alpha_0 + \alpha_1(\text{Number of Trades})_{i,t} + \sum_{j=2}^{6} \alpha_j(\text{Number of Trades})_{i,t+1-j} + \varepsilon_{i,t}$$

where i and t denote a security and day, respectively. It contains the coefficient estimates of number of daily transactions (α_1), five lagged variables of number of trades (α_2-α_6), and adjusted R^2, with their *t-statistics*. The *t-statistics* are heteroskedasticity and autocorrelation consistent estimators using a Bartlett kernel with an optimal bandwidth as in Andrews (1991). * denotes significance at the 0.05 level. We estimate the regression using GMM. *Chi-square statistic* comes from the model specification test with the *p-value* in parenthesis

Table 4. 2 Regression results of daily bid-ask spread on daily number of trades, volume, and their lagged variables for NASDAQ-NMS securities, 1990-1992 - *Continued*

	Parameter Estimates: Estimates	t-values	$\bar{R}^2$
Equation (a)			0.010
Equation (b)			0.142
α_0	0.6768	18.61*	
α_1	0.000057	0.64	
α_2	-0.278260	-8.66*	
β_0	-152.5332	-22.34*	
β_1	-0.105609	-0.21	
β_2	5.986755	4.88*	
β_3	1.684415	2.25*	
β_4	10.691193	25.42*	

Chi-sqaure statistic for the equation (a): 9.73 (0.0832)
Chi-square statistic for the equation (b): 78.38 (0.0000)

This table contains estimates of the following regression:

$$(\text{Bid - Ask Spreads})_{i,t} = \alpha_0 + \alpha_1(\text{Number of Trades})_{i,t} + \alpha_2(\text{Volume})_{i,t} + \varepsilon_{i,t}\,, \quad \text{(a)}$$

$$(\text{Number of trades})_{i,t} = \beta_0 + \beta_1(\ln(\text{Past - Price variability}))_{i,t-1} + \beta_2(\ln(\text{Market Informedness}))_{i,t-1} + \beta_3(\ln(\text{Firm Size}))_{i,t-1} + \beta_4(\ln(\text{Volume}))_{i,t} + \varepsilon_{i,t}\,, \quad \text{(b)}$$

where i and t denote a security and day, respectively. We estimate the regression using 2SLS. It contains the coefficient estimates of number of daily transactions (α_1), volume (α_2), past-price variability (β_1), market informedness (β_2), firm size (β_3), and adjusted R^2, with their t-statistics in parentheses. * denotes significance at the 0.05 level. *Chi-square statistics* come from the model specification tests for the equations (a) and (b) with each *p-value* in parenthesis

Table 4. 3 An incidental truncation corrected regression:
Regression results of daily bid-ask spread on daily number of trades, volume, and their lagged variables for NASDAQ-NMS securities, 1990-1992

	No. of Obs.	Bid-Ask Spread (in dollars)	Number of Trades	Volume
	11,194			
Average		0.06	15.33	181,160.7
Std. Dev.		1.51	42.61	299,125.7
Maximum		9.38	1,411.77	4,245,470.0
Q3		0.44	9.36	192,575.1
Median		0.28	6.05	84,224.8
Q1		0.19	2.90	40,073.4
Minimum		-9.86	1.00	2,336.6
Skewness		-5.26	19.16	4.6
Kurtosis		33.92	531.89	29.6

	Past-Price Variability (Std. Dev. of Bid Prices)	Market Informedness (Number of Institutions Holding a Firm's Shares)	Firm Size (Firm's Market Value)
Average	0.49	96.1	507,702.7
Std. Dev.	0.31	87.7	962,565.1
Maximum	9.78	833.0	15,989,643.0
Q3	0.74	118.0	104,340.9
Median	0.49	73.0	216,381.8
Q1	0.24	43.0	104,037.5
Minimum	0.00	3.0	21,416.3
Skewness	3.85	3.2	5.9
Kurtosis	98.14	15.1	47.6

** Correlation between past-price variability (std. dev. of bid prices) and market informedness (number of institutions holding a firm's shares): -0.10

This contains the average, standard deviation, quintile, skewness, and kurtosis for the variables: bid-ask spreads (in dollars), number of trades, volume, standard deviations of bid (or ask) prices (past-price variability), institutions holding a firm's shares (market informedness), and firm size (firm's market value) (in dollars). The descriptive statistics are based on monthly data for the entire sample period and are reported for all 11,194 securities that have a consecutive trading between January 1990 and December 1992. The individual-firm statistics are averaged across firms. The second column reports the total number of observations.

Table 4. 4 Monthly descriptive statistics for the variables -- past-price variability, market informedness, and firm size -- used in the regressions of number of trades (or bid-ask spread), 1990-1992

	OLS Estimates			2SLS Estimates		
	Estimates	*t-values*	R^2	Estimates	*t-values*	R^2
			0.146			0.146
β_0	-154.35	-26.07*		-153.84	-8.44*	
β_1	-0.04	-0.09		-0.04	-0.12	
β_2	3.57	3.53*		3.97	2.77*	
β_3	2.38	3.82*		2.39	2.18*	
β_4	10.99	29.27*		10.81	12.24*	

Chi-square statistic: 88.81 (0.0000)
Hausman *m-statistics*: 5.97 (0.31)
Degrees of freedom: 5

This table contains estimates of the following regression:

$$(\text{Number of trades})_{i,t} = \beta_0 + \beta_1(\ln(\text{Past - Price Variability}))_{i,t-1} + \beta_2(\ln(\text{Market Informedness}))_{i,t-1} + \beta_3(\ln(\text{Firm Size}))_{i,t-1} + \beta_4(\ln(\text{Volume}))_{i,t} + \varepsilon_{i,t}.$$

where i and t denote a security and day, respectively. We estimate the regression using OLS and 2SLS for Hausman specification test. It consists of the coefficient estimates of past-price variability (β_1), market informedness(β_2), firm size(β_3), volume(β_4), and adjusted R^2, with their *t-statistics*. The number in the parenthesis of the *Hausman m-statistic* denotes the *p-value* for the 2SLS to be better than the OLS. The *t-statistics* are heteroskedasticity and autocorrelation consistent estimators using a Bartlett kernel with an optimal bandwidth as in Andrews (1991). * denotes significance at the 0.05 level. We estimate the regression using GMM. *Chi-square statistic* comes from the model specification test with the *p-value* in parenthesis

Table 4. 5 Regression results of monthly average number of trades on monthly past-price variability, monthly market informedness, and monthly firm value for NASDAQ-NMS securities, 1990-1992, when the monthly trading volume is held constant

	OLS Estimates			2SLS Estimates		
	Estimates	*t-values*	R^2	Estimates	*t-values*	R^2
			0.005			0.004
γ_0	0.512	2.40*		0.531	2.55*	
γ_1	-0.005	-0.32		-0.005	-0.28	
γ_2	0.049	1.37		0.053	1.55	
γ_3	-0.044	-1.95*		-0.047	-1.77	
γ_4	-0.011	-0.84		-0.011	-0.74	

Chi-square statistic: 42.77 (0.0001)
Hausman *m-statistics*: 5.62 (0.30)
Degrees of freedom: 5

This table contains estimates of the following regression:

$$\text{(Bid - Ask Spreads)}_{i,t} = \gamma_0 + \gamma_1(\ln(\text{Past - Price Variability}))_{i,t-1} + \gamma_2(\ln(\text{Market Informedness}))_{i,t-1}$$
$$+ \gamma_3(\ln(\text{Firm Size}))_{i,t-1} + \gamma_4(\ln(\text{Volume}))_{i,t} + \varepsilon_{i,t}.$$

where i and t denote a security and day, respectively. We estimate the regression using OLS and 2SLS for Hausman specification test. It consists of the coefficient estimates of past-price variability (γ_1), market informedness(γ_2), firm size(γ_3), volume(γ_4), and adjusted R^2, with their *t-statistics*. The number in the parenthesis of the Hausman m-statistic denotes the p-value for the 2SLS to be better than the OLS. The *t-statistics* are heteroskedasticity and autocorrelation consistent estimators using a Bartlett kernel with an optimal bandwidth as in Andrews (1991). * denotes significance at the 0.05 level. We estimate the regression using GMM. *Chi-square statistic* comes from the model specification test with the *p-value* in parenthesis

Table 4. 6 Regression results of monthly bid-ask spread on monthly past-price variability, monthly market informedness, and monthly firm value for NASDAQ-NMS securities, 1990-1992, when the monthly trading volume is held constant

Chapter V. Measurement Error versus Traders' Strategic Behavior - Interpretation of an AR(1) Stock Return Volatility -

Empirical research shows that volatility is caused primarily by private information revealed through trading.[1] When French and Roll (1986) look into the puzzling difference in volatility between exchange trading hours and non-trading hours, they find that the behavior of returns around exchange holidays suggests that private information is the principle factor behind high trading-time variances. Barclay, Litzenberger, and Warner (1990) add the role of "volume" to the research of French and Roll (1986). They take samples from *the Tokyo Stock Exchange* to examine the determinants of stock returns variances and conclude that private information revealed through trading increases variance. There is additional research on this subject shedding more light on connections between private information and stock return volatility.

Barclay and Warner (1993) find that most of the cumulative stock-price change is due to medium-size trades in NYSE firms and interpret this as evidence that the price movements are due to private information from informed traders' stealth trading. The prevalence of the market microstructure perspective in recent finance research strengthens the theoretical and empirical result that price movements are caused primarily by the arrival of new information, mainly private information, and by the process that incorporates this information into market prices.[2]

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[1] There are many ways to measure stock market volatility. The most common is to use the conditional variance of stock returns, as done in this paper.

[2] There are some exceptions in which the public information is the major source of short-term returns volatility. For this, see Jones, Kaul, and Lipson (1994a).

Based on the effect of new information on the price movement, there is much research concerning stock return volatility, with respect to volume.[3] For any theoretical model considered, the basic premise of the research is that volume is the market statistic which is complementary in explaining the true value of the underlying security.

However, the results of the model in chapter III of this dissertation, based on the private information, show that volume may not be the only variable to reveal private information about the value of the underlying security. Chapter III states that, in a given trading period, the past-price variability of a security, market informedness, and the elasticity of informed traders' demand and supply schedules are related with both the number of trades and the bid-ask spread. Regression analyses in chapter IV reveal the following relationship dealing with this issue: (i) there exists a statistically significant relationship between the number of trades and market informedness even though the relationship between the bid-ask spread and market informedness is not statistically significant, and (ii) there is a positive relationship between bid-ask spread and number of trades when controlling for volume. From these relationships we may infer that market informedness influences the number of daily transactions, then indirectly the market maker's bid-ask spreads.[4] Thus, the number of trades combined with the market informedness and traders' strategic behavior might also be related with the stock return volatility.

Chapter V, after considering the stochastic volatility, examines first whether the stock return has a positive relationship with the number of trades. If it is positively related,

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[3] See Section II. 5. 1 in this dissertation.

then more transactions, given a daily fixed trading period, can be implicitly related with better information. Simply put, if someone has better information they can get a greater return. In this situation the better information would be that contained within the number of trades.

The second objective of this chapter is associated with the *non-synchronous trading effect* of the observed stock returns. This arises when time-series of stock prices are mistakenly considered to be measured at evenly spaced time-intervals, when in fact they are not. It is well known that this non-synchronous trading effect induces a serial correlation in stock returns, a stationary AR(1) process.[5]

However, if non-sychronicity is purposeful and informationally motivated, then the serial dependence it induces in stock returns should be considered to be genuine. It is the result of economic forces rather than measurement error. In such cases, purely statistical models of non-trading are clearly inappropriate and an economic model of strategic interactions is needed [see Campbell, Lo, and MacKinlay (1997)]. This is the motivation behind this pursuit. After considering the stochastic volatility, we compare stock returns either with a number of trades variable or with an AR(1) process. The former is associated with the informed traders' strategic behavior as in chapter III.

There is very little theoretical and empirical research relating to the number of trades, while theoretical and empirical research on the relationship between volume and volatility has been well documented. Since theoretical results about volume and its role in

[4] Empirically, bid-ask spreads have been found to be positively correlated with volatility [see Kothare and Laux (1995)].
[5] Bid-ask bounce and price discreteness can also lead to measurement errors and an autocorrelation in returns.

stock price movements are already reviewed in chapter II, their empirical results will be briefly described first, formulating ideas about the relationship between the number of trades and volatility. The empirical research related to the number of trades will then be studied. Moreover, since *the Exponential Generalized Autoregressive Conditional Heteroskedasticity* (E-GARCH) model is used for the stock market volatility in this chapter, the family of *the Autoregressive Conditional Heteroskedasticity* (ARCH) models are briefly summarized. Finally, we examine the relationship between stock returns and "number of trades". We then look at the behavior of stock returns with an AR(1) process, after considering their stochastic characteristics of its volatility.

V. 1. Empirical Studies on the Relationship between Volume and Volatility

There is an adage which explains very concisely the relationship between volume and stock price volatility: it takes volume to move prices.[6] Interest in this relationship has become part of the mainstream studies about market microstructure. It is assumed in this research that the volume (trade size) contains information about the true value of the underlying asset which the price cannot fully reveal by itself.

There are three approaches to specify the relationship between the daily trading volume and stock return volatility. One approach is *the two-step procedure*. It first estimates one regression equation to obtain the residuals. Then it uses the absolute residuals to estimate another regression equation for the relationship between the volatility and

[6] Here, as in chapter IV, volume is defined as the total shares (or dollar amounts) of stocks traded in a day [see footnote 1 in chapter IV].

volume. We can also use the autoregressive conditional heteroskedasticity (ARCH) family model. This approach notices that large and small residuals for time-series financial data tend to come in clusters, suggesting that the variance of an error may depend on the size of the preceding error. The ARCH family model not only considers this volatility clustering, but also conveniently makes variants of this model to adjust to other specific financial phenomena. Related with trading volume, the ARCH family of models take the volume variable either in the mean equation or in the variance equation. A final approach is associated with the *Mixture of Distribution Hypothesis* (MDH) which posits a joint dependence of stock returns and volume on an underlying latent event or information flow variable. The emphasis on a latent driving process separates the approach from an ARCH modeling strategy and points more towards a stochastic volatility representation.[7]

The empirical stylized facts about the relationship between volume and the stock price movements are as follows: (i) the correlation between volume and the absolute value of price change is positive in both equity and futures markets, (ii) The correlation between volume and the price changes is positive in equity markets, (iii) Expected trading volume is conditionally heteroskedastic, (iv) The autocorrelation of volume is positive, and (v) Volume is relatively heavy in bull markets and light in bear markets. These are noteworthy since they give insight into empirical research related to the number of trades in the security price.

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[7] For literature related with volume and stock volatility by the two-step procedure, see Jones, Kaul, and Lipson (1994b). For the ARCH model, see Lamoureux and Lastrapes (1990). For the MDH model, see

V. 2. Empirical Studies Related to Number of Trades

There are very few studies about the role of the number of trades in the stock price adjustment. Jones, Kaul, and Lipson (1994b) are the first researchers who draw our attention to the potential significance of the role of the number of trades. They use daily data of NASDAQ-NMS securities over the 1986-1991 period to investigate the relationship between volatility, volume, and the frequency (number) of trades.

To measure daily stock return volatility, they take a two-step procedure which is similar to the one in Schwert (1990). They first estimate the regression equation in which the stock return is a dependent variable and the 12 lagged returns are regressors. Then they use Ordinary Least Squares (OLS) to estimate the second regression equation in which the absolute residuals of the first regression equation become the dependent variable while volume, and frequency (number) of trades are independent variables.

They use this method to reflect an empirical regularity in finance: the stock returns are fat-tailed – leptokurtosis. Even though the time-varying variances of GARCH models reduce this leptokurtosis, they do not eliminate it.[8] Large events are still a concern, since the standard GARCH model computes the next period's variances by squaring the current period's shock, producing dramatic increases in variances. To alleviate this large events

Gallant, Rossi, and Tauchen (1992), Foster and Viswanathan (1995) and Andersen (1996).

[8] GARCH models are simply generalized ARCH models. The generalization is found in the assumption that the conditional variances are not only linked by squared innovations but also by past conditional variances. The extant GARCH models mostly focus on another empirical regularity in finance - asymmetry, which we will refer to in a later section. Thus far, the GARCH models themselves do not display obvious links between the leptokurtosis and the asymmetry in stock return. Some studies try to provide a unifying framework [see Nelson and Foster (1994)].

problem, i.e. an outliners problem, the two-step regression procedure specifies the conditional standard deviation as a moving average of lagged absolute residuals.

Jones, Kaul, and Lipson (1994) argue that it is the occurrence of transactions *per se*, and not their size, that generates volatility in the stock market. Trade size (volume) has no information beyond that contained in the frequency of transactions. However, as Engle (1990) points out, their two-step procedure may have some shortcomings. Analyzing by both too many orders of autoregression and a heteroskedastic error standard deviation together apparently bring about a lot of noise in the coefficients. In addition, this procedure cannot quantify the leverage effect which we will describe later.

V. 3. ARCH Family Models: ARCH and E-GARCH

It is well recognized that stock returns exhibit *volatility clustering*. Volatility clustering implies that big surprises in either direction will increase the probability of future volatility. Thus, in order to improve one's ability to forecast in time-series analysis, we should note the simple observation that forecasts, conditional on recent information, are more efficient than those which do not use this information. When dealing with this phenomenon in financial markets, the ARCH family of models have an advantage because they take into considerations the fact that forecasts of the variance at some future point in time can be improved by using recent information.

V. 3. 1. An ARCH model

A version of Engle's (1982) original ARCH(m) model is as follows: Suppose that an autoregressive process of order p [AR(p)] for an observed variable takes the form

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots\dots\dots + \phi_p y_{t-p} + u_t \ , \tag{5.1}$$

where u_t is white noise:

$$E(u_t) = 0 \tag{5.2}$$

$$E(u_t u_\tau) = \begin{cases} \sigma^2 & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases} \ . \tag{5.3}$$

In the ARCH (m) model, the conditional variance of u_t could change over time even though the unconditional variance of u_t is the constant σ^2. For this case, u_t follows an AR(m) process:

$$u_t = \sqrt{h_t} v_t \ , \tag{5.4}$$

where $h_t = \zeta + \alpha_1 u_{t-1}^2 + \dots\dots\dots\dots + \alpha_m u_{t-m}^2$ and $\{v_t\}$ is an i. i. d. sequence with zero mean and unit variance:

$$E(v_t)=0, \text{ and } E(v_t^2)=1. \tag{5.5}$$

V. 3. 2. An Exponential GARCH (E-GARCH) model

Engle's (1982) and later Bollerslev's (1986) original ARCH family of models have been fine-tuned extensively. The catalyst for the creation of the vast majority of these is for one reason. The United States equity returns show a strong asymmetry: negative returns are followed by larger increases in volatility than equally large positive returns.[9] The implications of such evidence are interrelated. Current negative surprises in the stock market raise *the debt-to-equity ratio*. This would begin a chain reaction, first raising the firm's riskiness, which in turn increases future volatility. It is termed *the leverage effect*.

Even though Black (1986) explains the asymmetry of stock return volatility by this leverage effect, he also recognizes its limitations. While a good theoretical description for this asymmetry is absent, the quest of many econometricians is to build a model that includes this asymmetry of stock return volatility.

In his seminal paper, Nelson (1991) notes that the linear generalized ARCH (GARCH) models fail to capture this kind of dynamic pattern. Thus, he considers the asymmetric influence between the positive and negative shocks on stock return volatility. His proposal to solve this dilemma is to model the evolution of the conditional variance of u_t as follows:

$$\log(h_t) = \zeta + \sum_{i=1}^{q} \alpha_i g(z_{t-i}) + \sum_{j=1}^{p} \gamma_j \log(h_{t-j})$$

$$g(z_t) = \theta z_t + \gamma[|z_t| - E|z_t|]$$

$$z_t = {u_t}/{\sqrt{h_t}} \qquad (5.6)$$

This model reflects not only statistical properties of the standard GARCH model, but also the leverage effect included in $g(z_t)$ function. Since the coefficient of the second term in $g(z_t)$ is set to be 1 ($\gamma = 1$) in our formulation, we should be careful to interpret θ. Suppose that $\theta < 1$. The innovation in variance, $g(z_t)$, is positive if the innovations z_t are less than $(2/\pi)^{1/2} / (\theta - 1)$.[10] Therefore, the negative innovations in returns, u_t, cause the innovation to the conditional variance to be positive if θ is positive and is much less than 1 -- let alone negativity.

There are other ARCH family models which have similar features with the E-GARCH model: *the threshold ARCH model* of Rabemananjara and Zakoian (1993), *the double-threshold ARCH model* of Li and Li (1996), the model of Glosten, Jagannathan, and

[9] See Christie (1982); French, Schwert, and Stambaugh (1987); Pagan and Schwert (1990); Schwert (1990); Nelson (1991); Bollerslev, Chou, and Kroner (1992); Campbell and Hentschel (1992); Hamilton (1994).

Runkle (1993), and *the Sign - and Volatility - Switching ARCH models* of Fornari and Mele (1997). All such models include the sign of past forecast errors as conditioning information for the current values of the conditional variance. This is the same as the E-GARCH model. In addition, they attempt to derive the asymptotic properties, useful in the estimation of continuous-time models in finance. They all have various advantages and disadvantages, generally making none more useful than another.[11]

Related with the asymmetric stock return volatility, Engle and Lee (1993), using *A Permanent and Transitory Component Model for the Conditional Variances*, provide a very intriguing result. They show that negative shocks predict higher volatility than positive shocks, but the effect is mainly temporary. This result implies *a mean reversion* of the stock return volatility. That is, a temporary asymmetric volatility exists in the stock market, while this reverts to a symmetric volatility, the trend, in the long run.

V. 4. Number of Trades, Measurement Error, and Stock Return Volatility

Now we examine our main issues. (i) Does the stock returns have a positive relationship with the number of trades after considering its stochastic characteristic of its volatility? (ii) Can the established time-series property of observed returns, (AR(1) process), come from

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[10] Note that $E|z_t| = (2/\pi)^{1/2}$ if $z_t \sim N(0,1)$. Substitute this, and $\gamma = 1$ into $g(z_t)$ in equation (5.6). For $g(z_t) = \theta z_t + \left[|z_t| - (2/\pi)^{1/2}\right] > 0$ given $\theta < 1$, $\theta z_t + |z_t| > (2/\pi)^{1/2}$. If $z_t > 0$ then $z_t > (2/\pi)^{1/2}/(\theta+1)$. If $z_t < 0$ then $z_t < (2/\pi)^{1/2}/(\theta-1)$. The negative innovations in returns, u_t, makes z_t negative.

the traders' strategical behavior rather than from simple measurement error in calculating the returns? For these, we first briefly look into the theoretical background, and then present the empirical model and its results.

V. 4. 1. Traders' strategic behavior: theoretical arguments about the number of trades' informational role as a source of uncertainty

For the relationship between number of trades and stock return volatility, Chapters II and III describe the theoretical background. In chapter II, *noisy rational expectations equilibrium* (NREE) models present the possibility that the number of trades can be a complementary market statistic since the market price cannot fully reveal the value of the underlying asset. Chapter III develops a sequential trade model which shows directly the important role of number of trades, when compared to that of volume, in a stock price adjustment process. To get some idea of the number of trades' role in a stock price adjustment process, we briefly describe the NREE model.

In an NREE model, traders conjecture that the equilibrium price system is the following linear function of private information, $\sum \tilde{y}_i$ and supply uncertainty, $\tilde{S}$:

$$p^* = \beta_0 + \beta_1 \sum \tilde{y}_i - \beta_2 \tilde{S} \quad . \qquad (5.7)$$

[11] Nelson (1990) also shows that the GARCH (1,1) model of Bollerslev (1986) and the AR(1) - E-GARCH model of Nelson (1991) approach continuous-time AR(1) processes, as the length of the sample frequency approaches zero.

Given this conjectured price functional, we can obtain the individual's optimal demand function conditional on a realized price, his private signal, and the uncertainty of the (aggregate) supply of the risky asset. After the market clearing, we can get the same equilibrium price, $p^* = \beta_0 + \beta_1 \sum \tilde{y}_i - \beta_2 \tilde{S}$, as the traders conjecture under the uniquely solved values for β_0, β_1, and β_2. Therefore, we have a NREE where there is no one-to-one relationship between the price of the risky asset and the sufficient statistic of traders' private information, $\bar{y}$. Price just partially reveals the private signals. [12]

Many researchers have tried to find the source of this model's uncertainty from volume. However, the sources of the uncertainty of the market are numerous. We can substitute the number of trades for this volume even in this NREE model.[13] When number of trades is considered as the source of uncertainty, if the stock returns has a positive relationship with the number of trades, then more transactions are implicitly related with better information. If someone has better information they can get a greater return. In this situation the better information would be that contained within the number of trades. This will give us an insight for the possibility that an AR(1) stock returns may not come from the measurement error but from the informed traders' strategic behavior.

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[12] See chapter II of this dissertation for the evolution of NREE models and their applications to trading volume.

[13] My dissertation belongs to this line of research, even though it does take a different model. For the research which refers the importance of the number of trades, see chapter III of this dissertation.

V. 4. 2. Interpretation of an AR(1) stock returns process: measurement error or traders' strategic behavior

The non-synchronous trading effect is one of the well-established time-series properties of observed stock returns [see Fisher (1966) and Lo and MacKinlay (1989) amongst others]. Up to this point it is considered as given that this arises from the measurement errors in calculating stock returns. For example, the daily prices of securities quoted in the financial press are usually closing prices. However, even though the last transaction of each price usually does not occur at the same time each day, we use them as daily prices. Thus, we have implicitly and incorrectly assumed that they are recorded at equal time intervals. Such an assumption can create a false impression of predictability in price changes and returns even if true price changes or returns are statistically independent. One of the implications of this non-synchronous trading effect is that the stock returns may follow a stationary AR(1) process.[14] This has become a truism widely accepted by the vast majority of researchers.

The premise of the extensive literature on non-synchronous trading is that non-trading is an outcome of institutional features such as lagged adjustments and non-synchronously reported prices.[15] However, if non-sychronicity is purposeful and informationally motivated, then this serial dependence it induces in stock returns should be considered genuine since it is the result of economic forces, rather than measurement error.

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[14] According to the general results about this effect, individual security returns may be positively autocorrelated and portfolio returns may be negatively autocorrelated, but these possibilities are unlikely, given empirically relevant parameter values. In addition, the effects of non-synchronous trading are more apparent in securities grouped by non-trading probabilities than in individual stocks [see Campbell, Lo, and MacKinlay (1997)].

[15] We can also consider *a mean reversion* of the stock returns with respect to the non-trading. According to Lo and MacKinlay (1990), for the non-trading process, the individual stock returns show the negative serial correlation. This negative serial correlation in individual stock returns can be explained by the fact that, during

In such cases, purely statistical models of non-trading are clearly inappropriate and an economic model of strategic interactions is needed [see Campbell, Lo, and MacKinlay (1997)].

Comparing stock return volatility between an AR(1) stock return and one with the number of trades variable can be the first step toward looking into the possibility that the serial dependence of stock returns may not come from simple measurement error, but from the traders' strategic behavior.

Another interpretation that relates to traders' strategic behavior based on information is possible.[16] The small firms generally are traded less frequently than larger firms. If there exists private market information which can influence both the large and the small firms, we can infer that the information pervasiveness for the larger firms is better than that for the smaller ones simply because there are more transactions. This better information pervasiveness of larger firms can be disseminated (or spill over) to the small firms. In this case, the price adjustment for the private market information and the informational dissemination (or spill-over) to the smaller firms through more transactions can be reflected in the mean equation of the stock returns by using the number of trades variable.

For the formal specification of this relationship between number of trades and stock returns, we can presume that returns are governed by a one-factor linear model:

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non-trading periods, the observed return is zero, and during trading periods, the observed return reverts back to its cumulated mean return. This mean reversion yields a negative serial correlation in this case.

$$R_{t,i} = a + bf_t + u_{t,i} \qquad (5.8)$$

where f_t is a zero-mean common factor and $u_{t,i}$ is zero-mean idiosyncratic noise that is temporally and cross-sectionally independent. Our focus is on number of trades as the only source of autocorrelation. For our purposes, the common factor f_t is i.i.d. and is independent of $u_{t,i}$ for all t and i. The return of each period is random. It retrieves movements precipitated by both information arrival and idiosyncratic noise.

V. 4. 3. The model specification

To estimate the relationship between stock returns and number of trades we estimate the following ARMA(p,q) exponential ARCH model (E-GARCH model) combined with return equation (5.8).[17] For the analysis of the AR(1) stock returns, we just substitute $R_{t-1,i}$ for the number of trades in the same equation:

$$R_{t,i} = a + bX_{t,i} + ch_{t,i} + u_{t,i}$$

$$\log(h_{t,i}) = \zeta + \sum_{j=1}^{q} \gamma_{j,i} \log(h_{t-j,i}) + \sum_{k=1}^{p} \alpha_{k,i} g(z_{t-k,i})$$

$$g(z_{t,i}) = \theta z_{t,i} + \gamma\left[\left|z_{t,i}\right| - E\left|z_{t,i}\right|\right]$$

[16] We consider the private market information which influences on the value of both large and small firms. Even though we could consider the public market information, we simply restrict our concern into the private market information for consistency.

[17] In ARMA(p,q), p denotes the order for autoregressive process while q denotes the order for the moving average process.

$$z_{t,i} = {u_{t,i}} \Big/ {\sqrt{h_{t,i}}} \ , \qquad (5.9)$$

where $R_{t,i}$ is the daily stock returns. We calculate them from bid returns, $(P_{b,t}-P_{b,t-1})/P_{b,t-1}$ where $P_{b,t}$ and $P_{b,t-1}$ are the bid prices for t (present) and t-1 (previous) periods respectively. We use this method in order to avoid negative serial correlation as the result of bid-ask bounce. $X_{t,i}$ denotes the number of trades or $R_{t-1,i}$ during t period for firm i, depending on the analyses.

Using the ARCH family model allows our analysis to capture persistent return volatility with parsimonious parameters. Moreover, an E-GARCH model allows previous returns to affect future volatility differently depending on their signs. This is clearly seen in the g function above. In equation (5.9), if θ is negative (or slightly positive in our formulation), then it reflects the leverage effect. As Nelson (1991) puts it, the theoretical justification for including the $h_{t,i}$ term in (5.9) is meager, since the required return on a portfolio is linear in its conditional variance only under very special circumstances.[18] Rather, the justification for including $h_{t,i}$ is pragmatic: a number of researchers using GARCH models have found a statistically significant positive relationship between conditional variance and returns on stock market indices, therefore, we adopt the form (5.9).

For our empirical analysis, we use daily returns for 317 firms from CRSP tapes. The data for the number of trades of each firm is taken from NASDAQ-NMS file of the

[18] In Merton's (1973) intertemporal CAPM model, for example, the instantaneous expected excess return on the market portfolio is linear in its conditional variance if there is a representative agent with a log utility. Adding $h_{t,i}$ allows us to partly consider the changing conditional mean in the E-GARCH model. Glosten, Jagannathan, and Runkle (1993) also show evidence in which a positive, as well as a negative, sign for the

CRSP tapes. The data span the period from January 2, 1990 to December 31, 1992, and contain 239,844 observations. We use the same criterion for the data selection of chapter IV in this dissertation.

We test our hypotheses in two ways. First, after testing all 317 firms on an individual firm basis, we set up 5 portfolios by the firm size, and average the individual firms' statistics for each portfolio: a test based on each individual firm. Since the average standard errors do not provide much information about the test statistics, we provide distributions of the *t-statistics* for each of the parameter estimates in the portfolios. Tables 5.1 and 5.2 show these results. Second, we set up 5 portfolios for the sample according to firm size first, then test each individually in order to find portfolio-specific volatility of each. Tables 5.3 and 5.4 show these results.

To select the order of the ARMA process for $\log(h_{t-i})$, we use the Schwarz Criterion [Schwarz (1978)], which provides consistent order-estimation in the context of linear ARMA models. For both the return with number of trades and an AR(1) stock returns, the Schwarz Criterion selects an ARMA(1,1) model for $\log(h_{t-i})$.[19]

V. 4. 4. Empirical results

Tables 5.1 and 5.2 give the parameter estimates and estimated standard errors for both the stock returns with the number of trades variable and one with an AR(1) stock returns.

covariance between the conditional mean and the conditional variance of the excess return on stocks would be consistent with the standard mean-variances theory in finance.

[19] We also use an AIC [Akaike's (1973) information criterion], but it has similar results. ARMA(1,2) is also selected by the Schwarz Criterion. The results remain very similar with ARMA(1,1).

They also show the distributions of *t-statistics* for each parameter estimates, which indicate the percentage of *t-statistics* over 2.0 in each portfolio. Recall that we test all 317 firms on an individual firm basis for these two tables.

First, we test whether, after considering the stochastic volatility, the stock returns has a positive relationship with the number of trades. Table 5.1 shows a positive relationship between stock returns and the number of trades. This positive relationship is prominent and statistically significant in the smaller firm size portfolios (portfolios 1 and 2). The coefficients of α_1 and γ_1 are positive and significant. Table 5.1 also shows that the sum of α_1 and γ_1 has a range of 0.607 to 0.860, implying that the extent of decrease in the volatility by a shock is relatively persistent.

Note that the parameter c (risk premium term) has a positive, as well as a negative, sign (though weakly). This term is controversial in the empirical world, since some researchers [see French , Schwert, and Stambaugh (1987) and Brock, Lakonishok, and LeBaron(1992)] report a significantly positive sign by using the standard GARCH-M model, but other studies, using alternative techniques, show a negative sign [Pagan and Hong (1988), Nelson (1991), and Glosten, Jagannathan, and Runkle (1989)]. Our results show that the asymmetric relationship between returns and changes in volatility, as represented by θ, is negative but statistically insignificant. A negative sign of θ indicates that volatility tends to rise (fall) when returns surprises are negative (positive). In summary, Table 5.1 shows that the number of trades could provide us with a hint of traders' strategical behavior with their own information. A comparison of these results with ones of AR(1) stock returns may clearly indicate the aspect of traders' strategic behavior reflected in the number of trades.

Table 5.2 shows parameter estimates of AR(1) returns related with their volatility. Both coefficients of α_1 and γ_1 are positive and significant. The sum of α_1 and γ_1 has a range of 0.697 to 0.997, implying that the extent of decrease in the volatility by a shock is also relatively persistent, like in Table 5.1. Parameter θ, is negative, but statistically insignificant. Parameter c is positive, but also statistically insignificant, except in portfolio 3. The coefficients for the first lag of stock returns, b are negative in all but the largest firm size portfolio.[20] They are, however, statistically insignificant. When we compare the panel B in Tables 5.1 and 5.2, the specification for the relationship between stock returns and number of trades, from a viewpoint of statistical significance, is slightly better than the one for an AR(1) stock returns.

Tables 5.3 and 5.4 give the parameter estimates and estimated standard errors for both the stock returns with the number of trades variable and one with an AR(1) stock returns. For these tables, we test each portfolio individually. The Ljung-Box (1979) tests significantly reject the null hypothesis of white noise for the residuals of stock returns. Thus, we can consider an ARCH family model for the analysis. Table 5.3 shows a positive relationship between stock returns and the number of trades for all portfolios except portfolio 2. However, the negative sign for the number of trades is not statistically significant. The parameter c has a statistically stronger positive, as well as a negative,

[20] The coefficient b in Table 5.1 is for number of trades while in Table 5.2 it is for the first lag of stock return.

sign than in Table 5.1. Parameter θ, is negative and statistically significant. This shows a strong asymmetric relationship between returns and changes in volatility.

Table 5.4 also shows that parameter θ, is negative and statistically significant. The parameter c is positive and statistically, significant except in portfolio 1. The coefficients for the first lag of the stock returns, b, are negative in small and medium portfolios (portfolios 1, 2, and 3) but positive in the larger firm size portfolios (portfolios 4 and 5).

The basic characteristics of all parameters by the test based on each individual firm are generally the same as those found by testing each portfolio. However, when we compare the results in Tables 5.3 and 5.4, we cannot clearly discern, from a viewpoint of statistical significance, which is more useful.

In summary, the results in Tables 5.1 and 5.2 clearly reveal a very similar relationship for the stock returns. Moreover, the specification for the relationship between stock returns and number of trades, from a statistical viewpoint, is slightly better than one for an AR(1) stock return. When we compare these on a portfolio by portfolio basis (Tables 5.3 and 5.4), however, the specification for the relationship between stock returns and number of trades is not as good as one for an AR(1) stock returns. Altogether by either tests based on each individual firm or tests based on each individual portfolio, it is plausible that the established AR(1) stock returns process might come from the traders' strategic behavior rather than from measurement errors.

Portfolio	A: Parameter estimates							
	a	b	c	ζ	γ_1	α_1	θ	R^2
1	0.057	0.000092	-1.745	-2.983	0.540	0.169	-0.938	0.026
(Smallest)	(0.0485)	(0.000036)	(1.2111)	(0.6754)	(0.1024)	(0.0484)	(3.3654)	
2	-0.030	0.000071	1.019	-3.989	0.402	0.205	-0.071	0.028
	(0.0660)	(0.000030)	(2.1228)	(0.8011)	(0.1235)	(0.0565)	(0.9829)	
3	0.018	0.000053	-0.401	-2.761	0.608	0.164	-2.078	0.025
	(0.0353)	(0.000034)	(1.1651)	(0.5634)	(0.0776)	(0.0581)	(1.1792)	
4	-0.006	0.000033	0.477	-3.020	0.581	0.185	-2.310	0.023
	(0.0133)	(0.000026)	(0.4881)	(0.6136)	(0.0831)	(0.0446)	(3.5818)	
5	0.009	0.000012	-0.343	-2.587	0.665	0.195	-0.132	0.016
(Largest)	(0.0163)	(0.000018)	(0.7118)	(0.6304)	(0.0541)	(0.0541)	(0.8702)	

Portfolio	B: Distributions of *t-statistics* for each parameter estimates (Percentage of *t-statistics* > 2.0)						
	a	b	c	ζ	γ_1	α_1	θ
1	40.90	65.09	31.81	77.27	77.27	77.27	40.90
2	29.54	72.72	34.09	81.80	70.40	72.72	43.18
3	27.27	65.09	25.00	65.09	77.27	70.40	55.20
4	20.45	65.09	25.00	70.40	86.36	72.72	36.36
5	22.50	45.00	27.27	72.72	81.80	84.09	34.09

Panel A contains estimates of the following E-GARCH model for 317 NASDAQ-NMS securities, 1990-1992:

$$R_{t,i} = a + bX_{t,i} + ch_{t,i} + u_{t,i}$$
$$\log h_{t,i} = \zeta + \gamma_1 \log h_{t-1,i} + \alpha_1 g(z_{t-1,i})$$
$$g(z_{t,i}) = \theta z_{t,i} + \gamma\left[|z_{t,i}| - E|z_{t,i}|\right]$$
$$z_{t,i} = u_{t,i} / \sqrt{h_{t,i}} ,$$

where $X_{t,i}$ denotes the number of trades. We test all 317 firms on an individual firm basis. Parameter estimates and standard errors in parentheses are average values for each portfolios. Panel B contains the percentage of each parameter's *t-statistics* greater than 2.0 within each portfolio.

Table 5. 1 The relationship between stock returns and number of trades related with their volatility: a test based on each individual firm

Portfolio	A: Parameter estimates							
	a	b	c	ζ	γ_1	α_1	θ	R^2
1	-0.019	-0.06482	0.532	-2.951	0.551	0.207	-1.476	0.019
(Smallest)	(0.0271)	(0.04418)	(0.7616)	(0.7354)	(0.1119)	(0.0569)	(0.9666)	
2	-0.044	-0.05962	1.318	-3.410	0.483	0.214	-0.223	0.020
	(0.0444)	(0.04483)	(1.3037)	(0.7270)	(0.1092)	(0.0587)	(0.3389)	
3	-0.010	-0.03328	0.418	-2.316	0.648	0.165	-2.819	0.013
	(0.0133)	(0.04119)	(0.0506)	(0.4994)	(0.0716)	(0.0471)	(2.6057)	
4	-0.008	-0.02646	0.314	-2.767	0.612	0.189	-1.217	0.009
	(0.0536)	(0.06987)	(0.3760)	(0.6755)	(0.3722)	(0.0821)	(3.5064)	
5	-0.007	0.06172	0.324	-1.782	0.768	0.209	-0.799	0.011
(Largest)	(0.0068)	(0.06826)	(0.3185)	(0.6050)	(0.0778)	(0.0579)	(1.1756)	

Portfolio	B: Distributions of *t-statistics* for each parameter estimates (Percentage of *t-statistics* > 2.0)						
	a	b	c	ζ	γ_1	α_1	θ
1	18.18	52.27	13.64	79.54	88.64	81.82	29.54
2	25.00	31.81	22.73	81.82	81.82	81.82	34.09
3	11.36	47.73	13.64	65.91	72.73	88.64	40.91
4	15.91	38.64	13.64	75.00	86.36	81.82	38.64
5	13.64	47.73	13.64	61.36	86.36	90.91	27.27

Panel A contains estimates of the following E-GARCH model for 317 NASDAQ-NMS securities, 1990-1992:

$$R_{t,i} = a + bR_{t-1,i} + ch_{t,i} + u_{t,i}$$
$$\log h_{t,i} = \zeta + \gamma_1 \log h_{t-1,i} + \alpha_1 g(z_{t-1,i})$$
$$g(z_{t,i}) = \theta z_{t,i} + \gamma\left[|z_{t,i}| - E|z_{t,i}|\right]$$
$$z_{t,i} = u_{t,i} / \sqrt{h_{t,i}} \, ,$$

We test all 317 firms on an individual firm basis. Parameter estimates and standard errors in parentheses are average values for each portfolios. Panel B contains the percentage of each parameter's *t-statistics* greater than 2.0 within each portfolio.

Table 5. 2 AR(1) returns related with their volatility: a test based on each individual firm

Portfolio	A: Parameter estimates							
	a	b	c	ζ	γ_1	α_1	θ	R^2
1	-0.0002	0.000014	-0.0331	-0.0431	0.1076	0.9913	-0.4433	0.040
(Smallest)	(0.0004)	(0.000003)*	(0.0116)*	(0.0035)*	(0.0038)*	(0.0006)*	(0.0179)*	
2	-0.0021	-0.000001	0.0755	-0.0743	0.0783	0.9875	-0.4595	0.040
	(0.0005)*	(0.000001)	(0.0185)*	(0.0070)*	(0.0036)*	(0.0011)*	(0.0271)*	
3	-0.0008	0.000006	0.0485	-0.0194	0.0750	0.9964	-0.3978	0.030
	(0.0003)*	(0.000001)*	(0.0121)*	(0.0024)*	(0.0025)*	(0.0004)*	(0.0235)*	
4	-0.0013	0.000009	0.0535	-0.0921	0.1063	0.9861	-0.2802	0.007
	(0.0003)*	(0.000001)*	(0.0136)*	(0.0074)*	(0.0047)*	(0.0011)*	(0.0126)*	
5	-0.0006	0.000000	0.0637	-0.0654	0.1190	0.9903	-0.2477	0.001
(Largest)	(0.0004)	(0.000000)	(0.0187)*	(0.0067)	(0.0040)*	(0.0009)*	(0.0164)*	

Portfolio	B: *The Ljung-Box test-statistics* (12)	C: *p-value*
1	559.53	< 0.001
2	177.35	< 0.001
3	132.32	< 0.001
4	157.01	< 0.001
5	297.25	< 0.001

Panel A contains estimates of the following E-GARCH model for 317 NASDAQ-NMS securities, 1990-1992:

$$R_{t,i} = a + bX_{t,i} + ch_{t,i} + u_{t,i}$$
$$\log h_{t,i} = \zeta + \gamma_1 \log h_{t-1,i} + \alpha_1 g(z_{t-1,i})$$
$$g(z_{t,i}) = \theta z_{t,i} + \gamma\left[|z_{t,i}| - E|z_{t,i}|\right]$$
$$z_{t,i} = {u_{t,i}}\Big/{\sqrt{h_{t,i}}} \, ,$$

where $X_{t,i}$ denotes the number of trades. We test each portfolio individually. Parentheses show the standard errors for each parameter. * denotes significance at the 0.01 level. Panel B contains *the Ljung-Box test statistics* with twelve lags. They follow a chi-square distribution. Their 5% critical value with twelve degrees of freedom is 3.07. *p-value* of *chi-square statistic* comes from the model specification test.

Table 5. 3 The relationship between stock returns and number of trades related with their volatility: a test based on each individual portfolio

Portfolio	A: Parameter estimates							
	a	b	c	ζ	γ_1	α_1	θ	R^2
1	-0.0000	-0.09139	-0.0154	-0.0478	0.1140	0.9906	-0.3819	0.094
(Smallest)	(0.0004)	(0.00283)*	(0.0110)	(0.0041)*	(0.0037)*	(0.0007)*	(0.0134)*	
2	-0.0021	-0.03032	0.0703	-0.0743	0.0790	0.9874	-0.223	0.020
	(0.0007)*	(0.00560)*	(0.0195)*	(0.0066)*	(0.0036)*	(0.0011)*	(0.0265)*	
3	-0.0011	-0.01976	0.0714	-0.0195	0.0759	0.9964	-0.3764	0.000
	(0.0003)*	(0.00448)*	(0.0118)*	(0.0026)*	(0.0026)*	(0.0004)*	(0.0222)*	
4	-0.0019	0.00957	0.1010	-0.0921	0.1058	0.9861	-0.2738	0.007
	(0.0004)*	(0.00473)*	(0.0172)*	(0.0072)*	(0.0046)*	(0.0010)*	(0.0192)*	
5	-0.0008	0.05266	0.0721	-0.0639	0.1167	0.9905	-0.2653	0.023
(Largest)	(0.0003)*	(0.00501)*	(0.0143)*	(0.0060)*	(0.0050)*	(0.0008)*	(0.0179)*	

Portfolio	B: *The Ljung-Box test-statistics* (12)	C: *p-value*
1	128.47	< 0.001
2	94.61	< 0.001
3	132.61	< 0.001
4	121.05	< 0.001
5	174.57	< 0.001

Panel A contains estimates of the following E-GARCH model for 317 NASDAQ-NMS securities, 1990-1992:

$$R_{t,i} = a + bR_{t-1,i} + ch_{t,i} + u_{t,i}$$
$$\log h_{t,i} = \zeta + \gamma_1 \log h_{t-1,i} + \alpha_1 g(z_{t-1,i})$$
$$g(z_{t,i}) = \theta z_{t,i} + \gamma\left[|z_{t,i}| - E|z_{t,i}|\right]$$
$$z_{t,i} = {u_{t,i}}\Big/{\sqrt{h_{t,i}}} ,$$

Parentheses show the standard errors for each parameter. We test each portfolio individually. * denotes significance at the 0.01 level. Panel B contains *the Ljung-Box test statistics* with twelve lags. They follow a chi-square distribution. Their 5% critical value with twelve degrees of freedom is 3.07. *p-value* of *chi-square statistic* comes from the model specification test.

Table 5. 4 AR(1) returns related with their volatility: a test based on each portfolio

Chapter VI. Concluding Remarks and Further Research

In chapter III, we develop a model to discover the roles of both number of trades and the trade size simultaneously, which differs from the previous theoretical literature. Using numerical examples, our analyses reveal that the significance of the number of trades for the market maker's pricing depends on the three variables: the market maker's prior probabilistic belief on the value of the underlying asset, the informedness of the market, and the elasticity of the informed traders' demand and supply schedules. Since the market maker's belief on the value of the underlying asset is conditional on the past and present information about the security price, we use the past-price variability as a proxy variable for his prior belief. We also find that the number of trades sends a better signal to the market maker in the following situations: given small past-price variability, when a small fraction of the informed traders exists in the market and the informed trader's demand schedule is perfectly inelastic, the bid-ask spread and the number of trades have a positive relationship. Given large past-price variability, when a large fraction of informed traders exists in the market and the informed trader's demand schedule is very elastic, the bid-ask spread and the number of trades have a negative relationship.

In chapter IV, we test the implications from the theoretical model using data from the NASDAQ-NMS of the CRSP file. The results indicate that the market maker generally raises the bid-ask spread when he experiences a relatively large number of (daily) transactions if gross volume is held constant for a given trading period. This positive effect of the number of trades on the market maker's bid-ask spread appears to be persistent. We also find that three variables -- past-price variability, market informedness, firm size -- can affect the number of trades, but have a statistically insignificant on the bid-ask spread.

In chapter V, after considering the stochastic characteristics of its volatility, we test whether the stock return has a positive relationship with the number of trades. We test whether the established time-series property of observed returns, AR(1) process, comes from the traders' strategic behaviors rather than from simple measurement error in calculating the returns. Our results show it to be plausible that this truly is the case.

Our theoretical results suggest that there is merit in contriving a comprehensive model to consider inventory and market power. They also suggest that we can generalize our model in two ways. One is related with the symmetric distribution of the market maker's prior belief. For robustness, we can use a more general one. Second is to extend our model into a continuos time framework.

Our empirical results suggest using a more cautious selection for sampling and better refined data for our proxy variables. The lack of an elasticity measure of the informed traders' demand and supply schedules prevents us from performing a direct test for our theoretical model. Regardless of the specific modeling, it is meaningful to investigate the elasticity of the informed traders' demand and supply schedules. These tasks await further research.

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