Economically justified quality assurance decision plans for variables and count data for tests prone to measurement error.

George A. Freestone

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ECONOMICALLY JUSTIFIED QUALITY ASSURANCE DECISION PLANS FOR VARIABLES AND COUNT DATA FOR TESTS PRONE TO MEASUREMENT ERROR

by

George A. Freestone

A Thesis

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Industrial Engineering

Lehigh University

1982
This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

(Date)

Professor in Charge

Chairman of Department
This thesis is dedicated to my wife, Carroll, whose patience and cooperation have facilitated the completion of this work.

I am deeply grateful to Air Products and Chemicals for its support of my Master's program. I thank Steven T. Kramm for his contribution to the original work.

I appreciate the helpful guidance from my thesis advisors, Professor Sutton Monro and Dr. Louis Plebani

Lastly, I acknowledge Carol R. McClellan whose perserverance in preparing this document is noteworthy.
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ECONOMICALLY JUSTIFIED QUALITY ASSURANCE DECISION PLANS FOR VARIABLES AND COUNT DATA FOR TESTS PRONE TO MEASUREMENT ERROR

by

George A. Freestone

ABSTRACT

An investigation of published quality assurance decision plan generation methodologies indicated that the bulk of the methodologies are analytic in nature and that many are devoid of consideration of measurement error, the producer's perspective, the customer's reaction to material rather than strictly the state of nature, the potential for contamination in storage decisions related to quality, and most importantly, economic factors.

As an employee of a firm facing these kinds of decisions, I sought a methodology which would both identify desirable decision plans and serve as a communications vehicle between analysts and manufacturing personnel. The methodology chosen was simulation. Two simulation models are presented: one for shipping decisions and one for storage decisions. The models were established only after analyzing the implications of alternate decisions. For this thesis the simulations were written in SLAM (Simulation Language for Alternative Modeling, Pritsker & Associates, 1980). An example of the inflexibility of the published plans is the difficulty surrounding the incorporation of the reroute decision into the shipping analysis. The reader will observe that failure to do so has a cost of approximately $600 per transaction in the shipping model.
In addition to incorporating measurement error into the two simulation models, an entire section of this thesis is devoted to measurement errors. Design of experiments plays an integral role in demonstrating potential for reduction in imprecision.

Two standard problems are defined: a shipping and a storage problem. The methodology developed in the thesis is used via "sectioning" to identify least cost decision criteria. Decision criteria are obtained from the other methodologies researched and then applied in a simulation modified to incorporate the peculiarities of each methodology's decision criteria.

The results are consistent. In both standard problems the methodology developed in the thesis is superior. In the shipping problem the difference is from a thesis "optimal" $1,156 to a next best $2,193 cost per transaction. In the storage problem, the difference is from a thesis "optimal" $63 to a next best $928 cost per transaction. Another full section is devoted to the application of the thesis results in a manufacturing environment.

The thesis concludes with a statement of recommendations, the most significant of which is that these decisions should be based on cost and, therefore, producer's and consumer's risks should be outputs rather than inputs to the plan generation.
I. INTRODUCTION

Imagine the plight of shipping supervisor Fred who is at this moment wrestling with the decision of whether or not to ship the 160,000 pounds of "stuff" in the railcar before him to customer XYZ as prime material. In his hands he carries the following quality assurance report from the plant analytical labs:

All results from Sample 1

<table>
<thead>
<tr>
<th>Specification</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attribute 1</td>
<td>.92 - .98</td>
</tr>
<tr>
<td>Attribute 2</td>
<td>&lt;100</td>
</tr>
<tr>
<td>Attribute 3</td>
<td>&gt;.6</td>
</tr>
<tr>
<td>Attribute 4</td>
<td>35 - 55</td>
</tr>
</tbody>
</table>

As shipping supervisor, he's less than enthused about shipping marginal product to customer XYZ. As a member of the manufacturing staff at the plant, however, he knows the importance of shipping the largest possible quantity of prime material - without rejection. It would not hurt the corporation to make the additional 5¢ per pound that selling as prime would allow. Chances are the customer testing the material on receipt would not see the marginal nature of attributes 1, 2, and 4. Fred decides to take his chances and ship it as prime - he will hear less grief today about it. Should this be the way this decision is made?
The same situation could have faced Production Supervisor Bill as he views the same set of results as did Fred for a 260,000 pound batch of product leaving the manufacturing process and now needing a home in the finished product silos. Bill has no completely empty silos: he can put this batch of product in with very "prime" material, he can put it in with definitely "offgrade" material, or he can defer making the decisions and put the material into two railcars that were planned to be used for other finished product shipments. He looks at the quality report and shakes his head because he knows of the testing procedures and the error inherent in these results. He also knows that he could be responsible for contaminating this batch of product - if it were saleable as prime. Bill also does not want to alienate his good drinking buddy Fred by using his railcars as auxiliary storage. Fred decides that it is prime material and therefore puts the 260,000 pounds in the prime silo. Is this the best way for the decision to be made?

Both Fred and Bill are intelligent guys. Fact is they have even tried to improve this decision making. Both have individually consulted textbooks in statistical quality control in search for the elixir of quality decision making. Both have become frustrated that the simple models do not represent the situation they have, and the more complex models seem to be incomprehensible. Their frustration led them to call in the Operations Researchers from headquarters. The OR analysts attempted to provide decision - making relief, but
their solution appeared more tedious and complex and insufficient when it was attempted in practice. So Fred and Bill are back to the historical way of making these decisions - minimizing personal regret.

This situation is at the core of the reason I am pursuing this topic in my thesis. I personally believe that a solution to these kinds of situations exists. I also believe that the implementation of economically based decision plans can successfully be achieved. I hope that his effort will convince myself, and others, that we can identify a solution methodology and implement results. This will be the focus of the work.
II. PROBLEM ADDRESSED

This effort will analyze quality decision plans for attributes measured on a continuous scale (variables) or attributes that are measured on an integer scale (count data). The decision plans under consideration will exclude attributes which are yes - no in nature (attributes). In the majority of the cases variables and count data arise in practice while measuring the quality of liquid or solid form sold in bulk containers (rail cars or tank trucks).

This analysis will incorporate measurement error because the vast majority of the quality assurance tests which are performed in industry are carried out be people. People tend to err. In addition, some technicians might translate test results slightly when pressure exists in the testing environment for the material to be classified as prime. Bias exists. Even in the case that a mechanised quality testing procedure is in place, some person had to instruct the machine in how to test. The machine can repeat the instructions - but only in the fashion that it was programmed. Random error is also likely in this mechanised case, it is just probable that the magnitude is reduced. Both bias and imprecision will be assumed to exist in all of the test under consideration in this analysis.

Since measurement error has just been noted as an important
consideration, an interesting question comes to mind. How do we objectively measure measurement error? Technicians may be wary of the enthusiastic analyst staring over the shoulder. Blind test results obtained on unmarked samples will also certainly generate questions of management's motivation in the technicians mind. This analysis will comment on methods for quantifying measurement error.

Textbook quality control test plans are usually set up on the notion of confidence of determining the correct or actual measurement. This of course, assumes that the recipient of the product will be measuring the actual quality. Chances are that the customer will try to do this, but he too is probably carrying the test out in a manual mode - subject to the same types of errors that the producer is. As a supplier of material it is imperative for the producer to predetermine his posture as a supplier: a vendor of quality material, a vendor of material which will suit the customer's requirements most of the time, or a vendor of random quality material. By varying the desired posture to be maintained the producer will drastically affect his quality assurance test parameters, or for that matter, the need to perform tests at all.
Frequently, quality control decision plans are set on the basis of appropriate producer's and consumer's risks. If these plans are to be meaningful in an industrial setting, these producer's and consumer's risks should be set on the basis of a cost analysis. Arbitrarily setting these risks will unattractively affect system operating costs unless carefully conceived. An analyst can prescribe certain risks in the hopes of attaining a particular level of customer service, but this methodology seems backwards. Actual customer dissatisfaction costs should be quantified, a model should be constructed that reflects these costs, and then the model should be solved so as to minimize costs. The producer's and consumer's risks will be determined based on the minimum cost guidelines for operating the system. Thus these risks will be an output rather than an input. The customer dissatisfaction or loss of goodwill costs may be difficult to identify, but it is the position of this analysis that an attempt should be made to properly quantify these costs (or a range of reasonable costs) and then apply a cost minimizing methodology to obtain desirable operating guidelines. In the experience of this analyst, it is usually the case that the costs are difficult to quantify until someone has taken the time to suggest an estimating method. From this perspective, what is superior to quality assurance guidelines that minimize cost?
This thesis will investigate decision plans for material that is assumed to be nonhomogeneous in quality attributes as well. For example, the quality of material at the top of the railcar may very well be drastically different than the quality of the material at the bottom of the railcar. This same situation is assumed to be possible in the manufactured lot situation.

Two illustrative networks (or models) follow. One network illustrates the consequences of potential actions regarding shipping decisions and the second traces the impact of storage decisions. These models are useful at this point for two reasons.

1) They simplify the conceptualization of the problem
2) After studying the networks, it becomes apparent that any sampling plan (single, double, or sequential) can be evaluated within this analytic framework

The networks presented here are only samples that attempt to convey the type of problem being addressed.
FIGURE 1:
ILLUSTRATIVE SHIPPING DECISION NETWORK
Illustrative Shipping Decision Model

This node by node description of a potential shipment decision model traces the various courses that a shipping transaction can take. It presupposes a given test procedure and a set of decision criteria referred to as DC2A, DC3A, and DC4A. The derivation of these "sample" costs used in this model are included in Table 1 following the node by node description. An assumption made in the illustrative networks is that one sample is taken and repeatedly tested.

NODE 1: The actual product quality of the current transaction is generated based on the particular test being considered. ABC CORP obtains one or more test result(s) from that that quality subject to the imprecision and bias associated with the test. Add the test cost ($10.50 for TEST 1) to the cost of this transaction.

NODE 2: The estimated probability of being within spec (EPWS) is calculated using the test result(s), the imprecision level for the test being considered, and the spec(s) for the product being considered. If the EPWS is greater than decision criterion DC3A go to NODE 5. If the EPWS is less than decision criterion DC2A go to NODE 4. If the EPWS is between DC2A and DC3A and the maximum number of tests, DC3N, has not been reached, go to NODE 3. If the maximum number of tests has been obtained go to NODE 4.

NODE 3: Product reaching this node is marginal product that ABC CORP does not feel confident enough to classify yet. Obtain one or more additional test results and add the test cost ($10.50 for TEST 1) to the cost for the transaction. Update the average of the tests results obtained so far and return to NODE 2.
NODE 4: Product reaching this node has been classified as off grade product. The fraction of the time the customer should obtain an in-spec test result is computed given the true quality of the product and the imprecision level of the test. Then a random number is generated and compared with that fraction to determine how the customer will classify the product. If the random number is lower than the fraction go to NODE 11, otherwise go to NODE 12.

NODE 5: Product reaching this node has been classified as prime product by ABC CORP. The customer then tests the product; if the customer determines it is prime, go to NODE 6, otherwise go the NODE 7.

NODE 6: Product reaching this node was classified by ABC CORP as prime and now has been called prime by the customer. If the true quality of the product is actually within spec go to NODE 17, otherwise go to NODE 18.

NODE 7: Product reaching this node was classified by ABC CORP as prime and now has been called off grade by the customer. Two test results are generated for the customer and two additional test results are generated for ABC CORP. The averages of the tests ABC CORP has taken so far and the customer's average result are then used to obtain a weighted average (weighted 4 to 1 in favor of the customer). If this combined average is within spec go to NODE 8, otherwise go to NODE 9.

NODE 8: If this node is reached ABC CORP has successfully refuted the customer's claim that the product is off grade. Add the administrative cost of $1000 to the cost of the transaction. If the product quality actually is within spec go to NODE 21. A otherwise go to NODE 22.
NODE 9: If this node is reached ABC CORP has failed to refute the customer's claim that the product is off grade. Add the administrative cost of $1000 to the cost of the transaction. Compute a new EPWS based on the updated average of ABC CORP's test results from NODE 7. If the EPWS is greater than decision criteria DC4A and at least DC4N tests have been obtained, ABC CORP has chosen to reroute the product for this transaction to another customer. We return to NODE 5 to determine how the next customer will classify the product and incur a reroute cost of $3000. If, however, the EPWS is less than the DC3A go to NODE 10.

NODE 10: ABC CORP has chosen to sell the product in this transaction to the current customer at a reduced price per pound. If the product quality actually is within spec go to NODE 13, otherwise go to NODE 14.

NODE 11: Product reaching this node was classified by ABC CORP as off grade and would have been called prime by the customer. If the true quality of the product is actually within spec go to NODE 19, otherwise go to NODE 20.

NODE 12: Product reaching this node was classified by ABC CORP as off grade and would have been called off grade by the customer. If the true quality of the product is actually within spec go to NODE 15, otherwise go to NODE 16.

NODE 13: The product reaching this node actually was prime product but has been sold as off grade product. Add an $8000 penalty cost to the cost for this transaction.

NODE 14: The product reaching this node actually was off grade and was sold as off grade product. No incremental cost is incurred.

NODE 15: The product reaching this node actually was prime product but has been sold as off grade product. Add a $3000 penalty cost to the cost for this transaction. (see Table 1 for explanation)

NODE 16: The product reaching this node actually was off grade and
was sold as off grade product. No incremental cost is incurred.

NODE 17: The product reaching this node actually was prime product and was sold as prime product. No incremental cost is incurred.

NODE 18: The product reaching this node actually was off grade and was sold as prime product. No incremental cost is incurred. A customer service penalty cost might be appropriate.

NODE 19: The product reaching this node actually was prime product but has been sold as off grade product. Add an $8000 penalty cost to the cost for this transaction.

NODE 20: The product reaching this node actually was off grade and was sold as off grade product, but could have been sold as prime product. Add an $8000 penalty cost for this transaction. (The $8000 cost could be dropped in the interest of customer service.)

NODE 21: The product reaching this node actually was prime product and was sold as prime product. No incremental cost is incurred.

NODE 22: The product reaching this node actually was off grade and was sold as prime product. No incremental cost is incurred. A customer service cost might be appropriate.
<table>
<thead>
<tr>
<th>ABC CORP Classification</th>
<th>Customer Classification</th>
<th>True Quality</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>prime</td>
<td>prime</td>
<td>prime</td>
<td>no cost</td>
</tr>
<tr>
<td>prime</td>
<td>off grade</td>
<td>off grade</td>
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</tr>
<tr>
<td>off grade</td>
<td>prime</td>
<td>prime</td>
<td>$8000</td>
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</table>

The $8000 figure represents the profit lost by selling a 160,000 lb. railcar of product at a 5¢/lb. price reduction.

The $3000 figure represents this same profit loss given that it would have taken on the average $5000 in refutation and rerouting costs to have sold the product (marginally prime in this case) as prime material. Since an average of $5000 would be spent in avoiding the $8000 incremental cost, the actual incremental cost incurred in $3000.
FIGURE 2:
ILLUSTRATIVE STORAGE DECISION MODEL

[Diagram of storage decision model with nodes and branches, illustrating decision-making processes in storage management.]
Illustrative Storage Decision Model

This node by node description of a potential storage decision model traces the various courses that a storage transaction can take. It presupposes a given test procedure and a set of decision criteria referred to as DC2A and DC3A. The derivation of the costs used in this model are included in Table 2 following the node by node description. An assumption made in this illustrative network is that one sample is taken and repeatedly tested.

NODE 1: The actual product quality of the current transaction is generated based on the particular test being considered. ABC CORP obtains one or more test results from a sample subject to the imprecision associated with the test. Add the test cost ($10.50/Test for Test 1) to the cost of this transaction.

NODE 2: The estimated probability of being within spec (EPWS) is calculated using the test result(s), the imprecision level for the test being considered, and the spec(s) for the product being considered. If the EPWS is greater than decision criteria DC3A go to NODE 5. If the EPWS is less than decision criteria DC2A go to NODE 4. If the EPWS is between DC2A and DC3A and the maximum number of tests, DC3N, has not been reached, go to NODE 3. If the maximum number of tests has been obtained go to NODE 4.

NODE 3: Product reaching this node is marginal product that ABC CORP does not feel confident enough to classify yet. Obtain one or more additional test results and add the test cost ($10.50/Test for 1) to the cost for the transaction. Update the average of the tests results obtained so far and return to NODE 2.
NODE 4: Product reaching this node has been classified as off grade product. Go to NODE 7.

NODE 5: Product reaching this node has been classified as prime product. Go to NODE 6.

NODE 6: The product mixes with off grade material in an off grade silo. The quality of the resultant mixture is the average of the quality of the lotsize and the quality of the product in the silo prior to mixing weighted by their respective quantities. If the resultant mixture is within spec go to NODE 8, otherwise go to NODE 9.

NODE 7: The product mixes with prime material in a prime silo. The quality of the resultant mixture is the average of the quality of the lotsize and the quality of the product in the silo prior to mixing weighted by their respective quantities. If the resultant mixture is within spec go to NODE 10, otherwise go to NODE 11.

NODE 8: The mixture of the lotsize and the prime silo has resulted in a mixture which a customer would consider prime. If the lotsize really was prime go to NODE 12, otherwise go to NODE 13.

NODE 9: The mixture of the lotsize and the prime silo has resulted in a mixture which a customer would consider off grade. If the lotsize really was prime go to NODE 14, otherwise go to NODE 15.

NODE 10: The mixture of the lotsize and the off grade silo has resulted in a mixture which a customer would consider prime. If the lotsize really was prime go to NODE 16, otherwise go to NODE 17.

NODE 11: The mixture of the lotsize and the off grade silo has resulted in a mixture which a customer would consider off grade. If the lotsize really was prime go to NODE 18, otherwise go to NODE 19.
NODE 12: Product reaching this node was prime and after mixing with a prime silo is still prime. No incremental cost is incurred.

NODE 13: Product reaching this node was off grade and after mixing with a prime silo is now prime. Subtract a savings of $8000 from the cost of this transaction.

NODE 14: Product reaching this node was prime and after mixing with a prime silo would now be off grade. This combination, however, is infeasible.

NODE 15: Product reaching this node was off grade and after mixing with a prime silo has made the silo off grade as well. Add a penalty of $12,500 to the cost of this transaction.

NODE 16: Product reaching this node was prime and after mixing with an off grade silo has made the silo prime as well. Subtract a savings of $12,500 from the cost of this transaction.

NODE 17: Product reaching this node was off grade and after mixing with an off grade silo has made the silo prime as well. This combination, however, is infeasible.

NODE 18: Product reaching this node was prime and after mixing with an off grade silo is now also off grade as well. Add a penalty of $8000 to the cost of this transaction.

NODE 19: Product reaching this node was off grade and after mixing with an off grade silo is still off grade. No incremental cost is incurred.
<table>
<thead>
<tr>
<th>ABC CORP Classification</th>
<th>Customer Classification</th>
<th>Lot True Quality</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>prime</td>
<td>prime</td>
<td>prime</td>
<td>no cost</td>
</tr>
<tr>
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<td>-$12,500*</td>
</tr>
</tbody>
</table>

* Negative costs reflect a cost avoided rather than an expense.

The $8000 figures represent the profit lost by selling a 160,000 lb. lotsize of product at a 5¢/lb. price reduction that was either incurred ($8000) or avoided (-$8000).

The $12,500 figures represent incurring or avoiding the same 5¢/lb. loss for 250,000 lbs. of product stored in a silo.
III. WHAT HAVE OTHERS SAID ON THIS TOPIC?

For each of the alternate methodologies discussed here, a standard format will be used. First, the article approach or methodology will be evaluated with regard to several specific criteria. A yes, no, or qualified response as to whether or not the approach accommodates certain features of the problem will be given. These criteria include

1. Does the methodology account for "variables" data?
2. Does the methodology account for "count" data?
3. Does the methodology accommodate more than one attribute at a time?
4. Does the methodology incorporate measurement error considerations?
5. Does the methodology deal with customer responses to material shipped to them where the customer's reaction is based on his perception of the material—not the true state of nature?
6. Does the methodology deal with the potential risk of contamination of material that exists in storage decisions?
7. Does the methodology provide analytical results?
8. Does the methodology provide results via simulation, or some other non analytic method?
9. Does the methodology provide single or double (non-sequential) sampling plans?
10. Does the methodology provide sequential sampling results?
11. Does the methodology deal with economic considerations?

In general, the larger the number of positive responses, the more relevant the article.
Second, a concise abstract of the reference will be provided. Last, then, the author of this thesis will offer some personal comment on article and make note of its relevance to the topic discussed here.
Reference: GAF001

Title: "Acceptance Sampling by Variables: Chapter 16", Quality Control and Industrial Statistics, Acheson J. Duncan

Specific Criteria Evaluation:

1. Yes
2. Yes, by extension
3. No
4. No
5. No
6. No
7. Yes
8. No
9. Yes
10. Yes
11. No

Abstract:

Duncan describes determining plans for X and S. The plans for X are most relevant. To determine a plan for X he suggests

1. Determine producer's risk a
2. Determine consumer's risk β
3. Determine process (or quality attribute) variability - S
4. Isolate specification limits
5. Compute:
   a) 'n' and 'X' threshold for non sequential plans
   b) sequential testing thresholds

Later, he relaxes the assumption that S is known with the general implication that more tests are required.
Comments and Relevance:

If the customer will truly be measuring the state of nature and if we have some rationale for assigning producer's and consumer's risk, then we are in business. Unfortunately, I do not believe the customer is any better at determining the true quality of product than we are. Nor do I think it inconsequential to assign good producer's and consumer's risks.

This reference will definitely yield decision plans to be compared with those I will later derive.
Reference: GAF002

Title: "A Note of the Relationship Between Measurement Error and Product Acceptance" IE AIIE Technical Notes, Thomas E. Diviney and Nasim A. David.

Specific Criteria Evaluation:

1. No Deals with attributes
2. No Deals with attributes
3. No
4. Yes
5. No
6. No
7. Yes
8. No
9. Yes
10. No
11. No

Abstract:

When measurement error is large, remedies are:

1. More precise measuring equipment;
2. Institution of an operator-measuring-training program; and,
3. The use of averages rather than single measurements.

When measurement error and product variability are independent:

\[ \sigma^2_{\text{observed}} = \sigma^2_{\text{measurement error}} + \sigma^2_{\text{product}} \]

Comments and Relevance:

One needs an organized plan for determining these quantities when \( \sigma^2 \)
product and $\sigma^2$ measurement error can confound the unwary analyst.

Relationship established between $\sigma^2_{\text{observed}}$ and $\sigma^2_{\text{product}}$ can be exceptionally useful in modifying the derivation of decision plans when measurement error exists.
This sampling plan is set up partially through the producer's perspective. He derives a plan for sampling which will minimize costs based on the ratio $K = \frac{K_2}{K_1}$ (where $K_2$ = the cost of shipping the customer a truly defective entity $K_1$ = the cost to test an entity of product) and $p$ (the fraction defective). He leads to the sampling plan.

- Complete inspection if $K_p > 1$
- No inspection if $K_p < 1$

It is unimportant, according to the author, as to the strategy adopted at $K_p = 1$. One only needs to know the "approximate" portion defective in production to set the plan in motion. A graph describing "optimal" plans is presented in Figure 3.
Comments and Relevance:

An attributes methodology can always be made to fit a variables or count data problem, but the converse is not true. As such, the Deming plan can very easily produce a plan to be contrasted with others.

Deming, owing to his success in the field of quality related problem, has several interesting quotes embedded in his text:

"The best way is to have no defectives at all"

"Loss of future business from a dissatisfied customer, and from potential customers that learned from his experience, may be enormous, and is unfortunately impossible to estimate."
Graph of $pk_3/k_3 = 1$. Any point on the curve is a point of indifference. One only needs to have rough values of $p$ and of $k_3/k_3$ to find which side of the curve is optimum. The choice is no inspection below the curve; 100% inspection above the curve. If by chance, the point determined by trial values of $p$ and $k_3/k_3$ lies close to the curve, choose 100% inspection, pending more information. This curve was suggested by Mr. Ronald A. Cristofano of the Nashua Corporation.

$n = N$

100% inspection above the curve

$n = 0$

No inspection below the curve

Ratio $k_3/k_3$
Specific Criteria Evaluation

1. Not relevent
2. Not relevent
3. Not relevent
4. Yes
5. Not relevent
6. Not relevent
7. Not relevent
8. Somewhat
9. No
10. Yes
11. No

Abstract:

The text dealing with measurement error discusses some fairly important considerations for incorporating measurement error into decision plans, specifically:

- Transformations are provided when the random error is not uniform.

- For experiments which lead to a comparison of means and which contain internal control the use of the Normal theory tables of significance will provide an adequate approximation even when fairly large departures from Normality occur.

The chapter or sequential decision plans follows Duncan's discussion.
Comments and Relevance:

The comments made with respect to measurement error will be useful in the approach developed by the author.
Comments and Relevance:

The comments made with respect to measurement error will be useful in the approach developed by the author.
Abstract:

They first define:

• specification limits to denote those limits within which the true value of the product should lie between if it is to be useful.

• test limits to denote those limits which should be applied to judge acceptability of the product based on measured values from tests.

The authors then provide guidance for determining plans under each of three situations.

1. Setting test limits such that the producer and the consumer both suffer equal risks of misjudging product. It is interesting to note that this would, in the example included in the article, lead to setting test limits outside specification limits.

2. Setting test limits such that the sum of the producer's and consumer's risk is a minimum.

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3. Setting test limits such that the cost of making wrong decisions is minimized. Investigation has shown that the cost of accepting a non-conforming unit would have to be at least six times as large as the cost of rejecting a conforming unit in order for test limits to be set inside specification limits.

The authors suggest that setting the policy on the basis of cost is the proper one to strive for.

Comments and Relevance:

There is exemplified in this article a very healthy attitude about costs and also enough meat to suggest decision plans for most problems. This will work for only two sided limits. It will be interesting to contrast a decision plan promoted by the author of the thesis with a decision plan generated via the third situation described above.
Reference: GAF006

Title: "On Estimating Precision of Measuring Instruments and Product Variability", Frank E. Grubbs

Specific Criteria Evaluation:

1. Yes
2. Yes
3. Not relevant
4. Yes
5. Somewhat
6. Not relevant
7. Yes - for quantifying error
8. No
9. Not relevant
10. Not relevant
11. Not relevant

Abstract:

The author provides an excellent mechanism for separating $\sigma$ error and $\sigma_{\text{product}}$ variability from $\sigma_{\text{observed}}$. Several fine examples are included in the article. The approach is used in Measurement Error - How to Quantify it.

Comments and Relevance:

The estimation of random error can be quantified in a more logical fashion than just computing $s$ for each operator without considering what other's results were on the same blind sample. This approach is useful when repeat measurements can not be made.
Abstract:

The author notes some properties of the minimization of expected total cost for quality control decision plans

1. This an unconstrained optimization problem.
2. There are two decision variables (in this case "n" and "c")
3. The expected total cost function is discrete, discontinuous (hence nondifferentiable), and convex containing many local optima.

The author assumes that all defectives will be identified by the consumer when used in a manufacturing process.

The expected total cost function is evaluated for alternate decision plans (ie "n" and "c" is a single sampling plan) using Bayes Theorem and EVPI (the expected value of perfect information). The cost function is a
Guthrie Johns model. Measurement error would make the cost equation more complex.

The emphasis on this paper is on the identification of the optimal $n^*$ and $C^*$ via pattern search. Pattern search is claimed to be an efficient method.

Comments and Relevance:

He offers meaningful comment on the use of mathematicl optimization on the cost equation.

He offers a meaningful search technique for identifying the optimum of an attribute plan.
Reference: GAF008

Title: GERT - Chapters 8, 9, 10, and 11 from Systems Analysis and Design Using Network Techniques, Gary E. Whitehouse

1. No
2. No
3. No
4. No
5. No
6. No
7. Yes
8. No
9. Yes
10. No
11. No

Abstract:

GERT is extended to use in single and double attributes sampling plans. Costs are not included here, but they could be.

Comments and Relevance:

GERTS and GERTS-Q provide SLAM like solutions to problems via simulation. The simulation approach offers clarity over the strictly analytical GERT analysis. The GERT approach could accommodate costs of sampling. GERT would have difficulty modelling the customer's response to material and costs of misclassified material. The major disadvantage with GERT, however, is that it is difficult to use as a modelling aid and would be extremely difficult to explain to those who would make use of the solution. Even Gary E. Whitehouse indicates that the use of GERT is
applicable to this type of problem, but that it is not a practical way to solve them. He drops back to GERTS solution of all sampling models.

GERT also seems to be more appropriate for modelling attributes plans rather than variables or count data plans. With attributes plans, only two states of nature exist: defective or non defective. With variables or count data plans an infinite number of states of nature exist, making the modelling virtually impossible.
Abstract:

The authors recommend writing a cost equation, much in the form of the Guthrie Johns model. The Guthrie Johns model is modified to take account of attributes sampling errors if the first and second kind. Conditional probabilities are used to accommodate measurement error considerations. The mathematical model of cost was minimized by an unsophisticated search technique.

Comment and Relevance:

Two quotes of interest show up in the article.

"Costs introduced through realistic error in the inspection process can be appreciable. This indicates that nontrivial
amounts of money spent to improve inspection accuracy may be worthwhile."

"Although the search procedure (to identify the optimal plan) was not sophisticated, neither was computer time a critical factor"

This methodology will offer an "attributes" solution of the standard problems being solved. The model used is identical with the succeeded article's approach.
Reference: GAF010

Title: An Interactive Computer Program for the Study of Attributes Acceptance Sampling," Kenneth Case.

Specific Criteria Evaluation

1. No
2. No
3. No
4. Yes
5. Yes
6. No
7. Yes
8. Yes
9. Yes
10. No
11. Yes

Abstract:

A free FORTRAN based interactive program was obtained from Oklahoma State University. The program allows the user to design a single sampling attributes decision plan under different scenarios.

1) Statistically Based Design
   a. no measurement error
   b. measurement error

2) Economically Based Design
   a. no measurement error
   b. measurement error

In addition to analytical results, the program allows the user to simulate particular decision plans of interest to study system response.

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Comments and Relevance:

It will be possible to restructure the standard problem as an attributes problem and derive solutions with this program. The results of the attributes version of the standard problem should be inferior to variables and count data formulations.

The program was installed, modified slightly, and is now operational at Air Products and Chemicals, Inc. Copies of the program can be obtained from Dr. Case or from the author of this thesis.
Reference: GAF011


Specific Criteria Evaluation:

1. No
2. No
3. Somewhat
4. Yes
5. Yes
6. No
7. Yes
8. No
9. Yes
10. Yes
11. Yes

Abstract:

The author constructs a decision tree of an attributes plan problem. Bayes theorem is used to determine the rolled back value of the branches. Measurement error is introduced.

Comments and Relevance:

If a probability table of possible outcomes can be constructed for variables and count data (as it was for attributes), then this approach may be useful. This limitation could not be overrun for the infinite possibilities in a variables plan and is the same kind of a liability as discussed to the GERT problem formulation.
The basic problem with a decision tree formulation of the problem would be that all possible outcomes would need to be enumerated. For the problem addressed in this thesis, that number would be infinite.
Abstract:

The expected value of perfect information (EVPI) concept is extended to sampling. An optimal sample size is computed by quantifying the value of the reduction in variance from prior to posterior distribution (both assumed to be normal).

The concept is illustrated below:
Comments and Relevance:

The results of this method of analysis would yield an optimal sample size, but no threshold value for making decisions. Thus, whereas the concept makes sense, the specific method of analysis will not be of use for the problem addressed here.
Abstract:

For a particular sequential environment involving attributes, a method of analyzing a least cost testing sequence with GERT is provided. The mean and variance of the total cost (testing, reworking and scrapping) are obtained. An optimal rule for determining the sequence is determined.

Comments and Relevance:

The problem modelled here is very specific, it does not have general utility. The model does not adequately represent the problem considered in this thesis, but it does represent another use of GERT in the quality control field.
As stated previously in the comments section of reference GAF008, GERT seems to be a tedious method for solving these problems.
Reference: GAF014

Title: "Bias and Imprecision in Variables Acceptance Sampling: Effects and Compensation," Wen-Haur Mei, Kenneth Case, and J. Schmidt

Specific Criteria Evaluation

1. Yes
2. Yes
3. No
4. Yes
5. Yes
6. No
7. Yes
8. No
9. Yes
10. No
11. No

Abstract:

A method is provided which yields "n" the number of samples (= test observations) and "DC \_x" the decision criteria to be used for the average of the test results obtained. The results are summarized below:
### TABLE 3: BIAS AND IMPRECISION COMPENSATION FOR VARIABLES PLANS

<table>
<thead>
<tr>
<th>Measurement errors</th>
<th>Compensating sampling plans</th>
<th>Upper specification</th>
<th>Lower specification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample size</td>
<td>$DC_x$</td>
<td>Sample size</td>
</tr>
<tr>
<td>Imprecision only</td>
<td>$n \left( \frac{h+1}{h} \right)$</td>
<td>$k\sigma$</td>
<td>$n \left( \frac{h+1}{h} \right)$</td>
</tr>
<tr>
<td>Bias only</td>
<td>$n$</td>
<td>$k\sigma - \mu_e$</td>
<td>$n$</td>
</tr>
<tr>
<td>Imprecision and bias</td>
<td>$n \left( \frac{h+1}{h} \right)$</td>
<td>$k\sigma - \mu_e$</td>
<td>$n \left( \frac{h+1}{h} \right)$</td>
</tr>
</tbody>
</table>

- $h$ above is defined to be $\sigma^2_{\text{lot}}$
  - $\sigma^2_{\text{error}}$
  - errors are assumed to be normally distributed
  - lot distribution of quality is assumed to be known and normal

**Comments and Relevance:**

This paper will yield results for the standard problem. The authors of this article indicate that is their experience it is common for $\sigma_{\text{error}} > \sigma_{\text{lot}}$ and that rarely is measurement error a function of dimension.
Reference: GAF015

Title: "Bayesian Decision Theory and Statistical Quality Control"
Morris Hamberg

Specific Criteria Evaluation:

1. No
2. No
3. No
4. No
5. No
6. No
7. Yes
8. No
9. Yes
10. Yes
11. Yes

Abstract:

Hamberg indicates a low percentage of literature published on this topic is cost oriented or associated with Bayesian analysis. This paper presents a method of analysis discussed earlier in GAF011 (Decision trees).

Comments and Relevance:

A decision tree solution to the standard problem will be tedious. All possible outcomes for each change occurrence would need to be enumerated and then modelled.
Reference: GAF016

Title: "Acceptance Inspection by Variables when Measurements are Subject to Error," Herbert David, Edward Frey, and John Walsh

Specific Criteria Evaluation

1. Yes
2. Yes
3. No
4. Yes
5. No
6. No
7. Yes
8. No
9. Yes
10. No
11. No

Abstract:

The results provide an analytical solution for one-sided acceptance criteria. The authors do provide a result for "N" the number of samples to be obtained and "n" the number of tests to be run for each sample. The procedure is routine, but tedious.

Comments and Relevance:

The article may provide meaningful results for the standard problem.
Reference: GAF017

Title: "On Economically Based Quality Control Decisions" E. Menipaz

Specific Criteria Evaluation:

1. No
2. No
3. No
4. No
5. No
6. No
7. Yes
8. No
9. No
10. Yes
11. Yes

Abstract:

For a specific manufacturing environment with parts being evaluated on the basis of attributes, a dynamic programming model is developed which minimized case-specific costs. The costs are evaluated after discounting.

Comments and Relevance:

The author has succeeded in applying mathematical programming techniques to quality control, but for a very specific problem. The discounting of costs in his analysis is amusing.
A summary of the ability of the selected articles to meet specific criteria is summarized in Table 4.

**TABLE 4: SUMMARY OF LITERATURE REVIEW**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>% Yes</th>
<th>% No</th>
<th>% Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>59</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>59</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>76</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>58</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>29</td>
<td>47</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>82</td>
<td>18</td>
</tr>
<tr>
<td>7</td>
<td>94</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>88</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>76</td>
<td>18</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>58</td>
<td>13</td>
</tr>
<tr>
<td>11</td>
<td>59</td>
<td>35</td>
<td>6</td>
</tr>
</tbody>
</table>
IV. THE AUTHOR'S APPROACH

A simulation model was selected by the author to solve the situation discussed in the Problem Addressed section. Simulation was selected because:

1) the problem discussed can be modelled
2) the results provided from the simulation can be analyzed in detail
3) alternate decision plans can easily be modelled and their performance readily identified
4) the method is straightforward enough to be explained to non-technical individuals
5) the determination of an optimal policy is straightforward

Two SLAM simulations were prepared: One which models the shipping decisions and a second which models the storage decisions. Appendix I depicts the shipping network and Appendix II depicts the storage model for one attribute under consideration. The first section of each Appendix (A) contains the SLAM network, the second section (B) contains the SLAM portion of the simulation model, the third section (C) certains the FORTRAN subroutines coded by the author. A description of each follows.

SLAM was selected as the modelling language since it can graphically depict the models and since it is easy to use--the analyst need not get bogged down with the details of maintaining files, etc. while simulating.
Shipping Network

The following attributes were established:

; ATRIB(1)=NOT USED
; ATRIB(2)=ACCUMULATED EPWS (ALL QUALITY ATTRIBUTES)
; ATRIB(3)=ACCUMULATED COST (ALL QUALITY ATTRIBUTES)
; ATRIB(4)=QUAL ATTRIBUTE 1:#OF TEST RESULTS OBTAINED
; ATRIB(5)=QUAL ATTRIBUTE 1:#OF GROUPS OF TEST RESULTS OBTAINED
; ATRIB(6)=QUAL ATTRIBUTE 1:COST ASSOCIATED WITH THIS QUAL ATRIB
; ATRIB(7)=QUAL ATTRIBUTE 1:EPWS THIS QUAL ATRIB
; ATRIB(8)=QUAL ATTRIBUTE 1:ACTUAL QUAL THIS QUAL ATRIB
; ATRIB(9)=QUAL ATTRIBUTE 1:CUST REACTION FLAG
; ATRIB(10)=QUAL ATTRIBUTE 1:ACTUAL FLAG THIS QUAL ATRIB
; ATRIB(11)=QUAL ATTRIBUTE 1:REFUTE FLAG THIS QUAL ATRIB
; ATRIB(12)=QUAL ATTRIBUTE 1:MEAN OF ABC TEST RESULTS TO DATE
; ATRIB(13)=QUAL Attribute 1:CUSTOMER MEAN OF TEST RESULTS
; ATRIB(14)=QUAL ATTRIBUTE 1:# OF REROUTES
; ATRIB(15)=# OF IN SPEC TEST RESULTS OBTAINED TO DATE

Several user functions were defined:

; USERF(1)= OBTAIN TEST RESULTS
; USERF(2)= DETERMINE CUSTOMER REACTION TO THIS ENTITY
; USERF(3)= DETERMINE ACTUAL QUALITY FLAG FOR THIS ENTITY
; USERF(4)= DETERMINE CUSTOMER REFUTATION FLAG
; USERF(5)= NOT USED
; USERF(6)= NOT USED
; USERF(7)= INCREMENT TEST COST

A node by node description of the SLAM shipping network (Appendix IA) follows:
Node(s) | Description
-----|-----
CREA | Creates transaction
AS01 | Assigns certain attributes to this transaction (this railcar of product)
| • The actual quality (average)
| • The number of test results to obtain on this pass
| • The EPWS for this attribute (assumes the measurements are normally distributed about the actual quality)
| • The cost associated with this transaction to date
G001 | A decision is made to either:
| • Classify this entity of products as prime and go to node G002, or
| • Classify this entity of product as offgrade and go to node AS04, or
| • Retest the entity of product and go to node AS02
AS02 | More test results are obtained. Costs are incremented and the estimate of the EPWS for this entity of product is updated. Control passes to node G001.
G002 | The customer's reaction to product we classified as prime is obtained.
AS03 | The customer obtains four test results (under the same level of imprecision and product variability as us) and then decides if the product is prime or offgrade strictly on the average of his test results. If the entity of product is deemed prime, the control passes to node G003, if not control passes to node AS06.
G003 | If the entity was truly prime, go to C001; if the entity was truly offgrade, go to AS05
CO01 | Collect statistics on this transaction
CO02 | • costs
CO03 | • number of tests
| • actual quality
<table>
<thead>
<tr>
<th>Node(s)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A050</td>
<td>Assign a customer service penalty cost for shipping truly offgrade material to a customer who mistakenly classified it prime. Collect statistics on this transaction:</td>
</tr>
<tr>
<td>C004</td>
<td>• costs</td>
</tr>
<tr>
<td>C005</td>
<td>• number of tests</td>
</tr>
<tr>
<td>C006</td>
<td>• actual quality</td>
</tr>
<tr>
<td>A060</td>
<td>An administrative cost to cover refutation is assigned to this entity since we classified product as prime and the customer has classified it as offgrade. We automatically attempt to refute the customer's claim. We successfully refute control passes to node G004, of not control passes to node G005.</td>
</tr>
<tr>
<td>G004</td>
<td>If the true quality of this entity was prime control passes to node C007, otherwise control passes to node A007.</td>
</tr>
<tr>
<td>C007</td>
<td>Collect statistics on this transaction.</td>
</tr>
<tr>
<td>C008</td>
<td>• costs</td>
</tr>
<tr>
<td>C009</td>
<td>• number of tests</td>
</tr>
<tr>
<td>C010</td>
<td>• actual quality</td>
</tr>
<tr>
<td>A007</td>
<td>A customer service penalty cost is assigned since we failed to refute statistics are collected for</td>
</tr>
<tr>
<td>C011</td>
<td></td>
</tr>
<tr>
<td>C012</td>
<td>• costs</td>
</tr>
<tr>
<td>G005</td>
<td>• number of tests</td>
</tr>
<tr>
<td>G006</td>
<td>• actual quality</td>
</tr>
<tr>
<td>G005</td>
<td>A decision is made with regard to diverting this material to another customer. This entity of product is not diverted (rerouted) to another customer if we only have one test result or if we are unsure it is prime. It is rerouted if we are confident it is prime and we have taken more than one test and we have not rerouted this entity more than four times yet. If we divert this entity control passes to node A008, otherwise control passes to node G006.</td>
</tr>
</tbody>
</table>

-58-
If the entity of product is prime, control passes to nodes AS09, otherwise control passes to node AS10.

A penalty cost is assigned since the misclassified prime product as offgrade and we believed him. Statistics are collected on:

- costs
- number of tests
- actual quality

A customer service penalty cost is assigned since we misclassified offgrade product as prime while the customer identified it as offgrade. Statistics are collected for:

- costs
- number of tests
- actual quality

The number of reroutes is incremented by one and the freight cost to divert this entity to another customer is assigned. Control passes to node G002.

The customer's reaction to material we have classified as offgrade is obtained. Again, the customer takes four test results, averages them, and if the average is within specification it would be classified as prime. If the customer classifies it prime, control passes to node G008, if not proceed to G009.

If this entity is truly prime, control passes to node AS11, of not go to AS12.

A customer service cost penalty is assigned to this transaction since it was truly prime, the customer classified it offgrade. Statistics are then collected for:

- cost
- number of tests
- actual quality

A cost penalty could be applied here since we could have sold offgrade material to the
<table>
<thead>
<tr>
<th>Node(s)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C023</td>
<td>customer as prime. Statistics are then collected for:</td>
</tr>
<tr>
<td></td>
<td>• cost</td>
</tr>
<tr>
<td></td>
<td>• number of tests</td>
</tr>
<tr>
<td></td>
<td>• actual quality</td>
</tr>
<tr>
<td>C024</td>
<td></td>
</tr>
<tr>
<td>G009</td>
<td>We have classified this entity of product as offgrade and the customer has agreed. If the actual quality of the product was pure then proceed to AS13, otherwise go to C028.</td>
</tr>
<tr>
<td>AS13</td>
<td>A cost penalty is assigned because although this entity of product was truly prime, both we and the customer classified it as offgrade.</td>
</tr>
<tr>
<td>C025</td>
<td>Statistics are then collected for:</td>
</tr>
<tr>
<td>C026</td>
<td>• cost</td>
</tr>
<tr>
<td>C027</td>
<td>• number of tests</td>
</tr>
<tr>
<td></td>
<td>• actual quality</td>
</tr>
<tr>
<td>C028</td>
<td>In reaching this node, we classified this entity as offgrade, the customer agreed, and it was truly offgrade collect statistics.</td>
</tr>
<tr>
<td>C029</td>
<td>• costs</td>
</tr>
<tr>
<td>C030</td>
<td>• number of tests</td>
</tr>
<tr>
<td></td>
<td>• actual quality</td>
</tr>
<tr>
<td>TR01</td>
<td>Terminates activity for this transaction: terminates the simulation after 10,000 transactions.</td>
</tr>
</tbody>
</table>
Storage Network

The following attributes are used:

- ATRIB(1) = USED FOR DUNCAN'S SEQUENTIAL TESTS
- ATRIB(2) = ACCUMULATED EPWS (ALL QUALITY ATTRIBUTES)
- ATRIB(3) = ACCUMULATED COST (ALL QUALITY ATTRIBUTES)
- ATRIB(4) = QUAL ATTRIBUTE 1: # OF TEST RESULTS OBTAINED
- ATRIB(5) = QUAL ATTRIBUTE 1: # OF GROUPS OF TEST RESULTS OBTAINED
- ATRIB(6) = QUAL ATTRIBUTE 1: COST ASSOCIATED WITH THIS QUAL ATRIB
- ATRIB(7) = QUAL ATTRIBUTE 1: EPWS THIS QUAL ATRIB
- ATRIB(8) = QUAL ATTRIBUTE 1: ACTUAL QUAL THIS QUAL ATRIB
- ATRIB(9) = QUAL ATTRIBUTE 1: CUST REACTION FLAG
- ATRIB(10) = QUAL ATTRIBUTE 1: ACTUAL FLAG THIS QUAL ATRIB FOR THIS LOT
- ATRIB(11) = QUAL ATTRIBUTE 1: ACTUAL FLAG FOR THE MIXTURE
- ATRIB(12) = QUAL ATTRIBUTE 1: MEAN OF ABC TEST RESULTS TO DATE
- ATRIB(13) = QUAL ATTRIBUTE 1: CUSTOMER MEAN OF TEST RESULTS
- ATRIB(14) = QUAL ATTRIBUTE 1: ACTUAL QUALITY OF QUALITY MIXTURE
- ATRIB(15) = QUAL ATTRIBUTE 1: # OF DEFECTS OBTAINED TO DATE
- ATRIB(16) = QUAL ATTRIBUTE 1: STD DEV OF RESULTS TO DATE
- ATRIB(17) = QUAL ATTRIBUTE 1: CALCULATED DC FOR WALSH PLAN

Again, several user functions were developed:

- USERF(1) = OBTAIN TEST RESULTS
- USERF(2) = DETERMINE CUST REACTION TO MIXTURE
- USERF(3) = DETERMINE ACTUAL QUALITY FLAG FOR THIS ENTITY
- USERF(4) = MIX PRODUCT IN OG SILO
- USERF(5) = MIX PRODUCT IN PRIME SILO
- USERF(6) = DETERMINE ACTUAL QUALITY FLAG FOR THE MIXTURE
- USERF(7) = INCREMENT TEST COST

A node by node description of the SLAM storage network (Appendix II A) follows:

<table>
<thead>
<tr>
<th>Node(s)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CREA</td>
<td>Creates transactions</td>
</tr>
<tr>
<td>ASO1</td>
<td>Assigns certain attributes to this transaction (this lot of product coming from the production line)</td>
</tr>
</tbody>
</table>

- The actual quality (average)
- The number of test results to obtain as this pass
- The EPWS for this attribute (assumes the measurements are normally distributed about the actual quality.
- The cost associated with this transaction to date
<table>
<thead>
<tr>
<th>Node(s)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GO01</td>
<td>A decision is made to either  &lt;br&gt;• Classify this lot of product as prime, put it into a prime silo, and go to node AS03; or  &lt;br&gt;• Classify this lot of product as offgrade, put it into an offgrade silo, and go to node AS04; or  &lt;br&gt;• Retest this lot of product and return to node AS02</td>
</tr>
<tr>
<td>AS02</td>
<td>More test results are obtained, costs are incremented and the estimated of the EPWS for this lot of product is updated. Control passes to node GO01</td>
</tr>
<tr>
<td>AS03</td>
<td>This manufactured lot of product is placed in a prime silo having an average prime quality. It is assumed to product in the silo mixes. The actual quality of the mixture is determined. The customer's potential reaction to the mixture is determined. The customer obtains four test results (under the same level of imprecision and product variability as us) and then decides if the product is prime or offgrade strictly as the average of his test results. If the mixture of product is deemed prime control passes to node GO04.</td>
</tr>
<tr>
<td>G003</td>
<td>If the mixture is truly prime proceed to node C001, if the mixture is truly offgrade proceed to node AS05.</td>
</tr>
<tr>
<td>C001</td>
<td>Collect statistics for this transaction:  &lt;br&gt;• costs  &lt;br&gt;• number of tests  &lt;br&gt;• actual quality of the lot</td>
</tr>
<tr>
<td>C002</td>
<td></td>
</tr>
<tr>
<td>C003</td>
<td></td>
</tr>
<tr>
<td>AS05</td>
<td>Assign a cost penalty for placing an offgrade lot of product into a silo prime product having the customer perceive the mixture as prime, and causing the mixture to become truly offgrade. Collect statistics on this transaction:</td>
</tr>
<tr>
<td>C004</td>
<td></td>
</tr>
<tr>
<td>C005</td>
<td></td>
</tr>
<tr>
<td>C006</td>
<td></td>
</tr>
<tr>
<td>C006</td>
<td></td>
</tr>
<tr>
<td>Node(s)</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>G004</td>
<td>If the mixture is truly prime, proceed to node AS06; if the mixture is truly offgrade proceed to node AS07.</td>
</tr>
<tr>
<td>AS06</td>
<td>A cost penalty could be applied here for placing a lot of product into a prime silo, having the mixture result in truly prime product, but having the customer perceive the mixture as offgrade material.</td>
</tr>
<tr>
<td>C007</td>
<td>Collect statistics on this transaction:</td>
</tr>
<tr>
<td>C008</td>
<td>- costs</td>
</tr>
<tr>
<td>C009</td>
<td>- number of tests</td>
</tr>
<tr>
<td>C012</td>
<td>- actual quality of the lot</td>
</tr>
<tr>
<td>AS07</td>
<td>Assign a cost penalty for placing an offgrade lot of product into a prime silo, having the customer perceive the mixture as offgrade, and causing the mixture to become truly offgrade. Collect statistics for this transaction:</td>
</tr>
<tr>
<td>C010</td>
<td>- costs</td>
</tr>
<tr>
<td>C011</td>
<td>- number of tests</td>
</tr>
<tr>
<td>C012</td>
<td>- actual quality of the lot</td>
</tr>
<tr>
<td>AS04</td>
<td>This manufactured lot of product is placed in an offgrade silo having an average offgrade quality. It is assumed the product in the silo mixes. The actual quality of the mixture is determined. The customer's potential reaction to the mixture is determined. The customer obtains four test results, averages them, and if the average is within specification, it is classified as prime. If the customer perceives the mixture as prime, proceed to node G008, otherwise, control passes to G009.</td>
</tr>
</tbody>
</table>
If the mixture is truly prime, proceed to node AS11, if the mixture is truly offgrade proceed to node AS12.

A truly prime manufactured lot is placed into an offgrade silo and the resulting mixture is customer prime. A contamination cost penalty could assigned if the mixture became truly prime via mixing. Collect statistics for this transaction:

- cost
- number of tests
- actual quality of the lot

A truly offgrade lot is placed into an offgrade silo and the resulting mixture is determined by the customer to be prime (probably an error). A benefit could be applied to reflect the ability of selling offgrade material as prime. Collect statistics for this transaction:

- cost
- number of tests
- actual quality of the lot

If the mixture is truly prime, proceed to node AS13, if the mixture is truly offgrade proceed to node CO28.

A truly prime manufactured lot is placed into an offgrade silo and the resulting mixture is perceived to be offgrade. Here a contamination cost penalty should be applied. Collect statistics for this transaction:

- cost
- number of tests
- actual quality of the lot

Collect statistics for this transaction:

- cost
- number of tests
- actual quality of the lot
<table>
<thead>
<tr>
<th>Node(s)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR01</td>
<td>Terminates activity for this transaction: terminates the simulation after 10,000 transactions.</td>
</tr>
</tbody>
</table>
Use of the Simulation Models

The SLAM shipping and storage simulation models just described can be used in two modes. In the first mode they can be used to isolate a least cost or "optimal" set of decision guidelines. This can be done by running several cases and noting the effect of alternate decision guidelines on the response variables--average incremental cost per transaction. By noting the effect of the response variable, one can identify the direction to move in setting input conditions to next evaluate. It is also possible to enumerate all possible cases. At any rate a least cost set of guidelines can be obtained.

In the second mode, these simulations can be run with guidelines of interest as input. System response can carefully be studied for any set of input conditions. This feature is extremely useful for evaluating decision guidelines that result from an alternate methodology. Very enlightening comparisons can be made.
V. COMMENT ON POSSIBLE SOLUTION TECHNIQUES

The material on the preceding pages here described several methodologies for solving quality assurance problems. This thesis will identify solutions to the standard problems from each of the methodologies and then rank the solution in terms of the response variable average cost per transaction.

The ranking will constitute a significant portion of the relative worth of these methodologies for the problem discussed here. The ranking will also be important when considering other problems for which the rigid mold of necessary assumptions for many of the previous methodologies does not fit. This will be discussed in the conclusions.
VI. MEASUREMENT ERROR - HOW TO MEASURE IT

This thesis suggests that measurement error is a significant factor in setting quality assurance discussion plans. This assumes that bias and random error can easily and accurately be determined.

Random Error

Random error can be calculated by using the methodology described here which is useful when repeated observations cannot be made or by observing technicians' deviation of repeated results on controlled samples. Bias can be computed by contrasting performance to average response on controlled samples.

The following example (extracted from the Grubb's article, "On Estimating Precision of Measuring Instruments and Product Variability" (and modified to illustrate computation of bias) depicts how these quantities could be computed:

In Table 5, there are listed the individual burning times of powder rain fuses as measured by each of three observers on 30 rounds of ammunition which were fired from a gun. The fuses were all set for a burning time of ten seconds.
The burning time of a fuse is defined as the interval of time which elapses from the instant the projectile leaves the gun muzzle until the fuse functions the projectile. The times given in Table 5 are measured by means of electric clocks. A switch on the gun muzzle starts three different electric clocks as the gun is fired and each observer stops his clock the instant he sees the flash or burst of an individual round. Each timer, of course, stops his clock independently of the other two timers. In Table 5 is given also the mean time of the three observers on individual rounds. The average time-to-burst of the 30 fuses is taken as the average of the 30 mean times with the effect of reaction time (which is known accurately) subtracted therefrom. The problem here is to determine whether the electric clock is a satisfactory instrument for measuring burning times, provided the electric clocks are properly calibrated with regard to average time so that the systematic error of measurement may be reconciled. This question is answered by comparing the variance in errors of measurement with the variance in burning times of the fuses.
In Table 5, there are given the fuse burning times by observer.

<table>
<thead>
<tr>
<th>Fuse Burning Times (seconds)</th>
<th>Mean Times (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observer A</td>
<td>Observer B</td>
</tr>
<tr>
<td>10.10</td>
<td>10.07</td>
</tr>
<tr>
<td>9.98</td>
<td>9.90</td>
</tr>
<tr>
<td>9.89</td>
<td>9.86</td>
</tr>
<tr>
<td>9.79</td>
<td>9.70</td>
</tr>
<tr>
<td>9.67</td>
<td>9.65</td>
</tr>
<tr>
<td>9.89</td>
<td>9.83</td>
</tr>
<tr>
<td>9.82</td>
<td>9.79</td>
</tr>
<tr>
<td>9.76</td>
<td>9.72</td>
</tr>
<tr>
<td>9.62</td>
<td>9.64</td>
</tr>
<tr>
<td>10.24</td>
<td>10.23</td>
</tr>
<tr>
<td>9.84</td>
<td>9.86</td>
</tr>
<tr>
<td>9.02</td>
<td>9.63</td>
</tr>
<tr>
<td>9.60</td>
<td>9.65</td>
</tr>
<tr>
<td>9.74</td>
<td>9.74</td>
</tr>
<tr>
<td>10.32</td>
<td>10.34</td>
</tr>
<tr>
<td>10.01</td>
<td>lost</td>
</tr>
<tr>
<td>9.65</td>
<td>9.64</td>
</tr>
<tr>
<td>9.50</td>
<td>9.50</td>
</tr>
<tr>
<td>9.56</td>
<td>9.55</td>
</tr>
<tr>
<td>9.54</td>
<td>9.54</td>
</tr>
<tr>
<td>9.89</td>
<td>9.88</td>
</tr>
<tr>
<td>9.53</td>
<td>9.51</td>
</tr>
<tr>
<td>9.86</td>
<td>9.84</td>
</tr>
</tbody>
</table>

In Table 6, there are given the algebraic differences between times of the three observers A, B, and C on each round.
It is to be noted that any one of the differences listed is not influenced by the level of burning time of an individual fuse and represents the actual difference in errors of measurement of the two observers involved. Thus, the variance of each of the columns headed A-B, B-C, and A-C, gives the variability of the difference in errors of measurement of the two observers compared. Therefore, taking the variances of the three columns in Table 6, the following three equations are arrived at.
where \( e_1 \), \( e_2 \) and \( e_3 \) represent the errors of measurement of the observers A, B, and C, respectively.

\[
\sigma_{e_1}^2, \sigma_{e_2}^2, \text{ and } \sigma_{e_3}^2 \text{ it is found that}
\]

\[
\text{est} \left( \sigma_{e_1}^2 \right) = \frac{1}{2} \left( S_{e_1-e_2}^2 + S_{e_1-e_3}^2 - S_{e_2-e_3}^2 \right)
\]

\[
\sigma_{e_1} = .0253 \text{ sec.}
\]

\[
\sigma_{e_1} = .0079 \text{ sec.}
\]

\[
\sigma_{e_2} = .0157 \text{ sec.}
\]

The above figures give a direct comparison of the ability or decision of each of the observers. It is seen that observer B is perhaps the best of the three since his precision of measurement is given by a standard deviation of only .0079 second, whereas observer A is the poorest timer of the three. As a matter of fact, one can look at the columns in Table II headed A-B and A-C and note that or the first 10 rounds or so observer A had a definite lag in stopping his clock as compared to the other two observers.

An estimate of \( \sigma_x^2 \) is obtained by using formula

\[
(\sigma_x^2) = S_x^2 + 1/3 \ e_1 + 1/3 \ e_2 + 1/3 \ e_3 - 1/18
\]

\[
(S_{e_1-e_2} + S_{e_1-e_3} + S_{e_1-e_2})
\]
That is, by subtracting \( \frac{1}{18} \) of the sum of the variances of differences (a) from the variance of mean times in column four of Table I. Thus,

\[
\text{est. } (\sigma_x^2) = \frac{1}{18} \left(0.0007030 + 0.0008878 + 0.0003108\right) = 0.04599
\]

or \( \sigma_x = 0.2145 \)

Thus, it is seen that the variation in burning times, \( \sigma_x' \), is about 8.5 times the variance in errors, \( \sigma_{e1} \), for the poorest operator and about 12 times the average variability in errors of measurement of the three operators combined. Consequently, it is found that the combination of electric clocks and operators provides an adequate measuring instrument.

Random error could also be determined by individually analyzing technician responses on homogeneous samples of unknown quality provided the quality being measured is easily replicated.

**Bias**

Bias (systematic measurement error) can be computed by first recording the performance of observers A, B, and C relative to the average response. Table 7 shows the necessary calculations. (This approach assumes the average is the best approximation of the true quality).
TABLE 7
DIFFERENCE BETWEEN OBSERVERS' AND AVERAGE TIMES
(seconds)

<table>
<thead>
<tr>
<th>A-AVG.</th>
<th>B-AVG.</th>
<th>C-AVG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>+.020</td>
<td>-.010</td>
<td>-.010</td>
</tr>
<tr>
<td>+.053</td>
<td>-.027</td>
<td>-.027</td>
</tr>
<tr>
<td>+.023</td>
<td>-.017</td>
<td>-.007</td>
</tr>
<tr>
<td>+.057</td>
<td>-.023</td>
<td>-.033</td>
</tr>
<tr>
<td>+.013</td>
<td>-.007</td>
<td>-.007</td>
</tr>
<tr>
<td>+.040</td>
<td>-.020</td>
<td>-.020</td>
</tr>
<tr>
<td>+.033</td>
<td>-.037</td>
<td>+.003</td>
</tr>
<tr>
<td>+.010</td>
<td>-.020</td>
<td>+.010</td>
</tr>
<tr>
<td>+.040</td>
<td>-.040</td>
<td>0</td>
</tr>
<tr>
<td>+.017</td>
<td>-.023</td>
<td>+.007</td>
</tr>
<tr>
<td>-.003</td>
<td>-.013</td>
<td>+.017</td>
</tr>
<tr>
<td>+.003</td>
<td>-.007</td>
<td>+.003</td>
</tr>
<tr>
<td>-.003</td>
<td>-.013</td>
<td>+.017</td>
</tr>
<tr>
<td>+.010</td>
<td>-.030</td>
<td>+.020</td>
</tr>
<tr>
<td>-.017</td>
<td>-.017</td>
<td>+.033</td>
</tr>
<tr>
<td>+.003</td>
<td>-.007</td>
<td>+.003</td>
</tr>
<tr>
<td>-.007</td>
<td>-.007</td>
<td>+.013</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-.010</td>
<td>0</td>
<td>+.010</td>
</tr>
<tr>
<td>+.003</td>
<td>-.007</td>
<td>+.003</td>
</tr>
<tr>
<td>+.003</td>
<td>-.007</td>
<td>+.003</td>
</tr>
<tr>
<td>+.003</td>
<td>+.003</td>
<td>-.007</td>
</tr>
<tr>
<td>+.003</td>
<td>-.007</td>
<td>+.003</td>
</tr>
<tr>
<td>+.003</td>
<td>+.003</td>
<td>-.007</td>
</tr>
<tr>
<td>+.010</td>
<td>0</td>
<td>-.010</td>
</tr>
<tr>
<td>-.003</td>
<td>-.003</td>
<td>+.007</td>
</tr>
<tr>
<td>0</td>
<td>-.010</td>
<td>+.010</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-.010</td>
<td>+.010</td>
</tr>
<tr>
<td>+.007</td>
<td>-.013</td>
<td>+.007</td>
</tr>
</tbody>
</table>
Estimates of bias are therefore:

Observer A = Σ (A-Avg)/30 = +.011
Observer B = Σ (B-Avg)/30 = -.012
Observer C = Σ (C-Avg)/30 = -.008

A summary of results from this analysis is shown in Table 8.

TABLE 8: SUMMARY OF ERROR DETERMINATION

<table>
<thead>
<tr>
<th>BIAS (Seconds)</th>
<th>RANDOM ERROR (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observer A</td>
<td>+.011</td>
</tr>
<tr>
<td>Observer B</td>
<td>-.012</td>
</tr>
<tr>
<td>Observer C</td>
<td>-.008</td>
</tr>
</tbody>
</table>

Product Variability = .2145 Seconds
VII. MEASUREMENT ERROR - HOW TO REDUCE IT

Assume an important quality assurance test is subject to a significant amount of "random" measurement error, indicated by historical auditing of the test procedure. The procedure in question involves three steps. The first step is to process a sample of the product and change its form by creating a film. The second step is to select a portion of that film for inspection. The third step involves a technician counting imperfections within the selected portion of film. A further complication in routine testing of product is that samples from the same aggregate may not be of homogeneous quality.

Management was extremely concerned with the performance of the testing procedure, in an attempt to assure successful application of the product in customers' processes. Therefore, management wanted to focus their attention in the areas of the testing procedure which would lead to the greatest improvement, or largest reduction in seemingly random test error.

Fortunately, the incremental expense associated with obtaining test results is minimal. This expense is extremely slight when compared to the benefit that would accrue to the company by improving the quality of decisions made with the test procedure.
Original Design

Since the important consideration during the analysis was the test procedure itself, the impact of non-homogeneous samples was divorced from the present investigation. In fact, all test results were obtained from a sample of material which was processed to assure product quality homogeneity during the original analysis.

After soliciting advice from product management personnel, the following factors were isolated as being important potential contributors to the seemingly random measurement error. These items were:

1. The processing of the sample to create a film.
2. The selection of an area from which to count imperfections.
3. Differences in counting techniques by different technicians.
4. Inconsistent application of counting techniques by the same technician.

The test results were obtained over a very short period of time, hence the effect of time was ignored. It was further assumed that technician performance did not change during the day.

A nested, full factorial design with replication was suggested. Pictorially the design is shown in Table 9:
Replication was accomplished by obtaining two observations within each cell (x). The total number of observations obtained was ninety; this was not an extreme expense and the design was accepted. The film location factor was nested within the processing run. The positions of "a" were generally the same for each processing run, but the selected films were not interchangeable and therefore the design was nested. Each of the factors (processing run, film location, technician, and replicate) were qualitative in nature.

The results of the test observations for this design are shown in Table 10.
TABLE 10: REPLICATED TEST RESULTS FOR FULL FACTORIAL DESIGN

<table>
<thead>
<tr>
<th>PROCESSING RUN</th>
<th>FILM LOCATION</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>21/21</td>
<td>10/11</td>
<td>10/14</td>
<td>12/15</td>
<td>3/6</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>14/14</td>
<td>13/11</td>
<td>12/8</td>
<td>11/10</td>
<td>10/9</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>21/20</td>
<td>8/9</td>
<td>9/13</td>
<td>8/13</td>
<td>9/6</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>10/10</td>
<td>19/16</td>
<td>22/21</td>
<td>21/17</td>
<td>8/11</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>21/21</td>
<td>17/13</td>
<td>20/19</td>
<td>18/14</td>
<td>10/15</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>14/15</td>
<td>12/10</td>
<td>12/12</td>
<td>10/9</td>
<td>5/7</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>14/24</td>
<td>12/14</td>
<td>9/13</td>
<td>12/14</td>
<td>8/10</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>23/21</td>
<td>21/23</td>
<td>17/20</td>
<td>19/18</td>
<td>16/20</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>21/14</td>
<td>14/14</td>
<td>17/19</td>
<td>13/16</td>
<td>15/15</td>
</tr>
</tbody>
</table>

KEY: Replicate 1 /Replicate 2

Original Analysis

Because the assumption underlying ANOVA that the experimental errors must be uniform was felt not to be true for the raw data, a transformation of the data was performed prior to the analysis. It was thought that the truely random error as measured by the standard deviation was proportional to the quality being measured, and consequently the logarithmic transformation was applied.

Historical goodness of fit tests on observed random errors have indicated that the null hypothesis that the errors are normally distributed could not be rejected. This was true even though the distribution in this case was discrete.

The calculations associated with the analysis of the results shown in Table
10 are depicted in Table 11 and Table 12. Table 11 is a hand calculation of the ANOVA table and Table 12 is a computer assisted analysis.

TABLE 11
MANUAL CALCULATIONS OF EFFECTS FOR FULL FACTORIAL DECISION

<table>
<thead>
<tr>
<th>Correction for the Mean = CM = (233.353**2)/90 = 605.040</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total SS = TOT SS = 618.395 - CM = 13.354</td>
</tr>
<tr>
<td>where 618.395 = the sum of every result squared</td>
</tr>
<tr>
<td>PR SS = (71.533<strong>2 + 79.326</strong>2 + 82.494**2)/30 - CM = 2.121</td>
</tr>
<tr>
<td>where 71.533 etc. are the sums by Processing Run</td>
</tr>
<tr>
<td>T SS = (52.516<strong>2 + 46.425</strong>2 + 47.682<strong>2 + 46.741</strong>2 + 39.989**2)</td>
</tr>
</tbody>
</table>
| /
| where 52.516 etc. are the sums by Technician            |
| FL(PR) SS = (23.814**2 + 24.006**2 + 23.712**2)/10 - (71.533**2/30) |
| + (28.199**2 + 27.967**2 + 23.160**2)/10 - (79.326**2/30) |
| + (25.209**2 + 29.792**2 + 27.493**2)/10 - (82.494**2/30) |
| = 2.668                                                 |
| where 23.814 etc. are the Film Location Sums and        |
| where 71.533 etc. are the Processing Run sums           |
| REP SS = (115.369**2 + 117.984**2)/45 - CM = .076       |
| where 115.369 etc. are the sums by Replicate            |
| + 17.428**2 + 15.903**2 + 17.046**2 + 15.907**2 + 13.043**2 |
| + 17.681**2 + 16.582**2 + 16.369**2 + 16.296**2 + 15.566**2) |
| /
| where 17.407 etc. are the sums for a Processing Run and  |
| Technician combination.                                 |

The computational method for the T*FL(PR) SS could not be found or it would have been included. Consequently, the residual or error sum of squares is:

<table>
<thead>
<tr>
<th>RES SS = TOT SS - PR SS - T SS - FL(PR) SS - REP SS - PR*T SS</th>
</tr>
</thead>
</table>

-80-
**TABLE 12: COMPUTER CALCULATION OF EFFECTS FOR FULL FACTORIAL DESIGN**

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>DF</th>
<th>SUM OF SQUARES</th>
<th>MEAN SQUARE</th>
<th>F VALUE</th>
<th>PR &gt; F</th>
<th>PHASES</th>
<th>C.V.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODEL</td>
<td>21</td>
<td>2,121,327.26</td>
<td>105,340.29</td>
<td>19.67</td>
<td>0.001</td>
<td>0.77</td>
<td>0.344</td>
</tr>
<tr>
<td>REPS</td>
<td>3</td>
<td>2,683.00</td>
<td>894.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COMBINATION TOTAL</td>
<td>44</td>
<td>2,123,910.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SOURCE</td>
<td></td>
<td>ANOVA SS</td>
<td>F VALUE</td>
<td>PR &gt; F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UTPS</td>
<td>7</td>
<td>2,121,327.26</td>
<td>20.41</td>
<td>0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIRCS</td>
<td>6</td>
<td>2,683.00</td>
<td>9.07</td>
<td>0.104</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>4</td>
<td>894.33</td>
<td>2.71</td>
<td>0.371</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T2</td>
<td>4</td>
<td>894.33</td>
<td>2.71</td>
<td>0.371</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCP</td>
<td>1</td>
<td>894.33</td>
<td>2.71</td>
<td>0.371</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Revised Problem Description

It did occur to the author that it would be of interest to design a set of experiments to achieve the same goal, but under the assumption that test results were extremely expensive to obtain. The potential payoff from rectifying the test procedure would still favorably balance the cost of experiments and analysis, however.

Revised Design

The original design called for ninety observations. The intent of the revised design is to identify the major contributors to the suggested random error at the lowest level of required test results. Replication is not investigated.

Two designs are proposed - one where the film location factor is assumed to be non-nested and a second where the nesting is observed. The two designs are provided to discuss the extra tests required to quantify the nested effect and because the nested design seems to be a logical extension of the non-nested design. In both cases only the three remaining main effects are of interest.

A fractional factorial design is proposed for the non-nested case. The following design is based upon suggestions from Davies' book and from an understanding of the necessary calculations provided from Duncan's "Quality Control and Industrial Statistics." This design is depicted pictorially in Table 13.
TABLE 13
NON-NESTED REVISED DESIGN

<table>
<thead>
<tr>
<th>PROCESSING RUN 1</th>
<th>PROCESSING RUN 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>FILM LOC A</td>
<td>FILM LOC A</td>
</tr>
<tr>
<td>FILM LOC C</td>
<td>FILM LOC C</td>
</tr>
<tr>
<td>Tech 1</td>
<td>-</td>
</tr>
<tr>
<td>Tech 2 X3</td>
<td>X1 X2</td>
</tr>
<tr>
<td></td>
<td>X2</td>
</tr>
</tbody>
</table>

The ANOVA calculations would then become:

TABLE 14: COMPUTATIONS REQUIRED FOR EFFECTS OF NON-NESTED REVISED DESIGN

Correction for the Mean = CM = (X1+X2+X3+X4)/4
Between Tech SS = T SS = ((X1+X2)**2 + (X3+X4)**2)/2 - CM
Between Processing Run SS = PR SS = ((X2+X4)**2 + (X1+X3)**2)/2 - CM
Between Film Location SS = FL SS = ((X2+X3)**2 + (X1+X4)**2)/2 - CM
Total SS = TOT SS = X1**2 + X2**2 + X3**2 + X4**2 - CM
Residual(error) SS = TOT SS - T SS - PR SS - FL SS

The second revised design shown in Table 14 is a factorial that allows the calculation of the nested effects, similar to the original design. Again, this design is prompted by an understanding of the method for calculating the effects.

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TABLE 15: NESTED REVISED DESIGN

<table>
<thead>
<tr>
<th>PROCESSING RUN 1</th>
<th>PROCESSING RUN 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>FILM LOC A</td>
<td>FILM LOC C</td>
</tr>
<tr>
<td>Tech 1</td>
<td>X1  X2</td>
</tr>
<tr>
<td>Tech 2</td>
<td>X5  X6</td>
</tr>
<tr>
<td>Tech 1</td>
<td>X3  X4</td>
</tr>
<tr>
<td>Tech 2</td>
<td>X7  X8</td>
</tr>
</tbody>
</table>

the ANOVA calculations for the second revised design would be:

TABLE 16: COMPUTATIONS REQUIRED FOR EFFECTS OF NESTED REVISED DESIGN

Tech 1 Sum = Sum1 = X1+X2+X3+X4
Tech 2 Sum = Sum2 = X5+X6+X7+X8

Correction For the Mean = CM = (X1**2 + ... X8**2)/8

Between tech SS = T SS = (Sum1**2 + Sum2**2)/4 - CM

Processing Run 1 Sum = Sum3 = X1+X2+X5+X6
Processing Run 2 Sum = Sum4 = X3+X4+X7+X8

Between Processing Runs
= PR SS = (Sum3**2 + Sum4**2)/4 - CM

Between Film Locations Within Processing Run SS
= FL(PR) SS = (((X1+X5)**2 + (X2+X6)**2
- (X1+X2+X5+X6)**2/4)
+((X3+X7)**2 + (X4+X8)**2
- (X3+X4+X7+X8)**2/4))

Total SS = TOT SS = X1**2 ... X8**2 - CM

Residual(error) SS = TOT SS - T SS - PR SS - FL(PR) SS

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Conclusions From Original Design Results

The ANOVA associated with the original design (shown in Tables 11 and 12) indicated that the following significant factors contributed to the test error. They are listed in order of decreasing importance as ranked by mean squares:

1. Between Technician
2. Between Processing Runs
3. Between Film Locations Within a Processing Run

The replicate F-value was not significant at the .001 level, nor was the F-value for the interaction of technicians and processing runs. Management could reduce the test error by focusing their attention on the three factors listed above in the order of their appearance.

The revised nested design reduced the necessary experiments from ninety to eight. The main effects (neglecting replication) can still be calculated.
VIII. IDENTIFICATION OF RELEVANT COSTS

The ability to use any approach which deals with costs requires some reflection, and possible analysis, to determine the necessary cost values. There are some such as W. Edward Deming who feel that the costs of "loss of future business from a dissatisfied customer...may be enormous, and is unfortunately impossible to estimate."

This is exactly the mentality that needs to be challenged. In the amount of time that is spent arguing about the ability to determine the required costs, the necessary costs, and any interesting sensitivities could be resolved.

Costs will be used in the author's approach. The derivation of the costs used will be covered in the description of the shipping and storage standard problem.
IX. PRODUCT VARIABILITY

The notion of product variability ($\sigma_{\text{LOT}}$ in many of the articles) is very important when setting decision plans. It is almost certain that material at the top of a bulk rail car (or manufactured lot) will be of a different composition than the material at the bottom of the rail car (or manufactured lot). When dealing with material in a large bulk container, the contents of the container will either be accepted or rejected. As such components of the large container cannot be dealt with individually.

One mechanism for dealing with the product variability concern is to make use of continuous sampling. This technique periodically takes small quantities of product and then forms a composite sample by aggregating the small portions of product. This perspective would mean that

$$\sigma_{\text{OBSERVED}} = \sigma_{\text{RANDOM ERROR}}$$

and that there is no product variability. The sample is assumed to be totally representative.

Another perspective which can be taken is that product variability is inherent in the observations. In this, grab samples would likely be the mechanism for obtaining product samples for testing. In this case, the observed test results would be prone to variability

$$\sigma^2_{\text{OBSERVED}} = \sigma^2_{\text{RANDOM ERROR}} + \sigma^2_{\text{LOT}}$$

The beauty of the SLAM simulation models is that either of the perspectives described above can easily be modelled. In this analysis, the latter situation will be assumed.
X. STANDARD PROBLEM 1

A representative shipping quality assurance decision problem was formulated and will be described here.

The following input considerations apply:

- Specifications: 80-120 is prime, offgrade otherwise (variables)

- Error of Measurement:

<table>
<thead>
<tr>
<th>Bias</th>
<th>( \sigma ) Random Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC Corp.</td>
<td>0</td>
</tr>
<tr>
<td>Customer(s)</td>
<td>0</td>
</tr>
</tbody>
</table>

- \( \sigma_{LOT} \) (product variability) = 15

- \( \sigma_{OBSERVED} = \sqrt{\sigma^2_{LOT} + \sigma^2_{RANDOM ERROR}} = 21.21 \)

- Actual product quality varies between 50 and 150, uniformly

- A summary of costs:
  - $10.00 per test result obtained
  - Node AS05: $50,000.00
    There a customer mistakenly accepted as prime a 170,000 lb. rail car of truly offgrade material that ABC Corp. classified as prime. The $50,000 results from 100 manhours being spent (@ $100/hr.) plus $15,000 of new equipment plus $25,000 of customer goodwill loss. The $25,000 is the margin associated with 125,000 lbs. (@$.20/lb) of lost business.
  - Node AS06: $1,000.00
    The $1,000 covers the approximately ten hours (@ $100/hr) spend to refute the customer's response that material ABC Corp. classified as prime was offgrade.
• Node AS07: $10,000.00
  The customer will react to being misled into accepting truly offgrade product as prime by
  1) Blending the product with truly offgrade, and
  2) Diverting 50,000 lbs (@ $.20/lb) to other producers.

• Node AS08: $3,000.00
  $2,000 of freight will be incurred and $1,000 of management's time (@ $100/hr).

• Node AS09: $17,000.00
  ABC Corp. lost the differential between the priced prime and offgrade product ($.10/lb) on a 170,000 lbs. railcar.

• Node AS11: $17,000.00
  ABC Corp lost the differential between the price of prime and offgrade product ($.10/lb) or a 120,000 lb. rail car.

• Node AS13: $17,000.00
  ABC Corp lost the differential between the price of prime and offgrade product ($.10/lb) on a 100,000 lb. rail car.

The results of interest are:

DC2A: the prime threshold.
DC3A: the offgrade threshold.
DC3N: the maximum number of tests allowed before a decision is made.
DC4A: the reroute threshold.
DC5N: the maximum number of reroutes allowed.
II1: the number of test results to obtain initially
II2: the number of test results to obtain on a subsequent sample.

Since simulation was the technique developed for solution by the author, certain simulation issues needed to be addressed such as:

1. How long to simulate? (in number of transactions)
2. How should seeds be used?
3. Should any transactions be thrown out before statistics are collected?
4. How should cases (scenarios) be evaluated in order to identify the optional plan?
Length of Simulation/Use of Seeds

How long should the simulation be run? If it is desirable to estimate the average cost per transaction within ± σ/50 with a probability of 0.95.

\[ n = \frac{(1.96\sigma)^2}{\frac{\sigma}{50}} \]  

\[ n = \frac{(50)^2 (1.96)^2 \sigma}{\sigma^2} \]  

\[ n = 9604 \]

Or, assume we wish to estimate \( \sigma^2 \) within 5% with a probability of 0.95

\[ n = 1 + 2 \frac{(1.96)^2}{.05^2} \]  

\[ n = 3074 \]

Another approach would suggest finding such that the average cost per transaction is estimated to within $70 with a probability of .95. A prior simulation experiment has indicated that σ for the cost per transaction is approximately 4,000. Therefore,
\[ n = \frac{(\sigma Z a / 2)^2}{d^2} \quad \text{Shannon 5.2} \]

\[ n = 12,544 \]

By properly controlling the seeds though we may be able to reduce the variance of the response variable, though. Correlated sampling is a technique for doing just that. Since we will be comparing average cost per transaction from one case to another, the variance of the difference will be:

\[ \text{Var (Diff)} = \text{Var (Case 1)} + \text{Var (Case 2)} - 2 \cdot \text{Cov (Case 1, Case 2)} \]

Now 2 Cov (Case 1, Case 2) will be:

\[ 2 \cdot \rho \cdot S (\text{Case 1}) \cdot S (\text{Case 2}) \]

Therefore, we want to drive \( \rho \) as close to 1 as we can. In order to accomplish this, the seed used to control the actual quality of the transactions being simulated was used solely for that purpose -- insuring the same actual quality transactions would be simulated for each case. This will make our previous estimates for \( n \) conservative.

From a practical point of view you want to simulate until stable results appear. To judge this a particular case was run for a
variety of numbers of transactions to be simulated. For this case, the following input parameters were used.

\begin{align*}
\text{DC2A} &= 0.10 \\
\text{DC3A} &= 0.50 \\
\text{DC4A} &= > 1.00 \\
\text{II}_1 &= 10 \\
\text{II}_2 &= 10 \\
\text{DC3N} &= 500 \\
\text{DC4N} &= 5 \\
\text{DC5N} &= 3
\end{align*}

The results were:

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{n} & \textbf{Avg. Cost Per Transaction} & \textbf{Std. Dev. of Cost per Transaction} \\
\hline
100 & 2378 & 5285 \\
1,000 & 2287 & 5226 \\
2,500 & 2373 & 5312 \\
5,000 & 2403 & 5344 \\
7,500 & 2419 & 5351 \\
10,000 & 2380 & 5313 \\
\hline
\end{tabular}
\caption{Shipping Model Results for Various Simulation Lengths}
\end{table}

The results appeared stable across the range of \textbf{n}.

One final consideration, also a practical one, was the computer execution time and cost of a case. The time and cost of simulation run for 10,000 transactions was acceptable. When the number of transactions exceeded this level, given the feedback potential of the network (testing and reroutes), the cost and time began to
become unwieldy.

**Truncation of Initial Conditions**

There are no aspects of this simulation (outside of the response variable) which need to reach a "steady state." Therefore, statistics will be collected starting with the first transaction.

**Determination of Optimum Conditions**

Two basic approaches exist for locating optimum conditions with a simulation model

1) Evaluation of independent variables.
   
   a) Mathematically naive techniques
      
      1. Heuristic search
      2. Complete enumeration
      3. Random search

   b) Methods appropriate to unimodal objective functions:
      
      1. Coordinate search
      2. Pattern search

2) Response surface methodology using experimental design and steepest ascent.

The least cost decision criteria were isolated via "sectioning," a heuristic search technique. The independent factors were ranked in terms of perceived importance, least cost guidelines were
identified for the most important two factors, then three factors, then four factors... until all factors had been considered.
Results

The least cost decision criteria for this standard problem were found to be:

\[
\begin{align*}
DC2A &= 0.03 \\
DC3A &= 0.90 \\
DC4A &= 0.70 \\
II_1 &= 10 \\
II_2 &= 10 \\
DC3N &= 500 \\
DC4N &= 4 \\
DC5N &= 3 \\
\end{align*}
\]

The results were found via sectioning and a summary of the results is found in Appendix III. Approximately one hundred cases were run. The search began by isolating best combinations of DC2A and DC3A. An initial set of cases yielded the following response surface.
FIGURE 5: SHIPPING MODEL RESPONSE SURFACE

PRIME VS. OFFGRADE THRESHOLD
IMPACT ON AVERAGE TRANSACTION COST

GEORGE A. FREESTONE JUNE 1982
A detailed analysis of the lower left corner indicated the following situation.

FIGURE 6: DETAILED SHIPPING MODEL RESPONSE SURFACE

PRIME VS. OFFGRADE THRESHOLD DETAIL

The least cost guideline summary results are:

- Average cost per transaction: $1,156
- Standard deviation of cost per transaction: $3,241
- Percentage of material classified as prime that was truly prime: 96%
- Percentage of material classified: 2%
as offgrade that was truly prime
Number of transactions rerouted 664

In no rerouting is allowed, then the least cost guidelines have the following results:

- Average cost per transaction $1,775
- Standard deviation of cost per transaction $4,851
- Percentage of material classified as prime that was truly prime 96%
- Percentage of material classified as offgrade that was truly prime 2%

Results from Other Methodologies

Duncan Variables Acceptance Sampling Plan

- Specifications: 80-120
- \( \sigma_{\text{LOT}} = 15 \)
- Solve for \( X_U, L, \) and \( n \)
  Assume, as Duncan does, that \( \alpha = .05, \beta = .10 \)

Then by writing equations for the probability of accepting a bad lot.

\[
\begin{align*}
(1) \quad X_U - 120 &= -1.282 \\
\frac{\text{---}}{15/\sqrt{n}}
\end{align*}
\]

\[
\begin{align*}
(2) \quad X_L - 80 &= 1.282 \\
\frac{\text{---}}{15/\sqrt{n}}
\end{align*}
\]
And by writing equations for the probability of accepting a good lot.

(3) \[ X_U - 100 = -1.960 \]
\[ \frac{\text{---}}{15/\sqrt{n}} \]

(4) \[ X_L - 100 = -1.960 \]
\[ \frac{\text{---}}{15/\sqrt{n}} \]

Now, solving the four equations for the three unknowns:
(1) + (2): \[ X_U - 120 + X_L - 80 = 0 \]
\[ X_U + X_L = 200 \]

(1) + (3): \[ \sqrt{n} = \frac{(1.282 + 1.960)}{20} \]
\[ n = 6 \]

Substitute \( n = 6 \) into (1)
\[ X_U - 120 = -1.282 \]
\[ 15/\sqrt{6} \]
\[ X_U = 120 - 7.8 \]
\[ X_U = 112 \]

\[ X_L = 88 \]

The plan \( X_U = 112, X_L = 88, n = 6 \) is GAFOOl (1)
The OC-curve for this plant is shown below in Figure 7.

FIGURE 7: OPERATING CHARACTERISTIC CURVE FOR DUNCAN VARIABLES PLAN
This plan was simulated for 10,000 transactions and without allowing any reroutes. A summary of the results is:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$3,485</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$6,784</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>93%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>23%</td>
</tr>
</tbody>
</table>

A Duncan plan with \( \alpha = \beta = .10 \) would also be of interest since we cannot be sure \( \alpha = .05, \beta = .10 \) is correct.

(1) \( X_U - 120 = -1.282 \)

\[
\frac{15}{\sqrt{n}}
\]

(2) \( X_L - 80 = -1.282 \)

\[
\frac{15}{\sqrt{n}}
\]

(3) \( X_U - 100 = -1.645 \)

\[
\frac{15}{\sqrt{n}}
\]

(4) \( X_L - 100 = -1.645 \)

\[
\frac{15}{\sqrt{n}}
\]
Again, \( X_U + X_L = 200 \)

\[
\sqrt{n} = \frac{(1.282 + 1.645)}{20} \tag{15}
\]

\( n = 5 \)
\( X_U = 111 \)
\( X_L = 89 \)

The plan \( X_U = 111, X_L = 89, n = 5 \) is GAF001(2)

<table>
<thead>
<tr>
<th>Average cost per transaction</th>
<th>$3,774</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$6,985</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>92%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>25%</td>
</tr>
</tbody>
</table>

A Duncan sequential sampling plan will also be setup. Since our case is a two-sided test, we need to compute:

\[
T = h_2 + sn \\
T = -h_2 - sn \\
T = -h_1 + sn \\
T = -h_1 - sn
\]
Let $\alpha = .05$, $\beta = .10$

Then:

$$a^* = 2.3026 \log_{10} (.90/.025) = 3.5835$$
$$b^* = 2.3026 \log_{10} (.975/.10) = 2.2773$$

$$\frac{(2.2773) (15)^2}{(120-20) 12} = \frac{h_1}{12} = 25.62$$

$$\frac{(3.5835) (15)^2}{(120-20) 12} = \frac{h_2}{12} = 40.32$$

$$s = \frac{120 - 80}{4} = 10$$

The relevant decision thresholds would be.

$\text{Reject}_1 (n) = h_1 + 10n$
$\text{Reject}_2 (n) = -h_2 - 10n$

$\text{Accept}_1 (n) = h_2 - 10n$
$\text{Accept}_2 (n) = -h_1 + 10n$

The rule is:

Compute $TR = \sum_i (\text{OBS}_i - 100)$

If $TR > \text{Reject}_1 (n)$ or $TR < \text{Reject}_2 (n)$, reject
if $TR > \text{Accept}_1 (n)$ and $TR < \text{Accept}_2 (n)$, accept
if neither, sample again

Call this plan GAF001(3). The plan is depicted in Table 16.
FIGURE 8: SEQUENTIAL VARIABLES PLAN

SHIPPING PROBLEM SEQUENTIAL DUNCAN PLAN

GEORGE A. FREESTONE JUNE 1962
The Duncan sequential plan was simulated for 10,000 transactions and without allowing any reroutes. The results are:

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$6,914</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$8,351</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>80%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>41%</td>
</tr>
</tbody>
</table>

The results were obtained with truncation of the testing at 500 tests for each transaction.

Deming Plan

The plan was set up following the advice of Deming. First, the relevant costs were estimated:

\[
K_2 = \text{cost of shipping a defective unit as prime} = $10,000 + \\
K_1 = \text{cost to test an entity} = $10
\]

The fraction of defective units produced in the simulation was 50% = p.

Then:

\[
K = \frac{K_2}{K_1} = \frac{$10,000}{$10} = 1,000
\]

and \( K_p = 500 \)
Since $K_p > 1$, then a total inspection of the 50 lb. components of the railcar should be conducted. Call this plan GAF003(1).

A summary of the simulation results for a case of 1,000 transactions and no reroutes follows: (the number of transactions was curtailed due to the large number of tests)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$35,430</td>
</tr>
<tr>
<td>Standard deviation of cost per</td>
<td>$4,824</td>
</tr>
<tr>
<td>transaction</td>
<td></td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>100%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>1%</td>
</tr>
</tbody>
</table>

The value of this plan is that you do not make mistakes, the price is that unnecessary tests are performed. The alternate Deming plan, no testing, was also simulated. The results from that case for 10,000 transactions and no reroutes is:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$4,274</td>
</tr>
<tr>
<td>Standard deviation of cost per</td>
<td>$5,442</td>
</tr>
<tr>
<td>transaction</td>
<td></td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>41%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>--</td>
</tr>
</tbody>
</table>

This case suggests the worst cost case, really. No attempt is made to discriminate, all material is shipped as prime. This plan is GAF003(2).
Grubbs and Coon Plans

The first Grubbs and Coon criteria used to derive a plan is that of setting test limits such that producer and consumer suffer equal risks.

\[
\sigma_{LOT} = 15
\]

\[
\sigma_{ERROR} = 15
\]

\[
r = \frac{\sigma_{LOT}}{\sigma_{ERROR}} = 1
\]

\[
k = 1.5 \quad \text{(This is determined from the specification limits 80-120 and the process average of 100.}
\]

\[
100 + K \sigma_{LOT} = 120
\]

\[
100 - K \sigma_{LOT} = 80
\]

\[
K = \frac{20}{15} = 1.33 = 1.5
\]

Arbitrarily, we desire to have the sum of the producer's and consumer's risks \( \leq .10 \). In order to achieve this we used \( r = 2 \). This can be accomplished by reducing \( \sigma_{\text{error}} \) by obtaining more test results. We are given that \( \sigma_{ERROR} = \sigma_{LOT} \) but require \( \sigma_{ERROR} < .5\sigma_{LOT} \).

\[
\sigma_{ERROR} = \frac{1}{\sqrt{n}} \quad < \quad \frac{\sigma_{LOT}}{2}
\]

\[
\frac{1}{\sqrt{n}} \quad < \quad \frac{1}{2}
\]

\[
\sqrt{n} \quad > \quad 4
\]

From a Grubbs/Coon table included below, we know that \( b = -.3541 \)
TABLE 18: GRUBBS AND COON TABLE TO EQUATE

PRODUCER’S AND CONSUMER’S RISKS

<table>
<thead>
<tr>
<th>Specification Limits = μ±kσ, Test Limits = μ±(k±b)σ</th>
<th>A = Risk of accepting nonconforming unit (Consumer’s Risk)</th>
<th>B = Risk of rejecting a conforming unit (Producer’s Risk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b = -k (\sqrt{2n-1} - r) = that value of b which will make A = B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>0.5</td>
<td>-1.2709</td>
<td>-1.2335</td>
</tr>
<tr>
<td>1.0</td>
<td>-1.0651</td>
<td>-0.9581</td>
</tr>
<tr>
<td>2.0</td>
<td>-1.0052</td>
<td>-0.9338</td>
</tr>
<tr>
<td>1.0</td>
<td>-2.1596</td>
<td>-1.9990</td>
</tr>
<tr>
<td>2.0</td>
<td>-1.8795</td>
<td>-1.7410</td>
</tr>
<tr>
<td>3.0</td>
<td>-1.6938</td>
<td>-1.5723</td>
</tr>
<tr>
<td>4.0</td>
<td>-1.5815</td>
<td>-1.4553</td>
</tr>
<tr>
<td>5.0</td>
<td>-1.4851</td>
<td>-1.3590</td>
</tr>
<tr>
<td>6.0</td>
<td>-1.3941</td>
<td>-1.2320</td>
</tr>
<tr>
<td>7.0</td>
<td>-1.3045</td>
<td>-1.1277</td>
</tr>
<tr>
<td>8.0</td>
<td>-1.2168</td>
<td>-1.0261</td>
</tr>
<tr>
<td>9.0</td>
<td>-1.1312</td>
<td>-0.9261</td>
</tr>
<tr>
<td>10.0</td>
<td>-1.0474</td>
<td>-0.8261</td>
</tr>
</tbody>
</table>

Decision criteria become:
Test limits = \( u \pm (K \sigma_{LOT} - b \sigma_{ERROR}) \)
Test limits = 100 + ((1.5)(15) - (.3541)(15))
Test limits = 100 + 28

The plan \( n=4 \), \( X_u=128 \), \( X_L=72 \) is called GAF005(1). A simulation of this plan for 10,000 transactions and no reroutes allowed follows:

- Average cost per transaction: $2,193
- Standard deviation of cost per transaction: $5,189
- Percentage of material classified as prime that was truly prime: 67%
- Percentage of material classified as offgrade that was truly prime: 6%

The second criteria that Grubbs and Coon suggest levels to plan that minimizes the sum of the producer's and consumer's risks.

Assuming we still arbitrarily set the sum of the producer's and consumer's risks < .10, we can determine \( n = 4 \) by the same logic used for the previous decision plan. From the Grubbs and Coon table shown below, we know that \( b = -.75 \).
TABLE 19: GRUBBS AND CONN TABLE TO MINIMIZE SUM OF PRODUCER’S AND CONSUMER’S RISKS

<table>
<thead>
<tr>
<th>Specification Limits = ( a \leq k_a )</th>
<th>Test Limits = ( u \leq (k_a - b_u) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = Risk of accepting nonconforming unit (Consumer’s Risk)</td>
<td>B = Risk of rejecting a conforming unit (Producer’s Risk)</td>
</tr>
<tr>
<td>b = ( k/r ) = that value of b which will minimize ((A+B))</td>
<td></td>
</tr>
<tr>
<td>( f ) = ( a/s_a )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( k )</th>
<th>1.3</th>
<th>2.0</th>
<th>2.5</th>
<th>2.8</th>
<th>3.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>-2.000</td>
<td>-2.500</td>
<td>-2.800</td>
<td>-3.000</td>
<td>-3.200</td>
<td>-3.500</td>
</tr>
<tr>
<td>( a ) = 0.1373</td>
<td>0.151</td>
<td>0.171</td>
<td>0.190</td>
<td>0.207</td>
<td>0.224</td>
<td>0.240</td>
</tr>
<tr>
<td>( A ) = 0.081</td>
<td>0.093</td>
<td>0.105</td>
<td>0.117</td>
<td>0.129</td>
<td>0.140</td>
<td>0.150</td>
</tr>
<tr>
<td>( A + B ) = 0.215</td>
<td>0.165</td>
<td>0.126</td>
<td>0.099</td>
<td>0.081</td>
<td>0.066</td>
<td>0.052</td>
</tr>
<tr>
<td>( A + B + (A+B) ) = 0.351</td>
<td>0.320</td>
<td>0.292</td>
<td>0.266</td>
<td>0.243</td>
<td>0.222</td>
<td>0.203</td>
</tr>
</tbody>
</table>

-110-
Decision criteria become:

Test limits = \( n \pm (K \sigma_{\text{LOT}} - b \sigma_{\text{ERROR}}) \)

Test limits = \( 100 \pm ((1.5)(15) - (-.75)(15)) \)

Test limits = \( 100 \pm 33.75 \)

The plan \( n=4, X_u=134, X_L=66 \) is called GAF005(2). A simulation of this plan for 10,000 transactions and no reroutes allowed follows:

| Average cost per transaction | $2,225 |
| Standard deviation of cost per transaction | $5,012 |
| Percentage of material classified as prime that was truly prime | 59% |
| Percentage of material classified as offgrade that was truly prime | 3% |

The third Grubbs and Coon criteria is perhaps the best conceptually. The third criteria creates decision plans which are economically based.

Let:

\( C_\alpha = \text{cost of accepting a non-conforming unit} = $10,000 \)

\( C_\beta = \text{cost of rejecting a conforming unit} = $17,000 \)
Test limits are set by solving the following expressions for $b$:

\[
\int_{\frac{rb + k}{1 + r^2}}^\infty e^{-\frac{t^2}{2}} \frac{1}{2\pi} \, dt = \frac{C_\beta}{C_\alpha + C_\beta}
\]

(This was obtained by finding the relationship for)

\[
\frac{d}{dt} \left(\alpha \cdot C_\alpha + \beta \cdot C_\beta\right) = 0
\]

This is the normal density function and we know that

\[
\frac{C_\beta}{C_\alpha + C_\beta} = \frac{\$17,000}{\$27,000} = 0.6296
\]

Therefore from the Normal Tables:

\[
\frac{rb + k}{1 + r^2} = 0.33
\]

\[
b = -1.03
\]

This provides us with all necessary information to construct the decision criteria for this plan.

Test limits = $u \pm (K \sigma_{LOT} - b \sigma_{ERROR})$

Test limits = $100 \pm ((1.5)(15) - (-1.03)(15))$

Test limits = $100 \pm 38$
The plan $n=1, X_u=62, X_L=138$ was simulated.

Plans with increased sample sizes of $n=2$ and $n=4$ were also attempted. The least cost of these ($n=4$) is reported here as GAF005(3). No reroutes were allowed.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$2,306</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$4,968</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>54%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>10%</td>
</tr>
</tbody>
</table>

**Case Program Plans**

First, an attributes sampling plan is derived on statistical grounds. The traditional binomial distribution equations are used to derive an $n$ and a $c$. An iterative approach is used that leads to the generation of four separate plans. The recommended plan is found by taking the better (lesser) acceptance number and averaging the sample sizes. The inputs for this problem were:

\[
\alpha = .05 \quad \beta = .10
\]

\[
p_1 = .01 \quad p_2 = .10
\]

lot size = 3400 (50 lb bags)
The resulting plan, called GAF010(1), is \( n=36, c=1 \). This plan was simulated for 10,000 transactions without allowing reroutes. The results are:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$3,658</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$6,726</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>97%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>21%</td>
</tr>
</tbody>
</table>

Second, a plan was generated which sets \( \alpha = \beta = .10 \) (all other input the same. The resulting plan, called GAF010(3), is \( n=45, c=1 \). This plan was simulated for 10,000 transactions without allowing reroutes. The results are:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$3,710</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$6,700</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>98%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>2%</td>
</tr>
</tbody>
</table>

Third, the Case plan GAF010(1) was modified to incorporate measurement error. Inspection error rates were set as follows:

\[
e_1 = \text{probability that a good item will be classified as bad} = .05 \\
e_2 = \text{probability that a bad item will be classified as good} = .05
\]
All other input was identical to the first Case plan. The resulting plan, called GAF010(2), is n=107, c=10. This plan was simulated for 10,000 transactions without allowing reroutes. The results are:

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$2,917</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$5,324</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>98%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>9%</td>
</tr>
</tbody>
</table>

Fourth, the GAF010(3) plan (α=β=.10) was modified to incorporate the $e_1$, $e_2$ error rates just discussed. The resulting plan, called GAF010(4), is n=81, c=7. This plan was simulated for 10,000 transactions without allowing reroutes. The results follow:

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$2,859</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$5,559</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>98%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>11%</td>
</tr>
</tbody>
</table>

Fifth, an attributes plan will be constructed via an economic basis. The model is a Guthrie-Johns and uses the following costs:
$S_1 = 10.00 = \text{cost per item of sampling}$

$S_2 = 0.00 = \text{cost per defective item found while sampling}$

$A_1 = 0.10 = \text{cost per item not inspected in an accepted lot}$

$A_2 = 5.50 = \text{cost per defective item found in an accepted lot}$

$R_1 = 0.20 = \text{cost per item not inspected in a rejected lot}$

$R_2 = 4.50 = \text{cost per good item found in a rejected lot}$

In addition, a mixed binomial distribution was used to describe process conditions:

<table>
<thead>
<tr>
<th>Percent Chance</th>
<th>Process Percent Defective</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>20%</td>
<td>5%</td>
</tr>
<tr>
<td>60%</td>
<td>1%</td>
</tr>
</tbody>
</table>

The resulting plan, called GAF010(S) is n=1, c=0. This plan was simulated for 10,000 transactions without allowing reroutes. The results are:

- Average cost per transaction: $3,905
- Standard deviation of cost per transaction: $6,849
- Percentage of material classified as prime that was truly prime: 62%
- Percentage of material classified as offgrade that was truly prime: 3%

The sixth and final Case plan is the previous plan modified to incorporate error rates ($e_1 = .05$, $e_2 = .05$). All other inputs were identical to GAF010(S). The resulting plan, called GAF010(6),
is also n=1, c=0. The results are the same as for GAF010(5).

Mei/Case/Schmidt Plans:

Mei/Case/Schmidt suggest a method for converting a Duncan-like plan to one that incorporates measurement error. In the situation with imprecision only, the sample size is multiplied by a factor j, where j is:

\[
\begin{align*}
    j &= \frac{h + 1}{h} \\
    h &= \frac{\sigma^2_{\text{LOT}}}{\sigma^2_{\text{ERROR}}} = \frac{15^2}{15^2} = 1 \\
    j &= \frac{1 + 1}{1} = 2
\end{align*}
\]

Both Duncan plans (α=.05, β=.10 and α=β=.10) were adjusted. The first was n=6, \(X_H=112, X_L=88\). The adjusted plan would be n=12, \(X_H=112, X_L=88\). This plan is called GAF014(1).
This plan was simulated for 10,000 transactions allowing no reroutes. The results are:

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$3,340</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$6,657</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>98%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>22%</td>
</tr>
</tbody>
</table>

The second Mei/Case/Schmidt plan would be $n=10, X_H=111, X_L=89$. This plan, called GAF014(2), was simulated for 10,000 transactions without allowing reroutes with the following results:

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$3,609</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$6,871</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>97%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>24%</td>
</tr>
</tbody>
</table>
X. STANDARD PROBLEM 2

A representative storage quality assurance problem was formulated and will be described here.

The following input considerations apply:

- Specifications: 100 or less is prime, offgrade otherwise (variables)

- Error of Measurement

<table>
<thead>
<tr>
<th></th>
<th>Bias</th>
<th>$\sigma$ Random Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC Corp</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Customer(s)</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

- $\sigma_{LOT} = 14.142$

- $\sigma_{OBSERVED} = \sqrt{\sigma_{LOT}^2 + \sigma_{RANDOM\ ERROR}^2} = 15$

- Actual product quality varies between 50 and 150 uniformly.

- A summary of costs:
  - $10.00 per test result obtained
  - $50,000.00

- Node AS05: $50,000.00

  We have placed a truly offgrade lot of material into a 400,000 lb. prime silo and caused the mixture to become offgrade. This would result in a loss of the delta between the prime and offgrade margin on the 400,000 contaminated lbs. ($0.10 \text{ lb.} \times 400,000 \text{ lbs.} = $40,000). In addition, the mixture's quality is marginal and the customer of this product would have been misled into accepting the material as prime. Recognizing this, he will divert 50,000 lbs. of business to other producers (@ $0.20/\text{lb.})$.
Thus, the total cost is $50,000.00

- Node AS07: $40,000
  The penalty here is identical to Node AS05, except the mixture would have been perceived as offgrade by ABC Corp. and the customer and, therefore, there would be no loss of customer goodwill.

- Node AS11: $17,000.00
  A truly prime lot was placed in an offgrade silo, and the customer would have reacted to the mixture by classifying it as prime (probably mistakenly). Since we contaminated the prime lot with the offgrade material in the silo, the differential in margin between prime and offgrade material ($0.10/lb.) is assessed to the 170,000 lbs. which were contaminated.

- Node AS13: $17,000.00
  A truly prime lot was placed in an offgrade silo and the customer would have perceived the mixture as offgrade. The lot was contaminated and, therefore, the delta between the margin of prime and offgrade product ($0.10/lb) will be assessed to this transaction.

- The manufactured lot size = 170,000 lbs.
- The average silo inventory (before mixing) = 400,000 lbs.
- The average quality of product in prime silo = 90.
- The average quality of product in an offgrade silo = 120.

The results of interest are:

- DC2A - the prime threshold
- DC3A - the offgrade threshold
- DC3N - the maximum number of tests allowed before a decision is made
- II\textsubscript{1} - the number of test results to obtain initially
- II\textsubscript{2} - the number of test results to obtain an subsequent samples
**Length of Simulation/Use of Seeds**

From a theoretical point of view, the same calculations as for Standard Problem 1 can be made (again assuming \(a=4000\) for the response variable of interest - cost per transaction) this leads to desirable simulation run length of approximately 10,000 transactions.

Also on theoretical grounds, as was the case in the shipping problem, correlated sampling will be used to reduce the variance of the response variable.

From a practical perspective, there is no potential for feedback in the storage model after the testing is complete. The length of execution time and corresponding cost for a simulation of 20,000 transactions was acceptable. A sample case was run for a variety of numbers of transactions. The following input parameters were used.

\[
\begin{align*}
\text{DC2A} &= 0.10 \\
\text{DC3A} &= 0.90 \\
\text{DC3N} &= 500 \\
\text{II}_1 &= 1 \\
\text{II}_2 &= 1
\end{align*}
\]
The results were

### TABLE 20: STORAGE MODEL RESULTS FOR VARIOUS SIMULATION LENGTHS

<table>
<thead>
<tr>
<th>n</th>
<th>Avg. Cost per Transaction</th>
<th>Std. Dev. of Cost per Transaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>235</td>
<td>1,748</td>
</tr>
<tr>
<td>1,000</td>
<td>354</td>
<td>2,610</td>
</tr>
<tr>
<td>2,500</td>
<td>324</td>
<td>2,325</td>
</tr>
<tr>
<td>5,000</td>
<td>315</td>
<td>2,209</td>
</tr>
<tr>
<td>10,000</td>
<td>357</td>
<td>2,332</td>
</tr>
<tr>
<td>20,000</td>
<td>350</td>
<td>2,331</td>
</tr>
</tbody>
</table>

For these reasons 20,000 transactions were run for purposes of identifying the least cost guidelines.

**Truncation of Initial Conditions**

As was the case in the shipping problem, statistics will be collected from the first transaction, there are no startup conditions to be concerned about.

**Determination of Optimum Conditions**

The least cost decision criteria were isolated via "sectioning," a heuristic search technique. The independent factors were ranked in perceived importance, least cost guidelines were identified for the two most important, then three factors, until all factors had been considered.
Least cost guidelines were achieved after roughly 80 computer simulation.

Results

The least cost decision criteria for the storage standard problem were found to be:

\[ DC2A = 0.02 \]
\[ DC3A = 0.30 \]
\[ DC3N = 300 \]
\[ I_1 = 4 \]
\[ I_2 = 1 \]

The least cost guideline summary results are:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$63</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$571</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>87%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>0%</td>
</tr>
</tbody>
</table>

A summary from the least cost case can be observed in Appendix IV.

Results from Other Methodologies

Duncan Variables Acceptance Sampling Plans

- Specifications: <100
- \[ \sigma_{LOT} = 14.14 \]

Solve for \( X_u \) and \( n \); assume, as Duncan does, that \( \alpha = 0.05 \) and \( \beta = 0.10 \).

By writing the equation for the probability of accepting a bad lot.

\[
(1) \quad \frac{X_u - 90}{14.14 / \sqrt{n}} = +1.645
\]
And by writing the equation for the probability of accepting a good lot

(2) \[
\frac{X_u - 100}{14.14/\sqrt{n}} = -1.282
\]

We have two equation for two unknowns. Solving them yields

\[
\frac{X_u - 90}{1.645} = \frac{X_u - 100}{-1.282}
\]

\[
279.93 = 2.927 X_u
\]

\[
X_u = 95.6
\]

Substituting yields

\[
\frac{1}{\sqrt{n}} = \frac{95.6 - 90}{(1.645) (14.14)}
\]

\[
n = 17
\]

The plan \(X_u = 95.6, n=17\) is GAF001(1). The OC curve for this plan is shown below:
FIGURE 9: OPERATING CHARACTERISTIC CURVE FOR DUNCAN VARIABLES PLAN

STORAGE PROBLEM DUNCAN VARIABLES PLAN
OPERATING CHARACTERISTIC CURVE

GEORGE A. FREESTONE AUGUST 1982

-125-
This plan, GAF001(1), was simulated for 20,000 transactions with the following results.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$ 928</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$3,508</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>99%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>8%</td>
</tr>
</tbody>
</table>

A Duncan plan with $\alpha = \beta = .10$ would also be interest since we cannot be some $\alpha = .05$, $\beta = .10$ is correct.

(1) \[ \frac{X_u - 90}{14.14 / \sqrt{n}} = 1.282 \]

(2) \[ \frac{X_u - 100}{14.14 / \sqrt{n}} = -1.282 \]

\[ \frac{X_u - 90}{1.282} = \frac{X_u - 100}{-1.282} \]

\[ 243.58 = 2.564 X_u \]

\[ X_u = 95 \]

Substituting yields

\[ \frac{1}{\sqrt{n}} = \frac{95-90}{(1.282)(14.14)} \]

\[ n = 13 \]
The plan $X_u = 95$, $n=13$ is called GAF001(2). The plan was simulated for 20,000 transactions with the following results:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$1,024</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$3,795</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>99%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>10%</td>
</tr>
</tbody>
</table>

A Duncan sequential plan will also be determined. We need to compute

$$T = h_2 + Sn$$
$$T = -h_2 + Sn$$

Letting $\alpha = .05$, $\beta = .10$, and $\sigma_{LOT} = 14.14$, compute:

$$a = 2.3026 \log_{10} \left( \frac{1-\beta}{\alpha} \right) = 2.8904$$
$$b = 2.3026 \log_{10} \left( \frac{1-\beta}{\alpha} \right) = 2.2513$$

$$h_1 = \frac{b \sigma^2_{LOT}}{100-90} = 45.026$$
$$h_2 = \frac{a \sigma^2_{LOT}}{100-90} = 57.808$$

$$S = \frac{90 + 100}{2} = 95$$

So therefore:

$$T = 57.808 + 95n = \text{Reject (n)}$$
\[ T = -45.026 + 95n = \text{Accept} \ (n) \]

The rule is:

\[ \text{Compute TR = } \sum \text{Obs}_i \text{, then} \]

\[ \begin{align*}
& \text{if TR > Reject} \ (n), \text{ reject} \\
& \text{if TR < Accept} \ (n), \text{ accept} \\
& \text{if neither, sample again.}
\end{align*} \]

This plan is depicted below:

**FIGURE 10: SEQUENTIAL VARIABLES PLAN**

**STORAGE PROBLEM SEQUENTIAL DUNCAN PLAN**

GEORGE A. FREESTONE JUNE 1982
This plan, called GAF001(3), was simulated for 1,000 transactions (due to potential run length of simulation) with the following results:

<table>
<thead>
<tr>
<th>Deming Plans</th>
<th>Define:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_2 =$ Cost of storing a truly defective lot = $50,000</td>
<td></td>
</tr>
<tr>
<td>$K_1 =$ Cost to sample = 10</td>
<td></td>
</tr>
<tr>
<td>$p =$ proportion defective = 50%</td>
<td></td>
</tr>
</tbody>
</table>

Let:

$K = K_2/K_1$

$K = $50,000/$10 = 5,000

Since $Kp = 5,000(.5) = 2,500 > 1$, we should conduct a complete inspection.

Assuming 50 lb. entities, the plan $n=3400$, DC2A=.10, DC3A=.90, is called GAF003(1). This plan was simulated for 1,000 transactions with the following results.

<table>
<thead>
<tr>
<th>Deming Plans</th>
<th>Define:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_2 =$ Cost of storing a truly defective lot = $50,000</td>
<td></td>
</tr>
<tr>
<td>$K_1 =$ Cost to sample = 10</td>
<td></td>
</tr>
<tr>
<td>$p =$ proportion defective = 50%</td>
<td></td>
</tr>
</tbody>
</table>

Let:

$K = K_2/K_1$

$K = $50,000/$10 = 5,000

Since $Kp = 5,000(.5) = 2,500 > 1$, we should conduct a complete inspection.

Assuming 50 lb. entities, the plan $n=3400$, DC2A=.10, DC3A=.90, is called GAF003(1). This plan was simulated for 1,000 transactions with the following results.

| Average cost per transaction | $7,407 |
| Standard deviation of cost per transaction | $8,430 |
| Percentage of material classified as prime that was truly prime | 100% |
| Percentage of material classified as off grade that was truly prime | 45% |

| Average cost per transaction | $34,000 |
| Standard deviation of cost per transaction | $34,000 |
| Percentage of material classified as prime that was truly prime | 100% |
| Percentage of material classified as off grade that was truly prime | 0% |
This plan, called GAF001(3), was simulated for 1,000 transactions (due to potential run length of simulation) with the following results:

Average cost per transaction $7,407
Standard deviation of cost per transaction $8,430
Percentage of material classified as prime that was truly prime 100%
Percentage of material classified as offgrade that was truly prime 45%

Deming Plans
Define:

\[ K_2 = \text{Cost of storing a truly defective lot} = \$50,000 \]
\[ K_1 = \text{Cost to sample} = 10 \]
\[ p = \text{proportion defective} = 50\% \]

Let:

\[ K = \frac{K_2}{K_1} \]
\[ K = \frac{\$50,000}{\$10} = 5,000 \]

Since \( Kp = 5,000(.5) = 2,500 > 1 \), we should conduct a complete inspection.

Assuming 50 lb. entities, the plan \( n=3400, DC2A=.10, DC3A=.90 \), is called GAF003(1). This plan was simulated for 1,000 transactions with the following results:

Average cost per transaction \( \$34,000 \)
Standard deviation of cost per transaction $-0-
Percentage of material classified as prime that was truly prime 100%
Percentage of material classified 0%
as offgrade that was truly prime

If $K_p$ had been less than 1, Deming would have suggested no inspection (everything stored as prime). That plan, called GAF003(2) was simulated for 20,000 transactions with the following results.

<table>
<thead>
<tr>
<th>Case Program Plans</th>
</tr>
</thead>
</table>

The computer program developed by Ken Case and acquired from Oklahoma State University was used to generate some attribute plan representatives of the storage standard problem. The first plan was derived with $\alpha = .05$ and $\beta = .10$. The lot size was assumed to be 3400 (50 lb. entities). The resulting plan, $n=339$ and $c=6$, was called GAF010(1). This plan was simulated for 1000 transactions with the following results:

---

<table>
<thead>
<tr>
<th>Average cost per transaction</th>
<th>$11,322</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$12,130</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>50%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>0%</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Average cost per transaction</th>
<th>$3,790</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$3,333</td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>100%</td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>7%</td>
</tr>
</tbody>
</table>

---

-130-
A similar plan was derived for the same set if input conditions, with the exception that \( \alpha = \beta = .10 \). This plan, called GAF010(3), was \( n = 311 \), \( c = 5 \). A simulation of 1,000 provided the following results.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average cost per transaction</strong></td>
<td>$3,790</td>
</tr>
<tr>
<td><strong>Standard deviation of cost per transaction</strong></td>
<td>$3,333</td>
</tr>
<tr>
<td><strong>Percentage of material classified as prime that was truly prime</strong></td>
<td>100%</td>
</tr>
<tr>
<td><strong>Percentage of material classified as offgrade that was truly prime</strong></td>
<td>7%</td>
</tr>
</tbody>
</table>

These two previous plus were modified to incorporate inspection error rates of:

\[
e_1 = \text{probability of classifying a good entity as bad} = .03 \\
e_2 = \text{probability of classifying a bad entity as good} = .15
\]

This variation for GAF010(1), called GAF010(2), suggested \( n = 1311 \) and \( c = 61 \). The plan was simulated for 100 transactions with the following results:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average cost per transaction</strong></td>
<td>$13,450</td>
</tr>
<tr>
<td><strong>Standard deviation of cost per transaction</strong></td>
<td>$2,391</td>
</tr>
<tr>
<td><strong>Percentage of material classified as prime that was truly prime</strong></td>
<td>100%</td>
</tr>
<tr>
<td><strong>Percentage of material classified as offgrade that was truly prime</strong></td>
<td>4%</td>
</tr>
</tbody>
</table>

The variation for GAF010(3), called GAF010(4), determined \( n \) to be 1015 and \( c \) to be 46. This plan (with \( \alpha = \beta = .10 \)) was simulated.
for 100 transactions with the following results:

Average cost per transaction $10,320
Standard deviation of cost per transaction $1,699
Percentage of material classified as prime that was truly prime 100%
Percentage of material classified as offgrade that was truly prime 2%

Two attributes plans were generated on an economic basis for the storage problem. The first—without measurement error, the second—with measurement error. The following cost parameters pertain to the Guthrie-Johns model:

\[ S_1 = 10.00 = \text{cost per item of sampling} \]
\[ S_2 = 0.00 = \text{additional sampling cost per defective item} \]
\[ A_1 = 0.10 = \text{cost per item not inspected in an accepted lot} \]
\[ A_2 = 12.00 = \text{cost per defective item found in an accepted lot} \]
\[ R_1 = 0.20 = \text{cost per item not inspected in a rejected lot} \]
\[ R_2 = 11.50 = \text{cost per good item found in a rejected lot} \]

The resulting plan, called GAF010(5), was \( n=1 \) and \( c=0 \). This plan was simulated for 20,000 transactions with the following results:

Average cost per transaction $1,181
Standard deviation of cost per transaction $4,816
Percentage of material classified as prime that was truly prime 88%
Percentage of material classified as offgrade that was truly prime 12%

Introduction of measurement error \( (e_1=.03, e_2=.15) \) did not change the
decision criteria (n and c) from GAF010(5).

Mei, Case, Schmidt Plans

The plans proposed are variations of common variables acceptance plans (ala Duncan). To deal with imprecision they suggest modifying the sample size in the following manner:

\[ n_{\text{Adj}} = n \left( \frac{h}{h+1} \right) \]

where

\[ h = \frac{s^2_{\text{LOT}}}{s^2_{\text{ERROR}}} = \frac{14.142}{5^2} = 8 \]

Bias is addressed in the Application of Results Section. Therefore, the two non-sequential Duncan plans should have their sample sizes increased as follows:

\[ n \cdot \left( \frac{h+1}{n} \right) = n_{\text{Adj}} \]

\[ n_{\text{Adj}} = 16 \left( \frac{9}{8} \right) = 18 \text{ (approximately)} \]
The first plan, the variation of GAF001(1), is n=18 and $X_u = 95.6$.

The adjusted plan (GAF014(1)) was simulated for 20,000 transactions with following results:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$1,030</td>
<td></td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$3,714</td>
<td></td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>10%</td>
<td></td>
</tr>
</tbody>
</table>

The second plan is the variation in GAF001(2) the adjusted plan, called GAF014(2) is n=14 and $X_u = 95.0$ and is as simulated for 20,000 transactions with the following results:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$1,160</td>
<td></td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$4,047</td>
<td></td>
</tr>
<tr>
<td>Percentage of material classified as prime that was truly prime</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>Percentage of material classified as offgrade that was truly prime</td>
<td>11%</td>
<td></td>
</tr>
</tbody>
</table>

**David, Fry, and Walsh Plans**

A tedious methods in provided to determine two plans from this methodology. The following inputs pertain.

\[
\alpha = .05, \beta = .10 \\
P_1 = \text{acceptable percentage of defections} \\
P_2 = \text{unacceptable percentage of defectives} \\
R = \frac{\sigma_{\text{LOT}}}{\sigma_{\text{ERROR}}} = 3 \\
R = \text{known upper limit of } R = 4
\]

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Then from Normal Tables

\[ K_{p1} = 2.326 = K_1 \]
\[ K_{p2} = 1.880 = K_2 \]
\[ K_\alpha = 1.645 \]
\[ K_\beta = 1.282 \]

We will consider two combinations of \( R \) and \( m \) (\( m \) is the number of observations from each sample):

\[ (R, m) = (4, 1), (4, 10) \]

For these two cases the preliminary steps in the calculation are shown in Table 210. In both cases we attempt to verify that \( 1 < d; \lambda < 0; d < c \). Since \( d > c \) a solution does not exist.
TABLE 21: PRELIMINARY CALCULATIONS FOR DAVID, WALSH, AND FREY PLANS

\[ \bar{R} = 4 \]

<table>
<thead>
<tr>
<th></th>
<th>( m = 1 )</th>
<th>( m = 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 )</td>
<td>-.66809</td>
<td>-.35798</td>
</tr>
<tr>
<td>( d )</td>
<td>.67653</td>
<td>.06783</td>
</tr>
<tr>
<td>( c )</td>
<td>.50693</td>
<td>.05069</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>-.54516</td>
<td>-.54516</td>
</tr>
</tbody>
</table>

- \( K_1 = K_2 \)
- \( d = K_1 \frac{\bar{R}S - \bar{R}S}{S - S} \)
- \( s = R^2 + m^{-1} \)
- \( S = R^2 + m^{-1} \)
- \( c = \frac{K_2}{K_1} \)
- \( p = \frac{K_2}{mR} \)
- \( \lambda = K_1 R - K_2 \bar{R} \)

Rather than not present a solution from this methodology, an assumption will be made that \( R=1 \). If this is done, a solution does exist and six combinations of the solution are presented (directly from the paper).
Combination of:

TABLE 22: SUMMARY OF DAVID, WALSH, AND FREY PLANS

<table>
<thead>
<tr>
<th>(R,m)</th>
<th>Plan name</th>
<th>N</th>
<th>m</th>
<th>K₀</th>
<th>L*</th>
<th>V₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>a,1</td>
<td>GAF016(1)</td>
<td>649</td>
<td>1</td>
<td>2.21</td>
<td>104.82</td>
<td>.96360</td>
</tr>
<tr>
<td>a,2</td>
<td>GAF016(2)</td>
<td>306</td>
<td>2</td>
<td>2.15</td>
<td>102.61</td>
<td>.52282</td>
</tr>
<tr>
<td>a,3</td>
<td>GAF016(3)</td>
<td>236</td>
<td>3</td>
<td>2.13</td>
<td>101.80</td>
<td>.35989</td>
</tr>
<tr>
<td>4,1</td>
<td>GAF016(4)</td>
<td>429</td>
<td>1</td>
<td>2.42</td>
<td>106.58</td>
<td>1.31714</td>
</tr>
<tr>
<td>4,2</td>
<td>GAF016(5)</td>
<td>260</td>
<td>2</td>
<td>2.27</td>
<td>103.43</td>
<td>.68624</td>
</tr>
<tr>
<td>4,3</td>
<td>GAF016(6)</td>
<td>214</td>
<td>3</td>
<td>2.21</td>
<td>102.33</td>
<td>.46527</td>
</tr>
</tbody>
</table>

* U = 100, σERROR = 5

The one sided acceptance criteria is of the form

Accept the lot, if an only if \( \bar{x} + k_{o} s < L \)

where \( \bar{x} = \) mean obtained from the N•m items sampled
\( S = \) standard deviation obtained from the N•m items sampled
\( L = \) limit = Offgrade threshold + \( V_{o} \) * σERROR

Each of these policies was simulated only for a very small number of transactions (100) since their economic performance will be poor since a large number of test results is obtained in all cases.
The results for GAF016(1) are:

- Average cost per transaction: $11,580
- Standard deviation of cost per transaction: $7,830
- Percentage of material classified as prime that was truly prime: 100%
- Percentage of material classified as offgrade that was truly prime: 37%

The results for GAF016(2) are:

- Average cost per transaction: $11,220
- Standard deviation of cost per transaction: $8,212
- Percentage of material classified as prime that was truly prime: 100%
- Percentage of material classified as offgrade that was truly prime: 43%

The results for GAF016(3) are:

- Average cost per transaction: $12,180
- Standard deviation of cost per transaction: $8,212
- Percentage of material classified as prime that was truly prime: 100%
- Percentage of material classified as offgrade that was truly prime: 43%
The results for GAF016(4) are:

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$9,390</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$8,212</td>
</tr>
<tr>
<td>Percentage of material classified as prime</td>
<td>100%</td>
</tr>
<tr>
<td>Percentage of material classified as truly</td>
<td>43%</td>
</tr>
</tbody>
</table>

The results for GAF016(5) are:

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$10,300</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$8,212</td>
</tr>
<tr>
<td>Percentage of material classified as prime</td>
<td>100%</td>
</tr>
<tr>
<td>Percentage of material classified as truly</td>
<td>43%</td>
</tr>
</tbody>
</table>

The results for GAF016(6) are:

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cost per transaction</td>
<td>$11,520</td>
</tr>
<tr>
<td>Standard deviation of cost per transaction</td>
<td>$8,212</td>
</tr>
<tr>
<td>Percentage of material classified as prime</td>
<td>100%</td>
</tr>
<tr>
<td>Percentage of material classified as truly</td>
<td>43%</td>
</tr>
</tbody>
</table>
The results of this thesis can provide specific guidelines for the shipping and storage decisions made on a routine basis. Descriptions of three cases illustrating their use follow.

Case 1: Shipment decision (Shipping network)
Case 2: Reroute decision (Shipping network)
Case 3: Storage decision (Storage network)

Case 1

Assume that
- customer specifications are .92 and .98 (two sided)
- random error = .03
- bias = .01
- decision criteria are (.10,.20,.90) (DC2A, DC3A, DC4A)
- test result #1 = .94 (plant result)

Since the test has both an upper and a lower specification, a two-sided test is required. Compute:

\[
Z_1 = \frac{\text{upper spec} - \text{adjusted test result}}{\text{random error}} \quad \text{and} \quad Z_2 = \frac{\text{lower spec} - \text{adjusted test result}}{\text{random error}}
\]

upper spec = .98 \quad lower spec = .92
adjusted test result = test result - bias = .94 - .01 = .93
random error = .03

\[
Z_1 = \frac{.98 - .93}{.03} = 1.67 \quad Z_2 = \frac{.92 - .93}{.03} = -.33
\]

Then EPWS = $\Phi(Z_1) - \Phi(Z_2)$ (Obtained from Normal Tables)
EPWS = .9525 - .3707 = .5818

Since EPWS is greater than the prime cutoff of .20, ship the material as prime.
Case 2

The following is an example of how the reroute criteria could be applied. Assume that

-- customer specifications = .92 and .98
-- random error = .03
-- bias = .01 (at plant)
-- decision criteria are (.10, .20, .90)
-- test result #1 = .94 (plant result)
-- test result #2 = .91 (corporate result)
-- test result #3 = .92 (corporate result)

This case can be considered a continuation of Case 1 where the customer in question has called the product off grade on the basis of his test results. The test is still two-sided, but is slightly more involved because of the multiple test results.

First compute the average adjusted result = \[
\frac{\text{sum of adjusted test results}}{\text{number of test results}}
\]

\[
= \frac{(.94-.01) + .91 + .92}{3}
\]

\[
= \frac{2.76}{3} = .92
\]

Notice that only the plant test results are adjusted for bias in this example. Then calculate adjusted random error = random error \[\frac{1}{\sqrt{n}}\]

\[
\text{adjusted random error} = \frac{.03}{\sqrt{3}} = .017
\]
Now compute

\[ Z_1 = \frac{\text{upper spec} - \text{average adjusted result}}{\text{adjusted random error}} \]

\[ Z_1 = \frac{.98 - .92}{.017} = 3.53 \]

\[ Z_2 = \frac{\text{lower spec} - \text{average adjusted result}}{\text{adjusted random error}} \]

\[ Z_2 = \frac{.92 - .92}{.017} = 0.000 \]

Then \( EPWS = \Phi(Z_1) - \Phi(Z_2) \) (Obtained from Normal tables)

\[ .9999 - .5000 = .4999 \]

Since the calculated EPWS is less than the decision criteria, do not reroute the shipment, but sell it as off grade to that customer.

Case 3

Assume that

-- customer specification is 50
-- random error = 20 (at plant)
-- bias = 0 (for plant test results)
-- decision criteria are (.10,.70)
-- test result #1 = 45

The test is one sided so we compute
Since there is no bias, adjusted test result = 45 - 0 = 45, and since 
n=1, the adjusted random error = the random error = 20.

\[
\begin{align*}
z & = \frac{50 - 45}{20} \\
& = 0.25
\end{align*}
\]

Then \( EPWS = \Phi(Z) \)

\[
EPWS = 0.5987 \quad \text{(Obtained from Normal Tables)}
\]

Since the calculated \( EPWS \) is less than the prime cutoff of .70 and
higher than the off grade cutoff of .10, retest.

Assume that test result #2 = 30.

Compute the average adjusted result = \( \frac{(45 - 0) + (30 - 0)}{2} = 37.5 \)
and the adjusted random error = \( \text{random error} \sqrt{n} = 20 / \sqrt{2} = 14.1 \)

Now recompute

\[
\begin{align*}
z & = \frac{50 - 37.5}{14.1} \\
& = 0.88
\end{align*}
\]

Then \( EPWS = \Phi(Z) \)

\[
EPWS = 0.8106 \quad \text{(Obtained from Normal Tables)}
\]

Since \( EPWS \) is greater than the prime cutoff of .70 store the
material as prime.

It is worth noting that it would be possible to obtain a calculator to
assist in these calculations. The calculator of interest could automate
the adjustment process and the calculation of \( EPWS \).
XV. CONCLUSIONS

The following conclusions have emanated from work in this thesis.

They are listed in order of importance:

1) While determining quality assurance decisions plans, producer's and consumer's risks should be outputs rather than inputs. Arbitrarily setting these values can lead to poor plan performance.

2) The tendency to set quality assurance decision plans based on producer's and consumer's risks comes from a reluctance to quantify the relevant costs.

3) Many published quality assurance decision plans ignore:
   - the producer's perspective
   - the potential for contamination in storage situations
   - the customer's reaction to the product rather than strictly the state of nature
   - measurement error
   - economic considerations

This omission leads to poor plan performance in economic terms.

4) Analytical solutions seem inferior from the perspective that the analyst can not understand implications of a particular plan readily and from the perspective that practitioners can not (and will not) comprehend them.

5) Simulation solutions seem inferior from the perspective that analysts need to search for "optimal" decision plans.
6) The thesis methodology provided the best decision plan in the shipping problem (see Table 23). The difference in cost is significant on theoretical grounds. The standard deviation of the difference in plan's average cost per transaction is approximately 35 ($\rho$ was calculated).

\[ \sigma_{\text{DIFF}}^2 = \sigma_{\text{DIFF}}^2 = \sigma_1^2 (\text{COST}_1) + \sigma_2^2 (\text{COST}_2) - 2 \rho \sigma_1 \sigma_2 (\text{COST}_1) (\text{COST}_2) \]

\[ \sigma_{\text{DIFF}}^2 = 35^2 + 35^2 - 2 (.5)(35)(35) \]

\[ \sigma_{\text{DIFF}} = 35 \]

The difference in cost is significant from the practitioner's perspective. Which plan would he choose?

7) The thesis methodology provided the best decision plan in storage problem (see Table 24). Here the difference is more striking, but the standard deviation of the difference is approximately 28. (Here again, $\rho$ was calculated)

\[ \sigma_{\text{DIFF}}^2 = \sigma_{\text{DIFF}}^2 = \sigma_1^2 (\text{COST}_1) + \sigma_2^2 (\text{COST}_2) - 2 \rho \sigma_1 \sigma_2 (\text{COST}_1) (\text{COST}_2) \]

\[ \sigma_{\text{DIFF}}^2 = 29^2 + 4^2 - 2 (.2)(4)(29) \]

\[ \sigma_{\text{DIFF}} = 28 \]

From a practitioner's viewpoint, the differences here are large.

8) This methodology can very easily be used for count data by substituting a POISSON distribution routine for the NORMAL distribution routine used in the simulation. NORMAL was used in both cases because the routine existed.
<table>
<thead>
<tr>
<th>Rank</th>
<th>Methodology</th>
<th># of Reroutes</th>
<th>Avg. Cost($) Per Trans</th>
<th>Std. Dev. of Cost($) Per Trans</th>
<th>ABC Prime Truly Prime</th>
<th>ABC Offgrade Truly Prime</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thesis</td>
<td>664</td>
<td>1,156</td>
<td>3,241</td>
<td>96%</td>
<td>2%</td>
</tr>
<tr>
<td>2</td>
<td>Thesis</td>
<td>0</td>
<td>1,775</td>
<td>4,851</td>
<td>96%</td>
<td>2%</td>
</tr>
<tr>
<td>3</td>
<td>GAF005(1)</td>
<td>0</td>
<td>2,193</td>
<td>5,189</td>
<td>67%</td>
<td>6%</td>
</tr>
<tr>
<td>4</td>
<td>GAF005(2)</td>
<td>0</td>
<td>2,225</td>
<td>5,012</td>
<td>59%</td>
<td>3%</td>
</tr>
<tr>
<td>5</td>
<td>GAF005(3)</td>
<td>0</td>
<td>2,306</td>
<td>4,968</td>
<td>54%</td>
<td>10%</td>
</tr>
<tr>
<td>6</td>
<td>GAF010(4)</td>
<td>0</td>
<td>2,859</td>
<td>5,559</td>
<td>98%</td>
<td>11%</td>
</tr>
<tr>
<td>7</td>
<td>GAF010(2)</td>
<td>0</td>
<td>2,917</td>
<td>5,324</td>
<td>98%</td>
<td>9%</td>
</tr>
<tr>
<td>8</td>
<td>GAF014(1)</td>
<td>0</td>
<td>3,340</td>
<td>6,657</td>
<td>98%</td>
<td>22%</td>
</tr>
<tr>
<td>9</td>
<td>GAF001(1)</td>
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<td>3,485</td>
<td>6,784</td>
<td>93%</td>
<td>23%</td>
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<td>10</td>
<td>CAF014(2)</td>
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<td>3,609</td>
<td>6,871</td>
<td>97%</td>
<td>2%</td>
</tr>
<tr>
<td>11</td>
<td>CAF010(1)</td>
<td>0</td>
<td>3,658</td>
<td>6,726</td>
<td>97%</td>
<td>21%</td>
</tr>
<tr>
<td>12</td>
<td>GAF010(3)</td>
<td>0</td>
<td>3,710</td>
<td>6,700</td>
<td>98%</td>
<td>2%</td>
</tr>
<tr>
<td>13</td>
<td>GAF001(2)</td>
<td>0</td>
<td>3,774</td>
<td>6,985</td>
<td>92%</td>
<td>25%</td>
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<tr>
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<td>GAF010(5)&amp;(6)</td>
<td>0</td>
<td>3,905</td>
<td>6,849</td>
<td>62%</td>
<td>3%</td>
</tr>
<tr>
<td>15</td>
<td>GAF003(2)</td>
<td>0</td>
<td>4,274</td>
<td>5,442</td>
<td>41%</td>
<td>0%</td>
</tr>
<tr>
<td>16</td>
<td>GAF001(3)</td>
<td>0</td>
<td>6,914</td>
<td>8,351</td>
<td>80%</td>
<td>41%</td>
</tr>
<tr>
<td>17</td>
<td>GAF003(1)</td>
<td>0</td>
<td>35,430</td>
<td>4,824</td>
<td>100%</td>
<td>1%</td>
</tr>
</tbody>
</table>

Note: Plan 15 represents no testing. Plans ranking lower than that do worse than if no testing had been performed.
### TABLE 23

**RANKING OF METHODOLOGIES PERFORMANCE ON THE SHIPPING PROBLEM**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Methodology</th>
<th># of Reroutes</th>
<th>Avg. Cost($) Per Trans</th>
<th>Std. Dev. of Cost($) Per Trans</th>
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<td>5,559</td>
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<td>0</td>
<td>4,274</td>
<td>5,442</td>
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<td>6,914</td>
<td>8,351</td>
<td>80%</td>
<td>41%</td>
</tr>
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<td>GAF003(1)</td>
<td>0</td>
<td>35,430</td>
<td>4,824</td>
<td>100%</td>
<td>1%</td>
</tr>
</tbody>
</table>

**Note:** Plan 15 represents no testing. Plans ranking lower than that do worse than if no testing had been performed.
<table>
<thead>
<tr>
<th>Rank</th>
<th>Methodology</th>
<th>Avg Cost($)</th>
<th>Std. Dev. of Cost($)</th>
<th>ABC Prime</th>
<th>ABC Offgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Per Trans.</td>
<td>Per Trans.</td>
<td>Truly Prime</td>
<td>Truly Prime</td>
</tr>
<tr>
<td>1</td>
<td>Thesis</td>
<td>63</td>
<td>571</td>
<td>87%</td>
<td>0%</td>
</tr>
<tr>
<td>2</td>
<td>GAF001(1)</td>
<td>928</td>
<td>3,508</td>
<td>99%</td>
<td>8%</td>
</tr>
<tr>
<td>3</td>
<td>GAF001(2)</td>
<td>1,024</td>
<td>3,795</td>
<td>99%</td>
<td>10%</td>
</tr>
<tr>
<td>4</td>
<td>GAF014(1)</td>
<td>1,030</td>
<td>3,714</td>
<td>100%</td>
<td>11%</td>
</tr>
<tr>
<td>5</td>
<td>GAF014(2)</td>
<td>1,160</td>
<td>4,047</td>
<td>100%</td>
<td>11%</td>
</tr>
<tr>
<td>6</td>
<td>GAF010(5)</td>
<td>1,181</td>
<td>4,816</td>
<td>88%</td>
<td>12%</td>
</tr>
<tr>
<td>7</td>
<td>GAF010(1)&amp;(3)</td>
<td>3,790</td>
<td>3,333</td>
<td>100%</td>
<td>7%</td>
</tr>
<tr>
<td>8</td>
<td>GAF001(3)</td>
<td>7,407</td>
<td>8,430</td>
<td>100%</td>
<td>45%</td>
</tr>
<tr>
<td>9</td>
<td>GAF016(4)</td>
<td>9,390</td>
<td>8,212</td>
<td>100%</td>
<td>43%</td>
</tr>
<tr>
<td>10</td>
<td>GAF016(5)</td>
<td>10,300</td>
<td>8,212</td>
<td>100%</td>
<td>43%</td>
</tr>
<tr>
<td>11</td>
<td>GAF010(4)</td>
<td>10,320</td>
<td>1,699</td>
<td>100%</td>
<td>2%</td>
</tr>
<tr>
<td>12</td>
<td>GAF016(2)</td>
<td>11,220</td>
<td>8,212</td>
<td>100%</td>
<td>43%</td>
</tr>
<tr>
<td>13</td>
<td>GAF003(2)</td>
<td>11,322</td>
<td>19,130</td>
<td>50%</td>
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</tr>
<tr>
<td>14</td>
<td>GAF016(6)</td>
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<td>100%</td>
<td>43%</td>
</tr>
<tr>
<td>15</td>
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<td>7,830</td>
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<td>37%</td>
</tr>
<tr>
<td>16</td>
<td>GAF016(3)</td>
<td>12,180</td>
<td>8,212</td>
<td>100%</td>
<td>43%</td>
</tr>
<tr>
<td>17</td>
<td>GAF010(2)</td>
<td>13,450</td>
<td>2,391</td>
<td>100%</td>
<td>4%</td>
</tr>
<tr>
<td>18</td>
<td>GAF003(1)</td>
<td>34,000</td>
<td>0</td>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Note: Plan 13 represents no testing. Plans ranking lower than that do worse than if no testing had been performed.
TABLE 24
RANKING OF METHODOLOGIES PERFORMANCE ON THE STORAGE PROBLEM

<table>
<thead>
<tr>
<th>Rank</th>
<th>Methodology</th>
<th>Avg Cost($) Per Trans.</th>
<th>Std. Dev. of Cost($) Per Trans.</th>
<th>ABC Prime Truly Prime</th>
<th>ABC Offgrade Truly Prime</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thesis</td>
<td>63</td>
<td>571</td>
<td>87%</td>
<td>0%</td>
</tr>
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<td>3,508</td>
<td>99%</td>
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</tr>
<tr>
<td>3</td>
<td>GAF001(2)</td>
<td>1,024</td>
<td>3,795</td>
<td>99%</td>
<td>10%</td>
</tr>
<tr>
<td>4</td>
<td>GAF014(1)</td>
<td>1,030</td>
<td>3,714</td>
<td>100%</td>
<td>10%</td>
</tr>
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<td>5</td>
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<td>1,160</td>
<td>4,047</td>
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<tr>
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<td>GAF010(1)&amp;(3)</td>
<td>3,790</td>
<td>3,333</td>
<td>100%</td>
<td>7%</td>
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<td>8</td>
<td>GAF001(3)</td>
<td>7,407</td>
<td>8,430</td>
<td>100%</td>
<td>45%</td>
</tr>
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<td>8,212</td>
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<td>10,300</td>
<td>8,212</td>
<td>100%</td>
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</tr>
<tr>
<td>11</td>
<td>GAF010(4)</td>
<td>10,320</td>
<td>1,699</td>
<td>100%</td>
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<td>GAF016(2)</td>
<td>11,220</td>
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<td>19,130</td>
<td>50%</td>
<td>-</td>
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</tr>
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<td>11,580</td>
<td>7,830</td>
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</tr>
</tbody>
</table>

Note: Plan 13 represents no testing. Plans ranking lower than that do worse than if no testing had been performed.
XVI. RECOMMENDATIONS

I recommend the following based in the work in this thesis:

1) Quality assurance plans be economically based.

2) Simulation be more extensively used to identify desirable quality assurance plans, to promote understanding of the derivation of the selected plans, and to promote an understanding of the implications of the selected plan.

3) Analysts first model the situation at hand prior to rushing to textbook "cookbook" formulae.

4) Some simple results of this thesis be applied before enormous amounts of time are poured into subsequent research along these specific lines.
EPWS - the estimated probability that the product is within specification given one or more test results, the imprecision level of the test, (and product variability, if any) and the prime test specifications for the product.

Blind test result - a test result obtained by a technician who had no knowledge of prior test results which might influence his results.

Random measurement error (imprecision) - that discrepancy in measured test results that occur in a random fashion. Technician counting fatigue can result in random measurement error. Random measurement error is defined in Phase I to be the standard deviation of repeated measurements of the sample.

Systematic measurement error (bias) - that inaccuracy in measured test result(s) when compared to the true quality being measured that occurs in a systematic fashion. Testing equipment which is out of adjustment can cause bias.

Prime test specification - the endpoints of a range of values for quality measurements that is considered prime or acceptable for a given product, test, and customer.

Decision criteria - cutoff values for the estimated probability that product is within spec derived from the test result(s) for that ct. If this estimated probability within spec (EPWS) is above some cu point, the product is considered prime. A sufficiently low EPWS ts in an offgrade classification while a midrange EPWS indicates a need for further testing. Additional test results allow further refinement to the EPWS which is then again compared against the cutoff points.

DC2A - decision criterion which serves as the EPWS cutoff between marginal and off grade product in both the storage and shipping networks (the offgrade cutoff).

DC3A - decision criterion which serves as the EPWS cutoff between prime and marginal product in both the storage and shipping networks (the prime cutoff).

DC4A - decision criterion which serves as the EPWS cutoff between rerouting a shipment that has been called off grade by the customer.
and selling the shipment at a reduced price. The criterion is used in the shipping network (the reroute cutoff).

Observed Variability - the variability in test results observed by technicians = \( \sigma_{OBS} \) in this thesis

\[ \sigma^2_{OBS} = \sigma^2_{PRODUCT} + \sigma^2_{RANDOM \ ERROR} \]

DC3N - a variable representing the maximum allowable number of test results that can be obtained in both the storage and shipping networks.

DC4N - a variable denoting the minimum number of test results required before a shipment can be rerouted in the shipping network.
BIBLIOGRAPHY


Hamberg, Morris, "Bayesian Decision Theory and Statistical Quality Control," Industrial Quality Control, Volume 19, No. 6, December 1962,
pp. 10-14.


APPENDIX IA

Shipping Network - SLAM Network
APPENDIX IB
Shipping Network - SLAM Code

//STEP1 EXEC PGM=H3D019GF,TIME=15
//STEP1 LIB DD DSN=SCILIB,DISP=SHR
//FT06F001 DD SYSOUT=A
//FT05F001 DD *

GEN,G.A.FREESTONE,QA.NETWORK.SHIP,04/14/82,1;
LIMITS,1,15,100;
PRIORITY/1,FIFO;
INITIALIZE,0.0,20000.0;

NETWORK;

; ATRIB(1)=NOT USED
; ATRIB(2)=ACCUMULATED EPWS (ALL QUALITY ATTRIBUTES)
; ATRIB(3)=ACCUMULATED COST (ALL QUALITY ATTRIBUTES)
; ATRIB(4)=QUAL ATTRIBUTE 1:#OF TEST RESULTS OBTAINED
; ATRIB(5)=QUAL ATTRIBUTE 1:#OF GROUPS OF TEST RESULTS OBTAINED
; ATRIB(6)=QUAL ATTRIBUTE 1:COST ASSOCIATED WITH THIS QUAL ATRIB
; ATRIB(7)=QUAL ATTRIBUTE 1:EPWS THIS QUAL ATRIB
; ATRIB(8)=QUAL ATTRIBUTE 1:ACTUAL QUAL THIS QUAL ATRIB
; ATRIB(9)=QUAL ATTRIBUTE 1:CUST REACTION FLAG
; ATRIB(10)=QUAL ATTRIBUTE 1:ACTUAL FLAG THIS QUAL ATRIB
; ATRIB(11)=QUAL ATTRIBUTE 1:REFUTE FLAG THIS QUAL ATRIB
; ATRIB(12)=QUAL ATTRIBUTE 1:MEAN OF ABC TEST RESULTS TO DATE
; ATRIB(13)=QUAL ATTRIBUTE 1:CUSTMEAN OF TEST RESULTS
; ATRIB(14)=QUAL ATTRIBUTE 1:# OF REROUTES
; ATRIB(15)=# OF IN SPEC TEST RESULTS OBTAINED TO DATE

USERF(1)= OBTAIN TEST RESULTS
USERF(2)= DETERMINE CUSTOMER REACTION TO THIS ENTITY
USERF(3)= DETERMINE ACTUAL QUALITY FLAG FOR THIS ENTITY
USERF(4)= DETERMINE CUSTOMER REFUTATION FLAG
USERF(5)= NOT USED
USERF(6)= NOT USED
USERF(7)= INCREMENT TEST COST

CREA CREATE,1.0,1.0,1,20000,1; CREATE 10000,TRANSACTIONS
ASO1 ASSIGN,II=10,
    ATRIB(8)=UNFRM(50.0,150.0,2),
    ATRIB(4)=0.0,
    ATRIB(7)=USERF(1),

-155-
ATRIB(5) = ATRIB(5) + 1,
ATRIB(6) = USERF(7),
ATRIB(10) = USERF(3);

GO01 GOON, 1;

ACT,, ATRIB(7).GT.XX(1).AND.ATRIB(7).LT.XX(2).AND.ATRIB(4).LT.500, AS02;
ACT,, ATRIB(7).GT.XX(2).AND.ATRIB(4).LE.500, CO34;
ACT,, ATRIB(7).LE.XX(1).OR.ATRIB(4).GE.500, CO35;

AS02 ASSIGN, II = 10,
    ATRIB(7) = USERF(1),
    ATRIB(5) = ATRIB(5) + 1,
    ATRIB(6) = USERF(7);

ACT,,, GO01;

AS03 ASSIGN, ATRIB(9) = USERF(2), 1;

CO34 COLCT, ATRIB(10), ABC P TR P;
GO02 GOON, 1;

CO01 COLCT, ATRIB(6), COST 1, 10, 0.0, 100.0;
CO02 COLCT, ATRIB(4), NO OF TESTS 1, 10, 0.0, 2.0;
CO03 COLCT, ATRIB(8), ACTUAL QUAL 1, 10, 0.0, 10.0;

AS05 ASSIGN, ATRIB(6) = ATRIB(6) + 10000.0;
CO04 COLCT, ATRIB(6), COST 2, 10, 0.0, 100.0;
CO05 COLCT, ATRIB(4), NO OF TESTS 2, 10, 0.0, 2.0;
CO06 COLCT, ATRIB(8), ACTUAL QUAL 2, 10, 0.0, 10.0;

AS06 ASSIGN, ATRIB(11) = USERF(4),
    ATRIB(6) = ATRIB(6) + 1000.0, 0.1;

CO07 COLCT, ATRIB(6), COST 3, 10, 0.0, 100.0;
CO08 COLCT, ATRIB(4), NO OF TESTS 3, 10, 0.0, 2.0;

CUSTOMER'S REACTION

ACT,, ATRIB(9).EQ.1, GO03;
ACT,, ATRIB(9).EQ.0, AS06;

GO03 GOON, 1;

ACT,, ATRIB(10).EQ.1, CO01;
ACT,, ATRIB(10).EQ.0, AS05;

AS05 ASSIGN, ATRIB(6) = ATRIB(6) + 10000.0;

CO04 COLCT, ATRIB(6), COST 2, 10, 0.0, 100.0;
CO05 COLCT, ATRIB(4), NO OF TESTS 2, 10, 0.0, 2.0;
CO06 COLCT, ATRIB(8), ACTUAL QUAL 2, 10, 0.0, 10.0;

ACT,, AS14;

AS06 ASSIGN, ATRIB(11) = USERF(4),
    ATRIB(6) = ATRIB(6) + 1000.0, 0.1;

ACT,, ATRIB(11).EQ.1, GO04;
ACT,, ATRIB(11).EQ.0, GO05;

GO04 GOON, 1;

ACT,, ATRIB(10).EQ.1, CO07;
ACT,, ATRIB(10).EQ.0, AS07;

CO07 COLCT, ATRIB(6), COST 3, 10, 0.0, 100.0;
CO08 COLCT, ATRIB(4), NO OF TESTS 3, 10, 0.0, 2.0;

ADMIN COST

SUCCESSFUL REFUTE
FAIL TO REFUTE

TRULY PR
TRULY OG
CO09  COLCT, ATRIB(8), ACTUAL QUAL 3, 10, 0.0, 10.0;
     ACT,, AS14;
AS07  ASSIGN, ATRIB(6) = ATRIB(6) + 10000.0;        TRULY OG, WE REFUTED, CUST OKD
CO11  COLCT, ATRIB(4), CO TESTS 4, 10, 0.0, 2.0;
CO12  COLCT, ATRIB(8), ACTUAL QUAL 4, 10, 0.0, 10.0;
     ACT,, AS14;
G005  GOON, 1;
     ACT, ATRIB(7).GE.0.90. AND. ATRIB(4).GT.4. AND. ATRIB(14).LT.3, AS08;
     ACT, ATRIB(7).LT.0.90. OR. ATRIB(4).LE.4. OR. ATRIB(14).GE.3, GO06;
AS08  ASSIGN, ATRIB(6) = ATRIB(6) + 3000.0,
     ATRIB(14) = ATRIB(14) + 1;
     ACT,, GO02; REROUTE TO NEXT CUSTOMER
GO06  GOON, 1;
     ACT,, ATRIB(10).EQ.1, AS09;  TRULY PRIME, SOLD AS OG, ABC PR
     ACT,, ATRIB(10).NE.1, AS10;  TRULY OG, SOLD AS OG, ABC PR
AS09  ASSIGN, ATRIB(6) = ATRIB(6) + 17000.0;
CO15  COLCT, ATRIB(8), ACTUAL QUAL 5, 10, 0.0, 10.0;
     ACT,, AS14;
AS10  ASSIGN, ATRIB(6) = ATRIB(6) + 0.0;
CO16  COLCT, ATRIB(6), COST 6, 10, 0.0, 100.0;
CO17  COLCT, ATRIB(4), NO OF TESTS 6, 10, 0.0, 2.0;
CO18  COLCT, ATRIB(8), ACTUAL QUAL 6, 10, 0.0, 10.0;
     ACT,, AS14;
; ;
CO35  COLCT, ATRIB(10), ABC OR TR P;
AS04  ASSIGN, ATRIB(9) = USERF(2);
G007  GOON, 1;
     ACT,, ATRIB(9).EQ.1, GO08; CUST PR, ABC OG
     ACT,, ATRIB(9).NE.1, GO09; CUST OG, ABC OG
G008  GOON, 1;
     ACT,, ATRIB(10).EQ.1, AS11; CUST PR, ABC OG, TRULY PR
     ACT,, ATRIB(10).NE.1, AS12; CUST PR, ABC OG, TRULY OG
AS11  ASSIGN, ATRIB(6) = ATRIB(6) + 17000.0;
CO19  COLCT, ATRIB(6), COST 7, 10, 0.0, 100.0;
CO20  COLCT, ATRIB(4), NO OF TESTS 7, 10, 0.0, 2.0;
CO21  COLCT, ATRIB(8), ACTUAL QUAL 7, 10, 0.0, 10.0;
     ACT,, AS14
AS12  ASSIGN, ATRIB(6) = ATRIB(6) + 0.0;  MISS CLASS COST?
CO22  COLCT, ATRIB(6), COST 8, 10, 0.0, 100.0;
CO23 COLCT, ATRIB(4), NO OF TESTS 8, 10, 0.0, 2.0;
CO24 COLCT, ATRIB(8), ACTUAL QUAL 8, 10, 0.0, 10.0;
       ACT,, AS14;
GO09 GOON, 1;
       ACT,, ATRIB(10).EQ.1, AS13;
       CUST OG, ABC OG, TR PR
       ACT,, ATRIB(10).EQ.0, CO28;
       CUST OG, ABC OG, TR OG
AS13 ASSIGN, ATRIB(6)=ATRIB(6)+17000.0;
CO25 COLCT, ATRIB(6), COST 9, 10, 0.0, 100.0;
CO26 COLCT, ATRIB(4), NO OF TESTS 9, 10, 0.0, 2.0;
CO27 COLCT, ATRIB(8), ACTUAL QUAL 9, 10, 0.0, 10.0;
       ACT,, AS14;
CO28 COLCT, ATRIB(6), COST 10, 10, 0.0, 100.0;
CO29 COLCT, ATRIB(4), NO OF TESTS 10, 10, 0.0, 2.0;
CO30 COLCT, ATRIB(8), ACTUAL QUAL 10, 10, 0.0, 10.0;
       ACT,, AS14;
       AS14 ASSIGN, XX(99)=XX(99)+1,
       XX(100)=XX(100)+ATRIB(6);
CO31 COLCT, ATRIB(6), TRANSACTION COST, 10, 0.0, 1000.0;
CO32 COLCT, ATRIB(14), NO OF REROUTES, 10, 0.0, 1.0;
CO33 COLCT, ATRIB(4), NO OF TESTS, 10, 0.0, 10.0;
TR01 TERMINATE, 10000;
    ENDNETWORK;
FIN;
//FT07F001 DD DSN=&TAPE7,UNIT=SYSDA,DISP=(NEW,PASS),
// DCB=(RECFM=FB,LRECL=80,BLKSIZE=6160),SPACE=(TRK,(5,1),RLSE)
//FT09F001 DD *
    0.0 21.213 1 80.0 1 120.0 10.0
/*
//FT10F001 DD DUMMY
//FT19F001 DD DUMMY
APPENDIX IC
Shipping Network - FORTRAN Subroutines

SUBROUTINE INTLC
COMMON/GFCOM1/ BIA,S,SREV,ILB,XLB,IHB,XHB,TSTCST
READ(9,100) BIA,S,ILB,XLB,IHB,XHB,TSTCST
100 FORMAT(2F10.2,I10,F10.2,I10,F10.2,F10.2)
WRITE(6,200) BIA,S,ILB,XLB,IHB,XHB,TSTCST
200 FORMAT('BIAS FORQUAL ATRIB 1= ',F10.2,/,A
SX,'STD DEV-OBSERVED ERROR FOR QUAL ATRIB 1= ',F10.2,/,B
SX,'LOWER BOUND INDICATOR FOR QUAL ATRIB 1= ',I10,/,C
SX,'LOWER BOUND FOR QUAL ATRIB 1= ',F10.2,
D SX,'UPPER BOUND INDICATOR FOR QUAL ATRIB 1= ',I10,/,E
SX,'UPPER BOUND FOR QUAL ATRIB 1= ',F10.2,
F SX,'COST PER TEST FOR ATRIB 1= ',F10.2)
RETURN
END

FUNCTION USERF(IX)
COMMON/SCOM1/ ATRIB(100),DD(100),DDL(100),DTNOW,II,MFA,MSTOP,NCLNRSC1
1,NCRDR,NPRNT,NNRUN,NNSET,NTAPE,SS(100),SSL(100),TNEXT,TNOW,XX(100)SC1
2 COMMON/GFCOM1/ BIA,S,SREV,ILB,XLB,IHB,XHB,TSTCST
GO TO (1,2,3,4,99,99,7,8),1X

C *** CALCULATE/REVISE THE EPWS FOR THE FIRST OR SUBSEQUENT TEST
C *** RESULT FOR A ONE OR TWO SIDED TEST ('II' TEST RESULTS ARE
C *** OBTAINED)
C
1 EPROB=1.0
IF(II.EQ.0) GO TO 666
DO 100 I=1,II
C
100 ATRIB(4)=ATRIB(4)+1.0
X1=ATRIB(8)-BIA
TR=RNORM(X1,S,1)
ATRIB(1)=ATRIB(1)+((TR-XX(18))
XX(19)=10.0*ATRIB(4)
XX(11)=40.32+XX(19)
XX(12)=40.32-XX(19)
XX(21)=25.62-XX(19)
XX(22)=-25.62+XX(19)
ATRIB(12)=((ATRIB(12)*(ATRIB(4)-1.0))+TR)/ATRIB(4)
WRITE(19,210) TNOW,I,TR,ATRIB(8),ATRIB(12),BIA,X1
210  FORMAT(15X,'TNOW,I,TR,ATRIB(8),ATRIB(12),BIA,X1 = ',F10.2,I5,
     A 5F10.2,/)  
     SREV=S/SQRT(ATRIB(4))  
     X=ATRIB(12)  
     Q=SPEC(X)  
     IF(Q.EQ.0.0) XD=1.0  
     IF(Q.EQ.1.0) XD=0.0  
     ATRIB(15)=ATRIB(15)+XD  
     EPROB=EPWS(X)  
     WRITE(19,120) TNOW,ATRIB(12),S,SREV,ATRIB(4),EPROB
120  FORMAT(10X,'TNOW,ATRIB(12),S,SREV,ATRIB(4),EPWS= ',6F10.2,/)  
100 CONTINUE  
666 USERF=EPROB  
RETURN
C
C *** DETERMINE CUSTOMER'S REACTION  I IF CUST PRIME, 0 OTHERWISE
C *** CUST TAKES 4 OBS, IF AUG IS IN, THEN CUSTPRIME
C
2  X1=RNORM(ATRIB(8),S,1)  
  X2=RNORM(ATRIB(8),S,1)  
  X3=RNORM(ATRIB(8),S,1)  
  X4=RNORM(ATRIB(8),S,1)  
  AVG=(X1+X2+X3+X4)/4.0  
  USERF = SPEC(AVG)  
  WRITE(19,115) TNOW,X1,X2,X3,X4,Avg,USERF
115  FORMAT(10X,'TNOW,X1,X2,X3,X4,Avg,SPEC= ',7F10.2,/)  
     ATRIB(13)=AVG  
RETURN
C
C *** ACTUAL QUAL FLAG 1 IF ACTUALLY PRIME, 0 OTHERWISE
C
3  USERF= SPEC(ATRIB(8))  
  WRITE(19,110) TNOW,ATRIB(8),USERF
110  FORMAT(10X,'TNOW,ATRIB(8),SPEC= ',3F10.2,/)  
RETURN
C
C *** REFUTATION FLAG 1 IF SUCCESSFUL, 0 OTHERWISE
C *** 4XCUST AVG+ ABC AVG/5 IS MEASURE USED TO DETERMINE IF
C *** IN SPEC
C
4  AVG=(4*ATRIB(13)+ATRIB(12))/5.0  
  USERF=SPEC(AVG)  
  WRITE(19,105) TNOW,Avg,USERF
105  FORMAT(10X,'TNOW,Avg,USERF')
105 FORMAT(10X,'TNOW,AVG,SPEC=',3F10.2,/)  
RETURN
C
C *** INCREMENT TEST COST
C
    7 USERF=ATRIB(6)+TSTCST*I1  
RETURN
C
C *** WRITE OUT TRANSACTION COST
C
    8 ITNOW=TNOW+.001  
WRITE(10,55) ITNOW,ATRIB(6)  
55 FORMAT(I10,F12.2)  
RETURN
99 RETURN
END
FUNCTION EPWS(X)
COMMON/GFCOM1/ BIA,S,SREV,ILB,XLB,ILB,XHB,TSTCST
REAL*4 IMP
C
C *** DECIDE ON TYPE OF TEST
C
    IF((ILB.EQ.1).AND.(IHB.EQ.1)) GO TO 100  
    IF((ILB.EQ.1).AND.(IHB.NE.1)) GO TO 150  
    IF((ILB.NE.1).AND.(IHB.EQ.1)) GO TO 200
C
C *** TWO SIDED TEST
C
100  Z1=(XLB-X)/SREV  
    Z2=(XHB-X)/SREV  
    PROB1=ERF(Z1)  
    PROB2=ERF(Z2)  
    EPWS=PROB2-PROB1  
RETURN
C
C *** ONE TAILED TEST, LOWER LIMIT
C
150  Z=(XLB-X)/SREV  
    EPWS=1.0-ERF(Z)  
RETURN
C
C *** ONE TAILED TEST, UPPER LIMIT
C
200  Z=(XHB-X)/SREV
-161-
FUNCTION SPEC(X)
C ***
RETURNS A 1 IF X IS IN SPEC, A 0 OTHERWISE
C
COMMON/GFCOML/ BIA,S,SREV,ILB,XLB,ILB,ILB,H-XHB,TSTCST
SPEC=0.0
C
C *** DECIDE ON TYPE OF TEST
C
IF((ILB.EQ.1).AND.(IHB.EQ.1)) GO TO 100
IF((ILB.EQ.1).AND.(IHB.NE.1)) GO TO 150
IF((ILB.NE.1).AND.(IHB.EQ.1)) GO TO 200
C
C *** TWO_SIDED TEST
C
100 IF((X.GE.XLB).AND.(X.LE.XHB)) SPEC=1.0
RETURN
C
C *** ONE_SIDED TEST, LOWER LIMIT
C
150 IF(X.GE.XLB) SPEC=1.0
RETURN
C
C *** ONE_SIDED TEST, UPPER LIMIT
C
200 IF(X.LE.XHB) SPEC=1.0
RETURN
END
FUNCTION ERF(Z)
C *** FUNCTION SUBROUTINE TO INTEGRATE NORMAL DISTRIBUTION FUNCTION
C *** A SOLUTION BY HASTINGS GIVES THE APPROXIMATION
C *** THE MAXIMUM ERROR OF THE APPROXIMATION IS .0000003
C
IF(Z.GT.4.17) GO TO 104
IF(Z.LT.-4.17) GO TO 105
ZZ=Z
IF(Z.LT.0) ZZ=-Z
T=ZZ/1.4142142
D=((((((.430638E-4*T+.2765672E-3)*T+.1520143E-3)*T+.92705272E-2)*T
1+.42282012E-1)*T+.70523078E-1)*T+1.0)**2
D=D*D
D=D*D
D=D*D
ERF=.5-.5/D
IF(Z) 101,102,103

101 ERF=.5-ERF
GO TO 106

102 ERF=.5
GO TO 106

103 ERF=.5+ERF
GO TO 106

104 ERF=1.0
RETURN

105 ERF=0.0
106 CONTINUE
RETURN
END

SUBROUTINE OUTPUT
COMMON/SCOM1/ ATRIB(100),DD(100),DDL(100),DTNOW,II,MFA,MSTOP,NCLNRSC1
1,NCRDR,NPRNT,NNRUN,NNSET,NTAPE,SS(100),SSL(100),TNEXT,TNOW,XX(100)SC1
2
AVCOST=XX(100)/XX(99)
WRITE(6,100) XX(99),XX(100),AVCOST

100 FORMAT(1X,23HXX(99),XX(100),AVCOST=,3E16.5)
RETURN
END
APPENDIX IIA
Storage Network - SLAM Network
APPENDIX IIA
Storage Network - SLAM Network
APPENDIX IIA
Storage Network - SLAM Network
APPENDIX IIB
Storage Network - SLAM Code

//STEP1 EXEC PGM=H3D019CF,TIME=15
//STEP2 DD DSN=SCILIB,DISP=SHR
//FT06F001 DD SYSOUT=A
//FT05F001 DD *
GEN,G.A.FREESTONE,QA.NETWORK.STORAGE,04/14/82,1;
LIMITS,1,17,100;
PRIORITY/1,FIFO;
INTLC,XX(1)=0.02,XX(2)=0.30;
INITIALIZE,0.0,25000.0;
NETWORK;
;
ATRIB(1)=USED FR DUNCAN'S SEQUENTIAL TESTS
ATRIB(2)=ACCUMULATED EPWS (ALL QUALITY ATTRIBUTES)
ATRIB(3)=ACCUMULATED COST (ALL QUALITY ATTRIBUTES)
ATRIB(4)=QUAL ATTRIBUTE 1:#OF TEST RESULTS OBTAINED
ATRIB(5)=QUAL ATTRIBUTE 1:#OF GROUPS OF TEST RESULTS OBTAINED
ATRIB(6)=QUAL ATTRIBUTE 1:COST ASSOCIATED WITH THIS QUAL ATRIB
ATRIB(7)=QUAL ATTRIBUTE 1:EPWS THIS QUAL ATRIB
ATRIB(8)=QUAL ATTRIBUTE 1:ACTIONAL QUAL THIS QUAL ATRIB
ATRIB( 9)=QUAL ATTRIBUTE 1:CUST REACTION FLAG
ATRIB(10)=QUAL ATTRIBUTE 1:ACTIONAL FLAG THIS QUAL ATRIB FOR THIS LOT
ATRIB(11)=QUAL ATTRIBUTE 1:ACTIONAL FLAG FOR THE MIXTURE
ATRIB(12)=QUAL ATTRIBUTE 1:MEAN OF ABC TEST RESULTS TO DATE
ATRIB(13)=QUAL ATTRIBUTE 1:CUSTOMER MEAN OF TEST RESULTS
ATRIB(14)=QUAL ATTRIBUTE 1:ACTIONAL QUALITY OF QUALITY MIXTURE
ATRIB(15)=QUAL Attribute 1: # OF DEFECTS OBTAINED TO DATE
ATRIB(16)=QUAL Attribute 1: STD DEV OF RESULTS TO DATE
ATRIB(17)=QUAL Attribute 1: CALCULATED DC FOR WALSH PLAN

USERF(1)=OBTAIN TEST RESULTS
USERF(2)=DETERMINE CUSTOMER REACTION TO MIXTURE
USERF(3)=DETERMINE ACTUAL QUALITY FLAG FOR THIS ENTITY
USERF(4)=MIX PRODUCT IN OG SILO
USERF(5)=MIX PRODUCT IN PRIME SILO
USERF(6)=DETERMINE ACTUAL QUALITY FLAG FOR THE MIXTURE
USERF(7)=INCREMENT TEST COST

CREA CREATE,1.0,1.0,1,22000.1;
CREATE TRANSACTIONS
ASO1 ASSIGN,II=4,
ATRIB(8)=UNFRM(50.0,150.0,2),
ATRIB(7)=USERF(1),
ATRIB(5)=ATRIB(5)+1,
ATRIB(6)=USERF(7),
ATRIB(10)=USERF(3);

G001 GOON;

ATRIB(7)=USERF(1),
ATRIB(5)=ATRIB(5)+1,
ATRIB(6)=USERF(7),
ATRIB(10)=USERF(3);

G001 GOON;

ACT,,ATRIB(7).GT.XX(1).AND.ATRIB(7).LT.XX(2).AND.ATRIB(4).LT.300,AS02;
ACT,,ATRIB(7).GT.XX(2).AND.ATRIB(4).LE.300,C034;
ACT,,ATRIB(7).LE.XX(1).OR.ATRIB(4).GE.300,C035;

AS02 ASSIGN,II=1,
ATRIB(7)=USERF(1),
ATRIB(5)=ATRIB(5)+1,
ATRIB(6)=USERF(7);

ACT,,GO01;

C034 COLCT,ATRIB(10),ABC P TR P;
G002 GOON;
AS03 ASSIGN,ATRIB(14)=USERF(5),
ATRIB(11)=USERF(6),
ATRIB(9)=USERF(2);

ACT,,ATRIB(9).EQ.1,G003;
ACT,,ATRIB(9).EQ.0,G004;

G003 GOON;

ACT,,ATRIB(11).EQ.1,CO01;
ACT,,ATRIB(11).EQ.0,AS05;

C001 COLCT,ATRIB(6),COST 1,10,0.0,100.0;
C002 COLCT,ATRIB(4),NO OF TESTS 1,10,0.0,2.0;
C003 COLCT,ATRIB(8),ACTUAL QUAL 1,10,0.0,10.0;

G003 GOON;

ACT,,ATRIB(11).EQ.1,CO01;
ACT,,ATRIB(11).EQ.0,AS05;

AS05 ASSIGN,ATRIB(6)=ATRIB(6)+50000.0; ADD CUST SERV PENALTY TO $40K
C004 COLCT,ATRIB(6),COST 2,10,0.0,100.0;
C005 COLCT,ATRIB(4),NO OF TESTS 2,10,0.0,2.0;
C006 COLCT,ATRIB(8),ACTUAL QUAL 2,10,0.0,10.0;

G004 GOON;

ACT,,ATRIB(11).EQ.1,CO07;
ACT,,ATRIB(11).EQ.0,AS07;

C007 COLCT,ATRIB(6),COST 3,10,0.0,100.0;
C008 COLCT,ATRIB(4),NO OF TESTS 3,10,0.0,2.0;
C009 COLCT,ATRIB(8),ACTUAL QUAL 3,10,0.0,10.0;
ACT,,,AS14;

AS07 ASSIGN, ATRIB(6) = ATRIB(6) + 40000.0;
CO10 COLCT, ATRIB(6), COST 4, 10, 0.0, 100.0;
CO11 COLCT, ATRIB(4), NO OF TESTS 4, 10, 0.0, 2.0;
CO12 COLCT, ATRIB(8), ACTUAL QUAL 4, 10, 0.0, 10.0;
    ACT,,,AS14;

    

CO35 COLCT, ATRIB(10), ABC O TR P;
AS04 ASSIGN, ATRIB(14) = USERF(4),
    ATRIB(11) = USERF(6),
    ATRIB(9) = USERF(2);
G007 GOON;
    ACT,, ATRIB(9).EQ.1,G008;
    ACT,, ATRIB(9).EQ.0,G009;
G008 GOON;
    ACT,, ATRIB(10).EQ.1,A S11;
    ACT,, ATRIB(10).EQ.0,A S12;
AS11 ASSIGN, ATRIB(6) = ATRIB(6) + 17000.0;
CO19 COLCT, ATRIB(6), COST 7, 10, 0.0, 100.0;
CO20 COLCT, ATRIB(4), NO OF TESTS 7, 10, 0.0, 2.0;
CO21 COLCT, ATRIB(8), ACTUAL QUAL 7, 10, 0.0, 10.0;
    ACT,,,AS14

AS12 ASSIGN, ATRIB(6) = ATRIB(6) + 0.0 ;
    MISS CLASS COST?
CO22 COLCT, ATRIB(6), COST 8, 10, 0.0, 100.0;
CO23 COLCT, ATRIB(4), NO OF TESTS 8, 10, 0.0, 2.0;
CO24 COLCT, ATRIB(8), ACTUAL QUAL 8, 10, 0.0, 10.0;
    ACT,,,AS14;
G009 GOON;
    ACT,, ATRIB(10).EQ.1,AS13;
    ACT,, ATRIB(10).EQ.0,CO28 ;
AS13 ASSIGN, ATRIB(6) = ATRIB(6) + 17000.0;
CO25 COLCT, ATRIB(6), COST 9, 10, 0.0, 100.0;
CO26 COLCT, ATRIB(4), NO OF TESTS 9, 10, 0.0, 2.0;
CO27 COLCT, ATRIB(8), ACTUAL QUAL 9, 10, 0.0, 10.0;
    ACT,,,AS14;
CO28 COLCT, ATRIB(6), COST 10, 10, 0.0, 100.0;
CO29 COLCT, ATRIB(4), NO OF TESTS 10, 10, 0.0, 2.0;
CO30 COLCT, ATRIB(8), ACTUAL QUAL 10, 10, 0.0, 10.0;
    ACT,,,AS14;

;
AS14  ASSIGN,XX(99)=XX(99)+1,
       XX(100)=XX(100)+ATRIB(6);
CO31  COLCT,ATRIB(6),TRANSACTION COST,10,0.0,1000.0;
CO33  COLCT,ATRIB(4),NO OF TESTS,10,0.0,10.0;
TR01  TERMINATE,20000;
       ENDNETWORK;
FIN;
//FT07F001 DD DSN=&amp;TAPE7,UNIT=SYSDA,DISP=(NEW,PASS),
   DCB=(RECFM=FB,LRECL=80,BLKSIZE=6160),SPACE=(TRK,(5,1),RLSE)
//FT09F001 DD *
   0.0   15.0  0   0.0   1  100.0 10.0
/*
//FT10F001 DD *
 &STORE LOTSIZ=170000.,SILSIZ=400000.,XPRIME=90.0,
    XLOFF=0.0,XHOF=120.0,&END
//FT11F001 DD DUMMY
//FT19F001 DD DUMMY

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SUBROUTINE INTLC
COMMON/GFCOM1/ BIA,S,SREV,ILB,XLB,IHB,XHB,TSTCST
COMMON/GFCOM2/ LOTSIZ,SILSIZ,XPRIME,XLOFF,XHOFF
REAL*4 LOTSIZ
NAMELIST/STORE/ LOTSIZ,SILSIZ,XPRIME,XLOFF,XHOFF
READ(9,100) BIA,S,ILB,XLB,IHB,XHB,TSTCST
100 FORMAT(2F10.2,I10,F10.2,I10,2F10.2)
WRITE(6,200) BIA,S,ILB,XLB,IHB,XHB,TSTCST
200 FORMAT(1',4X,'BIAS FOR QUAL ATRIB 1= ',F10.2,/, 
A 5X,'STD DEV-OBSERVED ERROR FOR QUAL ATRIB 1= ',F10.2,/, 
B 5X,'LOWER BOUND INDICATOR FOR QUAL ATRIB 1= ',I10,/, 
C 5X,'LOWER BOUND FOR QUAL ATRIB 1= ',F10.2,/, 
D 5X,'UPPER BOUND INDICATOR FOR QUAL ATRIB 1= ',I10,/, 
E 5X,'UPPER BOUND FOR QUAL ATRIB 1= ',F10.2,/, 
F 5X,'COST PER TEST FOR ATRIB 1= ',F10.2)
C
C *** READ STORAGE NETWORK DEPENDENT INFO
C
READ(10,STORE)
WRITE(6,STORE)
RETURN
END

FUNCTION USERF(IX)
COMMON/SCOM1/ ATRIB(100),DD(100),DDL(100),DTNOW,II,MFA,MSTOP,NCLNRSC1 1
1,NCRDR,NPRNT,NNRUN,NNSET,NTAPE,SS(100),SSL(100),TNEXT,TNOW,XX(100)SC1 2
COMMON /GFCOM1/ BIA,S,SREV,ILB,XLB,IHB,XHB,TSTCST
COMMON/GFCOM2/ LOTSIZ,SILSIZ,XPRIME,XLOFF,XHOFF
REAL*4 LOTSIZ
REAL*8 SUMNXB,SUNX2
GO TO(1,2,3,4,5,6,7,8),IX
C
C *** CALCULATE/REVISE THE EPWS FOR THE FIRST OR SUBSEQUENT TEST
C *** RESULT FOR A ONE OR TWO SIDED TEST ('II' TEST RESULTS ARE
C *** OBTAINED)
C
1 EPROB=1.0
IF(II.EQ.0) GO TO 666
DO 100 I=1,II
C
C *** OBTAIN TEST RESULT

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C

ATRIB(4)=ATRIB(4)+1.0
TR=RNORM(ATRIB(8)-BIA,S,1)
IF(TR.LT.0.0) TR=0.0
ATRIB(1)=ATRIB(1)+TR
XX(19)=95.0*ATRIB(4)
XX(11)=57.808+XX(19)
XX(12)=-45.026+XX(19)
ATRIB(12)=((ATRIB(12)*ATRIB(4)-1.0)+TR)/ATRIB(4)
STDDEV=9999.9.
IF(ATRIB(4).LT.1.1) SUMX2=TR*TR
IF(ATRIB(4).GT.1.0) SUMX2=SUMX2+TR*TR
SUMNXB=ATRIB(4)*ATRIB(12)*ATRIB(12)
TSUM=SUMX2-SUMNXB
IF(TSUM.LT.0.0) TSUM=0.0
IF(ATRIB(4).LT.1.5) GO TO 777
STDDEV=SQRT(1./(ATRIB(4)-1.0)*TSUM)
WRITE(19,223) SUMX2,SUMNXB,STDDEV
223 FORMAT(7X,,','SUMX2,SUMNXB,STDEV= ',,3F14.3)
777 ATRIB(16)=STDDEV
WRITE(19,210) TNOW,I,TR,ATRIB(8),ATRIB(12),STDDEV
210 FORMAT(15X,',TNOW,I,TR,A(8),A(12),STD= ',,F10.2,I5,4F10.2,/,)
SREV=S/SQRT(ATRIB(4))
X=ATRIB(12)
Q=SPEC(X)
IF(Q.EQ.0.0) XD=1.0
IF(Q.EQ.1.0) XD=0.0
ATRIB(15)=ATRIB(15)+XD
EPROB=EPWS(X)
WRITE(19,120) TNOW,ATRIB(12),S,SREV,ATRIB(4),EPROB
120 FORMAT(10X,',TNOW,ATRIB(12),S,SREV,ATRIB(4),EPWS= ',,6F10.2,/,)
100 CONTINUE
666 USERF=EPROB
RETURN

C

*** DETERMINE CUSTOMER'S REACTION 1 IF CUST PRIME, 0 OTHERWISE
*** CUST TAKES 4 OBS, IF AUG IS IN, THEN CUSTPRIME

2 X1=RNORM(ATRIB(14),S,1)
X2=RNORM(ATRIB(14),S,1)
X3=RNORM(ATRIB(14),S,1)
X4=RNORM(ATRIB(14),S,1)
AVG=(X1+X2+X3+X4)/4.0
USERF = SPEC(AVG)
WRITE(19,115) TNOW,X1,X2,X3,X4,AVG,USERF
115 FORMAT(10X,'TNOW,X1,X2,X3,X4,AVG,SPEC=',7F10.2,/)  
ATRIB(13)=AVG  
RETURN  

C
C  *** ACTUAL QUAL FLAG 1 IF ACTUALLY PRIME, 0 OTHERWISE
C
3 USERF= SPEC(ATRIB(8))  
WRITE(19,110) TNOW,ATRIB(8),USERF  
110 FORMAT(10X,'TNOW,ATRIB(8),SPEC=',3F10.2,/)  
RETURN  

C
C  *** STORE PRODUCT IN OFFGRADE SILO, DETERMINE IF MIXTURE QUALITY
C
4 IF((ILB.EQ.1).AND.(IHB.EQ.1)) GO TO 320  
IF((ILB.EQ.1).AND.(IHB.NE.1)) GO TO 300  
IF((ILB.NE.1).AND.(IHB.EQ.1)) GO TO 310  

C
C  *** ONE SIDED TEST, LOWER LIMIT
C
300 SIQUAL=(SILSIZ*XLOFF+LOTSIZ*ATRIB(B))/(SILSIZ+LOTSIZ)  
WRITE(19,556) ATRIB(B),SIQUAL  
556 FORMAT(25X,'ATRIB(B),SIQUAL=',2F14.3)  
USERF=SIQUAL  
RETURN  

C
C  *** ONE SIDED TEST, UPPER LIMIT
C
310 SIQUAL=(SILSIZ*XHOFF+LOTSIZ*ATRIB(B))/(SILSIZ+LOTSIZ)  
WRITE(19,556) ATRIB(B),SIQUAL  
USERF=SIQUAL  
RETURN  

C
C  *** TWO SIDED TEST, DECIDE IF ABC PERCEIVED HI OR LO BEFORE MIXING
C
320 IF(ATRIB(12).LE.XLB) GO TO 303  
C
C  *** ABC PERCEIVED QUALITY IS HIGH
C
SIQUAL=(SILSIZ*XLOFF+LOTSIZ*ATRIB(B))/(SILSIZ+LOTSIZ)  
WRITE(19,556) ATRIB(B),SIQUAL  
USERF=SIQUAL  
RETURN  

C
C *** ABC PERCEIVED QUALITY IS LOW

303 SIQUAL=(SILSIZ*XHOFF+LOTSIZ*ATRIB(8))/(SILSIZ+LOTSIZ)
WRITE(19,556) ATRIB(8),SIQUAL
USERF=SIQUAL
RETURN

C *** STORE PRODUCT IN PRIME SILO, MIX

5 SIQUAL=(SILSIZ*XPRIME+LOTSIZ*ATRIB(8))/(SILSIZ+LOTSIZ)
WRITE(19,556) ATRIB(8),SIQUAL
USERF=SIQUAL
RETURN

C *** ACTUAL QUALITY FLAG FOR SILO MIXTURE

6 USERF=SPEC(ATRIB(14))
RETURN

C *** SET TEST COST

7 USERF=ATRIB(6)+II*TSTCST
RETURN

C *** WRITE OUT TRANSACTION COST

8 ITNOW=TNOW+.001
WRITE(11,55) ITNOW,ATRIB(6)
55 FORMAT(I10,F12.2)
RETURN
END

FUNCTION EPWS(X)
COMMON/GFCOM1/ BIA,S,SREV,ILB,XLB,IHB,XHB,TSTCST
COMMON/GFCOM2/ LOTSIZ,SILSIZ,XPRIME,XLOFF,XHOFF
REAL*4 LOTSIZ
REAL*4 IMP

C *** DECIDE ON TYPE OF TEST

IF((ILB.EQ.1).AND.(IHB.EQ.1)) GO TO 100
IF((ILB.EQ.1).AND.(IHB.NE.1)) GO TO 150
IF((ILB.NE.1).AND.(IHB.EQ.1)) GO TO 200

C *** TWO SIDED TEST
**C**

100  
Z1=(XLB-X)/SREV  
Z2=(XHB-X)/SREV  
PROB1=ERF(Z1)  
PROB2=ERF(Z2)  
EPWS=PROB2-PROB1  
RETURN

**C**  
*** ONE TAILED TEST, LOWER LIMIT  
**C**

150  
Z=(XLB-X)/SREV  
EPWS=1.0-ERF(Z)  
RETURN

**C**  
*** ONE TAILED TEST, UPPER LIMIT  
**C**

200  
Z=(XHB-X)/SREV  
EPWS=ERF(Z)  
RETURN

FUNCTION SPEC(X)  
**C**  
*** RETURNS A 1 IF X IS IN SPEC, A 0 OTHERWISE  
**C**

COMMON/GFCOM1/ BIA,S,SREV,ILB,XLB,IHB,XHB,TSTCST  
COMMON/GFCOM2/ LOTSIZ,SILSIZ,XPRIME,XLOFF,XHOFF  
REAL*4 LOTSIZ  
SPEC=0.0

**C**  
*** DECIDE ON TYPE OF TEST  
**C**

IF((ILB.EQ.1).AND.(IHB.EQ.1)) GO TO 100  
IF((ILB.EQ.1).AND.(IHB.NE.1)) GO TO 150  
IF((ILB.NE.1).AND.(IHB.EQ.1)) GO TO 200

**C**  
*** TWO SIDED TEST  
**C**

100  
IF((X.GE.XLB).AND.(X.LE.XHB)) SPEC=1.0  
RETURN

**C**  
*** ONE SIDED TEST, LOWER LIMIT  
**C**

150  
IF(X.GE.XLB) SPEC=1.0  
RETURN
ONE SIDED TEST, UPPER LIMIT

IF(X.LE.XHB) SPEC=1.0
RETURN
END

FUNCTION ERF(Z)

FUNCTION SUBROUTINE TO INTEGRATE NORMAL DISTRIBUTION FUNCTION

A SOLUTION BY HASTINGS GIVES THE APPROXIMATION

THE MAXIMUM ERROR OF THE APPROXIMATION IS .0000003

IF(Z.GT.4.17) GO TO 104
IF(Z.LT.-4.17) GO TO 105
ZZ=Z
IF(Z.LT.0) ZZ=-Z
T=ZZ/1.4142142
D=((((.430638E-4*T+.2765672E-3)*T+.1520143E-3)*T+.92705272E-2)*T
  +.42282012E-1)*T+.70523078E-1)*T+1.0)**2
D=D*D
D=D*D
D=D*D
ERF=.5-.5/D
IF(Z) 101,102,103
101 ERF=.5-ERF
GO TO 106
102 ERF=.5
GO TO 106
103 ERF=.5+ERF
GO TO 106
104 ERF=1.0
RETURN
105 ERF=0.0
106 CONTINUE
RETURN
END

SUBROUTINE OUTPUT

COMMON/SCOM1/ ATRIB(100),DD(100),DDL(100),DTNOW,II,MFA,MSTOP,NCLNRSC1
1,1,NCRDR,NPRNT,NNRTT,NNSET,NTAPE,SS(100),SSL(100),TNEXT,TNOW,XX(100)SC1
2
AVCOST=XX(100)/XX(99)
WRITE(6,100) XX(99),XX(100),AVCOST
100 FORMAT(1X,23HXX(99),XX(100),AVCOST=,3E16.5)
RETURN
END
APPENDIX III

Shipping Problem Least Cost Guidelines

**STATISTICS FOR VARIABLES BASED ON OBSERVATION**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean Value</th>
<th>Standard Deviation</th>
<th>Coefficient of Variation</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arc 1</td>
<td>0.4920</td>
<td>0.1940</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.194E+00</td>
<td>0.019E+00</td>
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<tr>
<td>Cost 1</td>
<td>0.1662</td>
<td>0.1860</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.186E+00</td>
<td>0.018E+00</td>
</tr>
<tr>
<td>Actual Use 1</td>
<td>0.1620</td>
<td>0.1260</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.126E+00</td>
<td>0.012E+00</td>
</tr>
<tr>
<td>Cost 2</td>
<td>0.1362</td>
<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
<tr>
<td>No of Tests 2</td>
<td>0.1032</td>
<td>0.1260</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.126E+00</td>
<td>0.012E+00</td>
</tr>
<tr>
<td>Actual Use 2</td>
<td>0.1762</td>
<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
<tr>
<td>Cost 3</td>
<td>0.2140</td>
<td>0.1400</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.140E+00</td>
<td>0.014E+00</td>
</tr>
<tr>
<td>No of Tests 3</td>
<td>0.2342</td>
<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
<tr>
<td>Actual Use 3</td>
<td>0.1240</td>
<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
<tr>
<td>Cost 4</td>
<td>0.7260</td>
<td>0.1400</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.140E+00</td>
<td>0.014E+00</td>
</tr>
<tr>
<td>No of Tests 4</td>
<td>0.1040</td>
<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
<tr>
<td>Actual Use 4</td>
<td>0.1040</td>
<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
<tr>
<td>Cost 5</td>
<td>0.1240</td>
<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
<tr>
<td>Actual Use 5</td>
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<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
<tr>
<td>Cost 6</td>
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<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
<tr>
<td>No of Tests 6</td>
<td>0.1240</td>
<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
<tr>
<td>Actual Use 6</td>
<td>0.1240</td>
<td>0.1040</td>
<td>0.994E+00</td>
<td>0.0</td>
<td>0.104E+00</td>
<td>0.010E+00</td>
</tr>
</tbody>
</table>

**FILE STATISTICS**

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APPENDIX IV
Storage Problem Least Cost Guidelines
V I T A

Personal History

Name: George Andrew Freestone
Birthplace: Lakewood, Ohio
Birthdate: 14 April 1954
Parents: George Dudley and Barbara Seaman Freestone
Spouse: Carroll Siket Freestone
Children: Andrea Michelle Freestone

Educational Background

Lehigh University 1972-1976
Bachelor of Science in Industrial Engineering
Minor in Economics (magna cum laude)

Lehigh University 1977-1982
Candidate for Master of Science
in Industrial Engineering (with emphasis in
Operations Research)

Honors

Alpha Pi Mu, Tau Beta Pi, Delta Chi Scholarship Awards

Professional Experience

Air Products and Chemicals, Inc. 1975-
Trexlertown, Pennsylvania
Presently Systems Manager - Operations Research

Professional Societies

Institute of Industrial Engineers