An Angled End Tap for Fiber Optic Communications Systems.

Charles D. Sallada

Follow this and additional works at: http://preserve.lehigh.edu/etd
Part of the Electrical and Computer Engineering Commons

Recommended Citation
An Angled End Tap for Fiber Optic Communications Systems

by

Charles D. Sallada

A Thesis
Presented to the Graduate Committee
of Lehigh University
in candidacy for the Degree of
Master of Science
in
Electrical and Computer Engineering Department

Lehigh University
1982
This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

May 15, 1982

(date)

Professor in Charge

Chairman of the Department
ACKNOWLEDGEMENTS

The author wishes to express his appreciation to several individuals who's contributions have made completion of this project possible: Dr. M. DiDomenico Jr., M. A. Karr, and T. C. Rich for the tap conception, Dr. H. P. Hsu for his analytical contributions, Professor W. Dahlke of Lehigh University for his guidance and encouragement, and J. L. Hokanson who reviewed the manuscript.
# Table of Contents

**Abstract** | 1  
---|---  
**Section 1 - Introduction** | 3  
1.1 Optical Coupler and Taps | 3  
1.2 Project Goals | 4  
1.3 Technical Contributions | 5  
1.4 Physical Description of the Angled End Optical Tap | 5  
**Section 2 - Technical Background** | 6  
2.1 Tap Transmission and Coupling Coefficients | 6  
2.2 Step versus Graded Index Profiles | 7  
2.3 Analysis Approach | 9  
2.4 Fiber Characteristics | 10  
**Section 3 - Angled End Tap Analysis with Step Index Approximation** | 10  
3.1 Limits of Tap Angle $\theta$ | 10  
3.2 Tap Transmission and Coupling versus Ray Angle | 14  
3.3 Tap Transmission and Coupling for Gaussian and Uniform Light Distributions | 19  
3.4 Tap Transmission and Coupling versus Fiber Spacing | 21  
**Section 4 - Analysis of the Graded Index Fiber Tap** | 24  
4.1 Ray Propagation in Graded Index Fibers | 24  
4.2 Maximum Guided Ray Angle Over the Fiber Core | 25  
4.3 Analytical Differences | 26  
4.4 Calculated Transmission and Coupling | 28  
4.5 Determination of Minimum Collection Width | 28  
**Section 5 - Experimental Results and Conclusions** | 29  
5.1 Description of Experimental Tap | 29
| Figure 1 | Angled End Optical Tap |
| Figure 2 | Refractive Index Profiles of Step and Graded Index Fibers. |
| Figure 3 | Tap Model Showing Angles of Incidence at the Angled and Side Boundary. |
| Figure 4 | Tap Operation as a Function of Tap Angle. |
| Figure 5 | Determination of Total Transmitted Power from Ray A with Power P(A). |
| Figure 6 | Tap Transmission and Coupling vs. Ray Angle. |
| Figure 7 | Tap Transmission and Coupling vs. Ray Angle for Uniform and Gaussian Distribution of Ray Power. |
| Figure 8 | Ray Propagation with Finite Fiber Spacing. |
| Figure 9 | Transmitted and Coupled Outputs of the Tap for Rays Originating at Several Points Along the Fiber Cross Section. |
| Figure 10 | Tap Transmission and Coupling vs. Fiber Spacing for Various Tap Angles Theta. |
| Figure 11 | Ray Path in Medium with a Quadratic Index Variation. |
| Figure 12 | Maximum Guided Ray Angle vs. Fiber Radius for a Typical Graded Index Fiber. |
| Figure 13 | Graded Index Tap Transmission and Coupling vs. Fiber Spacing for Various Tap Angles, Theta. |
| Figure 14 | Comparison of 62° Graded Index Tap Results with Those Obtained Using the Step Index Approximation. |
| Figure 15 | Tap Coupled Output vs. Collection Width. |
| Figure 16 | Comparison of Theoretical and Experimental Results of 63 Degree Graded Index Angled End Tap. |
## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Fiber core radius</td>
</tr>
<tr>
<td>c</td>
<td>A constant describing ray propagation in graded index fibers.</td>
</tr>
<tr>
<td>d</td>
<td>Fiber spacing</td>
</tr>
<tr>
<td>g</td>
<td>Profile parameter</td>
</tr>
<tr>
<td>k, k₂</td>
<td>Fiber propagation constants</td>
</tr>
<tr>
<td>N&lt;sub&gt;c&lt;/sub&gt;</td>
<td>Refractive index of the fiber cladding</td>
</tr>
<tr>
<td>N₀</td>
<td>Refractive index at the center of the fiber</td>
</tr>
<tr>
<td>N(r)</td>
<td>Refractive index as a function of fiber radius</td>
</tr>
<tr>
<td>P(A)</td>
<td>Light power in the input fiber</td>
</tr>
<tr>
<td>P(c)</td>
<td>Light power reflected at the angle boundary</td>
</tr>
<tr>
<td>P(C)</td>
<td>Light power at the coupled output</td>
</tr>
<tr>
<td>P(T)</td>
<td>Light power in the output fiber</td>
</tr>
<tr>
<td>(\frac{P(T)}{P(A)})</td>
<td>Tap coupling coefficient</td>
</tr>
<tr>
<td>(\frac{P(T)}{P(A)})</td>
<td>Tap transmission coefficient</td>
</tr>
<tr>
<td>R</td>
<td>Reflection coefficient of a ray at the angle boundary</td>
</tr>
<tr>
<td>r</td>
<td>Radial position on the fiber</td>
</tr>
<tr>
<td>r&lt;sub&gt;₀&lt;/sub&gt;</td>
<td>Initial position of the ray</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$r'_0$</td>
<td>Initial direction of the ray</td>
</tr>
<tr>
<td>$r(z)$</td>
<td>Radial position of a ray along the fiber axis</td>
</tr>
<tr>
<td>$r'(z)$</td>
<td>Propagation direction along the fiber axis</td>
</tr>
<tr>
<td>$T$</td>
<td>Transmission coefficient of a ray at the angle boundary</td>
</tr>
<tr>
<td>$T_B$</td>
<td>Transmission coefficient of a ray at the side boundary</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Ray angle</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Angle of incidence at the angle boundary</td>
</tr>
<tr>
<td>$\beta'$</td>
<td>Refracted ray angle at the angle boundary</td>
</tr>
<tr>
<td>$\beta_{\text{crit}}$</td>
<td>Angle of total internal reflection at the angled boundary</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Index difference parameter</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Angle of incidence at the side boundary</td>
</tr>
<tr>
<td>$\psi'$</td>
<td>Angle of the refracted ray at the side boundary</td>
</tr>
<tr>
<td>$\psi_{\text{crit}}$</td>
<td>Angle of total internal reflection at the side boundary</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Tap angle</td>
</tr>
</tbody>
</table>
An Angled End Tap for Fiber Optic Communications Systems

by

Charles D. Sallada

ABSTRACT

A new type of optical coupler called an angled end tap has been designed for use in fiber optic communications systems. An optical tap is a device that allows the light propagating in an optical fiber to be sampled. The sampled portion of the optical signal, called the coupled output, can be used as a monitor or feedback control signal. Tap action is achieved by creating a beam splitter in the optical path. The beam splitter is made by cutting the optical fiber at an appropriate angle and creating an air gap in the fiber path.

A ray optics analysis of the tap has been made by the author. The analysis determines the tap performance as a function of the tap angle, the fiber spacing, and the fiber index profile. The analysis predicts that tap action occurs when the tap angle is between 34° and 71°, and that the tap outputs are nearly independent of light power distribution when the tap angle is about 63° (for a tap made with a typical graded index fiber). The analysis also shows that the coupled output of the tap is nearly constant for fiber spacings less than 15 μm.
At a tap angle of 63° and 15μm fiber spacing, the analysis predicts 86% transmission and 6.7% coupled output.

Seventeen experimental 63° graded index angled end optical taps have been constructed and characterized by the author. The fiber spacing on the experimental taps ranged from 10μm to 30μm. The measured transmission and coupling on these taps is in close agreement with theoretical results.
Section 1 - INTRODUCTION

Section 1.1 - Optical Couplers and Taps

The advent of low loss fibers and suitable optical sources have made optical communications systems a reality in recent years. Today’s optical systems incorporate a laser or light emitting diode (LED) source. The output of the laser or LED is transmitted via an optical cable to a detector. Typical transmission distances are 5 to 10 km. To extend the capacity of future optical systems, techniques such as wavelength multiplexing and bidirectional transmission will be required. These more advanced techniques will require additional optical components. Many of these fall into the general classification of branching networks called optical couplers. Couplers provide the functions in optical systems which divide or combine optical transmission paths.

There are three general types of optical couplers. The first is called a tap. A tap is a three port device that splits the transmitted signal into two channels, usually for monitoring purposes. The tapped portion of the signal is normally only a small percentage of the transmitted portion. The second type of coupler is called a splitter which is a special case of a tap where equal amplitudes are obtained at both outputs. The final type of coupler is called a combiner and is
used to merge two or more signals onto a single fiber.

Several techniques have been used to make optical couplers or branching networks.\textsuperscript{2,3} Couplers using integrated optics are not presently practical due to their high insertion loss and interface complexities with optical fibers. Several forms of fiber couplers have been constructed by changing the guidance properties of the fiber.\textsuperscript{4} These couplers generally have the disadvantage of being difficult to reproduce.

This thesis discusses the analysis of a new type of optical coupler, called an angled end fiber tap. This tap design avoids many of the problems associated with the integrated optics and guidance type designs. This is primarily due to the fact that the tap is created by a simple beam splitter in the fiber path and does not involve any complex structures. This is the first time that the beam splitter concept has been applied to an optical fiber tap. The angled end optical tap has two main advantages. First, it can be reproduced with good uniformity, and second, it can be easily integrated in the fiber path.

Section 1.2 - Project Goals

The angled end tap project has two main objectives. First, to develop relationships which will aid in determining the optimum tap design for various applications and fiber parameters. To accomplish this, we will develop a physical model of the tap. From this model we will derive equations which will
allow us to determine the optimum tap design for particular applications.

The second goal of the project is to develop experimental models of the tap which can be used in a feedback control loop of the laser transmitter. We will also compare the measured results of these taps to the theoretical results.

Section 1.3 - Technical Contributions

Contributions of many individuals were required to develop the tap. The tap was conceived by Dr. M. DiDomenico, Jr., M. A. Karr, and T. C. Rich of Bell Telephone Laboratories in 1976. These three individuals along with Dr. H. P. Hsu derived the basic equations used to determine the transmission and coupling for a step index fiber tap discussed in Section 3.1 and 3.2. The author has extended the basic analysis to include the effects of fiber spacing and light distribution on the tap performance discussed in Section 3.3 and 3.4. The analysis of the graded index fiber tap (Section 4) was done by the author with guidance from Dr. Hsu. The design of the experimental taps was primarily done by the author.

Section 1.4 - Physical Description of the Angled End Optical Tap

The basic idea of the angled end tap is to form a beam splitter in the fiber path. This is done by cutting the fiber at angle with respect to the direction of propagation and creating an air gap in the optical path. The tap output is realized by monitoring the reflections from the two faces of the angular cut
allow us to determine the optimum tap design for particular applications.

The second goal of the project is to develop experimental models of the tap which can be used in a feedback control loop of the laser transmitter. We will also compare the measured results of these taps to the theoretical results.

Section 1.3 - Technical Contributions

Contributions of many individuals were required to develop the tap. The tap was conceived by Dr. M. DiDomenico, Jr., M. A. Karr, and T. C. Rich of Bell Telephone Laboratories in 1976. These three individuals along with Dr. H. P. Hsu derived the basic equations used to determine the transmission and coupling for a step index fiber tap discussed in Section 3.1 and 3.2. The author has extended the basic analysis to include the effects of fiber spacing and light distribution on the tap performance discussed in Section 3.3 and 3.4. The analysis of the graded index fiber tap (Section 4) was done by the author with guidance from Dr. Hsu. The design of the experimental taps was primarily done by the author.

Section 1.4 - Physical Description of the Angled End Optical Tap

The basic idea of the angled end tap is to form a beam splitter in the fiber path. This is done by cutting the fiber at angle with respect to the direction of propagation and creating an air gap in the optical path. The tap output is realized by monitoring the reflections from the two faces of the angular cut
fibers as shown in Figure 1. The figure shows a two dimensional model of the tap in longitudinal cross-section.

Section 2 - TECHNICAL BACKGROUND

Section 2.1 - Tap Transmission and Coupling Coefficients

The tap is characterized by two quantities. The first is called the tap transmission coefficient and defines how much of the input optical power is transmitted to the output fiber. The second quantity is called the tap coupling coefficient and defines what portion of the input signal is available at the coupled output, i.e.,

\[
\text{Transmission Coefficient} = \frac{P(T)}{P(A)} \quad (2.1-1)
\]

\[
\text{Coupling Coefficient} = \frac{P(C)}{P(A)} \quad (2.1-2)
\]

where:  
P(A) is the light power in the input fiber  
P(T) is the light power in the output fiber  
P(C) is the light power at the coupled output

The transmission (coefficient) and coupling (coefficient) of the tap are dependent on the fiber index profile, the tap angle \( \theta \), and the fiber spacing which are also shown in Figure 1. For example, we will find that the amount of light reflected out the side of the fiber is dependent on the tap angle. Consequently, the coupled output of the tap will vary as \( \theta \) is changed. In this
report we will find the tap transmission and coupling as a function of $\Theta$, $d$, and the fiber index profile. The information we obtain from this analysis will be helpful in determining the best angle and fiber spacing for various tap applications.

Section 2.2 - Step Versus Graded Index Profiles

We will use a ray optics approach to analyze the tap. Consider first a fiber with the step index profile shown in Figure 2A where $N_0$ is the core index of refraction. The cladding index is $N_0 (1-\Delta)$, where $\Delta$ determines the index difference between the core and cladding and is usually a few parts per 100. Within the core, the light rays propagate in straight lines. If the angle of propagation of the ray, $\alpha$ (ray angle), is sufficiently small, the ray will experience total internal reflection at the side boundaries, and the ray moves through the fiber in a zigzag fashion. The ray propagates without attenuation provided the ray angle $\alpha$ is less than a critical angle $\alpha_{\text{max}}$. Rays propagating at $\alpha > \alpha_{\text{max}}$ will experience partial refraction into the cladding. The portion of the ray that is refracted into the cladding is lost. This process continues as the ray moves along the fiber until the entire ray is eventually dissipated in the cladding.

In multimode fibers there exist an angular spread in the light propagating in the fiber between $\pm \alpha_{\text{max}}$. Rays with $\alpha$ near zero will propagate through a
length of fiber faster than rays with $\alpha$ near $\alpha_{\text{max}}$ because each ray must travel a different path length. This effect results in pulse dispersion in multimode fiber systems. To minimize the dispersion in long distance optical systems, a multimode fiber whose index profile is shown in Figure 2B is used. This fiber has a graded index profile\(^5\) described by:

\[
N(r) = N_0 \left[1 - \Delta \frac{r}{a}\right]^g
\]  

\(2.2-1\)

where:
- \(N(r)\) = refractive index along the fiber radius
- \(N_0\) = refractive index at the center of the fiber
- \(r\) = radial position on the fiber
- \(a\) = fiber core diameter
- \(g\) = profile parameter
- \(\Delta\) = index difference parameter

In graded index fibers the rays move in sinusoidal paths (rather than straight lines) because of the continuously changing index medium in which the rays propagate. The profile parameter \(g\) can be chosen such that the total transit time for all rays is about the same regardless of path length. This results in a fiber that is nearly dispersion free. This condition is possible since rays propagating near the center of the fiber travel largely in a denser medium and move at slower velocities than rays propagating near the cladding interface.

An analysis\(^6\) by D. Gloge of Bell Telephone Laboratories shows that \(g \approx 2\) produces minimum dispersion. Consequently, fibers used in medium to long distance transmission systems have an approximate quadratic profile.
Section 2.3 - Analysis Approach

The final objective of the analysis is to determine the transmission and coupling for an angle end optical tap made with a typical graded index fiber. To accomplish this, we will divide the analysis into two sections. The first section describes the basic tap operation. For this part of the analysis we will approximate the actual fiber index profile with a step index profile. This approximation will allow us to analyze the tap operation without the use of the complex equations related to ray propagation in graded index fibers. In the second part of the analysis we will determine the transmission and coupling characteristics for a tap made with a typical graded index fiber.

The text of this paper includes a discussion of the important concepts and an explanation of the governing equations. Definitions of the symbols are listed with each equation. A complete list of the symbols used in this report is given at the beginning of the text. Also, many of the mathematical details which are not necessary to understanding the tap operation have been omitted.
Section 2.4 - Fiber Characteristics

One goal of this project is to develop a tap which can be used with an optical transmitter designed by Bell Telephone Laboratories. The system uses a graded index fiber which is similar to commercially available fibers. So that quantitative results can be obtained, the analysis is directed toward this graded index fiber which has the following characteristics.

Fiber Diameter = 110\mu m
Core Radius = a = 27.5\mu m
Numerical Aperture = .23

Quadratic Index Profile \( N(r) = N_0 \left[ 1 - \Delta \left( \frac{r}{a} \right)^2 \right] \)

Index Difference, \( \Delta = .01225 \)
\( N_0 = \) refractive index at center of fiber = 1.475
\( N_c = \) refractive index of cladding = 1.457

Section 3 - ANGLED END TAP ANALYSIS WITH STEP INDEX APPROXIMATION

Section 3.1 - Limits on Tap Angle \( \Theta \)

For the tap to be useful, most of the optical power in the input fiber must be transmitted into the output fiber and some optical power must be coupled out the side of the fiber. Consider a paraxial, meridinal ray impinging upon the fiber end as shown in Figure 3. For propagating rays to leave the input fiber,
the angle of incidence at the angled end ($\beta$) must be less than the critical angle of total internal reflection ($\beta_{\text{crit}}$). If $\beta$ is less than the critical angle some of the light will be refracted into the output fiber and the rest reflected toward the side boundary. The reflected light will exit the fiber providing it impinges on the side of the fiber at an angle ($\psi$) less than the critical angle at that interface ($\psi_{\text{crit}}$). These two critical angles $\beta_{\text{crit}}$ and $\psi_{\text{crit}}$ place upper and lower bounds on the tap angle $\Theta$.

The two critical angles can be determined by using Snell's Law\(^7\) of refraction across a dielectric boundary, i.e.,

$$N_i \sin(I) = N_r \sin(R) \tag{3.1-1}$$

where: $I$ and $R$ are the incident ray angle and angle of the refracted ray respectively.

and $N_i$ and $N_r$ are the refractive indices on the respective sides of the dielectric boundary.

For these calculations, the core index is assumed constant and equal to the cladding index of 1.457. We will also assume the fibers are in an air ambient with an index of refraction assumed to be equal to the free space index of 1.

Using 3.1-1 we find that:

$$\psi_{\text{crit}} = \beta_{\text{crit}} = \sin^{-1} \left( \frac{1}{N_c} \right) = 43.3^\circ \tag{3.1-2}$$
where: $\beta_{\text{crit}} = \text{critical angle at the angled surface}$

$\psi_{\text{crit}} = \text{critical angle at the side boundary}$

$N_c = \text{core index of refraction} = 1.457$

According to equation 3.1-2, two conditions must be filled for tap operation. First, $\beta$ must be less than $43.3^\circ$ to guarantee transmission into the output fiber. Second, $\psi$ must be less than $43.3^\circ$ to guarantee a coupled output. We can express equations 3.1-2 in terms of $\alpha$ and $\theta$ by using the trigonometric relationships derived in Appendix A, i.e.,

\[
\beta = \frac{\pi}{2} - \alpha - \theta \tag{3.1-3}
\]

\[
\psi = 2\theta + \alpha - \frac{\pi}{2} \tag{3.1-4}
\]

Combining 3.1-2 with 3.1-3 and 3.1-4, we find that tap operation is guaranteed if:

\[
\theta \geq 46.7 - \alpha \tag{3.1-5}
\]

and

\[
\theta \leq 66.6 - \frac{\alpha}{2} \tag{3.1-6}
\]

where: $\theta = \text{tap angle}$

$\alpha = \text{ray angle}$

The maximum variation in ray angle within the fiber must be known to determine the limits on the tap angle $\theta$. The maximum possible ray angle in
the fiber can be calculated from the numerical aperture of the fiber. The numerical aperture of the fiber, by definition, is equal to the sine of the maximum ray angle (in air) that can be guided by the fiber, i.e.

\[
\sin \alpha_{\text{max}}(\text{air}) = \text{Numerical Aperature} = .23
\]  

(3.1-7)

\[
\alpha_{\text{max}}(\text{air}) = 13.3^\circ
\]

By using Snell's Law, we can then determine the maximum ray angle in the fiber i.e.,

\[
\frac{\sin \alpha_{\text{max}}(\text{fiber})}{\sin \alpha_{\text{max}}(\text{air})} = \frac{1}{1.457}
\]

\[
\alpha_{\text{max}}(\text{fiber}) \approx \pm 9^\circ
\]

If we consider all possible rays whose angles (or modes) are between \( \pm 9^\circ \), Equations 3.1-5 and 3.1-6 define five regions of tap operation. Figure 4 is a graphical representation of tap operation over these five operating regions.

According to Equations 3.1-5 and 3.1-6, we find that for \( \theta < 33.7^\circ \) (Region 1) none of the rays propagating in the input fiber have a transmitted component because of total reflection at the angle boundary. We also see that for \( \theta > 71.1^\circ \) (Region 5), none of the permitted input rays have a coupled output because of total internal reflection at the side boundary. Additional analysis of Equations 3.1-5 and 3.1-6 shows that when \( \theta \) is between 55.7 and
62.1 (Region 3), all guided rays in the fiber will have both a transmitted and
coupled component. When \( \Theta \) is between 33.7 and 55.7 or between 62.1 and
71.1 (Region 2 and 4 respectively), some rays have a transmitted component,
and some rays have a coupled output. The tap operation as a function of \( \Theta \) is
summarized in the following table.

<table>
<thead>
<tr>
<th>Region 1</th>
<th>( \theta \leq 33 )</th>
<th>No transmission</th>
<th>All rays have coupled output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 2</td>
<td>33.7 &lt; ( \theta \leq 55.7 )</td>
<td>Some rays have transmission,</td>
<td>All rays have coupled output</td>
</tr>
<tr>
<td>Region 3</td>
<td>55.7 &lt; ( \theta \leq 62.1 )</td>
<td>All rays have transmission</td>
<td>All rays have coupled output</td>
</tr>
<tr>
<td>Region 4</td>
<td>62.1 &lt; ( \theta \leq 71.1 )</td>
<td>All rays have transmission</td>
<td>Some rays have coupled output</td>
</tr>
<tr>
<td>Region 5</td>
<td>( \theta &gt; 71.1 )</td>
<td>All rays have transmission</td>
<td>No coupled output</td>
</tr>
</tbody>
</table>

Clearly regions 1 and 5 are of no interest. Intuitively we might expect that the
best tap performance is in region 3, but tap operation is still possible in regions
2 and 4.

**Section 3.2 - Tap Transmission and Coupling Versus Ray Angle**

To determine the tap transmission and coupling, we will first consider a
single ray (A) with light power \( P(A) \) approaching the tap interface at angle \( \alpha \) as
shown in Figure 5. If the angle of incidence at the angled surface is less than
the critical angle, part of the light will be refracted and part reflected. That
portion of the ray which is refracted will then move across the air gap, strike the second fiber interface, and correspondingly be reflected and refracted. The portion of the ray remaining in the air gap will continue to be reflected and refracted until all power is transmitted into the two fibers. If the two fibers have the same tap angle all of the transmitted rays \( A', A'', A''', \ldots \) will have the same angle of incidence at the fiber ends. This means that the transmission and reflection coefficients, at both interfaces for each generated ray \( A', A'', A''', \ldots \), are the same. Thus, the total power transmitted into the second fiber \( P(T) \) from ray \( A \) is:

\[
P(T) = (T^2 + T^2R^2 + T^2R^4 + T^2R^6 + \ldots)P(A) \quad (3.2-1)
\]

where:
- \( P(A) \) = power in ray \( A \)
- \( P(T) \) = total power transmitted
- \( T \) = transmission coefficient at the angle boundary
- \( R \) = reflection coefficient at the angle boundary.

We can simplify Equation 3.2-1 by noting that the right hand side of the equation contains a geometric series in \( R^2 \). With some manipulation, we find that the tap transmission coefficient, \( P(T)/P(A) \), is determined by the transmission and reflection coefficients at the angle boundary, i.e.,

\[
\frac{P(T)}{P(A)} = \frac{T^2}{1-R^2} \quad (3.2-2)
\]
A similar analysis can be used on the portion of the ray that is available for the tap output. In this case the total reflected power \( P(c) \) from ray A is:

\[
P(c) = (R + T^2R^3 + T^2R^5 ...)P(A)
\]

The total power coupled out the side of the fiber, \( P(C) \) is the reflected power \( P(c) \) times the transmission coefficient at the side boundary \( T_B \). Therefore:

\[
P(C) = (RT_B + T^2T_BR + T^2T_BR^3 + T_BT^2R^5 ...)P(A) \tag{3.2-3}
\]

where:
- \( P(A) \) = power in ray A
- \( P(C) \) = total coupled output
- \( T_B \) = transmission coefficient at the angle boundary
- \( T \) = transmission coefficient at the angle boundary
- \( R \) = reflection coefficient at the angle boundary

Equation 3.2-3 can also be expressed in closed form. With some manipulation, we find that the tap coupling coefficient, \( P(C)/P(A) \) is expressed in terms of the transmission and reflection coefficients at both fiber boundaries, i.e.,

\[
\frac{P(C)}{P(A)} = RT_B + \left[ \frac{T^2T_BR}{1-R^2} \right] \tag{3.2-4}
\]

We can find the transmission and reflection coefficients \( (T, R, T_B) \) by using Fresnel's laws of reflection. i.e.
\[ T_{\parallel} = \frac{\sin 2\beta \sin 2\beta'}{\sin^2(\beta+\beta') \cos^2(\beta-\beta')} \]  
(3.2-5)

\[ R_{\parallel} = \frac{\tan^2(\beta-\beta')}{\tan^2(\beta+\beta')} \]  
(3.2-6)

\[ T_{\parallel\perp} = \frac{\sin 2\psi \sin 2\psi'}{\sin^2(\psi+\psi') \cos^2(\psi-\psi')} \]  
(3.2-7)

\[ T_{\perp} = \frac{\sin 2\beta \sin 2\beta'}{\sin^2(\beta+\beta')} \]  
(3.2-8)

\[ R_{\perp} = \frac{\sin^2(\beta-\beta')}{\sin^2(\beta+\beta')} \]  
(3.2-9)

\[ T_{\perp\parallel} = \frac{\sin 2\psi \sin 2\psi'}{\sin^2(\psi+\psi')} \]  
(3.2-10)

The subscripts \( \parallel \) and \( \perp \) refer to the parallel and perpendicular polarization components of the optical field in the plane defined by the ray propagation and dielectric surface normal vectors. The angles \( \beta, \beta', \psi, \) and \( \psi' \) are the angles of the reflected and refracted rays with respect to the surface normal vector.

According to Equations 3.2-5 thru 3.2-10, we see that the resulting transmission at a dielectric boundary is dependent on the amount of power in each polarization. For our calculations, we will assume that 1/2 the total power is in each polarization.

By employing equations 3.1-3 and 3.1-4 and Snell's Law, we can find \( \beta, \beta', \psi, \psi' \),
\( \psi \), and \( \psi' \) and hence the transmission and reflection coefficients, as a function of the ray angle \( \alpha \) and the tap angle \( \Theta \). Where:

\[
\beta = \frac{\pi}{2} + \alpha - \Theta \quad (3.1-3)
\]

\[
\psi = 2\Theta - \alpha - \frac{\pi}{2} \quad (3.1-4)
\]

\[
\beta' = \sin^{-1}[N_0 \cos(\alpha+\Theta)] \quad (3.2-11)
\]

\[
\psi' = -\sin^{-1}[N_0 \cos(2\Theta+\alpha)] \quad (3.2-12)
\]

Derivations of Equations 3.1-3, 3.1-4, 3.2-11 and 3.2-12 are given in Appendix A.

A computer program was written by the author to calculate the transmitted and coupled outputs of the tap as a function of \( \Theta \) for all possible rays between \( \pm 9^\circ \). Figure 6 shows plots of the transmitted and coupled outputs of the tap as a function of ray angle for several values of \( \Theta \). For \( \Theta = 55^\circ \) (Figure 6A), the transmission drops drastically with decreasing tap angles while the coupled power correspondingly increases. For \( \Theta = 65^\circ \) (Figure 6B), the transmission is relatively constant, but the coupled power approaches zero for large positive ray angles. Figure 6C shows the tap performance at \( \Theta = 62^\circ \). At this point the transmission is nearly constant while still maintaining a coupled output over all
possible ray angles.

Section 3.3 - Tap Transmission and Coupling for Gaussian and Uniform Light Distributions

From Figure 6, it is apparent that the total transmitted and coupled output of the tap is dependent on the angular (modal) distribution of power in the input fiber light rays. This distribution can vary greatly depending on the tap application. If the tap is used at the end of a long fiber, the power distribution with ray angle is approximately Gaussian and very stable. For the feedback control application, the tap is to be located near a laser source so that the laser output can be precisely controlled. The power distribution in the fiber at this position is not Gaussian and can change with laser age and temperature.

To determine the effect of changes of the light power distribution on the tap performance, the author has expanded basic computer program to allow weighting of the various input rays. Figure 7 is a plot of the transmitted and coupled outputs of the tap as a function of tap angle θ, assuming a uniform distribution (solid and dashed curves) and a Gaussian distribution (points) of power in the input light rays. Although these distributions represent only two power distributions, they are sufficient to show qualitatively how the tap outputs vary with light distribution. We can use these results to determine the proper tap angle for various tap applications. Also, we can summarize many of the tap characteristics with the use of this graph.
1. As θ gets increasingly large, the tap transmission increases and the coupled output decreases. This behavior is expected since the rays are striking the angled surface at angles approaching normal incidence where the transmission is a maximum. This results in increasing amounts of transmitted power and less power for the coupled output.

2. As θ decreases the transmission goes to zero. This also is expected since at some angle all rays are totally reflected at the angle boundary.

3. At a tap angles greater than 62°, the transmission and coupling become nearly independent of mode distribution. With θ equal to 62°, the transmission is about 90% and the coupling is about 8%. Considering the magnitudes of transmission and coupling and the modal stability, tap angles near 62° should produce good tap performance.

4. Larger coupled outputs can be achieved by selecting a tap angle below 62°. This may be desirable, but increased modal sensitivity results. Tap angles below 62° should only be used when the modal pattern is nearly constant.

5. A tap angle selection above 62° produces taps with increasing immunity to modal variations. This is desirable when the tap is used in an application such as laser feedback control where the modal pattern may vary. The penalty incurred with increasing the tap angle is less coupled output from the tap. The level of the coupled output must be taken into account
when selecting the tap angle.

6. A 50% beam splitter can be made with a tap angle of about 48°, but the split ratio will be very dependent on the tap angle.

Section 3.4 - Tap Transmission and Coupling versus Fiber Spacing

So far in this analysis we have assumed that all rays associated with the tap undergo multiple reflections until all power is either transmitted into the output fiber or reflected into the input fiber. This assumption requires the fiber spacing to be infinitely small. In reality the fiber spacing must be finite and some rays are lost. Figure 8 shows that as a ray moves across the air gap a vertical displacement occurs with each reflection. At some point the ray misses the output fiber and is lost. As the fiber spacing is increased the ray displacement per reflection increases which results in increasing amounts of lost power. In order to characterize these effects, we will expand the tap analysis to account for variations with fiber spacing.

We have assumed in our earlier analysis that the total light power in the input fiber is represented by a single set of rays between ± 9° and that they hit the angle end near the center of the fiber. In reality it is possible for the input fiber rays to exit the fiber at any position along the fiber cross section. We will discover that the position of the input ray is important in determining the amount of transmitted and coupled power derived from the ray.
Figure 9 shows the transmitted and reflected components for rays at several positions along the fiber cross section. For rays striking the angled end, in the region D1\( ^+ \) to D2, shown in Figure 9A, we see that only two of the secondary rays are transmitted into the output fiber. We also see that only three secondary rays are coupled out the side of the fiber. The transmitted and coupled outputs from an input ray in the region D1\( ^+ \) to D2 are:

\[
\frac{P(T)}{P(A)} = T^2(1+R^2) \tag{3.4-1}
\]

\[
\frac{P(C)}{P(A)} = RT_B + RT_B^2 + R^3T^2T_B \tag{3.4-2}
\]

Correspondingly the transmission and coupling for rays between D2\( ^+ \) and D3 shown in Figure 9B are:

\[
\frac{P(T)}{P(A)} = T^2 \tag{3.4-3}
\]

\[
\frac{P(C)}{P(A)} = RT_B(1+T^2) \tag{3.4-4}
\]

and between D3\( ^+ \) and D4 shown in Figure 9C are:

\[
\frac{P(T)}{P(A)} = 0 \tag{3.4-5}
\]

\[
\frac{P(C)}{P(A)} = RT_B \tag{3.4-6}
\]
A computer program was written by the author to calculate the transmission and coupling outputs of the tap taking into account the changes in the transmitted and coupled outputs defined by Equations 3.4-1 to 3.4-6. In this program, a position is selected along the fiber cross section. At this position, the transmission is calculated for rays with ray angles of -9, -8, -7 .... +8, +9 degrees. An effective transmission coefficient is then obtained by averaging the transmission components of all the rays. After these calculations, a new position is selected and a new effective transmission coefficient is calculated at the new position. This process is repeated until effective transmission values have been calculated over the entire cross section. The positions selected in the program were taken at 1 micron increments over the entire fiber cross section. Finally, the total tap transmission is found by averaging all the effective transmission values. A similar procedure is used to determine the total tap coupling.

Figure 10 is a plot of the tap transmitted and coupled outputs as a function of fiber spacing for various tap angles. As the fiber spacing d approaches zero and θ moves toward 90°, the transmission is limited to about 92%. Also, as the tap angle approaches 90°, the transmission becomes less dependent on fiber spacing. This is to be expected since the transmitted rays move across the gap with less displacement.

The coupled output is relatively independent of gap spacing. This is due to the fact that the major part of the coupled output is derived from the first
reflection at the angle boundary. Subsequent reflections are small and as a result the gap spacing does not have a major effect on the coupled output.

Section 4 - ANALYSIS OF THE GRADED INDEX FIBER TAP

Section 4.1 - Ray Propagation in Graded Index Fibers

A. Yariv has derived equations which define the ray propagation in graded index profiles of the form:

\[ N(r) = N_0 \left[ 1 - \frac{k_2}{2k} r^2 \right] \]  \hspace{1cm} (4.1-1)

Following Mr. Yariv's solutions, we find that the quadratic index profile causes the rays to move in sinusoidal like paths, as shown in Figure 11. Equations 4.1-2 and 4.1-3 describe the ray motion along the fiber axis \( z \) in terms of the initial ray position \( r_0 \) and direction \( r'_0 \), i.e.,

\[ r(z) = \cos \sqrt{\frac{k_2}{k}} z \ r_0 + \sqrt{\frac{k}{k_2}} \sin \sqrt{\frac{k_2}{k}} z \ r'_0 \]  \hspace{1cm} (4.1-2)

\[ r'(z) = -\sqrt{\frac{k}{k_2}} \sin \sqrt{\frac{k_2}{k}} z \ r_0 + \cos \sqrt{\frac{k_2}{k}} z \ r'_0 \]  \hspace{1cm} (4.1-3)
where:  
\( r(z) \) = the radial position of the ray along \( z \)  
\( r'(z) = \frac{dr(z)}{dz} \), the propagation direction (or slope) of a ray along \( z \)  
\( k \) & \( k_2 \) = propagation constants related to the fiber  
\( r_0 \) & \( r'_0 \) = the initial position and direction of the ray respectively.

Comparing Equation 2.2-1 with Equation 4.1-1, we find that

\[
\sqrt{\frac{k_2}{k}} = \frac{\sqrt{2\Delta}}{a} = C \tag{4.1-4}
\]

where:  
\( C = 5.69 \times 10^3 \mu \text{m}^{-1} \) for fiber described in Section 3.1.

The propagation equations can then be reduced to:

\[
r(z) = \left[ \cos(cz) \right] r_0 + \left[ \frac{1}{c} \sin(cz) \right] r'_0 \tag{4.1-5}
\]

\[
r'(z) = -c \left[ \sin(cz) \right] r_0 + \left[ \cos(cz) \right] r'_0 \tag{4.1-6}
\]

Section 4.2 - Maximum Guided Ray Angle Over the Fiber Core

In the case of a step index profile, the maximum ray angle of the guided rays is independent of the radial position, \( r \). This is not true in the graded index case. We can show this by analyzing Equation 4.1-5. For rays to be guided, \( r(z) \) must always be less than the fiber core radius. From Equation
4.1-5, it is apparent that the maximum ray angle in the fiber occurs at \( r = 0 \). As the initial position of the ray moves away from \( r = 0 \) the maximum slope for guided rays must decrease to maintain \( r(z) \) less than the fiber core radius.

Figure 12 is a plot of maximum guided ray angle versus initial ray position. The maximum guided ray angle is defined as the maximum angle of a ray, which can be guided in the fiber at a given radial position on the fiber. Since the maximum angle for guided rays is related to the numerical aperture, the fiber can be viewed as having a varying numerical aperture with radius. At the center \( (r = 0) \) the numerical aperture is defined by 3.1-7. Near the cladding interface the numerical aperture approaches zero, i.e., only the rays with \( r'(z) = 0 \) will be guided.

Section 4.3 - Analytical Differences

The total tap transmission and coupling in the graded index case are found by the averaging techniques used in the step index solution. Three major changes are required in the step index analysis to obtain numerical solutions for the graded index case. First, the ray propagation in the fibers follows Equations 4.1-5 and 4.1-6 rather than straight lines. The solution of these equations at various positions along the fiber are required to find \( \beta \) and \( \psi \) at the angled end and side boundaries, respectively. Since these equations are transcendental, the solutions are obtained by numerical techniques.
The second difference in the analysis involves the secondary rays generated by reflections in the air gap. In the step index case all secondary rays are parallel, thus the total transmission and coupling are defined by 3.2-2 and 3.2-4. In the graded index case, the index profile causes the secondary transmitted and coupled rays to propagate at different angles. Thus, the transmission and reflection coefficients are different for each generated ray, and Equations 3.2-2 and 3.2-4 are not valid in the graded index case. As a result of this, the tap transmission and coupling are calculated by summing the transmission and coupling components respectively. In these calculations, it is assumed that the majority of the power was transmitted into the fibers within the first four gap reflections and as a result all subsequent rays are neglected. Also note that the condition for transmission into either fiber is not only that the ray must hit the fiber core (as in the step index case) but it must also hit at an angle less than the critical angle of total reflection at that point on the fiber.

The final difference in the analysis relates to the aperturing effects of rays near the cladding interface as discussed in Section 4.2. The local numerical aperture requires that guided rays propagating near the cladding travel at ray angles near $0^\circ$. This means that the assumption of uniform ray angle distribution over the fiber cross section (used in calculating the effects of fiber spacing on tap performance in the step index case) is not valid in the graded index case. In the graded index case, the computer program is designed to include only guided rays at a given position in the fiber.
Section 4.4 - Calculated Transmission and Coupling

Figure 13 shows the calculated transmission and coupling vs. fiber spacing (with tap angle θ as a parameter) for a tap made from the graded index fiber whose index profile is defined in Section 3.1. We see from the plot that the tap transmission is 90% and the tap coupling is about 9% with a tap angle of 60°. It is important to note that the transmission decreases with increasing fiber spacing indicating that the fibers should be positioned as close as possible to maintain maximum transmission. The coupling below 15μm, however, is nearly independent of the fiber spacing.

A comparison of the calculated results obtained for a 62° graded index tap with the tap results using the step index approximation are shown in Figure 14. The results are very similar which indicates the conclusions drawn in Section 3.3 are valid in this case. At large fiber spacings, the graded index solution indicates less transmitted and coupled output. This is to be expected since some of the rays generated in the gap will not be coupled into the fibers because of the aperturing in the graded index case. We should note that at other tap angles, where the transmission and coupling are more dependent on the ray angle, the step index approximation may produce larger errors.

Section 4.5 - Determination of Minimum Collection Width

The width of the light exiting the coupled output of the tap ultimately determines the minimum size collector that can be used effectively with the
tap. The author has extended the tap analysis to find the amount of the coupled output power which can be collected within a given detection width. A plot of the detected power as a function of the detector width is shown in Figure 15. The plot indicates that virtually all of the coupled output is collected for detection widths >80μm. This result suggests that a fiber similar to the tap fibers cannot be used effectively to collect the coupled output since the core radius is less than the necessary 80μm.

Section 5 - EXPERIMENTAL RESULTS AND CONCLUSIONS
Section 5.1 - Description of the Experimental Tap

Several experimental models of the tap were designed and constructed by the author at Bell Telephone Laboratories, Allentown, PA. The taps were made from fibers whose profile is given in Section 3.1. The tap models were to be used in a feedback control loop of a laser transmitter.

Transmitter design personnel were contacted by the author to determine appropriate design objectives for the experimental taps. Two major objectives were determined from these discussions. First, the transmitter design requires that the tap transmission and coupling be greater than 75% and 5% respectively. Second, the tap outputs should be nearly independent of the modal distribution of the light propagating in the tap fibers. Taps with this characteristic will produce the most accurate control of the laser source.
The analysis presented in this paper was used by the author to select a tap angle of 63 degrees. At this tap angle the theoretical results indicate that the tap performance is nearly independent of the modal distribution, and the tap transmission and coupling are greater than required. Fiber spacings were chosen between 10 and 30 µm. This was done to determine the minimum practical fiber spacing consistent with a large transmission coefficient.

The discussions with the transmitter designers also lead to the selection of a large area photodetector to collect the coupled output of the tap. The active area of the photodetector is 500 microns. The results of Section 4.6 indicate that this detector will collect all of the light available at the coupled output of the tap.

Suitable angled end fibers were produced by using grinding and polishing techniques developed by Dr. Hsu. Assembly fixturing and techniques were developed by the author. Briefly speaking, the tap assembly involved first aligning the fibers in either a glass capillary tube or a silicon "V" grooved substrate. The detector is then positioned at the point of maximum coupled output and bonded in place with lens cement. Finally, the entire device is potted in epoxy.

Section 5.2 - Comparison of Experimental and Theoretical Results

Figure 16 shows a comparison of the results obtained from 17, 63° experimental taps with the theoretical data. The calculated transmission and
coupled outputs are shown by the solid lines. The measured data are shown by the asterisks. The measured transmission is typically within 4% of the theoretical values. A few of the experimental taps have transmission coefficients which were somewhat lower. This was found to be related to poor alignment of the tap fibers when silicon V-grooved substrates were used. Three taps with 10μm fiber spacing had transmission coefficients greater than theoretical. Studies on these taps revealed that the fiber angles are not precisely 63°. This caused the effective fiber spacing to be less than the expected 10μm.

The coupled output of most of the experimental taps is also somewhat lower than theoretical. This discrepancy is due to unaccounted losses in the coupled output optical path which are inherent in the capillary tube and silicon V-groove structures. Several taps were also found to have coupling coefficients greater than expected. Here again the discrepancies were caused by variations in the fiber angle.

The experimental taps have shown that the mechanical alignment and fiber angle tolerances are very important. Subsequent thermal tests have also shown that the fibers may move with temperature which can cause unwanted transmission and coupling changes. These results suggest that a more stable fiber alignment method must be developed before the tap is acceptable for the laser feedback application. Moreover, these new alignment methods must be simple and viable in a manufacturing environment.
Section 5.3 - Conclusions

A ray optics approach was used to analyze the tap. The analysis required the use of two basic optical laws (Snell and Fresnel). A. Yariv's equations of ray motion in dielectrics with a quadratic index profile were required in the graded index case. The analysis required a prudent use of trigonometry and attention to all possible rays within the tap. The measured results of the experimental taps are in good agreement with the theoretical results.

Several of the experimental taps have been used successfully in laser transmitters equipped with the feedback loop. Additional taps have also been assembled for several other monitor type applications. These taps have also been used successfully which indicates that the tap may have additional uses beyond its primary design objective. A decision has been made to delay tap production until the mechanical problems discussed in Section 5.2 have been resolved. It is the author's belief that additional engineering effort can resolve the mechanical problems and that taps of this type will eventually find use in future fiber optic systems.
REFERENCES


APPENDIX A - DETERMINATION $\beta, \beta', \psi, \psi'$

To find $\theta$,

\[ \angle AOB = \frac{\pi}{2} = \theta + \alpha + \beta \]

\[ \beta = \frac{\pi}{2} - \alpha - \theta \]  \hspace{1cm} (3.1-3)

To find $\psi$,

\[ \psi = \frac{\pi}{2} - \angle CDO \]

\[ \angle CDO + \theta + \angle COD = \pi \]

where

\[ \angle COD = \frac{\pi}{2} - \beta \]

then

\[ \angle CDO + \theta + \frac{\pi}{2} - \beta = \pi \]

\[ \angle CDO = \frac{\pi}{2} + \beta - \theta \]
and  \[ \psi = \frac{\pi}{2} - \left[ \frac{\pi}{2} + \beta - \Theta \right] \]
\[ \psi = \Theta - \beta \]

Substituting
Equation 3.1-3  \[ \psi = 2\Theta + \alpha - \frac{\pi}{2} \]  \hspace{1cm} (3.1-4)

\( \beta' \) and \( \psi' \) are determined from Snell's Law
\[ N_0 \sin \beta' = (1) \sin \beta' \]
then \[ \beta' = \sin^{-1} \left[ N_0 \sin \beta \right] \]

Substituting
Equation 3.1-3  \[ \beta' = \sin^{-1} \left[ N_0 \sin \left( \frac{\pi}{2} - \alpha - \Theta \right) \right] \]
\[ \beta' = \sin^{-1} \left[ N_0 \cos \left( \alpha + \Theta \right) \right] \]  \hspace{1cm} (3.2-11)

Similarly
\[ N_0 \sin \psi = (1) \sin \psi' \]
then \[ \psi' = \sin^{-1} \left[ N_0 \sin \psi \right] \]
Substituting
Equation 3.1-4  \[ \psi' = \sin^{-1}\left[N_0 \sin \left(2\theta + \alpha - \frac{\pi}{2}\right)\right]. \]

then  \[ \psi' = -\sin^{-1}\left[N_0 \cos \left(2\theta + \alpha\right)\right] \quad (3.2-12) \]
BIOGRAPHICAL SKETCH

CHARLES DONALD SALLADA


GRADUATE

Exeter Township High School, Reiffton, PA, June, 1964
DeVry Technical Institute, Chicago, ILL., Associate Degree in Electronics Technology, Honor Graduate, October, 1966.
Lafayette College, Easton, PA, B.S.E.E., Magna Cum Laude, June, 1975.

EXPERIENCE

Employed by Bell Telephone Laboratories from December, 1966 to present. Past assignments have been related to the development of Microwave Integrate Circuits for radio relay applications. Current work assignments are related to the development of an optical transmitter for a trans-Atlantic submarine system. Currently a Member of Technical Staff in the High Bit Rate Lightwave Transmitter Group at Allentown, PA.

MEMBER

Tau Beta Pi
Eta Kappa Nu
Phi Beta Kappa
TAP OR COUPLED OUTPUT, P;

— TAP ANGLE

— FIBER SPACING

/ INPUT, PL;

TRANSMITTED

OUTPUT, PL/;

OPTICAL FIBER;

ANGLED END OPTICAL TAP;

FIGURE 1
TAP OR COUPLED OUTPUT, \( P(C) \)

TAP ANGLE

FIBER SPACING, \( d \)

INPUT, \( P(A) \)

TRANSMITTED OUTPUT, \( P(T) \)

OPTICAL FIBERS

ANGLED END OPTICAL TAP

FIGURE 1
REFRACTIVE INDEX PROFILES
OF STEP AND GRADED INDEX FIBERS

FIGURE 2
\( \alpha = \text{RAY ANGLE} \)
\( \beta = \text{ANGLE OF INCIDENCE AT ANGLED BOUNDARY} \)
\( \Psi = \text{ANGLE OF INCIDENCE AT SIDE BOUNDARY} \)

AT \( \beta \geq \beta_{\text{Crit}} \) NO TRANSMITTED OUTPUT
\( \Psi \geq \Psi_{\text{Crit}} \) NO COUPLED OUTPUT

TAP MODEL SHOWING ANGLES OF INCIDENCE
AT ANGLED AND SIDE BOUNDARY

FIGURE 3
\( \alpha = \text{RAY ANGLE} \)

\( \beta = \text{ANGLE OF INCIDENCE AT ANGLED BOUNDARY} \)

\( \psi = \text{ANGLE OF INCIDENCE AT SIDE BOUNDARY} \)

\[ \beta \geq \beta_{\text{CRIT}} \text{ NO TRANSMITTED OUTPUT} \]

\[ \psi \geq \psi_{\text{CRIT}} \text{ NO COUPLED OUTPUT} \]

TAP MODEL SHOWING ANGLES OF INCIDENCE AT ANGLED AND SIDE BOUNDARY

FIGURE 3
TAP PERFORMANCE AS A FUNCTION OF TAP ANGLE

FIGURE 4
TAP PERFORMANCE AS A FUNCTION OF TAP ANGLE

FIGURE 4
DETERMINATION OF TOTAL TRANSMITTED
POWER FROM RAY A WITH POWER P(A)

FIGURE 5
FIGURE 6A—TAP TRANSMISSION AND COUPLING
Vs RAY ANGLE WITH TAP ANGLE
THETA EQUAL TO 55 DEGREES
FIGURE 6A-TAP TRANSMISSION AND COUPLING VS RAY ANGLE WITH TAP ANGLE THETA EQUAL TO 55 DEGREES
FIGURE 6B—TAP TRANSMISSION AND COUPLING Vs RAY ANGLE WITH TAP ANGLE THETA EQUAL TO 65 DEGREES
FIGURE 6B-TAP TRANSMISSION AND COUPLING VS RAY ANGLE WITH TAP ANGLE THETA EQUAL TO 65 DEGREES
FIGURE 6C—TAP TRANSMISSION AND COUPLING
Vs RAY ANGLE WITH TAP ANGLE
THETA EQUAL TO 62 DEGREES
THE RAY ANGLE V.S. RAY ANGLE WITH TAP ANGLE
FiguRe 6C-1AP TRANSMISSION AND COUPLING
FIGURE 7-TAP TRANSMISSION AND COUPLING VS RAY ANGLE FOR UNIFORM AND GAUSSIAN DISTRIBUTIONS OF RAY POWER
FIGURE 2-TAP TRANSMISSION AND COUPLING VS.
RAY ANGLE FOR UNIFORM AND GAUSSIAN
DISTRIBUTIONS OF RAY POWER

TRANSMISSION (%) — — —

COUPLING (%) —

TAP ANGLE, THETA (DEG.)

0 20 40 60 80 100

0 20 40 60 80 100
RAY PROPAGATION WITH FINITE FIBER SPACING

FIGURE 8
TRANSMITTED AND COUPLED OUTPUTS OF THE TAP FOR RAYS ORIGINATING AT SEVERAL POINTS ALONG THE FIBER CROSS SECTION

FIGURE 9
FIGURE 10—TAP TRANSMISSION AND COUPLING VS FIBER SPACING FOR VARIOUS TAP ANGLES, \( \theta \)
FIGURE 10-TTP TRANSMISSION AND COUPLING

THETA VS FIBER SPACING FOR VARIOUS
FIBER SPACING (MICRONS)
\[ \tan^{-1} r'(z) = a(z) \]

Position along fiber

\[ r(z) = \text{Radial position of ray} \]
\[ r'(z) = \text{Propagation direction} \]
\[ a(z) = \tan^{-1} r(z) \]

Ray path in medium with a quadratic index variation

Figure 11
RAY PATH IN MEDIUM WITH A QUADRATIC INDEX VARIATION

\[ r(z) = \text{RADIAL POSITION OF RAY} \]
\[ r'(z) = \text{PROPAGATION DIRECTION} \]
\[ \alpha(z) = \tan^{-1} r'(z) \]

FIGURE 11

RADIUS

z=0

z=1
FIGURE 12-MAXIMUM GUIDED RAY ANGLE VS FIBER RADIUS FOR A TYPICAL GRADED INDEX FIBER
Figure 12: Maximum guided ray angle vs Fiber radius for a typical graded index fiber.
FIGURE 13-GRADDED INDEX TAP TRANSMISSION AND COUPLING Vs FIBER SPACING FOR VARIOUS TAP ANGLES, THETA
For various gap angles, Theta and coupling vs fiber spacing

Figure 13-Graded Index Gap Transmission
FIGURE 14—COMPARISON OF 62° GRADED INDEX TAP RESULTS WITH THOSE OBTAINED USING THE STEP INDEX APPROXIMATION
THE STEP INDEX APPROXIMATION RESULTS WITH THOSE OBTAINED USING
FIGURE 4-COMPARISON OF 62° GRADED INDEX TAPER

FIBER SPACING (MICRONS)

COUPLING (%)

TRANSMISSION (%)

-- -- -- Step Index -- -- -- Graded Index
FIGURE 15—TAP COUPLED OUTPUT Vs COLLECTION WIDTH
FIGURE 16—COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS OF SEVERAL 63° GRADED INDEX ANGLED END TAPS
FIGURE 16—COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS OF SEVERAL 63° GRANTED INDEX ANGLED END TRAPS