Fluidized bed bubble eruptions in the presence of a single row of horizontal tubes.

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FLUIDIZED BED BUBBLE ERUPTIONS IN THE PRESENCE OF
A SINGLE ROW OF HORIZONTAL TUBES

by

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Professor in Charge

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NOMENCLATURE

\( a \) - cavity width \( \pm 2 \) (cm)

\( A_t \) - total horizontal bed area (cm\(^3\))

\( D_i \) - bubble diameter (cm)

\( D_e \) - equivalent spherical bubble diameter (cm)

\( D_{eo} \) - initial equivalent diameter (cm)

\( d_p \) - particle diameter (cm)

\( g \) - acceleration due to gravity (cm/s\(^2\))

\( H \) - distance of undisturbed free surface above the tubes (cm)

\( H_b \) - maximum bulge height (cm)

\( H_w \) - maximum wake eruption height (cm)

\( L \) - distance of undisturbed free surface above the distributor (cm)

\( L_t \) - distance of tube centerline above the distributor (cm)

\( N_d \) - number of holes in the distributor

\( U_b \) - bubble rise velocity (cm/s)

\( U_{br} \) - relative bubble velocity (cm/s)

\( U_{mf} \) - velocity of minimum fluidization (cm/s)

\( U_o \) - actual gas velocity (cm/s)

\( U^* \) - non-dimensional gas velocity, \( U_o/U_{mf} \) (cm/s)

\( V_{b} \) - cavity bottom rise velocity (cm/s)
\( V'_o \) - velocity at wake center of symmetry (cm/s)

\( X' \) - horizontal coordinate (cm)

\( Y' \) - vertical coordinate (cm)

Greek

\( \rho_p \) - particle mass density (g/cm\(^3\))
ABSTRACT

An experimental study was carried out to measure the effects of tubes on bubble behavior in a gas fluidized bed and on the nature of the bubble eruptions at the free surface. All of the experiments were performed with a single row of horizontal tubes inserted into a large three-dimensional fluidized bed containing 0.264 mm diameter glass beads. The tubes forced bubbles to coalesce at the gaps by diverting bubbles incident upon tube centers into the gaps, thus effectively reducing the bed area at the tube level. If the free surface was in the vicinity of the tubes and the tubes were not too far above the distributor, this forced coalescence led to higher double bubble fractions than without the tubes.

The tubes also reduced the bubble diameters as the bubbles rose past the tube level by (a) splitting the bubbles and (b) elongating the bubbles as they traversed the gaps. The splitting caused smaller bubbles in the region just above the tubes, but coalescence caused them to grow again rather quickly so that, provided the bed was deep enough, they eventually approached the diameters they would have had were there no tubes in the bed. The elongated bubbles (or cavities) produced significant wake eruption events, such as wake spikes, jet sprays, or explosions. These events occurred with a very high frequency (20 to 60 percent of all single bubbles), whereas they were almost non-existent for single bubbles in the absence of tubes.

In the cavities that formed as the bubbles traversed the gaps,
maximum wake velocities were large compared to the maximum bulge velocities, whereas, in bubble eruptions with no tubes in the bed, the maximum wake and bulge velocities were nearly equal.

Average wake eruptions for double bubbles reached greater heights than for single bubbles, partly because the wake spikes were larger for double bubbles, but primarily due to the higher fraction (80 to 100 percent vs. 20 to 60 percent for single bubbles) of the double bubbles that exhibited wake spike events.
1. INTRODUCTION

Fluidized beds have been in use for nearly six decades with the first commercial application being the Winkler gas producer patented in 1922 and beginning operations in 1926 [1]. Some consider the Fluid Catalyst Cracking (FCC) process [2] for the cracking of petroleum in 1942 the first large scale commercial application. This process was so successful that it awakened the interest of operators of other chemical processes using fixed bed gas-solid contacting equipment and led to hundreds of patents for fluidized bed processes. Incorporation into the commercial processes has not occurred so rapidly, however, for mostly technological and economical reasons. Geldart [3] in 1967 summarized the twenty-five years since the FCC process began (which he considered the first twenty-five years of fluidized bed applications), citing applications such as catalytic and non-catalytic hydrocarbon processing, coal carbonization and combustion, reduction roasting of iron oxide and uranium processing.

In the seventeen years since then, additional applications have resulted from the experiences of commercial operators and research results from various organizations. Recent applications include municipal refuse incineration [4], gasification of coal and lignite [5], continuous granulation and coating in medicines, foods, fertilizers, etcetera [6], and chlorination of metals [7].

Through the years, numerous investigators have examined characteristics of the fluidization processes. Of special interest have been
such interactions as particle mixing, gas contacting, solids elutria-
tion and heat transfer. Recently, researchers have looked at some
fundamental hydrodynamic phenomena. Experimental correlations and
mathematical models have been published for both two- and three-
dimensional beds in the areas of bubble growth [8], particle elutria-
tion [9], bubble frequency [10], and mechanisms for solids ejection
[11], to cite a few.

Many publications have also appeared discussing the effects of
horizontal tubes and tube bundles upon the heat transfer characteris-
tics of a fluidized bed. However, little has been published concerning
the effects of horizontal tubes upon the fluidization process and bubble
characteristics. A short communication by Nguyen, et al. [12] concluded
basically that with a bundle of 12 staggered rows of tubes the "bubble
eruption diameters are much less in the presence of the tubes than
without them" and that unlike the case without tubes, the "bubbles are
distributed more or less uniformly over the whole cross-section of the
bed". Colakyan, et al. [13] measured attrition rates and elutriation
rate constants and concluded the presence of a tube bundle above the
distributor plate did not affect elutriation rates above the transport
disengaging height. Horsler and Thompson [14] and others have experi-
 mented with vertical and horizontal baffles to the extent they found
that such baffles solved such problems as unreacted material resulting
from high velocity jets of reactants. Still, most research involving
internals has concentrated on heat transfer rather than fluidization
characteristics.
Dille [15] performed experiments in two- and three-dimensional fluidized beds in which he established several parameters of the bubble eruption process. For example, he defined four distinct types of eruption mechanisms, the bulge burst, mid-layer burst (for double bubbles), the wake spike, and the jet spray. Alkan [16] looked at these same features for the large bed used in the present research project. In addition, he studied bubble eruption frequency and other behavior. Edelstein [17] examined the theoretical aspects of wake eruptions for double bubbles in two- and three-dimensional beds, showing the similarities to previous studies of bubble eruptions in liquids. These three works and bubble growth correlations by others (i.e., Kato and Wen [18]) established the bases for comparison in determining the outcomes of the present experiments.

The purpose of this research was to determine the effects of horizontal tubes upon the bubble eruption process. Specifically, it was desired to find how the tube presence affects bubble growth, bulge and wake ejection heights, double bubble fraction, and the frequencies of the various wake eruption types. In addition, studies were made of the velocities of the bulge, wake center, and wake edges of the bubbles at various times in the eruption process. The purpose of these measurements was to determine how the various eruption phenomena might be similar to or different from each other or the situations with no tubes in the bed. Finally, this research was to point out any other affects the tubes might have upon the fluidization or elutriation processes.
2. **EXPERIMENTAL APPARATUS**

These experiments were conducted in a 0.76 m square by 2.44 m high fluidized bed (Figure 1). The bed was terminated by a hood leading to a cyclone for recovering material carried in the airstream at high flow rates. The outlet of the cyclone was in turn connected to an air bag to collect fines which could possibly escape the cyclone.

The side walls of the bed were of 6.35 mm thick steel plate and the front and back were 19 mm thick Plexiglas plates to enable it to support a metric ton of glass beads. Plexiglas was used to permit flow visualization studies. The corners of the bed were 6.35 mm x 102 mm steel angle iron bolted to the steel and Plexiglas plates. These angle iron corners were also welded into square frames at the top and bottom for structural strength and stability.

The fluidized bed was mounted on top of a 0.76 m square by 0.305 m high plenum box which had three 0.15 m diameter flexible hoses attached at each of three sides to deliver air to the box. The fourth side was removable for access to make modifications and add instrumentation as desired. The distributor mounted on top of the plenum box was a 6.35 mm thick steel plate with 480 holes of 4.0 mm diameter uniformly spaced over its surface. A wire mesh of 40 micrometer mesh size was glued to the top of the distributor plate to prevent bed material from seeping into the plenum box.

Air was supplied from twin 75 kilowatt Ingersoll-Rand compressors capable of delivering 0.42 cubic meters per second at 600 kPa pressure.
Two thin plate orifice meters of ASME design were used to measure flow rates. The larger meter was mounted in a pipe of ID 102.3 mm and had an orifice 71.6 mm in diameter. The ratio of the orifice diameter to the pipe inside diameter, beta, was 0.70. The smaller orifice meter was mounted in a pipe of ID 40.9 mm and had an orifice diameter of 30.7 mm (beta = 0.75). The larger meter was used to measure air flow velocities greater than about 0.10 m/s and the smaller meter was used for rates less than this to provide greater accuracy at low flow rates. The air distribution system was designed to provide the straight lengths of pipe required by ASME criteria (see Figure 2).

Pressure taps were mounted at one pipe diameter upstream and one-half pipe diameter downstream and were connected to differential manometers using colored water as a manometer fluid. In addition, absolute pressures were measured upstream of the orifice meters using mercury manometers. Finally, a mercury manometer was used to measure the pressure in the plenum box.

The bed material was 0.264 mm mean diameter glass beads of density $2.5 \text{ g/cm}^3$. The particle size distribution is shown in Table 1. The manufacturer (Flex-O-Lite) specified the beads to have minimum sphericity of 0.80.

The tube bundle was designed in two sections for ease of handling, one for the front half with the tube ends abutting the front Plexiglas panel and the other in the back such that the rows of tubes appeared in continuation of those in front. Each section had nine tubes of 1.5 inch
PVC Schedule 40 plastic pipe (outside diameter 4.76 cm) spaced 7.6 cm apart, centerline to centerline, and running parallel to each other, the distributor, and the bed surface. The experiments were carried out with the tube centerlines at two locations, first at 18 cm above the distributor and then at 40 cm above the distributor.

Frames consisting of 6.4 mm diameter threaded rods and steel shelf channel were used to hold the tubes in place. Each 38 cm long tube had two rods running vertically through it with nuts on the top and bottom of the tube to hold it in place. The frames were a minimum of 15 cm from the front of the bed so as not to interfere with bubble patterns in that front portion. Figure 3 shows the tube bundle schematically.

A Videologic Instar IV high speed video system taped the front surface phenomena at 120 frames per second. The camera was equipped with a close-up zoom lens making it possible to view the phenomena at distances as small as 15 cm, but most of the taping was done with the camera at distances greater than 1.0 m to provide a full view of the bed width.

Lighting consisted of two synchronized Instrobe 90 strobe lights which served to enhance the quality when viewing the tapes in slow motion.
3. EXPERIMENTAL AND DATA ANALYSIS PROCEDURES

In preparing to do some video taping, the bed was first fluidized at a fairly high rate to ensure uniform bubbling. The flow rate was then adjusted to the desired value by monitoring the manometers attached to the orifice meters. Values from $1.1 \ U_{mf}$ to $3.0 \ U_{mf}$ ($6.2 \text{ cm/sec}$ to $17.0 \text{ cm/sec}$) were used for the experiments. Higher rates resulted in excess turbulence, making it impossible to view individual bubbles, and lower rates produced very few bubbles. The depth of the bed material, $L$, was determined by noting the position of the free surface at each flow rate when there were no bubbles erupting. This measurement was quite accurate except when the free surface was in the immediate vicinity of the tubes. In that case, the bed depth could be in error as much as a centimeter due to the continuous fluctuations of the surface between the tubes.

The strobe lights were directed at the front surface of the bed and their positions were adjusted to provide uniform lighting. The video camera was positioned slightly more than one meter from the front surface of the bed to provide the proper field of view.

Most flow rates and bed depths were taped for 60 seconds (7200 frames) to provide an adequate number of bubbles for analysis. Lower flow rates needed longer times since fewer bubbles were produced.

For each trial the following information was recorded:

- Sequence number as seen on the video screen and video tape.
• Manometer reading from the orifice pressure taps to determine the flow rate.
• Manometer reading from the tap upstream of the orifice to determine the actual line pressure.
• Manometer reading from the tap in the plenum box to measure plenum box pressure.
• Bed depth for the actual flow rate being used in the trial.
• Tube spacing and distance of tubes above the distributor.

In most cases, viewing of the tapes was carried out one frame at a time with faster speeds between major "events". (The speed was continuously variable on the video monitor.) A grid with horizontal and vertical markings had been taped to the Plexiglas front plate to serve as a guide for determining bubble sizes, bed depth, bulge heights, aspect ratios, and wake eruption heights. When viewing the bubbles and their wakes, the following information was recorded:

• Bubble diameter at the free surface, \( D_1 \) (Figure 4).
  When the free surface was located somewhere within the region extending from just below to just above the tubes, the diameter was measured just below the tubes.
• Height of void region, \( H \) (Figure 4), which made it possible to determine aspect ratios, \( 2H/D_1 \) (or \( H/a \) as defined by Edelstein [17]). Since the height of the
void area was constantly changing, the maximum height obtained before the bulge erupted through the free surface was used as H. The bubbles in Figure 4 and later sketches were drawn by tracing their outlines on the video screen.

- Type of bubble. The bubbles were classified as single bubbles or double bubbles as done by Dille [15] and others. A double bubble was defined as an event in which two single bubbles coalesced vertically near the free surface in such a way that the nose of the trailing bubble affected the wake eruption of the leading bubble. Figures 5a and 5b illustrate the formation of double bubbles both above and below the tubes. Most bubbles were single bubbles but there was also a significant number of double bubbles.

- Type of eruption. Most single bubble eruptions were of the bulge burst type, with the surface only rising about half the bulge height when the wake emerged and no material appearing to break contact with the main bed material. However, many of the single bubbles had significant wakes in the form of wake spikes, jet sprays, and some that were recorded as "explosions". Wen [19] discussed the exploding regime of bubble eruptions and attributed it to a case where the bubble diameter was
increasing at a greater rate than the rise.

velocity. The criterion he gave is thus \( \frac{d(D_i)}{dt} > U_b \),

where \( U_b \) is the bubble rise velocity.

The work he cited (Catipovic, et al. [20]) dealt

with much larger particles than used here and involved

substantial bubble coalescence which caused rapid

growth in the bubble diameter.

- Bulge height, \( H_b \) (Figure 6). This was the maximum
  height reached above the free surface of the bulge
  material.

- Wake height, \( H_w \) (Figure 7), or other height indicating
  the maximum height at which significant material
  could be observed during the eruption process.

After the data was recorded, it was necessary to reduce it to

parameters that would allow meaningful interpretation and comparisons.

For each series the following were calculated separately for single and

double bubbles:

- Mean bubble diameter, \( D_i \), as measured at the free
  surface.

- Mean diameter below the tubes for the deeper bed depths.

- Mean bulge heights.

- Mean wake heights.

These mean values were obtained from samples ranging from approxi-
mately 40 to 160 bubbles. The means recorded are simply the arithmetic
means, separately calculated for single and double bubbles in each series.

In addition, the percentage of double bubbles was calculated for each series. The flow rate, bed depth, and tube orientation were used in the analysis, also.

To enable comparison with Edelstein's cavity formation mechanism [17], velocities were determined for the bulge, wake center, and wake edges for eighteen bubbles. Also, the aspect ratio, $H/a$, was calculated for each of these bubbles.

To determine the flow rate (or superficial gas velocity), a calibration curve was plotted for each of the orifice meters (Figures 8 and 9) and the gas velocity was obtained by measuring the differential pressure on the meter taps.

Since previous research in this area has been very limited, it was not known what parameters might be important. Therefore, several dozen plots were made from the data, relating actual and non-dimensional parameters such as bubble diameter, bulge height, and wake height to flow rate and bed depth. These plots were analyzed, looking for patterns that might be present. As potential relationships became apparent, additional experiments were conducted to fill in the gaps and the new data added to the old. Those areas where systematic behavior appeared are included in the "EXPERIMENTAL RESULTS" section.
4. EXPERIMENTAL RESULTS

4.1 DISCUSSION

In many ways the observations in this experiment resembled those of Alkan [16] and Dille [15] in three-dimensional beds without tubes. On the other hand, the addition of a single row of horizontal tubes to the fluidized bed caused some significant differences in the bubble growth and eruption phenomena.

Presented here are typical results of this work, with comparisons made with no-tube cases where feasible. The characteristics that are examined include velocities of the bulge and wake; bubble diameters as a function of bed depth, tube location, and flow rate; bulge and wake eruption heights as a function of bed depth, tube location, and flow rate; and fraction of the bubbles for each situation that can be classified as double bubbles.

It will be shown in particular that under certain circumstances the tubes increased the double bubble fraction, decreased bubble diameters, and/or increased wake eruption heights. It will also be shown that unlike the no-tube cases, a large fraction of the wake eruptions led to wake spikes, jet sprays, or explosions in the "splash region", the region including the free surface and most of the wake eruption area. Finally, an explanation of what causes the differences will be presented.

4.2 VELOCITY RESULTS

A generally accepted relationship for bubble rise velocity first
proposed by Davidson and Harrison [21]

\[ U_b = U_{br} + (U_o - U_{mf}), \]  

where \( U_b \) is the bubble rise velocity, \( U_{br} \) the relative bubble velocity with respect to the superficial gas velocity, and \( U_o - U_{mf} \) the superficial gas velocity. The relative bubble velocity can be obtained from

\[ U_{br} = 0.71 \left( gD_e \right)^{1/2}, \]

The equivalent diameter, \( D_e \), is estimated by the Kato and Wen [18] correlation (discussed later in this paper), since Alkan [16] showed it gave a good approximation for the bubble diameters for the bed material used here. Also, the velocity difference, \( U_o - U_{mf} \) can be written in terms of the non-dimensional velocity \( U^* \) as \( U_{mf} (U^* - 1) \). The bubble rise velocity can thus be expressed as

\[ U_b = 0.71 \left( gD_e \right)^{1/2} + U_{mf} (U^* - 1). \]

This is of course a constant for any given bed depth and flow rate.

Figure 10 shows this theoretical velocity along with the actual velocities of the bulge, wake center, and wake edges of a typical bubble as it moved through the tubes. The velocities were determined by finding the slopes for various points in Figure 11, which represent the trajectory of this bubble. The bubble exhibited Dille's bulge burst mechanism [15] and was accompanied by a symmetrical wake rising as high as the bulge. Note how the wake slowed down as it traversed the gap, then accelerated again upon exiting. The free surface was at
52.5 cm and the tubes at 40 cm for this eruption, with \( U^* = 1.75 \). The retarding action of the tubes tended to increase coalescence at the tubes as the trailing bubbles caught up. The subsequent acceleration as the bubbles left the gap contributed to higher wake trajectories with tubes in the bed. Note the bulge velocity prior to entering the tubes agrees well with the theoretical value, although a large number of bubbles would need to be examined to generate statistics necessary for a meaningful comparison.

The drawings of Figures 12 and 14 and the plots of Figures 13 and 15 show the similarity of behavior for bubbles with small wakes for the cases with and without tubes. The plots are of the positions of the bulge tops, wake centers and wake edges of typical bubbles at various times as shown on the time scales. Each frame represented 0.0083 seconds. Figure 15 only covers the rise of the bubble after it has left the tube gap. The maximum bulge velocities are not significantly different from the wake center maximum velocities for these two bubbles and thus the wakes do not erupt as high into the freeboard region as the bulges, since they reach their maximum velocities lower in the bed. For example, in Figure 15, the maximum bulge velocity occurs at about 33 cm above the distributor, whereas the maximum wake velocity occurs at about 26 cm above the distributor. The maximum velocity of the wake for this bubble type is also considerably less than for the wake spike events discussed later in this section. For example, the maximum wake velocity in Figure 15 is 135 cm/sec, whereas in Figures 22, 25, and 27 it reaches velocities of 275, 173 and 215 cm/sec, respectively.
Figure 16 shows the velocities of the bulge, wake center, and wake edges of the bubble of Figures 12 and 13 as determined from slopes on Figure 13. Here the velocities appear substantially higher than the predicted values but no meaning is attached to this difference as it represents one bubble only. Comparing with Figure 10, we see only a small velocity increase as the bubble erupts through the free surface, not the very large acceleration as caused by the tube presence.

Edelstein [17], when working with double bubbles, showed that the large wake eruptions that frequently accompany double bubbles result from elongated cavities. These cavities were actually the form taken by the trailing bubble as it stretched to try to fill the void created by the leading bubble. He pointed out that the wake jet behavior resulting from the cavity formation resembled that discussed by Klenzler, et al. [22], MacIntyre [23], and many others in explaining jetting action of single bubbles in liquids and suggested it should be possible to observe the same phenomena for single bubbles in fluidized beds. This idea was not pursued further by Edelstein, however, because jets rarely formed with single bubbles in his work without tubes.

Figure 17 is Edelstein's idealized cavity, where \( H \) is the distance from the free surface to the cavity bottom and \( 2a \) the cavity (bubble) diameter. Figure 18 shows the definition of the quantities \( V'_o \) and \( V'_b \), the velocities of the wake center and wake edges, respectively, when the wake center starts separating from the wake edges (the bifurcation point of Figure 18). The velocity of the wake center becomes quite high in these jets and leads to very high wake eruptions in the form of
wake spikes, jet sprays, or explosions. Edelstein showed that the wake center velocities (and thus the wake eruption heights) are a function of the aspect ratio, \( H/a \), of the cavity. Specifically, his work can be expressed by the relationship

\[
v_o'/(ag)^{1/2} = 1.4(H/a) + 1.3
\]

obtained from the theoretical line of Figure 19. The ratio \( v_o'/(ag)^{1/2} \) is a non-dimensional wake center velocity and 1.3 is the empirical constant for the non-dimensional wake edge velocity, \( v_b'/(ag)^{1/2} \), taken from his plot. The constant 1.4 was developed theoretically but agrees quite well with his experimental data.

Double bubbles occurred with tubes in the bed, also, and the wake eruptions were affected in much the same way by the cavity formation. The important difference in the current work, however, was the formation of elongated cavities as single bubbles traverse the tube gaps, although these cavities rarely have the rectangular shape of Figure 17. Most are wider at the top than at the bottom, having the shape shown in Figure 20, since the bubbles tended to spread out as they left the gaps. The variations in the shapes of the cavities resulted in a range of aspect ratios from 1.71 to 5.93 (Table 5). Since the cavity diameters, \( 2a \), tend to be larger for these single bubbles, the aspect ratios might be expected to be smaller for given wake velocities. This is not the case, as shown in Figure 21, which compares the current single bubble data with Edelstein's double bubble cavities. The least squares regression line for 18 single bubbles (of which fifteen had large wake spikes, one was classified as a wake explosion and two as jet sprays) can be
Expressed by

\[ \frac{v^*}{(ag)^{1/2}} = 0.62(H/a) + 1.7. \]  

(5)

Note the slope is considerably less than the 1.4 given by Edelstein for double bubble jet formation. However, the correlation coefficient is only 0.57 and not much significance can be given to the slope.

The wake velocities were obtained by plotting positions of the various points of the bubbles as a function of time, then finding the slopes at the bifurcation point as done by Edelstein (Figure 18). The many different types of wake eruption phenomena occurring in this experiment might be due to the different velocities of the bubble points and the variations from one type of eruption to the next. These variations might be a function of the bubble shapes and interactions as they leave the tube gaps.

In Figure 18, a plot of a typical trajectory for a double bubble symmetrical cavity, Edelstein [17] has shown how the maximum velocities of the wake center and wake bottom, \( V_o^* \) and \( V_b^* \), respectively, are determined. Figure 22 shows the trajectories for a single bubble eruption that started as a nearly rectangular cavity as it left the tubes and ended in a jet spray (see Figure 23). There is a strong similarity in the trajectories of the double bubble of Figure 18 and the single bubble of Figure 22, and this suggests that they are closely related phenomena. Figure 23 shows sketches of the sequence of events as the bubble erupts. These were obtained by tracing the outline of an
erupting bubble from the video screen. In 23a, the bubble assumes the rectangular cavity shape as it exits the tube gap. In Figures 23b and 23c we see the bulge bursting into the freeboard region. Figure 23d shows the bulge material collapsing and the wake erupting into the freeboard region. Finally, Figure 23e shows the spray of wake material resulting from the high velocities reached by the wake.

Figure 24 shows sketches of an eruption sequence that ended in a large wake spike. Again the cavity assumed a rectangular shape as it left the tubes (Figure 24c), but the wake did not burst into a spray. The trajectories for this eruption are shown in Figure 25. Figure 26a shows sketches of another nearly rectangular cavity exiting the tubes. In this eruption, however, the wake exploded (Figures 26e and 26f), throwing a large cloud of bed material into the freeboard region. The trajectories of the jet spray, wake spike, and explosion, Figures 22, 25, and 27, respectively, closely resemble each other, leading one to conclude the three events may be closely related, with the differences in their wake eruptions possibly resulting from the nature of the interactions of each at the tubes.

4.3 EFFECTS OF THE TUBES UPON DOUBLE BUBBLE FRACTION

One would expect the tubes to affect the double bubble fraction because the bubbles are slowed down as they hit the tubes, allowing trailing bubbles to catch up. In addition, as the bubbles are diverted into the gaps, the effective bed area for bubble eruptions is reduced.
If the height of the bed material above the tubes is large enough, however, this effect should disappear. As a matter of fact, the double bubble fraction might be expected to be less than in the no-tube case due to prior forced coalescence of adjacent bubbles at the tubes. Figure 28 shows the percentages of all bubbles which were classified as double bubbles for each series from Table 3 for the 40 cm tube location. Figure 29 does the same for the 18 cm tube location, with data taken from Table 2. Although Figures 28 and 29 indicate a tendency toward higher double bubble fraction with tubes than without, the scatter of the data precludes any firm conclusions.

Figures 30a-e show the same data as Figures 28 and 29 but with flow rate as the independent variable. In Figure 30d we see a systematic 30 to 50 percent greater double bubble fraction than without tubes. Here the tubes are at 18 cm and the free surface is between zero and five cm above the tubes. When the tubes were at 40 cm there was not as obvious a difference (Figure 30b), due to the lower bubble frequency (and thus a lesser likelihood of coalescence). In all other cases of Figure 30 there did not appear to be significant differences in double bubble fraction. Finally, Figure 31 shows the double bubble fraction is higher when the free surface is zero to fifteen cm above the tubes and reduces considerably by the time H = 40 cm, as expected due to the prior forced coalescence at the tubes and additional coalescence through added depth of bed material.
4.4 COMPARISON OF BUBBLE DIAMETERS AND WAKE HEIGHTS FOR SINGLE AND DOUBLE BUBBLES

When quoting the diameters of double bubbles, the diameter of the leading bubble was used (as done by Alkan [16] and Dille [15] previously). For this reason, one would not expect any differences between single and double bubble diameters. For each of the tests run, Figures 32a and 32b show the ratios of single to double bubble diameters to be approximately 1.0. Each data point represents a single sequence of bubbles, all taken at the same bed operating conditions (see Tables 2 and 3). The abscissa is the mean single bubble diameter and the ordinate the mean double bubble diameter for the test sequence.

When comparing the mean maximum wake eruption heights for single and double bubbles for each series, however, we see that the mean double bubble wake eruptions were significantly higher. In general, Figures 33a and 33b show the mean heights to be 1.5 to 3.0 times as high for double bubbles as for single bubbles. This result is expected when we examine the fraction of eruptions characterized by jet sprays, wake spikes, or explosions. It is shown later (Figures 36a and 36b) that the fraction ranges from 20 to 60 percent for single bubbles. Table 6 shows that about 80 to 100 percent of the double bubbles resulted in significant wake eruptions, with a mean fraction of 87 percent.

4.5 EFFECT OF TUBES UPON WAKE TYPES FOR SINGLE BUBBLES

It has been pointed out that the tubes led to elongation of the
bubbles in some instances as they passed through the gaps. This elonga-
tion led to cavity formation which often resulted in wake eruptions of the wake spike, jet spray, or explosion type. It is to be expected, therefore, that the frequency of such events would be greater in the presence of the tubes than otherwise.

Dille [15] found that 90 percent of the bubbles in the three-
dimensional bed with no tubes were single bubbles and virtually all of the single bubbles exhibited the bulge burst mechanism type of eruption. In these eruptions, the wake reached to a height between 0.5 and 1.0 times the bulge height. It is difficult to discriminate between a large bulge burst mechanism eruptions and a small wake spike eruption since there is a continuum of events. The single bubble wake spikes simply travel higher into the freeboard than the bulge burst mechanism wake eruptions. To determine the frequency of the events, it was somewhat arbitrarily decided to call those where the wake eruption height was more than 1.0 times the bulge height a spike (Figure 34). This of course included those wake events classified as jet sprays or explosions but it is justifiable since they appear to result from a very similar mechanism and are of low frequency of occurrence. Excluded by this classification were the definite wake spikes that did not reach the bulge height. However, it seemed a good compromise to reach an estimate of approximate wake spike frequency.

An alternate definition was applied as a rough check in which a wake spike was declared if a void area could be seen below the "wings"
of a mushroom-shaped wake eruption (Figure 35). This led to an even higher frequency of wake spike events which confirms that the previous definition led to a conservative estimate rather than overestimating the frequency.

Figures 36a and 36b show that for the cases where the free surface was above the tubes, the frequency of wake spike, jet spray and explosion eruptions range from 20 to 60 percent of all the single bubbles in each series. On the other hand, Dille [15] found that the "wake spike phenomenon only occurred when two or more bubbles coalesced at the free surface". In other words, single bubbles do not exhibit wake eruptions when there are no tubes in the bed. Alkan [16] also did not report any significant wake spikes with single bubbles when working with this bed material with no tubes in the bed.

Perhaps more surprising is the fact that even with the free surface about 40 cm above the tubes, the fraction of wake spike, jet spray, or explosion events for single bubbles remains at 20 to 50 percent. This implies that the effects of the tubes have not completely disappeared although the bubbles looked about the same as with no tubes as they approached the surface. The maximum velocities of wake material in these large eruptions were 29 to 85 percent higher than the maximum bulge velocities, as seen in Table 5 (series 308 and 310), whereas in the typical eruptions without tubes the velocity maxima were about the same for the bulge and wake (see Figure 13, for example, where the maximum wake velocity is only about 7 percent higher than the maximum bulge
Apparently what happens is that as the bubbles traverse the gaps, they are elongated, as discussed previously. As the bubbles continue to rise, the bottom of the wake catches up to the bulge so that the bubble appears to resume the shape it would have had had not the tubes elongated it. However, in order to catch up, the wakes had to reach the higher velocities which produced the large-wake eruptions noted.

4.6 EFFECTS UPON THE BUBBLE DIAMETERS OF SINGLE BUBBLES

Because of the tube interference, the question of how to measure the bubble diameters when the free surface was in the region of the tubes was difficult to answer. When the free surface was at the tube centerline, the bubble diameter was obviously just the gap width, since nearly all bubbles were larger than the gap width when they entered. However, for the sake of comparing results with previous work it was felt a more realistic approach would be to use the bubble diameters just before they entered the gaps as an estimate of bubble diameters. As the bubbles left the gap, they expanded again but were elongated into a variety of shapes, again leaving the determination of an equivalent diameter in question. Thus, for bed depths from 15 to 21 cm for the lower tube location and 40 to 46 cm for the higher tube location, there was uncertainty as to what should be used for the bubble diameters. However, this uncertainty does not negate the validity of the conclusions since they are not dependent upon bubble diameters in these regions to any
great extent.

To determine the effects of the tubes on the bubble diameters one must first establish a standard for comparison without tubes. Alkan [16] showed that this bed material produces bubbles that match the Kato and Wen [18] correlation quite well, if one applies a shape factor of 1.15. This factor was obtained by calculating cross-sectional areas of bubbles and using that information to find their volumes. Then an "equivalent diameter" of a spherical bubble was determined. The ratio of the measured frontal diameter, \( D_f \), to this equivalent spherical diameter was considered to be the shape factor. The Kato and Wen correlation gave equivalent diameters which needed to be multiplied by the shape factor before comparing with the observed diameters. Comparisons in this work are made, therefore, with Alkan's limited data as well as the Kato and Wen correlation.

The Kato and Wen correlation is given as

\[
D_e = 1.4 d_p \rho_p U/LU_{mf} + D_{eo},
\]  

(6)

where

\[
D_{eo} = 0.347 \left\{ A_t (U - U_{mf})/N_\text{d} \right\}^{0.4},
\]  

(7)

Since \( U_{o/mf} = U^* \), the non-dimensional flow rate, and since \( U_{o/mf} \) can be written as \( U_{mf}(U/O_{mf} - 1) = U_{mf}(U^* - 1) \), we can rewrite this as

\[
D_e = 1.4 d_p \rho_p U^* L + 0.347 \left\{ A_t U_{mf}(U^* - 1)/N_\text{d} \right\}^{0.4}.
\]  

(8)

Substituting the known values of \( d_p = 2.64 \times 10^{-2} \text{ cm} \), \( \rho_p = 2.5 \text{ g/cm}^3 \), \( U_{mf} = 5.68 \text{ cm/s} \), \( A_t = 5.76 \times 10^{-3} \text{ cm}^3 \), \( N_\text{d} = 484 \) and \( L = \text{depth of bed} \)
material in cm leads to

\[ D_e = 9.17 \times 10^{-2} \ U^* L + 1.87 (U^* - 1)^{0.4}. \]  

(9)

A simple computer program was written to generate tables for the equivalent diameters in the ranges of \( U^* \) and \( L \) used in these experiments. The theoretical diameters were then obtained by multiplying these values by the shape factor of 1.15.

If one were to hypothesize how the tubes would affect the bubbles, one would guess that large bubbles would likely sometimes split, producing more but smaller bubbles. In addition, as bubbles traverse the gaps, they would be squeezed together and elongated, again resulting in smaller bubble diameters as the bubbles leave the gaps.

Figures 37a–e show the average single bubble diameters for each series as a function of bed depth with tubes and without tubes as well as the Kato and Wen predicted diameters for the tubes in the 18 cm location. Figure 38 shows the same for the tubes in the 40 cm location. Figures 39 and 40 show theoretical and experimental diameters as a function of flow rate, \( U^* \), for the two tube locations, respectively.

No obvious differences exist between the bubble diameters with and without tubes, possibly due to the scatter of the data. Figure 40 shows the bubble diameters for the 52.5 cm bed depth were consistently below those for the 40 cm and 45 cm depths (where the measurements of diameters were made below the tubes, at about 35 cm). This result most likely is due to the splitting and elongation by the tubes, since the diameters at 52.5 cm would normally be significantly higher than at 35 cm.
Figures 41a and 41b illustrate the splitting of a large bubble and of a small bubble which were incident upon a tube center. Figure 42 illustrates the elongation of a bubble as it traverses the gap, giving it a smaller diameter above the tubes than it had below, contrary to the behavior in the absence of tubes. The incidence of these events is sufficient to cause a reduction in mean bubble diameters in the region just above the tubes.

4.7 EFFECT OF THE TUBES UPON BULGE AND WAKE ERUPTION HEIGHTS OF SINGLE BUBBLES

As the bubbles go through the tubes the bubble diameters are reduced under certain circumstances, as discussed above. No significant differences in bulge eruption heights can be seen for these bubbles, however, although the scatter of the data may obscure the facts, Figures 43a–f show the mean bulge eruption heights for all series of experiments done in this work. Looking at Figures 43a–e, one can conclude only that (1) the bulge eruption heights increase with bed depth (Figures 43a, b, and d) and to a lesser extent (2) with flow rate (Figures 43c and 43e).

Figures 44a–g show the mean wake eruption heights for single bubbles for each series in this work. The existence of the elongated cavities (previously discussed), which lead to large wake eruption phenomena, would lead one to expect these mean heights to be significantly greater with tubes in the bed. This difference appears quite obvious with the 40 cm tube location (Figures 44f and 44g), but does
not show up with the tubes at 18 cm, possibly due to a lack of comparable data. When one compares the ratios of mean bulge eruption height, $H_b$, to mean wake eruption height, $H_w$, for all the series (as shown in Figure 45), the effect of the tubes becomes apparent. Dille [15] reported the bulge eruption heights to be about two-thirds the bubble diameter and the mean wake eruption heights to be about one-third the mean bubble diameter for single bubbles. This would indicate a ratio of mean bulge heights to wake eruption heights of 2.0. Calculating these same ratios from Alkan's data (Table 4) leads to ratios ranging from 1.2 to 1.95 and a mean of about 1.5. With tubes in the bed, however, the ratio ranges from about 0.7 to 1.7 with most values in the 0.9 to 1.3 range.

Figure 45 shows the distribution of these ratios for data from Alkan's work and the current work. The smaller ratios resulted from the large wake eruption events caused by the tubes, even when the bed material was as much as 40 cm above the tubes.

4.8 EFFECTS OF THE TUBES UPON DOUBLE BUBBLES

It has already been shown that the double bubble diameters were about the same as the single bubble diameters (Figure 32). It has also been shown that the diameters decreased as the bubbles exited the gaps, which holds for double bubbles, also simply because of the manner in which the diameters were defined. It has further been shown that the mean wake eruption heights for double bubbles average 1.5 to 3.0 times
the means for single bubbles under the same conditions of flow rate and bed depth (Figures 33a and 33b). This is due to the very high fraction (averaging 87 percent, Table 6) of wake spikes, jet sprays or explosions accompanying double bubble eruptions. Figures 46a and 46b show the mean wake eruption heights of double bubbles for each series (from Tables 2b and 3b, respectively).

Figures 47a and 47b show the ratios of mean bulge heights to mean wake heights for the double bubbles. Here the ratios ranged between 0.4 and 0.8, reflecting the large wake eruptions of the double bubbles. The data for these ratios was taken from Table 3. The ratio can be seen to decrease as the flow rate increases. This is caused by the greater wake heights at higher flow rates, as can be seen in Figure 46.
CONCLUSIONS AND RECOMMENDATIONS

Summarizing the observations leads one to conclude that the tubes greatly affect the bubble growth and eruption processes, especially when the free surface is in the vicinity of the tubes. Bubbles are elongated as they traverse the gaps and split if they are incident upon a tube center or are larger than the tube spacing as they reach the tubes. Forced coalescence at the tubes increases the double bubble fraction when the tubes are near the free surface. Changes in bubble shape induced by the tube interactions result in greater wake eruption heights for single bubbles than in the case without tubes.

The observed effects of a single row of horizontal tubes upon the bubbles in this fluidized bed using 0.264 mm glass beads can be summarized as follows:

- The mean bubble diameters were reduced as the bubbles went through the gaps due to (a) splitting and (b) elongation of the bubbles. The tubes caused the bubbles to first slow down, then accelerate to higher velocities than they would have had without the tubes.

- Due to the formation of elongated cavities as the bubbles traversed the tube gaps, wake velocities for single bubbles were often large compared to the superficial gas velocity and bubble bulge velocity. These higher velocities often led to single bubble wakes characteristic of those observed when jets form, such as large mushroom-shaped
wake spikes, jet sprays or explosions. Because of the large wake eruptions, single bubbles were less "bulge-dominated" in the presence of tubes than without tubes. The average ratio of bulge height to wake eruption height, $H_b/H_w$, without tubes was about 1.5 (from Alkan's data) for this bed material and bed. In the current work, however, the ratio ranged from 0.9 to 1.3 for most series, depending upon the relative location of the tubes and free surface.

- The double bubble fraction is greater in the presence of tubes if the free surface is in the vicinity of the tubes and the tubes are not too far above the distributor, due to forced coalescence by the tubes. The wake behavior of the double bubbles was not noticeably different from the no-tube situations, although a lack of double bubble reference data makes firm conclusions impossible. The major differences appeared to be in the single bubble eruptions.

- The maximum wake velocities for individual single bubbles exhibiting wake spikes seems to vary with aspect ratio in much the same way as Edelstein's double bubble wake spikes (Figure 21). However, the slope of the least squares regression line is 0.62 for single bubbles, compared to 1.4 for Edelstein's double bubbles. Further work is necessary to model the single bubble cavity
formation to see if this difference in slope can be accounted for.

- Double bubble wakes erupt an average of 1.5 to 3.0 times as high into the freeboard region as single bubble wakes under the same bed conditions. This can be attributed to the fact that 87 percent of the double bubbles result in wake spikes (or similar events), compared to only 20 to 60 percent of the single bubbles.

- The mean ratios of bulge eruption height to wake eruption height for double bubbles ranges from 0.4 to 0.8, significantly below the ratios for single bubbles under the same conditions (0.7 to 1.7). This reflects the higher wakes that result from the jets formed when bubbles coalesce vertically into double bubbles.

Before the scope of this project was determined, preliminary observations were made with two horizontal rows of tubes, both in-line (vertically) and staggered. In addition, video tapes were made with a tube bundle containing seven horizontal, staggered rows of tubes. Additional work should be initiated to find the effects of two or more rows of tubes on the fluidization and elutriation processes.
Table 1. Size Distribution of Glass Beads (sieve analysis)

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<th>Diameter Range (mm)</th>
<th>Weight Fraction</th>
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<td>0-.090</td>
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<td>.090-.150</td>
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<td>.300-.425</td>
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<td>.425-.600</td>
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</table>

Mean diameter as determined by microscope on a sample of 150 beads = 0.264 mm.

All data taken from Alkan's thesis [16].
Table 2a. Data summary for single bubbles, $L_t = 18 \text{ cm}$

<table>
<thead>
<tr>
<th>Series</th>
<th>$L$ (cm)</th>
<th>$U^*$</th>
<th>$\bar{D}_l$ (cm)</th>
<th>$H_b$ (cm)</th>
<th>$H_w$ (cm)</th>
<th>$N$</th>
<th>$\bar{H}_b/\bar{H}_w$</th>
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* Data not plotted due to small sample size

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210  34.5  3.00  27  15  11.35  6.63  13.74
211  32.8  2.00  24  23  9.91  5.72  9.60
212  32.5  1.65  27  34  9.86  6.96  14.73
213  32.3  1.43  13  14  9.09  6.88  12.93
215  25.7  1.10  15  16  7.06  6.43  13.67
216  26.4  1.43  19  20  7.26  5.69  11.99
217  27.2  1.65  21  22  8.31  6.07  11.71
218  27.7  2.00  26  24  8.86  6.86  16.15
219  28.4  3.00  27  25  9.60  6.43  14.40
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229  22.4  2.50  25  18  8.84  6.91  11.79
230  22.9  3.00  43  30  8.28  7.54  12.47
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36
Table 3a. Data summary for single bubbles, $L_t = 40$ cm

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Table 3b. Data summary for double bubbles $L_t = 40$ cm.

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Table 4. Alkan's data. $\bar{H}_b$ and $\bar{H}_w$ are determined by multiplying his non-dimensional heights, $H/D_i$, by $D_i$.

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Table 5. Data for eighteen bubbles needed for Figure 21.

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Table 6. Fraction of double bubbles showing significant wake spike or other large wake event.

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<tr>
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<td>88</td>
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</table>

* only 4 in sample
Figure 1. Fluidized bed - cyclone arrangement (reprinted from Alkan's thesis [16]).
Figure 2. Sketch of the air system (reprinted from Alkan's thesis [16]).
Figure 3a. Front view of tube bundle through plexiglass

Figure 3b. Side view with steel plates removed

Figure 3. Tube bundle
Figure 4. Bubble diameter, $D_i$, and height of void, $H$, for (a) free surface well above tubes and (b) in the vicinity of the tubes.
Figure 5a. Sequence showing bubble coalescence above the tubes, leading to a double bubble.
Figure 5b. Sequence showing bubbles coalescing at the tubes, leading to an elongated single bubble above the tubes.
Figure 6. Definition of bulge height, $H_b$ (measured at maximum height of the eruption).
7a. Bulge burst mechanism

7b. Wake spike

Undisturbed free surface

7c. Wake spike

7d. Wake spike

Undisturbed free surface

7e. Explosion

7f. Jet spray

Undisturbed free surface

Figure 7. Definition of wake height, $H_w$, for various eruption types.
Figure 8. Small orifice calibration curve (reprinted from Alkan's thesis [16]).
Figure 8. Small orifice calibration curve (reprinted from Alkan's thesis [16]).

PRESSURE DROP (cm. H₂O)

U₀ (cm/sec)

SMALL ORIFICE
D₀ = 0.75 in.
Dₚ = 3.068 cm.
Figure 9. Large orifice calibration curve (reprinted from Alkan's thesis [16]).
Figure 9. Large orifice calibration curve (reprinted from Alken's thesis [16]).

\[ U^0 \text{ (cm/sec)} \]

FLOW RATE (cm/sec)

PRESSURE DROP (cm. H\textsubscript{2}O)

\[ D_p = 7.258 \text{ cm.} \]

\[ H = 0.070 \]

LARGE ORIFICE
Figure 10. Velocities of the bulge, wake center, and wake edges as they move through the tubes.
Figure 10. Velocities of the bulge, wake center, and wake edges as they move through the tubes.
Figure 11. Position of bubble points as a function of time
Figure II. Position of bubble points as a function of time.
Figure 12. Typical bubble eruption sequence without tubes. The dashed lines indicate the bulge burst. In Figure 12f, the bulge has collapsed and only the wake is seen.
Figure 13. Position vs. time for a bubble with no tubes. The maximum velocities are estimated from the slopes of the lines.
Figure 13. Position vs. time for a bubble with no tubes. The maximum velocities are estimated from the slopes of the lines.
Figure 14. Eruption sequence showing a bulge burst with a small wake. In Figure 14f (next page), the bulge has collapsed.
Figure 14. (Continued)
Figure 15. Bubble resembling the no-tube case rising from a tube gap. There is no wake spike. The bubble is illustrated in Figure 14.
Figure 15. Bubble resembling the no-tube case rising from a tube gap. There is no wake spike. The bubble is illustrated in Figure 14.

Maximum velocity (cm/s)

Time (sec)

Distance above distributor (cm)

Wake edges
Wake center
Bubble

Tubes

Free surface
Undisturbed
Figure 16. Velocities of the bubble of Figure 12, no tubes in the bed.
Figure 16. Velocities of the bubble of Figure 12, no cubes in the bed.
Figure 17. Edelstein's cavity idealization (reprinted from Edelstein's thesis [17]).
Figure 18. Definition of cavity bottom velocity $v_b'$, jet initial velocity $v_o'$ and cavity depth $H$. (reprinted from Edelstein's thesis [17]).
Figure 18. Definition of cavity bottom velocity \( v_b \). See integral.
Figure 19. Comparison between the mathematical model and the experimental results. (Reprinted from Edelstein's thesis [17]).
Figure 19. Comparison between the mathematical model and the experimental results.

Key:
- ▼: Video system 30
- ▲: Video system 20
- ◯: High-speed photography 30

Note: Reproduced from Davenport's thesis (177).
Figure 20. Typical elongated bubbles leaving the tube gaps.
Figure 21. Non-dimensional wake velocity vs. aspect ratio for double bubbles and single bubbles (data for double bubbles taken from Figure 19).
Figure 22. Trajectories for an elongated bubble ending with a jet spray.
Figure 22. Trajectories for an elongated bubble ending with a jet spray.
Figure 23. Wake eruption sequence ending in a jet spray.
Figure 24. Wake eruption sequence ending in a wake spike. The bulge has collapsed in Figure 24f and is not shown.
Figure 25. Trajectories for the bubble of Figure 24 (large wake spikes)
Figure 25. Trajectories for the bubble of Figure 24 (Large Wake Spikes)

Tubes
Undisrupted Free Surface
Wake Edges
Wake Eruption
Bulge

Maximum Velocities

Distance above distributor (cm)

Time (sec)
Figure 26. Sequential sketches of an exploding wake (continued on next page).
Figure 26. (Continued)

26d. 0.142 sec.

26e. 0.217 sec.

26f. 0.283 sec.
Figure 27. Trajectories showing the exploding wake of Figure 26.
Figure 27. Trajectories showing the explosive wake of Figure 26.
Figure 28. Double bubble fraction for tubes at 40 cm.
Figure 28. Double bubble fraction for tubes at 40 cm.

- **I (cm)**
  - 0
  - 10
  - 20
  - 30
  - 40
  - 50
  - 60
  - 70
  - 80

- **Percent double bubbles**
  - 0
  - 10
  - 20
  - 30
  - 40
  - 50
  - 60

Key:
- ▼ 2.0-2.2
- ■ 1.7-1.9
- ◆ 1.4-1.6
- ● 1.1-1.3
- ○ No Tubes
Figure 29. Double bubble fraction for tubes at 18 cm.
Figure 29. Double bubble fraction for tubes at 18 cm.
Figure 30a. Double bubble fraction for each series at 18 cm tube location and no-tube cases.
Figure 30a. Double bubble fraction for each series at 18 cm tube location and no-tube cases.

Percent double bubbles
Figure 30b. Double bubble fraction for each series at 40 cm tube location and no-tube cases
Figure 30b. Double bubble fraction for each series at 40 cm tube location and no-tube cases.
Figure 30c. Double bubble fraction with tubes at 18 cm, $L = 14-18$ cm.
Figure 30c. Double bubble fraction with tubes at 18 cm, $L = 15-18$ cm.
Figure 30d. Double bubble fraction vs. flow rate with tubes at 18 cm, L = 19-23 cm.
Figure 30d: Double bubble fraction vs. Flow rate with tubes at 18 cm, L = 19-25 cm.
Figure 30e. Double bubble fraction for tubes in 18 cm location, \( L = 24-38 \text{ cm} \)
Figure 30c. Double bubble fraction for tubes in 18 cm location.

Percent double bubbles

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<tr>
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</tr>
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</table>
Figure 31. Double bubble fraction for various depths of bed material above the tubes for 18 and 40 cm tube locations.
Figure 31. Double bubble formation for various depths of bed material above the tubes for 18 and 40 cm tube locations.
Mean single bubble diameter (cm)

Mean double bubble diameter (cm)

Figure 32a. Comparison of bubble diameters for double and single bubbles with tubes at 18 cm. and $0 < H < 20$ cm.
Figure 724. Comparison of bubble diameters for double and single bubbles with tubes at 18 cm. and 0 < H < 20 cm.

Mean single bubble diameter (cm)

Mean double bubble diameter (cm)
Figure 32b. Comparison of bubble diameters for double bubbles with tubes at 40 cm and $0 < H < 40$ cm.
Figure 3.2b. Comparison of bubble diameters for single and double bubbles.

Mean single bubble diameter (cm)

Mean double bubble diameter (cm)
Figure 33a. Double bubble vs. single bubble mean wake eruption heights with $L_t = 18$ cm and $0 < H < 20$ cm.
eruption heights with $L_e = 18$ cm and $0 < H < 20$ cm.

Figure 33a. Double bubble vs. single bubble mean wake

Mean wake height for double bubbles (cm)

Mean wake height for single bubbles (cm)
Figure 33b. Double bubble vs. single bubble wake eruption heights with $L_t = 40$ cm and $0 < H < 40$ cm
Figure 33b. Double bubble vs. single bubble wake eruption
Mean wake height, single bubbles (cm)

Mean wake height, double bubbles (cm)
Figure 34a. Single bubble bulge burst mechanism.
Figure 34b. Single bubble wake spike eruption.
Figure 35. Alternate definition of wake spike.
Figure 36a. Fraction of single bubbles exhibiting large wake eruptions, $L_e = 18$ cm.
Figure 3.6a. Fraction of single bubbles exhibiting large wake effects, $L = 18$ cm.

Percent wake spikes
Figure 36b. Fraction of single bubbles exhibiting large wake spikes, $L_t = 40$ cm.
Figure 36b. Fraction of single bubbles exhibiting large wake spikes, \( L = 40 \text{ cm} \).
Figure 37a. Experimental and theoretical bubble diameters for single bubbles, \( L_t = 18 \) cm. Solid lines are from the Kato and Wen correlations.
Figure 37A. Experimental and theoretical bubble diameters for single bubbles.
Figure 37b. Experimental and theoretical bubble diameters for single bubbles. Solid line is from the Kato and Wen correlation. Tubes are at 18 cm.
Figure 37b. Experimental and theoretical bubble diameters for

\[ \text{L (cm)} \]

\[ \text{Bubble diameter (cm)} \]

\[ u = 1.43 \]

\[ \text{Tubes} \]

\[ \text{No Tubes} \]

\[ \text{1.4 - 1.6} \]
Figure 37c. Experimental and theoretical bubble diameters for single bubbles. Solid line is from the Kato and Wen correlation. Tubes are at 18 cm.
Figure 3.7c. Experimental and theoretical bubble diameters for when correlation tubes are at 18 cm.

Single bubbles. Solid line is from the Kato and

\[ \text{Bubble diameter (cm)} \]

\[ \text{L (cm)} \]

\[ \text{Bubble diameter (cm)} \]

\[ \text{L (cm)} \]

\[ u = \text{1.75} \]

\[ \text{No tubes} \]

\[ u \]

\[ \text{Tubes} \]
Figure 37d. Experimental and theoretical bubble diameters for single bubbles, $L = 18$ cm. Solid lines are from the Kato and Wen correlation.
Solid lines are from the Kato and Wen correlation.

Figure 37d. Experimental and theoretical bubble diameters for single bubbles, $l = 18$ cm.
Figure 37e. Experimental and theoretical bubble diameters for single bubbles, \( L_t = 18 \text{ cm} \). Solid lines are from the Kato and Wen correlation.
Solid lines are from the Kato and Wen correlation. Experimental and theoretical bubble diameters for single bubbles, \( L = 18 \text{ cm} \).
Figure 38. Experimental and theoretical bubble diameters for single bubbles, $L_t = 18 \text{ cm}$. Solid lines are from the Kato and Wen correlations.
Figure 38. Experimental and theoretical bubble diameters for single bubbles.

Mean bubble diameter (cm)

L = 18 cm. Solid lines are from the Kao and Ven correlations.

- ▼: 2.0-2.2
- ■: 1.7-1.9
- ○: 1.1-1.3

No Tubes

L (cm)

Mean bubble diameter (cm)

U = 1.25

U = 2.25
Figure 39. Experimental and theoretical bubble diameters for tubes in 18 cm location. Solid lines are from the Kato and Wen correlation.
Karo and Wen correlation. Lubes in 18 cm location. Solid lines are from the

Figure 39. Experimental and theoretical bubble diameters for

\[ L = 25 \text{ cm} \]
\[ L = 15 \text{ cm} \]
\[ L = 35 \text{ cm} \]

Legend:
- 39-43
- 34-38
- 29-33
- 24-28
- 19-23
- 14-18

No Tubes

(cm)
Figure 40. Experimental and theoretical bubble diameters for single bubbles. Solid lines are from the Kato and Wen correlation. Tubes are at 40 cm.
Figure 40. Experimental and theoretical bubble diameters for Meniscus bubbles. Solid lines are from the Kato and Singh models. Tubs are at 40 cm.

Mean bubble diameter (cm)

L = 40 cm
L = 52.5 cm
L = 80 cm

60-80
49-53
44-49
39-43

No Tubs

L (cm)
a. Bubble approaches tubes.  
b. Two bubbles erupt from adjacent gaps.  
c. Bulges collapse and wakes begin erupting.  
d. Wakes protrude into the freeboard.  
e. One wake starts collapsing.  
f. Both wakes are collapsing.  

Figure 41a. Splitting of a bubble as it encounters the tubes.
Figure 41b. Small bubble splitting as it reaches the tubes incident upon a tube center. Bulge eruption is quite small and virtually no wake results.
Figure 42. Elongation of a bubble as it traverses the gap.
Figure 43a. Bulge eruption heights for single bubbles, tubes at 18 cm. 
$U^* = 1.1-1.9$
Figure 4.4a. Bulge eruption heights for single bubbles, tubes at 18 cm.

$U_L = 1.1 - 1.9$

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</table>

$U_L$ (cm)
Figure 43b. Bulge eruption heights for single bubbles, tubes at 18 cm, $U^* = 2.0-3.1^\circ$
Figure 4.3b. Bulge eruption heights for single bubbles, tubes at 18 cm. *u = 2.0-3.1 cm.

Mean bulge height (cm)

Tubes

No tubes

U

Tubes

2.0-2.2

2.3-2.5

2.9-3.1
Figure 43c. Bulge heights for single bubbles, tubes at 18 cm.
Figure 4.4c. Bulge heights for single bubbles, 3 cm at 18 cm.
Figure 43d. Bulge eruption heights of single bubbles, tubes at 40 cm
Figure 43e. Bulge heights for single bubbles, tubes at 40 cm.
Figure 43e. Bulge heights for single bubbles, tubes at 40 cm.

Mean bulge height (cm)

Tubes (cm)
49-53
44-48
39-43
60-80

No Tubes
Figure 44a. Wake eruption heights for tubes at 18 cm and $L = 14-23$ cm.
Figure 44a. Wake eruption heights for tubes at 18 cm and $L = L'$ = 14-23 cm.
Figure 44b. Wake eruption heights for tubes at 18 cm, L = 24-43 cm
Figure 4.4b. Wake eruption heights for tubes at 18 cm, L = 24-43 cm
Mean wake height (cm)
Figure 44c. Wake eruption height for tubes at 18 cm. $U^* = 1.1-1.6$
Figure 44c. Wake eruption height for tubes at 18 cm. * 1.1 - 1.6

Mean wake height (cm)
Figure 44d. Wake eruption height for tubes of 18 cm, $U^* = 1.7-2.2$
Mean wake height (cm)

Figure 4.4d. Wake eruption height for tubes of 18 cm, \( U^* = 1.7-2.2 \).
Figure 44e. Wake eruption height for tubes at 18 cm, $U^* = 2.3-3.1$
Figure 44e: Wake eruption height for tubes at 18 cm, $U^* = 2.3-3.1$. 

Mean wake height (cm)
Figure 44f. Wake eruption heights for single bubbles, $L_t = 40$ cm
Figure 4: Wake generation heights for single bubbles. \( L = 70 \text{ cm} \)
Figure 44g. Wake eruption heights for single bubbles, $L_t = 40$ cm
Figure 4.4. Wake eruption heights for single bubbles. L = 40 cm
Figure 45. Ratio of bulge to wake eruption on heights vs. bed depth for single bubbles, $L_t = 18$ cm and 40 cm.
Ratio of mean bulge height to mean wake eruption height

Figure 4.5: Ratio of bulge to wake eruption on heights vs. bed depth for single bubbles, $L_t = 18 \text{ cm}$ and $40 \text{ cm}$. 

- **Tubes**: $U^* = 1.1-1.3$
- **No Tubes**: $2.9-3.1$
- **2.0-2.2**
- **1.7-1.9**
- **1.4-1.6**
Figure 46a. Mean wake eruption heights for double bubbles, $L_t = 18$ cm.
Mean wake height (cm)

Figure 6. Mean wake eruption heights for double bubbles, $L = 18$ cm.
Figure 46b. Mean wake eruption heights for double bubbles, $L_t = 40$ cm
Figure 46b. Mean wake eruption heights for double bubbles, $L = 40$ cm.

Mean wake height (cm).

Tubes

0 15 20 25

1.7-1.9
2.0-2.2
1.1-1.3 $u^*$
Figure 47a. Ratio of bulge height to wake height for double bubbles, $L_t = 18$ cm
Figure 4.74. Ratio of bulge height to wake height for double bubbles, $L = 18$ cm.
Figure 47b. Ratio of bulge height to wake height for double bubbles, $L_t = 40\, \text{cm}$
Mean bulge height + mean wake eruption height

Figure 47b. Ratio of bulge height to wake height for double bubbles. $L_c = 40$ cm.
REFERENCES


VITA

Rex A. Freeman was born on a farm near Avilla, Indiana on January 31, 1938, the oldest boy of the seven children of Ray and Ruth Freeman. He graduated as valedictorian of the 1956 class of Garrett High School, Garrett, Indiana. From there he went to Manchester College, North Manchester, Indiana, receiving a B.S. degree in education in 1960, having majored in chemistry. After teaching high school chemistry and physics for four years at Marion High School, Marion, Indiana, he went to Brown University, Providence, Rhode Island, on a National Science Foundation Academic Year Institute, receiving an M.A.T. degree in physics in 1966.

Rex moved to Key West, Florida, in 1965 to teach physics and mathematics at Florida Keys Junior College. In 1968, he moved to Bethlehem, Pennsylvania, to assume a position at Northampton County Area Community College, where he is now a professor of physics and mathematics.

In addition to his degree work at Manchester College, Brown University, and Lehigh University, he has also earned credits in physics and engineering classes at the University of Colorado, Pennsylvania State University and Bucknell University.

Rex resides at 2430 Winston Road, Bethlehem, Pennsylvania, with his wife Irene and four children, Angela, Julie, Randy and Kenneth.