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PREDICTION OF THE SHEAR CONE GEOMETRY

SURROUNDING HEADED ANCHOR STUDS

by

Mark S. Bennett

A Thesis

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Civil Engineering

Lehigh University

1979

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This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

May 4, 1979

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1. INTRODUCTION

With the increased use of headed anchor studs by the nuclear power industry in the construction of nuclear power plants, there has been a resurgance of interest in accurately describing their behavior under load. For the past twenty years state-of-the-art thinking has revolved around the assumption of a 45° failure cone for calculating the ultimate capacity of this type of anchor when loaded in pure tension.

Recent tests conducted at Lehigh University and sponsored by Ebasco Services, Inc. of New York along with some previously existing data compiled by the Nelson Stud Welding Company of Lorain, Ohio indicate that the assumption of a 45° failure cone may be in error. Furthermore, these results indicate that the geometry of the shear cone may be a function of one or more of the stud dimensions and/or the compressive strength of the concrete in which they are cast.

Therefore, after first reviewing the current design procedure, a comparison between the predicted ultimate capacity and the actual ultimate capacity of several different size headed anchor studs was made. After doing so a formula was developed based on the stud dimensions and the concrete strength that enables the designer to better determine the geometry of the failure cone. This allows the designer to not only better predict the ultimate capacity of this type of anchor but also, and more importantly, to estimate the effects of grouping and insufficient edge distances. Limitations and implications of the resulting formula will also be discussed. This paper concludes by presenting the need for further research before the phenomenon of conical failure in concrete can be completely understood.

2. CURRENT DESIGN PROCEDURE

Current design practice for determining the ultimate capacity of a headed anchor stud is based on two possible modes of failure. The first mode of failure, is failure of the anchor itself due to surpassing the ultimate tensile strength of the steel. This mode of failure is better understood and more desirable as it provides some warning of impending failure. In an article published in the AISC Engineering Journal[1] MacMackin, Slutter and Fisher suggest the following equation for determining the ultimate capacity of a headed anchor stud when considering this mode of failure:

$$P_u' = A_s f_s \qquad (1)$$

where:

P_u' = tensile capacity of the anchor in newtons

A_s = cross-sectional area of the anchor shank in square meters

f_s = tensile strength of the anchor steel in pascals

The second possible mode of failure, is failure of the concrete in an area surrounding the anchor stud, as shown in Fig. 1. This mode of failure is less understood and less desirable because little warning is provided prior to failure. The Prestressed Concrete Institute Manual for Structural Design of

Architectural Precast Concrete[2] suggests the following equation to determine the ultimate capacity of a headed anchor stud when considering this mode of failure. It is based on the assumption of a conical failure along a 45° failure plane.* See Fig. 2.

$$P_{u,cap} = \phi_c \, k \, 0.332 \sqrt{f'_c} \; A_o \qquad (2)$$

where:

$P_{u,cap}$ = pullout capacity as a function of concrete strength, (MN)

ϕ_c = 0.65

k = 1.00 for normal weight concrete
0.85 for sand light weight concrete
0.75 for all light weight concrete

f'_c = compressive strength of concrete (MPa)

A_o = lateral surface area of failure cone
$= \sqrt{2}\, \pi \, \ell_e (\ell_e + d_h)$, $(m)^2$

This equation assumes this is an isolated anchor and group effects or effects of insufficient edge distances can be ignored.

Incorporated in this equation are several factors presently contained in the American Concrete Institute Building Code.[3] The $\phi_c = 0.65$ is a capacity reduction factor for bending in non-reinforced concrete. The k value is a reduction factor applied to

*Since the formula is actually presented in the reference in English units a change of constants was necessitated by the change to SI units in this paper.

the allowable ultimate stress and is equal to the values given depending on the weight of the concrete in which the anchors are cast. The value of 0.332 $\sqrt{f'_c}$ is the allowable shearing stress for concrete in two-way slabs and footings where punching shear is involved. In an article for the American Concrete Institute[4], Peter Courtis draws an analogy between the problem of punching shear in slabs and footings and the problem of conical failure associated mechanical anchorage devices. Figure 3 illustrates the basis on which this analogy was drawn.

<u>3. TEST DATA</u>

The results of recent tests involving headed anchor studs conducted at Lehigh University and sponsored by Ebasco Services, Inc. of New York along with test data supplied by the Nelson Stud Welding Company of Lorain, Ohio are summarized in Table 1. Included in this data are the results of 6 tests involving ASTM-A325 bolts. Since these bolts are of the same basic geometry as a headed anchor stud it will be assumed the shape of the shear cone surrounding them is of the same basic shape as that of a shear cone surrounding a headed anchor stud of the same dimensions. In all of these tests it was the strength of the concrete that determined the ultimate capacity of the anchorage. In other words, Eq. 2 was the governing equation. Also shown in Table 1 are the predicted ultimate capacities based on the suggested PCI equation (Eq. 2). As can be seen for the anchors with the shorter embedment lengths the predicted ultimate capacity provides conservative values when compared to the actual ultimate capacity. On the other hand the predicted ultimate capacity for the studs with the longer embedment lengths is higher than what was actually obtained.

A graph showing the actual ultimate load versus the predicted ultimate load is shown in Fig. 4.

3.1 Discussion of Results

The fact that the predicted ultimate capacity is sometimes 50% in error indicates one of the underlying assumptions in Eq. 2 may be invalid. Since reliable values for ϕ_c and the allowable stress have been obtained and both remain constant, it must be assumed that the A_o term is providing the error in estimating the ultimate capacity. The implications of this are many. Not only does an invalid A_o term lead to erroneous predicted ultimate capacities but since factors for grouping and edge distances are also based on the assumption of a 45^o shear cone they too may be in error. Table 2 shows the predicted A_o for each stud along with the actual A_o. By knowing the allowable ultimate stress and capacity reduction factor, the actual A_o could be calculated. Also shown are the values of θ as back calculated from the actual A_o. Since none of the tests were run until the actual shear cone was removed this method of back calculating θ is necessary. From Table 2 it is easily seen that the assumption of a 45^o shear cone may provide an adequate average value of θ, but that better values of θ may be obtained by forming a relationship between one or more of the stud dimensions and/or the compressive strength of the concrete. Figure 5 illustrates the relationship between θ and A_o. As can be seen as little as a 5^o discrepancy from the assumed value of 45^o may provide as much as a 28% error in the calculation of A_o.

Therefore the rest of this paper will concern itself with developing a formula to more accurately describe the shape of the shear cone. Once this relationship is known, it will be a simple task to predict the ultimate capacity of a headed anchor stud and along with it suggested spacing and edge distances.

4. DEVELOPMENT OF FORMULA

This chapter deals with the derivation of an equation to better describe the geometry of the shear cone. The word derivation may be a little misleading as an equation will actually be formulated using the technique of linear regression. There are, of course, certain limitations to this method as opposed to a strict theoretical derivation. These limitations will be discussed at the end of the chapter. There are some benefits to be derived using this method and they will be discussed at appropriate points throughout the chapter.

The organization of this chapter will be to first discuss the underlying assumptions that are made. Next the technique of linear regression as it applied to this problem will be discussed. Third, an equation to better describe the geometry of the shear cone will be developed using the techniques of linear regression. Finally the chapter will conclude with a discussion of the resulting formula and its limitations.

4.1 Assumptions

Two major assumptions must be made in order to proceed. The first assumption is that the capacity reduction factor, ϕ_c, and the allowable stress, $0.332\sqrt{f_c'}$, are both valid. Both of these values were used in the back calculation of θ. Should either of these values be incorrect the calculated value of θ will also be incorrect. Since a great deal of research has gone into the calculation of these values, and as mentioned before, they are both presently incorporated in the ACI Building Code, they will for the purpose of this paper, be assumed correct.

The second assumption is that all of the anchors tested and presented in Table 1 were cast in an area of concrete where group effects and effects of edge distances could be ignored. This assumption may be assumed valid by noting that none of the failures involved splitting of the concrete block, in which the anchors were cast, in any manner that would suggest that insufficient edge distances had been provided. Furthermore, no anchors were spaced any closer than their corresponding edge distances, therefore it may be assumed that effects of grouping may also be ignored.

4.2 Linear Regression Techniques

The technique of linear regression has several advantages[5]. First, any relationship, simple or complex, between a dependent variable, in this case θ, and any number of independent variables can be established. Second, the nature of the independent variables is not constrained. Therefore, they may be in any form and any independent variable may be included in the analysis. Third, since regression analysis has its own indicies of measurement not dependent on units, units don't have to be dealt with directly. This is particularly useful in this case where values such as $\sqrt{f'_c}$ are often dealt with. The fourth advantage of regression analysis is that it is particularly applicable to computer applications. Calculations in this paper were performed using the Lehigh Amalgamated

Package for Statistics (LEAPS) computer program presently available to users of the Lehigh University CDC 6400 computer system.

4.3 Initial Calculations

The first step in developing an equation for θ is to determine which variables θ may be a function of. There are several possible stud dimensions and, of course, the compressive strength of the concrete that may be contributing factors. There were three stud dimensions thought to be most influential. First, the embedment length, ℓ_e. By visual inspection it was apparent that θ seemed to increase as the embedment length increased. The second factor thought possibly to influence θ was the head diameter, d_h. Although no relationship was apparent by inspection, it was decided to include the head diameter in the analysis as a possible factor. The third possible factor was the shank diameter, d_s. Like the head diameter, although no relationship seemed to exist, it was included in the analysis. The final factor thought possibly to influence θ was the compressive strength of the concrete, f'_c. Again no apparent relationship was visible by inspection, but it can easily be rationalized that θ may well be a function of the concrete strength. Therefore, there were a total of four independent values thought possibly to influence θ, the dependent variable.

The next step was to formulate a correlation matrix.[6] This matrix will put a quantitative value on the possible linear relationship that may exist between any two of the variables, dependent or independent. Using the 21 data points listed in Table 2 as the sample population and including θ, ℓ_e, d_h, d_s, f_c' as the variables the following matrix was calculated.

	θ	ℓ_e	d_h	d_s	f_c'
θ	1.000				
ℓ_e	0.828	1.000		Symmetric	
d_h	0.750	0.764	1.000		
d_s	0.747	0.855	0.890	1.000	
f_c'	-0.139	-0.574	-0.158	-0.267	1.000

A value of 1.0 indicates a perfect linear relationship between the corresponding two variables. A negative sign indicates that the linear relationship that exists between the corresponding two variables is inverse in nature. The actual values indicate the linear rate of change per unit of one variable as compared to another.

Two things are of note in this matrix. First, there is a strong correlation between θ and all three stud dimensions. The correlation between θ and the compressive strength of the concrete was noticeably less. The second noticeable characteristic is the strong correlation between the stud dimensions themselves. In other words ℓ_e, d_s and d_h are all strongly correlated with each other. This of course should be true by the nature of the stud design and this fact will be made use of later in the paper. Scatter plots showing all four independent variables versus θ, the dependent variable, are shown in Figs. 6 through 9.

A technique known as stepwise regression was next employed to determine the relative importance of each independent variable in contributing to increasing the value of the multiple correlation coefficient, R. This coefficient is the standard measure of accuracy of the resulting equation in predicting θ and ranges from 0.0 to 1.0. It is a measure of the linear relationship between the dependent variable and any number of independent variables. Also of importance is the value R^2. It measures the proportion of change in the dependent variable that can be predicted using the developed relationship between the independent variable and the dependent variable. In other words, if $R^2 = 0.70$ then 70% of the change in the dependent variable can be estimated by the developed relationship. The other 30% may come from a variety of other factors such as scatter in the test data or a variable not included in the analysis. There are no units associated with the multiple correlation coefficient.

The stepwise regression package available to the users LEAPS solves for the coefficients of an equation in the following form:

$$Y = b_o + b_1 X_1 + b_2 X_2 \ldots . b_n X_n \tag{3}$$

where:

Y = dependent variable

X_i = independent variable

b_o = Y-intercept

b_i = regression coefficient corresponding to independent variable.

The program solves for these coefficients in a stepwise fashion. That is, the program first decides which independent variable makes the largest contribution to R. After which it solves for the constants b_o and b_i. The program next proceeds to determine the second most influential variable. Then new constants b_o, b_i, b_{i+1} are determined using the two most influential variables. This procedure is continued until all of the independent variables have been exhausted or no contribution to R is being made. The sample data was analysed in this manner and the results are shown in Table 3.

As can be seen the addition of the terms containing the embedment length and the concrete strength contributed largely to the increase of the multiple correlation coefficient whereas the addition of the head and shank diameters had little effect on the correlation coefficient. A correlation coefficient of 0.925 after the first two independent variables were included in the regression indicates an extremely high correlation between the embedment length, the concrete strength, and θ. The low contribution to the correlation coefficient obtained by adding the d_s and d_h terms indicates they provide little new information to the equation. This may have been predicted earlier by noting the high correlation between all of the stud dimensions. The result is, that even though the concrete strength has a low correlation coefficient compared to the other independent variables, it does provide new information to the equation that the additional stud dimensions do not.

The next step is to determine if the calculated coefficients are agreeable with the presently accepted shear cone theory.

4.4 Comparison with Shear Cone Theory

The best equation as determined by the step-wise regression technique is as follows:

$$\theta = 14.93 + 161.9\ \ell_e + 0.5312\ f'_c - 900.1\ d_s + 220.2\ d_h \tag{4}$$

Even though the magnitude of the coefficients reflect conversion factors and undetermined constants the sign of each factor should coincide with the presently accepted shear cone failure theory. Thus each term will be analyzed to determine its agreement with the shear cone theory.

Referring to Fig. 10 it is seen that for a certain allowable ultimate stress in the concrete and a constant applied load, P, the area needed to resist that force must remain constant. If the embedment length is increased and the area is to remain constant then θ must increase. Therefore as the embedment length increases, θ must increase and the positive sign associated with ℓ_e in the developed equation is appropriate.

Similarly Fig. 11 shows, for the same assumption of a constant force, P, a constant allowable stress, f_c', and a constant area, A_o, that as the head diameter, d_h, is increased the area is also increased. If the area is to remain the same then θ must increase as d_h increases and the positive sign associated with the head diameter is agreeable with the shear cone theory.

If the allowable ultimate concrete stress is allowed to change then the area needed to resist a constant force, P, must change inversely. Therefore increasing the allowable concrete stress, f_c', will necessitate a decrease in the area, A_o, over which the stress acts. Of course, as the area decreases, θ increases, and the positive sign associated with the concrete strength, f_c', in the equation is agreeable with the shear cone theory.

Since the shank diameter is not directly related to the conical surface area or the concrete strength, another approach must be used to rationalize the relationship between d_s and θ. If the shank diameter is increased, the allowable load as determined by considering the strength of the stud, Eq. 1, is also increased. If the connection is to withstand this increase in load for a given allowable ultimate stress, the area over which this stress acts must increase. If the area is to increase then θ must decrease. Therefore as d_s increases θ must decrease and the negative sign associated with the d_s term in the developed equation is agreeable with the shear cone theory.

The only remaining term to be discussed is the constant. Although not presented in this report, there is test data available which indicates that θ may decrease to about 20°.[2] The constant of +14.930 allows this value of 20° to occur within certain limits of stud dimensions and concrete strength and is therefore a reasonable value.

It has now been shown that all of the terms in the developed equation are agreeable with the presently accepted shear cone theory.

4.5 Refinements

Presently the developed equation is as follows:

$$\theta = 14.93 + 161.9\, \ell_e + 0.5312\, f'_c - 900.1\, d_s + 220.2\, d_h \tag{4}$$

It is completely linear and includes all four independent variables. It may be possible by introducing non-linear terms to improve the accuracy of this equation. To determine where the use of non-linear terms would be most advantageous, error plots were made as a function of each independent variable (Figs. 12 - 15). Plotted on the vertical axis is the error in θ when predicted using Eq. 4. A unique plot is made for each independent variable. Should any of the error plots show a strong continuous curvilinear tendency then the possibility exists that θ may be better predicted by use of a non-linear function of that particular variable.

The plots involving the embedment length and the head diameter show no continuous curvilinear tendencies whereas those plots involving the shank diameter and the concrete strength show some signs of a curvilinear relationship as indicated by the dashed lines.

By making use of logarithms, the following equations can be linearized as follows:

$$\theta = C_1 f_c'^{\alpha} \qquad \theta = C_2 d_s^{\beta}$$

$$\log \theta = \log C_1 + \alpha \log f_c' \qquad \log \theta = \log C_2 + \beta \log d_s$$

The optimum values for α and β can now be calculated using the technique of multiple linear regression. They were calculated to be - 0.0567 and 0.6865 respectively.

Next, three stepwise regression analyses were made using the four independent variables. In the first analysis only $d_s^{0.6865}$ was substituted for d_s. In the second analysis only $f_c'^{\,0.0567}$ was substituted for f_c'. The final analysis were performed making both substitutions. The results are shown in Tables 4 through 6.

As can be seen the accuracy was not noticeably improved. It even diminished in some cases indicating the assumption of a curvilinear relationship was not valid. Therefore it was decided to proceed with the originally developed equation in its linear form.

The next possible step was to eliminate some of the more trivial terms in the equation and round off the remaining terms so that a more useful form could be obtained. Since little contribution to R was made by adding the terms containing the head and shank diameters these were eliminated from the equation. The decreased accuracy from R = 0.941 to R = 0.925 is actually a trivial one. The constants calculated after including only the embedment length and the concrete strength in the regression were then rounded off so that the following equation was obtained:

$$\theta = 13.0 + 118.1\ \ell_e + 0.435\ f_c' \qquad (5)$$

A plot of θ as estimated via this equation versus the actual θ is shown in Fig. 16. The value of R using this equation was calculated to be R = 0.925, showing no significant decrease due to round off error.

It was decided that since more values could be forced into the 10% error band a constant of 14.0 should be tried. Therefore, the following equation was tried:

$$\theta = 14.0 + 118.1\ \ell_e + 0.435\ f_c' \qquad (6)$$

A plot showing the result of that trial is shown in Fig. 17. As can be seen only one value falls outside the 10% error band. Changing the value of the constant does not change the value of the multiple correlation coefficient.

Since 95% of the values fall within the 10% band this was deemed an acceptable equation, considering under good laboratory testing conditions, the coefficient of variation in concrete testing may itself range up to 7.0%.[7]

4.6 Predicting Ultimate Load

After θ has been determined by Eq. 6 the ultimate capacity of the anchor stud can be predicted as follows. First, the conical surface area, A_o, can be calculated using the following formula:

$$A_o = \frac{\pi \ell_e}{\sin \theta} \left(\frac{\ell_e}{\tan \theta} + d_h\right) \tag{7}$$

After determining A_o, the predicted ultimate load can be calculated using the formula suggested by the PCI, (Eq. 2). This was done for the available data and the results are shown in Fig. 18. As can be seen the results are significantly more accurate than those that were obtained using a constant value of 45^o for θ. This can be shown by examining the following.

	Ultimate Predicted Load w/θ = 45°	Ultimate Predicted Load w/θ = Variable
Mean % Error in Prediction of Ultimate Load	± 21	± 9
Maximum % Error in Prediction of Ultimate Load	+ 56	+ 22
Standard Derivation of % Error in Prediction of Ultimate Load	16	6

As can be seen the variable θ formula not only reduces the mean % error and the maximim % error but also reduces the standard deviation of the % error. Therefore, more values of the % error are grouped closer to 0.0 and the probability of having one or two excessively large errors in the prediction of the ultimate load is thus reduced.

5. SIGNIFICANCE TEST

Statisticians refer to the possibility of making a Type I error. That is, the probability that even though a high correlation coefficient is obtained that in actuality there is no correlation between the independent and depepdent variables, and the high correlation factor occurred only by coincidence. To calculate this probability an F-Test is performed. The value of F is calculated via the following formula which relies on the correlation coefficient, the number of data points, and the number of independent variables.

$$F = \frac{R^2 (n - k - 1)}{(1 - R^2) k} \qquad (7)$$

where

R^2 = square of multiple correlation coefficient

n = number of data points

k = number of independent variables.

Therefore, in this case:

$$F = \frac{(0.855)(21 - 2 - 1)}{(1 - 0.855) 2} = 53.07$$

There are tables available[5] that give the minimum value for F for a desired probability. In this case a probability of 0.01 was desired. It was determined that to meet this criteria F must equal or exceed 6.01. Since the calculated F exceeds this value, the 0.01 criteria is met. Therefore there is less than 1/100 chance that a correlation coefficient of R = 0.925 could have been attained if no actual correlation is present.

6. LIMITATIONS

There are several precautions that must be noted. First, the number of data points is relatively small. This can lead to a sample population which may not have the exact same properties as the population in general. Second, there is a high degree of correlation between the stud dimensions. This is, of course, by design but may lead to invalid results as to the contribution of each stud dimension to θ when regression techniques are employed. Third, there is noticeable grouping in the data points at either end of the sample population. If there are not adequate data points in between the two groups the correlation coefficient can be artifically inflated. The final limitation is that since regression techniques were used, extension of this formula beyond the limits of those data points found in the sample population may lead to erroneous results.

7. OTHER RELEVANT RESEARCH

In a design guide[8] for concrete anchorages the Tennessee Valley Authority (TVA) suggests a variable θ equation. The suggested equation, however, is to be used for determining θ when expansion anchors are used. An expansion anchor is an anchor which is "expanded laterally against the sides of a drilled hole in hardened concrete". The equation suggested is as follows:*

$$\theta = 28 + 133.9\ \ell_e \qquad (8)$$

The equation developed in this paper can be rewritten in the same form as the TVA equation as follows:

$$\theta = 14 + 118.1\ \ell_e + 0.435\ f_c'$$

If an average concrete strength of $f_c' = 27.6$ MPa is assumed and substituted into the developed equation it then becomes:

$$\theta = 26 + 118.1\ \ell_e \qquad (9)$$

As can be seen this is very similar to the formula suggested by the TVA (Eq. 8) for determining θ when using expansion anchors.

*Again the constants have been changed from those presented in the reference reflecting the change from English to SI units.

8. CONCLUSIONS

The following conclusions can be made:

1. Test results indicate that there is a high correlation between the embedment length and the geometry of the shear cone.
2. The same results also indicate the concrete strength also effects the shear cone geometry.
3. The equation developed, although having certain limitations, does predict the geometry of the shear cone within a 10% error.
4. The signs associated with the coefficients in the developed equation agree with those expected from shear cone theory.
5. The developed equation does agree with research of a similar nature conducted independently of research presented in this paper.
6. More research must be done before the exact relationship between the head and shank diameters and the geometry of the shear cone can be established.

9. SUMMARY

Essentially this paper discusses three inter-related topics. First, the present design criteria for headed anchor studs was presented. Its origins and the assumptions it made were also noted. Second, recent relevant research was discussed. The deviation of this research from the accepted design practice was noted. Finally, an equation was developed using regression techniques that attempted to better predict the shear cone geometry.

By no means is this the final solution for describing the shear cone geometry for headed anchor studs. The effects of the weight of the concrete in which the anchors are cast on the shear cone geometry must also be explored. There is evidence that suggests that the weight of the concrete may greatly effect the shear cone geomety. Also the exact allowable ultimate concrete stress, assumed in this paper to be $0.332\sqrt{f'_c}$ must be determined and it may be this is also a function of θ. Of course, further supportative data must be acquired before the formula in this paper can be accepted or refuted.

TABLE 1 SUMMARY OF TEST RESULTS

Type	ℓ_e (m)	d_h (m)	f'_c (MPa)	P_u^{cap} Predicted (kN)	P_u^{cap} Actual (kN)
Headed	0.0889	0.0238	21.20	44.3	100.0
Anchor	0.0889	0.0278	21.20	45.5	94.5
Studs	0.0889	0.0318	21.20	47.5	94.5
	0.0921	0.0138	35.71	65.8	82.3
	0.0921	0.0138	35.71	65.8	82.3
	0.0921	0.0318	35.71	65.8	82.3
	0.0921	0.0318	35.71	65.8	77.0
	0.0921	0.0318	34.68	64.8	81.0
	0.0921	0.0318	34.68	64.8	80.1
	0.0921	0.0318	34.68	64.8	71.2
	0.0921	0.0318	34.68	64.8	82.7
	0.1905	0.0318	16.89	167.2	149.9
	0.1905	0.0318	16.89	167.2	149.0
	0.1937	0.0349	33.78	246.9	191.3
ASTM	0.2032	0.0356	25.17	233.4	226.0
A-325	0.2032	0.0356	25.17	233.4	231.3
Bolts	0.2032	0.0356	25.17	233.4	195.7
	0.2032	0.0413	25.17	238.9	204.6
	0.2032	0.0413	25.17	238.9	222.4
	0.2032	0.0413	25.17	238.9	222.4
Headed	0.2223	0.0381	16.89	228.1	206.0
Anchor	0.2223	0.0381	16.89	228.1	169.0
Studs					

TABLE 2 SUMMARY OF CALCULATED Θ's

Type	ℓ_e (m)	d_h (m)	A_o Predicted (m^2)	A_o Actual (m^2)	Θ (degrees)
Headed	0.0889	0.0238	0.0445	0.1006	30
Anchor	0.0889	0.0278	0.0458	0.0948	31
Stud	0.0889	0.0318	0.0477	0.0948	31
	0.0921	0.0318	0.0510	0.0639	40
	0.0921	0.0318	0.0510	0.0639	40
	0.0921	0.0318	0.0510	0.0594	42
	0.0921	0.0318	0.0510	0.0639	40
	0.0921	0.0318	0.0510	0.0632	40
	0.0921	0.0318	0.0510	0.0561	43
	0.0921	0.0318	0.0510	0.0652	40
	0.1905	0.0318	0.1884	0.1677	48
	0.1905	0.0318	0.1884	0.1690	47
	0.1937	0.0349	0.1968	0.1523	51
ASTM	0.2032	0.0357	0.2155	0.2084	46
A-325	0.2032	0.0357	0.2155	0.2135	45
Bolt	0.2032	0.0357	0.2155	0.1806	49
	0.2032	0.0413	0.2206	0.1890	48
	0.2032	0.0413	0.2206	0.2052	47
	0.2032	0.0413	0.2206	0.2052	47
Headed	0.2223	0.0381	0.2568	0.2323	47
Anchor Stud	0.2223	0.0381	0.2568	0.1903	52

TABLE 3 RESULTS OF 1ST STEPWISE REGRESSION

	Variable	b_i	R	R^2
Step #1	ℓ_e	90.90	0.828	0.686
	constant	29.41		
Step #2	ℓ_e	118.1	0.925	0.855
	f'_c	0.435		
	constant	12.79		
Step #3	ℓ_e	165.85	0.939	0.882
	f'_c	0.554		
	d_s	-692.82		
	constant	17.05		
Step #4	ℓ_e	161.86	0.942	0.887
	f'_c	0.531		
	d_s	-900.12		
	d_h	220.21		
	constant	14.93		

TABLE 4

RESULTS OF 1ST STEPWISE REGRESSION WITH NONLINEAR TERMS

	Variable	b_i	R	R^2
Step #1	ℓ_e	90.90	0.828	0.686
	constant	49.41		
Step #2	ℓ_e	118.1	0.925	0.855
	f'_c	0.435		
	constant	12.79		
Step #3	ℓ_e	165.68	0.939	0.882
	f'_c	0.554		
	$d_s^{.6865}$	-293.55		
	constant	23.18		
Step #4	ℓ_e	162.08	0.941	0.885
	f'_c	0.534		
	$d_s^{.6865}$	-368.76		
	d_h	190.40		
	constant	22.91		

TABLE 5

RESULTS OF 2ND STEPWISE REGRESSION WITH NONLINEAR TERMS

	Variable	b_i	R	R^2
Step #1	ℓ_e	90.90	0.828	0.686
	constant	29.41		
Step #2	ℓ_e	117.69	0.906	0.820
	$f_c'^{-0.0567}$	-211.55		
	constant	201.36		
Step #3	ℓ_e	162.66	0.921	0.848
	$f_c'^{-0.0567}$	-278.49		
	d_s	-716.91		
	constant	264.48		
Step #4	ℓ_e	157.77	0.925	0.855
	$f_c'^{-0.0567}$	-263.06		
	d_s	-967.63		
	d_h	270.83		
	constant	248.19		

TABLE 6

RESULTS OF 3RD STEPWISE REGRESSION WITH NONLINEAR TERMS

	Variable	b_i	R	R^2
Step #1	ℓ_e	90.90	0.828	0.686
	constant	29.41		
Step #2	ℓ_e	117.69	0.906	0.820
	$f_c'^{-0.0567}$	-211.55		
	constant	201.36		
Step #3	ℓ_e	161.88	0.920	0.846
	$f_c'^{-0.0567}$	-276.85		
	$d_s^{0.6865}$	-299.32		
	constant	269.25		
Step #4	ℓ_e	157.28	0.923	0.851
	$f_c'^{-0.0567}$	-262.93		
	$d_s^{0.6865}$	-390.77		
	d_h	236.12		
	constant	256.54		

Fig. 1 Typical Shear Cone Failure

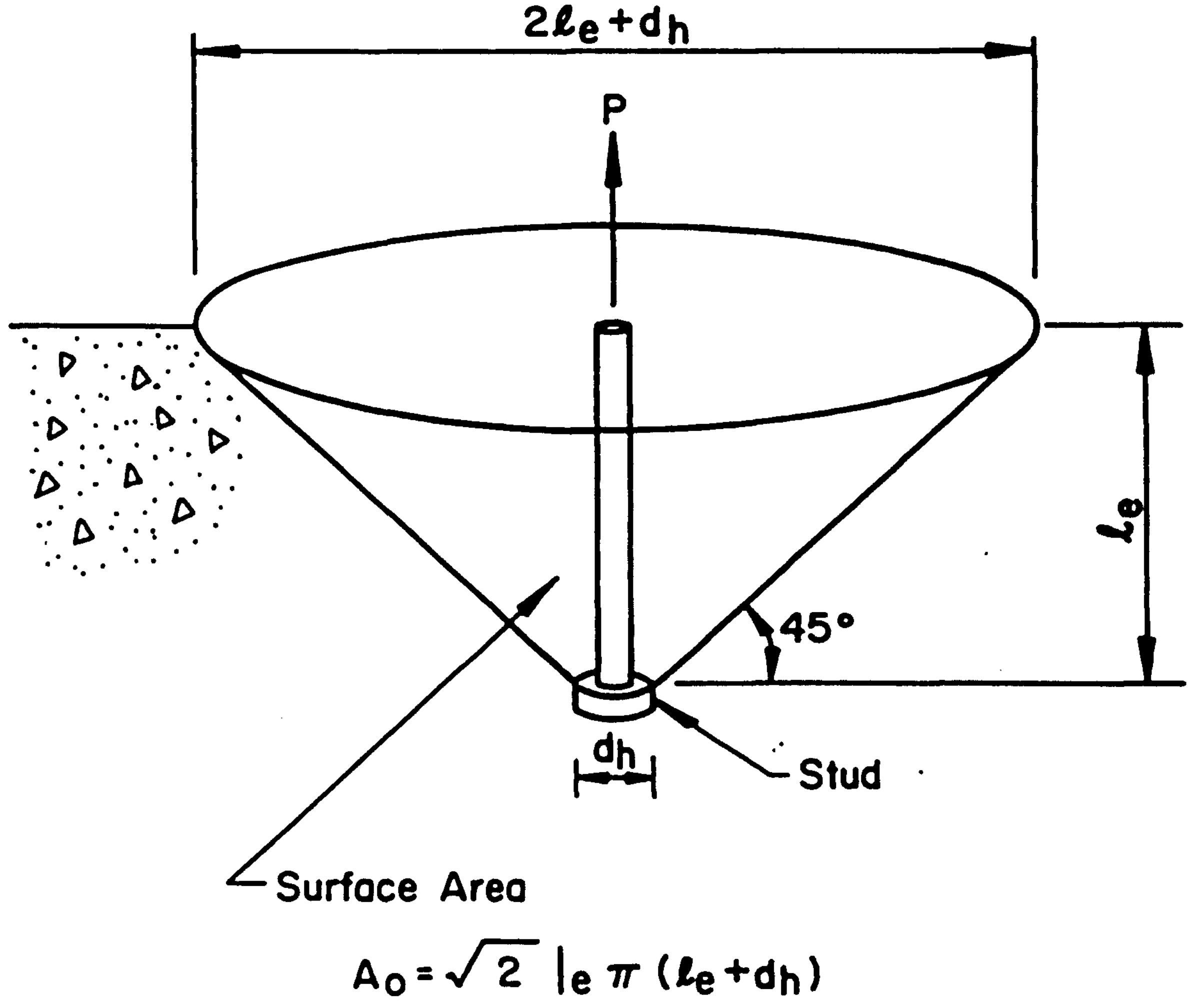

Fig. 2 Theoretical Shear Cone

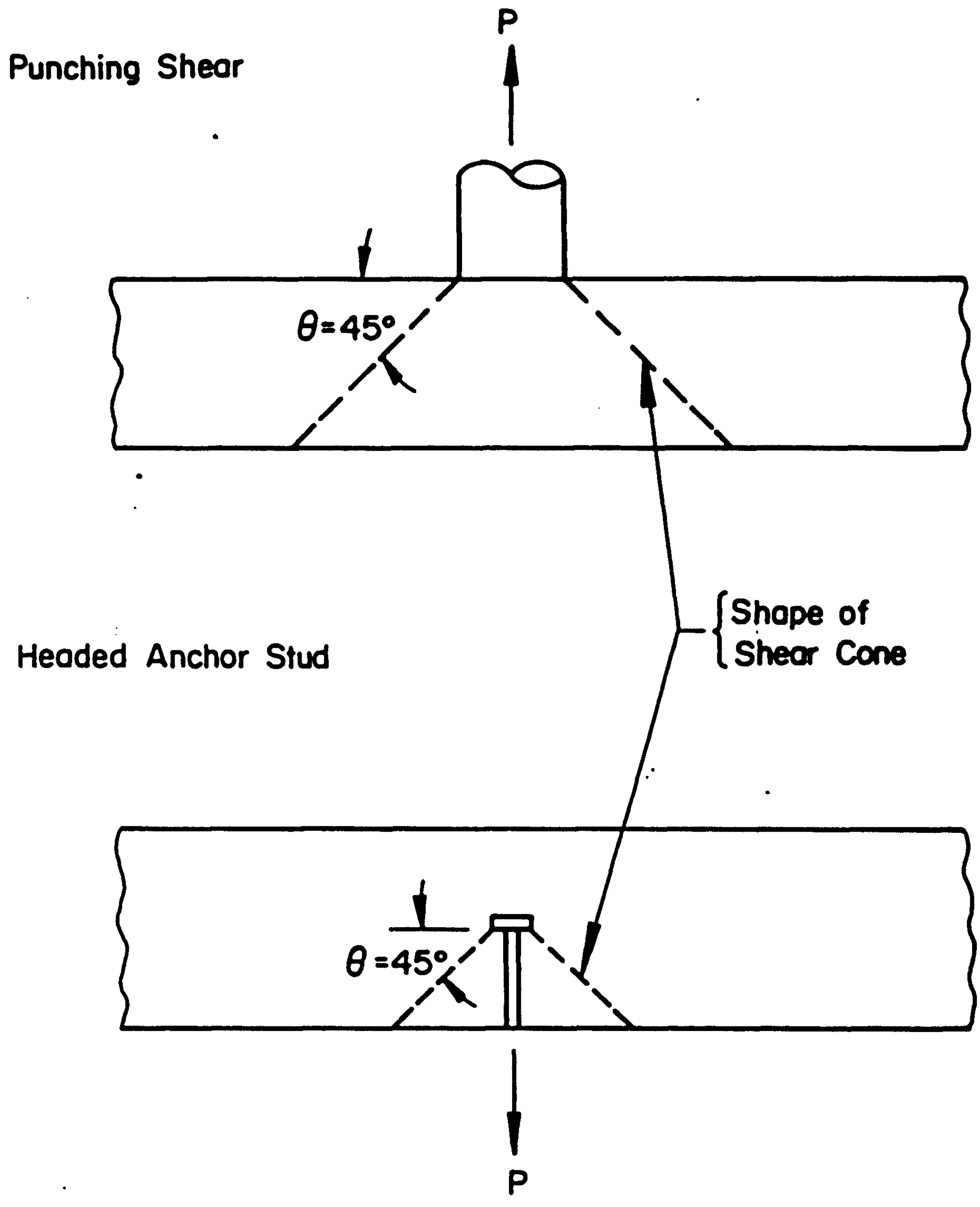

Fig. 3 Punching Shear Analogy

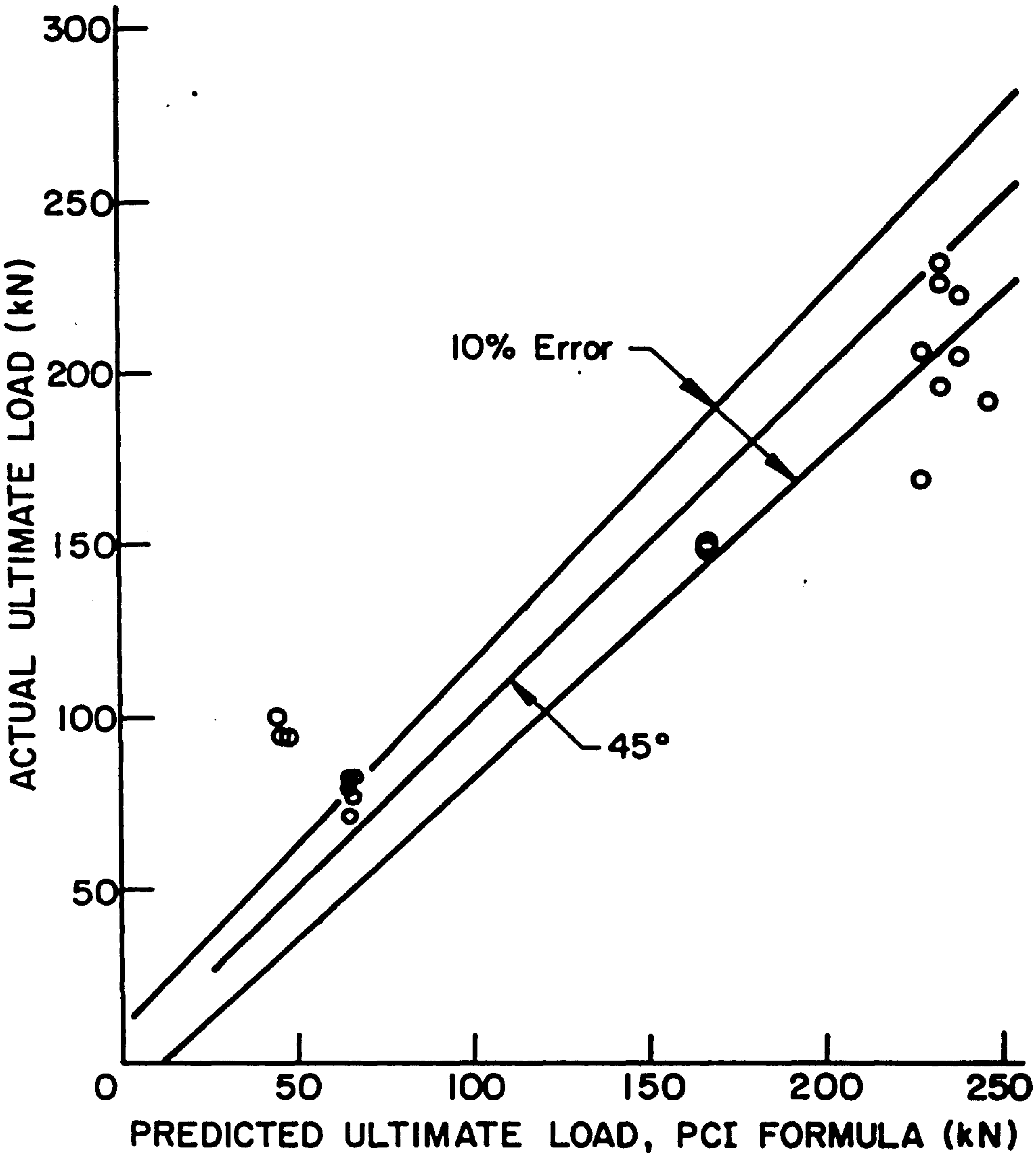

Fig. 4 Graph of Actual $P_{u_{cap}}$ versus Predicted $P_{u_{cap}}$

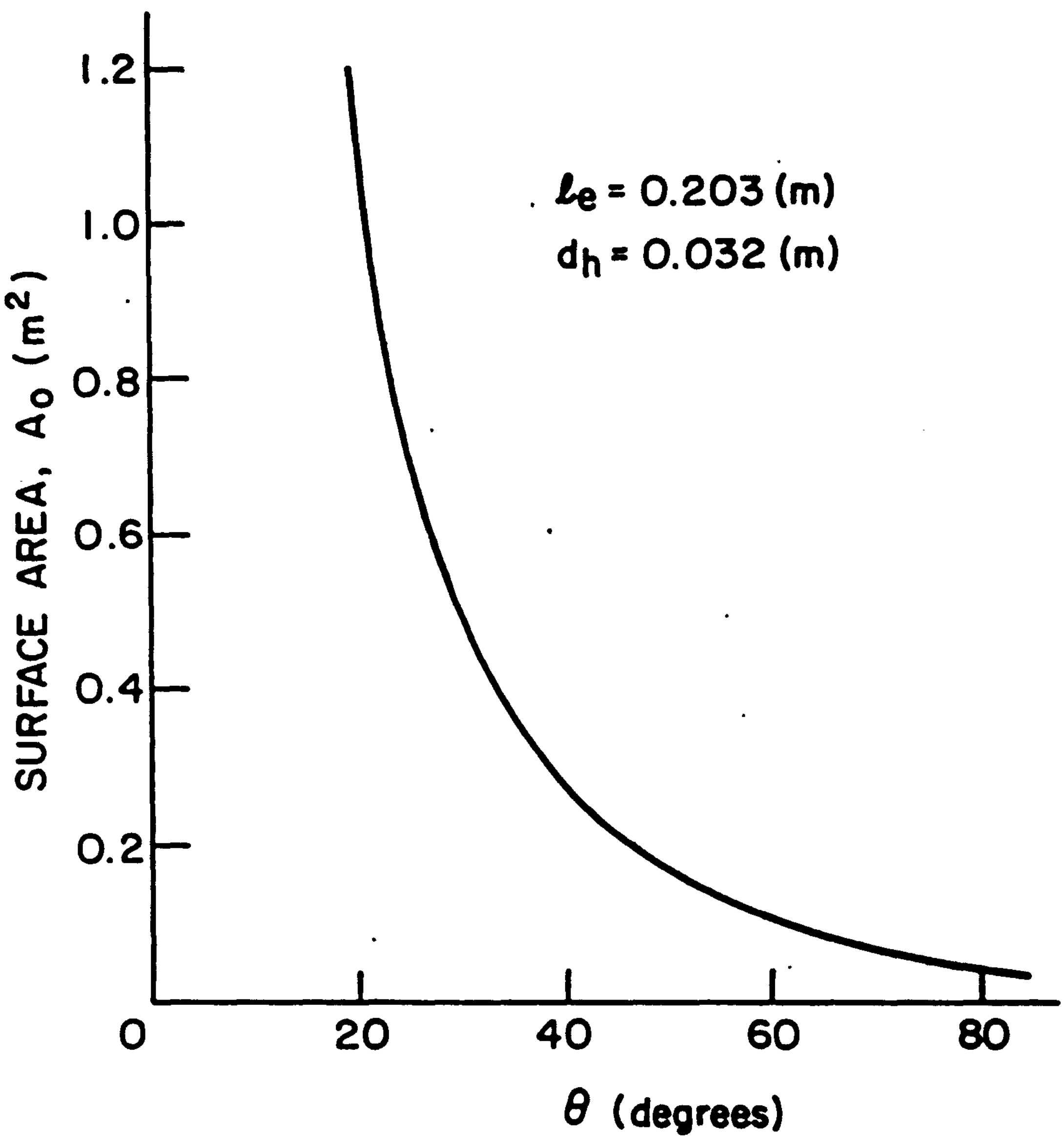

Fig. 5 Graph of A_o versus θ

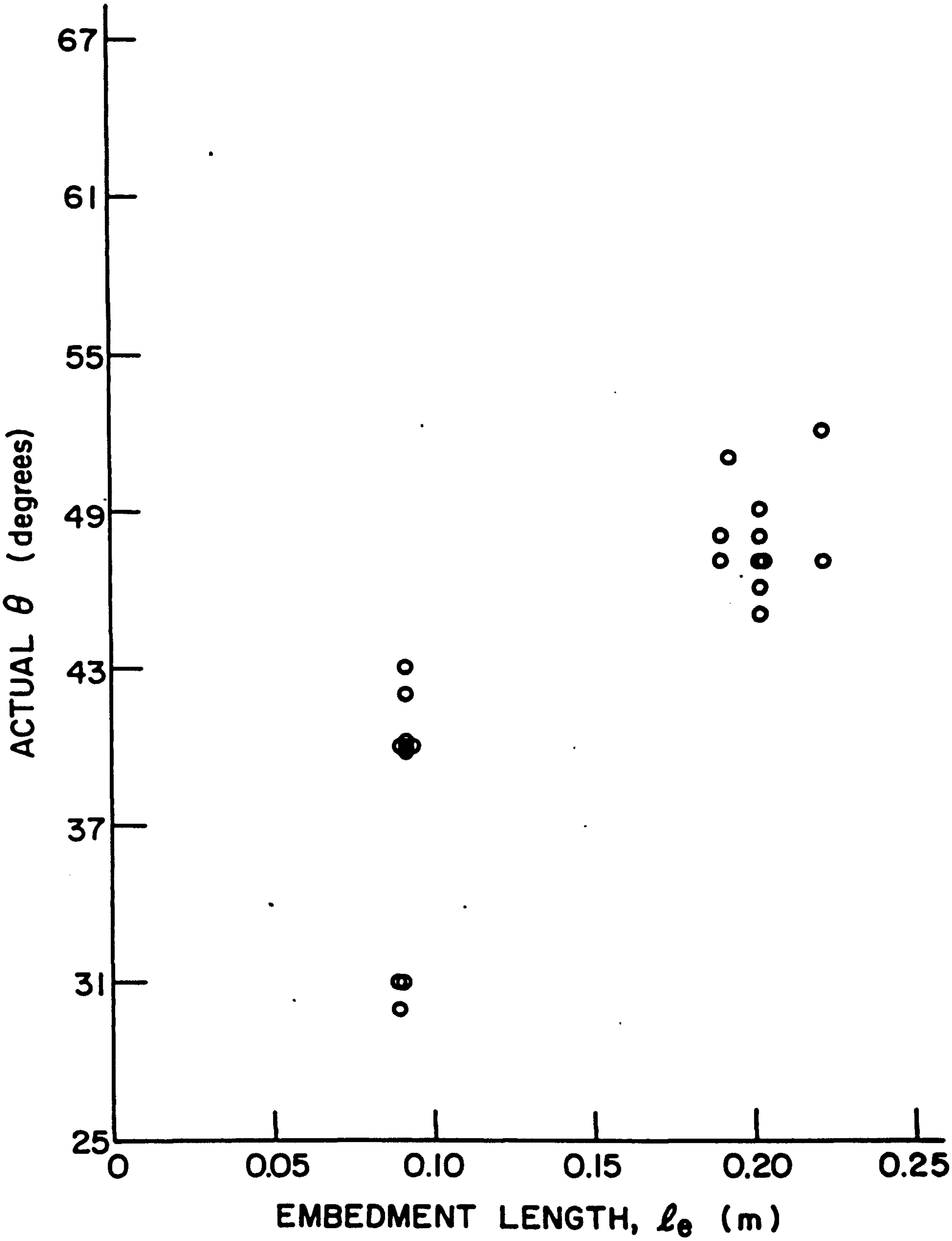

Fig. 6 Graph of θ versus ℓ_e

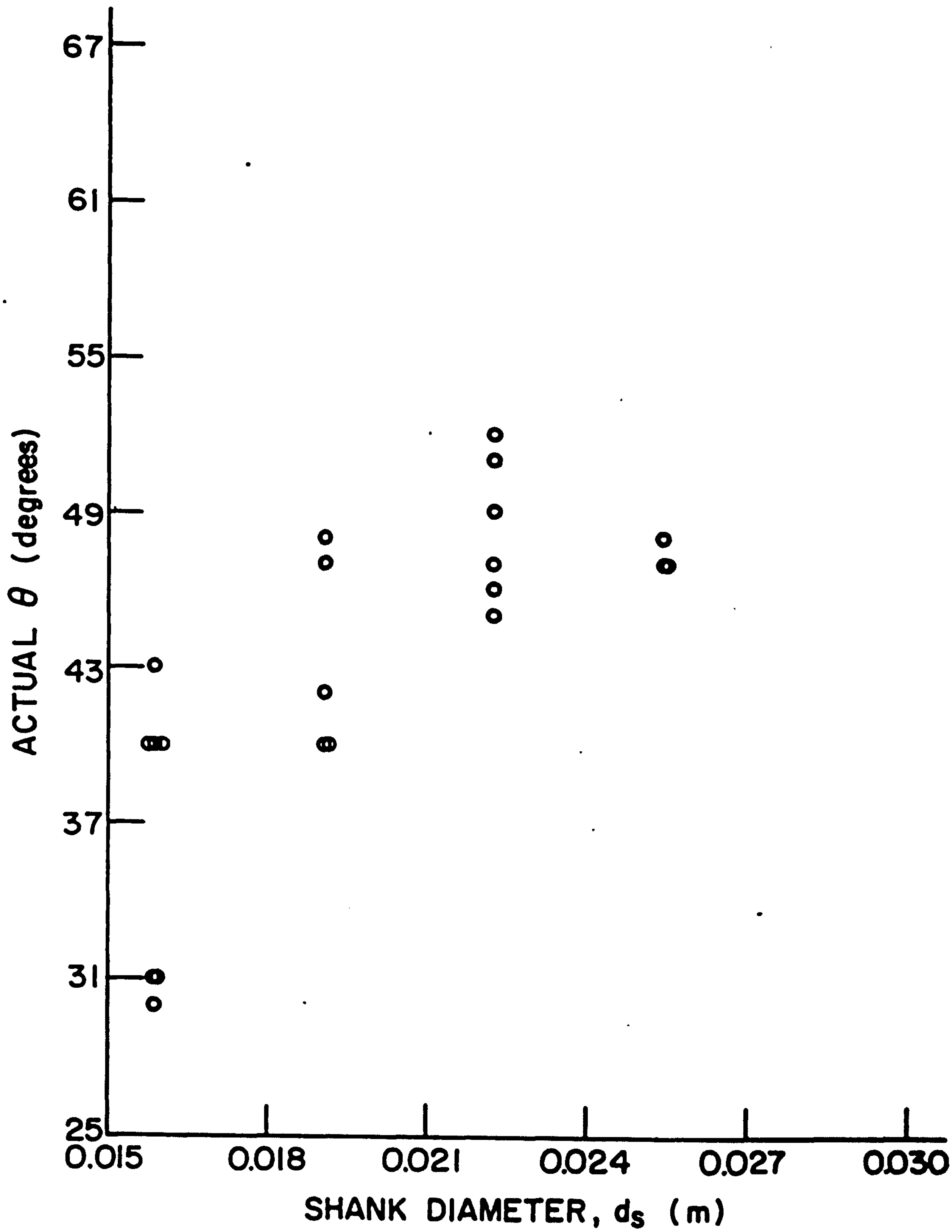

Fig. 7 Graph of θ versus d_s

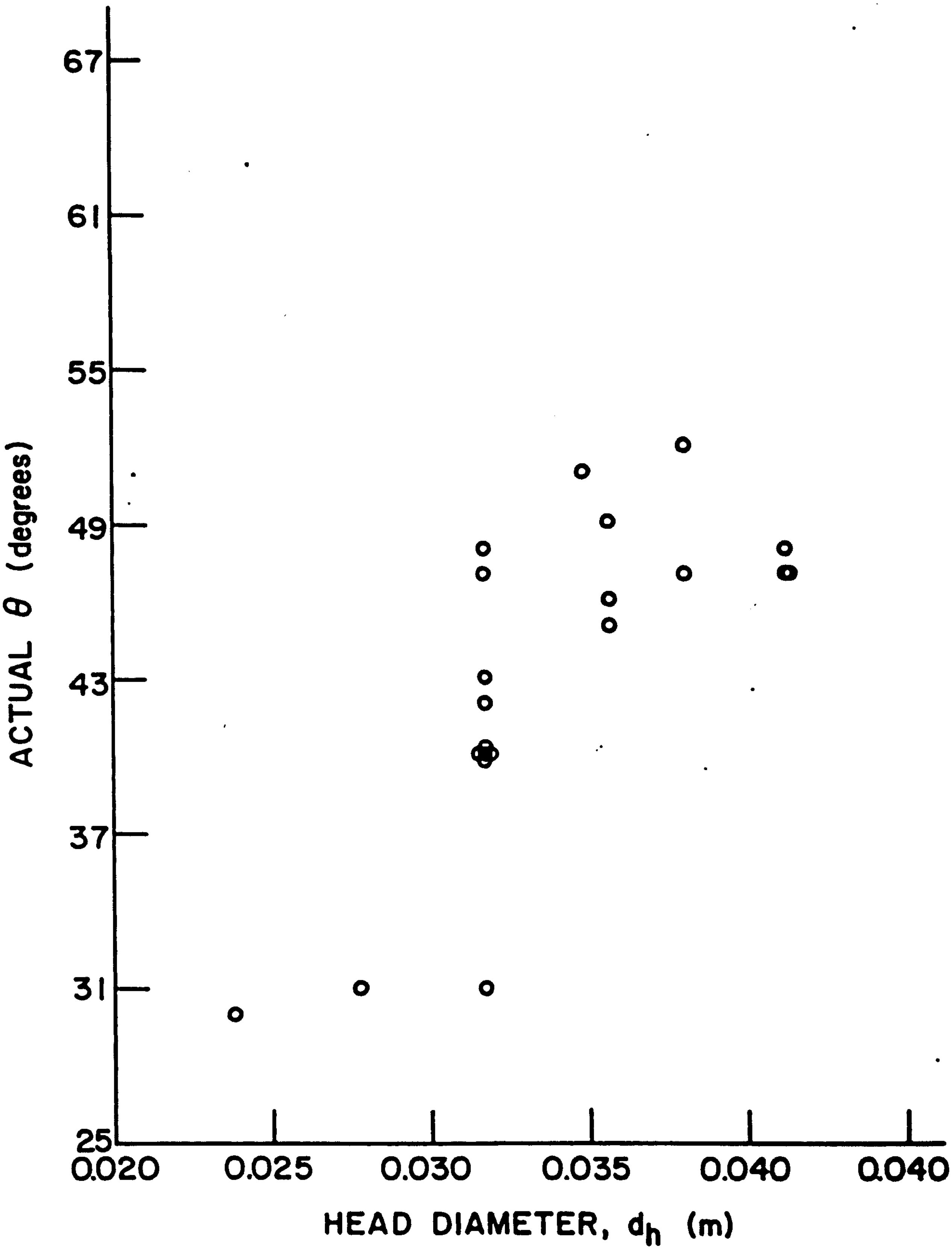

Fig. 8 Graph of θ versus d_h

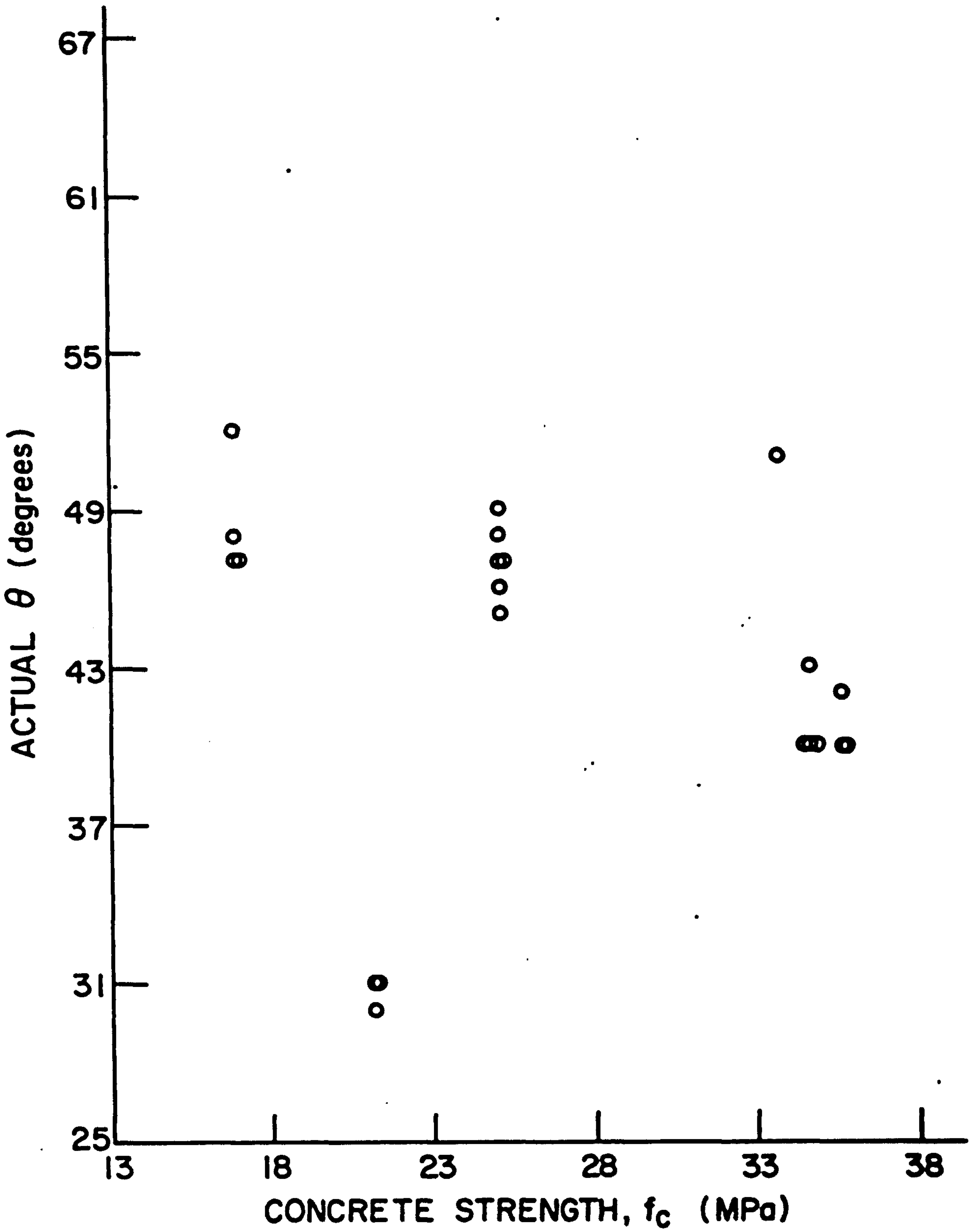

Fig. 9 Graph of θ versus f_c'

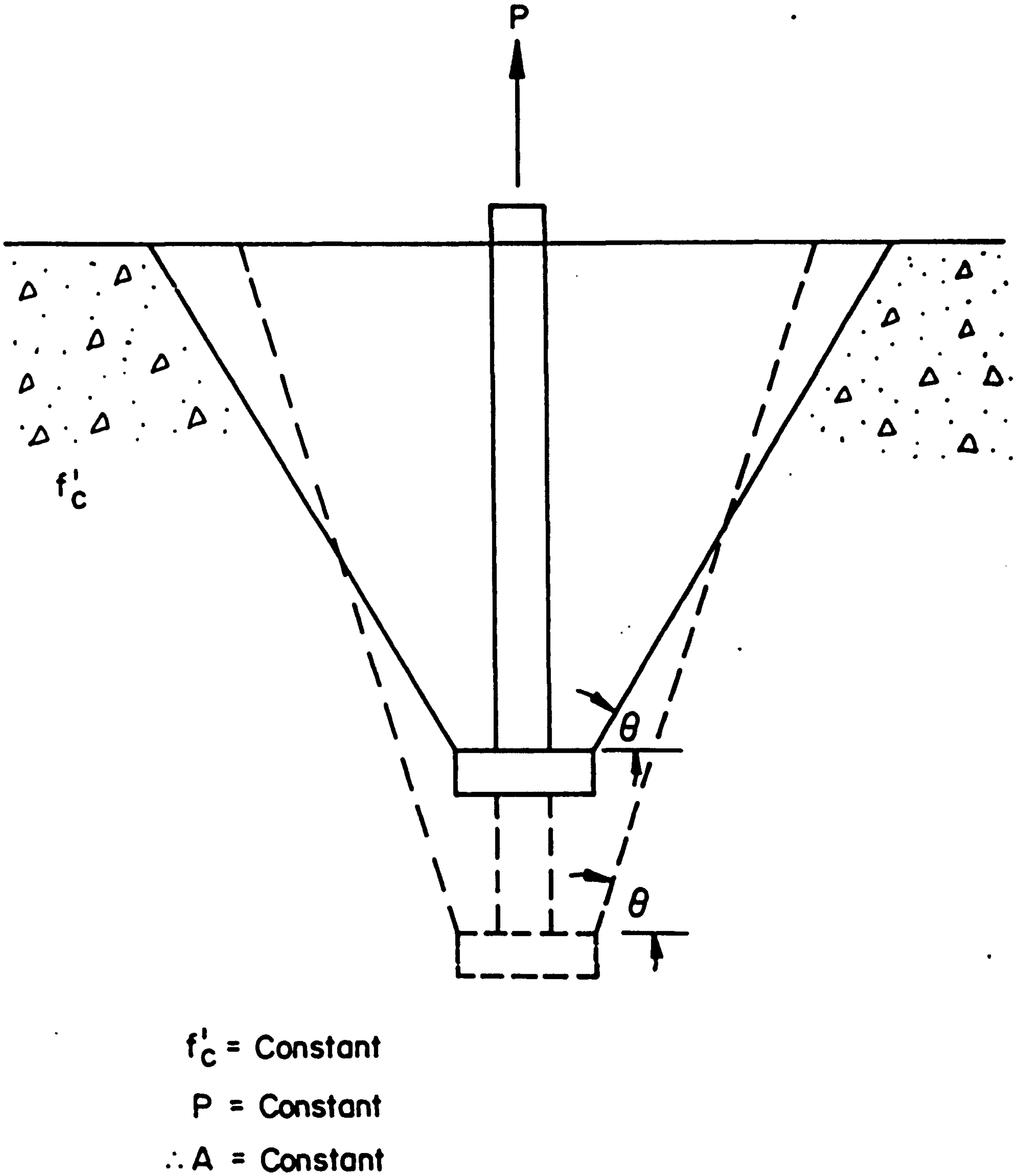

Fig. 10 Shear Cone Theory Relationship between ℓ_e and θ

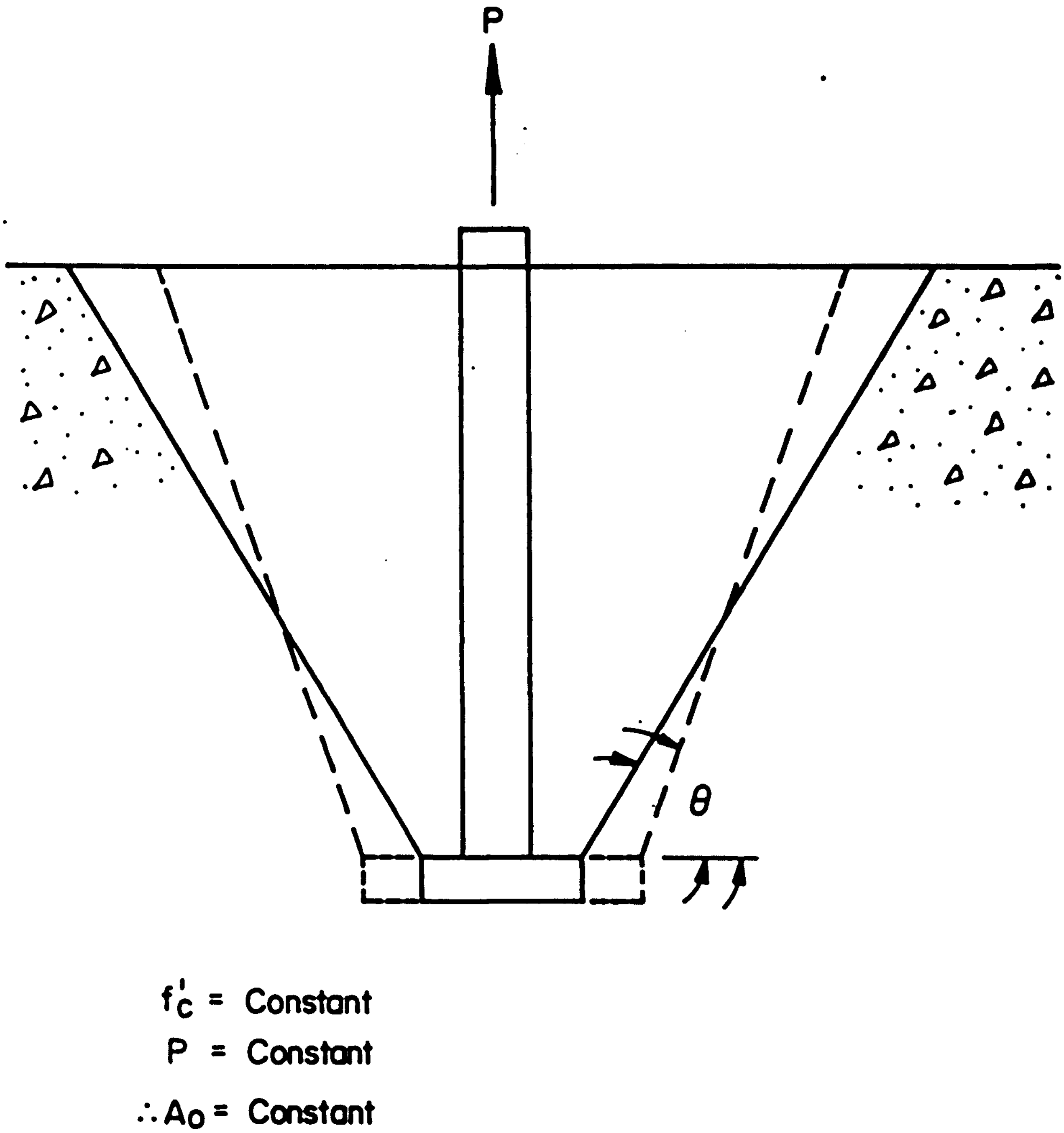

Fig. 11 Shear Cone Theory Relationship between d_h and θ

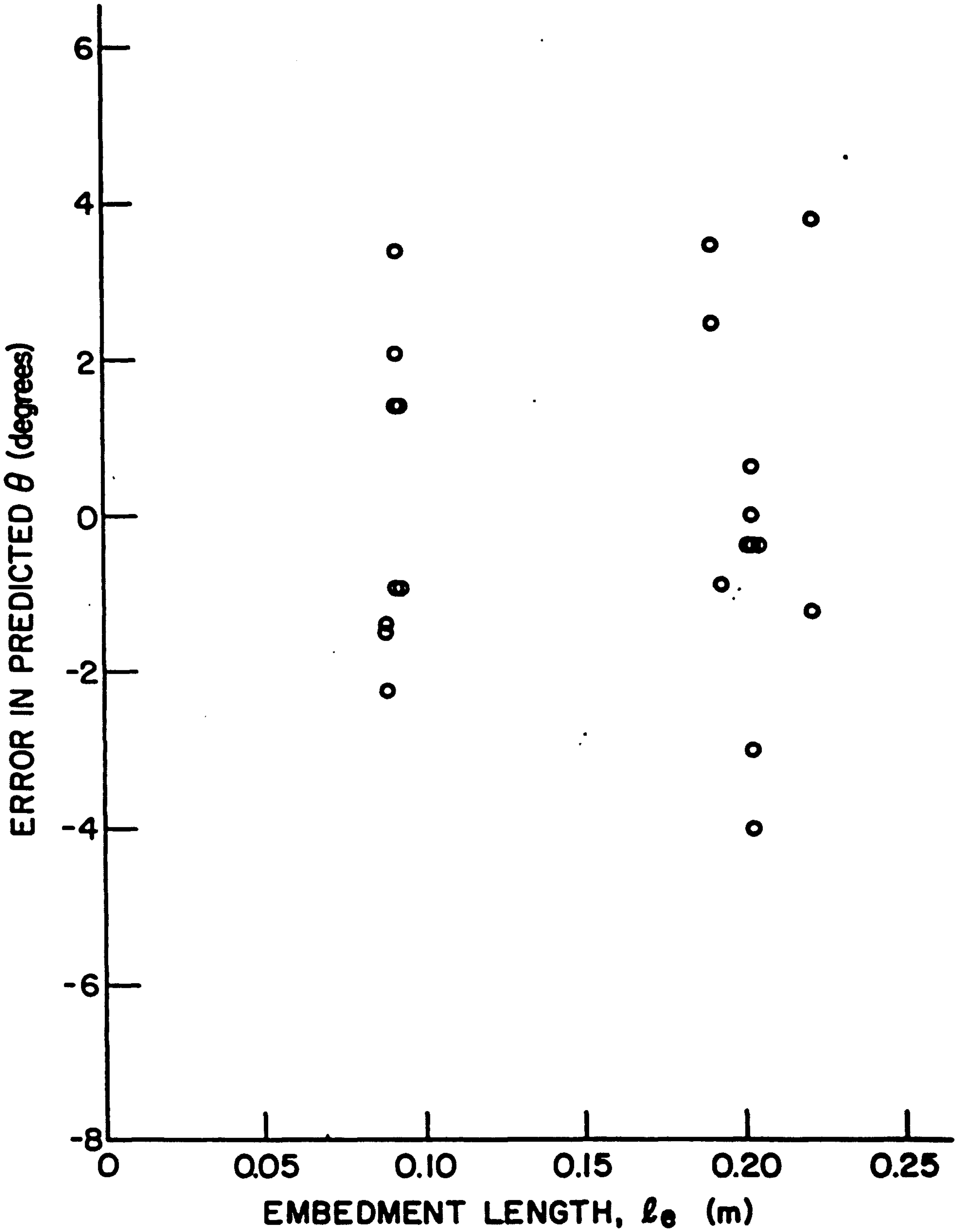

Fig. 12 Error in Predicted θ versus ℓ_e

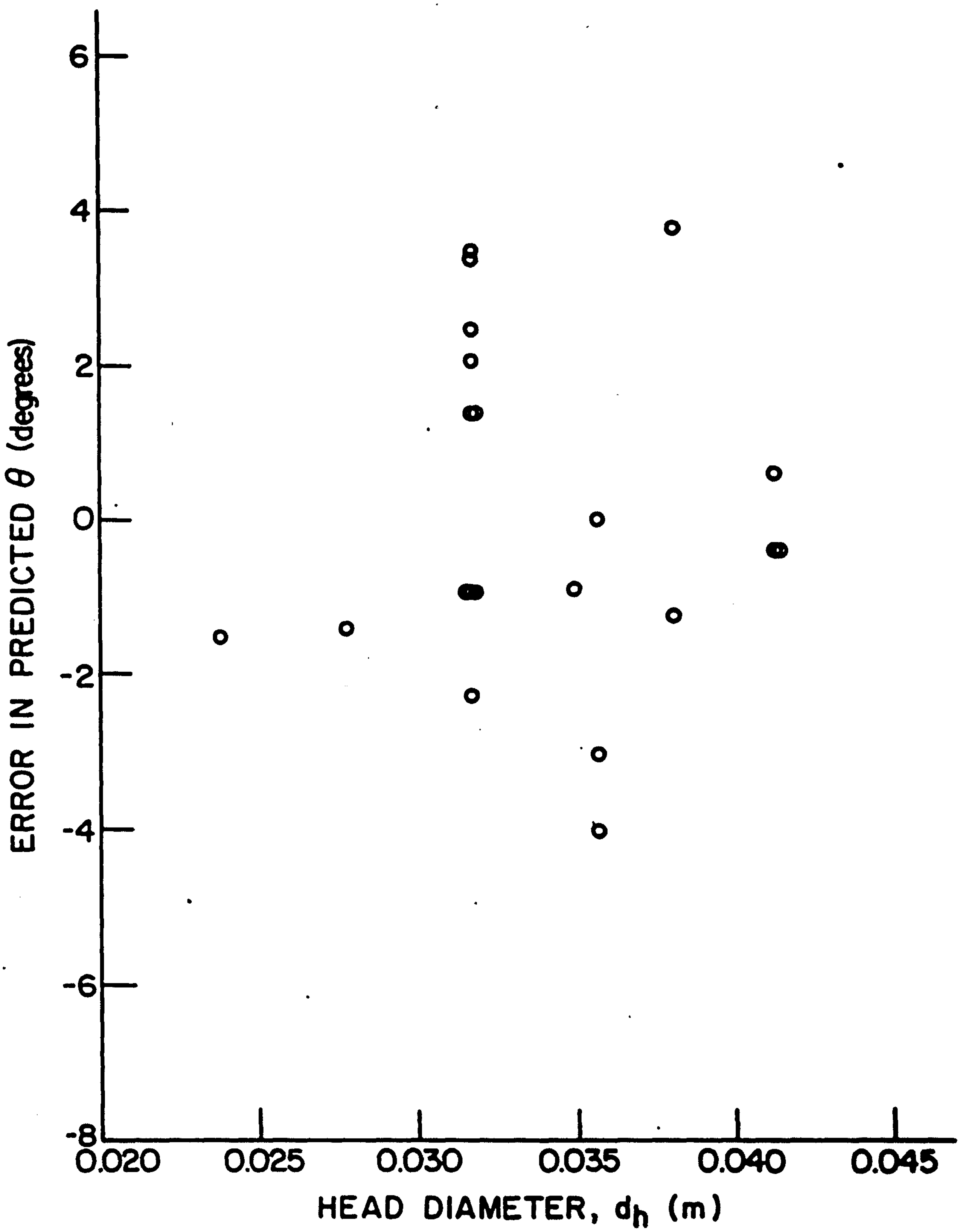

Fig. 13 Error in Predicted θ versus d_h

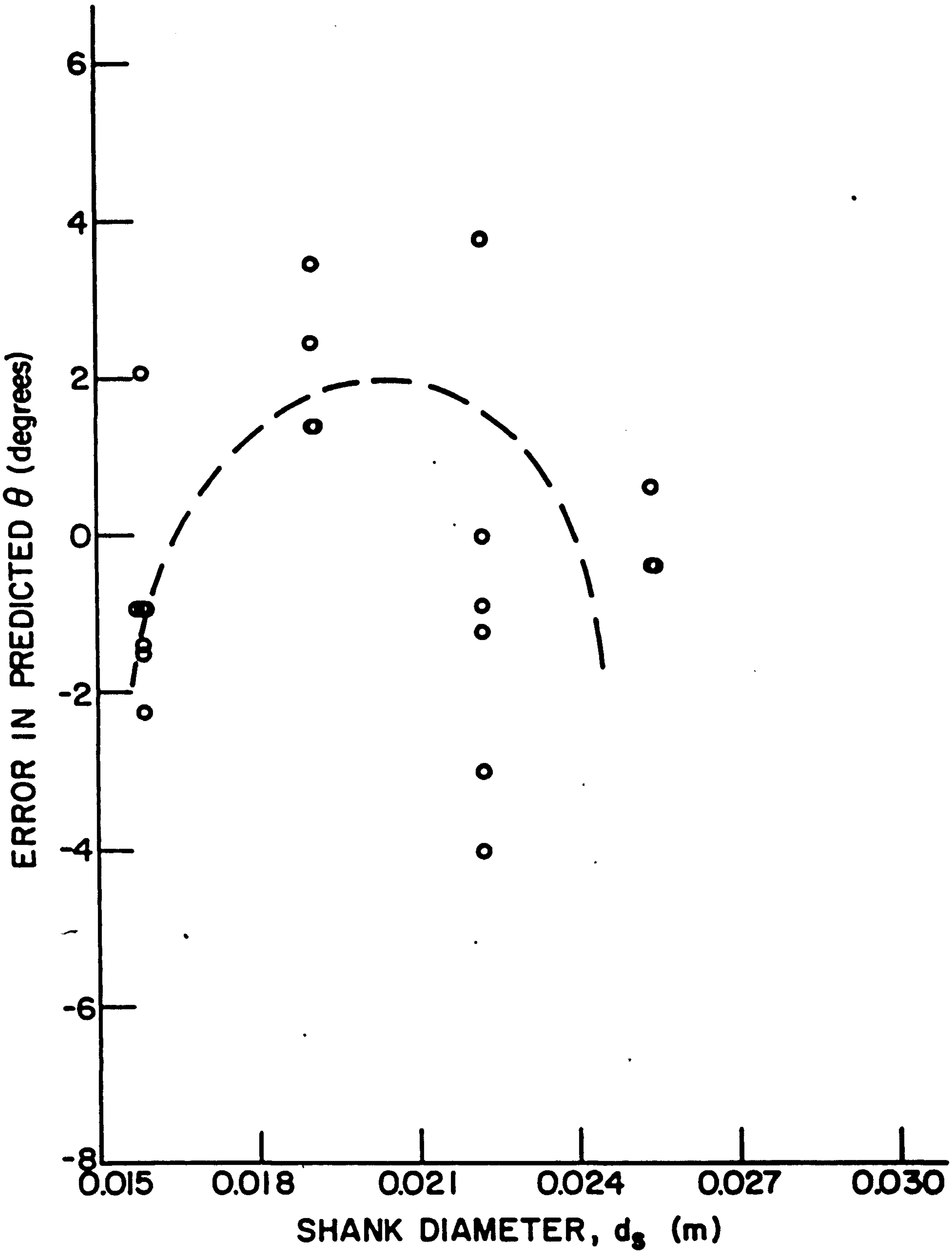

Fig. 14 Error in Predicted θ versus d_s

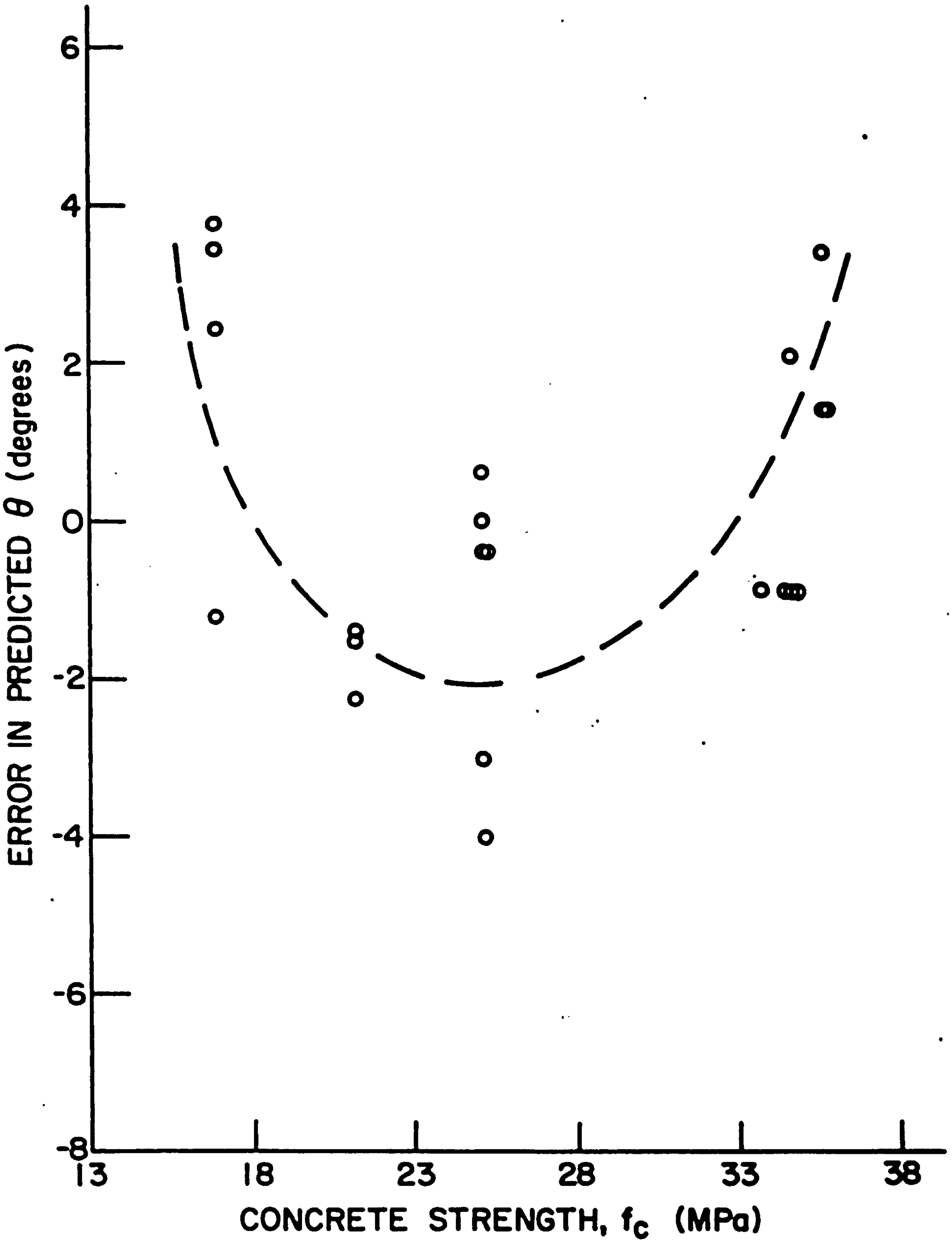

Fig. 15 Error in Predicted θ versus f_c'

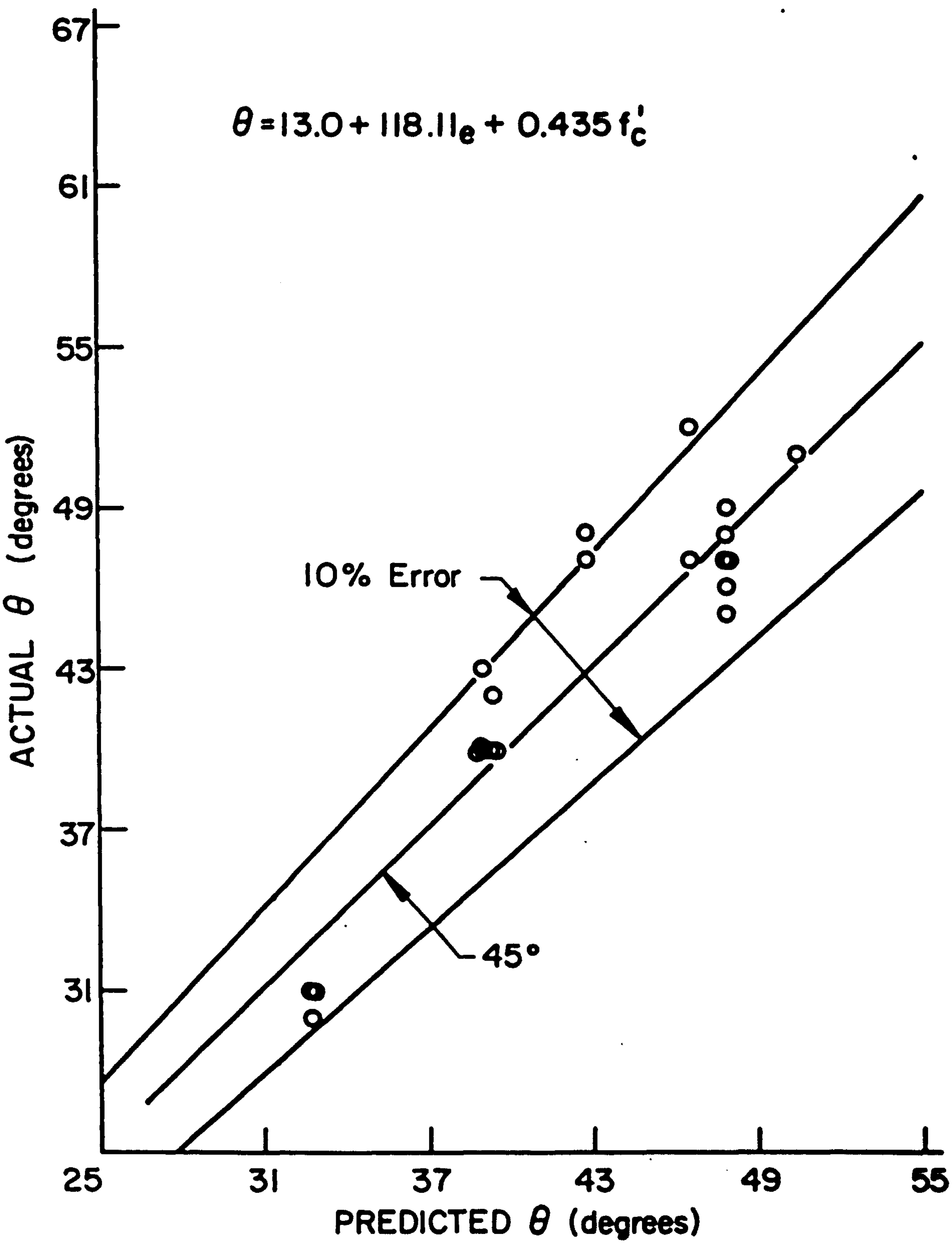

Fig. 16 Predicted θ (Eq. 5) versus Actual θ

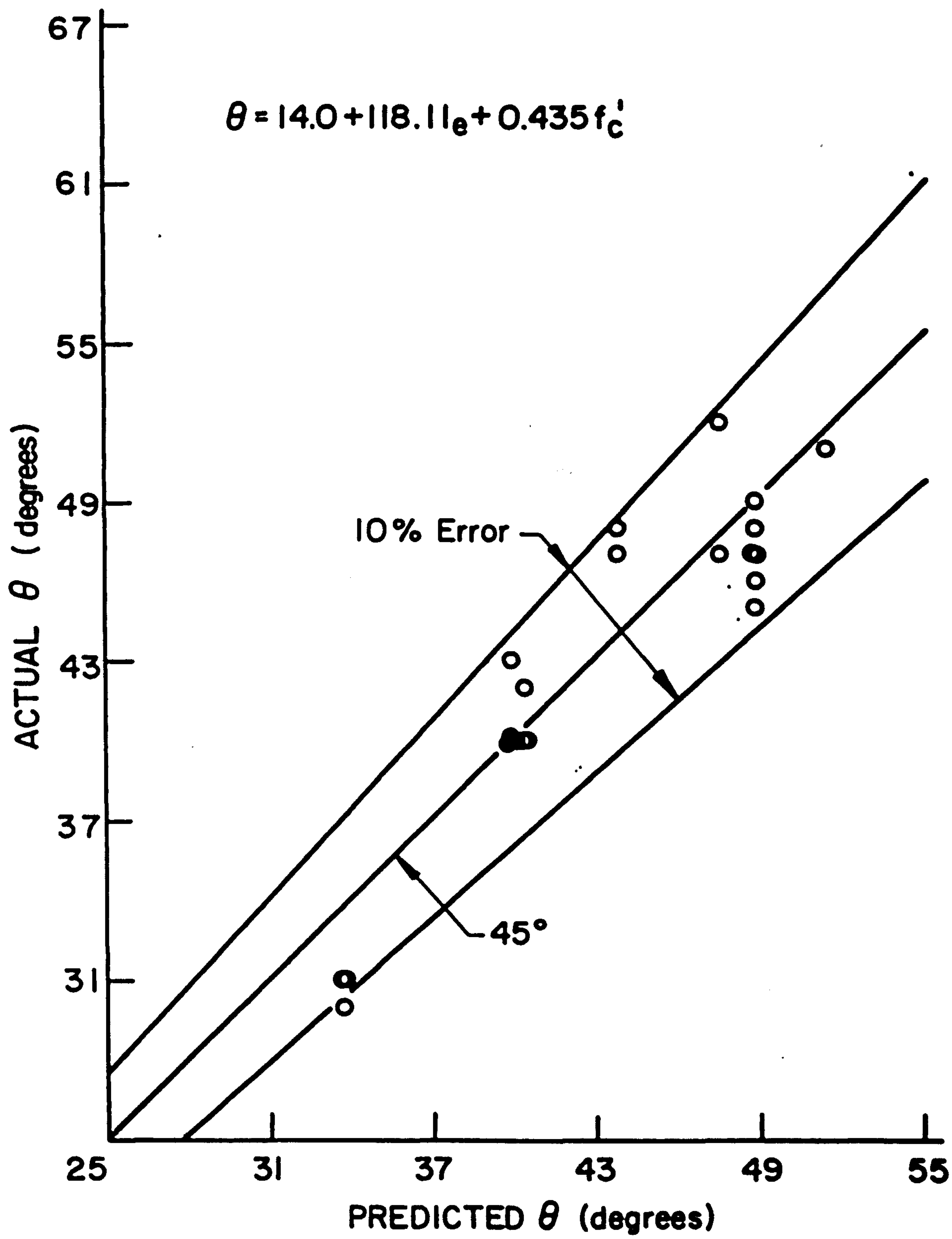

Fig. 17 Predicted θ (Eq. 6) versus Actual θ

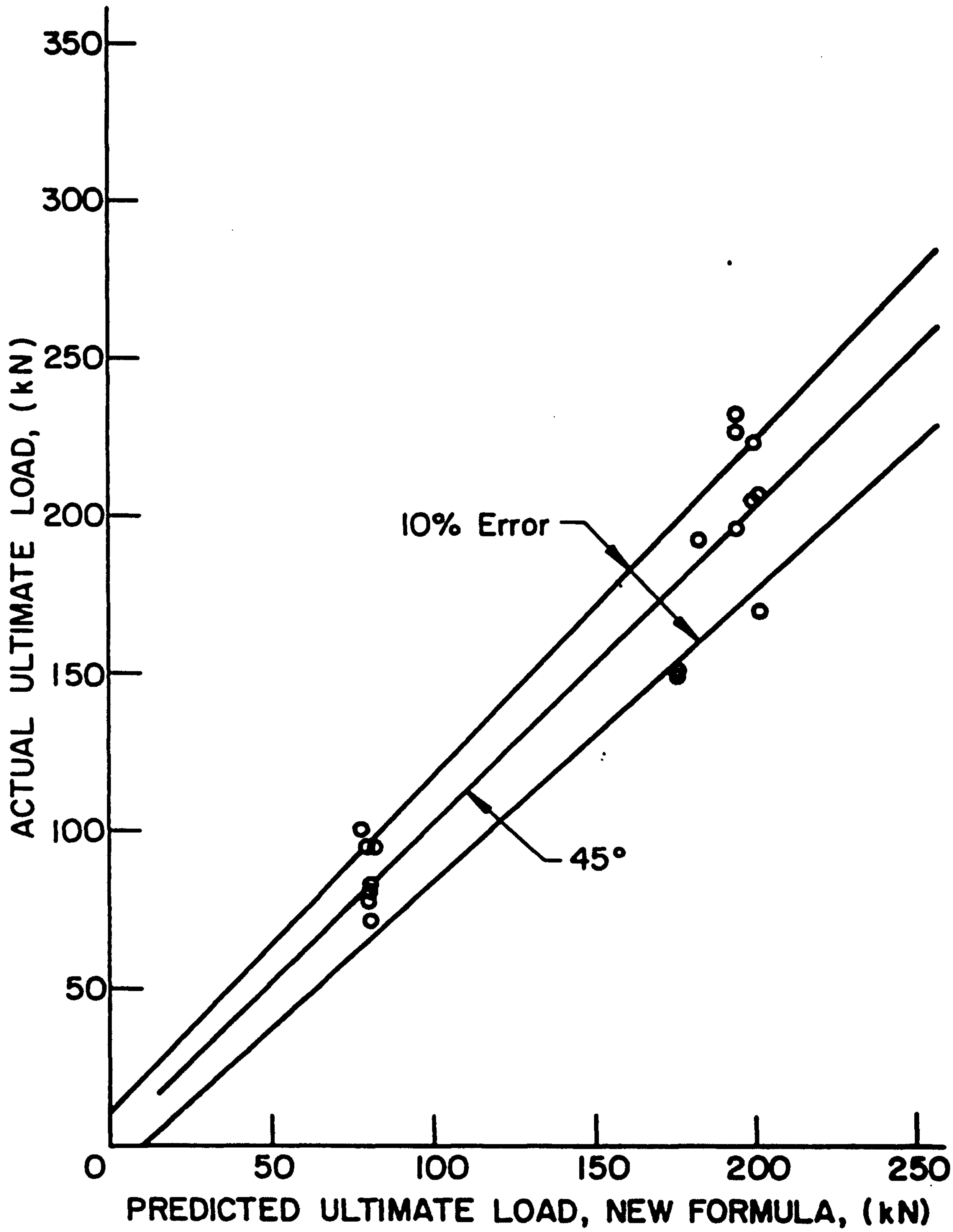

Fig. Actual $P_{u_{cap}}$ versus Predicted $P_{u_{cap}}$

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APPENDIX

Since those using headed anchor studs often deal with English units the equation developed in this paper can be converted to English units with the following results:

$$\theta = 13 + 3\,(\ell_e + f'_c) \qquad \text{(A.1)}$$

where

ℓ_e is expressed in inches

f'_c is expressed in ksi

and

θ is expressed in degrees.

The ultimate capacity of the steel can then be calculated as follows:

$$P_{u_{cap}} = \phi\, k\, 4\sqrt{f'_c}\; A_o$$

where

f'_c is expressed in psi

A_o is expressed in in.^2

and

$P_{u_{cap}}$ is expressed in lbs.

ϕ and k remain unchanged from Equation 2.

The accuracy of the formulas remain unchanged after conversion.

VITA

The author was born in Greenfield, Massachusetts on February 26, 1955. He is the son of Mr. and Mrs. Gordon G. Bennett. The author's primary and secondary education was received in the Greenfield Public School System, and in June 1973 the author graduated from Greenfield High School.

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