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Optimal control of tank turrets in the presence of stochastic disturbances.

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3 May 1979
(date)

Professor in Charge

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Abstract

Optimal control theory is applied to a dynamic model of a tank to investigate potential improvements in its "fire-on-the-move" stabilization. The action of rolling over rough ground is simulated by stochastic (random) inputs to the tank model. The performances of the optimal and conventional controllers are evaluated and compared through numerical simulation on a digital computer.

The conventional control system used as a basis of comparison is based on classical feedback concepts implemented with analog controllers. The optimal control systems use Kalman filters, which contain a linear model of the tank dynamics and the ground beneath the tank, to estimate the complete state from the feedback measurements. This includes knowledge of the ground contour beneath the tank, so the controller can act in anticipation of disturbances on the second through sixth wheels. Augmenting the conventional feedback signals with front wheel velocity measurements improves the control action by allowing the Kalman filter to make a better estimate of the ground contour.

When a linearized model of the tank is assumed, the optimal controller reduces the gun pointing error by a factor between 31 and 99. The smaller number includes the assumption of significant measurement noise and model errors. When the nonlinearities are inserted into the tank model, but a perfect observer is assumed,
the performance deteriorates but remains markedly superior to the conventional system. However, when a realistic linear Kalman observer is used, the system becomes unstable as a result of filter divergence. This problem presumably could be overcome through use of a nonlinear dynamic model within the observer which recognizes, in particular, coulomb friction in the suspension and the gun drives.
1. INTRODUCTION

This study investigates the potential of stochastic optimal control concepts for improved accommodation of a tank to disturbances caused by rolling over rough ground. Specifically, the objective is improvement of the gun aiming capability under these conditions. Conventional control systems employ classical feedback concepts using standard analog controllers. Optimal control is best implemented with digital controllers; small and inexpensive microprocessors are recent developments.

The design and testing of the optimal control schemes is carried out on the digital computer, using a mathematical model to describe the tank dynamics. While many simplifications of the physical complexities of a tank have been made, they are recognized partly by consideration of measurement and modeling errors.

The inputs to the linear sixth order conventional controller consist of absolute and relative angular rates of the gun in elevation and in azimuth. Two cases are investigated using stochastic optimal control theory, one assuming conventional measurements and the other assuming conventional measurements plus measurements of the front wheel velocities relative to the hull. All measurements are assumed to be contaminated with noise. In each case the optimal controller consists of an observer and a set of control coefficients. The observer has within it a dynamic model of the tank including the hull and tun dynamics, the ground shape and the delay of the ground disturbances from axle to axle. From the
feedback measurements the observer deduces an optimal estimate of the complete state. The set or matrix of optimal feedback coefficients, derived from deterministic optimal control theory, is multiplied by the optimal state estimate to yield the optimal control torques in elevation and in traverse.

The optimal controller utilizes knowledge of the time delays to gain, in effect, a preview of disturbances to the second through sixth axles. The front wheel measurements are added in an attempt to estimate these disturbances better.

All ground and noise disturbances are assumed to be random. An optimal controller designed for a deterministic disturbance would assume that all inputs will have the same shape, and therefore would not be optimal for other shaped disturbances. Likewise, if no model or measurement noise were assumed, the optimal controller would infer significant information from minute and likely meaningless variations in the measurements.

The development of the models is discussed in chapters 2, 3 and 4. Chapter 2 describes the nonlinear model and its associated simplifying assumptions. The terrain model is given in section 2.4. The linearization and discretization of the equations are discussed in chapters 3 and 4 respectively.

The deterministic optimal controller is developed in section 5.1 and the optimal observers in section 5.2. The performances of the different control schemes are calculated and compared in section 6.2 by means of performance indices or loss functions com-
prising the weighted sums of the variances of interest. Full simulations of each controller are carried out in section 6.3 for the linear tank model. In section 6.4, the sensitivity of the optimal schemes to nonlinearities is investigated.
2. TANK MODEL

The purpose of this study is the development of a control scheme for tank turrets. However, to achieve this, a mathematical model of the tank is needed. This model should be as simple as possible while maintaining sufficient detail for accuracy. For the purposes of this investigation a more complex model from an earlier study is adapted and simplified.

2.1 Source and Description of Tank Model

The M60A1 tank has been modeled in a study of hit probability for moving targets. The model consists of about sixty non-linear differential equations describing tank and target motion, gunner reactions, controller dynamics, ballistic computer, filters and reticle servos. While it is very complicated, it contains many simplifications of the complexities of the physical realities of the tank.

My study is concerned with the control of the aiming of the gun, so much of the information in the Hitpro model is of no interest. In addition, the simplest model possible which describes the tank accurately is desired. Consequently, some simplifying assumptions have been made.

2.2 Simplifications of the Model

The objective of this study is accommodation of ground disturbances by the turret and gun controllers. The model is built

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around these objectives. The first simplifying assumption is
that the tank moves only in a straight line at constant speed,
thus eliminating accelerations from these sources. It is assumed
that the target remains at a fixed position, relative to the
tank, eliminating the need for the ballistic computer to trace
and allowing the desired elevation and traverse space angles to
be specified as constant. The angles were arbitrarily chosen as
45 degrees left traverse and 30 degrees elevation. It is assumed
that the gun is not fired and that the gunner has no input (al-
though he is called upon to make some minor drift corrections).
Assumptions about the ground input are detailed in section 2.4.

The assumptions above allow the original model to be simpli-
...ied considerably. The state variables of concern are those de-
scribing hull motion, controller dynamics, gun motions and ground
contour. The hull motions are of four types: 1) linear vertical
motion, 2) pitch angular motion, 3) roll angular motion, and
4) yaw angular motion. There are two gun motions, elevation
angular motion and traverse angular motion. These are depicted
in fig. 2.1.

There are two orders for each type of motion, and so two
differential equations. Detailed developments of the equations
for hull motion can be found in the Hitpro report¹ and the report

¹. Ibid. Vol I
Fig. 2.1 Full and Gun Motions
2.3 Controller Description

The conventional controller system is part of the General Electric All-Electric Optimum Ratio Stabilized Drive system described in the Hitpro report. There are two controllers, one in elevation and one in traverse. Each is of third order and has as an input the desired angular rate. In our model the desired rates are zero. In addition, the elevation controller has as inputs the feedback signals consisting of gun elevation space rate or elevation gyro, gun elevation rate relative to the hull or elevation tach, and the hull angular rate in the direction that the gun is pointed. This last signal is a function of the pitch and roll angular rates and the traverse angle. The traverse controller has only one feedback input, that being the angular space rate in traverse or traverse gyro. The controller configurations are shown in figures 2.2 and 2.3.

The gyro feedbacks in each case are compared with the desired space rates to give the net error. This error is integrated so that the system can remember input commands and eventually follow them even if it is temporarily prevented from doing so due to torque or speed limitations. In our case this degen-

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Fig. 2.2  Conventional Controller - Elevation
Fig. 2.3 Conventional Controller - Traverse
erates to a position feedback. In the elevation controller an inner feedback loop compares the hull angular rate in the gun direction to the gun rate relative to the hull. They cancel one another when the space rate of the gun in elevation is zero.

These feedbacks are integrated, filtered and amplified to give the desired motor torque for control of elevation and traverse. The torque available, however, is limited by the actual motor as a function of speed (see Appendix B); and the actual motor torques can differ from what is directed by the controllers.

In later analysis the controller is considered as simply a black box with inputs of the four feedback signals and outputs consisting of two control efforts or desired motor torques.

2.4 The Ground Model

The ground disturbance employed is assumed to be random in nature. This precludes the controller from capitalizing unrealistically on knowledge of the ground contour in front of the tank, which would happen with a deterministic disturbance and an optimal controller. In addition, the ground is assumed to be rigid, that is, it is not compacted by the passage of the successive wheels. This assumption, coupled with those of constant speed and straight line travel, imposes the same displacement on each successive wheel on one side of the tank, with time delays equal to the distance from the front wheel divided by the speed. This time delay is taken to be 0.1 second per axle, which corresponds to a forward speed of 8.47 m/s.
The ground contour is generated from a Gaussian white sequence with a variance of 0.0058. The sequence describes a disturbance signal where successive points are placed at intervals of 0.01 second, and adjacent points are connected by straight lines as shown in fig. 2.4a. This stochastic disturbance signal is the input to a simple linear lag filter with transfer function \((ts + 1)^{-1}\). The output of this filter is the ground displacement in meters. The time constant \(t\) is chosen to be 0.1 sec, resulting in an exponentially decaying correlation between ground heights with a characteristic length of 0.85 meters apart. The input signals for the two treads are assumed to be completely independent.

The M60 tank has twelve wheels, so twelve orders are added to the model to give the ground simulation. There are two independent inputs of triangular pulses generated from random sequences. The time delays of these signals from axle to axle are handled by shift registers such that ten shifts occur between successive axles.

### 2.5 Description of the Nonlinear Model

The nonlinear model consists of a set of twenty-eight ordinary differential equations: eight for hull motions, four for gun and turret, six for control, and twelve to describe ground motion. The definition of these state variables and constants associated with the physical properties of the M60 tank can be found in Appendix A. The twenty-eight equations describing
the time derivatives of the state variables are shown in Appendix B.

The nonlinearities in this model are of four types. Gyroscopic terms occur due to several axes of rotation. They appear in the equations as the product of two state variables. A second type is trigonometric terms arising, again, from angular motion. Saturation of motors occurring when the controllers demand more torque than the motors are capable of delivering are included in the third type of nonlinearity. Also included are similar discontinuities in spring rate due to the helper springs. They come into play only for very large disturbances. The fourth nonlinearity in these equations is coulomb friction. Coulomb or dry friction exerts a constant force or moment opposing motion regardless of velocity. This type of friction occurs, according to Hitpro, between the turret and the hull, between the gun and gun trunnion, and on the first, second and sixth wheels on either side of the tank.

For simulation purposes, the equations representing the nonlinear tank are integrated using a simple fourth-order Runge-Kutta algorithm. The input sequences for these simulations are generated by a Gaussian random number algorithm and scaled appropriately.
3. THE LINEARIZED TANK MODEL

Most optimal control schemes have been developed for linear systems. The only schemes applicable for systems of the magnitude in this study are for linear systems. Therefore, the equations describing the dynamics of the system must be linearized. Linearization also permits the benefits of superposition and discretization, as is described in later sections.

3.1 Linearization of Equations

The equations of motion for the nonlinear system contain four basic types of nonlinearities noted earlier: gyroscopic, trigonometric, saturation or discontinuity, and coulomb friction. Linearization of the first two types involves a choice of equilibrium. The linearization of the other two is not as clear-cut and involves some assumptions about the magnitudes of the input disturbances. The coulomb friction, sometimes called an "essential nonlinearity," is especially critical.

The gyroscopic terms occur in equations for the derivatives of pitch, roll, yaw, elevation and traverse rates. In general these equations contain terms of the form:

\[ \frac{DZB}{dt} = ZA \ast ZC \]  

(3.1)

where \( DZB \) is the derivative of state variable "ZB", and "ZA" and "ZC" are state variables. The linearization of this equation results from looking at incremental changes in each of the variables about some equilibrium:

\[ \Delta DZB = AZA \ast \Delta ZC + AZA \ast AZC \]  

(3.2)
where \( \Delta DZB \) is the incremental change in the derivative "DZB", "AZA" and "AZC" are the equilibrium or average values of "ZA" and "ZC", and "\( \Delta ZA \)" and "\( \Delta ZC \)" are the deviations or incremental changes from these average states. The equilibrium values should be chosen such that the deviations occur equally above and below the chosen value. These average values are now constants as far as the equations are concerned, and the "\( \Delta \)" terms become the new state variables. Similarly trigonometric terms of the form:

\[
DZB = ZA \times \sin(ZC) 
\]

(3.3)

become upon linearization:

\[
\Delta DZB = \Delta ZA \times \sin(AZC) + AZA \times \cos(AZC) \times \Delta ZC 
\]

(3.4)

The chosen equilibrium states for all state variables are chosen as zero with three exceptions: the average turret traverse angle is chosen as 45° or 0.785 radians. The average elevation angle is 30° or 0.524 radians and a control variable influencing elevation torque has an equilibrium value that supplies torque sufficient to maintain this 30° elevation. These linearized equations should show good agreement with the nonlinear equations as long as the deviations are small, a few degrees or so. This is the case with all of the variables.

The other two nonlinearities are a bit more difficult to linearize. Saturation and spring discontinuities are handled similarly. The saturation of elevation and traverse torques imposes a restriction on desired torques as shown in fig. 3.1a.
Fig. 3.1 Nonlinearities in the Tank Model
These saturation values are a function of angular rate as shown in Appendix B. The suspension springs have helper springs on some wheels to stiffen the suspension as it approaches total compression and then a very high spring rate as the stop is encountered. These effects are shown in fig. 3.1b. In both cases, torque and spring rate, there is a linear portion of the curve below the nonlinear discontinuities. The input signals used are chosen such that the variables rarely exceed the points of discontinuity. Thus, the linear relationship is assumed to hold for all cases.

Coulomb friction is also a discontinuous nonlinearity but differs from the above in that no linear region exists about the origin or average as seen in fig. 3.1c. The coulomb friction is approximated nevertheless by linear viscous friction. One choice for the damping coefficient is the value which, for the level of disturbance assumed, dissipates the same amount of energy as the coulomb dampers. This is almost equivalent to equating the variance of the force (or torque) exerted by the viscous damper to the square of the coulomb friction force (torque). Through trial and error, values of viscous damping constants for each of the coulomb frictions was determined which approximately satisfy this criterion. They are shown in Appendix A as "CFE" for the coefficient of friction in elevation, "CFT" for traverse, and "b" for suspension dampers on the first, second and sixth wheels. (The other wheels have no dampers.)
Each of these linearizations imposes restrictions on the validity of the linear model which must be kept in mind as the study progresses. The model is only valid for reasonably small deviations from the equilibrium state. This means that if the gun or turret is left to wander more than a few degrees from the average angles the results may be questionable. The model is valid only for a specified level of random ground disturbance of a specific nature. The sensitivity of the model to these parameters is uncertain and may be a matter of concern. The level of torque exerted by the motors is assumed to remain below the saturation levels. This is of importance in the design of the optimal controllers.

3.2 Other Changes in the Model

The feasibility of optimal control decreases with increasing order of the system and with increasing bandwidth of its eigenvalues. For this reason the eigenvalues of the basic twelfth order system (excluding controllers and ground) were examined following a preliminary linearization. It was found that the natural frequency of the yaw mode is considerably higher than those of the other modes. Consequently the effect of yaw motion is small, and the two state variables representing it are assumed to have zero values. This eliminates two orders from the system equations.

On the other hand, it was discovered that a non-observability problem exists for the absolute elevation and absolute
traverse angles. This condition occurs because there is no position feedback from these angles. Thus, noise induces the gun to drift from the desired angles. The solution to this problem requires human gunner to make corrections for the otherwise inevitable gradual drift (in effect providing position feedback), but requires the control system to make the corrections for more rapid deviations of the gun from the desired angles. This is achieved through the addition of two filters to the control system, adding one order apiece to the overall system.

The two states added, called "ZSE" and "ZST", represent filtered absolute elevation and traverse, respectively. Their derivatives are defined as follows:

\[
\begin{align*}
DSE &= \text{Gyro Elevation} - \frac{ZSE}{\tau} \\
DST &= \text{Gyro Traverse} - \frac{ZST}{\tau}
\end{align*}
\]

The gyro signals in elevation and traverse are the respective absolute or space rates, and "\(\tau\)" is the time constant of the filters which was chosen to be 2 sec. It should be noted that "ZSE" and "ZST" are not actual physical states, but rather are conceptual states created for the evaluation and comparison of alternative controller configurations.

3.3 Comparison of Nonlinear and Linear Models

The linear model including the "ZSE" and "ZST" states, hull motions excluding yaw, gun and turret motions, and the conventional controller is of eighteenth order. The same twelfth order ground model as in the nonlinear system (it is linear) makes the complete linear model thirtieth order.
linear equations representing this system are shown in Appendix C.

A full simulation of the linear and nonlinear systems using the same input for each was done to determine whether the linearizing assumptions had any major detrimental effects on the linear system. The input consisted of independent sequences of 600 random numbers for each of the two treads. The equations were integrated utilizing identical fourth order Runge-Kutta algorithms to obtain the response for six seconds of running time. The results are shown in figures 3.2, 3.3, 3.4, and 3.5. It can be seen that the pitch and roll of the two systems in fig. 3.2 are in close agreement. The same can be said for the vertical position in fig. 3.3, which also shows the left and right input signals. In fig. 3.4 the absolute traverse of the two systems appears in good agreement, but their absolute elevation curves are markedly different. The curve for the nonlinear model shows more pronounced perturbations than the curve for the linear model. This presumably is a virtually inevitable result of the approximation of linear elevation damping in the face of fast response (compared with hull motions or the slower traverse motion). The correspondence nevertheless is believed to be adequate for the basic statistical purposes for which it is used, although these differences contribute to filter divergence in one optimal scheme discussed later. Fig. 3.5 shows the motor torques exerted by the two models. Again marked differences are apparent. Interestingly, the elevation torque for the nonlinear
Note: The mean values of each curve are offset from zero for clarity only.

Fig. 3.2 Simulation with Conventional Control: Ground Contours and Vertical Displacement of Hull - 23 -
Note: The mean values of each curve are offset from zero for clarity only.

Fig. 3.3 Simulation with Conventional Control: Roll and Pitch Angles
Fig. 3.4 Simulation with Conventional Control Gun Traverse and Elevation Angles
Note: The mean values of each curve are offset from zero for clarity only.
system resembles a square wave while the curve for the linear system resembles a sine wave. As with the angles the traverse torque curves show greater correspondence than those for elevation, but here the correspondence is restricted to general shape and, to a lesser degree, amplitude.
4. THE DISCRITIZED TANK MODEL

The optimal control strategy employed in this study requires either continuous-time differential equations or discrete-time state transition equations. An important part of the information in the tank dynamics is the time delays of the ground input, which are accommodated best by a discrete model. For this reason the continuous differential equations are discretized through a numerical algorithm to create the state transition matrix. Discretization also eliminates the need to integrate differential equations, once the state transition matrices have been established. Instead, time response can be calculated by relatively simple, cheap and rapid matrix multiplication.

4.1 Changes Made for Discretization

The purpose of discretization is to facilitate the design of an optimal controller. The conventional controller therefore is removed from the model at this point. In the conventional system, the equations are of the form

\[ \dot{x} = Ax + Bw \]  \hspace{1cm} (4.1)

where the \( x \)-vector represented the state including hull motions, gun and turret motions, ground displacements and the controller dynamics, and \( w \) is the vector of length 12 representing the stochastic inputs. \( A \) and \( B \) are matrices containing coefficients describing the differential equations. The six orders representing the controllers are removed and replaced with two controller inputs representing elevation and traverse torques. This gives the equations...
the form:

$$\dot{x} = Cx + Du + Ew$$  \hspace{1cm} (4.2)

Now $x$ is the hull, gun and ground states; $u$ is the controller inputs, and $w$ is still the stochastic inputs. The matrices $C$, $D$ and $E$ describe the effect each of the terms has on the derivative vector.

A problem exists in determining the terms in the $C$ matrix of equation 4.2 to describe the effect of the suspension dampers on the hull motion. This is because no variable represents wheel velocity (rate of change of ground height). The solution is to replace the ground height variables with variables representing the forces on the hull caused by wheel motion. The equation describing one of these forces is

$$t \frac{df}{dt} + f = b \frac{dw}{dt} + kw$$  \hspace{1cm} (4.3)

where $f$ is the force due to wheel motion, $b$ is the linear damper coefficient (found on first, second and sixth wheels only), $k$ is the suspension spring constant, $w$ is the stochastic input, and $t$ is the time constant of the stochastic filter. The time derivative of $w$ is constant between successive random points as seen in fig. 2.4a. This scheme leads to two separate equations:

$$\dot{x} = Gx + Hf + Ju$$ \hspace{1cm} (4.4a)

$$\dot{i} = \frac{1}{t} (I f + b \frac{dw}{dt} + k w)$$ \hspace{1cm} (4.4b)

For the full simulation the stochastic input $w$ to the successive wheels is handled by two time delay shift registers of length
fifty-one. In the discrete system the content of each location in the shift register is a state variable. To limit the size of the state only one side of the tank is excited at a time. The principle of superposition permits the two resulting responses to be summed to give the total response.

In the process of discretization, it is useful to have an expression for the forces in terms of the discrete points of the input function $w(t)$. This expression can be developed from 4.4b and shown to be

$$f(t) = \left< f(0) - \left[k \left[1 + \frac{t}{\Delta t} \right] - \frac{b}{\Delta t} \right] w(0) + \left(\frac{tk}{\Delta t} - \frac{b}{\Delta t}\right) w(1) \right> e^{-t/\tau}$$

$$+ \left[ w(1) - w(0) \right] \frac{kt}{\Delta t} + \left[ k \left(1 + \frac{\tau}{\Delta t}\right) - \frac{b}{\tau} \right] w(0) - \left[ \frac{tk}{\Delta t} - \frac{b}{\Delta t} \right] w(1)$$

(4.5)

assuming that the points $w(0)$ and $w(1)$ are connected by a straight line. At time $t=0$, force and input have the values $f(0)$ and $w(0)$, respectively. The input as a function of time in this case is

$$w(t) = w(1) - w(0) \frac{t}{\Delta t}$$

(4.6)

The shift registers supply the time delays for these input points between the six forces of the f-vector in equation 4.4b.

The model to be discretized now has twelve orders representing hull, gun and turret motions (including ZSE and ZST), six orders representing forces due to ground motion on one side, and fifty-one orders representing delay registers for the stochastic input on the excited side.

3.2 Discretization: The State Transition Matrix

A discrete system describes the state at discrete time inter-
vals. The value of the state at the next discrete time is determined from knowledge of the present state, motor torques and stochastic inputs. This process is described by

\[ x_{k+1} = P \cdot x_k + Q_u \cdot u_k + Q_w \cdot w_k \]  

(4.7)

where \( x_k \) is the present state and \( x_{k+1} \) is the state at the next discrete time; \( P \) is the state transition matrix and \( Q_u \) and \( Q_w \) describe the effects of the present stochastic input and motor torques on the change of state variables. The \( x \)-vector described in equation 4.7 consists of fifty-one time-delay variables, six force variables, and the twelve state variables describing tank motion for a total of sixty-nine. The \( u \)-vector of length two represents elevation and traverse torques. The last term, \( w \), is a single variable representing the stochastic input for the first axle; inputs for successive axles are in the \( x \)-vector at positions 10, 20, 30, 40 and 50 of the fifty-one time-delay variables.

The dimensions of the matrices \( P \), \( Q_u \) and \( Q_w \) are 69 x 69, 69 x 2, and 69 x 1, respectively. They are relatively sparse matrices, as shown in fig. 4.1. Six specific sub-matrices are indicated within the \( P \)-matrix. The 51 x 51 sub-matrix represents the delay operations with ones on the subdiagonal. The upper right sub-matrix is empty since the forces and tank state have no effect on delays and the tank state does not affect forces.

The 6 x 57 sub-matrix describes the discrete force equation
Fig. 4.1 Structure of $Z$, $Q_u$, and $Q_w$
\[
\begin{align*}
    f_{i+1} &= e^{-\Delta t / t} f_i \\
    &\quad + \left\{ \left[ k \left( 1 + t / \Delta t \right) - b / \Delta t \right] \left( 1 - e^{-\Delta t / t} \right) - k \right\} w_{i-1} \\
    &\quad + \left\{ k - \frac{tk - b}{\Delta t} \left( 1 - e^{-\Delta t / t} \right) \right\} w_i 
\end{align*}
\]

which can be derived from equation 4.5, by substituting \( \Delta t \) for \( t \). The time interval of the inputs, \( \Delta t = 0.01 \), is also chosen as the time interval of the discrete system. The six terms having the factor \( \exp(-\Delta t / t) \), one for each force, appear on the diagonal. The coefficients of the \( w_{i-1} \) terms for the forces appear in the \( x(1), x(11), x(21), x(31), x(41), x(51) \) positions for forces one through six, respectively. The coefficients for \( w_i \) terms are in positions \( x(10), x(20), x(30), x(40), \) and \( x(50) \) for forces two through six. (The \( w_i \) term for force one appears in the \( Q_w \) matrix.)

The 12 x 12 sub-matrix is nearly full. The sub-matrix contains coefficients for the variables \( x(58) \) to \( x(69) \) of equation 4.5, which are the twelve states of the \( x \)-vector in equation 4.4a. The values of the elements in the 12 x 12 section are calculated through multiple integrations of equation 4.4a using a variable step, variable order integration algorithm. \(^1\) The first column of this sub-matrix corresponds to \( x(58) \), the hull vertical position. To determine the values in this column,

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equation 4.4a is integrated over the interval $\Delta t$ with initial conditions of unity for the hull vertical position and zero for all other variables. The $f$ and $u$ vectors of equation 4.4a are also zero for this integration. The resulting values of the $x$-vector at $t=\Delta t$ are the values of the first column of the $12 \times 12$ sub-matrix.

To obtain values for the second column, corresponding to hull vertical velocity, the procedure is repeated assuming initial conditions of unity for the hull vertical velocity and zero for all other $x$'s, $f$'s and $u$'s. The procedure is repeated twelve times until the sub-matrix is completed. Some elements in the matrix are zero, as shown in fig. 4.1.

The values of the $12 \times 6$ sub-matrix of the $P$-matrix are calculated in a similar manner. However, instead of unity initial conditions for members of the $x$-vector of equation 4.4a, the elements of the $f$-vector of 4.4b are, in turn, set equal to unity. Thus, for the first column, $f(1)$ (corresponding to the force at wheel one) has an initial condition of unity while all other $f$'s, $x$'s, $u$'s and $w$'s are set to zero. After integrating both equations simultaneously to $t=\Delta t$, the resulting $x$-vector is the first column of the $12 \times 6$ sub-matrix, and so on.

The $12 \times 51$ sub-matrix contains non-zero columns at positions corresponding to the $w(0)$'s and $w(1)$'s of equation 4.5. This is due to $\Delta t$ not being vanishingly small. The values of these columns are also found by simulation. These simulations are run by obtaining an $f(t)$ from equation 4.5 for the appro-
appropriate force assuming $f(0) = 0$ and either $w(1)$ or $w(0)$ equals unity and the other equals zero. This $f(t)$ is used in equation 4.4a for the member of the $f$-vector under consideration. Equation 4.4a is integrated to $t=\Delta t$ with all initial conditions set equal to zero. The resulting values of the $x$-vector are placed in the column that corresponds to the non-zero input. Columns 1, 11, 21, 31, 41 and 51 correspond to $w(0)=1$ for forces one through six, respectively. Columns 10, 20, 30, 40 and 50 correspond to $w(1)=1$ for forces two through six, respectively. The values of the $x$-vector obtained by setting $w(1)=1$ for force one appear as the values of the bottom twelve elements of $Qw$.

The $Qw$-matrix describes the effect of the present motor torques on the full state at $t=\Delta t$. Obviously, motor torques do not affect the ground input variables, so the first 57 rows are zero. It is assumed that the values of the torques remain constant during the time interval $\Delta t$, which is the easiest strategy to implement in practice. The proper integration of equation 4.4a to obtain the change in $x$, over the interval $\Delta t$, due exclusively to $u_1$ and $u_2$ (i.e., with $x(0)=0$ and $f(t)=0$) is:

$$\Delta x = \int_0^t e^{G(\Delta t-t)} \int u(t) \, dt$$

(4.9).

Since $u(t)$ is chosen as constant, $u(t)=u$; this becomes

$$\Delta x = \int_0^t (1 - e^{G\Delta t}) \int u$$

(4.10)

in which $\exp(G\Delta t)$ equals the 12 x 12 state transition sub-matrix described above. These $\Delta x$-vectors become the elements in the
Qu-matrix.

This entire procedure is carried out twice, once for each side. The coefficients in the 12 x 6 and 12 x 51 sub-matrices and the last 12 coefficients of $Q_j$ depend on the right or left side. All other coefficients are unaffected by the location of the excitation.
5. OPTIMAL CONTROL

An optimal controller seeks to minimize a chosen performance index in the course of dynamic response. Optimal control theory states that if a quadratic performance index is defined as described below, and the structure and parameters of an assumed linear model are known, the optimal controller is of zeroth order, comprising an array of constants. The input to the controller must include all state variables, however. Its output is the control efforts, in our case gun drive motor torques. If it is assumed that all state variables are known without error, either by direct measurement or by indirect deduction with a perfect observer, the resulting control scheme is called deterministic optimal control. It is depicted schematically in fig. 5.1. Deterministic optimal control is the ideal with which all other more practical schemes are compared.

In a real control system only a few feedback measurements are made. Measurement of as few as one state variable can, in theory at least, be used in conjunction with a dynamic model, called an observer, to generate the unmeasured variables. Thus, the observer reconstructs the state from the measured variables. This reconstructed state is then the input to the same zeroth order controller used in deterministic optimal control. Measurement noise and model errors introduce errors into the deduced state, however. The observer may do a poor job on some states and a good job on others, depending on the coefficients employed. To find the best possible observer, the measured signal can be
assumed to be contaminated with random noise. The resulting scheme is called stochastic optimal control, and is shown schematically in fig. 5.2.

Stochastic optimal control is applied below twice. First, the same four feedback signals as used in the conventional controller are assumed, and second, these four signals plus measurements of the velocities of the two front wheels relative to the hull are assumed.

5.1 Deterministic Optimal Control

A quadratic performance index is defined for the controller in the discrete system described in chapter 4, and is minimized via a matrix Ricatti equation. The solution of the Ricatti equation gives the coefficients of the 2 x 69 zeroth order optimal controller.

5.1.1 The Performance Index

The objective of the controller is to minimize the excursions of the gun from the desired absolute elevation and traverse coordinates. This objective is represented in the performance index by the sum of the squares of ZSE and ZST, the state variables conceived expressly to characterize these rapid excursions. The two terms are weighted equally, thereby assuming that they are of equal importance (unequal weighting could be used if one was deemed more important than the other).

The linear system does not account for the saturation of control torques. Therefore, unless somehow limited by the per-
Fig. 5.1 System with Deterministic Optimal Control
Fig. 5.2 System with Stochastic Optimal Control
formance index, the optimal controller will exert infinite torque to minimize ZSE and ZST. This is prevented by adding the weighted squares of the elevation and traverse drive torques to the performance index. The resulting index is

\[ J = (ZSE)^2 + (ZST)^2 + CET (TQEL)^2 + CTT (TQTR)^2 \]  

(5.1)

where CET and CTT are the coefficients for elevation and traverse torques, TQEL and TQTR, respectively. The values for these coefficients were determined through trial and error to limit the levels of torques for the level of ground roughness assumed below their respective saturations. They are CET = 2.21E-17, and CTT = 1.50E-22.

5.1.2 The Matrix Ricatti Equation for Optimal Control

Given the system of equation A.7 and the performance index cast in standard matrix form

\[ J = \sum_k (x_{k+1}^T F_x x_{k+1} + u_{k+1}^T F_u u_{k+1}) \]  

(5.2)

(superscript T means transpose), the optimal control can be written

\[ u_k = -G x_k \]  

(5.3)

\[ G = (F_u + Q_u H Q_u)^{-1} Q_u^T H P \]  

(5.4)

in which \( H \) is the final steady state solution of the matrix Ricatti equation

\[ H_k = P^T H_{k+1} P + P^T H_{k+1} Q_u (F_u + Q_u^T H_{k+1} Q_u)^{-1} Q_u^T H_{k+1} P + F_x \]  

(5.5)

Almost any initial condition \( H_N \) can be used (\( H_N = F_x \) has particular meaning if other than steady state control is of interest),
and the steps are taken backwards \((k = N-1, N-2, N-3, \text{etc.})\).

(See any text on modern control for elucidation of this result.)

The entire scheme was run for the left side and separately for the right side for 800 iterations \((k = N-1 \text{ to } N-800)\) using their respective \(P\), \(Q_u\) and \(Q_v\), but the same performance index.

Since the state transition matrix contains information about the delays, the optimal controller can take advantage of a preview of the inputs to the second through sixth axles, evidenced by the profile of \(G\)-matrix values for the first 51 columns shown in fig. 5.3. These rows are the coefficients that are multiplied by the ground disturbance input variables to give contributions to the elevation and traverse torques. Recall that these ground inputs at positions 1, 10, 11, 20, 21, 30, 31, 40, 41, 50 and 51 cause forces on the hull, making it accelerate vertically, in pitch and in roll. The controller, anticipating the effect of these forces, exerts torques to counteract them. Note that the largest torque contributions come from the wheels with dampers (i.e. positions 1, 10, 11, 50, 51) and that these torques occur in doublets. The effects of right and left inputs on elevation torque are very similar, and there is little difference between the spikes at the first, second and sixth wheels for the two inputs, indicating that vertical acceleration is the primary consideration. The primary consideration in traverse is a combination of pitch and roll about the axis in the X-Z plane in the direction of the gun. These effects are as expected.
Fig 5.3 Ground Disturbance Coefficients in $G$ Matrix - 43 -
5.1.3 Simulation of Time Response for the Deterministic System

The discrete linear tank equation 4.7 combined with the optimal control equation 5.3 yields

\[ x_{k+1} = (P - Q_u C) x_k + Q_w w_k \] (5.6)

with different coefficients for left and right side excitations. Simulation of one response to an input \( w \) involves simply carrying out the matrix multiplications.

Two types of inputs, \( w \), are considered. The random sequences from section 3.3 are used for simulation purposes. Also, the response to a singular triangular pulse input is run. The variances of variables or combinations of variables that would occur for a stochastic input can be calculated from the response to this single pulse. (This method is discussed in section 6.1.) In each case the left and right simulations are done separately.

For the full simulation the responses of the two sides are summed to yield the total response. For evaluation of the performance the variances of each side are determined separately and then the variances are summed to yield the variances for a full stochastic input. Results are discussed in section 6.2.

5.2 Stochastic Optimal Control

Given a set of signal measurements with known noise levels defined in terms of the state variables, and knowing the level of input disturbance, \( w \), stochastic optimal estimation theory minimizes the errors in the estimated state. These estimates, multiplied by the optimal feedback coefficients, yields the optimal drive torques that control the gun. Thus the stochastic...
optimal controller consists of two parts. The observer or state estimator, called a Kalman filter, has the same order as the tank model. The optimal controller, comprising 138 feedback coefficients, has zeroth order. The input to the Kalman filter is the feedback signal measurements, and its output is the synthesized or Kalman state which is then the input to the optimal controller.

The estimate of the state generally improves with more or better feedback measurements. Better measurements are characterized by higher signal to noise ratios.

In practice the Kalman filter used herein is overly sensitive to modeling errors (described in section 6.4.2), despite its complexity. It is, however, a great deal closer to a practical controller than deterministic optimal control, and permits the potential effectiveness of various measurement transducers to be compared.

5.2.1 Conventional Feedback Measurement

Measurement devices on the M60A1 tank provide the feedback for the conventional control system described in section 2.3. They are retained for the first of two versions of the Kalman filter. In section 5.2.4 two additional measurements, the velocities of the front wheels relative to the hull, are added in an attempt to improve the preview capabilities.

The four conventional feedback signals are, in terms of the state variables,

\[
elevation \ tach = y_1 = \frac{Z21 - (\cos \theta _{A24}) Z9 - (\sin \theta _{A24}) Z10}{45}
\] (5.7)
elevation gyro = \( y_2 = Z21 \) \hspace{1cm} (5.8)
\[ \omega_{HE} = y_3 = (\cos AZ24) Z9 + (\sin AZ24) Z10 \] \hspace{1cm} (5.9)
traverse gyro = \( y_4 = (\cos AZ23) Z22 - (\sin AZ23) (\cos AZ24) Z10 + (\sin AZ23) (\sin AZ24) Z9 \) \hspace{1cm} (5.10)

where \( \omega_{HE} \) is the angular hull rate about the horizontal axis perpendicular to the vertical plane containing the axis of the gun barrel. These signals are assumed to be contaminated by noises of two basic types. The first type is noise due to signal measurement error, the actual physical noise that appears in electrical signals. It is assumed to be Gaussian white sequences added directly to the measurement. The second type is noise due to modeling error which arises due to unavoidable differences between the actual physical tank and the assumed tank model. This second type also is represented by white noise, this time added to the state variables comprising the measured signals. Its interpretation as model error represents a vague substitution rather than a precise equivalence, but its use is thought to improve the result.

All of the noises are assumed to be independent. In each case the variance of the white noise is set at one percent of the variance of the corresponding signal or variable which occurs in deterministic optimal control. This level is rather arbitrary, but was chosen so that the noises have a significant effect on the controller response for evaluation of noise sensitivity. If more realistic noise estimates become available they can be readily incorporated into the analysis.
The measured signals, $v_k$, are expressed in terms of the state, $x_k$, and the assumed noise, $v_k$:

$$v_k = C x_k + v_k \quad (5.11)$$

The noise, $v_k$ (consisting of measurement and modeling noise), and the ground roughness input, $w_k$, are characterized by the noise covariance matrix,

$$E\left\{ \begin{bmatrix} w_k \\ v_k \end{bmatrix} \begin{bmatrix} w_k \\ v_k \end{bmatrix}^T \right\} = \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix} \quad (5.12)$$

The $W$-matrix is just a single term on the diagonal, and the $V$-matrix has both diagonal and off-diagonal terms. The off-diagonal terms arise exclusively from the modeling noise. The true measurement noises, being uncorrelated, cause only diagonal terms in the covariance matrix. These are constant matrices, since it is assumed any transients have died out. These matrices are calculated for both the right and left sides.

5.2.2 The Matrix Riccati Equation for Optimal Estimation

Denoting the optimal estimate of $x_k$ as $\hat{x}_k$ and assuming any transients from incorrect conditions at start-up have decayed, it is well known that

$$\hat{x}_{k+1} = P \hat{x}_k + Q_u u_k + K [y_{k+1} - C(P \hat{x}_k + Q_u u_k)] \quad (5.13)$$

where $K_k$, the Kalman matrix, is given below as the asymptotic value of $K_k$ as $k \to \infty$.

$$K_k = M_k C^T (C M_k C^T + V)^{-1} \quad (5.14)$$

$$M_{k+1} = P (I - K_k C) M_k P^T + Q_u w Q_u^T, \quad M_0 = E\{x_0, x_0^T\} \quad (5.15)$$
This matrix Ricatti equation is iterated for increasing values of $k$. The equation is solved twice for conventional feedback, left and right side, using the respective $P$, $Q_w$, and $V$-matrices. They share common $C$, $Q_u$, and $W$-matrices. The two resulting $K$-matrices have no common terms. The equations appear to converge at 800 iterations on the right side and 340 iterations on the left. (The left side appears to converge and subsequently diverge for unknown reasons. The values at 340 iterations are used with excellent results.)

5.2.3 Simulation of Time Response for the Stochastic System

The control vector, $u_k$, in optimal stochastic control is derived from the $\hat{x}_k$-vector:

$$u_k = -G \hat{x}_k \quad (5.16)$$

By combining equations 4.7, 5.13 and 5.16, the complete state equations become

$$
\begin{bmatrix}
  x_{k+1} \\
  \hat{x}_{k+1}
\end{bmatrix} =
\begin{bmatrix}
  P & -Q_u G \\
  KCP & P - Q_u G - KCP
\end{bmatrix}
\begin{bmatrix}
  x_k \\
  \hat{x}_k
\end{bmatrix}
+ 
\begin{bmatrix}
  I \\
  KCP
\end{bmatrix}
Q_w u_k + 
\begin{bmatrix}
  0 \\
  K
\end{bmatrix}
\begin{bmatrix}
  v_k \\
  w_k
\end{bmatrix} 
\quad (5.17)
$$

These equations are used in the same ways as equation 5.6 with respect to the $u_k$ input. In addition, however, eight simulations per side are run to compute the response to the eight independent noises comprising $\nu$. These noise simulations are run using single square pulse inputs in the same way as the triangular pulses for $w$. Thus equation 5.17 is solved nine times per side for single pulse inputs in $\nu$ and $w$. The full simulations, two
per side, are run using \( \omega \) inputs only and no noise. The results are discussed in section 6.2.

5.2.4 Incorporation of Front Axle Measurement

The optimal controller takes advantage of the known time delays between the identical disturbances on successive axles. In a second application of optimal observer theory two additional feedback signals, left and right wheel velocities relative to the hull, are added to the four existing feedback signals to enable the observer to estimate the disturbances better. These signals could be measured readily by velocity transducers, and they provide more primitive information than relative position feedback.

These relative velocities are not state variables, but they must be described in terms of state variables. The components of these velocities due to the direct effect of ground roughness are, in operator notation \((D \rightarrow \frac{d}{dt})\),

\[
\frac{Dw(t)}{tD+1} = \omega - \frac{1/\ell}{k-b/\ell} (F - \frac{b}{\ell} \omega) = \frac{F-k\omega}{b-k\ell}
\]

(5.18)

where \( F \) is the force exerted on the excited front axle due to ground disturbances only. (Recall this is \( x(52) \).) The \( \omega \) in equation 5.18 is \( x(1) \). This velocity component plus those due to hull motions yield the equations

\[
y_5 = \frac{k x(1) - x(52)}{b-k\ell} + xW(1) Z9 + Z15 \div CT16 Z10
\]

(5.19)

\[
y_6 = 0 + xW(1) Z9 + Z15 \div CT16 Z10
\]

(5.20)

for the relative velocities, \( y_5 \) for the excited side, \( y_6 \) for the other. The upper signs apply when the right side is excited and
the lower signs apply when the left side is excited.

These signals, like all other feedback signals, are assumed to be contaminated by noise. Using the same premise for noise as before, two more measurement noises and three additional quasi-modeling noises would be added. Due to coincidental cancellation of terms, however, the attempts at representing the modeling noises for $x(52)$ and $x(1)$ lead to effective noise levels of 5% of expected signal variance instead of the intended 1%. This unrealistic and essentially inconsistent situation is avoided by substituting, instead, an additional 1% true measurement noise for the modeling errors of $x(52)$ and $x(1)$.

Thus only three additional noises are introduced: measurement noises for each of the relative velocity signals, and a modeling noise for $Z15$. The modeling noise and measurement noise for the unexcited side are given a variance of 1% of the corresponding nominal signal variances from the deterministic system. The measurement noise for the excited side is given a noise variance of 2% of the nominal signal level.

These two feedbacks are incorporated into the C-matrix (now $6 \times 69$) and the noises into the V-matrix (now $6 \times 6$). The matrix Ricatti equation (equation 5.15) is rerun to obtain the K-matrix for the augmented set of feedback signals. The entire procedure is executed twice, one for left and one for right side excitation, with the Ricatti equations run for 660 iterations.

Simulations for the system using this observer are similar
to those run for the four-channel feedback system. Equation 5.17 is used with the $C$- and $K$-matrices calculated above and all other matrices unchanged. Full simulations with random ground inputs but no noise are run for the left and right sides. Simulations using pulse inputs for the ground input and each of the eleven noises are run for both the left and right sides to evaluate variances, a total of 24 pulse simulations. The results of these simulations are discussed in Chapter 6.
6. COMPARISON OF RESULTS

Each of the systems described in the previous chapters is evaluated with respect to the design objective of minimizing deviations from the desired elevation and traverse angles within the control effort constraints of the drive systems. Systems evaluated in a like manner can be compared to determine their relative effectiveness in achieving the design objectives.

The two methods used in this study are 1) the calculation of various indices, including the optimal design performance index, from the variances of the index parameters, and 2) graphical comparisons of systems excited by the same random ground inputs. The variances for the calculation of indices are computed from the time responses of the various pulse inputs as described below. Sensitivity of the linear optimal schemes to nonlinearities also is investigated.

6.1 Determination of Variances

The mean square value of a stochastic variable is known as its variance. The variances of certain variables are used to evaluate the performance of the various control configurations. The weighted sums of these variances comprise the performance index.

Variances can be estimated by direct computation based on the results of a simulation using Gaussian white noise inputs over a sufficient time period to insure decay of transients. Although this method is indicated for nonlinear models, a much
cheaper and more accurate method was employed for the linear models. This can be viewed as a degenerate case of the method presented by Astrom. 1

The variance of the response of a linear system to a unity white Gaussian input signal equals the time integral of the square of the response of the same system to a unit impulse. We are dealing with the terrain model described in section 2.4 for which the input is a superposition of triangular pulses of randomly distributed heights at distinct time intervals. The variance of the response to this random excitation equals the time integral of the corresponding response to a single triangular pulse of unit height, divided by the time integral of the triangular pulse. In practice the signal decays rapidly enough in time to permit truncation of the integral with little error.

If the time integral of the square of the response is estimated in discrete time by summing the squares of the response with the same interval $\Delta t$ as is employed in the stochastic signal and triangular wave, then the variance can be approximated simply by the sum of the squares of the response to the unity height triangular pulse. This procedure is accurate so long as the response does not change rapidly during any interval, which is the case for these performance evaluations.

6.2 Comparison of Performance

The four control strategies described in the previous chapter (conventional, deterministic optimal, stochastic optimal with conventional measurements and stochastic optimal with additional front axle measurements) are compared on the basis of their performance indices. The values of these indices are calculated from variances determined, using the method described in section 6.1 for ground disturbance, measurement noise and quasi-modeling error noise inputs.

The same noise amplitudes are assumed in all systems (except for the deterministic system where noise is not considered). The stochastic optimal system with front axle measurements is affected by three noise terms which do not influence the other systems. Each of these noises is based on the amplitude of the corresponding signal in the performance of the deterministic optimal system. If the noise levels in each system were based upon the signal variances in the individual systems, the conventional system would have greater noise. Thus the assumption employed does not favor the non-conventional systems.

Tables 6.1, 6.2 and 6.3 show the results for each of the systems. The variances resulting from ground (rows labeled w) and noise inputs (NS1 to NS11) for both the right and left tread are given. The noises are defined as follows:

- NS1 Measurement noise of $y_1$ (elevation tachometer)
- NS2 Measurement noise of $y_2$ (elevation gyro)
Table 6.1  PERFORMANCE OF CONVENTIONAL CONTROL

<table>
<thead>
<tr>
<th>Input</th>
<th></th>
<th>Var (ZSE)</th>
<th>Var (ZST)</th>
<th>Var (u&lt;sub&gt;n1&lt;/sub&gt;)</th>
<th>Var (u&lt;sub&gt;tr&lt;/sub&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;/w&gt;</td>
<td>Right</td>
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<td>9.580 E-7</td>
<td>2.083 E+2</td>
<td>3.367 E+4</td>
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<tr>
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<td>Left</td>
<td>9.219 E-9</td>
<td>6.152 E-7</td>
<td>6.297 E+2</td>
<td>2.150 E+4</td>
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<tr>
<td>NS1</td>
<td>Right</td>
<td>6.977 E-8</td>
<td>1.490 E-10</td>
<td>9.099 E2</td>
<td>8.399</td>
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<tr>
<td>Left</td>
<td>1.027 E-6</td>
<td>2.195 E-9</td>
<td>1.340 E4</td>
<td>1.237 E2</td>
<td></td>
</tr>
<tr>
<td>NS2</td>
<td>Right</td>
<td>4.175 E-7</td>
<td>3.189 E-13</td>
<td>1.371 E1</td>
<td>1.9 E-2</td>
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<tr>
<td>Left</td>
<td>3.396 E-6</td>
<td>2.593 E-12</td>
<td>1.115 E2</td>
<td>1.6 E-1</td>
<td></td>
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<td>NS3</td>
<td>Right</td>
<td>6.852 E-8</td>
<td>1.463 E-10</td>
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<td>8.248</td>
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<td>1E-8</td>
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<td>1E-10</td>
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<td>1.102 E-6</td>
<td>1E-7</td>
<td>2.918 E4</td>
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<td>4.781 E-7</td>
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<td>7.386 E-6</td>
<td>29,613</td>
<td>195,630</td>
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<td>TOTAL NS and w's</td>
<td>9.935 E-6</td>
<td>8.959 E-6</td>
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<td>250,800</td>
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<td>PERF IND</td>
<td>1.587 E-6</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>w's</td>
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<tr>
<td>PERF IND</td>
<td>1.731 E-5</td>
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<td>NS's</td>
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<td></td>
</tr>
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<td>TOTAL PERF IND</td>
<td>1.890 E-5</td>
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</table>

Note: For all control systems:

\[
\text{PERF IND} = \text{var (ZSE)} + \text{var (ZST)} + [\text{var (u}_{\text{n1}})] \times 10^{-19} + [\text{var (u}_{\text{tr}})] \times 10^{-14}.
\]

\[
u_{\text{n1}} = \text{TQEL/C27}, \quad u_{\text{tr}} = \text{TQTR/C28}
\]

- 55 -
### Table 6.2 PERFORMANCE OF OPTIMAL CONTROL SYSTEMS

**Deterministic Optimal:** gives performance index of 6.953 E-10

<table>
<thead>
<tr>
<th>Input</th>
<th>Var (ZSE)</th>
<th>Var (ZST)</th>
<th>Var (u₁)</th>
<th>Var (u₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>Right</td>
<td>2.984 E-12</td>
<td>8.057 E-11</td>
<td>2.110 E+3</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.793 E-12</td>
<td>1.735 E-11</td>
<td>1.880 E+3</td>
</tr>
</tbody>
</table>

**Stochastic Optimal with Conventional Feedback Measurements:**

<table>
<thead>
<tr>
<th>Input</th>
<th>Var (ZSE)</th>
<th>Var (ZST)</th>
<th>Var (u₁)</th>
<th>Var (u₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>Right</td>
<td>2.857 E-11</td>
<td>3.638 E-10</td>
<td>3.783 E+2</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.009 E-11</td>
<td>7.220 E-11</td>
<td>7.488 E+2</td>
</tr>
<tr>
<td>NS1</td>
<td>Right</td>
<td>1.466 E-12</td>
<td>4.598 E-11</td>
<td>8.690 E00</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>2.291 E-13</td>
<td>8.886 E-12</td>
<td>1.643 E+1</td>
</tr>
<tr>
<td>NS2</td>
<td>Right</td>
<td>1.654 E-9</td>
<td>5.785 E-10</td>
<td>2.753 E+3</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.142 E-9</td>
<td>4.296 E-10</td>
<td>1.712 E+3</td>
</tr>
<tr>
<td>NS3</td>
<td>Right</td>
<td>4.498 E-11</td>
<td>3.415 E-11</td>
<td>5.133 E+1</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.522 E-11</td>
<td>1.448 E-11</td>
<td>2.5  E-1</td>
</tr>
<tr>
<td>NS4</td>
<td>Right</td>
<td>2.670 E-14</td>
<td>1.021 E-11</td>
<td>2.100 E-1</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>9.855 E-15</td>
<td>5.665 E-14</td>
<td>1.5  E-3</td>
</tr>
<tr>
<td>NS5</td>
<td>Right</td>
<td>9.143 E-12</td>
<td>1.853 E-9</td>
<td>4.040  -</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>2.404 E-13</td>
<td>3.896 E-11</td>
<td>7.946  -</td>
</tr>
<tr>
<td>NS6</td>
<td>Right</td>
<td>1.205 E-11</td>
<td>4.609 E-9</td>
<td>9.479 E+1</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>6.370 E-12</td>
<td>3.661 E-11</td>
<td>9.5  E-1</td>
</tr>
<tr>
<td>NS7</td>
<td>Right</td>
<td>1.672 E-9</td>
<td>6.144 E-10</td>
<td>2.812 E+3</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.140 E-9</td>
<td>4.290 E-10</td>
<td>1.747 E+3</td>
</tr>
<tr>
<td>NS8</td>
<td>Right</td>
<td>5.369 E-11</td>
<td>1.617 E-9</td>
<td>1.071 E+2</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>2.693 E-11</td>
<td>5.221 E-11</td>
<td>2.854  -</td>
</tr>
<tr>
<td>TOTAL NS's</td>
<td></td>
<td>5.778 E-9</td>
<td>1.037 E-8</td>
<td>9.319</td>
</tr>
<tr>
<td>TOTAL NS w's</td>
<td></td>
<td>5.817 E-9</td>
<td>1.081 E-8</td>
<td>10.447</td>
</tr>
<tr>
<td>PERF IND w's</td>
<td></td>
<td>1.126 E-9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PERF IND NS's</td>
<td></td>
<td>1.639 E-8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL PERF IND</td>
<td></td>
<td>1.751 E-8</td>
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<td></td>
</tr>
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Table 6.3 STOCHASTIC OPTIMAL WITH FRONT-AXLE FEEDBACK

<table>
<thead>
<tr>
<th>Input</th>
<th>Var (ZSE)</th>
<th>Var (ZST)</th>
<th>Var ($u_{e1}$)</th>
<th>Var ($u_{tr}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>Right</td>
<td>1.663E-11</td>
<td>1.224E-10</td>
<td>1.849E+3</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>6.897E-12</td>
<td>3.129E-11</td>
<td>1.926E+3</td>
</tr>
<tr>
<td>NS1</td>
<td>Right</td>
<td>7.304E-14</td>
<td>4.046E-11</td>
<td>7.711E-3</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>9.641E-14</td>
<td>9.937E-13</td>
<td>2.67E-2</td>
</tr>
<tr>
<td>NS2</td>
<td>Right</td>
<td>1.630E-9</td>
<td>6.497E-10</td>
<td>8.145E+2</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.136E-9</td>
<td>3.995E-10</td>
<td>5.781E+2</td>
</tr>
<tr>
<td>NS3</td>
<td>Right</td>
<td>6.279E-11</td>
<td>9.762E-11</td>
<td>2.961E+3</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.488E-11</td>
<td>6.432E-12</td>
<td>5.653 -</td>
</tr>
<tr>
<td>NS4</td>
<td>Right</td>
<td>1.866E-14</td>
<td>4.178E-12</td>
<td>1.250E-2</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.250E-14</td>
<td>3.765E-14</td>
<td>3.916E-3</td>
</tr>
<tr>
<td>NS5</td>
<td>Right</td>
<td>4.545E-12</td>
<td>1.340E-9</td>
<td>1.757E-2</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.696E-12</td>
<td>1.394E-11</td>
<td>5.999E-1</td>
</tr>
<tr>
<td>NS6</td>
<td>Right</td>
<td>8.421E-12</td>
<td>1.886E-9</td>
<td>5.641 -</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>8.078E-12</td>
<td>2.433E-11</td>
<td>2.531 -</td>
</tr>
<tr>
<td>NS7</td>
<td>Right</td>
<td>1.629E-9</td>
<td>6.212E-10</td>
<td>8.155E+2</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.134E-9</td>
<td>3.985E-10</td>
<td>5.789E+2</td>
</tr>
<tr>
<td>NS8</td>
<td>Right</td>
<td>4.762E-11</td>
<td>7.037E-10</td>
<td>1.799E+1</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.994E-11</td>
<td>1.402E-11</td>
<td>6.945 -</td>
</tr>
<tr>
<td>NS9</td>
<td>Right</td>
<td>1.906E-11</td>
<td>9.002E-10</td>
<td>1.561E+1</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>3.051E-12</td>
<td>7.981E-12</td>
<td>1.236 -</td>
</tr>
<tr>
<td>NS10</td>
<td>Right</td>
<td>2.057E-12</td>
<td>2.213E-10</td>
<td>3.178E+1</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>1.017E-12</td>
<td>2.299E-12</td>
<td>2.482E-1</td>
</tr>
<tr>
<td>NS11</td>
<td>Right</td>
<td>2E-23</td>
<td>2E-21</td>
<td>3E-10</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>6.044E-13</td>
<td>2.055E-11</td>
<td>2.026E+1</td>
</tr>
</tbody>
</table>

TOTAL NS's 5.723E-9 5.225E-9 2.925 11.780
TOTAL NS and w's 5.746E-9 5.379E-9 6.700 70.230
PERF IND w's 7.617E-10
PERF IND NS's 1.109E-8
TOTAL PERF IND 1.186E-8

- 57 -
The individual variances are summed to determine the total variances. (This is valid because all signals, ground disturbances, measurement noises and modeling errors are assumed to be uncorrelated.) Summary data at the bottom of each table shows total variances resulting from noise inputs only (TOTAL NS's) and the total variances resulting from all inputs (TOTAL NS's and ω's), and the components and totals of the performance index as noted.

Some noises in each system cause variances larger than those caused by the ground disturbances, while others have a negligible effect. The troublesome noises in the conventional system are not the same as in the optimal systems. The two stochastic optimal controllers are sensitive to the same noises and, as expected, the front axle measurements reduce this sensitivity. The performance of the systems for noise levels other than those assumed can be calculated by scaling the variances proportionately. The stochastic optimal systems thus scaled do not remain strictly optimal,
but in most cases their performances remain exemplary compared with the conventional system.

Table 6.4 shows summary data in a different and more condensed form. The results are in terms of indices defined by

\[
INDEX = \frac{\text{standard deviation of composite angle error}}{\text{average inverse fraction of saturation torque}}
\]

in which the numerator is the square root of the sum of the appropriate variances including both elevation and traverse, and the denominator is the inverse of the average of the ratios of the standard deviations of torque for elevation and traverse to the respective saturation torques. Three versions are given:

INDEX I: angles resulting from the \( w \)'s only, and torques resulting from the \( w \)'s only

INDEX II: angles resulting from all disturbances, and torques resulting from the \( w \)'s only

INDEX III: angles and torques resulting from all disturbances

INDEX I describes comparative behavior when no noise is present. It applies also to the deterministic optional control. Note that the conventional control is about 99 times worse than the deterministic optimal control, 53 times worse than the stochastic optimal control with conventional measurements, and 77 times worse than the stochastic optimal control with front axle measurement added.

INDEX II may be the most meaningful overall index, since the angle errors from all sources are of ultimate interest whereas it may not be necessary to prevent torque saturation due to the
Table 6.4 SUMMARY COMPARISONS OF CONTROL SYSTEMS

<table>
<thead>
<tr>
<th>Type of Control System</th>
<th>INDEX I</th>
<th>INDEX II</th>
<th>INDEX III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>$2.96 \times 10^{-4}$</td>
<td>$1.02 \times 10^{-3}$</td>
<td>$2.93 \times 10^{-3}$</td>
</tr>
<tr>
<td>Stochastic optimal, conventional measurements</td>
<td>$5.64 \times 10^{-6}$</td>
<td>$3.33 \times 10^{-5}$</td>
<td>$5.13 \times 10^{-5}$</td>
</tr>
<tr>
<td>Stochastic optimal, augmented measurements</td>
<td>$3.86 \times 10^{-6}$</td>
<td>$3.06 \times 10^{-5}$</td>
<td>$3.59 \times 10^{-5}$</td>
</tr>
<tr>
<td>Deterministic optimal</td>
<td>$2.98 \times 10^{-6}$</td>
<td>not computed</td>
<td></td>
</tr>
</tbody>
</table>
measurement and model noises. Torques produced by model noises, in particular, are in a sense a fiction, and noise-induced torques are at higher frequencies than ground-induced torques. With this index, the conventional control is about 30.7 times worse than the stochastic optimal control with conventional measurement, and 33.5 times worse than the stochastic optimal control with front axle measurement added.

INDEX III is similar to comparison of the various performance indices. With it the conventional system is about 57 times worse than the stochastic optimal control with conventional measurement, and 82 times worse than the stochastic optimal control with front axle measurement.

6.3 Simulation of Optimal Systems with Random Inputs

The discrete linear systems utilizing deterministic optimal control (equation 5.6) and stochastic optimal control (equation 5.17) are subjected to the same random ground disturbances used to generate figures 3.2 to 3.5. Two simulations are required in each case, one for the right side and one for the left, and the responses of the two sides are summed to obtain the total response. Plots generated from this response data are compared to the corresponding plots for the conventional system.

Since all simulations were run using the same ground input, all graphs for hull motions and input have essentially the same shapes as figures 3.2 and 3.3. Because they are so similar, the graphs of hull motion are not shown for subsequent systems. The differences lie in the graphs for gun angles and torques.
The time response for angle deviations using deterministic optimal control is shown in fig. 6.1. Note the difference in scale as compared with fig. 3.4. If the curves of fig. 6.1 were plotted on the scale of fig. 3.4, the deviations in both elevation and traverse would become imperceivable. Qualitatively, deterministic optimal control causes short jerky motions of the gun barrel. The cause of these rapid motions is evident in the torque curves for deterministic optimal control shown in fig. 6.2. High frequency torque oscillations appear to be superimposed on lower frequency signals. These lower frequency signals resemble the linear system curves of fig. 3.4. The high frequency oscillations apparently are caused by the doublet action described in section 5.1.2 and fig. 5.3.

The simulation for stochastic optimal control is shown only for the system with front axle measurement and no noise input. The absolute elevation and traverse angular deflections are shown in fig. 6.3. The rapid angular motion evident in the deterministic system appears again, but in addition there are somewhat slower and larger random deviations. These occur due to the inobservability of the absolute positions in the observer of the stochastic system. The torques for this system are shown in fig. 6.4. The traverse torque curve closely resembles the corresponding curve for the deterministic system. The elevation torque curve also resembles the corresponding curve for the deterministic system, but contains more of the high frequency oscillations. These results are consistent with those obtained in section 6.2.
Fig. 6.1 Simulation with Deterministic Optimal Control: Gun Traverse and Elevation Angles
Fig. 6.2 Simulation with Deterministic Optimal Control: Elevation and Traverse Motor Torque
Fig. 6.3 Simulation with Stochastic Optimal Control
Gun Traverse and Elevation Angles
Fig. 6.4 Simulation with Stochastic Optimal Control: Elevation and Traverse Motor Torque
6.4 Sensitivity of Optimal Control to Nonlinearities

While the deterministic and stochastic optimal control systems performed extremely well in the linear tank system, their behavior deteriorates substantially when put in place of the conventional controller in the nonlinear tank model. This sensitivity to nonlinearities is investigated through alterations of the nonlinear tank equations of Appendix B to accommodate the optimal controllers.

First, the states representing the conventional controller are removed and replaced by control torques in elevation and traverse, much the same as the procedure for discretizing the linear equations. These control torques remain torque/speed limited, however.

Second, the feedback signals required by the controller (for the deterministic case, all variables represented by the discrete state variables; for the stochastic case, the six feedback signals) must be expressed in terms of the state variables of the nonlinear system.

Since the equations are nonlinear, superposition is no longer valid. The left and right side controllers must therefore be combined to accommodate left and right side inputs. The controllers operate in discrete time while the tank equations are integrated from the differential equations, thus requiring a hybrid simulation.

The deterministic optimal control scheme requires that a discrete state of length 126 be synthesized from the state variables and delay registers of the nonlinear equations to represent 51 left side delay states, 51 right side delay states, 6 left side forces
due to ground disturbance, 6 right side forces due to ground dis-
turbance, and the 12 tank state variables common to left and right
sides, a total of 126. A 2 x 126 G-matrix is constructed from the
left and right G-matrices as follows:

Columns 1 to 51 = columns 1 to 51 from $G_{\text{left}}$
Columns 52 to 102 = columns 1 to 51 from $G_{\text{right}}$
Columns 103 to 108 = columns 52 to 57 from $G_{\text{left}}$
Columns 109 to 114 = columns 52 to 57 from $G_{\text{right}}$
Columns 115 to 126 = columns 58 to 69 common to $G_{\text{right}}$ & $G_{\text{left}}$

This is the G-matrix that is multiplied by the total discrete state,
x(1) to x(126), described above to obtain the optimal torques in
elevation and traverse. Recall that these torques remain constant
over the period $\Delta t$. The nonlinear equations are integrated to the
next discrete time and, again, the discrete state is determined.
This state is multiplied by the G-matrix to give the optimal torques
for the next $\Delta t$ period. The process continues and the time re-
sponse for the random input is computed. The results for the ran-
dom ground disturbance used in section 3.3 are shown in figures 6.5
and 6.6. Note that while the absolute angles are now considerably
greater than for the linear deterministic control, they are still
exemplary compared to the conventional control. Fig. 6.6 shows
that considerably more torque is being used in both elevation and
traverse. This is the only scheme where the elevation torque has
reached saturation levels. The traverse torques are at saturation
28 percent of the time. These high levels of torque occur because
Fig 6.5 Nonlinear Simulation with Deterministic Optimal Control: Gun Traverse and Elevation Angles
Fig 6.6 Nonlinear Simulation with Deterministic Optimal Control: Elevation and Traverse Motor Torque
the optimal control is based on the assumption of viscous damping, whereas the actual damping is coulomb friction.

Deterministic control schemes are impractical, as noted before. There is no way to measure or determine without error states, such as delayed random inputs, that do not exist physically. Implementing stochastic optimal control, however, presents a problem. While the left and right G-matrices can be combined, the K-matrices cannot be combined because the bottom twelve rows are not common between left and right. This dissimilarity occurs due to different multiplications of the P-matrix in the deterministic and stochastic matrix Ricatti equations. Attempts at using the bottom twelve rows from one or the other, or their average, proved to be unstable, even for the linear case. This investigation cannot be carried out, therefore, unless a K-matrix is found that can be used to synthesize the total state from the feedback measurements.

The investigation is carried out for a single sided input, instead. The linear suspension dampers, however, are designed for two sided inputs. The linear dampers of the Kalman state on the unexcited side will exert considerably less force than their nonlinear counterparts on the tank model. This effect is reduced by removing altogether the nonlinear dampers on the unexcited side. The effect of the torque saturation nonlinearity is accounted for in the Kalman filter, since the torques demanded by the controllers are considerably beyond the limits. These remedies are to no avail, however, and the simulation continues to be unstable because of the filter divergence caused by the approximation of coulomb friction by
linear viscous friction.

Nonlinearizing the Kalman filter might work rather well. The friction terms in the linear equations to be discretized can be removed, thus eliminating the suspected source of filter divergence from the $P_\cdot$, $Q_u$, and $Q_w$-matrices. The $K$-matrix for a single side can be used unchanged to approximate corrections to the Kalman state. Further corrections would then be made to account for coulomb friction based on the velocities existing after correction by the $K$-matrix. The approximation is far from perfect but may eliminate the filter divergence experienced using linear frictions nevertheless.

6.5 Conclusions and Recommendations

A considerable improvement over conventional control was achieved through the application of classic optimal control theory for linear systems. The ideal observer assumed in deterministic optimal control combined with the optimal feedback coefficients takes advantage of the time delay states to gain a preview of disturbances that will be encountered by successive wheels. As the bumps are encountered, large doublets of torque are exerted by the gun drives in azimuth and in elevation to counteract the effects of hull accelerations on gun imbalance. These doublets effectively counter the ground disturbance, but the rapid reversals of high level torque may not be physically realizable. The response of the traverse drive motor torque to command changes has a lag time of only 0.004 sec, which is so fast relative to the command changes them-
selves in the conventional system that it was neglected altogether. The effect of this neglected lag in the optimal system should be investigated in view of the sharper control changes therein and, if necessary, incorporated into the model. Backlash in the drive train also may be significant, but more specific information about the anti-backlash mechanisms is needed in order to determine its effect.

The optimal observer indicated by stochastic optimal control theory for the conventional feedback signals takes into account limited state measurement and, roughly, the contamination of those measurements by noise. This control configuration also is a great improvement over the conventional control system.

The front wheel velocity measurements are added to the feedback signals to improve the ability of the observer to estimate time delay states and forces on the hull. (The preview feature already exists in the optimal feedback coefficients.) The marginal improvement in the performance index may not justify the cost and complexity of these signals. However, other potential benefits, such as reduced sensitivity, may become important and should be considered. The measurements also may assume increased importance in cruder more practical sub-optimal controllers of the same general type.

Both stochastic optimal systems inherit the high frequency torque switching from the deterministic optimal controller, a characteristic which may affect the feasibility of the optimal schemes.
Moreover, the amplitude of this rapid switching increases to saturation levels when the deterministic controller is exposed to nonlinearities. I suspect that, if the filter divergence problems are solved, the stochastic optimal systems may exhibit similar behavior.

In order to obtain a K-matrix for the two-sided input, the matrix Ricatti equation for the stochastic system will have to be rerun for P-, Q-w-, V-, and W-matrices describing a dual tread input system. With the current ground model the dimensions would be 126 x 126 for P, 126 x 2 for Q_w, 126 x 6 for K, etc., too large for the memory and computer time available at Lehigh University. Since the ground model used is somewhat arbitrary, one possible solution is halving the number of delay states, doubling Δt and adjusting the time constant for the stochastic filters appropriately. The assumption that motor torques remain constant over Δt may have to be changed if the period is doubled.

If filter divergence still persists when the V-matrix for dual tread inputs is used, the nonlinearization of the Kalman filter suggested in section 6.4 should be tried. Sensitivity of the controller to nominal gun angles and vehicle forward speed also should be investigated, bearing in mind the justification of added cost and complexity of the front wheel measurements.
BIBLIOGRAPHY


HITPRO computer job output provided by N. Coleman, Picatinney Arsenal, job name STRAHLKS dated 9 March 1978.

APPENDIX A - Symbol Lists and Parameter Values

STATE VARIABLES

<table>
<thead>
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<th>EQUIVALENT SYMBOLS</th>
<th>DESCRIPTION</th>
<th>UNIT</th>
<th>DERIVATIVE SYMBOL</th>
</tr>
</thead>
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<tr>
<td>x(1)</td>
<td>Time delay variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(discrete system only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x(51)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Force variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(discrete system only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x(57)</td>
<td></td>
<td></td>
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</tr>
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<td>Hull pitch angular rate</td>
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<td>D9</td>
</tr>
<tr>
<td>x(62)</td>
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<td>Z9</td>
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<td></td>
<td></td>
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<td>D11</td>
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**Elevation zero speed torque limit**  
N·m  
6997.  A

**Traverse zero speed torque limit**  
N·m  
15877.  A

Source Code for Values:

A: HITPRO VOL. III
   G.E. All-Electric Drive System.

B: HITPRO Run supplied by Dr. Coleman of Picatinny Arsenal.

C: Changed from 48 that appeared in A.

D: Arrived at by assuming that VAR (friction torque) in linear model should equal square of Coulomb friction value of nonlinear model.

E: Value assumed in Research Report by Brown, Johnson & Barr
APPENDIX B: Nonlinear Equations

\[ D_9 = [M_{SZ} + M_{DZ} + (CT24-CT25)(Z10)(Z11)]/[CT36 + CT26 - (\sin^2 Z24)(CT27)] \]

\[ D_{10} = [M_{SX} + M_{DX} + (CT25-CT26)(Z11)(Z9) - (CT32)(Z10)]/[CT36 + CT24 \\( \sin^2 Z24 \)(CT27)] \]

\[ D_{11} = [M_{SY} + M_{DY} + (CT26-CT24)(Z9)(Z10) - (CT19)(Z11) - (CT20)(Z14)]/CT25 \]

\[ D_{12} = [Z9 - (Z14)(Z10)]/(\cos Z13) \]

\[ D_{13} = Z10 + (Z14)(Z9) \]

\[ D_{14} = Z11 - (\sin Z13)(D_{12}) \]

\[ D_{15} = (F_{SY})(CT23) - 9.81 \]

\[ D_{16} = Z15 \]

\[ D_{21} = C41[(\text{FRICTION TQ EL}) + \text{MOTOR TQ EL}) + (\text{DISTURBANCE TQ EL})] \]

\[ \text{FRICTION TQ EL} = -(C49)[\text{SIGN}\{Z21 - (Z9)(\cos Z24) + Z10(\sin Z24)\}] \]

\[ \text{MOTOR TQ EL}^* = C27[Z33 + C25(Z31 - C33(Z21) + (C33 - C35) \\( \sin Z24 \)) + Z9(\cos Z24)\}] \]

\[ \text{DISTURBANCE TQ EL} = (C43 - CT14(C45))[D9(\cos Z24) + D10(\sin Z24)] \]

\[ - C45[9.81(\cos Z12)(\cos Z13) + D15 - D9(CT28)] - C57(Z23) + C47 \]

\[ D_{22} = C42[(\text{FRICTION TQ TR}) + (\text{MOTOR TQ TR}) + (\text{DISTURBANCE TQ TR})] \]

\[ \text{FRICTION TQ TR} = -(C50)[\text{SIGN}(Z22 - Z11)] \]

\[ \text{MOTOR TQ TR}^* = C28[Z34 + C26(Z32)] \]

\[ \text{DISTURBANCE TQ TR} = C44(D11) - C48(\text{ACCEL } X_{tu}) + C46(\text{ACCEL } Z_{tu}) \]

\[ + C58(Z9(\cos Z24) + Z10(\sin Z24))(Z10(\cos Z24) - Z9(\sin Z24)) \]

\[ \text{ACCEL } X_{tu} = [9.81 \{(\sin Z12) + (\cos Z12)(\sin Z13)\} + CT28(D11)] \]

\[ (\sin Z24) - CT29 D9(\cos Z24) + D10(\sin Z24) \]

*Motor torques are limited as in fig. B1
\[ \text{ACCEL } Z_{tu} = [9.81 \left\{ (\sin Z_{12}) + (\cos Z_{12}) (\sin Z_{13}) (Z_{14}) \right\} (\sin Z_{24}) + [9.81 \left\{ (\sin Z_{12}) (Z_{14}) - (\cos Z_{12}) (\sin Z_{13}) + CT(D_{11}) \right\}] (\cos Z_{24}) + CT29 \left\{ D_{10}(\cos Z_{24}) - D_{9}(\sin Z_{24}) \right\} \]

\[ \text{M}_{DX} = -(\text{MOTOR TQ EL})(\sin Z_{24}) \]
\[ \text{M}_{DZ} = -(\text{MOTOR TQ EL})(\cos Z_{24}) \]
\[ \text{M}_{DY} = -(\text{MOTOR TQ EL}) \]
\[ D_{23} = Z_{21} - Z_{10}(\sin Z_{24}) - Z_{9}(\cos Z_{24}) \]
\[ D_{24} = Z_{22} \]
\[ D_{29} = -C_{11}(C_{29})Z_{21} \]
\[ D_{30} = -C_{12}(C_{30}) \left[ Z_{22}(\cos Z_{23}) - \left\{ Z_{10}(\cos Z_{24}) - Z_{9}(\sin Z_{24}) \right\} (\sin Z_{23}) \right] \]
\[ D_{31} = Z_{29} \]
\[ D_{32} = Z_{30} + (C_{16}/C_{12})D_{30} \]
\[ D_{33} = Z_{21}[Z_{31} - C_{33}(Z_{21}) + (C_{33} - C_{35}) \left\{ Z_{9}(\cos Z_{24}) + Z_{10}(\sin Z_{24}) \right\} ] \]
\[ D_{34} = C_{22}(Z_{32}) \]
\[ D_{W}(i) = (w_{i} - ZW(i))/t \quad i = 1, 2, ..., 12 \]
\[ w_{i} = \text{STOCHASTIC INPUT, Mean} = 0 \quad \text{variance} = (0.305/4)^2 \]
\[ F(i) = [-Z_{12}(XW^{i}) - Z_{13}(CT{16}) - Z_{16} + ZW(i)]k \]
\[ \times \text{SIGN}\left[ Z_{9}(XW^{i}) + Z_{10}(CT{16}) + Z_{15} - DW(i) \right]^{b} \quad \text{for } i = 1 \text{ to } 6 \]
\[ F(i) = [-Z_{12}(XW^{i}) - Z_{13}(CT{16}) - Z_{16} + ZW(i)]k \]
\[ \times \text{SIGN}\left[ Z_{9}(XW^{i}) - Z_{10}(CT{16}) + Z_{15} - DW(i) \right]^{b} \quad \text{for } i = 7 \text{ to } 12 \]

\* \( k \) is a function of the displacement given in the preceding brackets [].

\** \( b \) is non-zero for wheels 1, 2, 6, 7, 8, and 12 only

\[ M_{SZ} = \sum_{i=1}^{12} F(i)(XW^{i}) \]
\[ M_{SX} = \sum_{i=1}^{6} F(i) (CT16) - \sum_{i=7}^{12} F(i) (CT16) \]

\[ F_{SY} = \sum_{i=1}^{12} F(i) \]

\[ M_{SY} = -(\sin Z12) M_{SX} \]
Elevation | Traverse
---|---
$T_{\text{max}}$ (N-m) | 6997 | 15877
$A$ (N-m) | 13990 | 31687
Slope (Nms/ rad) | -8961 | -32966

Fig. B1 Torque Limits
APPENDIX C: Linearized Equations

The following are the equations after they have been linearized about a zero equilibrium for all variables except the following:

\[ Z_{24} = AZ_{24} \]
\[ Z_{23} = AZ_{23} \]
\[ Z_{33} = AZ_{33} \]

AZ24 and AZ23 are the desired directions of the gun in traverse and elevation respectively. A non-zero AZ33 is required to maintain the desired elevation angle (it is related to elevation torque) and is given by the expression:

\[ AZ_{33} = \frac{[C_{45}(9.81) - C_{47} + C_{57}(AZ_{23})]}{C_{27}} \]

Also, in linearizing, all hull yaw motion was eliminated since its effects were determined to be minimal. Thus:

\[ Z_{11} = Z_{14} = D_{11} = D_{13} = 0. \]

All "Z" variables are now changes from their equilibrium positions.

\[ D_{9} = \frac{(M_{SZ} + M_{DZ})}{[CT_{36} + CT_{26} - (SIN^{2}AZ_{24})CT_{27}]} \]
\[ D_{10} = \frac{[M_{SX} + M_{DX} - CT_{32}(Z_{10})]}{[CT_{36} + CT_{24} - (SIN^{2}AZ_{24})CT_{27}]} \]
\[ D_{12} = Z_{9} \]
\[ D_{13} = Z_{10} \]
\[ D_{15} = (F_{SY})CT_{23} \]
\[ D_{16} = Z_{15} \]
\[ D_{21} = C_{41}[(\text{FRICTION TQ EL}) + (\text{MOTOR TQ EL}) + (\text{DISTURBANCE TQ EL})] \]

FRICTION TQ EL = -CFE[\[Z_{21}\cos(AZ_{24}) + Z_{10}\sin(AZ_{24})\]^-]
MOTOR TQ EL = C27[Z33+C25Z31-C33(Z21)+(C33-C35)
       \[\{Z10(SIN AZ24)+Z9(COS AZ24)\}\]  
DISTURBANCE TQ EL = (C43-C14(C45))[D9(COS AZ24)+D10(SIN AZ24)]
       -C45(D15-D9(CT28))-C57(Z23)  
D22 = C42[(FRICTION TQ TR)+(MOTOR TQ TR)+(DISTURBANCE TQ TR)]  
FRICTION TQ TR = -CFT(Z22)  
MOTOR TQ TR = C28[Z34+C26(Z32)]  
DISTURBANCE TQ TR = -C48(\Delta ACCEL X_{tu})+C46(\Delta ACCEL Z_{tu})  
\[\begin{align*}  
\Delta ACCEL X_{tu} &= 9.81(Z12)(COS AZ24)+9.81(Z13)(SIN AZ24) 
   -CT29[D10(COS AZ24)-D9(SIN AZ24)] \\
\Delta ACCEL Z_{tu} &= 9.81(Z12)(SIN AZ24)-9.81(Z13)(COS AZ24) 
   +CT29[D10(COS AZ24)-D9(SIN AZ24)] 
\end{align*} \]  
\[\begin{align*}  
M_{DX} &= -(MOTOR TQ EL)(SIN AZ24)-C27(AZ33)(COS AZ24)Z24 \\
M_{DZ} &= -(MOTOR TQ EL)(COS AZ24)+C27(AZ33)(SIN AZ24)Z24 
\end{align*} \]  
D23 = Z21-Z10(SIN AZ24)-Z9(COS AZ24)  
D24 = Z22  
D29 = -C11(C29)Z21  
D30 = -C12(C30)[Z22(COS AZ23)-\{Z10(COS AZ24)-Z9(SIN AZ24)\}](SIN AZ23)  
D31 = Z29  
D32 = Z30+(C16/C12)D30  
D33 = C21[Z31-C33(Z21)+(C33-C35)\{Z9(COS AZ24)+Z10(SIN AZ24)\}]  
D34 = C22(Z32)  
DW(i) = (w_i-ZW(i)) \quad i = 1,2,\ldots,12  
  
w_i = STOCHASTIC INPUT, variance = (0.305/4)^2, \quad mean = 0.
DSE = -ZSE/TAU + \left[ Z_{21} - \left\{ Z_{9}(\cos AZ_{24}) + Z_{10}(\sin AZ_{24}) \right\} \right]

DST = -ZST/TAU + \left[ Z_{22}(\cos AZ_{23}) - \left\{ Z_{10}(\cos AZ_{24}) - Z_{9}(\sin AZ_{24}) \right\} \right](\sin AZ_{23})

F(i) = \left[ -Z_{12}(XW_{i}) \pm Z_{13}(CT_{16}) - Z_{16} + ZW(1) \right]k

- \left[ Z_{9}(XW_{i}) \pm Z_{10}(CT_{16}) + Z_{15} - DW(i) \right]b

Upper signs for i=1 to 6; lower for i=7 to 12.

*k is a constant, the main spring gradient.

**b is non-zero for wheels 1, 2, 6, 7, 8, 12 only.

M_{SZ} = \sum_{i=1}^{12} F(i)(XW_{i})

M_{SX} = \sum_{i=1}^{6} F(i)(CT_{16}) - \sum_{i=7}^{12} F(i)(CT_{16})

F_{SY} = \sum_{i=1}^{12} F(i)

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Michael C. Barr was born in Reading, Pa. on July 13, 1955, second of four sons born to Mr. and Mrs. Ronald G. Barr, Hamburg, Pa. He grew up in Hamburg, graduating from Hamburg Area High School in June, 1973, with honors. He graduated with honors from Lehigh University in May, 1977, receiving a Bachelor of Science degree in Mechanical Engineering. He entered Lehigh University graduate school in September, 1977, working toward a Master of Science degree in Mechanical Engineering in May, 1979. During this period he worked as a research assistant under Drs. Forbes T. Brown and Stanley H. Johnson, on a U.S. Army contract investigating the optimal control of tank turrets.

During the summer of 1976, he was employed as a summer student engineer by AMP Incorporated, Harrisburg, Pa. For several months in early 1979, he was employed by Bethlehem Steel in a temporary position as a consultant in the process control group.

He is a member of Pi Tau Sigma (1976) and Tau Beta Pi (1977).