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LRFD

A Comparison with Allowable Stress Design and Plastic Design

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STRUCTURAL DESIGN OBJECTIVES

- FUNCTIONAL REQUIREMENTS
- SUPPORT LOAD , PROVIDE STIFFNESS
- ECONOMICAL REQUIREMENTS

Fig. 1. Structural Design Objectives. The main objectives of Structural Design do not change with the design method.

Steps in Design

1. Function
2. Structure and Loading
3. Loading Conditions
4. Preliminary Design
5. Analysis
6. Selection of Section
7. Secondary Design Check

Fig. 2. The steps in design are also independent of the method that is used.

Allowable Stress Design
Working Stress Design
Elastic Design

Plastic Design
Ultimate Strength Design
Limit Design

Load and Resistance Factor Design
Load Factor Design
Limit States Design

Fig. 3. There are three groups of design concepts: The "allowable stress" group, the "plastic design" group, and the "load and resistance factor" group.

$$F_{\ell} L \leq F_{r} R$$

$$\gamma Q \leq \phi R$$

Fig. 4. The LRFD formulation is simple: The load factor times the load effect must be less than the resistance factor times the resistance of the member. (The format shown in the second line of the formulation is typical of that which appears in research papers.)

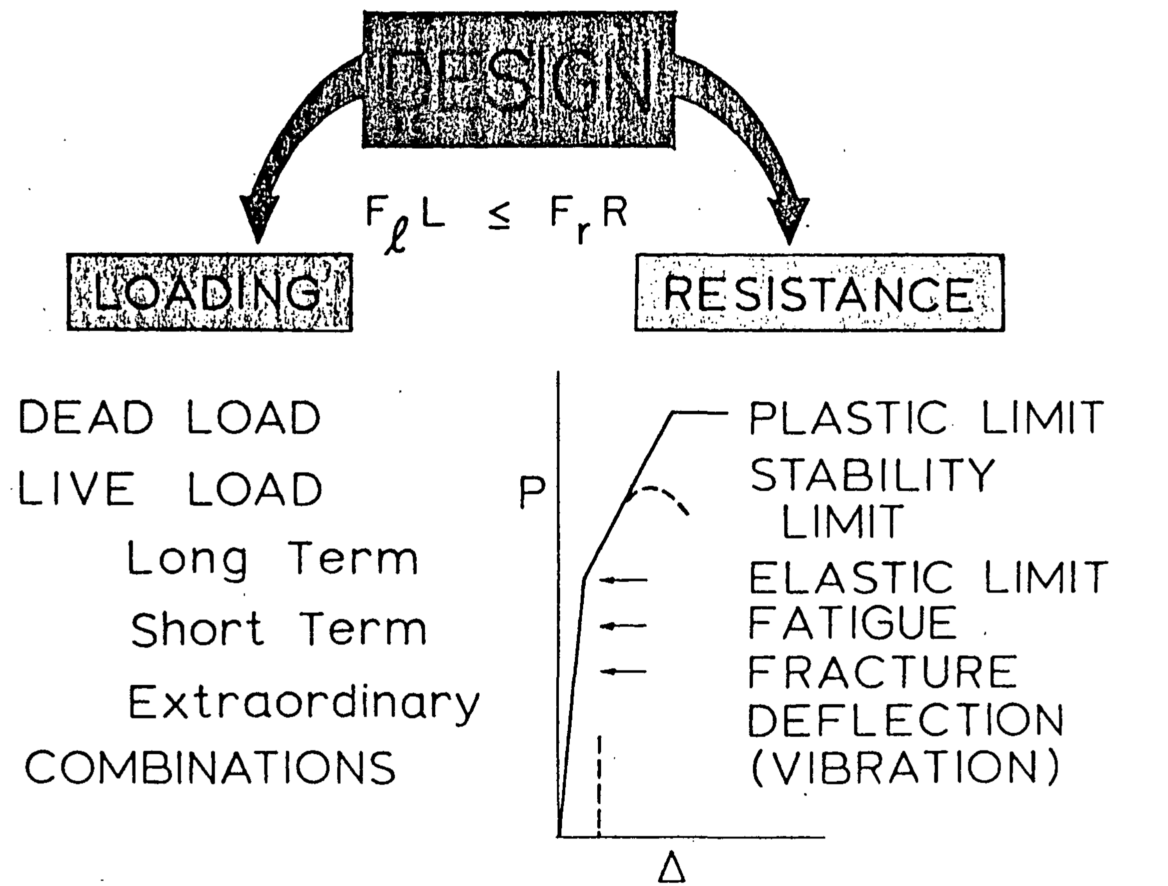


Fig. 5. LRFD involves the examination of the loading function (left) and the resistance function (right). Design is equating the two through analytical techniques and the use of the basic LRFD equation.

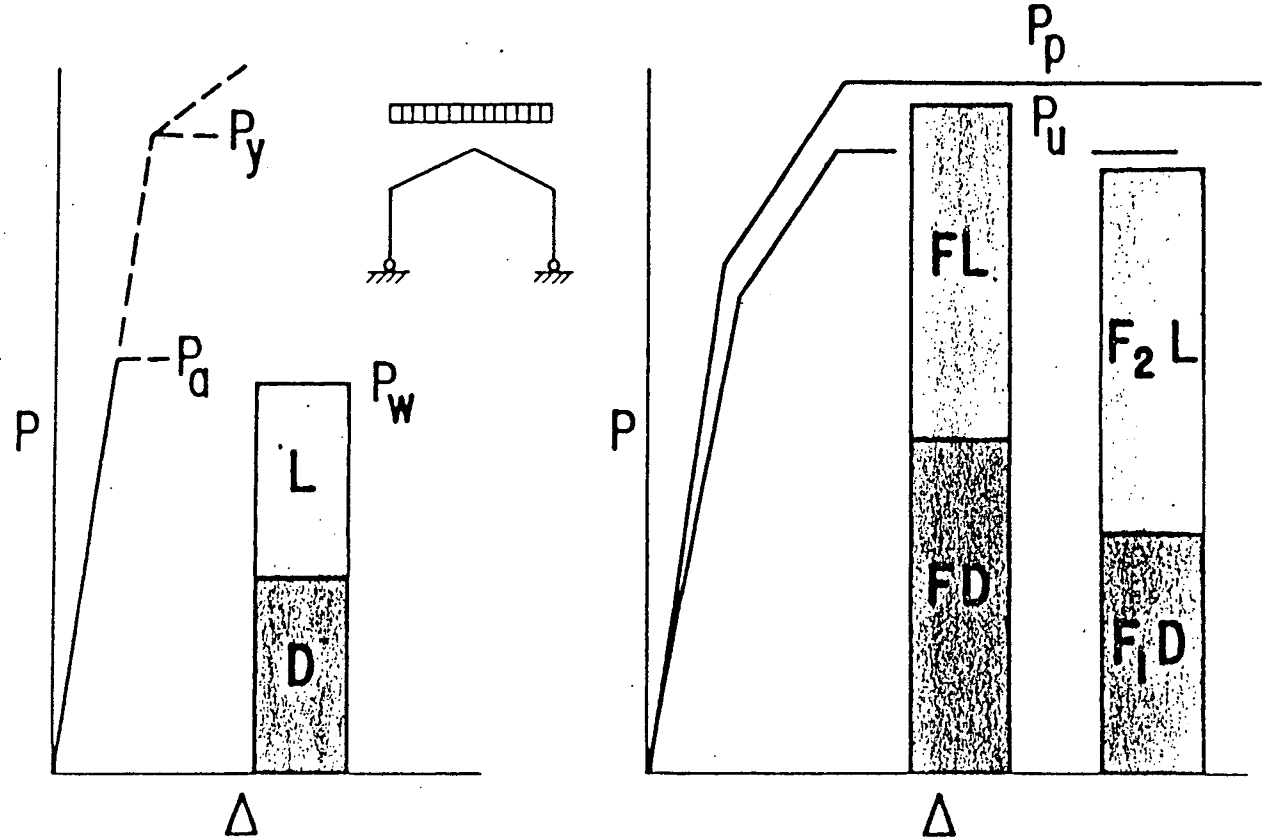


Fig. 6. LRFD is compared with allowable stress design at the left. At the right it is compared with a form of load factor design, first with single load factors and then with multiple load factors. The comparison is, in fact, with a plastic design, except that the use of multiple load factors can lead to lighter members.

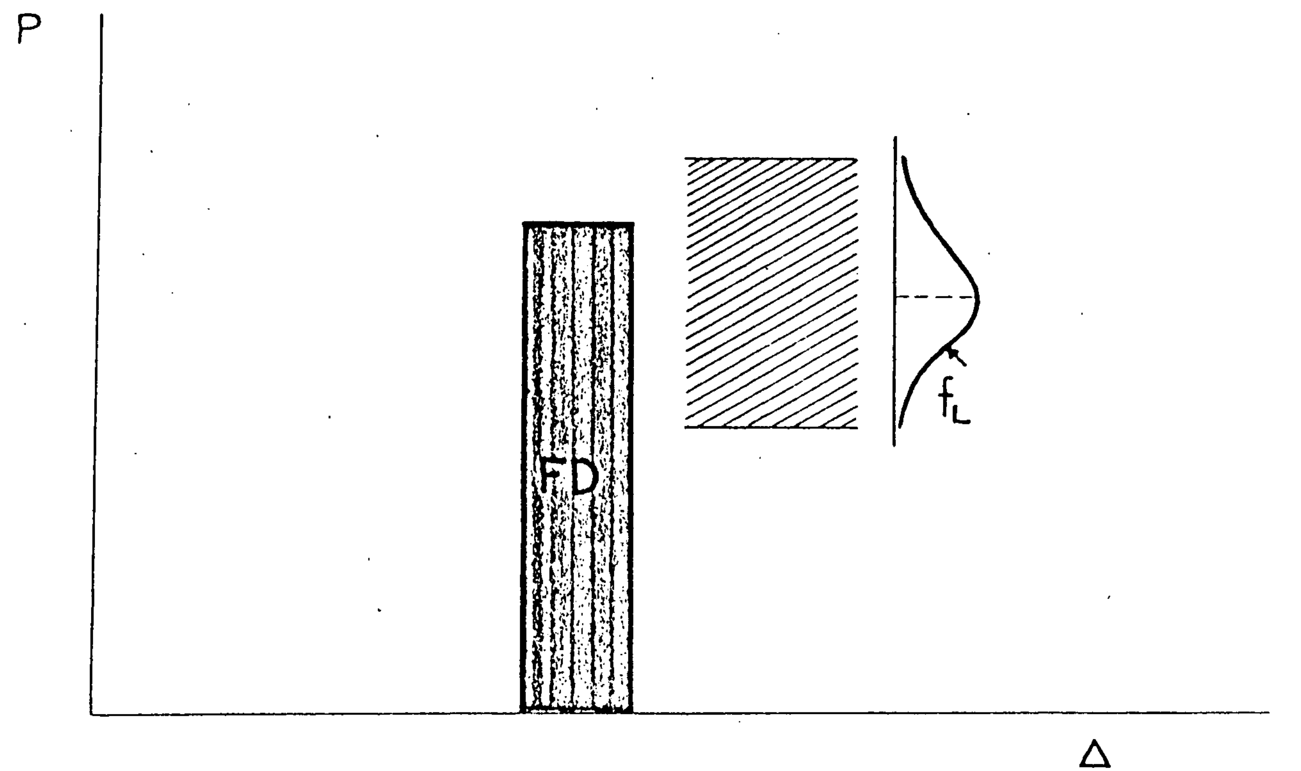


Fig. 7. The load deflection curves and load bars of Fig. 6 are simplistic. Actually there is uncertainty. This figure shows an example of uncertainty in the loading function f_{ℓ}.

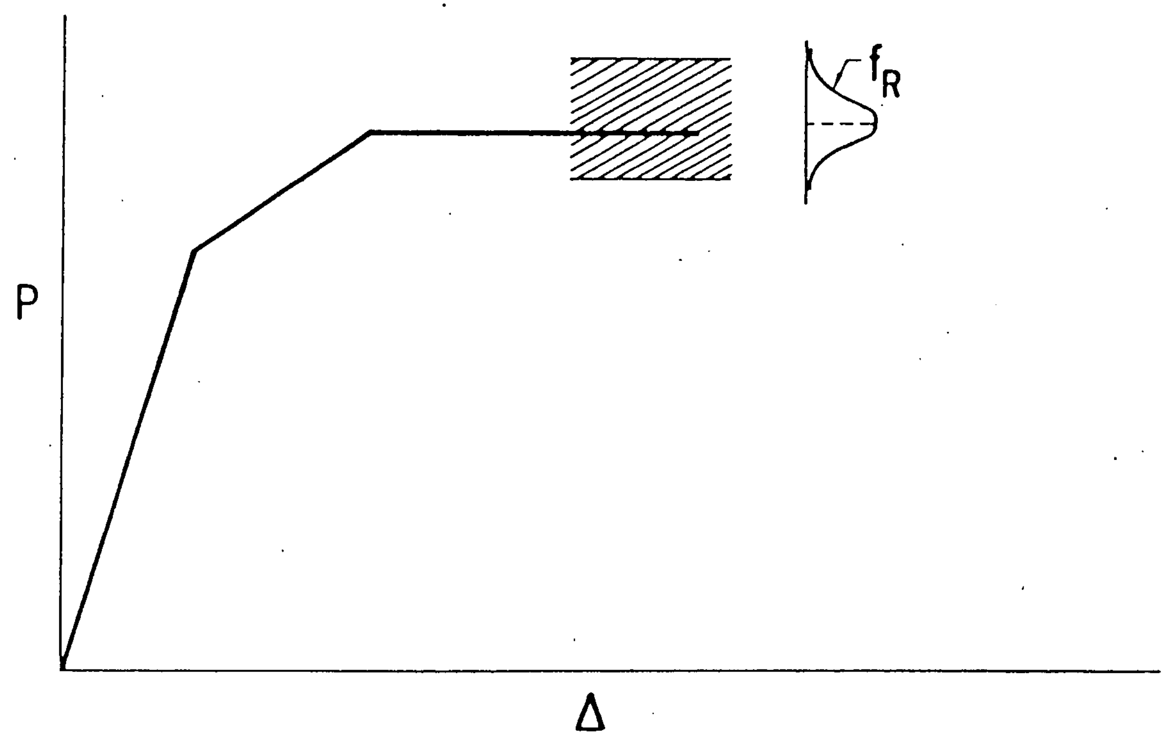


Fig. 8. The uncertainty in response is shown (f_r). Comparing with Fig. 7, there is less variation in response or resistance than there is in load.

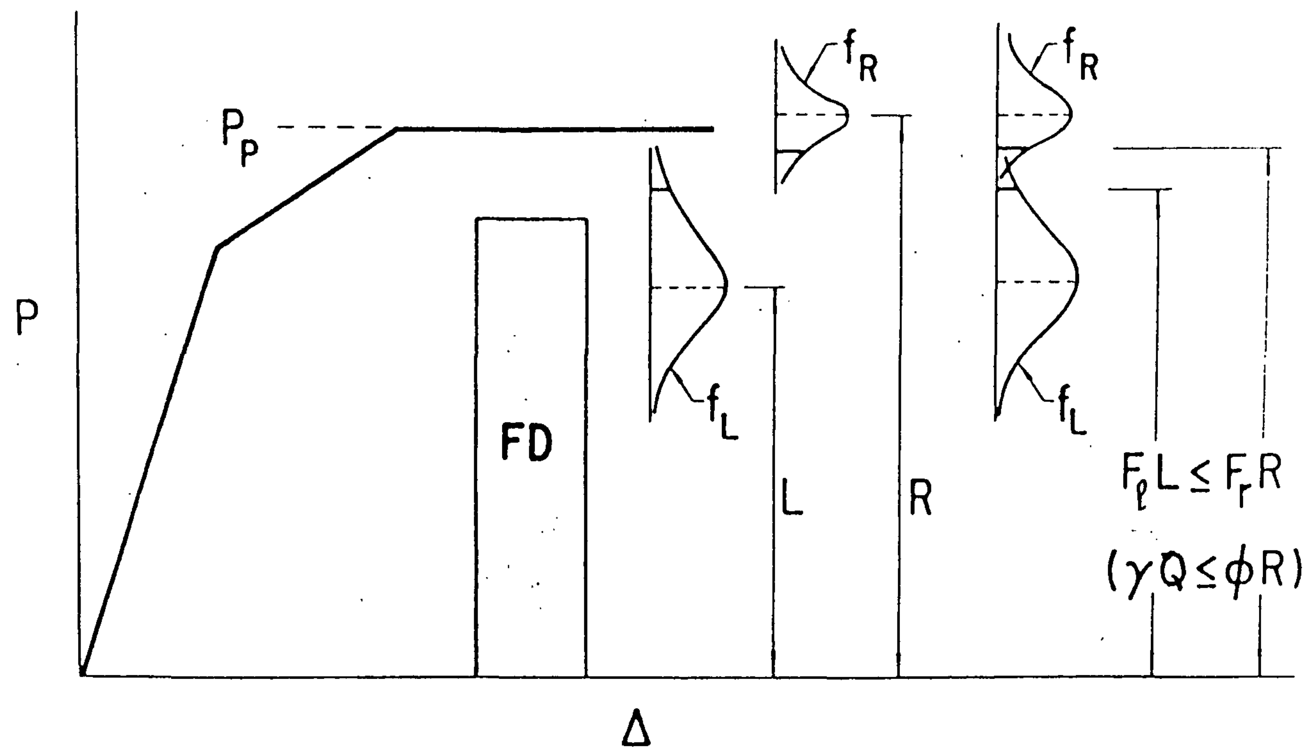


Fig. 9. Failure is defined according to the following criterion: The maximum load, f_ℓ is less than the minimum possible resistance (f_r x R).

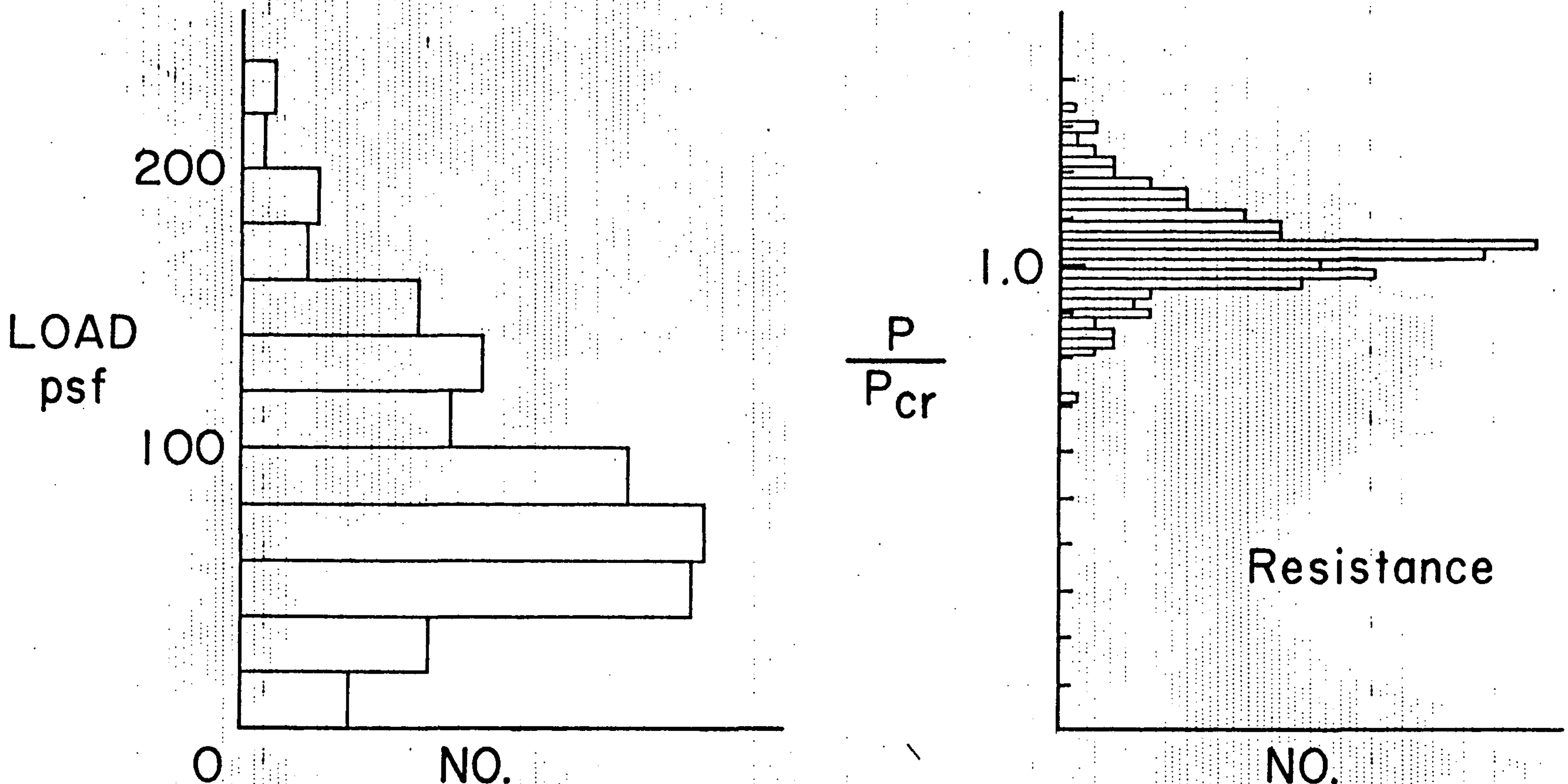


Fig. 10. Actual measured load and resistance data. To the left is shown the variation in floor load. To the right, the variation in resistance of continuous beams. These observations illustrate the greater scatter in load as compared to that of resistance.

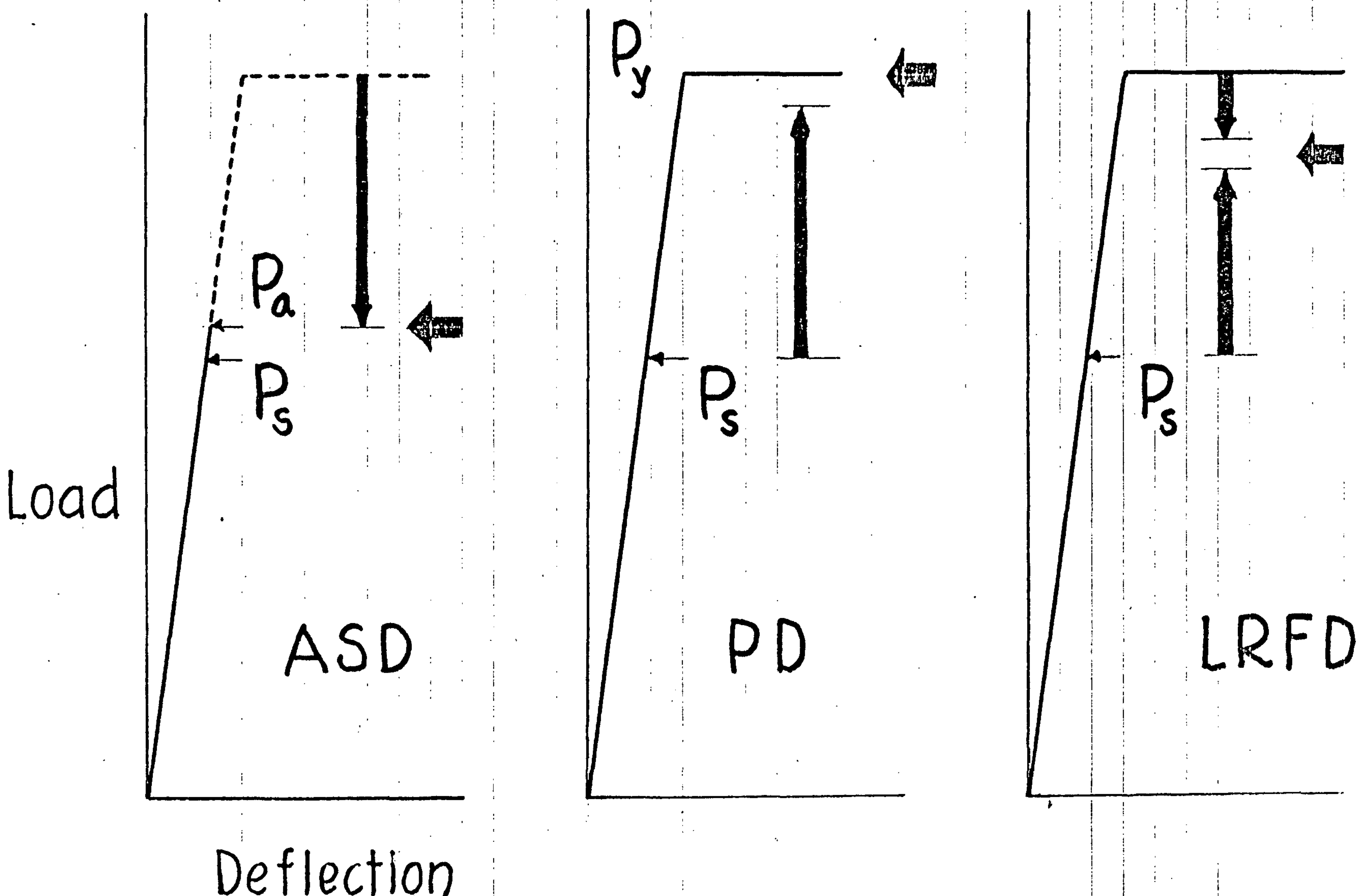


Fig. 11. How safety is achieved in the three design methods. Allowable Stress Design: Start with yield and come down to allowable. Plastic Design: Start with the service load and factor up to design ultimate load. LRFD: Factor up from service load and factor down from nominal resistance of the structure. The arrows show where design attention is focussed in each of the three methods.

APPROXIMATIONS IN ANALYSIS
APPROXIMATIONS IN DESIGN
STRESS CONCENTRATIONS AND RESIDUALS
VARIATION IN PROPERTIES
VARIATION IN DIMENSIONS
WORKMANSHIP
LOCATION
VARIATION IN LOAD -- TYPE
COMBINATION OF LOADS
INTENDED USE

Fig. 12. Tabulation of the approximations and uncertainties in design, workmanship, and loading. These factors must be accommodated in any design method.

		Load Factor
Tension	$F_t = 0.60 F_y$	1.67
Bending	$F_b = 0.66 F_y$	1.70
Compression	$F_a = \dfrac{\left[1 - \dfrac{(Kl/r)^2}{2C_c^2}\right] F_y}{\dfrac{5}{3} + \dfrac{3(Kl/r)}{8C_c} - \dfrac{(Kl/r)^3}{8C_c^3}}$	1.67-1.92

Fig. 13. How safety is achieved in Allowable Stress Design for three types of loading. To the right is shown the corresponding load factors.

1.3	GRAVITY PLUS LATERAL LOAD
1.7	TENSION MEMBERS SHORT COLUMNS BEAMS PLASTIC DESIGN
1.9	LONG COLUMNS
2.2	RIVETS BOLTS WELDS SHEAR CONNECTORS

Fig. 14. Uncertainty in Plastic Design is accommodated by load factors. Note the rational progression of load factors depending on the importance of member or uncertainty of loading or response.

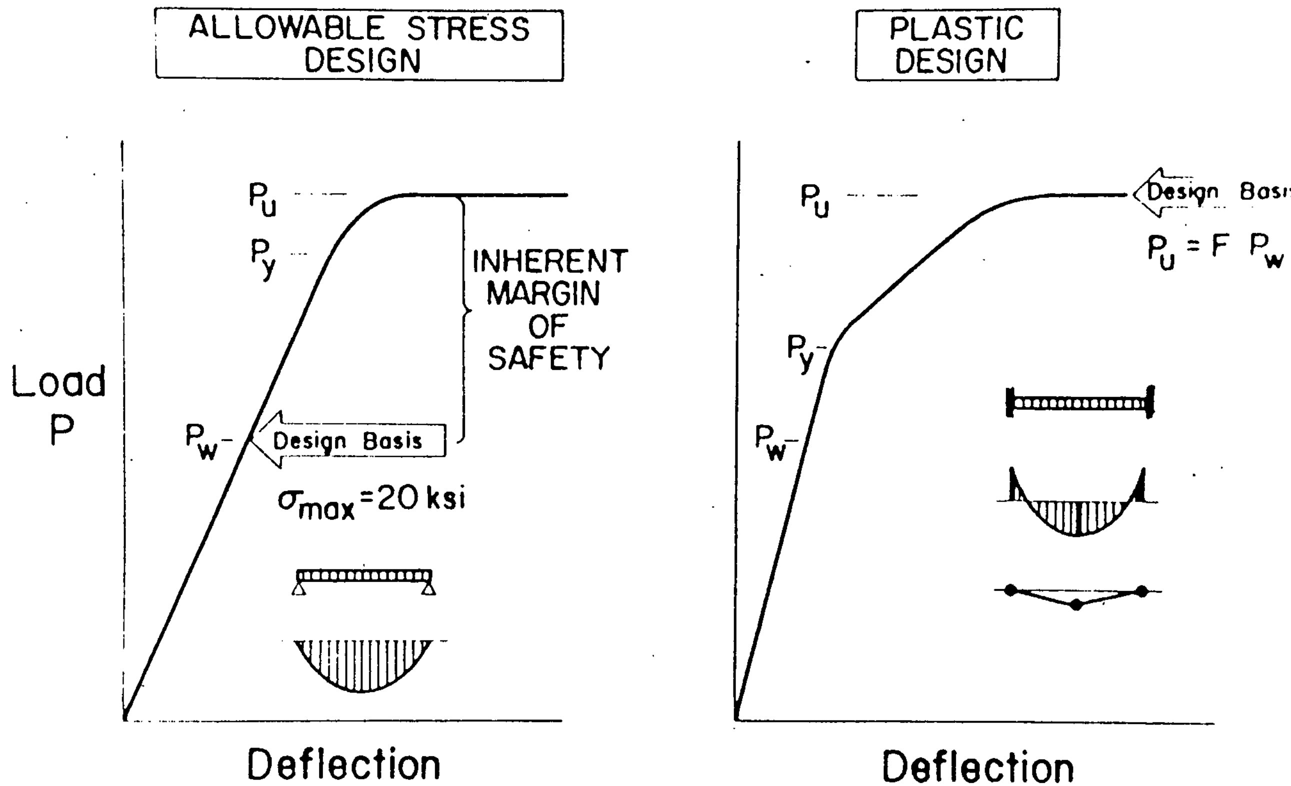


Fig. 15. The philosophy behind selection of load factor F in Plastic Design is this: The same safety in continuous beams as inherent in the past ASD of simple beams.

$$F = \frac{P_p}{P_a} = \frac{M_p}{M_a} = \frac{\sigma_y Z}{\sigma_a S}$$

$$\sigma_a = 0.66\,\sigma_y$$

$$\frac{Z}{S} = f \cong 1.12$$

$$F = \frac{\sigma_y}{0.66\,\sigma_y}\,1.12 = \cancel{1.68} \qquad \boxed{1.70}$$

Fig. 16. The calculation of F based on concept of Fig. 15.

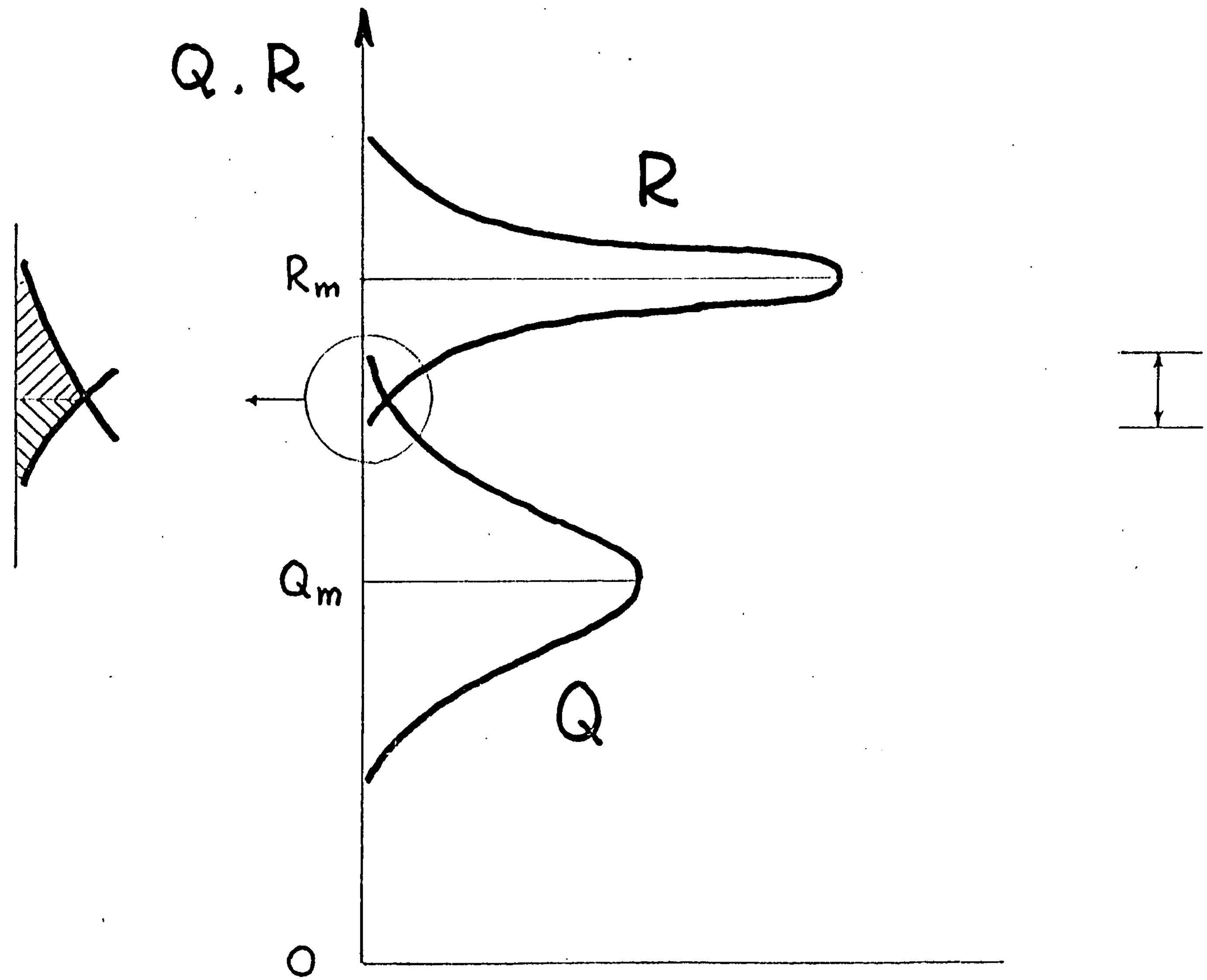


Fig. 17. Examining safety in LRFD. Variations of load and resistance indicated in Fig. 9 are shown. Resistance at top, load at bottom. What is failure? When Q is greater than R. Area under curves (see shaded at left) is related to probability of failure.

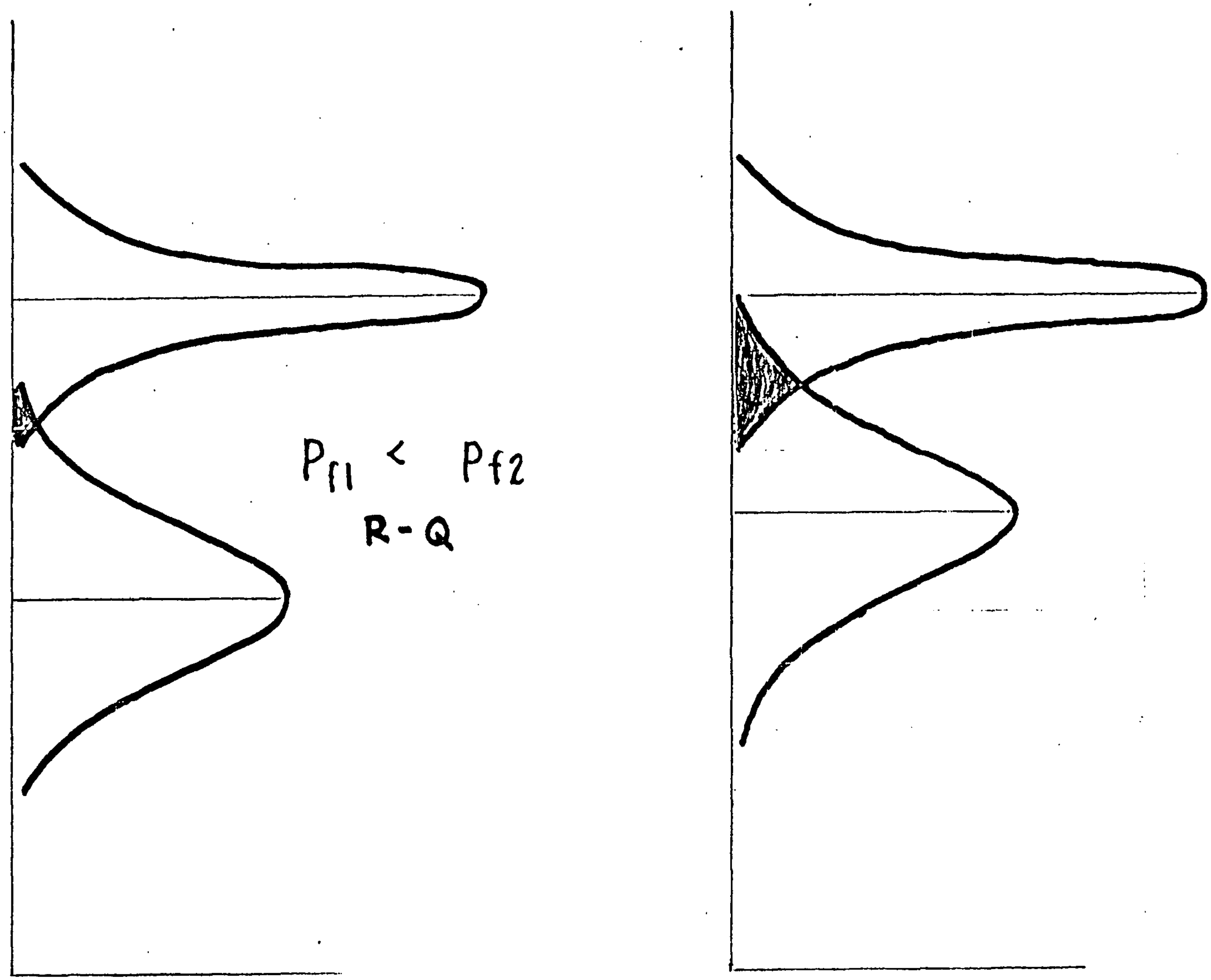


Fig. 18. Safety depends upon two things: The difference in Q and R and the variability of Q and R. This figure illustrates the first.

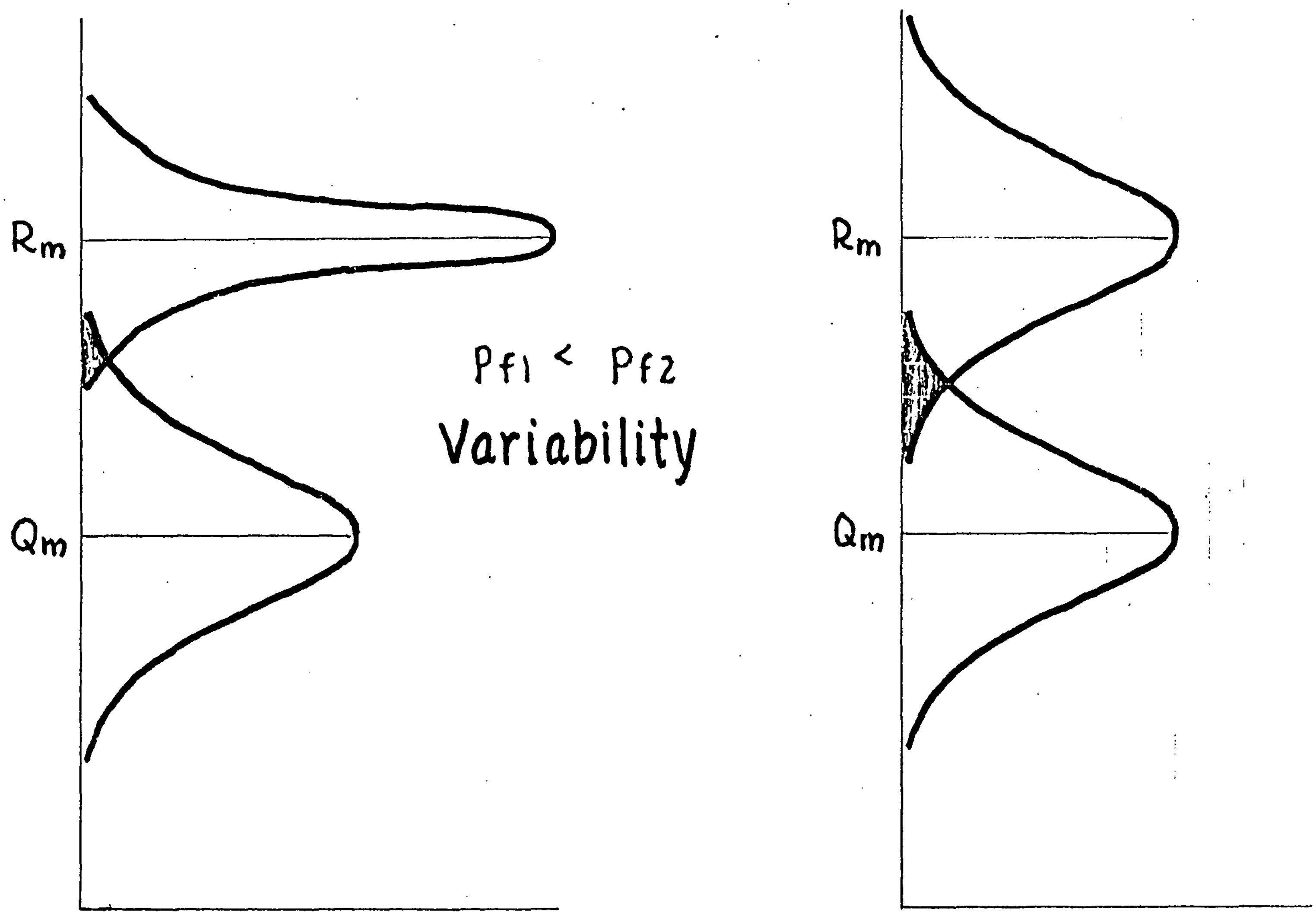


Fig. 19. An illustration of how an increase in the variability (in this case variability of load) increases the probability of failure.

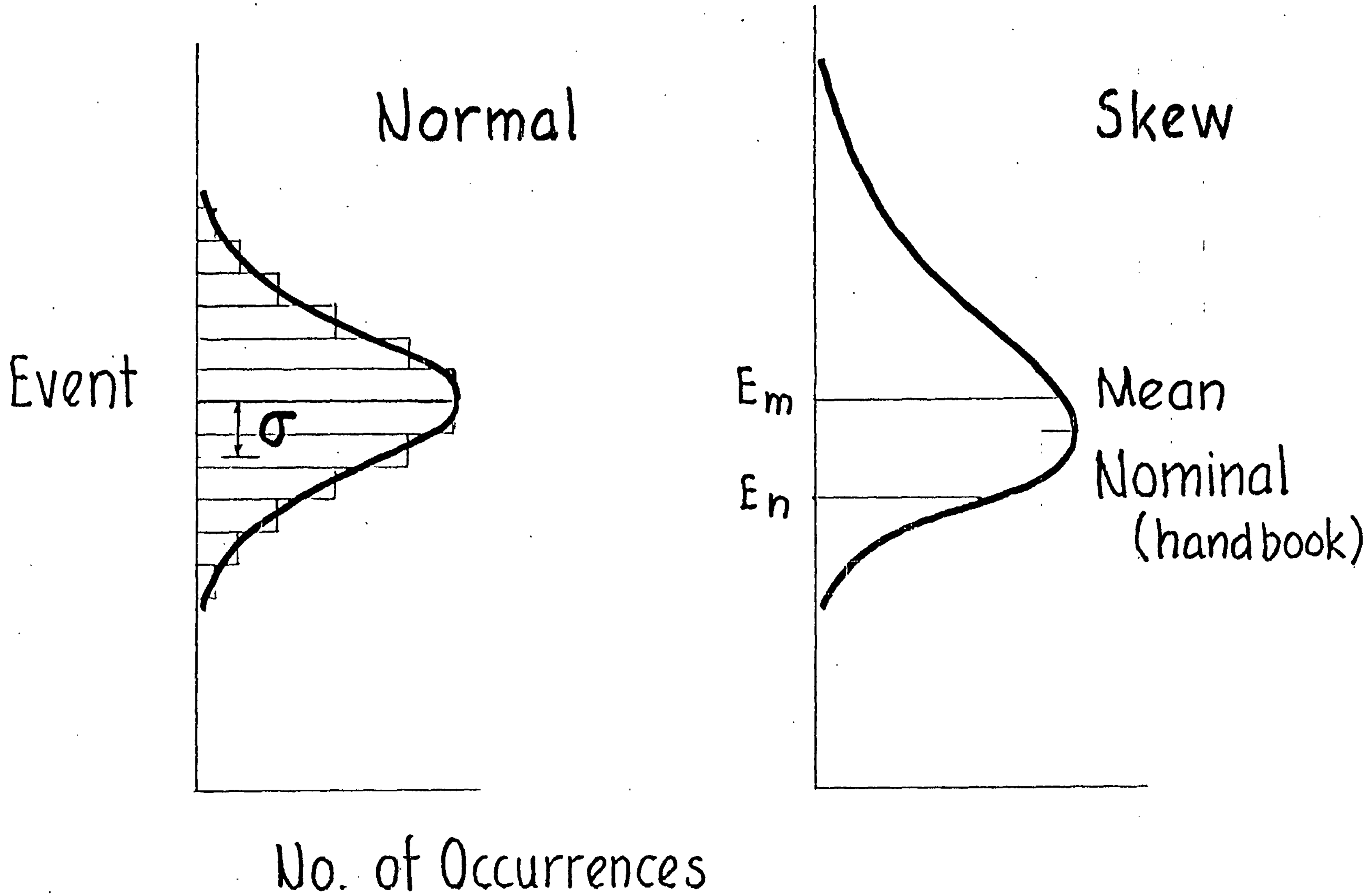


Fig. 20. Some of the terms and functions associated with measuring and evaluating variability: Normal and skew distributions. Standard deviations σ. Mean and nominal (handbook) events (E_m and E_n, respectively).

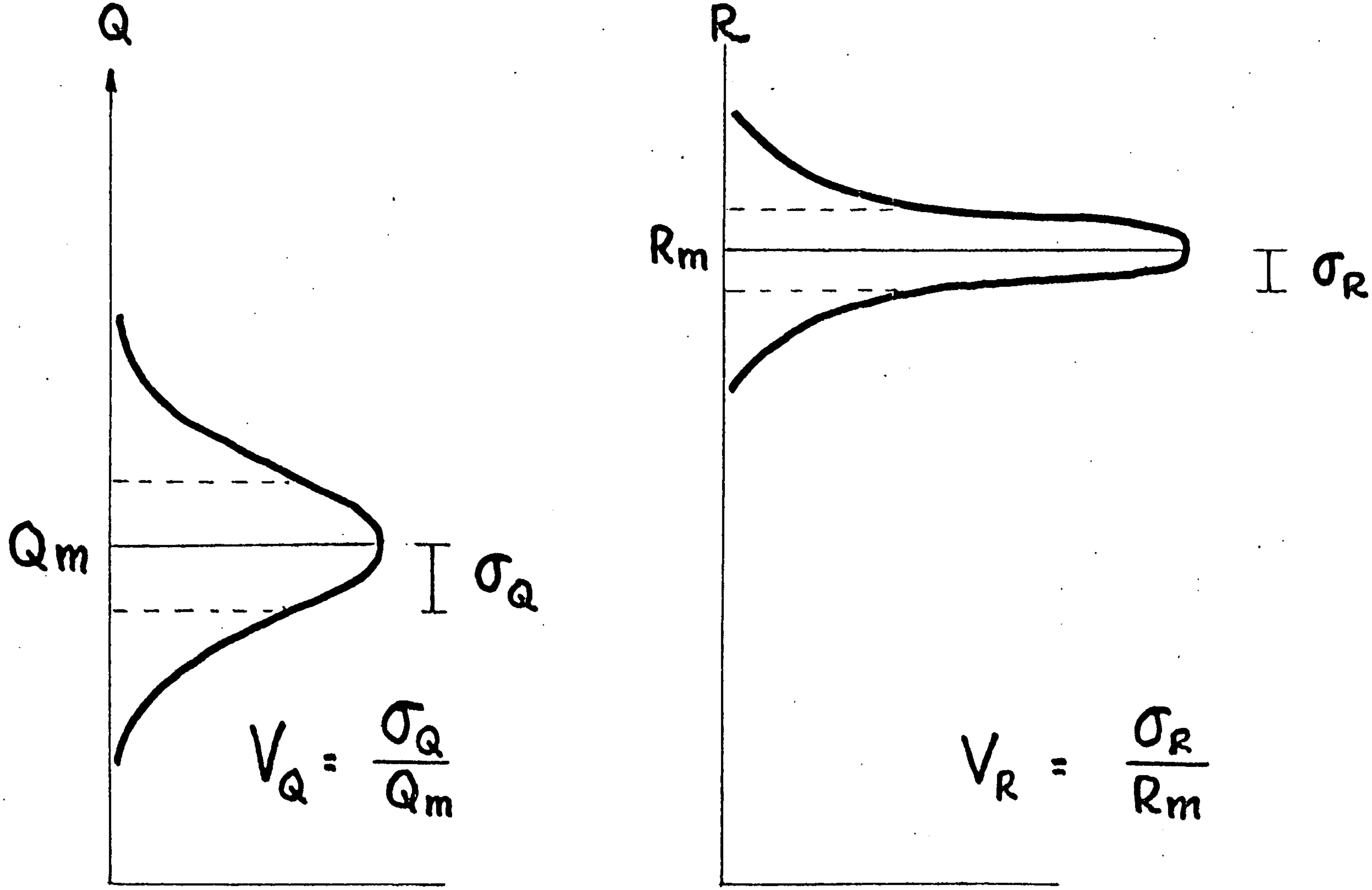


Fig. 21. Functions shown are the mean value of load, Q_m standard deviation of load σ_Q, mean value of resistance R_m, and standard deviation of resistance σ_R. Coefficient of variation is ratio of standard deviation to mean value.

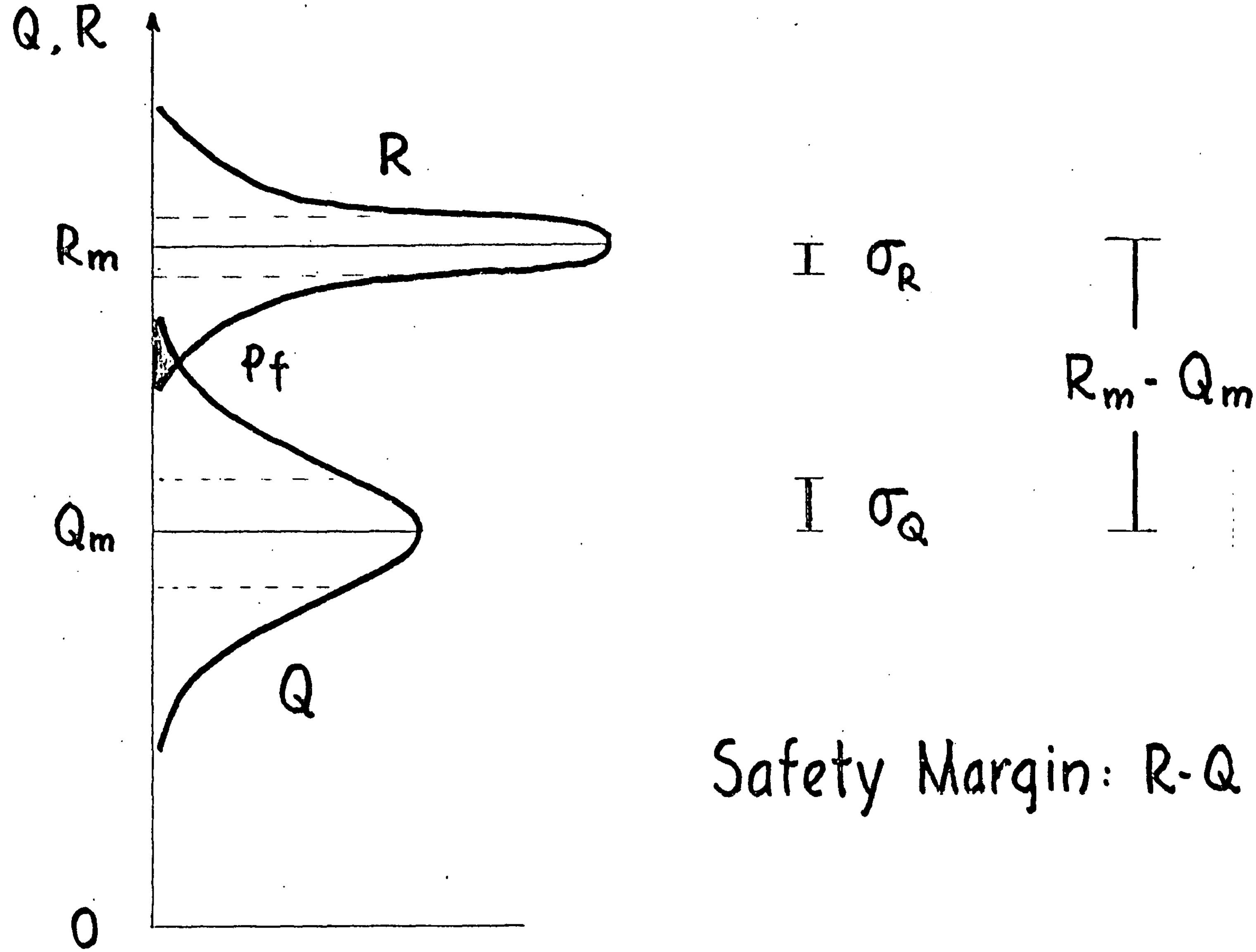


Fig. 22. Failure can now be defined more specifically. The safety margin is R − Q, so safety will depend on two things: $R_m - Q_m$ and on σ. Area under curve is probability of failure.

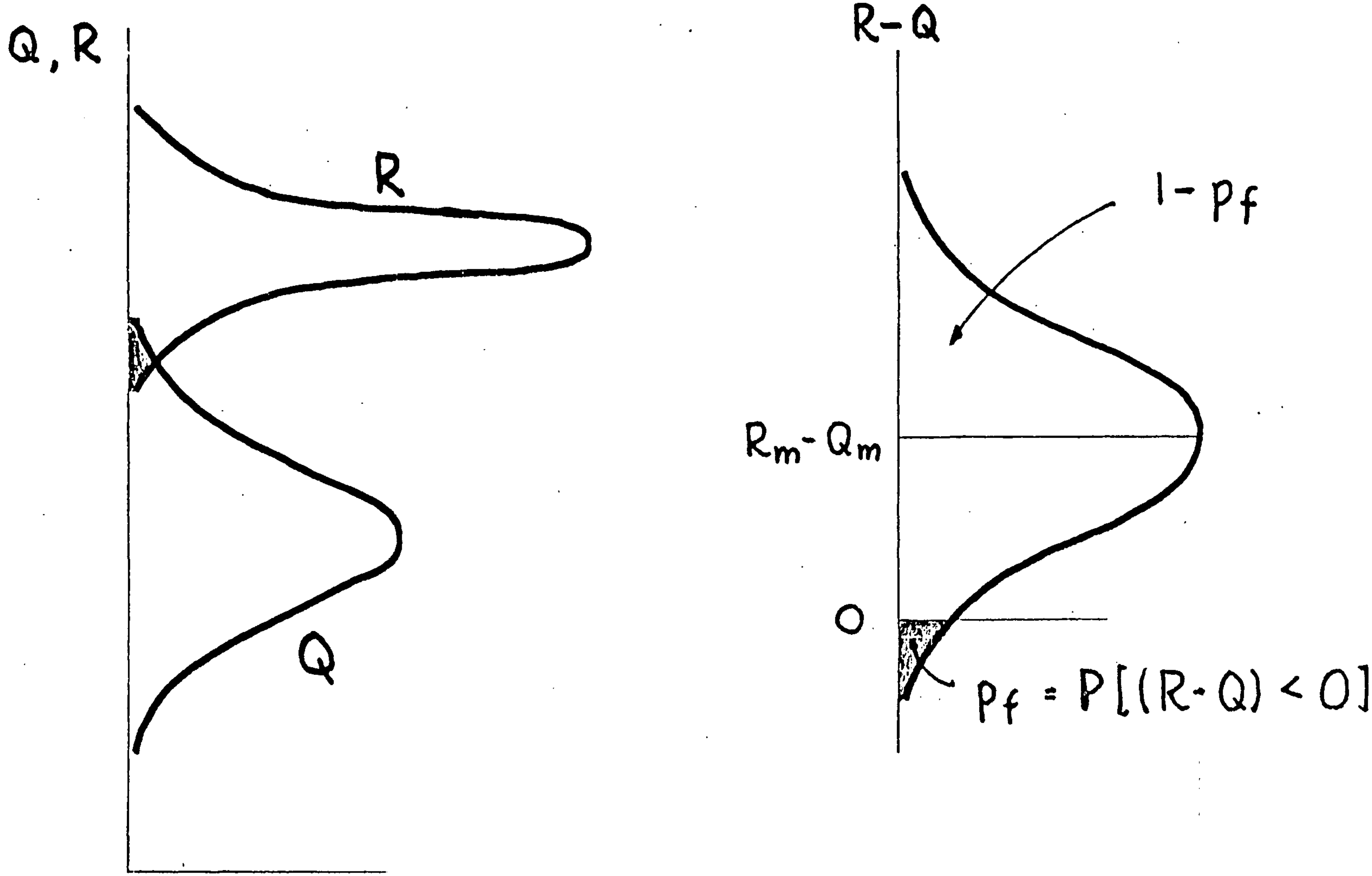
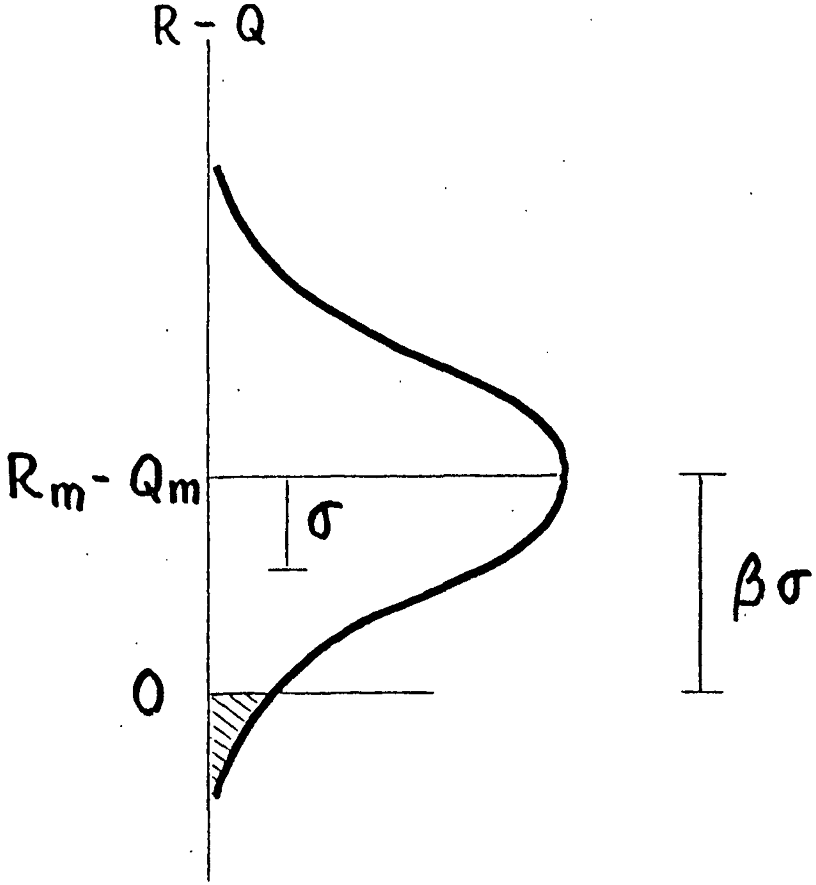


Fig. 23. If safety depends on R − Q it can be plotted that way. See on right.

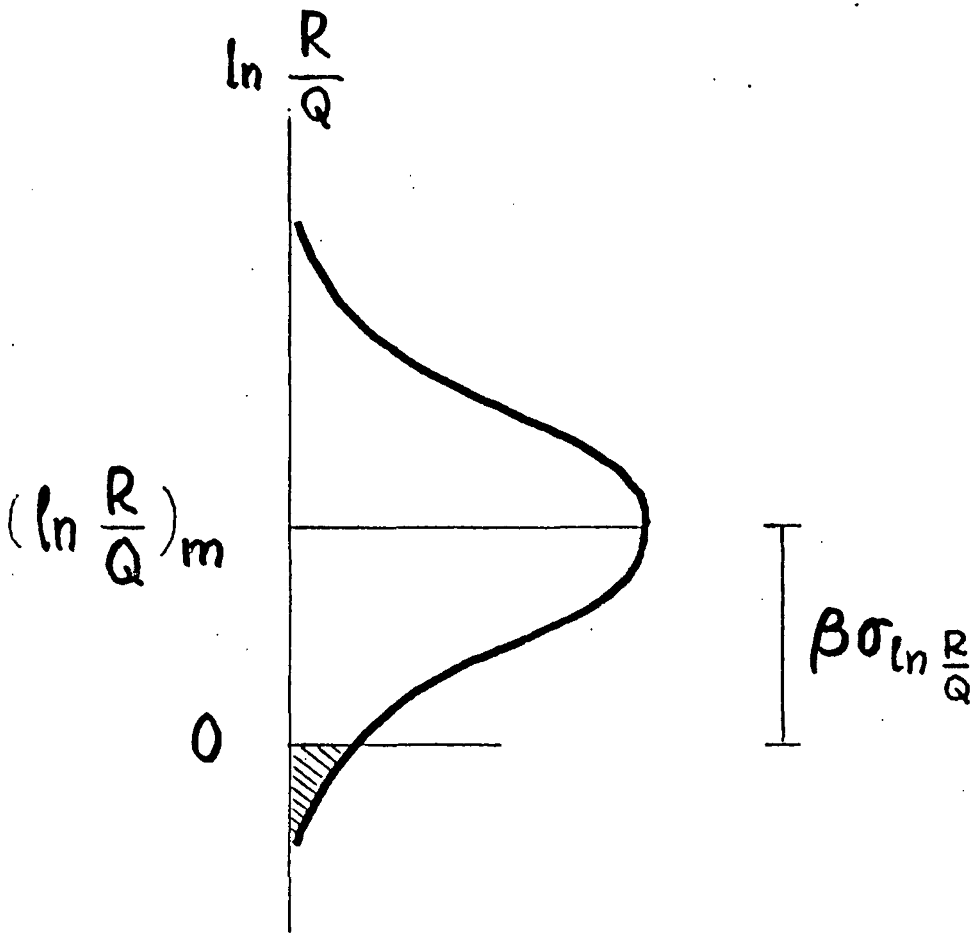


$$\beta\sigma = R_m - Q_m$$

$$\beta = \frac{R_m - Q_m}{\sigma}$$

$$\beta = \frac{R_m - Q_m}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$

Fig. 24. β is defined as the "safety index" or "reliability index". The relationship shown is only true for normal distributions.



$$\beta = \frac{(\ln \frac{R}{Q})_m}{\sigma_{\ln \frac{R}{Q}}}$$

$$\boxed{\beta = \frac{\ln \frac{R_m}{Q_m}}{\sqrt{V_R^2 + V_Q^2}}}$$

Fig. 25. The relationship of β to R_m, Q_m, and standard deviation can also be expressed in terms of logarithmic functions of R and Q.

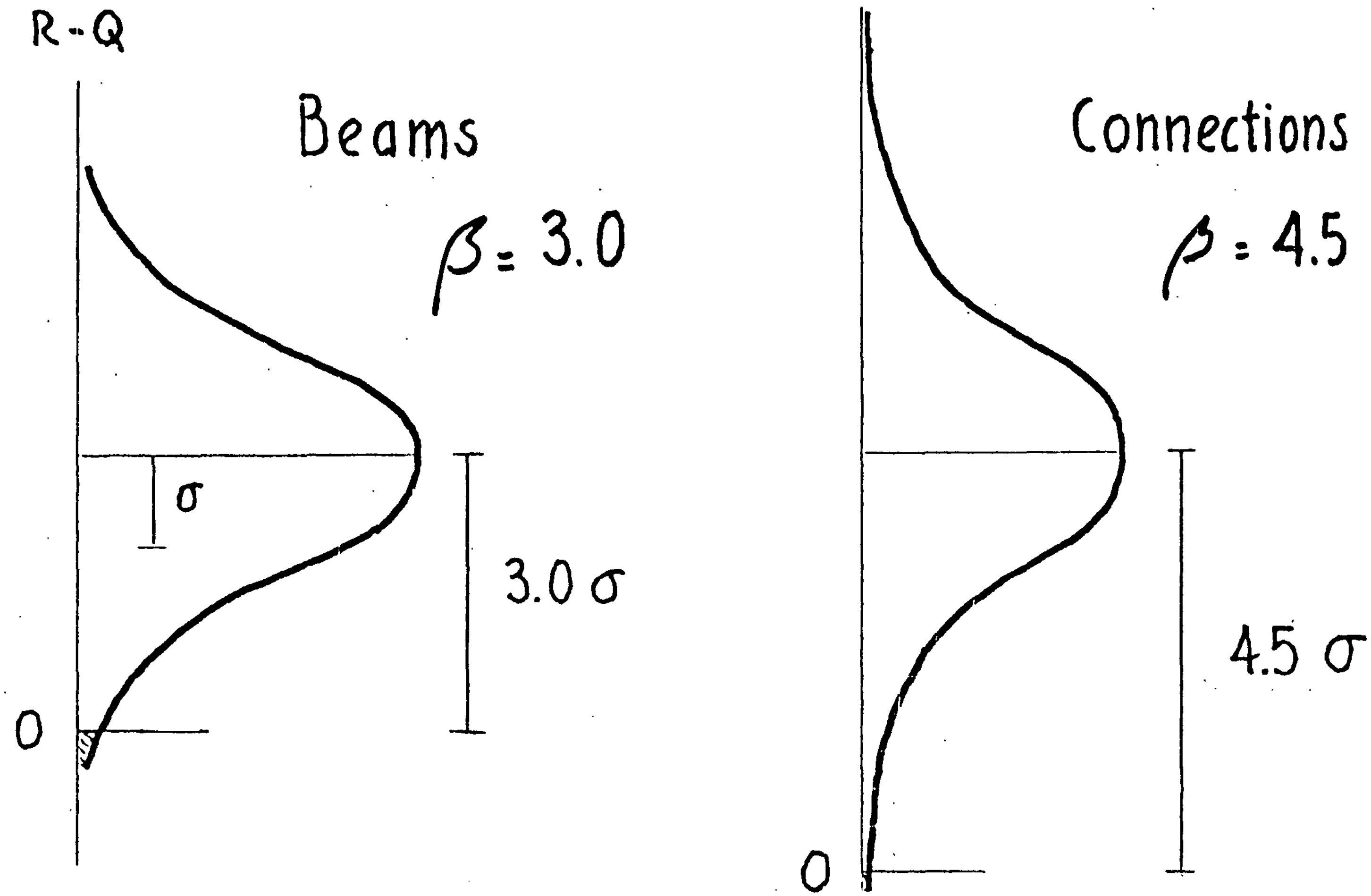


Fig. 26. "Calibration" is achieved by comparing the β-values with what would be obtained in the design of a beam by Allowable Stress Design. Two cases are shown.

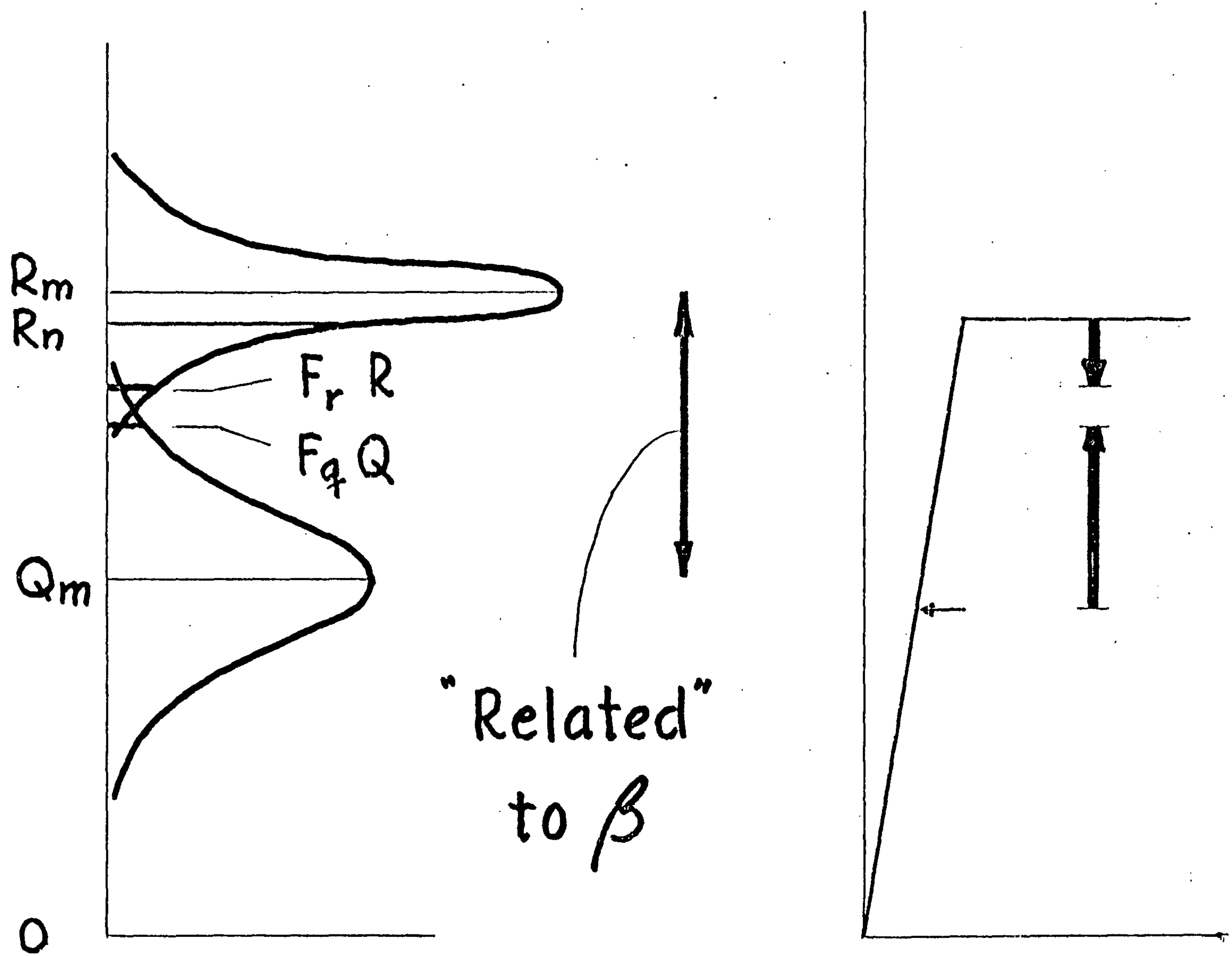


Fig. 27. Although the specific comparison is not indicated, this illustrates the fact that F_r x R and F_q x Q are related to β.

$$F_{\ell A} = e^{0.55 \beta V_A} \qquad \gamma_A$$

$$F_{\ell D} = 1 + 0.55 \beta V_D \qquad \gamma_D$$

$$F_{\ell L} = 1 + 0.55 \beta V_L \qquad \gamma_L$$

$$F_r = \frac{R_m}{R_n} e^{-0.55 \beta V_R} \qquad \phi$$

Fig. 28. The relationship of load and resistance factors with β is shown mathematically.

$$F_l L \leq F_r R$$

$$F_A \sum_i F_i L_i \leq F_r R$$

$$F_A \left(F_D L_D + F_L L_L + F_S L_S + F_W L_W + F_E L_E \right) \leq F_r R$$

Fig. 29. The simplification of the LRFD format shown earlier, is expanded here to show the multiple-load-factor aspect.

Load Combinations -- Load Factors

Dead	Live	Snow	Wind	Earthquake
1.4	-	-	-	-
1.2	1.6	-	-	-
1.2	0.5	1.6	-	-
1.2	-	1.6	0.8	-
1.2	0.5	-	1.3	-
1.2	0.5	-	-	1.5
1.2	-	0.2	-	1.5
- 0.9	-	-	1.3	-
- 0.9	-	-	-	1.5

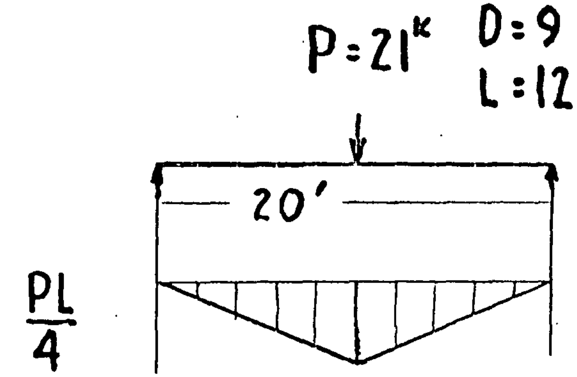
Fig. 30. A possible set of load combinations and load factors for use in LRFD.

Resistance Factors

Element		Limit State	F_r (ϕ)
Tension Member		Yield	0.9
		Fracture	.8
Beam		Bending (M_p, M_{cr})	.9
		Shear (V_p, V_{cr})	.9
Column	I - shape	Stability	.8
	Other	Stability	.8
Weld	Groove	Fracture	.9
	Fillet	Shear Fracture	.8
Bolt		Shear Fracture	.7

Fig. 31. Possible resistance factors for use in LRFD. These have been simplified and rounded off.

$$F_D Q_D + F_L Q_L \leq F_r R_n$$



$$F_D = 1.2 \qquad F_L = 1.6 \qquad F_R = 0.9$$

$$Q_D = M_D \qquad Q_L = M_L \qquad R_n = \sigma_y Z$$

$$1.2 M_D + 1.6 M_L \leq 0.9\, \sigma_y Z$$

$$1.2\frac{(9)(20)(12)}{4} + 1.6\frac{(12)(20)(12)}{4} \leq 0.9(36)Z$$

$$Z_{req} = 55.6 \text{ in}^3 \qquad \boxed{\text{use W16}\times 36} \quad (Z = 63.9 \text{ in}^3)$$

Fig. 32. Some of the essentials of the LRFD method are shown in this design example.

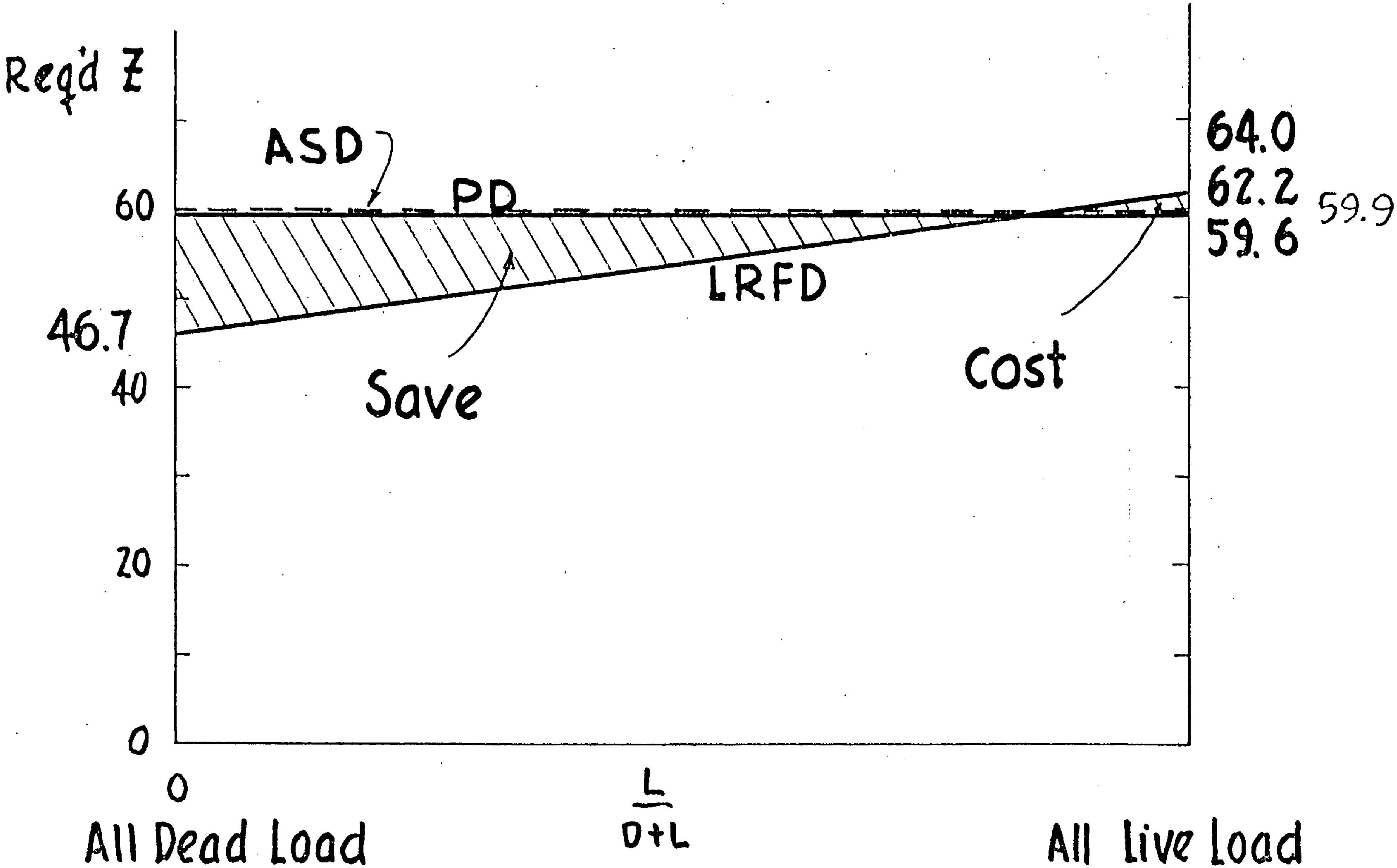


Fig. 33. The required plastic modulus according to three different designs: ASD (59.9 in.^{3}), PD (59.6 in.^{3}). LRFD (46.7 to 62.2 in.^{3}, depending on ratio of dead load to live load). Evidently the multiple load factor aspect of LRFD has a great deal to do with whether material will be saved or not.