Initial study of characterization parameters for a three-ended crack in a welded i beam during fatigue, March 1971.

H. Tada

Follow this and additional works at: http://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports

Recommended Citation
http://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports/1999
Low-Cycle-Fatigue

INITIAL STUDY OF CHARACTERIZATION PARAMETERS
FOR A THREE-ENDED CRACK IN A WELDED I BEAM DURING FATIGUE

by

H. Tada

March 1971

Fritz Engineering Laboratory Report No. 358.11
# Table of Contents

ABSTRACT

1. INTRODUCTION

2. BASIC EQUATIONS IN FRACTURE MECHANICS

3. ANALYSIS OF THE THREE-ENDED CRACK
   3.1 Assumptions
   3.2 Determination of the Interaction Forces
      3.2.1 The Interaction for the Maximum Load
      3.2.2 The Cyclic Range of the Interaction
   3.3 Estimation of the Crack Opening Stretch
      3.3.1 The Maximum Crack Opening Stretches
      3.3.2 The Interpretation of the Cyclic Ranges of the Crack Opening Stretches
      3.3.3 The Cyclic Ranges of the Crack Opening Stretches
   3.4 Expressions for the Openings and the Stress Intensity Factors in Sections 3.2 and 3.3

4. APPLICATIONS
   4.1 The Description of Data Input
   4.2 Numerical Calculations

5. RESULTS AND DISCUSSION
   5.1 The Maximum Crack Opening Stretches
      5.1.1 The Combination of $\delta_f$ and $\delta_w$
      5.1.2 The Effects of Interactions, Residual Stresses and Plasticity on $\delta_f$ and $\delta_w$
Table of Contents (continued)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2 The Ranges of the Crack Opening Stretches</td>
<td>36</td>
</tr>
<tr>
<td>5.2.1 The Combination of $\Delta \delta_f$ and $\Delta \delta_w$</td>
<td>36</td>
</tr>
<tr>
<td>5.2.2 The Effects of Interactions, Residual Stresses and Plasticity on $\Delta \delta_f$ and $\Delta \delta_w$</td>
<td>37</td>
</tr>
<tr>
<td>5.3 The Crack Growth Rates</td>
<td>40</td>
</tr>
<tr>
<td>5.4 The Interaction Forces</td>
<td>44</td>
</tr>
<tr>
<td>5.5 Others</td>
<td>45</td>
</tr>
<tr>
<td>6. SUMMARY</td>
<td>48</td>
</tr>
<tr>
<td>7. ACKNOWLEDGMENTS</td>
<td>49</td>
</tr>
<tr>
<td>8. NOMENCLATURE</td>
<td>50</td>
</tr>
<tr>
<td>9. FIGURES</td>
<td>54</td>
</tr>
<tr>
<td>10. REFERENCES</td>
<td>74</td>
</tr>
</tbody>
</table>
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
</tr>
<tr>
<td>3</td>
<td>56</td>
</tr>
<tr>
<td>4</td>
<td>57</td>
</tr>
<tr>
<td>5</td>
<td>58</td>
</tr>
<tr>
<td>6</td>
<td>59</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
</tr>
<tr>
<td>8</td>
<td>61</td>
</tr>
<tr>
<td>9</td>
<td>62</td>
</tr>
<tr>
<td>10</td>
<td>63</td>
</tr>
<tr>
<td>11</td>
<td>64</td>
</tr>
<tr>
<td>12</td>
<td>65</td>
</tr>
<tr>
<td>13</td>
<td>66</td>
</tr>
<tr>
<td>14</td>
<td>67</td>
</tr>
<tr>
<td>15</td>
<td>68</td>
</tr>
<tr>
<td>16</td>
<td>69</td>
</tr>
</tbody>
</table>

1. The I-Beam with a Three-Ended Crack
2. Crack Openings and the Interaction Force
3. A Crack and Coordinates
4. Analysis Model Crack Length \( a = a_0 + r_y \) and Crack Opening Stretch \( \delta \)
5. The Cracked Section of the Beam
6. The Geometric and Loading Configurations of the Flange and the Web \( 2a_f \) and \( a_w \) are \( r_y \)-corrected Crack Lengths
7. Variation of Crack Opening Stretch \( \Delta \delta \): Range of C.O.S. (with the crack length unchanged)
8. Basic Configurations and Approximation Models
9. Periodic Co-Linear Cracks with Pairs of Splitting Forces of Strength \( P \) Located Symmetrically
10. Combinations of Crack Lengths in the Flange and the Web
11. Combinations of Maximum Crack Opening Stretches in Flange and Web
12. Interaction for the Maximum Load
13. Maximum Crack Opening Stretches in Flange
14. Maximum Crack Opening Stretches in Web
15. Combinations of Ranges of Crack Opening Stretches in Flange and Web
16. Range of Interaction
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>Range of Crack Opening Stretches in Flange</td>
<td>70</td>
</tr>
<tr>
<td>18</td>
<td>Range of Crack Opening Stretches in Web</td>
<td>71</td>
</tr>
<tr>
<td>19</td>
<td>Crack Growth Rates in Flange and Web</td>
<td>72</td>
</tr>
<tr>
<td>20</td>
<td>Relations Between Crack Growth Rates (Observed) and Ranges of Crack Opening Stretches (Calculated)</td>
<td>73</td>
</tr>
</tbody>
</table>
ABSTRACT

In this report, an approximate analysis for the three-ended crack in a welded I-shaped beam under cyclic bending, typical of a crack which extends through the center of a girder flange and into the web, is made by using fracture mechanics technique.

The flange and web cracks are treated as separate plane crack problems and the interaction between these cracks is obtained from the compatibility of openings of the flange and the web cracks.

The idea of the crack opening stretch is applied to the analysis and understanding of crack growth behavior during cyclic loading (high and low cycle fatigue).
1. INTRODUCTION

One of the most serious crack flaws found in structures is a three-ended crack, which is a through-crack in the center of the flange and also progressed into the web, as shown in Fig. 1.

The flange crack is considered to be a central crack in a finite-width plate under uniform tension (with or without residual stresses) and the web crack is treated as half of a centrally cracked sheet with the flange on the bisection centerline of the sheet (Fig. 1). The effect of curvature of this line due to beam bending may be considered small.

The main interaction requirement between the flange and web cracks may be regarded as the equality of opening-direction displacements of the flange and of the web along the interaction centerline. However, the complexity is such that the representation of this assumption actually employed must be considerably simplified.

In the approximate analysis made in this report, the interaction force is assumed to be distributed only on the plane of the cracks and the openings are taken at the center of this span. The magnitude of the interaction force is adjusted to make this center point opening the same for the flange crack and the web crack.
It is expected that this first approximation will furnish a good estimation of the influence of the interaction between the cracks. An additional set of interaction forces would be required to establish a complete matching of the displacements of the flange and the web along the interaction centerline. However, these forces will be smaller than the interaction force used in the first approximation, and will be partially self-canceling in their effect on it due to their greater separation from the cracks.

Thus, the flange and the web may be treated as separate problems with an adjustable central closing force being added to one crack and the same force, as opening force, to the other crack. Noting that the elastic crack opening for each geometry and loading configuration is proportional to each applied load, this interaction force is obtained by simply equating the center openings of the cracks, that is, no iterative procedure, as was made by Madison (1), is needed.

In real structures subjected to low-cycle-fatigue, the yielding of the material frequently develops across the large sections of components. For this reason a plasticity type characterization, which can be employed after general yielding as well as before, and which is indicative of severity of the cyclic straining near the leading edge of the crack is needed. The crack opening stretch, \( \delta \), is a simple characterization having such capabilities. A close relationship can be retained to characterizations using the stress intensity factor, \( K \), and the crack extension force, \( \phi \), because of a simple proportionality between the plasticity, \( r_y \), corrected \( \phi \) values and \( \delta \) up to stress levels nearly large enough for general yielding.²
2. BASIC EQUATIONS IN FRACTURE MECHANICS

It is convenient to use Westergaard type stress functions\(^{(4)}\) to analyze the two-dimensional crack problems, when the external loads are applied symmetrically with respect to the x-axis, along which cracks are located (Fig. 3).

For the crack problems of the tensile opening mode, which is the most common mode of fracture and is called Mode I fracture,\(^{(4)}\) the Airy's stress function, \(F (\nabla F = 0)\), is assumed as

\[
F = \Re \bar{Z} + \gamma \Im \bar{Z} \tag{1}
\]

The stresses are calculated by

\[
\sigma_x = \Re \bar{Z} - \gamma \Im \bar{Z}' \\
\sigma_y = \Re \bar{Z} + \gamma \Im \bar{Z}' \\
\tau_{xy} = -\gamma \Re \bar{Z}' \tag{2}
\]

and the displacements in the x- and y-directions are, for plane stress condition,

\[
\begin{align*}
\text{Eu} & = (1 - \nu) \Re \bar{Z} - (1 + \nu) \gamma \Im \bar{Z} \\
\text{Ev} & = 2 \Im \bar{Z} - (1 + \nu) \gamma \Re \bar{Z} \tag{3}
\end{align*}
\]

where \(Z(z)\) is a function of \(z = x + iy\) and \(\frac{d\bar{Z}}{dz} = \bar{Z}, \frac{d\bar{Z}}{dz} = \bar{Z}, \text{ and } \frac{dZ}{dz} = Z'\).

The proper choice of the Westergaard stress function, \(Z(z)\), gives solutions for a variety of situations. Once the function \(Z(z)\)
is known, the crack tip stress field is determined by the Equations (2) as functions of the coordinate with a field parameter, K, which is a constant for a given geometry and loading configuration and termed the "stress intensity factor". (5)

The stress intensity factor, K, is calculated from $Z(z)$ as follows:

$$K = \lim_{z \to 0} \sqrt{2 \pi z} Z(z)$$  \hspace{1cm} (4)

where $z$ is a complex position vector measured appropriately from the tip of the crack (Fig. 3).

Irwin (5) treated the Griffith theory (6) strain energy release rate, $\phi$, as a generalized force, the "crack extension force", and developed the relationship between K and $\phi$. For plane stress

$$\phi = \frac{K^2}{E}$$  \hspace{1cm} (5)

The linear elastic fracture mechanics analysis described above does not include the effect of plasticity which occurs near the crack tip. The treatment of this effect by correction to the crack length in the following way has been helpful to extend the range of usefulness of the linear analysis and to estimate its applicability limits.

The addition of $r_Y$ to the crack length gives the leading edge of the analysis model a central position within the crack border plastic zone, where $r_Y$ is, from Equations (4) and (2), given by
\[ r_Y = \frac{1}{2\pi} \left( \frac{K}{\sigma_Y} \right)^2 \] (6)

and \( \sigma_Y \) is the yield strength of the material (Fig. 4).

The linear elastic equation for the displacement, \( v \), at a distance \( r \) from the leading edge of the analysis model crack is, from Equations (4) and (3),

\[ v = \frac{2}{\pi} \frac{K}{E} \sqrt{2 \pi r} \quad \text{(for plane stress)} \] (7)

A length factor, \( \delta \), defined as the value of \( 2v \) when \( r = r_Y \),

\[ \delta = \frac{4}{\pi} \frac{K^2}{E \sigma_Y} \]

or

\[ \delta = \frac{4}{\pi} \frac{\phi}{\sigma_Y} \quad \text{(for plane stress)} \] (8)

is termed the "crack opening stretch" (Fig. 4).

The validity of Equation (8) was found to be satisfactory up to stress levels nearly large enough for the general yield, provided that the \( K \) (or \( \phi \)) values are calculated using the crack size correction factor, \( r_Y \), given by Equation (6). (2)

In other words, although the elastic stress intensity factor, \( K \), itself will lose its clear physical meaning for such large scale yielding, the expression for \( K \) obtained by the elastic analysis remains useful to estimate the severity of the straining near the edge of the crack.
3. ANALYSIS OF THE THREE-ENDED CRACK

The configuration of the problem to be analyzed is shown in Fig. 1.

The basic ideas of the analysis described in Chapter 1 are summarized as follows:

1. Treat the flange and the web separately as plane problems.

2. Find the crack openings in the flange and the web for the specified crack lengths.
   (a) The cracked sections in both the flange and the web are elastic.
   (b) The cracked section of the flange is in the general yield condition.

3. From the compatibility of the openings (an approximate boundary condition), find the interaction force between them.

4. Calculate the crack opening stretches ($\delta_f$ and $\delta_w$) corresponding to the maximum load and the ranges of the crack opening stretches ($\Delta \delta_f$ and $\Delta \delta_w$) corresponding to the range of the load.
5. $\delta$'s are expected to be related to the static failure or the final instability of the crack, and $\Delta \delta$'s are expected to be closely related to the cyclic behavior (the crack growth rates).

3.1 Assumptions

The three-ended crack, shown in Fig. 1, was analyzed using the simple assumptions, described in Chapter 1, for the interaction between the flange crack and the web crack.

The further assumptions made are as follows:

1. The welded I-shaped beam analyzed is subjected to a constant maximum bending moment and a constant cyclic range of bending moment resulting in an applied maximum skin stress, $\sigma$, and a cyclic range of skin stress, $\Delta \sigma$. Thus, $\sigma$ and $\Delta \sigma$ correspond to the maximum uniform tension and the cyclic range of uniform tension, respectively, for the flange.

2. Both the flange and the web plates are in the plane stress condition.

3. The cracks, which started at the center of the tension flange and propagated into the web, are propagating symmetrically in the flange, as shown by the shaded area in Fig. 5.

4. The ratio of the crack length to the plate width in the web remains small even after the general yielding occurs
in the flange, so that no movement of the neutral axis of the beam due to the effect of the cracks and plasticity occurs, and the compression side of the beam is not included in the analysis.

5. The residual stress distributions by welding both in the flange and the web are simple, self-balanced rectangular patterns with the tensile residual stress equal to the yield strength, \( \sigma_y \), (Fig. 6). By this assumption, the residual stresses effect only on the static behavior of the beam and have no effect on range of \( \delta \) for cyclic behavior. (The portions under the tensile residual stress \( \sigma_y \) will carry the compressive load in the same manner as the elastic portions).

6. After the general yielding occurs in the cracked section of the flange, this section can carry only the load equal to the yield strength times the net section area and the rest of the load should be transmitted to the web. When this case occurs, the analysis will be made only for the web (which remains elastic) with this interaction force distributed on the plane of the crack, and the crack opening dislocation of the flange crack is assumed equal to the central opening of the flange crack (i.e., the opening of the web crack at the edge).

7. For the cyclic loading, since the most of the cracked section of the web remains elastic even after the general yielding occurs in the flange for the maximum load (the
assumptions 5, 6) nothing related to the cyclic behavior of the beam (i.e., the crack propagation rate etc.) will be different from the situation before the general yielding.

8. Since, for most ordinary structural materials, \( \sigma_{\text{ult}}/2\sigma_{\text{Y}} < 1 \), the final failure will occur before compressive yielding occurs in the cracked section of the flange for the minimum loading. Thus, the analysis of this case will be excluded.

9. The effect of the residual stress and strain behind the growing crack is not taken into account.

3.2 Determination of the Interaction Forces

The interaction forces may be (as described in Chapter 1) determined approximately by the consistency of the crack openings at the center of the flange crack (or at the edge of the web crack).

3.2.1 The Interaction for the Maximum Load

This condition for the maximum load (or for the static behavior) is expressed as

\[
U_f = U_w \tag{9}
\]

where \( 2U_f \) and \( 2U_w \) are the total openings of the flange and the web crack, respectively, at the center of the flange crack.

The \( U_f \) and \( U_w \) are given in the following forms including the unknown interaction forces in the third terms in both expressions.
\[ U_f = U_{f1} + U_{f2} + U_{f3} \]  
\[ U_w = U_{w1} + U_{w2} + U_{w3} \]

where the first terms \( U_{f1} \) and \( U_{w1} \) are the openings due to the applied load (bending moment), which are related to the residual stress distribution patterns under the assumption (5) in Section 3.1, the second terms \( U_{f2} \) and \( U_{w2} \) are the openings due to the residual stresses, and the third terms \( U_{f3} \) and \( U_{w3} \) are the openings due to the unknown interaction forces. All the U's above are the functions of the geometry (crack lengths) and the loading configurations, the expressions of which will be given below in Section 3.4.

The interaction forces before the general yielding occurs in the flange, \( P \), are assumed to be distributed uniformly on the area \( 2B_f \cdot 2B_w \) at the central portion of the flange crack (Fig. 5), i.e.,

\[ P = p \cdot (2B_f \cdot 2B_w) \]

where \( 2B_f \) and \( 2B_w \) are the thicknesses of the flange and the web, respectively. In the following analysis, \( p \) will be used in place of \( P \) as the interaction between the flange and the web.

To determine the interaction \( p \), it is convenient to express the Equations (10) in the following way by taking out the scaling factors \( (\sigma W_f/E) \) and \( (\sigma W_w/E) \), i.e.,
\[
U_f = \left( \frac{\sigma W_f}{E} \right) V_f \\
U_w = \left( \frac{\sigma W_w}{E} \right) V_w
\]

and

\[
V_f = V_{f1} + \left( \frac{\sigma_Y}{\sigma} \right) V_{f2} + \left( \frac{P}{\sigma} \right) V_{f3} \tag{13-f}
\]

\[
V_w = V_{w1} + \left( \frac{\sigma_Y}{\sigma} \right) V_{w2} - \left( \frac{P}{\sigma} \right) V_{w3} \tag{13-w}
\]

where \(W_f\) and \(W_w\) are the widths of the flange and the web, respectively.

Then the condition (9) becomes

\[
V_f = \left( \frac{W_w}{W_f} \right) V_w \tag{14}
\]

From Equations (14) and (13), the ratio of the interaction, \(p\), to the applied skin stress, \(\sigma\), corresponding to the maximum bending load is calculated by

\[
\left( \frac{P}{\sigma} \right) = \frac{\left( \frac{W_w}{W_f} \right) \left( V_{w1} + \left( \frac{\sigma_Y}{\sigma} \right) V_{w2} \right) - \left( V_{f1} + \left( \frac{\sigma_Y}{\sigma} \right) V_{f2} \right)}{V_{f3} + \left( \frac{W_w}{W_f} \right) V_{w3}} \tag{15}
\]

After the general yielding occurs in the cracked section of the flange, the expression of the interaction should be modified. Assuming the cracked section of the flange can carry only the load equal to the yield strength times the net section area, and the rest of the load corresponding to the maximum bending load, i.e.,

\[
P = \sigma (2 B_f \cdot W_f) - \sigma_Y \cdot 2 B_f (W_f - 2 a_{f0}) \tag{16}
\]
should be transmitted to the web, where $2 a_{fo}$ is the length of the flange crack. This interaction force is assumed again to be distributed on the area $2 B_f \cdot 2 B_w$ at the edge of the web crack.

Once the interaction $p$ is determined, then the separate analyses can be made for the flange and web.

### 3.2.2 The Cyclic Range of the Interaction

For the cyclic loading, under the assumption (5) in Section 3.1, replacing $\sigma$, $p$, and $V_f$, $V_w$ by the cyclic range of them, i.e., by $\Delta \sigma = \sigma - \sigma_{\text{min}}$, $\Delta p = p - p_{\text{min}}$ and $\Delta V_f = V_f - V_{f\text{min}}$, $\Delta V_w = V_w - V_{w\text{min}}$, respectively, and dropping the terms related to the residual stresses, the ratio of $\Delta p$ to $\Delta \sigma$ is, in quite a similar manner, calculated by,

$$
\left( \frac{\Delta p}{\Delta \sigma} \right) = \frac{\left( \frac{W_w}{W_f} \right) \Delta V_{w1} - \Delta V_{f1}}{\Delta V_{f3} + \left( \frac{W_w}{W_f} \right) \Delta V_{w3}}
$$

The expressions of $\Delta V$'s will be given later in Section 3.4 together with those of $V$'s.

For the minimum load, by the assumption (8) in Section 3.1, since the cracked section of the flange does not go into the compressive general yielding, no calculation corresponding to Equation (6) is needed.
3.3 Estimation of the Crack Opening Stretch

As was mentioned in Chapters 1 and 2, the expression of the linear elastic stress intensity factor, $K$, calculated for the $r_Y$ corrected crack length is useful to estimate the severity of the straining near the crack tip for the large scale yielding as well as for the small scale yielding.

Since a length factor, called the "crack opening stretch", which will be used as a characterization of the strain field near the crack tip, can be calculated directly from this $K$ (Equation (8)), the elastic analysis of the problem should be made first.

In the following analysis, the common subscripts used in the previous section will again be employed.

3.3.1 The Maximum Crack Opening Stretches

The total stress intensity factors for the flange, $K_f$, and for the web, $K_w$, at the maximum load (which will be related to the static behavior) are given in the forms corresponding to Equations (10),

$$K_f = K_{f1} + K_{f2} + K_{f3}$$  \hspace{1cm} (18-f)

$$K_w = K_{w1} + K_{w2} + K_{w3}$$  \hspace{1cm} (18-w)

or the taking out the scaling factors $(\sigma \sqrt{W_f})$ and $(\sigma \sqrt{W_w})$, i.e.,

$$K_f = (\sigma \sqrt{W_f}) \ k_f$$  \hspace{1cm} (19-f)

$$K_w = (\sigma \sqrt{W_w}) \ k_w$$  \hspace{1cm} (19-w)
and

\[ k_f = k_{f1} + \left( \frac{\sigma_Y}{\sigma} \right) k_{f2} + \left( \frac{p}{\sigma} \right) k_{f3} \]  
(20-f)

\[ k_w = k_{w1} + \left( \frac{\sigma_Y}{\sigma} \right) k_{w2} - \left( \frac{p}{\sigma} \right) k_{w3} \]  
(20-w)

Substituting the values of \( \frac{p}{\sigma} \) determined by Equation (15) into Equations (20), the maximum crack opening stretches for the flange, \( \delta_f \), and for the web \( \delta_w \), in plane stress conditions are calculated by Equation (8) as

\[ \delta_f = \frac{4}{\pi} \frac{K_f^2}{E \sigma_Y} = \frac{4}{\pi} \left( \frac{\sigma_Y}{\sigma} \right) \left( \frac{\sigma Y_f}{E} \right) k_f^2 \]  
(21-f)

\[ \delta_w = \frac{4}{\pi} \frac{K_w^2}{E \sigma_Y} = \frac{4}{\pi} \left( \frac{\sigma_Y}{\sigma} \right) \left( \frac{\sigma Y_w}{E} \right) k_w^2 \]  
(21-w)

In the preceding calculations, the \( \sigma_Y \)-corrected crack lengths should be used, i.e.,

\[ a_f = a_{fo} + r_{Yf} \]  
(22-f)

\[ a_w = a_{wo} + r_{YW} \]  
(22-w)

where \( 2a_{fo} \) and \( a_{wo} \) are the original (actual) crack lengths in the flange and the web, respectively, and the plasticity corrections \( r_{Yf} \) and \( r_{YW} \) are calculated by Equation (6) as

\[ r_{Yf} = \frac{1}{2\pi} \left( \frac{K_f}{\sigma_Y} \right)^2 = \frac{1}{2\pi} \left( \frac{\sigma_Y}{\sigma} \right)^2 \sigma Y_f k_f^2 \]  
(23-f)

\[ r_{YW} = \frac{1}{2\pi} \left( \frac{K_w}{\sigma_Y} \right)^2 = \frac{1}{2\pi} \left( \frac{\sigma_Y}{\sigma} \right)^2 \sigma Y_w k_w^2 \]  
(23-w)
Since Equations (23) include \( r_Y \)'s implicitly, to obtain the \( r_Y \)'s, the iterative procedures between Section 3.2 (Equation (15)) and this section (Equations (20) and (23)), starting with the original crack lengths, are necessary.

Once the \( r_Y \)'s are determined for the given loading and geometry configurations, the interactions \( p \), by Equation (15), and the crack opening stretches \( \delta \)'s, by Equations (21), are calculated. To calculate \( \delta \)'s from \( r_Y \)'s, it is convenient to use the relations given below by Equations (21') which are obtained by comparing Equations (21) and (23).

\[
\delta_f = \frac{8 \sigma_Y}{E} r_{Yf} \quad (21'-f)
\]

\[
\delta_w = \frac{8 \sigma_Y}{E} r_{Yw} \quad (21'-w)
\]

When the general yielding occurs in the flange at the cracked section, the analysis will be made only for the web using the interaction force given by Equation (16) and by the assumption (6) in Section 3.1. The crack opening stretch for the flange \( \delta_f \) is given by

\[
\delta_f = 2 U_w \quad (21''-f)
\]

while the \( \delta_w \) is given by Equation (21-f).

The expressions of \( K \)'s needed in the calculations above will be given in Section 3.4.
3.3.2 The Interpretation of the Cyclic Ranges of the Crack Opening Stretches

Before calculating the cyclic ranges of the crack opening stretches, $\Delta \delta$, some interpretation of it should be given here.

Regarding unloading as superposition of a reversal load with the yield strength equal to twice the initial yield strength\(^{(7)}\), the Equation (8) gives a prediction of the hysteresis loop of the crack opening stretch on unloading and reloading with the crack length unchanged.\(^{(8)}\) By this concept, on unloading the crack tip opening closes only by a fraction of the opening on the initial loading and on reloading it goes back to the initial opening. This amount of cyclic closing is defined as the (cyclic) "range of the crack opening stretch", which is expected to be related to the fatigue crack propagation rate.

This assumption extends the applicability of Equation (8) for the cyclic behavior because of the two times yield strength in the reverse direction.

A schematic interpretation of $\Delta \delta$ for a simple cyclic load is given with the relations to $\Delta \sigma$ and $\Delta K$ in Fig. 7.

3.3.3 The Cyclic Ranges of the Crack Opening Stretches

For the cyclic loading, the ranges of the stress intensity factors, $\Delta K_f$ and $\Delta K_w$, are calculated similarly
in the form corresponding to Equation (18) with the terms related to the residual stresses being dropped,

\begin{align}
\Delta K_f &= \Delta K_{f1} + \Delta K_{f3} \\
\Delta K_w &= \Delta K_{w1} + \Delta K_{w3}
\end{align}

(24-f)

(24-w)

using the scaling factors \((\Delta \sigma/W_f)\) and \((\Delta \sigma/W_w)\),

\begin{align}
\Delta K_f &= (\Delta \sigma/W_f) \Delta k_f \\
\Delta K_w &= (\Delta \sigma/W_w) \Delta k_w
\end{align}

(25-f)

(25-w)

and

\begin{align}
\Delta k_f &= \Delta k_{f1} + \left(\frac{\Delta P}{\Delta \sigma}\right) \Delta k_{f3} \\
\Delta k_w &= \Delta k_{w1} - \left(\frac{\Delta P}{\Delta \sigma}\right) \Delta k_{w3}
\end{align}

(26-f)

(26-w)

Substituting \((\Delta P/\Delta \sigma)\) obtained by Equation (17) into Equation (26), the ranges of the crack opening stretches are given by

\begin{align}
\Delta \delta_f &= \frac{4}{\pi} \frac{\Delta K_f^2}{E(2 \sigma_Y)} = \frac{4}{\pi} \left(\frac{\Delta \sigma}{2 \sigma_Y}\right) \left(\frac{\Delta \sigma \cdot W_f}{E}\right) \Delta k_f^2 \\
\Delta \delta_w &= \frac{4}{\pi} \frac{\Delta K_w^2}{E(2 \sigma_Y)} = \frac{4}{\pi} \left(\frac{\Delta \sigma}{2 \sigma_Y}\right) \left(\frac{\Delta \sigma \cdot W_w}{E}\right) \Delta k_w^2
\end{align}

(27-f)

(27-w)

For the reason given in 3.3.2 the yield strength \(2 \sigma_Y\) is used in place of \(\sigma_Y\) in Equation (27).
In the calculations, the $\Delta r_Y$-corrected crack lengths should be used, i.e.,

$$a_f = a_{f0} + \Delta r_{Yf} \quad (28-f)$$

$$a_w = a_{w0} + \Delta r_{YW} \quad (28-w)$$

where the ranges of the plasticity corrections are given by, from Equation (6) with $\sigma_Y$ being replaced by $2\sigma_Y$,

$$\Delta r_{Yf} = \frac{1}{2\pi} \left( \frac{\Delta K_f}{2\sigma_Y} \right)^2 = \frac{1}{2\pi} \left( \frac{\Delta \sigma}{2\sigma_Y} \right)^2 w_f \Delta k_f^2 \quad (29-f)$$

$$\Delta r_{YW} = \frac{1}{2\pi} \left( \frac{\Delta K_{wY}}{2\sigma_Y} \right)^2 = \frac{1}{2\pi} \left( \frac{\Delta \sigma}{2\sigma_Y} \right)^2 w_w \Delta k_w^2 \quad (29-w)$$

To obtain $\Delta r_Y$'s the iterative procedures between Equation (17) and Equations (26) and (29) are again needed, and $\Delta \delta$'s are obtained from these $\Delta r_Y$'s by

$$\Delta \delta_f = \frac{8(2 \sigma_Y)}{E} \Delta r_{Yf} \quad (27'-f)$$

$$\Delta \delta_w = \frac{8(2 \sigma_Y)}{E} \Delta r_{YW} \quad (27'-w)$$

The expressions used in the above calculations will be given in Section 3.4.

### 3.4 Expressions for the Openings and the Stress Intensity Factors

In Sections 3.2 and 3.3

Since the configurations to be analyzed (Fig. 6) are made up by the combinations of the four basic configurations shown in Fig. 8 (a)-(d), though (a) and (c) are special cases of (b) and (d)
respectively, the analyses of these four patterns may be sufficient to
give the expressions required in Sections 3.2 and 3.3.

The Westergaard stress function approach given in Chapter 2
is used and the finite width effect of the plate is replaced approxi-
mately by the periodic co-linear crack problem which satisfies one
of the two boundary conditions on the free edge, \( r_{xy} = 0 \), identically.
The replacements are expected to be good approximations for the
patterns (a) and (b) and somewhat crude but still good for the patterns
(c) and (d) when the crack lengths are small compared to the widths
of the plates.

The Westergaard stress functions given for the configuration
in Fig. 9 by Irwin\(^{(5)}\)

\[
Z(z) = \frac{2P}{W} \frac{\cos \frac{nL}{W} \sqrt{(\sin \frac{n\pi}{W})^2 - (\sin \frac{nL}{W})^2}}{[(\sin \frac{n\pi}{W})^2 - (\sin \frac{nL}{W})^2] \sqrt{1 - (\sin \frac{n\pi}{W}/\sin \frac{nL}{W})^2}}
\]

and

\[
\overline{Z}(z) = \int z(z) \, dz = -\frac{2P}{\pi} \arctan \frac{\sqrt{1 - (\cos \frac{n\pi}{W}/\cos \frac{nL}{W})^2}}{(\cos \frac{n\pi}{W}/\cos \frac{nL}{W})^2 - 1}
\]

or the expressions of the crack opening at the center and the stress
intensity factor derived from the above functions using Equations (3)
and (4),
\[ U = \frac{4}{\pi} \frac{P}{E} \frac{\arccosh}{\tan} \left( \frac{\frac{\pi v}{W}}{\tan} \frac{\frac{\pi v}{W}}{\tan} \right) \]  \hspace{1cm} (32)

and

\[ K = \frac{2}{W} \frac{P}{W} \sqrt{\tan} \frac{\frac{\pi v}{W}}{\tan} \frac{\frac{\pi v}{W}}{\tan} \frac{\cos}{\sqrt{\left( \sin \frac{\frac{\pi v}{W}}{W} \right)^2 - \left( \sin \frac{\frac{\pi v}{W}}{W} \right)^2}} \]  \hspace{1cm} (33)

are used as the Green's functions of the problem.

Only the results calculated for the approximate analysis models are given below without the details of the calculation.

(a) The case shown in Fig. 8(a)

The crack opening at the center:

\[ U_\alpha (\sigma, W, A) = \left( \frac{\sigma}{W} \right) \frac{\sqrt{\pi}}{A} V_\alpha (A) \]

\[ V_\alpha (A) = \frac{2}{\pi} \arccosh (\sec \frac{\pi}{2} A) \]  \hspace{1cm} (34)

The stress intensity factor:

\[ K_\alpha (\sigma, W, A) = (\sigma \sqrt{W}) k_\alpha (A) \]

\[ k_\alpha (A) = \sqrt{\tan} \frac{\pi}{2} A \]  \hspace{1cm} (35)

where

\[ A = \frac{2a}{W}. \]
(b) The case of Fig. 8(b)

The crack opening at the center:

$$U_b (\sigma, W, A, B) = \left(\frac{\sigma \varepsilon}{E}\right) V_b (A, B)$$

$$V_b (A, B) = \frac{2}{\pi} \int_0^B \frac{\tan \frac{\pi A}{2}}{\tan \frac{\pi}{2} X} \arccosh \left(\frac{\tan \frac{\pi}{2} A}{\tan \frac{\pi}{2} X}\right) dX \quad (36)$$

The stress intensity factor:

$$K_b (\sigma, W, A, B) = (\sigma \sqrt{W}) k_b (A, B)$$

$$k_b (A, B) = \frac{2}{\pi} \arcsin \left(\frac{\sin \frac{\pi B}{2}}{\sin \frac{\pi}{2} A}\right) \sqrt{\tan \frac{\pi}{2} A} \quad (37)$$

where $A = \frac{2a}{W}$ and $B = \frac{2b}{W}$.

(c) The case of Fig. 8(c)

The crack opening at the center:

$$U_c (\sigma, W, A) = \left(\frac{\sigma \varepsilon}{E}\right) V_c (A)$$

$$V_c (A) = \frac{2}{\pi} \int_0^A (1 - 2X) \arccosh \left(\frac{\tan \frac{\pi}{2} A}{\tan \frac{\pi}{2} X}\right) dX \quad (38)$$

The stress intensity factor:

$$K_c (\sigma, W, A) = (\sigma \sqrt{W}) k_c (A)$$
\[ K_c(A) = \sqrt{\tan \frac{\pi}{2} A} \int_0^A (1 - 2x) \frac{\cos \frac{\pi}{2} x}{\sqrt{(\sin \frac{\pi}{2} A)^2 - (\sin \frac{\pi}{2} x)^2}} \, dx \quad (39) \]

\[ = \sqrt{\tan \frac{\pi}{2} A} \left\{ 1 - \frac{4}{\pi} \int_0^1 \text{arc sin} \left( \sin \frac{\pi}{2} A \cdot \sin \frac{\pi}{2} t \right) dt \right\} \quad (39') \]

where \( A = \frac{2a}{w} \)

(d) The case of Fig. 8(d)

The crack opening at the center:

\[ U_d (\sigma, W, A, B) = (\sigma E) U_d (A, B) \]

\[ V_d (A, B) = \frac{2}{\pi} \int_0^B (1 - 2x) \text{arc cosh} \left( \frac{\tan \frac{\pi}{2} A}{\tan \frac{\pi}{2} x} \right) \, dx \quad (40) \]

The stress intensity factor:

\[ K_d (\sigma, W, A, B) = (\sigma \sqrt{W}) k_d (A, B) \]

\[ k_d (A, B) = \sqrt{\tan \frac{\pi}{2} A} \int_0^B (1 - 2x) \frac{\cos \frac{\pi}{2} x}{\sqrt{(\sin \frac{\pi}{2} A)^2 - (\sin \frac{\pi}{2} x)^2}} \, dx \]

\[ = \sqrt{\tan \frac{\pi}{2} A} \left\{ \frac{4}{\pi} \int_0^1 \text{arc sin} \left( \sin \frac{\pi}{2} A \cdot \sin \frac{\pi}{2} t \right) dt \right\} \quad (41') \]

where \( A = \frac{2a}{w} \) and \( B = \frac{2b}{w} \).

The integrations in \( V_b, V_c, V_d \) and \( k_c, k_d \) are calculated numerically.
The expressions of the openings and the stress intensity factors to be given in Sections 3.2 and 3.3 are as follows:

For the flange (Fig. 6(a)):

The openings are

\[ V_{f1} = \begin{cases} 
0 & \text{if } A_f \leq B_{rf} \\
(1 + \alpha_f)\{V_A (A_f) - V_b (A_f, B_{rf})\} & \text{if } A_f > B_{rf}
\end{cases} \]  

(42-f)

\[ V_{f2} = \begin{cases} 
V_A (A_f) & \text{if } A_f \leq B_{rf} \\
(1 + \alpha_f) V_b (A_f, B_{rf}) - \alpha_f V_A (A_f) & \text{if } A_f > B_{rf}
\end{cases} \]  

(43-f)

\[ V_{f3} = V_b (A_f, B_{wf}) \]  

(44-f)

and

\[ \Delta V_{f1} = V_A (A_f) \]  

(45-f)

\[ \Delta V_{f3} = V_b (A_f, B_{wf}) \]  

(46-f)

The stress intensity factors are

\[ k_{f1} = \begin{cases} 
0 & \text{if } A_f \leq B_{rf} \\
(1 + \alpha_f)\{k_A (A_f) - k_b (A_f, B_{rf})\} & \text{if } A_f > B_{rf}
\end{cases} \]  

(47-f)

\[ k_{f2} = \begin{cases} 
k_A (A_f) & \text{if } A_f \leq B_{rf} \\
(1 + \alpha_f) k_b (A_f, B_{rf}) - \alpha_f k_A (A_f) & \text{if } A_f > B_{rf}
\end{cases} \]  

(48-f)

\[ k_{f3} = k_b (A_f, B_{wf}) \]  

(49-f)

and

\[ \Delta k_{f1} = k_A (A_f) \]  

(50-f)

\[ \Delta k_{f3} = k_b (A_f, B_{wf}) \]  

(51-f)
where

\[ A_f = \frac{2\alpha_f}{W_f}, \quad B_{rf} = \frac{2b_{rf}}{W_f} = \frac{\alpha_f}{1 + \alpha_f} \quad \text{and} \quad B_{wf} = \frac{2b_w}{W_f} \]

For the web (Fig. 6(b)):

The openings are

\[ V_{w1} = \begin{cases} 
0 & A_w \leq B_{rw} \\
V_c (A_w) - V_d (A_w, B_{rw}) + \alpha_w (1 - B_{rw}) \{V_c (A_w) - V_d (A_w, B_{rw})\} & A_w > B_{rw}
\end{cases} \]

\[ V_{w2} = \begin{cases} 
V_a (A_w) & A_w \leq B_{fw} \\
(1 + \alpha_w) V_b (A_w, B_{fw}) - \alpha_w V_a (A_w) & A_w > B_{fw}
\end{cases} \]

\[ V_{w3} = V_b (A_w, B_{fw}) \]

and

\[ \Delta V_{w1} = V_c (A_w) \]

\[ \Delta V_{w3} = V_b (A_w, B_{fw}) \]

The stress intensity factors are

\[ k_{w1} = \begin{cases} 
0 & A_w \leq B_{rw} \\
k_c (A_w) - k_d (A_w, B_{rw}) + \alpha_w (1 - B_{rw}) \{k_c (A_w) - k_d (A_w, B_{rw})\} & A_w > B_{rw}
\end{cases} \]

\[ k_{w2} = \begin{cases} 
  k_0 (A_w) & A_w \leq B_{rw} \\
  (1 + \alpha_w) k_b (A_w, B_{rw}) - \alpha_w k_0 (A_w) & A_w > B_{rw}
\end{cases} \]

\( k_{w3} = k_b (A_w, B_{fw}) \) \hspace{1cm} (49-w)

and

\[ \Delta k_{w1} = k_c (A_w) \] \hspace{1cm} (50-w)

\[ \Delta k_{w3} = k_b (A_w, B_{fw}) \] \hspace{1cm} (51-w)

where

\[ A_w = \frac{a_w}{W_w}, B_{rw} = \frac{b_{rw}}{W_w} = \frac{1}{2} \frac{\alpha_w}{1 + \alpha_w} \text{ and } B_{fw} = \frac{B_f}{W_w}. \]

The calculations can be made for any combinations of the dimensions of the beam, the sizes of the cracks and the residual stress distribution patterns (when no residual stresses exist, \( \alpha' \)'s are simply replaced by zero) and the loading conditions. If the interactions \( p \) and \( \Delta p \) are taken as zero, the analyses are made for the two independent plates. That is, the effect of the interaction is estimated by comparing the cases with the interaction included with the independent analyses of the flange and the web, and the effect of the residual stresses is estimated from the analysis with and without \( \alpha' \)'s.
4. APPLICATIONS

The analysis made using the fracture mechanics technique in
Chapter 3 is applied to the welded I-shaped beam with a three-ended
 crack studied by Marek et al. (9)

4.1 The Description of Data Input

The following input data are given.

1. The properties of the material (ASTM A514 steel)
   \[ E = 30 \times 10^3 \text{ ksi} \]
   \[ \sigma_Y = 110.2 \text{ ksi} \]
   \[ \sigma_{ult} = 118.2 \text{ ksi} \]

2. The dimensions of the beam section (Fig. 1(a))
   \[ w_f = 6.82 \text{ in.} \]
   \[ W_f = 13.47 \text{ in.} \]
   \[ 2B_f = 0.394 \text{ in.} \]
   \[ 2B_w = 0.30 \text{ in.} \]

3. The loading condition (Fig. 1(b))
   \[ \sigma = 36.2 \text{ ksi} \]
   \[ \Delta \sigma = 22.6 \text{ ksi} \]

4. The residual stress distribution pattern (Fig. 1(c))

Simulating the data observed by Marek (9) and
Lozano (10) assume \( \alpha_f \) and \( \alpha_w \) as follows:
\[ \alpha_f = 1/6 \]
\[ \alpha_w = 1/8 \]

5. The combinations of the crack lengths \( 2\alpha_{fo} \) and \( \alpha_{wo} \) (Fig. (Fig. 1(a)).

The beam tested had an initial three-ended crack as a result of a fatigue test done previously under a different loading condition.

The combinations and the number of cycles given below are taken from the report by Marek et al.(9) and shown in Fig. 10.

<table>
<thead>
<tr>
<th>( A_{fo} = \frac{2\alpha_{fo}}{w_f} )</th>
<th>( A_{wo} = \frac{\alpha_{wo}}{w_w} )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.227</td>
<td>0.080</td>
<td>0</td>
</tr>
<tr>
<td>0.250</td>
<td>0.085</td>
<td>~ 4,000</td>
</tr>
<tr>
<td>0.30</td>
<td>0.093</td>
<td>~ 11,000</td>
</tr>
<tr>
<td>0.35</td>
<td>0.102</td>
<td>~ 19,000</td>
</tr>
<tr>
<td>0.40</td>
<td>0.111</td>
<td>~ 26,500</td>
</tr>
<tr>
<td>0.45</td>
<td>0.120</td>
<td>~ 34,000</td>
</tr>
<tr>
<td>0.50</td>
<td>0.127</td>
<td>~ 40,000</td>
</tr>
<tr>
<td>0.55</td>
<td>0.133</td>
<td>~ 45,000</td>
</tr>
<tr>
<td>0.60</td>
<td>0.137</td>
<td>~ 48,500</td>
</tr>
<tr>
<td>0.65</td>
<td>0.141</td>
<td>~ 51,500</td>
</tr>
<tr>
<td>0.70</td>
<td>0.143</td>
<td>~ 53,500</td>
</tr>
<tr>
<td>0.75</td>
<td>0.1445</td>
<td>~ 54,800</td>
</tr>
<tr>
<td>0.80</td>
<td>0.146</td>
<td>~ 56,000</td>
</tr>
<tr>
<td>~ 0.82</td>
<td>~ 0.150</td>
<td>~ 56,900*</td>
</tr>
</tbody>
</table>

*The final failure of the flange
It is noted that the flange crack is not propagating symmetrically\(^{(9)}\) and that it is regarded as a symmetrical crack of the total length \(2\alpha_{fo}\).

4.2 Numerical Calculations

The numerical calculations are carried out by the use of a CDC 6400 computer for the following cases.

1. Independent analyses of the flange and the web
   (without interaction, i.e., \(p = 0\) and \(\Delta p = 0\))
   (a) With no residual stresses present \((\alpha_f = \alpha_w = 0)\)
   (b) With residual stresses (under the assumption (5) in Section 3.1)
   (c) With residual stresses unchanged during cyclic loading.

2. Combined analyses of the flange and the web (with the interaction included)
   (a) Without residual stresses \((\alpha_f = \alpha_w = 0)\)
   (b) With residual stresses (under the assumption (5) in Section 3.1)
   (c) With residual stresses unchanged during cyclic loading.

It should be noted that the combinations of the crack lengths for the cases (2a) and (2c) may be somewhat different from those observed, which are given in the previous section and expected to be close to the case (2b). However, the difference is expected to be
small between the cases (2a) and (2b) considering the cyclic behavior under the assumption (5) in Section 3.1. Therefore, the direct comparison between (2a) and (2b) for the maximum load will give the effects of the residual stresses on the static nature. (The case (2c) is exactly the same as the case (2b) for the maximum load).

Under the same assumption ((5) in Section 3.1), the cyclic behaviors are not influenced by the presence of residual stresses, that is, there is no difference in cyclic nature between (2a) and (2b) and between (1a) and (1b). It is felt, however, that there may be some effects of the residual stresses, and the possible maximum effects may be given in the case where the residual stress distribution patterns remain unchanged during loading and unloading. This case, of course, seems to be unrealistic and again will make the combinations of the crack lengths different from those observed (for the case (2b)), but will give some information with the effects of residual stresses on the cyclic behavior (crack growth rates). That is, the direction of the effects of residual stresses on crack growth rates will be given by comparing (2b) and (2c). (The case (2a) is exactly the same as the case (2b) for the cyclic behavior).

By the reason given above, the calculation of the case (2c) is added, and for the sake of comparison the case (1c) is also added. It is noted that, for the maximum load, (1b) and (1c) are the same, and that, for the cyclic behavior, (1a) and (1b) are the same.
The calculations of (lc) and (2c) for the cyclic behavior are not included in the analyses in Chapter 3, but can be made simply replacing $\sigma$ and $\sigma_y$ by $\Delta \sigma$ and $2\sigma_y$, respectively, in the analyses done for the maximum load.

The results of the numerical calculations are given in Figs. 11 through 18.
5. RESULTS AND DISCUSSION

The results of the numerical calculations are given in Figs. 11 through 18. The discussion will mainly be given on the flange crack.

5.1 The Maximum Crack Opening Stretches

The maximum crack opening stretches may be related to the static behavior or the final failure of the beam.

5.1.1 The Combination of $\delta_f$ and $\delta_w$

The maximum crack opening stretches in the flange, $\delta_f$, and the corresponding values of the web, $\delta_w$, are shown in Fig. 11. The effects of the interaction, the residual stresses and the plasticity are included, that is, the calculations were made for the case (2b) or (2c) in Section 4.2.

Although the calculations of $\delta_f$ can be made for somewhat larger crack lengths than $2a_{foY}$ corresponding to the general yielding, $\delta_f$ should be calculated, after the general yielding occurs in the cracked section of the flange, by the assumption (6) in Section 3.1 (i.e. by Equations (16) and (21'-f) in place of Equations (15) and (21-f)). The results of the calculation shows large discrepancy at $a_{fo} = a_{foY}$ between the values of $\delta_f$ before and after
general yielding in the flange, that is, the $\delta_f$ value after general yielding is about twice as large as the $\delta_f$ value before.

Since it is expected that Equations (15) and (21-f) will give a good estimation of the crack opening stretch of the flange, $\delta_f$, at $a_{fo} = a_{foY}^{(2)}$ and that, after general yielding, the larger the flange crack length is, the better the assumption (6) in Section 3.1 may be applied, the discrepancy between the two curves may be eliminated by connecting them smoothly.

In the experiment, $2a_{foY}/W_f \sim 2/3$ since $\sigma/\sigma_Y \sim 1/3$, and the final failure occurred at $2a_{foY}/W_f \sim 0.82$. The range where the assumption (6) should be applied is comparatively small and the transition between the two curves may not be clear enough.

For the web, the maximum crack opening stretch, $\delta_w$, remains nearly constant until the general yielding occurs in the flange and since the cracked section of the web remains elastic, the transition occurs smoothly comparing with the flange.

5.1.2 The Effects of Interactions, Residual Stresses and Plasticity on $\delta_f$ and $\delta_w$

The interactions between the flange and the web are calculated for the cases (2) in Section 4.2, and the results are shown in Fig. 12. The effects of the residual stresses and the plasticity (the $r_Y$-correction) on the interaction are demonstrated by the difference between (2a) and (2b), (2c) and between the solid lines and the dashed lines, respectively.
The effect of the interaction on the crack opening stretches of the flange and the web are shown in Figs. 13 and 14, respectively, together with other effects.

The maximum crack opening stretch of the flange for each case described in Section 4.2 is shown in Fig. 13. In the figure, the solid lines for the cases (2b), (2c) correspond to the case with total effects included.

The effect of the residual stresses is given by the difference between the cases (2b), (2c) and the case (2a). The presence of the residual stresses gives more severity of the straining near the leading edges of the crack and its effect reduces with the increase of the crack length, i.e., the curve for (2b), (2c) approaches the curve for (2a) as the crack length increases.

The effect of the interaction is given by the difference between the cases (2) and (1). In the cases (1), the flange was treated as an independent plate under the same loading condition as the cases (2) except that the interaction is zero. Comparing the curve for (2b), (2c) with the curve for (1b), (1c), the effect of interaction, as is expected from Fig. 12, is first in the direction to increase the severity of the straining near the crack tips and then changes the direction to decrease it as the crack length increases.
As for the difference between the cases (1a) and (1b), (1c) which is the effect of the residual stresses, the similar explanation that given for the cases (2a) and (2b), (2c) may be given.

The effect of the plasticity is replaced by the $r_Y$-correction to the crack lengths. Since $r_Y$ is proportional to the crack opening stretch (Equations (21')), the change of which the crack length is similar to that of the crack opening stretch. In the flange, the effect of $r_Y$ is mostly in the direction to increase the severity of the straining near the leading edges of the crack.

Noting that the independent analysis of the flange with the residual stresses included, i.e., the case (1b), (1c), gives first somewhat smaller values of $\delta_f$ than the cases (2b), (2c), and then gives the largest values of $\delta_f$ (the severest condition) when the crack length becomes close to the general yielding condition, for the sake of the conservative design, the separate analysis of the flange with residual stresses may be sufficient for the static behavior of the beam (the flange).

The maximum crack opening stretches of the web, corresponding to Fig. 13 for the flange, are shown in Fig. 14. Since quite the similar discussion to that given to the curves in Fig. 13 for the flange may be given to the curves in Fig. 14 for the web there may be no need to give much discussion on the web.
It should be noted that the ratio of the length of the web crack to the width of the web, $a_{wo}/W_w$, did not change much and remains small (0.08 to 0.15) while the crack ratio of the flange, $2a_{fo}/W_f$, changed from 0.227 to the final failure of the flange (see Fig. 10) and that most of the cracked section of the web was always in elastic state during the cyclic loading.

5.2 The Ranges of the Crack Opening Stretches

The ranges of the crack opening stretches are expected to be closely related to the cyclic behavior of the beam (e.g., crack growth rates).

5.2.1 The Combination of $\Delta \delta_f$ and $\Delta \delta_w$

The ranges of the crack opening stretches in the flange, $\Delta \delta_f$, and the corresponding values of the web, $\Delta \delta_w$, are shown in Fig. 15. The effects of the interaction and the plasticity are included, and under the assumption (5) in Section 3.1 there is no effects of the residual stresses on the $\Delta \delta_f$ and $\Delta \delta_w$, that is, the results correspond to the case (2b) (or (2a)) in Section 4.2.

Considering that the cyclic range of the applied stress, $\Delta \sigma/\sigma_y$, is only about $1/5$ and that the web remains elastic even after the general yielding occurs in the cracked section of the flange, $\Delta \delta_f$ and $\Delta \delta_w$ may be calculated by the
same equations (Equations (27)) throughout the test and the curves of $\Delta \delta_f$ and $\Delta \delta_w$ are smooth as shown in Fig. 15.

The values of $\Delta \delta_f$ are larger than the corresponding values of $\Delta \delta_w$ and the difference is increasing with the number of the load cycles (or the crack growths). The actual combinations of the crack lengths of the flange and the web shown in Fig. 10 may be equivalent to the combinations of $\Delta \delta_f$ and $\Delta \delta_w$ in Fig. 15. The discussion on the crack growth rates will be given later in Section 5.3.

5.2.2 The Effects of Interactions, Residual Stresses and Plasticity on $\Delta \delta_f$ and $\Delta \delta_w$

The ranges of the interactions between the flange and the web during cyclic loading are calculated for the cases (2) in Section 4.2, and the results are shown in Fig. 16. The range of the interaction force is calculated by Equation (11) replacing $p$ by $\Delta p$. The influence of the residual stresses and the plasticity (the $\Delta R_Y$-correction) on the range of the interaction may be seen from Fig. 16. It is noted that, by the assumption (5) in Section 3.1, the cases (2a) and (2b) give the same results and that the effect of the residual stresses is in the opposite direction from that for the maximum crack opening stretches shown in Fig. 12. That is, the presence of the residual stresses will reduce the severity of the cyclic straining near the tips of the flange crack or $\Delta \delta_f$ while it increases the severity of the straining $\delta_f$ for the maximum load. The
plasticity effect is small because the yield strength is
doubled and the cyclic range of the applied load, \( \Delta \sigma \), is
small.

The effect of the cyclic range of the interaction on
\( \Delta \delta_f \) and \( \Delta \delta_w \) are shown in Figs. 17 and 18, respectively, to­
gether with other effects.

The cyclic range of the crack opening stretch of the
flange, \( \Delta \delta_f \), for each case described in Section 4.2 is shown
in Fig. 17. In the figure, the curve for the cases (2a), (2b)
is considered to correspond to the actual behavior under the
assumption (5) in Section 3.1.

The direction of the effect of the residual stresses
may be given by comparing the curves for the case (2c) and those
for the cases (2a), (2b). That is, the residual stresses
will reduce the cyclic range of the crack opening stretch
of the flange, \( \Delta \delta_f \), throughout the test. This effect is in
the opposite direction from that for the maximum load and reduces
as the crack length increases.

The influence of the interaction is demonstrated by the
difference between the cases (2) and (1). In the cases (1), the
flange was treated as an independent plate under the same
loading condition as the cases (2) except that the interaction
is taken as zero. Comparing the curve for (2a), (2b) with the
curve for (1a), (1b), it is seen that the interaction acts in
the direction to reduce \( \Delta \delta_f \) or the severity of the cyclic
straining near the leading edges of the flange crack for most range of the crack length.

As for the comparison of the curve for (1c) with the curve for (1a), (1b), quite the similar explanation to that given for the curves for (2c) and for (2a), (2b) may be made.

The plasticity effect on $\Delta \delta_f$, which may be given by the $\Delta r_Y$-correction to the crack lengths (Equations (28)), is much smaller comparing with that on $\delta_f$, because of the relatively small cyclic range of the applied load ($\Delta \sigma / \sigma_Y \sim 1/5$, while $\sigma / \sigma_Y \sim 1/3$) and the doubled yield strength. Since the results of calculations without the $\Delta r_Y$-correction is very close to the curves shown in Fig. 17, they are not demonstrated separately.

Noting that the separate analysis of the flange without the effect of residual stresses (the cases (1a), (1b)) gives the largest values of $\Delta \delta_f$ for most range of the crack length (the severest cyclic straining condition), the conservative estimation of the crack growth rate may be made by the analysis for this simplest case (a plate with a central crack under the cyclic uniform tension, without the effect of the residual stresses and the interaction).

The cyclic ranges of the crack opening stretch of the web, corresponding to Fig. 17 for the flange, are shown in Fig. 18. Discussion on the curves in Fig. 18 for the web, may be made in quite a similar manner to that done on the curves in Fig. 17 for the flange.
5.3 The Crack Growth Rates

The fatigue crack growth rates in the flange, \( \frac{da_{fo}}{dN} \), and the corresponding values for the web, \( \frac{da_{wo}}{dN} \), observed by Marek et al(9) are demonstrated in Fig. 19 by the two solid lines. In Fig. 20, these crack growth rates are plotted to the ranges of the crack opening stretches calculated for the case (2b) (or (2a)), that is, Fig. 20 is the combination of Fig. 15 and Fig. 19.

In actual fracture or fatigue testing the crack growth rates of both ends of the crack are sometimes quite different. In fact, the flange crack was observed to be growing unsymmetrically(9) and, though the flange should be regarded as an eccentrically cracked strip during the test, for simplicity, regard the flange as a strip with a central crack of the total length observed, \( 2a_{fo} \). Therefore, the flange crack growth rates, \( \frac{da_{fo}}{dN} \), plotted in Figs. 19 and 20 are actually the average value of both ends, i.e., \( \frac{1}{2} \left\{ \frac{d(2a_{fo})}{dn} \right\} \).

Considering that the cyclic range of the applied load is small \( (\Delta \sigma/\sigma_Y \sim 1/5) \) and that, though the maximum applied load \( (\sigma/\sigma_Y \sim 1/3) \) gives the general yield condition to the cracked section of the flange for \( 2a_{fo}/W_f \geq 2/3 \), the cracked section of the web remains elastic even after the general yielding occurs in the flange, and that the final failure occurred before the net section of the flange came into the compressive general yielding (it would occur for \( 2a_{fo}/W_f \geq 0.9 \) because the yield strength is doubled for the reversal load, i.e., \( \Delta \sigma/2\sigma_Y \sim 1/10 \)), the fatigue test by Marek et al(9) is considered to have
been done under the high-cycle-fatigue condition rather than the low-cycle-fatigue condition most of the time.

Paris (11) found the close proportionality between the fatigue crack growth rate, \( \frac{da}{dN} \), and the fourth power of the cyclic range of the stress intensity factor, \( \Delta K \), that is,

\[
\frac{da}{dN} = c(\Delta K)^4
\]  

(52)

For steels the numerical value of the constant of proportionality, \( c \), in Equation (52) is in the following range when \( \frac{da}{dN} \) and \( \Delta K \) are in units of inch/cycle and ksi \( \sqrt{\text{in.}} \), respectively.

\[
c = (1.0 \sim 2.0) \times 10^{-12}
\]  

(53)

c = 2.0 \times 10^{-12} \quad \text{gives the conservative estimation of the crack growth rate on all steels and Equations (52) and (53) gives a reasonable estimate for crack growth rates from } 10^{-6} \quad \text{in/cycle up to final failure, if } \Delta K < K_{IC} \text{ and } \Delta \sigma < \sigma_y, \text{ where } K_{IC} \text{ is the plane strain fracture toughness of the material.} (12)

The last statement in the previous paragraph may give the justification of the use of the relation given by Equation (52) even after the net section yielding of the flange occurs on the loading portion of load cycles. That is, even if the maximum applied load, \( \sigma \), is large enough to make the net section yield or \( \sigma_{\text{net}} \sim \sigma_y \), when the cyclic range of the applied load, \( \Delta \sigma \), is small enough to keep the net section elastic or to keep it from going into the compressive yielding, Equation (52) will not lose its validity. Noting that, for most practical structural materials, \( \sigma_{\text{ult}} < 2\sigma_y \), and that the yield
strength is doubled for reversal load, the final failure will occur before the net section goes into the compressive yielding. In other words, as long as the final failure does not occur on loading (or until the final stage of the fatigue crack growth, where the crack growth rate increases rapidly to the final failure, is reached), Equation (52) may give good estimation of the crack growth rates.

On the role which the maximum applied load may play on the fatigue crack growth rates, discussion will be given later.

There have not been many works done on the low-cycle-fatigue crack growth rate approached by the fracture mechanics technique. Yoshiki et al.\(^{(13)}\) made the low-cycle-fatigue test on the central notched plate specimen under the loading condition \(\sigma \sim \Delta \sigma \sim \sigma_Y\) and observed that the low-cycle-fatigue crack growth rates might be expressed by Equation (52). On the crack growth rates on the final stage of the fatigue test, Gopala-Krishna\(^{(14)}\) has been doing the fatigue test on the double cantilever beam specimen giving a severe straining condition near the leading edge of the crack and the results show that Equation (50) does not hold.

In this report, the crack opening stretch concept is used as the strain field parameter near the crack tip in place of the stress intensity factor. By the simple relation between the cyclic range of the stress intensity factor, \(\Delta K\), and that of the crack opening stretch, \(\Delta \delta\), and that of the crack opening stretch, \(\Delta \delta\), given by (from Equation (8) with replacements \(\delta, K\) and \(\sigma_Y\) by \(\Delta \delta, \Delta K\) and \(2 \sigma_Y\))
\[ \Delta \delta = \frac{4}{\pi} \frac{(\Delta K)^2}{E(2 \sigma_Y)} \]  

(54)

the relation between \( \frac{da}{dN} \) and \( \Delta K \), Equation (52), is converted into the relation between \( \frac{da}{dN} \) and \( \Delta \delta \) as

\[ \frac{da}{dN} = c' (\Delta \delta)^2 \]  

(52')

where

\[ c' = c \left( \frac{\pi}{2} E \sigma_Y \right)^2 \]

Thus, for the steel tested (\( \sigma_Y \approx 110 \text{ ksi}, E \approx 30 \times 10^3 \text{ ksi} \)) Equation (53) is converted into, taking the units of \( \frac{da}{dN} \) and \( \Delta \delta \) as inch/cycle, and in., respectively,

\[ c' = 26.8 \approx 53.6 \]  

(53')

Noting that, while the dependency of \( \Delta K \) on \( \sigma_Y \) is only the small plasticity adjustment to the crack length, \( \Delta \delta \) depends directly on \( \sigma_Y \), \( \Delta \delta \) may have the better meaning as a plasticity characterization of the severity of the cyclic straining near the crack tip.

From the sloped portion of the curve of the flange crack growth rate in Fig. 20, \( c' \) in Equation (52') is taken as about 70 and the crack growth rate is given by

\[ \frac{da}{dN} \sim 70 (\Delta \delta)^2 \]  

(55)

Using the calculated values of \( \Delta \delta \) for the flange, \( \Delta \delta_f \), and for the web, \( \Delta \delta_w \), (shown in Fig. 15), the curves of Equation (55) are inserted by dashed lines in Fig. 10. Both curves are monotonically increasing in the whole range where the calculations were made. The flat portion of the curve of \( \frac{da}{dN} \) observed for the small crack lengths,
$2 \frac{a_f}{W_f} \lesssim 0.5$, as was discussed by Marek et al.\textsuperscript{(9)} may be caused by the unsymmetry of the crack and the ambiguousness of the crack length which was measured only on the outer surface of the flange. The shape of the crack front changed as the test progressed and became stationary for $2 \frac{a_f}{W_f} \gtrsim 0.5$. In this range, Equation (52') seems to be applicable though the value of $c' (\sim 70)$ is somewhat larger than the value given by (53'). Hertzberg et al.\textsuperscript{(15)} showed that the larger ratio of $\lambda = \frac{K_{\text{max}}}{\partial K}$ gives the larger crack growth rate (four times larger values of $\lambda$ may give two times of the crack growth rates).

Comparing Fig. 11 with Fig. 15, since $\lambda \sim \frac{\delta}{\Delta \delta}$, for the flange $\lambda_f$ is decreasing from about 4 with the crack length for $2 \frac{a_f}{W_f} \lesssim 0.5$, this may also have effect to make the curve flat in that range. For the web $\lambda_w$ decreases from about 4.5 with the crack length for most of the range. This, together with the ambiguousness of the crack lengths, may give the flatness to the curve of the crack growth rates (the curve $d \frac{a_w}{W_w}$ in Figs. 19 and 20).

In Fig. 20, for the sake of comparison, the approximate locus of the experimental data obtained by Gopalakrishna\textsuperscript{(14)} and Hertzberg et al.\textsuperscript{(15)} for the same steel are shown by the dashed lines.

### 5.4 The Interaction Forces

There have not as yet been many works done on the interaction between the cracks in structures.

An interesting analysis on the interaction between the flange crack and the web crack has been done by Smith et al.\textsuperscript{(16)} by the use of
the simplified mathematical model composed of lumped volumes which are arranged at discrete points. Computations of the force transferred after unloading \( (\sigma = \Delta \sigma) \) are made. The results may correspond to

\[
P - \Delta P = (p - \Delta p) \left( 2B_f \cdot 2B_w \right)
\]

in this report with \( \Delta \sigma \) replaced by \( \Delta \sigma = \sigma \). However, in their analysis, although the actual mechanical properties and the average residual stresses measured for the beam of the same steel with the same dimensions as the beam treated in this report were considered, the combinations of the crack lengths of the flange and the web were specified arbitrarily. Since the crack lengths of the flange and the web may not be independent of each other but be determined uniquely by the geometric and loading configuration, the direct comparison of the values of interaction forces can not be done.

It is hoped that the computations of the interaction force will be made for the observed combinations of the crack lengths of the flange and the web.

5.5 Others

By the present analysis, though the experimental data are limited and some simplified (or probably oversimplified) assumptions have been made, it seems to have been shown that the fracture mechanics technique, using the crack opening stretch concept as the strain field parameter near the crack tips, may be applied to the structural components with cracks which may interact each other.
Considering the nature of the approximations of the crack openings and the stress intensity factors for the web crack by the periodic crack problems, described in Section 3.4, the length of the web crack may be restricted to be small compared with the width of the web (say $a_{w_0}/W_w \lesssim 0.2$). In other words, this may correspond to restricting the shape of the beam to have relatively large width of web (say $W_w/W_f \gtrsim 2$).

The negligence of the translation of the neutral axis of the beam on the cracked section after the general yield of the net section may not be significant for the case which satisfies the conditions mentioned above because the curvature along the interaction centerline will remain small during the cyclic loading, unless the loading condition is severe enough to make the net section of the flange yield from the early stage of the fatigue loading (or unless $\sigma$ is the order of $\sigma_Y$).

However, in a more strict sense, the beam is in the low-cycle fatigue condition when $\sigma \sim \Delta \sigma \sim \sigma_Y$, it is felt that, for this severe loading condition, the analysis had better be modified and that the corresponding experimental data will be needed.

The analysis shows that in the flange the residual stresses due to welding may reduce the crack growth rates (or $\Delta \delta_f$). It is hoped that the experiments and comparisons of the flange crack growth rates will be made on beams with residual stresses and beams without residual stresses (roll out beams or welded beams with residual stresses relieved by heat treatment) of the same shapes under the same loading conditions.
When the loading condition is not severe and the beam is under the high-cycle-fatigue condition rather than the low-cycle-fatigue condition for most part of the test, as the case analyzed in the present report, the simplest case, that is, the independent analysis of the flange plate with no residual stresses present, may give the rough but most conservative estimation of the flange crack growth rate.
6. SUMMARY

An approximate analysis for the three-ended crack in a welded I-beam subjected to a cyclic bending was done by using the fracture mechanics technique, in which as the strain field parameter near the crack tips, \( \delta \), termed the crack opening stretch is used.

The interaction between the cracks of the flange and the web are estimated by a simple approximate boundary condition that the central opening of the flange crack is equal to the edge opening of the web crack.

The experimental data analyzed shows the applicability of the method, but the loading condition may not be severe enough as the low-cycle-fatigue test in a strict sense, the applicability of the method to the low-cycle-fatigue crack problem in the structural components may still not be quite clear. Since the experimental data are limited, it is hoped that some experiments will be added.
7. ACKNOWLEDGMENTS

This investigation is one phase of a major research program designed to provide information on the behavior and design of joined structures under low-cycle fatigue.

The investigation was conducted at Fritz Engineering Laboratory and Packard Laboratory, Lehigh University, Bethlehem, Pennsylvania. The Office of Naval Research, Department of Defense sponsored the research under contract N0014-68-A-0514; NR064-509. The program manager for the overall research project is Lambert Tall.

Guidance from the project Advisory Committee on Low-Cycle Fatigue is gratefully acknowledged. Special thanks are due to Dr. G. R. Irwin who directed the work of this report; to project co-investigators who provided discussions and comments; to Mrs. Dorothy F. Fielding, who typed and edited the manuscript; and to Mr. John Gera who prepared the drawings.

Lynne S. Beedle is Director of Fritz Engineering Laboratory, and Joseph F. Libsch is Vice President for Research, Lehigh University.
8. NOMENCLATURE

Material Constants

\[ E \] Young's modulus
\[ \nu \] Poisson's ratio
\[ \sigma_Y \] tensile yield strength
\[ \sigma_{\text{ult}} \] ultimate tensile strength

Notations in Fracture Mechanics Equations

\[ Z(z) \] Westergaard type stress function
\[ z \] complex coordinates, \( z = x + iy \)
\[ a \] crack size factor
\[ 2U \] crack opening
\[ K \] stress intensity factor for the leading edge stress field of a crack
\[ \mathcal{E} \] crack extension force
\[ r_Y \] crack size plasticity correction factor
\[ \delta \] crack opening stretch

Subscripts

\[ f \] for flange
\[ w \] for web

Dimensions of Beam Section

\[ W_f \] width of flange
\[ W_w \] width of web
\[ 2B_f \] thickness of flange
\[ \begin{align*}
2B_w &= \text{thickness of web} \\
B_{wf} &= \frac{2B_w}{W_f} \\
B_{fw} &= \frac{B_f}{W_w}
\end{align*} \]

**Crack Sizes**

\[ \begin{align*}
2a_{fo} &= \text{actual length of flange crack} \\
a_{wo} &= \text{actual length of web crack} \\
2a_f &= \text{analysis model (r\textsubscript{Y}-corrected) crack length of flange} \\
a_w &= \text{analysis model (r\textsubscript{Y}-corrected) crack length of web} \\
A_f &= \frac{2a_f}{W_f} \\
A_w &= \frac{a_w}{W_w}
\end{align*} \]

**Applied Stresses**

\[ \begin{align*}
\sigma &= \text{applied maximum skin stress (uniform tension in flange)} \\
\Delta \sigma &= \text{cyclic range of applied skin stress}
\end{align*} \]

**Interaction Forces**

\[ \begin{align*}
P &= \text{interaction force for maximum load} \\
\Delta P &= \text{cyclic range of interaction force} \\
p &= \frac{P}{(2B_f \cdot 2B_w)} \\
\Delta p &= \frac{\Delta P}{(2B_f \cdot 2B_w)}
\end{align*} \]

**Crack Openings**

\[ \begin{align*}
2U_f &= \text{maximum opening of flange crack at center} \\
2U_w &= \text{maximum opening of web crack at edge} \\
2\Delta U_f &= \text{cyclic range of opening of flange crack at center} \\
2\Delta U_w &= \text{cyclic range of opening of web crack at edge}
\end{align*} \]
\[ v_f = \frac{U_f}{\sigma W_f} \]
\[ v_w = \frac{U_w}{\sigma W_w} \]
\[ \Delta v_f = \frac{\Delta U_f}{\sigma W_f} \]
\[ \Delta v_w = \frac{\Delta U_w}{\sigma W_w} \]

**Stress Intensity Factors**

\[ K_f = (K_{f1}, K_{f2}, K_{f3}) \]
maximum stress intensity factor of flange crack

\[ K_w = (K_{w1}, K_{w2}, K_{w3}) \]
maximum stress intensity factor of web crack

\[ \Delta K_f = (\Delta K_{f1}, \Delta K_{f2}) \]
cyclic range of stress intensity factor of flange crack

\[ \Delta K_w = (\Delta K_{w1}, \Delta K_{w2}) \]
cyclic range of stress intensity factor of web crack

\[ k_f = \frac{K_f}{\sigma \sqrt{W_f}} \]
\[ k_w = \frac{K_w}{\sigma \sqrt{W_w}} \]
\[ \Delta k_f = \frac{\Delta K_f}{\sigma \sqrt{W_f}} \]
\[ \Delta k_w = \frac{\Delta K_w}{\sigma \sqrt{W_w}} \]

**Plasticity Corrections**

\[ r_{Yf} \]
maximum plasticity correction to flange crack

\[ r_{YW} \]
maximum plasticity correction to web crack

\[ \Delta r_{Yf} \]
cyclic range of plasticity correction to flange crack

\[ \Delta r_{YW} \]
cyclic range of plasticity correction to web crack

**Crack Opening Stretches (C.O.S.)**

\[ \delta_f \]
maximum C.O.S. in flange

\[ \delta_w \]
maximum C.O.S. in web

\[ \Delta \delta_f \]
cyclic range of C.O.S. in flange

\[ \Delta \delta_w \]
cyclic range of C.O.S. in web
Residual Stress Distributions (Simplified)

- $\alpha_f$: ratio of compressive residual stress to $\sigma_y$ in flange
- $\alpha_w$: ratio of compressive residual stress to $\sigma_y$ in web
- $2b_{rf}$: width of portion under tensile residual stress in flange
- $b_{rw}$: width of portion under tensile residual stress in web (on tension side)

$B_{rf} = \frac{2b_{rf}}{W_f} = \frac{\alpha_f}{1 + \alpha_f}$

$B_{rw} = \frac{b_{rw}}{W_w} = \frac{\alpha_w}{2(1 + \alpha_w)}$
Fig. 1 The I-Beam with a Three-Ended Crack
Consistency of Crack Openings

\[ V_f = V_w \]

Fig. 2 Crack Openings and the Interaction Force
Fig. 3  A Crack and Coordinates

$z_1 = z - \alpha$
Fig. 4  Analysis Model Crack Length 
( \( a = a_0 + r_\gamma \) ) and Crack Opening 
Stretch ( \( \delta \) )
Fig. 5  The Cracked Section of the Beam
Fig. 6  The Geometric and Loading Configurations of the Flange and the Web (2τf and aw are r_Y-corrected crack lengths).
\[ \Delta \sigma = (1-\beta) \sigma_{\text{max}} \]
\[ \sigma_{\text{min}} = \beta \sigma_{\text{max}}. \]

**Stress Intensity Factor**
- **with \( r_t \)-correction**
- **without \( r_t \)-correction**

\[ \Delta K = c_0 (1-\beta) K_{\text{max}} \; ; \; c_0 = f_0 \left( \frac{\sigma_{\text{max}}}{\sigma_t} \right) \leq 1 \]

**Crack Opening Stretch**
- **with \( r_t \)-correction**
- **without \( r_t \)-correction**

\[ \Delta \delta = c_0 \cdot \frac{1}{2} (1-\beta)^2 \delta_{\text{max}} \; ; \; c_0 = f_0 \left( \frac{\sigma_{\text{max}}}{\sigma_t} \right) \lessapprox 1 \]

**Fig. 7 Variation of Crack Opening Stretch**

\[ \Delta \delta : \text{Range of C.O.S.} \]

(with the crack length unchanged)
Basic Patterns

Models for Approximation

Fig. 8 Basic Configurations and Approximation Models
\[ Z(z) = x + iy \]

\[ Z(z) = \frac{2P}{W} \frac{\cos \frac{\pi b}{W} \sqrt{(\sin \frac{\pi a}{W})^2 - (\sin \frac{\pi b}{W})^2}}{(\sin \frac{\pi a}{W})^2 - (\sin \frac{\pi b}{W})^2} \sqrt{1 - \left(\frac{\sin \frac{\pi a}{W}}{\sin \frac{\pi b}{W}}\right)^2} \]

\[ \bar{Z}(z) = -\frac{2P}{\pi} \arctan \sqrt{1 - \left(\frac{\cos \frac{\pi a}{W}}{\cos \frac{\pi b}{W}}\right)^2} \]

\[ K = \frac{2P}{W} \sqrt{W} \tan \frac{\pi a}{W} \frac{\cos \frac{\pi b}{W}}{\sqrt{(\sin \frac{\pi a}{W})^2 - (\sin \frac{\pi b}{W})^2}} \]

\[ \nu(0,0) = \frac{4P}{\pi E} \arccosh \left(\frac{\tan \frac{\pi a}{W}}{\tan \frac{\pi b}{W}}\right) \]

Fig. 9 Periodic Co-linear Cracks with Pairs of Splitting Forces of Strength P located Symmetrically
Fig. 10 Combinations of Crack Lengths in the Flange and the Web
Fig. 11 Combinations of Maximum Crack Opening Stretches in Flange and Web
Fig. 12 Interaction for the Maximum Load
Fig. 13  Maximum Crack Opening Stretches in Flange
Fig. 14  Maximum Crack Opening Stretches in Web
Fig. 15 Combinations of Ranges of Crack Opening Stretches in Flange and Web
Fig. 16 Range of Interaction
Fig. 17 Range of Crack Opening Stretches in Flange
Fig. 18 Range of Crack Opening Stretches in Web
Fig. 19 Crack Growth Rates in Flange and Web
Fig. 20 Relations between Crack Growth Rates and Ranges of Crack Opening Stretches (observed)
10. REFERENCES

1. Madison, R. B.
APPLICATION OF FRACTURE MECHANICS TO BRIDGES,
Fritz Engineering Laboratory Report No. 335.2,
Lehigh University, June 1969

2. Irwin, G. R., Lingaraju, B. and Tada, H.
INTERPRETATIONS OF THE CRACK OPENING DISLOCATION CONCEPT,
Fritz Engineering Laboratory Report No. 358.2,
Lehigh University, June 1969

3. Westergaard, H. M.
BEARING PRESSURES AND CRACKS,

4. For example
Irwin, G. R.
FRACTURE - ENCYCLOPEDIA OF PHYSICS, Springer,
Vol. VI, pp. 551-590, 1958
Paris, P. C. and Sih, G. C.
STRESS ANALYSIS OF CRACKS,
Fracture Toughness Testing and Its Application,
ASTM STP 381, pp. 30-83, 1965

5. Irwin, G. R.
ANALYSIS OF STRESSES AND STRAINS NEAR THE END OF A CRACK
TRaversing A PLATE,

6. Griffith, A. A.
THE PHENOMENON OF RUPTURE AND FLOW IN SOLIDS,
168, 1920

ELASTIC PLASTIC STRESS AND STRAIN DISTRIBUTION AROUND SHARP
NOTCHES UNDER REPEATED SHEAR,
9th International Congress of Applied Mechanics, Vol. 8,
University of Brussels, p. 51, 1957

8. McClintock, F. A. and Irwin, G. R.
PLASTICITY ASPECTS OF FRACTURE MECHANICS,
Fracture Toughness Testing and Its Applications,
ASTM STP 381, pp. 84-113, 1965
9. Marek, P., Perlman, M., Pense, A. W. and Tall, L.  
CRACKED BEAM TEST,  
Fritz Engineering Laboratory Report No. 358.4,  
Lehigh University, May 1969 (Revised April 1970, No. 358.4a)  

10. Lozano, S., Marek, P. and Yen, B. T.  
RESIDUAL STRESS REDISTRIBUTION IN WELDED BEAMS SUBJECTED  
TO CYCLIC LOADING,  
Fritz Engineering Laboratory Report No. 358.5,  
Lehigh University, November 1969  

11. Paris, P. C.  
THE FRACTURE MECHANICS APPROACH TO FATIGUE,  
Fatigue - An Interdisciplinary Approach, Syracuse  
University Press, p. 107, 1965  

BASIC ASPECTS OF CRACK GROWTH AND FRACTURE,  
Naval Research Laboratory, Report 6598, 1967  

13. Yoshiki, M. and others  
EFFECT OF LOW CYCLE FATIGUE ON THE BRITTLE FRACTURE  
INITIATION CHARACTERISTICS OF SHIP STEELS,  

14. Gopalakrishna, V.  

15. Hertzberg, R. W. and Nordberg, H.  
FATIGUE CRACK PROPAGATION IN A514 STEEL,  
Fritz Engineering Laboratory Report No. 358.7, Lehigh  
University, November 1969  

16. Smith, R. J., Marek, P. and Yen, B. T.  
REDISTRIBUTION OF STRESS AND STRAIN IN A BEAM WITH A CRACK,  
Fritz Engineering Laboratory Report No. 358.9, Lehigh  
University, November 1969