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J. Hartley Daniels

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ELASTIC-PLASTIC ANALYSIS OF UNBRACED FRAMES

by

Sung-Woo Kim

A Dissertation
Presented to the Graduate Faculty
of Lehigh University
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Professor David A. VanHorn is Chairman of the Civil Engineering Department and Professor Lynn S. Beedle is Director of Fritz Engineering Laboratory. The manuscript was typed with great care by Miss Karen Philbin and the drawings were prepared by Mr. John M. Gera.
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An exact analytical procedure is presented for determining the complete elastic-plastic behavior of unbraced multi-story steel frames which are subjected to nonproportional combined loading. The procedure is called the sway increment method of analysis and is based on determining the values of the applied lateral loads consistent with prescribed finite sway deflections of a story when the frame is also subjected to constant gravity loads. The analytical method utilizes a second-order elastic-plastic method of analysis, an incremental procedure, and a technique to predict the sway increments for next hinges. The procedure includes the effects of axial shortening, hinge reversal and residual stresses.

The sway increment method is used to study the lateral-load versus sway-deflection behavior of several multi-story frames under nonproportional combined loading. These studies indicate that the effect of axial shortening on the maximum lateral load capacity is not considerable and the primary effect of axial shortening is to induce lateral deflections. Plastic hinge reversals hardly occur in a frame before reaching its maximum lateral load unless there are any plastic hinges in the non-ewayed position with the gravity loads only. A number of plastic hinges in a frame is subjected to hinge reversals after failure. However, the effect of the hinge reversals on unloading behavior of the frame is very small.
Based on the sway increment method, an approximate method, which is called the one-story assemblage method, is developed to determine the approximate lateral-load versus sway-deflection behavior of a story of an unbraced frame. This method which is programmed for computer solution is very useful for performing the trial analyses associated with preliminary frame designs. The individual story behavior obtained using the one-story assemblage method has been compared with the story behavior determined from a sway increment analysis for several stories in two frames studied. The comparison indicates that the one-story assemblage method gives a reasonably good approximation to the load-deflection behavior of a story located in the middle and lower regions of an unbraced frame subjected to nonproportional combined loads.

The results of both the sway increment method of analysis and the one-story assemblage method of analysis are compared with experimental results. The agreement between the experimental results and the theoretical predictions is good.
1. INTRODUCTION

1.1 Problem Statement

With recent technological developments plastic design methods which were limited to one and two story rigid frames have been extended to both braced and unbraced multi-story frames. Paralleling the developments, the 1969 AISC Specification has been extended to include the complete design of planar frames in high-rise buildings, provided they are braced to take care of any lateral loading. (1)* Systematic procedures for the application of plastic design in proportioning the members of such frames have been developed and are available in the current literature. (2-4) The design method has successfully been used in designing multi-story apartment buildings. (5)

With the trend in modern building designs toward light curtain-wall construction, larger areas of glass and movable interior partitions, architectural considerations often make it desirable to omit bracing against sidesway. In such a case, the bare frame alone must supply the lateral stiffness and a design method for unbraced frames is required. In addition, a plastically designed braced multi-story frame requires an adequate bracing system that may result in the use of a disproportionate amount of steel, offsetting any savings resulting from the plastic analysis of the frame.

*The numbers in parenthesis refer to the list of references.
Therefore, there has been an increasing demand for the development of a practical method for the analysis of unbraced multi-story frames, on which a sound design method would be based.

In an unbraced multi-story frame, the $P\Delta$ effect caused by large vertical loads $P$ acting through lateral displacements $\Delta$ is significant and complicates the development of analytical and design techniques. The problem of overall instability due to the combination of the $P\Delta$ effect and the applied lateral loads also becomes of primary importance. For the development of a practical design method which considers such problem, it is essential to understand the complete behavior of an unbraced frame under the combined loading condition (gravity plus lateral loads).

In general, there are two different analytical approaches which have been proposed to predict the behavior and strength of an unbraced frame. The first is referred to in Ref. 2 as the compatibility analysis, where consideration is given to the zones of partial plastification; thus, the true moment-axial force-curvature ($M$-$P$-$\phi$) relationship is used in the analysis. The second is described as the second-order elastic-plastic analysis, where it is assumed that plastic hinges form at discrete points, and other portions of the members remain elastic.

Due to its complexity the compatibility analysis has been applied only to very simple frames, mainly to determine the buckling and stability limit loads. A compatibility analysis, even for a pinned base portal frame, is very tedious and requires considerable computer time. Although in principle the compatibility analysis
for a large multi-story frame can be formulated, it would also make a
tremendous demand on the capacity of computers and the actual computa­
tional efforts would likely be prohibitive. Furthermore, it remains
to be demonstrated that for a multi-story frame such an analysis
will always converge to the correct solution and determine the com­
plete load-deformation behavior of a frame up to the stability limit
load and beyond this limit load.

For second-order elastic-plastic analyses, the rigorous
requirement of compatible strains in the members is relaxed. The
analysis requires that numerous second-order elastic analyses be
performed on a structure having a steadily deteriorating stiffness
due to the formation of discrete plastic hinges. The analysis is
still quite involved but considerably simpler than a compatibility
analysis.

In both of these methods of analysis, the displacement
method is usually adopted for determining the deformations under
the given combined loads. As is the usual procedure in the displace­
ment method, the deformations are calculated from the equilibrium
equations. The applied loads are considered as the principal variables
and the compatible deformations are computed corresponding to the
applied loads. With this approach, there is always a singularity at
the stability limit load, where the determinant of the stiffness
matrix becomes zero. Convergence problems also occur well before the
singularity is reached. Therefore, the load which can be obtained
using this approach is at most a load close to the stability limit load.
Beyond the stability limit load, the determinant of the stiffness
matrix becomes negative and no solution can be obtained. Hence, the load-deformation behavior of an unbraced multi-story frame using this procedure can be determined only up to a point near but not at the stability limit load. Therefore, the behavior of the frame at the stability limit load, as well as at the formation of mechanism which is usually beyond the stability limit load and the subsequent unloading behavior of the frame cannot be determined. In order to understand the complete behavior of an unbraced multi-story frame, it is necessary to develop an analytical method which will give the complete load-deflection curve, up to the stability limit load and beyond the stability limit load.

The behavior of a frame will depend on the sequence of the load application. According to Vogel the mechanism load is unique for any loading path. It has been shown, however, that the stability limit load, which is usually greater than the mechanism load, depends on the loading path and there is no known relationship between the different stability limit loads obtained from different loading paths. Almost all research on unbraced multi-story frames subjected to combined loading has been devoted to determining the load-deformation behavior of frames under proportional loads. In most practical cases, however, the gravity loads will vary much more slowly with time than the lateral loads produced by wind or earthquake. The sequence of loading, which involves constant gravity loads and varying lateral loads, therefore will serve as a good approximation to the practical loading condition for most frames. Consequently, a comprehensive knowledge of the behavior of unbraced frames under this nonproportional
loading condition is necessary to better understand the complete practical behavior of unbraced frames.

In summary, because of the lack of understanding of the complete loading and unloading behavior of unbraced frames due to the limitations of existing analytical methods, the need exists for the development of an analytical method which can predict the complete elastic-plastic behavior of an unbraced multi-story frame up to the stability limit load and beyond the stability limit load, under the nonproportional loading condition consisting of constant gravity loads and varying lateral loads.

1.2 Objective and Scope

The objective of this dissertation is to develop an exact analytical method for predicting the complete elastic-plastic loading and unloading behavior of an unbraced multi-story frame subjected to nonproportional combined loading where gravity loads are constant and lateral loads vary. Based on the analytical approach developed in this dissertation, a procedure will be developed for determining the approximate load-deflection behavior of a single story in the middle and lower regions of an unbraced multi-story frame. In those regions, the combined load conditions usually control the selection of the beams and columns as shown in Fig. 1.1. It has previously been shown that gravity loads alone control the selection of members in the upper stories.\(^{(2)}\) As shown in Fig. 1.1 a transition zone occurs between these two regions where either gravity or combined loads may control.
Since the analytical approach is based on determining the value of the applied lateral loads consistent with a prescribed finite sway deflection of a story, the approach is referred to as the "sway increment method of analysis". The numerical procedure was programmed for a CDC 6400 electronic digital computer.

The scope of the study in this dissertation is as follows:

1. The development of the "sway increment method" of analysis which can determine the complete load-deflection behavior (loading and unloading) of an unbraced multi-story frame having constant gravity loads and with or without initial hinges (plastic or real) in the members.

2. The development of the approximate "one-story assemblage method" of analysis.

3. Theoretical studies on the load-deformation behavior of six unbraced multi-story frames under nonproportional loading.

4. The effects of column axial shortening on the load-deformation behavior of the frames studied in item 3.

5. The effects of hinge reversals on the load-deformation behavior of the frames studied in item 3.

6. Comparative studies of the behavior of a single story in a frame determined by the one-story assemblage method of analysis with the exact behavior of that story in the whole frame.

1.3 Review of Prior Work

In the absence of secondary failures (local buckling, etc.) unbraced frames will exhibit two basic modes of failure. When the frame is symmetrical and is under symmetrical gravity loads only, it can fail by buckling as shown in Fig. 1.2a. When the frame is subjected to combined gravity and lateral loads, it can fail by instability as shown in Fig. 1.2b. The corresponding load versus lateral-deflection behavior of each frame is shown in Fig. 1.2c. In the figure, the "load" refers to gravity loads in the case of buckling and lateral loads in the case of instability failure. No lateral deflection of the frame occurs prior to frame buckling. However, failure by instability is characterized by gradually increasing lateral deflection, first under an increasing lateral load and later, after reaching the stability limit load, under a decreasing lateral load. In the review of prior work which follows, a distinction is made between research on the buckling of frames and research on frames which fail by instability on the basis of the above described failure modes.

The buckling of symmetrical frames under symmetrical gravity loads has been studied by a number of researchers. The methods for the elastic buckling analysis of entire frameworks are well known and have been summarized by Bleich (7) and Horne and Merchant (8). A thorough study of the literature concerning the buckling of frames in
the elastic-plastic range was presented by Lu.\(^{(9)}\) A good survey of this work is also found in Chapter 15 of Ref. 2. Since this investigation is concerned with the instability mode of failure, no further attention will be given in this dissertation to the problems concerning frame buckling.

In 1954, Merchant\(^{(10)}\) discussed the effect of overall instability on the load-carrying capacity of rigid frames. Although Merchant's paper deals only with some fundamental concepts of instability, it stimulated other researchers to study the various inelastic instability problems of frames. Merchant proposed the use of a Rankine-type formula to determine the stability limit load of a rigid frame under the proportional loading condition. According to this procedure, the stability limit load is the harmonic mean of the rigid-plastic load and the elastic buckling load. The theoretical justification of the formula was provided by Horne.\(^{(11)}\) Results of experiments on model frames and more exact methods of analysis have shown Merchant's predictions to be on the conservative side in most cases, and thus safe. The objections against its use, however, are that in many cases it is too conservative and the evaluation of the rigid-plastic load and the elastic buckling load is not very easy for a frame of practical importance.

Wood\(^{(12)}\) observed that the collapse loads predicted by the simple plastic theory did not agree with the loads obtained from model tests of multi-story frames and that actual collapse load (stability limit load) of a multi-story frame was the deteriorated
critical load of the frame after yielding and plastic hinges occurred at several locations. Wood suggested finding the stability limit load by performing a buckling analysis on the frame after the formation of each successive plastic hinge under proportional loads. Although Wood clearly described the phenomena of frame instability in the plastic range in his work, his method is not particularly suitable for actual computation for large multi-story frames. Moreover, it is not possible to predict the deteriorated state of a frame without advance knowledge of the actual history of the order of plastic hinge formation in the frame.

In the early stage of the development of plastic design techniques for unbraced multi-story frames, considerable effort was made to formulate general procedures for computing the mechanism load. Heyman (13) proposed a plastic method which led to a design which is subsequently checked for instability effects. In this method a pattern of plastic hinges is assumed which involves both beams and columns. From this assumed hinge pattern the full plastic moments of the beams and columns are calculated. Beam sections are then chosen so that their full plastic moments correspond to the calculated values. The column sections are selected to remain elastic when the calculated values of full plastic moment and axial thrust are applied. Stevens (14) suggested limiting the deformations in order to control stability. A design method is proposed by which possible unfavorable changes in geometry are allowed for in the initial stages of a design when strength requirements are being satisfied. In the method, a conservative solution is obtained by ensuring that the initially
assumed deformations are overestimates of the actual deformations developed at the mechanism load. Holmes and Gandhi (15) presented a design method similar to Heyman's, (13) but the effects of instability are included at the onset by introducing a magnification factor. In their method, the sections selected by simple plastic theory are checked and increased, if necessary, to allow for instability effects. The columns are allowed to develop plastic hinges only at the mechanism load. Therefore, at any intermediate stage of loading, the columns are elastic. Lind (16) introduced a method of analysis in which correction moments determined by iteration are applied to the results of a first-order rigid-plastic analysis. Thus, Lind's method is also based on the attainment of the mechanism load. All of the proposed methods are based on the oversimplified assumptions of a mechanism and/or deflection shape and have not been shown to be practical for large multi-story frames.

Several authors have obtained solution for small structures by using the compatibility analysis in which more exact moment-curvature relationships are used in place of the plastic hinge assumption. Ang (17) presented a technique for the inelastic stability analysis of a hinge-based portal frame subjected only to lateral loading. He determined the lateral force consistent with an assumed sidesway displacement. He considered residual stresses in the beams and columns, strain-hardening and semi-rigid connections, but the effect of axial force on the beams and columns was ignored. Although simple portal frames subjected only to lateral loading were considered in his paper, his method introduced an analytical procedure which involves computing the applied load that is associated with a pre-
scribed finite sway deflection. A similar elastic-plastic procedure will be developed in this dissertation but for multi-story frames.

Chu and Pabarcius\textsuperscript{(14)} presented a trial and error method for obtaining the load versus sidesway behavior of an inelastic portal frame subjected to both vertical and horizontal loads. In their procedure, the value of the horizontal load consistent with an assumed sidesway displacement was determined by an iteration method. The beam was assumed to remain elastic, and the magnitude and the effect of axial force on beam stiffness was neglected. The effect of axial force on the curvature of the columns was incorporated into the analysis through the use of the $M$-$P$-$\Phi$ (moment-axial force-curvature) relationships developed by Ketter, Kaminsky and Beedle.\textsuperscript{(15)} Similar procedures for portal frames with hinged bases were also presented by Adams\textsuperscript{(16)} and Moses.\textsuperscript{(17)} The influence of gradual yielding in the beam was considered in the latter work. The nonproportional loading condition was considered in the above methods. The above procedures are certainly the most rigorous attempts to investigate frame instability and behavior in the inelastic range. However, the procedures presented are restricted only to the analyses of portal frames and the performance of a compatibility analysis, even for a pinned base portal frame, is very difficult.

Few researchers have attempted to extend the compatibility analysis to unbraced multi-story frames. Wright and Gaylord\textsuperscript{(18)} presented a computer iteration procedure suitable for performing a form of compatibility analysis of unbraced multi-story frames under nonproportional loads. In their procedure, each member of the frame
is divided into discrete intervals and the effective stiffness of each segment is computed, using the best available estimate of moments and axial forces in each cycle of iteration. With the calculated stiffness, the displacement method is used to obtain an improved estimate of moments and axial forces. Iteration continues with each load increment until the solution converges. The M-P-∅ relationships developed in Ref. 15 were also used in their work. In order to avoid the divergence of the solution near the stability limit load and to obtain a solution after the point of instability, a "partial correction technique" was developed. This technique makes use of fictitious elastic springs and a fictitious support distribution system. To avoid the problem of nonconvergence, the stiffness of the fictitious spring should have different values, depending on the frame geometry and vertical loads. More recently, Alvarez and Birnstiel(19) have presented a numerical procedure for the determination of the elastic-plastic behavior of plane unbraced rigid frames under proportional or nonproportional loading. An incremental load procedure was developed. The unknown incremental displacement vector is determined from the stiffness matrix and the incremental load vector. The method includes the effects of joint displacement, strain reversal, axial force in beams and columns, and spread of the inelastic zones in the members. To incorporate the M-P-∅ relationships for the column, a numerical procedure was developed for the determination of curvature for given values of axial force and bending moment. With the method, fixed base portal frames were analyzed for several loading conditions, and the application of their procedure for analyzing multi-story frames
was demonstrated by the analysis of a two-bay, two-story unbraced frame under nonproportional loading. In their procedure the incremental displacement vector is calculated from the equilibrium equations, as is the usual procedure in the displacement method. The determinant of the stiffness matrix becomes zero at the stability limit load, the increment of loads must be very small to avoid convergence problems, which results in a considerably increased number of iterations in the analysis. Although in principle the compatibility analysis for a large multi-story frame can be formulated, the actual computational efforts would likely be prohibitive.

In the second-order elastic-plastic method, the lateral-load versus lateral-deflection behavior of a frame is obtained up to the stability limit load. The method requires that numerous second-order elastic analyses be performed on a structure having deteriorated stiffness due to the formation of plastic hinges. In this approach the moment-curvature relationship for a given value of the axial load is assumed to be elastic-perfectly plastic. Harrison (24) has presented a second-order elastic analysis procedure for planar rigid frames. The displacement method is used where the member stiffness matrix is determined from the coefficients of the slope-deflection equations. In his procedure, the effects of axial forces and axial deformations have been accounted for, but the analysis is limited only to the elastic range.

Jennings and Majid (25) have developed a computer program which can analyze unbraced planar frames which are loaded by static,
proportional, concentrated loads. The analysis is performed by a matrix displacement method. Basically, the non-linear equations are solved by temporary linearization of the equations with an assumption of the axial loads in the columns. Hinge predictions are based on linear extrapolation of two successive points on the load-deflection curve. When assumed axial loads and load parameters at a hinge have converged, a new hinge is recorded and the iterative process is performed on a structure with deteriorated stiffness until the determinant of the stiffness matrix becomes negative. In their program, the effect of axial loads on the plastic moment capacity of a section was not considered. Horne and Majid\(^{(26,27)}\) have developed a computer based design method incorporating the second-order elastic-plastic analysis described above. The preliminary frame design is analyzed and redesigned, if necessary, to meet a minimum collapse load criterion. In addition, the following design criteria are placed on the formation of plastic hinges: 1) No plastic hinge should develop in a beam below the load factor of unity and the frame should be entirely elastic under the working load, 2) no plastic hinge should develop in a column below the design ultimate load either for combined loading or vertical loading. The method considers the reduced plastic moments in the columns in the presence of axial loads. The analysis has been developed to determine the elastic-plastic design of unbraced frames entirely by computer. Majid and Anderson\(^{(28)}\) have extended the method, originally developed by Jennings and Majid\(^{(25)}\) and later refined by Horne and Majid,\(^{(27)}\) for the elastic-plastic analysis of large structures. In their procedure, in order to make full use of the computer storage, all the properties of the stiffness
matrix were utilized, such as the symmetrical feature of the matrix and a large number of its zero elements. In addition, a more accurate procedure for predicting the next plastic hinge was adopted and facilities for the insertion of more than one hinge at a time were included in their program to reduce the computer time required for large structures. They have also developed a fully automatic computer design method for the elastic-plastic design of unbraced frames, based on the method originally described by Horne and Majid. The same design criteria in Ref. 27 were used in their procedure. Full details of the design of a fifteen story frame by the method have been presented.

Parikh also developed a computer based analysis method for the second-order elastic-plastic analysis of unbraced multi-story frames under proportional loading by using the slope-deflection equations. He also included the effect of axial deformations and residual stresses in the columns. The slope-deflection equations are extended to consider the behavior of members after the formation of one or more plastic hinges in each member. For this purpose, the slope-deflection equations are derived to accomodate all possible hinge patterns in a frame initially without real hinges. Instead of using a linear prediction technique for each successive plastic hinge, the structure is entirely re-analyzed at each load level by using the Gauss-Seidel iteration procedure for solving the slope-deflection equations. Plastic hinges are allowed to occur until the iteration procedure diverges and no further increase in load is possible. Parikh's iteration procedure for solving the slope-deflection equations makes it possible to use the computer storage space effectively.
The program can accommodate concentrated loads as well as uniform loads on the beams. Korn\(^{(32)}\) has used the same application of the slope-deflection equations for the study of the elastic-plastic behavior of multi-story frames. He considered the effects of axial deformation and curvature shortening due to bending but neglects the residual stresses in the columns. Loading is restricted to proportional, concentrated loads. Linear predictions of successive plastic hinges are used in the manner developed by Horn and Majid\(^{(27)}\) and the slope-deflection equations are solved by using the Gauss-Seidel iteration method. As the criterion for determining the stability limit load, the determinant of the stiffness matrix, before and after each plastic hinge formation, is calculated and when the determinant passes from a positive value to a zero or negative value, it is assumed that the stability limit load has been obtained and the analysis is terminated. Seventeen fixed base frames have been analyzed using the program and the behavior of each frame has been discussed. The behavior of eight unbraced multi-story frames analyzed by Korn are also discussed in a paper by Korn and Galambos.\(^{(33)}\) The same frames used by Korn and Galambos for the study of frame behavior have been analyzed by Adams, Majundar, Clark and MacGregor\(^{(34)}\) under uniformly distributed beam loads. The value of the uniform load on each beam is calculated as equal to the sum of the three equivalent concentrated beam loads used in Ref. 33 divided by the beam span. It has been shown that in spite of the difference in the distribution of the beam loads, the general trends of the frame behavior were similar to that obtained with concentrated loads. All of the above analytical procedures based on the second-order elastic-
plastic method enable the determination of the load-deformation behavior of a frame up to a point near the stability limit load and are suitable for the analysis of multi-story frames by a computer. However, as mentioned in Sect. 1.1, convergence problems occur near the stability limit load and the behavior of a frame at the subsequent unloading behavior of the frame cannot be determined by the above analytical procedures. The above procedures are still limited to the consideration of proportional loading.

\[ \text{Davies (35) extended the method of analysis of Jennings and Majid (25) to include hinge reversals and also to consider the response of plane frameworks to cyclic or programmed loading. His procedure can also take into account the effect of strain-hardening on the plastic hinges by considering the increased plastic moment capacity. The inclusion of the above effects considerably adds to the scope of second-order elastic-plastic methods of analysis.} \]

\[ \text{The sway subassemblage method of analysis was developed by Daniels (36, 37, 38) to determine the approximate second-order elastic-plastic behavior of individual stories of an unbraced multi-story frame. In the method, individual stories are isolated by assuming that all columns are bent into symmetrical double curvature. The resulting one-story assemblage is subdivided into sway subassemblages. Each sway subassemblage can then be analyzed by a second-order elastic-plastic analysis technique either manually with the help of prepared charts (36, 39) or by means of a computer. (40) The lateral-load versus sway-deflection curve of the one-story assemblage is determined by combining the resulting load-deflection curves of the component} \]
sway subassemblages. This curve gives the complete load-deflection behavior of the one-story assemblage up to and beyond the stability limit load. Nonproportional or proportional loading can be treated. In the latter case several analyses are performed each assuming a level of proportional loading. The axial forces in each column can be assumed to remain constant or they can be allowed to vary in any manner. In the latter case, several analyses are performed, each assuming a constant value of column axial loads. A design method for unbraced multi-story frames has been developed at Lehigh University,\(^2,41,42\) which employs the sway subassemblage method in one phase of the design. The method has suggested the following three-step design procedure: preliminary design, load-deflection analysis and revision. Tentative beam and column sizes are selected using the plastic moment balancing method.\(^43\) Initially estimated plastic moments are included when equilibrium is established. Following the preliminary design of the frame, a sway subassemblage analysis is performed at each story to verify the initial sway estimates and to assure compliance with maximum sway tolerances at working loads. In the third step of the design process, revisions to the preliminary design are made, based on strength, deflection and economy. The sway subassemblage method of analysis enable the determination of the approximate load-deflection behavior of a one-story assemblage. However, it remains to be demonstrated how well the individual story behavior determined from the sway subassemblage method of analysis agrees with the story behavior in the context of total frame.

Recently Emkin and Litle\(^44\) have developed a computer oriented method for plastic design of both braced and unbraced multi-
story frames including a consideration of elastic stress and elastic deflection constraints. For unbraced frames, the method proportions individual beams and columns according to a certain optimization procedure to minimize material cost. A minimum rolled section configuration is determined for a frame on the basis of beam mechanism failures due to factored gravity loads. Factored lateral loads and the PA effect are then applied to the frame on a story-by-story basis in an incremental fashion. After each application of an increment of story shear a new force distribution is determined and a redesign of those members which experience force changes is executed. After the first cycle of design of the frame with initially assumed deflections calculated for each story. If these deflections satisfy the convergence criterion, the design is completed. Otherwise, the calculated deflections become those of the PA effect in the next cycle of design. The employment of the story-by-story design procedures makes the method very efficient. However, the method is based on the oversimplified assumptions of a collapse mechanism.

In order to investigate the experimental behavior of unbraced frames, some model tests have been conducted. Law has presented a series of thirty-four model frame tests designed to investigate the effects of overall instability on load carrying capacity. All the models had a span length of 14 in. and a story height of 7.5 in. and were constructed of mild steel bars 0.25 in. wide having a thickness ranging from 0.198 to 0.250 in. Three, five and seven-story single-bay models were tested as plane frames free to sway. Vertical loads were applied to the beams at the quarter points, and some frames were also subjected to small horizontal loads at all the panel points. The test results
show that the Rankine type of empirical formula for estimating the inelastic instability load of frames suggested by Merchant\(^{(10)}\) is rather conservative and the reduction of ultimate load due to frame instability is larger for taller frames. Wakabayashi\(^{(46)}\) has conducted model tests on steel portal frames to investigate the effect of the axial force on the elastic-plastic behavior of tall steel buildings subjected to the earthquake, particularly on the horizontal ultimate strength and deformation property. His test results show the considerable effect of the axial force of columns on the horizontal maximum load. Wakabayashi, Nonaka and Morino\(^{(47)}\) also presented an experimental study on the behavior of single-bay three-story model frames, cut from a sheet of mild steel. A constant vertical load was applied on the columns and a monotonically increasing horizontal load at the top of a model frame. It has been shown that the experimental results are in good agreement with the second-order elastic-plastic type of analysis in the range of small displacements. However, there is considerable difference between the two values of the maximum horizontal force in the large deformation range. It has been also shown that an approximate elastic-strain-hardening analysis gives a reasonable agreement with experimental results in the large displacement range.

Very few tests have been performed on full size frames, related to the unbraced multi-story frame. Yarimci\(^{(48)}\) has described three tests on full-scale three-story unbraced steel frames with the specific purpose of verifying the validity of frame strength predictions in the presence of axial loads effects. Three specimens, two one-bay three-story and one two-bay three-story, were fabricated from A36 rolled
steel shapes with the bay length of 15 ft. and the story height of 10 ft. The bases of columns were fully fixed. The frames were tested under nonproportional loading conditions: the vertical loads were applied initially and maintained constant during the subsequent application of horizontal loads. It was shown that a second-order elastic-plastic analysis can adequately predict the inelastic response and the failure mechanism of such frames under nonproportional combining loading condition. Arnold, Adams and Lu \cite{49} reported on an experimental study of a full-scale hinged base portal frame. The objectives of the study were to test a simple frame which exhibits behavior similar to that of a multi-story frame and to examine the behavior of the high-strength steel columns used with the structural carbon steel beam under sway conditions. The test result shows a good agreement with the theoretical prediction from a second-order elastic-plastic analysis with strain-hardening.

For an experimental evaluation of the sway subassemblage method of analysis, two full-scale one-story assemblages were tested by Kim and Daniels.\cite{50} The two-bay one-story assemblages were subjected to nonproportional loading. The tests indicate good correlation between the experimental results and the sway subassemblage analysis of a one-story assemblage.
2. SWAY INCREMENT METHOD OF ANALYSIS

2.1 Introduction

Frame behavior can be characterized by the relationship between the applied loads, as they vary during the loading history, and the resulting deformations. The load-deformation relationship depends considerably on the sequence of the load application. The loading condition considered in this dissertation is nonproportional and is described as follows: the full value of the gravity load is applied first and then the lateral load is allowed to vary from zero to maximum and then allowed to decrease. Figure 2.1a shows an unbraced multi-story frame, subjected to constant distributed gravity load w and variable lateral load H at its deflected position. A typical load-deflection behavior of the unbraced frame under this loading condition is shown in Fig. 2.1b. The relationship shown in Fig. 2.1b is nonlinear from the start because of second-order geometric effects. After the elastic limit is reached, the slope of the curve is further reduced due to material yielding and finally the slope becomes zero at the attainment of the stability limit load $H_{\text{max}}$. Beyond the stability limit load, the slope becomes negative, that is, the lateral deflection continues to increase with decreasing lateral load. It can be seen from Fig. 2.1b that two equilibrium configurations of the frame are possible for a given value of lateral load. Stable equilibrium is possible prior to the stability limit load whereas only unstable equilibrium is possible afterwards. In general the mechanism load
will occur after the attainment of the stability limit load in unbraced multi-story frames.

As discussed in Chapter 1, the two general approaches for the determination of the lateral load versus sway deflection behavior of unbraced multi-story frames are the compatibility analysis and the second-order elastic-plastic analysis. In both analyses, the displacement method is usually used to calculate the lateral deflections of the frame from known lateral loads. In the method, a displacement vector \( \mathbf{U} \) is related to a load vector \( \mathbf{F} \) by the matrix equation

\[
K \mathbf{U} = \mathbf{F} \tag{2.1}
\]

where \( K \) is the symmetrical stiffness matrix of the structure. The main difference between a compatibility analysis and a second-order elastic-plastic analysis results from the formulation of this stiffness matrix \( K \). Whichever analyses is used, the unknown displacement vector can be obtained from

\[
\mathbf{U} = K^{-1} \mathbf{F} \tag{2.2}
\]

where \( K^{-1} \) is the flexibility matrix of the structure. Instead of matrix inversion, iteration techniques can also be used for the solution of Eq. 2.1. Having solved Eq. 2.1 all of the displacements are known and the bending moments and member axial forces are evaluated in the usual way.

In almost all analyses performed to date the applied load vector \( \mathbf{F} \) is considered the independent variable. In this case, there are, in general, three solution ranges, as indicated by Wright and
Gaylord.\textsuperscript{(22)} As shown in Fig. 2.2, the unknown displacements can easily be obtained initially. However, as the stability limit load is approached, the solution becomes more difficult because of the divergence of the solution as the determinant of the stiffness matrix approaches the singular point. The solution corresponding to the stability limit load cannot be attained unless special techniques are used, such as the technique described in Ref. 22. Beyond the stability limit load, the solution does not converge. Therefore, the load-deflection behavior after the stability limit load cannot be determined and the lateral deflection at the mechanism load, which usually occurs beyond the stability limit load cannot be calculated.

The convergence problems associated with the above approach can be avoided if the displacement vector $U$ is taken as the independent variable. In this case the lateral loads $H$ in Fig. 2.2 are to be calculated consistent with a specified lateral displacement $\Delta$ of the frame. Using this approach, there is only one possible lateral load for any given lateral displacement as shown in Fig. 2.3. It is evident by examining the figure that for every $\Delta$, there exists a unique $H$. The problem of the nonconvergence of solution can be avoided and the complete solution up to the stability limit load and beyond can be readily obtained. In fact, convergence is more rapid in the vicinity of the stability limit load. This method of solution will be developed in detail in this dissertation. The second-order elastic-plastic method of analysis will be used and the unknown displacement vector $F$ will be calculated from Eq. 2.1 using an iteration technique.
2.2 Strength of Sections

2.2.1 Plastic Moment Capacity

In the second-order elastic-plastic method of analysis, plastic hinges are assumed to occur at certain cross-sections when the plastic moment capacity of the cross-section is reached. The member is considered elastic between plastic hinge locations and it is further assumed that a plastic hinge may be replaced by a real hinge for analysis purposes.

In the absence of axial load, a plastic hinge is considered to form when the moment at a cross-section reaches the plastic moment capacity, $M_p$, of the section. However, in the presence of axial loads, the cross-section will exhibit a reduced plastic moment capacity, $M_{pc}$. For a wide flange section bent about its major axis, $M_{pc}$ will be taken as follows: (51)

$$M_{pc} = M_p \quad 0 < \frac{P}{P_y} \leq 0.15$$

(2.3)

$$M_{pc} = 1.18 \left(1 - \frac{P}{P_y}\right) M_p \quad 0.15 < \frac{P}{P_y} \leq 1.0$$

where $P$ is the applied axial load and $P_y$ is the axial yield load of the member.

2.2.2 Moment-Curvature-Thrust ($M$-$\phi$-$P$) Relationship

The $M$-$\phi$-$P$ relationship is determined from the resulting curvature $\phi$ of a cross-section under an applied moment $M$ in the presence of an axial load $P$. In general, the $M$-$\phi$ relationship is a function of the cross-section properties, material properties, applied axial load and residual stresses. In the absence of axial load, the
relationship for different shapes with and without residual stresses is as shown in Fig. 2.4, which is reproduced from Ref. 2. The solid curves show the true behavior and dashed line represents the idealized M-φ relationship used in this dissertation. The difference between using the actual M-φ curves or the idealized curve has been shown by others to be negligible for practical frames. (31) The idealized M-φ relationship shown in Fig. 2.4 can be expressed as follows:

\[ M = EI\phi \quad \text{for} \quad 0 < \phi < \phi_p \]  
\[ M = M_p \quad \text{for} \quad \phi > \phi_p \]  

(2.4)

where \( E \) is the modulus of elasticity and \( I \) is the moment of inertia of a cross-section about its major axis and \( \phi_p = M_p / EI \).

The relationship is much more complicated in the presence of axial loads. Figure 2.5, reproduced from Ref. 31, shows the M-φ-P relationship for a W8x31 shape with the residual stresses as shown in the inset and with different axial load ratios, \( P/P_y \). The curves shown by the solid lines have been obtained by the method of sectioning, using a numerical iterative procedure. (31) It has been shown that the W8x31 shape is a representative cross-section for most wide-flange column shapes and the residual stress pattern assumed is also a representative pattern for rolled shapes. (2) From the nondimensionalized curves in Fig. 2.5, it can be noticed that the idealized M-φ curve given by Eq. 2.4 is still a fairly good approximation to the true curves if \( M_{pc} \) is substituted for \( M_p \) when \( P/P_y \leq 0.7 \). If the same idealization is used for \( P/P_y > 0.7 \), significant error is observed.
In this case, some parts of the cross-section have already yielded under the combination of the stresses due to the applied axial load and the residual stresses prior to the application of bending moment, thus causing a considerable reduction in stiffness. The reduction can be readily seen in Fig. 2.6, reproduced from Ref. 31. This figure which can be constructed from Fig. 2.5 shows \( P/P_y \) versus \( \frac{M/M_{pc}}{\varphi/\varphi_{pc}} \) when \( \varphi/\varphi_{pc} = 0.1 \) and \( \sigma_{rc}/\sigma_y = 0.3 \). The same curve can also represent \( P/P_y \) versus \( I_e/I \) by considering that

\[
\frac{M/M_{pc}}{\varphi/\varphi_{pc}} = \frac{M/\varphi}{M_{pc}/\varphi_{pc}} = \frac{E I_e}{E I} = \frac{I_e}{I} \quad (2.5)
\]

where \( I_e \) is the effective moment of inertia of a cross section under axial load and containing residual stresses. As can be seen from the solid line of Fig. 2.6, the value of \( I_e/I \) varies from 1.0 for \( P/P_y \leq 0.7 \) to zero when \( P/P_y = 1.0 \). The dashed curve is an approximation of the relationship for \( P/P_y \geq 0.7 \). This approximation is used in this dissertation for the effective moment of inertia, \( I_e \) under the given residual stresses and axial loads. That is,

\[
\begin{align*}
\text{for } P/P_y & \leq 0.7 & \frac{I_e}{I} &= 1.0 \\
\text{for } P/P_y & \geq 0.7 & \frac{I_e}{I} &= \sqrt{\frac{\sigma_y}{\sigma_{rc}}} (1.0 - \frac{P}{P_y}) 
\end{align*} \quad (2.6)
\]

where \( \sigma_y \) is yield stress and \( \sigma_{rc} \) is the maximum compressive residual stress.

Using the above value for the effective moment of inertia, the \( M-\varphi \) relationship for \( P/P_y > 0.7 \) is more closely approximated. Figure 2.5 shows the idealized approximations for \( P/P_y = 0.8 \) and 0.9.
2.3 Incremental Procedure

2.3.1 One-Step Versus Incremental Procedure

There are two alternate procedures for determining the unknown lateral load which will produce a given lateral deflection. The first may be designated the one-step procedure, the second the incremental procedure. Consider the fixed base portal frame subjected to the loading shown in Fig. 2.7a. In the one-step procedure, as shown in Fig. 2.7b, the value of the unknown lateral load $H_n$ corresponding to the given lateral deflection $\Delta_n$ is determined directly by a one-step calculation in the analysis. In the incremental procedure as shown in Fig. 2.7c, the value of $H_n$ is obtained from the equation

$$H_n = \sum_{n=1}^{n} \delta H_n$$

(2.7)

where $\delta H_n$ is the increment of lateral load corresponding to the increment of lateral deflection $\delta \Delta_n$.

Both of these procedures when used in a second-order elastic-plastic analysis will result in the same load-deflection behavior of the frame only if the elastic unloading of a plastic hinge is not considered. However, in an unbraced multi-story frame, plastic hinge reversal can occur before the stability limit load is attained and it is generally believed that a number of plastic hinge reversals can occur following the stability limit load. In this case, the one-step procedure, in which the total deformations are assumed and the total loads are calculated cannot be used to determine the correct frame behavior in the presence of plastic hinge reversal. If unloading
of plastic hinges is to be considered in the analysis, the incremental procedure should be used. In addition the information obtained in each cycle using the incremental procedure can be conveniently used for the prediction of the formation of the next plastic hinge and to detect the unloading of a plastic hinge.

### 2.3.2 Sign Convention

The sign convention adopted in this dissertation is as follows:

1. Moments and rotations at the ends of members are positive when clockwise.
2. Moments acting at a joint are positive when counterclockwise.
3. Moment in the interior of a beam is positive when it produces tension on the bottom.
4. Moment in the interior of a column is positive when it produces tension on the right.
5. Horizontal shear in a column is positive if it causes a clockwise moment about the end of the column.
6. Axial forces in beams and columns are positive when they produce compression.
7. Vertical deflection of joints are positive when downward.
8. Lateral deflections of joints are positive when towards the right.

### 2.3.3 Incremental Slope-Deflection Equations

Referring to Fig. 2.8, the general slope-deflection equations considering the effect of axial loads on the bending moments are as follows:
\[
M_A = \frac{EI}{L} \left[ C\theta_A + S\theta_B - (C+S) \frac{\Delta_B - \Delta_A}{L} \right] + F_{MA}
\]

\[
M_B = \frac{EI}{L} \left[ C\theta_B + S\theta_A - (C+S) \frac{\Delta_B - \Delta_A}{L} \right] + F_{MB}
\]

where,

\[
\phi = \sqrt{\frac{P}{EI_e}}
\]

\[
s = \frac{1}{\phi} \left( \frac{\phi}{\sin \phi} - 1 \right)
\]

for compression

\[
c = \frac{1}{\phi} \left( 1 - \phi \cot \phi \right)
\]

\[
s = \frac{1}{\phi} \left( 1 - \frac{\phi}{\sin h \phi} \right)
\]

for tension

\[
c = \frac{1}{\phi} \left( \phi \cot h \phi - 1 \right)
\]

\[
S = \frac{c}{c^2 - s^2}
\]

\[
C = \frac{s}{c^2 - s^2}
\]

In Eq. 2.8, \( I \) is the moment of inertia of the cross-section, \( I_e \) is the effective moment of inertia, that is, the moment of inertia of the remaining elastic portion considering residual stresses and \( E \) is the modulus of elasticity. The parameter \( \phi \) is usually called the stability factor, while \( S \) and \( C \) are designated the stability functions. The fixed end moment terms, \( F_{MA} \) and \( F_{MB} \), are functions of the vertical
loading \( w \), which can either be uniformly distributed loads or concentrated loads.

Equations 2.8 can also be expressed in incremental form as follows:

\[
\delta M_A = \frac{EI}{L} \left[ C\theta_A + S\theta_B - (C+S) \frac{\delta \Delta_B - \delta \Delta_A}{L} \right] + \delta F M_A
\]

\[
\delta M_B = \frac{EI}{L} \left[ C\theta_B + S\theta_A - (C+S) \frac{\delta \Delta_B - \delta \Delta_A}{L} \right] + \delta F M_B
\]  

(2.9)

Using Eqs. 2.9, the increment in moment corresponding to a given increment of deformation can be calculated. The stability functions, \( C \) and \( S \), in Eqs. 2.9 are to be taken as the calculated values in the increment being considered.

The slope-deflection equations presented in Eqs. 2.8 or 2.9 are for the case of a continuous member, that is, prior to the formation of a plastic hinge. However, in the general elastic-plastic formulation, any member may contain from one to three plastic hinges located anywhere in the member and at any stage of loading. For purposes of analysis, these plastic hinges can be considered to form at three locations. A plastic hinge can form at either end of the member with an additional plastic hinge at an interior point where the bending moment is a maximum. In all, eight different combinations of plastic hinges in any member are possible as shown in Fig. 2.9. A different incremental slope-deflection equation can be written for each of the eight combinations shown in Fig. 2.9. The incremental slope-deflection equations for each combination are derived in Appendix 1.
Similar slope-deflection equations were derived in Ref. 31 but considered total deformations. Using the incremental slope-deflection equations, all end moments can be obtained as functions of the deformations (joint rotations, joint vertical deflections and sway deflections) and independent of plastic hinge rotations. Thus, for a multi-story frame containing a certain number of members, the number of slope-deflection equations and the number of unknowns will be constant regardless of the number of plastic hinges which can occur.

2.3.4 Incremental Inelastic Hinge Rotation

If hinge reversals are to be considered in the analysis of a multi-story frame, inelastic hinge rotation histories must be known and the hinge rotations must be examined at all stages to detect if unloading occurs. The numerical values of the plastic hinge rotations are also required for use in secondary design considerations such as local buckling and lateral-torsional buckling analyses. The incremental hinge rotation angles for a given lateral deflection or sway increment of the frame can be computed using the incremental slope-deflection equations discussed in the previous article. The total plastic hinge angle at any stage is the summation of the incremental values up to that stage. The equations for the calculation of incremental plastic hinge angles are derived in Appendix 2 for the various configurations of possible plastic hinges shown in Fig. 2.9.

2.4 Sway Increment Analysis

2.4.1 Assumptions

The basic assumptions on which the sway increment analysis has been formulated are as follows:
1. The material is homogeneous and elastic-perfectly plastic.
2. The frame is planar and rectangular.
3. All out-of-plane instabilities and local buckling are inhibited by providing adequate bracing and placing appropriate restrictions on member geometry.
4. No bracing or cladding is used in the plane of the frame to resist lateral deflection.
5. Member length is from center-of-connection to center-of-connection.
6. The column bases are assumed to be fixed in the plane of the frame.
7. The frame is loaded only by planar, static, nonproportional loads. The frame and initial gravity loading may be symmetrical or unsymmetrical.
8. Wind pressure can have any type of distribution over the building surface but it is carried to the framework as concentrated lateral loads applied horizontally at floor levels.
9. The frame is completely unstressed before the load is applied, except for cooling residual stress of rolled steel sections.
10. Axial loads in the beams are negligible.
11. Shearing strains are neglected.
12. A plastic hinge is replaced by a real hinge for the additional sway increments.
13. A joint mechanism is a valid failure mechanism.
Assumption 1 indicates that the effects of strain-hardening are not considered in this analysis. In actual frames, wind loads are distributed lateral loads applied by the exterior wall system and the value of wind loads depends on the magnitude of wind pressure on the exterior wall surface. Assumption 8 specifies that the distribution of the wind pressure over the building surface can have any shape (for example, uniform distribution or triangular distribution) but the wind load is carried to the framework as concentrated lateral loads applied at the exterior joints. Assumption 13 is necessary for any type of analysis which considers continually reduced plastic moments in the presence of axial loads unless the frame contains no more than two members at any joint. In the analysis, the joint mechanism is considered as a local failure which represents one limit of usefulness of the whole frame. As a demonstration of the typical formation of a joint mechanism in a multi-story frame refer to Fig. 2.10. Depending on the applied loads, frame geometry and sectional properties of members, it is possible to reach stage 1, say, in the analysis where plastic hinges have formed at the leeward end of the beam and at the bottom of the upper column at an leeward exterior joint as shown in Fig. 2.10a. In this case, the lower column moment $M_{L1}$ is determined solely by the joint equilibrium as shown. As the axial loads in the columns above and below the leeward joint increase the reduced plastic moment capacity of each column is decreased. At stage 2 shown in Fig. 2.10b $M_{pcU2} < M_{pcU1}$ and $M_{L2} > M_{L1}$. However, $M_{L2}$ is again determined by joint equilibrium as shown. Finally, stage 3 is reached as shown in Fig. 2.10c at which the lower column moment capacity is exhausted and a new plastic hinge develops. All members
now contain plastic hinges at the joint, denoting a state of instability which is a limit of usefulness of the frame. In reality, this state of instability is not possible due to the effects of strain-hardening and possible strain reversal. Since these effects are neglected in this analysis, the joint mechanism must be considered.

2.4.2 Convention for Numbering and Lateral Load Index

The general frame configuration considered in this dissertation is as shown in Fig. 2.11. Also shown are the distributed gravity loads and lateral loads. The numbering system for a level (boundary between two stories) starts from the top of the frame with one and proceeds downward as shown in the figure. Stories are numbered from the bottom and proceed upward. Thus, level 3 is the boundary between stories above and below level 3.

The value of the lateral load at an exterior joint (which coincides with a level) depends on the magnitude of the wind pressure, its distribution and the heights of the adjacent stories above and below the level in question. The magnitude of the wind pressure (that is, the magnitude of the lateral load) is continually changing during the analysis but the assumed shape of the wind pressure distribution is constant throughout the analysis. Therefore, the ratios of the lateral loads at the exterior joints must be constant at any stage of the analysis. Since the total shear acting in a story is equal to the sum of the lateral loads above that story, the total resisting shear in that story cannot be explicitly related to the absolute magnitude of the lateral load acting at any particular joint.
For convenience in the analysis, therefore, the constant ratio of the lateral load at a given level to the lateral load at a particular level is used. This ratio will be referred to as "lateral load index" of the given level. As shown in Fig. 2.11 if the lateral load $H_n$ at level $n$ is chosen for the particular lateral load, the lateral load indices of other levels are the ratios of the lateral loads at those levels to that at level $n$. Since the applied lateral loads are in equilibrium with the story shear forces, the total story shear in any story can be related to the lateral load indices and the particular lateral load. For example, consider story $m+1$ in Fig. 2.11. The total shear force in the story $Q_{m+1}$ can be expressed as

$$Q_{m+1} = \frac{1}{\sum_{i=n-1}^{1} H_i}$$

$$= \frac{1}{\sum_{i=n-1}^{1} (H_n \times \frac{H_i}{H_n})}$$

$$= H_n \frac{1}{\sum_{i=n-1}^{1} \frac{H_i}{H_n}}$$

$$= H_n \frac{1}{\sum_{i=n-1}^{1} I_i}$$

(2.10)

where $H_n$ is the particular lateral load and $I$ is the lateral load index. Thus, the total resisting shear in any story at any stage of the analysis is equal to the particular lateral load multiplied by the summation of the lateral load indices above the story in question. Therefore, the change in the value of the particular lateral load, as lateral deflections are increased, is considered
as the variation in "lateral load intensity" or "load intensity". Once the shear forces of individual stories are calculated, the shear forces can be expressed in terms of the load intensity. In the example frame of Fig. 2.11, the load intensity \( H \) is equal to \( H_n \) and its value can be calculated from the total shear force in the story \( m + 1 \) as

\[
H = H_n = \frac{Q_{m+1}}{\sum_{i=n-1}^{1} I_i}
\]

(2.11)

The load intensities calculated for the individual story shear forces must be same throughout the frame at any stage of the analysis. The lateral load \( H_i \) at level \( i \) is then expressed as

\[
H_i = H \times I_i
\]

(2.12)

where \( H \) is the load intensity and \( I_i \) is the lateral load index at level \( i \). The level at which the load intensity (that is, the particular lateral load) is chosen is arbitrary.

By plotting the relationship between the variation in load intensity and the corresponding sway deflection at the top of a frame, the load-deflection behavior of the frame can be examined. Also the load-deflection behavior of individual stories can be studied from the relationships between the variation in the load intensity and the corresponding sway deflection of the individual stories. In the proposed sway increment method of analysis, the increment of load intensity will be determined for a given sway increment in one story (initially the bottom story and after the stability limit load
is reached, the failed story). In other stories the sway increment, which results in the same increment of load intensity with that calculated in the story with the given sway increment, will be determined in each story by iteration.

2.4.3 Statical Equilibrium and Geometrical Compatibility

The conditions of statical equilibrium and geometrical compatibility must be satisfied for any member, any joints, any part of a frame or for the whole frame.

In deriving the incremental slope-deflection equations discussed in Sect. 2.3.3, equilibrium was maintained for part of a member as well as for the whole member. The axial and transverse deflections of a member must be continuous throughout the length of a member. Similarly, the rotations should be continuous, although with the one exception that after the formation of a plastic hinge, this rotational continuity may be violated. The incremental slope-deflection equations satisfy these conditions.

A joint is subjected to the stress resultants from all members framing into it. In general there are four members framing into the interior joints of a rectangular frame. For the exterior joints, there are usually only two or three members framing into the joint. The forces acting on a joint are shown in Fig. 2.12. To maintain equilibrium, the forces must satisfy the following equations:

\[ M_r + M_L + M_U + M_L = 0 \]  
(2.13)
The incremental form of Eq. 2.13 is given, for the purpose of the later use, as

\[ H_L - H_r + H_U - H_L = 0 \]  
(2.14)

\[ R_r + R_L + P_U - P_L = 0 \]  
(2.15)

The joints are assumed to be rigid acting as a point. The horizontal and vertical deformations of the ends of members framing into a joint must be the same in order to maintain continuity. Similarly the rotations of these ends must be the same unless there is a plastic hinge on one or more of these ends in which case this condition may be violated for those ends. These conditions will be satisfied in the analysis.

Another important condition of equilibrium to be considered in the analysis of unbraced frames is the story shear equilibrium. Figure 2.13 shows a typical story in a multi-story frame. To satisfy the equilibrium condition for the deformed structure, the following equation must be satisfied.

\[ \Sigma M_C + h \Sigma H + \Sigma P \Delta = 0 \]  
(2.17)

where \( \Sigma M_C \) is the sum of the column end moments, \( h \) is the story height, \( \Sigma H \) is horizontal shear, \( \Sigma P \) is the sum of applied gravity loads and \( \Delta \) is the sway deflection. The incremental equilibrium requirement for the story shear is then given by
In Eqs. 2.17 and 2.18 \( \Sigma P \) is to be taken as the total gravity load above the story under consideration which contributes to the \( P_\Delta \) effect. This concept illustrated in Fig. 2.14. The gravity load contributing to the \( P_\Delta \) effect is computed used the tributary area of the unbraced frame. The tributary area of an unbraced frame is computed using a width equal to the width of the unbraced frame between exterior columns and a length equal to the average spacing of the adjacent unbraced frames (\( s/2 + s/2 \) in Figs. 2.14a and 2.14b).

The axial shortening of columns is considered in the analysis so the joints at the same level can go through differential vertical deflections. On the other hand, since the axial forces in the beams are neglected, there is no change in the length of the beams. As a result the relative horizontal deflection of column tops with respect to the column bottom must be identical for all the columns in a story.

2.4.4 Analytical Procedure

The sway increment method of analysis proposed in this dissertation is based on the concept that for a given sway deflection of a story in an unbraced multi-story frame, the resulting internal stress resultants and the lateral load consistent with the sway deflection can be determined using an iterative incremental procedure. The basic concept of the method is further described in Fig. 2.15. In the two-bay three-story example frame shown in Fig. 2.15a, the

\[
\Sigma \delta M_C + h \Sigma \delta H + \Sigma P_\delta \Delta = 0 \quad (2.18)
\]
distribution and direction of the lateral loads applied at the joints are assumed to be as shown. In this case, a uniformly distributed wind loading is being assumed. Then, the lateral load indices are 0.5 at the first level and unity at levels 2 and 3. The lateral load at level 3 is chosen for the particular lateral load. Therefore, the variation in load intensity is the change of the value of H in the figure for this case.

In the analysis the sway deflection of the bottom story of the frame is initially incremented and the corresponding incremental load intensity is computed. As shown in Fig. 2.15b, at a particular stage of the analysis (after the (n-1)th sway increment) the total sway deflection of the bottom story is $\Delta_{n-1}$ and the load intensity $H_{n-1}$. As the first step of the next increment, the incremental sway of the bottom story $\delta \Delta_{n1}$ (that is, the nth sway increment) is applied. The magnitude of the incremental sway is determined from the prediction for the next plastic hinge, which will be discussed in detail in the next article. Then, the incremental load intensity $\delta H_n$ corresponding to the sway increment $\delta \Delta_{n1}$ is calculated from equilibrium equations by iteration in the bottom story. The total load intensity $H_n$ corresponding to the total sway deflection of the bottom story $\Delta_{n1}$ is obtained by adding the calculated incremental load intensity to the total load intensity at the beginning of the current increment $H_{n-1}$, as shown in Fig. 2.15b. Then, in the middle story, the sway increment $\delta \Delta_{n2}$, which results in the same incremental load intensity determined in the bottom story, is computed by iteration, as shown in Fig. 2.15c. The
same computation is performed in the top story, as shown in Fig. 2.15d.
In computing the incremental sway of a story, the latest available information on the incremental member end rotations and sway deflections of the adjacent stories are utilized. After the computation at the top story, the first cycle of the iteration is over. The incremental load intensity is computed again at the bottom story with the same given sway increment, using the latest information on deformations of the adjacent story. The new sway increment, which yields the newly computed incremental load intensity, is determined by iteration in the middle story and then in the top story. This cycle is repeated until the incremental load intensity is converged with the given sway increment at the bottom story.

In computing the incremental load intensity for a given sway increment of a story, the statical equilibrium and geometrical compatibility as discussed in Article 2.4.3 must be satisfied.
In the general slope-deflection method, the end moments and reactions in beams and columns are expressed in terms of unknown joint rotations, joint vertical deflections and sway deflections of stories plus the independent quantities like fixed end moments. These are the slope-deflection equations derived in Appendix 1. Then, the values of end forces are substituted in equilibrium equations. This results in linear simultaneous equations, with joint rotations, joint vertical deflections and sway deflections of stories as unknown quantities. The number of equations and the number of unknowns are identical. These equations are solved for the unknown quantities with the known
applied loads. Thus the deformations are known and the end forces are calculated.

However, in the proposed sway increment method of analysis, the sway deflections of stories are considered to be known variables and the lateral loads corresponding to the sway deflections are to be determined. Furthermore, in the proposed method the calculation is carried out story by story, not for the whole frame at once. The general approach of the slope-deflection method is simplified and modified for the purpose of the new analytical method. The general joint equilibrium equations expressed in Eqs. 2.13, 2.14 and 2.15 and the story shear equilibrium equation of Eq. 2.17 are still required to be satisfied at any stage in the analysis. In the proposed method, since the axial shortening of beams is neglected, Eq. 2.14 need not be considered. Compatibility in this respect will be maintained irrespective of the magnitude of the horizontal axial forces in the beams at the same level. To reduce the number of simultaneous equations to be solved at a time, Eq. 2.15 is treated separately. Then, the equilibrium equations to be considered at the same time are Eqs. 2.16 and 2.18, which are given in incremental terms. By substituting the values of end moments from incremental slope-deflection equations in Eq. 2.16 at each joint of one story, a series of linear simultaneous equations is resulted in, with joint rotations as unknown quantities. The simultaneous equations are solved for the unknown joint rotations of the story by the Gauss-Seidel iteration method. In those equations, the sway deflection of the story is a known value and for the joint vertical deflections, the values obtained from the
previous iteration are used in the current iteration. The joint vertical deflections are determined with the axial forces computed from Eq. 2.15. Also, the latest available information on the deformations (joint rotations and sway deflections) of the adjacent stories are incorporated in solving the simultaneous equations. Then, from Eq. 2.18, the shear force of the story is calculated with the calculated values of joint rotations. From the resulting shear force, the incremental load intensity can be determined. In the bottom story, the incremental load intensity is calculated for a given sway deflection. In other stories the incremental load intensity is calculated with a trial sway increment and compared with the value calculated in the bottom story. These procedures are repeated in each story except the bottom story until the calculated incremental load intensity of the story is equal to that of the bottom story. Then, the resulting sway increments of individual stories, which result in the same incremental load intensity with that calculated in the bottom story, are compatible with the given sway increment of the bottom story.

In the analysis of a multi-story frame, the shear capacity of a particular story is exhausted at a certain stage of the analysis and no more lateral load can be resisted by the story. In other stories there is still some reserved strength. However, due to the failure of one story, the whole frame cannot support additional lateral load and the lateral load at the failure of the story becomes the stability limit load of the frame. In this analysis, up to the stability limit load, the sway deflection of the bottom story is
incremented and the corresponding incremental load intensity is computed. After the stability limit load is reached, since no more lateral load can be resisted by the failed story, the sway deflection of the failed story is incremented. In this case, the corresponding incremental load intensity becomes negative and the lateral load decreases with the increasing sway deflection in the failed story.

Once all the incremental deformations (joint rotations, joint vertical deflections and sway deflections of stories) throughout the frame are known for a given sway increment of the bottom story or the failed story, the incremental end forces on members are calculated using the same incremental slope-deflection equations mentioned before. After the end forces are known, the stiffness coefficients can be revised and the moments on beams and columns can be checked for new plastic hinges.

The step-by-step procedure of the proposed analytical method may be briefly described as follows:

1. Assume that the structure is initially unloaded and unstrained.

2. Apply the working gravity loads at first and then on, an increment of the gravity loads.

3. Calculate joint rotations and joint vertical deflections at non-swayed position and then all member end forces.

4. Check all moments. If there is any cross-section where the moment exceeds its plastic moment capacity, insert a plastic hinge.
5. Repeat steps 2 and 4 until the factored gravity loads are reached.

6. Set sway increment. Initially, an arbitrary small increment is chosen and from then on, the increment is determined in steps 13 and 18.

7. Determine stability factors and stability functions of incremental slope-deflection equations for all members with the known member end forces.

8. Apply the sway increment which is fixed in step 6, to the bottom story (to the failed story, after the stability limit load is reached) and calculate the corresponding incremental load intensity.

9. Determine the sway increment which results in same incremental load intensity with that calculated in step 8 at each story except the story used in step 8, by iteration.

10. If the sway increment cannot be obtained at a particular story in step 9, this means that no more lateral load can be resisted by the story and the stability limit load is reached. Now on the sway deflection of the failed story is incremented.

11. Calculate incremental moments, axial forces and axial shortening of columns from the deformations determined in the calculations of steps 8 and 9.

12. Repeat steps 6 to 12 inclusive until the value of the incremental load intensity calculated in the bottom story
(in the failed story, after the stability limit load is reached) converges for the given sway increment.

13. Predict the minimum sway increment required in the bottom story (in the failed story, after the stability limit load is reached) for next hinge. Repeat steps 6 to 12 until prediction converges within tolerance. (In the numerical studies in Chapter 3, if the difference between the past and present values of the predicted sway increment is within 5 percent, then the prediction is considered to have converged).

14. Calculate inelastic hinge angles and test if there are any hinge reversals. If there are any hinge reversals, lock the hinges that have been unloaded. Return sway increment to the value at the start of the current increment and do again from step 6.

15. Calculate the total lateral load and the total sway deflection at each story by adding the current incremental values to the previous subtotals.

16. Test the ratio of the maximum lateral load (corresponding to the stability limit load) and the present lateral load. If the ratio reached the desired range on unloading part, stop the program and terminate the calculation.

17. Check all moments and insert a new hinge at the location where the moment reaches its plastic moment of the section.
18. Predict the minimum sway increment of the bottom story (of the failed story) required for the next hinge.

19. Repeat steps 6 to 18 until the condition in step 16 is satisfied.

2.4.5 Prediction of Plastic Hinges

The linear predictions are used in selecting the sway increment for the next hinge in the manner given by Jennings and Majid. The increments of load intensity at all the possible hinge locations are calculated for plastic hinges from the linear predictions and then the smallest increment is selected for the next hinge. The smallest increment of load intensity is converted to the necessary sway increment of the bottom story (of the failed story, after the stability limit load is reached).

Since the axial loads in beams are neglected in this analysis, the plastic moment capacity of a beam is constant. In Fig. 2.16, the linear prediction of the increment of load intensity $\delta H_n$ for the next hinge is illustrated for a particular possible hinge location of a member with a constant moment capacity in a structure. In the figure, $M_{n-2}$ is the total moment at the possible hinge location at the previous sway increment with load intensity $H_{n-2}$, $M_{n-1}$ is the moment at the current increment with load intensity $H_{n-1}$ and $\delta H_n$ is the necessary increment of load intensity to form a plastic hinge. From Fig. 2.16 it is observed that the value of $\delta H_n$ can be evaluated from

$$\delta H_n = \frac{H_{n-1} - H_{n-2}}{M_{n-1} - M_{n-2}} (M_p - M_{n-1})$$
The effect of axial loads on the plastic moment capacity is considered in columns and the plastic moment of a column is changed in accordance with Eq. 2.3, depending on the variation in its axial load. However, the axial load does not vary appreciably within an increment and Eq. 2.19 can also be used to predict the next hinges in columns with $M_{pc}$ instead of $M_p$. That is,

$$
\delta H_n = \frac{\delta H_{n-1}}{M_{n-1} - M_{n-2}} (M_p - M_{n-1})
$$

(2.19)

$M_{pc}$ is the reduced plastic moment of a column at the current increment.

It is important to note that the direction of the linear extension must specify in the direction of positively increasing load intensity. In this manner, correct hinge predictions are obtained for members whose moments vary with positive sign or negative sign or both signs. In the analysis, the sign of the moment at the current increment ($M_{n-1}$ in Fig. 2.16) is allocated to $M_p$ or $M_{pc}$ for the current prediction. The calculated increment of load intensity from Eq. 2.19 or 2.20 can be either positive or negative.

In the above procedure, all possible hinge locations are analyzed and the minimum increment of load intensity of the set
is determined and the necessary sway increment of the bottom story is calculated from the value by using the following equation:

\[ \delta A_n = \frac{\delta A_{n-1}}{\delta H_{n-1}} \delta H_n \]  

(2.21)

where \( \delta A_{n-1} \) is the sway increment of the bottom story at the current increment and \( \delta A_n \) is the necessary sway increment for the next hinge. Several iterations result in convergence of the nonlinear relationship at the correct next hinge. After passing the stability limit load, the prediction is carried out only in the failed story. In this case, the required smallest sway increment for the next hinge likely becomes large, causing the large drop in the load intensity with the increment. Therefore, if the required increment exceeds a specific value, the increment is not taken. Instead, the specific value is adopted for the next increment. In the numerical studies in Chapter 3, a tenth of the total sway deflection of the failed story is arbitrary selected for the specific value.

2.4.6 Hinge Rotation and Hinge Reversal

The incremental inelastic hinge rotation at each plastic hinge for a given sway increment can be calculated from the equations derived in Appendix 2. The total hinge rotation of a plastic hinge is determined by summing the incremental hinge rotations.

It has been recognized that when a structure is subjected to proportional loading or nonproportional loading it is possible that a cross-section of a member of the structure may undergo inelastic
deformation and subsequently return to an elastic condition.\(^{(32,35,54)}\)

When a complete elastic-plastic analysis is carried out, the stability limit load calculated under the assumption that the insertion of a plastic hinge is an irreversible process can lead to quite erroneous solutions. When the direction of rotation at the hinge location is reversed, the plastic hinge starts to unload elastically. Therefore, if the hinge reversal is not considered, the subsequent calculation will contain an incorrect rotation which may result in the wrong failure mechanism. Furthermore, ignoring the occurrence of a hinge reversal may result in an incorrect deflected shape which will result in the incorrect stability limit load. When an elastic-plastic analysis is carried out beyond the stability limit load of a multi-story frame, a number of plastic hinges likely unload elastically, especially in the stories which have reserve strength to resist additional lateral load at the limit of stability. Therefore, in order to understand the true load-deflection behavior of unbraced multi-story frames, it is necessary to study the effects of hinge reversals occurring in the analyses. In this dissertation, the hinge reversal will be considered in several example frames subjected to nonproportional loading.

In taking account of the hinge reversal, the approach utilized by Davies\(^{(35)}\) is employed in this dissertation. In Davies approach it is assumed that the cessation of plastic rotation at one section of a structure occurs simultaneously with the formation of an additional plastic hinge at some other section. As discussed
by Davies, this assumption is not rigorously justifiable because
the structure will generally be nonlinear between the formation of
successive plastic hinges. However, since this nonlinearity is
small, the assumption is reasonable within the assumptions of
second-order elastic-plastic analysis and satisfactory for all prac­
tical purposes.

In the process, the plastic hinge rotation is examined for
any reversal of the direction of rotation at each increment at each
plastic hinge. If there are any hinge reversals occurred, the hinges
are elastically locked and at the same time the computation is re­
turned to the start of the current increment. Therefore, the program
returns the sway increment to the value it had at the start of the
current increment and analysis is continued with the elastic response
of the unloaded hinge.

2.5 Computer Program

The sway increment method of analysis for the second-order
elastic-plastic analysis of unbraced multi-story frames, as described
in this chapter was programmed in Fortran IV language for a CDC6400
electronic digital computer. Many variations of this program are
possible to handle changes in loading conditions, geometrical con­
ditions, etc. One important variation used is a program for frames
with concentrated loads.

2.5.1 Program Scope and Limitation

In the computer program, the following factors can be
considered:
1. The influence of axial force on the stiffness of members.

2. Column shortening.

3. Hinge reversals.

4. Residual stresses in columns.

5. Uniformly distributed or concentrated beam loads.

The limitation imposed on the number of bays and the number of stories is that dictated only by the computer time and the storage capacity of the particular computer employed. In the program, the deformation history is recorded at each successive hinge and the information between hinge formations is not recorded. Also, the deformations are calculated only at the joints.

2.5.2 Input and Output

The first card of the input requires the specification of the number of stories, number of bays, modulus of elasticity, yield stresses of beams and columns, residual stress level and load factor. Following this card, bay lengths, story heights, working gravity loads on beams and columns, and lateral load index at each level are read. Next, the information on member sizes such as moment of inertia, plastic modulus and area are read.

All the input quantities are printed out for future reference at the beginning of output. Then, the complete member moments and axial forces with gravity loads only are printed out. Following pages record the following information at each successive hinge: the required sway increment for next hinge, the process of iteration for load intensity by specifying the number of iterations and its
value at each iteration, the process of hinge predictions by specifying the number of predictions needed before the next hinge is located, the number of hinge reversals occurred in the current increment and their locations, if any, the total load intensity and the total sway deflection or sway deflection index at each story at the current increment, the locations of plastic hinges and complete member moments after inserting new hinge, the axial forces in columns and the axial shortening of columns. When the ratio of the maximum load intensity, which is corresponding to the load intensity at the stability limit load, and the present load intensity reached the desired range on the unloading part, the program is stopped.

2.6.3 Flow Diagram

The schematic flow diagram of the program is presented in Fig. 2.17. In order to enable the program to be more rapidly followed, the various steps discussed in the previous section are simplified in the flow diagram. The flow diagram is self explanatory.

The whole program was divided into a main program and nine subroutines for ease in compiling.
3. NUMERICAL STUDIES

3.1 Introduction

The computer program described in Chapter 2 has been used to analyze six frames under combined gravity and lateral loads having either concentrated or distributed loads on the beams. The results of the analyses are presented in this chapter and compared with previous results where applicable.

The six frames initially considered fully fixed at the bottom of the columns and all joints are assumed to be fully rigid. For comparative purposes the results for two of the frames which have been pinned at the bottom of the columns are also shown. The dimensions of the frames are given from the center lines of the members.

The loading sequence is nonproportional in all cases: the factored gravity load (load factor of 1.3, unless otherwise stated) is applied first and then the lateral load is allowed to vary from zero to maximum and then allowed to decrease. The gravity loads on the beams are uniformly distributed loads in each case except for one frame in which concentrated loads are used. Lateral loads are assumed to be concentrated at the joints.

3.1.1 Frame Geometries, Members and Loads

The frame geometries, member sizes and loads of the six example frames considered in this numerical study are presented in Figs. 3.1 through 3.5 and Tables 3.1 through 3.3. The summary description of the six frames is as follows:
1. Frame 1 (Fig. 3.1a): a one-bay three-story frame with concentrated loads on beams and composed of steel having a 36 ksi yield stress level.

2. Frame 2 (Fig. 3.1b): a one-bay four-story frame with distributed loads and composed of steel having a 34.2 ksi (15.25 tsi) yield stress level.

3. Frame 3 (Fig. 3.2): a two-bay three-story frame with distributed loads and steel with a 36 ksi yield stress level.

4. Frame 4 (Fig. 3.3): a four-bay six-story unsymmetrical frame with distributed loads and steel with a 34.2 ksi yield stress level.

5. Frame 5 (Fig. 3.4): a three-bay ten-story unsymmetrical frame with distributed loads and steel with a 36 ksi yield stress level.

6. Frame 6 (Fig. 3.5 and Tables 3.1, 3.2 and 3.3): a two-bay 26-story unsymmetrical frame with distributed loads and steels with a 36 ksi yield stress level for beams and with a 50 ksi yield stress level for columns.

3.1.2 Source of the Frames

The six frames have been obtained from five different sources. Frame 1 has been taken from Ref. 48. This frame is one of the test frames described in the reference. The experimental behavior of the frame will be compared in Chapter 5 with the theoretical behavior determined by the sway increment analysis. Frame 1 is the only frame with concentrated loads on beams in this numerical study.
Frames 2 and 4 are taken from Ref. 13. The design of these frames was based on an assumed configuration of plastic hinges at the mechanism load. The beams were designed plastically and the columns elastically. The frames are assumed to be composed of steel having 34.2 ksi yield stress level. In the design of these frames, a load factor of 1.4 was used. The gravity loads shown in Figs. 3.1b and 3.3 are the factored gravity loads, using the load factor of 1.4. In the analyses of the two frames in this dissertation, plastic moments of members are assumed to be in accordance with Eq. 2.3 and the standard residual stresses for rolled shapes (2) are considered in the analysis.

Frame 3 has been taken from Ref. 31. In the reference, the frame was analyzed under proportional loading conditions. The same frame is analyzed under nonproportional loading in this dissertation. The maximum gravity loads attained under the proportional loading condition in Ref. 31 are used for the factored gravity loads. This frame is also analyzed with pinned bases to compare the effect of the column end condition on the lateral load capacity of a frame. In this case, it is assumed that the columns are hinge-supported at the bottoms.

Frame 5 has been taken from Ref. 55. In the reference, the frame was designed to meet working load drift limitation of $\Delta/h = 0.0025$. The frame is analyzed to study the load-deflection behavior under nonproportional loading. The frame is also analyzed with pinned bases as in the case of Frame 3.
The geometry and loads of Frame 6 has been taken from Ref. 56. In the reference, the original frame which was designed by an allowable-stress method was used as an example for the design of the wind bracing. From the original complex frame, a 2-bay 26-story bent was isolated for Frame 6 and was designed by a plastic design method.\(^{41,42}\) In the design, all beams were assumed to be composed of steel having a 36 ksi yield stress level and all columns were chosen on the basis of A441 steel with a yield stress level of 50 ksi. The geometry of Frame 6 is given in Fig. 3.5. Tables 3.1 and 3.2 list the sections used for beams and columns and Table 3.3 gives the working loads. The design ultimate load is equal to 1.3 times the working load, corresponding to the combined loading condition. In the design of Frame 6, beams were designed using the clear span and the live load reduction was considered.

3.2 Presentation of Results from Frame Analyses

3.2.1 Frame Behavior

The results of the investigation on load-deflection behavior of the frames are presented in Figs. 3.6 through 3.18. In the figures, the lateral load $H$ (which is also the lateral load intensity) is plotted against the deflection index $\Delta/h$ of the frame which is the lateral deflection of level 1 divided by the total frame height. The solid circles on the load-deflection curves indicate the formation of plastic hinges. The frames were analyzed considering the effects of axial shortening of columns as well as neglecting axial shortening. The effects of axial shortening of columns will be discussed in Sect. 3.3 in this Chapter. All frames except Frames 5 and 6 were also
analyzed with and without the effects of elastic unloading of plastic hinges (hinge reversals). The effects of hinge reversals will be discussed in Section 3.4.

In the computer program used for this numerical study, the test employed for convergence of a hinge prediction requires that the difference between the prediction in the previous cycle and in the current cycle should be within 5 percent of the predicted sway increment of the previous cycle. As the criteria for the formation of a plastic hinge, the difference between the calculated moment at a section and the plastic moment of the section should be within 1 percent. In some cases, the above criteria are satisfied simultaneously at more than one point in the frame. In such a case it is assumed that more than one plastic hinge forms at the same sway increment.

Figure 3.6 shows the load-deflection curve for Frame 1. After the formation of the first plastic hinges in the lower leeward column, the stiffness of the frame is reduced considerably. The failure of the frame was caused by instability due to reduction in stiffness upon formation of hinge number 5. The order of plastic hinge formation and hinge locations for Frame 1 are also shown in Fig. 3.6.

In Ref. 13 Heyman designed Frame 2 as an illustration for his design method. In his design method, the distributed beam loads shown in Fig. 3.1b were replaced by concentrated loads acting at mid-span of the beams as shown in Fig. 3.7a and a pattern of plastic hinge was assumed which involved collapse in both beams and columns. From this assumed hinge pattern the full plastic moments of beams
and columns were calculated. Beam sections were then chosen so that their full plastic moments corresponded to the calculated values. The column sections were designed to remain elastic when the calculated values of full plastic moment and axial thrust were applied. Subsequently Heyman calculated the deflection of the frame at the formation of the last plastic hinge. He used simple plastic theory and a virtual work method for this purpose. He obtained a total deflection at the top of the frame equal to 11.2 inches at the formation of the failure mechanism as shown in Fig. 3.7b. The load-deflection curve for Frame 2 obtained from the sway increment analysis is given in Fig. 3.8. The frame failed at a lateral load of 15.0 kips by a sway mechanism at the first story. The design ultimate load (D.L.) of the frame is 10.1 kips. The lateral deflection at top of the frame at failure is 6.85 inches compared with 11.2 inches obtained by simple plastic theory. The plastic hinges in Frame 2 and their sequence of formation are also shown in Fig. 3.8.

As mentioned in the previous section, Frame 3 was taken from Ref. 31. In the reference, the frame was analysed under proportional loading with the gravity loads shown inside parenthesis in Fig. 3.2. In the present analysis, the same frame is analyzed under nonproportional loading with the factored gravity loads equal to the maximum gravity loads attained under the proportional loading. Figure 3.9 shows the load-deflection curve of Frame 3 obtained from the sway increment analysis. The frame failed by instability at the formation of the 10th plastic hinges. The plastic hinges which form in Frame 3 are also shown in Fig. 3.9. In Fig. 3.10, the load-deflection
curve from the present analysis under nonproportional loading is compared with the result obtained under proportional loading which is reproduced from Ref. 31. It is observed that the maximum lateral loads for both cases are very close. However, due to the different loading sequences, the deformation characteristics are considerably different. The plastic hinges formed at failure in both cases are compared in Fig. 3.11. The numbers of plastic hinges formed at failure are same in both analysis. However, the orders of plastic hinge formation are different because of the different loading conditions. Frame 3 is also analyzed with the pinned bases. The result of the analysis of the pin-based frame is presented in Fig. 3.12 together with that of the fixed-based frame. It is noticed from the figure that there is a significant difference in the lateral load capacity of the frame depending on the column end condition. The maximum lateral load attained in the pin-based frame is 2.46 kips which is compared with 7.04 kips in the fixed-based frame.

For Frame 4, the load-deflection curve is given in Fig. 3.13. This frame was designed by Heyman along with Frame 2 in Ref. 13. The same plastic design method used for the design of Frame 2 was also used for this irregular frame. The design ultimate load (D.L.) is 4.2 kips as indicated also in Fig. 3.13. The maximum lateral load from the analysis of the frame is 4.01 kips, which is smaller than the design ultimate load in this case. The plastic hinges at failure are presented in Fig. 3.14.
Figure 3.15 shows the load-deflection behavior of Frame 5. The maximum lateral load at failure is 13.05 kips, which is considerably larger than the design ultimate load of 9.73 kips. In Ref. 55, this frame was designed to meet the deflection index limitation of $\Delta/h = 0.0025$ at working load. In the present analysis, since the factored gravity loads are applied at the start of the analysis, the working load deflection index cannot be determined. However, as an approximate estimation, the deflection index at the lateral load equal to the working load (W.L.) could be used, which is $\Delta/h = 0.00237$ for this case. The actual working load deflection index is apparently smaller than the approximate value. Therefore, Frame 5 satisfies the design limitation on the working load deflection index of the frame. The plastic hinges at the maximum lateral load and their sequence of formation in Frame 5 are given in Fig. 3.16. Frame 5 is also analyzed with pinned bases. Figure 3.17 shows the load-deflection curve of the pin-based frame. After the stability limit load is reached, the deflection index of the first story is increased very rapidly. However, the deflection indices of all other stories start to decrease. As a result, the deflection index of the frame remains almost constant after the stability limit load as shown in Fig. 3.17. Comparing with the load-deflection curve of the fixed-based frame, a considerable reduction in the lateral load capacity is observed as in the case of Frame 3. The maximum lateral load of the pin-based frame is 6.55 kips compared with 13.05 kips of the fixed-based frame.

The load-deflection curve for Frame 6 is given in Fig. 3.18. In the design of Frame 6, beams were designed using the clear span and the live load reduction was considered. However, in the present
analysis, the center-to-center span is used and the live load reduction is not considered. The maximum lateral load attained during the analysis is 9.12 kips with the consideration of axial shortening of columns and 9.51 kips without the consideration of axial shortening, which are compared with the design ultimate load of 9.35 kips. The plastic hinges formed in different parts of the frame at the maximum lateral load are shown in Fig. 3.19. Since the design is a weak-beam type design, few plastic hinges have formed inside columns. All the other plastic hinges are located inside beams. The formation of plastic hinges has reduced the stiffness of the frame considerably. This in turn has made the frame unstable and caused the failure. There is not complete mechanism.

In unsymmetrical frames, such as Frames 4, 5 and 6, the value of lateral load at non-swayed position may not be zero. The lateral load at each joint could have a different initial value, depending on geometry and applied gravity loads. However, the initial lateral loads at non-swayed position do not affect the maximum lateral load capacity of a frame. Also, since the lateral load consistent with a given sway deflection is determined in the sway increment analysis, the initial lateral loads do not have to be considered in the analysis. The results from the analysis are exact regardless of the initial lateral loads. The initial loads of Frames 4, 5 and 6 were found to be negligible and in Figs. 3.13, 3.15, and 3.18 the first points of load-deflection curves were connection to the origines.
The summary of the maximum lateral loads and the corresponding values of deflection index of frame of all example frames is presented in Table 3.4.

3.2.2 Behavior of Individual Story

The load-deflection characteristics of an unbraced multi-story frame can be studied in detail by examining the relative load-deflection behavior of individual stories. Figures 3.20 through 3.25 show the relative load-deflection curves of individual stories of the example frames. In the figures, the lateral load $H$ (which is also the load intensity) is plotted against the deflection index $\Delta/h$ of the story which is the relative deflection of the upper joint to the lower joint of a story divided by the story height.

The load-deflection curve of each story of Frame 1 is presented in Fig. 3.20. As shown in the figure, previously the stiffnesses of the first and second stories had been reduced considerably during the formation of successive plastic hinges. Immediately after reaching the stability limit load the relative lateral deflections of the second and third stories decreased, while the deflection of the first story increased rapidly. However, due to the formation of a new plastic hinge in the beam at level 3 (No. 6 in Fig. 3.6), the relative deflection of the second story started to increase again at much reduced rate. The frame failed by the instability failure of the first story.

Figure 3.21 shows the load-deflection behavior of the individual stories of Frame 2. As discussed in the previous article,
the frame failed due to the formation of a sway mechanism of the first story. It is noticed from Fig. 3.21 that the behavior of the third and fourth stories were almost linear because of no plastic hinges in those stories. Most of non-linearity of the overall behavior shown in Fig. 3.7 resulted from the deformations of the first and second stories. The unloading curve of the fourth story closely followed its loading curve since the effects of the formation of plastic hinges at other stories on the top story was very small.

The individual story behaviors of Frame 3 are presented in Fig. 3.22. In contrast to Frames 1 and 2, the lateral deflection of each story in Frame 3 continued to increase with the decreasing lateral load after failure. In this case, the failure of the frame can be attributed to the reduction in stiffness of the whole frame.

Figure 3.23 shows the load-deflection curves for the 1st, 2nd, 4th and 6th stories of Frame 4. After failure, the deflection of the second story increased rapidly, while the deflections of other stories started to decrease. The frame failed by the instability failure of the second story.

Figure 3.24 shows the load-deformation curves for the selected stories of Frame 5. The general behavior discussed above can be observed also in the figure. The frame failed by the instability failure of the first story. In Frame 5, the relative deflections of the 5th, 6th, 7th and 8th stories were larger than that of the first story at the maximum lateral load. However, after failure, the deflec-
tion of the first story increased rapidly, while the deflections of other stories started to decrease.

The load-deflection curves for the selected stories of Frame 6 are presented in Fig. 3.25. As in Frame 3, the deflection of each story continued to increase after failure in this frame. At the maximum lateral load, the largest relative lateral deflection occurred at the 21st story and the smallest at the 1st story. The frame failed by instability due to reduction in stiffness of the whole frame.

By studying the load-deflection behavior of individual stories of the frames, the behavior of the frames at failure can be grouped into two generalized categories. In the case of a frame in the first category, a particular story in the frame reaches its stability limit load or mechanism load during the analysis, while other stories still have additional load carrying capacity. Due to the occurrence of instability or mechanism in one story, the frame cannot support additional lateral load and consequently the load must decrease in order to satisfy the equilibrium condition as the sway deflection of that story is increased. After unloading begins, the lateral deflection of the story increases rapidly, while the lateral deflections of the other stories decrease.

In a frame in the second category, the failure of the frame results from the reduction in stiffness of the whole frame. As a result, after passing the stability limit load, the lateral deflection of each story of the frame continued to increase with the decreasing lateral load.
Frames 1, 2, 4 and 5 behaved in the manner described for the first category of frames. Frames 3 and 6 behaved in the manner described for the second frame category.

3.3 Effect of Axial Shortening of Columns

The effects of axial shortening are shown in Figs. 3.13, 3.15 and 3.18 for Frames 4, 5 and 6. In Frames 1, 2 and 3, the effects were negligible.

In general, the load-deflection behaviors of the frames were not subjected to drastic changes due to the effect of axial shortening. The initial parts of the load-deflection curves with and without axial shortening were almost identical in Frames 4 and 5. However, in Frame 6 (26-story frame), the amplification of lateral deflection at the first level due to axial shortening became considerable even in the early stage of the analysis. The amplifications of lateral deflection due to axial shortening were 6.0%, 8.0% and 12.3% at failure in Frames 4, 5 and 6, respectively. In contrast, the values of the maximum lateral loads of the frames were less sensitive to the effects of axial shortening. The maximum lateral load with axial shortening was higher in Frame 4 and lower in Frames 5 and 6 than the load without axial shortening. The changes in maximum lateral load due to axial shortening were 3.7%, 0.8% and 4.1% in Frames 4, 5 and 6, respectively. The summary of the maximum lateral loads corresponding deflection index of frame are given in Table 3.5.
Within the range of this study, it is seen that the primary effect of column axial shortening at failure was a change in lateral deflections rather than lateral loads. It is also noted that possible large changes in lateral load capacity and large increases in lateral deflection never materialized as a result of axial shortenings.

The effects of axial shortening generally resulted in a change of the order of hinge formations and the addition of few extra hinges at failure. Figure 3.14 shows the order of hinge formations and plastic hinges at maximum lateral load with and without axial shortening of Frame 4. From the figure, it can be seen that the order of hinge formation altered by axial shortening and one more hinge formed at failure due to axial shortening. Frames 5 and 6 had three more and four more hinges respectively at failure due to axial shortening as shown in Figs. 3.16 and 3.19.

3.4 Effect of Hinge Reversal

In order to examine the effects of hinge reversals, Frames 1, 2, 3 and 4 have been analyzed with and without considering hinge reversals. In Frames 5 and 6, only the analysis with hinge reversals has been carried out. In the analyses with hinge reversals, values of all incremental inelastic hinge angles were computed at each increment of sway and the directions of hinge angles were checked for any hinge reversals. Once any hinge reversals were detected, the hinges are elastically locked. In the analyses without hinge reversals, it was assumed that the insertion of a plastic hinge is
an irreversible process. That is, analyses proceeded on the basis that a reversal has not occurred.

In all frames analyzed both with and without hinge reversals, a number of plastic hinges was subjected to reversals in the direction of hinge angles after passing the maximum lateral load. However, only one frame (Frame 4) was subjected to a hinge reversal before reaching its maximum lateral load. In Frame 4, a hinge reversal was developed on a beam hinge in the analysis with axial deformation. The hinge in question as indicated with A in Fig. 3.14a had developed as the 6th hinge and reversal was noted after the formation of the 9th hinge. The frame failed after the formation of the 11th hinge. The point of the occurrence of the hinge reversal is indicated on the load-deflection curve shown in Fig. 3.13.

The load-deflection curve generated without considering hinge reversals is compared with the curve with hinge reversals in Fig. 3.6 for Frame 1. Three hinges were subjected to reversals after the formation of the 5th hinge at which the maximum lateral load was attained. The locations of the three hinges are given in Fig. 3.26a as open circles. The effects of hinge reversals on the unloading part of the overall load-deflection curve were very small. The effects of hinge reversals on the load-deflection curves of individual stores of Frame 1 are compared in Fig. 3.20. In the first and second stories, the effects were negligible. However, there was a considerable difference on the unloading curves of the
third story with and without hinge reversals. The difference would be explained by considering the fact that all hinges (two column hinges) in the third story underwent reversals and the large deformation at the failed first story affects less on the behavior of the third story than on that of the second story. The slope of the unloading curve of the third story became stiffer with hinge reversals than without, since the stiffness of the story was increased with hinge reversals.

Frame 2 developed three hinge reversals after the attainment of the maximum lateral load. The locations of the hinges are given in Fig. 3.26b. The effects of hinge reversals on unloading curves were negligible in overall behavior as well as individual story behaviors.

In Frame 3, the lateral deflections continued to increase after reaching the maximum lateral load without having any hinge reversal. After the formation of the 11th plastic hinge (Fig. 3.9), one plastic hinge was subjected to hinge reversal. The location of the hinge is shown in Fig. 3.26c. The effects of the hinge reversal on unloading curve were negligible.

The effects of hinge reversals on the behavior of Frame 4 are shown in Fig. 3.13. As discussed before, in Frame 4, a hinge was subjected to reversal before the attainment of the maximum lateral load. However, the effect of the hinge reversal was totally negligible in altering the maximum lateral load capacity of the frame and the
corresponding lateral deflection at the first level. After passing
the maximum lateral load a number of plastic hinges was subjected
to reversals. The locations of the hinges which were experienced
hinge reversals are shown in Fig. 3.27 for Frame 4. In this case,
the unloading curve was also insensitive to hinge reversals.

It has been seen in this study that almost all plastic hinges
except those in the failed story were subjected to reversals in the
direction of hinge angles after failure of the frames. However, the
effects of hinge reversals on unloading curve were not considerable.
Only one hinge reversal was detected before reaching the maximum
lateral load in this study and the effects of the hinge reversal
were totally negligible in altering the load-deflection behavior of
the frame.

In all frames considered in this numerical study except
Frame 6, there were no plastic hinges formed at non-swayed position
with the gravity loads only. In Frame 6, two plastic hinges formed
at non-swayed position. The first plastic hinge formed at the wind-
ward end of the beam at level 1 in the leeward span with the working
gravity loads. The second plastic hinge formed at the windward end
of the beam at level 4 in the leeward span with the gravity loads
equal to 1.1 times the working gravity loads. Both hinges were sub-
jected to reversals when the sway deflection was incremented. The
locations of the unloaded plastic hinges are shown in Fig. 3.19 as
cross marks. In Frames 5 and 6, a number of plastic hinges was sub-
jected to reversals after the attainment of the maximum lateral load.
However, no hinge reversal was detected before reaching the maximum lateral load except at those plastic hinges formed at non-swayed position in Frame 6.

3.5 **Position of Inflection Points**

The locations of inflection points of Frames 4, 5 and 6 have been examined from the bending moments in columns at several stages in the analyses. The bending moment diagram for a column is non-linear because of the presence of secondary moment caused by axial load and lateral deflection. In the computation of the positions of inflection points, however, it was assumed that the end moments vary linearly from one end to another. Almost all columns were bent in double curvature, so the magnitude of the secondary moments was considered to be negligibly small.

All of the inflection points of the three frames were very close to mid-height of the stories under factored gravity loads except in the bottom stories. In the bottom stories, the average position of the inflection points was 0.353 h from the bottom. The variation in the position of the inflection point across a story was relatively small.

The average position of the inflection points of stories did not vary appreciably as the deformations of the frames were increased. Table 3.6 shows the average position of the inflection points of each story under factored gravity load only, at two-third of maximum lateral load and at maximum lateral load. It is noticed that there were no appreciable variations in the positions throughout the
analyses and almost all of the inflection points maintained the points near the mid-height of the stories. Very few columns were subjected to single curvature bending during the analyses. However, in the bottom stories, the variations were considerable. The positions of the inflection points in the bottom stories became further from the bottom as the lateral deflections were increased. Two columns in Frame 5 and three columns in Frame 6 experienced single curvature bending other than the bottom stories during the analysis, as indicated in Table 3.6.

In general, the positions of inflection points are near the mid-height of stories under factored gravity load and the variations in the positions are not appreciable as the deformations are increased.
4. ONE-STORY ASSEMBLAGE METHOD OF ANALYSIS

4.1 Introduction

In an unbraced multi-story frame, the gravity load conditions govern the selection of beam and column sizes for a limited number of stories near the top of the frame. The combined load conditions control the selection in the middle and lower stories of the frame. Between the regions controlled by the gravity load and combined load conditions there is a transition zone where either condition may govern in any one story. The number of stories comprising these three regions is not definite and depends on many factors such as frame geometry, material properties and load factors. Figure 1.1 shows a typical distribution of the three regions for an unbraced multi-story frame.

The one-story assemblage method of analysis presented in this chapter can be used for the determination of the approximate lateral-load versus lateral-deflection behavior of a story in the middle and lower stories of an unbraced multi-story frame which is subjected to combined loads. As to be shown in Article 4.4.2 in this chapter, for a limited number of stories at the top of a frame and for the bottom story the method is not suitable for the approximate prediction of the load-deflection behavior of a story. The method is based on the concept of the sway increment method of analysis described in Chapter 3. Since it is intended that the method be suitable
for rapid computer computation, certain assumptions and approximations are made which could be avoided in a more refined but inevitably more complicated approach. In Sect. 4.4 the accuracy of this approximate method will be tested by comparing its results with those obtained by analyzing the structure as a whole by the sway increment method presented in Chapter 3.

4.2 One-Story Assemblages

It has been shown in the previous chapter that the positions of the inflection points in the columns of a story do not vary appreciably in a regular unbraced multi-story frame during loading. The inflection points are also very near to the midheight of the stories in most cases. Based on this information it is assumed that all columns are bent into symmetrical double curvature. A particular level can therefore be separated from a frame by passing cuts through the inflection points of columns above and below the level. The strength and deformation of the one-story assemblage thus produced will be assumed to closely approximate that of the frame in the vicinity of that level. At this point a one-story assemblage is identical to that described in Refs. 36, 37 and 38.

A typical unbraced multi-story frame with distributed gravity loads and wind loads has been shown in Fig. 2.11. The separation of a one-story assemblage at level \( n \) of the frame is shown in Fig. 4.1. Also shown in the figure are the various forces acting on the members and the resulting deformations of the level. In the figure, \( \Sigma H_{n-1} \)
is the total shear between levels \( n \) and \( n-1 \) and \( \Sigma H \) is the total shear between levels \( n \) and \( n+1 \), which can be expressed as

\[
\Sigma H_n = \Sigma H_{n-1} + H_n \tag{4.1}
\]

where \( H_n \) is the concentrated wind load at level \( n \). The lateral displacement of the top of each column above level \( n \) relative to level \( n \) is assumed to be equal to \( \Delta_{n-1/2} \). Similarly, the lateral displacement of each column below level \( n \) is equal to \( \Delta_{n/2} \). The axial forces in the columns above level \( n \) (upper columns of a one-story assemblage) are designated as \( P_{n-1} \) and the axial forces in the columns below level \( n \) (lower columns of a one-story assemblage) are \( P_n \).

The load-deflection behavior of story \( m \) in Fig. 2.11 can be described with the relationship of the horizontal shear \( \Sigma H_n \) and the deflection index \( \Delta_n/h_n \). In the isolated one-story assemblage at level \( n \) of Fig. 4.1 it is observed that

\[
\frac{\Delta_n}{2} / \frac{h_n}{2} = \frac{\Delta_n}{h_n} \tag{4.2}
\]

Since columns in story \( m \) are assumed to be in double curvature, \( \Delta_n/h_n \) will be the deflection index of that story. Therefore, the relationship between \( \Sigma H_n \) and \( \Delta_n/h_n \) of the one-story assemblage at level \( n \) can be used for the approximate load-deflection behavior of the story \( m \). The analysis of the frame at story \( m \) is then reduced to the analysis of the one-story assemblage at level \( n \) and it will no longer be necessary to consider the analysis of frame as a whole. Instead, the load-deflection behavior of the one-story assemblage is considered.
4.3 One-Story Assemblage Method

4.3.1 Assumptions

Including the assumptions made in the previous article to separate a one-story assemblage from a frame, the assumptions on which the proposed approximate method has been formulated are as follows:

1. The basic assumptions on material, geometry and loading made in Article 2.4.1 for the sway increment method are also applicable to the one-story assemblage method.

2. The point of inflection in each column above and below level \( n \) is at midheight of the column.

3. The tops of upper columns of a one-story assemblage undergo equal lateral deflections and the lower columns are pin-based at the bottoms.

4. Initially the axial loads in upper columns of the one-story assemblage at level \( n \) are calculated from the total gravity loads coming from the tributary length of each beam connected to the columns above the level \( n \). The axial load in an upper column varies in proportion to the variation in the shear force at the joint which comes from a beam or beams connected to the column at level \( n \) with the proportional constant of

\[
K = (n-1) \times \frac{\text{Average Beam Stiffness above level } n}{\text{Beam Stiffness at level } n}
\]

The axial loads in lower columns are calculated from the axial loads in upper columns and the shear forces which come from beams connected to the columns.
5. Axial shortening of column is not considered.
6. Hinge reversals are not considered.

Assumptions 2, 5 and 6 can be justified with the information obtained from the numerical studies of the previous chapter. From Chapter 3, it has been known that the average position of the inflection points in a story of an unbraced multi-story frame is near midheight of the story and the position does not vary appreciably throughout an analysis. The same tendency has also been observed in other frames by previous researchers. (31,38) The effects of axial shortening and hinge reversals on the load-deflection behavior of a frame have been shown to be very small under the nonproportional loading condition, which is considered in this dissertation. In addition, a number of one-story assemblages were analyzed with the consideration of possible hinge reversals, using the proposed one-story assemblage method. The results of the analyses have indicated that no hinge reversals occurred during the analyses.

Assumption 3 specifies that the tops of the upper columns are assumed to be connected with struts to maintain equal lateral deflections throughout the analysis. From assumption 2 above and assumption 9 in Article 2.4.1, where axial forces in the beams are assumed to be negligible, assumption 3 is easily justified as follows. For simplicity, consider a story of a two-bay frame as shown in Fig. 4.2. In a story the axial force at the top and the bottom of a column is the same. Since axial shortening is not considered, the
distances between inflection points, \( l_{AB} \) and \( l_{BC'} \) would be the same as the span lengths, \( L_{AB} \) and \( L_{BC} \), respectively, if the shear forces at the top and the bottom of each column are equal. Although they are not the same, the difference in the shear force would not cause an appreciable change in distance.

The column axial loads in a frame vary with increasing lateral loads and deformations. Since the variation in axial load for a particular story column is the summation of the variations in each column directly above the story column in question, assumption 4 is not fully justified. However, studies show that the variation in axial loads in the lower columns of a one-story assemblage using assumption 4 is reasonably close to that of the corresponding columns in a frame. Table 4.1 for example shows the comparison of axial loads from a sway increment frame analysis with those obtained from a one-story assemblage analysis, using assumption 4. In the table, the axial loads of each column of the selected stories of Frames 5 and 6 are compared corresponding to the non-swayed position (i.e. at the start of analysis) and to the two-third of the maximum lateral load capacity of the frames. It is noticed that the values from the two different analyses are very close at both cases. Moreover, since the total axial loads of a story are same in both analyses at any stage during analysis, the variation in axial loads does not result in any change on the overall \( P-Δ \) effects of the story. Thus, assumption 4 is reasonable and is safe with other assumptions.
4.3.2 Outline of the Method

Based on the assumptions discussed in the previous article, a one-story assemblage is isolated from a multi-story frame. A typical one-story assemblage is shown in Fig. 4.3a in its deformed position. In a one-story assemblage, the magnitudes of the applied lateral loads \( \Sigma H_{n-1} \) and \( H_n \) in Fig. 4.3a depend on its location in a frame and the lateral loads must be balanced with the resisting lateral force of the assemblage. Therefore, the results from one-story assemblage analyses of individual stories of a frame cannot be conveniently compared with each other for their strengths and deformation behaviors with the absolute value of resisting lateral forces. In order to avoid this, the load intensity discussed in Article 2.4.2 is used to indicate the magnitude of resisting lateral force in a one-story assemblage. The relationship of load intensity versus deflection index (joint lateral deflection divided by height of lower columns of one-story assemblage) then describes the load-deflection behavior of the one-story assemblage, and the strength and the behavior of each one-story assemblage can be conveniently compared. The load intensity is determined from the resisting lateral forces of a one-story assemblage divided by the summation of the lateral load indices (discussed in Article 2.4.2) above the corresponding story in the frame from which the one-story assemblage has been separated.

The proposed one-story assemblage method is for the generation of the load intensity versus sway-deflection curve of a one-story assemblage with rapid computer computation. The computational pro-
The procedures of the method is based, in many details, on the sway increment method described in Chapter 3, and the one-story assemblage method is an application of the sway increment method. As in the sway increment method, the second-order elastic-plastic analysis and the incremental approach are employed. Also, the same prediction technique used in the sway increment method is utilized for the one-story assemblage method. Referring to Fig. 4.3b, the generation of load-deflection curve of one-story assemblage is described. At a particular stage of the analysis (after the (i-1)th sway increment), the total sway deflection at the joint is \( \delta \left( \frac{A}{2} \right)_{i-1} \) and the load intensity \( H_{i-1} \). As the first step to obtain the next point on the load-deflection curve, the incremental sway \( \delta \left( \frac{A}{2} \right)_{i} \) (that is, the ith sway increment) is applied at the joint, which is determined from the prediction for the next hinge (discussed in Article 2.4.5). The corresponding incremental load intensity \( \delta H_{i} \) is calculated. Then, the resulting total sway deflection \( \delta \left( \frac{A}{2} \right)_{i} \) and load intensity \( H_{i} \) dictate the next point. Those procedures are repeated until the load intensity reaches at a desired value beyond the maximum load intensity. The schematic flow diagram of the one-story assemblage method is presented in Fig. 4.4.

The procedure for the one-story assemblage method was also programmed in Fortran IV language for a CDC 6400 electronic digital computer. The computer program consists of a main program and five subroutines. The input data of the program are number of bays, modulus of elasticity, yield stresses of beams and columns, residual stress level, bay lengths, story heights, the summation of lateral load indices above the corresponding story in a frame and factored
gravity load on beams and columns. The information on member properties such as moment of inertia, plastic modulus and area, also consists of the input data. All the input data are printed out at the beginning of output. All member moments and axial forces in columns are printed out at non-swayed position. Then, the load intensity, sway deflection, member moments, axial forces, locations of plastic hinges are printed out at the formation of each plastic hinge. When the load intensity reduced to the desired value after passing its maximum value, the analysis of a one-story assemblage is stopped and the next one-story assemblage is analyzed.

4.3.3 Numerical Example

A number of load-deflection curves of one-story assemblages obtained from the one-story assemblage method will be presented in the next section for the comparison with the corresponding story behaviors in the context of total frame obtained from the sway increment method. In this article, the analysis of a one-story assemblage is presented as an illustrative example.

Figure 4.5a shows a three-bay one-story assemblage separated from Frame 5 in Chapter 3 at level 8. The member sizes and factored gravity loads of the one-story assemblage are also shown in the figure. The axial loads on upper columns have been computed from the algebraic sum of the factored gravity loads coming from the tributary length of each beam connected to the columns above level 8, plus the weight of columns for interior columns and the weight of columns and wall loads for exterior columns above level 8.
The load-deflection curve of the one-story assemblage obtained from the proposed method is given in Fig. 4.5b. In the figure, the load intensity is plotted against the deflection index of the story $\Delta/h$. Points on the load-deflection curve at the formation of plastic hinges are indicated by solid circles. The locations of plastic hinges are shown in the inset of Fig. 4.5b. The number at each plastic hinge indicates the sequence in which the hinge formed with increasing sway. The deflection index at the formation of each plastic hinge can be determined from the curve at the corresponding point.

Due to the asymmetry of the illustrative one-story assemblage, the load intensity was -0.21 kips in the non-swayed position. The negative sign indicates that there must be a holding force in the opposite direction to the applied direction of the lateral loads to maintain the one-story assemblage in a non-swayed position. Because of this initial lateral force, the load-deflection curve started below the origin. The one-story assemblage failed by instability before the formation of a failure mechanism. The maximum load intensity was 14.4 kips and the corresponding absolute maximum resisting lateral force of the one-story assemblage was 108.0 kips. The deflection index of the story at the maximum load intensity was 0.0619.

4.4 Comparison of the One-Story Assemblage Method with the Sway Increment Method

4.4.1 Presentation of Results from Both Analyses

Each story of Frames 5 and 6 described in Chapter 3 have been analyzed for its load-deflection behavior with the one-story
assemblage method, by separating the corresponding one-story assem-
blage from the frames.

Figures 4.6 through 4.15 contain load-deflection curves for
selected stories of Frames 5 and 6. The load intensity $H$ is plotted
against the deflection index $\Delta/h$ for each story. In the figures,
the solid line represents the behavior of a story obtained by the
one-story assemblage method and the dashed line indicates the be-
havior of the story from a sway increment analysis. Due to the
asymmetry of the one-story assemblages isolated from Frames 5 and 6,
there were initial holding lateral forces in the non-swayed position
for some assemblages. In these cases, the load-deflection curves
started at the corresponding initial load intensity.

The load intensity corresponding to the maximum resisting
lateral force of each story of the frames obtained by the one-story
assemblage is presented in Table 4.2. The maximum load intensity
of Frame 5 from the sway increment analysis was 13.05 kips and 9.12
kips for Frame 6.

4.4.2 Comparative Studies

Story behavior obtained from the one-story assemblage
method is compared with the behavior of the corresponding story in
the context of the total frame, determined by a sway increment
analysis.

As discussed in Article 3.2.3, Frame 5 failed by the
instability failure of the first story. At failure, the first story
reached its stability limit load, while other stories still had additional load carrying capacity. As a result, after failure, the lateral deflections of all stories except the first story started to decrease. The magnitudes of the reserved capacities of the stories are unknown and depend on complex interactions between frame geometry, member properties and member strength of the stories.

However, in the one-story assemblage analysis, the absolute capacity of the resisting lateral force of a story is obtained and the complete load-deflection curve of the story is generated without the inter-relationship with other stories. In Frame 5, therefore, the comparison of the results from both analyses beyond the stability limit load of the frame would not be as meaningful as the comparison of the results up to the limit load.

The failure of Frame 6 resulted from the reduction in stiffness of the whole frame, as also discussed in Article 3.2.2. As a result, the lateral deflections of each story continued to increase with the decreasing lateral load, after passing the stability limit load. In this case, the results of the frame analysis and the one-story assemblage analysis can be compared for its strength and complete load-deflection behavior.

For convenience, the story behaviors from both analyses are compared in three parts of a frame; upper stories, middle and lower stories and bottom story.
(a) Upper Stories

The load-deflection curve of a typical upper story of each frame is presented in Figs. 4.6 and 4.10 for Frames 5 and 6, respectively. It is noticed from the figures that there is a considerable difference between the load-deflection behaviors from both analyses. The maximum load intensities as well as the deformation characteristics of the stories from both analysis are noticeably different. The maximum load intensities of upper stories obtained by the one-story assemblage method were generally much larger than the maximum load intensity of the frame, as can be observed from Table 4.2.

From this comparison, it can be said that the one-story assemblage method is not suitable for the approximate prediction of the load-deflection behavior of a story for a limited number of stories at the top of a frame. The number will depend on many factors such as loading, frame geometry and material properties.

It has been known in the design of multi-story frames that strength to carry gravity loads controls the top few stories of unbraced frames. As a result, these stories usually are much stronger and stiffer than required under combined loading conditions. Therefore, the one-story assemblage analysis of the top few stories of unbraced frames under combined loads is not important and may not even be necessary in reality.

(b) Middle and Lower Stories

The combined loading conditions control the selection of member sizes in design of unbraced frames in this region. Therefore
a final design of a frame must be analyzed under combined loads to check for strength and deformation behavior of this region.

Figures 4.7 and 4.8 show the load-deflection curves of two stories of Frame 5, selected from the middle and lower region of the frame. In Frame 5, the load-deflection behaviors from both analyses are in very good agreement on the initial part of loading curves as can be observed in the figures. Beyond the maximum load intensity of the frame, the lateral loads and sway deflections of one-story assemblages continued to increase up to the absolute capacity of the resisting lateral force of the assemblages, while the sway deflections of the corresponding stories from frame analysis started to decrease with the decreasing lateral load. The maximum load intensity obtained from one-story assemblage analysis was always larger than that from frame analysis for a story in this region, as shown in Table 4.2.

Considering that a meaningful comparison of the results can be made only up to the maximum load intensity of the frame in Frame 5, the load-deflection behaviors from both analyses compare very well.

The load-deflection curves of selected stories in the middle and lower region of Frame 6 are shown in Figs. 4.11 through 4.14. In this case, the load-deflection curves from the two different analysis do not compare as well as in Frame 5. However, the one-story assemblage method gave a reasonably good approximation of the overall load-deflection behavior of a story in this region. The maximum load intensity obtained from one-story assemblage analysis
was usually smaller than that from frame analysis in this region, except a few stories near the bottom (Table 4.2). The one-story assemblage method, in general, gave less sway deflection than that of the frame analysis for the same load intensity.

Based on the above comparative study, it can be said that the one-story assemblage method predicts a reasonably good approximate load-deflection behavior of a story in the middle and lower region of a multi-story frame.

(c) Bottom Story

Figures 4.9 and 4.15 shows the load-deflection curves of the bottom stories of Frames 5 and 6, respectively. It is noticed from the figures that the load-deflection curves from the two analyses were rather significantly different.

In Frame 5, the one-story assemblage method gave very smaller load intensity than that of the frame analysis for the same sway deflection. The maximum load intensity obtained from one-story assemblage analysis was 10.73 kips, compared with 13.05 kips from frame analysis. In the frame analysis, the failure of this bottom story resulted in the unloading of the frame.

In Frame 6, the maximum load intensity from one-story assemblage analysis was 10.99 kips compared with 9.12 kips from frame analysis. There was also a considerable discrepancy between the two load-deflection curves.
It has been discussed in Sect. 3.5 that the inflection points in the bottom story were near the lower ends of the columns in the non-swayed position in frame analysis. As sway deflection was increased, the inflection points kept moving up toward the upper ends of the columns. In Frame 5, the windward column of the bottom story started to be subjected to near single curvature bending at about a half of the maximum lateral load of the frame. In Frame 6, the average position of inflection points of the bottom story was 0.778 h from the bottom at the two-third of the maximum lateral load and all columns bent in near single curvature near the maximum lateral load.

However, in the one-story assemblage method, the inflection points are assumed to be at the middle of the columns throughout the analysis. The effect of the assumed position of the inflection points in the method would be one of the possible causes for the large discrepancy between the load-deflection behaviors of the bottom story from both analyses. The use of the one-story assemblage method for the bottom story of a frame would required more refined assumptions on the method to consider the effect of rather large variation in the position of inflection points during frame analysis.

4.5 Practical Use of One-Story Assemblage Method

The one-story assemblage method described in this chapter will enable the determination of the approximate lateral-load versus sway-deflection behavior of a story in the middle and lower stories of an unbraced multi-story frame which is subjected to combined
loads. Such an analysis will enable a designer to determine

a) The load-deflection behavior of the story,

b) the maximum shear resistance and the corresponding deflec-
tion index of the story,

c) the shear resistance at formation of a mechanism and the
corresponding deflection index, and

d) The deflection index corresponding to the factored lateral
load.

The one-story assemblage method is extremely rapid with
a digital computer. Thus it lends itself to incorporation into a
program for the automatic design of multi-story frames. In a design
process a preliminarily designed frame can be rapidly analyzed story
by story for the verification of the sway estimates used in the pre-
liminary design phase and the maximum lateral load capacity. The
method can be also utilized for the determination of the working
load sway deflection by applying the working gravity load. A final
check of the design can be made by the sway increment method of analysis
described in Chapter 3 in this dissertation.
5. COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

5.1 Comparison of Sway Increment Solution and Test Results

The results of three tests conducted at Lehigh University on full-size multi-story steel frames were presented in Ref. 48. In this section, the experimental behavior of each test frame is compared with the results obtained by the sway increment method of analysis. The geometries, member sizes and loadings of the three test frames, Frame A, B and C, are shown in Figs. 5.1a, 3.1a and 5.1b, respectively.

In Figs. 5.2, 5.3 and 5.4, the lateral load versus sway deflection relationships obtained by the sway increment method are compared with the experimental load-deflection curves. In the analysis of the frames, measured properties rather than nominal properties were used. Plastic hinges at ends of beams were assumed to form at the column faces rather than at the centers of a connection. In columns, plastic hinges were assumed to form at the centers of connections. A standard residual stress pattern for rolled shapes was also considered in the analysis. (2)

It can be observed from Figs. 5.2, 5.3 and 5.4 that the agreement between the predicted and experimental results is generally quite good for all three frames. The maximum discrepancy can be observed for Frame A in Fig. 5.2. It has been reported in Ref. 48 that the initial phase of the testing of Frame A was interrupted and resumed several times over a period of one month, because of
equipment failures. It has also been reported that a total permanent
deflection of 3 inches had been produced at the top of the frame
during that period (dash-and-dotted line of the experimental curve
in Fig. 5.2). Considering the interruptions of the test, there is
some room for doubt about the slope of the initial part of the
experimental curve and the excess of the test load over the load
obtained by the sway increment analysis. Yarimci suggests in Ref.
48 that the excess of the test load over the predicted load can
probably be attributed to strain-aging effects.

For Frames B and C, the agreement between the predicted
and experimental results is quite good, considering the possible
sources of divergence such as the effect of strain-hardening, which
is ignored in the sway increment analysis. The effect of the
locations of plastic hinges at ends of beams would be one of other
possible sources of divergence. In the analysis, plastic hinges
at ends of beams were assumed to form at the column faces. However,
it has been observed that due to the restraining action of the con­
nections, plastic hinges tend to form not at the column faces, but
at some distance away from them. (48,50) Since the effects of strain­
hardening and the restraining action of the connections are neglected
in the analysis, the result from the sway increment analysis would
be always conservative.

The full scale frame test results can be considered as
an important experimental verification of the sway increment method
of analysis.
5.2 Comparison of One-Story Assemblage Solution and Test Results

Two tests on full-size one-story assemblages were also conducted at Lehigh University and the results of the tests were presented in Ref. 50. The objective of the one-story assemblage tests was to compare the experimental behavior of a one-story assemblage with the behavior predicted by the sway subassemblage method. (36) In addition, it was also the objective of the tests to investigate the effects of the variation in axial loads in columns on the load-deflection behavior of a one-story assemblage. The test results are utilized to compare with the results obtained by the one-story assemblage method.

Figure 5.5 shows the geometries, member sizes and loadings of the two one-story assemblages, SA-1 and SA-2. During the tests, the values of the axial loads in upper columns were changed with increasing lateral deflections at the joint within the range indicated in Fig. 5.5, following the predetermined load program. The load program was designed to vary the axial load ratios in the windward restrained column (lower windward column) from 0.4 to 0.2 and in the leeward restrained column from 0.4 to 0.6 and to remain the axial load ratio in the interior restrained column constant at 0.4. In the tests, the applied axial loads at the upper columns were maintained constant through a certain lateral deflection before they were adjusted for the next set of axial loads in the load program. For details of the load program and the loading sequence used in the tests, Ref. 50 is referred to.
In the analysis of the test assemblages by the one-story assemblage method, measured member properties and material properties were used. A plastic hinge at end of a beam was assumed to form at the column face. In columns, a plastic hinge was assumed to form at the center of a connection. A standard residual stress pattern for rolled shapes was used for all columns.\(^{(2)}\) The axial loads applied to the upper columns during the tests were used for the column axial loads in the upper columns at the corresponding lateral deflection in the analysis.

In this comparative study, it should be noticed that the main assumptions used in the one-story assemblage method (Article 4.3.1) are almost satisfied in the test assemblages. That is, in the tests, each column was pinned at its end and the tops of upper columns were maintained to undergo equal lateral deflections. Also, the variation in the axial loads in upper columns of one-story assemblages do not have to be assumed in this case. From the values of axial loads applied in the tests, the exact axial loads in the upper columns can be used at each stage of analysis. Therefore, the results from the one-story assemblage analysis of the test assemblages are not approximate, but exact within the other basic assumption discussed in Article 4.3.1

In Figs. 5.6 and 5.7, the lateral load versus sway deflection relationships obtained by the one-story assemblage method are compared with the experimental load-deflection curves. The agreement between the predicted and experimental results is very good on the initial part of the loading curve in both test assemblages. Near the
maximum lateral load and on the subsequent unloading part of the load-deflection curve, test results show an excess of strength over that predicted in both assemblages.

The underestimation of strength in predictions can be explained by considering the additional factors which were neglected in the analysis, such as the effect of strain-hardening and the effect of restraining action of the connections, as discussed in the previous section. Since those effects are neglected in the analysis, the predictions are conservative near the maximum load and on the unloading curve.

In general, the results from one-story assemblage analysis compare very well with the experimental results. The comparisons indicate that the one-story assemblage method gives a good prediction of the load-deflection behavior of a one-story assemblage which satisfies the boundary conditions assumed in the method, as in the test assemblages. The comparisons also give an important experimental verification of the method.
6. EXTENSION OF STUDIES

6.1 Sway Increment Method of Analysis

The use of the sway increment method proposed in this dissertation has been restricted to rectangular, regular frames under nonproportional loading. The joint mechanism has been considered as a possible failure mechanism, and the effect of strain-hardening has not been incorporated in the method. Furthermore, the computations have been based on centerline distances for all members. By making some modification of the proposed method, reasonably irregular frames could be considered. The effect of strain-hardening could be incorporated in the method in the manner used by Davies. (35) Davies has used a linear relationship between hinge angle rotation and increased plastic moment capacity due to strain-hardening. If the effect of strain-hardening is considered in the method, the joint mechanism which is considered as a failure mechanism in the present analysis need not be considered. The clear span lengths of beams and columns can be treated in the method by the consideration of finite joint sizes. Such an extension may be advisable for some frames with short bay widths or some stories in the lower region of certain frames. The effect of joint deformation can be studied with the method if the above extension is made. The increased analysis potential resulting from the above added considerations is limited only by the capacity and time requirement of available computers.
The application of the sway increment method of analysis can also be extended for the study of the following areas:

1. A study of working load sway deflection of a frame. The load-deflection curve obtained by the sway increment method with working gravity loads will give the working load sway deflection at the lateral loads corresponding to the working lateral load.

2. An investigation of the influence of design methods in producing frames which behave in the manner described for the first category and the second category as discussed in Article 3.2.2. In the case of a frame in the first category, the failure of the frame resulted from the failure of a particular story by instability or mechanism. In a frame in the second category, the frame failed by instability due to the reduction in stiffness of the whole frame.

3. A study of the maximum hinge angles at plastic hinges required for the attainment of the factored lateral load or the maximum lateral load in an unbraced multi-story frame. This study will enable the check of the adequacy of bracing spacings and sectional properties (i.e., width-thickness ratios for the flange and web) of members used in design.

4. A development of optimum design solutions, where both frame instability effects and deflection limitations under working load are considered.
6. An application to frames with composite steel-concrete beams.

6.2 One-Story Assemblage Method of Analysis

The comparison of the results of the one-story assemblage method and experimental behavior has indicated that the effect of strain-hardening is rather substantial on the unloading part of the load-deflection curve for a one-story assemblage. The effect of strain-hardening could be incorporated in the proposed one-story assemblage method in the manner discussed in the previous section.

It has been shown that the one-story assemblage method is not suitable for the approximate prediction of the load-deflection behavior for the bottom story of a frame. By making some modification of the assumptions used in the one-story assemblage method, the method could be used to predict a reasonably good approximate load-deflection behavior of the bottom story. One of the possible modifications can be from the consideration of the variation in the position of inflection points at the bottom story.

The studies undertaken in this dissertation with the one-story assemblage method can also be extended for the study of the following areas:

1. An extensive investigation of the behavior of one-story assemblages isolated from frames in the first category and the second category as discussed in Article 3.2.2, in order to verify the trends disclosed by studies in this dissertation. The one-story assemblage method gave a larger maximum load intensity than that of the frame.
for a story isolated from the middle and lower region of a frame in the first category. The method gave generally a smaller maximum load intensity than that of a frame for a story isolated from a frame in the second category.

2. An investigation of the influence of the number of bays in one-story assemblages on the correlation of results from the one-story assemblage analysis and the frame analysis.

3. An application to the optimum design of frames.

4. An application to one-story assemblages with composite steel-concrete beams.
7. SUMMARY AND CONCLUSIONS

It has been the purpose of this dissertation to develop an analytical approach for predicting the complete elastic-plastic behavior of unbraced multi-story frames subjected to nonproportional loading. The analytical approach, which has been referred as the "sway increment method of analysis", is based on determining the values of the lateral loads consistent with prescribed finite sway deflections of a story. Second-order elastic-plastic analysis and incremental procedure have been incorporated in the proposed analytical method. Techniques to predict the sway increments for next hinges and to account for the hinge reversals have also been introduced into the proposed method.

The numerical procedure of the sway increment method was programmed in Fortran IV language for a CDC 6400 electronic digital computer. In the computer program, the following factors can be considered:

1. The influence of axial force on the stiffness of members.
2. Column shortening.
3. Hinge reversals.
4. Residual stresses in columns.
5. Uniformly distributed or concentrated beam loads.
Numerical studies have been carried out to examine the lateral load versus sway deflection behavior of unbraced multi-story frames under nonproportional loading by using the sway increment method of analysis. Six frames were taken from the literature for this numerical study. No specific attempts were made to assess the influence of design methods on resulting frame behaviors. The frames were loaded either by uniformly distributed loads or by concentrated loads on the beams. The six frames were initially considered fully fixed at the bottom of the columns. For comparative purposes two of the frames have been also analyzed with pinned-base at the bottom of the columns. Every frame has been analyzed with and without the consideration of axial deformations. In four frames, analyses with and without considering the effects of hinge reversals have been also performed. The load-deflection behavior of each frame has been presented and the behaviors of individual stories of the frames have been examined.

Based on the analytical results of the unbraced multi-story frames under nonproportional loading investigated in this dissertation, it can be concluded that:

1. The effect of column axial shortening is not considerable in altering the maximum lateral load capacity. The primary effect of axial shortening is to induce lateral deflections.

2. The effect of axial shortening generally result in a change of the order of plastic hinge formations and the addition of few extra hinges at failure.
3. Plastic hinge reversals hardly occur in a frame under nonproportional loading before reaching its maximum lateral load unless there are any plastic hinges in the non-swayed position with the factored gravity loads.

4. A number of plastic hinges in a frame is subjected to hinge reversals after failure. However, the effect of the hinge reversals on unloading behavior of the frame is not considerable.

5. The end condition at the bottom of the columns in a frame effects significantly on the maximum lateral load capacity as well as the lateral deflection behavior. In a frame, the maximum lateral load capacity was reduced by 65% by changing from the fixed end condition to the pinned end condition.

6. Except for bottom stories, the positions of inflection points are, in general, near the mid-height of stories under factored gravity loads and the variation in the positions is not appreciable as the lateral deflections are increased. Very few columns are subjected to single curvature bending during the analysis.

7. In bottom stories, the positions of inflection points are near the lower ends of the columns under factored gravity loads only. As the lateral deflections are increased, the positions of inflection points become further from the lower ends of the columns. The variation is significant and one or two columns can be subjected to single curvature bending.
Based on the sway increment method of analysis, the one-story assemblage method has been developed to determine the approximate lateral-load versus sway-deflection curve of a story of an unbraced frame with rapid computer computation. The computational procedures of the one-story assemblage method is based, in many details, on the sway increment method. As in the sway increment method, a second-order elastic-plastic method of analysis, an incremental approach and a technique to predict the sway increments for next hinges have been employed in the one-story assemblage method. The numerical procedure for the method was also programmed for a CDC 6400 electronic digital computer.

The load-deflection curve obtained by the one-story assemblage method enables the determination of the maximum lateral load capacity and the corresponding deflection index of a story, and the deflection index corresponding to the factored lateral load. Because of its speed of execution on the computer, the one-story assemblage method can be very useful for performing the trial analyses associated with preliminary design, even in the special cases where a more careful analysis is justified for the final design.

Story behavior obtained from the one-story assemblage method has been compared with the behavior of the corresponding story in the context of the total frame, determined by the sway increment method. Based on the comparative study, it can be concluded that:
1. The one-story assemblage method gives a reasonably good approximate load-deflection behavior of a story in the middle and lower region of an unbraced frame under combined loads.

2. For a limited number of stories at the top of a frame and for the bottom story, the use of the one-story assemblage method is not suitable.

The results of the sway increment method of analysis have been compared with experimental results of three full-size multi-story frames. The agreement is good for all three frames. The results of the one-story assemblage method have also been compared with results of two tests on full-size one-story assemblages. The agreement between the predicted and experimental results is good for the two one-story assemblages. The comparisons with the results of full-scale tests give an important experimental verification of the sway increment method and the one-story assemblage method proposed in this dissertation.
APPENDIX 1

Derivation of Incremental Slope-Deflection Equations for Beams and Columns

The second-order elastic-plastic analysis method used in this dissertation is dependent on the formulation of end moments in terms of joint rotations, member rotations and fixed end moments for varying configurations of plastic hinges in each member. The incremental slope-deflection equations corresponding to the eight possible combinations shown in Fig. 2.9 are derived in the following:

1. Beams

The beams in a multi-story frame are loaded by uniformly distributed loads or concentrated loads. Under the assumption of the non-proportional loading used in this dissertation, the vertical beam loads are constant. By computing the initial fixed end moment of a beam and the location of a plastic hinge at an interior point, either distributed loads or concentrated loads can be considered. The general incremental slope-deflection equations are the same for both cases. Axial force in the beams will generally be very small in comparison with the strength and stiffness of the cross-sectional area in resisting the axial force and its effect is neglected in this analysis. The ends of a beam may deflect by unequal amounts because of the axial shortening of columns. This will cause additional moments on the beam. The effect of the axial shortening of columns is incorporated in the slope-deflection equations. The various configurations of possible plastic hinges in any beam are shown in Fig. 2.9.
Combination (1)

This is the elastic condition and the general incremental slope-deflection equations are:

\[
\begin{align*}
\delta M_A &= \frac{4EI}{L} \delta \theta_A + \frac{2EI}{L} \delta \theta_B - \frac{6EI}{L} \left( \frac{\delta \Delta_B - \delta \Delta_A}{L} \right) + \delta F M_A \\
\delta M_B &= \frac{4EI}{L} \delta \theta_B + \frac{2EI}{L} \delta \theta_A - \frac{6EI}{L} \left( \frac{\delta \Delta_B - \delta \Delta_A}{L} \right) + \delta F M_B
\end{align*}
\]

With constant vertical beam loads, the increment of fixed end moments of a beam becomes zero after the initial stage of the application of beam loads. Therefore, the terms for the incremental fixed end moments are deleted from the above equations. Then the required equations become:

\[
\begin{align*}
\delta M_A &= \frac{4EI}{L} \delta \theta_A + \frac{2EI}{L} \delta \theta_B - \frac{6EI}{L} \left( \frac{\delta \Delta_B - \delta \Delta_A}{L} \right) \\
\delta M_B &= \frac{4EI}{L} \delta \theta_B + \frac{2EI}{L} \delta \theta_A - \frac{6EI}{L} \left( \frac{\delta \Delta_B - \delta \Delta_A}{L} \right)
\end{align*}
\]

Combination (3)

There is no increment of moment at a plastic hinge in a beam. Therefore, in this case \( \delta M_A \) is zero. Eliminating \( \delta \theta_A \) from Eqs. Al.2a and Al.2b, the incremental slope-deflection equations become:

\[
\begin{align*}
\delta M_A &= 0 \\
\delta M_B &= \frac{3EI}{L} \delta \theta_B - \frac{3EI}{L} \left( \frac{\delta \Delta_B - \delta \Delta_A}{L} \right)
\end{align*}
\]

Combination (5)

This case is similar to the above mentioned combination. Here \( \delta M_B \) is zero and \( \delta \theta_B \) can be eliminated. The resulting equations are,
Combination (2)

Referring to Fig. 8.1, the results of combinations (3) and (5) can be applied to part AC and CB separately.

\[
\begin{align*}
\delta M_A &= \frac{3EI}{x} \delta \theta_A - \frac{3EI}{x} \left( \frac{\delta \Delta_B - \delta \Delta_A}{x} \right) \\
\delta M_B &= 0
\end{align*}
\]

(A1.4)

To eliminate \(\delta \Delta_C\), the equation of end shears is used.

\[
\delta R_A + \delta R_B = 0
\]

(A1.6)

It is seen from Fig. 8.1 that the above equation can be rewritten in terms of end moments, resulting in

\[
-\frac{\delta M_A}{x} + \frac{\delta M_B}{y} = 0
\]

(A1.7)

Substituting the values of \(\delta M_A\) and \(\delta M_B\) from Eqs. A1.5 and Eq. A1.7 and solving for \(\delta \Delta_C\),

\[
\delta \Delta_C = \frac{x^3 y^3}{x^3 + y^3} \left[ \frac{\delta \theta_A}{x^2} - \frac{\delta \theta_B}{y^2} + \frac{\delta \Delta_A}{x^3} + \frac{\delta \Delta_B}{y^3} \right]
\]

(A1.8)

Substituting the value of \(\delta \Delta_C\) into Eq. A1.5 and rearranging the required incremental slope-deflection equations for this combination become

\[
\begin{align*}
\delta M_A &= \frac{3EI}{x^3 + y^3} \left[ x^2 \delta \theta_A + x y \delta \theta_B - x (\delta \Delta_B - \delta \Delta_A) \right] \\
\delta M_B &= \frac{3EI}{x^3 + y^3} \left[ y^2 \delta \theta_B + x y \delta \theta_A - y (\delta \Delta_B - \delta \Delta_A) \right]
\end{align*}
\]

(A1.9)
Combinations (4), (6), (7)

In these cases there are two plastic hinges in a beam and there are no moment changes at the plastic hinges. Hence, there is no moment change throughout the beam and the incremental moment is zero for each of the combinations (4), (6), and (7).

Combination (8)

In this case there are three plastic hinges in the beam. Three such hinges are considered to result in a local failure, a beam mechanism. In reality, such is not the case for second order analysis, since large geometric re-adjustments can be accommodated. In this analysis, the attainment of a beam mechanism is considered as the maximum load of the whole frame reached.

2. Columns

Figure 8.2 shows a column with applied end forces and deformations in their positive directions. For columns, there are no lateral loads. Unlike the case of beams, there are considerable axial forces in columns. The effects of axial forces in columns are incorporated into the slope-deflection equations by means of the stability functions. In addition, to simulate the true behavior of columns as nearly as possible, the effective moment of inertia discussed in Article 2.2.2 is used.

The plastic hinges in the column can form at one or both ends or at a point inside the column. Among the configurations of plastic hinges given in Fig. 2.9, the combinations (1), (2), (3), (5) and (7) are the possible cases for columns and cases (4), (6) and (8) represent impossible behavior.
The reduced plastic moment capacity of a column varies as the axial force of the column is changed. Thus, the moments at plastic hinges of columns are not constant. The variation of the plastic moments are considered in the incremental slope-deflection equations. For the possible combinations the incremental slope-deflection equations are derived below:

Combination (1)

This is the elastic case. To include the effect of axial loads on the bending moments, the stability functions $C$ and $S$ discussed in Article 2.3.3 are used. The incremental slope-deflection equations for this case are,

$$\delta M_A = \frac{CEI}{L} \delta \theta_A + \frac{SEI}{L} \delta \theta_B - (C + S) \frac{EI}{L} \frac{(\delta \Delta_B - \delta \Delta_A)}{L}$$

$$\delta M_B = \frac{CEI}{L} \delta \theta_B + \frac{SEI}{L} \delta \theta_A - (C + S) \frac{EI}{L} \frac{(\delta \Delta_B - \delta \Delta_A)}{L}$$

Combination (3)

In this case $\delta M_A$ is the change in the reduced plastic moment of the hinge, $\delta M_{pc}$, at $A$ and $\delta \theta_A$ can be eliminated from the equation written above. The resulting equations are:

$$\delta M_A = \delta M_{pc}$$

$$\delta M_B = \frac{S}{C} \delta M_A + \frac{C^2 - S^2}{C} \frac{EI}{L} \delta \theta_B - \frac{C^2 - S^2}{C} \frac{EI}{L} \frac{(\delta \Delta_B - \delta \Delta_A)}{L}$$

Combination (5)

In this case $\delta M_B$ is the change in the reduced plastic moment and $\delta \theta_B$ can similarly be eliminated. The resulting equations are:

$$\delta M_A = \frac{S}{C} \delta M_B + \frac{C^2 - S^2}{C} \frac{EI}{L} \delta \theta_A - \frac{C^2 - S^2}{C} \frac{EI}{L} \frac{(\delta \Delta_B - \delta \Delta_A)}{L}$$

$$\delta M_B = \delta M_{pc}$$
Combination (2)

In an unbraced multi-story frame, columns are forced into primarily double curvature configuration and the maximum moments occur at the ends. However, depending on the axial load, end moments and slenderness ratio of a column, the maximum moment can occur inside the column. In this case a plastic hinge will form inside the column only.

For a general case shown in Fig. 8.2, the maximum moment will occur at a distance \( x \) from \( A \) which is defined by

\[
\tan k x = \frac{-M_B - M_A \cos k L}{M_A \sin k L} \quad (A1.13)
\]

where

\[
k = \sqrt{\frac{P}{EI}}
\]

If the value of \( x \) obtained from Eq. A1.13 lies inside the column, the maximum moment occurs inside the column at that point. The maximum moment is equal to

\[
M_{\text{max}} = \sqrt{M_A^2 + M_B^2 + 2M_A M_B \cos k L} \quad \sin k L \quad (A1.14)
\]

If \( x \) is outside the column, the maximum moment is at one or both ends of the column. After the plastic hinge formation at a distance \( x \) from \( A \), the incremental slope-deflection equations are derived as follows:

Referring to Fig. 8.3, the results of combinations (3) and (5) can be applied to part AC and CB separately. For the part AC,

\[
S_k = \frac{x}{\delta M_A} = \frac{S}{\delta M} + K_1 \delta \theta_A - \frac{K_1}{\delta A_C} \quad (A1.15)
\]
where \[ K_1 = \frac{C_x^2 - S_x^2}{C_x} \frac{EI}{x} \] (A1.16)

and \( C_x \) and \( S_x \) are the stability functions for the part AC. For the part CB,

\[ \delta M_B = \frac{S_y}{C_y} \delta M_{pc} + K_2 \delta \theta_B - \frac{K_2}{y} (\delta \Delta_B - \delta \Delta_C) \] (A1.17)

where \[ K_2 = \frac{C_y^2 - S_y^2}{C_y} \frac{EI}{y} \] (A1.18)

and \( C_y \) and \( S_y \) are the stability functions for the part CB. From the equilibrium of moments for the part AC,

\[ \delta M_A + \delta R_A x + \delta P \delta \Delta_C - \delta M_{pc} = 0 \] (A1.19)

or \[ \delta R_A = -\frac{1}{x} (\delta M_A + \delta P \delta \Delta_C - \delta M_{pc}) \] (A1.20)

Similarly from the equilibrium of moments for the part CB

\[ \delta M_B - \delta R_B y + \delta P (\delta \Delta_B - \delta \Delta_C) + \delta M_{pc} = 0 \] (A1.21)

or \[ \delta R_B = \frac{1}{y} [\delta M_B + \delta P (\delta \Delta_B - \delta \Delta_C) + \delta M_{pc}] \] (A1.22)

From the equilibrium of end shears,

\[ \delta R_A + \delta R_B = 0 \] (A1.23)

Substituting the values of \( \delta R_A \) and \( \delta R_B \) into Eq. A1.23,

\[ -\frac{1}{x} [\delta M_A + \delta P \delta \Delta_C - \delta M_{pc}] + \frac{1}{y} [\delta M_B + \delta P (\delta \Delta_B - \delta \Delta_C) + \delta M_{pc}] = 0 \] (A1.24)

Substituting Eqs. A1.15 and A1.17 into Eq. A1.24 and solving for \( \delta \Delta_C \),
\[ \delta \Delta_c = \frac{1}{K_3} [K_1 y \delta \Theta_A - K_2 \delta \Theta_B + \frac{K_2}{y} \delta P \times (\frac{S_x}{C_x} y + \frac{S_y}{C_y} x + L) \delta M_{pc}] \]

(A1.25)

where \[ K_3 = K_1 \frac{y}{x} + K_2 \frac{x}{y} - \delta PL \]  

(A1.26)

Substituting Eq. A1.25 into Eqs. A1.15 and A1.17 and after some simplification, the necessary equations become,

\[ \delta M_A = K_1 (1 - \frac{y}{x} K_1) \delta \Theta_A + \frac{K_1 K_2}{K_3} \delta \Theta_B + \frac{1}{K_3} (\delta P - \frac{K_2}{y}) \delta \Delta_B \]

\[ + \frac{K_1}{K_3} (\frac{S_x}{C_x} y - \frac{S_x}{K_1} x + \frac{S_y}{C_y} x + L) \delta M_{pc} \]  

(A1.27a)

\[ \delta M_B = K_2 (1 - \frac{x}{y} K_3) \delta \Theta_B + \frac{K_1 K_2}{K_3} \delta \Theta_A [\frac{K_2}{y} - \delta P \times (\frac{S_x}{C_x} y - \frac{S_y}{C_y} x + L) \delta \Delta_B \]

\[ - \frac{K_2}{K_3} (\frac{S_x}{C_x} y - \frac{S_x}{K_2} x + \frac{S_y}{C_y} x + L) \delta M_{pc} \]  

(A1.27b)

where \[ K_1, K_2 \text{ and } K_3 \] are as defined in Eqs. A1.16, A1.18 and A1.26.

**Combination (7)**

In this case, none of the end moments depend upon the end rotations or translations. Only the variation in the reduced plastic moment causes the change of the end moments. Hence

\[ \delta M_A = \delta M_{pc} \]

(A1.28)

\[ \delta M_B = \delta M_{pc} \]
APPENDIX 2

Derivation of Equations for Incremental Hinge Angles

The equations necessary to compute hinge angle are derived from the incremental slope-deflection equations of Appendix 1 for the possible plastic hinge combinations of beams and columns. The combinations are the same as used in the derivation of slope-deflection equations. Positive values of the hinge angles indicate rotations in accordance with positive end moments.

1. Beams

Combination (1)

This case is for a fully continuous member. Therefore, the joint and member end rotations are identical. No hinge angles are present.

Combination (3)

There is a plastic hinge at the end A and the incremental hinge angle at the hinge, $\delta \theta_{HA}$, is obtained from

$$
\delta \theta_{HA} = \delta Q_A - \delta \theta_A
$$

(A2.1)

where $\delta Q_A$ is the incremental joint rotation at A and $\delta \theta_A$ is incremental member end rotation at A. Using Eq. A1.2a to calculate $\delta \theta_A$ for this case,

$$
\delta M_A = \frac{4EI}{L} \delta \theta_A + \frac{2EI}{L} \delta \theta_B - \frac{6EI}{L} \frac{\delta \Delta_B - \delta \Delta_A}{L} = 0
$$
Then, $\delta \theta_A$ becomes,

$$
\delta \theta_A = - \frac{1}{2} \delta \theta_B + \frac{3}{2} \left( \frac{\delta \Delta_B - \delta \Delta_A}{L} \right)
$$

(A2.2)

Substituting this into Eq. A2.1, the required equation for the incremental hinge angle becomes,

$$
\delta \theta_{HA} = \delta Q_A + \frac{1}{2} \delta \theta_B - \frac{3}{2} \left( \frac{\delta \Delta_B - \delta \Delta_A}{L} \right)
$$

(A2.4)

Combination (5)

In the similar way, the incremental hinge angle of the plastic hinge at B is given by

$$
\delta \theta_{HB} = \delta Q_B + \frac{1}{2} \delta \theta_A - \frac{3}{2} \left( \frac{\delta \Delta_B - \delta \Delta_A}{L} \right)
$$

(A2.4)

Combination (7)

In this case, the member rotation is caused only by unequal deflections of the ends of the beam due to the axial shortening of columns. Therefore, the required equations are

$$
\delta \theta_{HA} = \delta Q_A - \left( \frac{\delta \Delta_B - \delta \Delta_A}{L} \right)
$$

(A2.5)

$$
\delta \theta_{HB} = \delta Q_B - \left( \frac{\delta \Delta_B - \delta \Delta_A}{L} \right)
$$

(A2.6)

Combination (2)

With the reference to Fig. 8.1, the incremental hinge angle is given by

$$
\delta \theta_{HC} = \delta \theta_{CA} - \delta \theta_{CB}
$$

(A2.7)

where $\delta \theta_{CA}$ and $\delta \theta_{CB}$ are the incremental member end rotation of the part CA and that of the part CB at C, respectively.
The results of combinations (3) and (5) are applied to the part AC and CB separately. Then, the following equations can be obtained.

\[ \delta \theta_{CA} = -\frac{1}{2} \delta \theta_A + \frac{3}{2} \frac{\delta \Delta_C - \delta \Delta_A}{x} \]  
(A2.8a)

\[ \delta \theta_{CB} = -\frac{1}{2} \delta \theta_B + \frac{3}{2} \frac{\delta \Delta_C - \delta \Delta_A}{y} \]  
(A2.8b)

Rewriting Eq. A1.8 for \( \delta \Delta_C \),

\[ \delta \Delta_C = \frac{x}{x^3 + y^3} \left[ \frac{\delta \theta_A}{x} - \frac{\delta \theta_B}{y} + \frac{\delta \Delta_A}{x} + \frac{\delta \Delta_B}{y} \right] \]  
(A1.8)

Substituting Eqs. A2.8a and A2.8b into Eq. A2.7 and Eq. A1.8 into the resulting equation, the required incremental hinge angle is,

\[ \delta \theta_{HC} = \frac{1}{2} \frac{(2y^3 + 3x^2 - x^3) \delta \theta_A + (y^3 - 3y^2x^2 - 2x^3) \delta \theta_B + 3(x^2 - y^2)(\delta \Delta_C - \delta \Delta_A)}{(x^3 + y^3)} \]  
(A2.9)

Combination (4)

In this case, the incremental hinge angles is given by

\[ \delta \theta_{HA} = \delta \theta_A - \delta \theta_A \]  
(A2.10)

\[ \delta \theta_{HC} = \delta \theta_{CA} - \delta \theta_{CB} \]  
(A2.11)

The result of combination (7) can be applied to the part AC. Then it can be shown that

\[ \delta \theta_A = \delta \theta_{CA} = \frac{\delta \Delta_C - \delta \Delta_A}{x} \]  
(A2.12)

and in this case, \( \delta \Delta_C \) turns out to be

\[ \delta \Delta_C = \delta \Delta_B - y \delta \theta_B \]  
(A2.13)
Substituting Eq. A2.13 into Eq. A2.12,

$$\delta \theta_A = - \frac{x}{k} \delta \theta_B + \frac{\delta \Delta_B - \delta \Delta_A}{x}$$  \hspace{1cm} (A2.14)

Substituting the value into Eq. 2.10,

$$\delta \theta_{HA} = \delta Q_A + \frac{\delta \theta_B}{x} - \frac{\delta \Delta_B - \delta \Delta_A}{x}$$  \hspace{1cm} (A2.15)

Using Eq. A2.8b for the part CB and substituting Eq. A2.13 for \( \delta \Delta_C \), it can be shown that

$$\delta \theta_{CB} = \delta \theta_B$$  \hspace{1cm} (A2.16)

Substituting Eqs. A2.14 and A2.16 into Eq. A2.11 with the relationship of Eq. A2.12,

$$\delta \theta_{HC} = - \left( \frac{Y}{x} + 1 \right) \delta \theta_B + \frac{\delta \Delta_B - \delta \Delta_A}{x}$$  \hspace{1cm} (A2.17)

Equations A2.15 and A2.17 are the required equations for this combination.

**Combination (6)**

This case is similar to the combination (4) above. The required equations are,

$$\delta \theta_{HB} = \delta Q_B + \frac{x}{y} \delta \theta_A - \frac{\delta \Delta_B - \delta \Delta_A}{y}$$  \hspace{1cm} (A2.18)

$$\delta \theta_{HC} = \left( \frac{x}{y} + 1 \right) \delta \theta_A + \frac{\delta \Delta_B - \delta \Delta_A}{y}$$  \hspace{1cm} (A2.19)

**Combination (8)**

In this case, theoretically the deformations become infinitive and such beam mechanism is considered as the failure of the frame.
2. Columns

Combination (1)

No hinge angles are present.

Combination (3)

The incremental hinge angle is given by Eq. A2.1 in this case. The member end rotation $\delta \theta_A$ is obtained as follows:

$$\delta M_A = \delta M_{pc} = \frac{EI}{L} \left[ C \delta \theta_A + S \delta \theta_B - (C + S) \frac{\delta \Delta_A - \delta \Delta_B}{L} \right]$$

or

$$\delta \theta_A = -\frac{S}{C} \delta \theta_B + \frac{C + S}{C} \frac{\delta \Delta_B - \delta \Delta_A}{L} - \frac{L}{CEI} \delta M_{pc}$$

(A2.20)

Then, the required equation becomes

$$\delta \theta_{HA} = \delta Q_A + \frac{S}{C} \delta \theta_B - \frac{C + S}{C} \frac{\delta \Delta_B - \delta \Delta_A}{L} - \frac{L}{CEI} \delta M_{pc}$$

(A2.21)

Combination (5)

In the similar way,

$$\delta \theta_B = -\frac{S}{C} \delta \theta_A + \frac{C + S}{C} \frac{\delta \Delta_B - \delta \Delta_A}{L} + \frac{L}{CEI} \delta M_{pc}$$

(A2.22)

and

$$\delta \theta_{HB} = \delta Q_B + \frac{S}{C} \delta \theta_A - \frac{C + S}{C} \frac{\delta \Delta_B - \delta \Delta_A}{L} - \frac{L}{CEI} \delta M_{pc}$$

(A2.23)

Combination (7)

The required equations can be derived by considering the combinations (3) and (5). From Eqs. A2.20 and A2.22, the member end rotations become

$$\delta \theta_A = \delta \theta_B = \frac{\delta \Delta_B - \delta \Delta_A}{L} + \frac{1}{C + S} \frac{L}{EI} \delta M_{pc}$$

(A2.24)

Then,
\[ \delta_{HA} = \deltaQ_A - \delta\theta_A \]
\[ = \deltaQ_A - \frac{\delta\Delta_B - \delta\Delta_A}{L} - \frac{1}{C + S \cdot EI} \deltaM_{pc} \]  
(A2.25)

and \[ \delta_{HB} = \deltaQ_B - \delta\theta_B \]
\[ = \deltaQ_B - \frac{\delta\Delta_B - \delta\Delta_A}{L} - \frac{1}{C + S \cdot EI} \deltaM_{pc} \]  
(A2.26)

Combination (2)

As in the combination (2) of beams, the incremental hinge angle can be obtained from Eq. 2.7, that is,

\[ \delta\theta_{HC} = \delta\theta_{CA} - \delta\theta_{CB} \]  
(A2.7)

Referring to Fig. 8.3 and applying the results of combinations (3) and (5) to part AC and CB separately,

\[ \delta\theta_{CA} = -\frac{S}{C_x} \delta\theta_A + \frac{C_x + S}{C_x} \frac{\delta\Delta_C}{x} - \frac{x}{C_x \cdot EI} \deltaM_{pc} \]  
(A2.27)

\[ \delta\theta_{CB} = -\frac{S}{C_y} \delta\theta_B + \frac{C_y + S}{C_y} \frac{\delta\Delta_C}{y} + \frac{y}{C_y \cdot EI} \deltaM_{pc} \]  
(A2.28)

And from Eq. A1.25

\[ \delta\Delta_C = \frac{1}{K_3} \left[ K_1 y \delta\theta_A - K_2 x \delta\theta_B + \left( \frac{K_2}{y} - \deltaP \right) \cdot \delta\Delta_B - \left( \frac{S}{C_x} y + \frac{S}{C_y} x + L \right) \deltaM_{pc} \right] \]  
(A1.25)

where \( K_1, K_2 \) and \( K_3 \) are as defined in Eqs. A1.16, A1.18, and A1.26, respectively. Substituting Eqs. A2.27, A2.28 and A1.25 into Eq. A2.7, the required equation becomes,
\[ \delta \theta_{HC} = \left( \frac{K_4}{K_3} \right) \left( K_1 x - \frac{S_x}{C_x} \right) \delta \theta_A + \left( \frac{S_y}{C_y} - \frac{K_4}{K_3} K_2 x \right) \delta \theta_B \]

\[ - \left[ \frac{K_4}{K_3} \left( \frac{S_x}{C_x} \frac{y}{x} + \frac{S_y}{C_y} x + L \right) + \frac{1}{EI} \left( \frac{S_x}{C_x} + \frac{S_y}{C_y} \right) \right] \delta M_{pc} \]

\[ + \left[ \frac{K_4}{K_3} \left( \frac{S_z}{C_z} x - \delta P \right) - \frac{C_y + S_y}{C_y y} \right] \delta \Delta_B \]

(A2.29)

where

\[ K_4 = \frac{C_x + S_x}{C_x x} + \frac{C_y + S_y}{C_y y} \]  

(A2.30)
9. NOTATIONS

C Stability function;
E Modulus of elasticity;
F Load matrix;
FM Fixed end moment;
H Lateral load; load intensity;
$H_{\text{max}}$ Maximum lateral load;
I Moment of inertia; lateral load index;
$I_{e}$ Effective moment of inertia;
K Stiffness matrix; constant used in computation; proportional constant used for the variation in axial forces;
L Span length;
M Moment;
$M_{\text{max}}$ Maximum moment;
$M_{p}$ Plastic Moment;
$M_{pc}$ Reduced plastic moment;
P Axial force; concentrated gravity load;
$P_{y}$ Axial yield load in a column;
Q Joint rotation; total shear force of a story;
R Reaction;
S Stability function;
U Displacement matrix;
c Stability function;
h Column height;
k A function of axial load, moment inertia and modulus of elasticity;
Distance between inflection points; 

Story; 

Level; 

Stability function; distance between unbraced frames; 

Distributed gravity load per unit length; 

Distance from one end of a member to a plastic hinge in the interior; 

Distance from one end of a member to a plastic hinge in the interior, complement of $x$; 

Relative deflection; sway deflection; 

End rotation of a member; 

Hinge angle; 

Finite summation; 

Increment; 

Stress; 

Maximum compressive residual stress; 

Yield stress; 

Curvature, stability factor; 

Curvature at $M_p$; 

Curvature at $M_{pc}$. 
10. **TABLES**
<table>
<thead>
<tr>
<th>LEVEL</th>
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TABLE 3.3 FRAME 6 - WORKING GRAVITY LOADS

(a) Beam Loads (Unit k/ft)

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<tr>
<td>23</td>
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(b) Joint Loads (Unit kips)

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<th>C</th>
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### TABLE 3.4 SUMMARY OF MAXIMUM LATERAL LOAD AND CORRESPONDING DEFLECTION INDEX OF FRAME

<table>
<thead>
<tr>
<th>Frame</th>
<th>Maximum Lateral Load (kips)</th>
<th>Deflection Index of Frame</th>
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<tbody>
<tr>
<td>1</td>
<td>1.62</td>
<td>0.01020</td>
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<tr>
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<td>15.00</td>
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<td>4</td>
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<td>0.00351</td>
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<tr>
<td>5</td>
<td>13.05</td>
<td>0.00702</td>
</tr>
<tr>
<td>6</td>
<td>9.12</td>
<td>0.02199</td>
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### TABLE 3.5 EFFECTS OF AXIAL SHORTENING AT MAXIMUM LATERAL LOAD

<table>
<thead>
<tr>
<th>Frame</th>
<th>Maximum Lateral Load (kips)</th>
<th>Deflection Index of Frame</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With axial shortening</td>
<td>Without axial shortening</td>
</tr>
<tr>
<td></td>
<td>With axial shortening</td>
<td>Without axial shortening</td>
</tr>
<tr>
<td>4</td>
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<td>3.91</td>
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<tr>
<td>5</td>
<td>13.05</td>
<td>13.25</td>
</tr>
<tr>
<td>6</td>
<td>9.12</td>
<td>9.51</td>
</tr>
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</table>
### TABLE 3.6 LOCATION OF INFLECTION POINT

<table>
<thead>
<tr>
<th>Story</th>
<th>Average Distance Below Level Above</th>
<th>At Two-Third of Maximum Lateral Load</th>
<th>At Maximum Lateral Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Under Gravity Load</td>
<td></td>
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</tr>
<tr>
<td>Frame 4</td>
<td></td>
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</tr>
<tr>
<td>6</td>
<td>0.487 h</td>
<td>0.488 h</td>
<td>0.487 h</td>
</tr>
<tr>
<td>5</td>
<td>0.490 h</td>
<td>0.492 h</td>
<td>0.495 h</td>
</tr>
<tr>
<td>4</td>
<td>0.488 h</td>
<td>0.494 h</td>
<td>0.509 h</td>
</tr>
<tr>
<td>3</td>
<td>0.497 h</td>
<td>0.500 h</td>
<td>0.488 h*</td>
</tr>
<tr>
<td>2</td>
<td>0.490 h</td>
<td>0.500 h</td>
<td>0.500 h</td>
</tr>
<tr>
<td>1</td>
<td>0.348 h</td>
<td>0.537 h**</td>
<td>0.684 h</td>
</tr>
<tr>
<td>Frame 5</td>
<td></td>
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<tr>
<td>10</td>
<td>0.476 h</td>
<td>0.475 h</td>
<td>0.482 h</td>
</tr>
<tr>
<td>9</td>
<td>0.473 h</td>
<td>0.466 h</td>
<td>0.448 h</td>
</tr>
<tr>
<td>8</td>
<td>0.474 h</td>
<td>0.440 h</td>
<td>0.429 h</td>
</tr>
<tr>
<td>7</td>
<td>0.487 h</td>
<td>0.519 h</td>
<td>0.481 h</td>
</tr>
<tr>
<td>6</td>
<td>0.488 h</td>
<td>0.480 h</td>
<td>0.485 h</td>
</tr>
<tr>
<td>5</td>
<td>0.492 h</td>
<td>0.507 h</td>
<td>0.534 h*</td>
</tr>
<tr>
<td>4</td>
<td>0.497 h</td>
<td>0.492 h</td>
<td>0.469 h</td>
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<tr>
<td>3</td>
<td>0.474 h</td>
<td>0.467 h</td>
<td>0.534 h***</td>
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<tr>
<td>2</td>
<td>0.518 h</td>
<td>0.516 h</td>
<td>0.456 h***</td>
</tr>
<tr>
<td>1</td>
<td>0.343 h</td>
<td>0.592 h*</td>
<td>0.537 h</td>
</tr>
<tr>
<td>Frame 6</td>
<td></td>
<td></td>
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<tr>
<td>25</td>
<td>0.518 h</td>
<td>0.371 h</td>
<td>0.202 h*</td>
</tr>
<tr>
<td>23</td>
<td>0.436 h</td>
<td>0.436 h</td>
<td>0.338 h***</td>
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<tr>
<td>21</td>
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<td>0.438 h</td>
<td>0.609 h</td>
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<tr>
<td>19</td>
<td>0.482 h</td>
<td>0.530 h***</td>
<td>0.648 h</td>
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<tr>
<td>17</td>
<td>0.494 h</td>
<td>0.533 h***</td>
<td>0.053 h</td>
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<tr>
<td>15</td>
<td>0.491 h</td>
<td>0.577 h</td>
<td>0.548 h</td>
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<td>0.493 h</td>
<td>0.537 h</td>
<td>0.521 h</td>
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<td>0.564 h</td>
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<td>0.525 h</td>
<td>0.556 h</td>
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<td>0.494 h</td>
<td>0.518 h</td>
<td>0.470 h</td>
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<td>0.530 h</td>
<td>0.505 h</td>
<td>0.528 h</td>
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<td>1</td>
<td>0.368 h</td>
<td>0.778 h</td>
<td>0.910 h</td>
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*Excluding a column bent in near single curvature
**Excluding two columns bent in single curvature
***Excluding a column bent in single curvature
### TABLE 4.1 COMPARISON OF AXIAL LOADS

(a) Frame 5 (3-bay 10-story)

<table>
<thead>
<tr>
<th>Story</th>
<th>Column</th>
<th>Initial Axial Load in Non-Swayed Position (kips)</th>
<th>Axial Load at 2/3 of Max. Lateral Load Capacity of Frame (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sway Increment Analysis</td>
<td>One-Story Assemblage Analysis</td>
</tr>
<tr>
<td>7</td>
<td>A</td>
<td>268.7</td>
<td>264.9</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>408.4</td>
<td>410.7</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>360.1</td>
<td>363.4</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>223.7</td>
<td>221.5</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>417.2</td>
<td>412.5</td>
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<tr>
<td></td>
<td>B</td>
<td>625.7</td>
<td>629.1</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>552.1</td>
<td>555.8</td>
</tr>
<tr>
<td></td>
<td>D</td>
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<td>A</td>
<td>564.9</td>
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<td>D</td>
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*With reference to Fig. 3.4*
TABLE 4.1 (Continued)

(b) Frame 6 (2-bay 26-story)

<table>
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<th>Story</th>
<th>Column</th>
<th>Initial Axial Load in Non-Swayed Position (kips)</th>
<th>Axial Load at 2/3 of Max. Lateral Load &amp; Capacity of Frame (kips)</th>
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</thead>
<tbody>
<tr>
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<td>Sway Increment Analysis</td>
<td>One-Story Assemblage Analysis</td>
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11. **FIGURES**
GOVERNING DESIGN CONDITIONS

Gravity Load Conditions

Transition Zone

Combined Load Conditions

Fig. 1.1 DESIGN CONDITIONS FOR UNBRACED FRAMES
Fig. 1.2 BUCKLING AND INSTABILITY MODES OF FAILURE OF UNBRACED FRAMES
Fig. 2.1 TYPICAL LOAD-DEFLECTION BEHAVIOR OF AN UNBRACED FRAME UNDER NONPROPORTIONAL COMBINED LOADING
Fig. 2.2 SOLUTION RANGES WHEN H IS THE INDEPENDENT VARIABLE

Fig. 2.3 UNIQUE Δ VERSUS H RELATIONSHIP WHEN Δ IS THE INDEPENDENT VARIABLE
Fig. 2.4  MOMENT-CURVATURE RELATIONSHIPS WITHOUT AXIAL LOAD
Fig. 2.5 MOMENT-CURVATURE-THRUST RELATIONSHIPS
\[ \frac{P}{P_y} = 1.0 - \frac{\sigma_{rc}}{\sigma_y} \left( \frac{I_e}{I} \right)^2 \]

\[ \frac{W8 \times 31}{\phi \phi_{pc}} = 0.1 \]

\[ \frac{\sigma_{rc}}{\sigma_y} = 0.3 \]

From M-\(\phi-P\) Results

Assumed Residual Stress Pattern

Fig. 2.6 EFFECTIVE MOMENT OF INERTIA
Fig. 2.7 ONE-STEP VERSUS INCREMENTAL PROCEDURE
Fig. 2.8 LOADING AND END CONDITIONS OF A MEMBER
Combination No. | Locations | Hinge Locations
--- | --- | ---
1 | A <--- B | NONE
2 | A <--- C <--- B | C
3 | A <--- B | A
4 | A <--- C <--- B | A,C
5 | A <--- B | B
6 | A <--- C <--- B | C,B
7 | A <--- B | A,B
8 | A <--- C <--- B | A,B,C

Fig. 2.9 PLASTIC HINGE COMBINATIONS
Fig. 2.10 ILLUSTRATION OF THE FORMATION OF A JOINT MECHANISM
Fig. 2.11 CONFIGURATION, LOADING AND CONVENTION FOR NUMBERING
Fig. 2.12 FORCES AT A JOINT

Fig. 2.13 STORY SHEAR EQUILIBRIUM
Fig. 2.14 DETERMINATION OF TRIBUTARY AREA
OF AN UNBRACED FRAME
Fig. 2.15 COMPUTATIONAL PROCEDURE OF SWAY INCREMENT METHOD
Fig. 2.16 PREDICTION OF NEXT HINGE
Read frame geometry, loads and section properties

1

Set sway increment $\delta$ 

2

Calculate stability factors, stiffness functions and fixed end moments

Apply $\delta$ to the bottom story (the failed story, after the stability limit load is reached) and calculate the incremental load intensity

Calculate sway increment at other stories which results in the same incremental load intensity with that calculated in the bottom or the failed story

Calculate all incremental end moments, axial loads and axial shortenings

Test convergence of incremental load intensity. Within tolerance?

1

Yes

Adjust sway increment

1

No

2

Prediction within tolerance?

1

Yes

Calculate hinge angles and apply hinge rotation test

1

Yes

Any hinge reversal?

2

No

Return $\delta$ to value at start of the current increment and lock hinge that has unloaded

At first increment $\delta=0$. At second increment, $\delta$=arbitrary small value. Then on, $\delta$ from prediction

If sway increment cannot be obtained at a particular story, the story failed. Return $\delta$ to value at the start of the current increment. Now on sway deflection of the failed story is incremented.

Fig. 2.17 FLOW DIAGRAM--SWAY INCREMENT METHOD
Calculate total lateral load and total sway deflection at each story

Maximum load intensity attained? Yes Set maximum load intensity
No

Test ratio of maximum and current load intensities. Does the ratio reach the desired value? Yes Call exit end
No

Test moments and store new hinge

Write member end moments, axial forces and locations of hinges

Predict the smallest sway increment for next hinge

Fig. 2.17 (cont'd)
Fig. 3.1 FRAMES 1 and 2—GEOMETRY, MEMBER SIZES AND FACTORED GRAVITY LOADS

(a) Frame 1 (Ref. 48)

(b) Frame 2 (Ref. 13)
Fig. 3.2 FRAME 3--GEOMETRY, MEMBER SIZES AND FACTORED GRAVITY LOADS

(Ref. 31)
Story | Level | Beam Section | Internal Column | External Column
--- | --- | --- | --- | ---
1 | 0.5H | W10 x 21 | W6 x 25 | W6 x 25
2 | H | W14 x 30 | W6 x 25 | W6 x 25
3 | H | W14 x 30 | W8 x 35 | W6 x 25
4 | H | W14 x 30 | W8 x 35 | W6 x 25
5 | 1.13H | W14 x 30 | W8 x 40 | W6 x 25
6 | | W14 x 30 | W10 x 60 | W8 x 35

Ref. 13

Fig. 3.3 FRAME 4--GEOMETRY, MEMBER SIZES AND FACTORED GRAVITY LOADS
Fig. 3.4 FRAME 5--GEOMETRY, MEMBER SIZES AND LOADINGS
Fig. 3.5 FRAME 6—GEOMETRY AND LATERAL LOADS
Fig. 3.6 LOAD-DEFLECTION CURVE -- FRAME 1
(a) Working Loads Used in Design

(b) Failure Mechanism from Simple Plastic Theory

Fig. 3.7 HEYMAN'S FRAME
Fig. 3.8 LOAD-DEFLECTION CURVE--FRAME 2
Fig. 3.9 LOAD-DEFLECTION CURVE--FRAME 3
Fig. 3.10 COMPARISON OF LOAD-DEFLECTION CURVES OF FRAME 3' WITH DIFFERENT LOADING CONDITIONS

Under Proportional Loading (Ref. 31)

Under Non-Proportional Loading
Fig. 3.11 COMPARISON OF ORDERS OF PLASTIC HINGE FORMATION IN FRAME 3 WITH DIFFERENT LOADING CONDITIONS

(a) Under Proportional Loading (Ref. 31)

(b) Under Nonproportional Loading
Fig. 3.12 COMPARISON OF LOAD-DEFLECTION CURVES OF FRAME 3 WITH DIFFERENT COLUMN END CONDITIONS
Hinge Reversal at A

DEFLECTION INDEX OF FRAME

Fig. 3.13 LOAD-DEFLECTION CURVE -- FRAME 4
Fig. 3.14 PLASTIC HINGES AT MAXIMUM LATERAL LOAD AND THEIR SEQUENCE OF FORMATION--FRAME 4
Fig. 3.15  LOAD-DEFLECTION CURVE—FRAME 5
Fig. 3.16 PLASTIC HINGES AT MAXIMUM LATERAL LOAD AND THEIR SEQUENCE OF FORMATION -- FRAME 5
Fig. 3.17 COMPARISON OF LOAD-DEFLECTION CURVES OF FRAME 5 WITH DIFFERENT COLUMN END CONDITIONS

DEFLECTION INDEX OF FRAME

With Fixed Base

With Pinned Base
Fig. 3.18 LOAD-DEFLECTION CURVE--FRAME 6

DEFLECTION INDEX OF FRAME

H (KIPS)

With Axial Shortening of Columns
Without
Fig. 3.19 PLASTIC HINGES AT MAXIMUM LATERAL LOAD--FRAME 6
Fig. 3.20 LOAD-DEFLECTION INDEX CURVE OF EACH STORY -- FRAME 1
Fig. 3.21 LOAD-DEFLECTION INDEX CURVE OF EACH STORY--FRAME 2
Fig. 3.22 LOAD-DEFLECTION INDEX CURVE OF EACH STORY--FRAME 3
Fig. 3.23 LOAD-DEFLECTION INDEX CURVES OF SELECTED STORIES -- FRAME 4
Fig. 3.24 LOAD-DEFLECTION INDEX CURVES OF SELECTED STORIES--FRAME 5
Fig. 3.25 LOAD-DEFLECTION INDEX CURVES OF SELECTED STORIES--FRAME 6
○ Hinge Subjected to Elastic Unloading
● Plastic Hinge

Fig. 3.26  LOCATIONS OF HINGES SUBJECTED TO ELASTIC UNLOADING -- FRAMES 1, 2 and 3
- Hinge Subjected to Elastic Unloading

- Plastic Hinge

Fig. 3.27 LOCATIONS OF Hinges Subjected to Elastic Unloading--Frame 4
Fig. 4.1 ONE-STORY ASSEMBLAGE

Fig. 4.2 DISTANCE BETWEEN INFLECTION POINTS
(a) One-Story Assemblage

(b) Generation of Load-Deflection Curve

Fig. 4.3 GENERATION OF LOAD-DEFLECTION CURVE FOR ONE-STORY ASSEMBLAGE
Read geometry, loads and section properties

1. Set sway increment $\delta \Delta$ at joint

2. Calculate stability factors, stiffness functions and fixed end moments

Calculate incremental load intensity and sway increment at top of upper columns

Calculate all incremental end moments and axial loads

Test convergence of incremental lateral load intensity, within tolerance?

Yes

Predict for next hinge

No

Adjust sway increment

Prediction within tolerance?

Yes

Calculate total lateral load and total sway deflection at joint

Test ratio of maximum and current lateral loads. Does the ratio reach the desired value?

Yes

Call exit end

No

At first increment $\delta \Delta = 0$

At second increment, $\delta \Delta =$ arbitrary small value. Then on, $\delta \Delta$ from prediction

Fig. 4.4 FLOW DIAGRAM—ONE-STORY ASSEMBLAGE METHOD
Test moments and store new hinge

Write member end moments, axial forces and locations of hinges

Predict the smallest sway increment for next hinge

Beam mechanism or joint mechanism

---

Fig. 4.4 (cont'd)
Weight of Column: 3.9 k (Factored)
Wall Load on Exterior Column: 16.9 k (Factored)
(a) Geometry, Member Sizes and Factored Gravity Loads

(b) Load - Deflection Curve

Fig. 4.5 ANALYSIS OF ILLUSTRATIVE EXAMPLE
Fig. 4.6 LOAD-DEFLECTION CURVE OF STORY 7--FRAME 5

Fig. 4.7 LOAD-DEFLECTION CURVE OF STORY 4--FRAME 5
Fig. 4.8 LOAD-DEFLECTION CURVE OF STORY 2--FRAME 5

Fig. 4.9 LOAD-DEFLECTION CURVE OF STORY 1--FRAME 5
Fig. 4.10 LOAD-DEFLECTION CURVE OF STORY 22--FRAME 6

Fig. 4.11 LOAD-DEFLECTION CURVE OF STORY 13--FRAME 6
Fig. 4.12 LOAD-DEFLECTION CURVE OF STORY 10--FRAME 6

Fig. 4.13 LOAD-DEFLECTION CURVE OF STORY 7--FRAME 6
Fig. 4.14 LOAD-DEFLECTION CURVE OF STORY 4--FRAME 6

Fig. 4.15 LOAD-DEFLECTION CURVE OF STORY 1--FRAME 6
(a) Frame A

(b) Frame C

Fig. 5.1 YARIMCI'S TEST FRAMES (From Ref. 48)
Fig. 5.2 FRAME A - COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

**Experimental**

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**Sway Increment Analysis**

- LATERAL DEFORMATION AT TOP OF FRAME (IN.)
- LATERAL LOAD $H$ (KIPS)

---

*Fig. 5.2 FRAME A - COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS*
LATERAL DEFLECTION AT FIRST STORY (IN.)

Fig. 5.3 FRAME B-COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS
Fig. 5.4 FRAME C - COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS
Fig. 5.5 ONE-STORY ASSEMBLAGE TEST SPECIMEN AND LOADING (From Ref. 50)
Fig. 5.6 ONE- STORY ASSEMBLAGE SA-1-COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS
Fig. 5.7 ONE- STORY ASSEMBLAGE SA-2—COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS
Fig. 8.1 COMBINATION (2) IN BEAM

Fig. 8.2 LOADS AND DEFORMATIONS OF A COLUMN
Fig. 8.3 COMBINATION (2) IN COLUMN
12. REFERENCES

1. American Institute of Steel Construction

2. Driscoll, G. C., Jr. et al
   PLASTIC DESIGN OF MULTI-STOREY FRAMES, Fritz Engineering Laboratory

3. Lu, Le-Wu
   DESIGN OF BRACED MULTI-STOREY FRAMES BY THE PLASTIC METHOD,

4. American Iron and Steel Institute
   PLASTIC DESIGN OF BRACED MULTI-STOREY STEEL FRAMES, New York
   1968.

5. Allison, H.
   PLASTIC DESIGN CUTS COST OF PROTOTYPE HIGHRISE, Engineering
   News-Record, July 1967.

6. Vogel, U.
   DIE TRAGLASTBERECHUNG STAHLERER RAHMENTRAGWERKE NACH DER
   PLASTIZITATSTHEORIE II. ORDUNG Heft 15, Stahlbau-Verlag, Köln,
   1965.

7. Bleich, F.
   BUCKLING STRENGTH OF METAL STRUCTURES, McGraw-Hill Book Co.,


9. Lu, Le-Wu
   A SURVEY OF LITERATURE ON THE STABILITY OF FRAMES, Welding
   Research Council, Bulletin No. 81, September 1962.

10. Merchant, W.
    THE FAILURE LOAD OF RIGID JOINTED FRAMEWORKS AS INFLUENCED

11. Horne, M. R.
    INSTABILITY AND THE PLASTIC THEORY OF STRUCTURES, Transactions

12. Wood, R. H.
    THE STABILITY OF TALL BUILDINGS, Proceedings of the Institution
    of Civil Engineers, Vol. 11, September 1958.
13. Heyman, J.

14. Stevens, L. K.

15. Holmes, M. and Gandhi, S. N.

16. Lind, N. C.

17. Ang, A.

18. Chu, K. H. and Pabarcius, A.


20. Adams, P. F.
LOAD DEFORMATION RELATIONSHIPS FOR SIMPLE FRAMES, Fritz Engineering Laboratory Report No. 273.21, Lehigh University, December 1964.

21. Moses, F.

22. Wright, E. W. and Gaylord, E. H.

23. Alvarez, R. J. and Birnstiel, C.

24. Harrison, H. B.
AN ELASTIC-PLASTIC ANALYSIS BY COMPUTER FOR FRAMED STRUCTURES LOADED UP TO COLLAPSE, The Structural Engineer, Vol. 43, December 1965.

The Design of Sway Frames in Britain
PLASTIC DESIGN OF MULTI-STORY FRAMES-GUEST LECTURES, Fritz Engineering Laboratory Report No. 273.46, Lehigh University, August 1965.


28. Majid, K. I. and Anderson, D.

29. Majid, K. I. and Anderson, D.

AUTOMATIC ELASTIC-PLASTIC DESIGN OF SWAY FRAMES BY COMPUTER Joint Committee of the Institute of Structural Engineers and The Institute of Welding on "Fully Rigid Multi-Story Welded Steel Frames", February 1969.

31. Parikh, B. P.

32. Korn, A.

33. Korn, A. and Galambos, T. V.

34. Adams, P. F., Majundar, S.N.G., Clark, N. J. and MacGregor, J. G.
35. Davies, J. M.
THE RESPONSE OF PLANE FRAMEWORKS TO STATIC AND VARIABLE
REPEATED LOADING IN THE ELASTIC-PLASTIC RANGE, The Structural

36. Daniels, J. H. and Lu, L. W.
SWAY SUBASSEMBLAGE FOR UNBRACED FRAMES, ASCE Meeting Preprint

37. Daniels, J. H.
A PLASTIC METHOD FOR UNBRACED FRAME DESIGN, American Institute
of Steel Construction Engineering Journal, Vol. 3, No. 4,
October 1966.

38. Daniels, J. H.
COMBINED LOAD ANALYSIS OF UNBRACED FRAMES, Ph.D. Dissertation
Lehigh University, 1967, University Microfilms, Inc., Ann
Arbor, Michigan.

39. Daniels, J. H. and Lu, L. W.
DESIGN CHARTS FOR THE SUBASSEMBLAGE METHOD OF DESIGNING
UNBRACED MULTI-STORY FRAMES, Fritz Engineering Laboratory


41. Driscoll, G. C., Jr., Armacost, J. O., III and Lu, L. W.
PLASTIC DESIGN OF MULTI-STORY-UNBRACED FRAMES, Fritz Engineering Laboratory Report No. 345.2, Lehigh University, June 1968.

42. Driscoll, G. C., Jr., Armacost, J. O., III and Hansell, W. C.
PLASTIC DESIGN OF MULTI-STORY FRAMES BY COMPUTER, Journal
of the Structural Division, ASCE, Vol. 93, ST1, January 1970.

43. Hansell, W. C.
PRELIMINARY DESIGN OF UNBRACED MULTI-STORY FRAMES, Ph.D.
Dissertation, Lehigh University, 1966, University Microfilms,
Inc., Ann Arbor, Michigan.

44. Emkin, L. Z. and Litle, W. A.
PLASTIC DESIGN OF MULTI-STORY STEEL FRAMES BY COMPUTER,
Journal of the Structural Division, ASCE, Vol. 96, ST11,
November 1970.

45. Low, M. W.
SOME MODEL TESTS ON MULTI-STORY RIGID STEEL FRAMES, Proceedings
46. Wakabayashi, M.
THE RESTORING FORCE CHARACTERISTICS OF MULTI-STORY FRAMES,
Bulletin of the Disaster Prevention Research Institute, Kyoto
University, Kyoto, Japan, Vol. 14, Part 2, No. 78, February
1965.

47. Wakabayashi, M., Nonaka, T. and Morino, S.
AN EXPERIMENTAL STUDY ON THE INELASTIC BEHAVIOR OF STEEL
FRAMES WITH RECTANGULAR CROSS-SECTION SUBJECTED TO VERTICAL
AND HORIZONTAL LOADING, Bulletin of the Disaster Prevention
Research Institute, Kyoto University, Kyoto, Japan, Vol. 18,

48. Yarimci, E.
INCREMENTAL INELASTIC ANALYSIS OF FRAMED STRUCTURES AND SOME
EXPERIMENTAL VERIFICATIONS, Ph.D. Dissertation, Lehigh
University, 1966, University Microfilms, Inc., Ann Arbor,
Michigan.

49. Arnold, P., Adams, P. F. and Lu, L. W.
STRENGTH AND BEHAVIOR OF INELASTIC HYBRID FRAME, Journal
of the Structural Division, ASCE, Vol. 94, ST1, January 1968.

50. Kim, S. W. and Daniels, J. H.
EXPERIMENTS ON UNBRACED ONE-STORY ASSEMBLAGES, Fritz Engineering
Laboratory Report No. 346.4, Lehigh University.

51. Beedle, L. S.
PLASTIC DESIGN OF STEEL FRAMES, John Wiley & Sons, Inc., New
York, 1958.

52. Galambos, T. V.
STRUCTURAL MEMBERS AND FRAMES, Prentice-Hall Inc., Englewood

53. Salvadori, M. G. and Baron, M. L.
NUMERICAL METHODS IN ENGINEERING, Prentice-Hall, Inc. Englewood

54. Neal, B. G.
THE PLASTIC METHODS OF STRUCTURAL ANALYSIS, Second Edition,

55. Yoshida, H.
MINIMUM WEIGHT DESIGN OF FRAMES USING SWAY SUBASSEMBLAGE

56. Gaylord, E. H., Jr. and Gaylord, C. H.
DESIGN OF STEEL STRUCTURES, McGraw-Hill Book Co., New York,
New York, 1957.

57. Heyman, J.
ON THE ESTIMATION OF DEFLECTIONS IN ELASTIC-PLASTIC FRAMED
STRUCTURES, Proceedings of the Institution of Civil Engineers,
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