Recent developments in plastic design practice, August 1968 (69-7)

Lynn S. Beedle
Le-Wu. Lu
Lee Chong Lim

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ERRATA SHEET
for
"RECENT DEVELOPMENTS IN PLASTIC DESIGN PRACTICE"
by
Lynn S. Beedle
Le-Wu Lu
Lee Chong Lim

Page 2 Paragraph 2 Line 4 reads "----emphasis on the load factor and the need to account for ----". Should read "----emphasis on the load factor and the potential advantages in accounting for ----".

Page 3 Item 3: Change "beam-and-columns" to "beam-and-column".

Page 14 Paragraph 2 Line 1: Delete "a" from "----development is a load-factor design ----".

Page 14 Paragraph 2 Line 8: Correct "Europse" to "Europe".

Page 15 Paragraph 1 Line 5: Add "limiting" so that it reads as "----from the one, a limiting resistance function ----".

Page 16 Paragraph 1 Line 5: Correct "statical" to "statistical".

Page 19 Paragraph 3 Line 3: Change "Table 5" to "Table 6".

Page 22 Item 4 Line 3: Delete "to a certain extent in the AREA, but".

Table 2 (Page 25): Revised table attached.

Table 4 (Page 29): Move "8.8 Details with Regard to Bolting" in column 1 and "Brief description" in Column 2 up to the level of "Nominal Tension =" in column 3.

Table 5 (Page 31): Revised table attached.

Page 38 Ref. 45: Change "Korn, N. and ----" to "Korn, A. and ----".

Page 39 Ref. 58: Change "---- Little, W. A." to "---- Litle, W. A.".

Page 41 Ref. 78: Renumber it as Ref. 79.

Page 41 Insert: Ref. 78
Vincent, G. S.
TENTATIVE CRITERIA FOR LOAD FACTOR DESIGN OF STEEL HIGHWAY BRIDGES, American Iron and Steel Institute, New York, February, 1968.
<table>
<thead>
<tr>
<th>Country</th>
<th>Low Building Design</th>
<th>Multi-story Frames</th>
<th>Plastic Design Specification or Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
<td>Extensive application</td>
<td>A few</td>
<td>AISC - Part 2</td>
</tr>
<tr>
<td>Austria</td>
<td>Beams and Girders</td>
<td>Aware of none</td>
<td>ÖNORM B4600(1964) (general provisions)</td>
</tr>
<tr>
<td>Australia</td>
<td>Used for portal frames</td>
<td>Aware of none</td>
<td>A.S. CAL SAA, 1968</td>
</tr>
<tr>
<td>Belgium</td>
<td>Little application</td>
<td>Aware of none</td>
<td>Addendum to NBN1 (detailed spec.)</td>
</tr>
<tr>
<td>Canada</td>
<td>Extensive application</td>
<td>A few</td>
<td>CISC, 1967</td>
</tr>
<tr>
<td>Czechoslovakia</td>
<td>Parts of buildings</td>
<td></td>
<td>CSN 73-1401(68) (general provisions)</td>
</tr>
<tr>
<td>Denmark</td>
<td>A few</td>
<td>Aware of none</td>
<td>Hungarian Design Code (draft form)</td>
</tr>
<tr>
<td>France</td>
<td>None</td>
<td>None</td>
<td>&quot;Not Prohibited&quot;</td>
</tr>
<tr>
<td>Germany</td>
<td>Beams and Girders</td>
<td></td>
<td>DIN 4114, Vol. 2 (general provisions)</td>
</tr>
<tr>
<td>Hungary</td>
<td>Under current consideration (most buildings are of reinforced concrete)</td>
<td></td>
<td>Hungarian Design Code (draft form)</td>
</tr>
<tr>
<td>India</td>
<td>A few</td>
<td>Aware of none</td>
<td>I.S. 800</td>
</tr>
<tr>
<td>Italy</td>
<td>Aware of none</td>
<td>Aware of none</td>
<td>Not yet</td>
</tr>
<tr>
<td>Japan</td>
<td>Aware of none (a few pedestrian bridges)</td>
<td>Aware of none</td>
<td>Recommendations in draft form</td>
</tr>
<tr>
<td>Norway</td>
<td>Aware of none</td>
<td>Aware of none</td>
<td>NS424A (alternate method) (general provisions)</td>
</tr>
<tr>
<td>Portugal</td>
<td>Aware of none</td>
<td>Aware of none</td>
<td>BABS(1968) (alternate method)</td>
</tr>
<tr>
<td>Sweden</td>
<td>Beams and Girders</td>
<td></td>
<td>SIA No. 161 (alternate method)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>Beams and Girders</td>
<td></td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td>Nearly every portal frame</td>
<td>A few multi-story frames on the basis of &quot;open&quot; specifications, &quot;up to the designer&quot;</td>
<td>BSS 449 (general provisions)</td>
</tr>
<tr>
<td>Yugoslavia</td>
<td>Some</td>
<td>Aware of none</td>
<td>Under study</td>
</tr>
<tr>
<td>Europe</td>
<td>Task Group on Plastic Design is working on draft of a specification</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 5
LOAD FACTORS FOR PLASTIC DESIGN IN VARIOUS COUNTRIES

<table>
<thead>
<tr>
<th>Country</th>
<th>Assumed Shape Factor</th>
<th>Dead Load + Live Load</th>
<th>Dead Load + Live Load + Wind or Earthquake Forces</th>
<th>No. of Load Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
<td>1.12</td>
<td>1.70</td>
<td>1.30</td>
<td>2</td>
</tr>
<tr>
<td>Australia</td>
<td>1.15</td>
<td>1.75</td>
<td>1.40</td>
<td>2</td>
</tr>
<tr>
<td>Belgium</td>
<td>1.12</td>
<td>1.68</td>
<td>1.49</td>
<td>3</td>
</tr>
<tr>
<td>(1.12 for extreme wind)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>1.12</td>
<td>1.70</td>
<td>1.30</td>
<td>2</td>
</tr>
<tr>
<td>Germany</td>
<td></td>
<td>1.67f</td>
<td>1.46f</td>
<td>2</td>
</tr>
<tr>
<td>India</td>
<td>1.15</td>
<td>1.85</td>
<td>1.40</td>
<td>2</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.15</td>
<td>1.75 (Portal Frames)</td>
<td>1.40</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.50 (Multi-story Braced Frames)</td>
<td></td>
</tr>
</tbody>
</table>

MULTIPLE LOAD FACTORS

<table>
<thead>
<tr>
<th>Country</th>
<th>Assumed Shape Factor</th>
<th>Dead Load + Live Load</th>
<th>Dead Load + Live Load + Wind or Earthquake Forces</th>
<th>No. of Load Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Czechoslovakia</td>
<td>1.20 (max.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hungary*</td>
<td>1.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proposal 1: (single load factor) 1.2 - 1.5 depending on combinations of D, L, and I</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proposal 2: (multiple load factor) Many possible combinations</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.2D + 2.1L or 1.4(D+L) (normal condition)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.2(D+L) + 1.5S₁ (under snowfall)</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(D+L) + 1.5K; (D+L+S₂)+1.5K (under earthquake)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(D+L) + 1.5W₁ (under typhoon)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(D+L+S₂) + 1.5W₂ (under whirlwind)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yugoslavia*</td>
<td>1.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>D = 1.49, L = 1.68+Additional Combinations</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Under study

D  Dead Load  I  Irregular Live Load  S₂  Mean Snow Load
L  Live Load  f  Shape Factor  W₁  Wind Force (under typhoon)
K  Earthquake Force  S₁  Maximum Snow Load  W₂  Wind Force (under whirlwind)
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for
"RECENT DEVELOPMENTS IN PLASTIC DESIGN PRACTICE"
(Journal of the Structural Division, ASCE, 95 (ST9), September 1969)
by
L. S. Beedle, L. W. Lu, L. C. Lim

Page 1913
Line 27: Change "---- width-thickness ratios ----" to read "---- width-to-thickness ratios ----".

Page 1917
Table 1, Column 9: Delete "1963 specification Part 2".

Page 1918
Table 2: Delete "contemporary". Correct "little" to "a few". Should read "Hungarian Design Code (draft form)" instead of "Hungary Design Code (draft form)." Insert between Mexico and South Africa "Netherlands Under Study Detailed Specification A few A few None".

Line 8 from bottom: Change "---- the U.S.A., the United Kingdom and Canada also have ----" to read "---- the U.S.A., the United Kingdom, Canada and Mexico also have ----".

Line 7 from bottom: change "20" to read "21".

Line 3 from bottom: Insert "the U.S.A., the United Kingdom, Canada and Mexico also have".

Page 1919
Line 8 from bottom: Change "---- Tables 3 and 4. " should be read as "---- Table 3."

Page 1923
Table 5, Local Buckling, Column 3: Replace the b/t formula by:

Steel | b/t
-------|------
A36   | 17
A441  | 14
A572(65) | 12

Change \( \frac{d}{w} = 70 - 100 \frac{P}{P_y} \) when \( P/P_y \leq 0.27 \)

to read \( \frac{d}{w} = (70 - 100 \frac{P}{P_y}) \sqrt{\frac{36}{\sigma_y}} \) when \( P/P_y \leq 0.27 \)

Page 1924
Table 4: Delete "related to design" in the heading of Column 2.

Page 1925
Table 5, Local Buckling, Column 3: Change \( \frac{L_{cr}}{\sigma_y} = \left( \frac{1375}{\sigma_y} + 25 \right) r_y \)

to \( L_{cr} = \left( \frac{1375}{\sigma_y} + 25 \right) r_y \)

Page 1926
Table 5, Lateral Buckling, Column 2: Add

Moment Gradient: \( L_{cr} \leq \left( 60 + 40 \frac{M}{M_p} \right) r_y \)

for \( -0.625 < \frac{M}{M_p} \leq +1.0 \)

Uniform Moment: \( L_{cr} \leq 35 r_y \)

Table 5, Lateral Buckling, Column 3: Change

\( L_{cr} = \frac{374}{\sigma_y} r_y \) to \( L_{cr} = \left( \frac{1375}{\sigma_y} + 25 \right) r_y \)

\( L_{cr} = \frac{274}{\sigma_y} r_y \) to \( L_{cr} = \left( \frac{1375}{\sigma_y} + 25 \right) r_y \)

Page 1927
Table 5: Details with regard to Bolting, Column 3: Change 0.56 to 0.60 in the formula for computing nominal tension.

Page 1928
Line 7: "---- gravity load ranges from 1.70 (U.S.A., Canada and Mexico) to ----" should be read as "---- gravity load ranges from 1.67 (Mexico) to -----".

Page 1931
Table 6(a): Insert "Mexico 1.12 1.67 1.22 2".
Table 6(b): Insert "U.S.S.R. -- P_1 + P_2 or 1.2 L_3 1.40 several".
Table 6(b): Add to the footnote "L_3 = movable concentrated load."
Design Recommendations for Multi-Story Frames

RECENT DEVELOPMENTS IN PLASTIC DESIGN PRACTICE

by

Lynn S. Beedle
Le-Wu Lu
Lee Chong Lim

This work has been carried out as part of an investigation sponsored by the American Iron and Steel Institute.

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Fritz Engineering Laboratory
Department of Civil Engineering
Lehigh University
Bethlehem, Pennsylvania

August 1968

Fritz Engineering Laboratory Report No. 345.8
ANNOUNCEMENT

Additions to this preprint, promptly forwarded to the authors, will result in immediate consideration for possible use in the second edition of the "Commentary on Plastic Design in Steel". Revisions in this ASCE Manual (No. 41) are now being studied by an ASCE Ad Hoc Committee formed for this purpose.
RECENT DEVELOPMENTS IN PLASTIC DESIGN PRACTICE

By Lynn S. Beedle, F. ASCE, Le-Wu Lu, A.M. ASCE, and Lee Chong Lim

INTRODUCTION

During the past decade a number of major developments have taken place in the area of plastic analysis and design. One of these is the growth of research interest, with research on multi-story frames beginning at Lehigh University in 1958, and of similar activities elsewhere in the United States and in many other countries around the world. The recognition of plastic design in the specifications of many countries is another significant advance. There has been extensive use of the plastic method in the design of industrial buildings and, in some countries, high-rise office and apartment buildings. The resulting savings in material and design time have been substantial.

In 1961, the ASCE published a manual entitled "Commentary on Plastic Design in Steel," which contains much of the information on the subject accumulated up to that time.

The major research effort has been concerned with the application of plasticity concepts to the design of building frames with high strength steel members and to multi-story frames in which instability effects play a major role in influencing the load-carrying capacity. New steels with yield stress up to 65 ksi can now be included in plastic design. Experiments have been conducted on full scale braced and unbraced multi-story frames to study their...
ultimate strength. Design methods for multi-story frames, including the effect of instability in separate members as well as that of the entire structures, are available. Plastic strength has also been utilized extensively in the design of earthquake-resistant structures.

The substantial amount of research outlined above resulted in a summer conference held at Lehigh University in 1965 which brought into focus a number of the new problems and many of their solutions. A set of lecture notes and design aids (1,2) was issued during the conference, summarizing the new information and design techniques. Since 1965, several braced multi-story frames have been designed based on the methods presented in the lecture notes. It is expected that more extensive use of plastic design in multi-story frame design will be forthcoming.

The research on plastic analysis and design has resulted in some changes in design philosophy. It has necessitated a more precise definition of the limits of usefulness upon which the plastic method is based. It has brought additional emphasis on the load factor and the need to account for different values of this factor for different types of load.

To summarize these new developments in research and applications, the Committee on Plastic Design of the Structural Division of the ASCE took steps to prepare a revision to the Commentary on Plastic Design. Considerable amount of new information will be added together with new design recommendations. In addition to covering low unbraced frames, the Commentary will be expanded to include braced multi-story frames. A brief treatment of unbraced multi-story frames will also be included.

The new results obtained in recent years are being incorporated in the forthcoming edition of the AISC Specification. As in the earlier
(1963) edition, plastic design is formally recognized in "Part 2" of the Specification. The revised Specification will extend the application of plastic design to braced multi-story frames and to frames with high strength steel members. Many of the provisions in Part I (allowable-stress design) of the Specification continue to be affected by the research on the plastic behavior of structures. Several other countries have revised or are in the process of reviewing their specifications to permit more extensive use of plastic design in their specifications. International cooperation among various countries is rapidly increasing.

The purpose of this paper is to present a review of these recent developments and to indicate the future research needs and trend of design practice.

RESEARCH

Recent research on plastic analysis and design covers a very wide range of problems. The following are some of the areas of research which have received major attention:

1. Mechanical properties of high strength steels in the inelastic range.
2. Behavior and strength of individual components, such as beams, columns and connections.
3. Strength of sway and non-sway beam-and-column subassemblages or "limited frames".
4. Behavior and design of braced multi-story frames.
5. Behavior and design of unbraced multi-story frames.
6. Optimum (minimum weight) design.
7. Response of structural members and frames subjected to repeated and reversed loading.

A complete survey of the research work is beyond the scope of this paper.
However, most of the new information and results will be included in the second edition of the Commentary. Only a brief summary is given here.

Mechanical Properties of High Strength Steels

In recent years many types of high strength steel, with more favorable strength-to-price ratio than structural carbon steel, have become available. It is therefore desirable to extend the applicability of plastic design methods to these steels. Studies have been made on the mechanical properties of high strength steel with yield stress ranging from about 42 to 65 ksi (3,4). In these studies emphasis has been placed on those properties which are important in the application of plastic design. Some of these are: static yield stress level, strain at the onset of strain hardening and strain hardening modulus. Cooling residual stress distribution in wide-flange shapes has also been studied (3,5). Based on the results obtained from these investigations, it appears that plastic design can be extended to the new steels.

Studies on Component Behavior

Numerous experiments were performed on wide-flange beams under uniform moment (6) and moment gradient (7,8), with various types of lateral bracing (9), and on beams of high strength steel (3,10) to study the post-yield behavior. Theoretical models, based on the concept that failure results when local and lateral-torsional buckling occur simultaneously permitted a prediction of the limits of inelastic rotation capacity, and a definition of the required maximum flange and web width-thickness ratios and maximum bracing spacing (11,12,13,14). For example, the maximum flange width-thickness ratios for steels with yield points of 36 and 50 ksi were found to be 17 and 14 respectively. The corresponding maximum unbraced lengths for beams under uniform moment were determined to be, respectively, $38\gamma_y$ and $28\gamma_y$, where $\gamma_y$ is the weak axis radius of gyration.

The work on beam-columns has been concerned with the theoretical determination of the in-plane end moment versus end rotation curves, extending
the work of Chwalla (15) to wide-flange members containing residual stress (16,17), a summary of this work is given in Ref. 18. The solution of the problem was achieved by numerical integration procedures, and non-dimensional curves for use in design are presented in Ref. 2. Experiments have given excellent verification of the theoretically obtained curves over a wide-range of the relevant parameters (19,20). Theoretical studies on inelastic lateraltorsional buckling of unbraced beam-columns bent about their major axis have also shown good agreement with experiments (21,22). Design procedures, based on this research, have been developed. These are summarized in Ref. 23.

Extensive experimental programs were performed on various types of rigid corner connections and beam-to-column connections. A review of this work is presented in Ref. 24. Design procedures, based on this work, were developed to assure that connections have adequate rotation capacity and a greater moment capacity than the members to be joined. These procedures are summarized in Ref. 25. A problem which is being investigated currently is concerned with the influence of large axial forces on the strength of beam-to-column connections and the influence of strain-hardening in connection webs.

**Strength of Subassemblages**

The basic design element for braced multi-story frames has been found to be a "subassemblage" consisting of a column and its adjacent beams (26,28). The load-deformation behavior of such a subassemblage can be determined, using equilibrium, compatibility and, the moment-rotation relationships of its component members (28). Good correlation has been found between theoretically predicted and experimentally measured behavior (29). The tests also provided experimental confirmation of the behavior of individual beams, beam-columns and connections.
Similar studies have also been made on subassemblages with laterally unsupported beam-columns (30). Lateral-torsional buckling tends to influence the load-carrying capacity of the columns, but only to a limited extent. Additional studies aimed at the development of a practical design procedure are currently underway.

The subassemblage concept of design can also be applied to unbraced multi-story frames (31). In this case, a different type of subassemblage, consisting of a sway column and its adjacent beams, must be considered. These subassemblages are analyzed by a procedure that was developed for restrained columns permitted to sway (32). The basic concept is currently being checked by experiments.

Braced Frames - Behavior and Design Methods

A design method based on the weak-beam, strong-column concept (33) has been developed for braced multi-story frames (1,34). In this method, beams are designed to develop three-hinge mechanisms in the clear span between column flange faces under full gravity load. Columns are then proportioned to have sufficient capacity to resist the bending moment transmitted from the adjacent beams and also the axial thrust from stories above. Instability effects can be readily included in the design process by using the available charts. (2). The bracing system (x-or k-type bracing) is assumed to carry all lateral shear and to resist all shears due to the \( P_A \) effect in simple stress action without assistance from the frame. Additional considerations in the selection of bracing sizes include the maximum permissible slenderness ratio of the braced and the resulting sway deflection in each story at the working load.

A series of tests has been conducted on three-story, two-bay frames to verify the design procedure and to study the interaction between the frame and bracing system in resisting lateral load (35,36). The frames were loaded
by full gravity load, checkerboard gravity load, full gravity and lateral loads, and checkerboard gravity and lateral loads. The experimental ultimate load reached or exceeded the maximum load predicted by plastic theory with an average discrepancy of 4 percent. Diagonal bracing was found to carry most of the lateral load and the frame was required to resist only 14 to 26 percent of the total lateral load.

In the United Kingdom, a design method for braced frames has been proposed by a joint committee organized by the Institute of Welding and the Institution of Structural Engineers (37). In this method, all beams are designed plastically, but the columns are proportioned on a limit somewhat less than the elastic limit. The design permits the use of rigidly connected floor beams and takes into account the additional bending moment transmitted from the floor beams to the main members. The effect of biaxial bending must therefore be considered in the design. A load factor of 1.50 has been recommended in applying this method. The design procedure has been checked by full-scale tests on a three-story, two-bay x one-bay frame and is found to be conservative (38).

Unbraced Frames - Behavior and Design Methods

Among all the research work reviewed in this paper, the most extensive is on unbraced multi-story frames. This research covers the following areas:

1. Tests on unbraced frames to study their failure behavior,
2. Development of computer programs for determining the elastic and elastic-plastic range response of such frames, and
3. Development of design methods.

The first two areas of research have proceeded simultaneously. The behavior observed during the tests has been incorporated in the computer program,
and, conversely, the validity of the computer analysis has been checked by comparing the predicted response with the observed behavior. Several series of unbraced frames have been tested to observe the extent of the over-all instability effect caused by the $P_\Delta$ moment and the behavior at the maximum load and beyond. The results are reported in Refs. 39, 40 and 41. The results from one of the tests are compared with theoretical predictions in Fig. 1. A theory considering the influence of $P_\Delta$ moment is shown to yield close correlation with test. All the tests show conclusively that unbraced frames are likely to fail by over-all instability before the formation of a plastic mechanism and that any rational analysis and design procedure should attempt to include this effect.

Numerous computer programs have been developed for analyzing unbraced frames (42,43,44,45,46,47). Some of these programs are quite complete and are capable of handling relatively large frames. It is possible in these programs to include: the instability effects of individual members and of the entire frame, the bending moment caused by relative shortening of the columns, spread of yielding near the plastic hinges, and the influence of strain hardening. It is expected that further research in this area will produce computer programs which can provide solutions for very complex frames and include more secondary effects.

The design of an unbraced multi-story frame is considerably more complicated than that of a braced frame. Because of the over-all instability effect in an unbraced frame, its load-carrying capacity may become dependent on the resulting deflections. This interdependence would make a direct design almost impossible. Recent research has made available a three-step design procedure which can be used either manually or with the aid of a computer. In the first step of the procedure, tentative beam and column sizes are selected
using the plastic moment balancing method (48). This method is ideally suited because it can include an approximate $P\Delta$ effect. An initial sway deflection estimate is made and then the resulting $P\Delta$ moments are included when equilibrium is established. Using the member sizes obtained in the preliminary design, a sway analysis is then performed in the second step to verify the initial sway estimates and to check the load-carrying capacity. A method, known as the "Sway Subassemblage Method", has been developed specifically for this purpose (31,49,50). It will give the complete lateral load versus sway relationship for each story, from which the deflection at the service load can also be estimated. The final step in the design process is to revise the member sizes based on the results of the load-deflection analysis, or on other factors such as economy. This method is being tested on several multi-story, multi-bay frames. Further improvement and simplification appear to be possible.

Optimum (Minimum Weight) Design

Considerable success had been achieved in applying linear programming and dynamic programming techniques to obtain minimum weight designs for continuous beams and low building frames (51,52,53). In most of the work, it is assumed that there is an infinite range of sections available from which to choose member sizes. An approach considering the discrete nature of the available sections has been developed and applied to the design of low frames (54). Another recent work incorporated the AISC Specification in the formulation of the optimization process (55).

Only a few attempts have been made to develop optimum design solutions which consider the frame instability effects (56,57). An attempt has also been made to consider both frame instability effects and deflection limitations under service load (58). Further development in this area is forthcoming.
Structures Subjected to Repeated and Reversed Loading

It has long been recognized that the inelastic deformation capacity of a structure is one of the most important properties in earthquake-resistant design. In order to evaluate the deformation capacity of an entire structure, it is first necessary to determine the response of its components under repeatedly applied loads. Numerous experiments have been performed on structural components to study their inelastic range behavior. A program of study involving cyclic loading tests on cantilever and simply supported beams has been described in Ref. 59. Extensive tests on beam-columns subjected to a constant axial thrust and reversed bending moments have been performed by Japanese investigators (60,61). The behavior of various types of beam-to-column connections, including both bolted and welded connections and members made of high strength steel, has been studied (62,63,64). Further work on the behavior of the panel zone inside the connection is underway.

Repeated and reversed loading tests on single and multi-story frames have been carried out by several investigators in the U. S. and in Japan (65,66,67). Both braced and unbraced frames have been included in these studies. One of the significant findings from these studies is that the hysteretic loops are extremely stable even at very large lateral displacements.

The results of these studies have been used in analytical calculations for determining dynamic response and are being incorporated in design specifications.

APPLICATION

To assess the extent of use of plastic theory in structural design, a survey was made in 1960 (68). Up to that time plastic design had its
greatest application in low buildings in the United States and the United Kingdom. Today plastic design has gained wider acceptance and large numbers of plastically designed structures have been built in many parts of the world.

As a result of the completion of major research and in order to present the latest findings to the design profession, a conference on Plastic Design of Multi-Story Frames was held at Lehigh in 1965. At this time a set of lecture notes and design aids was distributed, containing the theoretical basis and the techniques developed for the plastic design of multi-story frames (1,2). There was a good representation at this conference from foreign countries. Numerous delegates from abroad participated as speakers in a special lecture series organized as a part of the conference (69).

In addition to the lecture notes a number of recently published books deal in parts, if not exclusively, with the plastic theory of structural analysis and design (70-75). In the United States, the AISI in collaboration with AISC, has recently published a manual dealing exclusively on the plastic design of braced multi-story frames (76).

The 1965 Summer Conference marks the beginning of the complete application of the plastic theory to the design of high-rise building in the United States. Already three major buildings, namely, the Stevenson Apartments (77), the Phillips Building, and the Hungerford Plaza, all in Maryland (see Table 1), have been built based on the design methods presented in the Summer Conference lecture notes. It is understood that several more braced multi-story frames are now in the design stage.

Elsewhere in the world, there is an increasing trend toward a recognition of the plastic theory in practice, as reflected in the specifications of several countries. Table 2 summarizes the extent of the applicability of the plastic
method of design in some countries. This is the result of a recent survey conducted in connection with the Commentary revision. In addition to the United States as mentioned earlier, the United Kingdom and Canada also have multi-story buildings which were designed by the plastic method. Not less than ten countries (see Table 2) have or will have building specifications that formally approve the use of plastic technique for designing steel structures. Among these countries are the United States, Australia, Belgium, Canada, Czechoslovakia, Hungary, India, Japan, the United Kingdom and Yugoslavia.

It is believed that more extensive recognition of the plastic method of design in other countries will be forthcoming. Evidence of this is the recent formation of a European Task Group on Plastic Design whose objective is to work out a common specification for the European countries.

DESIGN PHILOSOPHY

During its early development one of the major arguments in favor of plastic design was the simplicity it brought to the design process. In contrast with the trial-and-error method frequently required for allowable-stress design, "direct design" was possible for indeterminate structures. The continuity condition gave way to the mechanism condition. Plastic design was a method based on the ultimate load--a load which corresponded to the formation of a mechanism--a load termed the "plastic limit load".

But in a multi-story frame the ultimate load may not correspond to the plastic limit load. Especially in an unbraced frame, the structure may become unstable prior to reaching this limit condition, and it begins to unload before all the plastic hinges have formed that would be involved in a mechanism. This is illustrated in Fig. 1. According to first order theory the limit of usefulness is the plastic limit load. When second-order effects are taken into account the limit of usefulness is the stability limit load.
This does not mean that one must abandon maximum load as the design criterion for multi-story frames nor does it mean that one cannot utilize the plastic strength of steel in design. True, some of the simplicity is lost because certain of the design checks require a consideration of the continuity condition. But for these cases charts have been developed \((49)\), and more recently, computer programs have been prepared to make the resulting design process a viable one \((50)\).

Rather than having one maximum (or ultimate) load, there are two: the plastic limit load (associated with a mechanism) and a somewhat lower "stability-limit load". Depending on the type of structure, these are the appropriate limits of structural usefulness (based on strength).

This discussion gives rise inevitably to the question of terminology. Should the term "plastic design" be retained? A study of the definition contained in the first edition of the Commentary shows a consistent logic. Further plastic design is a unique term well known throughout the world as a method of design of steel frames based on maximum load-carrying capacity. Therefore the designation should be kept, albeit with a more precise definition of several terms. The following are provided for consistent terminology:

1. **Plastic Design**: A design method for continuous steel beams and frames which defines the limit of structural usefulness as the "maximum load". (The term, "plastic" comes from the fact that the maximum (or ultimate) load is computed from a knowledge of the strength of steel in the plastic range).*

2. **Plastic Limit Load**: The maximum load obtained when a sufficient number of yield zones have formed to permit the structure to deform plastically without further increase in load. It is the largest load a structure will support when perfect plasticity is assumed and when such factors as instability, strain hardening, and fracture are neglected.*

*These are essentially the same definition as included in the 1961 edition of the Commentary.
3. Stability Limit Load: The maximum load a structure can support when second order instability effects are taken into account.

As pointed out in Ref. 1, the general all inclusive term is "maximum load design". Plastic design is that aspect of maximum load design as applied to steel frames considering maximum component strength and a plastic analysis based either on the plastic limit load or the stability limit load.

Load-Factor Design

Another recent development is a load-factor design—a method of proportioning structural members for multiples of service loads. In this method the maximum design load is obtained by multiplying the service loads by a load factor; due account is taken of deflections, fatigue, stability, and other secondary design considerations. The limit of usefulness can be either the elastic limit, the stability limit, or the plastic limit load. It is used to a certain extent in concrete design, and it is gaining increasing attention in the United States and in Europe. A recent article in this country in the field of bridges is Ref. 78. One senses in Europe a significant shift away from allowable-stress design and toward load-factor design. (In Europe it is mostly termed "limit design"—not to be confused with limit design of reinforced concrete as applied in the U.S.A.).

In France the process of change to this design approach has been gradual. At first there were two specifications, and the designer could use either allowable-stress design or "limit design". It is understood that currently most building designs are carried out on a limit design basis.

Through the efforts of the European Convention of Constructional Steelwork Associations, involving 12 nations of Europe *, studies are currently underway for the uniform application of load-factor design for buildings. This study is in addition to the work of the European committee on Plastic Design previously mentioned. Austria adopted limit design in 1963.

*Austria, Belgium, Britain, France, Germany, Holland, Italy, Norway, Spain, Sweden, Switzerland, Yugoslavia.
The advance in design philosophy as expressed in the Czechoslovakian specifications is illustrative. All building designs there are carried out on a load factor basis. Their specifications are divided into two separate documents: a specification for loads and a specification for limit (or resistance) conditions. A loading function is determined from the one, a resistance function is determined from the other, and the two are equated in the design process.

The specification for loads ("Loading of Building Structures" CSN 730035) includes the different construction materials. It describes the categories of dead loads and of live loads ("long-term", "short-term", and "extraordinary"). It specifies the load factors to be used in each instance. Its provisions are quite detailed, including some 17 different categories of live loads. There are provisions for auto and truck loads, for cranes, for snow loads, for wind loads, for earthquakes. There is provision for the case when the live load is in the reverse direction of the dead load. In general the load factor is lower for dead load than for live load.

There is a separate specification for the resistance function ("Design of Steel Structures" CSN731401). This specification covers the two major limit conditions--adequate strength, and adequate deformation control. Under the strength provision the limit is usually the elastic limit, but the specification also includes a strength limit expressed in terms of the maximum plastic strength. It covers provisions for various steels. It includes limit stresses for combined bending, torsion, and axial thrust. It specifies the values of the resistance function, R, expressed in terms of yield point.

The net result of this separation of the loading function and the resistance function--which is the essence of load-factor design--is a savings of material when the structure is under high dead load. In some cases there is a minor increase in required material when the live load is high. It also
requires consideration of more load combinations because of the multiple load factor aspect.

However, the use of load-factor design permits one to take into account in an orderly way the differing load factors that should be applied to the different kinds of loads that can act on a structure and therefore leads to a more rational design. It also opens a way by which the designer can take advantage of the application of static analysis of the various factors that influence design—as these techniques become more and more available.

It is not the function of this article to deal in any depth with current allowable-stress design practice. However, research in the plastic behavior of structures already has opened the way to many design advances in the allowable-stress method (in improving the resistance function, for example). Further improvements might be possible by substituting load-factor design for allowable-stress design, and perhaps this should be examined in this country as has been done abroad—with due regard being given to the increase in design complexity. The load factor approach might well enhance the design of steel structures whose selected or assumed limit of usefulness is not the maximum load but a limit load arrived at through an elastic analysis. The concept of using multiple load factors could be applied in plastic design as well since the plastic method is, in fact, a load-factor design technique. It would be a parallel application.

REVISIONS IN THE PLASTIC DESIGN COMMENTARY

An Ad Hoc Committee consisting of thirty-five members was formed by ASCE in July 1967 to revise the 1961 edition of the Commentary on Plastic Design in Steel. The membership list is given in Table 3. This second edition of the Commentary is prepared under the auspices of the Subcommittee of Welding Research Council and of the ASCE Structural and Engineering Mechanics Divisions.
The entire membership of these three major committees is also given in Table 3.

At the first committee meeting (July 21, 1965 in New York City) it was agreed that the scope of the revision to the Commentary would be along the following lines:

1. The revision is to be a modest one.
2. The basic approach is still the simple plastic theory but with modifications where necessary to extend its applicability.
3. The Commentary is expanded to include braced multi-story frames.
4. Steels with a well-defined yield plateau are considered. The upper limit of the yield stress is 65 ksi.
5. The scope is limited to planar structures only.
6. Primarily static loading is considered. However, some attention is given to repeated loading effects.

Having established this scope, the committee went forward with the revisions and had, within a year, completed revisions of the first eight chapters. The remaining two chapters were revised or drafted shortly thereafter. Presently (August, 1968) the revision and drafts are being reviewed and subsequently will be submitted to the entire Joint Committee.

The second edition will have a new chapter on Multi-story Frames. Because of their increasingly important role in the plastic method of designing steel structures, three new articles have been added as follows: 1) the role of strain hardening, 2) moment balancing method, and 3) column deflection curves.

Table 4 outlines the major changes in the Commentary. Article 2.3 on "Strain-hardening" has been added because of its importance in assuring that the structure will remain stable and its relevance in stability solutions. Article 3.4 (Moment Balancing) is essential for the plastic design of multi-story frames. In Chapter 4, the scope is expanded to cover tall buildings.
The article on "Materials" is expanded to include A36, A441, and A572 (Grade 65) Steels. Formulas are given for computing the plastic moment of composite concrete and steel beams. Load factors are modified as a result of increased knowledge about stability problems (1). The load factors in use in different countries are included in the Commentary and the latest available information is contained in Table 5.

Results of recent tests on high-strength (A441 and A572) members and frames are included in Chapter 5. The major additions are the test results of five multi-story frames, one clad frame and on hybrid frame. In addition to the test results of structures, the revised Chapter 5 will have stress-strain relationships for the three steels mentioned above.

Some portions of Chapter 6 have been revised substantially. In Art. 6.1 (Shear Force) there is no change in the recommendation, but a revision in the derivation of the interaction equations has been made. Major changes have been made in Arts. 6.2 and 6.3 on the phenomena of local and lateral buckling of beams. The original article on "Local Buckling" resulted from a theoretical solution using an orthotropic plate model. The revised article is essentially based on the torsional buckling of a plate element (11). The present Art. 6.3 on "Lateral Buckling" is being expanded to include the latest work of Lay and Galambos (12,13,14). In Art. 6.4 (Variable Repeated Loading) results of recent tests on steel members and frames subjected to repeated inelastic strains of large magnitude are included.

Article 7.4 "Rotation Capacity" has been replaced by the new article "Column Deflection Curves" because of the latter's importance in the design of beam-columns in multi-story frames. The column deflection curve concept provides not only the means to determine the ultimate strength of columns but also their rotation capacity. It is now possible to provide formulas for
checking the possible occurrence of lateral-torsional buckling for laterally unbraced beam-columns. Article 7.6 "Frame Stability" has been moved to Chapter 10.

Article 8.6 "Details with Regard to Welding" has been revised substantially. There are new recommended design values for fillet welds. Article 8.8 "Details with regard to bolting" has been expanded and a formula for computing the prying force in a fastener is also given. Design values for bolts under tension, shear, or combined tension and shear are also included.

Possible design guides to limit deflection have been added to Chapter 9. "Multi-Story Frames" is the new chapter. It will contain a detailed description of the techniques available for designing braced multi-story frames, and some discussion is also included of unbraced multi-story frames under gravity and combined loads.

**REVISIONS IN AISC SPECIFICATION**

Concurrently with the changes in the Commentary, a revision of the AISC Specification has been under study. This work is also nearing completion, and the major changes in Part 2 are summarized in Table 6, subject, perhaps, to minor revisions since the Specification has not yet been finally adopted. The changes are compatible with the revisions currently prepared for the Commentary.

Briefly, the scope is expanded to permit the use of the plastic method for designing braced multi-story frames. Load factors of 1.85 and 1.40 have been reduced to 1.70 and 1.30 respectively, and steels of yield stress levels in the range of 36 to 65 ksi are permitted. A new section on vertical bracing system has been added. In lieu of tabulated values of B, G, H and J, columns can now be designed with a formula similar to Formula 7a in the present specification. The section on web crippling is moved to Part 1 of the Specification.
The maximum permitted b/t ratio has been revised so that the new specification will have a list of maximum permissible b/t ratios for the different grades of steel. The permitted depth-to-thickness ratio of beam and girder webs has also been revised. The new specification will have \( \frac{d}{w} \) ratio for the case of \( \frac{P}{p} \leq 0.27 \) and that of \( \frac{P}{p} > 0.27 \). Finally the revised specification will have new lateral bracing rules for beams under moment gradient and uniform moment.

Another revision—this one in procedure—will be the issuing of annual supplements to the AISC Specification, a procedure that will permit even more rapid incorporation of research results into design.

**FUTURE NEEDS**

**Research**

The needs in research for the decade 1966-75 were outlined by the ASCE Structural Division Committee on Research in its recent survey (79). In the section on plastic design, that report touched briefly on extensions to space frameworks and to multi-story frames; and in both of these areas work is underway. In a forthcoming ASCE manual, the current research projects in plastic design will be included as part of a larger survey, which is an updating of a similar listing published in 1965 (80).

Since nearly all research in steel structures is concerned with a more complete exploration of the entire load-carrying range, a complete list of needed solutions would be repetitive and prohibitively long. However, in addition to current efforts already mentioned, the following areas of study appear to be of particular present importance:

1. More precise determination of spacing of lateral supports. The present provisions seem too conservative in some cases—so much so that in a few instances a plastic design could be less economical than the corresponding allowable-stress design. This contradiction points to the need for a more complete study of the problem.
2 Bracing requirements. The question of what constitutes adequate lateral support is still not completely answered.

3. Local buckling as it interacts with lateral buckling. Further study may well lead to more liberal provisions than those presently available -- values that now place severe restriction on the applicability of the method to the higher strength steels.

4. An evaluation of rotation requirements. This is a major study of the amount of inelastic rotation required for different portions of a structure. It could well lead to a greater bracing spacing and also to larger limiting width-thickness ratios for flange and web.

5. More complete application of composite action. Local buckling limitations are involved here because of the greater rotations that now seem to be required at hinges in the negative moment region in this form of construction.

6. Box-shaped members. More information is needed with respect to bracing, local buckling, and crippling.

7. Post buckling behavior. Some reliance is already placed on post buckling strength, but further advances might be significant as a result of a more intensive study related to local, lateral, and general post-buckling strength.

**Design**

Some of the future opportunities for studies of the design process have already been mentioned under "DESIGN PHILOSOPHY". Some additional needs are the following:

1. The magnitude of the load factor and its uniformity for essentially identical conditions.

2. The magnitude of the loads that should be assumed in design.

3. Appropriate statistical designation of the yield stress level for various grades of steel.

It is of interest to note that these topics will be dealt with, among other items, at the forthcoming IABSE Symposium on Safety of Structures to be held in London in 1969.
The continued close cooperation of research workers, designers and specification writing groups will be important in assuring the further development and application of the plastic design technique. In addition continued international cooperation will be particularly advantageous.

SUMMARY

This review of recent developments in plastic design practice has shown that:

1. Considerable new information has become available in the last decade, not only in the area of research results concerning the inelastic behavior of steel structures, but also in experience coming out of design applications.

2. This new information is being used by the design profession, both with respect to plastic design and in allowable-stress design and in design to withstand earthquakes.

3. There is increased interest in the significance of load factors, in their magnitude, and in the possible use of multiple load factors in design.

4. The second edition of the Plastic Design Commentary, due to be completed this year will reflect the latest knowledge. Specifications are reflecting these changes to a certain extent in the AREA, but especially in the AISC Specification, currently being revised.

5. Areas of additional study have been identified that will make possible further improvements in the design of steel structures. Topics such as local and lateral buckling, inelastic rotation requirements, composite and box members, space frameworks, and evaluation of structural safety can all profit from additional examination. These studies will make possible a yet more complete utilization of the strength of steel in the plastic range.

ACKNOWLEDGMENTS

The work that led to this paper was conducted as part of a general study on "Design Recommendations for Multi-Story Frames" which is being
carried out at the Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University. The study is sponsored by the American Iron and Steel Institute and the Welding Research Council. The purpose of this study is to develop new or improved design recommendations for building frames, utilizing the results of research that has recently been completed or is currently underway. Technical advice to this study is provided by the Lehigh Project Subcommittee of the Structural Steel Committee of the Welding Research Council. Dr. T. R. Higgins is Chairman of the Lehigh Project Subcommittee.

Much of the information presented in this paper was obtained as background material for revising the ASCE Manual "Commentary on Plastic Design in Steel". The authors gratefully acknowledge the help they received from many members of the Ad Hoc Committee which was organized by the ASCE to revise the Commentary. A number of overseas members and representatives supplied the information presented in Tables 2 and 5 and in the section on "DESIGN PHILOSOPHY". Additional material was received from many of the authors' present and former colleagues at the Fritz Laboratory. The authors also would like to thank the AISC for permission to publish the material contained in Table 6.

The entire manuscript was typed with care by Mrs. D. Fielding and Miss K. Philbin. Their cooperation is appreciated.
### TABLE 1

**SOME MULTI-STORY FRAMES DESIGNED BY PLASTIC THEORY IN U.S.A.**

<table>
<thead>
<tr>
<th>Year</th>
<th>Identification</th>
<th>Tons, Struct Steel</th>
<th>Cost/sf</th>
<th>Stories</th>
<th>Bays</th>
<th>Spans</th>
<th>Design Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>Tower Building, Little Rock, Arkansas</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1956 AISC Proceedings</td>
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<tr>
<td>1967</td>
<td>Stevenson Apts., Maryland</td>
<td>369, 6.3</td>
<td>$1.17</td>
<td>11</td>
<td>15</td>
<td>3</td>
<td>1963 Spec. Part Two</td>
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<tr>
<td>1968</td>
<td>Phillips Building, Maryland</td>
<td>340, 8.6</td>
<td>$1.42</td>
<td>11</td>
<td></td>
<td></td>
<td>1965 Summer Conference Lecture Notes</td>
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<tr>
<td>1967</td>
<td>Hungerford Plaza, Maryland</td>
<td>168, 6.9</td>
<td>$1.21</td>
<td>4 (Office Building)</td>
<td></td>
<td></td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60, (O.W.J.)</td>
<td></td>
<td>1 (Shopping Complex)</td>
<td></td>
<td></td>
<td>&quot;</td>
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</table>

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## TABLE 2

<table>
<thead>
<tr>
<th>Country</th>
<th>Low Building Design</th>
<th>Multi-Story Frames</th>
<th>Plastic Design Specification</th>
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<tr>
<td>U. S. A.</td>
<td>Extensive application</td>
<td>A few</td>
<td>AISC - Part 2</td>
</tr>
<tr>
<td>Australia</td>
<td>Used for portal frames</td>
<td>Aware of none</td>
<td>A.S. CA1 SAA, 1968</td>
</tr>
<tr>
<td>Belgium</td>
<td>Little application</td>
<td>Aware of none</td>
<td>Addendum to NBN1 (detailed spec.)</td>
</tr>
<tr>
<td>Canada</td>
<td>Extensive application</td>
<td>A few</td>
<td>CISC, 1967</td>
</tr>
<tr>
<td>Czechoslovakia</td>
<td>Aware of none</td>
<td>Aware of none</td>
<td>CSN 73-1401(66) (general provisions)</td>
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<tr>
<td>France</td>
<td>None</td>
<td>None</td>
<td>Not yet</td>
</tr>
<tr>
<td>Germany</td>
<td>Aware of none</td>
<td>Aware of none</td>
<td>Not yet</td>
</tr>
<tr>
<td>Hungary</td>
<td>Under current consideration</td>
<td></td>
<td>Hungarian Design Code (draft form)</td>
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<tr>
<td></td>
<td>(most buildings are of</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>reinforced concrete)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>A few</td>
<td>Aware of none</td>
<td>I.S. 800</td>
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<tr>
<td>Japan</td>
<td>Aware of none</td>
<td>Aware of none</td>
<td>Recommendations in draft form</td>
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<td>Switzerland</td>
<td>Aware of none</td>
<td>Aware of none</td>
<td>Not yet</td>
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<tr>
<td>United Kingdom</td>
<td>Nearly every portal frame</td>
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<td>BSS 449 (general provisions)</td>
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<td>Yugoslavia</td>
<td>Some</td>
<td>Aware of none</td>
<td>Under study</td>
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<td>Europe</td>
<td>Task Group on Plastic Design is working on draft of a specification</td>
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### Table 3

#### Ad Hoc and Joint Committees

**Ad Hoc Committee**

| H. Allison | Y. Fujita | L. C. Lim |
| J. F. Baker | T. V. Galambos | L. W. Lu |
| S. C. Batterman | K. Gerstle | C. Massonnet |
| E. R. Bryan | J. A. Gilligan | B. M. McNamee |
| W. F. Chen | W. J. Hall | W. H. Munse |
| J. H. Daniels | W. C. Hansell | A. Ostapenko |
| G. C. Driscoll, Jr. | J. Heyman | E. P. Popov |
| D. C. Drucker | T. R. Higgins | D. Sfintesco |
| T. C. Fan | M. R. Horne | B. Thurlimann |
| M. Fialkow | B. C. Johnston | M. Wakabayashi |
| J. W. Fisher | M. G. Lay | L. S. Beedle, Chairman |
| G. F. Fox | S. L. Lee | |

**Joint Committee (Second Edition)**

<table>
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<tr>
<th>Structural Steel Committee</th>
<th>Engineering Mechanics Division</th>
<th>Structural Division</th>
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<td>Lehigh Project Subcommittee</td>
<td>Committee on Plasticity Related to Design</td>
<td>Committee on Plastic Design</td>
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<tr>
<td>J. A. Adams</td>
<td>Alfredo H. S. Ang</td>
<td>P. F. Adams</td>
</tr>
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<td>A. Amirikian</td>
<td>R. E. Ball</td>
<td>Lynn S. Beedle</td>
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<td>L. W. Lu, Chairman</td>
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<td></td>
<td>Theodore G. Toridis</td>
<td></td>
</tr>
<tr>
<td></td>
<td>George Winter</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ching-Yi Yang</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kurt H. Gerstle, Chairman</td>
<td></td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>----------------------</td>
<td>----------------------------------------------------------</td>
</tr>
<tr>
<td>2.3 Role of Strain Hardening</td>
<td>Not included</td>
<td>New Article added</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Increased importance of its role)</td>
</tr>
<tr>
<td>3.4 Moment Balancing Method</td>
<td>Not included</td>
<td>New Article added</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Important for Multi-story frames)</td>
</tr>
<tr>
<td>4.2 Types of Construction</td>
<td>Low buildings and continuous beams in braced multi-story frames</td>
<td>Expanded to include full plastic design of braced multi-story frames</td>
</tr>
<tr>
<td>4.3 Material</td>
<td>Only A7 steel (33 ksi)</td>
<td>A36, A441, A572 (36 ksi to 65 ksi)</td>
</tr>
<tr>
<td>4.6 Plastic Moment</td>
<td>For steel sections only</td>
<td>Expanded to cover composite section strength in simple beams</td>
</tr>
<tr>
<td>4.8 Load Factors</td>
<td>Gravity 1.85</td>
<td>Gravity 1.70</td>
</tr>
<tr>
<td></td>
<td>Gravity + Wind 1.40</td>
<td>Gravity + Wind 1.30</td>
</tr>
<tr>
<td>5.1 Basic Concepts</td>
<td>Properties of A7</td>
<td>Properties of new steels</td>
</tr>
<tr>
<td>5.3 Frames</td>
<td>Tests using A7 steel</td>
<td>Added: Structures tested since first edition</td>
</tr>
<tr>
<td>6.1 Shear Force</td>
<td>$V = 18,000 , \text{wd}$</td>
<td>Revised derivation of interaction equation</td>
</tr>
</tbody>
</table>

\[ V = \frac{\sigma}{\sqrt{3}} \, w \, d \]
## Major Changes in Commentary (continued)

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>6.2 Local Buckling</td>
<td>Theory was based on orthotropic plate model</td>
<td>New theory uses a shear strain-hardening modulus based on discontinuous $\sigma - \varepsilon$ relationship. Also considers moment gradient</td>
</tr>
<tr>
<td></td>
<td>$b/t = 17$</td>
<td>$b/t = \frac{3.6}{\sqrt{\frac{\varepsilon}{\sigma_y}}} \sqrt{\frac{1}{(3 + \frac{\sigma_{0u}}{\sigma_y})(1 + \frac{E}{5.2E_{st}})}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d/w = 43 \sqrt{\frac{36}{\sigma_y}}$ when $P/P_y \leq 27$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d/w = 70 - 100P/P_y$</td>
<td>$= 70 - 100 \frac{P}{P_y}$ when $P/P_y &gt; 27$</td>
</tr>
<tr>
<td></td>
<td>(See also AISC Spec., Table 5 Sect. 2.7)</td>
<td></td>
</tr>
<tr>
<td>6.3 Lateral Buckling</td>
<td>Solution based on single model</td>
<td>Added an additional model:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recognized two cases:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>moment gradient</td>
</tr>
<tr>
<td></td>
<td></td>
<td>uniform moment</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Method for computing Rotation Capacity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lateral Bracing Requirements</td>
</tr>
<tr>
<td>---------</td>
<td>---------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>6.4 Variable Repeated Loading</td>
<td>Brief description</td>
<td>Added Part E - repeating inelastic strain of large magnitude (important for building frame subjected to earthquake vibration)</td>
</tr>
<tr>
<td>7.4 Rotation Capacity Column Deflection Curves</td>
<td>CDC's not treated</td>
<td>New article replaced the old article. (Important for the design of beam-columns in multi-story frames)</td>
</tr>
<tr>
<td>7.5 Influence of Lateral-Torsional Buckling</td>
<td>Brief description</td>
<td>Suggested equation:</td>
</tr>
<tr>
<td>8.7 Details with regard to Welding</td>
<td>Fillet: $\tau_f = \frac{\sigma_v}{\sigma_w} \times $ allowable stress</td>
<td>Butt: (no change) Develop tensile yield of base material Fillet: $\tau_f = 0.5 \sigma_u$ Nominal Tension = $0.56 \times$ tensile strength on stress area Shear = $0.45 \times$ tensile strength on stress area Interaction formula presented Prying force: $Q = \left[ \frac{3b}{8a} - \frac{3}{20} \right] F$</td>
</tr>
<tr>
<td>10.5 Frame Stability</td>
<td>$2 \frac{P}{P_y} + \frac{L}{70r} \leq 1.0$</td>
<td>General treatment presented</td>
</tr>
</tbody>
</table>

*See ERANTA SHEET*
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>9.7 Deflection</td>
<td>Methods for computing deflection and rotation capacity</td>
<td>Added design guides to limit deflection: $L = \frac{882}{d \sigma_y}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discussed computer applications</td>
</tr>
<tr>
<td>Ch. 10 Multi-Story Frames</td>
<td>Not treated</td>
<td>New chapter added.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Applicability of plastic method of design to multi-story frames now available)</td>
</tr>
<tr>
<td>Glossary</td>
<td>Ultimate load</td>
<td>Plastic Limit Load (Analysis)</td>
</tr>
<tr>
<td></td>
<td>Ultimate load</td>
<td>Design Ultimate Load (Design)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stability Limit Load (Analysis)</td>
</tr>
</tbody>
</table>
**TABLE 5**

LOAD FACTORS FOR PLASTIC DESIGN IN VARIOUS COUNTRIES

<table>
<thead>
<tr>
<th>Country</th>
<th>Assumed Shape Factor</th>
<th>Dead Load + Live Load</th>
<th>Dead Load + Live Load + Wind or Earthquake Forces</th>
<th>No. of Load Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
<td>1.12</td>
<td>1.70</td>
<td>1.30</td>
<td>2</td>
</tr>
<tr>
<td>Australia</td>
<td>1.15</td>
<td>1.75</td>
<td>1.40</td>
<td>2</td>
</tr>
<tr>
<td>Belgium</td>
<td>1.12</td>
<td>1.68</td>
<td>1.49 (1.12 for extreme wind)</td>
<td>3</td>
</tr>
<tr>
<td>Canada</td>
<td>1.12</td>
<td>1.70</td>
<td>1.30</td>
<td>2</td>
</tr>
<tr>
<td>Czechoslovakia</td>
<td>1.20 (max.)</td>
<td>D: 1.10f - 1.30f</td>
<td>L: 1.30f - 1.40f</td>
<td>17</td>
</tr>
<tr>
<td>Hungary*</td>
<td>1.05</td>
<td></td>
<td>Proposal 1: (single load factor) 1.2 - 1.5 depending on combinations of D, L, and I</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Proposal 2 (multiple load factor) Many possible combinations</td>
<td>4</td>
</tr>
<tr>
<td>India</td>
<td>1.15</td>
<td>1.85</td>
<td>1.40</td>
<td>2</td>
</tr>
<tr>
<td>Japan*</td>
<td></td>
<td>[1.2D + 2.1L or 1.4(D+L)] (normal condition)</td>
<td>[1.2(D+L) + 1.5S₂] (under snowfall)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(D+L) + 1.5K; (D+L+S₂) + 1.5K (under earthquake)</td>
<td>(D+L) + 1.5W₁ (under typhoon)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(D+L+S₂) + 1.5W₂ (under whirlwind)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.15</td>
<td>1.75</td>
<td>1.40</td>
<td>2</td>
</tr>
<tr>
<td>Yugoslavia*</td>
<td>1.12</td>
<td>D = 1.49, L = 1.68+ Additional Combinations</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Under study

D Dead Load | I Irregular Live Load | S₂ Mean Snow Load
L Live Load | f Shape Factor        | W₁ Wind Force (under typhoon)
K Earthquake Force | S₁ Maximum Snow Load | W₂ Wind Force (under whirlwind)
**TABLE 6**

**AISC REVISIONS PART 2**

<table>
<thead>
<tr>
<th>Section</th>
<th>1963</th>
<th>1968</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.1 Scope</strong></td>
<td>Plastic design of continuous beams in multi-story frames</td>
<td>Plastic design in braced multi-story frames</td>
</tr>
<tr>
<td><strong>Load Factor</strong></td>
<td>D.L. + L.L. 1.85</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>D.L. + L.L. + Wind or Earthquake 1.40</td>
<td>1.30</td>
</tr>
<tr>
<td><strong>2.2 Structural Steel</strong></td>
<td>A7, A373, A36</td>
<td>A36, A242, A441, A529, A572, A588 (up to 65 ksi)</td>
</tr>
<tr>
<td>(not included)</td>
<td></td>
<td>New section added</td>
</tr>
<tr>
<td><strong>2.3 Vertical Bracing System</strong></td>
<td>Column strength in terms of B, G, H, J</td>
<td></td>
</tr>
<tr>
<td><strong>Frame Stability</strong></td>
<td>$\frac{P}{2P_y} + \frac{L}{70r} \leq 1.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use K factor in the above formula</td>
</tr>
<tr>
<td>(not included)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2.4 Columns</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2.5 Shear</strong></td>
<td></td>
<td>No major changes</td>
</tr>
<tr>
<td><strong>2.6 Web Crippling</strong></td>
<td></td>
<td>No major changes</td>
</tr>
</tbody>
</table>

-32-
<table>
<thead>
<tr>
<th>Section</th>
<th>1963</th>
<th>1968</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Minimum</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.7 Thickness</td>
<td>$b \leq 8.5$ (A36)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{b}{2t} &lt; 8.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{d}{w} = 410 \left(1 - 1.4 \frac{P}{P_y}\right)$ when $\frac{P}{P_y} \leq 0.27$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{d}{w} = \frac{255}{\sqrt{F_y}}$ when $\frac{P}{P_y} &gt; 0.27$</td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Connections</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.8 Lateral Bracing</td>
<td>$\kappa = (60 - 40 \frac{M}{M_p}) r_y$</td>
<td>$\kappa = \frac{1375}{F_y}$ when $\frac{M}{M_p} \geq 0.5$</td>
</tr>
<tr>
<td></td>
<td>$\kappa = 35 r_y$</td>
<td>$\kappa = \frac{1375}{F_y} + 25$ when $\frac{M}{M_p} &lt; 0.5$</td>
</tr>
</tbody>
</table>
Fig. 1 RESULTS OF AN UNBRACED FRAME TEST
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