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Preliminary

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A DIRECT METHOD FOR THE ESTIMATION
OF PRESTRESS LOSSES

by
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A DIRECT METHOD FOR THE ESTIMATION OF PRESTRESS LOSSES

Ti Huang

Abstract

A rational method for the estimation of prestress losses in pre-tensioned concrete members is presented. The method is based on the linking of experimentally developed stress-strain-time relationships of the steel and concrete materials. It enables a direct determination of stress and strain distributions in a member at any time within the service life of the member, and avoids the need for step-by-step accumulation. Wide ranges of variation for the concrete material characteristics and other design parameters are permitted.

The new method is illustrated by a practical design problem. Comparisons with two other recently proposed procedures show good agreement of results.

A DIRECT METHOD FOR THE ESTIMATION OF PRESTRESS LOSSES

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Introduction

The method for the estimation of prestress losses in prestressed concrete members varies widely in different specifications. At one time, a simple provision of a flat percentage or a flat value was used in many design codes (1,2). On the other hand, there exist today complicated procedures which involve the use of numerous equations, tables and charts, and require a step-by-step method of calculation (3,4). Neither extreme is satisfactory to the design engineers who desire to have a method which is simple, accurate, and flexible to accommodate variations of the numerous design and fabrication parameters of prestressed concrete members.

In this paper is presented a rational method which enables a direct and accurate prediction of prestress losses throughout the service life of the member. Variations in material properties, geometrical design and fabrication schedule are allowed for and step-by-step calculations are avoided. In the present form, this new method applies only to pretensioned members, but the basic concept involved is equally valid for post-tensioned members.

To facilitate discussion, prestress is defined to be the stress in steel or concrete, when all external loads, including the weight of the member, are temporarily and instantaneously removed. Consequently, the actual stress under a loaded condition is the sum of prestress and the stress caused by all prevailing loads. For the loss of prestress, reference is made to the

initial steel stress after anchorage. Therefore, the prestress loss for a pretensioned member includes the effects of elastic shortening, shrinkage, creep and relaxation. But friction and anchorage losses are not considered.

Basic Concept

The basic concept of the new method involves the use of stress-strain-time relationships to represent the elastic as well as the long-term rheological behavior of the steel and concrete materials. In the most general form, these relationships are given by the following equations:

$$f_s = f(\epsilon_s, t_s) = f_{s,el} - f_{s,rel} \quad (1)$$

$$\epsilon_c = g(f_c, t_c) = \epsilon_{c,el} + \epsilon_{c,sh} + \epsilon_{c,cr} \quad (2)$$

Eq. 1 shows the steel tensile stress f_s as a function of steel strain ϵ_s and time after tensioning t_s , and as the difference between the elastic stress $f_{s,el}$ and relaxation loss $f_{s,rel}$. Similarly, the concrete compressive strain ϵ_c is shown in Eq. 2 as a function of concrete fiber stress f_c and time after end of curing t_c . It is the sum of the elastic, shrinkage and creep components. Here it is assumed that transfer of prestress occurs immediately upon end of curing, hence shrinkage and creep are controlled by the same time parameter.

For a pretensioned concrete member, the stress-strain-time relationships of the concrete and steel materials are linked by three sets of conditions:

1. Time compatibility

$$t_s - t_c = k_1 \quad (3)$$

2. Strain compatibility, at the location of each prestressing strand

$$\epsilon_s + \epsilon_c = k_2 \quad (4)$$

3. Equilibrium conditions over the cross section

$$\int f_c dA_c - \sum f_s a_{ps} = P \quad (5)$$

$$\int f_c x dA_c - \sum f_s x a_{ps} = -M \quad (6)$$

In the above linking conditions,

k_1 = Time interval from tensioning of steel to transfer of prestress
(this includes the time for form setting, casting, and curing)

k_2 = Initial tensioning strain in steel

A_c = Area of net concrete section

a_{ps} = Area of individual prestressing element

x = Distance to elementary area from the centroidal horizontal axis

P = Thrust on section, caused by external loads

M = Bending moment on section caused by external loads

The positive directions of x , P and M are shown in Fig. 1. In Eqs. 5 and 6 the integrations are over the entire net concrete area, and the summations are over all pretensioning elements. All of the quantities defined for Eqs. 3 to 6 are design or fabrication parameters and are known or specified for the estimation of prestress losses. Thus, the Eqs. 1 through 6 represent a set

of six conditions for the two time variables t_s and t_c , and the four stress and strain variables (ϵ_s , ϵ_c , f_s and f_c) which are functions of the location parameter x . A reasonable assumption was made that the concrete stress varies linearly across the section

$$f_c = g_1 + g_2 x \quad (7)$$

With this condition added, there are now sufficient equations for all unknowns to be evaluated for any given time, i.e., the time-variations of the stresses and strains can be determined. Thus, once the member design and the initial conditions (k_1 and k_2) are known, a complete solution of the stress and strain distribution can be obtained by repeatedly solving the equation set 1 through 7, for different values of time. It is important to note that for any specified time, the solution is direct and not dependent upon the solution at preceding times. Thus, step-by-step accumulation is not needed. It should also be pointed out that the stresses f_s and f_c in the aforementioned equations include the effects of applied loads, and therefore are not the prestresses as defined earlier. By definition, the steel prestress and the prestress loss are evaluated by the following equations:

$$f_p = f_s - f_{s\ell} \quad (8)$$

$$\Delta f_p = f_{si} - f_p = f_{si} - f_s + f_{s\ell} \quad (9)$$

where: f_p = Steel prestress

$f_{s\ell}$ = Steel stress caused by applied loads including member weight
and all permanent loads

Δf_p = Loss of prestress

f_{si} = Initial steel stress immediately upon anchorage

Stress-Strain-Time Relationships

The functions f and g in Eqs. 1 and 2 were developed experimentally based on observations on the elastic, relaxation, creep and shrinkage behaviors of simple steel and concrete specimens. Steel relaxation data were obtained from strand specimens tested in fixed length loading frames under various initial tensile stresses. For information on concrete strains, concentrically pretensioned rectangular concrete specimens were used, in conjunction with similar specimens containing untensioned strands.

In the selection of time functions for regression analyses of the relaxation, shrinkage and creep data, special emphasis was placed on the suitability of these functions for extrapolation, since long term projection based on short term observations would undoubtedly be necessary. For this purpose, analyses were made using data covering different periods of time, and the projected values at some future time (arbitrarily taken at one hundred years after tensioning) were compared. The lack of sensitivity of the projected "final value" to the amount of experimental data used in the analysis was used as a primary criterion in the selection of the time functions (5). A modified form of the logarithmic function was chosen because of its simplicity and also satisfying this criterion of being "insensitive".

The relaxation loss data from steel strand specimens were first analyzed with respect to time and initial stress. The resulting expression was then combined with the elastic stress-strain relationship to form the stress-strain-time equation. The form of this equation is as the following.

$$f_s = f_{pu} \{A_1 + A_2 \epsilon_s + A_3 \epsilon_s^2 - [B_1 + B_2 \log(t_s + 1)] \epsilon_s - [B_3 + B_4 \log(t_s + 1)] \epsilon_s^2\} \quad (1a)$$

where f_s = Steel stress, in ksi (MN/m²)

f_{pu} = Specified ultimate tensile strength of steel, in ksi (MN/m²)

ϵ_s = Steel strain, in 10⁻²

t_s = Steel time, starting from initial tensioning, in days

The applicability of Eq. 1a is restricted because of the limited test ranges of controlled parameters. These ranges are

$$0.5 \leq f_s / f_{pu} \leq 0.8$$

$$1 \leq t_s \leq 36500$$

The experimental work dealt with 270 K (1860 MN/m²) grade stress-relieved seven-wire strand specimens of both 7/16 in. and 1/2 in. sizes. No significant size effect was found. The values of the regression coefficients are listed in Table 1.

Equation 2 for concrete characteristics was developed in a similar manner by combining expressions representing the elastic, shrinkage and creep strains (6). The elastic strains were directly measured at the time of pre-stress transfer. The shrinkage strain of a prestressed member was defined to be the same as that of plain concrete containing no reinforcement. The creep strain was obtained from the measured total strain by deducting the elastic, shrinkage as well as the elastic rebound strains. Time function for shrinkage and creep strains was selected using the same criteria as used for the relaxation behavior, and coincidentally, the same function was chosen. The functional form of the concrete stress-strain relationship is as following:

$$\begin{aligned} \epsilon_c = & C_1 f_c \\ & + [D_1 + D_2 \log (t_c + 1)] \\ & + [E_1 + E_2 \log (t_c + 1)] + f_c [E_3 + E_4 \log (t_c + 1)] \end{aligned} \quad (2a)$$

where: ϵ_c = Concrete strain, in 10^{-2}
 f_c = Concrete stress, in ksi (MN/m^2)
 t_c = Concrete time, in days, starting from the time of transfer,
 taken as the same as the end of curing period

In the experimental study, two concrete mixes were used, both satisfy the same minimum strength requirements [5.0 ksi ($34.5 \text{ MN}/\text{m}^2$) at transfer and 5.5 ksi ($37.9 \text{ MN}/\text{m}^2$) at 28 days]. Their composition, and manufacturing procedure were sufficiently different, however, that their rheological behaviors differed significantly. Two sets of regression coefficients were developed, therefore, to reflect this wide variation. They are given

in Table 2. The range of applicability of Eq. 2a, on account of the test range of the controlled parameters, is

$$0 \leq f_c \leq 3.3 \text{ ksi (22.8 MN/m}^2\text{)}$$

$$1 \leq t_c \leq 36500$$

Formulation of Procedure

For any specified time, Eq. 1a reduces to a simple quadratic function of ϵ_s , and Eq. 2a is linear in terms of f_c . Their combination with Eqs. 3 through 7 results in a pair of simultaneous quadratic equations in g_1 and g_2 . The solution of g_1 and g_2 then enables the evaluation of steel and concrete stresses and strains over the entire cross section. Note that a general solution in this manner would result in different losses in the several prestressing elements, thus causing a gradual shift of the centroid of prestressing.

For practical purposes, all prestressing steel is usually regarded as concentrated at one point, the c.g.s., for stress calculations. When this simplification is used, the simultaneous quadratic equations can be simplified into one single quadratic equation in terms of the concrete fiber stress at c.g.s., f_{cs} , as follows

$$(R_1 - \beta f'_{cl}) + (R_2 + 1 - \beta) f_{cs} + R_3 f_{cs}^2 = 0 \quad (10)$$

where: f_{cs} = Concrete fiber stress at c.g.s., in ksi (MN/m²)
 (= $g_1 + g_2 e_g$)

f'_{cl} = Nominal concrete fiber stress at c.g.s. caused by applied loads, in ksi (MN/m²)

$$= -\frac{P}{A_g} + \frac{Me_g}{I_g} \quad (\text{tension positive})$$

β = A dimensionless geometrical parameter

$$= \frac{1}{A_{ps} \left(\frac{1}{A_g} + \frac{e_g^2}{I_g} \right)} = \frac{A_g I_g}{A_{ps} (I_g + A_g e_g^2)}$$

where: A_g = Area of gross cross section, in sq. in. (sq. cm.)

I_g = Moment of inertia of gross cross section, in in.⁴ (cm.⁴)

e_g = Eccentricity of prestress with reference to gross cross section, in in. (cm.)

A_{ps} = Total area of prestressing steel, in sq. in. (sq. cm.)

The equilibrium Eqs. 5 and 6 can also be simplified to yield the value of steel stress at any arbitrary time:

$$f_s = (\beta - 1) f_{cs} + \beta f'_{cl} \quad (11)$$

The derivation of Eqs. 10 and 11, as well as the definitions of the coefficients R_1 , R_2 and R_3 , are found in the Appendix.

In summary, the procedure for an analysis of prestress losses in a pretensioned member is as following:

1. Material, geometry and fabrication parameters are known or specified for the problem. (These include the concrete characteristics, β , f'_{cl} , k_1 and k_2 .)

2. Evaluate R_1 , R_2 and R_3 for each specified time t_c . (See Appendix)
3. Solve Eq. 10 for f_{cs} .
4. Evaluate the steel stress f_s by Eq. 11.
5. Calculate the concrete and steel strains, ϵ_c and ϵ_s , by Eqs. 2 and 4, respectively.
6. Evaluate steel prestress by Eq. 8, and prestress loss by Eq. 9.

Example and Comparison

As explained earlier, the new method enables a direct solution of the prestress loss at any time during the service life of the member without the need of a step-by-step accumulative technique. However, to determine the complete history of prestress variation in a member, the tasks 2 through 6 enumerated above must be repeated many times for different values of t_c , and the amount of calculations involved is considerable. A computer program has been developed to carry out these calculations.

An example is presented herein to illustrate the calculations according to the new procedure, and to compare the results with those from several other procedures. This example deals with a PennDOT standard 20/33 I-beam (2), spanning 60 ft. (18.3 m) c.c., and prestressed with thirty-four 1/2 in. (1.27 cm) stress-relieved strands of the 270 K (1860 MN/m²) grade. The concrete used is the one corresponding to the lower bound of prestress losses. $e_g = 7.95$ in. (20.2 cm.) $k_1 = 2.3$ days.
 $f_{si} = 183.6$ ksi (1266 MN/m²) = 0.68 f_{pu} . This beam is part of a highway bridge, where the deck slab is 7-1/2 in. (19.05 cm) thick, cast-in-place

140 days after transfer. The spacing between beams is 6 ft. 10 in. (208 cm) c.c. An additional dead load of 30 psf (1440 N/m²) is applied later to be resisted by the composite section.

For the sake of simplicity, the 30 psf superimposed load is treated as applied together with the deck gravity load at 140 days. From the geometry of the given section, it is calculated that $\beta = 50.5$. Before application of superimposed loads, $f'_{cl} = 0.417$ ksi (2.88 MN/m²) and $f_{sl} = 1.93$ ksi (13.3 MN/m²). Afterwards, $f'_{cl} = 1.171$ ksi (8.07 MN/m²) and $f_{sl} = 5.4$ ksi (37.4 MN/m²).

Detailed calculations according to the new procedure are illustrated for the time just prior to the application of deck and other superimposed loads. At that time, $t_c = 140$ days, $t_s = 142.3$ days, $f'_{cl} = 0.417$ ksi, (2.88 MN/m²) and $f_{sl} = 1.93$ ksi (13.3 MN/m²). With reference to Tables 1, 2 and the Appendix, the coefficients R_1 , R_2 and R_3 in Eq. 10 are evaluated as follows.

From the steel stress-strain relationship, (coefficients A_1 , A_2 and A_3), for the initial tensioning stress $f_{si} = 0.68 f_{pu}$, $k_2 = 0.65509$

$$P_1 = -0.04229 (270) = 11.4$$

$$P_2 = [1.21952 - (-0.05867) - 0.00023 \log (142.3 + 1)] (270) = 345.0$$

$$P_3 = [-0.17827 - 0.11860 - 0.04858 \log (142.3 + 1)] (270) = -108.4$$

$$Q_1 = -0.00066 - 0.00664 + (0.01500 - 0.00331) \log (140 + 1) = 0.0178$$

$$Q_2 = 0.02105 - 0.00371 + 0.01409 \log (140 + 1) = 0.0476$$

$$k_2 - Q_1 = 0.655 - 0.0178 = 0.637$$

$$R_1 = -11.4 + 345.0 (0.637) - 108.4 (0.637)^2 = 164.4$$

$$R_2 = -0.0476 [345.0 - 2 (108.4) (0.637)] = -9.85$$

$$R_3 = -108.4 (0.0476)^2 = -0.246$$

Substituting into Eq. 10

$$(164.4 - 50.5 \times 0.417) + (-9.85 - 49.5) f_{cs} - 0.246 f_{cs}^2 = 0$$

Simplifying $143.3 - 59.3 f_{cs} - 0.246 f_{cs}^2 = 0$

The solution for f_{cs} is 2.39 ksi From Eq. 11

$$f_s = 49.5 (2.39) + 50.5 (0.417) = 139.5 \text{ ksi}$$

Hence $f_p = 139.5 - 1.93 = 137.6 \text{ ksi}$

$$\Delta f_p = 183.6 - 137.6 = 46.0 \text{ ksi } (317 \text{ MN/m}^2)$$

It should be re-emphasized here that the prestress loss is calculated directly from the initial and present conditions, without any reference to the

intervening loading history. Fig. 2 shows the computer results of similar calculations at other times.

From Fig. 2, it is easily seen that the growth of prestress loss is nearly linear with respect to $\log t_c$, as long as the load remains unchanged. It would be reasonable, therefore, to simplify the calculating procedure, by taking advantage of this phenomenon. Direct solution will be needed only at a few key stages, and prestress loss at intermediate time can be easily estimated by means of this linear semi-logarithmic relationship.

Also shown in Fig. 2 are estimates based on a step-by-step procedure recommended by the PCI Committee on Prestress Losses (4), and by the latest proposed revision to the AASHTO Bridge Specification (8). Calculation according to the 1973 AASHTO Specification (7) resulted in an extremely high loss estimate of nearly 80 ksi (552 MN/m²) and was not shown in Fig. 2. It should be pointed out that both the PCI and the AASHTO methods appear to have implicitly defined prestress to include the stress caused by applied loads. In order that the comparison will be meaningful, all estimates shown in Fig. 2 have been adjusted to conform to the definition given earlier in this paper.

Very good agreement is noted between the PCI method and the new method being presented here, particularly during the initial period before the increase of external load. The low estimate of the "final" loss by the PCI method (55.5 ksi, 383 MN/m²) is believed to be a reflection of a relatively short assumed service life.

The AASHTO method (8) deals with the final loss only, and does not yield as much information as the other two methods. While the final loss

predicted by AASHTO method (62.6 ksi, 432 MN/m²) appears to agree quite well with the prediction by the new method, (61.1 ksi, 421 MN/m²) there are indications that AASHTO also considered a service life shorter than 100 years. Consequently, it would be more appropriate to recognize the difference and conclude that the AASHTO method results in slightly higher loss predictions than the new method. It should be reiterated that in this example, concrete corresponding to the lower bound losses is considered. While the new method is very sensitive to the characteristics of concrete, the AASHTO method is not, as only the elastic loss is affected. When the same example was repeated using the high loss concrete, the new method yielded a final loss of 76.9 ksi (530 MN/m²) at 100 years, while the AASHTO method resulted in a significantly lower loss of only 65.4 ksi (451 MN/m²) at an unspecified time. Similar comparisons have been observed in other examples. In general, it can be stated that the AASHTO procedure yield predicted final loss values lying within the range predicted by the new method, but much closer to the lower bound.

In conclusion, the new method for the estimation of prestress losses is seen to be a viable alternative to the several methods currently available. It allows for wide ranges of variation for the material characteristics as well as other design parameters, and enables the direct determination of the prestress loss at any time during the service life of the member.

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APPENDIXDerivations of Equations

The set of equations used in the development of the basic analytical procedure includes the two stress-strain-time relationships, the four linking relationships and the linear relationship defining concrete stress distribution in the member section.

$$f_s = f_{pu} \{A_1 + A_2 \epsilon_s + A_3 S^2 - [B_1 + B_2 \log(t_s + 1)] \epsilon_s - [B_3 + B_4 \log(t_s + 1)] S^2\} \quad (1a)$$

$$\epsilon_c = C_1 f_c + [D_1 + D_2 \log(t_c + 1)] + \{[E_1 + E_2 \log(t_c + 1)] + f_c [E_3 + E_4 \log(t_c + 1)]\} \quad (2a)$$

$$t_s - t_c = k_1 \quad (3)$$

$$\epsilon_s + \epsilon_c = k_2 \quad (4)$$

$$\int f_c dA_c - \sum f_{x ps} a_{ps} = P \quad (5)$$

$$\int f_c x dA_c - \sum f_s x a_{ps} = -M \quad (6)$$

$$f_c = g_1 + g_2 x \quad (7)$$

In these equations, f_c , f_s , ϵ_c and ϵ_s are functions of x , and in Eqs. 5 and 6, the integrations are over the net concrete section area and the summations cover all prestressing steel elements. Substituting Eq. 7 into 5 and 6, and performing the integrations,

$$A g_1 - \sum (f_s + f_{cs}) a_{ps} = P \quad (5a)$$

$$I g_2 - \sum (f_s + f_{cs}) x_s a_{ps} = -M \quad (6a)$$

where: f_{cs} = Concrete fiber stress at the level of prestress steel

x_s = x distance for an individual prestressing element

Therefore
$$f_{cs} = g_1 + g_2 x_s \quad (A-1)$$

To simplify further derivation, a group of parameters are introduced.

$$P_1 = A f_{pu}$$

$$P_2 = [A_2 - B_1 - B_2 \log(t_s + 1)] f_{pu}$$

$$P_3 = [A_3 - B_3 - B_4 \log(t_s + 1)] f_{pu}$$

$$Q_1 = D_1 + E_1 + (D_2 + E_2) \log(t_c + 1)$$

$$Q_2 = C_1 + E_3 + E_4 \log(t_c + 1)$$

Then
$$f_s = P_1 + P_2 \epsilon_s + P_3 \epsilon_s^2 \quad (A-2)$$

$$\epsilon_c = Q_1 + Q_2 f_c \quad (A-3)$$

Substituting into Eq. 4:

$$\epsilon_s = k_2 - Q_1 - Q_2 f_{cs} \quad (A-4)$$

Substitute into Eq. A-1:

$$\begin{aligned} f_s &= P_1 + P_2 (k_2 - Q_1 - Q_2 f_{cs}) + P_3 (k_2 - Q_1 - Q_2 f_{cs})^2 \\ &= R_1 + R_2 f_{cs} + R_3 f_{cs}^2 \end{aligned} \quad (A-5)$$

where $R_1 = P_1 + P_2 (k_2 - Q_1) + P_3 (k_2 - Q_1)^2$

$$R_2 = -Q_2 [P_2 + 2P_3 (k_2 - Q_1)]$$

$$R_3 = P_3 Q_2^2$$

Substituting Eqs. A-1 and A-5 into the equilibrium conditions 5a and 6a

$$A_{g_1} - \Sigma [R_1 + (R_2 + 1) (g_1 + g_2 x_s) + R_3 (g_1 + g_2 x_s)^2] a_{ps} = P \quad (A-6)$$

$$I_{g_2} - \Sigma [R_1 + (R_2 + 1) (g_1 + g_2 x_s) + R_3 (g_1 + g_2 x_s)^2] x_s a_{ps} = -M \quad (A-7)$$

These equations are simultaneous quadratic equations in g_1 and g_2 .

In the simplified case when prestressing steel is regarded as concentrated at one level, x_s becomes a constant for all elements, and is equal to e_g by definition.

Replacing x_s by e_g in Eqs. A-6, A-7 and A-4,

$$A_{g_1} - [R_1 + (R_2 + 1) (g_1 + g_2 e_g) + R_3 (g_1 + g_2 e_g)^2] A_{ps} = P \quad (A-6a)$$

$$I_{g_2} - [R_1 + (R_2 + 1) (g_1 + g_2 e_g) + R_3 (g_1 + g_2 e_g)^2] A_{ps} e_g = -M \quad (A-7a)$$

$$f_{cs} = g_1 + g_2 e_g \quad (A-4a)$$

Multiply Eq. A-6a by I_g , Eq. A-7a by $(A_g e_g)$, add these two equations, and substitute Eq. A-4a

$$A_g I_g f_{cs} - [R_1 + (R_2 + 1) f_{cs} + R_3 f_{cs}^2] A_{ps} (I_g + A_g e_g^2) = P I_g - M A_g e_g$$

Therefore

$$f_{cs} - [R_1 + (R_2 + 1) f_{cs} + R_3 f_{cs}^2] A_{ps} \left(\frac{1}{A_g} + \frac{e_g^2}{I_g} \right) = \frac{P}{A_g} - \frac{M e_g}{I_g} \quad (A-10)$$

Two parameters are now introduced

$$\beta = \frac{1}{A_{ps} \left(\frac{1}{A_g} + \frac{e_g^2}{I_g} \right)}$$

$$f'_{cl} = - \frac{P}{A_g} + \frac{M e_g}{I_g}$$

Eq. A-10 is then transformed into Eq. 10

$$(R_1 - \beta f'_{cl}) + (R_2 - \beta + 1) f_{cs} + R_3 f_{cs}^2 = 0 \quad (10)$$

It is important to note that f'_{cl} is the nominal concrete stress caused by the applied loads, based on gross section properties, and using a tension positive sign convention. The dimensionless geometrical parameter β is closely associated with the ratio of steel prestress to concrete prestress.

Equation 11 for steel stress is obtained by subtracting Eq. 10 from Eq. A-5

$$\beta f'_{cl} + (\beta - 1) f_{cs} = f_s \quad (11)$$

Table 1 Coefficients for Steel Stress-Strain-Time Relationship

Table 2 Coefficients for Concrete Stress-Strain-Time Relationship

TABLE 1: COEFFICIENTS FOR STEEL STRESS-STRAIN-TIME
RELATIONSHIPS

Elastic Coefficients	A_1	-0.04229
	A_2	1.21952
	A_3	-0.17827
Relaxation Coefficients	B_1	-0.05867
	B_2	0.00023
	B_3	0.11860
	B_4	0.04858

All coefficients are dimensionless,
and remains same in SI units

TABLE 2: COEFFICIENTS FOR CONCRETE STRESS-STRAIN-TIME
RELATIONSHIPS

Concrete Mix		Upper Bound Loss	Lower Bound Loss
Elastic Strain	C_1	0.02500	0.02105
Shrinkage	D_1	-0.00668	-0.00066
	D_2	0.02454	0.01500
Creep	E_1	-0.01280	-0.00664
	E_2	0.00675	-0.00331
	E_3	-0.00060	-0.00371
	E_4	0.01609	0.01409

For SI units, C_1 , E_3 and E_4 should be multiplied by 0.145 .

(to be combined with f_c in MN/m^2)

Fig. 1 Sign Convention for Applied Loads

Fig. 2 Predicted Prestress Losses - Example Problem

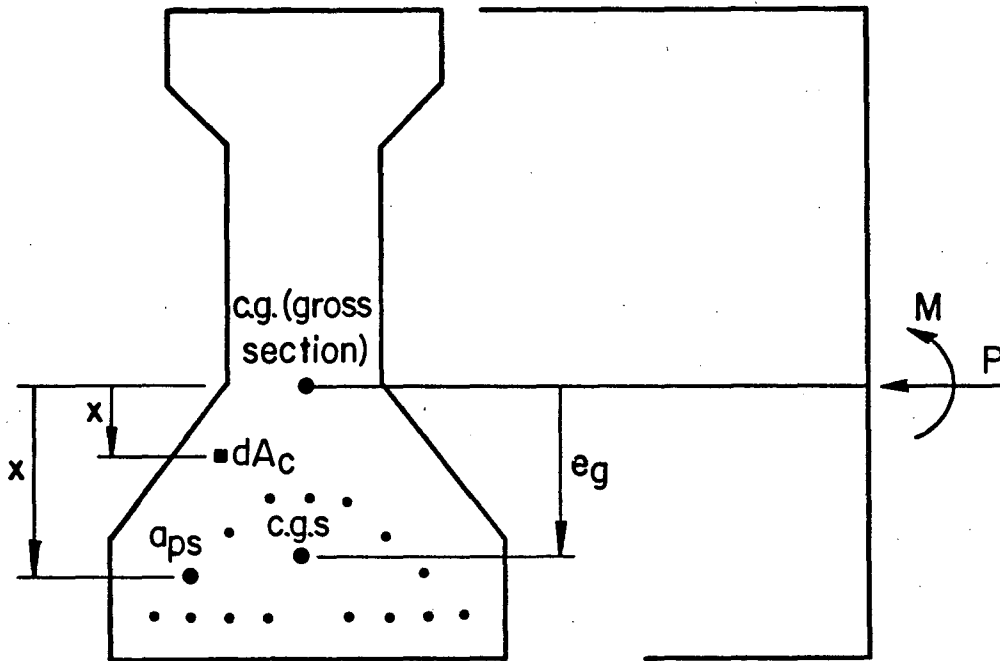


Fig. 1 Sign Convention for Applied Loads

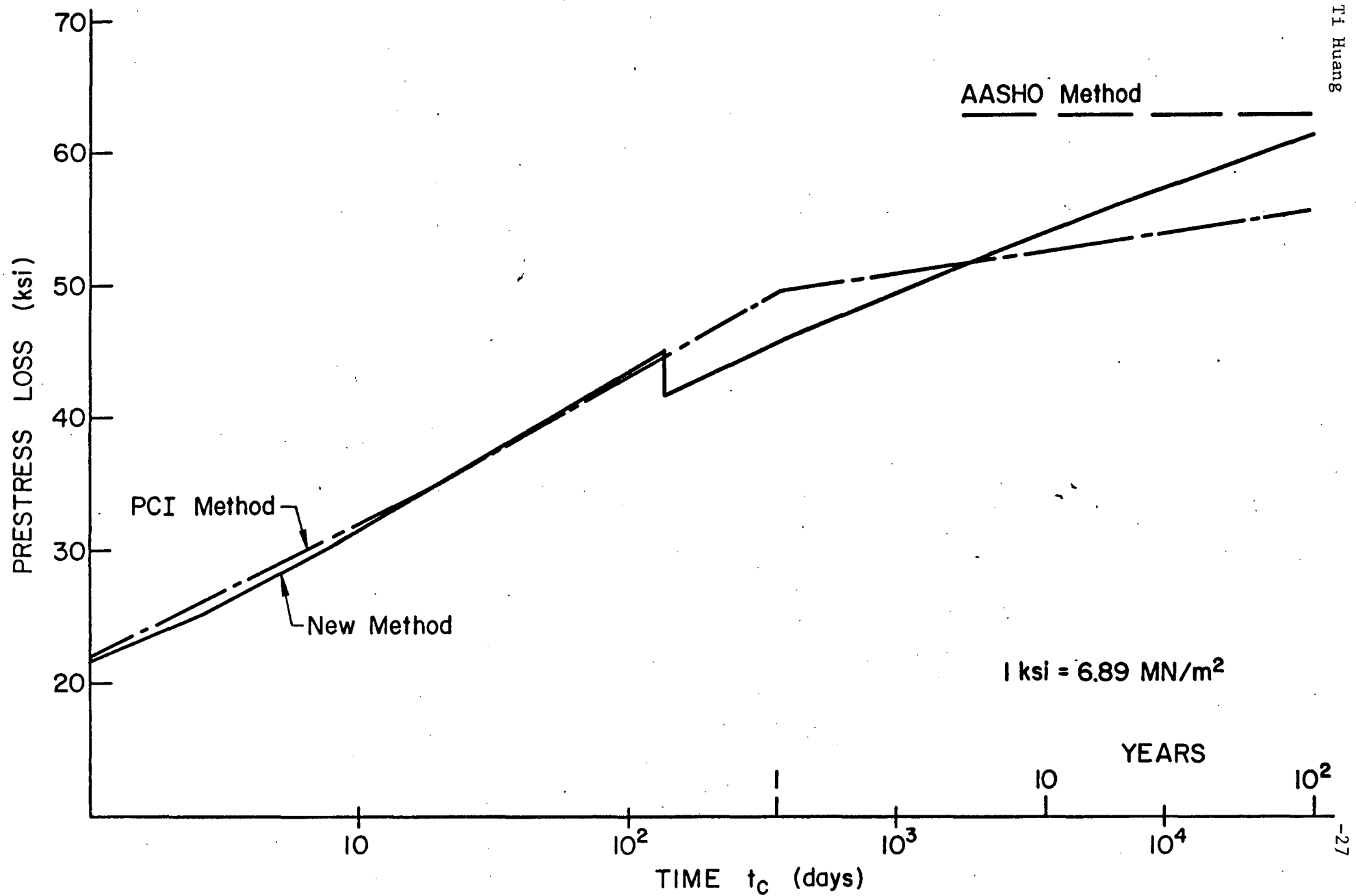


Fig. 2 Predicted Prestress Losses - Example Problem