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Hybrid steel columns, May 1969 (72-37)

N. R. NagarajaRao

P. Marek

L. Tall

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Welded Columns

HYBRID STEEL COLUMNS

by

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ABSTRACT

This report presents the analysis and results of a theoretical and experimental investigation to determine the strength of hybrid steel columns.

The investigation was made on centrally-loaded welded H-shaped columns with high-strength steel flanges and low-strength steel webs.

The tangent modulus and ultimate load, the mechanical properties of the materials, the actual residual stress distribution and local buckling were taken into consideration for the theoretical analyses of column curves. The predictions were verified by tests.

The experimental study included five hybrid shapes, fabricated from flame-cut or universal-mill plates. The following tests were conducted: tension specimen coupon, residual stress measurements, stub column tests and pinned-end column tests with a slenderness ratio of 65.

The study showed that the column strength of
hybrid shapes can be predicted from the actual residual stress distribution by assuming a hypothetical residual stress in the web equal to the difference in yield strength of flange and web.

The investigation was completed by a discussion of approximate estimation of residual stress distribution and its magnitude, local buckling considerations and elastic stress of webs at the "working" load.

For slenderness ratios up to 70, the A514 homogeneous steel columns are usually the most economical; for ratios from chart 50 up to 70, hybrid columns may lead to a price reduction.
1. INTRODUCTION

The hybrid steel column is a special type of component structural member that consists of higher-strength steel flanges and a lower-strength steel web. Much information is already available about the use of hybrid beams, (1) which frequently offer significant material cost savings over homogeneous beams, but there has not yet been extensive theoretical and experimental analysis of the behavior of hybrid steel columns.

This paper is a contribution to hybrid section analysis and gives a description of the theoretical analysis and tests executed at Lehigh University and the comparison of results and discussion about the usefulness of compression members composed of two different types of steel. The study is described in greater detail in Ref. 2.

The concept of hybrid shapes has been applied to structural members in bending, by placing a stronger material in a position where it can resist higher stresses, thus using materials according to their strengths. Such explanations are not satisfactory for centrally loaded ideal columns, with no region of lower
stresses; already in the case of beam-columns the hybrid cross section may offer some savings. Another interesting problem that suggests the study is the reinforcing of old columns to carry heavier loads by adding high-strength steel cover plates.

A knowledge of residual stresses in hybrid sections and information about the behavior of centrally loaded columns are necessary in considering design recommendations for both columns and beam-columns. For these reasons, an investigation was made on welded H-shapes, with high-strength steel flanges and low-strength steel webs, with respect to:

a. The difference in the yield stresses of the materials used,
b. The residual stress distribution caused by fabrication of plates and by welding,
c. The local buckling characteristics,
d. The out-of-straightness of the column.
2. THEORETICAL ANALYSIS

Strength of an Ideal Column

The strength of a column may be defined by its bifurcation or buckling load, (at which a theoretically straight column is indifferent to its deflected shape), and by its ultimate load (the maximum load that a column can carry, defining the transition from a stable to an unstable configuration). A review of the past work is provided in several papers.\(^{(3,4)}\) Research in the last decade has underlined the great influence of residual stress distribution on column strength.\(^{(5)}\) The tangent modulus and reduced modulus buckling theories are applicable with some modification to columns containing residual stresses.\(^{(3,6)}\)

In determining the column curve for a hybrid steel column, the main problems are caused by the difference in yield stresses of the component material and by the residual stresses. A simple method presented here, uses the residual stress itself as a tool to transform a hybrid shape into a homogeneous shape.

It is assumed that the stress-strain curve of the web steel and flange steel are both idealized as shown in Fig. 1.
An ideal hybrid H-shape is shown in Fig. 2.

The yield stresses are \( \sigma_f \) for the flange, \( \sigma_w \) for the web, and \( \frac{\sigma_w}{\sigma_f} = \alpha \) (<1.0). The entire web yields when the average stress is \( \sigma_w \) and further loading is transferred to both flanges only. Column curves for this ideal column as shown in Fig. 3 are derived as follows:

For the flange area \( A_f \) and web area \( A_w \), the load to cause yielding on the complete section,
\[
P_y = \frac{1}{2} A_f \sigma_f + A_w \sigma_w
\]
and the moment of inertia
\[
I = \frac{1}{12} A_f \cdot r_f^2 + A_w \cdot r_w^2 = A r^2.
\]

The Euler buckling stress,
\[
\sigma = \frac{\pi^2 E}{(L/r)^2} \tag{1}
\]
and
\[
\frac{\sigma_f}{\sigma} = \frac{\pi^2 E}{\sigma_f} \frac{1}{(L/r)^2}
\]
with \( \frac{\pi^2 E}{\sigma_f} = c^2 \)

then
\[
\frac{\sigma}{\sigma_f} = \left( \frac{c}{L/r} \right)^2 \tag{2}
\]

For \( \sigma < \sigma_w \)
\[
\frac{P}{P_y} = \frac{\sigma}{\sigma_f} \tag{3}
\]
At \( \sigma = \sigma_w \), \( L/r = C \frac{c}{\sqrt{\sigma}} = C_1 \) 

At the load corresponding to \( \sigma = \sigma_w \), the web yields completely, and \( I_x \) and \( I_y \) reduce to \( I_{ex} \) and \( I_{ey} \) respectively, and \( \frac{P}{\sigma_y} \) reduce to \( \frac{P}{\sigma_f} \).

**X-axis Buckling**

\[
\frac{I_{ex}}{I} = \frac{A_f \cdot r_f^2}{A \cdot r^2} = \beta 
\]

\[
\frac{P}{\sigma_f} = \frac{\pi^2 \cdot E}{\sigma_f L^2} \cdot \frac{I_{ex}}{I} \cdot Ar^2
\]

\[
\frac{\sigma}{\sigma_f} = \frac{\pi^2 E}{\sigma_f} \cdot \frac{\beta}{(L/r)^2} = \beta \left( \frac{c}{L/r} \right)^2
\]

At \( \sigma = \sigma_w \), \( L/r = c \sqrt{\beta / \alpha} = C_2 \)

For \( \sigma = \sigma_w \), the column curve is given by Eq. 6. The maximum value of \( \sigma/\sigma_f \) is

\[
\frac{\sigma}{\sigma_f} = \frac{A_f \sigma_f + A_w \sigma_w}{A \sigma_f} = \gamma < 1.0
\]

and the corresponding slenderness ratio,

\[
L/r = c \sqrt{\beta / \gamma} = C_3
\]

For convenience, to have \( P/P_y \) reach 1.0 when the entire shape yields, the \( P/P_y \) ordinates are divided by 

Eqs. 2 and 6 rewritten as

\[
\frac{P}{P_y} = \frac{C^2}{(L/r)^2} \text{ for } \sigma < \sigma_w
\]
\[
\frac{P}{P_y} = \frac{C_m^2 \cdot \beta}{(L/r)^2} \quad \text{for } \sigma > \sigma_w
\] (10)

where \( C_m = \frac{C}{\sqrt{Y}} \) (11)

Y-axis Buckling

\[
I_{ey} \leq I \quad \text{1.0 when the web yields so that although}
\]

\[
\frac{\sigma}{\sigma_f} \neq \frac{P}{P_y}
\]

\[
\frac{\sigma}{\sigma_f} = \left( \frac{c}{L/r} \right)^2 \quad \text{for } \sigma < \sigma_f
\] (2)

\[
\frac{P}{P_y} = \left( \frac{c_m}{L/r} \right)^2 \quad \text{for } \sigma < \sigma_f
\] (6)

Tangent Modulus Strength

Similar to homogeneous columns, the tangent modulus load can be found by two methods: (1) using the tangent modulus obtained by a stub column test and (2) using the residual stress distribution. (3,5)

Although the stub column method has not been used to predict the tangent modulus curve in this investigation, stub column tests were conducted to obtain other important characteristics of hybrid columns.
The residual stress distribution of hybrid shapes is described in Section 3. Some modifications were made to the measured distributions in order to use them for computing column curves with the help of a digital computer.

The following assumptions were made for the mechanical properties:
- an idealized stress-strain relationship
- the material is homogeneous
- plane cross-sections remain plane after deformation
- the residual stress is constant along each fiber in the cross section and axial symmetry of cross section and residual stress exists
- the hybrid shape may be transformed into a homogeneous shape as described above.

The tangent modulus curves obtained by a computer program \(^2\) are shown in Figs. 4 (a) through (e). The portions of curves shown dotted are for the shapes without residual stresses.

In Fig. 5, the tangent modulus column curve
for buckling about the y-axis for the hybrid column is shown with the slenderness ratio non-dimensionalized. Also shown are the CRC column curve and tangent modulus curve for A7 steel H-shapes welded from UM plates. Figure 5 further demonstrates the fact that in shapes with A514 flanges, the role of the web material is not very important for y-axis buckling. Furthermore, it shows that the curves for columns with A441 flanges are significantly above those of A7 welded columns and that the column curve for the shape with A441 flame cut flanges is very close to the CRC curve. For comparison, also a column curve for a flame-cut welded shape, 12H79, of A36 steel grade is included.\(^7\) The comparison of all shown column curves confirms the effect of \(\frac{\sigma_r}{\sigma_y}\), as \(\frac{\sigma_r}{\sigma_y}\) decreases, column strength increases.

**Ultimate Load and the Load Deflection Curve**

The theoretical load deflection curves, such as the one shown in Fig. 6, were obtained after simplifying the computation by the assumption of a single sine wave initial deflection curve and after modifying the actual measured residual stress distribution for perfectly symmetrical distribution about the geometrical axes of the section. The maximum value of
the load in each of the load-deflection curves is the theoretical ultimate load of that column for the corresponding slenderness ratio. No initial out-of-straightness was assumed. The ultimate strength column curves obtained by plotting these ultimate loads and slenderness ratios are shown in Figs. 7 (a) to (e).

**Eccentrically Loaded Columns**

The research program was concerned with the investigation of centrally loaded columns. Due to fabrication error, unsymmetrical residual stress distribution, or eccentricity of the load, the actual column may have significant out-of-straightness and the method of computing the load-deflection curve must be modified. The computer program for centrally loaded columns was adjusted to compute the load-deflection curve, taking eccentricity into consideration. Figure 8 shows the computed load-deflection curves for shape No. IV, with \( e = 0, e = 0.1 \) in. and \( e = 0.20 \) in. and \( L/r = 65 \). These curves indicate that the ultimate loads are less than the tangent modulus load by about 15% and 20% respectively.

**Local Buckling**

In hybrid shapes, web buckling may be of some
concern because the web may be partially or wholly inelastic at working load. Flange buckling can be considered without difficulty, since it will be mostly elastic. The width-thickness ratio of the web must be such that the web does not buckle even if it has yielded completely.

Plate buckling in the inelastic range has been investigated in the past in connection with plastic design, see Ref. 11. The buckling of plates containing residual stresses has been analyzed in Refs. 8 and 9. According to the conclusions, local buckling in the plastic range is prevented when $\lambda \leq 0.62$ if the ends are fixed and $\lambda \leq 0.68$ if the ends are simply supported.
3. EXPERIMENTAL STUDIES

Description

The study included five hybrid shapes, which are described in Table 1. The hybrid columns were fabricated from flame-cut plates or UM plates (No. II) which were not subjected to any cold-bending or straightening. The component plates were 20 ft. long. The shapes after welding also were not straightened or cold-bent or trimmed in any way. The cross section is shown in Fig. 2.

The tests conducted on these shapes were:
tension specimen coupon tests, residual stress measurements, stub column tests, and pinned-end tests.

Tension Specimens

Coupon tests were used to obtain mechanical properties and to ascertain that the right type of steel had been used. The results of tests are given in Table 2. As shown, all steels except the A441 steel web of No. IV showed more than the specified stress level. The coupon dimensions were in accordance with ASTM specifications. (10)
Residual Stress

Residual stress measurements were made by the method of sectioning.\(^{(12)}\) The specimens were 3 ft. long and one specimen was taken from each hybrid shape. The results for all shapes are shown in Figs. 9 and 10. The shape of the residual stress distribution corresponds to the usual distribution in homogeneous columns and in the flange tips of universal mill plates and of flame-cut plates.

Stub-Column

Stub column tests provide an average stress-strain curve of the whole cross section, including the effect of residual stresses. Stub columns are short columns long enough to retain the residual stress distribution in normal column and short enough not to fail by column buckling. The length of such columns is prescribed in Ref. 13; the test columns were 24 inches long. Strains were measured after alignment by 0.0001 in. dial gages over a gage length of 10 inches. Stub columns shapes Nos. I, IV, V, were tested in a 5 million pound hydraulic testing machine, while those with A441 flanges (No. II and III) were tested in an 800,000 pound mechanical testing machine. The results of the stub column tests are given in Table 3 and in Figs. 11 (a) to (c).
Pinned-End Columns

Pinned-end column tests verify the prediction of column strength made on the basis of the residual stress distribution or of stub column tests, and provide information on the behavior of the shapes used as columns. One column of each shape was tested as a pinned-end column. With a length of 8 ft. and a slenderness ratio of 65, axial load was applied through special fixtures which simulate a pinned-end condition about the y-axis and a fixed-end condition about the x-axis (Ref. 14). Strains were measured by means of SR-4 A-I type strain gages of 1" gage length placed at the mid-height of the column and near both ends. They were also used to align the columns so that the load would be exactly central at mid-height and as close as possible to central at the ends. The columns had a slight initial out-of-straightness and were bent in single curvature. The maximum out-of-straightness measured was at the mid-height. (See Table 4) Load was applied in small increments and the lateral deflection at mid-height was measured by a 0.001 in. dial gage and also with a transit and 0.01 in. scales at quarter points on the length of the column. The rotations at the ends were measured by a 0.0001 in. dial gage and level bar.
The results of the pinned-end column tests are shown in Table 4. The test results exceed the predicted values by 1% (No. I and No. IV), by 10% (No. III) and by 6% (No. V). The test result is 4% below the predicted value for shape No. II. Based on the limited number of tests, a reasonably good prediction can be made with the measured residual stress distribution and the computer program.

The load-deflection curves of the column tests are shown in Fig. 12. The ultimate load was reached when the lateral deflection was about 0.1" for all columns, except shape No. IV.

A comparison of the predicted and actual load-deflection curves can be made with the help of Fig. 13. The prediction of the load-deflection curve is complicated, since in an actual column such factors as out-of-straightness, eccentricity of load and unsymmetrical residual stress distribution, all influence the behavior considerably.

Local Buckling

Flanges - The width-thickness ratio for the webs and flanges of the H-shapes used in this
study were 16 and 12 respectively. These are comparatively low values, and therefore, local buckling did not occur in the stub column tests until strains were well in the plastic range. It was also observed that local buckling occurred in the flange first, and by the time the web started to buckle, the flanges were severely buckled. Thus it was found for the test specimens that the width-thickness ratio of the flange was the criterion for strength.

Webs - The stress-strain curves for the webs obtained from pinned-end column tests are shown in Fig. 14. The strains were measured at the mid-height of columns and in the middle of the web on both sides and averaged. From the load-deflection curves of the columns shown in Fig. 12, it is seen that the mid-height deflection is negligible up to 60% of yield load. Therefore, it can be assumed that the cross section at mid-height is subjected to a negligible amount of bending stress and the strains recorded are due essentially to the axial load.
4. DISCUSSION OF RESULTS

Ultimate Load

The theoretical tangent modulus column curves and ultimate load curves for the hybrid shapes have shown that the strength of these columns is much higher than that of welded homogeneous A36 columns.

The computation of the tangent modulus load is simple compared to the ultimate load calculation. As shown for columns with A514 flanges (Fig. 7), \( P_u / P_t \approx 1.0 \), and it is therefore not necessary to compute the ultimate load. For columns with A441 steel flanges, \( P_u / P_t = 1.05 \), whereas for welded columns of A36 steel \( P_u / P_t \) may be as high as 1.25. The hybrid columns tested in this study had ultimate loads of about 75 per cent of the yield load. Welded H columns of A36 steel of comparable slenderness ratio have such a high value. (3)

Column Tests

The hybrid columns with A514 flanges failed suddenly after reaching the ultimate load.

In Section 2, the local buckling considerations of the columns tested were mentioned.
thickness ratio specified by AISC for allowable stress design may be used for flanges since the working stresses in the flanges can be expected to be within the elastic limit; the validity of these ratios has not been confirmed by tests.

As far as the webs are considered, the column tests results described in Section 3 lead to the following comments:

If it is assumed that the factor of safety for these columns is 2.0, the columns of Shapes Nos. I, IV, V, II and III would have working loads of $0.38 P_y$, $0.37 P_y$, $0.42 P_y$, $0.37 P_y$ and $0.42 P_y$ respectively, based on pinned-end column tests. This shows that all shapes except No. I will have their webs well within the elastic limit. If it is desired that the average stress in the web be within the yield stress, then A36 and A514 steels should not be used in the same shape.

The values of width-thickness ratio of web as used in the AISC specification are not very restrictive for A36 or A441 steels. The limiting ratios would be 42 and 34 respectively. These values were intended for the condition that the webs are
allowed to strain-harden. The webs of hybrid shapes would have strained not more than $2\varepsilon_y$ when the entire shape is in plastic state. Hence, the recommended values are conservative.

**Price-Strength Ratio**

A relationship between price-to-strength ratio and slenderness ratio can be developed with the aid of column curves and cost data. Figure 15 shows such curves which are based on the theoretical cross-sectional areas and the average net mill prices, 1969 level. The prices used in the calculation are: A514, 15 cents/lb; A441, 12 cents/lb; A36, 9 cents/lb. Figure 15 shows that A514 steel columns should be economical for low slenderness ratios. For slenderness ratios larger than about 70, A36 steel is progressively suitable. The combination of A514 and A441 with A441 and A36 steel respectively, not only reduces the dead load to be carried, but may also bring economical savings. For a reliable comparison, further factors should be taken into consideration, for example fabrication, transportation and errection, any of which may change the price relationship shown in Fig. 15.

The price relationship of the hybrid shape No. 5 in comparison with the homogeneous shapes is
shown in Figure 16. The price of the hybrid section (6" x 1/2" flanges, A514 steel and 6" x 3/8" web, A441 steel) is compared for each slenderness ratio with the prices of homogeneous columns, all having the same allowable load. The ultimate load for hybrid section No. 5 was obtained from Fig. 7c; the factor of safety and allowable stresses for A36, A441 and A514 steel grades are those of the AISC specification. As shown in Fig. 15B for very low slenderness ratios up to about 45, A514 steel is most economical; for slenderness ratios above 75, A36 steel is the cheapest. For slenderness ratios from 45 up to 75, use of the hybrid section may reduce the price by more than 10 per cent.
5. CONCLUSIONS

The typical tests conducted on hybrid shapes were coupon tests, residual stress measurements, stub column tests and pinned-end column tests. Residual stress measurements made on welded plates of A7, A36, A441 and A514 steels were used to estimate the residual stresses in hybrid shapes. The actual residual stresses in the hybrid shapes were used to compute the tangent modulus column curve and the ultimate load curve by a digital computer. These curves were verified by tests.

The following are the important conclusions of this study:

1. The residual stress distributions are similar in all three shapes with A514 steel flanges. The tensile residual stress at the flame-cut flange tips ranges from 30 ksi to 70 ksi, and is about 25 ksi at the welds (average through the thickness). The compressive residual stress is about 20 ksi. The webs have high tensile residual stress in the immediate vicinity of the welds and a compressive residual stress of about 10 ksi over the remaining area.
2. The column strength of hybrid shapes can be predicted from the actual residual stress distribution and by assuming a hypothetical residual stress in the web equal to the difference in yield strength of flange and web.

3. The tangent modulus column curves show that the column strength reduction due to residual stresses is small (no more than 10%) for shapes with A514 steel flanges; also, the hybrid column with A441 flame-cut flanges is stronger than the column with A441 Universal Mill flanges, and the hybrid column with A441 flame-cut flanges has a tangent modulus curve very close to the CRC curve.

4. For shapes with A514 steel flanges, the ultimate load is only slightly higher (up to one per cent) than the tangent modulus load. The ultimate load for the shape with A441 flame-cut flanges is about 5% greater than the tangent modulus load. For the shape with A441 Universal Mill flanges, the ultimate load is considerably greater than the tangent modulus load at the low slenderness ratios (up to 20 per cent).

5. Local buckling considerations require that the webs must not buckle up to a strain of twice the yield strain ($2\varepsilon_y$). The width-thickness ratios of webs must not exceed 42 for A36 steel. The flanges may
be designed with respect to requirements of allowable stress design. Further research into the plastic behavior of A441 and A514 steels are necessary to make definite recommendations.

6. The pinned-end test columns carried the predicted ultimate load within 5 per cent.

7. The column tests showed, that at the "working" load the webs of all shapes except No. I (A514 flanges and A36 webs) were elastic.

8. For slenderness ratios up to about 70, the A514 homogeneous steel columns are usually the most economical; for slenderness ratios from about 50 up to 70, hybrid columns with A514 flanges may bring a price reduction and can be taken into consideration.

9. The information obtained on the behavior of the hybrid shapes should be taken into consideration in future column curve studies. This will be particularly true if different column design curves for different shapes and steel grades will be developed. Only then will the more reliable and logical price comparison be made.
6. ACKNOWLEDGEMENTS

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Task Group 1 of the CRC under the Chairmanship of John A. Gilligan provided valuable guidance. Special thanks are due to Lynn S. Beedle, Director of Fritz Laboratory for his helpful suggestions throughout the program.

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figures were drawn by J. Izquierdo, J. M. Gera and Mrs. Sharon Balogh. Special thanks are due to Miss Joanne Mies for typing the report.
7. NOTATION

A

Area of cross section

A_f

Area of flanges

A_w

Area of web

c, c_1, c_2, c_3, c_m

Constants

E

Young's Modulus

E_t

Tangent Modulus

I, I_x, I_y

Moment of inertia of a section

I_e, I_{ex}, I_{ey}

Moment of inertia of the elastic area of a partly yielded cross section

L

Length of a column

K_L

Effective length of a column

P

Load on a column

P_y

Yield load

P_t

Tangent modulus

P_u

Ultimate load

r

Radius of gyration of entire section

t

Thickness of flange

\sigma_{cr}

Buckling stress = P_{cr}/A

\sigma_f

Yield stress of flange

\sigma_w

Yield stress of web

\sigma_y

Yield stress in general

\alpha, \beta, \gamma

Constants
\( \varepsilon \)  Strain

\( \varepsilon_y \)  Yield strain = \( \sigma_y / E \)

\( \lambda \)  Non-dimensional slenderness ratio of a plate or column
8. TABLES AND FIGURES
# TABLE I

Hybrid Shapes in Test Program

<table>
<thead>
<tr>
<th>No.</th>
<th>Flange</th>
<th>Web</th>
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<tbody>
<tr>
<td>I</td>
<td>A514</td>
<td>A36</td>
</tr>
<tr>
<td>II</td>
<td>A441(UM)</td>
<td>A36</td>
</tr>
<tr>
<td>III</td>
<td>A441</td>
<td>A36</td>
</tr>
<tr>
<td>IV</td>
<td>A514</td>
<td>A441</td>
</tr>
<tr>
<td>V</td>
<td>A514</td>
<td>A441</td>
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</table>

UM - Universal Mill Plates
**TABLE II**

TENSILE SPECIMEN RESULTS OF STEELS USED
IN HYBRID SHAPES

(Laboratory Tests)

<table>
<thead>
<tr>
<th>Shape No.</th>
<th>Flange</th>
<th>$\sigma_y$ (ksi)</th>
<th>$\sigma_{ult}$ (ksi)</th>
<th>Web</th>
<th>$\sigma_y$ (ksi)</th>
<th>$\sigma_{ult}$ (ksi)</th>
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<td>110</td>
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<td>39</td>
<td>67</td>
</tr>
<tr>
<td>III</td>
<td>A441</td>
<td>51</td>
<td>78</td>
<td>A36</td>
<td>39</td>
<td>66</td>
</tr>
<tr>
<td>IV</td>
<td>A514</td>
<td>106</td>
<td>117</td>
<td>A441</td>
<td>49</td>
<td>72</td>
</tr>
<tr>
<td>V</td>
<td>A514</td>
<td>104</td>
<td>117</td>
<td>A441</td>
<td>53</td>
<td>75</td>
</tr>
</tbody>
</table>
### TABLE III

**RESULTS OF STUB COLUMN TESTS**

<table>
<thead>
<tr>
<th>Shape No.</th>
<th>Flange</th>
<th>Web</th>
<th>Yield Load From Coupons (kip)</th>
<th>Yield Load From Stub Column (kip)</th>
<th>$\frac{p_y}{p^*_y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>A514</td>
<td>A36</td>
<td>750</td>
<td>774</td>
<td>1.03</td>
</tr>
<tr>
<td>II</td>
<td>A441UM</td>
<td>A36</td>
<td>388</td>
<td>393</td>
<td>1.01</td>
</tr>
<tr>
<td>III</td>
<td>A441FC</td>
<td>A36</td>
<td>394</td>
<td>403</td>
<td>1.02</td>
</tr>
<tr>
<td>IV</td>
<td>A514</td>
<td>A441</td>
<td>746</td>
<td>779</td>
<td>1.04</td>
</tr>
<tr>
<td>V</td>
<td>A514</td>
<td>A441</td>
<td>743</td>
<td>735</td>
<td>0.99</td>
</tr>
</tbody>
</table>

*FC = Flame-Cut Plate*
### TABLE IV

RESULTS OF PINNED-END COLUMN TESTS

<table>
<thead>
<tr>
<th>Shape No.</th>
<th>Maximum Out-of-Straightness + (in.)</th>
<th>Predicted Load</th>
<th>Test Results</th>
<th>(P_u) Test (P_u) Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pu (kips)</td>
<td>Py (kips) Pu/Py</td>
<td>Pu/Pu Pu/Pu**</td>
<td>Pu/Pu**</td>
</tr>
<tr>
<td>I</td>
<td>0.06</td>
<td>572 0.75</td>
<td>581 0.78 0.75</td>
<td>1.01</td>
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<tr>
<td>II</td>
<td>0.05</td>
<td>294 0.76</td>
<td>283 0.73 0.72</td>
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<tr>
<td>III</td>
<td>0.02</td>
<td>315 0.80</td>
<td>334 0.85 0.83</td>
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<tr>
<td>IV</td>
<td>0.10</td>
<td>574 0.77</td>
<td>580 0.78 0.74</td>
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</tr>
<tr>
<td>V</td>
<td>0.09</td>
<td>561 0.75</td>
<td>619 0.83 0.84</td>
<td>1.10</td>
</tr>
</tbody>
</table>

+ This occurred at mid-height of columns.

* Based on coupon strengths of flange and web.

** Based on stub column tests.
Fig. 1  Ideal Stress-Strain Curve

Fig. 2  Cross Section of a Hybrid Steel Column
Fig. 3 Column Curves for a Hybrid H-Shape Without Residual Stress
Fig. 4  Tangent Modulus Column Curves (A514 Flanges)
Fig. 4  Tangent Modulus Column Curves (A441 Flanges)
<table>
<thead>
<tr>
<th>Shape</th>
<th>Flange</th>
<th>Web</th>
<th>Curve</th>
<th>Test Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A514</td>
<td>A36</td>
<td></td>
<td>○</td>
</tr>
<tr>
<td>2</td>
<td>A441</td>
<td>A36</td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>3</td>
<td>A441</td>
<td>A36</td>
<td></td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>A514</td>
<td>A441</td>
<td></td>
<td>□</td>
</tr>
<tr>
<td>5</td>
<td>A514</td>
<td>A441</td>
<td></td>
<td>△</td>
</tr>
<tr>
<td></td>
<td>A7</td>
<td>A7</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td></td>
<td>A36</td>
<td>A36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5 Non-dimensional Column Curves for Y-Axis

Buckling
Fig. 6 Theoretical Load-Deflection Curves

Shape #1
A514 & A36
b = 6"
Fig. 6  Theoretical Load-Deflection Curves
Fig. 7 Ultimate Load Column Curves (A514 Flanges)
Fig. 7  Ultimate Load Column Curves (A441 Flanges)

Fig. 8  Load-Deflection Curve for Centric and Eccentric Loaded Column
Fig. 9 Residual Stress Distribution in Hybrid H-Shapes (A514 Flanges)
Fig. 10. Residual Stress Distribution in Hybrid H-Shapes (A441 Flanges)
Fig. 11  Stub Column Stress-Strain Curves
Fig. 12 Experimental Load-Deflection Curves
Fig. 13 Theoretical and Experimental Load Deflection Curves
Fig. 14 Stress-Strain Curves of Webs in Pinned-End Column Tests
Fig. 15  Price-to-Strength versus Slenderness Ratio.

(Homogeneous Shapes)

Fig. 16  Price of the Hybrid Shape No. 5 versus Price of Homogeneous Shapes.
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