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INELASTIC COLUMNS
WITH RESIDUAL STRESSES

by
Ching-Kuo Yu

A Dissertation
Presented to the Graduate Committee
of Lehigh University
in Candidacy for the Degree of
Doctor of Philosophy
in
Civil Engineering

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ABSTRACT

This dissertation presents the results of an investigation of the buckling strength of centrally loaded columns and of the ultimate strength and load-deformation behavior of beam-columns.

Although emphasis is placed on H-shapes made of A514 constructional alloy steel, those both rolled heat-treated and welded built-up, the column strength of welded heavy shapes of A36 steel is considered also. The method developed can be applied to other materials as long as the prerequisite information, sectional properties, residual stress distribution and mechanical properties of the material, is available.

The dissertation is divided into three parts. First, the stress-strain relationship and other relevant mechanical properties of A514 steel as well as the residual stress distribution in the shapes were investigated. A representative stress-strain curve for A514 steel and typical patterns and magnitudes of residual stress distribution for rolled and welded H-shapes were determined by averaging all the experimental results available.

Secondly, the buckling strength of pinned-end columns was calculated with special consideration given to the
effects of residual stresses and the non-linearity of the stress-strain relationship. Two approaches were taken for the computation of the buckling load, one based on a given residual stress pattern and mechanical properties, employing a finite area numerical integration procedure, and another based on stub column test results, that is, a semi-empirical method. A computer program was developed which can be used to calculate the tangent modulus load for any kind of combination of mechanical properties and residual stress distribution. In addition, the effects of geometric properties of the cross-section, shape of stress-strain curve, pattern of residual stress distribution, and variation of residual stresses across the thickness of the component plates on the buckling strength of columns were also investigated.

Finally, the ultimate strength and the load-deformation behavior, as well as the local buckling phenomenon of A514 steel beam-columns, were studied. The study of A514 steel beam-columns was carried out both on rolled heat-treated and on welded built-up shapes. Because of the non-linearity of the stress-strain relationship of the steel and the particular pattern of residual stress, the moment-curvature-thrust curves of A514 steel shapes could differ from those of low carbon steel shapes. Therefore, a new set of moment-curvature-thrust curves, which also includes the strain reversal effect, was prepared. The end moment vs. end rotation relationship
of a beam-column was determined by a numerical method. Special attention was given to the unloading effect of reversed curvatures.

The theoretical analyses were compared with their corresponding full scale experiments. Comparisons were made between the theoretical results and the experimental results of the buckling strength of columns and the ultimate strength of beam-columns. It was shown that a good correlation exists.

A number of conclusions were drawn, and the more important were that:

1. The stress-strain relationship of A514 steel can be described well by a set of simulated mathematical equations which includes a linear equation, elastic range; a polynomial equation, transition range; and another linear equation, strain hardening range.

2. For a given cross-section and residual stress distribution, the non-dimensional tangent modulus strength of an A514 steel column is greater than that of its counterpart of low carbon steel, if the columns are in the range of low slenderness function; but smaller, if in the range of intermediate slenderness function.
3. The distribution and magnitude of the residual stresses, and thus the column strength, are influenced by the mode of manufacture of the column sections. Rolled shapes appear to exhibit the least reduction in strength due to the effects of residual stresses; shapes fabricated from sheared plates appear to exhibit the greatest reduction. Shapes fabricated from flame-cut plates appear to have a more favorable distribution of residual stresses than do those fabricated from sheared plates, but are still not as strong as rolled shapes.

4. For heavy columns, in which residual stresses vary considerably across the thickness of the component plate, the tangent modulus load analysis based on the average values of the residual stresses on the inside and outside surfaces of the cross-section gives an inaccurate prediction of column strength. The variation of residual stress in the plate elements must be considered.

5. The column strength can be predicted by the test results of stub columns provided that the relationship between the effective moment of inertia and the effective tangent
6. The ultimate strength and the load-deformation characteristics of an A514 steel beam-column can be determined by a computer program which includes the considerations of the effects of residual stress, strain reversal, shape of the stress-strain curve and unloading effects of reversed curvatures.

7. The strain reversal effect on moment-curvature-thrust relationships is more pronounced for non-linear materials than for materials which have an elastic-perfectly-plastic stress-strain curve if other conditions, such as residual stresses in the section and applied thrust are identical.

8. The unloading effects generally are insignificant for load-deformation relationships except that portion of the descending part of the moment-rotation curve which corresponds to large end rotations.

9. The critical strain at which local buckling may occur can be predicted well by performing a test on a stub column cut from the
same length from which the column or the
beam-column is fabricated.

10. The "regional criterion" provides a
significant basis for the prediction of
flange local buckling in beam-columns
bent with respect to the strong axis.

11. For a given slenderness ratio, the inter­
action curve of A514 steel beam-columns
lies above that extrapolated from the
interaction curve of A36 beam-columns;
however, the difference between then is
not pronounced.
I. INTRODUCTION

Due to the increasing demands for higher stresses in, and larger sizes of structures such as long-span bridges, heavy-welded vessels, hydrospace and space vehicles and multi-story buildings, the development and the use of weldable quenched and tempered constructional alloy steels has been greatly accelerated. High strength steels with yield strengths in excess of 200 ksi have been developed. In this study, special attention is given to A514 steel which has a yield stress of about 110 ksi and which has shown the good weldability, toughness, and strength required for use in steel structures.

A514 steel shows mechanical properties considerably different from those of the conventional low carbon steels. It has a non-linear stress-strain relationship and a yield stress approximately three times higher than that of carbon steels. Additionally, previous research shows that the magnitude of residual stresses essentially is independent of the yield stress of steels if steels are not heat-treated after rolling. Heat-treatment can reduce the residual stresses in the steel shape. Consequently, the residual stress to yield stress ratio for heat-treated A514 steel

* The word non-linear is used throughout the dissertation to describe a stress-strain curve which is partly or completely curved, to distinguish it from the conventional idealized elastic-perfectly-plastic stress-strain relationship.
shapes would be much less than that for low grade steels. The use of this high strength steel in structural members may cause somewhat different behavior than that observed in the structural carbon steel members toward which most previous investigations have been directed.

The dissertation is divided into three parts: (1) mechanical properties and residual stresses, (2) the buckling strength of centrally loaded columns, and (3) the ultimate strength of A514 steel beam-columns.

Strictly speaking, the buckling phenomenon can occur only for a theoretically perfectly straight or flat compression member. Although no such member exists in reality, it is considered that the buckling strength is the most fundamental characteristic of the compression member, and that on which the strength of practical members is dependent. Therefore, it provides the basis for column design formulas, as well as serving as the "anchor point" for beam-column interaction curves.

Extensive studies have been made on topics related to elastic-perfectly-plastic columns and to aluminum columns. In the first case, it is considered that the material has a stress-strain relationship that can be represented by two straight lines whose slopes are equal to the modulus of elasticity, and zero, respectively, and residual stresses in the section are generally included. For the second case, even though columns are loaded into
the inelastic range, these shapes generally are considered as free of residual stresses, since aluminum structural shapes are stretched after quenching to achieve straightness, and the stretching removes most of the residual stresses. For A514 constructional alloy steel columns, both the residual stresses in the section and the non-linearity of the stress-strain relationship must be considered in the buckling strength analysis. Consequently, the results of previous investigations can not be applied directly to the present study.

Furthermore, in the previous studies residual stresses are considered to be constant across the thickness of the component plates of the column. This assumption is reasonable for light thin-walled shapes, however, it is untrue for heavy shapes. Therefore, a study of the residual stresses in, and the column strength of, heavy welded built-up A36 steel and A441 steel columns, has been included also.

When a "non-linear" column containing residual stress is subjected to thrust, it will behave elastically if the material does exhibit an elastic range until the thrust reaches a certain value which causes fiber stresses in some portions of the cross section to exceed the proportional limit stress. Under a thrust less than this value and for certain geometrical limitations, the column may buckle elastically. When the thrust exceeds this
value, some parts of the cross-section enter into the transition range, or inelastic range, due to the presence of compressive residual stresses, whereas the remaining portion of the section remains in the elastic range.

As thrust is increased, it reaches a value which brings the cross section all into the inelastic range, or parts into the elastic range and the rest into the strain hardening range, and eventually all into the strain hardening range. In this study, elastic buckling refers to the column buckling when the cross section remains in the elastic range, whereas in the other cases, it is called inelastic buckling. This is to distinguish the buckling behavior of non-linear columns from the elastic perfectly-plastic columns whose buckling phenomena generally are categorized as elastic, elastic-plastic, and plastic buckling.

The combined effect of both the shape of the stress-strain relationship and the residual stress distribution on the column strength is investigated. In addition, other factors such as sectional properties, variation of residual stress across the thickness of component plates, and pattern of residual stress distribution are also discussed in detail. A computer program written in FORTRAN IV language is presented to carry out the numerical

* The stress-strain relationship of the material is assumed to consist of a linearly elastic range, a curved transition range, and a linear strain hardening range, and as that of A514 constructional alloy steel.
computation. Factors that may influence column strength can be added into the main program in subroutine subprogram forms, separately or in combination.

The buckling strength of columns can also be predicted by using stub column test results. A stub column is defined as a column long enough to retain the original magnitude of residual stresses in the section and short enough to prevent any premature failure from occurring before the yield load of the section is obtained.

For material with stress-strain relationship which can be idealized as elastic perfectly-plastic, the ratio of effective tangent modulus (or the slope of the stub column curve) to the modulus of elasticity is equal to the ratio of remaining elastic area to the total area of the cross-section. (1) Generally, for a certain given residual stress pattern and shape of the cross-section; such as that for rolled shapes of A36 steel and of small size, the moment of inertia of the remaining elastic area of the cross-section is a simple function of the elastic area itself since it is of a constant shape through the loading. Therefore, stub column results can be used directly in determining the column buckling strength.

However, for a non-linear material, the effective tangent modulus \* depends on the effective area \* instead.

\* The definition of "effective tangent modulus", "effective area" and "effective moment of inertia" is given in Section 3.6, Chapter 3, and in Chapter 6, Nomenclature and Definitions.
of the elastic area remaining. The relationship between the effective moment of inertia* and the effective tangent modulus is quite involved since the shape of the effective area may change from load to load. Explicit equations to relate the effective tangent modulus to the effective moment of inertia are not possible because of their complexity. Instead the relationship between the effective tangent modulus and effective moment of inertia may be developed by a numerical method, and presented in the form of charts, for different types of residual stress patterns and mechanical properties.

The ultimate strength and load-deformation behavior of A514 steel beam-columns are also studied. The term beam-column denotes a member which is subject simultaneously to axial force and bending moment. The bending moment in the member may be caused by externally applied end moments, eccentricity of longitudinal forces, initial out-of-straightness of axially loaded columns, or transverse forces in addition to axial forces and end moments. In this study, only the types of beam-columns which are subject to constant axial force and varying end moments are investigated. Furthermore, the beam-columns studied are assumed to be laterally supported, that is, they fail in the bending plane without twisting.

The determination of the ultimate strength of a beam-column is a problem in which inelastic action must be considered. Extensive research has been carried out in the study of the behavior of laterally supported wide-flange
shapes under combined moment and axial force, including the effect of residual stresses. Again, the methods and solutions previously developed are applicable only to elastic perfectly-plastic material and are restricted to one particular pattern of residual stress distribution.

In this study, the ultimate strength, the load-deformation behavior and the local buckling phenomenon of A514 steel beam-columns are investigated. The non-linear property of the stress-strain curve, various patterns of residual stress due to cooling after either rolling or welding, and the strain reversal effect are included in determining the moment-thrust-curvature relationship.

Instead of using the column deflection curves*, a direct "stepwise" integration procedure is used. At each change of end moment, the numerical integration is carried out at fixed stations on the deflected beam-column to determine the load-deflection relationship. The present internal moment at an integration station is recorded and compared to its loading history, and hence it can be decided whether the loading or the unloading path of the moment-thrust-curvature curve is followed in order to continue the numerical integration to the next station. Therefore, the unloading effect due to reversed internal moments can be included.

* The abbreviation CDC is used to represent column deflection curves throughout the dissertation.
The numerical computation was achieved by means of an IBM 360/65 computer. The program was written in FORTRAN IV language.

The most severe local effect that normally occurs in an H-shaped section is buckling of a localized portion of a flange or the web. Local buckling could cause considerable changes in the cross-section geometry and stress distribution. In this study, attention has been given to flange local buckling which is the more critical mode for beam-columns.

In a previous investigation, Galambos and Lay (2) have predicted that local buckling in beam-columns will occur when a critical region is strain hardened. This is the so-called "regional criterion" for local buckling. However, their study assumed the idealized elastic, perfectly-plastic and strain hardening stress-strain curve in which the strain at strain-hardening is considerably larger than the yield strain. Therefore, the comparatively small amount of residual strains are neglected. For A514 steel beam-columns, the strain at strain-hardening is approximately one and one-half times the yield strain and the residual stress effects must be considered in determining local buckling. The critical width-thickness ratio and the critical strain must be determined by separate theoretical analyses. Previous studies (3) on plate buckling with consideration of residual stresses provided an approximate solution for the present
problem, and this is verified in this study by tests of stub columns cut from the same pieces from which the beam-column specimens were fabricated. Flange local buckling of beam-columns was observed in the experiments. It is found that the "regional criterion" gives a good prediction of the occurrence of local buckling when the critical strain is known.

This dissertation presents an extensive analysis of the buckling strength of centrally loaded columns and the ultimate strength of beam-columns made of A514 steel. Both residual stresses and the non-linearity of the stress-strain relationship of material have a pronounced influence on the prediction of column or beam-column strength.
2. MECHANICAL PROPERTIES AND RESIDUAL STRESS

2.1 Stress-Strain Relationship

Stress-strain curves and the related mechanical properties are the basic means of determining the quality and the usefulness of metals and of providing fundamental knowledge for use in the design of metal structures and parts. The structural metals that are widely used at present may be divided into four categories: (1) structural carbon steels, (2) high strength low-alloy steels, (3) constructional alloy steels, and (4) aluminum alloys. These metals have two different types of stress-strain curves -- those indicating a yield point, and those not indicating this. For structural carbon steels and high strength low-alloy steels, the stress-strain curves are of the first type; that is, the stress is linearly proportional to strain up to the yield point and thereafter is constant or nearly constant over a large range of strain. Therefore, their mechanical properties can be characterized simply, by such items as modulus of elasticity, upper yield point, static yield level, strain-hardening strains, and strain hardening modulus as shown in Fig. 2.1. For constructional alloy steels and aluminum alloys, the stress-strain curves are of the second type; that is, the stress deviates from a
linear relationship with strain at stresses below the yield strength and usually does not exhibit a region in which the stress remains constant over a large range of strains. There is no apparent yield point or yield stress level in this second type of stress-strain curve. Usually, a nominal yield point is determined by the 0.2% strain offset method. This stress-strain relationship with no apparent yield point will be described as a "non-linear" relationship in this dissertation.

To describe the non-linear curves, Ramberg and Osgood have developed a set of curves in terms of three parameters: namely, the modulus of elasticity and two secant strengths. The comparison of these curves with those from tests of aluminum alloy, stainless steel and chromium nickel steel sheets, shows a satisfactory agreement. The dimensionless Ramberg-Osgood curves (from Ref. 5) are shown in Fig. 2.2. The Ramberg-Osgood equation is:

$$\frac{E\epsilon}{\sigma_1} = \frac{\sigma}{\sigma_1} + \frac{1-m}{m} (\frac{\sigma}{\sigma_1})^n$$

where:

E, \sigma and \epsilon are modulus of elasticity, stress and strain, respectively,

n is a constant dependent on the shape of the stress-strain curve,
\( \sigma_1 \) is the secant yield strength which is equal to the stress of the intersection with the stress-strain curve of a line through the origin having a slope equal to \( mE \). It is usually chosen in such a way that it is equal to \( \sigma_y \) (static yield stress determined by 0.2% offset method).

Although the Ramberg-Osgood representation fits the stress-strain curves of most metals used in aircraft construction, it cannot be used to describe the stress-strain relationships of constructional alloy steels, such as A514 steel, simply because the stress-strain curves of these steels usually approach a straight line with a very small slope after the "knee" portion of the curve. In this dissertation, a new type of mathematical equation was developed to represent the stress-strain relationship of A514 steel. Details of the mechanical properties of A514 constructional alloy steel are described in the next section, Section 2.2.

All of the stress-strain curves used in this study were presented in dimensionless form. This was done to eliminate the variations in modulus of elasticity and yield stress, so that only the characteristic shape of the curve will remain. Therefore, as long as the shape of the stress-strain curves are the same, even though their yield stress and/or modulus of elasticity may be different, one dimensionless equation can be used to describe all of them.
Furthermore, such simplification enables research to be carried out on more generalized bases and the results thus obtained can later be applied to many other materials, even for those still undeveloped, so long as the shape of the stress-strain curve is the same.

2.2 Mechanical Property Tests of A514 Steel

A total of fifty-eight tension coupon tests and eight compression coupon tests were conducted on specimens taken from various rolled H-shapes and plates made of A514 steel. The tension coupon dimensions were determined according to the ASTM Standards for the tension test specimen. The speed of testing for tension coupons was within the recommended ASTM limits, that is, the crosshead speed did not exceed 1/16 in. per minute per inch of gage length. The load-elongation curve was plotted by an automatic recording device. After exceeding the elastic limit, the testing machine was stopped at appropriate strain intervals to determine the stress-strain relationship at a zero strain rate. A typical stress-strain curve obtained from this type of test is shown on Fig. 2.3. Table 2.1 gives the results of all the tension coupon tests. Figure 2.4 shows the histogram plots for the mechanical properties of A514 steel.

From the test curves, it can be observed that the proportional limit ranges from 0.65 \( \sigma_y \) to 1.0 \( \sigma_y \) with an average
value of 0.825 $\sigma_y$ and that the curve is a straight line after the yield stress, the yield stress being obtained by the 0.2% offset method.\(^{(4)}\) In order to determine a representative stress-strain curve from all the test results, a method suggested by the Column Research Council \(^{(1)}\) was used. From the proportional limit to the yield stress, the strain departures from the modulus line (Fig. 2.3) for various fixed percentages of individual yield stress were recorded. For the case when not enough static points were taken in the transition part of the stress-strain curve, a method developed by Cozzone and Melcon\(^{(7)}\) was used to determine the transition portion of the static stress-strain curve; as shown in Fig. 2.5, a line OA' is drawn from the origin to the static yield point (determined by 0.2% offset) and extended to intersect the "dynamic" stress-strain curve at point A. To obtain the static stress-strain curve several lines were drawn as OB, OC, OD and the corresponding static points B', C', D' were determined. For example, the point B' was determined by means of the equation OB' = OB x OA'/OA. The curve through B', C', and D' is the static stress-strain curve in the transition region. Dividing the measured strain by $\varepsilon_y$, where $\varepsilon_y$ is equal to $\sigma_y/E$, and averaging all the offset values at the same stress level, a representative stress-strain curve in dimensionless form was obtained as shown in Fig. 2.6a.
The stress-strain equation in its transition portion ("non-linear" part) was determined as shown in Fig. 2.6b. It was found that a polynomial equation of the form

\[
\frac{\sigma}{\varepsilon_y} = A_o + \sum A_n \left( \frac{\varepsilon}{\varepsilon_y} - \frac{\varepsilon_{st}}{\varepsilon_y} \right)^n
\]

(2.2)

\( n = 1, 3.5 \)

fitted experimental results satisfactorily. The constants \( A_o \) and \( A_n \) can be determined by fitting the experimentally obtained points in the transition region of the stress-strain curve. Since the slope of the stress-strain curve is constant after the transition range, strain-hardening may be assumed to occur immediately after the yield stress, that is,

\[
\frac{\varepsilon}{\varepsilon_y} = \frac{\varepsilon_{st}}{\varepsilon_y}, \quad \frac{\sigma}{\varepsilon_y} = 1 \quad \text{and} \quad \frac{E_t}{E} = \frac{E_{st}}{E}
\]

Therefore, \( A_o = 1.0 \), \( A_1 = \frac{E_{st}}{E} \)

Other constants can be determined similarly. However, it was found that, just by satisfying the conditions at the proportional limit, that is,

\[
\frac{\varepsilon}{\varepsilon_y} = \frac{\varepsilon_p}{\varepsilon_y} \quad \text{then} \quad \frac{\sigma}{\varepsilon_y} = \frac{\varepsilon_p}{\varepsilon_y} \quad \text{and} \quad \frac{E_t}{E} = 1,
\]

a 5th order polynomial is sufficient to describe the transition portion of the actual stress-strain curve. Therefore,
for any A514 steel shape or plate, the constants in Eq. 2.2 can be determined if coupon test data of $E$, $E_{st}$, and $\varepsilon_y$, $\varepsilon_{st}$, $\sigma_y$ are available.

The stress-strain curve for the A514 steel can be, therefore, described by the following three equations:

\[
\frac{\sigma}{\sigma_y} = \frac{\varepsilon}{\varepsilon_y} \quad \text{when} \quad 0 \leq \frac{\sigma}{\sigma_y} \leq 0.8
\]

\[
\frac{\sigma}{\sigma_y} = 1.0 + 0.005 \left( \frac{\varepsilon}{\varepsilon_y} - 1.52 \right) + 0.3647 \left( \frac{\varepsilon}{\varepsilon_y} - 1.52 \right)^3
\]
\[
\quad + 0.3276 \left( \frac{\varepsilon}{\varepsilon_y} - 1.52 \right)^5
\]
\[
\text{when} \quad 0.8 \leq \frac{\sigma}{\sigma_y} \leq 1.0 \quad (2.3)
\]

and \[
\frac{\sigma}{\sigma_y} = 1.0 + 0.005 \left( \frac{\varepsilon}{\varepsilon_y} - 1.52 \right)
\]
\[
\text{when} \quad 1.0 \leq \frac{\sigma}{\sigma_y}
\]

A comparison with the experimentally obtained typical stress-strain curve and tangent modulus curve in the transition region is shown in Fig. 2.6b. The accuracy of Eq. 2.3 appears to be adequate.

Several compression coupon tests were conducted and the results are shown on Table 2.2. The size of specimen used for compression tests is in accordance with the Recommendation of Column Research Council (1) and no lateral supports were used since the coupon itself was sufficiently stocky.
It was observed that both the modulus of elasticity and the yield stress determined by compression coupon tests are nearly the same as those obtained from tension coupon tests. However, from the compression specimen tests, the transition portion of the stress-strain curve has a sharper knee and a higher strain hardening modulus than those of tension coupon tests. Presumably, the end bearing effect of the compression test and the tempering and quenching treatment could have caused the steeper stress-strain curves obtained from the compression tests.

The difficulty of preparation of specimens and alignment make the compression coupon test less desirable. In this study, because of the limited number of compression coupon tests conducted, no statistical analysis could be performed and the results obtained from tension coupon tests were taken as representative of the mechanical properties of A514 steel for both tension and compression, even though actually they are slightly different from each other.

2.3 Residual Stress

Due to the importance of the effects of residual stress on the behavior of structural members, especially compressive members such as columns, much research has been carried out in this field during the past decade. Residual stresses appear as a consequence of the plastic deformation of material. The sources of this plastic deformation can be many, such as thermal stresses due to uneven cooling
of various parts of the structural shapes, cold bending or straightening of the member.

During the cooling process for a rolled section, the area of surface in contact with the cooling medium, either air or liquid, usually is more at the edges of the section than the center portion. This causes a faster cooling rate at the edges and normally forms compressive residual stress there and tensile residual stress at the center and junctions. For a welded built-up section, welding causes temperatures to rise rapidly in the region near the welding line while most of the remaining portion of the section is unaffected. However, when the weld-affected zone starts to cool, the rest of the section gradually rises in temperature. The whole section on cooling to the ambient temperature experiences non-uniform thermal changes that cause nonhomogeneous plastification, and thus the formation of residual stress. Residual stresses due to welding or cooling from rolling are simply thermal stresses remaining when the material has cooled to ambient temperature. (8)

Theoretical studies on the analysis of thermal residual stress have been made by Boulton and Lance Martin (9), Gruning (10), Rodgers and Fetcher (11), Weiner (12), Tall (13), and Estuar (14). However, the difficulty of theoretically predicting thermal residual stress in plates or shapes is due to the uncertainty of many
variables which affect the temperature distribution and thermal stresses. Therefore, to simplify the problem for theoretical analysis, quite a number of assumptions, sometimes far removed from reality, must be made. (Reference 13 has a detailed discussion of this). Consequently, theoretically obtained values of residual stress generally do not give satisfactory correlation with the actual measurements. Furthermore, other kinds of residual stresses such as those due to cold bending or rotary straightening may exist together with thermal residual stress, complicating the theoretical analysis even further. At present reliance on actual residual stress measurement is necessary.

Extensive experimental investigations of residual stress distribution in steel shapes of rolled and welded built-up sections, in both low carbon and high strength steels, have been conducted by Ketter, Huber, Fujita, Beedle, and later by Tall, Estuar, Rao, and Nishino (3), (13), (14), (15), (16), (17), (18), (19). Several conclusions may be drawn from their studies:

1. If consideration is restricted to residual stress distribution, a welded built-up cross-section may be considered separately as plates with welds, provided the relative sizes of plate elements are nearly the same. Otherwise, the effect of restraint of the adjacent plates has to be taken into consideration.
2. Flame cutting, as well as edge welding, causes tensile residual stress at the flame-cut or welded edge.

3. Weld sizes, electrode strength, whether manual or automatic welding, and number of weld passes have no significant effect on the shape and magnitude of residual stress in plates or welded built-up sections provided their geometric shapes are the same.

4. Welded built-up shapes have residual stresses considerably higher than those in rolled shapes.

5. For rolled steel shapes, the magnitudes of the residual stresses are independent of the yield stress of the material, that is, regardless of which steel a shape was made, the magnitude of the compressive residual stress at the flange edges is approximately the same. Heat-treatment may lower the magnitude appreciably as often occurs in A514 rolled steel shapes which have a compressive residual stress at the flange edges of about 5 ksi. Patterns of residual stress distribution in rolled steel shapes may be represented by straight lines as shown in Fig. 2.7a.

6. Welding residual stresses in steel shapes may be approximated by several straight lines, with the
tensile residual stress approaching the yield stress of the weld metal at the weld. For flame-cut plates, tension residual stresses often exist at the flange tips. However, for universal mill plates or plates with sheared edges, compressive residual stresses generally exist at the flange tips. The approximate patterns of residual stress distribution in welded built-up shapes are shown in Fig. 2.7b.

7. For plates or sections with a maximum thickness of less than one inch, residual stresses may be assumed to be the same across the thickness. For sections with a thickness greater than one inch, variation of residual stress through the thickness must be considered in the prediction of the column strength.

8. The variation of residual stress in any fiber along a member which has not been cold bent or otherwise mechanically straightened is small.
3. **BUCKLING STRENGTH OF CENTRALLY LOADED COLUMNS**

3.1 **Scope and Purpose of the Study**

Since the publication of Shanley's papers\(^{(20)(21)}\), the tangent modulus load has been accepted as the smallest value of the axial load at which bifurcation of equilibrium can occur, and has therefore been considered as the design criterion for a centrally loaded column, although the actual ultimate load of a perfectly straight column should always be slightly higher than the tangent modulus load.

In the past decade, a great deal of research has been carried out on the column buckling strength analysis. The most significant contribution of previous investigations is the discovery and recognition of the importance of the influence of residual stresses on column strength. However, almost all of the previous research was restricted in application to materials having an elastic perfectly-plastic stress-strain curve. For other materials which exhibit a non-linear type of stress-strain relationship, residual stress effects were either neglected or accounted for by means of an empirical formula. Nevertheless, residual stresses do influence the buckling strength of columns made of materials with a non-linear type stress-strain curve. This chapter is devoted mainly to a discussion of the combined effects
of residual stresses and the shape of the stress-strain curve on the buckling strength of columns.

In addition, other factors such as patterns of distribution, magnitude and variation of residual stresses across the thickness and geometric properties of the cross-section, which may have a pronounced influence on the strength of intermediate length columns, are also discussed.

A numerical method using a digital computer was developed to compute the buckling strength of columns considering a number of influencing factors such as those mentioned above. All the considerations, such as the stress-strain relationship and residual stress distribution, were put into a subroutine subprogram form. Therefore, by combining different subroutine programs with the main program, those influencing factors, alone or in combination, were included. Although it can be used for sections of other shapes as well; with slight modification, the computer program presented is limited to H- and box-sections.

A method of predicting buckling strength based on stub column test results is presented. Certain charts were designed to permit computations of buckling strength directly from the stub column test results.
3.2 **Fundamental Assumptions**

The following assumptions serve as the basis for the subsequent studies:

1. The column is initially perfectly straight.

2. Plane sections remain plane before and after bending.

3. The stress-strain relationship in any "fiber" of the column is the same as that observed in a tensile coupon.

4. The magnitude and pattern of distribution of residual stress are the same at any cross-section of the column.

5. The effect of shear deformation is neglected.

6. The external load is applied axially to the centroid of the cross-section causing uniform strain over the cross-section and through the whole length before bifurcation.

7. The cross-section is constant along the length of the column.

8. The section is such that the possibility of torsional buckling can be precluded.

3.3 **Basic Theory**

An initially straight axially loaded prismatic column will maintain its straight configuration up to a critical
load at which it can be in equilibrium in either a straight or slightly bent position. Based on the Engesser-Shanley theory, at the instant of bifurcation, the requirements of equilibrium of internal and external forces are (see Fig. 3.1)

\[ \int_A \Delta \sigma dA = 0 \]

and

\[ \int_A \Delta \sigma y dA = P v \] (3.1)

where \( \Delta \sigma \) is the increment of stress located a distance \( y \) from the neutral axis (or the axis of constant strain). \( P \) is the external axial force and \( v \) is the deflection in the direction of the axis \( y \). For an infinitesimal amount of bending the increments in stress predicted by small deflection theory are

\[ \Delta \sigma = 0 \quad E_t \cdot y = - \frac{d^2 v}{dx^2} \quad E_t y \] (3.2)

Here \( E_t \) is the tangent modulus corresponding to the axial stress at the point. Substitution of Eq. 3.2 into 3.1 yields the expressions

\[ \int_A E_t y dA = 0 \] (3.3a)

and

\[ \frac{d^2 v}{dx^2} \int_A E_t y^2 dA + P v = 0 \] (3.3b)

in which \( x \) is the coordinate along the length of the column.
From Eq. 3.3a, the location of the neutral axis is determined. For a symmetrical section with a symmetrical residual stress distribution, this axis coincides with one of the principal axes of the section.

The critical load $P_{cr}$ is obtained from Equation 3.3b as

$$P_{cr} = \pi^2 \int_A E_t y^2 dA$$

where $L$ is the effective length of the column.

It should be apparent that the buckling strength of a column depends on the tangent modulus of each elemental area and therefore is a function of residual stress distribution as well as material properties.

If the column remains in the elastic range up to the critical load, then $E_t = E$ over the whole cross-section and the critical load is the Euler load.

$$P_{cr} = \pi^2 EI/L^2$$

For inelastic material without residual stress in the section, inelastic behavior is homogeneous through the column and the critical load is given by the expression

$$P_{cr} = \frac{\pi^2 E_t I}{L^2}$$
In the presence of residual stresses, the tangent modulus may vary from point to point on the section for a stress-strain relationship of either elastic-perfectly-plastic or non-linear type. The calculation of critical loads becomes much more complicated. If the notation $I_m$, effective moment of inertia, is introduced,

$$I_m = \int_A \frac{E}{E} y^2 dA \quad (3.7)$$

then,

$$P_{cr} = \frac{\pi^2 EI_m}{L^2} \quad (3.8)$$

The computation of $I_m$ is discussed in Section 3.4.

3.4 Numerical Procedure

The numerical method of computation is developed for columns of H- or box cross sections, containing either cooling or welding type residual stresses. The stress-strain curve of the material is assumed to be of either the non-linear type or the elastic-perfectly-plastic type. However, the method is, by its nature, applicable for columns with any kind of residual stress distribution and stress-strain curve, and it is suitable for computation by a digital computer.

As shown in the foregoing section, in order to evaluate the buckling load $P_{cr}$, the effective moment of inertia
I_m must first be calculated. Here I_m depends on the residual stress distribution, the magnitude of applied force, and the stress-strain relationship of the material. Generally, it will not be practical to calculate P_{cr} directly (22); instead, the equivalent length L is determined. The numerical computation is accomplished as follows:

1. Divide the section into a sufficient number of finite area meshes as shown in Fig. 3.2.

2. Record the residual strain at the center of each mesh (assuming the residual stresses distributed on each mesh are uniform and the same as that at the center point of the mesh).

3. Assume a uniform strain applied to the column. The total strain at a point is equal to the residual strain plus the applied uniform longitudinal strain.

4. From the tangent modulus-strain equation and the stress-strain equation, determine the tangent modulus and the stress, respectively, corresponding to the total strain in each mesh determined in step 3.

5. Sum up the internal axial force on all the meshes \[ P = \int A \sigma dA \] and compute the modified moment of inertia I_m from Eq. 3.7.
6. Compute the equivalent column length for the calculated $P$ and $I_m$

$$L = \frac{\sqrt{EI_m}}{P} \cdot \beta$$

(3.9)

7. Increase the applied uniform longitudinal strain and repeat steps 1 through 6 until the entire column strength curve is obtained.

For dimensionless analysis, equation 3.9 can be rewritten in the form

$$\lambda = \frac{1}{\kappa} \cdot \sqrt{\frac{\sigma_y}{E}} \cdot \frac{L}{r} = \sqrt{\frac{I_m/I_p}{P_{cr}/P_y}}$$

(3.10)

The function $\lambda$ defined by Eq. 3.10 will hereafter be referred to as slenderness function. The dimensionless analysis in this fashion will eliminate the material properties, such as $\sigma_y$ and $E$ in the computation. Only the shape of the stress-strain curve and pattern and the ratio of residual stress distribution are left as variables.

Numerical computation was carried out by means of a digital computer. All of the programs were written in Fortran IV language. Programs were prepared for rectangular, box- and H-columns. For symmetric sections with symmetric residual stresses, the cases considered in this chapter, only one-quarter of the section need be worked in the computation of the buckling strength of columns. The flow charts of the main programs and the subroutine programs are shown in Appendix A.
3.5 Factors Influencing Buckling Strength of the Columns

Many factors, such as shape of the cross-section, pattern and magnitude of residual stress distribution, and variation of residual stress across the thickness, may effect the shape of a column buckling curve. Although any distribution of residual stress can be considered, only idealized patterns of residual stress distribution as shown in Fig. 2.7 were used for the illustrations. The triangular distribution (Fig. 2.7a) is similar to the pattern observed in A514 steel rolled heat-treated shapes whereas the other two (Fig. 2.7b) resemble the pattern in welded shapes of A514 steel; one for sections made of shear cut plates and the other, flame-cut plates.

Effect of the Geometric Properties of the Section

To demonstrate the effect of the geometric configuration of the section, two shapes, 8WF31 and 27WF94, which are quite different in their geometric properties, were selected. Material properties were selected to correspond to those of A514 steel. The column curves for both strong and weak axis buckling are shown in Fig. 3.3. Column curves for strong axis bending (Fig. 3.3a) are slightly different for the two columns. However, the maximum difference is only approximately 2% of the buckling strength. Column curves for weak axis buckling (Fig. 3.3b) are almost identical for the two sections. For strong axis buckling
the web area contributes a larger percentage to the total moment of inertia than it does for weak axis buckling, for which it is virtually negligible compared to the percentage contributed by the flange area. Therefore, sectional properties for a H-shaped section do not affect the column strength for weak axis buckling but do influence the column curve for the strong axis buckling. However, for the commonly used sections, even for strong axis buckling, the effect of geometric properties on column strength can be neglected. That is, for a given residual stress distribution and a stress-strain curve, one column curve in dimensionless form may be sufficient to describe the basic column strength for most commonly used sections.

Effect of the Stress-Strain Curve

The influence of the shape of the stress-strain curve on the column strength is illustrated in Figs. 3.4 and 3.5. Two types of stress-strain curves, the elastic-perfectly-plastic type and that of A514 steel, were used for comparison. The residual stress distribution assumed is shown on the graphs. For strong axis bending of rolled shapes (Fig. 3.4a), the elastic-perfectly-plastic material has a higher nondimensional column strength than does A514 steel. However, with the increase of residual stress magnitude, the effect of mechanical properties of the material diminishes, and the column curves for both materials approach each other. For weak axis bending of rolled shapes (Fig. 3.4b), elastic-perfectly-
plastic material has a higher strength for medium length columns, whereas for short columns, A514 steel rolled shapes have a higher column strength.

A comparison of column curves for a residual stress pattern similar to those existing in welded built-up shapes made of sheared edge plates is shown in Fig. 3.5. A pronounced difference for A514 steel is the absence of the sudden jumps (discontinuities) of the column curves due to abrupt yielding over a large portion of the cross section when an elastic perfectly-plastic stress-strain curve is assumed. Again, for both strong and weak axis bending, elastic-perfectly-plastic materials show higher strength for columns of medium length and lower strength for short columns.

It is apparent from the foregoing discussion that the shape of the stress-strain curves does have a pronounced influence on the buckling strength of columns. If a non-linear type stress-strain curve is modified by replacing its knee portion with two straight lines similar to the stress-strain curve for elastic-perfectly-plastic material, it will result in either an overestimate of column strength in the medium slenderness region or an underestimate of strength for short columns.
Effect of Pattern of Residual Stress Distribution

The pattern of residual stress distribution in a section is controlled by the manufacturing or fabricating process. Pronounced differences in column strength can occur due to the pattern of the residual stress distribution. To demonstrate its influence on the column strength, three typical residual stress patterns, as shown in Fig. 3.6a, were selected for the column curve computation. The maximum compressive residual stresses are the same, $0.1\sigma_y$, for all the cases, however, the pattern of residual stress distribution differs. These residual stress patterns were similar to those in (1) rolled columns (2) welded columns with sheared plates and (3) welded columns with flame-cut plates, all of A514 steel. The column curves obtained are shown in Figs. 3.6b and 3.6c. Among these, rolled columns provided the highest column strength; the welded columns with flame-cut plates next and the welded columns with sheared plates the lowest.

Generally speaking, the less spread are the compressive residual stresses over the cross-section, the less their influence on the column strength. Also, if the material farthest from the axis of bending is in a state of residual compression, column failure occurs
at a lower load than would otherwise be expected. It can be observed in Figs. 3.6b and 3.6c that the difference in column strength between columns with flame-cut plates and those with sheared edge plates was more pronounced for weak axis bending than for strong axis bending. However, no matter what the pattern of residual stress distribution is, it always will cause a reduction in column strength. Optimum column strength can be obtained for straight columns only in the absence of residual stress, leading to strengths close to the homogeneous* inelastic buckling load or to the Euler load, whichever is reached first.

Effect of Variation of Residual Stresses Across the Thickness of the Component Plates

With the progress of welding techniques and the requirement of large heavy sections in modern structures, welded heavy sections are frequently being used in construction. However, the structural behavior of such heavy members, especially if they are used as compressive members, still needs investigation. A study of the residual stresses in, and the column strength of heavy built-up A36 steel and A441 steel columns has been carried out at Fritz Laboratory, Lehigh University.

* Homogeneous here means that the axial stress at every point on the cross-section is the same.
Preliminary tests and residual stress measurements have been conducted on eight heavy 15H290 shapes where H designates a welded H-shape. Details of the test program and experimental results are given in Reference 23. The main findings in this pilot study are that the residual stresses vary appreciably across the thickness of the flanges of the heavy welded built-up shapes and their effect on the column strength might differ from that on commonly used thin sections, both rolled and welded built-up. Hence, it is necessary to investigate the effect of the variation of residual stress across the plate thickness on the column strength.

Previous investigations\(^{(14),(23)}\) indicated that the variation of residual stress across the thickness of the component plates of 15H290 sections is either a straight line or else slightly parabolic in nature. The penetration of the weld, that is, comparing groove or fillet welds, does not make any significant difference in the residual stress magnitude or distribution. Only the method of preparing the component plates, whether they are universal mill or flame-cut, will alter the pattern of residual stress distribution substantially. The influence of the grade of steel was also investigated. It was found that for H-shapes built up from universal mill component plates, those made of A441 steel and those of A36 steel do not show significant difference in residual stress
distribution. However, for welded built-up shapes with flame-cut plates, those made of A441 steel have tensile residual stresses at the flange tips higher than do those of A36 steel, nearly equal to the yield stress of the material. This may indicate that for UM plate welded H-shapes, regardless of the grade of steel of the component plates, the residual stress distribution is a similar type; but for flame-cut welded H-shapes, the different grades of steel must be treated separately.

In this study, the effect of different patterns of residual stress variation across the thickness on the strength of columns is investigated. The theoretical analysis is performed on a section which has the same dimensions as those of 15H290 shapes (Fig. 3.7) and of A36 steel. Patterns of residual stress variation considered are; constant, linear and parabolic, across the thickness of the flanges, as shown in Fig. 3.11. Several idealized residual stress patterns are used to represent the residual stress distribution in the direction of the width of the component plates.

In order to have a close resemblance between the assumed idealized residual stress distribution and the actual measurements, the significant residual stress measurements on the surface of the 15H290 shapes (measurements before the final sectioning along the thickness), are summarized and tabulated as shown in Table 3.1. The idealized residual stress patterns on the surface of the section
are obtained by fitting polynomial equations passing through the average measured residual stresses at several key locations, as shown in the figures in Table 3.1. Also, the idealized residual stress patterns on the outside and inside surfaces of one half of a flange are shown as lines 1 and 7 in Fig. 3.7 for A36 steel 15H290 shapes with UM plates and lines 1 and 10 in Fig. 3.8 for A36 steel 15H290 shapes with flame-cut plates. The residual stresses are assumed to be distributed parabolically on the web of the heavy shapes with maximum compressive residual stress at the center of the web and are constant through the web thickness. For linear variation of residual stresses across the flange of the section, the topographic plots of the residual stress distribution are also shown in Fig. 3.7 and 3.8, again for one-half of a flange only. The actual residual stress measurements are also shown in Fig. 3.9 and 3.10 for an A36 steel shape and an A441 shape, respectively. It is observed that the idealized residual stress distributions are similar to those experimentally obtained.

The advantage of representing the residual stress distribution by contour lines (Fig. 3.7 to Fig. 3.10) is that not only the nature of residual stress distribution, but also the penetration of yielding, as load is applied, can be shown clearly in the same figure. For example, in Fig. 3.7, if the applied $\frac{P}{P_y}$ is equal to 0.6, that portion of the flange on the right of the contour line
0.4 will be yielded and contour line 0.4 becomes the boundary line between the elastic area and the yielded area. If so desired, these topographic plots can be used directly to determine the moment of inertia of the remaining elastic area and the effective tangent modulus at each loading level.

It should be noted that the calculated residual stresses at the junctures of the flanges and web plates sometimes are greater than the yield stress of the parent material. This is because of the higher yield stress of the weld metal; however, this was assumed equal to the yield stress of the parent metal for the theoretical analysis. This simplification is regarded as justified because the weld metal covers only a negligible percentage of the total area of the cross section.

The column curves are calculated based on the idealized residual stress distribution on the inside and outside surfaces of the sections. For constant variation, that is, no variation of residual stresses across the thickness of component plates, the magnitude of residual stress at a location is determined as the average value of those on the inside and outside surfaces. For parabolic variation, residual stresses on the surfaces are in general raised \(0.2 \sigma_y\) in magnitude to achieve internal stress balance and to represent the most severe difference in measured residual stresses across the plate thickness. However, the pattern of the residual stress distribution on the surfaces remains
the same. Naturally, this is not an attempt to cope with the actual residual stress variation across the thickness in the section, in which the amplitudes of the parabolas could vary from location to location, but only to try to have a more severe representation which can clearly indicate the direction of the effect of the patterns of variation of residual stresses on the strength of columns.

The column curves for both shapes with UM plates and shapes with flame-cut plates are presented in Fig. 3.11a and 3.11b, respectively. It was observed that use of the parabolic or linear variation of residual stress across the thickness does not make a significant difference in the column strength, except in the high slenderness ratio range (Fig. 3.11). This may indicate that for those portions of the section which show a non-linear variation of residual stress in the direction of plate thickness, assuming a linear variation by reducing the surface residual stresses an equal amount but still maintaining the same "residual stress area", as shown in Fig. 3.11, may have no appreciable influence on the column strength prediction.

In Fig. 4.11, it is apparent that using an average value of the inside and outside surface residual stresses, assumed to be constant across the plate thickness, results in an overestimation of the column strength for strong axis bending for shapes with either UM or flame-cut plates.
However, for weak axis bending, it results in a lower column curve for shapes with UM plates but a higher column curve for shapes with flame-cut plates if they are compared with the corresponding column curves obtained by assuming linear variation of the residual stress through the thickness.

This phenomenon can be attributed to the different shape and size of the remaining elastic areas at a certain load, as shown in Fig. 3.12. From a numerical analysis, it was found that if the variation of residual stress across the thickness is assumed linear the remaining elastic area under a given load level is smaller than that if the residual stress is assumed constant and equal to the average of inside and outside surface measurements (Fig. 3.13). Also, the distribution of the elastic area, or the shape of the area is different. Figure 3.12a illustrates the penetration of yielding for shapes with UM plates; it shows that when constant residual stresses are assumed, more elastic area moved closer to the y-axis and away from the x-axis. Consequently, compared to the cases of varying residual stresses, at the same load level, the corresponding L/r generally is smaller for columns bent about the weak axis but larger for those bent about the strong axis. The increase of elastic area can overcome this effect somewhat at higher load levels, for weak axis bending, where there is a slightly higher strength for constant residual stress across the thickness (Fig. 3.11a). Furthermore, because the elastic area is usually farther
from the x-axis than the y-axis, this effect of the shapes of elastic area is more pronounced for strong axis bending than for weak axis bending. For the shapes with flame-cut plates, similar effects are observed. However, for weak axis bending, the increase of elastic area near the flange tips overcomes the effect of that portion of the elastic area which moves toward the y-axis, and in fact yields a larger $I_e$ compared to that based on the linearly varying residual stresses. Therefore, as contrasted to the case for shapes with UM plates, higher column strength is predicted in FC shapes for weak axis bending when residual stresses are constant across the thickness of the plates.

The advantage of using shapes with flame-cut plates can be clearly seen in Fig. 3.11; both weak axis and strong axis buckling strengths are higher than those corresponding to shapes with UM plates, especially for weak axis bending. In fact, for flame-cut preparation of the component plates, part of the column curve for weak axis bending is even higher than that for strong axis bending. The meaning of the terms "weak axis" and "strong axis" have lost their significance here. They have been retained as a matter of convenience to designate the principal axes parallel to the web and the flanges, respectively.

The study of the strength and behavior of heavy welded and rolled shapes is still underway at the present
3.6 Estimation of Column Buckling Strength from Stub Column Test Results

The computation of the buckling load for columns can be divided into two parts: (1) those based on measured or assumed residual stress patterns and (2) those based on stub column tests. The first method has been discussed in the previous sections. In this section, the second method is described.

A stub column is defined as a column long enough to retain the original magnitude of residual stresses in the section and short enough to prevent any premature failure from occurring before the yield load of the section is obtained\(^{(24)}\)(\(^{(25)}\)). A stub column test is performed in order to obtain an average stress-strain curve for the complete cross-section which takes into account the effects of residual stresses.

From a stub column test, the effective tangent modulus, which is a slope of the average stress-strain curve obtained from the stub column test, is defined as

$$E_m = \frac{d\sigma_{ave}}{d\varepsilon} = \frac{dP/A}{\int_A E_t dA} = \frac{\int_A E_t dA}{A} \quad (3.11)$$
If designate \[ \int \frac{E_t}{E} \frac{dA}{A} = A_m \] (3.12)

where, \( A_m \) is the area of the effective section which is defined as the equivalent elastic section that functions in the identical manner as the actual section in which part or all of the section is in the inelastic range; \( A \) is the original cross-sectional area; and \( E_t \) is the tangent modulus from the tangent modulus-strain curve, then, from equations (3.11) and (3.12),

\[ \frac{E_m}{E} = \frac{A_m}{A} \] (3.13)

It can be observed that the term \[ \int \frac{E_t}{E} y^2 dA \] (or \( I_m \)) in equation (3.7) is the moment of inertia of the effective section \( A_m \). Consequently,

\[ I_m = f(\frac{A_m}{A}) = f(\frac{E_m}{E}) \] (3.14)

where the function \( f \) will depend on the shape of the effective section.

For rolled or welded built-up "thin" H-shapes which are made of material having an elastic perfectly-plastic stress-strain relationship and have a piecewise smooth residual stress pattern with the maximum compressive residual stresses at the flange tips, the shape of the effective area, or elastic area in this case, remain
rectangular for flanges and web through the complete loading process \((6)\).

In Reference 6, the approximate relationships between \(\frac{I_e}{I} \) and \(\frac{E_m}{E} \) were presented as \((6)\)

\[
\frac{I_{ex}}{I_x} = \frac{E_m}{E}
\]

and

\[
\frac{I_{ey}}{I_y} = \left(\frac{E_m}{E}\right)^3
\]

where \(I_{ex} \) and \(I_{ey} \) are moments of inertia of the elastic portion of the section with respect to strong axis and weak axis, respectively.

However, for materials with other than an elastic perfectly-plastic stress-strain relationship or sections in which the maximum compressive residual stresses are not at the flange tips, the relationship between \(I_m/I \) and \(E_m/E \) is more complex, especially for the weak axis bending of an H-shape.

To illustrate this fact, the explicit solution was obtained for a rectangular section with a triangular type residual stress distribution and a stress-strain curve represented by two straight lines connected by a parabolic transition curve as shown in Fig. 3.14a. Only
one-dimensional residual stresses were considered in this example, and the bending axis parallel to the direction along which residual stress varies was designated as the x-axis and the other axis as the y-axis (Fig. 3.17b). For bending about the y-axis,

\[ I_{my} = \int_{A_m} \frac{E_t}{E} x^2 \, dA_m = \int_{A_m} x^2 \, dA_m \quad (3.16) \]

where \( dA_m = (b \frac{t}{E}) \, dx = \bar{d} \, dx \).

And for bending about the x-axis,

\[ I_{mx} = \int_{A_m} \frac{E_t}{E} y^2 \, dA_m = \int_{A_m} y^2 \, dA_m \quad (3.17) \]

where \( dA_m = (\frac{E_t}{E} \cdot dx) \cdot d = \bar{d} \cdot dx \).

Both \( \bar{d} \) and \( d \) are the width of a differential element of area of the effective sections (Fig. 3.14c) for y- and x-axis bending, respectively. The dimension of the cross-section in the direction perpendicular to the bending axis cannot be modified without affecting the integrations of Eqs. 3.16 and 3.17. Hence, the effective area must be obtained by changing only the width of the
original section. Therefore, the modified area for x-axis bending remains a rectangle without any change in its depth. The relationship between $I_{mx}$ and $A_m$ is

$$\frac{I_{mx}}{I_x} = \frac{A_m}{A} = \frac{E_m}{E}$$

Equation (3.18) is always valid irrespective of the type of stress-strain relationship and the pattern and magnitude of residual stress distribution, provided the residual stress does not vary through the depth of a rectangular section. The depth is defined as the dimension of the cross-section which is perpendicular to the bending axis.

However, for y-axis bending, the problem becomes more complex because the shape of the effective area could change from one load to the other. Not only the material properties, but the magnitude and pattern of residual stress distribution, as well, will influence the $\frac{I_{my}}{I_y}$ vs. $\frac{E_m}{E}$ relation. In general, it is difficult to find an explicit function which can include all of the different combinations of material properties and residual stresses; frequently it is easier to treat them separately. For this example, the transformations of the effective area are shown in Fig. 3.14c. The explicit expressions of $\frac{I_{my}/I_y}{I_y}$ vs. $\frac{E_m}{E}$ are shown in Appendix B.
For H-shapes made of non-linear materials, the analytical evaluation for \( I_m/I \) is even more complicated. Instead, a numerical approach is taken to evaluate the \( I_m/I \) in term of \( E_m/E \). For the strong axis (x-axis) bending, it is found that \( I_{mx}/I_x \) and \( E_m/E \) are practically equal for light H-shapes with constant residual stresses through the thickness. The relationship between \( I_{mx}/I_x \) and \( E_m/E \) for H-shapes is the same as that for rectangular sections. That is, for strong axis bending.

\[
\frac{I_{mx}}{I_x} = \frac{E_m}{E} \tag{3.19}
\]

This is true regardless the nature of the stress-strain relationship and the residual stress distribution.

The behavior of an H-shape bent with respect to the weak axis is equivalent to that of a rectangular section bent about the y-axis, therefore, the relationship between \( I_{my}/I_y \) and \( E_m/E \) is not as simple as that of strong axis bending.

A set of curves were developed for the \( I_{my}/I_y \) vs. \( E_m/E \) relationship for different types of residual stress distribution (Fig. 3.15 to Fig. 3.18). For light H-shapes, the material properties are assumed to be the same as those of A514 steel and the sectional properties the same as those of the 8WF31 column. These curves can be used for other light H-shapes made of A514 steel as
well, since the geometric properties of these sections do not have significant influence on the final results (Sect. 3.5).

Curves for $I_{mx}/I_x$ vs. $E_m/E$ and $I_{my}/I_y$ vs. $E_m/E$ for heavy built-up shapes of A36 steel are presented in Fig. 3.18. It can be observed that $I_{mx}/I_m$ is not equal to $E_m/E$, as are those for light H-shapes. This is because of the variation of residual stresses across the thickness of flanges is more pronounced in thick plates than for thin plates, which will alter the shape of effective area during the loading process.

Charts can be prepared for any shape as long as the mechanical properties of the material and pattern of residual stress distribution are known.

The application of the $E_m/E$ vs. $I_m/I$ charts for the determination of column strength is described generally as follows:

1. The approximate pattern of residual stress distribution in the cross-section and the stress-strain relationship of the material must be known; then, the corresponding chart can be selected.

2. The effective tangent moduli are determined by drawing lines tangent to the stub column stress-strain relationship at different values of $P/P_y$; the slopes of these lines define the corresponding effective tangent modulus.
3. To determine the maximum compressive residual stress, $\sigma_{rc}$, the proportional limit stress, $\sigma_{pm}$, of the stub column stress-strain curve must be first determined. This can be accomplished by locating the point on the stub column stress-strain curve where the slope starts to deviate from that of the modulus of elasticity, $E$. Then, $\sigma_{rc}$ is simply equal to the difference between $\sigma_p$ (stress at the proportional limit on the stress-strain curve of the material) and $\sigma_{pm}$. That is,

$$\sigma_{rc} = \sigma_p - \sigma_{pm}$$ (2.20)

4. Based on the $\sigma_{rc}$ determined, the corresponding chart and the exact or interpolated curve for $\sigma_{rc}$ are used. The corresponding $I_m/E$ for each measured $E$ can then be determined.

5. The corresponding slenderness function for a set of $P/E$, $E$, and $I_m$ can be obtained from

$$\lambda = \sqrt{\frac{I_m}{I_e}} \frac{1}{P} \frac{E}{P}$$ (2.21)
6. The slenderness function, $\lambda$, is calculated at different $P/P_y$ levels and the $P/P_y$ vs. $\lambda$ curve plotted; this is the column curve based on the stub column test.

The results of tests of two stub columns, one welded 7H28 shape with sheared edge plates and the other a welded 7H28 shape with flame-cut plates, both of A514 steel, are selected here as an example. The load-strain curves of the stub columns are shown in Fig. 3.1a. The column curves based on these two stub column tests are shown in Fig. 3.19b. Column curves based on the measured residual stresses obtained by means of a numerical analysis are also presented in Fig. 3.19b. The comparison of column curves obtained by these two different approaches shows that good correlation exists between them. The small difference can be accounted for in the error induced in the determination of the effective modulus; also the actual residual stress distribution in the section could be slightly different from that assumed for developing the $E_m/E$ vs. $I_m/I$ curves. The column test points are also shown on the same figure (Fig. 3.19b) to give some indications of the accuracy of the theoretical curves.

The advantage of using stub column tests to predict the column strength is that, if the stub column test is conducted carefully, there is a savings due to not having to perform full size column tests and residual stress
measurement. As long as the related $\frac{E_m}{E}$ vs. $\frac{I_m}{I}$ relationship is available, column strength can be predicted rather accurately from the results of stub column tests.

3.7 Comparison Between the Results of Theoretical Analysis and Tests

This section, even though it mainly presents the comparison of test results of A514 steel columns with the theoretical tangent modulus strength, has included in its contents some discussion on the pros and cons of comparing the test results with various theoretical solutions, which can be applied to columns made of steel other than A514 steel as well.

This comparison of the theory with the tests is limited in scope to "thin" walled H-sections made of A514 steel, either heat-treated rolled or welded built-up, with thickness no greater than 1 inch, and only "small size" with flange width of less than 10 inches because only in this range are sufficient data available to ensure the idealization of residual stresses from which the theoretical tangent modulus strength is determined.

A series of full scale column tests were conducted in order to verify the theoretical analysis. It was expected that from the correlation between theory and experimental results, a reliable column curve, if not for columns of all kinds, at least for a special case, could be
determined for design purposes. However, it is difficult to make a valid comparison between theory and experiments because of the fact that the theoretical analysis usually is based on many assumptions and idealizations which cannot be met by the test specimen. The column buckling equation was derived from the basic assumption that the compressive members were initially perfectly straight. In reality, not a single column was without any initial out-of-straightness. Bending deformations generally occur either at the instant of application of load or in the early stages of loading. Thus, this experimental buckling load should not be compared with the theoretical buckling load which is based on the assumption that the column is originally perfectly straight. Yet, test maximum loads cannot be compared with the ultimate load analysis either because the initial shape of a column is difficult to determine accurately. This dilemma may be resolved by either of the two compromises; (1) to compare the test ultimate load with the theoretical buckling load and to assume the effect of initial out-of-straightness to be small enough to be neglected, or (2) to compare the test results with the theoretical ultimate load analysis in which the assumed initial out-of-straightness is made as close as possible to that of the test specimen. Obviously, the first approach is simpler, but probably less accurate. The test maximum loads were compared with the theoretical buckling loads here.
In addition to the problem of the inevitable initial out-of-straightness of the columns, the residual stress distribution in the shapes differ from one specimen to another. Since both the magnitudes and the pattern of residual stresses influence the column strength, a better correlation between theory and tests can be obtained if the exact values of residual stresses in the test specimen are used in the theoretical analysis. However, if this approach is employed, every single column must be subject to an independent analysis. This is both tedious and impractical. The test results here are compared with the results of a theoretical buckling analysis in which residual stress distributions were determined from the average of all the actual residual stress measurements available.

The patterns and magnitude of the idealized residual stress distribution for welded built-up shapes are shown as (2) and (3) in Figure 3.6a. The idealized residual stresses in heat-treated, rolled shapes are shown in (1) in Fig. 3.6a; however, the maximum tensile and compressive residual stresses actually measured are close to 0.05 $\sigma$.

A total of eight column tests of A514 steel have been conducted; two rolled heat-treated columns of 8WF31 and six welded built-up H-columns; three with flame-cut plates and the other three with sheared plates. All the columns were tested with pinned-end supports about one
of the principal axes and completely restrained about
the other principal axis.

The maximum loads of all the tested columns are
compared with the theoretical buckling loads shown in
Fig. 3.6. For those columns bent about the weak axis,
that is, the principal axis parallel to the web plate,
good correlation exists between the results of the theo­
retical analysis and the tests as shown in Fig. 3.6b.
However, larger discrepancies are shown for the two
columns which were tested for strong axis bending. It
was observed during these two tests that twisting of
the column preceded the unloading, and therefore, caused
somewhat of a reduction of the column strength. Hence,
the theoretical buckling load cannot be used to compare
the test results directly.

Nevertheless, for weak axis bending, the theoreti­
cal column curves seem reasonably close to the test results
and perhaps can be used as the design criteria for small
size A514 steel columns. Of course, it will be more de­
sirable if more test points of different shapes and slender­
ness ratio can be provided to make a more adequate compari­
son between theoretical analysis and tests.
4. THE ULTIMATE STRENGTH OF BEAM-COLUMNS

4.1 Introduction

A beam-column may be defined as a member which is subject to forces producing significant amounts of both bending and compression. The bending moment in the member may be caused by externally applied end moments, eccentricity of longitudinal forces, initial out-of-straightness of axially loaded columns, or transverse forces in addition to axial forces and end moments. Several typical beam-columns are shown in Fig. 4.1. In this study, only the types of beam-columns which are subject to constant concentric axial forces and end moments are discussed.

The ultimate strength analysis of beam-columns was first treated as a stability problem by Von Karman\(^{(26)}\). He suggested a double integration procedure which was based on the equilibrium and the compatibility conditions of all the sections along the member, and this has established the theoretical background for all the subsequent analyses of beam-columns. However, Von Karman's exact concept was difficult to apply to practical problems without the facilities of fast calculating devices. Consequently, approximate solutions, either by assuming a certain function for the shape of a deflected member or by simplifying
the mechanical properties of the real material, were presented by Westergaard and Osgood\textsuperscript{(27)} and Jezek\textsuperscript{(28)}. Von Karman's work was extended by Chwalla who, in a series of papers published between 1928 and 1937\textsuperscript{(29)}, presented the results of analyses of beam-columns of several different cross-sectional shapes subjected to eccentric loads. Chwalla's most significant contributions were the establishment of the foundation for the concept of column deflection curves\textsuperscript{(30)}. Twenty years passed without significant progress beyond Chwalla's work. In the past decade, by means of electronic computers, investigations of the behavior of beam-columns have been extended to provide more extensive analyses which include the effect of residual stresses. The analysis of beam-columns has been accomplished essentially in two ways. One is to consider straight members subject to longitudinal loads with a constant eccentricity or initially crooked members subject to axial forces at the ends. Recent developments in this type of approach include the contributions of Batterman and Johnston\textsuperscript{(31)}, Malvick and Lee\textsuperscript{(32)}, and Birnстиel and Michalos\textsuperscript{(33)}. The other approach to beam-column problems is that in which the axial force is assumed to be held constant and the end moments or transverse loads are varied. Dealing with beam-column problems in this sense permits the use of the concept of column deflection curves for the determination of the load-deformation and other needed relationships.
in the design of beam-columns in multi-story frames. Extensive research on this subject has been carried out at Lehigh University. The main investigators were Ojalvo, Levi, Galambos, Lay, and Lu. Charts and tables are available and can be used directly in the design of beam-columns, with or without sway, in multi-story frames.

The previous investigations have been limited to materials having an elastic perfectly-plastic stress-strain relationship and restricted to sections with residual stresses of the cooling-after-rolling type. Also, it was assumed that during the entire loading history, no reversal of the strain of the plastified sections is permitted, and the reversal of curvatures after ultimate loads, that is, the unloading effect, is neglected. The present investigation is a study of the behavior of rolled and welded beam-columns made of A514 steel. Because of the non-linearity of the stress-strain curve and the different residual stress distributions, the behavior of A514 steel beam-columns could differ significantly from those that have previously been investigated. A computer program was prepared to include not only the true material properties and residual stresses in the section, but to include also the effects of strain reversal and unloading of moments. At present, the program covers only equal end moment cases. However, if desired, it can be modified for the cases of unequal end moments. Numerical solutions
thus obtained are compared with the full scale experiments and also with the analytical solutions obtained by the extrapolation procedure (30).

For a beam-column subjected to end reactions shown in Fig. 4.2a, the equation of equilibrium of moments may be written

\[ P v + M_L + \frac{M_R - M_L}{\bar{L}} x = M_i \]  

(4.1)

Here, \( M_i \) is the internal moment at a cross-section, \( x \) is the distance from the left end, \( v \) is the deflection, \( M_L \) and \( M_R \) are external moments at the left and right ends, respectively. \( P \) is the axial force and \( \bar{L} \) is the chord length of the deflected member.

All the external forces can be replaced by a single equivalent force, \( F \), the location of which can be determined by the geometric equilibrium conditions (42)(34). That is, as shown in Fig. 4.2b,

\[ \tan \tau = \frac{M_R - M_L}{P \bar{L}}, \quad \alpha = \frac{M_L}{M_R - M_L}, \quad F = \frac{P}{\cos \tau} \]  

(4.2)

The equilibrium equation rewritten in terms of the new coordinates \( w \) and \( u \) is

\[ F' u = M_i \]  

(4.3)
Of course, the integration of Eq. 4.3 need not be restricted to the length LR. It can be carried out from the origin 0 (See Fig. 4.2b) to the other end with u = 0 again. This curve is by definition a half wave-length of a column deflection curve(34). It is obvious that, if exact solutions are desired, the CDC's must be constructed by identically satisfying Eq. 4.2. In general, F is a function of end moments and chord length of the beam-column. Fortunately, deformations of beam-columns are generally small enough to assure the applicability of the small deflection theory. Therefore, Tan\(\zeta\) \(\approx\) Sin\(\zeta\) and Cos\(\zeta\) \(\approx\) 1, thus F \(\approx\) P, where the symbol \(\approx\) represents "approximately equal to." A CDC based on this assumption is independent of the external moments applied on the actual beam-column and is only a function of axial force P. Furthermore, a CDC can therefore accommodate an infinite number of equilibrium deflected shapes corresponding to beam-columns of different length and end conditions (30). The integration procedure and the application of CDC's have been described in detail elsewhere(30).

4.2 Moment-Curvature-Thrust Relationship for Beam-Columns

A prerequisite for performing ultimate strength analyses of beam-columns is a knowledge of the relationship existing between the bending moment and the axial force acting on a cross-section, and the resulting curvature.

* This is the abbreviation of the term" column deflection curves".
The basic equations are

\[ \int_A \sigma \cdot dA = P \quad (4.4a) \]

and \[ \int_A \sigma \cdot y \cdot dA = M_i \quad (4.4b) \]

Here, \( y \) is the distance of a finite element area \( dA \) from the bending axis and \( \sigma \) is the stress in this element (See Fig. 3.2). The stress at each element is a function of strain, and therefore the stress-strain relationship must be defined first. Generally, the monotonic stress strain relationship can be described well by the data obtained from a tension specimen test, and recorded or represented by a mathematical equation as

\[ \sigma = f(\varepsilon) \quad (4.5) \]

However, if the stress-strain relationships are history-dependent, equation 4.4 is invalid if the strain reverses. In this study, the incremental stress-strain relationship is given by (as shown in Fig. 4.3)

\[ \sigma = f(\varepsilon) \quad \text{for } \varepsilon = \varepsilon^* \]

\[ \sigma = \sigma^* - 2 \cdot f \left( \frac{\varepsilon^* - \varepsilon}{2} \right) \quad \text{for } -\varepsilon^* \leq \varepsilon \leq \varepsilon^* \quad (4.6) \]

\[ \sigma = -f(|\varepsilon|) \quad \text{for } \varepsilon < -\varepsilon^* \]
in which $\sigma^*$ and $\varepsilon^*$ are the largest compressive stress and strain to which the material of any element has been subjected. The sign convention used here is plus for compression, and minus for tension.

The total strain at any point in a loaded beam-column is composed of a residual strain ($\varepsilon_r$), a constant strain over the entire cross-section due to the presence of axial load ($\varepsilon_c$) and the strain due to curvature ($\varepsilon_\theta$). That is

$$\varepsilon = \varepsilon_r + \varepsilon_c + \varepsilon_\theta$$  \hspace{1cm} (4.7)

Here $\varepsilon_\theta = y \cdot \theta$ \hspace{1cm} (4.8)

where $\theta$ is the curvature at the section under consideration. When the stress-strain relationship is known, it is obvious that if $P$ is specified, and by assuming a value for the curvature $\theta$, the corresponding $M$ can be determined by satisfying both equations (4.4a) and (4.4b). If the axial force is applied first on the member and held constant through the whole loading process, a moment-curvature relationship can be established.

The numerical procedure for the determination of $M-P-\theta$ curve is a trial-and-error process (43). For a given residual stress distribution, $\varepsilon_r$ is known; and for the given curvature $\theta$, $\varepsilon_\theta$ is known. By assuming an $\varepsilon_c$

* "M-P-\theta" denotes moment-thrust-curvature.
value for the whole cross-section, the total strain, and therefore the stress at each element area is determined. The summation of total internal forces must be equal to the given \( P \), otherwise \( \varepsilon_C \) must be revised until Eq. 4.4a is satisfied. Then, the corresponding \( M_I \) can be evaluated by means of Eq. 4.4b. By increasing the value of \( \Phi \) and repeating the calculation, a complete moment-curvature relationship can be determined for a specified axial force, \( P \).

In this study, the stress-strain relationship of the material and residual stress distribution is programmed in subroutine subprogram forms. Both the material properties and the strain reversal effect are included.

In Fig. 4.4a, the progress of applied strain on a section for a given constant thrust is shown. There are two possibilities by which the strain reversal can influence the \( M-P-\Phi \) curves. First, as shown in Fig. 4.4b, if the \( (\varepsilon_p - \varepsilon_{rc}) \) line runs across the strain reversal zone (where \( \varepsilon_p \) is the strain at the proportional limit and \( \varepsilon_{rc} \) is the compressive residual strain at a point,) or \( \varepsilon_C \), the initial applied strain due to a given constant thrust, is larger than the minimum \( (\varepsilon_p - \varepsilon_{rc}) \), the strain reversal will effect the resulting \( M-P-\Phi \) curves. The strain reversal effected region is shown in the section in Fig. 4.4b. The second possibility is when the curvature
is very large, the tensile strain near the convex side can be larger than that at the proportional limit as shown in Fig. 4.4c. Then at further loading, reversed tensile strain influences the M-P-0 curves.

From this observation, it is obvious that if the axial thrust ratio, \( P/P_y \) (\( P \) is the axial force corresponding to yield stress level) is less than \((\sigma_p - \sigma_{rc})/\sigma_y\), the reversed strain does not affect the results since it is still within the elastic range, except when the curvature ratio \((\theta/\theta_{pc})\) is very large. However, when the applied thrust ratio is larger than \((\sigma_p - \sigma_{rc})/\sigma_y\), pronounced differences could occur if the strain reversal effect is neglected. To demonstrate the effect of strain reversal, a set of M-P-0 curves is presented in Fig. 4.5. The section is a welded A514 steel H-shape built-up from flame-cut plates. The M-P-0 curves were plotted for \( P/P_y \) varying from 0.5 to 0.9. It is clear that for \( P/P_y \) less than 0.7 (proportional limit \( \sigma_p/\sigma_y \) is 0.8 and maximum compressive residual stress \( \sigma_{rc}/\sigma_y = 0.1 \)), the case in which strain reversal is considered yields results which are identical with the corresponding one in which the stress-strain relationship is assumed to follow the monotonous stress-strain curve only. However, for \( P/P_y \) equal to 0.8 and 0.9, significant differences are shown for the two cases. Therefore, the influence of strain reversal is pronounced if the section exhibits a combination of compressive
residual stresses and thrust which cause yielding immediately after thrust is applied.

In addition to the effect of strain reversal, the pattern of distribution and magnitude of residual stress also change the shape of the M-P-Ø curve. Figure 4.6 presents three types of residual stress distributions which represent the idealized residual stresses in (A) rolled low carbon steel section, (B) rolled heat-treated A514 steel section and (C) welded built-up A514 steel shapes with flame-cut plates. If the material properties are assumed to be elastic-perfectly plastic, the M-P-Ø curves for the three types of residual stress distribution are curves (1), (2), and (4) in Fig. 4.6. It is noticed that there are significant differences among them in the elastic-plastic range. Generally speaking, the M-P-Ø curve for the rolled structural carbon-steel section, which has the largest compressive residual stress ratio ($\sigma_{rc}/\sigma_y$) among the three, exhibits a smoother knee whereas the rolled heat-treated A514 steel shapes, for which the compressive residual stress ratio is the smallest and thus residual stress effect the least, show a sharper knee.

Furthermore, aside from the effect of residual stresses, the material properties also play an important role on the M-P-Ø curve. Again in Fig. 4.6, curves (2) and (3) are the M-P-Ø curves for sections with identical residual stress distribution but different material
properties; one is of elastic perfectly-plastic type and the other representative of A514 steel. For material with a non-linear type of stress-strain curve, such as that of A514 steel, the M-P-Ø curve is lower in the knee portion than that for which an elastic perfectly-plastic stress-strain curve is assumed. However, for curvature greater than that at the end of the knee, curve (3) is above curve (2), due to the strain hardening property of the A514 steel. Curves (4) and (5) are also presented in Fig. 4.5 for welding type residual stresses and a similar behavior is observed.

It should be noticed here that all the values shown in Fig. 4.6 are in nondimensional form. For elastic perfectly-plastic materials, there are indeed a yield stress and a yield strain. However, for A514 steel, all the values are based on a nominal yield stress determined by a 0.2% offset and a yield strain that is equal to \( \frac{\sigma_y}{E} \). Naturally, the yield strain so defined is not the strain corresponding to the yield stress.

For most practically used beam-columns, the internal moments for a large portion of the member are within the knee range of the M-P-Ø curve during the loading process. Therefore, the shape of the knee has a pronounced influence on the load-deformation relationship and the ultimate strength of the beam columns. This leads to the emphasis on the basic assumptions of the residual stress distribution
as well as of the shape of the stress-strain curve and of the strain reversal phenomenon in the case when thrust is applied first and yielding before the application of moment. The assumption that thrust applied before the moment is approximate to the actual behavior of multi-story frames in which most of the axial forces in the columns are due to the dead load and moments to the live load.

4.3 Load-Deflection Relationship

In the general design practice for planar structures it is often sufficient to know the ultimate strength of beam-columns. However, in plastic design, especially for multi-story buildings, it is necessary to determine the maximum moment of a joint of a subassemblage (30). Therefore, not only the ultimate moment capacity but also the complete load-deformation curve of each individual beam-column must be known. The most practical and useful way of presenting the load-deflection relationship of a beam-column is the end moment vs. end rotation curve.

There are generally two types of numerical integration for the determination of load vs. deformation curves of a beam column. One of the two methods is Newmark's numerical integration procedure (44). The merit of Newmark's method is that it can be applied to any kind of end conditions and the interactive process converges
reasonably fast. However, Newmark's numerical integration diverges if the assumed end moment is larger than the ultimate load, and the descending branch of the M-θ curve becomes very difficult to obtain. The other numerical method is the so-called "stepwise" integration procedure \(^{(34)}\). This method has been used extensively in the development of CDC's. However, during the construction of these CDC's, it was assumed that reversed internal moments would still follow the monotonically increased M-P-θ curve. Therefore, M-θ curves obtained from these CDC's curves do not include the unloading effect. If this unloading effect is to be considered, at each integration station of the beam-column, the present moment must be compared with its history to determine the corresponding curvature. Consequently, if a series of CDC's are to be developed in the same manner, the location of the segment (which corresponds to a particular beam-column) of a CDC must be known beforehand so that the history of CDC's can be made identical to that of the beam-column in question. This is impossible for most cases except for a few particular end-loading conditions as shown in Fig. 4.7; (1) equal end moments (single curvature), where the mid-height of the beam-column is always at the peaks of the CDC's, or (2) equal end moments (double curvature) and (3) one end pinned (zero end moment) where for case (2) the mid-height and for case (3) the zero moment end are always at one end of the CDC's. Therefore, integration of CDC's for case (1) can always be initiated at the quarter points,
where the slopes are zero, and for cases (2) and (3), at zero deflection point, and then the unloading effect can be hence considered. Of course, this negates the advantage of using CDC's, that is, that they are assumed to be history independent and hence may be used for beam-columns of any end conditions and length.

To avoid the unnecessary extra integration, it is suggested that the unloading behavior of beam-columns can be included in the M-\(\Theta\) curve, if the following numerical integration procedure is employed.

1. Subdivide the length of the member which is under a constant thrust into \(n\) integration stations as shown in Fig. 4.8a. The distance between any two adjacent stations on the deflected member is \(\lambda (= \frac{L}{(n-1)})\) (approximately equal to the arc length within the segment).

2. Assume that the segment in each sublength is a circular arc.

3. Assume an end rotation and an end moment at station 1.

4. Determine the curvature \(\theta\) at station 1 from the M-P-\(\Theta\) curve. (If present \(M_1\) is less than the previous maximum \(M_1\), the unloading M-P-\(\Theta\) curve is to apply).

5. Deflection at station 2, \(v_2 = \lambda \cdot \sin (\theta - \frac{1}{2} \theta \cdot \lambda)\)

the slope at station 2, \(\theta_2 = \theta_1 - \theta \cdot \lambda\)
6. The moment at station 2 is \( M_2 = M_1 + P v_2 - \frac{M_1 - M_n}{L} \cdot \cos(\theta_1 - \frac{1}{2} \phi_1) \cdot \lambda \).

7. Determine \( \phi_2 \) from \( M-P-\phi \) curve, and carry on the integration in the same manner as from step (4) to (6). That is,

\[
\begin{align*}
\nu_i &= \lambda \cdot \sin (\theta_{i-1} - \frac{1}{2} \phi_{i-1} \cdot \lambda) + \nu_{i-1} \\
\theta_i &= \theta_{i-1} - \phi_{i-1} \cdot \lambda \\
M_i &= M_1 + P v_i - \frac{M_1 - M_n}{L} \cdot \lambda \cdot \sum_{1}^{i-1} \cos (\theta_{i-1} - \frac{1}{2} \phi_{i-1} \cdot \lambda)
\end{align*}
\]

8. If the assumed \( M_1 \) and \( \theta_1 \) is correct, then at the \( n \)th station, \( \nu_n \) should be zero. Otherwise decrease \( M_1 \) if \( \nu_n \) is negative, increase \( M_1 \) if \( \nu_n \) is positive, and repeat step (3) to (7) until \( \nu_n \) is within a certain allowable error.

9. Increase \( \phi_1 \) and increase or decrease \( M_1 \) of a certain amount and repeat the whole process as from step (1) to step (8) until the complete \( m-\phi \) as needed is obtained.

The numerical integration procedure suggested above is essentially the same as that used in the development of CDC's. The point of difference is the fact that the integration is carried out on the deflected shape of the member for fixed stations. Thus the history of every station can be recorded, and the unloading effect can be taken into account. For previous studies with CDC's, the numerical integration
was carried out independently for each CDC, and the resulting CDC's are not of the same length. Therefore, with this method there is no way of knowing the history of a certain point.

The M-θ curve for the equal end moments (single curvature) case is considered here. For this particular situation, the slope at mid-height point is always zero and the internal moment at this point always increases during the whole loading history. Numerical integration can be simplified by starting at the mid-height point and working toward the end with only one half of the member\(^{(45)}\). The example given here is for the M-θ curves of A514 steel; both rolled and welded type residual stresses are considered.

The actual moment-curvature relationship for a beam has been presented by Popov\(^{(46)}\), who obtained the resulting moment curvature hysteresis loops as shown in Fig. 4.9a. It can be seen that when the moment is reversed, the initial unloading portion for a moment curvature hysteresis loop is approximately linear (see Fig. 4.9a). In the present study the elastic unloading of moment is postulated. The M-P-θ relationship is therefore represented by the following equations (see Fig. 4.9b).

\[
\begin{align*}
\phi &= \phi^* \\
\phi &= f(M_i, P) \\
\phi &< \phi^* \\n\phi &= \phi^* - (M_i^* - M_i)/EI
\end{align*}
\]

Where \(\phi^*\) and \(M_i^*\) are the largest curvature and internal moment, respectively, to which the column has been subject at a station.
The M-θ curves for A514 steel beam-columns with slenderness ratios ranging from 20 to 40 are presented in Fig. 4.10 and 4.11. It is apparent that there is little difference between the M-θ curves including the unloading effect and excluding it. The reasons for this are that the portions of the member that do unload are the less highly loaded regions, for example when L/r = 20, the moments at the unloading region are around 0.8 M_{pc}, which is approximately on the start of the knee of the M-P-θ curve where the elastic unloading effect is not pronounced, and also most of the deformation of the column continues to come from the regions under monotonic loading. From Figs. 3.10 and 3.11, it can also be seen that the unloading effect is more pronounced for low slenderness ratio columns whose ultimate strength is generally higher than that of higher slenderness ratio columns. Also, on the descending portion of the M-θ curves, the larger the end rotation, the greater the difference between the loading and unloading curves. Naturally, if end conditions have been changed, for example, at one end the applied moment being kept zero, or the axial forces reduced, the unloading effect may be different.

The effect of mechanical properties and residual stresses as well as the unloading behavior on the M-θ curves are also important. In Fig. 4.12, M-θ curves for columns with L/r = 40 are presented. The axial force is constant, 0.55 P_y.
It is shown that both the shape of the stress-strain curves and residual stress distribution can influence the \( M-\theta \) curve. If the material properties are kept the same, the difference can be approximately 10\% in ultimate load if the residual stresses, which are nondimensionalized with respect to the yield stress, are different. If nondimensional residual stresses are held constant, differences in the material properties can introduce a difference of up to 10\% of the ultimate load. Therefore, accuracy of representation of the stress-strain relationship of the material and residual stress distribution in the section are needed in order to provide a good prediction of the strength of beam-columns.

The interaction curves between \( P/P_y \) and \( M/M_p \) for equal end moment conditions (symmetrical bending) are shown in Fig. 4.13 for slenderness ratios equal to 20, 40, and 60. Beam-columns of rolled heat-treated shapes show higher ultimate strength than those of welded built-up shapes. This can be understood as the consequence of the smaller effect of residual stresses on the \( M-P-\theta \) curves for rolled shapes than that for welded shapes.

An approximate solution for the case \( L/r \) equal to 20, which is obtained by extrapolating from the results obtained from A36 steel beam-columns, is also presented in Fig. 4.13. For beam-columns made of steel other than A36, the slenderness ratio must be adjusted according to the following formula\(^{30}\).
The interaction curve determined from this extrapolation procedure is also presented in Fig. 4.13, for the case \( L/r = 20 \). It is shown that the approximate solution is slightly lower than the corresponding "exact solution".

### 4.4 Experimental Investigations

An experimental investigation of the behavior of beam-columns made of A514 high strength alloy steel has been carried out. The program consisted of tests of two full scale beam-columns, one a rolled 8WF40 shape and the other an 11H71 shape. The members were tested in an "as-delivered" condition; no attempt was made to eliminate rolling or welding residual stresses by annealing. The magnitude and distribution of the residual stresses were determined by actual measurements \((47)\); it was found that they were close to the results of previous measurements (see Sect. 2.2) and hence the idealized residual stress distribution as shown in Fig. 3.6a was used for the determination of beam-column strength. The beam-columns were tested under equal end moment (single curvature) conditions.

#### 4.4.1 Test Procedure

The procedure for testing beam-columns has been described in detail previously \((48)(49)\). Only a brief outline is given here for review and completeness.
The general set-up of the beam-column specimen is shown in Fig. 4.14a. The horizontal moment arms are rigidly welded to the end of the column. The sizes of the beams are comparatively larger than that of the column so that the beam sections remain in the elastic range during the whole loading process. Pinned-end fixtures were utilized to ensure that there are no end moments other than those imposed by the moment arms, applied at the column ends. In Fig. 4.14a it can be seen that the axial force in the column is made up of the direct force applied by the testing machine, \( P \) and the jack force, \( F \). To simulate the situation existing in the lower stories of a multi-story frame and to be in accord with the assumptions for the theoretical analysis, the tests were performed with the axial load held constant. Thus at each increment of load or deformation, the direct force, \( P \), was adjusted so that the total force in the column remained at \( 0.55 P_y \), where \( P_y \) is the yield load of the column.

The direct axial force, \( P \), was first applied on the column; the beam-to-column joints were rotated by applying the jack force to the ends of the moment arms. The column was therefore forced into a symmetrical curvature mode of deformation. In order to preclude any deformation out of the plane perpendicular to the strong axis, the column was braced at the third points by two sets of lateral braces. The lateral braces used were designed for the
laboratory testing of large structures permitted to sway. In the early stages of loading, that is, in the elastic range, approximately equal increments of moment were applied to the column. In the inelastic range, comparatively larger deformations occur for the same amount of moment increment, therefore, end rotations instead of moment are used as a basis for loading in order to obtain a complete load-deformation curve with approximately evenly distributed test points.

At each increment of load or end rotation, the end rotations were measured by level bars (see Fig. 4.14b). The mid-height deflection, in the bending plane as well as out-of-plane, of the column was also measured by mechanical dial gages. SR-4 gages were mounted at the beam and column junctions as well as at several other locations along the column, as shown in Fig. 4.14b, to determine strain distribution in the column or to serve as a means for checking moments. Figure 4.15 shows the photographs taken at the beginning and end of the test. The occurrence of local buckling of the compressed flange was determined by measuring the out-of-plane deformations of the flanges at five locations in the vicinity of mid-height of the beam-column with an inside micrometer.

4.4.2 Test Results -- In-Plane Behavior

The results of the tests can best be presented in the form of end moment vs. end-rotation curves as shown in
Fig. 4.16 and Fig. 4.17. In Fig. 4.16 the $M-\theta$ curve for the 8WF40 A514 steel beam-column is shown. Fig. 4.17 contains the $M-\theta$ curve for the 11H71 welded A514 steel beam-column. The moments indicated by open points represent the total applied moment determined from the hydraulic jack load. The length of the moment arm is the distance from the centerline of the column to the center of the rod to which the hydraulic jack is connected. The end moments were also checked by the reading of the dynamometer which is inserted in series with the jack and by four sets of SR-4 strain gages which were affixed to the loading beam, near its junction with the column. The difference between the moment readings by these three means are shown in Fig. 4.18. It is apparent that they are rather consistent.

The length used to compute slenderness ratios of the columns were the distances between the points of intersection of the centerlines of the column and loading beams. For both beam-columns, the slenderness ratio, $L/r$, is 40. Because of the stiffness of the joint, the rigidity of the beam-column near the ends is greater than that of the remainder of the column. Therefore, the actual effective $L/r$ is slightly less than that measured. Comparison of the experimental results with the theoretical reveals that the testing points are above the theoretically obtained $M-\theta$ curve (Figs. 4.16 and 4.17). This discrepancy is due in part to the fact that the actual slenderness ratio has been
reduced somewhat by the installation of joint stiffeners and to the fact that the actual stress-strain relationship determined from tension coupon tests shows a slightly higher proportion limit than that of the average typical stress-strain curve on which the theoretical analysis was based. The tests are compared also to the theory in a plot of $\frac{M}{M_{pc}}$ vs. $L/r$ as shown in Fig. 4.19. The difference between theory and test is approximately 5% for both rolled and welded built-up shapes. From Fig. 4.19, it can also be observed that the difference of ultimate strength for rolled and welded shapes vanished for low slenderness ratios. This is apparently because of the fact that the internal moments in the greater portion of the member are within the strain hardening region at ultimate load, and hence the residual stress effect is insignificant.

4.4.3 Test Results -- Local Buckling

The local buckling deformations of the flanges of the beam-columns were measured during the process of testing. It was observed that hardly any web buckling ever occurred and the b/t ratio of the flange provided sufficient stiffness to ensure the development of rotation capacity of the beam-column. That is, local buckling of the flange occurred after the moment capacity had dropped five percent below its maximum value (ultimate strength).
It is difficult in general to analyze the buckling of plates in which residual stresses exist. The local buckling problem of columns or beam-columns with residual stresses becomes more involved because not only the compressive residual stresses could induce yielding before the local buckling stress is reached, but also because of the uncertainty of the coefficient of restraint at the junction of component plates\(^{(45)}\). In the case of H-sections, it was found that the assumption that the flange is divided into two cantilevers which have full deflection restraint but zero rotation restraint is close to reality but conservative\(^{(37)}\). In this study, this same restraint condition was assumed for the flanges of the beam-columns. It is not intended to perform an elaborate theoretical analysis with respect to the local buckling behavior of columns or beam-columns here. Experimental results are compared to the theoretical analysis available for the purpose of determining the maximum allowable b/t ratio for the flange such that local buckling does not occur before the attainment of yield stress and for the prediction of local buckling point for a beam-column. Nishino\(^{(3)}\) and Ueda\(^{(51)}\) presented the theoretical and experimental investigations of the plate buckling under different edge conditions and residual stress patterns. Figure 4.19 shows a plot of \(\frac{\sigma_{cr}}{\sigma_y} \text{ vs. } \frac{b}{2t} \cdot \sqrt{\frac{\sigma_y}{E}}\) curves for plates simply supported and free at unloaded edges, and simply supported on both loading edges. Even though the stress-strain law,
elastic perfectly-plastic, and residual stress patterns used in Refs. 3 and 51 are slightly different from those used in this study, their solutions provide a reasonable theoretical prediction for the buckling of A514 steel plates, if not overall, at least on the determination of the critical b/t ratio. Figure 4.20 shows that, even when the pattern of residual stress varies, the maximum \( \frac{b}{2t} \sqrt{\frac{\sigma_y}{E}} \) values when \( \frac{\sigma_{cr}}{\sigma_y} = 1.0 \) are nearly the same. It is therefore concluded that the maximum allowable b/t ratio for a yielded flange would be

\[
(\frac{b}{2t})_{\text{max.}} = 0.45 \times \frac{E}{\sqrt{\sigma_y}}
\]

For A514 steel with \( E = 29 \times 10^3 \) ksi, \( \sigma_y = 110 \) ksi.

\[
(\frac{b}{t})_{\text{max.}} = 14.5
\]

where \( b \) and \( t \) are the width and thickness of the flange, respectively. The b/t ratio of both sections tested are equal to 14.35 which is slightly less than the maximum b/t for a yielded flange.

In order to obtain some verification of the approximate theory and, in addition, to furnish some data relating mechanical properties, stub columns were tested under uniform axial load. The local buckling points were determined by the so-called "top of the knee method"\(^{(52)}\). Figure 4.21 shows the load vs. out-of-plane deflection...
curves of the flange plates at several measured points. The approximate local buckling point can be determined as shown in Fig. 4.22 and 4.23 as the solid points. For both cases, the flange buckling stress or critical stress, \( \sigma_{cr} \), is approximately equal to the yield stress of the section. Therefore, it seems reasonable to put \( b/t = 14.5 \) as the maximum allowable width-thickness ratio (or critical width thickness ratio) if the flange is to be designed to withstand the yield load, \( P_y \), without local buckling.

The critical strain, \( \varepsilon_{cr} \), is therefore nearly equal to the sum of the strain at the yield point and the maximum tensile residual strain. That is

\[
\varepsilon_{cr} = 1.52 \cdot \varepsilon_y + \varepsilon_{rt} = 0.006
\]

for rolled heat-treated shapes, and

\[
\varepsilon_{cr} = 3.04 \cdot \varepsilon_y = 0.0115
\]

for welded built-up shapes. The stub column tests show that the critical strains are close to these values, (Fig. 4.22 and 4.23) and hence these two critical strains are taken as the standard for determining the local buckling of beam-columns if \( b/t \) is less than 14.5.

The plate buckling problems in the strain hardening range have been studied by Haaijer\(^{53}\), and extended by
Galambos (36) and Lay (37) for application to beam, beam-column and column members. Lay stated "Local buckling will not be critical until a critical region is strain-hardened", and adopted an L/b value of 1.2 as the minimum criterion for local buckling where L is the half wavelength of the local buckle and b is the width of the flange. In this study the same L/b = 1.2 is used for the determination of local buckling point on the theoretical M-Θ curve. It is taken as the local buckling moment when the minimum applied strain in a length of 1.2b is equal to the critical strain. For beam-columns with thin component plates, it is generally sufficient, for the prediction of local buckling, to assume that the strain applied to the flange is uniform across the thickness and width if the beam-columns are bent about the strong axis. For beam-columns with equal end moments which cause symmetrical bending, the local buckling point can be determined theoretically as an end rotation at which the strain at the location 0.6b from the mid-height reaches the critical strain. The experimental local buckling point was determined again by the "top of the knee" method. In Fig. 4.16 and 4.17, local buckling points are shown as cross marks. It is observed that there is good correlation between the local buckling points determined theoretically and experimentally. Also, it is interesting to notice that for welded built-up shapes the occurrence of local buckling is at a comparatively larger end
rotation than that for rolled heat-treated shapes. Apparently, this is because of the higher tensile residual stresses in the welded shape which increase the value of the critical strain necessary to cause total yielding of the flange. This indicates that welding residual stresses can actually increase the rotation capacity of the beam-column, if the termination of rotation capacity is taken as the local buckling point.

Furthermore, the initiation of local buckling does not seem to reduce to strength of beam-columns dramatically. The M-θ curves still follow their original path for some distance until pronounced out-of-plane deflections of the flanges are observed. If further study on the post local buckling behavior confirms this in the future, the use of beam-columns may be extended beyond the local buckling point.
5. SUMMARY AND CONCLUSIONS

This dissertation presents the results of an investigation of the strength of centrally loaded columns and beam-columns. Particular attention has been given to members made of a non-linear material, namely, A514 constructional alloy steel; however, the analytical methods employed in this study can be applied to columns or beam-columns of any material, linear or non-linear, as long as the stress-strain relationship of the material is well defined. In particular, the following problems are investigated:

1. The stress-strain relationship and other relevant mechanical properties of quenched and tempered A514 low-alloy steel.


3. The ultimate strength, the load-deformation characteristics, and the local buckling behavior of rolled and welded A514 steel beam-columns.

The new contributions of this dissertation are as follows:
1. The representative stress-strain relationship of A514 steel is determined by experiments of tension specimen tests, and a mathematical model is developed to represent the nondimensional stress-strain curve of A514 steel.

2. A method is presented for the computation of the tangent modulus strength of columns made of non-linear materials. It is developed into a computer program that takes into account the shape of the stress-strain curve and the residual stress distribution. The program differs from those developed previously by its flexibility of use. All the prerequisite information such as stress-strain relationship and residual stress distribution are programmed in subroutine forms; thus, any stress-strain relationship and residual stresses, idealized or actual, can be considered.

3. The basic relationship between the stress-strain curve obtained from a stub column test and the basic column strength curve is established. Charts are prepared to accommodate the prediction of the buckling load of steel columns from stub column test results. This approach
simplifies the process of predicting column strength and eliminates the necessities of full-scale column test and residual stress measurement.

4. The strength of rolled and welded H beam-columns made of A514 steel is analyzed both theoretically and experimentally. The effect of the residual stresses due to welding in the welded shapes, and the effect of strain reversal and the mechanical properties of the steel are included in the determination of the moment-curvature-thrust relationships. The load-deformation behavior of beam-columns is determined by carrying out numerical integration on the fixed stations on the deflected shape of the beam columns; therefore, the unloading effects due to reversed curvatures can be included. A computer program is developed, which is able to carry out all the computations in a single run.

5. The critical strain and local buckling behavior of A514 steel beam-columns are investigated both experimentally and theoretically. A method for theoretical prediction is suggested, and theoretical results are close to those obtained by experiments.
The following conclusions may be drawn from the study of this dissertation:

1. The stress-strain relationship of A514 steel can be closely simulated by three equations; a fifth order polynomial equation for the transition range and two linear equations for the elastic and strain hardening range, respectively. The particular characteristics of this stress-strain relationship is that no obvious yield plateau is observed. Instead, strain hardening occurs immediately after the ending of the transition range, continues until the tensile strength, and then starts to unload. (Section 2.2, Fig. 2.6).

2. The typical pattern and magnitude of residual stresses in rolled A514 shapes can be represented by a triangular distribution with maximum compressive and tensile residual stress approximately equal to 5% of the yield stress. For welded shapes, the tensile residual stress at the welds is about the same as the yield stress, compressive residual stresses are about 10% of the yield stress; for flame-cut plates, tensile residual stress of 30% of the yield stress exist at the flange tips. The patterns of residual stress distribution
are of trapezoidal shape. (Section 2.3, Figs. 2.7 and 3.6a).

3. The sectional properties of an H-shaped section do not affect the column strength for weak axis buckling and influence the column curve for strong axis buckling insignificantly. For practical purposes, for a given residual stress distribution and a stress-strain curve, one column curve in dimensionless form may be sufficient to describe the basic column strength for most commonly used sections (Section 3.5, Fig. 3.3).

4. The shape of the stress-strain curve has a dominant influence on the final column strength curve. To assume the stress-strain relationships of A514 steel to be elastic perfectly-plastic overestimates column strength in the medium slenderness region or it underestimates the strength of short columns. (Section 3.5, Figs. 3.4 and 3.5).

5. The distribution and magnitude of the residual stresses, and thus the column strength, are influenced by the mode of manufacture of the column sections. Rolled shapes exhibit the least reduction in strength due to the
effects of residual stresses, and shapes fabricated from sheared-edge plates the greatest. Shapes fabricated from flame-cut plates have a more favorable distribution of residual stresses than do those fabricated from sheared plates, but are still not as strong as rolled shapes (Sections 3.5 and 3.7, Fig. 3.6).

6. For a heavy column the penetration of the yielded area in the sections is considerably different from that in a column of a light shape because the residual stresses vary considerably across the thickness of its component plates. Two 15H290 welded built-up shapes, one with universal mill plates and the other with flame-cut plates, all of A36 steel, are investigated. It is concluded that by assuming constant residual stress across the thickness, the buckling strength for shapes with UM plates will be overestimated for strong axis bending, but underestimated for weak axis bending; and for shapes with flame-cut plates, overestimated for strong and weak axis bending. (Section 3.5, Fig. 3.11).
7. It is misleading and incorrect to apply indiscriminately the approximate relationship between the effective tangent modulus (obtained from a stub column test) and the elastic moment of inertia on any shapes. For shapes which do not have an elastic-perfectly-plastic stress-strain curve and/or do not have the maximum compressive residual stress at flange tips and distributed with decreasing values towards the center of the flange, the relationship between the average tangent modulus and the "effective moment of inertia" with respect to the weak axis is very involved and must be treated individually. (Section 3.6).

8. For strong axis buckling of light H-shapes, the relationship between the effective tangent modulus and the effective moment of inertia is practically linear irrespective of the stress-strain relationship and the pattern of residual stresses (Section 3.6).

9. For weak axis buckling of light H-shapes of A514 steel and for strong and weak axis buckling of welded heavy 15H290 shapes of A36 steel, a numerical method is employed
to determine the relationship between the effective tangent modulus and the effective moment of inertia. The results are presented in the form of several charts. Each of these charts represents a combination of a stress-strain relationship and a certain pattern of residual stresses. By using these charts, the corresponding effective moment of inertia for a given effective tangent modulus can be easily determined. (Section 3.6, Fig. 3.15 to Fig. 3.19).

10. The mechanical properties of the material, pattern and magnitude of residual stresses and strain reversal effect are important in the final shape of M-P-Ø curves which in turn is the sole basis for the determination of the load-deformation characteristics of beam-columns. (Section 4.2, Figs. 4.5 and 4.6).

11. For beam-columns, the strain reversal effect is more pronounced for non-linear materials than for linear materials if other conditions, that is, residual stresses and thrust, are identical. (Section 4.2).

12. The unloading effect generally is not pronounced immediately after ultimate load. Significant differences between the inclusion or the exclusion of the unloading effect
can be shown only when a large portion of the descending part of the moment-rotation curve is plotted. (Section 4.2 Figs. 4.10 and 4.11).

13. Two full scale beam-column tests, one rolled 8WF40 shape and the other 11H71 welded shape were conducted. A comparison between the theoretical curves and the corresponding experimental M-θ curves has shown that the theory can predict not only the ultimate strength but also the complete history of a beam-column with excellent accuracy. (Section 4.4.2, Figs. 4.16, 4.17, and 4.19).

14. Comparing the direct integration solutions to the extrapolation solutions obtained from previous investigations in A36 steel shapes, it is shown that for A514 steel shapes, both rolled and welded built-up shapes, direct integration solutions provide higher ultimate strength. Hence, the extrapolation procedure may provide an approximate but conservative estimate of the strength of A514 steel shapes. (Section 4.3, Fig. 4.13).
15. The local buckling point of beam-columns is determined both experimentally and theoretically. It is found that the "regional criterion" provides a sufficient basis for the prediction of flange local buckling of beam-columns bent with respect to strong axis and the stub column tests supply vital data for the determination of critical strain when compressive flanges buckle locally. (Section 4.4.3).
6. APPENDICES
APPENDIX A: FLOW CHARTS FOR TANGENT MODULUS LOAD

COMPUTER PROGRAM
### SYMBOLS IN PROGRAMS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area of cross section</td>
</tr>
<tr>
<td>AL</td>
<td>Area of one flange element, nondimensional with A</td>
</tr>
<tr>
<td>AW</td>
<td>Area of one web element, nondimensional with A</td>
</tr>
<tr>
<td>B</td>
<td>Width of flange</td>
</tr>
<tr>
<td>D</td>
<td>Depth of section</td>
</tr>
<tr>
<td>DA, DB, DC</td>
<td>Constants in a polynomial equation</td>
</tr>
<tr>
<td>DW</td>
<td>Web depth</td>
</tr>
<tr>
<td>E</td>
<td>Strain</td>
</tr>
<tr>
<td>ECOE</td>
<td>Effective modulus nondimensional with modulus of elasticity</td>
</tr>
<tr>
<td>EE</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>EIXM</td>
<td>Effective moment of inertia with respect to strong axis (x-axis) nondimensional with the moment of inertia of the section</td>
</tr>
<tr>
<td>EIYM</td>
<td>Effective moment of inertia with respect to weak axis (y-axis) nondimensional with the moment of inertia of the section</td>
</tr>
<tr>
<td>EL</td>
<td>Total strain at a flange element, nondimensional with yield strain</td>
</tr>
<tr>
<td>EOEY</td>
<td>Applied strain nondimensional with yield strain</td>
</tr>
<tr>
<td>ET</td>
<td>Tangent modulus</td>
</tr>
<tr>
<td>ETL</td>
<td>Tangent modulus corresponding to a strain at a flange element, nondimensional with modulus of elasticity</td>
</tr>
<tr>
<td>ETW</td>
<td>Tangent modulus corresponding to a strain at a web element, nondimensional with modulus of elasticity</td>
</tr>
</tbody>
</table>
EW

Total strain at a web element, nondimensional with yield strain

FR1,FR2,etc.

Input residual stresses

FW

Residual stress at a web element

FY

Static yield stress

INPUT

Input tape number

IOUT

Output tape number

ISEC

Nominal depth of an H-shape

J

Number of subroutine subprogram RESID

L

Index constant

LSB

Nominal weight per unit length of column

M

Subscript

N

Subscript

NA

A constant in the Ramberg-Osgood representation

NB

A constant in Ramberg-Osgood representation

NUM

Total number of residual stresses which define the residual stress distribution in the section

PL

Proportional limit

POPY

Total applied load nondimensional with yield load of the cross section

RA

Increment of applied strain

RADX

Radius of gyration with respect to x-axis

Rady

Moment of inertia with respect to y-axis

RESID

Subroutine subprogram for residual stress distribution

RL

Residual strain at a flange element nondimensional with yield strain

RW

Residual strain at a web element, nondimensional with yield stress
<table>
<thead>
<tr>
<th>S</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIN</td>
<td>Number of strain increment</td>
</tr>
<tr>
<td>SL</td>
<td>Total stress at a flange element nondimensional with yield stress</td>
</tr>
<tr>
<td>SM</td>
<td>Secant modulus for a Ramberg-Osgood stress-strain curve</td>
</tr>
<tr>
<td>START</td>
<td>First applied strain</td>
</tr>
<tr>
<td>STREA</td>
<td>Subroutine subprogram for stress-strain relationship</td>
</tr>
<tr>
<td>SW</td>
<td>Total stress at a web element, nondimensional with yield stress</td>
</tr>
<tr>
<td>T</td>
<td>Flange thickness</td>
</tr>
<tr>
<td>TOP</td>
<td>Maximum applied strain</td>
</tr>
<tr>
<td>TRANS</td>
<td>Subroutine subprogram for transferring residual stress to residual strain</td>
</tr>
<tr>
<td>VALUE</td>
<td>Applied strain</td>
</tr>
<tr>
<td>W</td>
<td>Web thickness</td>
</tr>
<tr>
<td>XI</td>
<td>Moment of inertia with respect to x-axis</td>
</tr>
<tr>
<td>XL</td>
<td>Distance between centroid of a flange element and y-axis</td>
</tr>
<tr>
<td>XLAM</td>
<td>Slenderness function with respect to the strong axis (x-axis)</td>
</tr>
<tr>
<td>XW</td>
<td>Distance between centroid of a web element and y-axis</td>
</tr>
<tr>
<td>YI</td>
<td>Moment of inertia with respect to y-axis</td>
</tr>
<tr>
<td>YL</td>
<td>Distance between centroid of a flange element and x-axis</td>
</tr>
<tr>
<td>YLAM</td>
<td>Slenderness function with respect to the weak axis (y-axis)</td>
</tr>
<tr>
<td>YW</td>
<td>Distance between centroid of a web element to x-axis</td>
</tr>
</tbody>
</table>
Main Program
Main Program (continued)
DEW = (S(l)-S(l-l)) * (E(l)-E(l-l))

RFW = (E(l)-E(l-l)) * (E(l)-E(l-l))

ETW(M,N) = ET(l)-REW*(E(l)-ABS(EW(M,N)))

WS = S(l)-DEW*(E(l)-ABS(EW(M,N)))

POPY + SL(M,N)*AL*4.0
POFY = POTY + SW(M,N)*AW*4.0
EIXM = EIXM + ETLM(N)*YLM(N)
EYXM = EYXM + ETLM(N)*YLM(N)

**2*AAL*4.0/RADX**2
**2*AAL*4.0/RADY**2

EYXM = EYXM + ETLM(N)*YLM(N)

**2*AAL*4.0/RADY**2

Main Program (continued)
Subroutine Subprogram
"STREA" (Ramberg-Osgood Type Stress-Strain Curves)
Subroutine Subprogram
"TRANS"

Subroutine Subprogram
"RESID" (Rolled H-Shapes)
Subroutine Subprogram
"RESID" (Welded H-Shapes with Sheared Plates)

Subroutine Subprogram
"RESID" (Welded H-Shapes with Flame-Cut Plates)
Subroutine Subprogram
"RESID" (Welded Heavy H-Shapes of A36 Steel)
APPENDIX B: EFFECTIVE MOMENT OF INERTIA AND EFFECTIVE TANGENT MODULUS RELATIONSHIP FOR A RECTANGULAR SECTION

1. Stress-Strain Relationship of the Material (Fig. 3.14a):

\[
\frac{\sigma}{\sigma_y} = \frac{\varepsilon}{\varepsilon_y} \quad \text{Elastic Range}
\]

\[
\frac{\sigma}{\sigma_y} = -\frac{1}{8} + \frac{3}{2} \left(\frac{\varepsilon}{\varepsilon_y}\right) - \frac{1}{2} \left(\frac{\varepsilon}{\varepsilon_y}\right)^2 \quad \text{Transition Range}
\]

\[
\frac{\sigma}{\sigma_y} = 1 \quad \text{Perfectly Plastic Range}
\]

2. Residual Stress Distribution (Fig. 3.14b):

Triangular type with maximum compressive stress

\[\sigma_{rc} = 0.3 \sigma_y.\]

3. Average Stress-Strain and Tangent Modulus-Strain Equations (Stub Column Curves, Fig. 3.14a).

when \[\sigma \leq \frac{\varepsilon}{\varepsilon_y} \leq 0.2\]

\[
\frac{\sigma}{\sigma_y} = \frac{\varepsilon}{\varepsilon_y}
\]

\[
\frac{E_m}{E} = 1.0
\]
when \( 0.2 \leq \frac{\varepsilon}{\varepsilon_y} \leq 0.8 \)

\[
\frac{\sigma}{\sigma_y} = -\frac{1}{3.6} \left( \frac{\varepsilon}{\varepsilon_y} \right)^3 + \frac{1}{6} \left( \frac{\varepsilon}{\varepsilon_y} \right)^2 + \frac{29}{30} \left( \frac{\varepsilon}{\varepsilon_y} \right) + \frac{0.004}{1.8}
\]

\[
\frac{E_m}{E} = -\frac{1}{1.2} \left( \frac{\varepsilon}{\varepsilon_y} \right)^2 + \frac{1}{3} \left( \frac{\varepsilon}{\varepsilon_y} \right) + \frac{29}{30}
\]

when \( 0.8 \leq \frac{\varepsilon}{\varepsilon_y} \leq 1.2 \)

\[
\frac{\sigma}{\sigma_y} = -0.5 \left( \frac{\varepsilon}{\varepsilon_y} \right)^2 + 1.5 \left( \frac{\varepsilon}{\varepsilon_y} \right) - 0.14
\]

\[
\frac{E_m}{E} = -\left( \frac{\varepsilon}{\varepsilon_y} \right) + 1.5
\]

when \( 1.2 \leq \frac{\varepsilon}{\varepsilon_y} \leq 1.8 \)

\[
\frac{\sigma}{\sigma_y} = \frac{1}{7.2} \left( \frac{\varepsilon}{\varepsilon_y} \right)^3 - \frac{3}{4} \left( \frac{\varepsilon}{\varepsilon_y} \right)^2 + 1.35 \left( \frac{\varepsilon}{\varepsilon_y} \right) - 0.31
\]

\[
\frac{E_m}{E} = \frac{1}{2.4} \left( \frac{\varepsilon}{\varepsilon_y} \right)^2 - \frac{3}{2} \left( \frac{\varepsilon}{\varepsilon_y} \right) + 1.35
\]

and when \( 1.8 \leq \frac{\varepsilon}{\varepsilon_y} \)

\[
\frac{\sigma}{\sigma_y} = 1.0
\]

\[
\frac{E_m}{E} = 0.0
\]
4. $\frac{I_m}{I}$ vs. $\frac{E_m}{E}$ Relationship (y-axis bending, Fig. 3.17c)

**Elastic-Inelastic Range**

$0.2 \leq \frac{\varepsilon}{\varepsilon_y} \leq 0.8$

$$\frac{I_m}{I} = 1 - \frac{1}{4} (1 - \alpha)(1 - \beta)(\alpha^2 + 2\alpha + 3)$$

$$\alpha = 1 - \sqrt{\frac{10}{3} (1 - \frac{E_m}{E})}$$

$$\beta = 0.2 \cdot (2 + 3\alpha)$$

**Inelastic Range**

$0.8 \leq \frac{\varepsilon}{\varepsilon_y} \leq 1.2$

$$\frac{I_m}{I} = \frac{1}{4} (3\beta + \delta)$$

$$\delta = 0.3 + \frac{E_m}{E}$$

$$\beta = -0.3 + \frac{E_m}{E}$$

**Inelastic Range**

$1.2 \leq \frac{\varepsilon}{\varepsilon_y} \leq 1.8$

$$\frac{I_m}{I} = \frac{1}{4} \beta \alpha^3$$

$$\alpha = \frac{10}{3} \frac{E_m}{E}$$

$$\beta = 0.6 \alpha$$
7. NOMENCLATURE AND DEFINITIONS

\begin{itemize}
  \item \( A \) Area of cross section
  \item \( A_m \) Effective Area \((= \int_0^A \frac{E}{E} \, dA)\)
  \item \( b \) Width of flange
  \item \( \bar{d} \) Effective width
  \item \( d \) Depth of section
  \item \( E \) Modulus of elasticity
  \item \( E_m \) Effective tangent modulus
  \item \( E_{st} \) Strain-hardening modulus
  \item \( E_t \) Tangent modulus
  \item \( f \) a function
  \item \( I \) Moment of inertia - subscripts \( x \) and \( y \) refer to the \( x \) and \( y \) axes (strong and weak axes), respectively
  \item \( I_e \) Moment of inertia of elastic portion of cross section - subscripts \( x \) and \( y \) refer to the \( x \) and \( y \) axes, respectively
  \item \( I_m \) Effective moment of inertia \((= \int_\frac{E}{E} \bar{y}^2 \, dA) - 
  \) subscripts \( x \) and \( y \) refer to the \( x \) and \( y \) axes, respectively.
  \item \( k \) A constant in the Ramberg-Osgood representation
  \item \( L \) Column length
  \item \( \bar{l} \) Chord length of a deflected member
  \item \( M \) Bending moment - subscripts \( R \) and \( L \) refer to moments at the right and left ends, respectively, of a beam-column, \( i \) refers to internal moment
  \item \( M_p \) Plastic moment
  \item \( M_{pc} \) Reduced plastic moment
\end{itemize}
\( M_u \)  
Ultimate moment

\( m \)  
Slope of the straight line which defines the secant yield stress

\( n \)  
A constant in the Ramberg-Osgood representation

\( P \)  
Axial load

\( P_{cr} \)  
Buckling (critical) load

\( P_y \)  
Axial yield load in a column

\( r \)  
Radius of gyration - subscripts \( x \) and \( y \) refer to strong and weak axes radii.

\( t \)  
Thickness of flange

\( u,v,w \)  
Displacement in the \( x \), \( y \), and \( z \) directions, respectively

\( x,y,z \)  
Coordinate axes, coordinates of the point with respect to \( x \), \( y \), and \( z \) axes

\( \varepsilon \)  
Strain

\( \varepsilon_c \)  
Strain due to axial load

\( \varepsilon_{cr} \)  
Critical strain

\( \varepsilon_p \)  
Strain at proportional limit

\( \varepsilon_r \)  
Residual strain

\( \varepsilon_{rc} \)  
Maximum compressive residual strain

\( \varepsilon_{rt} \)  
Maximum tensile residual strain

\( \varepsilon_{st} \)  
Strain at start of strain hardening

\( \varepsilon_t \)  
Total strain

\( \varepsilon_y \)  
Yield strain \((\sigma_y/E)\)

\( \varepsilon_\theta \)  
Strain due to curvature

\( \varepsilon_x \)  
Largest strain any element area experienced

\( \Theta \)  
End rotation of a member

\( \lambda \)  
Slenderness function, distance between two adjacent integration stations
\[ \sum \] Summation
\[ \phi \] Curvature
\[ \phi_p \] Curvature at \( M_p \)
\[ \phi_{pc} \] Curvature at \( M_{pc} \)
\[ \sigma \] Stress
\[ \sigma_{cr} \] Critical stress
\[ \sigma_p \] Stress at proportional limit
\[ \sigma_{pm} \] Proportional limit stress determined from a stub column test
\[ \sigma_r \] Residual stress
\[ \sigma_{rc} \] Maximum compressive residual stress
\[ \sigma_{rt} \] Maximum tensile residual stress
\[ \sigma_y \] Yield stress (determined by 0.2% offset method for non-linear stress-strain relationship)
\[ \sigma^* \] Largest stress any element area experienced
\[ \sigma_l \] Secant yield stress
### TABLE 2.1 TENSION SPECIMEN TEST RESULTS

(A514 STEEL PLATES OR SHAPES)

<table>
<thead>
<tr>
<th>Shape</th>
<th>Tension Specimen No.</th>
<th>Proportional Limit (ksi)</th>
<th>Static Yield Stress (ksi)</th>
<th>Modulus of Elasticity (ksi)</th>
<th>Strain Hardening Modulus (ksi)</th>
<th>Ultimate Stress (ksi)</th>
<th>Reduction of Area (%)</th>
<th>Elongation in Gage Length (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW</td>
<td>0.86</td>
<td>107</td>
<td>28,500</td>
<td>172</td>
<td>119</td>
<td>46</td>
<td>14.6</td>
<td>11.6</td>
</tr>
<tr>
<td>AF-1</td>
<td>0.86</td>
<td>109</td>
<td>29,000</td>
<td>197</td>
<td>123</td>
<td>55</td>
<td>14.6</td>
<td>11.6</td>
</tr>
<tr>
<td>AF-2</td>
<td>0.78</td>
<td>113</td>
<td>27,700</td>
<td>122</td>
<td>126</td>
<td>58</td>
<td>14.6</td>
<td>11.6</td>
</tr>
<tr>
<td>AF-3</td>
<td>0.86</td>
<td>111</td>
<td>29,000</td>
<td>193</td>
<td>125</td>
<td>58</td>
<td>14.6</td>
<td>11.6</td>
</tr>
<tr>
<td>BF-1</td>
<td>0.76</td>
<td>120</td>
<td>27,000</td>
<td>110</td>
<td>139</td>
<td>57</td>
<td>15.0</td>
<td>12.1</td>
</tr>
<tr>
<td>EF-2</td>
<td>0.79</td>
<td>127</td>
<td>29,100</td>
<td>150</td>
<td>137</td>
<td>57</td>
<td>15.0</td>
<td>12.1</td>
</tr>
<tr>
<td>EF-3</td>
<td>0.75</td>
<td>130</td>
<td>28,100</td>
<td>120</td>
<td>140</td>
<td>55</td>
<td>15.0</td>
<td>12.1</td>
</tr>
<tr>
<td>EF-4</td>
<td>0.90</td>
<td>131</td>
<td>28,900</td>
<td>155</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>JW-20</td>
<td>0.85</td>
<td>113</td>
<td>30,200</td>
<td>172</td>
<td>124</td>
<td>46</td>
<td>10.0</td>
<td>9.0</td>
</tr>
<tr>
<td>JW-21</td>
<td>0.85</td>
<td>113</td>
<td>28,400</td>
<td>177</td>
<td>125</td>
<td>40</td>
<td>9.2</td>
<td>12.1</td>
</tr>
<tr>
<td>JW-22</td>
<td>0.85</td>
<td>115</td>
<td>27,900</td>
<td>116</td>
<td>125</td>
<td>46</td>
<td>11.0</td>
<td>9.0</td>
</tr>
</tbody>
</table>

* Second letter, "W" or "F", denotes specimen from web or flange coupon, respectively.

** The values of E should be regarded as indicative only since they were measured directly from the autographically recorded curve.

*** Gage length was 8 inches.
<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Source</th>
<th>Area (in²)</th>
<th>Static Yield Stress (ksi)</th>
<th>Modulus of Elasticity (ksi)</th>
<th>Strain Hardening Modulus (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-1</td>
<td></td>
<td>0.3725</td>
<td>108</td>
<td>30,800</td>
<td>300</td>
</tr>
<tr>
<td>C-2</td>
<td>1/2&quot; plates</td>
<td>0.3725</td>
<td>110</td>
<td>29,700</td>
<td>280</td>
</tr>
<tr>
<td>C-3</td>
<td>(as-delivered)</td>
<td>0.3724</td>
<td>110</td>
<td>31,200</td>
<td>310</td>
</tr>
<tr>
<td>C-4</td>
<td></td>
<td>0.3725</td>
<td>110</td>
<td>29,700</td>
<td>300</td>
</tr>
<tr>
<td>C-5</td>
<td></td>
<td>0.5991</td>
<td>110</td>
<td>29,700</td>
<td>320</td>
</tr>
<tr>
<td>C-6</td>
<td></td>
<td>0.5990</td>
<td>122</td>
<td>30,000</td>
<td>350</td>
</tr>
<tr>
<td>C-7</td>
<td>3/4&quot; plates</td>
<td>0.5991</td>
<td>121</td>
<td>31,100</td>
<td>360</td>
</tr>
<tr>
<td>C-8</td>
<td>(as-delivered)</td>
<td>0.5990</td>
<td>122</td>
<td>31,600</td>
<td>350</td>
</tr>
</tbody>
</table>
Table 3.1  RESIDUAL STRESS DISTRIBUTION  
IN WELDED HEAVY H-SHAPES

![Diagram of Welded Heavy H-Shapes]

<table>
<thead>
<tr>
<th>Specimen Grade</th>
<th>Plate of Weld Edge Preparation</th>
<th>RESIDUAL STRESSES (ksi)</th>
<th>Inside Face</th>
<th>Outside Face</th>
</tr>
</thead>
<tbody>
<tr>
<td>C2 A36 1/2&quot; F UM</td>
<td>$\sigma_1$</td>
<td>+23.3</td>
<td>+29.3</td>
<td>+31.5</td>
</tr>
<tr>
<td></td>
<td>$\tau_1$</td>
<td>+16.8</td>
<td>+10.2</td>
<td>+11.4</td>
</tr>
<tr>
<td>C4 A36 60G UM</td>
<td>$\sigma_3$</td>
<td>+30.9</td>
<td>+28.9</td>
<td>+28.5</td>
</tr>
<tr>
<td></td>
<td>$\tau_3$</td>
<td>+19.3</td>
<td>+18.1</td>
<td>+17.0</td>
</tr>
<tr>
<td>C6 A441 1/2&quot; F UM</td>
<td>$\sigma_5$</td>
<td>+8.6</td>
<td>+11.6</td>
<td>+13.7</td>
</tr>
<tr>
<td></td>
<td>$\tau_5$</td>
<td>+21.0</td>
<td>+23.0</td>
<td>---</td>
</tr>
<tr>
<td>C8 A441 60G UM</td>
<td>$\sigma_7$</td>
<td>+21.2</td>
<td>+19.8</td>
<td>+17.1</td>
</tr>
<tr>
<td></td>
<td>$\tau_7$</td>
<td>+11.8</td>
<td>+11.9</td>
<td>+14.1</td>
</tr>
<tr>
<td>C10 A36 1/2&quot; F FC</td>
<td>$\sigma_9$</td>
<td>-3.9</td>
<td>-3.80</td>
<td>-18.3</td>
</tr>
<tr>
<td></td>
<td>$\tau_9$</td>
<td>-10.1</td>
<td>-15.5</td>
<td>-20.9</td>
</tr>
<tr>
<td>C12 A36 60G FC</td>
<td>$\sigma_10$</td>
<td>-6.8</td>
<td>-3.8</td>
<td>-6.7</td>
</tr>
<tr>
<td></td>
<td>$\tau_10$</td>
<td>-17.0</td>
<td>-15.4</td>
<td>-7.9</td>
</tr>
<tr>
<td>C14 A441 1/2&quot; F FC</td>
<td>$\sigma_{11}$</td>
<td>-37.8</td>
<td>-36.0</td>
<td>-37.6</td>
</tr>
<tr>
<td></td>
<td>$\tau_{11}$</td>
<td>-47.9</td>
<td>-41.6</td>
<td>-50.3</td>
</tr>
<tr>
<td>C16 A441 60G FC</td>
<td>$\sigma_{12}$</td>
<td>-46.6</td>
<td>-45.2</td>
<td>-46.9</td>
</tr>
<tr>
<td></td>
<td>$\tau_{12}$</td>
<td>-40.1</td>
<td>-37.0</td>
<td>-34.4</td>
</tr>
</tbody>
</table>

**Notes:** First row for each specimen shows the residual stress on the outside face and second row, inside face. Average values are listed for web residual stresses.

UM - Universal Mill  
FC - Flame Cut  
F - Fillet Weld  
G - Groove Weld
9. FIGURES
Fig. 2.1 Diagrammatic Stress-Strain Curve for Structural Carbon Steel
Fig. 2.2 Dimensionless Ramberg-Osgood Stress-Strain Curve (From Reference 4)
Fig. 2.3  Typical Stress-Strain Curve for an A514 Steel Tension Specimen Test

Tension Specimen No. J12
Average Area 0.433 in.²
Static Yield Stress 115 ksi
Ultimate Tensile Stress 125 ksi
Modulus of Elasticity 28500 ksi
Strain Hardening Modulus 140 ksi
Fig. 2.4  Histograms for Mechanical Properties of A514 Steel
Fig. 2.5 Determination of the Static Stress-Strain Curve for Nonlinear Materials
(a) AVERAGE TYPICAL STRESS-STRAIN CURVE

(b) STRESS-STRAIN AND TANGENT MODULUS-STRAIN RELATIONSHIPS IN THE TRANSITION RANGE

Fig. 2.6 Stress-Strain Relationship for A514 Steel
(a) Rolled Steel Shapes

(b) Welded Built-up Steel Shapes

Fig. 2.7  Idealized Residual Stress Distribution
Distribution of Stress at Critical Load before Buckling.

Distribution of Stress at Critical Load in the Buckled State.

Fig. 3.1 Stress Diagram During Buckling
Fig. 3.2 Arrangement of Finite Area Elements
Fig. 3.3 Tangent Modulus Curves for Rolled Columns (Effect of the Size of the Section)
Fig. 3.4 Tangent Modulus Curves for Rolled Columns (Effect of Stress-Strain Curve)
Fig. 3.5 Tangent Modulus Curves for Welded Columns With Sheared Plates (Effect of Stress-Strain Curves)
Fig. 3.6 Column Curves for Various Patterns of Residual Stress Distribution and Comparison With Experimental Results
Fig. 3.7  Idealized Residual Stress Distribution in Welded Heavy H-Shapes
Fig. 3.8  Idealized Residual Stress Distribution in Welded Heavy H-Shapes
Fig. 3.9 Measured Residual Stress Distribution in a Welded Heavy H-Shape
Fig. 3.10 Measured Residual Stress Distribution in a Welded Heavy H-Shape
Fig. 3.11 Column Curves for 15H290 Shapes (Based on Idealized Residual Stress Distributions)
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Fig. 3.13 Relationship Between the Applied Axial Force and the Effective Area
Fig. 3.14  Progress of the Effective Area for a Rectangular Section
Fig. 3.15 Effective Modulus vs. Effective Moment of Inertia Relationship
Welded Built-Up H-Shapes With Sheared Edge Plates.
A 514 Steel

Fig. 3.16 Effective Modulus vs. Effective Moment of Inertia Relationship
Welded Built-Up
H-Shapes With
Flame-Cut Plates.
A514 Steel
Fig. 3.18 Effective Modulus vs. Effective Moment of Inertia Relationship for Welded Heavy H-Shapes
Fig. 3.19 Column Curves Based on Stub Column Tests
Fig. 4.1 Several Typical Beam-Columns
Fig. 4.2 Forces on a Beam-Column
Fig. 4.3 Unloading Stress-Strain Curve
Fig. 4.4 Typical Strain Diagram for a Beam-Column
Fig. 4.5  Moment-Curvature-Thrust Relationship
Fig. 4.6 Comparison of Moment-Curvature-Thrust Curves
Fig. 4.7 Loading Conditions for Beam-Columns
Fig. 4.8 Numerical Procedure for Calculating Load-Deflection Relationship
(a) MOMENT CURVATURE HYSTERESIS
(FROM REFERENCE 46)

(b) MOMENT CURVATURE CURVE
WITH UNLOADING

Fig. 4.9 Moment-Curvature Curves
Fig. 4.10 End-Moment vs. End Rotation Curves
Fig. 4.11  End-Moment vs. End Rotation Curves
Fig. 4.12 Comparison of End-Moment and End-Rotation Curves

- Elastic Perfectly Plastic $\sigma - \varepsilon$ Curve
- A514 Steel $\sigma - \varepsilon$ Curve

$P = 0.55P_y$

$\frac{L}{r_x} = 40$

$\frac{M}{M_{pc}}$

END ROTATION $\theta$ (RADIANS)
Fig. 4.13 Interaction Curves for A514 Steel Beam-Columns
(a) GENERAL LAYOUT

Fig. 4.14  Detail of the Beam-Column Specimen
(a) Beginning of Test

(b) End of Test

Fig. 4.15 Beam-Column in Testing Machine
Fig. 4.16 Load-Deformation Relationship and Test Results
Welded IIH71 With Flame-Cut Plates, A514 Steel
Braced

\[ \frac{M}{M_{pc}} \]

**END MOMENT**

**END ROTATION, \( \theta \) (RADIANS)**

Experiment
Theory

Welded IIH71 With Flame-Cut Plates, A514 Steel
Braced

\[ P = 0.55 P_y \]

\[ \frac{L}{r_x} = 40 \]

**Fig. 4.17 Load-Deformation Relationship and Test Results**
Fig. 4.18 Error of the Moment Readings

- $M_j$ Hydraulic Jack Reading
- $M_d$ Dynomometer Reading
- $M_s$ Strain Gage Reading
Fig. 4.19 Comparison Between Test Results and Theoretical Solutions
Fig. 4.20 Plate Buckling Curves (Plates Simply Supported and Free at Unloading Edges)
Load - Deflection Curves of Local Buckling Tests on an IIH 71 Shape

Fig. 4.21 Load-Local Deformation Curves
Fig. 4.22 A Stub Column Test
Fig. 4.23 A Stub Column Test

II H 7I
Welded Built-Up Shape
A514 Steel
Stub Column Test 290-I-H
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