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RE: Fritz Lab Report No. 276.9
TESTS ON THE STABILITY OF STEEL FRAMES
by Y. C. Yen, L. W. Lu, and G. C. Driscoll, Jr.

Gentlemen:

We are forwarding you a copy of the above paper for review and comment. This paper describes tests on three sets of pinned-base rectangular portal frames made from small size wide-flange shapes. These frames were tested to verify the theoretical solution for frame instability due to sidesway which was derived by Le-Wu Lu in his dissertation. The tests also verify the adequacy of the AISC rule for load and slenderness of columns in continuous frames where sidesway is not prevented.

We wish to submit this paper to ASCE for publication after making necessary editorial changes. We would appreciate any comments or suggestions you wish to make about this paper.

Please return the attached post card expressing your sentiments with respect to publication by December 15th.

Sincerely yours,

George C. Driscoll, Jr.
Associate Project Director

GCD:1m

cc: ASCE Committee on Plasticity Related to Design
K. H. Koopman
L. S. Beadle
Welded Continuous Frames and Their Components

TESTS ON THE STABILITY OF STEEL FRAMES

by

Yu-Chin Yen
Le-Wu Lu
George C. Driscoll, Jr.

This work has been carried out as a part of an investigation sponsored jointly by the Welding Research Council and the Department of the Navy with funds furnished by the following:

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Fritz Engineering Laboratory
Civil Engineering Department
Lehigh University
Bethlehem, Pennsylvania

September 1961

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ABSTRACT

In order to determine the inelastic buckling strength of pin-ended rectangular portal frames, three sets of frames with varying heights were tested. The frames were fabricated from a specially rolled mild steel fence post section having geometric properties similar to those of a wide flange rolled shape and were subjected to three concentrated loads on each beam and one at the top of each column.

Load was applied by a lever system and metal dead weights. In each test, observations were made to determine the critical load which would cause sidesway of the frame. Thus the maximum load carried by a frame was determined. The test results were compared with those obtained from theoretical predictions.
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1. INTRODUCTION

The method of plastic design in steel structures has been rapidly accepted in this country. So far several hundreds of plastically designed buildings have been erected. For a general introduction to the concepts involved in such methods, reference can be made to (2) and (3).

One of the important assumptions made in the plastic methods of designing structures is that no buckling of any type should occur prior to the formation of the plastic mechanism. Local buckling and the buckling of individual members are not the scope of this investigation. However, the maximum load which the structure as a whole can carry may be less than the load computed on the basis of the strength of its individual members if sidesway is not prevented. In this case the possibility exists that the frame, as a whole, becomes unstable before the plastic mechanism is formed. If this occurs the structure is said to have failed by "frame instability".

The phenomena of overall instability are illustrated in Fig. 1 for a portal frame which is not prevented from sidesway.

In case 1 the frame carries no primary bending moment, therefore the behavior of the frame is analogous to that of
a centrally loaded column, in which bifurcation of equilibrium is possible at a certain critical load. In the elastic range the problems are solved both theoretically and experimentally. However, not much work has yet been done in the inelastic range.

In case 2 of Fig. 1, the frame carries primary bending moments at the instant when the system passes from stable to unstable equilibrium. This is the more practical situation since rigid frames are primarily designed to support loads by bending action rather than by compression. The solution to this type of stability problem becomes very complicated and only a few attempts have been made to solve them. Among these are the investigations by Chwalla\(^{4,5}\), Puwein\(^6\) and Masur, et al\(^7\), in the elastic range and experimental work by Bolton\(^8\), Salem\(^9\), Gurney\(^10\) and Low\(^11\) in the inelastic range.

Until the completion of Lu's\(^12\) dissertation, there was no analytical method by which inelastic frame instability of this type could be predicted precisely. Lu's method is based on the modified moment distribution procedure due to Winter, et al\(^13\), in which stiffnesses are modified for the effect of axial force present in the members at a given load. In this analysis, all the required stiffness and carry over constants are modified not only for the effect of
axial force but also for the effect of yielding. The method of analysis is outlined as follows.

First the frame is assumed to be braced from sidesway instability. By a numerical integration process the moment vs. angular rotation curve of the beam due to vertical loading and end moment is obtained. The end moment vs. rotation curve of a column was developed by Ojalvo in Reference 14. Applying the boundary condition of equal beam and column rotation at the knee of the frame, the moment and rotation at the knee are determined at the assumed total vertical load.

Knowing the two end moments, the moment diagram of the beam may be easily constructed by statics. Since the effective flexural rigidity (EI)eff. of the section can be determined as the instantaneous slope on the M-Ø diagram corresponding to the applied moment, the width of the analogous column may be determined. By the method of column analogy, stiffness and carry over factors of beam may be determined. The stiffness factor of the column with a hinged end can be determined as the slope of the moment-rotation curve of a beam-column. Having the stiffness and carry over factors of the members, it is then possible to determine the effects of a small lateral displacement. By introducing arbitrary fixed end moments due to sidesway displacement of the column tops and performing a moment distribution computation for the frame, the moments at the knees of the columns are determined.
From the moments in columns, the horizontal shear force in the frame can be obtained. As a criterion of sidesway buckling, the critical condition will be reached when the sum of the resulting shears becomes zero. In other words, no lateral force is required to push the frame sidewise as it is in actual loading condition. The load corresponding to this critical condition determines the inelastic buckling strength of the frame. This paper presents the results of tests made to verify the theoretical solution for the instability of symmetrical frames loaded vertically only and having primary bending moment in the frame.

A third type of frame instability is shown in Case 3 of Fig. 1. The frame is subjected to a combination of horizontal and vertical forces. It deforms laterally from the first load application. The change in geometry introduces additional bending moment in the columns. The whole frame becomes unstable in this deformed position much like an eccentrically loaded column. At a certain critical loading, the structure continues to deform without an increase in load. This leads the frame to a failure. This problem is important in the design of multi-story buildings subjected to wind loads. Future work both analytical and experimental is required on this subject.
Since 1958 research on the problem of frame stability has been carried on at Fritz Engineering Laboratory as a part of the broad investigation titled "Welded Continuous Frames and Their Components". Several model frames of welded box sections were tested. Their results will be found in Reference 15.

In order to verify the inelastic buckling solution in Lu's work, three sets of model frames with column slenderness ratios of 40, 60 and 80 were proposed for test in June 1960. The frames simulated the first floor of a three story building. Fig. 2 shows the dimension and loading of the frames. The frames were loaded by dead weights magnified by a lever system so that there was proportional loading and the loading system could sway freely with the frame. After reaching a certain load the frames would sway sidewise and the horizontal deflection would increase continuously without additional load. Thus the ultimate loads of the frames were obtained and comparisons with those of the theoretical predictions were made.
2. DESCRIPTION OF FRAMES

2.1 Characteristic Features of Test Frames

The frames were single bay rectangular rigid frames as summarized in Table 1 and detailed in Fig. 3. Three sets of frames with span lengths of $87 \frac{19}{32}$ inches and heights of $43 \frac{13}{16}$, $65 \frac{11}{16}$ and $87 \frac{19}{32}$ inches respectively were designated as W-1, W-2 and W-3 in sequence of their height.

The following features of the frames distinguish the present investigation from its predecessors.

(1) Frames were subjected to a primary bending moment and were loaded into the inelastic range.

(2) The most practical structural shape of WF section was adopted.

(3) Members of a frame were subjected to strong axis bending, therefore, two frames with a bracing system between them were tested at the same time to eliminate premature lateral-torsional buckling.

(4) The load vs. slenderness ratio of the columns were chosen as variables in this investigation.
From the theory of eccentrically loaded columns, it is known that the effect of eccentricity upon buckling strength is considerable in the plastic range of buckling but tapers off in the elastic range with increasing slenderness ratio of the columns. This implies that the slenderness ratio of the columns of a frame plays an important role in the buckling strength of the frame in the inelastic range. Therefore, three sets of frames with different column heights were tested. The slenderness ratios of the columns were chosen such that inelastic buckling of a frame would occur prior to the formation of failure mechanism in simple plastic theory. The variables governing the buckling strength were the load $P$ on the column and slenderness ratio $\frac{h}{r}$ of the column, while the span length and cross section of the member were kept the same for the three frames.

2.2 Sectional Properties

The shape of the cross section of the member is one of the important factors affecting the buckling load. Some frame stability tests have been conducted using box sections, tubes or solid bars but so far no test of this kind has been conducted using WF section. In practical building frames, however, WF sections are commonly used. Therefore, the frames were fabricated from $2 \frac{5}{6} \times 1\frac{1}{2}$ WF shapes designated
No. M-2362 by Bethlehem Steel Company. The cross sectional dimensions were measured with the aid of a micrometer and actual properties of the section were compared with those given by Bethlehem Steel Company. A comparison of the actual and nominal properties is shown in Table 2.

2.3 **Material Properties**

The material properties of the member were determined by tension coupon test, stub column test and control beam test. Two coupons cut from the flanges and one from the web of the section were tested in a screw type testing machine. Load and elongation over a 6 in. gage length were measured and plotted by means of a Tinius-Olsen extensometer and a low-magnification automatic stress-strain recorder. The rate of application of load was about 0.025 inch per minute and 0.1 inch per minute after strain hardening. After the yielding region had been reached but before strain-hardening had commenced, the strain rate was reduced to zero for a period of a few minutes to allow the load to reach an equilibrium point. From this reading the lowest possible yield stress could be calculated, thus insuring that in the actual test structure the yield stress would be equal to this or greater. The results of coupon tests are summarized in Table 4.

A cross-section (stub-column) test was made to find the compressive stress-strain curve of a full cross-section of
the member of the frames. This was an axial compression test which gave the integrated effect of different web and flange strengths in one test. The main purpose of the test was to obtain a compressive yield value to use in predicting the theoretical buckling load. It should be pointed out that there is a substantial difference of yield stress level between flange and web, as shown in the results of coupon tests in Table 3. Therefore, the stub column test is necessary and the result of the test gives a more reasonable yield stress level. The specimen was 6 inches in length. Two SR-4 strain gages were provided for the measurements of strains. Prior to the stress-strain test, the specimen was aligned to insure concentric loading. The result of the test is shown in Fig. 4. The average yield stress level of flange coupons is very close to the result obtained from stub column test. Therefore, the average value of 42,693 ksi was adopted as the yield stress level.

In order to obtain the actual moment-curvature relationship of the section as shown in Fig. 5, a control beam test was necessary. The test setup is shown in Fig. 5. The beam was simply supported at its ends and loaded at its third points causing pure bending in the portion between the concentrated loads. Two optical mirrors were attached to rods welded perpendicular to the plane of the beam at the load points. The mirrors reflected the image of a graduates scale 10 feet away
from the center of the beam. When load was applied, the beam and the attached mirrors as well, rotated. Readings of the reflected image of the scales in each mirror were obtained through a transit. The increments of the scale readings were used to calculate the angular rotation between the points of the two mirrors. The unit angular rotation gave the curvature of the beam under the applied moment. The moment-curvature curve of the section is plotted in Fig. 6. SR-4 strain gages attached at the flanges also provided additional results for check. The plastic moment \( M_p \) from the test result was 47.6 kips-in. However, another \( M_p \) of 46.87 kips-in. was obtained by calculation based on the measured area of the section and the adopted yield stress level. Therefore it was decided that \( M_p \) value of 47 kips-in. would be adopted for theoretical predictions.

2.4 Loading Condition of Test Frames

As shown in Fig. 2 a uniform beam loading was approximated by three concentrated loads \( P_1 \). A concentrated load \( P \) at the top of the column represented the loads from the upper floors. This particular loading condition would simulate a condition that may be expected in the lower stories of a tall building. A parameter \( \alpha \) which relates the magnitude of the concentrated column load \( P \) to the beam load \( P_1 \) was kept constant for each case. This implied that the beam and column loads were assumed to increase simultaneously with a fixed
ratio $\alpha$ between them. The total number of stories was then $\alpha+1$. In this experiment $\alpha$ was about two, therefore, the frames could be considered as the first story of a three-story building.

2.5 Design of Test Frames

The frames were designed according to the method of plastic design outlined in References (2) and (3). Since the loading pattern and the size of the beam had been selected in advance, maximum moments and forces throughout the frame could be determined on the basis of a simple plastic analysis considering a beam mechanism. Knowing these moments and forces, base fittings and welded connections were designed.

The knees, column base plates and loading points on the beams were of all-welded construction. The frames were fabricated in the laboratory by welders and fitters whose regular jobs involve similar operations at the plant of a steel fabricator.

2.6 Lateral Bracing System

Past experience in testing rigid frames into the plastic range had shown that adequate lateral support was essential if the theoretical ultimate load was to be attained. This would prevent lateral-torsional buckling of the member. In this investigation two frames connected by a lateral bracing
system were tested at the same time. Thus eliminating possible frictional forces to be developed between the model frame and lateral supporting guide if a single frame would be tested.

The system was composed of welded purlins made from $1 \frac{1}{2}'' \times \frac{3}{4}''$ channels and cross braces. The cross braces were made of $\frac{1}{4}'' \varnothing$ threaded steel bar, $2 \frac{1}{2}''$ turn buckles and hooks. The cross sectional properties of the purlins are given in Table 3. The spacing of purlins was less than $45 \, r_\gamma$ of the main frame member which was well within the critical length for lateral torsional buckling in the inelastic range as suggested in Reference 16, where $r_\gamma$ is the radius of gyration about the weak axis of the section. The lateral bracing system can be seen in the photographs of the test frame in Figs. 7, 8 and 9.
3. **LOADING SYSTEM AND TEST APPARATUS**

3.1 **Requirements and General Arrangement**

There are several basic principles upon which the loading system and test apparatus were designed. These requirements, on the other hand, characterize the peculiarity of the test setup. Fig. 3 shows a general arrangement of test setup which was designed to meet the following requirements.

1. Proportional loading should hold.
2. Loading systems should not restrain the frame from sidesway movement.
3. Loading system, apparatus as well as the frames, should be symmetrical in both directions.
4. Nearly perfect pin-ended column support was required.
5. Deformation of the frames at every stage of loading should be measured precisely.

According to the above mentioned requirements, the load was applied on the frames by five sets of lever systems together with dead weight on loading baskets. One end of the multiplication lever was connected with the base beams (ground) by wire ropes and turnbuckles, while the loading basket was hung on the other end. The turnbuckles were used to adjust the lengths of wire rope so that the multiplication lever could be kept in level. The lever ratios of multiplication lever were designed to magnify the dead weight on the loading basket and also to produce a proportional increment of load.
on the frames. By proportional loading it is meant that the ratio of column load to the beam load was constant at the value and throughout the loading process. The frames together with loading system were fixed in position by four sets of column base fixtures on the base beams so that the frames could sway freely only in the plane of the frames. The two 14WF314 base beams and one 14WF61 beam in between were anchored to the concrete floor of the test bed by two bolts of four inches in diameter. The base beams were heavy enough to transmit the load to the floor without any appreciable deformation. The weights used for loading were 20 and 50 pound steel blocks and assorted round blocks with varying weights of 10 to 55 pounds as shown on the baskets in Figs. 10, 11 and 12. They were provided by Bethlehem Steel Company.

3.2 Beam Loading System

Section A-A of Fig. 3 indicates the type of loading device which transmits the loading weight on the basket to the beams of the frames. There are three identical devices hung on the middle of the third point of the beams. Since the systems were connected with the base beam by wire ropes, initiation of sidesway of the frames would not be prevented. At the six loading points on the beams, specially designed hangers connected the loading system to the frames. At the point of attachment of the 315.7 multiplication lever to the
315.7 spreader beam, a screw device was provided to allow equal distribution of the load to each frame. Any error in the lever ratio of the multiplication levers could produce an incorrect load on the beams. This could be corrected by putting a slightly different weight on the loading basket so that the six readings from the dynamometers were nearly equal. The reading from the dynamometers gave directly the magnitude of the concentrated load $P_1$ on the beam.

3.3 Column Loading System

Referring to the elevation of Fig. 3, there are two sets of column loading system, one at each end of the frame. Each column loading system applies a concentrated load to the pair of columns at its end of the frame. A multiplication lever made from a 6 B 12 with welded 1/4 inch cover plates was rested on a spreader beam of 6 B 12 section. A roller support between the spreader beam and the multiplication lever enabled the frames to sway freely without introducing serious frictional force. As shown in Fig. 13 a shaft and two sets of roller bearings were fitted into a pillow block and a pillow block was screwed to each spreader beam. A sample bearing was proof tested under vertical load of 20 kips to assure adequate rolling capacity of the bearing during the test.
One end of the multiplication lever was tied down to the base beam by a 1/2 inch diameter wire rope, a dynamometer and a turnbuckle. The turnbuckle was used to adjust the length of the wire rope to keep the multiplication lever in level. Otherwise a horizontal component of force from the loading beam would tend to push the frame sidewise. The column load was obtained by adding the dynamometer reading in the tie down wire rope to the weight of the loading beams plus the weight of the loading basket and added weights.

3.4 **Column Base Fixture**

Since a slight restraint at the column end tends to increase the buckling load considerably as shown in Ref. 17, a perfect pin-ended support free from any restraint was necessary. Fig. 14 shows the details of one of the column base fixtures used. The column base plate was connected to a shaft which was cut flat at the top so that the base plate could fully rest on the shaft. Four 3/16" Ø screw were used to fix the column to the shaft. The shaft was fitted into two roller bearings of 40 ton capacity each. The bearings were held in a pillow block which was screwed to a stiffened base box. The base box was then clamped to the flange of a base beam. In order to assure that no slip would occur between the base box and base beam during test, spot welding of about one inch in length was done after alignment of the frames was completed. Then the frames could only sway freely in the direction of the plane of the frame.
3.5 Deflection Measurement

The deflections of the beams and columns of the frame at the end of each loading increment were measured by a graduated scale and transit. The scale was about one foot in length. One end of the scale was pointed so that it could be inserted into punched holes on the frame. The holes were punched 6 inches apart throughout the outer surface of the flanges of the frame members. The pattern of the holes is shown in Fig. 15.

Three transits, one for each member, were set up in front of the frame for deflection measurement. Reference points were marked on the floor and wall to fix the positions of the transits and their directions of observation. The deflection was read accurately to 0.01 inch and estimated to the next digit.

As shown in Figs. 10, 11 and 12, there was a vertical rod beside each column. Horizontal lines on the rod were drawn at the same level as the punched holes on the columns. By holding the scale in the punched holes and along the line, horizontal deflection of the column was read through a transit which could only rotate in a vertical plane parallel to the column. In order to measure the vertical deflection on each beam, a triangular plate was used to assure the vertical position of the scale. The transit was fixed against vertical rotation but it could rotate in a horizontal plane along the beams for taking vertical deflection of the beams.
3.6 Strain Measurement

Strain in the frames was measured by attached strain gages. All strain gages were electric resistance SR-4 type A-1 linear gages. The location of the gages on the frame is shown in Fig. 16. There were 24 gages throughout the frames. The gage readings were used to align the point of application of the load. The alignment would not be perfect until the strain reading showed symmetrical figures in both directions.

A relatively small number of gages was used on the tests, because the elastic behavior of the frame was not being emphasized in the investigation. After the yield point has been reached at a gage location, knowledge of the exact magnitude of strain is not of prime importance; however, all of the strain readings were taken after each increment of load throughout the test. The strain gages were connected to a switch box and then to the strain indicators.
5. **TEST PROCEDURE**

5.1 **Test Setup**

After the frames were erected on the base beam, strain gages were mounted and wires were connected to the switch boxes and indicators. Then the frames were whitewashed with hydrated lime. Flaking of the whitewash during testing indicates the progression of yielding. Initial readings of strains and dynamometers were taken before the heavy loading beams and baskets were put on the frame. After the loading beams were set in position, turnbuckles were adjusted to keep the loading beams level.

5.2 **Alignment of Test Frames**

Around 200 pounds of weight was loaded on each loading basket. By taking increments of the load dynamometer reading, it was easy to figure out which side of the frames was overloaded. The weight was unloaded and the lever ratio of the multiplication lever was adjusted. The process was repeated until the difference was within 5%.

After alignment of the multiplication lever was finished, strain readings were taken. Unsymmetrical strain readings indicated that the point of application of the loading system was deviated. Again the weight was removed to adjust the position of the spreader beam on the columns. Alignment was continued until the increments of reading from strain indicators and dynamometers gave symmetrical values within 5% error.
It is of interest to point out that for the first and second sets of load increments, the readings were not quite satisfactory. However, when the load was increased more, the results tended to be more reliable. An explanation for this could be that the initial deformation in the wire and multiplication lever affected the initial reading very much.

5.3 Testing

Figures 10, 11 and 12 show the testing of frames W-1, W-2 and W-3 respectively. In order to keep the relationship of proportional loading, 80 pounds of weight was loaded initially on each of the three baskets of the beams. This was an adjustment to the different weight of multiplication levers for the columns. From then on, the same weight on each basket would produce proportional loading on the frames.

Two men were necessary for reading and recording the dynamometers and strain indicators. The applied loads on the beams were calculated by multiplying the measured strain increments by the constants obtained from dynamometer calibrations. In case the loading obtained from the dynamometers was not satisfactory, it could be adjusted by putting on the baskets a slightly different weight for the next loading.

Another couple of men took care of the deflection measurement. One held the scale, while the other took the readings through the transit. The scale was shifted from one position to another, and hence the deflections of the beams and columns were obtained.
After each increment of load, the deflections at the centers of the beams were measured and plotted on the predicted curve. This load vs. deflection curve would justify that both the testing apparatus and the frames were functioning satisfactorily.

For frame W-1, the first five sets of load increments were about 200 pounds in each basket. The increment was gradually decreased to about 10 pounds at the final loading, No. 19. After loading No. 10 visible yielding on the frames was observed at the inner surface of the right column at the corner. This was the first indication that the frames might sway to the right. The frames started to sway visibly at loading No. 11. From then on, it took more than 20 minutes of waiting for the frame to slow down the sidesway motion. Several jacks and wood blocks were put under the baskets to assure that no sudden failure of the frame would occur. At the final loading the frames swayed slowly and continuously, therefore, no deflection was taken and the test was finished.

Frame W-2 was 22 inches higher than frame W-1. The wire ropes on the loading beams were made 22 inches longer. The load increment was 200 pounds for the first five sets, then seven sets of 100 pound increments followed. Finally three sets of 30 pound increments concluded the test.
Frame W-3 was about 9 feet in height above the floor so it vibrated considerably after each loading process. At the end of test the frames swayed considerably so that the rotation at the knee caused the spreader beam at the top of the column to tilt too much. Eventually the spreader beam overturned and multiplication beam above slid down on the frame.
5. TEST RESULTS AND THEIR COMPARISON WITH THEORETICAL PREDICTIONS

5.1 Test Results

Generally speaking, the test frames and the apparatus behaved satisfactorily throughout the test.

The deformed shapes of the frames W-1 and W-2 are shown in Fig. 17 and 18 respectively. The last set of deflections was not taken due to overturning of one of the spreader beams at the top of the column, therefore, the deformed shape of frame W-3 is not shown.

The dotted lines in Fig. 17 and 18 show the deflected shape of the frames when the horizontal deflection of the column top first became noticeable. The load corresponding to this point is defined as critical load $P_{cr}$. The solid lines show the shape of the frames just before the ultimate load $P_{ult}$ was reached. Any further increase of the applied load would have caused continued sway of the frames. This is the maximum load the frame can carry and is defined as ultimate load $P_{ult}$.

The test results were plotted in Figs. 19, 20 and 21 as the load versus horizontal deflection curve at the column top. The predicted buckling loads $P_{pre}$ were shown as dotted lines on deflection curves of the Southeast columns of the frames.
Table 5 provides a summary of all the test results and of the theoretical analysis. The ultimate loads obtained from the tests are 11.172, 10.136 and 9.160 kips compared to the predicted buckling loads of 10.648, 10.181 and 8.611 kips for frames $W=1$, $W=2$ and $W=3$ respectively.

Because of the satisfactory lateral supporting system, the two frames were acting together as one frame and no lateral buckling of the members was observed before the ultimate loads were reached in any of the three tests.

5.2 Comparison of Test Results with Theoretical Prediction

The ultimate loads $P_{ult}$ obtained from the tests were plotted as points $W=1$, $W=2$ and $W=3$ in Fig. 22. The curves in the graphs show the predicted inelastic buckling loads based on the theory given in Ref. 12. For frames $W=1$ and $W=3$, the experimental loads are several percent higher than the predicted loads, while both experimental and predicted loads are about equal for frame $W=2$. The average error between the theoretical prediction and test results is less than 4%. As shown in Fig. 22, a different coefficient of proportionality $\alpha$ was used for frame $W=3$. It was believed from previous test results that frame $W=3$ would fail by elastic buckling. Therefore, $\alpha$ was reduced to 1.8 to assure that the frame would buckle in the inelastic range.
The experimental buckling load $P_{ult}$ was compared with the predicted buckling load $P_{pre}$, beam-column instability load $P_{um}$ and simple plastic load $P_u$ as summarized in Table 5.

5.3 Comparison with the AISC Formula

To safeguard against frame instability in plastically designed one- and two-story rigid frames, the following rule is recommended in the AISC Plastic Design Manual. (18)

"Columns in continuous frames where sidesway is not prevented (a) by diagonal bracing (b) by attachment to an adjacent structure having ample lateral stability or (c) by floor slabs or roof decks secured horizontally by walls or bracing systems parallel to the plane of the continuous frames shall be so proportioned that

$$2 \frac{P}{P_y} + \frac{h}{70r} \leq 1.0$$

where $P =$ the axial force in the column when the frame carries its maximum load.

$P_y =$ the axial yield load of the column

$\frac{h}{r} =$ slenderness ratio of the column."

The justification of this rule can be found in Ref.(3). A frame with a combination of axial load and column slenderness ratio within the triangular envelope in Fig. 23 can carry as much load as a braced frame. Test results indicated that Frame W-1 could carry 96.9% of the load of a braced frame.
The point is very close to the 100% line, therefore, the formula is quite accurate in the vicinity of the loading and slenderness ratio of frame W-1.
6. CONCLUSIONS

The following conclusions can be drawn from the results of the investigation presented in this paper.

(1) The predicted buckling loads are very close to the experimental buckling loads and are less than 4% on the safe side as could be observed in Table 5. This indicates both the theory and the test are very satisfactory and successful.

(2) The ultimate load $P_{ult}$ is about 84% of the simple plastic load. The reduction in load carrying capacity from simple plastic load is too large to be neglected. Therefore, a check against frame stability should be made in plastically designed frame.

(3) The designs of model frames and test setup are very satisfactory as evidenced by the test results. The success of this test could be considered as a cornerstone to a more complicated test of multi-story frame.

(4) The agreement of the test results with the AISC formula for the slenderness of columns in frames not braced against sidesway provides confidence that the AISC formula will safeguard against the frame stability problem in plastically designed frames. Since no tests were conducted in the regions of higher and lower axial load, a more precise design formula can not be recommended now. However, examination of
these results in the light of the approximate theoretical analysis given in Ref. 3 suggests that the AISC formula may be more conservative in these regions.

The inelastic buckling problem of single story rectangular frame has been properly solved and future work should be done on the stability of multi-story frames.
ACKNOWLEDGEMENTS

This report is based on a thesis prepared by Y. C. Yen in partial fulfillment of the requirements for the Master of Science degree.

The work contained in this report is part of an investigation on "Welded Continuous Frames and Their Components" being conducted under the direction of Dr. Lynn S. Beedle.

The project is sponsored jointly by the Welding Research Council and the Department of the Navy through the Institute of Research at Lehigh University. Funds are furnished by the American Institute of Steel Construction, American Iron and Steel Institute, Office of Naval Research, Bureau of Ships, and the Bureau of Yards and Docks. The Column Research Council of the Engineering Foundation acts in an advisory capacity.

The work was done at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania.

The assistance of Mr. Kenneth R. Harpel, Laboratory Foreman, and the Fritz Lab shop personnel in preparing the test setup and in conducting the tests is greatly appreciated.

The drawings were prepared by Mr. Stanley A. Gawlik and the manuscript was typed by Mrs. Lillian Morrow. Their cooperation is appreciated.
8. TABLES AND FIGURES
### Table 1. DIMENSIONS OF TEST FRAMES

<table>
<thead>
<tr>
<th>Frame No.</th>
<th>Span L in</th>
<th>Height of Column h in</th>
<th>$r_x$ in$^2$</th>
<th>$h/r_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-1</td>
<td>87 19/32</td>
<td>43 13/16</td>
<td>1.095</td>
<td>40</td>
</tr>
<tr>
<td>W-2</td>
<td>87 19/32</td>
<td>65 11/16</td>
<td>1.095</td>
<td>60</td>
</tr>
<tr>
<td>W-3</td>
<td>87 19/32</td>
<td>87 19/32</td>
<td>1.095</td>
<td>80</td>
</tr>
</tbody>
</table>

### Table 2. SECTIONAL PROPERTIES OF TEST SPECIMENS

<table>
<thead>
<tr>
<th></th>
<th>Area of Section A, in$^2$</th>
<th>Depth of Section d, in</th>
<th>Flange Width b, in</th>
<th>Flange Thickness t, in</th>
<th>Web Thickness w, in</th>
<th>$I_x$ in$^4$</th>
<th>$S_x$ in$^3$</th>
<th>$r_x$ in</th>
<th>Z in</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>1.085</td>
<td>2.625</td>
<td>1.840</td>
<td>0.201</td>
<td>0.156</td>
<td>1.236</td>
<td>0.942</td>
<td>1.062</td>
<td>1.086</td>
<td>1.16</td>
</tr>
<tr>
<td>Measured</td>
<td>1.043</td>
<td>2.625</td>
<td>1.813</td>
<td>0.207</td>
<td>0.156</td>
<td>1.251</td>
<td>0.953</td>
<td>1.095</td>
<td>1.067</td>
<td>1.12</td>
</tr>
</tbody>
</table>
Table 3. SECTIONAL PROPERTIES OF PURLIN

<table>
<thead>
<tr>
<th>Area of Section A, in²</th>
<th>Depth of Section d, in</th>
<th>Flange Width b, in</th>
<th>Flange Thickness t, in</th>
<th>Web Thickness w, in</th>
<th>Iₓ in⁴</th>
<th>Sₓ in³</th>
<th>rₓ in</th>
<th>rᵧ in</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.34</td>
<td>1.5</td>
<td>0.75</td>
<td>0.188</td>
<td>0.125</td>
<td>0.11</td>
<td>0.15</td>
<td>0.56</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Table 4. SUMMARY OF COUPON TEST RESULTS

<table>
<thead>
<tr>
<th>Location</th>
<th>ᵧ</th>
<th>ᵧult</th>
<th>ᵧY</th>
<th>ᵧst</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flange</td>
<td>42,110</td>
<td>54,710</td>
<td>0.00134</td>
<td>0.01336</td>
<td>32,047,000</td>
</tr>
<tr>
<td>&quot;</td>
<td>43,270</td>
<td>54,570</td>
<td>0.00125</td>
<td>0.01403</td>
<td>31,592,000</td>
</tr>
<tr>
<td>Web</td>
<td>48,280</td>
<td>60,350</td>
<td>0.00167</td>
<td>0.01086</td>
<td>30,483,000</td>
</tr>
</tbody>
</table>

where ᵧ = Static Yield Level  
ᵧult = Ultimate Tensile Strength  
ᵧY = Initial Yield Strain  
ᵧst = Strain at Initiation of Strain Hardening  
E = Modulus of Elasticity  
E mean = 31,374,000
Table 5. SUMMARY OF THE TESTS AND COMPARISON WITH THEORETICAL PREDICTIONS

<table>
<thead>
<tr>
<th>Frame No.</th>
<th>h/r</th>
<th>Experimental Buckling Loads lbs</th>
<th>Predicted Buckling Load lbs</th>
<th>Column Instability Load lbs</th>
<th>Simple Plastic Load lbs</th>
<th>$\frac{P_{cr}}{P_{pre}}$</th>
<th>$\frac{P_{ult}}{P_{pre}}$</th>
<th>$\frac{P_{ult}}{P_{u}}$</th>
<th>$\frac{P_{ult}}{P_{u}}$</th>
<th>$\frac{P_{um}}{P_{u}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-1</td>
<td>40</td>
<td>9,216</td>
<td>10,648</td>
<td>11,533</td>
<td>12,433</td>
<td>0.866</td>
<td>1.049</td>
<td>0.899</td>
<td>0.969</td>
<td>0.928</td>
</tr>
<tr>
<td>W-2</td>
<td>60</td>
<td>8,793</td>
<td>10,181</td>
<td>11,469</td>
<td>12,433</td>
<td>0.864</td>
<td>0.996</td>
<td>0.815</td>
<td>0.884</td>
<td>0.922</td>
</tr>
<tr>
<td>W-3</td>
<td>80</td>
<td>8,194</td>
<td>9,160</td>
<td>8,611</td>
<td>10,437</td>
<td>0.952</td>
<td>1.064</td>
<td>0.801</td>
<td>0.818</td>
<td>0.913</td>
</tr>
</tbody>
</table>
FIG. 1 TYPES OF FRAME INSTABILITY

FIG. 2 DIMENSIONS AND LOADING OF FRAMES
FIG. 3 MODEL FRAME AND TEST SET-UP
FIG. 4 STRESS-STRAIN CURVE FROM STUB-COLUMN TEST

FIG. 5 MOMENT-CURVATURE CURVE FROM CONTROL BEAM TEST
Fig. 6. SETUP FOR CONTROL BEAM TEST

Fig. 7. FRAME W-1 AFTER TESTING
Fig. 8. FRAME W-2 AFTER TESTING

Fig. 9. FRAME W-3 AFTER TESTING
Fig. 10. TESTING OF FRAME W-1

Fig. 11. TESTING OF FRAME W-2
Fig. 12 TESTING OF FRAME W-3
FIG. 13 ROLLER SUPPORT FOR MULTIPLICATION BEAM
Roller Brogs. to be Pressed Each End of Shaft Bore of Brogs. 1.250 In. +.0005 - .000

SHAFT

PILLOW BLOCK

FIG. 14 COLUMN BASE FIXTURE
FIG. 15 LOCATION OF DEFLECTION MEASUREMENT

FIG. 16 LOCATION OF STRAIN GAGES
FIG. 17. DEFORMED SHAPES OF W-1
FIG. 18. DEFORMED SHAPES OF W-2
FIG. 19  LOAD vs. HORIZ. DEFLECTION AT COL. TOP OF FRAME W-1
FIG. 20 LOAD VS. HORIZ. DEFLECTION AT COL. TOP OF FRAME W-2
FIG. 21 LOAD VS. HORIZ. DEFLECTION AT COL. TOP OF FRAME W-3
Simple Plastic Load

Beam Column Instability

Inelastic Buckling

Elastic Buckling

\[ P = \alpha \frac{3}{2} P_1 \]
\[ P = (1 + \alpha) \frac{3}{2} P_1 \]
\[ \alpha = 2.0 \]

\[ \alpha = 1.8 \]

FIG. 22 TEST RESULTS AND THEIR COMP. WITH THEOR. PREDICTIONS
FIG. 23 COMPARISON OF TEST RESULTS WITH THE AISC DESIGN RULE

\[ 2 \frac{P}{P_y} + \frac{1}{70} \frac{h}{r} = 1.0 \]

AISC Rule

- \( \frac{P}{P_y} \) vs \( \frac{h}{r} \)

- Points: W-1 (96.9%), W-2 (88.4%), W-3 (81.8%)
8. REFERENCES

1. Beedle, L. S.

2. Beedle, L. S.

3. WRC-ASCE Joint Committee
   COMMENTARY ON PLASTIC DESIGN IN STEEL, ASCE Manual No. 41, (1961)

4. Chwalla, E.
   DIE STABILITAT LOTRECHT BELASTETER RECHTECKRAHMEN, Der Bauingenieur, Vol. 19, p. 69, (1938)

5. Chwalla, E., and Kollbrunner, C. F.
   BEITRÄGE ZUM KNICKPROBLEM DES BOGENTRÄGERS UND DES RAHMENS, Der Stahlbau, Vol. 11, p. 94, (1938)

6. Puwein, M. G.
   DIE KNICKFESTIGKEIT DES RECHTECKRAHMENS, Die Bautechnik, Vol. 18, p. 32, (1940)


8. Bolton, A.
   STRUCTURAL FRAMEWORK, Ph.D. Dissertation, Manchester University, (1957)

9. Salem, A.
   FRAME INSTABILITY IN THE PLASTIC RANGE, Ph.D. Dissertation, Manchester University, (1958)

10. Gurney, T. R.

11. Low, M. W.
12. Lu, L. W.
   STABILITY OF ELASTIC AND PARTIALLY PLASTIC FRAMES,
   Ph.D. Dissertation, Lehigh University, (1960)

13. Winter, G., Hsu, P. T., Koo, B., and Loh, M. H.
   BUCKLING OF TRUSSES AND RIGID FRAMES, Cornell
   University, Engineering Experiment Station Bulletin
   No. 36, (1948)

14. Ojalvo, M.
   RESTRAINED COLUMNS, Journal of the Engineering

15. Lu, L. W. and Driscoll, G. C., Jr.
   BUCKLING TESTS ON MODEL FRAMES, Fritz Laboratory
   Report 276.3 Lehigh University, (in preparation)

16. Lee, G. C., and Galambos, T. V.
   THE POST-BUCKLING STRENGTH OF WIDE-FLANGE BEAMS,
   Fritz Laboratory Report 205 E.12, Lehigh University,
   (1961)

17. Galambos, T. V.
   INFLUENCE OF PARTIAL BASE FIXITY ON FRAME STABILITY,
   Journal of the Structural Division, ASCE, Vol. 86,
   No. ST5, (1960)

18. AISC
   PLASTIC DESIGN IN STEEL, AISC, 1959