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D. Feder

G. C. Lee

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Residual Stress and the Compressive Properties of Steel

Progress Report

RESIDUAL STRESSES IN HIGH STRENGTH STEEL

by

Diethelm K. Feder
George C. Lee

This work has been carried out as a part of an investigation sponsored jointly by the Column Research Council, the Pennsylvania Department of Highways, the Bureau of Public Roads, and the National Science Foundation.

Fritz Engineering Laboratory
Department of Civil Engineering
Lehigh University
Bethlehem, Pennsylvania

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SUMMARY

The report describes the test program comprising coupon tests, stub column tests, residual stress measurements, and column tests. The test results are discussed and the part played by residual stresses is investigated. A short theoretical treatment of the effect of residual stresses on column strength is presented. It is shown that the influence of residual stresses on column strength is smaller in high strength steel than in carbon steel.
1. INTRODUCTION

Residual stresses are the initial or "locked-up" stresses that exist in an unloaded structural member. They may arise from a number of causes, the major ones being

(a) differential cooling after hot rolling
(b) cold bending
(c) welding.

In this paper, only cause (a) is considered.

It can be shown that the part of the cross section which cools slowest will be in residual tension. For wide-flange sections this is at the connection of the flanges and the web. However, the details of the residual stress distribution will depend on the geometry of the cross section.

Within the elastic range, longitudinal residual strains superimpose with other strains in exactly the same way as strains caused by loading conditions. Therefore, residual stresses influence the average stress-strain relation of a structural member as a whole: Consider a short section consisting of ideally elastic-plastic material (Fig. 1) to be tested in compression. This is usually referred to as "cross section test" or "stub column test". If the specimen is short enough to prevent buckling and if it is free of residual stresses, the section as a whole will obey the stress-strain law of Fig. 1. But in the presence of residual stresses the parts of the section which are in residual compression will
reach the yield limit earlier. Young's modulus for these parts then becomes zero, which means that the stress-strain curve of the total section deviates from the straight line on which $E = \text{const.}$ (Fig. 2).

As may be gathered from Fig. 2, residual stresses do not affect the ultimate load of a stub column, but they have a considerable influence on column stability. The reason for this is that for the yielded areas Young's modulus is zero and therefore they will not take any increase in load. Consequently, the effective moment of inertia is reduced, which results in a lower buckling load (reduction up to 35\%\).

Based on these considerations column formulas have been developed and compared with test results 2, 4, 9, 12. The manner in which the formulas were derived is illustrated in Section 2, where an expression for the critical load is derived on the basis of the residual stress pattern encountered in this investigation.

2. OBJECTIVES

The papers referred to above resulted from a test program on WF-columns of structural carbon steel (ASTM designation A7). The tests on which this report is based were conducted on three WF-columns of high strength low alloy steel (ASTM designation A242 with chemical composition as shown in Table II) so that comparison might be made between residual stresses in high
strength steel and those in A7 steel. In particular, a major question had to be clarified as to whether a higher yield point will cause higher residual stresses. The three shapes selected were 8WF31, 12WF50 and 12WF65. The 8WF31 was chosen because it is the one most common in use and a large number of 8WF31 sections in A7 steel had been tested before. The other two shapes were selected because, in the A7 steel series, their residual stress patterns had the greatest difference from that of the 8WF31.

3. COLUMN FORMULAS

3.1 General Considerations

The theory of the stability of WF-columns containing residual stresses has been expounded in several of the references 2, 4, 9, 12, and only a brief review will be given here to acquaint the reader with the basic concepts and with the notations used.

As soon as the section of a WF-column under axial load is partially yielded due to the existence of residual stresses, the Euler buckling formula for a pin-ended column

\[ P_c = \pi^2 \frac{EI}{L^2} \quad \text{or} \quad \sigma_c = \frac{P_c}{A} = \frac{\pi^2 E}{(\frac{L}{r})^2} \quad (3.1) \]

is no longer valid. Since Young's modulus is zero for the yielded parts, so is their contribution to the flexural
rigidity $EI$, that is the moment of inertia is reduced and the buckling load is only

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad \text{or} \quad \sigma_{cr} = \frac{P_{cr}}{A} = \pi^2 \frac{E I_e}{(\frac{L}{n})^2} \quad (3.2)$$

where $I_e = \text{effective moment of inertia}$.

$P_{cr}$ as defined by Eq. 3.2 is usually referred to as the tangent modulus load since it may be computed from the tangent modulus of the stub column curve (see Section 3.2). The tangent modulus load signifies the load at which instability occurs, thus giving a lower limit for the load that the column can carry. The ultimate load is higher than the tangent modulus load (Fig. 3). A theoretical upper limit for the ultimate load can be determined by the reduced modulus theory which takes into account strain reversal. Since the reduced modulus theory becomes very complicated for structural steel shapes containing residual stresses, an approximate formula is presented in Section 3.4 for the estimation of the increase in carrying capacity beyond the tangent modulus load (Fig. 3).

For most wide-flange shapes the maximum compressive residual stresses are located at the flange tips and in the web center, so that yielding will spread from these points. Characterizing the amount of yield penetration by $x_0$ and $y_0$ respectively (Fig. 4), the effective moments of
inertia become

\[ I_{ey} = A_F \frac{b^2}{12} \left( \frac{2x_0}{b} \right)^3 \]  

\[ I_{ex} = A_W \frac{h^2}{12} \left[ 1 - \left( \frac{2y_0}{h} \right)^3 \right] + A_F \left( \frac{d^2}{4} - \frac{dt}{2} \right) \left( \frac{x_0}{b} \right)^3 \]  

where some small terms have been neglected. If a relationship between \( \sigma_{cr} \) of Eq. 3.2 and \( I_{ey} \), \( I_{ex} \) can be established, the critical slenderness ratio of a given wide-flange shape can be computed. There are two ways of relating \( \sigma_{cr} \) and \( I_e \), one is based on the stress-strain curve obtained from a stub column test (Section 1 and 4.5), the other is based on the measured residual stress distribution.

3.2 Column Curves from Stub Column Test

Given the average stress strain curve of a stub column test (Fig. 2) as defined in the introduction, it may be verified that

\[ \frac{E_t}{E} = \frac{A_e}{A} \]  

(3.5)

with

\[ E_t = \frac{d\sigma}{d\varepsilon} = \text{tangent modulus} \]

and \( A_e = \text{effective area} \), that is the area that is still elastic. \( A_e \) may be expressed in terms of \( x_0 \) and \( y_0 \) as follows

\[ A_e = A_W - 2wy_0 + 4tx_0 \]
Dividing by A and making use of Eq. 3.5

\[
\frac{E_t}{E} = \frac{A_w}{A} \left(1 - \frac{2\gamma_0}{h} + \frac{A_p}{A_w} \frac{2x_0}{b}\right)
\] (3.6)

**Case 1:** Yielding in flanges only

Then \(2x_0/b = 1\) and

\[
\frac{2x_0}{b} = \left(\frac{E_t}{E} \frac{A}{A_w} - 1\right) \frac{A_w}{A_f} = (\gamma - 1) \frac{A_w}{A_f}
\] (3.7)

where

\[\gamma = \frac{E_t}{E} \frac{A}{A_w}\]

**Case 2:** Yielding in web only

Then \(2x_0/b = 1\) and

\[
\frac{2\gamma_0}{h} = (1 - \frac{E_t}{E}) \frac{A}{A_w} = \frac{A}{A_w} - \gamma
\] (3.8)

Formulas (3.7) and (3.8) are perfectly independent of the type of residual stress distribution (linear, parabolic, hyperbolic) provided there is symmetry with respect to the x- and y-axis. The procedure for computing a column curve \(\sigma_{cr} = f(L/r)\) for case 1 or 2 is:

1) take arbitrary \(\sigma_{cr}\)

2) find corresponding \(E_t\) from stub column curve

3) determine \(2x_0/b\) or \(2\gamma_0/h\) respectively by (3.7) or (3.8)

4) compute \(I_{ex}, I_{ey}\) by (3.3) and (3.4)

5) find \(L/r\) corresponding to selected \(\sigma_{cr}\) from (3.2)
Case 3: Yielding in both web and flanges

In this case the computations become somewhat more complicated, since $E_t$ depends on $x_o$ as well as on $y_o$. The function that interrelates $y_o$ and $x_o$ is determined by the shape of the residual stress distribution and the relative magnitude of residual stresses in web and flange. If the residual stresses are known from measurements, it is possible to determine $x_o = f(y_o)$ graphically. However, the data obtained from the stub column test are sufficient to solve the problem, if an idealized stress pattern is assumed (see end of this section).

The residual stress distribution in the 12WF50 and 12WF65 as found in this investigation for A242 steel and in earlier tests for A7 steel 2, 4, 9 is essentially parabolic in both flange and web (Fig. 5b and 5c). With the simplification that the stresses at the ends of the web are equal to the average stress at the flange center, the idealized residual stress pattern sketched in Fig. 6b is obtained. The three "characteristic" values $\sigma_{rc}$, $\sigma_{ro}$, $\sigma_{rw}$ are sufficient to determine the residual stress parabolas

$$\sigma_r = \left(\frac{2x}{b}\right)^2 (\sigma_{rc} - \sigma_{ro}) + \sigma_{ro} \text{ in the flange}$$

and

$$\sigma_r = \frac{2y}{h} (\sigma_{ro} - \sigma_{rw}) + \sigma_{rw} \text{ in the web (3.10).}$$

The residual stresses at the borders between the yielded
areas and the areas still elastic are then

\[ \sigma_{rx_0} = \left(\frac{2x_0}{b}\right)^2 (\sigma_{rc} - \sigma_{ro}) + \sigma_{ro} \quad (3.11) \]

\[ \sigma_{ry_0} = \left(\frac{2y_0}{h}\right)^2 (\sigma_{ro} - \sigma_{rw}) + \sigma_{rw} \quad (3.12) \]

From Fig. 7 it may be seen that

\[ \sigma_{rx_0} = \sigma_{ry_0} \quad (3.13) \]

which leads to

\[ \left(\frac{2y_0}{h}\right)^2 - \left(\frac{2x_0}{b}\right)^2 \alpha = 1 \quad (3.14) \]

where

\[ \alpha = \frac{\sigma_{rc} - \sigma_{ro}}{\sigma_{ro} - \sigma_{rw}} \quad (3.15) \]

Combining Eq. 3.14 and Eq. 3.6 gives \( x_0 \) as a function of \( E_t \) and

\[ \frac{2x_0}{b} = \frac{A_w \cdot A_f}{\alpha A_w^2 - A_f^2} \left[ 1 - \gamma - \sqrt{1 - \alpha \gamma (2 - \gamma) \left(\frac{A_w}{A_f}\right)^2} \right] \quad (3.16) \]

where

\[ \gamma = \frac{E_t}{E} \frac{A_f}{A_w} \]

To determine the constant \( \alpha \) from the stub column test consider Fig. 2. The curve deviates from the straight line when the first yield occurs, i.e., the sum of highest
compressive residual stress and imposed stress reaches the yield limit. Naming this point in the stress-strain diagram (Fig. 2) $\varepsilon_p$

$$
\sigma_{rw} = \varepsilon_p E - \sigma_y \quad \sigma_{rc} = \varepsilon_p E - \sigma_y \quad \sigma_{rw} \quad \sigma_{rc} \quad (3.17)
$$

Web and flange will be completely yielded when the sum of highest tensile residual stress and imposed stress has reached the yield limit. Naming this point in the stress-strain diagram $\varepsilon_0$

$$
\sigma_{ro} = \varepsilon_0 E - \sigma_y \quad (3.18)
$$

The third of the characteristic residual stresses, i.e., $\sigma_{rc}$ or $\sigma_{rw}$ may then be found by equilibrium considerations. For the parabolic pattern in question $\int \sigma_{rdA} = 0$ leads to

$$
A_w (\sigma_{ro} = 2\sigma_{rw}) + A_f (\sigma_{rc} + 2\sigma_{ro}) = 0 \quad (3.19)
$$

With $\sigma_{rw}$, $\sigma_{ro}$, $\sigma_{rc}$ known $\alpha$ can be computed from Eq. 3.15.

3.3 Column Curves from Measured Residual Stress Distribution

Assume the residual stress distribution of a WF-column to be given. The shape of the residual stress pattern (linear, parabolic etc.) is of no importance in the following derivation, except that it must be symmetrical with respect to the x-axis and the y-axis. By use of Fig. 7 the following
equation for the average stress on the column may be obtained

\[ \sigma_{cr} = \sigma_y - \frac{A_e}{A} \sigma_{rxo} - \frac{A_r}{A} \int_{x_0}^{b} \sigma_{rdx} - \frac{A_w}{h} \int_{y_0}^{y} \sigma_{rdy} \] (3.20)

The procedure to find the critical slenderness ratio is in this case:

1) assume certain \( 2x_0/b \)
2) determine \( \sigma_{rxo} = \sigma_{ryo} \) and \( 2y_0/h \) \{ may be done graphically \}
3) evaluate stress integrals
4) compute \( I_{ex}, I_{ey} \) using \( 2x_0/b \) and \( 2y_0/h \) and find \( \sigma_{cr} \) from (3.20)
5) compute \( L/r \) from (3.2) and plot \( \sigma_{cr} = f(L/r) \)

For an idealized parabolic stress pattern steps 2) and 3) may be performed by making use of Eq. 3.11, 3.14, 3.9 and 3.10.

3.4 Estimation of Ultimate Load

In the straight configuration the cross section was assumed to be yielded according to Fig. 4. With \( I_e \) as effective moment of inertia, the loss in moment of inertia compared to a fully elastic cross section is

\[ \Delta I = I - I_e \]

When the column changes to a bent configuration the convex side is subjected to a tensile strain that brings the yielded parts on this side back into action. For an infinitesimal
lateral deflection the loss in moment of inertia compared to a fully elastic cross section is now only

$$\Delta I = \frac{1}{2} (I - I_e)$$  \hspace{1cm} (3.21)

From Eq. 3.21 a potential increase in stress of

$$\Delta \sigma_{cr} = \frac{1}{2} (\sigma_e - \sigma_{cr})$$  \hspace{1cm} (3.22)

can be derived, $\sigma_e$ being the Euler buckling stress. Due to the fact that there will be more yielding when the load is increased and that in the bent configuration the cross section is subjected to a moment in addition to the axial force, the potential increase in stress as given by Eq. 3.22 cannot be fully realized. A detailed investigation of the problem of the ultimate column load is presented in Ref. 3. A consideration of the formulas derived there and comparisons with test results have led to the conclusion that the approximate formula

$$\Delta \sigma_{cr} = \frac{1}{3} (\sigma_e - \sigma_{cr})$$  \hspace{1cm} (3.23)

allows a satisfactory estimation of the ultimate load of wide-flange columns. Either $\sigma_e$ or $\sigma_y$ is governing, depending on which one is smaller.
4. DESCRIPTION OF TESTS

4.1 Test Program

The test program for this investigation comprised the following test:

(A) 15 tension coupon tests
(B) 2 compression coupon tests
(C) 3 sets of residual stress measurements
(D) 3 stub column tests
(E) 4 axial column tests, where bending was allowed in the weak direction.

The shapes tested were 8WF31, 12WF50 and 12WF65. The fabricating process was observed by members of the project staff who saw to it, that the shapes were placed on the cooling bed individually in order to secure even cooling. Further, the shapes were not allowed to be rotarized or otherwise cold bent, but the straightest parts of the rollings were selected as test material.

For subdivision of specimens and test numbers see Table I and Fig. 8.

4.2 Tension Coupon Tests

All coupons were tested in a 120,000 lb. screw type universal testing machine with electronically operated load indicator and automatic recorder. The coupons were cut from different parts of the cross section to obtain a picture of
the variation of the material properties across the section. Since the upper yield point is a function of the strain rate, only the static yield level is listed in Table III. (The static yield level is observed by halting the testing machine and waiting till the load has stabilized.) Young's modulus was determined directly from the recorded load-strain diagram. The coupons were dimensioned and the elongations determined according to ASTM standards.

4.3 Compression Coupon Tests

Only two compression coupons were tested, the intention being to find out whether the steel has the same properties in compression as in tension. The coupons were cut from the tip of one flange and the center of the web of the 8WF31 section, as shown in Fig. 9. The dimensions followed the recommendations of Research Committee A of the Column Research Council. That is

\[
\begin{align*}
  b &> t \\
  L &\leq 4.5t \\
  L &\geq 2b + g \\
  b &< g + 2b
\end{align*}
\]

where \( g \) is the gage length. Strains were measured by means of an averaging compressometer whose measuring elements were SR-4 gages 1/2 inch long. The coupons rested on a bearing block and were provided with a spherical bearing on top; they
were aligned carefully, the maximum deviation from the average strain not being allowed to exceed 5%.

4.4 Residual Stress Measurement

Residual stresses were determined by the sectioning method \(^9\), i.e., the distance \((\approx 10\text"\) between gage points on an 11" length of the specimen was measured before and after cutting and sectioning into strips. The residual stress was computed from the residual strain released in this manner. Temperature changes were taken into account by relating all measurements to a standard 10" mild steel bar attached to the specimen. The test pieces were cut from the middle of long columns (see Fig. 8) and carefully selected to be free of yield lines.

4.5 Stub Column Tests

The stub column tests were carried out to obtain a direct average stress-strain curve which would show the effect of the residual stress distribution on the compressive properties of the section as a whole.

The length of the specimens was selected so that they would be short enough to prevent buckling and long enough not to disturb the residual stress distribution in the central gage section (boundary conditions at the ends require residual stress equal to zero). Strains were measured by 1/10,000 dial
gages over a 10 inch gage length; two gages placed on opposite sides of the cross section were used to compensate for uneven deformation. In test No. SC-1 strains were measured also by SR-4 gages of 1" gage length (type A-11). They gave the same results as the mechanical gages and since they have the disadvantage of being too sensitive to local yielding, they were not used in the other two stub column tests 10. Near the four flange tips the average shortening of the stub column was measured using 1/1000 dial gages. They also served for aligning; the alignment was considered satisfactory when the maximum deviation of each corner gage from the average was within 5%. The specimens were whitewashed with a solution of hydrated lime, so that the yielding process could be observed. For a picture of the test set-up showing a stub column in an advanced stage of yielding see Fig. 10.

4.6 Column Tests

Columns C-1 and C-2 were tested in an 800,000 lb. screw-type universal testing machine as was the corresponding stub column SC-1. The other shapes (12WF50 and 12WF65) were tested in a 5,000,000 lb. hydraulic universal testing machine.

The axial load was applied through a pair of special end fixtures, a detailed description of which is given in Ref. 8. Strains were measured by means of SR-4 gages of 1-inch gage length located at the center sections and near both ends of the columns. They were used also to check the alignment
which was required to be within 5%. The initial crookedness was measured and the results are plotted in Fig. 15, which also gives the maximum ec/r² values. By aligning the columns the effect of initial deflections was partially eliminated. During the test, the center line deflections were measured by means of a transit and a 1/100 inch scale clamped to the flange. The columns were whitewashed so that the yielding process could be followed during the test (see Fig. 11 and Fig. 12).

5. TEST RESULTS AND DISCUSSION

5.1 Coupon Tests

The results of the tension and compression coupon tests are listed in Table III. In Table IV, the properties of A242 and A7 steel are compared; both coupon series were taken from 8WF31 shapes.

The variation of the yield strength across the section was found to be quite marked. It can amount to 10% of the yield stress level, taking minimum and maximum values, as for instance, for the coupons T2 and T7. Though the differences are rarely as extreme as this, the yield strength of the flange tips and the web center tends to be higher than at the connection of web and flange. This is in accordance with the general observation that faster cooling will increase the yield strength, whereas the ultimate strength is not affected. The amount of variation of yield strength
is the same, percentage wise, in shapes of A342 steel as in shapes of A7 steel. With respect to the Young's modulus or other coupon data, no definite pattern of variation across the section is evident.

Contrary to most tension coupons, the compression coupons do not show an upper yield point (Fig. 9). Because of this fact and because the upper yield point ($\sigma_{yu}$) in tension coupons depends very much on the strain rate with which the coupon is tested, $\sigma_{yu}$ was not considered an essential material constant and has not been listed in the tables. The differences in yield strength between compression and tension coupons (see Table III) are small. Therefore only tension coupons were tested in the later part of the investigation. The value for Young's modulus determined from the compression coupons is slightly lower than for the equivalent tension coupons. This might be due to the inaccuracy of the gage factor of the compressometer, since the E-values obtained from the stub column tests and the weighted average of the tension coupons are in rather good agreement.

5.2 Residual Stress Measurements

The residual stress distributions were computed from the measured residual strains by using $E = 30.10^3$ ksi. The extreme values encountered in the three shapes are compiled in Table VII, the actual residual stress distributions are shown in Fig. 5.
The patterns are quite similar to those obtained for A7 steel, as indicated in Fig. 5. The 12WF50 has considerably higher residual compression in the web than the same shape in A7 steel. However, as earlier investigations on shapes of structural carbon steel have shown, this difference lies within the range of variations caused by different cooling conditions. Theoretically, the residual stresses in high strength steel should not be higher than in A7 steel, since the coefficient of expansion, Young's modulus and the variation of \( \sigma_y \) with the temperature are essentially the same in both types of steel. Also, in the other two shapes, the magnitude of residual stresses is approximately the same as encountered in A7 steel (Fig. 5). Though only a statistical study would prove conclusively if there is a tendency for higher residual stresses in A242 steel or not, it can be assumed that the residual stresses in structural carbon steel and in high strength steel are about equal in magnitude and in distribution. On account of the higher yield level, this reduces the effect of residual stresses on the strength of members of high strength steel.

5.3 Stub Column Test

The principal data from the stub column tests are listed in Table V which also gives a comparison with the weighted average of the coupon tests. The stress-strain curves as recorded during the tests are reproduced in Fig. 13, where
the correlation between theoretically (that is from the residual stress measurements) expected and observed first yield is also indicated. The stress at which first yield should occur in web or flange respectively is obtained by subtracting the highest compressive residual stress from the yield stress. The actual first yield is indicated by the formation of a yield line through flaking of the mill scale, as shown by the whitewash. In general, the correlation is quite good, discrepancies may be due to local stress concentrations, eccentric loading, or minor variations of the residual stresses.

The flanges of stub column SC-1 buckled, as indicated in Fig. 13, before the full yield stress level as compared with the weighted coupon average could be reached. The stress-strain diagram therefore dropped instead of maintaining a constant yield level. It has been found that this phenomenon can be expected if the ratio of flange width to thickness is greater than 17 (9). For the 8WF31 shape the value of b/t is 18.5.

The stress-strain curve for specimen SC-3 (12WF65) shows less deviation from the straight line than should be expected from the residual stress measurement. This may be due to the fact that SC-3 was only 32" long instead of 40" as planned. The residual stress may not have been preserved to the full extent over the entire central 10" gage length. The resulting
stress-strain curve leads to the prediction of a higher buckling load for the column than found later on in the test (see Fig. 14). If the prediction is based on the measured residual stress distribution (compare Section 3.3) it gives a buckling load which agrees with the test result (Fig. 3c and 14).

5.4 Column Tests

For a summary of the column test results see Table VI; the load versus center deflection curves are plotted in Fig. 14. Figure 3 compares the test results with the column curves.

During the tests, small deflections from the straight configuration were observed before the tangent modulus load was reached. This, no doubt, was caused by the imperfection of the alignment and by the initial crookedness of the columns (Fig. 15).

Some remarks on individual column tests might be in order: Column C-1 illustrates a border case. As its slenderness ratio is in the neighborhood of the point where the Euler curve and the yield stress meet, the column continued to deflect at a relatively fast rate once it had started to buckle. As a result it was not possible to obtain data of the buckling process; the center deflection changed suddenly from 0.07 to 2.5 inches. For shorter members this condition
was not so pronounced and unloading points could be obtained (C-2 and C-4 in Fig. 14).

For column C-3 the alignment was very good, and the deflection at the center remained below 1/100 of an inch up to a load very near the predicted tangent modulus load. The columns do not deflect extensively immediately after reaching the tangent modulus load. This is due to strain reversal on the convex side which brings into action again the yielded areas on this side, thus increasing the moment of inertia. A way of estimating the increase in load is developed in Section 3.4.

Column C-4 failed very close to the tangent modulus load computed from the stub column curve. But most probably the inaccuracy of the stub column curve is to be blamed for this (see Section 5.3). A computation of the tangent modulus load based on the measured residual stress distribution (see Section 3.3) leads to only 929 kips which is much closer to the onset of excessive deflection measured in the test.

6. CONCLUSIONS

The following conclusions are based on studies made of three WF shapes rolled to ASTM-A542 specifications:

1) High strength steel shows the same material properties in compression as in tension (Table III and Fig. 9).
2) The variation of most material properties across the section is negligible. Only the yield stress level of the flange tips is up to 10% higher than at the flange center (Table III).

3) Residual stresses in rolled sections of high strength steel have the same pattern and magnitude as those in structural carbon steel (Fig. 5).

4) Full scale column tests show that the data obtained from coupon tests and stub column tests provide sufficient information to predict the strength of columns with high accuracy (Fig. 3 and 14).

5) Because of their higher yield stress, the strength of columns of high strength steel is relatively less influenced by residual stresses as compared with structural grade ASTM A7 steel.
7. ACKNOWLEDGEMENTS

This investigation, jointly sponsored by the Column Research Council, the Pennsylvania Department of Highways, the Bureau of Public Roads and the National Science Foundation, has been carried out as a part of a project being directed by Dr. Lynn S. Beedle at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania. Professor William J. Eney is the Director of Fritz Engineering Laboratory and the Head of the Department of Civil Engineering.

The support of the members of Committee A of the Column Research Council (chairman John A. Gilligan) in carrying out this program is gratefully acknowledged.

Dr. Robert L. Ketter, formerly at Fritz Engineering Laboratory and now at the University of Buffalo, planned the program and directed a portion of it. Messrs. S. J. Errera, Engineer of Tests, and K. R. Harpel, Foreman of Fritz Engineering Laboratory, prepared the test set-ups and the tests. Messrs. Le Wu Lu, Akira Nitta and Lambert Tall assisted in the tests. Their help is sincerely appreciated by the authors.
8. NOMENCLATURE

A  Cross sectional area
A_f, A_w  Area of flanges or of web respectively
b  Flange width
d  Depth of section
E  Young's modulus
E_t  Tangent modulus
g  Gage length
h  Height of web
I  Moment of inertia
I_e  Effective moment of inertia
L  Length of column
L/r  Slenderness ratio
P  Axial load on column
P_cr  Buckling load determined from tangent modulus concept
P_e  Euler buckling load
Pult  Ultimate load
P_y  Load at yield level
r  Radius of gyration
t  Flange thickness
w  Web thickness
x_o  Yield penetration in flanges
y_o  Yield penetration in web
\[ \alpha = \frac{\sigma_{rc} - \sigma_{ro}}{\sigma_{ro} - \sigma_{rw}} \]

\( \varepsilon \) Strain

\( \sigma \) Stress, subscripts same as loads

\( \sigma_r \) Residual stress
9. TABLES
### TABLE 1

#### TABLE OF SPECIMEN NUMBERS

<table>
<thead>
<tr>
<th>TYPE OF TEST</th>
<th>8WF31</th>
<th>12WF50</th>
<th>12WF65</th>
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<tbody>
<tr>
<td>TENSION COUPON</td>
<td>T1, T2, T3, T4, T5, T6, T7, T8, T9</td>
<td>T10, T11, T12</td>
<td>T13, T14, T15</td>
</tr>
<tr>
<td>COMPRESSION COUPON</td>
<td>CC1, CC2</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>RESIDUAL STRESS MEASUREMENT</td>
<td>RS-1</td>
<td>RS-2</td>
<td>RS-3</td>
</tr>
<tr>
<td>STUB COLUMN</td>
<td>SC-1</td>
<td>SC-2</td>
<td>SC-3</td>
</tr>
<tr>
<td>COLUMN</td>
<td>C-1, C-2</td>
<td>C-3</td>
<td>C-4</td>
</tr>
</tbody>
</table>
### TABLE II

**CHEMICAL COMPOSITION OF A242 STEEL SPECIMENS**

<table>
<thead>
<tr>
<th>Shape</th>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
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<tr>
<td>8WF31</td>
<td>0.15</td>
<td>1.09</td>
<td>0.027</td>
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<tr>
<td>12WF50</td>
<td>0.13</td>
<td>1.18</td>
<td>0.027</td>
<td>0.023</td>
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<td>0.15</td>
<td>1.15</td>
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<td>0.031</td>
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</table>

<table>
<thead>
<tr>
<th>Si</th>
<th>Ni</th>
<th>Cr</th>
<th>Mo</th>
<th>Cu</th>
<th>Va</th>
<th>Ti</th>
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<td>0.22</td>
<td>0.08</td>
<td>0.05</td>
<td>0.02</td>
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<tr>
<td>12WF50</td>
<td>0.23</td>
<td>0.06</td>
<td>0.03</td>
<td>0.01</td>
<td>0.08</td>
<td>0.04</td>
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<tr>
<td>12WF65</td>
<td>0.28</td>
<td>0.10</td>
<td>0.05</td>
<td>0.03</td>
<td>0.28</td>
<td>0.05</td>
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### TABLE III

**COUPON TEST RESULTS**

<table>
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<tr>
<th>Location</th>
<th>Test No.</th>
<th>E $10^3$ ksi</th>
<th>$\sigma_{YS}$ ksi</th>
<th>$\sigma_Y$ Mill Test ksi</th>
<th>$\sigma_{ult}$ ksi</th>
<th>Elong. %</th>
<th>Reduct. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>8WF31</td>
<td>T1</td>
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<td>55.64</td>
<td>76.48</td>
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<tr>
<td></td>
<td>T2</td>
<td>30.6</td>
<td>51.41</td>
<td>76.48</td>
<td>28.15</td>
<td>58.93</td>
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</tr>
<tr>
<td></td>
<td>T3</td>
<td>31.6</td>
<td>54.62</td>
<td>76.01</td>
<td>26.35</td>
<td>53.65</td>
<td></td>
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<tr>
<td></td>
<td>T4</td>
<td>32.8</td>
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<td>74.62</td>
<td>20.5</td>
<td>53.15</td>
<td></td>
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<tr>
<td></td>
<td>T5</td>
<td>32.3</td>
<td>55.07</td>
<td>76.04</td>
<td>27.95</td>
<td>48.39</td>
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<td></td>
<td>T6</td>
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<td>53.19</td>
<td>75.16</td>
<td>28.19</td>
<td>52.53</td>
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<td></td>
<td>T7</td>
<td>32.0</td>
<td>53.76</td>
<td>75.68</td>
<td>30.40</td>
<td>58.14</td>
<td></td>
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<tr>
<td></td>
<td>T8</td>
<td>30.9</td>
<td>51.39</td>
<td>75.62</td>
<td>24.19</td>
<td>55.59</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T9</td>
<td>31.6</td>
<td>51.39</td>
<td>75.31</td>
<td>33.53</td>
<td>60.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T10</td>
<td>30.9</td>
<td>51.39</td>
<td>75.31</td>
<td>33.53</td>
<td>60.28</td>
<td></td>
</tr>
<tr>
<td>12WF50</td>
<td>T11</td>
<td>28.9</td>
<td>50.40</td>
<td>75.35</td>
<td>27.00</td>
<td>58.01</td>
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</tr>
<tr>
<td></td>
<td>T12</td>
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<td>53.55</td>
<td>77.40</td>
<td>24.62</td>
<td>57.13</td>
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<tr>
<td></td>
<td>T13</td>
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<td>54.59</td>
<td>75.79</td>
<td>22.62</td>
<td>45.72</td>
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<td></td>
<td>T14</td>
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<td>77.19</td>
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<td>56.76</td>
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<tr>
<td></td>
<td>T15</td>
<td>30.2</td>
<td>53.43</td>
<td>81.20</td>
<td>24.25</td>
<td>54.79</td>
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</tr>
</tbody>
</table>

Note: T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12, T13, T14, T15
### TABLE IVa - COMPARISON OF AVERAGE TENSION COUPON RESULTS
OF A-7 AND A-242 STEELS
(All Values in ksi)

<table>
<thead>
<tr>
<th>Type</th>
<th>Shape</th>
<th>E</th>
<th>σ_p</th>
<th>σ_y(u)</th>
<th>σ_y</th>
<th>% $\frac{σ_p}{σ_y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-7</td>
<td>Flange</td>
<td>30,010</td>
<td>32.0</td>
<td>39.1</td>
<td>37.4</td>
<td>85.5</td>
</tr>
<tr>
<td></td>
<td>Web</td>
<td>29,270</td>
<td>27.7</td>
<td>42.6</td>
<td>35.7</td>
<td>77.7</td>
</tr>
<tr>
<td></td>
<td>Weighted</td>
<td>29,820</td>
<td>30.9</td>
<td>39.9</td>
<td>37.0</td>
<td>83</td>
</tr>
<tr>
<td>A-242</td>
<td>Flange</td>
<td>31,180</td>
<td>44.3</td>
<td>58.4</td>
<td>54.4</td>
<td>81.6</td>
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<tr>
<td></td>
<td>Web</td>
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<td>45.9</td>
<td>58.5</td>
<td>54.6</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>Weighted</td>
<td>31,300</td>
<td>44.6</td>
<td>58.4</td>
<td>54.5</td>
<td>82</td>
</tr>
</tbody>
</table>

### TABLE IVb - COMPARISON OF AVERAGE COMPRESSION COUPON RESULTS
OF A-7 AND A-242 STEELS
(All Values in ksi)

<table>
<thead>
<tr>
<th>Type</th>
<th>Shape</th>
<th>E</th>
<th>σ_p</th>
<th>σ_y</th>
<th>% $\frac{σ_p}{σ_y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-7</td>
<td>Flange</td>
<td>28,940</td>
<td>30.4</td>
<td>39.6</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>Web</td>
<td>30,000</td>
<td>30.0</td>
<td>43.3</td>
<td>69.5</td>
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<tr>
<td></td>
<td>Weighted</td>
<td>29,200</td>
<td>30.3</td>
<td>40.5</td>
<td>75</td>
</tr>
<tr>
<td>A-242</td>
<td>Flange</td>
<td>29,850</td>
<td>45.8</td>
<td>54.2</td>
<td>84.4</td>
</tr>
<tr>
<td></td>
<td>Web</td>
<td>29,680</td>
<td>44.9</td>
<td>57.1</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td>Weighted</td>
<td>29,830</td>
<td>45.5</td>
<td>54.9</td>
<td>82.8</td>
</tr>
</tbody>
</table>

The A-7 Values are taken from Reference 9.
### TABLE V

**STUB COLUMN TEST RESULTS AND COMPARISON WITH WEIGHTED COUPON AVERAGE**

<table>
<thead>
<tr>
<th>Shape</th>
<th>8WF31</th>
<th>12WF50</th>
<th>12WF65</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test No.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E 10^3 ksi</td>
<td>31.3</td>
<td>31.2</td>
<td>31.8</td>
</tr>
<tr>
<td>1st yield ksi</td>
<td>--</td>
<td>39.2</td>
<td>30.5</td>
</tr>
<tr>
<td>σ_y ksi</td>
<td>54.5</td>
<td>56.4</td>
<td>55.0</td>
</tr>
</tbody>
</table>

### TABLE VI

**COLUMN TEST RESULTS**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8WF31</td>
<td>C-1</td>
<td>12' 0&quot;</td>
<td>72</td>
<td>398</td>
<td>380</td>
<td>400.3</td>
<td>56.4</td>
<td>21%</td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>9' 0&quot;</td>
<td>54</td>
<td>429</td>
<td>400</td>
<td>421.5</td>
<td>56.4</td>
<td>17%</td>
</tr>
<tr>
<td>12WF50</td>
<td>C-3</td>
<td>12' 3&quot;</td>
<td>75</td>
<td>584</td>
<td>600</td>
<td>655</td>
<td>52.6</td>
<td>15%</td>
</tr>
<tr>
<td>12WF65</td>
<td>C-4</td>
<td>15' 6&quot;</td>
<td>61.7</td>
<td>925</td>
<td>900</td>
<td>984</td>
<td>56.8</td>
<td>13%</td>
</tr>
</tbody>
</table>

*Note: All Loads in kip, all Stresses in ksi*
<table>
<thead>
<tr>
<th>Shape</th>
<th>Test No.</th>
<th>Flange Tips $\sigma_{rc}$</th>
<th>Flange Center $\sigma_{ro}$</th>
<th>Web Center $\sigma_{tw}$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>average</td>
<td>Outside Inside Average</td>
<td></td>
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<tr>
<td>8WF31</td>
<td>RS-1</td>
<td>-14.2</td>
<td>+6.5 +7.5 +9.3 +6.2</td>
<td>+2.1 +2.4 +1.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>maximum</td>
<td>+6.5 +7.5 +11.7 +6.2</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>minimum</td>
<td>+5.4 +6.9 +9.3 +6.2</td>
<td></td>
</tr>
<tr>
<td>12WF50</td>
<td>RS-2</td>
<td>-7.5</td>
<td>+19.5 +9.1 +13.0</td>
<td>-31.5 -31.7 -31.3</td>
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<td></td>
<td></td>
<td>maximum</td>
<td>+19.7 +11.5 +13.0</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>minimum</td>
<td>+19.2 +6.7 +13.0</td>
<td></td>
</tr>
<tr>
<td>12WF65</td>
<td>RS-3</td>
<td>-14.5</td>
<td>+17.6 +10.5 +13.2</td>
<td>-20.1 -20.3 -20.0</td>
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<td></td>
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<td>maximum</td>
<td>+18.4 +11.1 +13.2</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>minimum</td>
<td>+16.9 +9.6 +13.2</td>
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</tbody>
</table>
10. FIGURES
Fig. 1 - IDEALIZED STRESS-STRAIN CURVE FOR STRUCTURAL STEEL

Fig. 2 - TYPICAL STUB-COLUMN CURVE
Fig. 3a - COLUMN CURVE FOR 8WF31  
(TANGENT MODULUS LOAD)

\[ \sigma_y = 56.4 \text{ ksi} \]

\[ \sigma_p = 39.2 \text{ ksi} \]

Fig. 3b - COLUMN CURVE FOR 12WF50  
(TANGENT MODULUS LOAD)

\[ \sigma_y = 21.0 \text{ ksi} \]

\[ \sigma_p = 21.0 \text{ ksi} \]
Weak Axis Test Results

Indicates Estimated Ultimate Load

\( \Theta \) Weak Axis Test Results

\( \sigma_p = 31.9 \text{ ksi} \)

--- Column Curves Computed From Stub-Column Test

--- Column Curves Computed From Residual Stress Pattern

Fig. 3c - COLUMN CURVE FOR 12WF65 (TANGENT MODULUS LOAD)

Fig. 4 - NOTATIONS FOR YIELDED SECTION
Fig. 5 - RESIDUAL STRESS DISTRIBUTIONS
Fig. 6a - IDEALIZED LINEAR STRESS DISTRIBUTION (8WF31)

Fig. 6b - IDEALIZED PARABOLIC STRESS DISTRIBUTION (12WF50) (12WF65)
Fig. 7 - STRESS DIAGRAMS

Residual Stress

Nominal Stress
Due to
Imposed Strain

Added Stress

Actual Stress
Fig. 8 - POSITION OF INDIVIDUAL SPECIMENS DURING ROLLING

Explanations: Shaded Portions: discarded
Specimen Numbers: see Table I
T = Top of Ingot
B = Bottom of Ingot
Fig. 9 - TENSION AND COMPRESSION COUPON STRESS-STRAIN CURVES
Fig. 10 - STUB COLUMN TEST SET-UP

Note advanced yielding
Fig. 11 - COLUMN C4 AFTER TEST

Note yielding in web and parabolic yield distribution along concave side of flange.
Fig. 12A - YIELD-LINES IN FLANGES OF 12WF65 COLUMN

Fig. 12b - YIELD-LINES IN WEB OF 12WF65 COLUMN
Fig. 13 - STUB COLUMN CURVES

(a) First Yield in Flange Observed
(b) First Yield in Web Observed
(c) First Theoretical Yield in Flange From Residual Stress
(d) First Theoretical Yield in Web Measurement

--- Yield Level, Weighted Coupon Average
Fig. 14 - CENTERLINE DEFLECTIONS IN COLUMN TEST

Arrow Indicates Load Up To Which \( \delta_e \leq 1/100 \)-inch
Fig. 15 - INITIAL DEFLECTION CURVES OF COLUMNS

Scale: Length 1 inch \( \equiv \) .2 ft.
Deflection 1/20-inch \( \equiv \) 1/64-inch
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<p>| | | |</p>
<table>
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<tr>
<th></th>
<th></th>
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</table>
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