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Welded Built-Up Columns

RESIDUAL STRESSES IN WELDED PLATES

- A THEORETICAL STUDY -

by

Lambert Tall

This work has been carried out as a part of an investigation sponsored jointly by the Column Research Council, the Pennsylvania Department of Highways, the U.S. Department of Commerce Bureau of Public Roads and the National Science Foundation.

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A simple method is presented for the calculation of thermal and residual stresses produced in individual plates due to welding. Comparisons of the residual stress distribution are made with experimental results.

The method is a step-by-step method for the computation of the residual stresses arising in welded plates, based on a knowledge of the temperature distribution as obtained from the equations in the literature. The boundary of the plastic deformations is automatically used in the calculation and is not determined specifically. The method takes equilibrium into account at all stages of cooling, simulating the spontaneous equilibrium which actually occurs. The method is also applicable to edge-welded plates.

The accuracy of the method is limited by a knowledge of the variation of the material properties with temperature and by a knowledge of the heat losses during welding. Calculations are presented for both center-welded and edge-welded structural steel plates, and are compared with experimental results for the same plates. For certain examples, the thickness and width of the plates have been taken into account.
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1. INTRODUCTION

This paper presents the results of a theoretical investigation into the residual stress distribution set up in steel plates welded either at the center or at the edge.

Residual stresses due to welding or cooling from rolling are thermal stresses remaining when the material has cooled to ambient temperature.\(^{(1)}\) Thermal stresses have been investigated both analytically and experimentally for a number of years, the first theoretical studies being based incorrectly on the premise that the heated material remains wholly elastic.

Welded built-up members are being used more frequently in steel construction due to economy, convenience and aesthetics. It is only recently that a true insight into the behavior of columns under load has shown that the residual stress distribution inherent in the cross section plays a major role in the column strength characteristics.\(^{(2)(3)}\) Welded shapes have residual stress magnitudes and distributions different from those of rolled shapes, and yet design formulas prepared for rolled structural shapes are being applied to welded columns.

During the pioneer years of arc-welding a number of
decades ago, the analytical and experimental investigation of the welded joint had presumed an elastic behavior throughout the complete process of welding. It was not until 1936, when Boulton and Lance Martin\(^\text{(4)}\) published analytical and experimental results that it was shown that welding induced plastic deformation of material in and near the weld, and that the residual stresses resulting after cooling were due to these plastic deformations.

Fujita\(^\text{(5)}\) studied the thermal and residual stresses arising in a center-welded plate by both experimental and theoretical means. His tests indicated that the temperature distribution of a plate can be assumed to be that of an infinite plate if the width of the plate is wider than about 10".

The residual stress in any fiber of a material in the as-cooled condition may be regarded as the thermal stress in that fiber after cooling. Thermal stresses in plates under a temperature gradient have been studied to a considerable extent\(^\text{(6)}\) and are dependent on the temperature distribution in the plate. The derivation of the temperature distribution is the most important step towards the derivation of the thermal and residual stresses. When combined with the plastic deformation occurring, the welded plate problem becomes very formidable, so much so that early solutions made assumptions which cannot be accepted in the light of present knowledge.
This paper presents a simple step-by-step method for the computation of the residual stresses resulting in welded plates, based on a knowledge of the temperature distribution as obtained from the equations existing in the literature. The boundary of the plastic deformations is automatically used in the calculation and is not determined specifically. The method takes equilibrium into account at all stages of cooling, simulating the spontaneous equilibrium which actually occurs.

This paper presents the results of one phase of a general study on residual stresses and the strength of welded built-up columns. The purpose of this phase was to measure and predict the residual stresses arising from edge- and center-welds, simulating welds occurring in built-up sections. Built-up welded cross sections may be considered separately as plates with welds, provided the effect of restraint of adjacent plates is taken into account. On this basis, if the residual stress distribution in an individual plate due to a certain weld is known, then it will be possible to predict the residual stress distribution of any welded cross-sectional shape.

The investigation was concerned only with bars and universal mill plates of ASTM Designation A7 Structural Steel of sizes 6" x 1/4", 8" x 1/2" and 12" x 3/4". The plates considered for computation were single plates edge-welded, center-welded, or two plates butt-welded to simulate a center-welded plate. A single pass weld was considered, although the method is applicable to multi-pass welds.
The derivation of the analytical expressions for the distribution of temperature in a heated plate depends on many factors, such as thickness of plate, size of plate, speed of welding, position of weld, not to mention such factors as the thermal properties of the plate. The analysis which has the greatest use with the plates used in fabricated steel structures is one which considers an infinite plate of a thickness sufficiently small to regard the plate problem as two dimensional, and with the weld deposited in a bead which is uniform throughout the thickness of the plate. Such an analysis was first performed, simultaneously and independently, by Boulton and Lance Martin(4) and by Rosenthal(7). The derivation of the equations is assisted to a great extent by the unique and complete analyses into the conduction of heat in solids by Carslaw and Jaeger(8).

An insight into the problem of temperature distribution may be obtained from the outline of the analysis for temperature distribution given in Ref. 9.

The following assumptions were made for the analysis:

1. the heat losses through the surface of the plate are neglected,
2. the heat source is a point source, and any effects at the edge due to this assumption are neglected,

3. the temperature at a great distance from the source of heat remains unchanged, and

4. the physical properties of the material are constant.

Figure 1(a) illustrates the variation of temperature due to the heat source. (The temperature distributions shown at various cross sections are necessary for the computation of thermal stresses.)

It has been shown that the general equation for the temperature distribution is:

\[ T = \frac{Q_p}{\pi k} e^{-\frac{\pi^2}{2} \frac{K_0(\pi vr)}{h}} \]  \hspace{1cm} \text{(edge weld)} \hspace{1cm} (2.1)

where

\[ r = \sqrt{x^2 + y^2} \]

\[ h = \text{thickness of plate} \]

\[ K_0 = \text{modified Bessel function of second kind, zero order} \]

\[ k = \text{thermal conductivity} \]

\[ Q_p = \text{total heat input} \]

\[ v = \text{velocity of welding rod} \]

\[ \frac{1}{\gamma \rho} = \frac{k}{s \rho} = \text{thermal diffusivity} \]

\[ s = \text{specific heat of material} \]

\[ \rho = \text{density of material} \]
Equation 2.1 has been derived for a two dimensional plate which has an edge and a finite thickness, but is infinite in the other directions, as shown in Fig. 1(b). For a finite plate with an edge-weld, the equation is directly applicable for all but the narrowest widths. For narrow plates, the temperature distribution may be obtained from that for the infinite case, by use of the "mirror method"(7), shown in Fig. 2. (The mirror method assumes that the temperature distribution for a finite plate is the summation of the temperature distribution for an infinite plate with that of its mirror image.) The temperature distribution may be obtained for center-welded plates in a manner similar to that for edge-welded plates, provided that half the heat input is assumed to enter each plate on both sides of the weld. (For the two-dimensional case of relatively thin plates, butt-welded similar plates and center-welded single plates may be regarded as identical.) Equation 2.1, therefore, is most versatile, being used to compute the temperature distributions in edge-welded plates, center-welded plates, and by superposition, in plates welded on two edges simultaneously. All these cases are applicable for use in the calculation of temperature distribution for plates composing fabricated structural shapes, such as H- and box-shapes.

Equation 2.1 has been derived in a modified form by authors other than those given above. Analyses also exist for the temperature distribution in solids with different boundary conditions subjected to moving heat sources.
Studies also exist on the physical properties of steels and the effect of these on the heat transfer.\(^{(10,11,12,13,14,15)}\)

An example of the calculation of temperature distribution for a center-welded plate as well as the values to be used for the constants is given in Appendix A of Ref. 9.
3. MATERIAL PROPERTIES AT ELEVATED TEMPERATURES

The thermal and residual stress computation is dependent upon the three parameters, yield point, Young's modulus of elasticity, and the coefficient of linear expansion of the material under consideration (that is, \(\Delta \sigma = \alpha \cdot \Delta T.E\)).

Figure 3 shows the approximate variation of these three physical properties with temperature for mild steel\(^{16}\). The word "approximate" has to be used when describing these physical properties, since the variation is so great between materials differing only slightly in chemical content and because the measurement of these values at elevated temperatures poses problems great enough to render the correct application of some values difficult. For instance, although the yield point in tension is regarded as being of the same magnitude as that in compression for mild steel, this may not necessarily be so at elevated temperatures -- even the method of measurement of the elevated temperature is difficult. It is for this reason that there is such great variation in the values for the material properties obtained by different investigators.

For the purposes of this investigation, a literature survey was made and from the experimental values presented a value was obtained for each of the three parameters.
A number of the available papers were investigated (Refs. 16 to 27) in an effort to compare and evaluate the methods used in the measurement, and the average curves as obtained for the parameters for mild steel were based on four investigations, which, it was felt, reflected favorably with the type of steel used in this study. (16, 17, 18, 19) Table 1 enumerates the chemical composition of the plates and shapes used in the experimental study, and also that of the steels measured for physical properties at the elevated temperatures. Figures 4(a), (b) and (c) give the values obtained from the literature for the physical properties, $\sigma_y$, $E_t$ and $\alpha$; limits to the curves have been given, and the curves used in the computations for this investigation are also shown.

It was noted from the survey that the chemical composition had its greatest affect on the physical properties at room temperature, and that at the elevated temperatures the spread of values tended to become smaller. Furthermore, the stress-strain relationship at the elevated temperatures deviated considerably from the approximately idealized elastic-plastic relationship at room temperature. (28)

The yield point was defined in most cases by a 0.1% offset. The zero value for the yield point is at approximately 1500 °F.

It was noted that Young's modulus had an almost linear decrease from room temperature to about 1000 °F and that above 1400 °F no further measurements were possible due to
creep which could not be eliminated. There is a general conclusion, not proved however, that Young's modulus reaches zero in the range between 1500 and 2000 °F, depending on the composition of the steel. Furthermore, most measurements have been dynamic, and the comparison of dynamic and static test results is not known. (A bar of the material is vibrated transversely, the frequency measured, and Young's modulus determined.\(^{19,20}\)

The coefficient of linear expansion decreases to zero in the range between 1200 °F and 1500 °F and then increases again to the high value — this is due to a change (polymorphic transformation) in the phase diagram of steel at these temperatures. (The crystal composition of the steel changes at about 1400 °F from ferrite and pearlite to ferrite and austinite.) No data existed in the literature studied for this coefficient at a temperature higher than 1750 °F.
4. THERMAL CONSTANTS

Before Eq. 2.1 may be solved the values for the thermal constants must be known. These constants are not actual constants since they vary with temperature, but may be assumed as constant, since the inaccuracies involved by so doing are negligible. The values which are given in this study are taken from a number of sources\(^4,7,8,24,29\) and have been cross-checked for accuracy.

Equation 2.1 contains three thermal constants, namely, the total heat input \(Q_p\), the thermal conductivity \(k\), and the thermal diffusivity \(\frac{1}{2}k\).

The heat input \(Q_p\) is a function of the electrode type, its voltage and amperage, and its electrical properties, as well as the radiation from the surface of the plate and from the deposited weld. The heat deposited by an electrode during arc-welding is\(^30\)

\[
Q = A \cdot V \cdot t
\]  

(4.1)

where

\(A\) = current in amperes

\(V\) = voltage

\(t\) = time in seconds

\(Q\) = total heat generated in watts-sec

(or in 0.24 calories).
The net heat, or the heat input is the total heat less such factors as energy loss due to melting of electrode, energy supplied to the bead, radiation and other losses. Rosenthal (7,31) has suggested that 65% of the total heat goes into the plate and is influential towards the formation of thermal and residual stresses. This figure of 65% was assumed constant for all plates on the basis of a theoretical study and experimental verification for two edge-welded plates. (31) Jackson and Shrubsall (30) in a study concerned with the distribution of energy during electric welding showed that a variation occurred for all factors dependent upon the current, but it was shown that for a current of approximately 200 amps, 20 - 40% of the energy went into the melting of base metal and the melting of the electrode. Therefore, since the heat loss from the surface is small in proportion to other losses (this being one of the assumptions of the derivation of Eq. 2.1), the final heat input into the plate may be approximately that obtained by Rosenthal in his investigation. (31) Although the actual heat input may vary with the welding conditions, for convenience for this study, Rosenthal's value of 65% was assumed at first. (Since the experimental verification of this value was obtained from two edge-welded plates, it may be possible that smaller heat losses will occur for V-groove welds, or for fillet welds.)

As the theoretical investigation progressed, it became obvious that the differences between theoretical and experi-
mental results were too great to be explained by the variation of material properties at high temperatures. Finally, computations were made using various percentages for the proportion of total heat generated which actually went into the plate. This is discussed further in Section 5. Further information on the heat flow in arc-welding is available in the literature.\(^{32,33}\)

The thermal conductivity, \(k\), is a parameter defining the proportionality of the flow of heat between two surfaces and the difference of temperature of these two surfaces. Although it varies with temperature, it is approximately constant, and this can be seen in Fig. 5 which plots the parameter at varying temperatures and for different carbon contents.\(^{24,29}\) A value of 0.40 watt/cm.\(^{\circ}\)C appears to be average, and this value is 0.096 cal/cm.\(^{\circ}\)C.sec. and quite close to the figure of 0.1 suggested by Rosenthal\(^{7}\) and the A.S.M.\(^{34}\), and is adopted for this investigation.

The thermal diffusivity is a parameter relating the specific gravity, the specific heat and the thermal conductivity of a material. The value adopted for this investigation was \(\nabla = 6\) sec/cm\(^{2}\) being an average value obtained from the literature.\(^{7}\)

The units describing the above constants are in the metric system, and were transformed into the English system for use in computation.
5. THERMAL AND RESIDUAL STRESSES

As an example of the early elastic theories, Grünig\(^{(35)}\) assumed that the residual stress was equal to the thermal stress at the instant of welding, the thermal stress being limited by the yield strength of the material. This value for residual stress was approximately true since the thermal stresses at the instant of welding have the greatest influence on the resulting residual stresses.

Boulton and Lance Martin\(^{(4)}\) were the first to make a plastic analysis, and they assumed that the bead created a plastic region in the vicinity of the weld and an elastic region elsewhere. The width of the plastic region was determined as the result of careful experiments. The next noteworthy solution was by Rodgers and Fetcher\(^{(16)}\) who assumed an instantaneous thermit weld at the centerline of a plate and developed a method which gave the complete history of the stress variation during welding; however, it was not made clear whether the effect of heat loss was considered.

The following is a step-by-step method for the calculation of thermal and residual stresses resulting in a plate arc-welded at either the centerline or edge by a moving electrode, assuming that the bead is deposited at a uniform rate. At each step the plastic deformation is considered,
and equilibrium accounted for. This method is compared to others at the end of the description.

Equation 2.1 must first be solved for the temperature distribution, utilizing the parameters as indicated by test, such as speed of welding, current and voltage. (This was considered in the example of Appendix A in Ref. 9.) Since it is impossible to make a continuous computation for the thermal and residual stress calculation, the time between onset of welding and end of cooling must be divided into sufficient intervals so that the temperature and thermal stress for each increment may be regarded as being constant. Since the greatest changes in temperature occur immediately after the onset of welding, the time increments should be short at first and comparatively long for the time approaching infinity. For most calculations it was found that about five time intervals were sufficient, namely, with time taken at 0, 25, 50, 100, 300 and 00 seconds.

Starting with zero time the thermal stress is calculated for the initial temperature. The temperature increment for the next time interval is calculated and the thermal stress increment corresponding to it determined. This thermal stress increment is added to that existing at zero time, the addition being limited by the yield stress of the material at the actual temperature under consideration. At each step, the stresses should be in equilibrium. This step is repeated for each time interval until the last, the thermal
stress at the last step being that at the cooled temperature, that is, the residual stress.

This method is shown pictorially in Fig. 6 and described in detail in the Appendix. It is noted in the figure that the thermal stress at zero time is zero at the weld; this is due to the concentrated heat source and resulting high temperature there. Each increment has its thermal stress added to that already cumulated from the preceding steps, the addition must satisfy both equilibrium and the maximum yield stress existing at that temperature. The temperature defining the yield strength is that of the time interval, not that of the temperature increment. Very little accuracy is added to the computation by increasing the time intervals at any time other than close to zero time.

There are a number of assumptions inherent in the above method:

1. The thermal constants and the material properties at elevated temperatures are known. (Actually, the data available is very limited.)

2. The weld deposits metal at a uniform rate and uniformly throughout the thickness of the plate. (Actually, the rate of deposition is close to uniform with an experienced welder, or with automatic welding equipment, -- but the uniformity throughout the deposition does not usually exist.)
3. The time intervals are small enough to give a reasonably continuous and true thermal stress pattern. (Actually, the accuracy of the computation depends mainly on the time intervals at the onset of the welding.)

4. The effects of transverse stress has been neglected. (Actually, the weld while melting and cooling causes changes not only in the longitudinal direction, but also transversely — that is, a moment exists due to the transverse forces which is neglected in the computation. This moment can be increased or decreased if adjoining parts cause a non-uniform restraint, for example, at the junction of a flange and web. The effect, however, is of secondary importance.)

5. The fiber has no further resistance to load when its stress reaches the yield point value. This assumes an idealized elastic-plastic stress-strain relationship at all temperatures. (Actually, as described in Section 3, the stress-strain curve at elevated temperatures is not of the idealized elastic-plastic type, but rather a non-linear curve. The yield point is therefore defined by an offset so that considerable resistance to load can exist past the specified yield point.)

6. The heat input is known. (Actually, heat input
for this investigation was not determined experimentally. The heat input has a marked effect on the thermal and residual stress distribution.

In addition to the above assumptions, it was also assumed that plane sections remain plane, and that each fiber has the same properties. Also, it is assumed that the problem of yielding and elasticity is two-dimensional.

Finally, it is assumed that the conditions of compatibility are satisfied at all times throughout the process of the formation of thermal and residual stresses. The compatibility condition is that each transverse section of the plate remains plane and perpendicular to the line of weld for each time increment considered. In other words, the strains corresponding to the thermal stress at any time must be uniform across the complete section. The assumption of continuous compatibility is based on the fact that a length of plate being welded will be sufficiently long that any element in it may be regarded as having fixed ends; that is, the compatibility condition is enforced. Actually, a section under a thermal gradient is neither completely free, nor completely fixed. A very short section of plate would be completely free to expand, whereas a very long plate would have a restraining influence exerted on the heated portion by the cooled portion. Furthermore, the usual shop practice is to clamp the structural elements to a work table.
for fabrication. Figure 7 gives an example of a plate in a fixed-ended condition sustaining a compressive temperature force and a tensile equilibrium force.

It would be well to point out that the major step involved in this computational method is that of adding together the incremental thermal stress, the previous thermal stress and the equilibrium stress. As shown in the Appendix, there is no actual addition. In reality, the process of stress formation is simultaneous with that of equilibrium. The "addition" of the method is really a computational approach to the natural phenomenon of stress formation and equilibrium. The example shown in the Appendix illustrates the efficacy and accuracy of the method.

The equilibrium force is not actually distributed evenly over the cross section. Indeed, at the vicinity of the weld for the first few time increments the material is plastic and incapable of resisting any stress. The equilibrium force does not strictly imply uniform equilibrium stresses. The equilibrium stresses derive from the equilibrium strains which are uniform over the whole section. But, the Young's modulus is not uniform, due to the effect of elevated temperature. However, except for very hot material near the weld which is already plastic, the variation of $E$ leads to a negligible variation in the equilibrium stresses. For this reason the equilibrium stresses have been made uniform and act over the whole cross section except the plastic
material near the weld.

The above method for the calculation of residual stresses was prepared for this investigation and differs from all previous methods found in the literature, except that developed by Rogers and Fetcher\(^\text{15}\). As pointed out earlier in this section, other methods either did not consider plastic deformations, or else assumed an arbitrary position for the extent of plastic material during welding. This method automatically takes into account the extent of plastic deformations. The method differs from that of Rogers and Fetcher, not only that it has been applied to isotherms set up by a moving electrode, but also in the consideration of equilibrium and heat input. The method of Rogers and Fetcher applied equilibrium to the algebraic addition of incremental stress and the previous thermal stress. As shown above, this does not represent the actual natural phenomenon of spontaneous equilibrium. When considering such differences, one should not forget that all methods to compute residual stress are approximate, and vary only in the degree of approximation. The method of this investigation has been applied also to edge-welded plates, and this has not been considered hitherto in the literature; (this is discussed below).

Figures 8(a) and (b) give a plot of the temperature and thermal stress variation with time.\(^\text{9}\) Theoretical and experimental values for residual stress are compared in
Figs. 8(c), (d) and (e). Figure 8(c) also shows that cutting down the number of time increments has only a minor effect on the computed residual stress. Figure 9 shows further comparisons between computed and measured residual stresses in a center-welded plate. The computation for this plate considers the effect of including initial stresses, of considering the effect of preheating, the effect of yield strength modified to a higher value near the weld, and the effect of using the equation for "thin" plates (described further in Section 8). Although there is not a good correlation between computed and measured residual stresses for the computations using $Q_p = 1.40 \ Q$, the correlation does appear reasonable for computations using $Q_p = 0.95 \ Q$. This is shown in Fig. 8(e) where experimental results for residual stresses in a center-welded 8" x 1/2" plate are shown; also shown are computed values for 95% and 140% heat input and for effect of initial stresses and effect of modifying yield strength to a higher value in the vicinity of the weld. Figure 9 also compares the effects of the different heat inputs. The results of these figures indicates that a heat input of 95% of the experimental gives theoretical residual stress distributions which are quite close to those determined experimentally. (This is shown for Plates T-3-1 and T-5-4 and inferred for Plate T-1-1.)

The values of heat input greater than unity reflect the computations made for heat inputs before correct values for voltage were known. They do serve, however, to show the
effect of variation of heat input on the final residual stress distribution. The results of these limited number of computations indicate that the commonly accepted value of 35% heat losses may be too great.

The preceding has described center-welded plates. The edge-welded plate, on the other hand, requires a further step in the computation for residual stresses. This step is to take into account the equilibrium of moments due to the non-symmetry of the temperature stresses, the moment equilibrium check being made at the same time as the check for force equilibrium. This further step is required since the temperature distribution is not symmetrical. The comparison between the two cases, symmetrical and non-symmetrical temperature distribution for thermal stress, is shown diagrammatically in Fig. 7. The computation of thermal and residual stresses for edge-welded plates is outlined in the Appendix.

As seen from Fig. 7, symmetrical and non-symmetrical temperature distributions differ in their effect on the thermal stress. A symmetrical temperature distribution creates an equilibrium force which acts in the center of the plate resulting in uniform equilibrium stresses. The non-symmetrical temperature distribution, on the other hand, creates an equilibrium force which does not act in the center of the plate. That is, the equilibrium force for this latter case results in equilibrium stresses which are not uniform across the plate.
Figure 10 gives a comparison between residual stresses for an edge-welded plate, as determined from experiment, and by computation, using heat inputs of 120%, 60% and 40%. As concluded above for the center-welded plates, 65% heat input (or 35% heat loss) may be too conservative, and for an edge weld, where heat losses would be even greater, it would appear that 40% heat input (60% heat loss) is more realistic.

It should be noted that this whole section, and indeed, the complete paper, is concerned with two-dimensional residual stresses and yield criteria. It is assumed that the plates under consideration are sufficiently thin that the variation of residual stress through the thickness may be neglected. If the plates are in effect two-dimensional, then the yield criteria for plastic deformation is defined for each fiber by its yield strength. This assumption leads to no great inaccuracies for the usual structural plates.\(^{(7)}\) Indeed, without this assumption, the temperature distribution, thermal and residual stress computations would be almost impossible to compute. Weiner\(^{(36)}\) made the first analytical investigation of the thermal and residual stresses occurring in a free plate of elastic-plastic material subjected to a varying heat input over one face. The solution presented was rigorous and unique. The Mises' condition was applied as the criterion of yielding, but the variation of material properties with temperature was disregarded, otherwise the problem would have been insoluble. The Weiner solution indirectly shows the applicability for practical cases of the assumption of the simple yield criterion.
A number of aids are available for the computations required in the analytical investigation, outlined above. These depend upon the material and thermal properties of the material under investigation, and can only be calculated if certain values are assumed for these properties which may be regarded as being the same for all plates in the investigation.

Perhaps the most tedious computation involved is that of the temperature distribution. This can be reduced considerably if tables are constructed for \( r \) and \( \xi \), and for the term \( e^{-n\nu\xi} \cdot K_0(n\nu r) \) in Eq. 2.1. This latter term depends mainly upon \( \nu \), the velocity of the electrode, which varies somewhat depending upon conditions. Hence, provided a sufficient number of values are chosen for \( r \), \( \xi \), and \( \nu \), the computation of the temperature distribution for any plate with the same material properties as that for which the constants are calculated is reduced to the use of the table and the heat input and the thickness of the plate. In other words, the calculation of temperature at any point is reduced to a one-step computation.

A further aid to the computation can be made with the thermal and residual stress calculations. This is the
construction of tables showing the temperature stresses over the complete range of temperature, such tables giving the stress without the double measurement of the Young's modulus and the coefficient of linear expansion at any particular temperature. Hence, the value of the incremental temperature stress \([\Delta \sigma]\) may be read directly from the prepared tables.

Another aid in the computation may be the determination of the equilibrium stresses on the basis of an equilibrium check made with a planimeter, or by counting squares. However, the division of the plate into definite widths for the residual stress computation is simple and quick, any desirable accuracy being obtained by making the width of the strips narrower.

References 37, 38, 39 and 40 were found to be very helpful in the computation of Bessel functions required for the temperature distribution.
7. COMPARISON BETWEEN ANALYTICAL AND EXPERIMENTAL RESULTS

Comparison between the residual stress distribution for four welded plates, as calculated by the methods explained in Section 5 and as measured in the test program, are shown in Figs. 8, 9 and 10. Figure 11 shows typical residual stress distributions measured in the experimental program, which has been described in Ref. 41.

Figures 8(c), (d), (e), 9(a), (b), and 10 show residual stress distributions for four plates, and compare the experimental results against various computed values and for variation in assumed heat input. The computed values include the effect of initial residual stresses, do not include this effect, include the effect of preheating from the electrode, include the effect of more or less steps in the calculation, and include the effect of very small thickness of plate.

In general, the computation methods gave results which differed from the experimental values. The computed residual stress distribution was similar in shape to the experimental one and, in some cases, the differences between them could be regarded as negligible. (Plates T-3-1 and T-5-5, Figs. 8(e) and 10.) The major factor which
dominated the residual stress distribution was the heat input. While this was not specifically determined experimentally, comparison of theory and experiment for residual stress distribution indicated that heat inputs of about 95% of the total heat generated during welding occurred for center-welded V-grooved plates, and about 40% for edge-welded plates. The number of computations made was too limited in this study, as well as in the major study, (31) to make any definite conclusions as to heat losses and actual heat input contributing to the formation of residual stresses during welding. Previous studies have not compared the heat losses from V-groove welds and edge-welds.

The computation for the derivation of the residual stresses in Plate T-5-5 has been described in the Appendix, and for Plate T-3-1 in Ref. 9. It is seen from the plot in Fig. 8(c) that a reduction in the number of steps in the computation does not have any great effect on the computed residual stress distribution. (The number of steps considered are 6, 5 and 4.) Neither is there any great effect due to the inclusion of initial residual stresses for the calculation (Fig. 8(d)). Since the computation assumed a yield strength of 37 ksi, and the actual experimental measurement (41) showed a yield strength of 58 ksi at the weld, a computation was carried out assuming that the vicinity of the centerline had a yield strength varying from 58 ksi at the centerline to 37 ksi at a distance of 1.2 inches from the centerline. The extra forces brought into effect in
the computation resulted in a distribution with values of residual stress much higher than those of any other computed distribution for a heat input of 140%; the correlation was reasonable for $Q_p = 0.95 Q$. In general, the weld material has a yield point up to 50% higher than that of the parent material, and this should be taken into account in the computation. (See Figs. 8(d), (e), and 9(a).

For the heat losses giving best correlation, no reason could be given for the small discrepancies between computed and test results, except that the thermal and material properties for the material are not known, particularly at the higher temperatures. The effect of restraint of adjoining pieces must be discounted, in the light of the results of Ref. 9.

The plates of the experimental investigation, when welded, are clamped to heavy structural shapes. These shapes would absorb some of the heat from the plates. It is not known how this would affect the residual stress distribution of the plates.

The inclusion of the effect of pre-heating is shown in the curves for Plate T-5-4, Fig. 9(a). (The distribution for the effect of pre-heating actually coincides with that for the same calculation without this effect -- "no initial stresses". In other words, the effect is negligible.) Preheating of the plate during welding comes about since the portion of the plate immediately before the moving electrode is heated before
the electrode actually reaches there. The computation merely involves finding the temperature distribution in front of the moving electrode, as well as behind it, as is normally the case.

Plate T-1-1, Fig. 9(b), may be regarded as a "thin" plate, being only 1/4" thick. Then, according to Rosenthal, heat losses through the surface to surrounding atmosphere may have to be taken into account. (Rosenthal showed that this should be done for thickness less than about 0.4 inch.) The term $\lambda v$ in the Bessel function of Eq. 2.1 is modified to $\sqrt{(\lambda v)^2 + m}$, where $m = \frac{2k_1}{hk}$, and $k_1$ is the thermal surface transmission. Such a computation resulted in different curves, although again, only an approximation to the experimental results. (See Fig. 9(b) under "Equations modified for very small thickness of plate".)

For ease of comparison and computation, all the plates which were used for computation were center-welded in one pass or edge-welded in one pass. A computation for the residual stress distribution from a multi-pass weld would be similar to that given above. This is true also for the inclusion of the effects of initial residual stress. The initial residual stresses are taken into account when the summation of thermal and equilibrium stresses is made, (see Appendix). Similarly, to take into account multi-pass welds, the actual time difference between each pass must be known,
and when the temperature distribution for each pass is calculated, that of the preceding pass must be taken into account. Such a computation was not made.

It was mentioned in the Introduction that the temperature distribution of a plate can be assumed to be that of an infinite plate if the width of the plate is wider than about 10". A graphical method was also given to determine the temperature distribution for a finite plate from that for an infinite plate; see "Mirror Method", Fig. 2. For the computations of this investigation, all temperature distributions were computed on the assumption of an infinite plate, with no correction made for the finite width. The effect of finite width exists only for the temperature distributions at about \( t = 100 \) and \( t = 300 \) seconds; for these time increments the temperature at the far edge is finite. These corrections were not made since the temperature distribution at zero time has the greatest effect on the final residual stresses, any small variation in later distributions playing a negligible role in the final residual stress distribution.

In general, the calculated residual stress magnitude and distribution are quite close to the experimental values, provided the assumed heat losses given above were used.
8. SUMMARY

1. The investigation was concerned only with bars and universal mill plates of ASTM Designation A7 Structural Steel of sizes 6" x 1/4", 8" x 1/2" and 12" x 3/4". The plates considered for computation were single plates edge-welded, center-welded, or two plates butt-welded to simulate a center-welded plate. A single pass weld was considered, although the method is applicable to multi-pass welds.

2. The temperature distribution in a plate heated by welding depends upon the thickness and width of the plate, speed of welding, position of weld, the thermal properties of the material, and the heat input due to the welding. (Section 2.)

3. The equation for the temperature distribution in a thin semi-infinite plate (Eq. 2.1) was used to compute the temperature distribution in edge-welded plates and in center-welded plates, and, by superposition, may be used in plates welded on two edges similarly and simultaneously. (Section 2.) This equation may also be used along with the "mirror method" to give the temperature distribution for "narrow plates". (Figure 2.) A knowledge of the
temperature distribution enables the thermal and residual stresses to be computed.

4. The review of the literature showed that there is a considerable variation in the properties of materials at high temperatures. The variation may be considerable even for steel of approximately the same carbon content. Lack of knowledge of the exact variation is one factor preventing an accurate residual stress computation being made for a welded plate. (Figures 4 and 5.)

5. The heat input during welding is known only very approximately. The limited literature available shows that the heat losses from melting of electrode and of bead, radiation and other sources is about 35%. However, the results of this investigation, for three center-welded and one edge-welded plate, have indicated that assumptions of losses of about 5% and 60% respectively for center-welded V-grooved plates and for edge-welded plates result in computed residual stress distributions very similar to those obtained by experiment. (Figures 8(e), 9 and 10.)

6. The computations made for residual stress distribution in welded plates considered the effect of initial cooling residual stresses, the effect of very small thickness of plate, and the effect of more or less steps in the computation. All of these effects have
a noticeable influence on the residual stress magnitude and distribution. These effects vary in their magnitude, but are not as great as the effect of variation of heat input. (Figures 8(c) and (d), 9 and 10.) The effect of preheating from the electrode was negligible. (Figure 9(a).)

7. The computational method for the determination of residual stress distributions in welded plates gave results very close to those obtained by experiment provided the assumed heat losses given above were used. (Figures 8(c) and (d), 9 and 10.) Such computations can be carried out only when the welding conditions are known or can be accurately estimated. Furthermore the assumptions for the heat losses depend upon accurate measurements of the heat input obtained from a knowledge of the welding conditions.

8. The yield point of the material was assumed to be 37 ksi for the computations. The thermal and residual stresses reach the yield point value only at and in the vicinity of the weld. The material at the weld is under a tensile residual stress with a magnitude greater than the yield point of the parent material. Computations were made taking into account the higher yield point of the weld material; this effect was very noticeable, but
varied in its magnitude. (Figures 8(d), 9 and 10.)

9. The effect of heat input dominates all other effects considered in the computations. Until further work is conducted to study the heat losses and the actual heat input for V-groove welds, fillet welds, butt welds and edge welds, computational methods for obtaining residual stresses will not be reliable.
9. NOMENCLATURE

A  current in amps
E  Young's modulus of elasticity
K_0  modified Bessel function of second kind, zero order
Q  total heat generated
Q_p  total heat input
T  temperature at a point in a solid, depth of a weld
T_{ave}  average temperature of two temperature increments
T_{25}  temperature at a time increment of 25 seconds after onset of welding
V  voltage in volts
    symbol denoting single V-groove weld

d  width of a plate, symbol denoting a differential or an element of a quantity
h  thickness of plate
k  thermal conductivity of a material
m  a parameter, m = 2k_1/hk
r  radial ordinate, r = \sqrt{\xi^2 + y^2}

s  specific heat of a material
\( t \)  
- time

\( v \)  
- velocity of welding rod during deposition of weld

\( x, y, z \)  
- cartesian coordinates

\( \Delta T \)  
- increase in temperature \( T \), at a point or at an element of volume in a body

\( \Sigma \)  
- a symbol denoting a summation of terms

\( \alpha \)  
- coefficient of linear expansion

\( \epsilon \)  
- unit strain

\( \frac{1}{2\alpha} \)  
- thermal diffusivity \( = k/\rho c \)

\( \xi \)  
- longitudinal ordinate with respect to a moving heat source

\( \rho \)  
- density

\( \sigma \)  
- stress

\( \sigma = \Delta T. \alpha. E \)  
- temperature stress

\( \sigma_{eq} \)  
- equilibrium stress, used in thermal stress calculations

\( \sigma_m \)  
- moment equilibrium stress, used in thermal stress calculations

\( \sigma_r \)  
- residual stress

\( \sigma_y \)  
- yield point or yield strength
ACKNOWLEDGEMENTS

This report presents a part of the theoretical and experimental investigation made during the course of a research program on the influence of residual stress on the strength of welded built-up columns.

The investigation was conducted at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania. The Pennsylvania Department of Highways and the U.S. Department of Commerce Bureau of Public Roads, the National Science Foundation and the Engineering Foundation through the Column Research Council sponsored jointly the research program.

Lynn S. Beedle directed the project, and a Column Research Council Task Group under the chairmanship of John A. Gilligan has provided valuable guidance.

Acknowledgement is also due to Alan H. Cook, Fiorello R. Estuar, Diethelm K. Feder, Mogens P. Nielsen, N. R. Nagaraja Rao, George J. Tamaro and Robert G. Wagner whose advice and assistance with this investigation have been invaluable.
The method for computing the temperature distribution, thermal and residual stresses for a center-welded plate has been completely detailed in Appendix A of Ref. 9.

The following is a computation of thermal and residual stresses in an edge-welded plate. One step of the step-by-step method is illustrated in detail. The example also serves as a model for the computation of thermal and residual stresses in center-welded plates; for which case, the method is the same, except that there will be no moment equilibrium to take into account.

**Plate T-5-5**  
**Edge-Welded Plate**

- 12" x 3/4" plate  
- 1/4" edge weld  
- Current: 325 amp.  
- Voltage: 30 v.  
- Velocity: 0.10 in/sec.

The following computation for thermal stresses at the instant of welding does not include initial cooling residual stresses, and it is based on a 60% heat input (or 40% losses).
### Table: Stress Analysis

<table>
<thead>
<tr>
<th>Trial</th>
<th>Eq,</th>
<th>( \sigma_m/y )</th>
<th>( \sigma_m )</th>
<th>( \sigma_y )</th>
<th>( \Delta P )</th>
<th>( \Delta P_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.5</td>
<td>-0.3</td>
<td>-1.7</td>
<td>-1.5</td>
<td>-1.3</td>
<td>-1.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.6</td>
<td>0.6+1.2+1.8</td>
<td>0.6+1.2+1.8</td>
<td>0.6+1.2+1.8</td>
</tr>
<tr>
<td>2nd</td>
<td>1.0</td>
<td>+1.0</td>
<td>+5.6</td>
<td>+4.8</td>
<td>+4.4</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>0.0</td>
<td>0.0+2.0-4.0-6.0</td>
<td>0.0+2.0-4.0-6.0</td>
<td>0.0+2.0-4.0-6.0</td>
</tr>
<tr>
<td>3rd</td>
<td>1.5</td>
<td>+0.8</td>
<td>-18.5-21.4</td>
<td>+0.7</td>
<td>+5.0</td>
<td>+4.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.1</td>
<td>0.1+3.1+1.5-0.1-2.3-3.3</td>
<td>0.1+3.1+1.5-0.1-2.3-3.3</td>
<td>0.1+3.1+1.5-0.1-2.3-3.3</td>
</tr>
<tr>
<td>4th, 5th and 6th Trials</td>
<td>1.6</td>
<td>+0.8</td>
<td>+0.7</td>
<td>+1.6</td>
<td>+1.6</td>
<td>+1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.8</td>
<td>0.8+1.6+0.7</td>
<td>0.8+1.6+0.7</td>
<td>0.8+1.6+0.7</td>
</tr>
<tr>
<td>7th</td>
<td>1.7</td>
<td>+0.75</td>
<td>-18.5-20.5</td>
<td>+0.7</td>
<td>+5.0</td>
<td>+4.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.7</td>
<td>0.7+3.2+1.7</td>
<td>0.7+3.2+1.7</td>
<td>0.7+3.2+1.7</td>
</tr>
</tbody>
</table>

### Notes
- \( \delta_{eq} = 0.5 \) for the 1st trial.
- \( \delta_{eq} = 1.0 \) for the 2nd trial.
- \( \delta_{eq} = 1.5 \) for the 3rd trial.
- \( \delta_{eq} = 1.6 \) for the 4th to 6th trials.
- \( \delta_{eq} = 1.7 \) for the 7th trial.
- \( \sigma_m/y = +0.8 \) for the 3rd trial.
- \( \sigma_m/y = +0.7 \) for the 7th trial.
- \( \Delta P = 0.0 \) for the 2nd and 3rd trials.
- \( \Delta P = 0.0 \) for the 4th to 6th trials.
- \( \Delta P = 0.0 \) for the 7th trial.
- \( \sigma_y = 37 \text{ ksi} \) for all trials.
- **Room temperature = 70 °F**
- **\( I = 108 \text{ in}^2 \)**
In the above table, the values for the temperature at zero time come from the computation for temperature distribution, performed separately. \( \Delta T \) is the increment of temperature over the preceding temperature. The values for \( E \) and \( \alpha \) are the average temperature of the increment, and are taken from Figs. 4(b) and (c). The temperature stress \( [\sigma] \) is the stress resulting from the increase in temperature.

The temperature stresses are not in equilibrium, and equilibrium is made by obtaining the total force, \( \Sigma \Delta P \), and the total moment, \( \Sigma \Delta P \cdot y \), and opposing them by equal and opposite forces distributed over the complete elastic cross-section. (This is illustrated in Fig. 7.) For convenience, the cross-section has been divided into a number of widths, \( \Delta b \). In the case above, for \( t = 0 \) secs, the first two widths next to the weld are in a plastic condition, and hence play no role in resisting the equilibrium force.

Since the actual thermal stress for the time increment is a summation of the temperature stress and the equilibrium stresses, and since this summation is actually simultaneous, a trial and error method must be used in a computation.

The seven trials used above illustrate the process of trial and error operation required for equilibrium. The first trial is arbitrary, and is based on the knowledge that the summation of equilibrium and of moment equilibrium stresses \( (\Sigma \frac{\Delta P}{12} = -6.2, \Sigma \frac{\Delta P \cdot y}{1} = -3.8) \) are usually much greater than is the case for the final trial, although they
may be used for a first trial.

The first trial has assumed \( \sigma_{eq} = +0.5 \) and \( \sigma_m/y = -0.3 \), where \( \sigma_{eq} \) is the equilibrium stress due to equilibrium force, and \( \sigma_m \) is the equilibrium stress due to the equilibrium moment. \( \sigma_m = \frac{M}{I} \). The summation is made of temperature stress \([\sigma]\) and equilibrium stresses \( \sigma_{eq} \) and \( \sigma_m \). The thermal stress for the first trial, \( \sigma_o \), is obtained from this summation by modifying the values where needed by taking into account the maximum value of the yield strength at the temperature of the increment. The trial is unsatisfactory as seen from equilibrium check, since the thermal stress distribution does not result in force and moment equilibrium. 

\[
\sum \Delta P = -13.7 \quad \text{and} \quad \sum \Delta P \cdot y = -129.9.
\]

The final trial shows the actual values for thermal stress. Experience in the method will cut down the number of trials. The number of trials involved is about twice that for a comparable center-welded plate, which has only one variable, \( \sigma_{eq} \).

In these calculations, the thickness of the plate need not be considered in the equilibrium stresses. This is because the method is iterative, and the final equilibrium condition automatically accounts for thickness. However, if the thickness does vary, it must be taken into consideration. Also, the neutral axis for the partially plastic section is
no longer at the centerline of the plate, but is assumed so for the computation for simplicity, the error involved being negligible.

\[ t = 25 \]

Note: \[ T_{\text{ave}} = \frac{T_{25} + T_0}{2} + T_{\text{room}} \]

<table>
<thead>
<tr>
<th>( y )</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.8</th>
<th>1.2</th>
<th>1.6</th>
<th>2.0</th>
<th>4.0</th>
<th>6.0</th>
<th>8.0</th>
<th>10.0</th>
<th>12.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{e=25} )</td>
<td>1825</td>
<td>1775</td>
<td>1630</td>
<td>960</td>
<td>657</td>
<td>338</td>
<td>175</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>max ( \sigma_y )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>19</td>
<td>28.5</td>
<td>36</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>( T_{\text{ave}} )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>589</td>
<td>376</td>
<td>207</td>
<td>122</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>( \sigma_{\text{ave}} \cdot 10^3 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>8.1</td>
<td>7.3</td>
<td>6.7</td>
<td>6.3</td>
<td>6.1</td>
<td>6.1</td>
<td>6.1</td>
<td>6.1</td>
<td></td>
</tr>
<tr>
<td>( E_{\text{ave}} \cdot 10^3 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>270</td>
<td>29.0</td>
<td>29.7</td>
<td>30.0</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>-905</td>
<td>+1768</td>
<td>+1753</td>
<td>+562</td>
<td>+263</td>
<td>+105</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>([\sigma] )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-166</td>
<td>-119</td>
<td>-52.3</td>
<td>-19.9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Final Trial: \( \sigma_{eq} = +6.5 \), \( \frac{T_m}{y} = +2.3 \)

\[ \sigma_{25} = \sigma_0 + [\sigma] + \sigma_{eq} + \sigma_m \]  
limited by: \( \max \sigma_y \)

\[ \sigma_{25} \] | 0 | 0 | 0 | -19.0 | -285 | -307 | +0.5 | +14.3 | +8.2 | +2.1 | -4.0 | -10.1 | +0.4, -3.4 OK

The portion of the calculation above for \( t = 25 \) secs shows that the whole computational procedure is repeated again. In this case, there is one additional stress in the summation for the thermal stress. A summation is made of temperature, previous thermal and present equilibrium stresses, or \( \sigma_{25} = \sigma_0 + [\sigma] + \sigma_{eq} + \sigma_m \). The summation is limited by the maximum value for \( \sigma_y \) at any point.

In the case where initial cooling residual stresses
are included, these are added together with the summation of stresses for the first time increment, \( t = 0 \).

The summation used to obtain the thermal stress is a computational approach. No computational method can put into equilibrium the summation of temperature stress and preceding thermal stress, taking into account the limits of yield strength; the method must be trial and error.

### Summary of Thermal Stresses:

Plate T-5-5, Edge-Welded Plate (\( Q_p = 0.60 \) Q)

<table>
<thead>
<tr>
<th>( y )</th>
<th>( 0 )</th>
<th>0.2</th>
<th>0.4</th>
<th>0.8</th>
<th>1.2</th>
<th>1.6</th>
<th>2.0</th>
<th>4.0</th>
<th>6.0</th>
<th>8.0</th>
<th>10.0</th>
<th>12.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 0 )</td>
<td>0</td>
<td>0</td>
<td>-18.5</td>
<td>-20.8</td>
<td>+5.0</td>
<td>+4.7</td>
<td>+3.2</td>
<td>+1.7</td>
<td>+0.2</td>
<td>-1.3</td>
<td>-2.8</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>0</td>
<td>-19.0</td>
<td>-26.5</td>
<td>-30.7</td>
<td>+0.5</td>
<td>+14.3</td>
<td>+8.2</td>
<td>+2.1</td>
<td>-4.0</td>
<td>-10.1</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>+4.5</td>
<td>+5.5</td>
<td>+6.5</td>
<td>-14.5</td>
<td>-23.5</td>
<td>-31.7</td>
<td>-25.9</td>
<td>+13.8</td>
<td>+13.7</td>
<td>+3.6</td>
<td>-6.5</td>
<td>-16.6</td>
</tr>
<tr>
<td>100</td>
<td>+19.0</td>
<td>+19.0</td>
<td>+19.0</td>
<td>+19.0</td>
<td>-19.0</td>
<td>-26.5</td>
<td>-32.0</td>
<td>+14.3</td>
<td>+12.7</td>
<td>+4.1</td>
<td>-4.5</td>
<td>-13.1</td>
</tr>
<tr>
<td>300</td>
<td>+26.5</td>
<td>+26.5</td>
<td>+26.5</td>
<td>+26.5</td>
<td>+26.5</td>
<td>+26.5</td>
<td>+30.0</td>
<td>-8.9</td>
<td>+17.7</td>
<td>+8.1</td>
<td>-1.5</td>
<td>-11.1</td>
</tr>
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<td>( \infty )</td>
<td>+37.0</td>
<td>+37.0</td>
<td>+37.0</td>
<td>+37.0</td>
<td>+128.5</td>
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<td>-16.6</td>
<td>-4.5</td>
<td>+7.6</td>
<td>+19.7</td>
<td></td>
</tr>
</tbody>
</table>

The residual stresses computed for \( Q_p = 1.20 \) Q, 0.60 Q, and 0.40 Q are plotted in Fig. 10. Initial cooling residual stresses are also considered, as well as the effect of higher yield strength at the weld. (The initial cooling residual stresses were assumed as being parabolic in distribution with +12.5 ksi tension at the edges, and -5.0 compression
at the center. It is possible that stresses of too high a magnitude have been assumed. This is seen from a comparison of the stresses at the 12" point for the experimental results and those for $Q_p = 0.40 Q$; at other points, the correlation is very reasonable.) The effect of increase of heat input is markedly visible on the residual stress distribution.
12. TABLES AND FIGURES
### TABLE 1

**a) AVERAGE CHEMICAL COMPOSITION OF STEEL PLATES USED**

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>0.20</td>
<td>0.60</td>
<td>0.011</td>
<td>0.025</td>
</tr>
</tbody>
</table>

**b) CHEMICAL COMPOSITION OF STEEL USED IN ELEVATED TEMPERATURE MATERIAL PROPERTIES TESTS**

<table>
<thead>
<tr>
<th>Ref.</th>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0.20</td>
<td>not given</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.13</td>
<td>0.45</td>
<td>0.011</td>
<td>0.023</td>
</tr>
<tr>
<td>18</td>
<td>0.16</td>
<td>0.55</td>
<td>0.013</td>
<td>0.032</td>
</tr>
<tr>
<td>19</td>
<td>0.31</td>
<td>0.66</td>
<td>not given</td>
<td></td>
</tr>
</tbody>
</table>
## TABLE 2

### TEST PROGRAM: PLATE SIZES TESTED

<table>
<thead>
<tr>
<th>t</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>16</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4</td>
<td>x o</td>
<td>x o</td>
<td>x o</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3/8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x o</td>
<td>x o</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3/4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>o</td>
<td>o</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x o</td>
</tr>
</tbody>
</table>

- **t**: Thickness of plate
- **b**: Width of plate
- **x**: Residual stress before welding
- **o**: Residual stress after welding
- **+**: Coupon tests before welding
- *****: Coupon tests after welding

**Note**: All sizes given are in inches.
Isotherms Showing Temperature Distribution

(a) TEMPERATURE DISTRIBUTION DUE TO A MOVING SOURCE OF HEAT

(b) EDGE WELDED PLATE

Fig. 1 MOVING ELECTRODE AND TEMPERATURE DISTRIBUTION
Fig. 2 TEMPERATURE DISTRIBUTION FOR FINITE PLATES BY MEANS OF "MIRROR METHOD"

Fig. 3 MATERIAL PROPERTIES AT ELEVATED TEMPERATURES
Fig. 4 VARIATION OF MATERIAL PROPERTIES WITH TEMPERATURE
Fig. 5 THERMAL CONDUCTIVITY FOR STEELS OF DIFFERENT CARBON CONTENT

Fig. 6 THE FORMATION OF THERMAL AND RESIDUAL STRESS
Plate

Temperature Distribution

Temperature Stresses

Temperature Force

Equilibrium Stresses

Equilibrium Force

Thermal Stress

= Temp. Stress + Equil. Stress

(a) Symmetrical Temperature Distribution

Plate

Temperature Distribution

Temperature Stresses

Temperature Force

Equilibrium Stresses

Equilibrium Force

Thermal Stress

(b) Unsymmetrical Temperature Distribution

Fig. 7 FORMATION OF THERMAL STRESS
Fig. 8a THEORETICAL TEMPERATURE DISTRIBUTION CURVES

Fig. 8b THEORETICAL THERMAL AND RESIDUAL STRESSES
Theoretical and Experimental Residual Stress Distribution

**Fig. 8c**

- **Experimental Results**
- **Theoretical Curves**
- Number of Steps Used:
  - \( t = 0, 25, 50, 100, 300, 000 \)
  - \( t = 0, 25, 50, 200, 000 \)
  - \( t = 0, 25, 50, 000 \)

**PLATE T-3-1**
- 8" x 1/2" PLATE
- \( Q_p = 1.40 Q \)

**Fig. 8d**

- **Experimental Results**
- **Theoretical Curves**
  - No Initial Stresses (\( t = 0, 25, 50, 100, 300, 000 \))
  - With Initial Stresses (\( t = 0, 25, 50, 100, 300, 000 \))
  - Yield Point Modified Near Weld (\( t = 0, 25, 50, 100, 300, 000 \))

**PLATE T-3-1**
- 8" x 1/2" PLATE
- \( Q_p = 1.40 Q \)
Fig. 8e RESIDUAL STRESSES: EFFECT OF HEAT INPUT
PLATE T-5-4

Experimental Results

Theoretical Curves

No Initial Stresses (t=0, 25, 50, 100, 300, 00)

With Initial Stresses $Q_p = 0.95 Q$,
(t=0, 25, 50, 100, 300, 00)

With Initial Stresses (t=0, 25, 50, 100, 300, 00)

Yield Point Modified Near Weld
(t=0, 25, 100, 500, 00)

PLATE T-1-1

6" x $\frac{3}{4}$" PLATE

$Q_p = 2.00 Q$

THEORETICAL AND EXPERIMENTAL RESIDUAL STRESS DISTRIBUTION

Fig. 9a

Fig. 9b
Fig. 10 RESIDUAL STRESSES IN EDGE WELDED PLATE
PLATE T-1
6" x 1/4"

PLATE T-7
6" x 1/4"

PLATE T-2
10" x 1/2"

PLATES BEFORE WELDING

PLATES AFTER WELDING

Fig. 11 TYPICAL RESIDUAL STRESS DISTRIBUTIONS
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