1955

Box sections in plastic design, 1955

J. L. Storch
BOX SECTIONS IN PLASTIC DESIGN

by

John L. Storch

Research Methods, C.E. 400
Dept. of Civil Engineering and Mechanics
Lehigh University

1955
CONTENTS

I. Synopsis 1

II. Introduction 2

1. The Why and Wherefore of Plastic Design 2

2. Test Results Reveal Shortcomings of Present Shapes 2

3. Is the Box Section the Answer? 3

4. A Proposed Research Program 3

III. Plasticity and the Box Section 4

1. What is Plastic Theory? 4
   a) Basic Hypothesis
   b) Concepts

2. Validity of the Plastic Concept 5
   a) Shortcomings of Elastic Design Methods
   b) Safety Factors and Unpredictable Stresses

3. Essential Qualities and Characteristics of Shapes 8
   a) Plastic Hinge
   b) Rotation Capacity
   c) Shape Factor
   d) Lateral Stiffness
   e) Knees and Connections

4. Box Section vs. Wide Flange Shape 12
   a) Faults of the WF Shape
   b) The Box Section Offers Promise
   c) Tentative Design of a Comparable Box Section
   d) Design Criteria

5. Tentative Test Program 14
   a) Beam Program, Gaging, Procedure
   b) Portal-Knee Program, Gaging, Procedure

IV. Discussion and Summary 17

1. Plasticity, Its Promises and Its Demands 17

2. Conclusions from Completed Tests 17

3. Fields to be Investigated 17

4. Resumé 18

V. Acknowledgements 20

VI. References 21

VII. Nomenclature 22

VIII. Tables and Figures 23
The purpose of this paper is to present in a brief fashion some of the concepts of Plastic Design and/or Analysis methods, and to bring out one of the major difficulties met with during tests of Plastically-designed structures using run-of-the-mill structural shapes, namely the problem of lateral buckling.

Further, it is intended to present the preliminary work thus far completed which is to be expanded into a test program designed to study the behavior under loads of a built-up box section in the plastic range of stress. The objective of the program is to compare the results of tests on the box section to the known results of a comparable Wide-Flange shape, and to formulate conclusions as to the acceptability of such a section within the field of plastic behavior.
INTRODUCTION

1. The Why And Therefore Of Plastic Design.

With the advent of mathematical stress analysis methods, the tendency was for engineers to regard stress in individual members as the criterion of design, rather than the overall strength of the structure. This trend naturally detracted somewhat from the time-tested axiom of experience being the best of all methods in calculating the overall safety of a structure. An unbiased evaluation of the limitations and restrictions demanded by elastic-stress specifications in use today implies that less emphasis be put on individual stresses and more heed be paid to the safety of the mass as a whole. The present-day emphasis that is being put on research and study of Plastic Design methods is permitting a return to the more rational basis of considering the overall strength as the most valid basis for structural acceptability.4

2. Test Results Reveal Shortcomings Of Present Shapes.

It is the purpose of this paper to present to the reader some of the concepts of such Plastic Design methods (working stresses beyond the elastic range) that are receiving attention today, and to call specific attention to the work being done in this field at the Fritz Laboratory at Lehigh University under the direction of Prof. J. J. Eney.34512. One of the major difficulties encountered in the research program and the large-scale tests at the Fritz Laboratory is that of premature buckling of the compression flange and web of the Wide-Flange shapes subjected to stresses in the plastic range.
preventing full utilization of the reserve strength of the parent material.

3. Is The Box Section The Answer?

This premature failure of the geometry of the members in lateral buckling - a factor dependent upon the torsional properties of the shapes in question, has propagated a need for investigation of members of other shapes which might be suitable for use in plastically-designed structures. It is believed that a built-up box section might exhibit sufficient lateral-torsional stability under extreme stresses to warrant its use where certain WF shapes fail before reaching predicted values of load, rotational capacity, moment, etc.


A tentative design of a box section is presented in Part 4 of section III, which will be subjected to beam and portal-knee bending tests. The results of these future tests will be compared to those already obtained in Fritz Laboratory for 14WF30 members in like tests\(^{(3)(4)}\). Dimensions, properties, gage setup and the proposed test program are discussed, as well as the main factors to be determined and compared.
1. What Is Plastic Theory?

   a) Basic Hypothesis. In elastic design, the criterion of failure is the first attainment of yield or plasticity (neglecting residual and concentrated stresses), whereas in Plastic design, the failure criterion is the maximum capacity load. Of course, in either method, deflections may be the limiting factor. Plastic theory attempts to base a design on the concept of small incremental loads producing large permanent deformations. These permanent deformations must of course stress the metal fibers beyond their yield or elastic limit, and hence cause flows in what is termed a plastic manner. The basic hypothesis behind the reasoning of Plastic theory is the $M-\phi$, or Moment-Curvature relationship, the ability of a member to sustain considerable unit curvature while resisting a constant moment. This concept is called the Plastic Hinge.

   b) Concepts. Fig. 1 shows a typical $M-\phi$ curve for a WF shape with various stress distributions corresponding to like points on the curve. Point #1 corresponds to the extreme fiber just attaining yield-point stress. Point 2 shows yield penetrated to about $1/4$-depth, and point 3 shows the stress distribution when complete yield of the member has taken place, that condition being termed the Fully Plastic Moment.

   The attainment of a fully plastic moment would correspond to infinite curvatures, hence at a section where $M_p$ is presumed to be reached, finite changes in slope angle could occur in infinitesimal distances. Relative rotations across such a
section could occur freely at constant moment, thus the mem-
bers could be regarded as connected at that section by a hinge
which transmits only a constant moment, $M_p$. Obviously, infinite
curvatures would necessitate infinite strains, and cannot be
realized. Abrupt changes or kinks do not occur, but large
changes of curvature can be observed within relatively short
lengths under very small changes in bending moment. Hence,
'relatively' means by comparison with elastic behavior (10).

It is this ability of structural steel, a ductile
material, to deform considerably without collapse, upon which
Plastic Theory is founded. The great reserve of strength above
and beyond the elastic limit has to date been brushed aside,
by existing design specifications, but the disciples of
Plastic Theory are endeavoring through research to establish
a valid set of design specifications based on the overall strength
of the structure up to collapse loads, such rules of practice
being based on data proven to be correct through tests of
members in the plastic range of stress.

2. The Validity of the Plastic Concept.

a) Shortcomings of Elastic Design Methods. Two plain and
simple examples which follow will tend to illustrate why the
reserve strength of steel in the plastic range should be
utilized, hence - to prove that Plastic methods have a rightful
place in the engineer's store of design methods.

Fig. 2 shows a fixed-ended beam under increasing load
up to collapse at the formation of the last plastic hinge.
The first failure occurs at the left end, forming a plastic
hinge. Secondly, a hinge develops under the load causing the
right-hand portion to act as a cantilever which leads to
plastic collapse when the section at the wall on the right develops a plastic hinge.

The load at failure can be predicted by assuming plastic hinges to develop in order at points of maximum moment, as in part f of Fig. 2. The calculation of load at this condition gives a useful approximation of the observed failure load.

\[ P \cdot 2 \theta \cdot L/3 = 2\delta M_p + 3\eta M_p + 6M_p \]

hence: \[ P_{\text{max.}} = P_c = 9M_p/L \]

Now, if this same fixed-ended beam were designed elastically, the allowable load would have been

\[ P_{\text{all}} = (27 \cdot I \cdot \sigma_{\text{all}})/(4 \cdot c \cdot L) \]

the factor of safety being

\[ P_c/P_{\text{all}} = (4 \cdot M_p)/(3 \cdot s \cdot \sigma_{\text{all}}) \]

Likewise, for a simply supported beam of the same dimensions,

\[ P_{\text{all}} = (9 \cdot s \cdot \sigma_{\text{all}})/(2 \cdot L) \]

and, from part g of Fig. 2,

\[ P_c = 9 M_p / 2 L \]

Therefore, the factor of safety in the case of the simply-supported beam becomes

\[ F.S. = P_c / P_{\text{all}} = M_p / (s \cdot \sigma_{\text{all}}) \]

But, in both cases, the ratio of \( M_p / (s \cdot \sigma_{\text{all}}) \) is the same, or essentially so, showing that the fixed-ended beam was designed
with a factor of safety of 4/3 that of the simple beam, based upon existing elastic design codes\(^{(10)}\).

Likewise, Fig. 3 shows the load-deflection curves for three beams designed for the same given working load\(^{(1)}\). For the given length, an 18WF 50 shape is required to prevent yield-point stresses. With fixed ends, still using elastic design procedures, only a 16WF36 is required. Further still, by utilizing one of the available plastic design methods, the member can be reduced to a 14WF30 shape. The plastic design saves 17% weight of the fixed-ended beam, and 40% of the weight of the simple beam, with similar deflection requirements.

The significances of the comparison are that: 1) At the working load, all beams are within the elastic range. 2) Each design limit load is based on the commencement of uncontrollable deflections, and 3) That the deflections at working load of the plastically designed beam are less than that of the simply supported beam and only slightly greater than the fixed-ended beam — at a substantial saving in weight\(^{(1)}\).

Should not plastic design methods therefore be allowed when deflections are not of prime importance, if such economical advantages are evident through this practice?

b) Safety Factors and Unpredictable Stresses. In like manner, elastic design methods are based on such exacting factors of continuity, proper fixed ends, freedom from fabrication errors and other assumed conditions, whereas such ideal conditions seldom exist. Variations in these assumed conditions, such as settlement, temperature, initial stresses, residual stresses and stress concentrations have considerable effect on elastic
stresses, causing local plastic failure at many points in elastically designed structures. These unpredictables are merely overlooked by specifying certain ductility requirements of the steel used. Too, elastic safety factors are based on percentage increases in stresses. Such linearity between loads and stress is not always the case, and safety factors based on stress alone may be far less than calculated when related to loads.

On the other hand, plastic stresses are far less likely to be affected by the unpredictable variables mentioned, and safety factors are based on the ultimate loads as calculated at collapse.

On the basis of these last two criteria alone, plastic methods of design — where applicable — are seemingly justified.

3. Essential Qualities And Characteristics Of Shapes.

As in any method of design, the Plastic Method requires that certain physical and functioning characteristics be exhibited by the members, shapes and connections employed in any structure so designed. Just as elastic methods require minimum yield stresses, limited deflections and specific ductility requirements of structural steel, so does the Plastic Method deem essential various qualities such as rotation capacity.

* The term "Load Factor" is applied in the case of plastic design. \((\text{Working Load}) \times (\text{Load Factor}) = (\text{Collapse Load})\).

** Where deflections are of primary importance, elastic methods are generally better to use. Likewise, in the case of shake-down, or where fatigue or creep failures are the limiting factor, plastic methods would hardly be applicable.5
shape factor, development of plastic hinge, adequate unit rotation, lateral stiffness, and other qualities unique to design by plastic methods.

a) Plastic Hinge. Since the method is based on the assumption that members will yield under excessive stresses, yet sustain loads, the first requirement is that the shapes used in plastic design will develop the plastic hinge. In effect, this is to say that the member must sustain moments beyond the yield-stress moment. In doing so, the plastic strains must be absorbed by the member until the condition develops where the entire cross section becomes strained beyond the yield point. Under this condition the cross section continues to deform under a constant value of moment - that moment being designated as the fully plastic moment. Such a mechanism acts as a rusty hinge, so to speak, and resists motion until a certain value of moment is applied across it, and will resist no more moment than the value of the fully plastic moment. When this condition is attained, the section undergoing this ultra-elastic deformation is said to have become a plastic hinge.

b) Rotation Capacity. To expand on this requirement, the concept of rotation capacity is presented. It is not enough to require that sections develop the plastic hinge. We must also demand of them the ability to rotate through a considerable unit angle-change after the $M_p$ has been reached. Rotational capacity implies that a member exhibit the ability to deform without marked local or lateral buckling - the ability to continue to sustain the effect of the fully plastic moment while stresses are increasing at other sections, until the
structure fails as a mechanism, with "hinges" causing ultimate collapse, rather than local or lateral buckling.

c) **Shape Factor.** As can be seen from Fig. 1, the value of the fully plastic moment ($M_p$) is greater than the value of the moment at first yield ($M_y$). The ratio of these values, $M_p/M_y$, is called the shape factor of the section under consideration. Since $M_p = Z_p \cdot \sigma_y$ and $M_y = S \cdot \sigma_y$ the shape factor for any section can be readily calculated as the ratio of $Z_p/S$. Most WF shapes have a shape factor of about 1.14 about the strong axis. More compact sections have greater shape factors, that for a rectangular section beam being roughly 1.50. Standard I-beams with heavy webs and small flanges will have greater shape factors than WF sections with thin webs.

d) **Lateral Stiffness.** In plastic design lateral elastic or plastic buckling can cause failure long before the attainment of plastic hinges. Hence, it is essential that such failures be prevented if the plastic concept is to be applied. Local buckling of webs and flanges of WF sections due to excessive compressive stresses lead to failure in lateral buckling and must be prevented by suitable choices of shapes, stiffeners or lateral bracing systems. Proportions of WF shapes should be so chosen so as to prevent or eliminate the possibility of lateral buckling. If such shapes are unavailable or lead to excessive design, lateral bracing at critical points is

* Tentative requirements of shape dimensions to insure against elastic buckling:

\[
\begin{align*}
\frac{b}{t} & \leq 30 \\
\frac{d}{w} & \leq 50 \quad \text{(This may be too strict. A value of 60 might be better.)}
\end{align*}
\]
necessary to insure against premature failures. This require-
ment not only applies to elastic buckling, but to plastic
buckling as well, in that plastic lateral buckling can reduce
the moment-resistance value of the section during rotation
in the plastic range preventing attainment of the full plastic
strength \(^{5}\).

d) Knees and Connections. Any rigid frame (plastic theory
most applicable) exhibits maximum moments at the corners or
knees. It is essential, therefore \(^{5}\), that such connections be
investigated and designed to withstand the plastic moment
across their sections before hinges develop at other critical
points. If the connection is to hold up \(^{5}\) collapse loads
are reached, the same requirements of rotation capacity and
resistance to buckling will be expected of them as of a straight
member. Knees and haunches must be adequate to develop the
full \(M_p\) of the members joined. The average unit rotation of
the connection must not exceed and the rigidity must be at
least as great as that of an equivalent length of the rolled
beam. Likewise, the rotation capacity of the knee must be suf-
ficient to absorb further rotations after reaching the plastic
hinge condition \(^{7}\). Tests at Fritz Laboratory \(^{4}\) tend to show
that straight knees exhibit greater rotation capacity than
haunched knees, but require lateral supporting systems to
prevent premature buckling. Haunched knees with stiffeners
hold up well against lateral buckling but fail to provide ade-
quate rotational capacity across the knee. Deeply-haunched
knees, however, are a carry-over from elastic design methods,
and it is unlikely they would prove economical if forced into
4. **Box Section vs. Wide Flange Shape.**

a) **Faults of the WF Shape.** The failure of straight knees in lateral buckling, while displaying excellent rotational capacity during the tests at Fritz Laboratory, has prompted attention to be brought to the possibility of using a built-up box section in plastically designed frames. Lateral instability is related to the torsional rigidity of a section, and it is not strange that WF shapes fail in this manner. However, a box section of the same general dimensions and weight as a given WF shape will exhibit torsional stiffness greater than the WF many times over. This ability of the box section to resist torsion is readily taken advantage of in many applications of design within the elastic range. Should it not then also prove advantageous in plastic methods? The answer to that question is hoped to be found as the result of a proposed test program at Fritz Laboratory under the direction of Dr. F.W. Schutz.

Among the various WF shapes tested at Fritz Lab as beams and as portal frames was the 14WF30 section. This section, while exhibiting excellent rotational and hinge characteristics, showed marked tendency toward premature yield and lateral buckling. Severe local and lateral plastic buckling was observed in the 1/3-point loading beam test and the full value of \( M_p \) never was reached.

b) **The Box Section Offers Promise.** Since lateral bracing systems to guard against such failures sometimes amount to considerable percentage increases in weight of steel (and
hence, in cost), it is the intention of the proposed program to design a box section as nearly balanced against the 14WF30 section as is practicable, and to subject that box section to similar tests as were conducted on the WF shape. Being careful to hold all factors as nearly equal as possible, such as weight, overall dimensions, area, Moment of Inertia and Plastic Modulus, the results of the tests (it is hoped) will indicate whether or not an adequately designed box section will offset the cost of a system of lateral bracing and prove to be the answer to the question: "What shape is best suited for plastic design methods?".

c) Tentative Design of a Comparative Box Section. Fig. 4a shows a section of the 14WF30 shape and lists its various properties. Part b of Fig. 4 shows the tentative box design under consideration for comparison against the performance of the WF section. Its various properties are listed below it. It will be noted that, since bending of the sections is about the major axis, the Section and Plastic Modulii, overall dimensions and area of the box were held as close as possible to those of the WF section, to eliminate as many variables as possible. By so doing, effects of size, appreciable variances in area, bending stiffness, etc. are reduced to a minimum. Note also that the relative agreement in areas of the two shapes will warrant considerable attention, should the box section exhibit adequate restraint against lateral buckling without use of a bracing system. A factor in favor of the WF shape with bracing system is, however, the additional cost of fabricating the box. It is not the major purpose of the
proposed program, however, to weigh the merits of one shape over the other as regards economy. Our aim is to establish the validity of the box section in the plastic method, or vice versa, as the test results shall warrant.

d) **Design Criteria.** One might ask: "On what basis is our design seated - what specifications do we go by in proportioning the box?" There are no accepted plastic specifications in use today in the United States, although two somewhat broad clauses are written into specifications in use in Great Britain at this time(5). Even these, however, do not apply directly to the problem at hand. However, to insure against premature elastic buckling of the flanges and webs of the proposed box section, liberty was taken to resort to tentative rules of practice as set forth by Dr. L.S. Beedle in his Doctorate Dissertation(2) and in "Interim Report 26"(7), an unpublished paper which can be obtained through Fritz Laboratory at Lehigh University. Fig. 5 shows the investigation of geometry of the section and of the knee, according to the above-mentioned rules of practice. Investigation against failure by elastic buckling of the webs and flanges indicates adequate dimensions. A check of the knee section indicates adequate web thickness, but the diagonal stiffener has been included as extra insurance against shear failure of the web plates before attaining the fully plastic moment.

5. **Tentative Test Program.**

a) **Beam Program, Gaging, Procedure.** In Fig. 6 is shown the proposed set-up for the test of the beam, using 1/3-point loading to put the center portion of the beam under pure bending, as was done with the 14"TF30 section. The data secured
from gages in this center portion will be used to compute
the actual $M-\phi$ curve, for comparison with the computed curve.
The length of the test beam has not as yet been decided, in
that certain tests are under way at the present time by Dr.
B. Thurlimann at the Fritz Laboratory to establish a limiting
$L/b$ ratio for WF sections, beyond which known lateral buckling
will occur. It is intended in the test of the box section,
to choose a length which would cause buckling failure in a
14WF30 section, and to observe the effects of this "excess"
length on the box.

Measurements will be taken by means of SR-4 strain gages
on top and bottom flanges and on either web, to obtain the
distribution of stress in these plates. Likewise, dial gages
at intervals along the center-line of the lower flange will
measure deflections, referred to the N. A. at support level,
plus gages mounted as is shown in Fig. 7 on the compression
flange to pick up any wave formation that might develop.
Rotation at load points is to be measured by means of level
bars. Should lateral deflections occur, they will be detected
by means of a transit instrument by taking readings on in-
dicators mounted at various positions along top and bottom
flange edges. Readings will be taken for each increment of load
up to collapse.

b) Portal-Knee Program, Gaging, Procedure. The test position
and a drawing of the knee with gages in position is shown in
Fig. 8. Again, the length of the legs beyond the knee is still
under consideration, and the instrumentation is subject to
change. At the present time it is planned to mount strain
gages on webs, flanges, knee and stiffener plate as shown,
in order to obtain the strain distribution of all surfaces,
special regard being given to those critical surfaces in
and about the knee. To preserve the continuity of the external
portions of the knee, lead wires from the stiffener plate
SR-4 gages will be brought out through one of the legs at or
near the load points. Wave-action in the compression flange
and the compression side of the webs will be picked up by
suitably mounted dial gages, similar to the method to be used
on the beam test. Deflection of the load points toward each
other will also be measured by a dial gage, while twist of
the members, and lateral sway, if any, will again be read by
means of a transit and conveniently located scales or in-
dicators. Loads will be applied in increments and will be
carried through until collapse occurs.
1. Plasticity, Its Promises and Its Demands.

It is apparent that the shortcomings of elastic analysis methods make it essential that the behavior of structural steel within the plastic range be investigated in order that more economical designs can be attained. Research thus far shows promising results as regards the predictability of behavior of members beyond the yield-stress loads, and warrants further study in this field. At the same time, however, it is apparent that shapes designed for use within the elastic ranges exhibit certain undesirable properties and characteristics which make them unsuitable in plastic behavior. Further research regarding section geometry is necessary in order to establish a code upon which designer and fabricator may meet in agreement, in the event of the general acceptance of the plastic method.

2. Conclusions from Completed Tests.

The tests thus far completed at Fritz Laboratory have revealed certain of these detrimental traits common to most structural shapes in use today, as well as having established other features desirable - and present - in those same shapes. Certain of those faults, however, cannot be set aside lightly, and the major difficulty encountered throughout the test program was that of premature lateral buckling, propagated by various causes but of which the lack of torsional rigidity was of major importance.

3. Fields to be Investigated.

Continued investigation of the effect of shape of members
upon plastic performance will be effected in the near future at Fritz Laboratory, special emphasis being laid upon the performance of a box section, for which the test program has been outlined. The excellent characteristic of rigidity in torsion of such a section is well known, and it is the purpose of the oncoming program to determine the extent to which this characteristic will carry over into stress ranges beyond the elastic limits. While this isolated program is of necessity narrowed to a somewhat limited scope, many other factors involving a balance between two such members of different shape must also, in the long run, be evaluated and weighed. Among these being that of cost of fabrication, ease of erection, size effect, connection details, etc. It is readily agreed that no one factor on the basis of one test program is to be the decisive argument pro or con as regards the likelihood of box sections in plastic design, and that further extensive investigation of the matter is imperative.

4. Resume.

This paper has presented a brief argument in favor of plastic design methods as regards steel structures, and has endeavored to show that such methods deserve proper consideration by structural engineers, due to the sometimes illogical limits demanded by elastic range specifications. Mention has been made of a limited number of instances wherein plastic theory results in considerable savings of materials with equal factors of safety and comparable deflection limitations as are obtained through elastic design procedures, obviously confronting the specification writers with arguments that cannot be overlooked, as regards the reserve strength of steel
beyond the yield point.

Various qualities or traits of structural shapes that are essential to them if they are to be used in plastic design are presented and discussed. Among these the ability to resist premature lateral buckling was emphasized and has led to a proposed research program involving a built-up box section.

Comparisons have been made between a 14WF30 shape - on which results of a test program are available - and a tentative design of a comparable fourteen inch box section, as regards weight, areas, Section and Plastic Modulii, etc. The proposed test program, including gage layout and measuring systems have been presented, along with a discussion of the factors to be compared.
ACKNOWLEDGEMENTS

The author wishes to express his gratitude to Dr. W.F. Schutz for the assistance and patient recommendations he has offered regarding the proposed test program, and to Dr. L.S. Beedle for his helpful suggestions concerning the selection and organization of material. Acknowledgement is herein expressed for all information, drawings and data from all sources which have been used in the preparation of the paper.
REFERENCES

1. Beedle, L. S.  "Recent Tests of Rigid Frames"
   -Ph.D. Dissertation, Lehigh Univ., 1952
3. Beedle, Topractsoglou & Johnston  "Progress Report No. 4, Part 2"
   -Welding Research Supplement, Aug., 1952
4. Beedle, Topractsoglou & Johnston  "Progress Report No. 4, Part 3"
   -Welding Research Supplement, Nov., 1952
5. Beedle, Yang & Johnston  "Progress Report No. 8"
   -Welding Research Supplement, May, 1953
6. Greenberg & Prager  "Limit Design of Beams and Frames"
   -ASCE Proceedings 77, #59, Feb., 1951
7. Interim Report #26  "Rules of Practice in Plastic Design"
   -Fritz Laboratory, Lehigh University, (Unpubl.)
8. Johnston, B.G.  "Torsional Properties of Steel Beams"
   -Ph.D. Dissertation, Lehigh Univ., 1934
   -Engineer, Sept 5, 12 - 1952
10. Symonds & Neal  "Recent Progress in Plastic Methods of Stress Analysis"
11. Yang, C. H.  "Plastic Behavior of Continuous Beams"
    -Ph.D. Dissertation, Lehigh Univ., 1951
12. Yang, Beedle & Johnston  "Progress Report No. 5"
    -Welding Research Supplement, April, 1952
A ......... cross-sectional area
b .......... flange width
b_s .......... stiffener width
c .......... half-depth
d .......... depth of section
E .......... Modulus of Elasticity
f .......... shape factor, Z_p / S
I .......... Moment of Inertia of cross section
L .......... span length
M .......... moment
M_p .......... full plastic hinge moment, Z_p * σ_y
M_y .......... moment at initial yield, S * σ_y
P .......... concentrated load, or total load on span
P_all .......... allowable load corresponding to σ_all
P_c .......... collapse load
S .......... section modulus, I / c
t .......... flange thickness
w .......... web thickness
w_a .......... web thickness available across knee
w_r .......... web thickness required across knee
Z_p .......... Plastic Modulus of full cross section
σ .......... unit normal stress in bending
σ_all .......... allowable unit normal stress in bending
σ_y .......... lower yield point stress
φ .......... curvature of bent member
δ .......... deflection
FIGURE 3

Load

Deflection

II III I

P_W

\delta_L

16WF50

16WF36

14WF30
\[ I_x = 289.6 \quad I_y = 17.5 \]
\[ S_x = 41.8 \quad S_y = 5.2 \]
\[ r_x = 5.73 \quad r_y = 1.41 \]
\[ Z_{P_x} = 48.35 \quad Z_{P_y} = 4.59 \]
\[ A = 8.81 \]

\[ I_x = 275.7 \quad I_y = 56.4 \]
\[ S_x = 39.7 \quad S_y = 18.0 \]
\[ r_x = 5.12 \quad r_y = 2.31 \]
\[ Z_{P_x} = 48.35 \quad Z_{P_y} = 24.3 \]
\[ A = 10.52 \]

**Figure 4**
INVESTIGATION DATA

\[
\begin{align*}
\frac{b}{t} & \leq 30 \quad \frac{6.25}{0.3125} = 20 \\
\frac{d}{w} & \leq 60 \quad \frac{13.235}{0.25} = 53 \\
\text{Knee: } w_r &= \frac{23}{d^2} \quad \frac{2(39.75)}{(13.86)^2} = 0.414 \\
\text{wa} &= 2t \quad 2(0.25) = 0.500 \\
w_r - w_a &= -0.086
\end{align*}
\]

Thickness across knee is sufficient, but stiffener plate will be added for extra safety against shear failure.

\[
t_s = \frac{(w_r - w_a)d}{b_s 2^{\frac{1}{2}}} = 0 \quad \text{(required stiffener thickness)}
\]

\[
\frac{d}{b} \leq 3 \quad \frac{(13.86)}{(6.25)} = 2.22
\]

\[
\frac{L}{b} = ? \quad \text{(Contingent on results of tests under way by Dr. B. Thurlimann at Fritz Lab.)}
\]

FIGURE 5

Test set-up for beam loading

FIGURE 6

Moment Diagram