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Report on

TESTS OF A PRETENSIONED BOX SECTION BEAM

FOR A 60° SKEW BRIDGE

for

Concrete Products of America
(Division of American-Marietta Company)
Pottstown, Pennsylvania
S. L. Selvaggio, Chief Engineer

Fritz Engineering Laboratory
by

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Test Conducted by

Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pennsylvania

March, 1957
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INTRODUCTION

The release of prestress in a pretensioned prestressed beam causes a complicated local stress condition in the end block due to the fact that the prestressing force is applied to the concrete over a bonded length of strand at the end of the beam. Over this so called anchorage length there is a build-up of precompression in the concrete and bending moment on the cross section. The rate of increase of bending moment is quite high when the anchorage length is relatively short and therefore must be accompanied by high shearing stresses. In the case where the end of the beam is not perpendicular to the longitudinal axis of the beam as is the case in beams for skewed bridges, the prestress is applied in the end block eccentrically in both the horizontal and vertical planes. The shearing stresses within the anchorage length are higher than those in a squared-end beam. Tension stresses resulting from the shearing force often causes cracks to form at the ends of the beam when prestress is released. Additional stirrup reinforcement is required in the end blocks of skewed beams to carry the high shear stresses. Herein is reported the results of testing a pretensioned box section beam for a 60° skew bridge; both ends of the beam had an acute angle of 30° and abutments were placed parallel with the ends of the
beam. The beam was produced and the test sponsored by Concrete Products Company of America (A Division of American Marietta Company).

OBJECTIVES OF THE TESTS

In the end portion of the skewed box section beam there are two critical areas. The first of these is at the very end of the skewed face where additional stirrups are necessary to control cracks caused by principal tension stresses in the concrete when prestress is first released.
A theoretical analysis of the stresses in this portion of the beam is given in Appendix A. The second critical area is at the face of the abutment at the obtuse angle of the end block where the cross section changes from a solid end block to a box section. When the beam is loaded between midspan and the support, the larger portion of the vertical shear will be carried by the web on this side of the beam. Also at this section there may be additional shear stresses resulting from torsion in the beam, but stresses due to torsion will be higher in the test beam than in an actual bridge member.

The objectives of this test were to observe the behavior of the end portion of the skewed beam and to determine the following:
1. The number of stirrups required to take the tensile forces at the very end of the beam.

2. The adequacy of the web reinforcement at the face of the abutment.

3. The distribution of the bearing pressure on the support qualitatively.

4. The effectiveness of two layers of graphite-impregnated asbestos pads 1/4 in. in thickness for providing movement at the expansion end of the beam.

DESCRIPTION OF A BEAM

The test beam consists of a standard box cross section (Figure 1) presently being used for multibeam bridges of spans up to about 68 feet center to center of supports. The ends of the beam were skewed at an angle of 30° with the longitudinal axis of the beam. The span of the test beam had to be limited to 25' c. to c. of supports in order to test the beam conveniently. The web reinforcement normally used in this type of section consists of "U"-shaped stirrups, one of each pair inserted from the top and one from the bottom, spaced 4" c. to c. on one end and 8" c. to c. on the opposite end.
PLAN OF TEST BEAM

(Traced from Drawing of Concrete Products Company)
The stirrups in the end blocks were designed in this manner (1) to provide sufficient anchorage length on the bottom (2) to provide better torsional reinforcement and (3) to control the width of cracks in the end-block resulting from the eccentric prestress.

LOADING

The loads to be applied to the test beam were determined by assuming an actual bridge span of 68' c. to c. of supports, which is maximum span for this section, and computing the maximum shear under dead load plus live load plus impact for a 68' span (See Appendix B, Part I). All loads mentioned hereafter unless otherwise specified refer to design load for a beam of mean span of 68'-0". Since the stresses in the end-block are affected by shear and torsion, the application of the loading of a 68' beam to a beam of 25' span produces stresses which are more severe than conditions in an actual bridge member. It could be argued that the stresses due to torsion in a 68' test beam would be larger, but this would be far more severe than the case of a member in the bridge which is restrained from twisting by adjacent members. Although the longer beam would have a greater deflection under design load, the amount of rotation of the two beams at the
support will not be much different because both beams deflect at midspan an amount about equal to the amount of camber in the beams under the loads involved. It must be pointed out that in an actual bridge all forces are carried by group action of all members, and therefore the testing of a single member is a severe test regardless of the length of the test beam.

The number of repetitions of design load during the life of a bridge member has been estimated to be about 1 million cycles. It was therefore decided to apply at least 1 million cycles of loading which would vary between D.L. shear and full L.L. + D.L. + I shear on a 68' beam by a single concentrated load at midspan. The question of whether or not a heavy concentrated load on a bridge member near the face of the abutment where the stresses resulting from anchorage of strands exists has any detrimental effect on a beam was also investigated. In this portion of the testing, the load was applied at a point 6'-0" from the support, and its magnitude was 110 kips which is about 6% less than 2(D.L. + L.L. + I) in shear for a 68'-0" span.

SET-UP OF TEST BEAM

The beam was set on two abutments which were placed
FIG. 2 Plan View Showing Jack Positioned for Loading Near Support

FIG. 3 Jack Positioned for Loading at Midspan
FIG. 4  Detail View of One Abutment

FIG. 5  Position of Loading Beam for Loading at Midspan
on an angle of 30° with the longitudinal axis of the beam. Figure 2 shows a view of the test beam in position for loading near the support, and Figure 3 shows a side view of the beam with the Amsler equipment arranged for applying loading at midspan. The abutments consisted of pedestals with a bearing surface 1'-0" in width on which the beam was seated with grout; Figure 4 shows a close-up view of one of the abutments. Grouting was used because it was noticed upon placing the beams on the level abutments that the bottom surface of the beam was deformed due to the prestress so that the effective bearing area on the ends of the beam was less than 5% of the bearing area provided by the abutment.

After half a million cycles of loading at midspan, 2 layers of asbestos pad were placed between the grout and the steel support at one end of the beam (end having stirrups spaced at 8" c. to c.). The asbestos pads were impregnated on one side with graphite, and the pads were placed with the graphite surfaces together to provide for expansion at that end of the beam.

A loading beam was used to distribute the jack load uniformly across the section of the beam in a direction perpendicular to the longitudinal axis of the beam as shown in Figure 5.
Figure 6 - Cracks in end-block at beginning of testing
TEST PROCEDURE

The beam was examined upon being received at Fritz Laboratory and all existing cracks located. There were several small shrinkage cracks at various points on the beam, but the only cracks of importance were the ones on the end of the beam having stirrups spaced at 3" c. to c. (See Figure 6). The widths of these cracks were measured at several points along their length with a microscope, and Huggenberger Gage points were set opposite these points. A record of the change in width of these cracks at the gage points was kept throughout the testing. Figure 7 shows

FIG. 7  Huggenberger Gage Being Used

To Measure Change in Width of Crack
the Huggenberger Gage being read to determine change in crack width. Throughout the entire sequence of testing the total change in the crack widths was negligible as indicated by the graph of Figure 8 which shows the change in width of the more prominent of the two cracks at one of the gage points during the testing.

The test program is outlined on the following page. The load point for Tests 6 through 9 was near the end of the beam in which stirrups were spaced 8" c. to c.; this end of the beam also was supported on asbestos pads while the opposite end was not. The loads given in the table are jack loads rather than effective static loads for the dynamic testing. However, the difference between the effective load and jack load is only about 1%. The speed of dynamic testing was 250 cycles of loading per minute.

From the application of a very large concentrated load repeatedly at the same point, horizontal cracks formed in the shear keys under the loading beams. With the load applied at midspan, the cracks opened at about 4000,000 cycles of Test No. 2. In the case of the load applied near the support, the cracks were noticeable only at a load of 110 kips on the jack. The cause of these cracks is apparent if one considers a short segment of the beam under the loading
Figure 3 - Maximum width of crack at gage point "A" (See Figure 6)

Figure 8 - Maximum width of crack at gage point "A" (See Fig. 6)
## TEST PROGRAM

| Test | Load Point         | Description of Test                                      | Load Limits in Terms of % of Design Shear for a Span of 68' - 0"
|------|-------------------|----------------------------------------------------------|----------------------------------------------------------
| 1    | Midspan           | Static load increments of 10 kips to 80 kips.            | 97.6% (D.L. + L.L. + I) max.                             |
| 2    | Midspan           | Repetitive loading of 670,000 cycles of 15 kips to 75 kips. | 88.4% (D.L.) min.                                        |
|      |                   |                                                          | 91.5% (D.L. + L.L. + I) max.                             |
| 3    | Midspan           | Static load increments of 10 kips to 90 kips.            | 110% (D.L. + L.L. + I) max.                             |
| 4    | Midspan           | Repetitive loading of 1.1 million cycles of 15 kips to 82 kips. | 88.4% (D.L.) min.                                        |
|      |                   |                                                          | 100% (D.L. + L.L. + I) max.                             |
| 5    | Midspan           | Static load increments of 10 kips to 90 kips.            | 110% (D.L. + L.L. + I) max.                             |
| 6    | 6' from support   | Static load increments of 10 kips to 60 kips.            | 111% (D.L. + L.L. + I) max.                             |
| 7    | 6' from support   | Repetitive loading of 1.1 million cycles of 10 kips to 54 kips. | 60% (D.L.) min.                                        |
|      |                   |                                                          | 100% (D.L. + L.L. + I) max.                             |
| 8    | 6' from support   | Static load increments of 10 kips to 60 kips.            | 111% (D.L.) max.                                        |
| 9    | 6' from support   | Static load increments of 10 kips to 110 kips.           | 194% (D.L. + L.L. + I) max.                             |
beam as a rigid frame box carrying a heavy distributed load on the top member of the frame. The bending moments in the vertical member of the frame would be in such a direction as to cause tension on the outside of the box, and in the case of the beam result in horizontal cracks on the minimum section, i.e. the shear keys. These cracks formed only at loads much higher than would ever occur as a single concentrated load on a bridge and so should not be considered as being serious.

During Test No. 4 after about 390,000 cycles of loading, a crack formed at the end of the beam having stirrups spaced at 4". This crack was located in a position similar with respect to the end of the beam to the one on the opposite end of the beam which had formed at release of prestress. However, the 4" stirrup spacing limited the length of this crack to less than half the length of the one on the other end of the beam and the width of it never exceeded 0.004", the maximum width of the corresponding crack in the opposite end of the beam was 0.015" at the beginning of the testing. Neither of the cracks caused by eccentric prestress exhibited any tendency to increase in length or width during testing.

From observing the length of both of these cracks, each appeared to have crossed 2 stirrups being stopped short
of the third stirrup from the end of the beam. With reference
to Figure 1, it should be pointed out that at the level at
which cracks formed each stirrup consisted of 4 bars. This
should be kept in mind if any other arrangement of stirrups
is used in the end-blocks of skewed beams. It should also
be noted that with each stirrup consisting of 2 "U" shaped
bars placed from both top and bottom, there is positive
anchorage from both above and below the crack making each
stirrup more effective than if only one "U" shaped stirrup
were present at any section of the beam.

DEFLECTIONS

The means used to keep a record of the deflections
of the beam are shown in Figures 9 and 10. The Ames Dial was
used for static deflections while the spring steel cantilever
beam having electrical resistance strain gages mounted on it
was read by an oscilloscope (Figure 10) to detect any changes
in dynamic deflections. Oscilloscope readings were taken
every 100,000 cycles of loading by photographing the cathode
ray tube. No measurable change in dynamic deflections were
observed during any stage of the testing.

The load-deflection curves for the 3 static tests
with the load at midspan are shown in Figure 11. The
FIG. 9 Cantilever Beam and Ames Dial for Measuring Dynamic and Static Deflections

FIG. 10 Oscilloscope for Indicating Dynamic Deflections
Figure 11 - Load-deflection curve for loading at midspan

Deflection of beam at midspan in inches
Figure 12 - Load-deflection curves for loading near abutment

Deflection of beam at midspan in inches
The non-linearity of the curves for the second and third static tests is the result of the change in thickness of the asbestos pads under the end of the beam; a correction to the deflection readings was applied for an average change in thickness of the pads, but this correction would only be approximate.

The dashed line in Figure 12 is the calculated deflection curve for loads at a distance of 6'-0" from the support calculated by using the stiffness properties of the beam as derived from the static tests with the load at mid-span considering only bending and neglecting any possible deflection due to torsion. Comparison of the dashed curve with the measured load-deflection curves indicates that deflection due to torsion is negligible. The high resistance of a box section to torsional moments would lead one to expect this result.

It was observed that with loads on the beam at midspan both ends of the beam appeared to have higher bearing pressures near the obtuse angle of the skewed end block. This is to be expected since the distance from load to support along the edge of the beam leading to the obtuse angle of the skew is shorter than on the opposite edge of the beam, and this causes the shorter side to carry the large portion of the shear resulting in higher bearing pressure near the
obtuse angle of the skewed end. With the load applied near
the support, the bearing pressure at the support nearest
to the jack was distributed in the same manner as described
above, but at the opposite end of the beam it appeared that
the higher bearing pressure was near the acute angle of the
end block because the other edge of the beam was actually
off of the abutment during a portion of dynamic loading
cycles. Since a skewed slab or girder under load tends to
transmit load to its supports in proportion to the stiffness
of the various paths to the support and therefore should
always produce higher bearing pressures at the obtuse angle
of the end block, the observed behavior at the far support
indicates that the local deformation of the end of the beam
due to prestress must have a greater influence than the
twisting of the beam under load. On a longer span, however,
the reverse of this would probably occur. In any case it
seems quite certain that the bearing pressure will always be
higher near the obtuse angle of the end block except when
the load is near the far abutment and the average bearing
pressure is low.

EFFECTIVENESS OF ASBESTOS PADS

Incorporated into the test program was the evaluation
of asbestos pads for seating bridge beams on abutments.
Asbestos pads because of the inertness and good wearing resistance of the material have been suggested by Concrete Products Company of America as suitable material for seating bridge beams on abutments. Two pads of 1/4" thickness with the graphite impregnated surfaces facing each other were placed under one end of the beam. During the dynamic testing of the beam there was a very noticeable longitudinal movement between the two graphite surfaces. There was no noticeable transverse movement between the pads. The pads were in use through a total of 2.2 million cycles of design load reaction without showing any signs of wear.

During static tests of the beam, the change in thickness of the asbestos pads was measured near the edges.
of the beam. Figure 13 shows an Ames Dial in position for measuring the change in thickness of the pads; the edge of the pads is visible as a dark layer between the white bearing plate and the beam. Figure 14 shows a curve of reaction at support vs. change in thickness of pad for Test No. 3, and Figure 15 shows the same for Test No. 9. Figure 16 shows the change in thickness plotted against bearing pressure for a piece of the same material 9" x 12" tested in compression. Using the linear portion of the curve in Figure 16, as a calibration curve, it is possible to roughly estimate the bearing pressure along the face of the abutment caused by the reaction at the support from the jack loads. In Test No. 3 where the maximum jack load was 90 kips or \( 1.1(D.L. + L.L. + I) \), the bearing pressure along the face of the abutment (using Figure 14) varies from about 710 psi. at the obtuse angle to about 185 psi. at the acute angle. The use of grout and asbestos pads placed flush with the face of the abutment caused this bearing pressure to be much higher than under a beam in an actual bridge where the asbestos pad would not be placed flush with the face of the abutment and grout would not be used. Also the placing of a heavy load on a short beam such as the test beam, which has only a very small amount of camber, results in beam action with the clear span as the distance between points of reaction. On a beam
Figure 14 - Change in thickness of asbestos pad measured at face of abutment (Test No. 3)
Figure 15 - Change in thickness of Asbestos Pad measured at face of abutment (Test No. 9)

Change in thickness of pad in inches

Reaction at support in kips

Near acute angle of end-block

Average

Near obtuse angle of end-block
Figure 16 - Change in thickness vs. bearing pressure for asbestos pad

Slope = \frac{0.01''}{178 \text{ psi}}

Change in thickness of asbestos pad in inches
of 68' span which would carry these loads in practice, there is a greater slope of the beam at the ends due to camber and this results in the distance between points of reactions being greater than the clear span. This results in a lower edge bearing pressure at the face of the abutment. In a bridge the group action of several beams forces the bearing pressure along the face of the abutment to be more uniform. Thus the magnitudes of bearing pressure observed on the test beam can certainly be considered as a limiting case which could hardly be encountered in practice.

Although the maximum bearing pressure observed under the test beam only slightly exceeds the AASHO allowable bearing pressure of 700 psi, the need for some means of seating the beams on the abutment to make the bearing pressure as uniform as possible is desirable. From the performance of the asbestos pads on the test beam it is felt that these pads are satisfactory for seating beams and providing for expansion.

**SLIP OF STRANDS**

Ames dials for the purpose of measuring slip of the prestressing strands with respect to the concrete were mounted on a few strands. No slip of strands was recorded
CONCLUSIONS

1. The stirrups used in the test beam consisting of two "U" shaped stirrups inserted from top and bottom were sufficient reinforcement for the end-block of the test beam if spaced 8" c. to c.

2. The same type of stirrups spaced 4" c. to c. in the extreme end of the beam would virtually eliminate the cracks which are produced by release of prestress.

3. Cracks which form at the point of the skew upon release of prestress are not detrimental if stirrup reinforcement is equivalent to the minimum provided in the test beam.

4. Deformation of the end-block due to prestress makes seating of the beam using grout and/or some kind of compressible pad imperative to reduce high local bearing pressures.

5. Even with the use of both grout and compressible asbestos pads, the bearing pressure at the face of the abutment near the obtuse angle of the end-block exceeded 700 psi (See page 23), but in an actual bridge the transfer of load to adjacent beams by the shear keys would tend to make the bearing pressure more uniform.
6. The asbestos pads used with the test beam provided sufficient longitudinal movement of the beam, and after 2.2 million cycles of full design load shear, there was no evidence of wear in the material.

7. The application of a high concentrated load near the face of the abutment did not cause slip of strand or any other evidence of failure.
ACKNOWLEDGMENTS

This project was sponsored by the Concrete Products of America, a Division of American-Marietta Company. The test beam was designed and manufactured by Concrete Products of America at their Pottstown, Pennsylvania Plant under the direct supervision of Mr. Samuel L. Selvaggio, Chief Engineer. The test was carried out through the Institute of Research at Lehigh University by the staff of the Fritz Engineering Laboratory under the administrative direction of Professor William J. Eney.

The assistance of Mr. K. R. Harpel, Laboratory Foreman and his staff in preparing and assisting with the conducting of the tests, and the work of Mr. I. J. Taylor and his technicians in providing the instrumentation is gratefully acknowledged.

Members of the Fritz Laboratory Staff who assisted in the testing were Fuad Nuwaysir, George Nasr, and Cengiz Gokkent.
APPENDIX A

The prestressing force on any section near the end of a skewed beam is eccentric on a horizontal plane as well as on a vertical plane as can be seen by looking at Section A-A of Figure 1. This condition produces warping of the end block of the beam as shown in Sketch 1, resulting in an uneven distribution of bearing pressure when the beam is placed on level abutments.

![Sketch 1](image)

Across the Section A-A (See Figure 1), the stress in the steel varies from a maximum along the side of the beam to zero along the end face of the beam resulting in the center of gravity of the steel force in the horizontal direction being outside the kern of the cross section. This results in direct tension stress along the end face of the beam in the
same way that the eccentricity of prestress in the vertical direction produces tension in the top fibers of the beam. If now the shear stresses which occur as a result of the build-up of precompression over the anchorage length of the strands is considered, it will be found that a principal tension stress of considerable magnitude exists at points along the end face of the beam above the level of the steel. These principal tension stresses cause the cracks which were observed in the test beam. It is also possible that shear stresses which result from transverse variation of prestress may produce cracks in a longitudinal direction on the bottom surface of the beam making it desirable to have stirrups across the bottom of the beam near the end. The length of cracks will vary with the size and spacing of stirrups in the end of the beam.

A qualitative picture of the shear stresses in the end-block can be obtained by a method of analysis* which for beams other than sharp angled skewed beams also yields a fairly accurate quantitative values. Considering a section of the beam within the anchorage length of the bonded strands as in Sketch 2, the free body diagram of Sketch 3 is drawn.

NOTATION

F = effective prestressing force in the tendons
ΔF = increment of change in prestressing force
C = compressive stress
ΔC = increment of change in compressive stress
s = shear stress
Δl = increment of length along the beam
Q = statical moment of the area of the cross section above the level on which shear stress is computed
I = moment of inertia of cross section
A = gross area of concrete
a = area of portion of cross section above level on which shear stress is computed.

The increment of change in compressive stress at a distance "y" from the center of gravity of the concrete area is

\[ \Delta C = \frac{\Delta F}{A} - \frac{\Delta Fe}{I} \]

where compression is positive and tension negative. The equilibrium condition on any horizontal plane a distance "y" from the center of gravity of the concrete area is

\[ sb \Delta l = \text{total shear force} \]

\[ = \int_{y}^{\text{ht}} \Delta C dA = \frac{\Delta F}{A} \int_{y}^{\text{ht}} dA - \frac{\Delta Fe}{I} \int_{y}^{\text{ht}} ydA \]

\[ = \Delta F \left[ \frac{d}{A} - \frac{eQ}{I} \right] \]

or \[ S = \frac{\Delta F}{b \Delta l} \left[ \frac{a}{A} - \frac{eQ}{I} \right] \] (1)

Sketch 4 shows the distribution of the shear stresses over the depth of the member.

The maximum stress occurs just above the prestressing tendons, and as seen in Sketch 4, the direction of this maximum shear stress is the same as the direction of shear stress which results from load on the beam.
Returning now to the case of a skewed beam, the shear stress on a vertical plane caused by anchorage of the strands will be computed. Shear stress due to loads on the member and transverse variations of prestress will be neglected, but it can be seen that if these shear stresses were also considered the effect would be a higher principal tension stress along the end face of the beam.

Equation 1 is now applied to a section of the skewed end-block which is 8" in width to determine the order of magnitude of shear stresses resulting from anchorage of strands. The critical section is taken to be 8" from the bottom of the beam, this being the level of the crack in the test beam. The anchorage length of the strands was assumed to be about 15" -- a value which has been obtained in other tests with similar members.
\[ \triangle L = 1 \text{ in.} \]
\[ \Delta T = 14 \times (15) \frac{1}{15'} = 14 \text{ k/in.} \]
\[ b = 8 \text{ in.} \]
\[ A = 8(33) = 264 \text{ in}^2 \]
\[ a = 8(25) = 200 \text{ in}^2 \]
\[ e = 12.25 \text{ in.} \]
\[ I = \frac{1}{12} (8)(33)^3 = 24,000 \text{ in}^4 \]
\[ Q = 8(8)(12.5) = 800 \text{ in}^3 \]
\[ S = \frac{\Delta T}{b \Delta l} \left( \frac{a}{A} - \frac{Q e}{I} \right) \quad (1) \]
\[ = \left[ \frac{14,000}{8(1)} \frac{200}{264} - \frac{800(12.25)}{24,000} \right] = 613 \text{ psi} \]

The distribution of stresses on the section 16" from the end of the beam is shown in Sketch 5. It will be seen that point "x" is in a region of direct tension stress and a shear stress of at least 600 psi. The principal tension stress at this point will therefore be greater than 600 psi because of the shear stresses which have been neglected in the analysis. The principal tension stress will exceed the strength of the concrete in tension for points corresponding to point "x" for cross sections along most of the end-block if stirrups are not used.
APPENDIX B
Part I

DETERMINATION OF LOAD TO PRODUCE DESIGN SHEAR STRESS ON BEAM HAVING A SPAN OF 68' - 0".

End shear per lane (AASHO, 68' span) = 62.1 k

Maximum shear per beam (0.6)(0.5)(62.1) = 18.65 k

Impact = \(\frac{50}{68+125}\) = 25.9%

L.L. + I = 18.65(1.259) = 23.45 k

Correction for D.L. = (68-25)580 = 25 Kips

Correction for shear = 12.5 Kips

Wearing surface = 90(68) = 6.2 Kips

Shear = 3.1 Kips

Total shear
- L.L. + I = 23.45 k
- D.L. correction = 12.50 k
- Wearing surface = 3.10 k
- 1 solid bulkhead = 1.90 k

40.95 Kips

Effective load at midspan 2(41) = 82.0 Kips

Effective load at 6'-0" from support = \(\frac{25}{19}(41)\) = 54.0 Kips
APPENDIX B
Part II

LOAD REQUIRED 6'-'0" FROM SUPPORT TO PRODUCE SHEAR OF 2 (D.L.+L.L.+I) BETWEEN JACK AND NEARER SUPPORT

\[ 2(D.L.+L.L.+I) = 2(54) + \frac{25}{19} (D.L. \text{ of test beam}) \]
\[ = 2(54) + \frac{25}{19} (7.25) = 117.5 \text{ Kips} \]

Capacity of jack = 110 Kips

\[ (117.5 - 110) = 7.5 \text{ Kips} \]
\[ \frac{7.5 \times 100}{117} = 6.41\% \]

Maximum shear on test beam is about 6% lower than 2(D.L.+L.L.+I) shear for 68'-'0" beam.