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When a buyer of bridge or building steel receives from the mill a report indicating that his lot of material has a yield point greater than the specified minimum of 33 kips per sq. inch may conclude that the structure he recently had designed to a maximum stress of 20 kips per sq. inch will have a "factor of safety" of 1.65. The real factor of safety against large permanent distortion or collapse may be lower than this, but more frequently it will be much greater. In the extreme limits between dangerous weakness and gross overdesign it will probably range between 1.00 and 10.0, these limits being narrowed in proportion to the quality of the design, correctness of knowledge as to material properties, and accuracy with which the maximum applied loads have been estimated. In some applications questions of fatigue and corrosion will also play an important role.

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THE ACCEPTABILITY OF AN ENGINEERING STRUCTURE

To the ultimate user of a bridge or building the load-carrying structural framework will be acceptable if it is:

1. Strong enough to carry without damaging yield, fracture, or buckling all expected or accidental loads that may be applied throughout its lifetime.

2. Elastically rigid enough not to vibrate, or deflect to an extent that would reduce usefulness or cause discomfort to occupants.

3. Enduring enough to withstand any corrosive action and not to fail by fatigue.

4. Economical in choice and use of material.

The concern herein will be primarily related to stress-strain properties of the material in relation to the static strength of the structure.

The engineer of tomorrow will design with greater regard to
the overall usable strength of the complete structure, whereas today engineering specifications are primarily concerned with the separate strength of each structural component, i.e., the beam, the column, the tension member, and the connection. Present day design procedures lead to safe and enduring overall structures, hence changes from present methods may be shown incoming. Nevertheless, greater attention to the interaction between the components of a structure will in the future lead to greater economy in the use of material as well as in dollars and cents saved. Such a trend has long been felt in aeronautical engineering, wherein progress has been forced not so much by economy by the need to achieve the absolute minimum of weight.

In realizing more and more of the usable strength in a given structure, the designer must correct realistic strength analysis of the whole structure. Buckling problems must be given more exact attention, in both the elastic and inelastic range of material behavior.
The ultimate load capacity of a structure will be evaluated, utilizing the inelastic range of the stress-strain curve, and this load capacity will be divided by an overall factor of safety to determine the usable working load capacity. There may be an increase in the level of stress at the working loads when and if the ultimate load capacity justifies such an increase. The importance of "stress" calculations will decrease as the greater importance of "strength" calculations is fully realized.

At the same time it must be kept in mind that a structure may have much more strength than can be utilized if the deflections cause unsightly or uncomfortable distortion, lead to failure or malfunctioning of non-load carrying parts or machinery, or if the accompanying flexibility results in psychologically unbearable vibration.

Another counter to the trend of increasing ***permitted loads to take full advantage of usable static strength will be **
the necessity for careful attention to the possibility of fatigue failure. Any conclusions that may be reached herein will be applicable only in cases wherein fatigue failure is not a problem.

**STRUCTURAL FAILURE IN RELATION TO THE SCALE OF THE OBSERVATION**

In considering the relationship between local inelastic behavior of the material and the overall behavior of the structure, our conclusions will be markedly affected by the scale of the observation.

Suppose we are interested in the overall behavior of the Empire State Building but get our opinion as to the yielding of the material from a microscopic observer. If he were located near a point of high stress-concentration a slight overload on some part of the structure might easily create local plastic flow adjacent to our observer that would seem to be catastrophic. To be straddling an earthquake fault during its formation would be mild in comparison. But if our observer grew to a height of ten inches this localized yielding would
have been scarcely noticeable. Most of the beams and columns would have undergone greater yield while being straightened in the mill, fabricated in the shop, and forced into position during erection. It is during these three operations that the ductility of the metal beyond the yield point is called upon to the greatest degree. However, having permitted yielding in the mills, shops and field, there is no valid reason to prohibit it thereafter, provided such yielding has a negligible effect upon the usability of the structure. Permitting such yield makes it possible to avoid in design a complete analysis of local stress concentrations, which would be impractical if not impossible.

Suppose it is agreed to neglect the yield at local stress concentrations and our ten inch tall observer now concentrates his attention at the top of one of the steel beams where the calculated stress first reaches the yield point. Rather general yielding may now develop to a degree quite noticeable to our ten inch observer - yet the beam will carry much more load than
when it initially yielded. If the beam is part of a continuous frame, complete yielding at one location will not usually coincide with failure of the structure. As the load increases a general rearrangement of moments may result, yield may progress at the initial location and at still greater loads develop at new locations without serious increase in overall deflections.

In the field of bridge and building structures - particularly the latter - there is and will continue to be an important trend to utilize material to realize the best distribution to provide the optimum combination of economy and usable strength. The calculation of usable strength will more and more be based on analyses which at least locally take advantage of reserve strength accruing from the behavior beyond the elastic range.

In the earliest days of the art the structural engineer intuitively designed structures that, on the basis of repeated experiences, had requisite strength. As the engineer learned analytical methods of elastic stress analysis and coupled this with test information regarding strength of
materials his attention became more and more focused on local stress rather than overall strength of the structure. Now, attention is turning back to the whole structure - the exact value of stress loses some of its significance - as the engineer continued on page 8.
is leading the science of calculating the overall strength of the entire assemblage of members.

Theories for strength calculation to take account of the reserve strength beyond the yield point have been variously called "plastic", "collaps", or "limit" design. These propose to calculate the safe working load by formula (2) below, rather than by (1).

(1) Conventional Stress Design

\[
\text{Safe Load} = \text{Load at which maximum calculated stress equals yield point} \times \text{Stress factor of safety}
\]

(2) Plastic Strength Design

\[
\text{Safe Load} = \text{Maximum load that structure will carry without prohibitive deformation} \times \text{Strength Factor of Safety}
\]

Since, as has been shown, the yield point is always exceeded regardless of whether we adopt procedure (1) or (2), the choice is not one of whether or not any yielding is permitted - it is simply a matter of the degree to which it is to be allowed and the general approach that is to be made in design. More
often than not, the safe load computed by (2) will be less than the load (1) at which maximum calculated stress equals the yield point. Hence, except for local stress concentrations, the structure will usually remain elastic at working loads even though design procedure (2) is used. If this is not the case, it will be necessary to calculate the deflections in the early plastic range -- a difficult problem. The calculation of maximum plastic strength on the other hand is a simple process in those structures for which procedures are now available, i.e., simple continuous frames.

In bridges and buildings the principal load carrying stresses are largely uniaxial and the material used is primarily structural steel -- these two facts leading to simplicity in the plastic theory sufficient for the structural engineer and rendering fairly direct the relationship between the tensile and compressive stress-strain diagram of structural steel and the load deformation characteristics of steel structures.

What information does he need if he deliberately extends
his design and analysis into the plastic range and replaces
the consideration of working stress by predicting the load
carrying capacity of the structure? He must, of course, continue
to consider the overall deflections of the structure and its
component members and in designing into the plastic range
these impose the most important design criteria. The structural
ingineer will probably treat local stress concentration much
as he does at present in so-called elastic design, but it may
be necessary to give more weight to their possible effect in
influencing the overall plastic behavior of the structure.
The magnitude of the maximum stress, in plastic design, no
longer is a design criterion; its only use in elastic design
having been to insure against the plastic behavior that is now
considered permissible.

Certainly in the field of fully continuous steel building
frames designs based on calculated "collapse" or "limit" loads
appear to have a definite field of application. The many works
of J. F. Baker(1) and his associates in Great Britain and the writings of J. A. van den Broek(2), in this country, who coined the term "limit design", have done much to focus the attention of engineers on this subject.

To calculate the deflection of a steel structure as a function of load in the plastic range and thereby determine its useful capacity, the structural designer is primarily interested in the basic information needed to determine the overall behavior of a structural member under the primary or load carrying forces. For example, in a beam, the functional relationship between bending moment and change in slope per unit length of beam provides the information necessary to the calculation of the beam deflections caused by the bending moment distribution resulting from any given load distribution. Two alternatives


(2) J. A. van den Broek, "Theory of Limit Design" 1948, John Wiley and Sons, 144 pages.
may be used to obtain the needed information:

1. Bending tests of actual typical beams may be made to give directly the requisite moment-angle change relationship.

2. The moment-angle change relationship may be computed from the tensile and compression stress-strain diagrams of the material.

The first of the foregoing alternatives is hardly practicable because of the cost of making the tests. The second procedure is obviously the more desirable and mathematical procedures for making the necessary calculations have long been available.

The important question, therefore, is: "What shape of stress-strain curve shall be assumed?"

In elastic design there are accepted specification values for the elastic modulus and the yield point. What is now needed is an accepted "standard" for the stress-strain curve beyond the elastic range and the assurance that by use of this "standard" curve one may predict with sufficient accuracy the corresponding
inelastic behavior of any given structure. Even if no radical changes are made in overall design procedures, this information is needed to determine the buckling strength of compression members, the governing factor in any design procedure.

THE STRESS-STRAIN CURVE FOR STEEL

In the inelastic range there are no widely accepted standard stress-strain curves of the various steels available for use even in the case of the simplest of strength determinations, the tension test. In the elastic range, on the other hand, there are accepted values of the elastic modulus of elasticity Poisson's "E", and the approximate limit of elastic behavior, the "yield point" of steel or "yield strength" of a non-ferrous alloy. In the latter case, however, the "yield maximum strength" is determined by a specified degree of inelastic strain that places this value well beyond the elastic range.

If an accepted minimum standard stress-strain curve were available for each specified type of constructional metal it
would then be possible to predict the strength of beams, columns, frames, and other structural elements on the basis of fairly simple inelastic analysis procedures for problems wherein the principal strength-controlling stresses are mostly uniaxial; i.e., two of the three principal stresses in the region of maximum stress are at or near zero. The general case wherein two or three principal stresses are appreciably different from zero requires a more complex explanation of the limit of elastic behavior and the stress-strain development beyond this limit. Fortunately, the behavior of most structural members is primarily governed by uniaxial stress fields, except in regions of high shear stresses in which case there are two principal stresses of opposite sign and comparable magnitude.

In establishing a basic stress-strain curve that might be used in structural analysis, stress-strain curves of individual samples are of little use. There is needed a systematic statistical study over a wide range of samples to include all of the
variables that effect the shape of a stress-strain curve. To facilitate such a study, the stress-strain curve might be catalogued as shown in Figure 1, wherein seven items are shown, the numerical values of which would permit the replotting of a sufficiently accurate stress-strain curve in any given case. In a large mass of data each of these seven items could be studied statistically to determine its range of variation defined limits of probability and from such a statistical study a basic minimum curve might be arrived at. New studies would need to be made periodically as changes in raw materials and manufacturing processes developed.

The cataloguing of stress-strain curves might be justifiably simplified by ignoring the upper yield point entirely. The upper yield point is a condition of instability, is sensitive to surface roughness, rate of strain, and other variables. Furthermore, the contribution of the upper yield point to the strength of a member loaded into the plastic range is rather
negligible if it exists at all, and disappears entirely in a bent beam, for example, as the limit of complete plasticity is approached.

Turning again to Fig. 1, $\sigma_p$ has been noted as the stress at an offset strain of 0.0001. This is an arbitrary selection of strain, chosen to determine the general shape of the curve and the "true" proportional limit, a function of the sensitivity of the strain measuring apparatus, will be considerably lower for commonly available apparatus(3).

To predict the overall strength and deformation of a structural member it is most important to know the lower yield point $\sigma_{LY}$ and $\varepsilon_{LY}$, the strain at the lower yield point prior to general "strain-strengthening"#.

(3)

# More commonly called "strain-hardening" by metallurgists.
In many cases a structural member will deflect far beyond the limit of structural usefulness without entering the general strain-strengthening region. However, the initial rate of strain-strengthening will provide the additional information necessary in those cases where plastic strains are this large. Specifications usually do not require determination of any of these three important plasticity stress and strain measures, $\sigma_{LY}$, $\epsilon_{LY}$, and $\frac{d\sigma}{d\epsilon}$.

The simplification of the initial stress-strain curve as shown in Fig. 2 ignoring $\sigma_p$ and $\sigma_{uy}$ as relatively unimportant maximum will be considered as providing enough information for inelastic strength analysis. In the case of stainless steel and nonferrous metals, stress-strain diagrams similar in shape to that shown in Figure 3 could be catalogued for purposes of statistical analysis by recording the stress at several arbitrary offsets of strain.

The lower yield point in a tension or compression test may be defined as the minimum level of stress after initial yielding has started just sufficient to successively develop new planes.
of slip in the portions of the bar that remain in the elastic state. Average strain over a gage length of several inches gradually increases, but locally, the strain proceeds in spurts along knife like zones that show up on the surface as "Lueder's" lines(4). The local strain, therefore, varies markedly from point to point.

Although the lower yield point is most significant in determining strength, and therefore most important to the designer, it is the upper yield point that is determined by the mill in accordance with rules set forth by the standard specifications(5). The upper yield point is important, therefore, in that our present notions the yield strength of structural steel are conditioned by the presently as to available accumulation of data, most of which reports only the upper yield point. It is additionally unfortunate that the acceptance of structural steel

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(5) For example,
is based on mill test reports that often give a misleading estimate of steel strength even when tests are made strictly according to A.S.T.M. Standards (E8-46) for "Tension Testing of Metallic Materials". This is particularly the case in using the older beam and balance type testing machines for which the specification reads "When the yield point of the material is reached, the increase of load stops, but the operator runs the poise a trifle beyond the balance position ... the laboratory a well-defined upper yield point of 47,150 psi and an ultimate of 84,250 psi were determined. A great deal of similar information has been recorded and the practices are defended on the basis that all mills use a similar practice and that the test is therefore satisfactory for comparative purposes. Steel mills are changing to modern hydraulic testing machines in which case the maximum load at the "bolt in the gage" is accepted as the yield point. This is an improvement but a specification limiting the strain rate to a reasonably low value is needed together with a determination
of the lower yield point. It is obvious from the foregoing that most mill test records, while possibly suitable for comparative purposes, are not of much use in defining the yield point that might be the basis for plastic design.

The corresponding stress is taken as the "yield point". The italics have been inserted by the author. Furthermore, since there is no rate of strain specified the mill tests are made at a speed that raises the upper yield considerably. As an extreme example, we have on record a mill test report for a silicon structural steel quoting a yield point of 60,300 psi and an ultimate of 82,200 psi. Tested at a slow rate in