A study of wire sections for chain links, 1950

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A STUDY OF WIRE SECTIONS FOR CHAIN LINKS

Progress Report No. 3

for

The American Chain and Cable Company
American Chain Division

by

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INTRODUCTION

This study of chain links was undertaken for the American Chain Division of the American Chain and Cable Company in order to develop a new cross-section to be used in place of the traditional circular cross-section in common use. The desire for a different cross-section had a double motivation: to find a section which would be stronger in cross-bending and if possible stronger in straight pull than the circular section, and to find a section of distinctive shape for use with a high-strength steel alloy so that the alloy chain would not be confused with ordinary chain when used by men in the field.

This study presents the results of the investigation of ten suggested cross-sections, and gives a table of relative strength for each section for various types of loading from which one or more sections may be selected for further study.
1. BASIC THEORY

a. Notation  Referring to Figs. 1, 2, and 3, the following will be used in this study:

X-axis is the longitudinal axis of the chain link
Y-axis is the transverse axis in the plane of the link
Z-axis is the transverse axis perpendicular to the plane of the link
P is the force applied at the end of the link
\( \phi \) is the angle between P and the X-axis. It is the angle of cross-bending

\( P_x \) is the \( x \)-component of the applied force

We have \( P_x = P \cos \phi \)

\( P_z \) is the transverse force. This is the force of cross-bending, and is zero when cross-bending does not occur. We have \( P_z = Psin \phi \)

\( R \) is the transverse force which produces cross-bending. It is not necessarily applied at the center of the link

\( l \) is half the length of the straight part of the link

\( L \) is half the inside length of the link. \( L = l + R_1 \)

\( R_1 \) is the inside radius of the end of the link

\( d \) is the depth of the link

\( D \) is the width of the modified link or the diameter of the circular link

\( M_z \) is the bending moment about the Z-axis at the center of the straight part of the link. It is produced by the force \( P_x \)
M_y is the bending moment about the y-axis at the center of the link. It is produced by the force P_z and is the moment of cross-bending. μ is the coefficient of moment for the fixed-end moment produced by P_x. σ is the unit stress in bending at any section. R is distance from inside of wire to centroid of cross-section of wire.

b. Bending moments at the center

If a chain link is subjected only to straight pull, that is if P_x acts and P_z is zero, the bending moment in the center of the straight section is equal to \( \mu \frac{P_x}{R} (R_1 + \bar{R}) \) where \( \mu \) is a coefficient (to be determined later) which is a function of the dimensions of the link. \( \mu \) is introduced because the link is statically indeterminate in the XY plane. In order to resist this bending moment we need a large moment of inertia around the z-axis, roughly speaking, we would want the width D to be as great as possible, giving a large I_z.

If the link is subjected only to cross-bending, that is, if P_x is zero and only P_z acts, the bending moment at the center is \( P_z \left( \ell + R_1 + \bar{R} \right) \) or \( P_z \left( \ell + 2R_1 \right) \) depending on where P_z acts. In this case the link acts as a simple cantilever beam, supporting half of P_z. In order to resist this bending we require a large I_y, requiring a large depth, d.

In practice both P_z and P_y will be acting, so that we require large values of both I_y and of I_z. The relative
sizes of $I_y$ and of $I_z$ required depend on the bending moments, which bending moments in turn depend on the relative magnitudes of $P_x$ and of $P_z$ and on the physical dimensions of the link end of the wire from which it is formed. $P_x$ and $P_z$ in turn will depend on the conditions of loading, and these conditions of loading also may depend on link and wire sizes.

In the case of a circular section $I_z$ and $I_y$ are equal, which gives satisfactory proportions only if $M_y$ and $M_z$ are equal. A study of the bending situation shows that this condition will occur only when $P_x$ is considerably larger than $P_y$ for ordinary proportions of link and of wire.

Theoretically, the most efficient cross-section of the wire would not be a wire in the ordinary sense, but something in the nature of a hollow rectangular tube. This is obviously out of the question, and we are then faced with the problem of finding a solid cross-section which will give us values of $I_z$ and of $I_y$ in the proportion required. Since a a hollow section is not allowed, second choice would be in the form of a structural I-section or H-section, or an approximation of such a section, roughly hour-glass shaped as in Fig. 4, which was first proposed.

It was found that such a section would be open to several objections. It offers reentrant surfaces which make it difficult to handle in welding. It also offers four bearing surfaces, which may under certain conditions of extreme cross-bending be reduced to only two bearing surfaces. For certain proportions it was also found that
Note:
PLANES AT RIGHT ANGLES

Figure 5
such a section would offer a passing link.

c. Bearing surface

In the usual circular wire cross-section the radius of the wire is smaller than the radius of the end of the link. In any such case the bearing between the links will theoretically be a single point. In the proportions commonly used the radius of the end of the link is 1.42 times the wire radius. When the chain is stressed this point of contact will enlarge to an area, depending on the relative radii, the elasticity, the plastic flow of the chain and the forces involved. In all cases the area of contact will be substantially the same under all angles of cross-bending.

The most satisfactory way to increase this bearing area is to make the radius of the wire equal to the radius of the link along the surface of contact. In such a case the theoretical contact of two links will be approximately in the shape of a cross. The length of each leg will be the arc intercepted by the central angle $2\alpha$. (see Fig. 5) The legs actually will be arcs of circles bent in opposite directions so that the figure is saddle-shaped. As the chain is stressed the lines of contact get wider, retaining the original cross-shaped outline. Although no study has been made of the matter, it seems not unreasonable to estimate that such a bearing area would be many times the area available from the traditional small wire, even if the angle of contact is only half of the possible $180^\circ$. 
As the angle of cross-bending increases one of the four arms of the cross will be shortened until \( \alpha \) is reached, at which time only three arms will be in contact. Even in this, the worst possible situation, the bearing area is relatively quite large.

Because of the foregoing considerations it was decided that the portion of the wire on the inside of the link would be given the same radius as the link itself in all of the sections studied.

d. Dimensions of link and of wire

In order to obtain results which were comparable with current practices in the field, a standard proportion was used, where the inside width is 1.42D and inside length is 2.70D, D being the diameter of the wire. This gives a length to width ratio of the inside of the link of 2.70/1.42, or 1.90:1. In our notation, this means that \( \ell = 0.90R_1 \), or \( L = 1.90R_1 \).

Four considerations limited the wire dimensions used. (1) There must be clearance to turn the link while forming it. (2) There must be sufficient clearance to allow the welding operation. (3) It must be a bucket chain, that is, there must be some clearance so that the links will fall freely into position. (4) It must not be a passing link, so that both \( d \) and \( D \) must be greater than \( R_1 \).

e. Eccentricity of curved beams

In a straight beam the neutral axis (the axis along which there is no elongation or shortening of the beam)
Figure 6
lies at the centroid of the cross-sectional area of the beam. In a curved beam this is not true, since there is a greater amount of material outside of the centroidal axis than there is inside, and the elongation of the beam is affected by it. In a curved beam the neutral axis lies inside of the centroidal axis. The ratio of the distance from the centroid to the neutral axis to the centroidal radius of curvature of the beam is called the eccentricity. For a given area and shape of cross-section this eccentricity varies inversely as the radius of curvature of the beam, and for a given radius and area it varies with the shape of the cross-section. In the case of a standard chain link of circular cross-section the eccentricity is 0.0540D inside the centroidal axis. The theory and tables for standard sections may be found in the references cited.

Although the side-weld of the link is a straight beam, this eccentricity does enter indirectly into the picture, since it affects the bending coefficient $\mu$.

2. STRESSES AT THE WELD

a. Determination of $\mu$

Consider a quarter of a chain link, subjected to both straight pull and to cross-bending. (Fig. 6) The moment at the center of the weld caused by the cross-bending force $P_2$ is merely $\frac{1}{2}P_2$ ($L \neq R_1$), assuming that the force acts through the center of the circular portion of the section. The section shown is statically determinate in the vertical plane.
In considering the moment in straight pull, however, the problem is not so simple, since the existence of a bending moment at both ends of our quarter-section makes the link statically indeterminate. By applying Castigliano's Theorem (see Appendix I) we can obtain the following expression for the bending moment at the center of the link, due to the straight pull of a force $P_x$ applied at the end of the link:

$$M_x = \frac{P_x R}{2} \mu$$

where

$$\mu = \frac{\left[ \frac{1}{\pi} - 2 - \frac{2e}{R} \right]}{\left[ \frac{1}{\pi} + \frac{2e \ell}{K_x^2} \right]}$$

$R$ in this expression is the radius of curvature to the centroid, which in our notation is $(R_1 + \bar{R})$.

It will be seen that this fixed-end moment at the center of the weld is not only a function of the force and the lever arm, but is also a function of the eccentricity, the length, the radius of gyration of the cross-section and the radius of curvature of the end of the link, despite the fact that we are concerned only with the straight section. If the quarter section shown were a separate body with no connection (and hence no bending moment) at the point B, the moment would be $P_x (R_1 + \bar{R})$. In the actual case the moment is $\mu \frac{P_x}{2} (R_1 + \bar{R})$ where $\mu$ will usually be in the range between 0.25 and 0.3.

b. **Stress Determination**

The stress at the central weld will be made of three different terms, direct tension, bending from $P_x$
(about the Z-axis) and bending from \( P_z \) (about the Y-axis).

The direct tension will be uniformly distributed over the entire area, and will simply be \( \frac{P \times 0}{2A} \)

The bending moment from \( P_x \) will be \( \mu \frac{P_x}{2} (R_1 + \overline{R}) \), and the unit stress will be:

\[
\sigma_{ex} = \frac{\mu \frac{P_x}{2} (R_1 + \overline{R}) y}{I_z} \]

where \( y \) is the distance to the left or right of the neutral axis. (see Fig. 2). Tension is found to the right, and since the neutral axis is to the left of the section, the maximum stress is tension at the extreme right where \( y = \overline{R} \), and is (using the expression \( I_z = A k_z^2 \))

\[
M_{ox} \sigma_{Bx} = \frac{\mu \frac{P_x}{2} (R_1 + \overline{R}) \overline{R}}{A k_z^2} \]

The bending moment from \( P_z \) (cross-bending) is \( \frac{1}{2} P_z (L \neq R_1) \). It is assumed that the force \( P_z \) acts through the center of curvature of the arc on the face of the wire. The bending stress in general is

\[
\sigma_{By} = \frac{P_z}{I_y} (L + R_1) \frac{z}{I_y} \]

where \( z \) is the distance above or below the Y-axis. Tension is in the section above the Y-axis, because of the symmetry of the section, the maximum stress in tension at the top and in compression at the bottom are equal, and are given by (letting \( I_y = A k_y^2 \))

\[
M_{ox} \sigma_{By} = \frac{P_z}{I_y} (L + R_1) \frac{z}{A k_y^2} \]

The maximum compressive stress is found at the lower left corner, and is given by:
Because of the presence of the negative term (representing the direct tension), compression is not critical and may be generally neglected.

Maximum tension will occur either at the upper right corner of the section, or somewhere along the right curved edge. It will be given by the expression:

$$\sigma_{\text{tens}} = \frac{1}{2A} \left[ \frac{\mu P_x (R_i + R)}{K_z^2} (R_i + R) + \frac{P_z}{K_y^2} \frac{d}{Z} (L + R_i) - P_x \right]$$

where \(z\) and \(y\) are related by the fact that they are both on the arc of the circle.

It can easily be shown (see Appendix II) that the maximum stress will occur at the point on the arc where the slope is:

$$-\mu \frac{\frac{P_y (R_i + R)}{K_y^2} \frac{d}{Z}}{P_x} = -\mu \frac{(R_i + R) k_y^2}{(R_i + L) k_z^2} \cos \phi$$

where \(\phi\) is the angle of cross-bending (see Fig. 1).

In the common case of 45° cross-bending, the maximum stress is found where the slope is:

$$\mu \frac{(R_i + R) k_y^2}{(R_i + L) k_z^2}$$

If this value of the slope is not attained anywhere along the arc of circle, maximum tension will also take place at the upper right corner of the section.
3. FORCES IN CROSS-BENDING

A chain link in cross-bending may take one of four positions as illustrated in Fig. 7, 8, 9, and 10, and the forces on the link are different for each of these positions. All four possibilities exist whether the wire section is circular or the modified section used in this study. The position assumed by the link will depend on all of the dimensions of the link, but the most critical dimension is the length. Obviously the position will also depend on the shape of the cross section, and in the case of a circular link, on the point of contact between successive links. For our modified link we have assumed that the link seats itself completely.

Fig. 7 illustrates one case. Here the link is relatively quite short so that the corner of the load does not get an opportunity to touch the middle of the link. The forces acting on the link are then \( P_1, Q_1, P_2, \) and \( Q_2, \) and the friction forces \( fQ_1 \) and \( fQ_2 \) as illustrated on Fig. 7. The angle of the link will be very close to 45°, although some slight variation from 45° may be possible.

It can easily be seen that for a 45° angle, neglecting friction, \( P_1 = Q_1, \) and the link is statically determinate. \( P_1 \) and \( Q_1 \) (and \( P_2 \) and \( Q_2, \) for all four of these forces are equal) form a resultant force along the axis of the link whose magnitude is \( P_1 \sqrt{2}. \) Since this force is along the axis of the link, the link acts as though it were in straight pull only, and the allowable load is the ordinary allowable straight pull load multiplied by \( 1/\sqrt{2}. \)
The elimination of cross-bending in such a case is a very real advantage, but is to some extent offset by the fact that such a link must necessarily be a short one, and consequently the chain will be relatively heavy.

The second possibility is shown in Fig. 8, where the dimensions of the link are such that it just touches the corner of the load. The angle in this case will always be exactly 45°. The forces will include the reaction of the corner on the link, R, and an appropriate friction force, fR, in addition to the previous ones.

This link is statically indeterminate, since with one force given, say P₁, we still have four unknown forces, P₂, Q₁, Q₂, and R, which cannot be solved by the three equations of statics. This may be visualized by considering the chain made of a completely rigid material and allowing the corner to become softer. If the corner is rigid the force R is large, but as the corner softens R decreases and finally becomes zero.

To obtain exact condition of Fig. 8, the chain link would have to be very carefully designed. If there were any wear on the bearing surface between the links, or if the link was longer, the third case, illustrated in Fig. 9 would be found. The same forces act, and again the forces are statically indeterminate, depending on the elasticity of the link and of the corner. As outlined in Appendix 3, it is possible to determine the angle accurately, either by a trigonometric solution or by a graphic construction. The determination of the forces acting is theoretically possible by the use of the
Figure 9

Figure 10
theory of elasticity, but totally impracticable within a reasonable time, particularly since the coefficient of friction may vary, a slight change of shape at the corner will produce a serious change in forces, and particularly since the elastic limit of the corner may be exceeded.

With a longer link it is not necessary that both adjacent links lie flat against the load. In Fig. 9, allow the vertical link on the left to be pulled downward gradually. If the force \( P_2 \) is allowed to slack off temporarily the horizontal link may be lifted away from the load, and a situation produced as illustrated in Fig. 10. As the vertical link is pulled down the point of application of \( P_2 \) (the center of curvature of the upper end of the link) follows the path outlined. It rises to a maximum position, and then gradually moves down until both adjacent links are again touching the load in a lower position.

Imagine a chain sling over two successive corners of the load. If the chain on one corner takes a position such as in Fig. 9, there is no assurance that the chain on the other corner will take such a position. If the length of the side of the load between corners is not an exact multiple of the inner link length (\( L \) in Fig. 1 and 3) the link on the second corner may take a position as in Fig. 10, and remain stable under appropriate values of \( P_1 \) and \( P_2 \).

If the length of the vertical face of the load is such as to hold the chain in this critical position the forces acting on it are those shown in Fig. 10. \( Q_2 \) is zero, as is
the corresponding frictional force. $P_2$ acts to the right and produces a cross-bending moment around the corner of the load. $P_2$ has been assumed horizontal to simplify the calculations, in practice it may fall a little below the horizontal or assume any angle up to $\theta$.

The lever arm of cross-bending ($P_2$ horizontal) is the vertical distance between $P_2$ and the center of gravity of the section through $R$. This will vary from $(R_1 + D - \frac{d}{2} \cos \theta)$ when the right adjacent link is just touching the horizontal face of the load (Fig. 9) to some maximum value depending on the shape of the path followed by the point of application of $P_2$. In this case the other forces on the link do not affect the moment, since they are applied at or to the left of the section under consideration.

It is possible to find this critical angle for maximum lever arm by a simple geometric construction. The angle, of course, will vary with all of the dimensions of the link and the cross-section. From this angle it is possible to find the lever arm of cross-bending, and hence the cross-bending moment as well as the straight-pull moment in the section.

It might be added that in this case with $Q_2 = 0$, the link again becomes statically determinate, since with $P_1$ or $P_2$ known only three unknown forces remain, $Q_1$, $R$, and $P_2$ or $P_1$. The equations for these forces in terms of $P_1$ are very intricate, and depend on the coefficient of friction between the vertical link and the surface, and between the cross link and the corner. Since friction opposes attempted motion, it is necessary to know in which direction motion is impending.
A study of these equations shows that it is possible
to have a situation such as illustrated in Fig. 10 if the
relative sizes of $P_1$ and $P_2$ are appropriate. In some cases,
of course, the link will slip and assume one or the other of
the positions of Fig. 9. In such a case, however, the moment
of cross-bending is smaller than at the critical angle, so
that computations were made for the critical angle and the
link clear of the load, as in Fig. 10.

5. TABULATION OF RESULTS

The large tabulation gives the results of studies made
on a number of chain links with varied wire cross-sections. In
all of these cases the following specifications were adhered
to:

1. The inside of the link was made of two half-cir-
cular arcs connected by two straight sides. In the specifi-
cations of link given as standard, the inside width is $1.42D$
and the total inside length is $2.70D$, where $D$ is the wire
diameter. This gives as the corresponding section an inside
width of $2R_1$ and an inside length of $3.8R_1$.

2. The inside of the wire was made in the form of a
circular arc of radius $R_1$ so as to give maximum bearing sur-
face.

3. Clearances have been provided for forming, welding,
and for freedom of motion within the link to provide a bucket
chain. In addition, limitations on the depth and thickness
were imposed by the fact that a passing link is undesirable.
Preliminary studies showed that extending the section too far in the z-direction gave a relatively very small moment of inertia about the vertical axis, which moment of inertia resists the bending moment in straight pull. For this reason a semi-circle, although very strong in cross-bending, is relatively weak in straight pull, and was not considered. The semi-circle or any other section in which the entire semi-circular arc is used has the added disadvantage of fitting so snugly that it would be impracticable for general use. Instead, the circular arcs were cut off at 60° (sections 1 to 5) and at 50° (sections 11 to 15) with various widths and shapes of outer surface. Note that 1 and 11, 2 and 12, etc., correspond in width and in shape of outer surface.

The headings of the tabulation indicate the section considered. The circular cross-section of diameter $1.414R_1$ is the standard link with which all of the others were compared.

Immediately below the headings are listed the areas, radii of gyration about each axis and distance between the curved inside face and the centroid. These are all given in terms of the radius.

The next entry, $e$, is the eccentricity of the curved section of the link, the distance between the neutral axis of the curved part of the link and the centroid of the cross-section. It is always toward the inside of the link, which in this case is toward the circular arc, or to the left. It is expressed as a pure number; to get the actual distance it is necessary to multiply it by $(R_1 + \bar{R})$. (See Appendix I)
Since everything is to be compared to a circular link, and since the weight of the chain is to be an important factor, the next entry gives $W/W_0$ where $W$ is the weight of the indicated link and $W_0$ is the weight of our standard cross-section. It is, of course, a function of the area and the length of the line of centroids around the curved part of the link. Since it is given as a ratio of weights, the density does not enter into the picture. The inside length remains the same for all sections.

The next three entries, the clearances, are self-explanatory. One additional clearance should be noted: the distance between the lower right and the upper left corner of the cross-section cannot exceed $2R_1$. In sections (3) and (13) this distance is exactly $2R_1$, so that the link just has room to turn for welding. In the other sections there is no difficulty.

The next four entries are the most important items. The first gives the ratio of the strength of the actual section studied to the strength of the standard circular section for straight pull. That is, for a proportion as given for the typical link, and for cross section of wire as shown for section (1), a chain will support 0.761 times the load that the typical circular chain will support.

However, in a stricter sense this does not give a true picture of the relative values of the circular cross-section and the modified section, for the modified section will weigh only 0.884 as much as the circular section. If we make allowances for this difference in weight by comparing the $P/W$ terms,
where P is the allowable load and W the weight of a single link of chain (or the weight of a single foot of chain, since they are proportional) we get a different result. In the case of section (1) we find that the ratio of the load per weight of a foot of chain for the section is 0.858 what is would be for a circular cross-section chain of equal weight.

The meaning of these terms can best be explained by an example. Suppose a given chain of circular cross-section and given inside length of link can carry a load of 10,000 lb. in straight pull. Then a chain of the same inside length (and of necessity different weight) with a wire section (1) can carry a load of 7610 lb. in straight pull, and a chain of same size with a wire of section (2) can carry a load of 11,740 lb. in straight pull, and similarly across the table for the values of P/P₀.

The entry for P/W over P₀/W₀ requires a more careful consideration. In general, the load a chain can carry is almost directly proportional to the weight per foot of chain. A standard chain weighing 1 lb per ft. can carry 1700 lb., and a heavier chain can carry loads in approximately the same ratio. Thus if we have a circular section chain of known weight we can easily find the allowable load simply by multiplying the weight per foot by an appropriate factor, in this case 1700, the factor depending on the cross-section of the chain, the material and the proportions of the link. If we consider section (1) we find that a chain of this cross-section and the same inside link length as a given circular chain will weigh only 0.884
times as much. Taking the 1700 ratio for purposes of comparison, a circular chain weighing 10 lb per ft. will take a load of 10 times 1700, or 17,000 lb. A section (1) chain of the same weight per foot of chain, and therefore of different dimensions, will carry a load equal to this value multiplied by the \((P/W)/(P_o/W_o)\) ratio, that is, 1700 times 0.851 = 14,500 lb.

The \(P/W\) entry, then, is in the nature of an indication of what might be considered the efficiency of the chain. If it is greater than 1.00 we can apply a heavier load for a given weight of chain, as is the case in all of the sections (11) to (15).

The second set of entries is for 90° cross-bending, the most extreme case. To continue with numerical examples, suppose we have a 10 lb/ft. chain of circular cross-section which can take 17,000 lb. straight pull. This same circular section can take a load of 17,000 times 0.255 = 4330 lb. when subjected to 90° cross-bending. A chain of section (1) with the same inside length as the circular chain can take a 90° cross-bending pull of 17,000 times 0.303 = 5150 lb. in 90° cross-bending, and so across the table. It will be seen that for all sections except (11) the modified sections are more satisfactory in cross-bending than the circular section.

The \(P/W\) entries are handled similarly. Our 10 lb per ft. chain can take 10 times 1.00 times 1700 = 17,000 lb. in straight pull. This same chain, however, can take only 10 times 0.255 times 17,000 = 4330 lb. pull in 90° cross-bending.
A chain of section (1) with the same weight of 10 lb per ft. chain but of different dimensions can take the \((P/W)(P_0/W_0)\) ratio times 17,000 lb. pull in cross-bending, that is, 0.342 times 17,000 = 5800 lb. in 90° cross-bending.

It will be seen that all of the entries in this table are larger than the circular section entry, indicating that in terms of allowable load per weight per foot of chain the modified sections are all more efficient. If the chain were loaded only in cross-bending a section (1) chain could weigh \(0.255/0.342 = 0.730\) times the weight of a circular chain and still have the same strength in cross-bending.

The entries for 45° cross-bending follow the same pattern as previously outlined.

The next entry, designated \(\phi\) critical is for the situation outlined in Fig. 10, where one end of the link protrudes above the surface of the load. The \(\phi\) Critical entry gives the angle at which the maximum lever arm is found and the overhang entry is the actual lever arm, measured to the center of the circular arc. The circular section and sections (13) and (15) do not have entries, since in the circular cross-section there is triple contact (as in Fig. 8) and for (13) and (15) there is no contact at the corner, (as in Fig. 9). For (13) and (15) there is no cross-bending possible, and for the circular section the static indetermination makes computation impracticable.

The last entries give the values of the angle of triple contact. There are two complementary angles illustrated by the two sketches. Values for these angles are given.
5. **FURTHER STUDIES**

Five of the sections studied (sections 11 to 15) present an improvement over the circular link in strength to weight ratio for both straight pull and in cross-bending. Four others (sections 2 to 5), although slightly less efficient than the circular section in straight pull, are definitely superior in cross-bending. Two sections (13 and 15) avoid cross-bending over right angle corners.

In view of the number of sections which deserve attention, the authors have made no attempt to select one particular section for further study. To complete the work the following program is suggested:

1. Selection of two or more sections as being the most promising for further investigation.

2. Making up chains with the selected cross sections and testing them under various loading conditions, particularly to check their performance under plastic conditions.

3. Studying further the geometry of the selected links, and studying the mechanics of those links, particularly under various cross-bending conditions.

4. Studying the effect of varying $L/R_1$, that is, the relative length of the link as it changes the allowable load in straight pull, cross-bending, and the load to weight ratio. It is possible that with some of the sections a longer or shorter link will be more satisfactory.

5. In all of the present theory it has been assumed that the load was applied at a single point. Actually it is distributed over a considerable area, and a study of the effect of this may give fruitful results.
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<th>Properties of Cross Section</th>
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<th>Cross Section (3)</th>
<th>Cross Section (4)</th>
<th>Cross Section (5)</th>
<th>Cross Section (6)</th>
<th>Cross Section (7)</th>
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ACKNOWLEDGMENT

This report was written by Prof. Kleinschmidt. It summarizes the research work carried out from November 1 to December 15 by Prof. Beer and Prof. Kleinschmidt in consultation with Prof. W. J. Eney, Head of the Department of Civil Engineering and Mechanics.

The numerical results embodied in the table at the end of this report were obtained by Messrs. Jacobsen and Strausser under the direction of Prof. Beer.

REFERENCES

Curved beam theory is discussed in Vol. II of Timoshenko's Strength of Materials and in Sooey's Advanced Strength of Materials. In addition Sooey summarizes two out-of-print pamphlets on chain links.
APPENDIX I.

Derivation of \( \mu \).

(See Timoshenko, Strength of Materials, Vol. II, Page 83)

According to Castigliano's First Theorem, if we set up an expression for the strain energy in a structure and differentiate the strain energy expression with respect to one of the forces or moments acting on the structure, this partial derivative will give us the deflection of that force or moment. Thus if \( U \) is the total strain energy in a body, and the body is acted upon by an external force \( P \), the deflection of \( P \) is given by the expression \( \frac{\partial U}{\partial P} \).

Consider a quarter of a chain link, as in Fig. 6. The left end of this quarter link is subjected to a force of \( \frac{1}{2} P \), and a moment of \( M_z \) at the cut section. (We are not concerned with the moment in the vertical plane.) At any point in the curved portion of the beam,

the bending moment \( M = \frac{1}{2} P \cos \theta + M_z \)

the direct stress \( N = \frac{1}{2} P \sin \theta \)

the shear \( V = \frac{1}{2} P \cos \theta \)

The total strain energy of a curved bar is

\[
U_c = \int_{0}^{s} \left( \frac{M_z^2}{2AEeR} + \frac{N^2}{2AE} - \frac{MN}{AE} + \frac{\alpha V^2}{2AG} \right) ds
\]

For the straight part of the beam, \( M = M_z, N = \frac{1}{2} P \), and \( V = 0 \). The total strain energy for the straight beam is:

\[
U_s = \int_{0}^{L} \left( \frac{M_z^2}{2EI_z} + \frac{P_x^2}{B^2EA} \right) dx
\]

We shall now make the appropriate substitutions, differentiate the strain energy with respect to \( M_z \), and equate the resulting expression to zero. The expression \( \frac{\partial U}{\partial M_z} \) gives the angular deflection at the left end of the
quarter link (which is the center of the straight part of the complete link) and which from symmetry has an angular deflection of zero.

The total strain energy (giving on the terms involving $M_z$ and performing the integration) is:

$$
\left[ -\frac{M_z P_x R^3}{2AE e} + \frac{M_z^2 R^5}{2AE e} + \frac{P_x M_z \cos \theta R^3}{2AE R} \right]_{0}^{\pi} + \left[ \frac{M_z^2 \chi}{2EI_z} \right]_{0}^{\pi} = U
$$

Then by differentiation:

$$\frac{\partial U}{\partial M_z} = 0 = -\frac{P_x R \frac{\pi}{2}}{2AE e} + \frac{P_x R}{2AE e} + \frac{2M_z^2 \frac{\pi}{2}}{2AE e} + \frac{2M_z \frac{\pi}{2}}{2EI_z} - \frac{P_x}{2AE}
$$

If we let $I_z = A k_z^2$ and solve for $M_z$ we obtain

$$M_z = \frac{P_x R}{2} \left[ \frac{\pi - 2 + \frac{2e}{R}}{\pi + \frac{2e}{R}} \right]
$$

which gives the value of $\frac{M_z}{R}$ as perviously used.
APPENDIX II.

Location of Maximum tension.

From section 2-b we have for the maximum tension on the upper curved part of our cross-section

\[ \sigma = \frac{1}{2A} \left[ \frac{\nu P_x (R_i + \overline{R}) y}{K_y^2} + P_x + \frac{P_z (R_i + L) z}{K_y^2} \right] \]

where the variables are \( y \) and \( z \), the coordinates of a point on the curved section of the cross-section, and all other quantities are constants. To find the maximum we differentiate with respect to \( y \) and equate to zero,

\[ \frac{d\sigma}{dy} = \frac{1}{2A} \left[ \frac{\nu P_x (R_i + \overline{R})}{K_y^2} + \frac{P_z (R_i + L)}{K_y^2} \frac{dz}{dy} \right] = 0 \]

Solving this expression for \( \frac{dz}{dy} \), which is the slope of the curve, we find the expression given in section 2-b.
Figure A-3-1
APPENDIX III.

Determination of angle of cross-bending.

Cross-bending occurs when a link is placed at a corner and subjected to two forces at right angles, with a third force in the middle of the link, as shown in Fig. A-3-1. In certain cases where the wire is large in relation to the size of the link such cross-bending may be impossible, but in ordinary cases it does occur.

Because of the geometry of our proposed link it is possible to find an exact relationship for the cross-bending angle, that is the angle at which the long sides of the two adjacent links are against the edges of the load and the central part of the link touches the corner of the load. The situation is illustrated in Fig. A-3-2, in which all of the notation is as before. In addition the point of contact of corner and link is designated as being a distance aL from the inside of one end of the link.

It will immediately be apparent that there are two situations possible, the one illustrated and one where the cross-link is nearly vertical, with the point of contact near the upper right end of the link. It will also be apparent that this case gives us a value of \( \phi \) complementary to the \( \phi \) of first position.

The vertical \( R_1 \neq D \) can be developed in terms of \( aL, d \) and \( R_1 \) as shown in Fig. A-3-3, and the horizontal \( R_1 \neq D \) as shown in Fig. A-3-4. From these we get

\[
(aL + R_1) \sin \phi + \frac{d}{2} \cos \phi = R_1 + D
\]

\[
[(2 - a)L + R_1] \cos \phi + \frac{d}{2} \sin \phi = R_1 + D
\]
Figure A-3-5
Eliminating the unwanted distance $aL$ from these equations and by some trigonometric manipulation we get the equation:

$$(L + R_1) \sin 2\theta + \frac{d}{2} - (D + R_1) (\sin \theta + \cos \theta) = 0$$

in which the only unknown for a given a chain link is $\theta$. Unfortunately, this equation can be solved only by trial and error, but solutions agree with results obtained graphically.

For a graphic solution we may use Fig. A-3-5. $C$ is the corner over which the bending takes place. The center line of our cross chain will always be a distance $\frac{d}{2}$ away from this corner, that is, will always be tangent to a circle of radius $\frac{d}{2}$ from the corner. The upper end of the chain link (measured to the center of the arc of the cross-section) will follow a locus which is a horizontal line located a distance $(R_1 - D)$ above the corner, and the lower end will follow a vertical line $(R_1 - D)$ to the left of the corner. We may draw these lines ($XX$ and $YY$ in Fig. A-3-4). There are two possible lines of length $(L + 2R_1)$ whose ends lie on $XX$ and $YY$ which are at the same time tangent to the circle $C$, of radius $\frac{d}{2}$. These two lines give the angle of cross-bending, $\theta$.

It should be noted that this computation was based on the assumption that the links seated themselves completely. With the traditional circular wire the excessive wear of one link on another may prevent seating in the expected
place but it is unlikely that such mis-seating would happen with the modified cross-section suggested in this report.

If the load has angles other than right angles both the graphic and analytic method of finding $\phi$ may be modified accordingly. The graphic method is more practicable in such a case. Only right angle loads were considered in this study.

Submitted December 19, 1950