Combined bending and torsion of plate girders

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COMBINED BENDING AND TORSION
OF
PLATE GIRDER

Cooperative Investigation By
Lehigh University
and
Swarthmore College

Sponsored By
Pennsylvania Department of Highways
and
Bureau of Public Roads

Conducted By
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Bruce G. Johnston    William J. Eney

Final Report
Fritz Laboratory Project No. 215B
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"COMBINED BENDING AND TORSION OF PLATE GIRDERS"

CHAPTER I SYNOPSIS

An extensive investigation of plate girder behavior under torsion loading was undertaken by the Fritz Engineering Laboratory of Lehigh University. This report covers the third phase of the project, the experimental test program of which was carried out at Swarthmore College.

Full-size plate girder specimens, approximately 4 feet deep and 38 feet long, of bolted and riveted construction were subjected to combined bending and torsion loads. A concentrated vertical load was applied by a testing machine at midspan while the specimen was simply supported, free to warp but restrained against twisting at the ends. The behavior of the specimens under various combinations of lateral eccentricity and height of load application was observed. The transverse and longitudinal variation of the bending and shear stresses, the vertical and lateral displacements of the section, and the angular distortion of both web and flange were measured by SR-4 gages, dial gages and level bars respectively. The experimentally-determined stresses and distortions were compared with corresponding computed values obtained from representative theories.

The results in general confirmed expectations in that the linear theory, admittedly approximate, gave values on the low side, especially in the case of the critical flange normal stress. On the other hand the values based on the non-linear theories, rigorously incorporating the effect of deflections, checked satisfactorily with measured values.
CHAPTER II  INTRODUCTION

A. Program

A comprehensive investigation of the behavior of built-up structural members when subjected to loading conditions involving torsion was undertaken by Lehigh University at the Fritz Engineering Laboratory. The complete program was divided into three phases.

The first phase covered the basic uniform torsion case of a plate girder subjected to a shaft loading without end restraints. Both shallow and deep girders were tested by Chang and Johnston at Lehigh University as part of project No. 211.(10)

In the second phase a series of shallow plate girder specimens were placed under non-uniform torsion by means of a shaft loading with end restraint. These tests were conducted by Kubo, Johnston, and Eney at Lehigh University under project No. 215A.(11)

The third phase of this program is based on combined bending and torsion loading tests of plate girders of average proportions performed at Swarthmore College by Kubo, Carpenter, Johnston, and Eney. This project, designated No. 215B, is summarized in this report.

B. Sponsorship

All three phases of this program have been supported by grants from the Department of Highways of the Commonwealth of Pennsylvania in cooperation with the Federal Bureau of Public Roads. Both Lehigh University and Swarthmore College have made available their testing facilities without charge.

Note: References designated by a superscript number are summarized in the Bibliography, Appendix A
C. **Classification of Torsion**

A member subjected to twist is under uniform torsion if there is no restraint of the warping tendency. This condition is also known as the St. Venant or pure torsion. A member of constant torsional rigidity under such conditions will have only torsional shear stresses and will exhibit a constant unit angle of twist. Among the common cases of uniform torsion are the following:

1. Cylindrical member without end restraint under shaft loading
2. Cylindrical member with end restraint under shaft loading
3. Non-circular section without end restraint under shaft loading.

When the warping tendency of non-circular sections is restrained, either by physical means or by virtue of symmetry, then normal and shear stresses due to flange bending are produced in addition to the torsional shear stresses. The unit angle of twist is no longer constant along the span. The angular distortions are reduced in magnitude compared to the uniform torsion case. Representative cases of non-uniform torsion are the following:

1. Open section, restrained at one end, subjected to a shaft torque
2. Cantilever beam acted upon by an eccentrically applied load
3. Simply supported beam with a transverse load applied eccentrically at some intermediate point. This combined bending and torsion loading is the one most commonly met in practice.

D. **Objectives**

This third phase of the non-uniform torsion project was designed to investigate the behavior of full-size plate girders under combined bending and torsion loading. Although formulas based on superposition
of effects have been widely used to compute stresses and distortions, it is recognized that they are based on assumptions which are not strictly correct. The specific objectives of this investigation were as follows:

1. To determine the resulting distribution and variation of the normal and shear stresses in the web and flange of plate girders and to check against theoretical computed values.

2. To measure the angular and linear distortions and to compare the results with computed values obtained from the available theories.

3. To investigate the non-linearity of stress and distortion due to the influence of the deflections on the bending and torsional moments.

4. To compare the efficacy of the available theories in predicting the characteristic behavior of plate girders.

5. To examine the effect of section properties on the results, both experimentally and analytically.

E. Scope

A limited yet representative group of full-size plate girders of bolted and riveted construction were included in this investigation. The three specimens used were of average depth with bending and torsional section properties kept constant over the span length of 36' 8". A single, concentrated, eccentric load applied vertically at the center-line was the loading used to produce the combined bending and torsion condition.

The magnitude of the applied load was limited so that the maximum combined normal and shear stresses were well below the elastic limit. In this way the basic section of web plate and flange angles could be retested, after predetermined modifications, as another specimen.
CHAPTER III  COMBINED BENDING AND TORSION THEORY - SUMMARY

A. Analytical Development -- General

A common case of a structural member subjected to combined bending and torsion loading is a beam or girder under the action of a system of eccentrically applied transverse forces. As a close approximation, each force can be replaced by an equal parallel force acting through the shear center and a shaft torque of a magnitude equal to the product of the force and the initial eccentricity. The vertical bending effect of the transverse force at the shear center on the stresses and displacements are readily evaluated by elementary formulas. The non-uniform torsion effect of the shaft torque with restraint of warping is also readily predictable if the influence of the lateral displacement is neglected.

Several analytical solutions of this problem have been proposed. As is usually the case, the degree of precision improves as the number of assumptions are reduced. However, the complexity of the derivation also increases. In general the methods are basically linear or non-linear. The linear theory is based on the assumption of small deflections which can be neglected. In the non-linear theories, deflections are considered too large to be neglected.

Representative of the various available methods and selected for comparative purposes in this report were the following:

(1) Bethlehem Steel Co. Formulae
(2) Timoshenko Solution
(3) Sourouchnikoff's Method
(4) Pettersson's Theory.
B. Bethlehem Steel Co. Formulae -- Summary

The basic Timoshenko solution has been summarized and applied to several typical cases of uniform and non-uniform torsion.(1) The stress and rotation functions are expressed in terms of the critical values. The combined stress values are obtained by superimposing the vertical and torsion bending effects only. These formulae are included in this study since they represent the simplest available solution.

C. Timoshenko Solution -- Summary

The basic equilibrium equations for an I-beam of bisymmetrical cross section subjected to non-uniform torsion was solved by Timoshenko.(2) Essentially it is the linear, small deflection theory. For the combined bending and torsion case, the bending and torsional stresses are superimposed to obtain the resultant stresses. In the case of an eccentrically applied vertical load, the lateral, vertical and torsional bending stresses are considered. However, the lateral bending and torsional stresses are based on the initial eccentricity and do not include the effect of the induced displacements.

This solution has been extended to cover a large number of different combinations of loading and end conditions and appears in various forms.(3)(4)(5)(6)
D. Sourochnikoff's Method -- Summary

This non-linear solution\(^{(7)}\) is an extension of the basic linear theory in that it takes into consideration the influence of the deflections on the bending and torsional moments as well as the angular distortions.

The effect of a pure torque acting at the midspan of a simply supported beam of bisymmetrical section is first investigated. Four typical variations of torque along the span are considered. It was found that the ratio of the angle of twist to the maximum angle of twist at midspan was nearly parabolic in all four cases. Since the torque curve in the case of combined bending and torsion falls between the cases previously studied, it is assumed that the angle of twist curve can be represented by a second degree parabola. The bending moment in the lateral direction due to this assumed rotation leads to the lateral deflection by means of the flexural relationship. The actual torque at any point can now be evaluated. A simplifying assumption that the adjusted torque is constant yields an expression for the maximum angle of twist under combined bending and torsion alone. The maximum normal flange fiber stress at midspan is obtained by superimposing the vertical bending, lateral bending and torsion bending effects. The total stress is not proportional to the load, since the latter two components are functions of the maximum angle of twist which in turn is not proportional to the load.
E. Pettersson's Theory -- Summary

The combined bending and torsion case represented by a simply supported I-beam of bisymmetrical cross-section loaded at midspan by an oblique eccentric concentrated load has been solved by Pettersson.\(^8\)(9)

Since this is a stress problem where the law of superposition is not strictly applicable, deflections are taken into account. The differential equation for the angle of rotation has been deduced and solved by means of infinite power series. The angle of rotation and its second derivative, which are used to obtain the combined stress, have been transformed so as to obtain a direct expression of the effect produced by the risk of lateral buckling. The maximum angle of twist in combined bending and torsion is expressed in terms of the corresponding distortion produced by a twisting moment and a factor which is equal to unity when the concentrated load is infinitely small, and which increases indefinitely as the applied load approaches the critical value in lateral buckling.
F. Experimental Work -- Review

In contrast to the number of analytical solutions that have been published on the subject of combined bending and torsion of I-Beams, the number of reports on actual laboratory tests is quite limited. The experimental investigations that have been reported to date have utilized either rolled sections or welded I-Beams.

In a series of experiments conducted at the Royal Institute of Technology, two specimens of bisymmetrical and monosymmetrical cross-sections were in turn subjected to an eccentric vertical load at the center of a simply supported span. The sections were of welded construction approximately 10 inches deep and 20 feet long with a $\frac{gL}{L}$ ratio of 4.7 and 6.4 respectively. Weights were suspended from the beam specimen by means of a yoke. The theoretical end conditions were carefully maintained by means of vertical hinges, single-direction axial ball bearings and a system of pulleys and weights.

The strains were measured by electric resistance type gages at selected points, while the horizontal and vertical displacements were measured by means of a theodolite sighted on drawing pins attached to the flanges.

The comparison of observed results and calculated values based on the Pettersson Theory was exceptionally good. Direct comparison with the predicted values based on other theories were not included.
CHAPTER IV - EXPERIMENTAL PROGRAM

A. General

In the initial series of tests conducted at Lehigh University to investigate the uniform torsion behavior of built-up structural members,\(^{(10)}\) a wide range of specimens were tested. Included in this group were a series of shallow as well as relatively deep plate girders of bolted, riveted and welded construction, \(14\frac{1}{2}\)" and \(48\frac{1}{2}\)" back to back of angles respectively. The shallow specimens were used in the second phase of the experimental program at Lehigh University\(^{(11)}\) which was devoted to the study of non-uniform torsion as exemplified by shaft torque loading applied at midspan with the concomitant restraint of warping. The deeper girders were not tested at that time with the available equipment because of their limited length-to-depth ratio. It was felt that the desired typical non-uniform torsion behavior could not be produced in so short a span.

When it was decided to extend the program to cover the combined bending and torsion loading of plate girders, economy dictated the selection of sections which were similar to those for which torsional constants had been obtained experimentally in the first phase. Due to the limitation of time, the basic torsional constant test could not be conducted on the new specimens. Therefore, the previously-obtained experimental values were adapted for use in this report with reasonable assurance as to their applicability.
R. Test Specimens - Fabrication and Properties

All significant details of specimen T6A-4 and T6B-1 of the uniform torsion tests\(^{(10)}\) were duplicated in the design of the new specimen. In order to obtain the maximum possible reuse of material, it was decided to start with high tensile bolts as connectors and to achieve desired variations in torsional properties by changing the number of cover plates. Stresses were to be kept within reasonable limits.

Features of all three sections finally selected for this test program are summarized in Fig. 4:1. All sections are similar except for the number of cover plates or the mode of fabrication. Fabrication details of a representative specimen, T12-R, are shown in Fig.4:2.

For practical reasons some modifications in fabrication details were introduced in the new specimens. The basic pitch of holes for connectors was made the same for both the horizontal and vertical legs of the flange angles. The spacing of the intermediate stiffeners was increased slightly in order to maintain reasonably uniform spacing along the span. Provisions were also made for the contemplated addition of another set of stiffeners in each panel. These additional holes were not utilized since time did not permit the study of the effect of stiffener spacing.
Since this program was an extension of two previous investigations, the numerical sequence of specimen designation was continued. It is to be noted that specimen T6A-4 and T6B-1 of the first phase are similar to the new specimens T10-B and T12-R respectively. A specimen corresponding to T11-B without cover plates was not tested in the first phase. The letter designation following the specimen number refers to the mode of fabrication, B for bolted and R for riveted construction.

Shop fabrication procedures, as well as subsequent laboratory adjustments, were carefully standardized in order to obtain specimens structurally equivalent to those previously tested. The fabrication of the specimen and the auxiliary support frames was undertaken by the Bethlehem Steel Company at their Pottstown, Pennsylvania plant.

The high-strength bolts were tightened by means of a calibrated torque wrench to a uniform value of 300 lb.-ft. torque. No case-hardened washers had been used in the bolted specimens of the first phase. It was recognized that such washers would insure more uniform resultant bolt tension and that the bolt tension could also have been increased so as to conform to current specifications by tightening the nuts with a pneumatic gun. However, it was felt that such modifications would introduce an indeterminate change in the torsion constant of the new specimen. Consequently the bolting procedure used on the original specimen, T6A-4, was adopted for both specimens T10-B and T11-B.
The subsequent modification of specimen T10-B to T12-R involved the replacement of the bolts by rivets. In order to save the cost of transporting the 7-ton specimen to the shop and back, riveting by means of a pneumatic hammer at the Swarthmore College laboratory was considered. However, the original specimen, T6B-1, had been riveted by means of a bull riveter under standard shop conditions. Therefore, for the reason previously cited, the bolted plate girder specimen T10-B was shipped back to the fabricating shop at Pottstown for riveting and returned as specimen T12-R.

The torsion constant $K$ of a plate girder can be determined by both analytical and experimental means. The values used in this report are summarized in Fig. 4.3. The system of designation is patterned after the one used in the non-uniform torsion report. (11)

The experimental value, Test $(K)_{A}$, is based on the results of the uniform torsion tests conducted on similar specimens in the first phase. The torsional rigidity $KG$ is equivalent to the slope of the straight-line portion of the torque vs. unit angle of twist curve.

The constant $(K_{s})_{D}$ is the lower limit value based on the assumption that all elements of the cross-section twist through the same angle but separately. Due to the lack of cover plates in specimen T11-B, there is only limited unity of action between the elements. The separate-action constant was taken to be the representative value. No experimental value was available for this specimen.

As the upper limit the torsion constant of a built-up section would approach that of an equivalent solid section of the same external dimensions.
Since the elements of a bolted or riveted plate girder are actually connected along well-defined gage lines and interplane friction does exist, the so called integral-action constant $K_1$ must fall somewhere between the two extreme values. Based on the concept proposed by de Vries, the core portion in each flange acts as a solid section while the remainder of the section acts as separate rectangles when the section is twisted.

The computed value $(K_1)_B$ is the integral-action constant proposed for design in which the hump and end effects, which tend to neutralize each other, have been neglected. The $(K_1)_C$ value incorporates both corrections.
C. Test Set-up

The unusual magnitude of the space and force requirements of the full-size plate girder tests necessitated careful design of the handling, loading and support arrangements. Several possible proposals were studied from the standpoint of practicability as well as efficiency. The first problem was the choice of the loading system. It was recognized that a system of suspended dead loads would best fit the assumed conditions of the theory. However, such a system of weights would remain posed like the Sword of Damocles throughout the loading and unloading process. The ever-present possibility of an unfortunate accident was deemed sufficient grounds for the elimination of suspended weights in favor of the use of a universal testing machine for loading.

Since the specimen weighed approximately seven tons, an overhead crane of sufficient capacity and maneuverability would be required to place it in the proper position for testing. The magnitude of the forces that were contemplated were such that a universal testing machine of at least 100,000 pounds capacity in compression was indicated. The requirement that the specimen be supported at both ends meant that the effective bed of the testing machine had to be at least 38 feet long.

All three major specifications could be met by the available facilities in the Civil Engineering Laboratory at Swarthmore College. Arrangements were made in the summer of 1950 by Dr. Bruce Johnston, at that time Director of the Fritz Engineering Laboratory at Lehigh University, with Professor Samuel Carpenter, Chairman of the Civil Engineering Department at Swarthmore College, to handle the test program as a cooperative effort by both institutions.
Several possible loading patterns were considered. However, it was decided to concentrate on the fundamental case of a concentrated vertical load applied with an initial eccentricity at the centerline. A schematic sketch of the test set-up for a typical combined bending and torsion test is shown in Fig. 4-4.

In addition to the requirement of load capacity, it was necessary that the point of application of the concentrated load be adjustable. The eccentricity with respect to the longitudinal axis was to vary from five to twenty inches. The applied load was to be set at two levels: first, at the top of the flange and then at mid-height. Further, the load was to remain essentially vertical during the ensuing distortion of the specimen. Based on practical considerations the use of a single all-purpose loading bracket was abandoned in favor of two separate brackets, one for each level.

The first eccentric loading system, shown schematically in Fig. 4-5, was designed to apply a vertical eccentric load at the level of the top flange. A bracket was bolted on one side of the plate girder specimen. A strap plate served both a tie and as a support for a series of grooved plates placed at intervals of five inches. A surface-hardened knife edge was tack-welded to a steel block which in turn was welded to a rigid flat plate. A nest of three-inch diameter steel rollers, held together by spacer bars at both ends so as to be free to roll as a unit in either direction normal to their common axis, was placed under another smooth steel plate which was bolted to the underside of the testing machine head to serve as a bearing plate.
The roller nest was designed to permit the lateral movement of the lower plate relative to the upper plate, thereby reflecting the movement of the girder specimen without altering the initial parallel alignment. Thus the applied load distributed over a wide area of the lower plate for stability, would be transmitted to the girder as a concentrated, vertical load.

Since the loading device had to be kept at the centerline of the testing machine, the girder specimen was shifted laterally as a unit a specified distance when a different eccentricity was to be introduced.

To apply a concentrated vertical eccentric load at mid-height of the girder specimen, a second loading system, shown in Fig. 4:6, was devised. A new bracket with a line of horizontally-spaced holes replaced the former bracket at midspan. These reamed holes were located so that they would be at the proper distance from the longitudinal axis of the specimen.

A bracket, made up of a tee-section, was attached to the underside of the testing machine head. A loading post, fabricated from a long length of 4 x 4 inch square steel tubing, was connected at its upper and lower ends to the respective brackets by a pair of 1.25 inch round pins inserted through the matching holes. When the girder was properly aligned, the loading post would be vertical and at the desired eccentricity.

It was recognized that since the specimen would twist and be displaced vertically and horizontally by the load, the post would not remain vertical under this system. However a mitigating factor was
the relatively long length of 6 feet between pins. The lateral
displacement of the load point was measured directly by means of a
suspended plumb bob. The corresponding change in the direction of
the applied load from the vertical was small.

The requirements which governed the design of the end support
arrangement were as follows:

a. the specimen should be simply-supported at both ends for bending
   in the vertical plane

b. there should be no twisting of the section at either end

c. there should be no restraint of the warping tendency at the ends.

After investigating several alternative arrangements, the
system shown in Fig. 4:7 was adopted for the east end. A slightly
modified arrangement was used at the west end of the girder specimen.
Under each end of the girder was placed a welded frame from which the
necessary support features were projected. The structural frame was
designed as a rigid knee. At each end of the span a 3-inch diameter
roller was positioned between two base-plates placed under and normal
to the girder. Under certain loading conditions it was found that
friction alone could not supply the requisite restraint against lateral
movement of the ends. A collar was added to the roller by shrink
fitting a ring of rectangular cross section at the middle. Matching
grooves were machined on the contact face of the base plates. The
roller was free to roll but was positively restrained from shifting
sideways.

Due to the application of the eccentric load at midspan, the
girder will twist about the points of vertical support. It would
have been desirable to have the specimen twist about its longitudinal axis at mid-height. However, the fabrication of a suitable support fixture, adequate for the loads to be transmitted, would have been so complicated that this idea was bypassed in favor of a simpler arrangement. A knife-edge and matching grooved plate was placed parallel and directly under the girder web between the lower base plate and the top of the support frame. The contact surface of the knife-edge would be the point about which the girder would tend to rotate.

The restraint against this rotation was provided by a pair of tie rods. The upper and lower rods extended out horizontally from the girder and were connected to vertical posts of the support frame by suitable fixtures. A thrust bearing was placed under the locking nut so that it could be readily turned on the threaded rod even under pressure. Thus the end cross sections could be brought back to its original vertical position after the load had been applied. A plumb bob suspended from the top flange past a scale attached to the bottom flange served as an indicator of the magnitude of rotation at that end.

The resisting torque at each end could be evaluated from statics if the stress in each tie rod were known. A pair of electric strain gages was cemented to the machined throat section of the rods used at the east end. Calibration constants had been obtained from standard tension tests for each rod.

Since two calibrated ring dynamometers were available, they were inserted into the tie rods used at the west end. The dial readings
were a direct measure of the change in the linear dimension caused by the stress transmitted by the dynamometer.

D. Instrumentation

The behavior of the girder specimen under load was to be evaluated quantitatively in terms of strains and distortions. Since there were two symmetrical half spans, it was decided to use the west half of the specimen for a comprehensive strain survey and the east half to study the linear and angular movements.

An extensive system of electric strain gages was required to determine the normal and shear stress distribution along the span as well as around the section. Approximately 100 SR-4 gages of various types were cemented to the specimen at selected stations. Normal stresses in a given direction were measured by an uni-axial gage, shear stress combined with one normal stress by a bi-axial gage and principal stresses and their orientation at a given point by a tri-axial gage.

Since all three specimens were modifications of the same basic section, the strain gages used on one were usually available for use in the others. However, some replacements and additions were made necessary by accidents in handling or changes in section.

The major stations were spaced 5'-6" apart along the half span. The variation in normal and shear stresses at these stations across the flange and the web could be defined from the strain readings. For additional flange stress values along the axis, intermediate stations were established.
The technique of cementing gages and wiring leads followed standard laboratory practice. Switching units were used to bring the numerous leads into the portable strain indicators. Systematic checking procedures were instituted to minimize instrumental as well as human errors.

The variable nature of the anticipated distortions were such that a comprehensive network of linear and angular measurements was also required. The relative transverse and vertical displacements of points along the longitudinal axis as well as the angle of twist of the web and flange were desired. In line with the previously designated pattern, the system of dial gages and level bar supports, eventually selected, was placed over the east half of the girder specimen.

The angle of twist of both web and flange at selected stations was measured by a level bar with a 2-inch gage length which could be set up on special supports tack welded to the specimen. The arrangement and stationing of the level bar supports are shown schematically in Fig. 4:8. Since the angle of twist was variable along the span due to restraint of warping, as many stations as practicable were established.

Details of the adjustable type of level bar, improved by Mr. Kasten of the Swarthmore College Engineering Laboratory staff, are sketched in Fig. 4:9. A micrometer screw was used to bring the bar back to level as indicated by a spirit bubble after each load increment. The corresponding change in the Ames dial gages set at one of the gage points gave the desired measure of the angular distortion.
The range of the instrument was increased appreciably by means of a filler block of known thickness which could be inserted or removed from between the fixed points and the main bar. This adjustment was equivalent to a reset of the Ames dial by an amount equal to the thickness. The vertical positioning of the dial gage and the micrometer screw was also adjustable, which eliminated the need for resetting the adjustable plate which held the level bar support in place.

The lateral and vertical components of displacements of the stations along the longitudinal axis were to be measured with respect to vertical and horizontal reference planes respectively joining corresponding points at the ends of the specimen. The net displacement could be obtained by subtracting from the gross value a correction for the yielding of the supports, if any.

The universal testing machine at Swarthmore College was equipped with a specially-designed, welded platform frame made up of heavy rolled beams which made it possible to conduct flexure tests of long-span beams. In the conventional arrangement, load applied by hydraulic pressure to the loading ram under the platform is transferred by the lower crosshead to the tension screws, thence to the specimen by way of the upper crosshead and the measuring capsule. The upward-acting reaction under the platform is balanced by the sum of the downward-acting beam reactions on top of the platform. No unbalanced external forces are introduced under the base of the testing machine frame since the forces constitute a closed system. The full capacity of 600,000 pounds would therefore be available for loading.
The anticipated elastic deformation of the platform as a cantilevered frame under the beam reactions, although small, proved to be a troublesome factor in the design of a distortion-measuring system. After several schemes to compensate for this factor were tried out on a vertical bending test of a specimen, it was decided that a different arrangement should be explored.

The platform of the testing machine was blocked up at both ends by steel shims to eliminate the elastic deformations and the attendant support settlement at the ends of the specimen. Under load the beam end reactions would be transferred directly out of the system to the floor of the platform pit. The upward-acting reaction under the platform, no longer balanced by the beam reactions, would become an uplift force on the testing machine frame. Under this arrangement, the maximum load would be limited by the capacity of the anchor bolts and the weight of the frame itself. The safe load was approximately 100 kips. Since the maximum load contemplated in this program was also 100 kips, this arrangement was adopted for use.

Now the reference plane for vertical displacements could be the stable floor of the laboratory. A grid-work of light channel sections, tack-welded together, was placed under the east half of the girder specimen straddling the testing machine pit.

To measure the transverse displacement of points along the span under load, both the horizontal and vertical components would have to be obtained separately. Another complication was the fact that a dial measuring the horizontal component would theoretically have to shift vertically to compensate for the latter component of displacement.
This factor could be neutralized by fixing the dial at a considerable distance away from the specimen and connecting the plunger of the dial to the specimen with a taut length of fine flexible wire. The change in direction of the wire would be so small that the change in dial reading could be interpreted as the desired component of displacement.

A set of three dial gages was used at each station. Two gages were placed below the specimen symmetrically about the web. The gages were connected directly to outriggers tack-welded to the web at mid-height. The gage supports on each side were spaced along a steel angle-section which could be shifted parallel to the girder on top of the dial support frame. When properly aligned, the dial gages were directly below the point of attachment of the connecting wire to the outriggers. The wire was kept taut by means of a suspended weight attached to the lower end of the plunger. These dial readings could be combined to give the vertical deflection as well as another independent value of the angle of twist at the station. Fig. 4:8 shows schematically how the level bar and dial gage systems were combined on the south side of the web.

The third gage of each set was placed on a horizontal line directly opposite the mid-height of the specimen at each station. A vertical rod was cantilevered from the dial support frame. A small plate was positioned on the rod and held by a pair of locking nuts. The dial gage was clamped to the plate in a horizontal position. A flexible wire connected the forward end of the plunger to the specimen. The opposite end of the plunger was connected to a light weight by a wire riding over a pulley to keep the wire system taut. Since the forces
involved were very small, the dial support could be considered fixed in space. The change in dial reading gave the lateral displacement of the specimen.

Fig. 4:10 shows the battery of north side dial gages set up on the support frame. An overall view of the test set-up is pictured in Fig. 4:11.
E. Test Schedule

Vertical and lateral bending tests of each specimen were contemplated in order to obtain values of the flexural constants; however, the large number of combined bending and torsion tests that had been planned made for a very tight schedule. It was necessary at times to pass up a lateral bending test or to conduct a partial test by taking only a limited number of readings. Fig. 4:12 gives the summary of the test program as conducted.

Major problems in handling of the girder specimen were encountered in the course of the investigation. To move the specimen into the laboratory and then into the testing machine required a ten-ton overhead crane and careful maneuvering as indicated in Fig. 4:13.

After a combined bending and torsion test for a given eccentricity had been completed, the next eccentricity was introduced by shifting both end support frames the proper amount. This movement was accomplished by jacking against a block fixed to the floor at each end or taking up on a tension rod depending on the direction desired. The eccentric load had been removed so that only the friction between the base of the end support frame and the top of the platform had to be overcome.

When the girder specimen was to be tested in lateral bending, it had to be turned on its side. A special bracket with a large diameter rod as a handle was bolted to each end of the girder. The end support was removed after the specimen was jacked up. A ball bearing race supported on a grillage was slipped into place. When the jacks were lowered, the specimen could be turned quite readily. Fig. 4:14 shows the girder in the process of being turned on its side.
CHAPTER V  ANALYTICAL STUDY OF RESULTS

A. Preliminary Tests

A short segment of each main member of the plate girder specimen had been reserved for tensile tests. Flat coupons were prepared and tested for the usual tensile properties. The results are summarized in tabular form in Fig. 5:1. As noted, the material satisfied the standard specifications of the A.S.T.M. Designation A7-46 for structural steel. A common set of elasticity values was adopted and used in all of the subsequent computations so that the results would be comparable.

There are several ways in which the moment of inertia of a plate girder section can be computed. To indicate the range of variation three values of $I_x$ were computed for each specimen. The computed gross $I_x$ was based on the typical solid section. The net $I_x$, used in this report, was obtained on the basis of a section with holes deducted from both top and bottom flanges. A weighted value of $I_x$ was computed by proportion, considering the equivalent length of holes and the typical rivet pitch.

Two sets of experimental values were obtained from each vertical bending test; namely, deflections and flange normal stresses along the span. These experimental values were plotted at the corresponding points. A value of $I_x$ was found by trial which yielded a curve matching the plotted points quite closely. The two values of $I_x$ were averaged to obtain the effective $I_x$.

Typical results for the riveted specimen, T12-R, are reproduced in Figs. 5:2 and 5:3, based on the test with the concentrated load at
the centerline. As an independent check, another vertical bending test with concentrated loads at two symmetrical points was also conducted for this specimen. A similar set of curves was prepared for each of the three specimens. The bending stress variation for the bolted specimen, T10-B, is shown in Fig. 5:4.

The vertical bending properties are summarized in Fig. 5:5. It is of interest to note the following results which are based on a limited number of tests and are not represented to be comprehensive:

1. The values of effective $I_x$ based on measured deflections and normal stresses checked quite well for each specimen, the maximum difference being 6%.

2. The two vertical bending tests for the riveted specimen, T12-R, yielded almost identical values for the effective $I_x$.

3. The average effective value of $I_x$ for the riveted specimen, T12-R, was approximately 5% higher than that for the bolted version, T10-B.

The moment of inertia, $I_y$, about the weak axis was obtained in a manner similar to that outlined for $I_x$. A typical test curve depicting the variation in flexural stress at the extreme edges along the span is shown in Fig. 5:6.

A typical comparison of measured and computed values of deflection for the bolted specimen without cover plates is shown in Fig. 5:7.

The lateral bending properties are summarized in Fig. 5:8.

The lateral bending test for the riveted specimen, T12-R, was not conducted due to the lack of time. However, it appears reasonable
to assume that the difference between the $I_y$ test values for the riveted and bolted specimens of the same make-up should be less pronounced than in the previous case for $I_x$. The corresponding values for Specimen T10-B were adopted for use in the T12-R computations.

Since sections similar to those previously tested in pure torsion by Chang and Johnston\(^{(10)}\) were being used in this investigation of combined bending and torsion, no additional calibration tests were performed. The experimental and computed values of the torsion constant adopted for use in this report are summarized in Fig. 4:3.
B. Combined Bending and Torsion Tests

The plate girder specimen was carefully aligned in the testing machine so that the load would be applied with the proper eccentricity. Then the instrumentation for both strain and distortion measurements was installed. A couple of dry runs were made with certain critical gages under constant observation to detect any unpredicted behavior.

A small initial load was applied to take up any slack that might be in the system. Initial readings were taken of all gages. The design load was applied in increments and then removed. A complete set of readings was usually taken at the end of each increment. The maximum load was limited to a value such that all stresses were well within the elastic range.

In the first test the end tie-rods were adjusted after each increment of load to bring the end section back to its original vertical position, as indicated by the suspended plumb bob, before readings were taken. Later this operation was discontinued when it was noted that the end rotation was quite small.

Wherever possible, the test data was plotted against the load increments and the value corresponding to the maximum load was determined graphically. This procedure eliminated the possibility of a single erratic reading being used as a representative test value.

The SR-4 strain gage readings were converted to stresses in the prescribed manner, using the previously adopted elastic constants.

The level bar readings were reduced to angles of twist by dividing the change in dial reading by the gage length. The vertical dial gage readings were combined to give both the vertical displacement
and the angle of twist. The unit angle of twist at points midway between stations was determined by dividing the difference in the angle of twist by the distance between the stations.

It was recognized that since the flange does not twist as much as the web at any given station, those two values must not be combined. The angle of twist of the web form from both the level bar and dial gage readings checked quite well. The average value was used in the comparative curves.

In the computations for stress and distortions under combined bending and torsion, values of section properties and elastic constants were required. Experimental as well as computed values were usually available. For comparative purposes, it was decided to group the values into logical combinations which are summarized in Fig. 5:9. All of the test values were grouped together and used in combination "A". Approximate values that would normally be used in design computations were designated as Combination "B". More exact computed values incorporating certain refinements were used in Combination "C". The minimum values of the torsional and flexural constants were grouped as Combination "D".

It had been established in the second phase on non-uniform torsion of plate girders (11) that the theory for rolled beams could be extended to built-up sections if the proper constants were introduced. Of the various available analytical solutions covering the combined bending and torsion of beams, four representative ones were selected for comparative purposes. These were designated the Bethlehem Steel Co. Formulae (1), the Timoshenko solution (2), the Sourcchnikoff method (7),
and the Pettersson theory (8). For convenience the computations were
differentiated by a letter corresponding to the first letter in the
name. For example, all computations based on the Timoshenko equations
are preceded by the letter T. A fifth method, utilizing actual measured
distortions and equilibrium conditions, was designated computation "H".

Finally the computations were grouped into series according to the
method, the end conditions, and combination of constants used. Each
set of computed values would be represented by a method and series
designation such as T2.

The notation used is summarized in Fig. 5:9. For comparative
purposes the results of the analytical study are presented in terms of
either Series 1, 2 or 4.
C. Comparison of Analytical Methods

Since one of the objectives of this investigation was to compare the efficacy of the available theories in the prediction of the characteristic behavior of plate girders, computations were carried out by each of the four methods wherever possible. The same combination of constants was used in any one series.

The three components of flange stress are the vertical bending, lateral bending and torsional bending stresses. These are all additive at the south side of the top flange for specimens eccentrically loaded on the north side.

A breakdown of the components as determined by the four methods for two different combinations of constants for a given specimen and loading is tabulated in Fig. 5:10. The vertical bending stress remains the same, regardless of the method of computation. The lateral bending stress, when included, varies over a limited range. The largest and most sensitive component is the torsional bending stress.

The variation in the three components with load resulting from the use of the Pettersson formula for a given specimen is shown graphically in Fig. 5:11. The vertical bending stress varies linearly with the applied load, while the lateral and torsional bending components increase at an increasing rate with load. The relative magnitude of the components will of course vary as the load and initial eccentricity are changed. With greater eccentricities the torsional and lateral bending stresses will increase at the expense of the vertical bending component.
The manner in which the computed maximum combined flange stress varies with load for a typical specimen and loading is shown graphically in Fig. 5:12. The simplified Bethlehem Steel Co. Formula yields a straight line variation with load. The Sourouchnikoff and Pettersson methods exhibited the characteristic amplification of stress due to the influence of deflections, the latter usually giving slightly higher values than the former. The Timoshenko solution, though approximate, includes a lateral bending term which resulted in values intermediate between the maximum and the minimum. For the majority of cases investigated, this relative order of magnitude was maintained.

The increase in the effective torsional arm of an initially eccentric load applied above midheight of a bisymmetrical section due to twist results in a non-linear variation of the angle of twist with load. The deflection methods of Sourouchnikoff and Pettersson recognize this behavior and show an increasing rate of increase of the angular distortion with load. The approximate Bethlehem Steel Co. and Timoshenko solutions are not sensitive to this effect and a straight line variation results. Fig. 5:13 shows graphically how the computed values of the centerline angle of twist vary as a function of the magnitude of the eccentric load for a given specimen.

The superposition of the computed vertical bending and the torsional bending effects with or without the lateral bending component under combined bending and torsion loading yields resultant normal stresses which differ in value at each of the four flange corners. With the vertical eccentric load applied on the north side, the component stresses
at the south side of the top flange are all compressive and therefore additive while one or more of the components are subtractive at each of the other corners. Fig. 5.14 shows how the combined flange stress at each corner, computed by the Timoshenko formula, varies along the span for a typical specimen and load.

Another factor which was investigated was the effect of the variation of the eccentricity of the vertical load on the computed maximum flange stress. In order to establish a basis for comparison, the applied load was modified so that the combination of load and eccentricity produced the same computed maximum stress by use of the Bethlehem Steel Co. formulae. Then the corresponding stress was computed by each of the other three methods. The resulting values are summarized in Fig. 5.15. Under these conditions of decreasing load with increasing eccentricity, the combined flange stress computed by the Timoshenko, Pettersson, and Sourochnikoff methods showed a gradual decline in magnitude.

Both of the non-linear solutions of the combined bending and torsion problem take into account the effect of the change in height of eccentric load application. As an example the Pettersson solution for the maximum combined stress when a 50 kip load is applied with an initial eccentricity of 10 inches first at midheight (\( \bar{a} = 0 \)) and then at the top of flange (\( \bar{a} = 27.5'' \)) is depicted in Fig. 5.16. The flange stress is increased by approximately 8% as a result of the increase in the height of load application in this particular case. The angular distortions would also increase due to this change.
D. **Comparison of Computed and Experimental Values**

Using various combinations of constants in the selected analytical methods, the important stress and distortion functions were computed. Among these were the variation along the span of the normal stress at the flange corners, the web angle of twist, and the combined shear stress in the flange and web. These computed values were compared graphically with the results obtained from the experimental data. It is to be noted that the amount of calculations involved increased rapidly as refinements are introduced into the theory.

With each of the methods used, it was possible to obtain a computed value for the combined normal stress at midspan. These values are plotted at the centerline and identified by a distinctive symbol. However, only the Timoshenko theory could be used directly to compute values along the span. For comparative purposes a modified Pettersson curve was obtained by proportion. Values at intermediate points were computed by multiplying the Timoshenko value at that point by the ratio of the Pettersson to Timoshenko stress at midspan.

The experimental values were reduced from the test data obtained from SR-4 strain gages stationed along the span. A typical comparison of experimental versus computed flange normal stress is shown graphically in Fig. 5:17 for the bolted specimen T10-B. There was some scatter in the experimentally determined points. In general the modified Pettersson curve afforded a closer approximation to test values than the Timoshenko solution, which was consistently low in the region of high values. The approximate Bethlehem Steel Co. solution was also off appreciably on the
unsafe side at midspan. The Sourochnikoff curve would have fallen slightly under the Pettersson curve, based on previous studies. After making allowances for experimental variations, it appears that the approximate methods, although simpler to apply, give combined stresses which are incorrect on the unsafe side.

Computation "H", developed in connection with the investigation of non-uniform torsion of plate girders, provides another method of evaluating the veracity of the measured strains and the computed stresses. This method, based on measured angles of twist along the span, theoretically adjusts for possible deviations from assumed test conditions.

The experimental data from a typical riveted specimen test are plotted against the Timoshenko and Computation "H" solutions in Fig. 5:18. As in the previous case, the Timoshenko and Bethlehem Steel Co. solutions are undervalued. The Computation "H" curve fits the plot of measured values quite well.
The resultant shear stress in the flange is computed by superimposing the torsional and transverse shear components. Both are based on certain assumptions regarding the distribution of forces across the flange. The procedure for combined bending and torsion loading is essentially the same as for the non-uniform torsion case.\(^{(11)}\)

The torsional shear stress, based on the de Vries assumption, varies directly with the effective thickness of the flange. The transverse shear stress is assumed to vary parabolically across the flange.

Two longitudinal gage lines were selected for study: the flange centerline and a line between the rivet line and the edge of the cover plates. The Timoshenko formula was used to compute the variation along the span based on certain design constants. The curve for the flange centerline is shown solid, while that for the outside gage line is dashed.

The AX-5 strain gage data were converted into normal and shear stresses by the conventional relationships. The resultant shear stress was plotted as circled points at the respective stations. Typical results for the bolted and riveted specimen are shown in Figs. 5:19 and 5:20.

In spite of the relatively low range of measured stress and the approximations made in the theoretical computations, the check of computed and experimental values was reasonably good. It was concluded that the maximum shear stresses in the flange along the centerline can be predicted adequately by the Timoshenko formula.
The resultant shear stress in the web is made up of two parts; namely, the torsional and the transverse shear stresses. The first component is essentially a shear force per unit length which follows a tip-to-tail procession around the section. The torsional shear stress varies directly as the torsional shear torque which in turn varies with the unit angle of twist along the span. The transverse shear stress due to vertical bending is assumed to be uniformly distributed over the web area and proportional to the vertical shear.

For the specimens as loaded, the two shear stresses are additive on the north side of the web and subtractive on the south side. The Timoshenko method, based on certain design constants, was used to compute the torsional shear stress. The combined shear stress was then determined and plotted along the span.

The AX-5 strain gage readings were reduced to bending and shear stresses. The shear stresses were plotted at the respective stations as circled points. Fig. 5:21 shows the maximum combined shear stresses from a typical load test while the minimum values are depicted in Fig. 5:22.

Here again the check of experimental and computed values was good in view of the relatively low range of stress. Even in the case shown in Fig. 5:22 where the sign of the shear stress underwent a reversal, the theoretical variation was reasonably well matched.
CHAPTER VI CONCLUSIONS

A. Evaluation of Test Program

An extensive experimental investigation of built-up structural members subjected to torsion loading was laid out by Dr. Bruce Johnston while Director of the Fritz Engineering Laboratory. The test specimens ranged from a series of flat plates to full-size plate girders, which were fabricated as bolted, riveted or welded units. The loading conditions included pure torsion, non-uniform torsion and finally combined bending and torsion.

For rolled beams and equivalent welded sections, a small-scale model of the proper proportions could have been tested with reasonable assurance that correlation between prototype and model would be maintained. However, for built-up sections held together by connectors the scale effect would be difficult to control and evaluate. In order to by-pass this factor, full size specimens utilizing conventional modes of fabrication were used in this investigation.

On the other hand the use of full-size specimens in place of models introduces certain requirements and limitations. Larger capacity units must be provided for handling and testing. Material and operating costs are increased. The longer period per test reduces the number of set-ups that can be investigated.

The calibration tests of the plate girder specimens, conducted for the purpose of evaluating section constants, gave a good indication of results that could be expected later in the combined bending and torsion tests. The vertical bending test, for example, after several
modifications, yielded consistent results between computed and experimental values. The preliminary tests also served to check out the individual SR-4 strain gages as well as the dial gage method of measuring displacements. In general, the experimental data obtained from the combined bending and torsion tests were adequate and of acceptable quality.

B. Summary of Test Results

Comparison of observed and predicted behavior of the plate girder specimens led to the following conclusions:

1. The mode of fabrication has an influence on the effective value of section properties.

   For example, based on the vertical bending tests, the effective value of $I_x$ for the riveted specimen was approximately 5% higher than that for the bolted version of identical section. A more sensitive section property is the torsion constant of a plate girder. From the pure torsion performed by Chang and Johnston, (10) the torsion constant for the riveted girder was found to be about 18% higher than that for the corresponding bolted specimen. It must be pointed out that these differences were experimentally determined for certain specimens as fabricated. Since fabrication techniques, especially in bolted construction, can vary over a wide range, no single value would be applicable in every case.
2. The linear theory of combined bending and torsion can be used to predict the flange and web shear stresses.

When applied to the plate girders tested, the comparison of computed and measured shear stresses was favorable in spite of certain assumptions made regarding integral action behavior. It is to be noted that the shear condition was not critical in these specimens and the shear stresses were relatively low in magnitude.

3. The linear theory of combined bending and torsion yields undervalued predictions of flange normal stresses.

The magnitude of the measured normal stresses in the flange was generally larger than that computed by the approximate methods. This discrepancy on the unsafe side was in line with expectations. Both the Bethlehem Steel Co. and Timoshenko methods neglect the amplifying effect of deflections. These approximate methods are relatively simple to apply. However, it is to be expected that the difference between actual and computed values of maximum combined normal stress will increase as the section becomes deeper and more slender.

4. Analytical methods based on the non-linear theory of combined bending and torsion can predict the normal flange bending stress condition.

The computed values of the normal stress in the flange of the plate girders tested checked reasonably well with measured
values at corresponding points. This improved check was expected since certain provisions for the effect of deflections have been incorporated in both the Sourochaikoff and Pettersson solutions. For the particular combination of section properties and loading conditions used in this investigation, computed stresses by the two non-linear methods did not differ appreciably.
APPENDIX A

BIBLIOGRAPHY


(4) Timoshenko, S. "Theory of Bending Torsion and Bending of Thin-walled Members of Open Cross Sections." Journal of the Franklin Institute, Vol. 239, April, 1945.


APPENDIX B
ACKNOWLEDGMENTS

The investigation of plate girder behavior under combined bending and torsion loading was suggested by Dr. Bruce G. Johnston, while Director of the Fritz Engineering Laboratory of Lehigh University, as a logical extension of the uniform and non-uniform torsion study.

The test program was carried out at Swarthmore College with the cooperation and guidance of Professor Samuel Carpenter, Chairman of the Department of Civil Engineering. Mr. Ed Kasten and other members of the laboratory staff provided valuable technical assistance in many phases of the experimental study.

Professor William J. Eney, Head of the Department of Civil Engineering, Lehigh University, and present Director of the Fritz Engineering Laboratory, has lent invaluable aid as staff coordinator in all administrative and fiscal matters and in the preparation of the final report.

Consultant on various technical aspects of the project has been Mr. Karl de Vries of the Bethlehem Steel Company.

The Pennsylvania Department of Highways and the Bureau of Public Roads have been most cooperative as co-sponsors of all three phases of this project. The sponsors were represented by Messrs. H. G. Van Riper and W. R. Herman of the Department of Highways and Messrs. Raymond Archibald, E.L. Erickson, Jacob Ediehman, and Neil Van Eanam of the Bureau of Public Roads.

The experimental phase, the analytical study, and the compilation of the report have been supervised by Dr. Gerald G. Kubo acting as Project Director.
ILLUSTRATIONS
### SUMMARY OF TEST SPECIMENS

#### COMBINED BENDING AND TORSION TESTS

**Phase 3** Series B Deep Girders

<table>
<thead>
<tr>
<th>Mark</th>
<th>T10-B</th>
<th>T11-B</th>
<th>T12-R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sketch</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Sketch](image)

**Notes:**

- All specimens were 38' 0" long
- All specimens were 48 1/2" b. to b. of angles
- All specimens were made up of the same material
  - 1-Web Plate 48 x 1/2"
  - 4-Flange angles 6 x 6 x 5/8
  - Cover Plates 14 x 5/8 (where used)
- Holes for connectors were spaced at 2 1/2" c. to c. except at stiffeners
- Stiffeners were spaced 3' 8" apart

---

**Combined Bending and Torsion of Plate Girders**

Testing Program conducted at Swarthmore College
Lehigh University - Fritz Engineering Laboratory Project 215B
**Fig. 4.3**

**SUMMARY OF TORSIONAL PROPERTIES**

<table>
<thead>
<tr>
<th>Torsion Constant $K$ (in. $^4$)</th>
<th>Specimen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swarthmore College Mark</td>
<td>T10-B</td>
</tr>
<tr>
<td>Lehigh University Mark</td>
<td>T6A-4</td>
</tr>
</tbody>
</table>

![Sketch](image)

**A. Experimental Value**

(Based on Test)

1. Test $(K)_A$  
   - $59.60$  
   - $73.10$  
   - $--$

**B. Integral-Action Constant**

(Computed)

1. $(K_I)_B$  
   - $62.43$  
   - $62.43$  
   - $--$
2. $(K_I)_C$  
   - $66.74$  
   - $66.74$  
   - $--$

**C. Separate-Action Constant**

(Computed)

1. $(K_S)_D$  
   - $10.28$  
   - $10.28$  
   - $5.89$

Test $(K)_A$ - Experimental constant based on uniform torsion test

$(K_I)_B$ - Integral-action constant, neglecting edge and hump effects

$(K_I)_C$ - Integral-action constant, incorporating edge and hump effects.

$(K_S)_D$ - Separate-action constant, assuming elements act separately.

Computed values based on nominal dimensions

Uniform torsion tests conducted at Lehigh University by Chang and Johnston \(^{10}\) on Phase I specimens

Combined Bending and Torsion of Plate Girders
Lehigh University - Fritz Engineering Laboratory Project 215B
Fig. 4.5
Eccentric Loading System I
Concentrated Vertical Load at Midspan
Acting at Top of Flange
Section "C"

Combined Bending and Torsion of Plate Girders
Testing program conducted at Swarthmore College
Lehigh University - Fritz Engineering Laboratory Project 215B

Fig. 4.4
Schematic Sketch Test Set-Up

Combined Bending and Torsion of Plate Girders
Testing program conducted at Swarthmore College
Lehigh University - Fritz Engineering Laboratory Project 215B
FIG. 4:6

ECCENTRIC LOADING SYSTEM II

CONCENTRATED VERTICAL LOAD AT MIDSPAN
ACTING AT MIDHEIGHT
SECTION "C2"

Combined Bending and Torsion of Plate Girders
Testing Program conducted at Swarthmore College
Lehigh University - Fritz Engineering Laboratory Project 215B
Combined Bending and Torsion of Plate Girders
Testing program conducted at Swarthmore College
Lehigh University - Fritz Engineering Laboratory Project 215B
Combined Bending and Torsion of Plate Girders

Testing program conducted at Swarthmore College
Lehigh University - Fritz Engineering Laboratory Project 215B
Combined Bending and Torsion of Plate Girders
Testing program conducted at Swarthmore College
Lehigh University - Fritz Engineering Laboratory Project 215B
FIG. 10  DIAL GAGE SUPPORT FRAME
LOADING HEAD ON BRACKET

FIG. 11  OVERALL VIEW OF TEST SET-UP
### Fig. 4:12 COMBINED BENDING AND TORSION OF PLATE GIRDERS

#### SUMMARY OF TEST PROGRAM

<table>
<thead>
<tr>
<th>Specimen and Test No.</th>
<th>T10-B</th>
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<th>T12-R</th>
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<td>301 A &amp; B</td>
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<td>T6B-1 Chang</td>
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**Phase 1:** Tests at Lehigh University -- by Chang (Pure Torsion)

**Phase 2:** " " " -- by Kubo (Non-Uniform Torsion)

**Phase 3:** " " Swarthmore College -- by Kubo (Combined Bending and Torsion)
FIG. 4:13
MANEUVERING SPECIMEN
INTO LABORATORY

FIG. 4:14
ROTATING SPECIMEN FROM VERTICAL TO HORIZONTAL POSITION
Fig. 5:1

**SUMMARY OF TENSION TESTS OF COUPONS**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Coupon Location</th>
<th>Modulus Elasticity E (ksi)</th>
<th>Ultimate Stress (ksi)</th>
<th>% Elongation in 2 in.</th>
<th>% Reduction of Area</th>
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<td>Avg. Flange Angles</td>
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<tr>
<td>TA-1 Web Plate</td>
<td>30,600</td>
<td>58.70</td>
<td>46</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>TA-2 Web Plate</td>
<td>31,800</td>
<td>58.70</td>
<td>46</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>TA-3 Web Plate</td>
<td>30,050</td>
<td>58.40</td>
<td>--</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>TA-4 Web Plate</td>
<td>29,900</td>
<td>58.40</td>
<td>--</td>
<td>58</td>
<td></td>
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<tr>
<td>Avg. Web Plate</td>
<td>30,590</td>
<td>58.55</td>
<td>46</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>TC-1 Cover Plate</td>
<td>31,600</td>
<td>69.40</td>
<td>45</td>
<td>56</td>
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<tr>
<td>TC-2 Cover Plate</td>
<td>30,400</td>
<td>66.60</td>
<td>46</td>
<td>55</td>
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<tr>
<td>Avg. Cover Plate</td>
<td>31,000</td>
<td>68.00</td>
<td>45.5</td>
<td>55.5</td>
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<tr>
<td>TC-3 Cover Plate at 45°</td>
<td>32,700</td>
<td>65.60</td>
<td>44</td>
<td>64</td>
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<tr>
<td>TC-4 Cover Plate at 45°</td>
<td>31,400</td>
<td>67.70</td>
<td>43</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>Avg. Cover Plate at 45°</td>
<td>32,050</td>
<td>66.65</td>
<td>43.5</td>
<td>58.5</td>
<td></td>
</tr>
<tr>
<td>TSB-1 Stiffener Angles</td>
<td>32,100</td>
<td>64.00</td>
<td>42</td>
<td>60</td>
<td></td>
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<tr>
<td>TSB-2 Stiffener Angles</td>
<td>33,000</td>
<td>63.50</td>
<td>40</td>
<td>61.5</td>
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<tr>
<td>Avg. Stiffener Angles</td>
<td>32,550</td>
<td>63.75</td>
<td>41</td>
<td>61</td>
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<tr>
<td>Avg. of All Specimens</td>
<td>31,110</td>
<td>63.07</td>
<td>45</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>Avg. of Groups I, II, &amp; III</td>
<td>30,510</td>
<td>63.15</td>
<td>46.5</td>
<td>56.5</td>
<td></td>
</tr>
<tr>
<td>Avg. of Groups I, II, III &amp; IV</td>
<td>30,900</td>
<td>64.03</td>
<td>45.8</td>
<td>57.0</td>
<td></td>
</tr>
</tbody>
</table>

**ASTM - 2.7 - 46 Specifications**

---

Combined Bending and Torsion of Plate Girders
Tensile Tests conducted at Lehigh University
Lehigh University - Fritz Engineering Laboratory Project 215B
Fig. 5:2  VERTICAL DEFLECTION ALONG SPAN - EXPERIMENTAL vs. COMPUTED

VERTICAL BENDING TEST - CONCENTRATED LOAD AT CENTERLINE
Fig. 5.3  VERTICAL BENDING STRESS ALONG SPAN - EXPERIMENTAL vs. COMPUTED

VERTICAL BENDING TEST - CONCENTRATED LOAD AT CENTERLINE
Fig. 5:4  VERTICAL BENDING STRESS ALONG SPAN - EXPERIMENTAL VS. COMPUTED

VERTICAL BENDING TEST - CONCENTRATED LOAD AT CENTERLINE
**Fig. 5:5**

**SUMMARY OF VERTICAL BENDING PROPERTIES**

Moment of Inertia $I_x$ (in.4)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>T10-B</th>
<th>T12-R</th>
<th>T11-B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Swarthmore Test No.</strong></td>
<td>101</td>
<td>301A</td>
<td>301B</td>
</tr>
<tr>
<td><strong>Sketch</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bolted</td>
<td>Riveted</td>
<td>Bolted</td>
<td></td>
</tr>
</tbody>
</table>

**A. Computed Values**

1. Gross | 40,800 | 40,800 | 40,800 | 19,130 |
3. Weighted $I_x$ | 39,000 | 39,000 | 39,000 | 18,300 |

**B. Test Values**

1. Deflection Curve | 36,600 | 29,000 | 39,200 |
2. Normal Stress Curve | 35,700 | 37,000 | 37,500 |
3. Avg. Effective $I_x$ | 36,150 | 38,200 |

**Notes:**

Test 101 - Concentrated load at centerline
Test 301A - Concentrated load at centerline
Test 301B - Concentrated loads at two symmetrical points
Test 202 - Concentrated load at centerline

Combined Bending and Torsion of Plate Girders Testing Program conducted at Swarthmore College Lehigh University - Fritz Engineering Laboratory Project 215B
Fig. 5:6 LATERAL BENDING STRESS ALONG SPAN - EXPERIMENTAL vs. COMPUTED
LATERAL BENDING TEST - CONCENTRATED LOAD AT CENTERLINE
Fig. 5:7  VERTICAL DEFLECTIONS ALONG SPAN - EXPERIMENTAL vs. COMPUTED

LATERAL BENDING TEST - CONCENTRATED LOAD AT CENTERLINE
Fig. 5.8

SUMMARY OF LATERAL BENDING PROPERTIES

Moment of Inertia $I_y$ (in.²)

<table>
<thead>
<tr>
<th>Lehigh Mark</th>
<th>Specimen</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Swarthmore Test No.</td>
<td>T10-B</td>
<td>T12-R</td>
<td>T11-B</td>
</tr>
<tr>
<td></td>
<td>114</td>
<td>--</td>
<td>201</td>
</tr>
</tbody>
</table>

![Sketch]

<table>
<thead>
<tr>
<th></th>
<th>Bolted</th>
<th>Riveted</th>
<th>Bolted</th>
</tr>
</thead>
</table>

C. Computed

1. Gross
   - T10-B: 780
   - T12-R: 780
   - T11-B: 209
2. Net
   - T10-B: 604
   - T12-R: 604
   - T11-B: 150
3. Weighted $I_y$
   - T10-B: --
   - T12-R: --
   - T11-B: --

D. Test Values

1. Deflection Curve
   - T10-B: 706
   - T12-R: *
   - T11-B: 170
2. Normal Stress Curve
   - T10-B: 640
   - T12-R: *
   - T11-B: 156
3. Avg. Effective $I_y$
   - T10-B: 673
   - T12-R: *
   - T11-B: 163

Notes:

Test 114 - Concentrated load at centerline
Test 201 - Concentrated load at centerline

* No experimental values are available. Corresponding values from T10-B were used in the computations.

Combined Bending and Torsion of Plate Girders
Testing program conducted at Swarthmore College
Lehigh University - Fritz Engineering Laboratory Project 215B
**Fig. 5:9**

**SUMMARY OF NOTATION**

<table>
<thead>
<tr>
<th>Series</th>
<th>Method of Computation</th>
<th>End Condition</th>
<th>Comb. of Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>T</td>
<td>S</td>
</tr>
<tr>
<td>1</td>
<td>B1</td>
<td>T1</td>
<td>S1</td>
</tr>
<tr>
<td>2</td>
<td>B2</td>
<td>T2</td>
<td>S2</td>
</tr>
<tr>
<td>3</td>
<td>B3</td>
<td>T3</td>
<td>S3</td>
</tr>
<tr>
<td>4</td>
<td>B4</td>
<td>T4</td>
<td>S4</td>
</tr>
<tr>
<td>5</td>
<td>B5</td>
<td>T5</td>
<td>S5</td>
</tr>
<tr>
<td>6</td>
<td>--</td>
<td>--</td>
<td>--</td>
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</table>

**Notes:**

<table>
<thead>
<tr>
<th>Method of Computation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>B - B. S. Co. Formula</td>
<td>(Approximate - Linear theory)</td>
</tr>
<tr>
<td>T - Timoshenko</td>
<td>(    )</td>
</tr>
<tr>
<td>S - Sourouchnikoff</td>
<td>(Deflection - Non-linear theory)</td>
</tr>
<tr>
<td>P - Pettersson</td>
<td>(    )</td>
</tr>
<tr>
<td>H - Based on measured distortions</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Combination of Constants</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A - Test values of ((K_A))</td>
<td>Test (I_x) (\text{Text } I_y)</td>
</tr>
<tr>
<td>B - Computed Design ((K_I)_B)</td>
<td>Gross (I_x) Gross (I_y)</td>
</tr>
<tr>
<td>C - Computed Theoretical ((K_I)_C)</td>
<td>Weighted (I_x) Weighted (I_y)</td>
</tr>
<tr>
<td>D - Computed ((K_S)_D)</td>
<td>Net (I_x) Net (I_y)</td>
</tr>
</tbody>
</table>

Combined Bending and Torsion of Plate Girders
Testing Program conducted at Swarthmore College
Lehigh University - Fritz Engineering Laboratory Project 215B
### COMPARISON OF STRESS COMPONENTS

#### Maximum Combined Normal Stress at Centerline

**SUMMARY OF COMPUTED VALUES**

Specimen T10-B-103  \( e = 10'' \)
Bolted  \( P = 50 \) kip  \( a = 27.5'' \)

<table>
<thead>
<tr>
<th>Computation</th>
<th>Comb. of Constants</th>
<th>Normal Stress (k.s.i.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series</td>
<td>Method</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>B1</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>T1</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>B</td>
</tr>
<tr>
<td>II</td>
<td>B2</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>A</td>
</tr>
</tbody>
</table>

#### Method of Computation
- **B** - B. S. Co. Formula
- **T** - Timoshenko
- **S** - Souruchnikoff
- **P** - Pettersson

#### Combination of Constants
- **A** - Test  \( K_x, K_y \)
- **B** - Design  \( K_x, K_y \)
- \( \sigma_{b_x} \) - Vertical bending stress
- \( \sigma_{b_y} \) - Lateral stress
- \( \sigma_T \) - Torsional stress

Combined Bending and Torsion of Plate Girders
Testing Program conducted at Swarthmore College
Lehigh University - Fritz Engineering Laboratory Project 215B
Notes:
Computed Value
△ Pettersson Formula P1

Fig. 5:11 COMPONENTS OF FLANGE STRESS AT CENTERLINE VS. LOAD -- COMPUTED
Fig. 5:12 MAXIMUM COMBINED FLANGE STRESS VS. LOAD -- COMPUTED

Notes:
Computed Values
× B. S. Co. Formula Bl
- Timoshenko " T1
□ Sourouchnikoff " S1
△ Pettersson " P1

T10-B-102

$e = 5''$
$P = 79$ kip.
$\bar{a} = 27.5''$
Bolted

C. B. & T.
Fig. 5.13 LOAD VS. MAXIMUM ANGLE OF TWIST AT CENTERLINE -- COMPUTED

Notes:
Computed Values

- B. S. Co. Formula Bl
- Timoshenko " T1
- Sourochnikoff " S1
- Pettersson " P1

T10-B-102

- e = 5"
- P = 79 kip
- \( \bar{a} = 27.5" \)
- Bolted
- C. B. & T.
Fig. 5:14  COMPUTED COMBINED FLANGE STRESS AT EACH CORNER ALONG SPAN
FIG. 5:15

EFFECT OF VARIATION OF ECCENTRICITY ON COMPUTED MAXIMUM FLANGE STRESS

<table>
<thead>
<tr>
<th>ECCENTRICITY (e, in.)</th>
<th>VERTICAL LOAD (P, kip.)</th>
<th>BETHLEHEM STEEL CO.</th>
<th>TIMOSHENKO</th>
<th>PETTERSSON</th>
<th>SOUFOKNIKOFF</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B 2 h</td>
<td>T 2 h</td>
<td>P 2 h</td>
<td>S 2 h</td>
</tr>
<tr>
<td>5</td>
<td>79</td>
<td>16.29</td>
<td>18.76</td>
<td>21.55</td>
<td>21.61</td>
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<tr>
<td>10</td>
<td>48.5</td>
<td>16.29</td>
<td>18.20</td>
<td>19.82</td>
<td>19.89</td>
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<tr>
<td>15</td>
<td>35</td>
<td>16.29</td>
<td>17.73</td>
<td>18.82</td>
<td>18.92</td>
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<tr>
<td>20</td>
<td>27.3</td>
<td>16.29</td>
<td>17.39</td>
<td>18.24</td>
<td>18.37</td>
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</table>

Notes:

Specimen T 10 - B

a = 27.5"

Compression stress at top south corner
FIG. 5:16  
EFFECT OF CHANGE OF HEIGHT OF LOAD APPLICATION  
Comparison of Computed Maximum Flange Stress Along Span - Pettersson Theory
Fig. 5:17  Experimental vs. Computed $\sigma$ - Max. Normal Stress in Flange
Fig. 5:18  Experimental vs. Computed $\sigma$—Max. Normal Stress in Flange
Fig. 5:19  EXPERIMENTAL VS. COMPUTED  COMBINED FLANGE SHEAR
Fig. 5:20  EXPERIMENTAL VS. COMPUTED  COMBINED FLANGE SHEAR
L/2 = 220"

Notes:

* Exp. Value -- Avg. Web Shear
  North Side

Computed Value
  -- Timoshenko Formula T1

Fig. 5.21  EXPERIMENTAL VS. COMPUTED  COMBINED WEB SHEAR (MAX.)
Fig. 5-22. EXPERIMENTAL VS. COMPUTED COMBINED WEB SHEAR (MIN.)