Shell arch roof model under dead load and half side live load, September 1950

B. Thurlimann
B. G. Johnston

Follow this and additional works at: http://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports

Recommended Citation
http://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports/1462
PROGRESS REPORT 213D

SHELL ARCH ROOF MODEL UNDER SIMULATED DEAD LOAD AND HALF SIDE LIVE LOAD

by

Bruno Thürlimann
Bruce G. Johnston

to

Roberts and Schaefer Engineering Company

Fritz Engineering Laboratory
Department of Civil Engineering and Mechanics
Lehigh University
Bethlehem, Penna.

September 19, 1950
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Summary</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Part I: Theoretical Analysis</td>
<td>2</td>
</tr>
<tr>
<td><strong>Problem</strong></td>
<td>2</td>
</tr>
<tr>
<td>1. Membrane Solution for a Cylindrical Shell</td>
<td>3</td>
</tr>
<tr>
<td>Continuous Over Two Spans</td>
<td></td>
</tr>
<tr>
<td>2. Loading of the Ribs</td>
<td>4</td>
</tr>
<tr>
<td>3. Calculation of the Arches for Dead Load (symmetrical load case)</td>
<td>6</td>
</tr>
<tr>
<td>a.) Arches Under Shear Forces S</td>
<td>6</td>
</tr>
<tr>
<td>b.) Arches Under String Force Y</td>
<td>9</td>
</tr>
<tr>
<td>4. Calculation of the Arches for Anti-symmetrical Load</td>
<td>11</td>
</tr>
<tr>
<td>a.) Arches Under Shear Forces S and Horizontal Loads P</td>
<td>13</td>
</tr>
<tr>
<td>b.) Arches Under String Force Y</td>
<td>15</td>
</tr>
<tr>
<td>5. Deflection of the Arches</td>
<td>17</td>
</tr>
<tr>
<td>6. Approximate Calculation of the Shell Forces</td>
<td>17</td>
</tr>
<tr>
<td><strong>Part II: Numerical Analysis</strong></td>
<td>13</td>
</tr>
<tr>
<td>Effective Cross Section of the Ribs</td>
<td>18</td>
</tr>
<tr>
<td>List of Principal Dimensions and Datas</td>
<td>20</td>
</tr>
<tr>
<td>1. Membrane Solution</td>
<td>21</td>
</tr>
<tr>
<td>2. Loading of the ribs due to the Boundary Forces</td>
<td>22</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (cont'd)

3. Symmetrical Load Case (dead load) 23
4. Anti-symmetrical Load Case 25
5. Dead Load 25
   a.) Ribs 25
   b.) Shell Forces for $\omega = 0$ 27
6. Live Load over the Half Span 27
   a.) Ribs 28
   b.) Deflection of the Ribs 29

Part III: Experimental Investigation 31
1. Description of Model and Test Set-up 31
2. Test Procedure 32
3. Test Results 33
4. Comparison between Test Results and Analysis 34
5. Relation between the Model and an Actual Structure 36

Conclusion 38

Tables I to VI 39-50

Figures 1 to 17

Notations

List of References
Progress Report 213D

Shell Arch Roof Model under Simulated Dead Load and Half Side Live Load

by

Bruno Thürlimann and Bruce G. Johnston

Summary

The analysis of a shell arch roof model (Fig. 1) under two cases of vertical loads simulating (1) dead load and (2) uniformly distributed live load (e.g., snow) over the half span (Fig. 2), is developed. The analytical results are compared with experimental results obtained from tests performed on an actual model.

Introduction

Uniform load over the whole and over the half span are the two most important load cases for the present type of a structure. The half span load very nearly gives the maximum extreme fiber stress due to bending.

The actual loading of the model (Fig. 2) is not a uniformly distributed one as assumed in the analysis. This does not influence the stress distribution in the ribs to a great extent. However, local differences between the experimental and analytical shell forces, especially the bending moments, must be expected. The problem of the ribs is therefore treated in a more complete way. For the shell forces an approximate procedure is used, similar to the one in Progress Report 213C. The problem of the edge-member disturbance is not taken up
because a separation of the different influences for the present type of a model is too involved.

Part I: Theoretical Analysis

Problem:

The structure, as shown in Fig. 1, is highly statically indeterminate. A first step to a solution is the analysis of the shell as a continuous member over two spans, supported by three diaphragms (Fig. 3). The boundary forces (shear forces $S$) and the deformations along the boundaries (strains $\varepsilon_\omega$ in circumferential direction) give the loading conditions for the ribs. The condition of continuity between ribs and shell are fulfilled, if the effective cross-section* of the ribs is taken (See Ref. (1)\(^\dagger\) p. 28 and Ref. (2)\(^\dagger\) p. 3).

The following problems have to be studied:

1. Membrane solution of a cylindrical shell, continuous over two spans.
2. Ribs (effective section*) under the action of shear force $S$.
3. Ribs (effective section under the action of a string force $Y$, eliminating the relative difference

* By "effective section" of the rib, the cross section consisting of the rib and a flange of width equal to the effective width is meant. On the following pages the term is used with this meaning.

\(^\dagger\) (1), (2) See list of references at end of report.
in strain $\varepsilon_\omega$ between rib and shell due to the membrane solution.

The dead load presents a symmetrical case of loading. The ribs are twice statically indeterminate (Fig. 4).

Live load over the half span can be found by superposition of a symmetrical and an anti-symmetrical load case (Fig. 4). The advantage of this solution is the splitting of the three redundant forces of the rib in two independent groups of two and one redundant.

1.) Membrane solution for a cylindrical shell continuous over two spans:

Dischinger, Ref. (4) p. 227, treats the problem of the continuous stiffened cylindrical shell in a very complete way. Flügge, Ref. (3) p. 76, presents the case of a shell continuous over two spans. In the following direct use of the equations derived by Flügge, Ref. (3), is made. The notation is changed to conform with the notation used in the previous progress reports.

The membrane forces in a continuous shell, Fig. 3, under a load of $p$ lb/in$^2$ of shell surface are: (Flügge, Ref. (3), p. 79)

$$T_2 = -p R_s \cos \omega$$ (1)

$$S = -p \sin \omega \left(2 \bar{x} - \frac{\frac{5}{2} l^2 + 6(4+3\nu) R_s^2}{4 l^2 + 6(1+\nu) R_s^2} \right)$$ (2)

$$T_1 = \frac{P}{R_s} \cos \omega \left[ \bar{x}^2 - \frac{\frac{5}{2} l^2 + 6(4+3\nu) R_s^2}{4 l^2 + 6(1+\nu) R_s^2} \bar{x} + \frac{l^2}{4} \frac{l^2 - 6\nu R_s^2}{l^2 + 6(1+\nu) R_s^2} \right]$$ (3)
The shear force $S$ becomes zero for

$$
\overline{x}_o = \frac{L}{8} \frac{5 \ell^2 + 6(4+3v) R^2_S}{\ell^2 + 6(1+v) R^2_S}
$$

(4)

At the middle and outer ribs, the $S$-forces are:

- Middle rib: $S \mid \overline{x} = \bar{x}_o = 2p \bar{x}_o \sin \omega$
  
  (5)

- Outer rib: $S \mid \overline{x} = \bar{x} = -2p(\ell - \bar{x}_o) \sin \omega$
  
  (6)

The normal strain in circumferential direction is:

$$
E \varepsilon_\omega = \frac{T_2}{d} = -\frac{pR_o}{d} \cos \omega
$$

(7)

2.) Loading of the ribs:

The membrane solution resulted in boundary shear forces $S$, Eq. (5) and (6), and in normal strain $\varepsilon_\omega$ in circumferential direction, Eq. (7). The discontinuity caused by the $\varepsilon_\omega$ between the rib and the shell has to be eliminated.

The shear forces $S$ give the following loading condition:

From Eq. (5) and (6)

$$
S = \bar{S} \sin \omega
$$

(8)

Where: Outer rib: $\overline{S} \mid \overline{x} = \bar{x} = -2p(\ell - \overline{x}_o)$

Middle rib: $\overline{S} \mid \overline{x} = \bar{x}_o = 2p \overline{x}_o$

$\overline{x}_o$ as given by (4)

Applying a string force $Y$ at $\overline{x} = \ell$ and $\overline{x} = 0$, the membrane strains $\varepsilon_\omega$ may be eliminated in order to fulfill the continuity condition between shell and rib. The magnitude of $Y$ is readily determined.* The strain $\varepsilon_\omega$ as function of $T_2$ is:

* Ref. (5) gives a very complete treatment of the boundary problem of cylindrical shells for the boundaries $x = \text{const.}$
\[ E \varepsilon_\omega = \frac{T_2}{d} \]

\( T_2 \) at the point of application of the string force \( Y \) is (Ref. (2), Eq. (3)):

\[ (T_2)_Y = \frac{Y}{b} \]

Where: \( b \) = effective width

Subscript \( Y \) indicates that \( T_2 \) is taken at the point of application of \( Y \).

The strain \( \varepsilon_\omega \) due to \( Y \) is:

\[ (E \varepsilon_\omega)_Y = \frac{Y}{bd} \]  \( (9) \)

To eliminate the membrane strain \( \varepsilon_\omega \), \( Y \) has to be: (Eq. (7) and (9))

\[ - \frac{pR_S}{d} \cos \omega + \frac{Y}{bd} = 0 \]

\[ Y = pR_S b \cos \omega \]

Putting \( pR_S b = Y \)

\[ Y = Y \cos \omega \]  \( (10) \)

In summary, the membrane solution of the shell has the boundary forces \( S \), Eq. (8) and \( Y \), Eq. (10). Introduced in the reversed direction they are the loads for the ribs. As cross-section of the ribs the effective one is used to take account of the interaction of rib and shell. (See Ref. (2), p. 3).
3.) **Calculation of the arches for dead load (symmetrical load case)**

The arches in an actual structure are under the following 3 loads (compare Fig. 5).

Eq. (8)

a.) Shear Force: \( S = \overline{S} \sin \omega \)

Eq. (10)

b.) String Force: \( Y = \overline{Y} \cos \omega \)

c.) Dead load of the rib (cross section of the rib only).

The ribs are analysed for the first two loads. The dead load of the rib itself is not treated because this load does not enter into the problem under investigation and in a general design problem it will not offer any difficulty.*

a.) Archs under shear forces \( S \):  

In Fig. 6a.) the statically determinate base system is shown. The normal force \( N_o \) and the bending moment \( M_o \) in the base system may be obtained by integrating the contribution of the distributed shear load \( S \) that is applied by the shell. If \( \omega \) is the angle at which \( N_o \) and \( M_o \) are determined, let \( \overline{S} \) \( \alpha \) represent the shear load applied over an incremental shell distance at any angle \( \alpha \) between 0 and \( \omega \) (Fig. 6b). Then

\[
N_o = - \int \overline{S} \cos(\omega - \alpha) \, d\alpha
\]

Substituting from Eq. (8), \( S = \overline{S} \sin \alpha \) and performing the integration,

\[
N_o = - \frac{1}{2} \overline{S} \omega \sin \omega
\]  

* In the test the "dead load" was simulated for the shell only as a uniformly distributed "live load" acting over the entire shell. See Fig. 2 and p. 31.
Similarly, the contribution to the moment $M_o$ of incremental shear load is $SR_s d\alpha$ multiplied by the moment arm $y_s - R_e \left[ 1 - \cos (\omega - \alpha) \right]$ as shown in Fig. 6b. The integrated total of $M_o$ then is,

$$M_o = \int_0^\omega SR_s \left[ y_s - R_e (1 - \cos (\omega - \alpha)) \right] d\alpha$$

substituting as before and integrating,

$$M_o = SR_s R_e \left[ (1 - \frac{y_s}{R_e}) (1 - \cos \omega) - \frac{1}{2} \omega \sin \omega \right]$$  \hspace{1cm} (12)

The normal force $N$ and the bending moment $M$ in the arch are:

$$N = \rho_o - H_c \cos \omega$$  \hspace{1cm} (13)

$$M = M_o + H_c R_e (1 - \cos \omega) + M_c$$  \hspace{1cm} (14)

The boundary conditions for $\omega = \omega_k$ determine the redundants $H_c$ and $M_c$:

$$\omega = \omega_k : \delta_k = 0$$

$$\phi_k = - \kappa M_k$$

Where: $\delta_k$ = horizontal displacement of the support

$\phi_k$ = rotation of the support

$\kappa$ = coefficient of elastic restraint*

$M_k$ = moment $M$ for $\omega = \omega_k$

$\delta_k$ and $\phi_k$ are found by means of the work equation.

Fig. 6c shows the virtual load system used to find the above deformations.

For $\delta_k$:

$$\begin{align*}
M' &= -R_e (\cos \omega - \cos \omega_k) \\
N' &= -\cos \omega
\end{align*}$$  \hspace{1cm} (15)

For $\phi_k$:

$$\begin{align*}
M' &= 1 \\
N' &= 0
\end{align*}$$  \hspace{1cm} (16)

* See Progress Report 213C, p. 5 ff.
Applying the work equation
\[ \mathcal{J} = \int \frac{M'N'ds}{EI} + \int \frac{N'N'ds}{EA} \]

\( \delta_k \) and \( \phi_k \) are:

Eq. 13, 14 and 15:
\[ \delta_k = \int \frac{1}{EI} \left[ -R_0(\cos \omega - \cos \omega_k) \right] \left[ M_0 + H_cR_0(1-\cos \omega) + M_c \right] R_0 d\omega \\
+ \int \frac{1}{EA} \left[ \cos \omega (H_c \cos \omega - N_0) \right] R_0 d\omega \]

Eq. 13, 14 and 16
\[ \phi_k = \int \frac{1}{EI} \left[ M_0 + H_cR_0(1-\cos \omega) + M_c \right] R_0 d\omega \]

Assuming EI and EA constant, the integration can be performed:
\[ \delta_k \frac{EI}{R_0^2} = M_c(\omega_k \cos \omega_k - \sin \omega_k) + H_cR_0 \left[ \omega_k \cos \omega_k + \frac{1}{2} \omega_k - \frac{1}{4} \sin 2\omega_k - \sin \omega_k \right] \\
+ \frac{1}{AI_0^2} \left[ \frac{1}{2} \omega_k + \frac{1}{4} \sin 2\omega_k \right] \]
\[ + \frac{3}{8} \omega_k \cos^2 \omega_k + \frac{1}{8} \omega_k \sin^2 \omega_k \]
\[ + \frac{1}{10} \frac{R_0^2}{AI_0} (\sin 2\omega_k - 2\omega_k \cos 2\omega_k) \] (17)

\[ \phi_k \frac{EI}{R_0} = M_c \omega_k + H_cR_0(\omega_k - \sin \omega_k) + \frac{3}{8} \omega_k \cos^2 \omega_k + \frac{1}{8} \omega_k \sin^2 \omega_k \]
\[ + \frac{1}{2} \omega_k \cos \omega_k - \frac{1}{2} \sin \omega_k \] (18)

Introducing in Eq. (17) and (18) the boundary conditions for \( \delta_k \) and \( \phi_k \) at \( \omega = \omega_k \),
\[ \delta_k = 0 \]
\[ \phi_k = -\tau M_k = -\tau M_c + H_c R_0 (1 - \cos \omega_k) + \bar{S} R_0 \left(1 - \frac{y_s}{R_0}\right) \]
\[(1 - \cos \omega_k) = \frac{1}{2} \omega_k \sin \omega_k] \]
then,
\[ M_c (\omega_k \cos \omega_k - \sin \omega_k) + H_c R_0 \left[ (\omega_k \cos \omega_k + \frac{1}{2} \omega_k - \frac{1}{4} \sin 2\omega_k - \sin \omega_k) \right. \]
\[ + \frac{1}{AR_0^2} \left( \frac{1}{2} \omega_k + \frac{1}{4} \sin 2\omega_k \right) \]
\[ = -\bar{S} R_0 \left[ (1 - \frac{y_s}{R_0}) (\omega_k \cos \omega_k + \frac{1}{2} \omega_k - \frac{1}{4} \sin 2\omega_k - \sin \omega_k) - \frac{3}{16} \sin 2\omega_k + \frac{3}{8} \omega_k \right. \]
\[ \cos \omega_k + \frac{1}{2} \omega_k \sin \omega_k + \frac{1}{16} \frac{1}{AR_0^2} (\sin 2\omega_k - 2\omega_k \cos 2\omega_k) \]
\[ M_c (\omega_k + \bar{\kappa}) + H_c R_0 \left[ \omega_k - \sin \omega_k + \bar{\kappa} (1 - \cos \omega_k) \right. \]
\[ = -\bar{S} R_0 \left[ (1 - \frac{y_s}{R_0}) \left( \omega_k - \sin \omega_k + \bar{\kappa} (1 - \cos \omega_k) \right) + \frac{1}{2} \omega_k \cos \omega_k \right. \]
\[ - \frac{1}{2} \sin \omega_k (1 + \bar{\kappa} \omega_k) \] \] \[ (19) \]
\[ (20) \]

Where: \[ \bar{\kappa} = \frac{E I}{R_0} \kappa \]

Eq. (19) and (20) determine the redundants \( M_c \) and \( H_c \) for given values of \( S \) and \( \kappa \). The numerical example in Part II is an illustration of the procedure.

b.) Arches under string force \( Y \):

The base system is the same as under a.). The normal force \( N_o \) and the bending moment \( M_o \) are readily determined from Fig. 6d:

\[ N_o = -Y \]
\[ M_o = -y_s Y \]
Substituting from Eq. (10), \( Y = \bar{Y} \cos \omega \)

\[
N_0 = - \bar{Y} \cos \omega \quad (21)
\]

\[
M_0 = - y_s \bar{Y} \cos \omega \quad (22)
\]

The normal force \( N \) and the bending moment \( M \) in the arch are:

\[
N = N_0 - H_c \cos \omega \quad (23)
\]

\[
M = M_0 + H_c R_e (1 - \cos \omega) + M_c \quad (24)
\]

The same boundary conditions as in case a.) hold. Using a virtual load system as shown in Fig. 6c, the work equation gives the following expressions for \( \delta_k \) and \( \phi_k \):

\[
\delta_k = \int_0^{\omega} \frac{1}{EI} \left[ - R_e (\cos \omega - \cos \omega_k) \right] \left[ M_0 + H_c R_e (1 - \cos \omega) + M_c \right] R_e d\omega
\]

\[
+ \int_0^{\omega} \frac{1}{EI} \cos \omega (H_c \cos \omega - N_0) R_e d\omega
\]

\[
\phi_k = \int_0^{\omega} \frac{1}{EI} \left[ M_0 + H_c R_e (1 - \cos \omega) + M_c \right] R_e d\omega
\]

Replacing \( N_0 \) and \( M_0 \) by Eq. (21) and (22) and assuming \( EI \) and \( EA \) constant, then:

\[
\frac{EI}{R_e} \delta_k = M_c (\omega_k \cos \omega_k - \sin \omega_k) + H_c R_e \left[ (\omega_k \cos \omega_k + \frac{1}{2} \omega_k - \frac{1}{4} \sin \omega_k) \sin \omega_k \right]
\]

\[
+ \frac{1}{AR_e} \left[ \frac{1}{2} \omega_k^2 + \frac{1}{4} \sin \omega_k \right]
\]

\[
+ \bar{Y} \left[ y_s (\frac{1}{2} \omega_k - \frac{1}{4} \sin \omega_k) + \frac{1}{AR_e} (\frac{1}{2} \omega_k^2 + \frac{1}{4} \sin \omega_k) \right]
\]

\[
\frac{EI}{R_e} \phi_k = M_c \omega_k + H_c R_e (\omega_k - \sin \omega_k) - y_s \bar{Y} \sin \omega_k
\]

The boundary conditions for \( \delta_k \) and \( \phi_k \) are:

\[
\delta_k = 0
\]
\[ \phi_k = -\kappa M_k = -\kappa \left[ M_c + H_c R_e (1 - \cos \omega_k) - y_S \bar{Y} \cos \omega_k \right] \]

The equations for the 2 redundants \( M_c \) and \( H_c \) have the final form:

\[ M_c (\omega_k \cos \omega_k - \sin \omega_k) + H_c R_e \left[ (\omega_k \cos \omega_k + \frac{1}{2} \omega_k - \frac{3}{4} \sin 2 \omega_k - \sin \omega_k) + \frac{1}{AR_e} \left( \frac{1}{2} \omega_k + \frac{1}{4} \sin 2 \omega_k \right) \right] = - \bar{Y} \left[ y_S \left( \frac{1}{2} \omega_k - \frac{1}{4} \sin 2 \omega_k \right) + \frac{1}{AR_e} \left( \frac{1}{2} \omega_k + \frac{1}{4} \sin 2 \omega_k \right) \right] \] (27)

\[ M_c (\omega_k + \bar{E}) + H_c R_e \left[ \omega_k \sin \omega_k + \bar{E} (1 - \cos \omega_k) \right] = y_S \bar{Y} (\sin \omega_k + \bar{E} \cos \omega_k) \] (28)

Where: \( \bar{E} = \frac{EI}{R_e} \kappa \)

\( \bar{Y} \) and \( \kappa \) together with Eq. (27) and (28) determine the redundants \( M_c \) and \( H_c \).

4.) Calculation of the arches for anti-symmetrical load

Fig. 4 shows that uniformly distributed live load over the half span can be thought of as a superposition of a symmetrical and anti-symmetrical load.

For an anti-symmetrical load the structure undergoes an anti-symmetrical deformation with regard to the center line \( \omega = 0 \) (Fig. 7a). By replacing the action of the left part of the structure on the right one by a simple support (Fig. 7b) the conditions of the right part are not changed. It is therefore sufficient to investigate the simplified structure of Fig. 7b.

The shell transmits the following three loads to the arches:

a.) Shear Force: \( S = \bar{S} \sin \omega \)
b.) String Force: \( Y = \overline{Y} \cos \omega \)

c.) Horizontal Force \( P \) at \( \omega = 0 \)

a.) and b.) are the same loads as explained under 2.), Eq. (7) and (8). The horizontal load \( P \) is caused by the sudden jump of the membrane \( T_2 \)-Forces at the center line from compression to tension. Fig. 7c illustrates the situation. The free bodies 1 and 2 are acted upon on the right side by the forces \( T_2 \). On the left side the \( T_2 \)-Forces are zero. (Opposite situation for the free bodies 3 and 4.)

The bodies are kept in equilibrium by shear forces \( F \). The opposite Forces \( F \) are acting on the ribs. The problem may be simplified by replacing the action of the \( F \)-Forces on the ribs by concentrated forces \( P \) at the end of the ribs.

St. Venant's principle justifies the procedure for sections sufficiently far removed from the center line \( \omega = 0 \). In case of a rigid cross beam at \( \omega = 0 \), the above simplification would coincide with the actual situation.

The sum of the \( P \)-Forces is equal to the integral of the \( T_2 \)-Forces over the cross section:

\[
2P_1 + P_2 = -2T_2
\]

Basing the magnitude of \( P_1 \) and \( P_2 \) on the zero point of the membrane shear at \( \overline{x}_0 \) (see Eq. (4)), \( P_1 \) and \( P_2 \) are:

\[
\begin{align*}
P_1 &= - (l - \overline{x}_0) \cdot T_2 = (l - \overline{x}_0) R_{SP} \\
P_2 &= - 2\overline{x}_0 \cdot T_2 = 2\overline{x}_0 R_{SP}
\end{align*}
\]

In the following the shear force \( S \) and the horizontal load \( P \) will be treated as one load case a.), the string Force \( Y \) as a second load case b.).

* This procedure is an approximation only.
a.) Arches under shear forces $S$ and horizontal loads $P$:

A statically determinate base system for the structure in Fig. 8a is shown in Fig. 8b. The condition for finding the redundant reaction $V_c$ is $\delta_V = 0$, $\delta_V$ being the vertical displacement of the arch at the center $\omega = 0$.

Normal force $N_o$ and bending moment $M_o$ in the base system due to the action of $P$ are:

\[
(N_o)_P = P \cos \omega \\
(M_o)_P = -P R_e (1 - \frac{y_s}{R} - \cos \omega)
\]

Due to the shear force $S$, they are given by Eq. (11) and (12). Combining the parts due to $P$ and $S$ the results are:

\[
N_o = P \cos \omega - \frac{1}{2} SR_s \omega \sin \omega \\
M_o = -P R_e (1 - \frac{y_s}{R} - \cos \omega) + S R_e \left[ (1 - \frac{y_s}{R_e}) (1 - \cos \omega) - \frac{1}{2} \omega \sin \omega \right]
\]  

The normal force $N$ and the bending moment $M$ in the arch are:

\[
N = N_o + V_c \sin \omega \\
M = M_o + V_c R_e \sin \omega
\]  

The virtual load system for the determination of the deflection $\delta_V$ (Fig. 8c) causes the following normal force and bending moment:

\[
N' = \sin \omega \\
M' = R_e \sin \omega
\]

The application of the work equation for finding $\delta_V$ presents a special problem. For the work of the external forces consists not only of the work of the virtual load $P' = 1$ times the displacement $\delta_V$, but due to the elastic restraint of the arch the moment at the abutment $M_k$ (in the
virtual load system) times the rotation of the abutment \( \phi_k \) (in the actual load system) contributes to the external work. Hence,

\[
\int_{\phi_k} \left( M_k M_k \right) = \int_0^{\omega_k} \frac{1}{EA} \sin \omega (M_0 + V_c R_0 \sin \omega) \sin \omega d\omega \\
+ \int_0^{\omega_k} \frac{1}{EA} \sin \omega (N_0 + V_c \sin \omega) R_0 d\omega
\]

Making use of the expression,

\[
M_k = R_0 \sin \omega_k \\
\phi_k = -\kappa M_k = -\kappa (M_0 + V_c R_0 \sin \omega)
\]

\[
= -\kappa \left[ -PR_0 (1 - \frac{V_c}{R_0} \cos \omega_k) + \frac{\sin \omega_k}{\omega_k} \left( 1 - \frac{V_c}{R_0} \right) \right]
\]

and performing the integration by replacing \( N_0 \) and \( M_0 \) by Eq. (32) and (33), the expression becomes:

\[
\int_{\phi_k} \left( -\kappa R_0 \sin \omega_k \right) \left[ -PR_0 (1 - \frac{V_c}{R_0} \cos \omega_k) + \frac{\sin \omega_k}{\omega_k} \left( 1 - \frac{V_c}{R_0} \right) \right]
\]

\[
- \frac{1}{2} \sin^2 \omega_k + \frac{V_c R_0 \sin \omega_k}{\omega_k} = \frac{F_s^2}{ET} \left[ -PR_0 \left( 1 - \frac{V_c}{R_0} \right) \right]
\]

\[
- \frac{1}{2} \sin^2 \omega_k + \frac{\sin^2 \omega_k}{\omega_k} \left( 1 - \frac{V_c}{R_0} \right) \left( 1 - \frac{1}{2} \sin^2 \omega_k \right)
\]

\[
- \frac{1}{8} \left( \omega_k - \omega_k \sin 2 \omega_k - \frac{1}{2} \cos 2 \omega_k + \frac{1}{2} \right) + \frac{1}{2} V_c R_0 \left( \omega_k - \frac{1}{2} \sin^2 \omega_k \right)
\]

\[
+ \frac{V_c}{EA} \left[ \frac{1}{2} \sin^2 \omega_k - \frac{1}{8} \sin^2 \omega_k \sin 2 \omega_k - \frac{1}{8} \cos 2 \omega_k + \frac{1}{2} \right]
\]

The condition \( \int_{\phi_k} = 0 \) leads to an expression for the redundant \( V_c \):
\[
V_{cR_e}\left[\frac{1}{2}\omega_k - \frac{1}{4}\sin2\omega_k + \bar{K}\sin^2\omega_k + \frac{1}{AR_e^2}\left(\omega_k - \frac{1}{2}\sin2\omega_k\right)\right]
\]

\[
=PR_e\left[(1 - \frac{y_s}{R_e})(1 - \cos\omega_k) - \frac{1}{2}\sin\omega_k + \bar{K}\left((1 - \frac{y_s}{R_e})\sin\omega_k - \frac{1}{2}\sin2\omega_k\right)\right]
\]

\[
- \frac{1}{AR_e^2}\frac{1}{2}\sin^2\omega_k\] - \bar{K}R_e\left[(1 - \frac{y_s}{R_e})(1 - \cos\omega_k) + \frac{1}{2}\sin^2\omega_k\right]
\]

\[
- \frac{1}{8}(\omega_k^2 - \omega_k\sin2\omega_k - \frac{1}{2}\cos2\omega_k + \frac{1}{2}) + \bar{K}\left((1 - \frac{y_s}{R_e})\sin\omega_k - \frac{1}{2}\sin2\omega_k\right)
\]

\[
- \frac{1}{2}\omega_k\sin^2\omega_k\right)
\]

\[
- \frac{1}{8}\frac{1}{AR_e^2}\left(\omega_k^2 - \omega_k\sin2\omega_k - \frac{1}{2}\cos2\omega_k + \frac{1}{2}\right)
\] \quad (34)

Where: \( \bar{K} = \frac{EI}{R_e}K \)

Eq. (34) gives the redundant \( V_c \) as function of the loads \( P \) and \( S \) and of the coefficient of elastic restraint \( \bar{K} \).

b.) Arches under string force \( Y \):

Taking the base system the same as in case a. (Fig. 8b) normal force \( N_o \) and bending moment \( M_o \) are identical with the ones of 3, b.), p. 9:

\[
N_o = -Y\cos\omega \quad (35)
\]

\[
M_o = -y_sY\cos\omega \quad (36)
\]

If the action of \( V_c \) is superimposed the actual \( N \) and \( M \) are found:

\[
N = -Y\cos\omega + V_c\sin\omega \quad (37)
\]

\[
M = -y_sY\cos\omega + V_cR_e\sin\omega \quad (38)
\]

The work equation, using the virtual load system of a.) (Fig. 8c), determines the vertical displacement \( \delta v \) at the center. Again, the additional work of the virtual moment \( M_k \) at the abutment is taken into account:
Replacing

\[ M_k = \Re \sin \omega_k \]

\[ \phi_k = -\kappa M_k = -\kappa (-y_s \Re \cos \omega_k + V_c \Re \sin \omega_k) \]

and integrating, the above expression takes the form:

\[
\int V - \kappa \Re \sin \omega_k \left(-y_s \Re \cos \omega_k + V_c \Re \sin \omega_k\right) \]

\[ = \frac{\Re^2}{EI} \left[ -\frac{1}{2} y_s \Re \sin^2 \omega_k + V_c \Re \left(\frac{1}{2} \omega_k - \frac{1}{4} \sin 2 \omega_k\right)\right] \]

\[ + \frac{\Re}{EA} \left[ -\frac{1}{2} \Re \sin^2 \omega_k + V_c \left(\frac{1}{2} \omega_k - \frac{1}{4} \sin 2 \omega_k\right)\right] \]

Introducing the condition \( \int V = 0 \), the redundant \( V_c \) has the final form:

\[
\Re \left[\frac{1}{2} \omega_k - \frac{1}{4} \sin 2 \omega_k + \kappa \sin^2 \omega_k + \frac{1}{AK_0} \left(\frac{1}{2} \omega_k - \frac{1}{4} \sin 2 \omega_k\right)\right] \]

\[ = \frac{1}{2} \Re \left[ y_s \sin^2 \omega_k + \kappa y_s \sin 2 \omega_k + \frac{1}{AK_0} \sin^2 \omega_k\right] \]  \(39\)

Eq. (39) is an expression of the redundant \( V_c \) for the case of an anti-symmetrical string force \( Y \).

The different load cases treated under 3.) and 4.) allow the calculation of the normal forces and bending moments in the stiffening ribs of the shell roof model for dead load and uniformly distributed load over one half side (p lb/in^2 of shell surface). The stresses are determined by the well known formula

\[ \sigma = \frac{N}{A} + M \frac{Y}{I} \]
5. **Deflection of the Arches**

The calculation of the deflection of the arches does not offer any new problem. The most suitable way is the use of the work equation, placing a unit virtual load at the point for which the deflection should be determined. In Part II an illustrative example is given.

6. **Approximate Calculation of the Shell Force**

The disturbance of the membrane forces of the shell due to the action of the ribs can be considered in an approximate, but nevertheless very accurate way ($M_1$ moments and $T_2$ forces). The procedure was outlined in Ref. (2), p. 3.

The forces acting on the shell are:

1. Load $q$ (dead load or live load on the shell)
2. String force $Y$, eliminating the normal strain of the membrane solution, in the following part called $Y_\varepsilon$ (Eq. (10)).
3. String force $Y$, due to the interaction between rib and shell, called $Y_s$. In Ref. (2) $Y_s$ was derived (Eq. (3), p. 3)

$$Y_s = bd \sigma_s$$

(40)

where $b = $ effective width

d = thickness of shell

$\sigma_s = $ normal stress in rib in the fiber of the connecting line rib-shell.

In summary, the membrane forces of the shell have to be superimposed by the direct forces $T_2$ and bending moments $M_1$, caused by the action of the string force $Y = Y_\varepsilon + Y_s$. The
latter problem was already treated extensively in Ref. (1) and Ref. (2).

The above procedure does not hold in the edge-member region or in a region where the load is discontinuous (for example by half side live load around the center line \( \omega = 0 \)).

**Part II: Numerical Analysis**

The analysis of the model under simulated dead load or live load over the half span (Fig. 2) is made in the following steps:

1. Membrane solution, Eq. (1) to (4).
2. Loading of the ribs due to the boundary forces \( S \) and \( Y \) of the shell, Eq. (8) and (10).
3. Symmetrical load case:
   - Eq. (19) and (20) and Eq. (27) and (28)
4. Anti-symmetrical load case:
   - Eq. (34), Eq. (37)
5. Dead load: Fiber stresses of the ribs
6. Live load: Fiber stresses of the ribs

**Effective cross section of the ribs:**

See Progress Report 213C (Ref. (2)) for further explanation:

\[ \lambda - \text{Test: Approximate expression for } c \text{ in case of live load over the half span:} \]

\[ c \approx \frac{5\pi}{4\omega_k} = \frac{5\pi}{4 \cdot 0.5866} = 6.695^* \]

* See Fig. 12, where the stress \( \varepsilon_L \) has the variation of a sine function of have-wave length of about \( \omega = 0.45 \)

\[ c = \frac{\pi}{2} \frac{2}{0.45} \approx \frac{5\pi}{4\omega_k} \text{. In case of dead load, } c \text{ and } \lambda \text{ are still smaller.} \]
\[ \lambda = c \sqrt{\frac{d}{d_{RS}}} = 6.695 \sqrt{\frac{0.118}{108}} = 0.2213 \]

The \( K \)-values for the effective width for \( \lambda = 0 \) are therefore sufficiently accurate.

Ref. (1), p. 23 and Fig. 8:

\[
\begin{align*}
\text{Outer rib:} & \quad \lambda = 0 \\
& \quad (\beta_x) = 0 \\
& \quad K = 0.3799
\end{align*}
\]

\[
\begin{align*}
\text{Middle rib:} & \quad \lambda = 0 \\
& \quad (\beta_x) = \infty \\
& \quad K = 1.520
\end{align*}
\]

Effective width:

\[
\begin{align*}
\text{Outer rib:} & \quad b = K \sqrt{Rd} = 0.3799 \sqrt{108 \times 0.118} = 1.356'' \\
\text{Middle rib:} & \quad b = K \sqrt{Rd} = 1.520 \sqrt{108 \times 0.118} = 5.425''
\end{align*}
\]

Effective cross section:

Outer ribs:

\[
\begin{align*}
\text{Area:} & \quad A = 1.227 \text{ in}^2 \\
\text{Distance:} & \quad y_u = 1.186 \text{ in} \\
& \quad y_L = 0.9259 \text{ in} \\
& \quad y_s = 0.8669 \text{ in}
\end{align*}
\]

(See Ref. (2), p. 12)
Radius of effective rib-section: \( R_e = 108.926 \text{ in} \)

Moment of inertia: \( I = 0.5356 \text{ in}^4 \)

\[
\frac{I}{I \text{ (rib only)}} = \frac{0.5356}{0.3970} = 1.35
\]

Middle rib:

\[
\begin{align*}
\text{Centroid} & \quad \text{Radius} \quad 0.625 \\
2.113 & \quad \text{Area:} \\
R_e & \quad \frac{1}{2}b
\end{align*}
\]

Distance:

\[
\begin{align*}
\bar{y}_u & = 1.382 \text{ in} \\
\bar{y}_L & = 0.7308 \text{ in} \\
\bar{y}_s & = 0.6718 \text{ in}
\end{align*}
\]

Radius of effective rib-section \( R_e = 108.731 \text{ in} \)

Moment of inertia \( I = 0.9204 \text{ in}^4 \)

\[
\frac{I}{I \text{ (rib only)}} = \frac{0.9204}{0.4914} = 1.84
\]

List of principal dimensions and datas (Fig. 1)

| Outer ribs | Radius \( R_R = 109.06" \) | Height \( h_R = 2.113" \) | Thickness \( b_R = 0.505" \) | Angle \( \omega_k = 0.5866 \) |
| Middle rib | Radius \( R_R = 109.06" \) | Height \( h_R = 2.113" \) | Thickness \( b_R = 0.625" \) |
Shell:
- Radius \( R_s = 108" \)
- Thickness \( d = 0.118" \)
- Distance between ribs \( l = 12" \)

Loads:
- Dead load \( q = 8.222 \text{ lb/in}^2 \)
- Live load \( p = 4.933 \text{ lb/in}^2 \)

Measured support movements: Insignificant

1.) Membrane solution:

Eq. (4): \[ \bar{x}_o = \frac{l}{8} \frac{5l^2 + 6(4+3v)R_s^3}{l^2 + 6(1+v)R_s^2} = \frac{12}{8} \frac{5 \cdot 12^2 + 6(4+0.3)108^3}{12^2 + 6(1+0.3)108^2} \]

\[ \bar{x}_o = 5.657" \]

Membrane forces (Eq. (1) to (3)):

**Dead Load:** \( q = 8.222 \text{ lb/in}^2 \)

\[ T_2 = -qR_s \sin \omega = -8.222 \cdot 108 \sin \omega = -888.0 \sin \omega \]

\[ S = -2q \sin(\bar{x} - \bar{x}_o) = -2 \cdot 8.222(\bar{x} - 5.657) \sin \omega \]

\[ S = -16.444(\bar{x} - 5.657) \sin \omega \]

\[ T_1 = \frac{p}{R_s} \cos \omega \left( \bar{x}^2 - 2\bar{x}_o \bar{x} + \frac{l^2}{4} \right) \cdot \frac{l^2 - 6vR_s^3}{l^2 + 6(1+v)R_s^2} \]

\[ T_1 = \frac{8.222}{108} \cos \omega \left( \bar{x}^2 - 11.314\bar{x} + \frac{12^2}{4} \right) \cdot \frac{12^2 - 6 \cdot 0.3 \cdot 108^2}{12^2 + 6(1+0.3)108^2} \]

\[ T_1 = 0.07612 (\bar{x}^2 - 11.314\bar{x} - 8.238) \cos \omega \]

**Live Load:** \( p = 4.933 \text{ lb/in}^2 \)

\[ T_2 = -pR_s \cos \omega = -4.933 \cdot 108 \cos \omega = -532.8 \cos \omega \]

\[ S = -2q \sin(\bar{x} - \bar{x}_o) = -9.866(\bar{x} - 5.657) \sin \omega \]

\[ T_1 = \frac{p}{R_s} \cos \omega \left( \bar{x}^2 - 2\bar{x}_o \bar{x} + \frac{l^2}{4} \right) \cdot \frac{l^2 - 6vR_s^3}{l^2 + 6(1+v)R_s^2} \]

\[ T_1 = 0.04688 (\bar{x}^2 - 11.314\bar{x} - 8.238) \cos \omega \]
2.) Loading of the ribs due to the boundary forces S and Y of the Shell

**Shell Forces:**

**S - Forces:** Eq. (8) \( S = \bar{S} \sin \omega \)

**Dead Load:**

- Outer rib: \( \bar{S} \bigg|_{x=l} = -2q(l - \bar{x}_0) = -2 \cdot 8.222(12 - 5.657) = -104.3 \text{ lb/in} \)
- Middle rib: \( \bar{S} \bigg|_{x=0} = 2q\bar{x}_0 = 2 \cdot 8.222 \cdot 5.657 = 93.03 \text{ lb/in} \)

**Live Load:**

- Outer rib: \( \bar{S} \bigg|_{x=l} = -2p(l - \bar{x}_0) = -62.58 \text{ lb/in} \)
- Middle rib: \( \bar{S} \bigg|_{x=0} = 2p\bar{x}_0 = 55.81 \text{ lb/in} \)

**Y-Forces:**

Eq. (10) \( Y = \bar{Y} \cos \omega \)

**Dead Load:**

- Outer rib: \( \bar{Y} \bigg|_{x=l} = qRsb^* = 8.222 \cdot 108 \cdot 1.356 = 1204 \text{ lb/in} \)
- Middle rib: \( \bar{Y} \bigg|_{x=0} = qRsb = 8.222 \cdot 108 \cdot 5.425 = 4817 \text{ lb/in} \)

**Live Load:**

- Outer rib: \( \bar{Y} \bigg|_{x=l} = qRsb = 4.933 \cdot 108 \cdot 1.356 = 7224 \text{ lb/in} \)
- Middle rib: \( \bar{Y} \bigg|_{x=0} = pRsb = 4.933 \cdot 108 \cdot 5.425 = 2890 \text{ lb/in} \)

**Forces acting on the ribs:**

Taking the direction of Y and S acting on the rib positive as shown in Fig.5 (S positive if directed toward the abutments; Y position if tension in the string, the loads on the ribs are:

* For calculation of b see p. 19.
Dead load \( q \) (\( q = 8.222 \text{ lb/in}^2 \))

<table>
<thead>
<tr>
<th></th>
<th>Outer ribs</th>
<th>Middle rib</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q = 8.222 \text{ lb/in}^2 )</td>
<td>( \bar{s} = 12.69 q )</td>
<td>( \bar{s} = 22.63 q )</td>
</tr>
<tr>
<td></td>
<td>( \bar{y} = 146.4 q )</td>
<td>( \bar{y} = 585.9 q )</td>
</tr>
</tbody>
</table>

Live load \( p \) (\( p = 4.933 \text{ lb/in}^2 \))

<table>
<thead>
<tr>
<th></th>
<th>Outer ribs</th>
<th>Middle rib</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p = 4.933 \text{ lb/in}^2 )</td>
<td>( \bar{s} = 12.69 p )</td>
<td>( \bar{s} = 22.63 p )</td>
</tr>
<tr>
<td></td>
<td>( \bar{y} = 146.4 p )</td>
<td>( \bar{y} = 585.9 p )</td>
</tr>
</tbody>
</table>

Forces \( P_1 \) and \( P_2 \) (Eq. (29)), (For live load only):

Outer ribs: \( P_1 = (1-x_o)RS_p = (12-5.657)108 p \)

\[ = 685.0 \text{ p} = 53.98 \bar{s} \]

Middle rib: \( P_2 = 2x_oRS_p = 2 \cdot 5.657 \cdot 108 p \)

\[ = 1222 \text{ p} = 53.98 \bar{s} \]

3.) Symmetrical load case (dead load)

a.) Solution of Eq. (19) and (20) for the redundants

\( M_c \) and \( N_c \):

List: \( \omega = 0.5866 \)

\[ \sin \omega = 0.5535 \quad 326^* \quad \cos \omega = 0.8328 \quad 275 \]

\[ \sin^2 \omega = 0.3063 \quad 983 \quad \cos^2 \omega = 0.6936 \quad 016 \]

\[ \sin 2\omega = 0.9219 \quad 944 \quad \cos 2 \omega = 0.3872 \quad 033 \]

Outer rib:

\[ R_o = 108.926 \text{ in} \]
\[ R_S = 108 \text{ in} \]
\[ y_s = 0.8669 \text{ in} \]
\[ I = 0.5356 \text{ in}^4 \]
\[ A = 1.227 \text{ in}^2 \]
\[ \frac{I}{AR_o^2} = 3.679 \cdot 10^{-5} \]

Middle rib:

\[ R_o = 108.731 \text{ in} \approx 108.926 \text{ in}. \]
\[ R_S = 108 \text{ in} \]
\[ y_s = 0.6718 \text{ in} \]
\[ I = 0.9204 \text{ in}^4 \]
\[ A = 1.961 \text{ in}^2 \]
\[ \frac{I}{AR_o^2} = 3.970 \cdot 10^{-5} \approx 3.679 \cdot 10^{-5} \]

* The different terms of Eq. (19) and (20) are small differences of large numbers. Therefore 7 decimal places are taken.
By taking the values $R_e$ and $\frac{I}{AR_e^2}$ of the outer ribs for the middle rib also, the calculation of the outer and middle ribs can be done simultaneously. The error involved for the middle rib is insignificant.

Coefficient of elastic restraint:

$$\kappa = 0.05 \frac{R_e}{EI}$$

$$\bar{\kappa} = \frac{EI}{R_e} \kappa = 0.05$$

Using in Eq. (19) and (20) the above listed numerical values, they are written as:

(19) \[-0.064996 M_C - 0.23695 H_C = 0.09793 \]  
(20) \[0.6366 M_C + 4.5124 H_C = -5.5554 \]

The redundants are:

$$M_C = 6.3050$$

$$H_C = -2.1428$$

b.) Solution of Eq. (27) and (28) for the redundants $M_C$ and $H_C$

The calculations for the outer ribs and the middle rib are made separately. The difference between the $y_s$ of the outer ribs and the middle rib entering Eq. (27) and (28) are about 30%. Substituting in Eq. (27) and (28) above listed numerical values:

(27) \[-0.064996 M_C - 0.23695 H_C = -(0.06280 y_s + 0.00210) \]  
(28) \[0.6366 M_C + 4.5124 H_C = 0.5952 y_s \]

Outer ribs: $y_s = 0.8669$ in

$$M_C = 0.9324$$

$$H_C = -0.01725$$

* $\kappa$ differs from the value chosen in Progress Report 213C, Ref. (2). Possibly this is due to the difference in the force distribution in the end region.
Middle rib: \( y_s = 0.6718 \)
\[ M_c = 0.7377 \, \overline{Y} \]
\[ H_c = -0.01547 \, \overline{Y} \]

4.) Anti-symmetrical load case:

(34) \[ 8.5094 \, V_c = 1.3571 \, P - 2.9624 \, \overline{S} \]
From p. 31 \( P = 53.98 \, \overline{S} \)
\[ V_c = 11.60 \, \overline{S} \]

(39) \[ 8.5094 \, V_c = 0.1763 \, y_s \, \overline{Y} \]
\[ V_c = 0.02072 \, y_s \, \overline{Y} \]

Outer ribs: \( y_s = 0.8669 \)
\[ V_c = 0.01796 \, \overline{Y} \]

Middle rib: \( y_s = 0.6718 \)
\[ V_c = 0.01392 \, \overline{Y} \]

5.) Dead Load: \( q = 8.222 \, \text{lb/in}^2 \)

a.) ribs:

Normal force \( N \) and bending moment \( M \) in the ribs:

Eq. (13) and (14)
\[ N = N_o - H_c \cos \omega \]
\[ M = M_o + H_c R_e (1 - \cos \omega) + M_c \]

Loading by shear forces \( S \):

(11) \[ N_o = -\frac{1}{2} S R_S \omega \sin \omega \]
(12) \[ M_o = S R_S R_e \left[ (1 - \frac{y_s}{R_e}) (1 - \cos \omega) - \frac{1}{2} \omega \sin \omega \right] \]

p. 24 \( H_c = -2.1428 \, \overline{S} \)
\[ M_c = 6.3050 \, \overline{S} \]
Loading by string forces $Y$:

(21) $N_o = -Y \cos \omega$

(22) $M_o = -y_s Y \cos \omega$

From p. 24 and 25

Outer ribs: $H_o = -0.01725 Y$

$M_o = 0.9324 Y$

Middle rib: $H_o = -0.01547 Y$

$M_o = 0.7377 Y$

Loading forces $\bar{S}$ and $\bar{Y}$:

From p. 21

Outer ribs: $\bar{S} = 12.69 q$

$\bar{Y} = 146.4 q$

Middle ribs: $\bar{S} = 22.63 q$

$\bar{Y} = 585.9 q$

Total $H_o$ and $M_o$:

Outer ribs:

$H_o = -2.1428 \bar{S} - 0.01725 Y = -29.72 q$

$M_o = 6.3050 \bar{S} + 0.9324 Y = 216.5 q$

Middle rib:

$H_o = -2.1428 \bar{S} - 0.01547 Y = -67.55 q$

$M_o = 6.3050 \bar{S} + 0.7377 Y = 574.9 q$

All necessary calculations to find the normal force $N$ and
the bending moment $M$ due to dead load $q = 8.222$ lb/in$^2$ are
shown in Tables I and II. The fiber stresses $G_u$ and $G_L$
are calculated by using the well known formula $G = \frac{N}{A} + \frac{M}{I}$.

Fig. 11 is a graphical representation of the analytical
results. For comparison the test results are also plotted.
b.) **Shell Forces for \( \omega = 0 \):**

Knowing \( \sigma_u \) and \( \sigma_L \),

\[
\sigma_s = \sigma_L - 0.028 ( \sigma_L - \sigma_u )
\]

Eq. (40)  \( \gamma_s = bd \sigma_s \)

For center line \( \omega = 0 \):

**Outer rib:**  \( \sigma_s = 409.6 \text{ lb/in}^2 \)

\( b = 1.356" \)

\( \gamma_s = 1.356 \cdot 0.118 \cdot 409.6 = 65.54 \text{ lb.} \)

**Middle rib:**  \( \sigma_s = -1126.8 \text{ lb/in}^2 \)

\( b = 5.425" \)

\( \gamma_s = -5.425 \cdot 0.118 \cdot 1126.8 = -721.3 \text{ lb.} \)

The above force and the \( \gamma \)-force introduced in order to eliminate the circumferential strain \( \varepsilon_\omega \) are superimposed.

From p. 20:

**Outer rib:**  \( \gamma_\varepsilon = 1204 \text{ lb.} \)

**Middle rib:**  \( \gamma_\varepsilon = 4817 \text{ lb.} \)

**Total \( \gamma \)-forces:**  \( \gamma = \gamma_s + \gamma_\varepsilon \)

**Outer ribs:**  \( \gamma = 65.54 + 1204 = 1270 \text{ lb.} \)

**Middle rib:**  \( \gamma = -721.3 + 4817 = 4096 \text{ lb.} \)

In Table V the numerical calculation of the \( T_2 \) and \( M_1 \) is made. The procedure is exactly parallel to the one explained in Ref. (2), p. 17, 6. Fig. 13 is a graph of the analytical and experimental results.

6.) **Live load over the half-span:**  \( p = 4.933 \text{ lb/in}^2 \)

* See Ref. (2), p. 17, \( \sigma_u \) and \( \sigma_L \) are taken from Table II, column \( (21), (22) \) and \( (35), (36) \).

+ On p. 20 the \( Y \) is given without the subscript.
a.) **Ribs:**

The load is split in a symmetrical load $\frac{P}{2}$ and an anti-symmetrical load $\frac{P}{2}$ (as shown in Fig. 4).

The **symmetrical load case** is identical to the case of dead load. The dead load $q$ has to be replaced by the live load $\frac{P}{2}$.

The **anti-symmetrical load case** requires a separate treatment. The normal force $N$ and the bending moment $M$ in the ribs are given by Eq. (32) and (33):

$$N = N_0 + V_c \sin \omega$$

$$M = M_0 + V_c R_e \sin \omega$$

**Loading by shear forces $S$:**

(30) $N_0 = P \cos \omega - \frac{1}{2} S R_s \omega \sin \omega$

(31) $M_0 = -P R_e (1 - \frac{V_s}{R_e} - \cos \omega) + S R_s R_e \left[ (1 - 2 \frac{V_s}{R_e})(1 - \cos \omega) \right.$

From p. 21: $P = 53.98$ $\bar{S}$

$$N_0 = (53.98 \cos \omega - 54 \omega \sin \omega) \bar{S}$$

$$M_0 = \left[ -53.98 \left( 1 - \frac{V_s}{R_e} - \cos \omega \right) + 108 \left( 1 - \frac{V_s}{R_e} \right)(1 - \cos \omega) \right.$$

From p. 24: $V_c = 11.60 \bar{S}$

**Loading by string forces $Y$:**

(35) $N_0 = -Y \cos \omega$

(36) $M_0 = -y_s Y \cos \omega$

From p. 24:

**Outer ribs:** $V_c = 0.01796 \bar{Y}$

**Middle rib:** $V_c = 0.01392 \bar{Y}$
Loading forces $\bar{S}$ and $Y$:

From p. 21:

Outer ribs: $\bar{S} = 12.69 \text{ kips}$
$Y = 146.4 \text{ kips}$

Middle rib: $\bar{S} = 22.63 \text{ kips}$
$Y = 585.9 \text{ kips}$

Total $V_c$:

Outer ribs: $V_c = 11.60 \bar{S} + 0.01796 Y = 149.6 \text{ kips}$

Middle rib: $V_c = 11.60 \bar{S} + 0.01392 Y = 270.7 \text{ kips}$

In Table III the calculations for the normal force $N$ and the bending moment $M$ are made. By superposition of the symmetrical and the anti-symmetrical load case for the load $\frac{p}{2}$ (where $p$ is the live load), the case of the live load $p$ over the half span is found (Table IV). The fiber stresses $\sigma_u$ and $\sigma_L$ are found in the usual way (Table IV). Fig. 12 compares the analytical and experimental fiber stresses $\sigma_u$ and $\sigma_L$.

b.) Deflection of ribs:

Vertical deflection at point of dials 3 and 4 (Fig. 10).

Virtual load system: $P=1$

Simply supported beam.

Reactions, normal forces and moments.
Left side of P:

\[ R_l = \frac{36.5}{119.6} = 0.305 \]

\[ M'_l = R_l R_e (\sin \omega_k + \sin \omega) \]

\[ N'_l = R_l \sin \omega \]

Right side of P:

\[ R_r = \frac{83.1}{119.6} = 0.695 \]

\[ M'_r = R_r R_e (\sin \omega_k - \sin \omega) \]

\[ N'_r = -R_r \sin \omega \]

Table VI shows the calculations using a numerical integration procedure (Trapezoid Formula).

Work equation:

\[ \int = \int_{-\omega}^{\omega_k} \frac{M'M ds}{EI} + \int_{-\omega}^{\omega_k} \frac{N'N ds}{EA} \]

Numerical approximation:

\[ \int \approx \frac{\Delta s}{n} \left( \sum_{\omega_k} M'M + \frac{1}{A} \sum_{\omega_k} N'N \right) \]

The length of the interval \( \Delta s \) is constant between \( \omega = \pm 0.5796 \). The intervals at the two ends are shorter. At the center line \( \omega = 0 \) the normal force \( N \) and the bending moment \( M \) are discontinuous. For the summation the above mentioned points have to be considered.

<table>
<thead>
<tr>
<th>Path ( \omega )</th>
<th>( \Delta s )</th>
<th>( \Sigma M'M )</th>
<th>( \Sigma N'N )</th>
<th>( \Delta s \Sigma M'M )</th>
<th>( \Delta s \Sigma N'N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5866 to -0.5796</td>
<td>0.762&quot;</td>
<td>318</td>
<td>105.7</td>
<td>242</td>
<td>80.5</td>
</tr>
<tr>
<td>-0.5796 to -0.5796</td>
<td>6.318&quot;</td>
<td>114,305</td>
<td>273.2</td>
<td>722,179</td>
<td>1726.1</td>
</tr>
<tr>
<td>-0.5796 to -0.5866</td>
<td>0.762&quot;</td>
<td>-704</td>
<td>2.1</td>
<td>-536</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Multiplier

\[ \frac{P}{L} = \frac{4.933}{8} = 2.466 \text{ lb/in}^2 \]

\[ I = 0.5356 \text{ in}^4 \]

\[ \frac{I}{A} = \frac{0.5356}{1.2271} = 0.4365 \text{ in}^3 \]
Total integral: \[ \Delta s \sum_{k} \Delta s_{k} = 721,885 \text{ in} \]
\[ \frac{E}{A} \Delta s \sum_{k} N_{k} = 789 \text{ in} \]
\[ 722,674 \text{ in} \]

Deflection at the location of dial 3 and 4 (Fig. 10)

\[ E = 29.5 \times 10^6 \text{ lb/in}^2 \]

\[ \delta = \frac{722,674 \times 2.466}{29.5 \times 10^6 \times 0.5356} = 0.1128 \text{ in.} \]

Comparison to test results:

The following deflections were measured:

Dial 3*: \[ \delta = 0.1199 \text{ in.} \]
Dial 4*: \[ \delta = 0.0987 \text{ in.} \]

The average of Dials 3 and 4 is 0.1093 inches as compared with the theoretically determined value of 0.1128. The agreement is excellent, the difference in Dials 3 and 4 presumably being due to variation in model dimensions.

Part III: Experimental Investigation

1.) Description of Model and Test Set-up

The same model as described in Progress Report 213C (Ref. (2)) was used. The loads were applied by means of so called "tension pads"+ developed during the war by Martin Aircraft Corp. for the testing of aeroplanes. Fig. 15 is a photograph of the mounting of the pads.

The tension pad consists of a steel plate 5 x 5 x 1/8", with a central hole to which a sponge rubber layer of 1"

* From symmetry conditions, Dial 3 and Dial 4 should have the same deflections.
+ Tension pads are manufactured by the F. G. Schenuit Rubber Company, Baltimore, Maryland.
thickness is glued. The pad is glued to the object to be tested by a special process. A tensile load applied through a pin and a spherical washer to the center hole of the steel plate is uniformly distributed over the corresponding model area of the pad by the rubber.

Pilot tests made with rubber pads (Fig. 14) showed a very uniform distribution of the load. The increase in the shell stiffness, due to the tension pads was negligible. The ultimate carrying capacity of a pad was found to about 2100 lb. (84 lb/in² of pad). At 1200 lb. the steel plate of the pad started to deform plastically.

Eighty pads were fixed to the lower side of the shell as seen in Fig. 2 (also Fig. 17). Through a system of levers and equalizers 8 pads were combined into a single loading unit. The load was produced by water buckets hanging on the end of a lever arm of an advantage 1 to 10. Fig. 16 gives an overall view of the test set-up.

The applied load was very uniformly distributed over the area of the pads. But, as Fig. 2 may show, this load did not correspond to a uniform distribution over the entire shell surface as assumed in the theoretical analysis. The influence of this discrepancy will be discussed under point 4.

As recording devices SR-4 electrical strain gages and dial deflection gages were used (Fig. 9 and Fig. 10), as described already in Progress Report 213C, p. 19.

2.) Test Procedure:

Plastic flow, due to residual welding stresses and cold
rolling in fabricating the model, were eliminated by loading the model several times by loads 10 to 15% higher than the loads in the actual tests (See Ref. (2), p. 20).

"Dead load" was produced by 10 loads of 250 lb. each (water buckets), applied to the lever arm of 1 to 10 advantage. Eight pads were uniformly loaded up by one load. The total load on the model was $10 \times 10 \times 250 = 25,000$ lb. or per one pad $\frac{25,000}{80} = 312.5$ lb. This corresponds approximately to a uniformly distributed load $q = \frac{25,000}{\frac{24}{126.7}} = 8.222$ lb/in$^2$, which figure was used in the theoretical analysis.

"Live load" over the half span required the loading of all pads on the right side of the center line. Five loads of 150 lb. each gave a total of $5 \times 10 \times 150 = 7500$ lb. each. Each pad carried $\frac{7500}{40} = 187.5$ lb. The approximate uniform live load, used for the theoretical analysis, was $p = \frac{7500}{24 \times 63.35} = 4.933$ lb/in$^2$.

The readings were taken at the "zero load" (an initial load of about 70 lb. on each lever was applied to set the whole loading system) and at the "load" (250 lb. for "dead load", 150 lb. for "live load" on each lever respectively). No intermediate readings were taken. The procedure was repeated once, allowing a check of all readings. Very close correspondence between the 2 sets of readings were made.

3.) Test Results:

As test results the mean of the 2 sets of readings are recorded. They are tabulated in Appendix to Progress Report 213-D "Test Results for Simulated Dead Load and Uniformly Distributed Live Load over the Half-Span on a Model
of an Arch Roof".

On the basis of the test readings the experimental fiber stresses $\sigma_u$ and $\sigma_L$ of the ribs were worked out. Shell forces $T_2$ and moments $M_1$ for the center line $\omega = 0$ in case of dead load are also computed.

4.) **Comparison between Test Results and Analysis:**

Before making a comparison, it is of advantage to recall the simplifying assumptions made in the theoretical analysis.

1. The action of the edge-member at $\omega = \pm \omega_k$ is approximately taken into account by a coefficient of elastic restraint $\kappa$ for the ribs (see Ref. (2), p. 5; this report p. 7). No exact study of the edge-member problem was made.

2. The effective width of the ribs was determined by assuming the shell moment $M_2$ and Poisson's ratio $v$ equal to zero (Ref. (1), p. 6). The influence of the parameter $\lambda$ (force distribution factor) was disregarded in calculating the effective width (see p. 21).

3. The shell forces $T_2$ and moments $M_1$ for the case of dead load at the center $\omega = 0$ were computed by an approximate procedure. (See p. 17 and 26).

4. The torsional stiffness of the ribs were disregarded.

5. The dimensions of the model are assumed to agree perfectly with those shown on the drawing whereas minor deviations are known to exist.
due to the assumption of the horizontal loads $P$, caused by the discontinuity of the $T_2$-Forces in the membrane solution (Fig. 7c). The problem is further discussed on page 12. It may be seen, that the influence of the above assumption is of local character only, as can be expected on the basis of de St. Venant's principle.

The check of the deflections at the location of the dials 3 and 4 (see p. 30) is within the usual experimental accuracy.

5.) Relation between the Model and an Actual Structure

This point is discussed in Ref. (2), p. 23 to 25. Anticipating what was already said there, the following table gives a comparison between the stresses and the deflection of the Model and Structure.

$n$ is the scale factor between Model and Structure

<table>
<thead>
<tr>
<th>Material</th>
<th>Model</th>
<th>Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity</td>
<td>$E_{\text{Mod}}$</td>
<td>$E_{\text{St}}$</td>
</tr>
</tbody>
</table>

Given Relations:

- Length
  - $l_{\text{Mod}}$ (in)
  - $l_{\text{St}} = n \cdot l_{\text{Mod}}$

- Load per unit area
  - $P_{\text{Mod}}$ (lb/in$^2$)
  - $P_{\text{St}} = P_{\text{Mod}}$

- Stresses
  - $\sigma_{\text{Mod}}$ (lb/in$^2$)
  - $\sigma_{\text{St}} = \sigma_{\text{Mod}}$

- Deflections
  - $\delta_{\text{Mod}}$ (in)
  - $\delta_{\text{St}} = n \frac{E_{\text{Mod}}}{E_{\text{St}}} \delta_{\text{Mod}}$
"Dead Load": Fig. 11 is a graph of the experimental and theoretical results for the fiber stresses in the ribs. The discrepancy between test and analysis may be better understood by keeping in mind the following 2 points:

1. In general, the stresses are rather small (compare Fig. 11 to the stresses for live load on Fig. 12).*

2. In case of dead load, the line of thrust (parabola) is very near to the middle line of the ribs (circular shape). A small deviation of the actual shape of the middle line from the assumed theoretical one (circle) may lead to already considerable change in the bending moments. Fractions of an inch are sufficient to produce this effect. It is believed, that this latter point may explain sufficiently the observed discrepancy.

The experimental and theoretical shell forces $T_2$ and bending moment $M_1$ are in fairly good agreement considering the highly complicated nature of the stress distribution. (Fig. 13).

"Live Load": Test and theory agree fairly well for the fiber stresses $\sigma_u$ and $\sigma_L$ of the ribs (Fig. 12). In case of live load over the half span the line of thrust is far from the middle line of the ribs. The bending moment does not show the high sensitivity for a small variation of the shape of the middle line as exhibited in case of dead load. The discontinuity of the stresses at the center $\omega = 0$ is

* The scale of the two graphs was held constant in order to allow a better comparison.
Example:

Given:  Shell roff (e.g. Rapid City)

Scale factor $n = 30$

Loads:  Dead load $q = 8.222 \text{ lb/in}^2$

Live load $p = 4.933 \text{ lb/in}^2$

Stresses: The stresses of the model and the structure are equal at the corresponding points. The above load in terms of square feet are:

Dead load (of shell only): $q = 1184 \text{ lb/ft}^2$

Live load: $p = 710.4 \text{ lb/ft}^2$

For any variation in the loads, the stresses have to be changed proportionally.

Deflections: The deflections of the structure will be 

\[ n \cdot \frac{E_{\text{Mod}}}{E_{\text{St}}} \]

times greater than the deflection of the original. For other loads, the deflection have to be varied proportionally.

* Dead load as far as the weight of the shell itself is concerned. The weight of the ribs was not taken into account (See p. 6).
Conclusions

The present report gives a method for the analysis of an Arch Shell Roof Construction under dead load and uniformly distributed live load over the half span. The method is straightforward and suitable for actual design applications. The problem of the edge-member disturbance was not studied. This problem is of local character in the present type of structures.

Tests, performed on the model, verified the proposed analysis.
### Table I

**Symmetrical load case:** Normal force $N_o$ and bending moment $M_o$ of the rib in the base system.

#### a.) Under shear force $S$

\[ 1 - \frac{y_s}{R_e} = 0.9920 \ 41 \]

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$\sin\omega$</th>
<th>$\cos\omega$</th>
<th>$1 - \cos\omega$</th>
<th>$N_o$ (Eq. 11)</th>
<th>$M_o$ (Eq. 12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>0.50</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>0.80</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>1.10</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>1.40</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>1.70</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>2.00</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>2.30</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>2.60</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>2.90</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>3.20</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiplier</th>
<th>$\frac{5}{\omega}$</th>
<th>$\frac{5}{\omega}$</th>
<th>$\frac{y_s}{\omega}$</th>
<th>$\frac{y_s}{\omega}$</th>
</tr>
</thead>
</table>

#### b.) Under string force $Y$

\[ 1 - \frac{y_s}{R_e} = 0.9920 \ 41 \]

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$\sin\omega$</th>
<th>$\cos\omega$</th>
<th>$1 - \cos\omega$</th>
<th>$N_o$ (Eq. 21)</th>
<th>$M_o$ (Eq. 22)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>0.50</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>0.80</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>1.10</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>1.40</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>1.70</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>2.00</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>2.30</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>2.60</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>2.90</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
<tr>
<td>3.20</td>
<td>0.1579 67</td>
<td>0.9933 18</td>
<td>0.0066 82</td>
<td>- 0.18</td>
<td>- 0.15</td>
</tr>
</tbody>
</table>

\[ 1 - \frac{y_s}{R_e} = 0.9920 \ 41 \]
TABLE II

Dead load: \( q = 8.222 \text{ lb/in}^2 \)

a.) Outer ribs: \( y_s = 0.8669'' \)

\[
\begin{align*}
\omega & \quad (\frac{M_o}{S})_\omega \quad (\frac{M_o}{Y})_\omega \quad H_c \cos \omega \quad N \quad (\frac{M_o}{S})_y \quad (\frac{M_o}{Y})_y \quad H_c R_e (1 - \cos \omega) \quad M_c \\
0 & \quad 0.0580 & -2.3 & -146.4 & -29.72 & -116.7 & -0.0 & -126.9 & -0.0 & 89.6 & 82.5 & 71.0 & 60.8 & 50.5 & 40.8 & 30.8 & 20.9 & 11.0 & 0.0 \\
0.1159 & -9.1 & -145.1 & -29.52 & -125.1 & -6.9 & -126.1 & -21.7 & 61.8 & 51.4 & 41.1 & 30.5 & 20.9 & 11.0 & 0.0 \\
0.1739 & -20.7 & -142.2 & -23.27 & -135.6 & -12.2 & -125.0 & -48.3 & 30.5 & 20.9 & 11.0 & 0.0 \\
0.2319 & -36.5 & -142.5 & -28.92 & -150.1 & -13.8 & -123.5 & -86.7 & 11.0 & 0.0 \\
0.2898 & -56.7 & -140.3 & -28.48 & -168.5 & -6.0 & -121.7 & -135.9 & 216.5 & 46.2 & 30.8 & 20.9 & 11.0 & 0.0 \\
0.3478 & -81.2 & -137.7 & -27.94 & -191.0 & 19.2 & -119.3 & -193.8 & 77.4 & 0.0 \\
0.4057 & -109.3 & -134.6 & -27.31 & -217.1 & 70.3 & -116.6 & -262.7 & -92.5 & 0.0 \\
0.4637 & -142.1 & -131.0 & -26.58 & -246.5 & 153.0 & -113.5 & -341.8 & -30.8 & 0.0 \\
0.5216 & -178.0 & -127.0 & -25.77 & -279.2 & 224.2 & -110.1 & -430.5 & -23.9 & 0.0 \\
0.5796 & -217.5 & -122.5 & -24.57 & -315.1 & 492.2 & -106.2 & -528.7 & 73.8 & 0.0 \\
0.6386 & -222.5 & -121.9 & -24.75 & -319.6 & 521.1 & -105.7 & -641.2 & 90.7 & 0.0 \\
\end{align*}
\]

Multiplier \( q \) \( q \) \( q \) \( q \) \( q \) \( q \) \( q \) \( q \) \( q \) \( q \)
TABLE II (Dead load, cont'd)

Fiber stresses in the outer ribs: (lb/in$^2$)

\[
\sigma_A = \frac{222}{1.227} = 6.701
\]

\[
\frac{\sigma_{yu}}{\sigma_{yl}} = \frac{222}{0.5356} \cdot 1.186 = 13.21
\]

\[
\frac{\sigma_{yl}}{\sigma_{yl}} = \frac{222}{0.5356} \cdot 0.9259 = 14.22
\]

<table>
<thead>
<tr>
<th>1</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-782.7</td>
<td>1632</td>
<td>1274</td>
<td>2145</td>
<td>491</td>
</tr>
<tr>
<td>.0580</td>
<td>-796.1</td>
<td>1502</td>
<td>1173</td>
<td>2298</td>
<td>377</td>
</tr>
<tr>
<td>.1159</td>
<td>-658.3</td>
<td>1125</td>
<td>897</td>
<td>1963</td>
<td>41</td>
</tr>
<tr>
<td>.1739</td>
<td>-908.7</td>
<td>555</td>
<td>434</td>
<td>1464</td>
<td>475</td>
</tr>
<tr>
<td>.2319</td>
<td>-1006</td>
<td>-137</td>
<td>-107</td>
<td>369</td>
<td>-1113</td>
</tr>
<tr>
<td>.2898</td>
<td>-1129</td>
<td>-841</td>
<td>-657</td>
<td>238</td>
<td>-1786</td>
</tr>
<tr>
<td>.3478</td>
<td>-1220</td>
<td>-1409</td>
<td>-1101</td>
<td>129</td>
<td>-2331</td>
</tr>
<tr>
<td>.4057</td>
<td>-1455</td>
<td>-1634</td>
<td>-1315</td>
<td>229</td>
<td>-2770</td>
</tr>
<tr>
<td>.4637</td>
<td>-1652</td>
<td>-1471</td>
<td>-1149</td>
<td>161</td>
<td>-2801</td>
</tr>
<tr>
<td>.5216</td>
<td>-1871</td>
<td>-544</td>
<td>-425</td>
<td>1327</td>
<td>-2206</td>
</tr>
<tr>
<td>.5796</td>
<td>-2111</td>
<td>1344</td>
<td>1049</td>
<td>3455</td>
<td>-1062</td>
</tr>
<tr>
<td>.6366</td>
<td>-2142</td>
<td>1652</td>
<td>1290</td>
<td>3794</td>
<td>-282</td>
</tr>
</tbody>
</table>

lb/in$^2$  lb/in$^2$
TABLE II (Dead load, cont'd)

b.) Middle rib: \( y_s = 0.6718'' \)

\[
\begin{align*}
\bar{S} & = 22.63 \text{ q} \\
\bar{Y} & = 585.9 \text{ q} \\
y_s \bar{Y} & = 393.6 \text{ q}
\end{align*}
\]

\[
\begin{array}{ccccccccccc}
& \text{(N)}_b & (M)_b & H_c \cos \omega & N & (M)_b & H_c R_e (1 - \cos \omega) & M_c & M \\
1 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 \\
10 & 0 & -585.9 & -57.55 & -528.3 & 0 & -393.6 & 0 & 181.3 \\
0.0580 & -4.1 & -585.0 & -57.45 & -531.6 & -3.4 & -393.0 & -10.5 & 129.7 \\
0.1159 & -16.3 & -582.0 & -57.16 & -451.1 & -12.2 & -391.0 & -42.0 & 71.1 \\
0.1739 & -36.9 & -577.1 & -56.68 & -573.5 & -21.7 & -387.7 & 94.4 & 0. \\
0.2319 & -65.2 & -570.2 & -56.01 & -579.4 & -24.7 & -383.1 & -167.1 & 574.9 & 73.8 \\
0.2898 & -101.2 & -561.5 & -55.15 & -607.5 & -10.6 & -377.2 & -260.9 & 135.7 \\
0.3478 & -144.2 & -550.8 & -54.10 & -641.5 & -34.2 & -370.1 & -374.7 & 156.2 \\
0.4057 & -196.7 & -538.4 & -52.88 & -681.2 & 125.4 & -361.7 & -507.9 & 73.9 \\
0.4637 & -253.3 & -524.1 & -51.47 & -726.1 & 231.7 & -352.1 & -660.7 & -156.2 \\
0.5216 & -317.5 & -508.0 & -49.90 & -775.6 & 524.6 & -341.3 & -832.1 & 101.5 \\
0.5796 & -387.9 & -490.2 & -48.15 & -829.9 & 877.8 & -329.3 & -1021.9 & 130.3 \\
0.6566 & -493.7 & -448.0 & -47.93 & -836.2 & 929.2 & -327.8 & -1046.0 & 130.3 \\
\end{array}
\]

Multiplier \( q \)

\[
\begin{align*}
\text{Multiplier} & \quad q & \quad q & \quad q & \quad q & \quad q & \quad q & \quad q \\
0 & 181.3 & 168.0 & 129.7 & 71.1 & 574.9 & 73.8 & 135.7 \\
0.0580 & 129.7 & 71.1 & 0. \\
0.1159 & 71.1 & 0. \\
0.1739 & 0. \\
0.2319 & 574.9 & 73.8 \\
0.2898 & 135.7 \\
0.3478 & 156.2 \\
0.4057 & 73.9 \\
0.4637 & -156.2 \\
0.5216 & -73.9 \\
0.5796 & 101.5 \\
0.6566 & 130.3
\end{align*}
\]
TABLE II (Dead load, cont'd)

Fiber stresses in the middle rib: (lb/in²)

\[
\begin{align*}
\frac{q}{A} &= \frac{8.222}{1.961} = 4.193 \\
\frac{q}{A_Y} &= \frac{8.222}{0.9204} 1.382 = 12.35 \\
\frac{q}{A_Y L} &= \frac{8.222}{0.9204} 0.7308 = 6.528
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{N}{A})</td>
<td>(\frac{M}{I_Y U})</td>
<td>(\frac{M}{I_Y L})</td>
<td>(C_U)</td>
<td>(C_L)</td>
<td></td>
</tr>
<tr>
<td>(=26 \cdot \frac{q}{A})</td>
<td>(=31 \cdot \frac{q}{A_Y U})</td>
<td>(=31 \cdot \frac{q}{A_Y L})</td>
<td>(=32 \cdot 33)</td>
<td>(=32 \cdot 34)</td>
<td></td>
</tr>
<tr>
<td>0.380</td>
<td>-2239</td>
<td>911</td>
<td>481</td>
<td>-1020</td>
<td>-3028</td>
</tr>
<tr>
<td>0.2319</td>
<td>-2429</td>
<td>0</td>
<td>0</td>
<td>-2429</td>
<td>-2429</td>
</tr>
<tr>
<td>0.2898</td>
<td>-2547</td>
<td>-911</td>
<td>-481</td>
<td>-1636</td>
<td>-3028</td>
</tr>
<tr>
<td>0.3478</td>
<td>-2690</td>
<td>-1676</td>
<td>-886</td>
<td>-1014</td>
<td>-3576</td>
</tr>
<tr>
<td>0.4057</td>
<td>-2856</td>
<td>-2091</td>
<td>-1105</td>
<td>-765</td>
<td>-3961</td>
</tr>
<tr>
<td>0.4637</td>
<td>-3045</td>
<td>-1329</td>
<td>-1020</td>
<td>-1116</td>
<td>-4065</td>
</tr>
<tr>
<td>0.5216</td>
<td>-3252</td>
<td>-913</td>
<td>-482</td>
<td>-2339</td>
<td>-3734</td>
</tr>
<tr>
<td>0.5796</td>
<td>-3480</td>
<td>1254</td>
<td>663</td>
<td>-4734</td>
<td>-2817</td>
</tr>
<tr>
<td>0.5866</td>
<td>-3509</td>
<td>1609</td>
<td>851</td>
<td>-6115</td>
<td>-2658</td>
</tr>
</tbody>
</table>

lb/in²  lb/in²
TABLE III

Anti-symmetrical load case: (load p)
a.) Normal Force $N_o$ and bending moment $M_o$ of the rib in the base system.

1.) Under shear force $S$ and horizontal load $P$

$$R_e = 106.926$$

$$P = 53.93 \ \$$

$$1 - \frac{y_s}{R} = 0.9920 \ 41$$

<table>
<thead>
<tr>
<th>1</th>
<th>37</th>
<th>38</th>
<th>39</th>
<th>40</th>
<th>41</th>
<th>42</th>
<th>43</th>
<th>44</th>
</tr>
</thead>
</table>
| $\omega$ | $(N_o)_S$ | $P \cos \omega$ | $N_o$ | $(M_o)_S$ | $PR_e(1 - \frac{y_s}{R} \cos \omega)$ | $M_o$ | $N_o$ | $M_o$
| 5  | 5  | 37+38 | 6  | 40-41 | Eq.(30) | Eq.(31) | Eq.(35) | Eq.(36) |
| 0. | -0. | 53.98 | 53.98 | -0. | -46.8 | 46.8 | -1. | -1. |
| 0.0580 | -0.13 | 53.98 | 53.71 | -0.15 | -36.9 | 36.7 | -0.9933 | -0.9933 |
| 0.159 | -0.72 | 53.62 | 52.90 | -0.54 | -7.4 | 6.9 | -0.9933 | -0.9933 |
| 0.1739 | -1.63 | 53.16 | 51.53 | -0.96 | 41.9 | -42.9 | -0.9849 | -0.9849 |
| 0.2319 | -2.28 | 52.93 | 49.65 | -1.09 | 110.7 | -111.8 | -0.9732 | -0.9732 |
| 0.2898 | -4.47 | 51.73 | 47.26 | -1.47 | 198.5 | -199.0 | -0.9583 | -0.9583 |
| 0.3478 | -6.40 | 50.75 | 44.35 | -1.51 | 305.4 | -306.9 | -0.9401 | -0.9401 |
| 0.4057 | -8.65 | 49.60 | 40.95 | -5.54 | 430.8 | -436.3 | -0.9188 | -0.9188 |
| 0.4637 | -11.20 | 48.28 | 37.08 | -12.45 | 574.4 | -586.9 | -0.8944 | -0.8944 |
| 0.5216 | -14.03 | 46.80 | 32.77 | -23.13 | 735.5 | -738.7 | -0.8670 | -0.8670 |
| 0.5796 | -17.14 | 45.17 | 28.03 | -38.79 | 914.1 | -922.9 | -0.8367 | -0.8367 |
| 0.5866 | -17.53 | 44.95 | 27.42 | -41.06 | 936.7 | -977.8 | -0.8328 | -0.8328 |

Multiplier $S$ | $S$ | $S$ | $S$ | $S$ | $S$ | $Y$ | $y_sY$
TABLE III (Anti-symmetrical load case, cont'd)

b.) Outer ribs: \( y_s = 0.8669" \)

\[ \begin{align*}
V_c &= 149.6 \text{ p} \\
S &= 12.69 \text{ p} \\
V_cR_e &= 16295 \text{ p} \\
Y' &= 146.4 \text{ p} \\
y_sY &= 126.9 \text{ p}
\end{align*} \]

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>( \left( N_o \right)_S )</th>
<th>( \left( N_o \right)_Y )</th>
<th>( V_c \sin \omega )</th>
<th>( N )</th>
<th>( \left( M_o \right)_S )</th>
<th>( \left( M_o \right)_Y )</th>
<th>( V_cR_e \sin \omega )</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>685.0</td>
<td>146.4</td>
<td>0</td>
<td>538.6</td>
<td>594</td>
<td>-126.9</td>
<td>0</td>
<td>467</td>
</tr>
<tr>
<td>0.0580</td>
<td>681.6</td>
<td>146.2</td>
<td>8.67</td>
<td>544.1</td>
<td>466</td>
<td>-126.7</td>
<td>945</td>
<td>1284</td>
</tr>
<tr>
<td>0.1159</td>
<td>671.3</td>
<td>144.3</td>
<td>17.29</td>
<td>545.1</td>
<td>88</td>
<td>-126.1</td>
<td>1884</td>
<td>1460</td>
</tr>
<tr>
<td>0.1729</td>
<td>653.9</td>
<td>144.2</td>
<td>25.38</td>
<td>535.6</td>
<td>-544</td>
<td>-125.0</td>
<td>2819</td>
<td>2150</td>
</tr>
<tr>
<td>0.2319</td>
<td>630.1</td>
<td>142.5</td>
<td>34.38</td>
<td>522.0</td>
<td>-1419</td>
<td>-123.5</td>
<td>3745</td>
<td>2202</td>
</tr>
<tr>
<td>0.2898</td>
<td>599.7</td>
<td>140.3</td>
<td>42.75</td>
<td>502.2</td>
<td>-2525</td>
<td>-121.7</td>
<td>4655</td>
<td>2085</td>
</tr>
<tr>
<td>0.3478</td>
<td>562.8</td>
<td>137.7</td>
<td>50.99</td>
<td>475.1</td>
<td>-3895</td>
<td>-119.3</td>
<td>5554</td>
<td>1540</td>
</tr>
<tr>
<td>0.4057</td>
<td>519.7</td>
<td>134.6</td>
<td>59.05</td>
<td>444.2</td>
<td>-5537</td>
<td>-116.6</td>
<td>6432</td>
<td>778</td>
</tr>
<tr>
<td>0.4637</td>
<td>470.5</td>
<td>131.0</td>
<td>66.91</td>
<td>406.4</td>
<td>-7443</td>
<td>-113.5</td>
<td>7288</td>
<td>-274</td>
</tr>
<tr>
<td>0.5216</td>
<td>415.9</td>
<td>127.0</td>
<td>74.84</td>
<td>363.4</td>
<td>-9623</td>
<td>-110.1</td>
<td>8119</td>
<td>-1619</td>
</tr>
<tr>
<td>0.5796</td>
<td>355.7</td>
<td>122.5</td>
<td>81.93</td>
<td>315.1</td>
<td>-12092</td>
<td>-106.2</td>
<td>8925</td>
<td>-3273</td>
</tr>
<tr>
<td>0.6376</td>
<td>348.0</td>
<td>121.9</td>
<td>82.30</td>
<td>308.9</td>
<td>-12408</td>
<td>-105.7</td>
<td>9019</td>
<td>-3495</td>
</tr>
</tbody>
</table>

Multiplier \( p \) \( p \) \( p \) \( p \) \( p \) \( p \) \( p \) \( p \) \( p \)
TABLE III (Anti-symmetrical load case, cont'd)

c.) Middle rib: \( y_s = 0.6713'' \) \( V_c = 270.7 \text{ p} \)
\( S = 22.33 \text{ p} \) \( V_c R_e = 29433 \text{ p} \)
\( Y = 585.9 \text{ p} \)
\( y_s \gamma = 393.6 \text{ p} \)

\[
\begin{array}{cccccccccc}
\omega & (N_0)_x & (N_0)_y & V_c \sin \omega & N & (M_0)_x & (M_0)_y & V_c R_e \sin \omega & M \\
=39.5 & =43.5 & =2V_c & =39.5 & =43.5 & =2V_c & =39.5 & =43.5 & =2V_c & =39.5 \\
0.0580 & 1215 & -584.3 & 15.39 & 645.3 & 1059 & -393.6 & 0 & 665 \\
0.1159 & 1197 & -582.0 & 12.29 & 646.3 & 1056 & -391.0 & 3404 & 3169 \\
0.1739 & 1166 & -577.1 & 46.84 & 635.3 & -397.1 & -387.7 & 5093 & 3734 \\
0.2319 & 1124 & -570.2 & 62.21 & 616.0 & -2530 & -383.1 & 6765 & 3852 \\
0.2898 & 1069 & -561.5 & 77.36 & 584.9 & -4503 & -377.2 & 8411 & 3531 \\
0.3478 & 1004 & -550.8 & 92.25 & 545.5 & -6945 & -370.0 & 10032 & 2717 \\
0.4057 & 927 & -538.3 & 106.3 & 495.5 & -9973 & -361.3 & 11616 & 1381 \\
0.4637 & 839 & -524.0 & 121.1 & 436.1 & -13222 & -352.0 & 13164 & -470 \\
0.5216 & 742 & -508.0 & 134.3 & 368.8 & -17169 & -341.3 & 14666 & -2844 \\
0.5796 & 634 & -490.2 & 148.3 & 292.1 & -21564 & -329.3 & 16120 & -5773 \\
0.6366 & 621 & -487.9 & 149.8 & 282.9 & -22128 & -328.0 & 16292 & -6164 \\
\end{array}
\]

Multiplier \( p \) \( p \) \( p \) \( p \) \( p \) \( p \) \( p \) \( p \)
TABLE IV

Live load: $p=4.933$ lb/in²

a.) Outer ribs: 

\[
\frac{P}{2A} = \frac{4.933}{2.454} = 2.010 \quad \frac{P}{2I} y_L = \frac{4.933}{1.0712} 0.9259 = 4.264
\]

\[
\frac{P}{2I} y_u = \frac{4.933}{1.0712} 1.186 = 5.462
\]

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>N</th>
<th>M</th>
<th>N/M</th>
<th>$M y_u$</th>
<th>$M y_L$</th>
<th>$\sigma_u$</th>
<th>$\sigma_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5866</td>
<td>328.5</td>
<td>3586</td>
<td>-1263</td>
<td>19587</td>
<td>15291</td>
<td>-20350</td>
<td>140^3^8</td>
</tr>
<tr>
<td>-0.5796</td>
<td>430.2</td>
<td>3347</td>
<td>-1267</td>
<td>18281</td>
<td>14272</td>
<td>-19548</td>
<td>13005</td>
</tr>
<tr>
<td>-0.5216</td>
<td>1589</td>
<td>1392</td>
<td>-1292</td>
<td>8679</td>
<td>6775</td>
<td>-9971</td>
<td>5483</td>
</tr>
<tr>
<td>-0.4637</td>
<td>193</td>
<td>1312</td>
<td>-1292</td>
<td>8679</td>
<td>6775</td>
<td>-9971</td>
<td>5483</td>
</tr>
<tr>
<td>-0.4057</td>
<td>871</td>
<td>1329</td>
<td>-1292</td>
<td>8679</td>
<td>6775</td>
<td>-9971</td>
<td>5483</td>
</tr>
<tr>
<td>-0.3478</td>
<td>1617</td>
<td>1341</td>
<td>-1292</td>
<td>8679</td>
<td>6775</td>
<td>-9971</td>
<td>5483</td>
</tr>
<tr>
<td>-0.2898</td>
<td>2054</td>
<td>1348</td>
<td>-1292</td>
<td>8679</td>
<td>6775</td>
<td>-9971</td>
<td>5483</td>
</tr>
<tr>
<td>-0.2319</td>
<td>2210</td>
<td>1351</td>
<td>-1292</td>
<td>8679</td>
<td>6775</td>
<td>-9971</td>
<td>5483</td>
</tr>
<tr>
<td>-0.1739</td>
<td>2194</td>
<td>1351</td>
<td>-1292</td>
<td>8679</td>
<td>6775</td>
<td>-9971</td>
<td>5483</td>
</tr>
<tr>
<td>-0.1159</td>
<td>1784</td>
<td>1347</td>
<td>-1292</td>
<td>8679</td>
<td>6775</td>
<td>-9971</td>
<td>5483</td>
</tr>
<tr>
<td>-0.0580</td>
<td>1201</td>
<td>1332</td>
<td>-1292</td>
<td>8679</td>
<td>6775</td>
<td>-9971</td>
<td>5483</td>
</tr>
<tr>
<td>0</td>
<td>377</td>
<td>1317</td>
<td>-1292</td>
<td>8679</td>
<td>6775</td>
<td>-9971</td>
<td>5483</td>
</tr>
<tr>
<td>0.0580</td>
<td>557</td>
<td>855</td>
<td>848</td>
<td>7467</td>
<td>5829</td>
<td>-6612</td>
<td>6684</td>
</tr>
<tr>
<td>0.1159</td>
<td>1367</td>
<td>855</td>
<td>848</td>
<td>7467</td>
<td>5829</td>
<td>-6612</td>
<td>6684</td>
</tr>
<tr>
<td>0.1739</td>
<td>1908</td>
<td>844</td>
<td>10421</td>
<td>8366</td>
<td>9300</td>
<td>-11109</td>
<td>10104</td>
</tr>
<tr>
<td>0.2319</td>
<td>2194</td>
<td>748</td>
<td>11913</td>
<td>9355</td>
<td>9355</td>
<td>-12366</td>
<td>10103</td>
</tr>
<tr>
<td>0.2898</td>
<td>1962</td>
<td>671</td>
<td>10716</td>
<td>8366</td>
<td>9355</td>
<td>-12366</td>
<td>10103</td>
</tr>
<tr>
<td>0.3478</td>
<td>1463</td>
<td>573</td>
<td>7991</td>
<td>6233</td>
<td>7418</td>
<td>-7418</td>
<td>6811</td>
</tr>
<tr>
<td>0.4057</td>
<td>685</td>
<td>457</td>
<td>3741</td>
<td>2921</td>
<td>2921</td>
<td>3264</td>
<td>3378</td>
</tr>
<tr>
<td>0.4637</td>
<td>355</td>
<td>321</td>
<td>1939</td>
<td>1514</td>
<td>1514</td>
<td>2260</td>
<td>1133</td>
</tr>
<tr>
<td>0.5216</td>
<td>1649</td>
<td>169</td>
<td>9007</td>
<td>7031</td>
<td>9176</td>
<td>6362</td>
<td>9037</td>
</tr>
<tr>
<td>0.5796</td>
<td>0</td>
<td>0</td>
<td>17473</td>
<td>17473</td>
<td>17473</td>
<td>13641</td>
<td>13641</td>
</tr>
<tr>
<td>0.6866</td>
<td>-10.7</td>
<td>-3404</td>
<td>-22</td>
<td>-18593</td>
<td>-14515</td>
<td>18571</td>
<td>-14537</td>
</tr>
</tbody>
</table>

Multiplier $\frac{P}{2}$  $\frac{P}{2}$  lb/in²  lb/in²
TABLE IV (Live load, cont'd)

b.) **Middle rib:** \( p = 4.933 \, \text{lb/in}^2 \)

\[
\begin{align*}
\frac{p}{2A} &= 4.933 \\
\frac{p}{21\gamma u} &= 1.9408 \\
\frac{p}{21\gamma L} &= 1.8408 \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>( N )</th>
<th>( M )</th>
<th>( N A )</th>
<th>( \gamma u )</th>
<th>( \gamma L )</th>
<th>( C_u )</th>
<th>( C_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.5866</td>
<td>-1120</td>
<td>6294</td>
<td>-1409</td>
<td>23307</td>
<td>12324</td>
<td>-24716</td>
<td>10915</td>
</tr>
<tr>
<td>-.5796</td>
<td>-1122</td>
<td>5875</td>
<td>-1411</td>
<td>21755</td>
<td>11503</td>
<td>-23166</td>
<td>10092</td>
</tr>
<tr>
<td>-.5216</td>
<td>-1144</td>
<td>2770</td>
<td>-1439</td>
<td>10257</td>
<td>5424</td>
<td>-11696</td>
<td>3985</td>
</tr>
<tr>
<td>-.4637</td>
<td>-1162</td>
<td>314</td>
<td>-1462</td>
<td>1163</td>
<td>615</td>
<td>-2625</td>
<td>847</td>
</tr>
<tr>
<td>-.4057</td>
<td>-1177</td>
<td>-1550</td>
<td>-1481</td>
<td>-5740</td>
<td>-3035</td>
<td>4259</td>
<td>4656</td>
</tr>
<tr>
<td>-.3478</td>
<td>-1187</td>
<td>-2853</td>
<td>-1493</td>
<td>-10565</td>
<td>-5586</td>
<td>9072</td>
<td>7079</td>
</tr>
<tr>
<td>-.2998</td>
<td>-1192</td>
<td>-3605</td>
<td>-1500</td>
<td>-13349</td>
<td>-7059</td>
<td>11849</td>
<td>8559</td>
</tr>
<tr>
<td>-.2319</td>
<td>-1195</td>
<td>-3852</td>
<td>-1503</td>
<td>-14264</td>
<td>-7542</td>
<td>12761</td>
<td>9045</td>
</tr>
<tr>
<td>-.1739</td>
<td>-1193</td>
<td>-3663</td>
<td>-1501</td>
<td>-13564</td>
<td>-7172</td>
<td>12063</td>
<td>8673</td>
</tr>
<tr>
<td>-.1159</td>
<td>-1187</td>
<td>-5039</td>
<td>-1493</td>
<td>-11253</td>
<td>-5950</td>
<td>9760</td>
<td>7443</td>
</tr>
<tr>
<td>-.0580</td>
<td>-1177</td>
<td>-1976</td>
<td>-1481</td>
<td>-7317</td>
<td>-3869</td>
<td>5836</td>
<td>5350</td>
</tr>
<tr>
<td>0</td>
<td>-1164</td>
<td>-484</td>
<td>-1464</td>
<td>-1792</td>
<td>-948</td>
<td>328</td>
<td>2412</td>
</tr>
<tr>
<td>0</td>
<td>108</td>
<td>846</td>
<td>136</td>
<td>3133</td>
<td>1656</td>
<td>-2997</td>
<td>1792</td>
</tr>
<tr>
<td>0.0580</td>
<td>114</td>
<td>2312</td>
<td>143</td>
<td>8561</td>
<td>4527</td>
<td>-8418</td>
<td>4670</td>
</tr>
<tr>
<td>0.1159</td>
<td>105</td>
<td>3299</td>
<td>132</td>
<td>12216</td>
<td>6459</td>
<td>-12084</td>
<td>6591</td>
</tr>
<tr>
<td>0.1739</td>
<td>79</td>
<td>3805</td>
<td>99</td>
<td>14090</td>
<td>7450</td>
<td>-13991</td>
<td>7549</td>
</tr>
<tr>
<td>0.2319</td>
<td>37</td>
<td>3852</td>
<td>47</td>
<td>14264</td>
<td>7542</td>
<td>-14217</td>
<td>7689</td>
</tr>
<tr>
<td>0.2898</td>
<td>23</td>
<td>3457</td>
<td>29</td>
<td>12801</td>
<td>6769</td>
<td>-12830</td>
<td>6740</td>
</tr>
<tr>
<td>0.3478</td>
<td>96</td>
<td>2581</td>
<td>121</td>
<td>9557</td>
<td>5054</td>
<td>-9678</td>
<td>4933</td>
</tr>
<tr>
<td>0.4057</td>
<td>-166</td>
<td>1212</td>
<td>-234</td>
<td>4488</td>
<td>2373</td>
<td>-4722</td>
<td>2159</td>
</tr>
<tr>
<td>0.4637</td>
<td>-290</td>
<td>-626</td>
<td>-365</td>
<td>-2318</td>
<td>-1226</td>
<td>1953</td>
<td>1591</td>
</tr>
<tr>
<td>0.5216</td>
<td>-407</td>
<td>-3918</td>
<td>-512</td>
<td>-10805</td>
<td>-5713</td>
<td>10293</td>
<td>6225</td>
</tr>
<tr>
<td>0.5796</td>
<td>-533</td>
<td>-3671</td>
<td>-677</td>
<td>-21000</td>
<td>-11104</td>
<td>20323</td>
<td>11781</td>
</tr>
<tr>
<td>0.5866</td>
<td>-554</td>
<td>-6034</td>
<td>-697</td>
<td>-22344</td>
<td>-11815</td>
<td>21647</td>
<td>12612</td>
</tr>
</tbody>
</table>

**Multiplier** \( \frac{p}{2} \), \( \frac{p}{2} \) \( \text{lb/in}^2 \) \( \text{lb/in}^2 \)
TABLE V

**Dead load:** Shell forces \( T_2 \) and moments \( M_1 \) at the center line \( \omega = 0 \)

**Outer ribs:** \( Y = 1270 \text{ lb.} \)

**Middle rib:** \( Y = 4096 \text{ lb.} \)

### \( T_2 \) - Forces: (lb/in)

<table>
<thead>
<tr>
<th>( x ) (in)</th>
<th>( x=12'' )</th>
<th>11''</th>
<th>10''</th>
<th>9''</th>
<th>7.5''</th>
<th>6''</th>
<th>4.5''</th>
<th>3''</th>
<th>2''</th>
<th>1''</th>
<th>0''</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_{\text{outer}} )</td>
<td>936.2</td>
<td>604.0</td>
<td>331.6</td>
<td>138.8</td>
<td>-15.7</td>
<td>-61.3</td>
<td>(-60.0)</td>
<td>-33.3</td>
<td>(-20.0)</td>
<td>(-10.0)</td>
<td>-3.2</td>
</tr>
<tr>
<td>( Y_{\text{middle}} )</td>
<td>-11.1</td>
<td>(-10.0)</td>
<td>(-20.0)</td>
<td>-31.5</td>
<td>(0.0)</td>
<td>16.8</td>
<td>130.3</td>
<td>335.1</td>
<td>510.0</td>
<td>675.0</td>
<td>754.9</td>
</tr>
<tr>
<td>( T_2 - \text{Membr} )</td>
<td>-888.0</td>
<td>-888.0</td>
<td>-888.0</td>
<td>-888.0</td>
<td>-888.0</td>
<td>-888.0</td>
<td>-888.0</td>
<td>-888.0</td>
<td>-888.0</td>
<td>-888.0</td>
<td>-888.0</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>48.2</td>
<td>-294.0</td>
<td>-576.4</td>
<td>-780.7</td>
<td>-903.7</td>
<td>-932.5</td>
<td>-807.7</td>
<td>-686.2</td>
<td>-398.0</td>
<td>-223.0</td>
<td>-133.1</td>
</tr>
</tbody>
</table>

### \( M_1 \) - Moments: (in-lb/in)

<table>
<thead>
<tr>
<th>( y ) (in)</th>
<th>( 0 )</th>
<th>7.951</th>
<th>10.262</th>
<th>9.435</th>
<th>6.048</th>
<th>2.798</th>
<th>(-1.000)</th>
<th>-0.204</th>
<th>(-0.100)</th>
<th>(-0.050)</th>
<th>-0.367</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_{\text{outer}} )</td>
<td>-0.297</td>
<td>(0.250)</td>
<td>(0.500)</td>
<td>0.750</td>
<td>(2.000)</td>
<td>3.940</td>
<td>5.308</td>
<td>3.793</td>
<td>-0.832</td>
<td>-10.179</td>
<td>-25.719</td>
</tr>
<tr>
<td>( Y_{\text{middle}} )</td>
<td>(-0.297)</td>
<td>(0.250)</td>
<td>(0.500)</td>
<td>0.750</td>
<td>(2.000)</td>
<td>3.940</td>
<td>5.308</td>
<td>3.793</td>
<td>-0.832</td>
<td>-10.179</td>
<td>-25.719</td>
</tr>
<tr>
<td>( M_1 )</td>
<td>0</td>
<td>8.201</td>
<td>10.762</td>
<td>10.185</td>
<td>8.048</td>
<td>6.738</td>
<td>4.308</td>
<td>3.589</td>
<td>-0.932</td>
<td>-10.229</td>
<td>-25.719</td>
</tr>
</tbody>
</table>

**Note:** ( ) numbers are interpolated

/numbers are neglected
TABLE VI

<table>
<thead>
<tr>
<th>Live load: Deflection of outer ribs at location of dials 3 and 4:</th>
<th>$R_l = 0.305$</th>
<th>$\sin \omega_k = 0.5535$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_r = 0.695$</td>
<td>$R_e = 108.926$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$\sin \omega$</th>
<th>$\sin \omega_k + \sin \omega$</th>
<th>$M'$</th>
<th>$N'$</th>
<th>$M'M'$</th>
<th>$N'N'$</th>
<th>$\frac{1}{2}M'M'$</th>
<th>$\frac{1}{2}N'N'$</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>-0.5866</td>
<td>-0.5535</td>
<td>0.0</td>
<td>-0.169</td>
<td>3586</td>
<td>-628.5</td>
<td>-0</td>
<td>106.2</td>
<td>0.531</td>
</tr>
<tr>
<td>76</td>
<td>-0.5796</td>
<td>-0.5477</td>
<td>0.058</td>
<td>-0.167</td>
<td>3347</td>
<td>-630.2</td>
<td>636</td>
<td>105.2</td>
<td>318.526</td>
</tr>
<tr>
<td>77</td>
<td>-0.5216</td>
<td>-0.4983</td>
<td>0.052</td>
<td>-0.152</td>
<td>1589</td>
<td>-642.6</td>
<td>2908</td>
<td>97.7</td>
<td></td>
</tr>
<tr>
<td>78</td>
<td>-0.4637</td>
<td>-0.4473</td>
<td>0.106</td>
<td>-0.136</td>
<td>193</td>
<td>-652.9</td>
<td>681</td>
<td>88.8</td>
<td></td>
</tr>
<tr>
<td>79</td>
<td>-0.4057</td>
<td>-0.3947</td>
<td>0.158</td>
<td>-0.120</td>
<td>-871</td>
<td>-661.4</td>
<td>-4599</td>
<td>79.3</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>-0.3478</td>
<td>-0.3408</td>
<td>0.217</td>
<td>-0.104</td>
<td>-1617</td>
<td>-667.1</td>
<td>-11432</td>
<td>69.4</td>
<td></td>
</tr>
<tr>
<td>81</td>
<td>-0.2898</td>
<td>-0.2858</td>
<td>0.267</td>
<td>-0.087</td>
<td>-2054</td>
<td>-670.7</td>
<td>-18260</td>
<td>58.4</td>
<td></td>
</tr>
<tr>
<td>82</td>
<td>-0.2319</td>
<td>-0.2298</td>
<td>0.323</td>
<td>-0.070</td>
<td>-2210</td>
<td>-672.1</td>
<td>-23758</td>
<td>47.0</td>
<td></td>
</tr>
<tr>
<td>83</td>
<td>-0.1739</td>
<td>-0.1730</td>
<td>0.380</td>
<td>-0.053</td>
<td>-2119</td>
<td>-671.2</td>
<td>-26784</td>
<td>35.6</td>
<td></td>
</tr>
<tr>
<td>84</td>
<td>-0.1159</td>
<td>-0.1156</td>
<td>0.437</td>
<td>-0.035</td>
<td>-1784</td>
<td>-670.2</td>
<td>-25957</td>
<td>23.5</td>
<td></td>
</tr>
</tbody>
</table>

* Upper value from $\omega = -0.5866$ to 0.1739.
Lower value from $\omega = 0.2319$ to 0.5866.
Shell roof model showing measured dimensions.

All dimensions in inches.
**DEAD LOAD**

Theoretical
\[ q = 8.222 \text{ lbs/in}^2 \]

Actual, \( \text{Pad} = 312.5 \text{ lbs} \)

Tension pad (5'' x 5'')

1.26''

5''

1.38''

**LIVE LOAD**

Theoretical
\[ p = 4.933 \text{ lbs/in}^2 \]

Actual, \( \text{Pad} = 187.5 \text{ lbs} \)

Tension pad (5'' x 5'')


Fig 2
Membrane Forces on Shell Element

Stiffening Diaphragms

\[ \omega \]

\[ \omega_\lambda \]

\[ R \]

\[ l \]

\[ \bar{x} \]

\[ \bar{y} \]
1) Symmetric Load Case

2) Anti-Symmetric Load Case

Dead Load: Case 1, replacing P/2 by q

Live Load over Half Span: Case 1 + Case 2
Dead load of rib (cross section of rib only)

String Force $Y$

$Y = \bar{Y} \cos \omega$

Shear Force $S$

$S = \bar{S} \sin \omega$
Deformed Arch Axis

Anti-symmetric Load Case

Simplified Structure

Outer Rib

Middle Rib

Outer Rib

Membrane $T_z$ (tension)

Membrane $T_z$ (comp.)

$P = -(l - \bar{x}) T_z$

$P_2 = -2\bar{x} T_z$

Fig 7
a) \( S = S \sin \omega \)

b) Displacement \( \delta_v \)

(c) Displacement \( \delta_v \),

\[ P = 1 \]

\[ M = R_e \sin \omega \]

\[ \omega \]

Fig 8
For example, this is gage 03

- Single gage
+ Cross gage
☆ Rosette gage

LAY-OUT OF STRAIN

GAGE COORDINATE

SYSTEM

FIG. 9
VERTICAL DEFLECTION DIALS 1 TO 13

(+ L-2)

END ROTATIONS LEVEL BARS L-1 AND L-2

16.6" 18.5" 24" 24" 18" 18.5"

119.6"

LATERAL DISPLACEMENT DIALS 15 TO 19

LAY-OUT OF DIAL GAGES AND LEVEL BARS

FIG. 10

ELONGATION OF SPAN DIAL 14

L-1

L-2
Fig II

LOAD: dead load

p = 8.222 lb/sq in. of shell surface

σ_u - stress in upper fiber
σ_L - stress in lower fiber

DEAD LOAD:

FIBER-STRESS
(N + Mv) / A - I

AT UPPER AND LOWER EDGES OF RIBS
STRESS in psi

\[ \sigma = \frac{P}{A} \]

where:
- \( \sigma \) is stress in psi
- \( P \) is load
- \( A \) is cross-sectional area of rib

Fig 12

MIDDLE RIB

OUTER RIBS

LOAD, half-size, live-load

\( p = 4933 \) lb/4 in. of shell surface

LIVE LOAD

FIBER-STRESS AT UPPER AND LOWER EDGES OF RIBS

\( \frac{(N + M)}{L} \)

theoretical

experimental
NOTATIONS

Subscripts

P  due to the load P
s  due to the shear force S
S  refers to Shell
R  refers to Rib
u  refers to upper fiber of rib
L  refers to lower fiber of rib
ε  due to the membrane strain εω

Roman Alphabet

A  cross section area
bR  width of rib
c = \frac{\pi}{2\omega_k} coefficient as given in Progress Report 213-B
D  thickness of shell
E  modulus of Elasticity
H_C  horizontal thrust of middle or outer ribs
h  distance between the two outer ribs of the shell = 2l
h_R  height of rib
I  moment of inertia
K  coefficient determining the effective width, see Progress Report 213-B, Eq. (41), (42) and Fig. 8
l  distance between the ribs, Fig. 3
M  bending moment of the rib (effective section)
M_C  bending moment of the rib at the center, ω = 0
M_K  bending moment of the rib at the springing line, ω = ω_k
M_0  bending moment of the rib in the statically determinate base system
$M_1$ bending moment per unit width of the shell in axial direction

$M_2$ bending moment per unit width of shell in circumferential direction

$M'$ bending moment used in the work equation due to the virtual load system

$M'_k$ virtual bending moment of the rib at the springing line, $\omega = \omega_k$

$M'_l$ virtual bending moment of the rib on the left side of the virtual load $P' = 1$, p. 29

$M'_r$ virtual bending moment of the rib on the right side of the virtual load $P' = 1$, p. 29

$N$ normal force in the rib (effective section)

$N_0$ normal force of the rib in the statically determinate base system

$N'$ normal force used in the work equation due to the virtual load system

$N'_l$ virtual normal force of the rib on the left side of the virtual load $P' = 1$, p. 29

$N'_r$ virtual normal force of the rib on the right side of the virtual load $P' = 1$, p. 29

$p$ load per unit area on the shell, in the numerical example used as live load per unit area

$P, P_1, P_2$ horizontal forces due to anti-symmetrical loading, acting on the ribs at the center, $\omega = 0$, see p. 12 and Eq. (29)

$q$ dead load per unit area of the shell

$R_e$ radius of the effective section of the rib

$R_s$ radius of the shell
radius of the rib
reaction of the left support in the virtual load system of p. 29
reaction of the right support in the virtual load system of p. 29
tangential shear force per unit width of the shell
coefficient of the tangential shear $S$, defined by Eq. (8)
normal force per unit width of shell in axial direction
normal force per unit width of shell in circumferential direction
vertical reaction at the center, $\omega = 0$, in the anti-symmetrical load case
coordinate in axial direction, Fig. 3
distance from the middle rib at which the membrane shear force $S$ becomes zero, Eq. (4) and Fig. 7
string force as given in Progress Report 213-B, p. 7 and Fig. 4
coefficient of the string force $Y$, defined by Eq. (10)
distance between the lower fiber of the rib and the centroid of the effective section; see sketches p. 19 and 20
distance between the centroid of the effective section and the middle surface of the shell; see sketches p. 19 and 20
distance between the upper fiber of the rib and the centroid of the effective section; see sketches p. 19 and 20

Greek Alphabet

$\alpha$ angular coordinate, as used in Fig. 6
\( \beta \) coefficient depending on shell dimensions, Progress Report 213-B, Eq. (10) to (14)

\( \delta \) vertical deflection of the ribs, used in the numerical example for the deflection at the location of the dial gages 3 and 4

\( \delta_h \) horizontal displacement of the abutments, Fig. 6c

\( \delta_v \) vertical deflection at the center in the anti-symmetrical load case, Fig. 8b

\( \varepsilon \) strain

\( \varepsilon_\omega \) strain in the circumferential direction along the connecting line of rib and shell

\( \kappa \) coefficient of elastic restraint of the rib (effective section) by the abutment

\( \bar{\kappa} = \frac{EI}{Re} \cdot \kappa \)

\( \lambda = \kappa \sqrt{\frac{d}{R}} \) coefficient depending on shell dimensions and force distribution, Progress Report 213-B, p. 24

\( \nu \) Poisson's ratio

\( \sigma_L \) stress in the lower fiber of the rib

\( \sigma_n \) normal stress in the rib in the fiber of the connecting line rib-shell

\( \sigma_u \) stress in the upper fiber of the rib

\( \omega \) angular coordinate in circumferential direction, Fig. 3

\( \omega_k \) angle of opening of the shell structure, Fig. 3
LIST OF REFERENCES


(3) W. Flügge; "Statik und Dynamik der Schalen" Springer Verlag, Berlin 1934.
