1950

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"STRESS & STRAIN DISTRIBUTION AND ROTATIONS IN VERY SHORT-SPAN BEAMS OF WIDE-FLANGE SECTION"

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It is sometimes necessary that the stress and strain distribution in short beams be investigated. As a result of such an investigation, a load-rotation curve for the beam could be obtained. Such a curve in both the elastic and plastic range is of great value, especially for the corner (knee) of square rigid frames without diagonal stiffeners. The following discussion is limited to I-beam or wide-flange sections. Fig. 1 shows the symbols used.

For both elastic and elasto-plastic ranges the ordinary assumptions made for beam formulas will be assumed true, namely,

1. Rotations due to shear are added to those of flexure and they are independent of each other.

2. The region which has yielded due to shear cannot take any normal stress.

3. Normal strains are always proportional to the distance from the neutral axis.

4. Stress-strain relations are assumed as shown.

I. Shear Stresses and Rotations

It is assumed that as the external loading is increased the beam yields in shear at the neutral axis and that shear yielding takes place uniformly along the length of the beam (cantilever, or simple beam with load at mid-span). As the load is increased, the yielding proceeds towards the flanges. When rotation due to shear considered are these at the neutral axis.

A. Elastic Range

a. Stress and strain distribution.

The distribution of shear stress area shown in Fig. 2 with almost all the shearing force taken by the web and with parabolic distribution. While stresses are elastic

\[ \gamma = \frac{T_{\text{max}}}{G} \frac{V}{wGI} \left[ \frac{bh^2}{8} - \frac{h^2}{8} (b-w) \right] \tag{1} \]

Since shear \( V \) is constant along the beam, the rotation
is the same along the neutral axis. The rotation is maximum at the neutral axis and minimum at the end of the web.

B. Elasto-plastic Range

As the external load is increased \( T_{\text{max}} \) reaches the value of \( T_y \) and yielding starts. If the load at this instance is \( V_o \) we have

\[
T_{\text{max}} = T_y = \frac{V_o}{w} \left[ \frac{bh^2}{8} - \frac{h_i^2}{8} (b-w) \right]
\]

(2)

\[
T_{\text{min}} = \frac{V_o}{w} \left[ \frac{bh^2}{8} - \frac{bh_i^2}{8} \right]
\]

(3)

and

\[
T_y - T_{\text{min}} = \frac{V_o h_i^2}{8w}
\]

(4)

for any additional load \( V \) yielding will proceed toward the flanges.

It is assumed that as yielding proceeds the distribution of shears in the elastic portion is given by a family of curves similar to those in the elastic range. When

yielding starts at the neutral axis the equation of the parabola (considering \( x \)-axis along the neutral axis and the \( y \)-axis through the \( T_{\text{min}} \)) is

\[
x = \frac{V_o h_i^2}{8w} \left( 1 - \frac{4}{h_i^2} y^2 \right)
\]

(5)

Then

\[
\frac{dx}{dy} = -\frac{V_o}{I} y
\]

The equation of family of curves is: \( x = \frac{V_o}{2I} y^2 + C \) (6)

The depth of yielding \( y \), is given by the point of intersection of each curve with the line \( x = \frac{V_o h_i^2}{8w} \)
Therefore, when \( x = \frac{V_0 \cdot h_1^2}{8I} \)

\( y = y' \)

and by substituting in equation (6)

\[ C = \frac{V_0 \cdot h_1^2}{8I} + \frac{V_0}{2I} \cdot y_i^2 \]

And (6) becomes:

\[ x = \frac{V_0 \cdot h_1^2}{8I} + \frac{V_0}{2I} \cdot y_i^2 + \frac{V_0 \cdot h_1^2}{8I} = \frac{V_0}{2I} \left( \frac{y_i^2 - y^2}{2} \right) \]

in which \( y > y_i \)

For different values of \( y_i \) the area shown shaded in Fig. 4 will give the additional shearing force \( V \) above \( V_0 \). If point \( R(x_i, \frac{h_i}{2}) \) is the intersection of \( y_i = \frac{h_i}{2} \) and one of the curves we have from (7)

\[ x = \frac{V_0 \cdot h_1^2}{8I} + \frac{V_0}{2I} \cdot y_i^2 - \frac{V_0 \cdot h_1^2}{8I} = \frac{V_0}{2I} \cdot y_i^2 \]

Consequently

\[ \frac{V}{2} = w \int_0^x \left[ \frac{h_i}{2} - F(a) \right] dx + w \int_{x_i}^{\frac{V_0 \cdot h_i^2}{8I}} \left[ F(b) - F(a) \right] dx \]

but

\[ F(a) = \sqrt{\frac{2I}{V_0} \left( \frac{V_0 \cdot h_i^2}{8I} - x \right)} \]

\[ F(b) = \sqrt{\frac{2I}{V_0} \left( \frac{V_0 \cdot h_i^2}{8I} + \frac{V_0}{2I} \cdot y_i^2 - x \right)} \]

Taking as \( k_1 = \frac{V_0 \cdot h_i^2}{8I} \), \( k_2 = \frac{4}{2I} \cdot h_i^2 \), \( k_1 \), \( k_2 = \frac{V_0}{2I} \) and substituting in (8) we have

\[ \frac{V}{2} = w \int_0^x \frac{h_i}{2} dx + w \int_{x_i}^{k_i} \left( \sqrt{k_i \left( 1 + k_2 \cdot y_i^2 \right) - x} \right) dx - w \int_{x_i}^{k_i} \left( \frac{k_i}{k_1} \left( 1 - k_1 - x \right) \right) dx \]

\[ V = -w \left[ \frac{k_i}{3} \left( \frac{k_2 \cdot h_i^3}{4} - \frac{4}{3} \cdot h_i + \frac{4}{k_1} \right) \right] + k_i \cdot k_2 \cdot y_i^2 \cdot w \left( h_i - \frac{4}{3} \cdot y_i \right) \]

Substituting the constants and simplifying we have

\[ V = \frac{V_0}{2I} \left( h_i - \frac{4}{3} \cdot y_i \right) w \]

\[ \text{(9)} \]
From equation (5) we have \( \gamma = \frac{V_0 h_1^2}{8 IG} \left( 1 - \frac{y}{h_1} y^2 \right) \) (10)

When \( T_{max} = Ty \) at \( y = 0 \), \( \gamma = \frac{V_0 h_1^2}{8 IG} \)

From (10), we have \( \frac{d\gamma}{dy} = -\frac{V_0 y}{IG} \)

Therefore \( \gamma = C - \frac{V_0 y^2}{2IG} \) (11)

Where \( C \) is the constant of integration

When \( y = y_1 \), \( \gamma = \gamma_1 \), \( \therefore C = \gamma_1 + \frac{V_0 y_1^2}{2IG} \)

Substituting in (11)
\( \gamma = \gamma_1 + \frac{V_0}{2IG} (y_1^2 - \gamma^2) \) (12)

This equation is assumed valid for \( \gamma > y_1 \). For \( \gamma < y_1 \), that is in the plastic region it is assumed that \( \gamma \) varies as a straight line. The slope at \( R \) is

\[ \frac{d\gamma}{dy} |_{y = y_1} = \frac{V_0 y}{IG} \]

and
\[ \lambda = y_1 \times \frac{d\gamma}{dy} = \frac{V_0 y_1^2}{IG} \]

where \( \lambda = \) increase in rotation at the neutral axis

when yielding has advanced to \( \gamma_1 \)

The total rotation at the neutral axis is
\[ \gamma_N = \gamma_1 + \frac{V_0}{\omega IG} \left[ \frac{bh^2}{8} - \frac{h_1^2}{8} (b-w) \right] + \frac{V_0 y_1^2}{IG} \] (13)
II. Normal Stresses, strains and rotations

A. Elastic

Strains are proportional to the distance from the neutral axis and the material follows Hooke's Law. The rotations are given as:

\[ \phi = \frac{M}{R} = \frac{M}{EI} \]

where \( R \) = radius of curvature

\( EI \) = the flexural rigidity of the beam.

B. Elasto-plastic

Suppose shear yielding advanced to \( y = y' \), as shown in Fig. 6. It will be assumed that

\[ M = 2 \int_{y'}^{h} G y \, dy \text{ but } \varepsilon = \frac{y}{R} \text{ and } G = \varepsilon E = \frac{E y}{R} \]

Substitution: \( M = 2 \int_{y'}^{h} \frac{E y^2}{R} \, dy = E \frac{2}{R} y^2 \text{ (Momemt of inertia of plastic portion)} \)

and

\[ \phi = \frac{1}{R} \frac{M}{E(I - \frac{2}{3} wy_1^3)} = \frac{M}{EI_e} = \frac{M}{E(I - I_p)} \]

(14)

For the wide flange

\[ \phi = \frac{1}{R} \frac{M}{E(I - \frac{2}{3} wy_1^3)} \]

(15)

and

\[ G = \frac{M y}{I - \frac{2}{3} wy_1^3} \]

(15a)

where \( y > y' \)

When yielding reaches the flanges \( y_1 = \frac{h}{2} \)

and

\[ \phi = \frac{M}{E(I - \frac{1}{12} wh_1^3)} \]

(16)
After this point has been reached the two flanges will bend independently and obviously the increase in external load will be very small.

In a connection (knee) of the type 7 there could be an increase in loading above the load that causes complete shear failure in the web by considering the flanges and the stiffeners as a rigid box.

Consider a very short beam whose web has yielded in shear. It will be assumed that any increase in load beyond this point will be taken by the flanges alone in direct tension and compression in the two flanges.

If \( M = \) additional moment
\[ A_f = \text{flange area} \]
\[ d = \text{distance between flanges} = h - t \]
\[ P_f = \frac{M}{h - t} \quad \text{and} \quad G_f = \frac{P_f A_f}{(h - t) A_f} \quad (17) \]

From (15a) when \( y = \frac{h_f}{2} \) we have
\[ G = \frac{M h_f}{I - \frac{1}{2} w h_f^3} \quad (18) \]

Therefore complete collapse will occur when the sum \( G + G_f \) as given by (17) and (18) becomes equal to \( G \gamma \) for the material. The moment in (18) is caused by the force \( V_o \left[ l + \frac{h_f^3}{24 I} w \right] \) while in (17) is caused by any additional force.

Summary

Elastic range:

Rotation = Shear Rotation + Bending Rotation
\[ = \frac{V}{w G I} \left[ \frac{bh_f^2}{b} \frac{h_f^2}{b} (b - w) \right] + \frac{M}{EI} \]

Plastic Range:

When \( \gamma_{\max} = \gamma = \frac{V_o}{w I G} \left[ \frac{bh_f^2}{b} \frac{h_f^2}{b} (b - w) \right] \]

Maximum Shearing Force = \( V_o + V = V_o + \frac{V_o h_f^2}{8} \left( \frac{h_f}{b} \right) w \)
\[ = V_o \left[ l + \frac{h_f^3}{24 I} w \right] \]

Rotation due to shear = \( \gamma + \frac{V_o}{2IG} (\gamma - \gamma_f) \)

Total " " " " = \( \frac{V_o}{wIG} \left[ \frac{bh_f^2}{b} (h_f - h_f^2) + 3 w h_f^2 \right] \left[ h_f^2 - \frac{h_f^2}{b} (b - w) + w y_f^2 \right] \)

Max. Total Rotation due to shear = \( \frac{V_o}{wIG} \left[ \frac{bh_f^2}{b} (h_f - h_f^2) + 3 w h_f^2 \right] \)

Rotation (shear & Bending) = \( \frac{M}{E (I - \frac{1}{2} w y_f^2)} + \frac{V_o}{w IG} \left[ \frac{bh_f^2}{b} - \frac{h_f^2}{b} (b - w) + w y_f^2 \right] \)