Notes on behavior of i and wf beams in shear, October 1951

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REPORT

Notes on Behavior of "I" and "W" Beams in Shear

TO

BY

C. N. YANG
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Slat on a beam in elastic range.

\[ a \int_0^l f_s y_0 b dl + 2m_1 = \frac{2MAD}{I} \]  

\[ 2 \int_0^l f_s y_0 b dl + m_2 = \frac{2MAy_0}{I} + m_2 \]  

\[ (1)+2 = \frac{2MA}{I} \left(D+y_0\right) + 2m_1 + m_2 \]  

It is very apparent from the above diagram that:

\[ \frac{2M(A+y_0)}{I} \] \( \alpha \) equals the moment represented by \( \Delta ABCD \).
\( \omega M_1 \) represent the moment of area \( CC'B' \) (times two).

\[ M_2 = \ldots = C'D'E'F' \]

The summation of above terms of course represented by areas \( AB'HK \) i.e. the moment \( M \) at the action.

This checks the method of analysis is correct, and we can assume the radius of curvature of the action \( r = R = \frac{M}{E} \) and prove it analytically instead of using the diagrams.
\[ I = \frac{b h^3}{12} \]

\[ Q = \frac{(h - y)^2 b}{2 a} \]

\[ \sigma_s = \frac{V}{b h^3} \frac{(h - y)^2}{b h^3} \]

\[ \sigma_s = \frac{6V(h^2 - y^2)}{b h^3} \]

\[ y = 0 \quad \sigma_s = \sigma_{y,p} = \frac{6V(h^2)}{b h^3} = \frac{6V}{b h^3} \]

\[ V = \sigma_{y,p} \frac{b h}{6} \]

\[ \sigma_s = \sigma_{y,p} \left( 1 - \frac{y^2}{h^2} \right) \]

\[ \text{From Equation (11)} \]

\[ 2 \int_0^\infty f_s \, b \, dL + 2 m_1 \quad \text{(A)} \]
Where \( f_5 = g_{y,p} - g_{y,0} (1 - \frac{y_0^2}{h^2}) \)
\[ f_5 = g_{y,p} \frac{b_0^2}{h^2} \]
\[ 2 \int_0^l f_5 b_0 d\theta \Delta_m = 2 \left( \frac{g_{y,p} y_0^2}{h^2} b_0 \Delta_l + m_1 \right) \]

For part (2)
\[ 2 \int_0^l f_5 y_0^2 d\theta + m_2 = \frac{20 g_{y,p} y_0^3}{h^2} b_0 y_0 + m_2 \]
\[ = \frac{20 g_{y,p} y_0^3}{h^2} b_0 + m_2 \]  \[ \text{(B)} \]

We don't know what is \( m_2 \) except we know

Shear distribution

\[ b \int_{-h}^{h} dy \Delta_l = 0 \]
\[ \int_{-h}^{h} f_5 b_0 \Delta_l \, d\theta + \int_0^l \left( f_5 + \frac{df_5}{dy} \right) b_0 \, d\theta \]
\[ = \frac{2 y g_{y,p} x b_0^2}{h^2} \int_{-h}^{h} dy \]
\[ \therefore \Delta_N = \frac{2 y g_{y,p} x b_0^2}{h^2} \]
\[ m_2 = \bar{m} \int_0^y \frac{\delta y \, dy}{y_0} = \frac{2g \, y \, \rho \, l}{R^2} \int_0^y \frac{y^2 \, dy}{y_0} \]

\[ = \frac{2g \, y \, \rho \, l}{R^2} \frac{y_0^3}{12} \]

\[ m_2 = -\frac{8g \, y \, \rho \, b \, l \, y_0^3}{3R^2} \]

\[ m_2 = -\frac{8g \, y \, \rho \, b \, l \, y_0^3}{3R^2} \]

\[ (A11 : b) = M = V \cdot E \]

\[ \text{Set } M \rightarrow \bar{M} \]

\[ \frac{\partial}{\partial y} \left( \frac{d\alpha}{dy} \right)_A = \frac{1}{\frac{d\alpha}{dy}} \left( \frac{d\alpha}{dy} \right)_B \]

\[ \frac{\partial}{\partial y} \left( \frac{d\alpha}{dy} \right)_A = 2g \, y \, \rho \, y_0 / R^2 \]
Suppose \( m_1 = v_1 \epsilon \)

\[
\sigma_s = \frac{6 v_1 (D^2 - y^2)}{b D^3}
\]

\[
\frac{d\sigma_s}{dy} = - \frac{12 v_1 y}{b D^3} \quad \text{part } y = \frac{b}{2}
\]

\[
\left( \frac{d\sigma_s}{dy} \right)_b = - \frac{6 v_1}{b D^2}
\]

\[
\frac{b D^2}{6 v_1} = \frac{2 \sigma_{y.p. y_0}}{k^2}
\]

\[
v_1 = \frac{b k^2 D^2}{12 \sigma_{y.p. y_0}} \quad \text{(d')}
\]

\[M = V \epsilon = (A) + (B)\]

\[
V \epsilon = 2 \left( \frac{\sigma_{y.p. y_0}^2 y^2 b k^2}{k^2} + \frac{b k^2 y^2}{12 \sigma_{y.p. y_0}} \right)
\]

\[
+ \frac{2 \sigma_{y.p. y_0} y^2 b k^2}{12} \phi - \frac{8 \sigma_{y.p. y_0} b k^2 y^3}{3 k^2}
\]

\[\text{(c)}\]
From equation we know

\[ D = \frac{L - 24}{2} \]

If we know \( V \) we can solve the
equation of third degree to solve
for \( y_0 \).

By use of \( y_0 \) we'll have the equation
of deflection curve by means of equation
(16)

\[ \frac{d^2y}{dx^2} = -\frac{M_1}{EI} = -\frac{VL}{EI} \]
1. Discussion on I Beams.

- Solve the web part as discussed above.
- Before the shearing stress getting too high in flange, it can be regarded as separate beams.

2. Discussion on end condition.

Two separate beams rigidly connected at both ends under concentric load. The solution should be one equivalent to a rigid beam.
Shear strength of "I" sections

$$\tau_s = \frac{VQ}{2bh}$$

$$\sigma = \frac{Mc}{I} = \frac{V \times 1 \times c}{I}$$

Suppose we want the same bending strength:

$$V = 2kI$$

where $$k = \frac{\tau_s y_p}{\sigma}$$

$$\therefore \tau_s = k \frac{Q}{bh}$$

Therefore the shear stress is linearly proportional to ratio $$\frac{Q}{bh}$$
8 uw = 40

\[ a = 41.05 \]
\[ b = 3.711 \]

\[ \frac{a}{2b} = \frac{110.5}{8} = 13.8 \]

14 uw = 30

\[ Q = \frac{3.83 \times 6.733 \times 13.86 + (13.86 - 3.83)^2}{2} \times \frac{1}{2} \times 270 \]

\[ Q = 23.86 + 35.8 + 11.5 = 71.1 \]

\[ b = 0.27 \]

\[ \frac{Q}{b^2} = \frac{175}{0.27^2} = 125.5 \]

It is very obvious that 14 uw = 30 is by much lower shearing strength than that of 8 uw = 40.
Shear

1. Shear strength of "I" beams
2. Plastic behavior of beams due to shear failure
3. Contribution of beam deflection due to shear failure
4. Shear failure and bending strength of beams

Same sections above under same moment

Calculate the relation of bending strength

5. Consideration of shear failure in plastic design

a. Reluctant. Only happens in changing shape of the section. Rectangular could be considered equivalent to determine structure in design.

b. Usual "I" shape the contribution of deflection will be enormous before the section reached its developed its reluctant shape.

c. Bend if strength would be affected as the shear failure spread through the web.

d. It seems shear should be limited below its initial yield strength. The web thickness should be increased in the conventional sections.

e. In case of non-uniform shear the problem of changes to a problem of eccentricity.
(i.e. Plastic-Plastic shear) Assumptions made in course plastic shear analysis are not adequate. When only the max shear reaches the shear yield strength, of course, residual strength and defl. will not be as much affected as in the case of cont. shear

5. Procedure of shear plastic flow stress hardening in webs of "I" beams
Theoretical analysis of beam behaviour after it has been yielded by shear

March

April 29, 49
Along AB & CD there is no normal stress inside so after load applied point like E.F do not shift there places.

Assume after loading planes like Q.E & F.K still transformed to planes as G.E.F.H. (Fully shear force rigid in elastic region)

The normal stress distribution of course is the same proportion as line G.E & F.H.
From the previous proof we know that in the shearing failure region, normal stresses equal to zero.

General assumptions are made for the solution of the problem:

a. Shear deformation in the elastic region is zero. (Plan remains plane)

\[ \frac{1}{K} = \frac{d^2y}{dx^2} \]  

b. Geometric approximation

Stress distribution diagram under previous assumption.
\[\Sigma M = 0\]

\[M = 2 \int \frac{p}{L} E_y dA\]

\[M = 2 \int \frac{p}{L} (y - y_0) E_y dA\] \( \left[ \frac{\varepsilon_0}{2} = \frac{\varepsilon}{(y-y_0)} \right] \)

When \(\varepsilon_0 = \) fiber strain, \(E\varepsilon_0 = \) fiber stress = \(\sigma_0\).

\[M = \frac{4}{3} \int \frac{R}{L} (y - y_0) \sigma_0 y dA\]

\[M = \alpha \frac{I_2 T_0 - \frac{\sigma_0 L_1}{R}}{\frac{L}{2}} = \frac{2V_0}{R} \left[ I_2 - y_0 L_1 \right] \]

Where \(I_2\) = moment of inertia of area about \(N-A\).

\(L_1 = \) moment of plastic area about \(N-A\).
\[ R = \frac{L_0}{\frac{h}{2} - y_0} = \frac{J_0}{(h-y_0)E} \quad \text{where} \quad n = \frac{R}{h} \]

\[ J_0 = \frac{\frac{hM}{(I_{21} - y_0^2)}} {I_{21} - y_0^2} \]

\[ \frac{1}{R} = R = \frac{hM}{I_{21} - y_0^2} E (h-y_0) \]

In case of constant shear:
- \( I_{21}, y_0 \) are constant.
- Uniform load
- \( I_{21}, y_0 \) are functions of \( x \)

\[ \frac{d^2y}{dx^2} = -\frac{hM}{(h-y_0)(I_{21} - y_0^2)E} \quad \text{where} \quad m = f(x) \]

Solve this equation for general deflection curve.
The distribution of shearing stress

\[ \sigma = \frac{F \, w}{(I_2 - y_0^2)} \]

\[ \frac{d\sigma}{dx} = \frac{F}{(I_2 - y_0^2)} \frac{dw}{dx} \]

Suppose \( I_2 \leq y_2^2 \) unk.

\[ b \, T_{sy.p} = \int_{y_0}^h \frac{d\sigma}{dx} \, dA = \frac{dW}{dx} \]

In constant shear

\[ b \, T_{sy.p} = \int_{y_0}^h \frac{h}{(I_2 - y_0^2)} \frac{dw}{dx} \, dA = \frac{h}{y_0} \]

\[ b \, \sigma_{sy.p} = \frac{h}{(I_2 - y_0^2)} \frac{W \, A_0}{I_2 - y_0^2} = \frac{h \, w \, A_0}{I_2 - y_0^2} \]

\[ (I_2 - y_0^2) = \frac{h \, w \, 2u}{b \, \sigma_{sy.p}} \]

The only unknown in above eq. is \( y_0 \).

\[ y_0 = -\frac{h \, w \, A_0 + b \, \sigma_{sy.p}}{2 \, b \, \sigma_{sy.p}} \]

\[ y_0 = \frac{I_2}{2} \left( \frac{A_0 \, w}{2 \, b \, \sigma_{sy.p}} - \frac{I_2}{16 \, b \, \sigma_{sy.p}} \right) \]
Boundary Condition Discussion

1. Simply supported.

\[ W \]

\[ \begin{align*}
  x &= 0 \\
  m &= 0 \\
  x &= \frac{l}{2} \\
  \frac{dy}{dx} &= 0
\end{align*} \]

This can be considered in general case the moment at the boundary is zero.

2. Full restrained.

\[ W \]

\[ \frac{d^2 y}{dx^2} = -\frac{h m}{(I_2 - y_0^2) E (h - y_0)} \]

Solve the above eq put \( \frac{dy}{dx} = 0 \) at

\[ x = 0 \quad \& \quad x = l \]
3. Shear Rerains Boundary

Permanent Elongation = \( \delta_0 \) at \( V \) along

in plastic range

\[
R = \frac{hw}{(I_2 - V_0^2)(h - y_0)E}
\]

put \( (2I_2 - V_0^2) = S \)

\[
(h - y) = 0
\]

\[
R = \frac{hwx}{SDE}
\]

\[
\delta_0 = \int_{0}^{l} e \, dx \hspace{1cm} e = R(V - V_0)
\]

\[
\delta_0 = \int_{0}^{l} R(V - V_0) \, dx
\]

\[
\delta_0 = \frac{e^2 hw (V - V_0)}{2SDE}
\]

Suppose the slope at \( x = 0 \) equals \( \delta_0 \).
Elongation a $V = \delta = V\alpha_0$

Stress $\sigma = \frac{(\delta - \delta_0)E}{\varepsilon}$

At the elastic range $\delta_0 = 0$

$\sigma = \frac{E}{\varepsilon}$

$V = \frac{V\alpha_0}{E}$

$\Sigma = \mathbf{M} = 0$ at the current

$\int h \sigma dA = 0$

$\Sigma = \frac{V}{h} \frac{\delta - \delta_0}{\varepsilon} E dA + \int \frac{V \delta_0}{E} dA = 0 \
\Sigma = \frac{V}{h} \frac{\delta - \delta_0}{\varepsilon} E dA + \int \frac{V \delta_0}{E} dA = 0$

$\Sigma = \frac{V}{h} \frac{\delta - \delta_0}{\varepsilon} E dA + \int \frac{V \delta_0}{E} dA = \int \frac{V \delta_0}{E} dA$

$I_2 \alpha_0 + I_1 \alpha_0 = \frac{\Sigma}{E} [I_{-2} - V_0 I_2]$

$I_2 \alpha_0 = \frac{(I_2 - V_0 I_2) hW}{2(I_2 + I_0)(h-V_0)(I_{-2} - V_0 I_2)}$

$\frac{\Sigma}{E} = \frac{(I_2 - V_0 I_2) \alpha_0 hW}{2(I_2 + I_0)(h-V_0)(I_{-2} - V_0 I_2)}$

$\alpha_0 = \frac{\Sigma}{E}$
$d_o$ is determined when $W$, $e$, and cross-section area of the beam are given.

\[
\frac{d^2 y}{dx^2} = -\frac{h_m}{E(h-y)(y-y_0)}
\]

\[
\left(\frac{dy}{dx}\right)_{x=0} = 0
\]

Equation (1) can be written to

\[
2 \int v_b \delta y_E dA + \int v_1 \delta E dA = M
\]

For any fixed end of moment elastically restrained support.

Shear strain hardening

It is determined by $\left| \frac{dy}{dx} \right|_{\text{max}}$. 
Discussion on I section

1. Make the solution up to the yield line penetrating to all the web.

2. Consider the two flanges separate beams for additional loads.

3. Boundary condition No. 3 can be solved just in the same way illustrated.

Example in 8W40 restrained, ultimate shearing strength.
Conclusions from above analysis

1. Bending failure in beams changes the shearing stress magnitude and its distribution.

2. Shear failure in beams changes the normal stress distribution and deflection curves.

3. Max. shear is strength of a member is usually represented by \( V = T_s A \).

But actually beams under high shear are weakened first by shear failure then captured by bending.

4. Item 1 & 2 are solved under general beam stress assumptions for flexural formula.
The shear action near the support local yielded by shear at a low cost.

The cantilever strength is more significantly affected by shear at these sections.
Failed by bending

Failed by shear

End condition discussion

Discussion about resisting moment over the end

More strain should be observed than the central, constant moment section
Analysis of shearing stresses in beams after flanges are added by bending.

Jan. 2 / 1949
The flanges in section "A" are in strain-hardened region and the flanges in section "B" are in plastic region but before strain hardening. Everything in C section is in elastic condition.

We want to find the max. shearing stress.
a. Before strain hardening

(Take consideration of a rectangular beam)

\[ \int f_s \, b \, dx = \int y_2 \, dh \]

\[ \int y_2 \, dh = \frac{h}{2} \left[ \gamma_{y.p} \, y_1 + \gamma_{y.p.b} (y_2 - y_1) - \frac{1}{2} \gamma_{y.p} \, y_2 \right] \]

\[ = \frac{b \gamma_{y.p}}{2} (y_2 - y_1) \]

Now suppose \( y_2 = y_1 + \frac{dy}{dx} \, dx \)

\[ f_s = \frac{\gamma_{y.p}}{2} \frac{dy}{dx} \]

\[ \frac{dy}{dx} = f(V, \text{shape of section}) \]

It is of course very difficult to solve \( y = f(x) \) in irregular sections but in rectangular it seems possible.
b/ After strain hardening

Actually before \( \frac{dy}{dx} \) gets too high, in generally structural material sections, the fiber would get strain hardening.

\[
\begin{array}{c}
\text{H} \\
\text{Top}.
\end{array}
\]

It is clear that after strain hardening, the plastic region inside of the beam takes shearing stress again.

The analysis though not impossible, it would be too complicated.

I'm especially interested as the max. shear stress would happen at \( \text{the center of the web in the section where the} \) 

range but before strain hardening.
\[ M = \sigma_{y.p.} 2 l + 2b (h_0 - y) \sigma_{y.p.} x \left[ y + \frac{h_0 y}{2} \right] \]

\[ + 2x \frac{\sigma_{y.p.} x y b}{2} \nu \frac{2}{3} y \]

\[ \frac{dM}{dx} = -v = -2b \sigma_{y.p.} y \frac{dy}{dx} \]

\[ \frac{dy}{dx} = \frac{3V}{2y b \sigma_{y.p.}} \]

Suppose \( \frac{h_0}{y_0} = 15 = \text{Plastic Limit} \)

(Plastic strain = 15 x Elastic strain before strain hardening)

\[ \left( \frac{dy}{dx} \right)_0 = \frac{3V \times 15}{2 b \sigma_{y.p.}} = 3kV \]

\[ f_s = \frac{\sigma_{y.p.} \times 3V \times 15}{2 \times 2b \sigma_{y.p.}} = \frac{45 \nu}{4b b} \]

or \( f_s = \frac{3kV}{4b b} \)
It shows after the flanges are yielded
the max. shear stress equal to

\[ f_s = 0.75 k \frac{V}{h b} \]

Where

\[ k = \frac{\text{plastic strain}}{\text{elastic strain}} \]

\( f_s \) function of \( h, b, V, \) only

Not same thing for any kind of section

or rectangular section
General Conclusion

1. Suppose $Ac^2 = I$

\[ A = \text{Area of web flanges} \]

\[ f_s = \frac{VQ}{I_b} = \frac{V \times Ac}{Ac^2 b} \]

\[ \sigma = \frac{Mc}{I} = \frac{KVc}{Ac^2} \]

\[ \therefore V = KA_c \]

\[ \therefore f_s = K \frac{Ac^2}{K^2} = KA. \]

It is clear shear yielding is more significant in wide flange sections.

2. Deflections of the beams after the web shears to plastic range seems could be analyzed by the proposed methods.

3. The yielding strength and the ultimate strength of beams by bending are apparently raised while shear yielding strength is just about checked.

It may mean that...
Calc.

Shear Problemson

8 WF 40 Beams.
Octahedral Shear Hypothesis

\[ T_y = T_y P \times \frac{1}{N_3} = 39.5 \times \frac{1}{1.5} = 26.3 \text{ kips} \]

\[ W = \frac{T_y P \times b x t}{Q} = \frac{39.5}{0.371} \quad \text{Mean value strength of web material} \]

\[ b = 0.357 \]

\[ t = 0.05 \]

\[ I = 143.2 \]

\[ Q = 41.0 \text{ kips} \]

\[ W = 143.2 \times 3.71 \times 22.8 \times 0.05 = 39.6 \text{ kips} \]

Initial Yielding load at support = 34.7 k

Ultimate = 55.6 k

\[ W = \frac{34.8 \times 3.71 \times 22.8 \times 0.05}{41.0} = 52.5 \text{ kips} \]
Test B

The load is far from shear yielding

line at corners stress Concentration is also an factor

Test B2

1. We found shear yield lines at a load of 44,000 k. This is due to the residual stresses in the rolling section that made the beam look sheared earlier than expected.

2. The stress strain relation checks pretty close i.e., T1 & T2.

At load 28 k

\[
T_1 = 1.14 \times 10^7 \times 830 \times 10^{-6} (5-21, 3-23)
\]

\[
T_1 = 9.5 \text{ k/}^2
\]

\[
T_2 = \frac{28 \times 44.05/2}{143.2 \times .371} = 10.5 \text{ k/}^2
\]

3. General shear flow lines are seen at load 47 k \(\Rightarrow 50\) k.
No vertical shearing yielding lines seen, this may be due to the shear strength of the web along these two directions are not the same.
Yielding processing of the shear web.

The shearing stress in an I-section may be distributed as above. But we know

\[ \int_{A} f_s \, dA = F_s \]

where \( F_s \) = Shearing force

i.e. if there is stress concentration in the gillets then the max shearing stress at the center would be smaller than calculated conventionally.

Due to the stress concentration at gillets (Tensile press give the stress patter for WF section as below)

It wouldn't make too much error by assuming the shearing stress distributed uniformly in the web to simplify the method of analysis.

Actually in the web it is just the same way to proceed its yielding as we discussed before.
Suppose the shaded area are yielded by shear at Fig. A. Then the additional shear load will make the section act like two separate beams. Its shear stress will be distributed as Fig. B.

Combine the shear stress curves AB and CD. It is clear that the additional load will bring the yielding region from B to C.

The yield by shear is then proceeded...
Failure of the short High shear beam.

The ultimate shearing strength is

\[ W = A \cdot \frac{f}{\text{Typ}} \]

Where \( A = \text{cross sectional area} \)

But before the beam develops its full ultimate
ultimate strength \( f_{\text{shear}} \) it may fail
by bending in the following way.

Suppose \( W_0 \) is the load to
make the web of the beam yield
by shear.

Suppose the shear stress of the
beam at the root still can be
calculated by formula

\[ f_1 = \frac{M_c}{I} = \frac{W_0 L x C}{I} \]

Now we increase the load \( \Delta W \) and \( \Delta W + W_0 < f_{\text{Typ}} A \)

The additional shear stress
would be

\[ f_2 = \frac{\Delta W L x C'}{2I} \]
Where \( c' = \frac{1}{2} \text{ thickness of flange} \)

\[ I' = \text{Moment of inertia of the flange about its own centroid axis} \]

\( f_1 + f_2 \) may exceed \( f_{y.p.} \) before the web gets shear strain hardened.

This is of course a function of the shape of the section and end conditions at \( A \) and \( A' \).

A neat way to check is to assume the end condition:

\[ \text{(Assume the end condition)} \]

An easy way to check is to assume the end condition:

\[ (\text{Assume constant plastic}) \]

\[ 15 \times \frac{f_{y.p.}}{G} = 8 \text{ (strain)} \]

The web yield by shear may give a deflection at the tip of beams without any strain hardening.

\[ \text{Moment} = \frac{Wd}{2} \]

Substitute \( \frac{Wd}{2} \) to the

slope formula to find \( W \).

Then you can get moment and find \( f_2 \)

See if \( f_2 \) and above exceeds yield strength of the plate.
Procedure of strain hardening by shear in beams.

\[ W_0 = \text{load to make the web yield} \]
\[ W = \text{the additional load to make the plane given an angle of } \alpha_0 \text{ at the end} \]
\[ \alpha_0 = 15 \times \frac{J_y}{P} \]

Suppose the load is still increasing. Then part of the increasing load will be taken by the web at A'A' section due to strain hardening. But at B'B' section, the slope angle. The new slope at the end will then be \( \alpha \) where \( \alpha > \alpha_0 \). But at section B, the slope of the flange is \( \alpha_1 \) and it might be \( \alpha_0 < \alpha_1 < \alpha \).

It is clear that the additional shear taken by web at B'B' is smaller than...
at A'A section

Shear force at web, due to \( w_0 \), due to \( w \), due to \( w_0 \)

Shear force at flange

\[ 2(s_1 + s_2) \text{ at any section} = \Delta W \]

As \( \Delta W \) increase, CC' section moves inward. To the right of CC' section, the web is apparently strain hardened by shear.

We might assume \( s_2 \) load function on flange (straight line function). Otherwise, the problem becomes too complicated to be a special elastic support beam.
AB shows its neutral position.

AB' transformed in 

after the load is applied and assume that the beam is a rigid plate.

Suppose the mid portion cannot take any shear; the section would become A"c'd'B" and the shearing strain for the mid portion would be 

\[ \frac{c'd'd'}{cd} \]

\[ T_y p = J_0 = J_1 \]

\[ T_1 \] is the fiber shearing stress for the beam case and beams.

(A) \[ M = W \times \frac{L}{2} \] 

\[ M = W \times \frac{L}{2} - 2 b J_1 x \frac{d^2}{2} \]
Frictionless

The deflection curve of the beam under pure moment is a function of end condition.

Free

The additional moment is apparently

{\begin{align*}
\text{df} & = \text{dF} \\
\text{the end condition becomes} & \\
\text{Restrained}
\end{align*}}

\[ \text{Date to continue...} \]

\[ \sqrt{\text{A beam under shear and moment is taken different}} \]

The end condition still determines the moment taken.
When the end is free, it is very apparent that those shearing stresses make the two flanges ends remain the straight after bending since as long as this stress holds the moment of inertia of the section should be considered as rigid.

But when taken out this shear stress the two flanges should be considered as separately two beams. The ends of the two flanges are no longer in one line.

Eq (A) would take care the mere zero shearing stress in boundary of a beam.
It is very interesting to see that as a problem above.

When "W" increases, the bend of the beam AB of course turns as the beam deflects.

But after the web all yielded by shear, the flanges become two beams like below:

Due to M, the upper flange would under tension "T"

\[ T = \frac{M}{R} \]

The deflection of the angle of AB due to SW would be \( \delta \alpha \)

\[ \delta \alpha = \frac{T \times l}{AE} \]

Where \( A_u \) = cross area of upper flange.
The above beam, the end conditions are not the same as above. The turning of AB line after the web shear yielded is also a function of the mid portion of the beam.

This end condition changes the moment in shear section as changes the deflection in shear portion.

In this case you can't just consider flanges as separate beams even if the web is yielded. The end condition at AB must be taken in consideration.
Outline on shear

problems in 

from 4" I section beams

Tests Results
4.1 Assum. Basic Data

<table>
<thead>
<tr>
<th></th>
<th>Hand Book</th>
<th>Computer</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>6</td>
<td>5.81</td>
<td>6.14*</td>
</tr>
<tr>
<td>E</td>
<td>2.96 x 10^7 **</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Q</td>
<td>-</td>
<td>3.325/2</td>
<td>-</td>
</tr>
<tr>
<td>µ</td>
<td>2.97 **</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* Calculated from Test No. 9 assume E = 2.96 x 10^7

** Adopted from a paper by Dr. Johnston
1. Shearing Strength (by octahedral hypothesis)

\[ T_{y,p} = 40,300 \text{ lb/ft} \text{ web.} \]

\[ T_{y,p} = \frac{F_{y,p}}{N_B} = 40,300 \times \frac{1}{N_B} = 2.33 \text{ Kips/ft} \]

\[ V = \frac{F_s \times b \times I}{Q} \]

\[ I = 6 \]

\[ b = 0.19'' \text{ (manual)} \]

\[ Q = 3.25/2 \]

\[ V = 16 \text{ Kips} \]

One-third of loading \( W = 32 \text{ Kips} \)

2. Max. Shearing Strain and Stress Relations

\[ J = \frac{E}{2(1+\mu)} \epsilon_s = \frac{2.96}{2(1+0.297)} \epsilon_s = \frac{1140}{2(0.297)} \epsilon_s \times 10^7 \]

Max. Elastic Strain

\[ \epsilon_s(\text{max. stress}) = \frac{23.3 \times 10^3}{1.94 \times 10^3} = 12.040 \times 10^{-6} \]
3. Initial Yield Moment

\[ f_y.p. = \frac{M_c}{I} \]

\[ I = 6 \]

\[ c = 2.025 \]

\[ f_{yp} = 34.8 \times 10^3 \text{ upper} \]

\[ f_{yp} = 3.34 \times 10^3 \text{ lower} \]

\[ M = 3.34 \times 10^3 \times 6 / 2.025 = 100 \times 10^3 \text{ k-ft} \]

\[ Arm = 4'' \]

\[ = 6'' \]

\[ = 8'' \]

\[ W_{yp} = 50 \text{ kip.} \]

\[ W_{yp} = 33.3 \text{ kip.} \]

\[ W_{yp} = 25 \text{ kip.} \]

4. Max. Ultimate Bending Moment

\[ M = f_{yp} \times 2 = 33.4 \times 3.325 = 111 \times 10^3 \text{ k-ft} \]

\[ Arm = 4'' \]

\[ = 6'' \]

\[ = 8'' \]

\[ W_{ul} = 55.5 \text{ kips} \]

\[ W_{ul} = 37 \text{ kips} \]

\[ W_{ul} = 29.8 \text{ kips} \]
Test No. 1

1. Shear

Wey. due to shear = 28 k see (9) & (10)

Observed strain $\varepsilon = 1.750 \times 10^{-6}$ (mean of two sections)

$\bar{s}_1 = \varepsilon E = 1.14 \times 10^7 \times 1.750 \times 10^{-6} = 20.1 \times 10^3 \text{ kips/ft}^2$

$\bar{s}_2 = \frac{V_0}{b_2} = \frac{28 \times 5.325}{2} = 20.4 \times 10^3 \text{ kips/ft}^2$

a. It yields at a lower value than predicted 23.3 k/ft

b. The distribution of shear stress seems to check the assumption very well. The value of $\bar{s}_1$ and $\bar{s}_2$ are pretty close.

$\bar{s}_1$ could be regarded as the actual shearing stress measured in the beam.

$\bar{s}_2$ could be regarded as the calculated shearing stress.
2. Shear strains and beam deflection

We know elastic shearing strain has very small effect on deflection of a beam. But as soon as it reaches the plastic range, the effect on deflection of the beam becomes more important.

The shearing stress distribution of a wide flange section is as follows:

```
               T
```

When the web is yielded by shearing stress, the flange shearing stress may be ten times below yielding strength.

Suppose the two ends of the beam are in built-in conditions, then any further increase in shearing load to the beam would be taken by the two flanges. The two flanges would act like two separate beams.
Upper surface of flange

Lower surface of flange

Induced strain in web.

We suggest where calculate the "I" of flange use an effective area as below:

effective area

Actual shear stress distribution on a web is as follows:

due to stress

Conclusions of course it is a function of root radius and the shape in wide flange sections may like:
The above shear deformations include both vertical and horizontal shearing strain.

In a wide flange section, suppose the shear stress along the web could be regarded as uniformly distributed then after its failure by shear we can take the two flanges as individual beams.

But due to continuation of strain in an elastic body, some comp. or tensile strain induced in the web.
Elastic support of individual large beam

It is a very important factor to note the ultimate 
streaming strength of a beam section is that the 
load made the whole section distribute with 
uniform lower yield of shear and stress.

Some conclusions on test 1.

1. The beam is failed by shear. Theoretical 
deflection curve.

2. The shape of deflection curve would be a supported.

3. The initial shear yieldling checks the assumption 
   very well.
Test No. 2

No strain gages put on this specimen.

The beam yielded at about 28 k.

It went up to 48 kip still held by its yield strength due to bending.
Of course, due to the span of this kind of beam, it is a little bit more short. The stress distribution may not be exactly as assumed—see (57).
Test No. 3.

Shear

\[
\begin{align*}
\text{Wpd. by shear} &= 30 \text{ Kip} \\
\text{EYps} &= 2040 \times 10^{-6} \\
J_1 &= G \text{EYps} = 22.8 \text{ Kip} / \text{in} \\
J_2 &= \frac{VQ}{Ib} = 21.9 \text{ Kip} / \text{in}
\end{align*}
\]

1. The initial shearing yielding still close as calculated.

2. Deflection curve shows the beam yields at \( W = 30 \text{ Kip} \) by shear.

3. No normal stress strains, only yielded.

4. Highest load we got 44 K it is below the initial bending yield strength.
Test 4.

\[ \frac{\Delta}{\sigma} = \frac{0}{0.1} \leq 0.01 \]

No. strain gauges put on this specimen

1. Yielded at 30 k by shear

2. Load went up to 41 k far from yield strength by handup
Shear

\[ W_y = 0.4 \text{ k} \]

\[ G_y = 1650 \times 10^{-6} \]

\[ J_f = 1650 \times 10^{-6} \times 1.14 = 18.75 \times 10^3 \text{ in}^3 \]

\[ J_a = \frac{V^2}{h^3} = 17.5 \times 10^3 \text{ in}^3 \]

Shear yield happens 25% before predicted value.

This may due to special stress pattern, while on those longer moment arms, the shear yields check better (6.7.8).

Or may due to residual stress on the def. curve gives the yield if load of 32 k.
Test No. 6

\[ W_{yp} = 32 \text{ kip} \]

\[ E_{yp} = 2153 \times 10^6 \]

\[ J_1 = 24.5 \text{ kip} / \text{in.} \quad (J_1 = 6.6 \text{ kip} / \text{in.}) \]

\[ J_2 = \frac{V_0}{I_b} = 23.3 \text{ kip} / \text{in.} \]

1. The deflection curve shows the yielding at 32 k. Probably due to shear.

2. The picture Fig 1 can see there are vertical shear yield of lines.

3. The beam should be initially yield by bending at a load of \( W > 33.3 \text{ kip} \).

But normal stress strain graph shows no yield at a load of 40 k. That may prove that a stress gradient exist the yield strength of the material could be raised. (ed.)

4. The ultimate load should be 37 k. We get 40 k is apparently higher than estimates.
Those yields region might have been strain hardening.

Those transition zone may still yield slowly.

Those region still in elastic range and the yield strength is raised by stress gradient.

We have some other explanations on their higher ultimate yield strength.

5. The special defl. curve.

6. Fiber strain at $4350 \# (11, 12, 13,)$

\[ \varepsilon = 1413 \times 10^6 \]

\[ \sigma_1 = \varepsilon E = 2.96 \times 10^7 \times 1413 \times 10^{-6} = 4.2 \text{Kips/in}^2 \]

\[ \sigma_2 = \frac{Mc}{I} = \frac{4350 \times 3 \times 0.05 \times 6}{6} = 43.6 \text{ Kips/in}^2 \]

While data in coupon test is 348 upper.
Test No. 7

\[
\left\{\begin{array}{ll}
\Delta = & 8' \\
\Delta = & 8' - 8' = 8'
\end{array}\right.
\]

\[\frac{\Delta}{8'} = \frac{8'}{8'} = 1\]

\[
W_{yp.} = 28 \text{kN} \quad \text{See gage (21, 22)}
\]

\[
E_{yp.} = 1820 \times 10^{-6}
\]

\[
J_1 = E G = \frac{20.7}{k \text{kip/ft}^2}
\]

\[
J_2 = \frac{V_0}{b} = \frac{20.4}{k \text{kip/ft}^2}
\]

1. Calculated bending yield strength

\[\frac{25}{2} = 12.5 \text{kN} \]

The deflection curve shows yield at 28 kN. May be due to both.

2. Vertical shear yield is as clear at Fig 1 and Fig 8.

3. The cal. UI: load 27.8 kN.

We got 30.15 kN.

4. Show the special deflection curve.
Test 8.

\[
\begin{align*}
W_{y,p} &= 30 \\
6y_{p} &= 2023 \times 10^{-6} \\
J_{1} &= 23.1 \\
\bar{T}_{2} &= 21.9
\end{align*}
\]

1. The deflection curve shows the initial yield is commenced at a load of 2.8 k.
   It is apparently by bending.

2. Initial yield strength = 25 K
   Ultimate load = 278 K
   The beam is, however, overstrength.
   We reach a load of 33 K.

3. Shows the special defl. curve.
Test No. 9

Wy.p. by hand $S_y = 9K$ (calculator)

1. Shear gage didn't fail

2. Deflection curve shows a yield at 9.5 K

3. Ultimate $W_u = 8.9 K$
   We got 9.8 K
There is no shear stress gradient exist.

1. We could use the ultimate shearing strength \( T_y_p \) as the design load for shear.

   \[ T_y_p = \text{lower shearing yield point}. \]

   \[ A = \text{cross area of the section}. \]

   Under such a design load the deflection may be too large before the beam reaches its ultimate shearing strength.

5. The max. shear theory may be able to get a little more consistent result than octahedral shear theory as point 3 discusses.

6. Stress pattern near loading pits and support doesn't seem change much.

   The short moment arm didn't change much of the shearing stress pattern either.