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Optimum Quantization and Reconstruction of Power Flows from Voltage Measurements at Dispersed Buses

Xinda Ke
Lehigh University

Optimum Quantization and Reconstruction of Power Flows from Voltage Measurements at Dispersed Buses

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Xinda Ke

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Date

Prof. Rick S. Blum
(Thesis Advisor)

Prof. Filbert J.Bartoli
(Chairperson of Department)

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Abstract

Optimum quantization of data can transform the original continuous data into a finite number of discrete levels for transmission by digital communication methods. The design of the optimum nonuniform quantizer for a single data stream has been beautifully described in the seminal work of Max [1]. For other work on quantization see the references in [2] [3].

On the other hand, the electrical grid system in the United States is undergoing significant upgrades that will allow rapid monitoring, reconfiguration and control. Digital communication systems are expected to play a key role and so quantization methods are required. Here we consider optimum quantization and reconstruction under what appears to be the most commonly employed models [4] in the industry, the so called the DC power flow model. We have not seen previous studies on this problem. Further, we introduce a method for decomposing the problem into smaller problems which are easier to solve. this appears to be the first paper to propose this specific approach. In the DC power flow model, the assumptions of small phase angles, small resistances, and nearly unity amplitudes are employed to simplify the original nonlinear equations into a set of linear equations. These approximations are generally very reasonable in typical power systems as these restrictions are often enforced to promote stability and efficiency.

Chapter 1

Introduction

Optimum quantization of data can transform the original continuous data into a finite number of discrete levels for transmission by digital communication methods. The design of the optimum nonuniform quantizer for a single data stream has been beautifully described in the seminal work of Max [1]. For other work on quantization see the references in [2] [3].

On the other hand, the electrical grid system in the United States is undergoing significant upgrades that will allow rapid monitoring, reconfiguration and control. Digital communication systems are expected to play a key role and so quantization methods are required. Here we consider optimum quantization and reconstruction under what appears to be the most commonly employed models [4] in the industry, the so called the DC power flow model. We have not seen previous studies on this problem. Further, we introduce a method for decomposing the problem into smaller problems which are easier to solve. this appears to be the first paper to propose this specific approach. In the DC power flow model, the assumptions of small phase angles, small resistances, and nearly unity amplitudes are employed to simplify the original nonlinear equations into a set of linear equations. These approximations are generally very reasonable in typical power systems as these restrictions are often enforced to promote stability and efficiency. Extending the analysis here for the nonlinear equations is straightforward, although solution complexity can greatly increase. We will not consider this further due to space limitations.

Chapter 2

System Model and Problem Formulation

2.0.1 DC power flow model

Consider a power system with N buses. Assume that all buses are connected to the ground through some susceptances. The ground can be viewed as the $(N+1)$-th bus in the system and is used as the voltage reference. Let δ_i denote the voltage angle at bus i $(1 \leqslant i \leqslant N+1)$ and we have $\delta_{N+1} = 0$. Let $b_{i,j}$ $(i \neq j)$ denote the susceptance between bus i and bus j. Collect the voltage angles at the first N buses in the vector $\delta = [\delta_1, \delta_2, \ldots, \delta_N]^T$, and collect the active power injected to the first N buses in the vector $\mathbf{p} = [p_1, p_2, \ldots, p_N]^T$. Assuming the DC power flow model, a matrix formulation of equation between power flow and voltage angles can be expressed as

$$\begin{bmatrix} \mathbf{p} \\ p_{N+1} \end{bmatrix} = \begin{bmatrix} \mathbf{G} & & -b_{1,N+1} \\ & & \cdots \\ -b_{N+1,1} & \ldots & \sum_{i=1}^{N} b_{i,N+1} \end{bmatrix} \begin{bmatrix} \delta \\ 0 \end{bmatrix} \tag{2.1}$$

where $\mathbf{G}$ is the $N \times N$ bus susceptance matrix defined by

$$(i,j)\text{-th element } g_{i,j} = \begin{cases} \sum_{k=1,k\neq i}^{N+1} b_{i,k} & \text{if } i = j \\ -b_{i,j} & \text{if } i \neq j \end{cases} . \tag{2.2}$$

Under mild conditions $\mathbf{G}$ is nonsingular and from equation (2.1) we have

$$\mathbf{p} = \mathbf{G}\delta. \tag{2.3}$$

2.0.2 Optimum Quantization Problem Formulation

We define the mean square error in the power flow computation due to quantization as

$$e = \sum_{i=1}^{N} w_i e_i = \sum_{i=1}^{N} w_i E\{(p_i - \hat{p}_i)^2\} \tag{2.4}$$

where $w_i \geq 0$ is a weight to allow different emphasis to different desired (unquantized) power flows $p_i, i = 1, \ldots, N$ and $\hat{p}_i, i = 1, \ldots, N$ are the reconstructed power flows which are calculated based on quantized measurements. Each of the voltage angles $\delta_1, \delta_2, \ldots, \delta_N$ are generally measured at locations which are fairly far apart, while we generally want the power flows at different locations, the control centers. Thus we want to immediately quantize the voltage angles, where they are measured, using scalar quantizers and then digitally transmit the quantized values, each represented by an integer value, to the control centers. Thus, we define a set of N scalar quantizers to quantize the scalars $\delta_i, i = 1, \ldots, N$. The i^{th} scalar quantizer, which quantizes δ_i employs the $M_i + 1$ thresholds $t_{i,0}, \ldots, t_{i,M_i}$ where $t_{i,0} = -\infty$ and $t_{i,M_i} = \infty$. So if $t_{i,j-1} \leq \delta_i < t_{i,j}$ then the quantization symbol, j is transmitted to the control center to represent δ_i where j is an integer between 1 and M_i. In order to focus on the quantization, we assume noise free communication so δ_i is received correctly. We can denote the actual quantization symbol transmitted for the measurement δ_i as j_i. At the control center the quantization symbols from all measurements $j_1, \ldots, j_N$ are used to determine the representation (reconstruction) of the power flows based on the quantized measurements to yield $\hat{p}_i = \ell_{i,j_1,\ldots,j_N}, i = 1, \ldots, N$. Thus the quantization and reconstruction parameters needed to completely define this approach are $\{t_{i,j_1}, \ldots t_{i,j_N}, \ell_{i,j_1,\ldots,j_N}, \forall i, j_1, \ldots, j_N\}$. Next we describe necessary conditions for an optimum set of parameters.

Using the just described definitions and letting $f(\delta_1, \ldots, \delta_N)$ denote the joint probability density function of $\delta_1, \ldots, \delta_N$, the minimum square error (MSE) is calculated as

$$\begin{aligned} e &= \sum_{i=1}^{N} w_i \int_{\delta_1} \cdots \int_{\delta_N} (p_i - \hat{p}_i)^2 f(\delta_1, \ldots, \delta_N) d\delta_1 \cdots d\delta_1 \\ &= \sum_{i=1}^{N} w_i \sum_{j_1=1}^{M_1} \int_{t_{1,j_1-1}}^{t_{1,j_1}} \cdots \sum_{j_N=1}^{M_N} \int_{t_{N,j_N-1}}^{t_{N,j_N}} \\ &\quad (\ell_{i,j_1,\ldots,j_N} - \sum_{n=1}^{N} g_{i,n}\delta_n)^2 f(\delta_1, \ldots, \delta_N) d\delta_1 \cdots d\delta_N \end{aligned} \tag{2.5}$$

Chapter 3

Optimum Quantization and Reconstruction

From the MSE function in (2.5), except fixed thresholds $t_{i,0} = -\infty$ and $t_{i,M_i} = \infty$ we can find necessary conditions on the optimum quantization and reconstruction parameters $\{t_{i,j_1}, \ldots t_{i,j_N}, \ell_{i,j_1,\ldots,j_N}, \forall i, j_1, \ldots, j_N\}$ by computing

$$\frac{d}{d\ell_{i',j'_1,\ldots,j'_N}} e = 0 \text{ for } i' = 1, \ldots, N \text{ and } j'_k = 1, \ldots, M_k - 1 \tag{3.1}$$

and

$$\frac{d}{dt_{k,j'_k}} e = 0 \text{ for } k = 1, \ldots, N \text{ and } j'_k = 1, \ldots, M_k - 1 \tag{3.2}$$

For the general case where all the angles are statistically dependent random variables, calculating (3.1) yields

$$\begin{aligned} &\ell_{i',j'_1,\ldots,j'_N} = \\ &\frac{\int_{t_{1,j'_1-1}}^{t_{1,j'_1}} \cdots \int_{t_{N,j'_N-1}}^{t_{N,j'_N}} \left(\sum_{n=1}^{N} g_{i',n}\delta_n\right) f(\delta_1, \ldots, \delta_N) d\delta_1 \cdots d\delta_N}{\int_{t_{1,j'_1-1}}^{t_{1,j'_1}} \cdots \int_{t_{N,j'_N-1}}^{t_{N,j'_N}} f(\delta_1, \ldots, \delta_N) d\delta_1 \cdots d\delta_N} \\ &\text{for } i' = 1, \ldots, N \end{aligned} \tag{3.3}$$

Similarly, for the general case where all the angles are statistically dependent random variables, calculating (3.2) yields

$$
\begin{aligned}
& 2\sum_{i=1}^{N} w_i \sum_{j_1=1}^{M_1} \text{ except } j_k \sum_{j_N=1}^{M_N} \\
& (\ell_{i,j_1,\ldots,j_N}|_{j_k=j'_k} - \ell_{i,j_1,\ldots,j_N}|_{j_k=j'_k+1}) \\
& \int_{t_{1,j_1-1}}^{t_{1,j_1}} \text{ except } j_k \int_{t_{N,j_N-1}}^{t_{N,j_N}} \\
& (\sum_{n=1}^{N} g_{i,n}\delta_n) f(\delta_1,\ldots,\delta_N)|_{\delta_k=t_{k,j'_k}}] d\delta_1 \cdots \text{ except } j_k \cdots d\delta_N \\
= \; & \sum_{i=1}^{N} w_i \sum_{j_1=1}^{M_1} \text{ except } j_k \sum_{j_N=1}^{M_N} \\
& (\ell^2_{i,j_1,\ldots,j_N}|_{j_k=j'_k} - \ell^2_{i,j_1,\ldots,j_N}|_{j_k=j'_k+1}) \\
& \int_{t_{1,j_1-1}}^{t_{1,j_1}} \text{ except } j_k \int_{t_{N,j_N-1}}^{t_{N,j_N}} \\
& f(\delta_1,\ldots,\delta_N)|_{\delta_k=t_{k,j'_k}}] d\delta_1 \cdots \text{ except } j_k \cdots d\delta_N \\
& \text{ for } k = 1,\ldots,N \text{ and } j_k = 1,\ldots,M_k - 1.
\end{aligned}
\tag{3.4}
$$

By simultaneously solving (3.3) and (3.4), we can find the optimum parameters $t_{i,j_1}, \ldots t_{i,j_N}, \ell_{i,j_1,\ldots,j_N}$ $\forall i, j_1, \ldots, j_N$. One algorithmic description for this process is summarized as

- Initialize the values of $t_{i,j_1}, \ldots, t_{i,j_N}$.
- Update $t_{1,1}$ while keeping all other parameters unchanged to satisfy $\frac{d}{dt_{1,1}}e = 0$ in equation (3.4).
- Sequentially update the other parameters $t_{1,2}, \ldots t_{i,j_N}$ (one at a time) until all parameters $t_{i,j_1}, \ldots t_{i,j_N}$ approximately satisfy $\frac{d}{dt_{1,1}}e = 0$, $\frac{d}{dt_{1,2}}e = 0$,..., $\frac{d}{dt_{i,j_N}}e = 0$. In the process of updating, we always update one parameter at a time and keep all other parameter unchanged.

We note that many other solution approaches are possible.

3.0.3 Independent Phase Measurements

For the special case where all the angles are statistically independent random variables, calculating (3.1) yields

$$\ell_{i',j'_1,\ldots j'_N} = \sum\nolimits_{n=1}^{N} \frac{\int_{t_{n,j'_n-1}}^{t_{n,j'_n}} g_{i',n}\delta_n f(\delta_n) d\delta_n}{\int_{t_{n,j'_n-1}}^{t_{n,j'_n}} f(\delta_n) d\delta_n} \tag{3.5}$$

Thus, as implied by (3.5), we can decompose the entire quantization and reconstruction system into N subsystems. Each of these subsystems focuses only on quantization and reconstruction for one of the measured angles and so these systems are uncoupled from each other. Then we can compute each reconstructed power flow by summing the reconstructions from each of the subsystems as described in (3.5). We can make these ideas even more apparent with some manipulations. By defining the part of the level due to a given subsystem as

$$\ell_{i',j'_n} = \frac{\int_{t_{n,j'_n-1}}^{t_{n,j'_n}} g_{i',n}\delta_n f(\delta_n) d\delta_n}{\int_{t_{n,j'_n-1}}^{t_{n,j'_n}} f(\delta_n) d\delta_n}, \tag{3.6}$$

(3.5) can thus be expressed as

$$\ell_{i',j'_1,\ldots j'_N} = \sum\nolimits_{n=1}^{N} \ell_{i',j'_n} \text{ for } i' = 1, \ldots, N. \tag{3.7}$$

where we can now see clearly how each subsystem contributes to the reconstructed power flows.

Considering (3.7), solving (3.2) will ultimately lead to setting derivatives of (3.6) to zero with the ultimate solution of

$$t_{k,j_k'} = \frac{\ell_{i',j_k'} + \ell_{i',j_k'+1}}{2} \tag{3.8}$$

In fact, we end up needing to solve exactly the Lloyd-Max equations

$$\ell_{i',j_k'} = \frac{\int_{t_{k,j_k'-1}}^{t_{k,j_k'}} g_{i',k}\delta_k f(\delta_k) d\delta_k}{\int_{t_{k,j_k'-1}}^{t_{k,j_k'}} f(\delta_k) d\delta_k} \tag{3.9}$$

$$t_{k,j_k'} = \frac{\ell_{i',j_k'} + \ell_{i',j_k'+1}}{2} \tag{3.10}$$

Then we just need to plug the subsystem levels (3.10) into (3.7).

3.0.4 Statistically Dependent Groups of Voltage Angles

We can also simplify the problem of finding the optimum parameters for cases where some measurements are statistically dependent and some are not. Suppose the sets of measurements at different buses can be broken into several groups. If all the measurements in a given group must be modeled as being statistically dependent but if measurements from different groups can be approximated as independent of one another, then we can find some necessary conditions that are are uncoupled from group to group. The resulting approach significantly reduces the complexity of finding the optimum quantization and reconstruction parameters. Further, this approximation appears reasonable in modeling power networks, where measurements at neighboring nodes may require a statistically dependent model but nodes which are far apart may not.

While we can perform the above mentioned decomposition for any number of groups and any size groups, we explain the idea for the specific case where there are two groups of undermined size. From this example, it is straightforward to understand how to perform the decomposition for any other desired case. Consider the specific case where the voltage angles δ_1 to δ_l are statistically dependent and where the voltage angles δ_{l+1} to δ_N are also statistically dependent but δ_1 to δ_l are statistically independent from δ_{l+1} to δ_N. According to equation (3.1), the equation (3.3) can be rewritten as

$$\ell_{i',j_1'\ldots j_N'} = \tag{3.11}$$
$$\frac{\int_{t_{1,j_1'-1}}^{t_{1,j_1'}} \ldots \int_{t_{l,j_l'-1}}^{t_{l,j_l'}} \sum_{n=1}^{k} g_{i',n}\delta_n f(\delta_1,\ldots,\delta_l) d\delta_1,\ldots,d\delta_l}{\int_{t_{1,j_1'-1}}^{t_{1,j_1'}} \ldots \int_{t_{l,j_l'-1}}^{t_{l,j_l'}} f(\delta_1,\ldots,\delta_l) d\delta_1,\ldots,\delta_l} +$$
$$\frac{\int_{t_{l+1,j_{l+1}'-1}}^{t_{l+1,j_{l+1}'}} \ldots \int_{t_{N,j_N'-1}}^{t_{N,j_N'}} \sum_{n=l+1}^{N} g_{i',n}\delta_n f(\delta_{l+1}\ldots\delta_N) d\delta_{l+1}\ldots d\delta_N}{\int_{t_{l+1,j_{l+1}'-1}}^{t_{l+1,j_{l+1}'}} \ldots \int_{t_{N,j_N'-1}}^{t_{N,j_N'}} f(\delta_{l+1}\ldots\delta_N) d\delta_1\ldots\delta_N}$$

Thus, we have decomposed the entire optimum quantization and reconstruction system into the sum of two smaller quantization and reconstruction systems and the two smaller systems are uncoupled from each other. This can be seen more explicitly with some manipulation. In particular, (3.11) can thus be further expressed as

$$\ell_{i',j_1',\ldots j_N'} = \ell_{i',j_1',\ldots j_l'} + \ell_{i',j_{l+1}',\ldots j_N'} \tag{3.12}$$
$$\text{for } i' = 1,\ldots,N \text{ and } j_l' = 1,\ldots,M_l - 1 \text{ for } l = 1,\ldots,N.$$

and we can see the similarity to the completely independent case in (3.7).

From (3.11) and (3.12), we find

$$\ell_{i',j_1',\ldots,j_l'} = \tag{3.13}$$
$$\frac{\int_{t_{1,j_1'-1}}^{t_{1,j_1'}} \cdots \int_{t_{l,j_l'-1}}^{t_{l,j_l'}} \left(\sum_{n=1}^{N} g_{i',n}\delta_n\right) f(\delta_1,\ldots,\delta_l) d\delta_1 \cdots d\delta_l}{\int_{t_{1,j_1'-1}}^{t_{1,j_1'}} \cdots \int_{t_{l,j_l'-1}}^{t_{l,j_l'}} f(\delta_1,\ldots,\delta_l) d\delta_1 \cdots d\delta_l}$$
$$\text{for } i' = 1,\ldots,N$$

Further, calculating (3.2) yields

$$
\begin{aligned}
& 2\sum_{i=1}^{N} w_i \sum_{j_1=1}^{M_1} \text{ except } j_k \sum_{j_l=1}^{M_l} (\ell_{i,j_1,\ldots,j_l}|_{j_k=j'_k} - \ell_{i,j_1,\ldots,j_l}|_{j_k=j'_k+1}) \\
& \int_{t_{1,j_1-1}}^{t_{1,j_1}} \text{ except } j_k \int_{t_{l,j_l-1}}^{t_{l,j_l}} \\
& (\sum_{n=1}^{l} g_{i,n}\delta_n) f(\delta_1,\ldots,\delta_l)|_{\delta_k=t_{k,j'_k}}] d\delta_1 \cdots \text{ except } j_k \cdots d\delta_l \\
= \; & \sum_{i=1}^{N} w_i \sum_{j_1=1}^{M_1} \text{ except } j_k \sum_{j_l=1}^{M_l} (\ell^2_{i,j_1,\ldots,j_l}|_{j_k=j'_k} - \ell^2_{i,j_1,\ldots,j_l}|_{j_k=j'_k+1}) \\
& \int_{t_{1,j_1-1}}^{t_{1,j_1}} \text{ except } j_k \int_{t_{l,j_l-1}}^{t_{l,j_l}} \\
& f(\delta_1,\ldots,\delta_l)|_{\delta_k=t_{k,j'_k}}] d\delta_1 \cdots \text{ except } j_k \cdots d\delta_l \\
& \text{for } k=1,\ldots,l \text{ and } j_k=1,\ldots,M_k-1.
\end{aligned}
\tag{3.14}
$$

By simultaneously solving (3.13) and (3.14), we can find the optimum parameters $t_{i,j_1} \ldots t_{i,j_l}$, $\ell_{i,j_1 \ldots j_l}, \forall i, j_1, \ldots, j_N$.

Similarity, we can also find the optimum parameters $t_{i,j_{l+1}} \ldots t_{i,j_N}, \ell_{i,j_{l+1} \ldots j_N} \forall i, j_{l+1}, \ldots, j_N$ in a similar manner and find the overall solution using equation (3.12) The approach is similar to that in section

Chapter 4

Numerical Results

Next we present some numerical results to illustrate the theory previously described. We assume there are four angle measurements δ_i $i = 4$ which each follow a marginal Gaussian distribution with zero mean and variance 0.05. Consider the case where δ_1 and δ_2 and are partially correlated and δ_3 and δ_4 are also partially correlated. On the other hand the vector (δ_1, δ_2) is uncorrelated with (δ_3, δ_4). The correlation between δ_1 and δ_2 is described by the correlation coefficient $\rho = 0.5$. The correlation between δ_3 and δ_4 is also described by the correlation coefficient $\rho = 0.5$. Each angle measurement is quantized with a scalar quantizer which employs 4 thresholds. The 4×4 bus susceptance matrix G is

$$G = \begin{bmatrix} -18.46 & 15.26 & 0 & 0 \\ 15.26 & -29.35 & 4.78 & 5.12 \\ 0 & 4.78 & -8.85 & 3.21 \\ 0 & 5.12 & 3.21 & -37.34 \end{bmatrix}$$

which was chosen by selecting 4 buses from the IEEE fourteen bus model susceptance matrix. The final mean square error has been normalized by the square magnitude of the largest element in the susceptance matrix G is replaced by the biggest value within G.

Therefore, in this example, the measurements can be decomposed into two groups. Group one is composed of δ_1 and δ_2. Group two is composed of δ_3 and δ_4. In order to verify the

Simplified strategy				
Normalized minimum square error	$t_{1,0}$	$t_{1,1}$	$t_{1,2}$	$t_{1,3}$
0.0731	-1	-0.1320	0.1324	1
	$t_{2,0}$	$t_{2,1}$	$t_{2,2}$	$t_{2,3}$
	-1	-0.1325	0.1322	1
	$t_{3,0}$	$t_{3,1}$	$t_{3,2}$	$t_{3,3}$
	-1	-0.1322	0.1326	1
	$t_{4,0}$	$t_{4,1}$	$t_{4,2}$	$t_{4,3}$
	-1	-0.1328	0.1324	1
Unsimplified strategy				
Normalized minimum square error	$t_{1,0}$	$t_{1,1}$	$t_{1,2}$	$t_{1,3}$
0.0734	-1	-0.1318	0.1320	1
	$t_{2,0}$	$t_{2,1}$	$t_{2,2}$	$t_{2,3}$
	-1	-0.1323	0.1327	1
	$t_{3,0}$	$t_{3,1}$	$t_{3,2}$	$t_{3,3}$
	-1	-0.1326	0.1320	1
	$t_{4,0}$	$t_{4,1}$	$t_{4,2}$	$t_{4,3}$
	-1	-0.1322	0.1333	1

Table 4.1: comparison of simplified strategy and unsimplified strategy when each phase measurement employs 4 threshold

optimality of our simplified quantization and reconstruction approach, we compare the results with those obtained from directly solving the original unsimplified necessary conditions.

From table 1 we can see that the two solutions yield approximately the same performance and that the thresholds employed in the two strategies finally converge to approximately the same values. The time required to find the two solutions on a computer run was: 34 min for the simplified strategy and 9 hours 20 min for the unsimplified strategy This shows that our simplified strategy can greatly simply the entire problem while ensuring the same performance as the unsimplified strategy. Table 2 shows solutions for the identical problem but where each angle measurement is quantized with a scalar quantizer that employs 5 thresholds. As expected, we can see decreased minimum square error compared with the results in

Simplified strategy					
Normalized minimum square error	$t_{1,0}$	$t_{1,1}$	$t_{1,2}$	$t_{1,3}$	$t_{1,4}$
0.0708	-1	-0.2117	0.0344	0.2110	1
	$t_{2,0}$	$t_{2,1}$	$t_{2,2}$	$t_{2,3}$	$t_{2,4}$
	-1	-0.2084	0.0142	0.2086	1
	$t_{3,0}$	$t_{3,1}$	$t_{3,2}$	$t_{3,3}$	$t_{3,4}$
	-1	-0.2126	0.0416	0.2114	1
	$t_{4,0}$	$t_{4,1}$	$t_{4,2}$	$t_{4,3}$	$t_{4,4}$
	-1	-0.2149	-0.0217	0.2133	1
Unsimplified strategy					
Normalized minimum square error	$t_{1,0}$	$t_{1,1}$	$t_{1,2}$	$t_{1,3}$	$t_{1,4}$
0.0708	-1	-0.2075	0.0421	0.2126	1
	$t_{2,0}$	$t_{2,1}$	$t_{2,2}$	$t_{2,3}$	$t_{2,4}$
	-1	-0.2098	0.0100	0.2078	1
	$t_{3,0}$	$t_{3,1}$	$t_{3,2}$	$t_{3,3}$	$t_{3,4}$
	-1	-0.2113	-0.0374	0.2143	1
	$t_{4,0}$	$t_{4,1}$	$t_{4,2}$	$t_{4,3}$	$t_{4,4}$
	-1	-0.2117	-0.0210	0.2076	1

Table 4.2: comparison of simplified strategy and unsimplified strategy when each phase measurement employs 5 threshold

Table 1. Generally, more thresholds per angle measurement implies the reconstructed power measurements will be more accurate.

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Vita

Xinda Ke received his bachelor degree in electronic information engineering at Wuhan University of Science and Technology, Wuhan, China in 2006. He has been pursuing the Master degree in electrical engineering at Lehigh University, Bethlehem, PA since 2011. His research areas are statistical signal detection and estimation.