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Development and Evaluation of Simplified Design Procedures for the Analysis and Design of...

September 2005
DEVELOPMENT AND EVALUATION OF SIMPLIFIED DESIGN PROCEDURES
FOR THE ANALYSIS AND DESIGN OF BUILDINGS WITH
SHAPE MEMORY ALLOY WIRE DAMPER

by

Donghao Liu

A Thesis
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Abstract

This thesis examines the performance of a simplified design procedure (SDP) for preliminary seismic design of frame buildings with hysteretic dampers. The SDP uses elastic-static analysis and is applicable to structures with hysteretic dampers such as shape memory alloy (SMA) wire damper. The SMA wire damper (SMAWD) has its core energy dissipation component made from superelastic SMA. SMAWD exhibits a nonlinear hysteretic behavior, and the SDP idealizes the nonlinear hysteretic damping as linear viscous damping. As examples, designs for a 3-story steel moment resisting frame (SMRF) equipped with SMAWDs were carried out using the SDP. In order to validate the SDP for structures with SMAWDs in a statistical sense, Monte Carlo simulation of the seismic response of a 3-story SMRF building model with SMAWDs has been performed based on a large ensemble of artificially generated earthquake ground motions. In this study, a method based upon a modified Kanai-Tajimi filtered Gaussian white noise process is used to generate the simulated earthquake accelerograms, which were used for the excitation input in the Monte Carlo simulation study.

Through this simulation study, results obtained by the SDP for structures with hysteretic dampers, which determines the minimum damper capacity in accordance with the desired story drift ratio, appear to provide an upper bound response for artificial generated ground motions scaled to the design basis earthquake (DBE). sufficiently general to enable different structural limit states to be considered in the design criteria. The linear elastic analysis associated with the SDP, which uses
the equivalent lateral force method, is quite simple to apply. The effectiveness of SMAWD in controlling the seismic response of framed building structures during strong earthquakes is clearly demonstrated by the Monte Carlo simulation study. Structures equipped with the SMAWDs are observed to have a much lower likelihood of experiencing excessive inter-story drift during a strong earthquake than corresponding uncontrolled structures.
1.1 Seismic Design Methodology

It is well known that strong earthquakes may bring damages to civil engineering structures. Therefore it has become one of the most serious structural engineering concerns to develop effective design methods to protect structures, along with their occupants and contents, from the detrimental effects of strong earthquake loading.

Traditional approach to seismic design has been based upon providing a combination of strength and ductility to resist the imposed loads. The frequent goal of the traditional approach to seismic design is to prevent collapse of the structure in the event of an earthquake, i.e. to protect life. For major earthquakes, structural design engineers rely on the inherent ductility of carefully detailed structures to prevent catastrophic failures and keep the structure performance at the life safety level, while accepting a certain level of structural and non-structural damage. Often this ductility is provided through material nonlinearity such as yielding of steel and nonlinear response of concrete in compression. Additionally, carefully designed structural systems can achieve high energy dissipation capacity through optimal design.

Much of the traditional seismic design of structures relies upon yielding of certain members to limit the forces in the structure. Since the members that yield may also be part of the gravity load carrying system of the structure and may be difficult to repair for damages, there have been efforts made to introduce members or devices into the
structures which are not part of the gravity load bearing system. These additional members or devices will limit the forces and add damping and which do not need repair or can be easily repaired. Many ordinary braced systems are of this type.

By considering the actual dynamic nature of environmental disturbances, more dramatic improvements can be realized. As a result of this dynamical point of view, new and innovative concepts of structural protection have been advanced and are at various stages of development. Modern structural protective systems can be broadly divided into three groups as shown below:

A). Seismic Isolation
   (i) Elastomeric Bearings;
   (ii) Lead rubber Bearings;
   (iii) Sliding Friction Pendulum;

B). Passive Energy Dissipation
   (1) Hysteretic dampers;
   (2) Viscous dampers;

C). Semi-active and Active Control
   (i) Active Tendon Systems;
   (ii) Active Mass Dampers;
   (iii) Variable Stiffness or Damping systems;
   (iv) Smart Damper (e.g., MR damper):

1.2 Structure Control for Seismic Resistance

1.2.1 Seismic Isolation
Seismic isolation is now widely used in many parts of the world. As compared with the other two structural protection methods, base isolation can be considered as a more mature technology with wider applications. A seismic isolation system is typically placed near the foundation of a structure. By means of its flexibility and energy absorption capacity, seismic isolation system partially reflects and partially absorbs the earthquake input energy before this energy can be transferred to the superstructure. The net effect is a reduction of the energy dissipation demand on the superstructural system, resulting in an increase in its seismic survivability. A detailed review of seismic isolation technology can be found in the monograph by Skinner et al. (1993).

1.2.2 Semi-active and Active Control

The second group is the semi-active and active control systems. Semi-active and active structural control is an area of structural protection in which the motion of a structure is controlled or modified by means of the action of a control system through some external energy supply. However, semi-active systems require only nominal amounts of energy to adjust their mechanical properties and, unlike fully active systems; they cannot add energy to the structure. Considerable attention has been paid to both semi-active and active structural control research in recent years (books are published in 1997), with particular emphasis on the alleviation of wind and seismic response. The technology is now at the stage where actual systems have been designed, fabricated and installed in full-scale structures (Soong and Dargush 1997).
1.2.3 Passive Energy Dissipation Devices

Research and development of passive energy dissipation (PED) devices for structural applications have roughly a 25-year history. Similar to seismic isolation technology, the basic function of passive energy dissipation devices when incorporated into a structure is to absorb or consume a portion of the input energy, thereby reducing the energy dissipation demand on primary structural members and minimizing possible structural damage. Unlike seismic isolation, however, these devices can be effective against wind induced motions as well as those due to earthquakes. In contrast to semi-active and active systems, there is no need for external power supply in passive control systems (Soong, and Dargush 1997). Compared with active control systems, passive dissipation devices have the potential to provide slightly worse or equivalent seismic performance of structures without the need for sophisticated technology and high cost.

In recent years, serious efforts have been undertaken to develop the concept of energy dissipation or supplemental damping into a workable technology (Soong and Dargush 1997), and a large number of passive control systems or PED devices have been developed and installed in structures for performance enhancement under earthquake loading. In North America, PED devices have been implemented in approximately 103 buildings and many bridges, either for retrofit or for new construction. Figure 1 shows the distribution of these buildings along the years in which PED systems were installed (Soong and Spencer 2002).
PED devices can be categorized into the following major groups: viscous damping devices, hysteretic damper and dynamic vibration absorbers of either the tuned mass or tuned liquid oscillator type.

**Viscoelastic Devices**

Viscous types of energy-dissipating devices utilizing copolymers to dissipate energy in shear deformation. These devices significantly increase the capacity of the structure to dissipate energy, but have little influence on the natural vibration periods of the structure (Chopra 2001).

Viscoelastic solids: The application of viscoelastic solid devices to civil engineering structures appears to have begun in 1969 when 10,000 viscoelastic dampers were installed in each of the twin towers of the World Trade Center in New York to help resist wind loads. Analytical and experimental studies on the dynamic response of visco-elastic energy dissipators and on the seismic response of viscoelastically damped structures have been carried out in the past two decades (see, e.g., Soong and Dargush 1997; Fu and Kasai 1998.).

Viscoelastic materials used in civil engineering structures are typically copolymers or glassy substances. Viscoelastic solid devices dissipate energy through shear deformation of the viscoelastic layers, which also depends on the vibrational frequency, strain and ambient temperature (Hanson and Soong 2000).

Viscous fluids: Viscous fluid damping devices include viscous damping walls and viscous fluid dampers. The viscous wall developed by Sumitomo Construction Company consists of a plate moving in a thin steel case filled with highly viscous
fluid. Viscous fluid damper, widely used in the military and aerospace industry for many years, has recently been adapted for structural applications in civil engineering (Makris and Constantinou, 1990; Constantinou and Symans, 1992). A viscous fluid damper generally consists of a piston in the damper housing filled with a compound of silicone or similar type of oil, and the piston may contain a number of small orifices through which the fluid may pass from one side of the piston to the other (Constantinou and Symans, 1992). Thus, viscous fluid dampers dissipate energy through the movement of a piston in a highly viscous fluid based on the concept of fluid orificing (Hanson and Soong 2000).

**Hysteretic Damping Devices**

The energy dissipation of hysteretic damping devices depends primarily on the relative displacements within the device and not on their relative velocities. Metallic yielding devices and friction devices are examples of hysteretic damping devices.

Metallic Yielding Devices: Metallic yielding dampers dissipate energy through hysteretic behavior when deformed into their inelastic range. Metallic yielding devices have some desirable features such as stable hysteretic behavior, long term reliability and relative insensitivity to environmental temperature. Hence a wide variety of devices have been developed and tested that dissipate energy in flexural, shear, or extensional deformation modes. For example, the X-shaped or triangular added damping and stiffness (TADAS) device, buckling restraining brace. The earliest implementation of metallic yielding dampers in structures occurred in New Zealand and Japan, which are reported in Aiken and Kelly (1992). Skinner et al.
More recent applications include the use of ADAS dampers in the seismic upgrade of existing buildings in Mexico (Clark et al. 1999) and in the USA (Pall and Marsh 1982). It is the first time in the United States that earthquake energy dissipation devices have been used for the seismic upgrade of a building located in San Francisco, California. The subject building is a Wells Fargo Banking building in San Francisco, CA. It's a 2-story nonductile concrete frame structure built in 1976 and was damaged in the 1989 Loma Prieta earthquake. A total of seven ADAS devices were employed, each with a yielding force of 150 kips. Linear and nonlinear analyses were both used in the retrofit design process. 2-D nonlinear time-history analyses using Drain-2DX were performed to verify the final design. The comparison of the behavior of the original structure with the final upgraded building shows that the responses of the upgraded structure have been reduced significantly and the upgraded structure meets all seismic criteria.

A variation of the metallic yielding devices is the buckling restraining braces, also called tension/compression yielding brace or unbonded brace (Wada et al. 1999, Clark et al. 1999). A summary of much of the early development of unbonded braces which use a steel core inside a concrete-filled steel tube is provided in Wantanabe, et al. (1988). Since the 1995 Kobe earthquake, these elements have been used in numerous major building structures in Japan (e.g., Reina, and Normile. 1997). According to the records from the Building Center of Japan for the year 1997, approximately two-thirds of all tall buildings(taller than 60 meters) approved for design that year incorporate some form of passive damping system, and the majority of these used hysteretic dampers.(Building Center of Japan. 1997)
Friction Devices: Various types of energy-dissipating devices, utilizing friction as means of energy dissipation, have been developed and tested by researchers. They increase the capacity of the structure to dissipate energy but do not change the natural vibration periods significantly (Chopra 2001). Generally, friction devices generate rectangular hysteretic loops similar to the characteristics of Coulomb friction (Hanson and Soong 2000).

In recent years, there have been a number of structural applications of friction dampers aimed at providing enhanced seismic protection of new and retrofitted structures. This activity in North America is primarily associated with the use of Pall friction devices in Canada and the USA (Pall et al. 1982); and slotted-bolted connection in the USA (Grigorian 1993). For example, the applications of friction dampers to the McConnel Library of the Concordia University in Montreal, Canada are discussed in Pall (1993). A total of 143 dampers were employed in this case. A series of nonlinear analyses using DRAIN-TABS were conducted to establish the optimum slip load for the Pall devices, which ranges from 600 to 700 KN depending on the location within the structure. For the three-dimensional time-history analyses, artificial seismic signals were generated with a wide range of frequency contents and the peak ground acceleration scaled to 0.18 G to represent expected ground motion in Montreal. Under this level of excitation, an estimate of the equivalent damping ratio for the structure with frictional devices is approximately 50 per cent. In addition, for this library building, the use of friction dampers resulted in a net savings of 1.5% of the total building cost (Soong and Spencer 2002).
While the energy dissipation characteristics of hysteretic devices are generally considered to be rate independent, some of their material properties may be sensitive to velocities at which they operate (Hanson and Soong 2000).

1.3 Shape Memory Alloys and Its Application to Energy Dissipation Device

At present several applications of passive seismic protection exist all over the world, both in new and in existing constructions. Some recent destructive earthquakes (Northridge, 1995; Kobe, 1996) definitively proved their effectiveness. However, the increasingly demanding performance requirements push towards the development of new devices, exploiting the peculiar characteristics of new advanced materials. Indeed, current technologies present some limitations, such as problems related to aging and durability (e.g. for rubber components), to maintenance (e.g. for those based on fluid viscosity), to installation complexity or replacement and geometry restoration after strong events (e.g. those based on steel yielding or lead extrusion), to variable performances depending on temperature (e.g. polymer-based devices) (Dolce et al. 2000).

1.3.1 General Properties of Shape Memory Alloys

In the 1960s, Buehler and Wiley developed a series of nickel-titanium alloys, with a composition of 53 to 57% nickel by weight, which exhibited an unusual effect: severely deformed specimens of the alloys, with residual strain of 8-15%, regained their original shape after a thermal cycle. This effect became known as the shape-
memory effect, and the alloys exhibiting it were named shape-memory alloys. The nickel-titanium alloys were then commercialized under the trade name Nitinol.

It was later found that at sufficiently high temperatures such materials also possess the property of superelasticity, that is, the recovery of large deformations during mechanical loading-unloading cycles performed at constant temperature (Fugazza 2003).

1.3.2 Shape Memory Effect (SME)

The stress strain curve of Nitinol exhibiting the shape memory effect (SME) is shown in Figure 2. The SME is observed when the temperature of a piece of shape memory alloy is cooled to below the temperature $M_r$. At this stage the alloy is completely composed of Martensite which can be easily deformed. After distorting the SMA the original shape can be recovered simply by heating the wire above the temperature $A_f$. $M_s$ denotes the temperature where the structure starts to change from austenite to martensite upon cooling; $M_r$ is the temperature where the transition is finished. Accordingly, $A_s$ and $A_f$ are the temperatures where the reverse transformation from martensite to austenite start and finish, receptively.

1.3.3 Super-elasticity Characteristics (SEC)

Super-elasticity (or pseudo-elasticity) refers to the ability of SMA to return to its original shape upon unloading after substantial deformation. This occurs in SMA when the alloy is completely composed of Austenite (temperature is greater than $A_r$). The load on the shape memory alloy is increased until the Austenite becomes
transformed into Martensite simply due to the loading; this process is shown in Figure 3. The loading is absorbed by the softer Martensite, but as soon as the loading is decreased the Martensite begins to transform back to Austenite since the temperature of the alloy is still above $A_r$, and the SMA springs back to its original shape.

There are several advantages to apply SMA dampers to seismic protection of structures. (Dolce M. et al. 2000, DesRoches and Delemont 2002).

i). SMA materials can repeatedly absorb a large amount of strain energy under cyclic loading without permanent deformation. It is also possible to make supplemental energy dissipation device with self-centering capability.

ii). SMA materials such as nitinol has extraordinary fatigue resistance under large strain cycles, i.e. capability to undergo several hundreds of cycles with large strain amplitude, and therefore SMA dampers made from nitinol can resist destructive earthquakes many times, with little need of substitution or maintenance;

iii). SMA materials such as nitinol exhibit great durability and long-term reliability. For example, NiTi alloy has an excellent corrosion resistance and no degradation due to aging;

1.4 Shape-Memory Alloy Based Structural Control Devices

Due to the above-described unique features, SMA has attracted the attention of scientific and engineering communities. These desirable properties of SMA make them attractive candidates for applications in such diverse areas as aerospace and biomedical engineering. Additionally, SMA is finding new applications in the field of civil engineering, especially for seismic protection of civil engineering structures.
The next section gives a review of the major applications that exploit the two main properties of SMA - shape-memory effect and super-elastic behavior.

The majority of SMA applications have been done for biomedical applications. Recently, research efforts have been extended to using SMA-based devices for smart civil structures (e.g., Graesser and Cozzarelli 1991; Witting and Cozzarelli 1992; Clark et al. 1995; Whittaker et al. 1995; Higashino 1996; Dolce et al. 2000; Wilde et al. 2000; Castellano et al. 2001; Saadat et al. 2002; Ocel et al. 2004). Krumme et al. (1995) have introduced an important class of SMA damper -- the center-tapped (CT) device for passive control of the dynamic response of civil structures. The center-tapped device comprises a simple slider mechanism in which resistance to linear sliding is provided by two pairs of opposed SMA tension elements. The effectiveness of SMA-based damping device for passive control of civil structures was demonstrated through the experimental study of both concrete and steel frame buildings by Clark et al. (1995) and Whittaker et al. (1995). Two families of passive seismic control devices (special braces for framed structures and isolation devices for buildings and bridges) based on nickel-titanium shape memory alloys were developed by Dolce et al. (2000) and their effectiveness was verified through an extensive testing program. In Italy, superelastic SMA damping devices have also been implemented in several masonry cultural heritage structures to enhance their seismic resistance capacities during recent restoration (Castellano et al. 2001; Indirli et al. 2001). DesRoches and Delemont (2002) have tested the efficacy of superelastic NiTi bars as bridge restrainers to reduce the risk of collapse from unseating of bridge superstructures at the hinges. Additionally, two full-scale partially restrained steel
beam-column connections using SMA bars for providing additional energy dissipation were tested by Ocel et al. (2004). The connection consists of four large diameter NiTi SMA bars connecting the beam flange to the column flange and serve as the primary moment transfer mechanism. The connections exhibited a high level of energy dissipation, large ductility capacity, and no strength degradation after being subjected to cycles up to 4% drift.

1.5 Research Objectives and Scope

Very recently, a superelastic SMA WD with tunable hysteretic behaviors has been proposed. Preliminary research has shown that the SMA WD offers an effective energy dissipation device with a simple configuration that can be useful for a variety of passive structural control applications including seismic response reduction. However, more research work needs to be done before the SMA WD can be successfully implemented in real structures. To address this, the main objectives of this research are:

i). Develop a simplified design procedure to facilitate preliminary seismic design of framed buildings with hysteretic damping devices, with an emphasis on SMA WD.

In order to apply this new SMA WD to actual structures, it is necessary to formulate design guidelines and procedures, based upon knowledge gained from theoretical and experimental studies. It is important to ensure the preliminary selection and design of dampers for structural application as simple as possible.

SMA WD is essentially a type of hysteretic damping devices despite the obvious difference in geometric configuration between metallic yielding devices and
SMAWDs. The analysis and design procedures for the passively damped structure equipped with SMAWDs can closely follow those developed for hysteretic devices.

ii). Perform a statistical analysis of structures equipped with SMAWDs to validate the simplified design procedure as well as the energy dissipation capability of SMAWD.

Results from dynamic analysis were used as a baseline against which to compare the simplified design method. Dynamic analysis was performed using Matlab code in a state space formulation. In order to give statistically significant results, a 3-story steel frame structure with and without SMAWDs was subjected to a large ensemble of earthquake ground motion records containing 2,000 different artificial records. The procedure for generating these records is based on the modified Kanai-Tajimi filtered Gaussian white noise process.
Fig. 1.1 Implementation of Passive Energy Dissipation Device in North America for Seismic Applications. (Soong and Spencer 2002)

Shape Memory Effect

Figure 1.2 Typical Stress-strain Relationship Showing Shape-memory Effect
Figure 1.3 Typical stress-strain relationship showing super-elasticity characteristics.
2.1 Introduction

This chapter gives a brief description of a shape memory alloy (SMA) wire damper that can be used for a variety of passive structural control applications including seismic response reduction.

In order to effectively include the SMA wire damper (SMAWD) in the design of actual structures, one must be able to characterize their expected nonlinear force-displacement behavior under arbitrary cyclic loads. In this chapter, a constitutive model for SMA is presented. Since the SMAWD uses superelastic Nitinol wires as its core energy dissipation component, emphasis here is given only to uniaxial models, which are commonly employed for numerical tests. A constitutive law for uniaxial SMA elements based on the Ozdemir model is described in this chapter.

2.2 Constitutive Modeling of Uniaxial SMA Elements

2.2.1 Ozdemir Model

The Ozdemir model was originally developed by Ozdemir (1976) to study the mechanical response of materials showing hysteresis. Although this model does not
capture the superelastic behavior exhibited by SMAs, understanding of the Ozdemir model is necessary for us to derive the constitutive model for SMA.

The equations that describe the Ozdemir model are expressed as follows,

\[
\dot{\sigma} = E \left[ \dot{\varepsilon} - |\dot{\varepsilon}| \cdot \left( \frac{\sigma - \beta}{Y} \right)^n \right] \quad (2.1)
\]

\[
\dot{\beta} = \alpha \cdot E \cdot |\dot{\varepsilon}| \cdot \left( \frac{\sigma - \beta}{Y} \right)^n \quad (2.2)
\]

where \(\sigma\) = the one-dimensional stress; \(\varepsilon\) = the one dimensional strain; \(\beta\) = the one-dimensional backstress; \(E\) = the elastic modulus; \(Y\) = the upper (loading) plateau stress (i.e., the ‘yielding’-like plateau in the loading stage); \(\alpha\) = a constant controlling the slope of the \(\sigma - \varepsilon\) curve, given by \(\alpha = E_y/(E - E_y)\), where \(E_y\) = the slope of the \(\sigma - \varepsilon\) curve after yielding; \(n\) = a constant controlling the sharpness of transition between different phases; over dot denotes the ordinary time derivative;

By rearranging Eqn. (2.1), we will get:

\[
\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + |\dot{\varepsilon}| \cdot \left( \frac{\sigma - \beta}{Y} \right)^n \quad (2.3)
\]

Examination of this equation reveals that the total strain is made up of two separate components: (1) A linear elastic component \(\sigma/E\); and (2) a non-linear inelastic component, \(\varepsilon''\) which is described by the rate expression \(\dot{\varepsilon}'' = |\dot{\varepsilon}| \cdot [(\sigma - \beta)/Y]^n\). This inelastic component is a function of the strain rate \(\dot{\varepsilon}\) and the overstress \(\sigma - \beta\). It is possible to show that for the model described by Eqns. (2.1) and (2.2), the strain rate is independent of the applied rates of loading (i.e.,
not dependent on either $\dot{\sigma}$ or $\dot{\varepsilon}$. By subtracting Eqn. (2.2) from Eqn. (2.1), the following differential equation is obtained,

$$\dot{\sigma} - \dot{\beta} = E\dot{\varepsilon}[1 - (1 + \alpha)(\frac{\alpha - \beta}{Y})^n]$$  \hspace{1cm} (2.4)

Eqn. (2.4) can be rewritten as

$$d\varepsilon = \frac{d(\sigma - \beta)}{E\left[1 - (1 + \alpha)(\frac{\sigma - \beta}{Y})^n\right]}$$  \hspace{1cm} (2.5)

By taking integral of Eqn. (2.5), the expression for strain can be derived as,

$$\varepsilon = \frac{Y}{E(1 + \alpha)^{1/n}} \int_{\sigma - \beta}^{\sigma - \beta+y} \frac{d\xi}{1 - \xi^n}$$  \hspace{1cm} (2.6)

It can be seen that the integral in Eqn. (2.6) is a function of the overstress $\sigma - \beta$.

Through this derivation, it is now clear that Eqns. (2.1) and (2.2) represent rate-independent stress-strain behavior. For the special case where $n = 1$, the following result can be readily obtained:

$$\sigma = \frac{E\alpha}{1 + \alpha} \varepsilon + \frac{Y}{(1 + \alpha)^2} \left\{1 - \exp\left[-\frac{E(1 + \alpha)}{Y} \varepsilon\right]\right\}$$  \hspace{1cm} (2.7)

\textit{2.2.2 Modified Ozdemir Model}

The model used for this study is a modified version of the Ozdemir model proposed by Grasser and Cozzarelli (1991). Since this model is an extension of the rate-independent model by Ozdemir, it is called the modified Ozdemir model. The modified Ozdemir model simulates the one-dimensional macroscopic stress-strain relationship of superelastic SMA wires.
To modify Eqns. (2.1) and (2.2) for SMA twinning hysteretic and/or superelastic behavior, let us first examine the slope of the stress-strain curve. Dividing Eqn. (2.1) by $\dot{\varepsilon}$ yields the slope:

$$\frac{d\sigma}{d\varepsilon} = E \left[ 1 - \text{sgn}(\dot{\varepsilon}) \left( \frac{\sigma - \beta}{Y} \right)^a \right]$$

(2.8)

where the notation $\text{sgn}(x)$ is used to represent the Signum function defined below,

$$\text{sgn}(x) = \begin{cases} +1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

(2.9)

By examining Eqn. (2.8), we can see that the slope is constant during purely linear elastic loading and unloading, that is, when $(\sigma - \beta)/Y < 1$. It is possible to modify the shape of the inelastic portion of the stress-strain curve by altering the expression for the backstress. First, we combine Eqns. (2.1) and (2.2) and integrate to simplify the expression for backstress. Assuming that the zero initial conditions for $\sigma$, $\beta$, and $\varepsilon$ (i.e. an undeformed virgin material having no residual stresses), the integration operation yields:

$$\frac{\beta}{E} = \alpha \left( \varepsilon - \frac{\sigma}{E} \right) = \alpha \varepsilon''$$

(2.10)

It is seen from Eqn. (2.10) that the backstress is a linear function of the inelastic strain $\varepsilon''$.

By modifying Eqn. (2.10), it becomes possible to describe the various aspects of the shape memory alloy behavior. Such an expression can be derived by adding another term to the inelastic strain in Eqn. (2.10), i.e..
\[
\frac{\beta}{E} = \alpha \left( \varepsilon^\text{in} + f_T \cdot |\varepsilon|^c \cdot \text{erf}(a \varepsilon) \cdot [u(\varepsilon)] \right)
\]

(2.11)

where \( f_T, a, \) and \( c \) are material constants controlling the recovery of the inelastic strain upon unloading; \( \text{erf}(\cdot) \) and \( u(\cdot) \) are the error function and unit step function, respectively. The unit step function and the error function, are defined as follows,

\[
u(x) = \begin{cases} 
0 & \text{if } x < 0 \\
+1 & \text{if } x \geq 0
\end{cases}
\]

(2.12)

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt
\]

(2.13)

The error function expressed by Eqn. (2.12) is a smoothly changing function that passes through zero at the origin and has asymptotic limits of -1 and +1 at \( x = -\infty \) and \( x = +\infty \), respectively.

Moreover, by proper choice of \( f_T, a, \) and \( c \), the inelastic strain can be fully recovered upon complete unloading to \( \varepsilon = 0 \), which enables the description of the superelastic behavior of shape memory alloys. Therefore, a one-dimensional constitutive model that can produce the general macroscopic stress-strain character of SMA is given by Eqns. (2.1) and (2.11). These equations are rewritten as follows,

\[
\dot{\sigma} = E \left[ \dot{\varepsilon} - |\dot{\varepsilon}| \cdot \left( \frac{\sigma - \beta}{Y} \right)^n \right]
\]

(2.14)

\[
\beta = E \alpha \left( \varepsilon^\text{in} + f_T \cdot |\varepsilon|^c \cdot \text{erf}(a \varepsilon) \cdot [u(\varepsilon)] \right)
\]

(2.15)

In particular, this model has the following advantages (Fugazza 2003):

- Ability to predict both the superelastic and the shape-memory effect:

- Simplicity to be extended to the three-dimensional case:
• Ability to describe sub-loops (i.e. partial transformation paths).

On the other hand, the main drawbacks of the model are:

• Rate and temperature independence;
• Inability to predict the strain hardening behavior at large deformations.

2.3 Configuration of SMAWD

Selection of SMA

Although there exist many SMAs, such as Ti-Ni, Cu-Al-Ni, Cu-Zn-Al, Au-Cd, Mn-Cu, Ni-Mn-Ga, and Fe-based alloys, most of the practical SMAs are Ti-Ni-based alloys, since other SMAs are usually not ductile (or not ductile enough) or are of low strength and exhibit grain-boundary fracture (Otsuka and Kakeshita 2002). Ni-Ti based alloys are superior to other SMA materials in many aspects and Nitinol is the most widely used SMA material at present (Zhang and Zhu 2005).

Seismic application of superelastic shape memory alloys has been implemented using wrapped bundles of wires, instead of large bars, due to the uncertainty of size effects when large cross sections are used (Fugazza 2003).

The parameter values of the modified Ozdemir model calibrated with experimental data are listed in Table 1. It is seen from Figure 2.1 that the modified Ozdemir model represents the stress-strain relationship of superelastic Nitinol wires reasonably well. It is worth noting again that the modified Ozdemir model is independent of loading rate and temperature effects; furthermore, it can not predict the strain hardening behavior of SMA at large deformation levels (Zhang and Zhu 2005).
This section briefly describes the configuration of SMAWDs. Figure 2.2 shows the configuration of the SMAWD under investigation. The SMAWD utilizes superelastic SMA wires as its core energy dissipation component, which provides damping through its unique hysteretic behavior. As indicated in Figure 2.2-(b), the length of each SMA wire strand in the damper is denoted as $L$. The angle that are formed by two SMA wire strands placed on opposite sides of the damper is denoted as wire inclination angle, $\theta$. For simplicity of illustration, only one SMA wire strand (strand “A” or “B”) is shown on each side of the damper in Figure 2.2-(b); In practice, multiple SMA wires strands parallel to each other will be used on each side of the damper to achieve larger damper forces.

Effect of Pre-strain Level of SMA Wire (Zhang and Zhu 2005)

This section examines the effect of pre-strain level of SMA wires on the damper performance. Figure 2.3 shows the hysteresis loops of SMAWDs subjected to four different levels of damper displacement, which are equal to 1, 2, 4 and 6 mm, respectively. Two pre-strain levels – 0 and 3% are considered in Figure 2.3. The zero pre-strain means that SMA wires are not pre-stressed. The wire inclination angle $\theta$ is set as 30 deg. for all cases. The peak strains of SMA wires with no pre-strain, corresponding to Figure 3-(a) to (d), are 0.3, 0.6, 1.3 and 1.9 %, respectively, while the peak strains of SMA wires with 3% pre-strain are 3.3, 3.7, 4.3 and 5.0 %, respectively. As seen from Figure 2.3, dampers with 3% pre-strain have larger and
fatter hysteresis loops and thus can dissipate more energy than those with no pre-strain when subjected to the same amplitude displacement. At very small displacements, the damper with no pre-strain behaves like an elastic spring and thus no energy is dissipated. This is due to the initial linear stress-strain relationship of superelastic SMA wires as shown in Figure 2.1. Additionally, for dampers without any pre-strain, at any time instant only one of the opposed SMA wire strands is activated since the other wire strand is too slack to take any compressive load. However, for dampers with pre-stressed wires, hysteretic damping can be generated even at small displacements, and both wire strands are almost always activated for energy dissipation until the wire strand on one side of the damper goes into compression. Clearly, pre-stressing of SMA wires can enhance the damper’s ability of energy dissipation, especially at relatively small displacements. This feature might be useful for passive control of wind-induced vibration in structures, which typically has relatively small vibration amplitude.

Figure 2.4 shows the hysteresis loops of SMA wire dampers with four different pre-strain levels, i.e., 0, 0.3, 2 and 3%, respectively. The hysteresis loops in Figure 2.4 correspond to the first 6% strain cycle of SMA wires. The wire inclination angle of the damper is equal to 30 deg in all cases. 0.3% pre-strain corresponds to a point near the intersection of initial linear elastic curve and lower plateau in the stress-strain curve for superelastic SMA wires, as shown in Figure 1. In Figure 2.4, “Pinch” effect is observed in the hysteretic loops of SMA wire dampers with low pre-strain levels. Higher pre-strain levels result in “fat” hysteresis loops and thus increased energy dissipation capacity if compared at the same displacement level; however, the damper
stroke, which is defined as the damper displacement corresponding to a 6% strain of SMA wires, would get smaller for dampers with higher pre-strain levels. Additionally, the total energy dissipated in one hysteresis loop of SMA wires corresponding to a 6% strain cycle remains constant for dampers with different pre-strain levels. Therefore, a tradeoff exists between the damper stroke and energy dissipation capacity at small displacements for the design of the SMAWD under consideration. Increased energy dissipation at small displacements comes at the price of reduced damper stroke which is set by the 6% maximum allowable strain for superelastic SMA wires as assumed in this study. It is recommended that the pre-strain level should be kept below 3%, which is half of the assumed 6% maximum allowable strain. Otherwise, a smaller value for damper stroke will result. As described above, for dampers without any pre-strain, only the wire strand on one side is activated at small displacement. Therefore, the initial stiffness of dampers with pre-strain is twice that of dampers without pre-strain, as shown in Figures 2.3 and 2.4. As seen in Figure 2.4, during the loading stage, the force-displacement curve of the damper with 2% pre-strain can be divided into three distinct phases, between which drastic slope changes can be observed. During the initial loading phase, the strain of SMA wires on one side of the damper gets increased while the wire strand on the other side undergoes decreasing strain; when the wires on one side enter the linear elastic range from lower plateau (Figure 2.1), the damper stiffness gets increased again; upon further loading, one wire strand would be slack and deactivated, and the damper stiffness takes a shape turn to a very small value as evidenced by the top plateau in Figure 2.4. It is seen from Figure 2.4 that very minor differences exist between the
hysteresis loops of dampers with no pre-strain and 0.3% pre-strain, which implies that a low pre-strain level have no significant effect on the damper performance, except for the initial stiffness.

Figure 2.5 presents the relationship between the damper stroke and pre-strain levels of SMA wires. Three different wire inclination angles – 30, 90 and 150 deg. are considered. The damper stroke is defined as the displacement of the damper corresponding to 6% strain of SMA wires. The general trend for the damper stroke vs. pre-strain relationship as observed in Figure 2.5 is that as the pre-strain level increases, the damper stroke gets reduced. An almost linear relationship is observed between the damper stroke and pre-strain levels of SMA wires. Also observed from Figure 2.5 is that as the wire inclination angle gets larger, the damper stroke becomes larger too.

By pre-stressing the SMA wires to an appropriate level, SMA wires on both sides of the damper will be always in tension during the damper’s design operation range and thus dissipate more energy for a certain level of damper displacement, as evidenced by Figure 2.3. Additionally, its ability of energy dissipation at small displacements can be increased by using pre-tensioned SMA wires.
Table 2.1 Parameters of the modified Ozdemir model and test data for superelastic SMA wires

<table>
<thead>
<tr>
<th>Parameters for the Superelastic SMA Wires</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test Data</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Modified Ozedemir Model</strong></td>
</tr>
</tbody>
</table>

§ Test data is based on a quasis-static test of superelastic Nitinol wires with a diameter of 0.127 mm (0.005 inch) and a gage length of 305 mm (12 inch).
Figure 2.1 Stress-strain relationship of superelastic Nitinol wire

Figure 2.2 Schematics of SMAWD configuration

(a) 3-d view
(b) Front view (only one strand is shown for simplicity)

Figure 2.2 Schematics of SMAWD configuration

Figure 2.3 Comparison of hysteretic behavior of SMAWD at various strain levels.
pre-strain = 0 or 3%. $\theta = 30$ deg.
Figure 2.4 Comparison of hysteretic behavior with various pre-strain levels, 6% strain cycle, $\theta = 30$ deg

Figure 2.5 Damper stroke vs. pre-strain level curve of SMAWD
CHAPTER 3
A SIMPLIFIED SEISMIC DESIGN PROCEDURE FOR BUILDING STRUCTURES WITH SHAPE MEMORY ALLOY WIRE DAMPER

3.1 Introduction

As shown in Chapter 2, the analytical model developed for SMA can represent its hysteretic behavior reasonably well. However, this analytical model is too complex to use for conventional design procedures. A simplified design procedure (SDP) is needed to determine the damper capacity for preliminary design of structures. In this chapter, a SDP for preliminary seismic design of frame buildings with SMA wire damper is presented. The SDP uses elastic-static analysis and is applicable to hysteretic dampers such as the SMA wire damper. Design examples that illustrate the use of the SDP for sizing SMA wire dampers in a 3-story moment resistant frame building are also given in this chapter.

3.2 Design Methodology for Structures with Hysteretic Dampers

Intensive efforts have been undertaken to develop the concept of energy dissipation into a workable technology, and so far a number of these devices have been installed in structures throughout the world. Consequently, it is important to develop design methodologies applicable to more generic passively damped structural systems.
In the meanwhile, approximate displacement-based seismic design methods which are based on the use of non-linear static pushover analysis, have undergone a rapid increase in popularity and are starting to find their way into design guidelines and codes of practice. These methods are considered a step forward from the use of linear analysis and ductility-modified response spectra. However, it should be stressed that pushover analysis methods have no rigorous theoretical basis, and might lead to inaccurate results if the assumed load distribution is incorrect (Williams and Albermani 2003). Additionally, plastic analysis needs to be performed to obtain the push-over curve which demands more computation resources than that allowed in the preliminary design stage. Therefore, such pushover analysis method is not recommended for preliminary design of structures with passive dampers.

The SMA wire damper can be used for seismic response reduction of building structures. Although the SMA wire damper has different appearance and geometric configuration compared with metallic yielding devices, the fundamental dissipative mechanism in both cases results from hysteretic behavior of metal materials. Therefore, the analysis and design procedure for structures with SMA wire damper can follow those developed for hysteretic devices since SMA wire damper belongs to hysteretic dampers.

The simplified method presented here for preliminary design is based on the use of linear structural model and equivalent linearized model for hysteretic damping.

The simplified design procedure for structures with SMA wire dampers are based on a couple assumptions: (i) primary structure is modeled with an equivalent linear elastic, viscously damped system; (ii) Rigid floor is assumed so that in the principal
direction of building each floor is associated with one horizontal degree of freedom (DOF) and rotational DOFs are ignored; (iii) the hysteretic damping of SMA wire damper is idealized into equivalent linear viscous damping.

3.3 Simplified Design Procedure for Building Structures with SMA Wire Damper

A SDP is proposed herein for multi-degree of freedom (MDOF) frame building structures equipped with SMA wire damper. The SDP is based on an elastic-static analysis to determine the damper properties so that the damped frame building satisfies specified seismic performance objectives. The SDP consists of the following steps.

1. Establish the Target Seismic Performance and Design Criteria

Suppose the building under consideration is a framed structure located in California. The primary design goal is to keep the structure at operational or immediate occupancy level under design basis earthquake (DBE) with a 10% probability of exceedance within 50 years and at immediate occupancy and life safety level for the maximum credible earthquake (MCE) with a 2% probability of exceedance within 50 years. Design criteria to achieve the target performance level such as story drift limits, are established in this step.

Performance Requirement for Steel Moment Frames (FEMA 2001):

Conventional seismic design: Under DBE- The desired performance level is life safety performance level; Under MCE- The desired performance level is collapse prevention performance level.
Seismic design with structural dampers: Under DBE - The desired performance levels are immediate occupancy for the DBE. Interstory drift should be limited to 0.7% transient, and negligible permanent drift (i.e., minor local yielding at a few places; no fractures; minor buckling or observable permanent distortion of members).

2. **Idealize Hysteretic Damping into Linear Viscous Damping for SMA Wire Damper**

A bilinear model is first used to approximately represent the hysteresis of SMA wire damper. The hysteretic damping provided by the bilinear model is then converted into equivalent linear viscous damping plus an equivalent spring.

\[ k_e = \text{the initial elastic stiffness}; \]

\[ k_h = \text{the strain hardening stiffness}; \]

\[ k_d = \text{the equivalent stiffness taken as the secant stiffness at the maximum device displacement, } d_0; \]

\[ \alpha = k_h / k_e; \]

\[ A = \text{the total cross section area of SMA wires}; \]

\[ L = \text{the initial length of the SMA wires which is determined according to the target drift limit and the maximum allowable strain of the SMA wires}; \]

\[ d_0 = \text{the maximum device displacement corresponding to maximum structural displacement. It can be assigned based on the target design values}; \]

\[ d_c = \text{the characteristic displacement}; \]

\[ T_1 = \text{the period of the cyclic motion applied to the damper (when damper is placed in an actual structure, the } T_1 \text{ is thus the period of the dominant vibration mode of the structure)}; \]
The equivalent stiffness and viscous damping coefficient can be calculated as follows (Hanson and Soong 2000):

\[
k_d = \frac{k_e \cdot d_y + k_b (d_0 - d_y)}{d_0} \tag{3.1}
\]

\[
c_d = \frac{4(k_e - k_b) \cdot d_y (d_0 - d_y) T_i}{2\pi^2 d_0^2} \tag{3.2}
\]

3. Select a range of \( \beta \) Values and Damper Locations

\( \beta \) is the ratio of the initial elastic damper stiffness per story to the story stiffness without dampers and braces. After selecting a \( \beta \) value, the geometry of the SMA wire damper is determined at the end of the SDP.

4. Perform Elastic-static Analysis

As \( k_b \) goes to infinity in the case of rigid bracing (Hanson and Soong 2000)

\[
k = k_i + \alpha_d k_d \tag{3.3}
\]

\[
c = c_i + \alpha_d c_d \tag{3.4}
\]

<table>
<thead>
<tr>
<th>Damper Assembly</th>
<th>( \alpha_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagonal bracing</td>
<td>( \cos^2 \theta )</td>
</tr>
<tr>
<td>Chevron or cross bracing</td>
<td>1</td>
</tr>
<tr>
<td>Upper toggle bracing</td>
<td>( \gamma_1^2 )</td>
</tr>
<tr>
<td>Lower toggle bracing</td>
<td>( \gamma_2^2 )</td>
</tr>
<tr>
<td>Scissor-jack bracing</td>
<td>( \gamma_3^2 )</td>
</tr>
</tbody>
</table>
After replacing $k_s$ and $c_s$ of the primary structure with $k$ and $c$, linear analysis of structures with SMA wire damper can be carried out following conventional procedures. It is noted that there is a complication that the mode shapes for structures with significantly nonproportional damping are complex. However some researchers (Hanson and Soong, 2000) have shown that deviations of the exact mode shapes from those assuming proportional damping are usually not significant. Therefore it is assumed in this study that mode shapes calculated with the modified stiffness matrix can be used for subsequent response calculation. This way we avoid the need to deal with the complex mode shapes for the response spectrum analysis procedure.

With a range of $\beta$ values defined in last step, a series of elastic-static analysis are then performed using the elastic-static analysis procedure discussed in Section 3.4.

5. Compare Structural Response Obtained in the Last Step with Target Performance

6. Choose the Minimum $\beta$ Value that Satisfies Design Criteria and Provides Target Seismic Performance

7. Calculate the Cross Section Area of the SMA Wire Strands

8. Perform Dynamic Time History Analyses (DTHA) to validate the Simplified Design Procedure (optional)

3.4 Elastic-static Analysis Procedure for Damped MDOF Frame Systems
STEP (1) Estimate First-mode Deflected Shape of Damped Frame System, \( u \), by Analyzing the Frame under a Pattern of Equivalent Lateral Forces \( P \).

STEP (2) Estimate First-mode Period \( T_1 \) Using Raleigh's Method

STEP (3) Estimate First-mode Damping Ratio, \( \beta_{eq} \)

STEP (4) Determine Seismic Coefficients from Design Spectra

\( Cs = \) the seismic response coefficient

\( R = \) the response modification coefficient

STEP (5) Compute Equivalent Lateral Forces

The equivalent lateral force procedure in ASCE 7-02 (SEI/ASCE 2003) is used but the damping coefficients \( B_s \) and \( B_I \) as a function of effective damping \( \beta \) are included. Since the damping ratio of the damper added structure is higher than the design spectrum (5 per cent), \( Cs \) obtained from the design spectrum is divided by a damping reduction factor, \( B_s \) or \( B_I \). The value of \( B_s \) and \( B_I \) shall be taken from Table 3.3.

STEP (6) Perform Static Analysis under Equivalent Lateral Forces to Estimate Displacements, Internal Forces, and Deformations of Damped Frame System

3.5 Calculation of Seismic Response in Design Basis Earthquake

3.5.1. Example 1

A 3-story shear frames with lumped mass shown in Figure. 3.3. All floor masses are equal to \( m \), and all stories have the same height \( h \) and story stiffness \( k \).

Step (1) Estimate first-mode deflected shape of damped frame system. \( u \), by analyzing the frame under a pattern of equivalent lateral forces \( P \).
\[ M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad K_s = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \quad K_d = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \]

\[ k_e = \alpha k_h; \quad k_e = \gamma k_d; \quad k_e = \beta k_s \]

Estimate the maximum yield displacement of the damper and then from the hysteretic loops we can get

\[ d_0 = 0.0047633L; \]
\[ d_y = 0.022466L \]
\[ k_e = 6.3839e+01A/L; \]
\[ k_h = 1.1667e+009 A/L; \]
\[ k_d = 1.4455e+010A/L \]
\[ \gamma = 4.4165; \]

Let \( L = 1.50m \), then we will get

\[ h = 3.96 \text{ m} \]

\[ P = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} mg \]

\[ K = K_s + K_d \quad (\text{For Chevron or cross bracing, } \alpha_d = 1) \]

\[ u = K^{-1} P \]

Step (2) Estimate first-mode period \( T_1 \) using the Raleigh’s method.

\[ K_1 = u^T \cdot K \cdot u; \quad M_1 = u^T \cdot M \cdot u \]
\[ \omega_1 = \sqrt{\frac{K_1}{M_1}} \quad ; \quad T_1 = \frac{2\pi}{\omega_1} \]

Step (3) Estimate first-mode damping ratio, \( \beta_{eq} \)

\( \beta \) is the ratio of the initial elastic damper stiffness per story to the structure stiffness of each story without damper.

\[
\beta = \frac{k_e}{k_s} \quad A = \frac{\beta \cdot k_s \cdot L}{6.383e+010}
\]

A, \( \omega_1 \) and \( T_1 \) are functions of \( \beta \)

\[
c_d = \frac{4(k_e - k_h) \cdot d_y \cdot (d_0 - d_y) T_1}{2\pi^2 d_0^2}
\]

So \( C_d \) is a function of \( \beta \)

\[
C = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} c_d
\]

\( C \) is also a function of \( \beta \).

\[
\beta_{cq} = \frac{u^T \cdot C \cdot u}{2M1 \cdot \omega_1}
\]

So \( \beta_{cq} \) is also a function of \( \beta \).

STEP (4) Determine seismic coefficient from design spectrum

\[
C_s = \frac{S_{DS}}{R/I}
\]

\( C_s \) = seismic response coefficient

\( S_{DS} \) = the design spectral response acceleration parameter in the short period range:

\( R \) = the response modification coefficient.
\( I \) = the occupancy importance coefficient;

Alternatively, the seismic response coefficient, \((C_s)\), need not be greater than the following equation:

\[
C_s = \frac{S_{D1}}{T(R/I)}
\]

\( T \) is the approximate fundamental period of the structure

\[
T = C_i h^2
\]

but shall not be taken less than

\[
C_s = 0.044S_{D5} I
\]

nor for building and structures in Seismic Design Categories E and F

\[
C_s = \frac{0.5S_1}{R/I}
\]

As the damping ratio of the damper-added structures is higher than the design spectrum (5% critical damping), thus \( C_s \) obtained from the design spectrum is divided by a damping reduction factor, \( B_s \) and \( B_f \) (FEMA 2001).

Step (5) Compute equivalent lateral forces according to the IBC 2003

\[
V' = \frac{C_s}{B_s} W'
\]

\( W = \) the total dead load and applicable portions of other load

\[
F_X = C_{s'} V'
\]

Vertical distribution factor
For structure having a period of 0.5 sec or less, \( k = 1 \); for structures having a period of 2.5 sec or more, \( k = 2 \). For structures having a period between 0.5 and 2.5 seconds, \( k \) shall be 2 or shall be determined by linear interpolation between 1 and 2.

**STEP (6)** Perform static analysis under equivalent lateral forces to estimate displacements, internal forces, and deformations of damped frame system.

The deflections determined by an elastic analysis

\[
\delta_{xe} = K^{-1} F
\]

The deflection of level \( x \) at the center of the mass, \( \delta_x \), shall be determined in accordance with the following equation

\[
\delta_x = \frac{C_d \delta_{xe}}{I}
\]

\( C_d \) = the deflection amplification factor.

The values of \( C_d \) and \( R \) are taken as 1 since the structure is designed to remain elastic in under the DBE.

**Example 2**

This example presents detailed calculations in the application of the equivalent lateral force procedure (ELF) and response spectrum analysis (RSA) procedures to the analysis of a 3-story Steel MRF incorporated with SMA damping systems shown in Figure 3.4.
(1). Calculation of Response Spectrum with 5% Damping Ratio.

Mapped acceleration parameters:

Site Los Angeles: $S_s = 92.6548\%g; S_1 = 42.0889\%g$

The spectral response acceleration with 5% damping is shown in figure 3.5.

Table 3.2 Design Spectral Response Parameters with 5% damping

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$S_{DS}$</th>
<th>$S_{DI}$</th>
<th>$T_0$</th>
<th>$T_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>0.6976g</td>
<td>0.4430g</td>
<td>0.1270g</td>
<td>0.6351g</td>
</tr>
</tbody>
</table>

(2). Estimate First-mode Deflected Shape of Damped Frame System, $u$, by Analyzing the Frame under a Pattern of Equivalent Lateral Forces $P$.

$$M = \begin{bmatrix} 517790 & 0 & 0 \\ 0 & 478350 & 0 \\ 0 & 0 & 478350 \end{bmatrix}$$

$$K_s = \begin{bmatrix} 9.3407E+7 & -1.2888E+8 & 4.1527E+7 \\ -1.2888E+8 & 3.1342E+8 & -2.373E+8 \\ 4.1527E+7 & -2.373E+8 & 4.3645E+8 \end{bmatrix} \text{N/m}$$

$k_{s1} = 9.3407E+7 \text{ N/m}; k_{s2} = 1.8454E+8 \text{ N/m}; k_{s3} = 2.4068E+8 \text{ N/m};$

$$k_{ei} = \alpha k_{m}; k_{ei} = \gamma k_{s}; k_{ei} = \beta k_{u},$$

$$K_a = \begin{bmatrix} k_{s1} & -k_{s1} & 0 \\ -k_{s1} & k_{s1} + k_{s2} & -k_{s2} \\ 0 & -k_{s2} & k_{s2} + k_{s3} \end{bmatrix} \text{; } K_e = \begin{bmatrix} k_{s1} & -k_{s1} & 0 \\ -k_{s1} & k_{s1} + k_{s2} & -k_{s2} \\ 0 & -k_{s2} & k_{s2} + k_{s3} \end{bmatrix} \beta$$

Estimate the maximum yield displacement of the damper and then from the hysteretic loops we can get:
\[ d_0 = 0.0047633L; \quad d_r = 0.022466L \]

\[ k_e = 6.3839e+010A/L; \quad k_h = 1.1667e+009 A/L; \quad k_d = 1.4455e+010A/L \]

\[ \gamma = 4.4165; \]

\[ h = 3.96m \]

\[ P = \begin{bmatrix} 1 \\ 1 \end{bmatrix} mg \]

\[ K = K_s + K_d = K_s + \frac{\beta}{\gamma} K_o \quad \text{(For Chevron or cross bracing,} \quad a_d = 1) \]

\[ u = K^{-1}P \]

(3). Estimate first-mode period \( T_1 \) using Raleigh’s method.

\[ K_1 = u^T \cdot K \cdot u; \quad M_1 = u^T \cdot M \cdot u \]

\[ \omega_1 = \sqrt{\frac{K_1}{M_1}}; \quad T_1 = \frac{2\pi}{\omega_1} \]

The relationship between \( \beta \) and \( T_1 \) is illustrated in Figure 3.6.

(4). Estimate first-mode damping ratio, \( \beta_{eq} \)

\( \beta \) is the ratio of the initial elastic damper stiffness per story to the structure stiffness of each story without damper.

\[ \beta = \frac{k_e}{k_s}; \quad A = \frac{\beta \cdot k_s \cdot L}{6.3839e+010} \]

A. \( \omega_1 \) and \( T_1 \) are functions of \( \beta \)

\[ c_d = \frac{4(k_e - k_h) \cdot d_s \cdot (d_0 - d_s) T_1}{2\pi^2 d_0^2} \]

So \( c_d \) is a function of \( \beta \)
\[ C = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \]

\( C \) is also a function of \( \beta \).

\[ \beta_{eq} = \frac{u^T \cdot C \cdot u}{2M1 \cdot \omega_1} \]

So \( \beta_{eq} \) is also a function of \( \beta \).

The relationship between \( \beta \) and \( \beta_{eq} \) is illustrated in Figure 3.6.

(5). Perform linear elastic analysis

\[ C_s = \frac{S_{ds}}{R/I} \leq \frac{S_{pl}}{T(R/I)} \], but shall not be taken less than \( C_s = 0.044S_{ds}I \)

\( T \) can be estimated by \( T = C_s h_n^2 \) or using the fundamental mode shape.

As the damping ratio of the damper-added structures is higher than the design spectrum (5% critical damping), thus \( C_s \) obtained from the design spectrum is divided by a damping reduction factor, \( B_s \) and \( B_I \) (FEMA 2001).

Table 3.3 Damping Coefficients \( B_s \) and \( B_I \) as a function of effective Damping \( \beta \)

<table>
<thead>
<tr>
<th>Effective Viscous Damping ( \beta ) (percentage of critical damping)(^1)</th>
<th>( B_s )</th>
<th>( B_I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq 2 )</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>1.3</td>
<td>1.2</td>
</tr>
<tr>
<td>20</td>
<td>1.8</td>
<td>1.5</td>
</tr>
<tr>
<td>30</td>
<td>2.3</td>
<td>1.7</td>
</tr>
<tr>
<td>40</td>
<td>2.7</td>
<td>1.9</td>
</tr>
<tr>
<td>( \geq 50 )</td>
<td>3.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>
1. Damping coefficients shall be based on linear interpolation for effective viscous damping values other than those given.

(6). Compute equivalent lateral forces according to the IBC 2003

\[ V = \frac{C_d}{B_s} W \]

\[ F_x = C_{vx} V \]

Vertical distribution factor

\[ C_{vx} = \frac{\sum_{i=1}^{n} w_i h_x^k}{\sum_{i=1}^{n} w_i h_i^k} \]

\[ \delta_{sx} = K^{-1} F \]

The deflection of level x at the center of the mass, \( \delta_x \), shall be determined in accordance with the following equation

\[ \delta_x = \frac{C_d \delta_{sx}}{I} \]

\( C_d \) = the deflection amplification factor.

The values of \( C_d \) and \( R \) are taken as 1 since the structure is designed to remain elastic in under the OBE. And the drift ratio for 1\(^{st}\), 2\(^{nd}\), 3\(^{rd}\) story is given by Table 3.4. The relationship between \( \beta \) and drift ratio is illustrated in Figure 3.6.

Table 3.4. Selected \( \beta \) value and response obtained from SDP

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( T_i ) (sec)</th>
<th>( \beta_{eq} ) (%)</th>
<th>Story Drift Ratio (%)</th>
<th>Cross Section Area of SMA Wire Strand (mm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1(^{st}) ST</td>
<td>2(^{nd}) ST</td>
</tr>
<tr>
<td>1.39</td>
<td>0.815</td>
<td>22.44</td>
<td>0.61</td>
<td>0.70</td>
</tr>
</tbody>
</table>

ST = story
Figure 3.1 Linearization of non-linear hysteretic behavior

\[ k_x = \frac{A}{L} \tan(\alpha_2) \]

\[ k_x = \frac{A}{L} \tan(\alpha_1) \]

\[ d_0 = \varepsilon_0 L \]

\[ d_y = \varepsilon_y L \]

Figure 3.2 Damper Assembly

(a) Chevron Bracing  (b) Diagonal Bracing  (c) Cross Bracing
Figure 3.3 A 3-story lumped mass model
Figure 3.4 3-story benchmark building illustration (Ohtori et al. 2004)
Figure 3.5 Spectral Response Acceleration with 5% damping
Figure 3.6 Relationship between $\beta$ and $T_1$

Figure 3.7 Relationship between $\beta$ and $\beta_{eq}$-Equivalent damping ratio
Figure 3.8 Relationship between $\beta$ and drift ratio
CHAPTER 4

ARTIFICIAL EARTHQUAKE SIMULATION

AND STATISTICAL ANALYSIS OF STRUCTURES WITH

SHAPE MEMORY ALLOY WIRE DAMPER

4.1 Introduction

In this chapter, numerical simulation studies are carried out to verify the energy dissipation capability of shape memory alloy wire damper (SMAWD) and validate the simplified design procedure (SDP). In order to perform the numerical simulation, a large ensemble of earthquake accelerograms are required. In this study, a method based upon a modified Kanai-Tajimi filtered Gaussian white noise process is used to generate the artificial earthquake accelerograms. These artificial earthquake accelerograms will then be used for earthquake excitation in the simulation studies. Results from dynamic time history analysis indicate that the SDP estimates the seismic response of structures with SMAWD with sufficient accuracy for preliminary design.

4.2 Artificial Earthquake Record Simulation

An ensemble of artificial earthquake ground motion records consisting of 2,000 records have been generated for subsequent Monte Carlo simulation study. These artificial earthquake accelerograms are compatible with the IBC 2003 (ICBO 2003)
design spectrum. The procedure used here to generate the artificial ground motion is modified from that used by Zhang and Iwan (2003).

The power spectral density function of an actual earthquake ground motion is not constant but usually has one or more predominant frequencies associated with the source mechanism and local site effects. A representative form for the power spectral density has been suggested by Clough and Penzien (1993) and may be expressed as

\[ S(\omega) = S_0 |H_1(\omega)|^2 |H_2(\omega)|^2 \]  

(4.1)

Where \( S_0 \) is a constant and

\[ H_1(\omega) = \frac{1 + 2i \xi_1 \left( \frac{\omega}{\omega_1} \right)}{1 - \left( \frac{\omega}{\omega_1} \right)^2 + 2i \xi_1 \left( \frac{\omega}{\omega_1} \right)} \]

\[ H_2(\omega) = \frac{\left( \frac{\omega}{\omega_2} \right)^2}{1 - \left( \frac{\omega}{\omega_2} \right)^2 + 2i \xi_2 \left( \frac{\omega}{\omega_2} \right)} \]  

(4.2)

\( H_1(\omega) \) is the well-known Kanai-Tajimi filter function which amplifies the frequency content in the neighborhood of \( \omega = \omega_1 \) and attenuates the frequency content above \( \omega_1 \) (Kanai 1957, Tajimi 1960). The function of \( H_2(\omega) \) is to attenuate the very low frequency content of the signal. Parameters \( \omega_1 \) and \( \xi_1 \) appearing in \( H_1(\omega) \) may be thought of as the characteristic frequency and damping of ground motion, respectively. Housner and Jennings (1964) suggested that 15.6 rad/sec for \( \omega_1 \) and 0.64 for \( \xi_1 \) as being representative of large to moderate earthquakes and firm soil conditions, and these values are employed in this study. Values for the parameters \( \omega_2 \) and \( \xi_2 \) were determined to be 1.2 rad/sec and 0.84, respectively, by calibrating to the N-S component of the 1940 El Centro record. The general appearance of power spectral density function is illustrated in Figure 4.1.
For the simulation of artificial earthquake accelerograms with specific frequency content, two methods are commonly used. One is based on filtered Gaussian white noise. In another, accelerograms are directly synthesized by the superposition of sinusoidal waveforms. The latter method is used in this study. Note that a Gaussian process can be represented in terms of a harmonic series,

\[ x(t) = 2 \sum_{i=1}^{n} \sqrt{S(\omega_i)} \Delta \omega \sin(\omega_i t + \phi_i) \quad (4.3) \]

where \( \omega_i \) are the selected frequencies at equal spacing \( \Delta \omega \) and \( \phi_i \) are randomly generated phase angles with a uniform distribution over the interval \((0, 2\pi)\). A unilateral power spectrum \( S(\omega) \) is used herein. A typical Gaussian process and its power spectral density function are illustrated in Figure 4.2.

The envelope of recorded earthquake accelerograms are typically composed of three distinct phase in the time domain: the build-up phase, nearly constant phase, and decaying phase. A simple method for obtaining a non-stationary synthetic accelerogram is to multiply the stationary random process synthesized by the method described above with a deterministic envelope function. The envelope function proposed by Iyengar et al. (1969) based on a regression analysis of the 1940 El Centro record is adopted in this study and is shown in Figure 4.3. Although ground motion models which account for the time-varying nature of relative frequency content have also been proposed, time-invariant models in which \( S(\omega) \) reflects the frequency content during the most intense part of the ground motion are believed to be sufficiently accurate for most studies.
To obtain an earthquake accelerogram that is consistent in amplitude with a design response spectrum, the non-stationary accelerogram is normalized by a scaling factor corresponding to the design response spectrum specified in the 2003 IBC code (ICBO 2003). The parameters for the design spectrum are listed in Table 1.

The scaling factor is determined as the ratio between the design response spectrum and the average value of the actual response spectrum calculated from the artificial ground motion over a specified frequency range. This frequency range has been chosen to be the interval \([T_0, T_s]\) in which the spectral pseudo-acceleration is constant in the 2003 IBC code.

The accelerogram obtained from the random process simulation is multiplied by the scaling factor determined above to raise or lower the entire response spectrum. A typical artificial ground motion accelerogram obtained in this way and its response spectrum is illustrated in Figure 4.4.

### 4.3 Numerical Simulation-based Statistical Performance Analysis

A 3-story steel MRF structure equipped with SMAWD is used for the numerical simulation study, as shown in Figure 4.5. In this study, a state space-based numerical time integration algorithm is used for the earthquake response analysis.

#### 4.3.1. State-space-based Time History Analysis

The following assumptions are made in this study:

- Only response in the horizontal direction is considered.
• All system parameters are known in advance and do not change during the excitation duration. The structure remains within its linear elastic range.

• The mass of dampers is negligible compared to that of the structure.

Under these conditions, the equations of motion for the structure subjected to base excitation $\ddot{x}_g$ are expressed as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -ML\ddot{x}_g(t) + \Gamma u(t)$$  \hspace{1cm} (4.4)

where

$x(t)$ is the relative displacement vector of the structure

$M$ is the mass matrix

$C$ is the damping matrix

$K$ is the stiffness matrix

$L = [1 \hspace{0.5cm} 1 \hspace{0.5cm} 1]^T$, $\Gamma = [-1 \hspace{0.5cm} -1 \hspace{0.5cm} -1]^T$.

$u(t)$ is the damper force

Rewriting the above equations of motion in the state-space form yields

$$\dot{X}(t) = AX(t) + B_u u(t) + B_w w(t)$$  \hspace{1cm} (4.5)

where

$$X(t) = \{x(t) \hspace{0.5cm} \dot{x}(t)\}^T$$ is the state vector

$$A = \begin{bmatrix} \phi & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \hspace{0.5cm} B_u = \begin{bmatrix} \phi_v \\ M^{-1}\Gamma \end{bmatrix}, \hspace{0.5cm} B_w = \begin{bmatrix} \phi_v \\ -L \end{bmatrix}$$

$w(t) = \ddot{x}_g(t)$ represents the external excitation, and $\phi$, $\phi_v$ and $I$ denote the null vector, null matrix and identity matrix, respectively.
In structures, the displacements and velocities of the system masses are commonly used as state variables. The above linear state equation forms the basis for the formulation and solution of system dynamics in this study. By using the state equations, the well-known second-order differential equations characterizing the motion of a structural system can be rewritten as a set of first-order differential equations. Current techniques for the high-speed solution of first-order differential equations make the state-space approach particularly attractive.

To compute the dynamics of a structure with SMAWDs subjected to external disturbance, a numerical scheme in discrete time has to be employed because of the nonlinearity and history dependence associated with the damper-added structure. The state variable, \( X(t_{k+1}) \) at time \( t_{k+1} \) for the given initial condition \( X(t_k) \) at time \( t_k \), is given by the following convolution integral:

\[
X(t_{k+1}) = e^{A\Delta t}X(t_k) + \int_{t_k}^{t_{k+1}} e^{A(t_{k+1}-\tau)}[B_u u(\tau) + B_w w(\tau)]d\tau
\]  

(4.6)

where \( \Delta t = t_{k+1} - t_k \)

A partitioned predictor-corrector (PPC) method (Inaudi JA 1992) is employed for computing the integral in Eqn. (4.6). A brief description of the PPC method is given in Appendix B. Using this numerical scheme, \( X(t_{k+1}) \) can be expressed as

\[
X(t_{k+1}) = \Phi X(t_k) + B_{u1}u(t_k) + B_{u2}u(t_{k+1}) + B_{w1}w(t_k) + B_{w2}w(t_{k+1})
\]  

(4.7)

where

\[
\Phi = e^{A\Delta t}
\]

\[
B_{u1} = A^{-1}\left[\Phi + \frac{A^{-1}(I - \Phi)}{\Delta t}\right]B_u
\]
\begin{align*}
B_{u2} &= A^{-1} \left[ \frac{A^{-1}(\Phi - I)}{\Delta t} - I \right] B_u \\
B_{w1} &= A^{-1} \left[ \Phi - \frac{A^{-1}(I - \Phi)}{\Delta t} \right] B_w \\
B_{w2} &= A^{-1} \left[ \frac{A^{-1}(\Phi - I)}{\Delta t} - I \right] B_w \tag{4.8}
\end{align*}

and \( \tilde{u}(t_{s+1}) \) is the estimated value of the damper force vector as described in appendix B. Iterations were used in this study to improve the accuracy of state space-based numerical integration in the presence of hysteretic behavior of SMA wire dampers.

4.3.2. Numerical Example

To validate the SDP, a 3-story steel framed building described in Example 2 of Chapter 3 is employed for the numerical simulation study. This 3-story steel MRF building meets the seismic code and represents a typical low-rise building designed for Los Angeles, California region.

The numerical study will concentrate on an in-plane analysis of the structure. For computational considerations, static condensation is employed to eliminate all rotational and vertical DOFs from the dynamics analysis. It is assumed that the inertias associated with the rotational and vertical DOFs are negligible. Also, a rigid floor assumption is made and therefore only the horizontal DOF corresponding to each floor of the three story structure is retained. The accuracy of this reduced-order model has been confirmed by comparing the results for El Centro ground motion to those given by Spencer Jr et al. (2005). The first three natural periods of the
condensed model are 1.01, 0.33, and 0.17 s, respectively. These values agree quite
well with the results of a full-order model, which are given by Spencer Jr et al. (2005)
Two percent modal damping is assigned to each mode of the linear model. It is
assumed that the SMAWDs are installed in the planar frame described above.

Results and Discussion

Simulation results for the uncontrolled and controlled building are presented in
Figures 4.6 to 4.20. It is noted that in some figures individual story of the building are
identified when inter-story drift is considered. In other figures, when displacement,
velocity, acceleration and damper force are considered, individual floor is used to
identify these response values at each floor level.

Figures 4.6, 4.7 and 4.8 show the displacement, story drift ratio and acceleration
responses of the structures, respectively. These figures also compared the
displacement, story drift ratio and acceleration of the structure with SMAWDs with
those of the structure without damper. These figures clearly show the effectiveness of
SMAWD in suppressing the dynamic responses of structures such as displacement
and story drift. The addition of SMAWD also resulted in an increase in the
fundamental frequency of the structure.

Figure 4.9 shows the time history response of SMA wire damper forces and Figure
4.10 shows the hysteresis loops of SMA wire dampers during a typical artificially
generated earthquake. The results shown in these two figures indicate that vibrational
energy induced by the earthquake can be effectively dissipated with SMAWDs.
Histograms describing the probability distributions of the structure’s displacement, inter-story drift ratio and absolute accelerations at various floor levels are shown in Figures 4.11 to 4.16. The normalized frequency of occurrence \( P_i \) is defined as:

\[
P_i = \frac{N_i}{N \cdot \Delta_i}
\]

(4.9)

where \( N \) is the ensemble size; \( N_i \) is the ensemble of occurrence for the quantity falling within the interval \([x_i, x_i + \Delta_i]\); therefore, \( \sum N_i = N \), while \( \Delta_i \) is the corresponding interval length. Thus

\[
\sum_{i=1}^{N} P_i \cdot \Delta_i = 1
\]

(4.10)

In a discrete sense, \( P_i \) may be taken as the probability density function (PDF) for the represented quantity.

The root mean square (RMS) of a quantity is computed as

\[
\| \| = \sqrt{\frac{1}{t_f} \int_0^{t_f} \cdot^2 \, dt}
\]

(4.11)

where \( t_f \) is the duration of the excitation.

Figures 4.11 and 4.12 show the normalized occurrence frequency of the peak and RMS story displacement of the building structure, respectively. It is observed that SMAWD is capable of significantly reducing the displacement of the structure compared to the uncontrolled case. The device performance is quite robust in a statistical sense as the PDFs of both peak and RMS floor displacement of the controlled structure are concentrated in a much narrower range than the uncontrolled structures.
Figures 4.13 and 4.14 show the normalized occurrence frequency of the peak and RMS story drift ratio of the building structure, respectively. It is observed that SMAWD is capable of significantly reducing the interstory drifts of the structure compared to the uncontrolled case. For the structure with SMAWDs, the mean values of maximum story drift fall between 0.33% and 0.52%, which are much smaller than the drift ratio values ranging from 1.07% to 1.46% for the structure without damper. The device performance is quite robust in a statistical sense as the PDFs of both peak and RMS interstory drift ratios of the controlled structure are concentrated in a much narrower range than the corresponding uncontrolled structures.

In Figures 4.15 and 4.16, the normalized occurrence frequencies of the peak and RMS absolute accelerations at various floors of the structure are presented. It is seen that the controlled structure (i.e., with SMAWD) generally has smaller acceleration response than the uncontrolled case as evidenced by the normalized occurrence frequency of the absolute accelerations of the structure.

Figures 4.17, 4.18 and 4.19 show the envelopes of the displacement, interstory drift and absolute acceleration response of the building, respectively. The envelopes in those figures are calculated by averaging the absolute values of the ensemble of response time histories. The effectiveness of SMAWD in suppressing the displacement, interstory drift and absolute acceleration is evident in the results shown in Figure 4.17 to 4.19.

Based on the results presented, it is concluded that a structure equipped with SMAWD would have a much higher likelihood of surviving a strong earthquake than a corresponding uncontrolled structures.
Figure 4.20 shows the statistical results compared with the SDP results. Close agreement between the SDP and simulation-based statistical analysis results of the 3rd story is observed while the SDP gives slightly conservative results compared with the statistical analysis results in the first and second stories of the building. With the selected $\beta$ value (i.e., $\beta = 1.39$), the mean values of maximum story drifts would satisfy the 0.7% drift limit set by the design criteria. Therefore, the SDP provides a potentially effective method for preliminary design of structures with SMA WD.
Figure 4.1 General appearance of power spectral density function $S(\omega)$

Figure 4.2 A typical Gaussian Process and Its Power Spectral Density Function $S(\omega)$
Figure 4.3 Intensity Envelope Function $e(t)$ for Non-stationary Process $a(t)$

Figure 4.4 A Typical artificial ground motion accelerogram
Figure 4.5. (a) Three-story benchmark building equipped with SMAWD; (b) close-up view of SMAWD
Figure 4.6 Typical displacement responses of structures subjected to artificial earthquake records (controlled = structure with SMAWD; Uncontrolled = structure without SMAWD)
Figure 4.7 Typical acceleration responses of structures subjected to artificial earthquake records (controlled = structure with SMA WD; Uncontrolled = structure without SMA WD)
Figure 4.8 Typical drift ratio responses of structures subjected to artificial earthquake records (controlled = structure with SMAWD; Uncontrolled = structure without SMAWD)
Figure 4.9 Typical damper force responses of structures subjected to artificial earthquake records (damper force corresponds to two damper placed in one story of one frame)
Figure 4.10 Typical hysteresis loops of SMAWD in structures subjected to artificial earthquake records
Fig 4.11 Normalized occurrence frequency of peak displacement (controlled = structure with SMAWD; Uncontrolled = structure without SMAWD)
Figure 4.12. Normalized occurrence frequency of RMS displacement (controlled = structure with SMAWD; Uncontrolled = structure without SMAWD)
Figure 4.13. Normalized occurrence frequency of peak drift ratio (controlled = structure with SMAWD; Uncontrolled = structure without SMAWD)
Figure 4.14. Normalized occurrence frequency of RMS drift ratio (controlled = structure with SMAWD; Uncontrolled = structure without SMAWD)
Figure 4.15. Normalized occurrence frequency of maximum absolute acceleration (controlled = structure with SMAWD; Uncontrolled = structure without SMAWD)
Figure 4.16. Normalized occurrence frequency of RMS absolute acceleration (controlled = structure with SMAWD; Uncontrolled = structure without SMAWD)
Figure 4.17. Envelope of the story displacement of the building

(dash line = structure without SMAWD; solid line = structure with SMAWD)
Figure 4.18. Envelope of the drift ratio of the building

(dash line = structure without SMAWD; solid line=structure with SMAWD)
Figure 4.19. Envelope of the acceleration of the building

(dash line = structure without SMAWD; solid line = structure with SMAWD)
Figure 4.20. Comparison of drift ratio between DTHA and SDP with a series of selected $\beta$ value
Table 4.1 Parameters of the IBC design spectrum (IBC 2003)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$S_{DS}$</th>
<th>$S_{DL}$</th>
<th>$T_0$</th>
<th>$T_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>0.6976g</td>
<td>0.4430g</td>
<td>0.1270g</td>
<td>0.6351g</td>
</tr>
</tbody>
</table>

Table 4.2 Structural Properties and response obtained from SDP

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$T_1$</th>
<th>$\beta_{eq}$ (%)</th>
<th>Story Drift Ratio (%)</th>
<th>Cross Section Area of SMA Wire Strands</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1st ST.</td>
</tr>
<tr>
<td>1.18</td>
<td>0.875</td>
<td>20.10</td>
<td>0.64</td>
<td>6673</td>
</tr>
<tr>
<td>1.28</td>
<td>0.867</td>
<td>21.29</td>
<td>0.61</td>
<td>7239</td>
</tr>
<tr>
<td>1.38</td>
<td>0.815</td>
<td>22.44</td>
<td>0.59</td>
<td>7804</td>
</tr>
<tr>
<td>1.48</td>
<td>0.851</td>
<td>23.56</td>
<td>0.59</td>
<td>8370</td>
</tr>
<tr>
<td>1.58</td>
<td>0.843</td>
<td>24.64</td>
<td>0.58</td>
<td>8935</td>
</tr>
</tbody>
</table>

ST. = story
CHAPTER 5

CONCLUSIONS

5.1 Conclusions

A simplified design procedure (SDP) for passively controlled structures with hysteretic dampers such as the proposed shape memory alloy wire damper (SMAWD) is examined in this research. Monte Carlo simulation study of the seismic response of a three story steel-framed building equipped with the proposed passive damper is performed based on a large ensemble of artificially generated earthquake ground motions. Statistical analysis results of both uncontrolled and controlled structures (i.e., with SMA wire damper) have been obtained from the Monte Carlo simulation study. A procedure for generating code-compatible artificial earthquake accelerograms is also described in this report.

Based on the research findings of this simulation study, the following conclusions can be drawn,

- The effectiveness of the SMA wire damper in controlling the interstory drifts and acceleration responses of building structures during large earthquakes is clearly demonstrated. Structures equipped with the SMA wire damper are observed to have a much lower likelihood of experiencing excessive inter-story drifts in a strong earthquake than the corresponding uncontrolled structures.

- The SDP can be used to determine the damper properties in accordance with the design criteria specified by the user of the SDP. The SDP is sufficiently general to enable different structural limit states to be considered in the design...
criteria. The linear elastic analysis, associated with the SDP, uses the equivalent lateral force method, which is easy to apply. Slightly conservative results are obtained by the SDP compared with the statistical analysis results for this 3-story steel framed building structures. The SDP provides a potentially effective method for preliminary design of structures with SMAWDs.
References

17. Kanai K., Semi-empirical formula for the seismic characteristics of the ground, Bulletin of the Earthquake Research Institute, University of Tokyo, 1957; 35:309-325.


9.4.1.2.4 Site Coefficients and Adjusted Maximum Considered Earthquake Spectral Response Acceleration Parameters. The maximum considered earthquake spectral response acceleration for short periods \( S_{MS} \) and at 1-sec \( S_{SI} \), adjusted for site class effects, shall be determined by Eqs. 9.4.1.2.4-1 and 9.4.1.2.4-2, respectively.

\[
S_{MS} = F_u S_s
\]
(9.4.1.2.4-1)

\[
S_{SI} = F_v S_I
\]
(9.4.1.2.4-2)

Where

\( S_I \): = the mapped maximum considered earthquake spectral response acceleration at a period of 1-sec as determined in accordance with Section 9.4.1

\( S_s \) = the mapped maximum considered earthquake spectral response acceleration at short periods as determined in accordance with Section 9.4.1

where site coefficients \( F_u \) and \( F_v \) are defined in Tables 9.4.1.2.4a and b, respectively.

9.4.1.2.5 Design Spectral Response Acceleration Parameters. Design earthquake spectral response acceleration at short periods, \( S_{DS} \), and at 1-sec period, \( S_{DI} \), shall be determined from Eqs. 9.4.1.2.5-1 and 9.4.1.2.5-2, respectively.

\[
S_{DS} = \frac{2}{3} S_{MS}
\]
(9.4.1.2.5-1)

\[
S_{DI} = \frac{2}{3} S_{SI}
\]
(9.4.1.2.5-2)

9.4.1.2.6 General Procedure Response Spectrum. Where a design response
spectrum is required by these provisions and site-specific procedures are not used, the
design response spectrum curve shall be developed as indicated in Figure 9.4.1.2.6 and as follows:

1. For periods less than or equal to $T_0$, the design spectral response acceleration, $S_a$, shall be taken as given by Eq. 9.4.1.2.6-1:

$$S_a = S_{DS} \left( 0.4 + 0.6 \frac{T}{T_0} \right)$$  \hspace{1cm} (9.4.1.2.6-1)

2. For periods greater than or equal to $T_0$ and less than or equal to $T_s$, the design spectral response acceleration, $S_a$, shall be taken as equal to $S_{DS}$.

3. For periods greater than $T_s$, the design spectral response acceleration, $S_a$, shall be taken as given by Eq. 9.4.1.2.6-2:

$$S_a = \frac{S_{DL}}{T}$$  \hspace{1cm} (9.4.1.2.6-2)

where

$T = \text{the fundamental period of the structure (sec)}$

$T_0 = 0.2 S_{DL}/S_{DS}$ and

$T_s = S_{DL}/S_{DS}$

**9.5.5 Equivalent Lateral Force Procedure.**

**9.5.5.1 General.** Section 9.5.5 provides required minimum standards for the equivalent lateral force procedure of seismic analysis of structures. An equivalent lateral force analysis shall consist of the application of equivalent static lateral forces to a linear mathematical model of the structure. The directions of application of
lateral forces shall be as indicated in Section 9.5.2.5.2. The lateral forces applied in each direction shall be the total seismic base shear given by Section 9.5.5.2 and shall be distributed vertically in accordance with the provisions of Section 9.5.5.3. For purposes of analysis, the structure is considered to be fixed at the base. See Section 9.5.2.5 for limitations on the use of this procedure.

9.5.5.2 Seismic Base Shear. The seismic base shear \( V \) in a given direction shall be determined in accordance with the following equation:

\[
V = C_s W
\]  

(9.5.5.2-1)

where

- \( C_s \) = the seismic response coefficient determined in accordance with Section 9.5.5.2.1
- \( W \) = the total dead load and applicable portions of other loads as indicated in Section 9.5.3.

9.5.5.2.1 Calculation of Seismic Response Coefficient. When the fundamental period of the structure is computed, the seismic design coefficient (\( C_s \)) shall be determined in accordance with the following equation:

\[
C_s = \frac{S_{ds}}{R/I}
\]  

(9.5.5.2.1-1)

where

- \( S_{ds} \) = the design spectral response acceleration in the short period range as determined from Section 9.4.1.2.5
- \( R \) = the response modification factor in Table 9.5.2.2
\[ I = \text{the occupancy importance factor determined in accordance with section 9.1.4} \]

A soil-structure interaction reduction shall not be used unless Section 9.5.9 or another generally accepted procedure approved by the authority having jurisdiction is used.

Alternatively, the seismic response coefficient, \( (C_s) \), need not be greater than the following equation:

\[
C_s = \frac{S_{DI}}{T(R/I)} \quad (9.5.5.2.1-2)
\]

but shall not be taken less than

\[
C_s = 0.044S_{DS}I \quad (9.5.5.2.1-3)
\]

nor for buildings and structures in Seismic Design Categories E and F

\[
C_s = \frac{S_{DS}}{R/I} \quad (9.5.5.2.1-4)
\]

where \( I \) and \( R \) are as defined above and

\( S_{DI} = \text{the design spectral response acceleration at a period of 1.0 sec, in units of g-sec,} \)

as determined from Section 9.4.1.2.5

\( T = \text{the fundamental period of the structure (sec) determined in Section 9.5.5.3} \)

\( S_I = \text{the mapped maximum considered earthquake spectral response acceleration} \)

determined in accordance with Section 9.4.1

A soil-structure interaction reduction is permitted when determined using Section 9.5.9.

For regular structures 5 stories or less in height and having a period, \( T \), of 0.5 sec
or less, the seismic response coefficient, $C_s$ shall be permitted to be calculated using values of 1.5 g and 0.6 g, respectively, for the mapped maximum considered earthquake spectral response accelerations $S_s$ and $S_f$.

9.5.5.3 Period Determination. The fundamental period of the structure $(T)$ in the direction under consideration shall be established using the structural properties and deformational characteristics of the resisting elements in a properly substantiated analysis. The fundamental period $(T)$ shall not exceed the product of the coefficient for upper limit on calculated period $(C_u)$ from Table 9.5.5.3 and the approximate fundamental period $(T_a)$ determined from Eq. 9.5.5.3-1. As an alternative to performing an analysis to determine the fundamental period $(T)$, it shall be permitted to use the approximate building period, $(T_a)$, calculated in accordance with Section 9.5.5.3.2, directly.

9.5.5.3.1 Upper Limit on Calculated Period. The fundamental building period $(T)$ determined in a properly substantiated analysis shall not exceed the product of the coefficient for upper limit on calculated period $(C_u)$ from Table 9.5.5.3.1 and the approximate fundamental period $(T_a)$ determined in accordance with Section 9.5.5.3.2. shall be determined from the following equation:

$$T_a = C_i h_n^x$$  \hspace{1cm} (9.5.5.3.2-1)

where $h_n$ is the height in ft above the base to the highest level of the structure and the coefficients $C_i$ and $x$ are determined from Table 9.5.5.3.2.

Alternatively, it shall be permitted to determine the approximate fundamental
period \( (T_a) \), in seconds, from the following equation for structures not exceeding 12 stories in height in which the lateral-force-resisting system consists entirely of concrete or steel moment resisting frames and the story height is at least 10 ft (3 m):

\[
T_a = 0.1N
\]

Where \( N \) = number of stories

The approximate fundamental period, \( T_Q \), in seconds for masonry or concrete shear wall structures shall be permitted to be determined from Eq. 9.5.5.3.2-2 as follows:

\[
T_Q = 0.0019 \frac{h_n}{\sqrt{C_w}}
\]

(9.5.5.3.2-2)

where \( h_n \) is as defined above and \( C_w \), is calculated from Eq. 9.5.5.3.2-3 as follows:

\[
C_w = \frac{100}{A_B} \sum_{i=1}^{n} \left( \frac{h_n}{h_i} \right)^2 \frac{A_i}{1 + 0.83 \left( \frac{h_i}{D_i} \right)^2}
\]

(9.5.5.3.2-2)

where

\[ A_B = \text{the base area of the structure ft}^2 \]
\[ A_i = \text{the area of shear wall "i" in ft}^2 \]
\[ D_i = \text{the length of shear wall "i" in ft} \]
\[ n = \text{the number of shear walls in the building effective in resisting lateral forces in the direction under consideration} \]

9.5.5.4 Vertical Distribution of Seismic Forces. The lateral seismic force \( (F_x) \) (kip or
kN) induced at any level shall be determined from the following equations:

\[ F_x = C_{vx} V \]  

(9.5.5.4-1)

and

\[ C_{vx} = \frac{\sum w_i h_i^k}{\sum w_i h_i^k} \]  

(9.5.5.4-2)

where

- \( C_{vx} \) = vertical distribution factor
- \( V \) = total design lateral force or shear at the base of the structure, (kip or kN)
- \( w_i \) and \( w_x \) = the portion of the total gravity load of the structure (W) located or assigned to Level i or x
- \( h_i \) and \( h_x \) = the height (ft or m) from the base to Level i or x
- \( k \) = an exponent related to the structure period as follows: for structures having a period of 0.5 sec or less, \( k = 1 \) for structures having a period of 2.5 sec or more, \( k \) = 2 for structures having a period between 0.5 and 2.5 seconds, \( k \) shall be 2 or shall be determined by linear interpolation between 1 and 2

9.5.5.5 Horizontal Shear Distribution and Torsion. The seismic design story shear in any story \( (V_x) \) (kip or kN) shall be determined from the following equation:

\[ V_x = \sum_{i=x}^n F_i \]  

(9.5.5.5)

where \( F_i \) = the portion of the seismic base shear \( (V) \) (kip or kN) induced at Level i

9.5.5.5.1 Direct Shear. The seismic design story shear \( (V_x) \) (kip or kN) shall be
distributed to the various vertical elements of the seismic force-resisting system in the story under consideration based on the relative lateral stiffness of the vertical resisting elements and the diaphragm.

9.5.5.5.2 Torsion. Where diaphragms are not flexible, the design shall include the torsional moment \( M_t \) (kip or kN) resulting from the location of the structure masses plus the accidental torsional moments \( M_{ta} \) (kip or kN) caused by assumed displacement of the mass each way from its actual location by a distance equal to 5% of the dimension of the structure perpendicular to the direction of the applied forces. Where earthquake forces are applied concurrently in two orthogonal directions, the required 5% displacement of the center of mass need not be applied in both of the orthogonal directions at the same time, but shall be applied in the direction that produces the greater effect.

Structures of Seismic Design Categories C, D, E, and F, where Type 1 torsional irregularity exist as defined in Table 9.5.2.3.2, shall have the effect accounted for by multiplying \( M_{ta} \) at each level by a torsional amplification factor \( A_x \) determined from the following equation:

\[
A_x = \left( \frac{\delta_{\text{max}}}{1.2\delta_{\text{avg}}} \right)^2
\]  

(9.5.5.5.2)

where

\( \delta_{\text{max}} \) = the maximum displacement at Level \( x \) (in. or mm)

\( \delta_{\text{avg}} \) = the average of the displacements at the extreme points of the structure at Level \( x \) (in. or mm)
Exception: The torsional and accidental torsional moment need not be amplified for structures of light-frame construction.

The torsional amplification factor \( A_x \) is not required to exceed 3.0. The more severe loading for each element shall be considered for design.

**9.5.5.6 Overturning.** The structure shall be designed to resist overturning effects caused by the seismic forces determined in Section 9.5.4.4. At any story, the increment of overturning moment in the story under consideration shall be distributed to the various vertical elements of the lateral force-resisting system in the same proportion as the distribution of the horizontal shears to those elements.

The overturning moments at Level \( x \) \( (M_x) \) (kip-ft or kN-m) shall be determined from the following equation:

\[
M_x = \sum_{i=2}^{n} F_i (h_i - h_x)
\]

where

\( F_i \) = the portion of the seismic base shear \( (V) \) induced at Level \( i \)

\( h_i \) and \( h_x \) = the height (in ft or m) from the base to Level \( i \) or \( x \)

The foundations of structures, except inverted pendulum-type structures, shall be permitted to be designed for 75\% of the foundation overturning design moment \( (M_f) \) (kip-ft or kN-m) at the foundation-soil are determined using the equation for the overturning moment at Level \( x \) \( (M_x) \) (kip-ft or kN-m).

**9.5.7 Drift Determination and P-Delta Effects.** Story drifts and, where required,
member forces and moments due to P-delta effects shall be determined in accordance with this Section. Determination of story drifts shall be based on the application of the design seismic forces to a mathematical model of the physical structure. The model shall include the stiffness and strength of all elements that are significant to the distribution of forces and deformations in the structure and shall represent the spatial distribution of the mass and stiffness of the structure. In addition, the model shall comply with the following:

1. Stiffness properties of reinforced concrete and masonry elements shall consider the effects of cracked sections, and

2. For steel moment resisting frame systems, the contribution of panel zone deformations to overall story drift shall be included.

9.5.7.1 **Story Drift Determination.** The design drift (Δ) shall be computed as the difference of the deflections at the top and bottom of the story under consideration. Where allowable stress design is used Δ shall be computed using code-specified earthquake forces without reduction.

**Exception:** For structures of Seismic Design Categories C, D, E, and F having plan irregularity Types la or lb of Table 9.5.2.3.2, the design story drift, D, shall be computed as the largest difference of the deflections along any of the edges of the structure at the top and bottom of the story under consideration.

The deflections of Level x at the center of the mass (δx) (in. or mm) shall be determined in accordance with the following equation:
\[
\delta_x = \frac{C_d \delta_{ex}}{I}
\]  
(9.5.5.7.1)

where

\( C_d \) = the deflection amplification factor in Table 9.5.2.2

\( \delta_{ex} \) = the deflections determined by an elastic analysis

\( I \) = the importance factor determined in accordance with Section 9.1.4

The elastic analysis of the seismic force-resisting system shall be made using the prescribed seismic design forces of Section 9.5.3.4. For the purpose of this Section, the value of the base shear, \( V \), used in Eq. 9.5.3.2 need not be limited by the value obtained from Eq. 9.5.5.2.1-3. For determining compliance with the story drift limitation of Section 9.5.2.8, the deflections at the center of mass of Level \( x \) (\( \delta_x \)) (in. or mm) shall be calculated as required in this Section. For the purposes of this drift analysis only, the upper-bound limitation specified in Section 9.5.3.3 on the computed fundamental period, \( T \), in seconds, of the building does not apply for computing forces and displacements. Where applicable, the design story drift (\( \Delta \)) (in. or mm) shall be increased by the incremental factor relating to the P-delta effects as determined in Section 9.5.5.7.2. When calculating drift, the redundancy coefficient, \( \rho \), is not used.

9.5.5.7.2 P-Delta Effects. P-delta effects on story shears and moments, the resulting member forces and moments, and the story drifts induced by these effects are not required to be considered when the stability coefficient (\( \Theta \)) as determined by the following equation is equal to or less than 0.10:

- 99 -
\[ \theta = \frac{P_x \Delta}{V_x h_{sx} C_d} \]  

(9.5.5.7.2-1)

where

\( P_x = \) the total vertical design load at and above Level \( x \). (kip or kN); when computing

\( \Delta = \) the design story drift as defined in Section 9.5.3.7.1 occurring simultaneously with \( V_x \), (in. or mm)

\( V_x = \) the seismic shear force acting between Levels \( x \) and \( x-1 \), (kips or kN)

\( h_{sx} = \) the story height below Level \( x \), (in. or mm)

\( C_d = \) the deflection amplification factor in Table 9.5.2.2

The stability coefficient \( (\theta) \) shall not exceed \( \theta_{\text{max}} \) determined as follows:

\[ \theta_{\text{max}} = \frac{0.5}{\beta C_d} \leq 0.25 \]  

(9.5.5.7.2-2)

where \( \beta \) is the ratio of shear demand to shear capacity for the story between Level \( x \) and \( x-1 \). This ratio may be conservatively taken as 1.0. When the stability coefficient \( (\theta) \) is greater than 0.10 but less than or equal to \( \theta_{\text{max}} \), the incremental factor related to P-delta effects \( (a_d) \) shall be determined by rational analysis. To obtain the story drift for including the P-delta effect, the design story drift determined in Section 9.5.5.7.1 shall be multiplied by \( 1.0/(1 - \theta) \).

When \( \theta \) is greater than \( \theta_{\text{max}} \), the structure is potentially unstable and shall be redesigned. When the P-delta effect is included in an automated analysis, Eq. 9.5.5.7.2-2 must still be satisfied, however, the value of \( \theta \) computed from Eq.
9.5.5.7.2-1 using the results of the P-delta analysis may be divided by (1 + B) before checking Eq. 9.5.5.7.2-2.
<table>
<thead>
<tr>
<th>Site Class</th>
<th>$S_s \leq 0.25$</th>
<th>$S_s = 0.5$</th>
<th>$S_s = 0.75$</th>
<th>$S_s = 1.0$</th>
<th>$S_s \geq 1.25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>B</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>C</td>
<td>1.2</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>D</td>
<td>1.6</td>
<td>1.4</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>E</td>
<td>2.5</td>
<td>1.7</td>
<td>1.2</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>F</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
</tr>
</tbody>
</table>

Note: Use straight-line interpolation for intermediate values of $S_s$.

*a* Site-specific geotechnical investigation and dynamic site response analyses shall be performed except that for structures with periods of vibration equal to or less than 0.5-seconds, values of $F_a$ for liquefiable soils may be assumed equal to the values for the site class determined without regard to liquefaction in Step 3 of Section 9.4.1.2.2.
### Table 9.4.1.2.4b

Values of $F_v$ as a Function of Site Class and Mapped 1-second Period Maximum Considered Earthquake Spectral Acceleration

<table>
<thead>
<tr>
<th>Site Class</th>
<th>$S_f \geq 0.1$</th>
<th>$S_f = 0.2$</th>
<th>$S_f = 0.3$</th>
<th>$S_f = 0.4$</th>
<th>$S_f \geq 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>B</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>C</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
</tr>
<tr>
<td>D</td>
<td>2.4</td>
<td>2.0</td>
<td>1.8</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td>E</td>
<td>3.5</td>
<td>3.2</td>
<td>2.8</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td>F</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
</tr>
</tbody>
</table>

Note: Use straight-line interpolation for intermediate values of $S_f$.

A Site-specific geotechnical investigation and dynamic site response analyses shall be performed except that for structures with periods of vibration equal to or less than 0.5 seconds, values of $F_v$ for liquefiable soils may be assumed equal to the values for the site class determined without regard to liquefaction in Step 3 of Section 9.4.1.2.2.

### Table 9.5.5.3.1

Coefficient for Upper Limit on Calculated Period

<table>
<thead>
<tr>
<th>Design Spectral Response Acceleration at 1 Second, $S_{DI}$</th>
<th>Coefficient $C_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 0.4$</td>
<td>1.4</td>
</tr>
<tr>
<td>0.3</td>
<td>1.4</td>
</tr>
<tr>
<td>0.2</td>
<td>1.5</td>
</tr>
<tr>
<td>0.15</td>
<td>1.6</td>
</tr>
<tr>
<td>0.1</td>
<td>1.7</td>
</tr>
<tr>
<td>$\leq 0.05$</td>
<td>1.7</td>
</tr>
</tbody>
</table>
Table 9.5.5.3.2
Values of Approximate Period Parameters $C_{t}$ and $x$

<table>
<thead>
<tr>
<th>Structure Type</th>
<th>$C_{t}$</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment resisting frame systems of steel in which the frames resist 100% of the required seismic force and are not enclosed or adjoined by more rigid components that will prevent the frames from deflecting when subjected to seismic forces</td>
<td>0.028(0.068)$^a$</td>
<td>0.8</td>
</tr>
<tr>
<td>Moment resisting frame systems of reinforced concrete in which the frames resist 100% of the required seismic force and are not enclosed or adjoined by more rigid components that will prevent the frame from deflecting when subjected to seismic forces</td>
<td>0.016(0.044)$^a$</td>
<td>0.9</td>
</tr>
<tr>
<td>Eccentrically braced steel frames</td>
<td>0.03(0.07)$^a$</td>
<td>0.75</td>
</tr>
<tr>
<td>All other structural systems</td>
<td>0.02(0.055)</td>
<td>0.75</td>
</tr>
</tbody>
</table>

$^a$ - metric equivalents are shown in parentheses
Figure 9.4.1.2.6 Design Response Spectrum
APPENDIX B

Partitioned Predictor-Corrector Method

This appendix provides a derivation of the results given in Chapter 4. Reference [14] has been used extensively for this appendix. Consider the equation of motion for the controlled structure,

\[ \dot{X}(t) = AX(t) + B_xu(t) + B_ww(t) \]  

(B.1)

and the control force vector can be determined by the current state variables as follows:

\[ u(t) = G(X(t)) = G(x(t), \dot{x}(t)) \]

(B.2)

Because of the nonlinearity and history dependence of the control forces, a numerical scheme for integrating in discrete time has to be employed. Assume that the state of the system \( X(t_k) \) at step \( t_k \) and the external disturbance is known (as we can measure these by sensors installed within the structure). The state variable \( X(t_{k+j}) \) at time \( t_{k+j} \) is given by the following convolution integral:

\[ X(t_{k+j}) = e^{A\Delta t}X(t_k) + \int_{t_k}^{t_{k+j}} e^{A(t_j\tau - t_k)}[B_xu(\tau) + B_ww(\tau)]d\tau \]

(B.3)

where \( \Delta t = t_{k+j} - t_k \). Making the change of variable \( \xi = \tau - t_k \), we can write

\[ X(t_{k+j}) = e^{A\Delta t}X(t_k) + e^{A\Delta t}\int_{-T}^{T} e^{-A\xi} [B_xu(t_k + \xi) + B_ww(t_k + \xi)]d\xi \]

(B.4)
During the interval $0 < \xi < \Delta t$, the value of $u(t_k + \xi)$ is not known because Eqn. B.2 needs to be solved to find its value. Let's assume that the control force has a linear variation within the interval $0 < \xi < \Delta t$, i.e.,

$$u(t_k + \xi) = u(t_k) + \frac{\hat{u}(t_{k+1}) - u(t_k)}{\Delta t} \xi$$

(B.5)

where $\hat{u}(t_{k+1})$ represents the estimate of the control force at time $t_{k+1}$ using the following procedures.

First find the estimate of the state variable $\hat{X}(t_{k+1})$ at time $t_{k+1}$ by assuming zero-order holder for both $u(t_k + \xi)$ and $w(t_k + \xi)$ during the interval $0 < \xi < \Delta t$.

$$\hat{X}(t_{k+1}) = e^{A\xi} X(t_k) + \int_{t_k}^{t_{k+1}} e^{A\sigma} d\sigma [B_u u(t_k) + B_w w(t_k)]$$

$$= e^{A\xi} X(t_k) + A^{-1}(e^{A\xi} - I) B_u u(t_k) + B_w w(t_k)$$

(B.6)

Then the estimate of the control force $\hat{u}(t_{k+1})$ at time $t_{k+1}$ can be determined by

$$\hat{u}(t_{k+1}) = G(\hat{X}(t_{k+1}))$$

(B.7)

The external disturbance term is known at discrete time $t_k$, and a linear variation is also assumed during the interval $0 < \xi < \Delta t$.

$$w(t_k + \xi) = w(t_k) + \frac{w(t_{k+1}) - w(t_k)}{\Delta t} \xi$$

(B.8)

Substituting Eqns. (B.5) and (B.8) into Eqn. (B.4) yields
\(X(t_{k+1}) = \Phi X(t_k) + B_{u1}u(t_k) + B_{u2}\ddot{u}(t_{k+1}) + B_{w1}w(t_k) + B_{w2}w(t_{k+1})\)  \(\text{(B.9)}\)

where

\[
\Phi = e^{A\Delta t}
\]

\[
B_{u1} = A^{-1} \left[ \Phi + \frac{A^{-1}(I - \Phi)}{\Delta t} \right] B_u
\]

\[
B_{u2} = A^{-1} \left[ \frac{A^{-1}(\Phi - I)}{\Delta t} - I \right] B_u
\]

\[
B_{w1} = A^{-1} \left[ \Phi - \frac{A^{-1}(I - \Phi)}{\Delta t} \right] B_w
\]

\[
B_{w2} = A^{-1} \left[ \frac{A^{-1}(\Phi - I)}{\Delta t} - I \right] B_w
\]  \(\text{(B.10)}\)
Vita

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