Assignment analysis for a rental-truck network

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Assignment Analysis for a Rental-truck Network

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Abstract

In this study, we examine truck assignment policies in a rental-truck network. In this setting, trucks are distributed among nodes in a network and customers arrive according to Poisson processes requesting trucks. We model the problem of optimizing truck assignment decisions with Linear Programming (LP) and Mean Value Analysis (MVA) techniques. The objective is to find the best assignment policy for the rental-truck network by considering the truck utilization (operating costs) and fleet size (capital costs), while at same time minimizing empty movements and achieving customer satisfaction. The proposed framework will not only be helpful for the cost minimization and policy analysis of the rental-truck network, but may also be useful for other decision making considerations, such as the replacement analysis of the trucks.

Keywords: Assignment policy analysis, Network decomposition, Queuing network
Chapter 1

Introduction

1.1 Problem Description and Motivation

Normally, a truck-rental company may have hundreds or even thousands of trucks in operation. At the same time, the company may also have numerous locations around the country serving as origins and destinations for travel. In this huge and dynamic network, customers come and go. With the high demand for transportation vehicles, effective truck-rental policies are crucial for smoothing everyday operation and generating profit for a truck-rental company. While a customer may be indifferent to the choice of truck, different allocations of trucks to locations and different assignment policies give different operating expenses and utilizations of the trucks, which in turn affect the overall efficiency and profitability of the company. Ineffective decision-making not only adds costs but also causes customer dissatisfaction.

With the awareness of the importance of truck-rental policies, this study examines the truck-rental problem with queuing network analysis assuming multiple truck types. In this study, we assume that trucks provide the same service to all customers. However, the trucks are differentiated by age, and thus their operating costs are different. Compared with other existing queuing network analysis, this problem has the following special features and requirements:
(1) In traditional closed queuing network, when a unit has completed service at one station, it may proceed in zero time to any station in the system according to a fixed, stationary probability distribution associated with the station it is leaving. In this thesis, the problem is defined by a closed queuing network with transit time between pairs of stations, and the time taken for a unit to move from one station to another station is a random variable with exponential distribution (Posner and Bernholtz [1]). Therefore, at any given point in time, there are not only trucks idle at stations, but also in transit. The traveling time between stations makes the problem more complicated than traditional queuing networks.

(2) As the trucks are to satisfy demands at each station, there are in fact two queues at each station. One is the customer queue; the other is the truck queue. Therefore, the queuing system at each station can be viewed as two symmetric queues, one of trucks in line for loading, and one of customers awaiting trucks. This is called a Double-ended Queue (Kendall [2], Brigham [3], Foster [4], Dobbie [5], Sasieni [6]).

(3) The objective of the rental manager is to efficiently assign trucks for satisfying customer requirements. This includes meeting a customer satisfaction rate at each station (truck arrival rate to customer demand rate). In this study, we assume a rate of 90% - 95%, so that almost all of the customer requirements are satisfied. Meeting customer requirements is an important factor for business success, therefore, this requirement should be included when determining the truck fleet sizes.

(4) As the maintenance and operating costs and breakdown probabilities for new trucks are generally much lower than those for older trucks, this study assumes that new trucks should be used at a high rate. This will in turn decrease the utilization of older trucks and reduce overall maintenance and operating costs. Under the assumption that trucks are to be kept a given length of time, this is the lowest-cost policy, assuming it does not require excess trucks.

(5) As age/mileage and other features differentiate trucks, different policies are possible for assigning the trucks to the customers. Some truck-rental companies may elect to
assign the newest trucks to the coming customers, while others may prefer other arrangements based on distance, usage, or other concerns. This study aims at understanding different assignment policies given different decompositions of the rental-truck network. By decomposing the rental-truck network into networks defined by different age groups separately, different truck assignment policies can be analyzed. The queuing network techniques can then be applied to analyze the fleet sizing strategies and steady-state characteristics.

1.2 Related Research

Queuing network models abound in applications and appear in a wide range of important and diverse areas, such as communication networks and tele-traffic, computer time-sharing and multiprogramming systems, maintenance and repair facilities, production, assembly and inspection operations, air traffic control, and medical care delivery system (Lemoine [12]).

The study of network of queues starts with the work of Erlang [7] and Engset [8] in telephony. However, much of these work was concerned with special types of networks (Disney and Konig [9]). Most modern queueing network theory stems from Jackson[10][11]. In Jackson network, arrivals from the "outside" to node $i$ follow a Possion process and service times at each channel are independent and exponentially distributed.

Ever since the development of Jackson network (Jackson [10]), it has been extended in several ways. For example, Jackson himself, in his later study (Jackson [11]), allowed state-dependent arrival processes and state-dependent services for open networks. Posner and Bernholtz [1] firstly treated closed Jackson networks to include travel times between nodes of the network. Their important result was that the traveling times could always be modeled as another node, but most often they are ample-server nodes. In Posner and Bernholtz [13], the results of Posner and Bernholtz [1] were further generalized to permit different types of customers, with a different set of service rates, routing probabilities,
and travel time distribution for each type.

Another extension of Jackson networks deals with extensions to multiple classes of customer networks, namely, multiclass Jackson networks, where, in addition to each class of customers having its own routing structure, each class also has its own mean arrival rate, and the mean service times at a node may depend on the particular type (class to which the customer belongs) as well (Gross and Harris [14]). Baskett et al. [15] treated such multiclass Jackson networks, and obtained product-form solutions. The model also allowed the network to be open for some classes of customers and closed for others, where customers may switch classes after finishing at a node. Kelly [16][17] considered multiclass customer classes, and set up a notational structure which allowed for unique class service times at multiserver FCFS nodes. Kelly's conjecture that many of his results can be extended to include general service-time distribution was then proved by Barbour [18]. Gross and Ince [19] further applied Kelly's multiclass results to a closed network and obtained numerical solutions for an application in inventory control.

In order to analyze the closed queueing network with priority scheduling, Shalev-Oren, etc. [20] developed a model called Priority Mean Value Analysis (PMVA) which extends the mean value analysis of closed network of queues with multiple product types, various non-preemptive priority service disciplines, and with parallel machine stations. They later extended the model for the analysis of flexible manufacturing systems with distinct repeated visits (Kim, Schweitzer, and Seidmann [21]).

Although there have been studies for multiclass network analysis and priority queues, according to our understanding, there were no formal studies carried out for the analysis of assignment policies for the rental-truck network. Although both multiclass network and PMVA can be applied to analyze the rental-truck assignment network in different perspectives, they either don't satisfy the features of the rental-truck network or don't exactly match the objectives of the assignment policy analysis. The multiclass network analysis requires known routing probabilities, arrival rates and service rates for the different types of trucks. However, before the network is decomposed and the optimal policy
is found out, we don’t have that information. The PMVA can be a method used for the assignment analysis for the rental-truck network. However, although it gives priorities to the different types of trucks, it doesn’t decompose the network into new truck network and old truck network. There are still possibilities that the utilization of the new trucks is not optimized because the PMVA does not guarantee the assignment of new trucks only to busy nodes.

With the awareness of the limitations of the existing queuing network techniques, this study analyzes the truck-rental assignment policies by decomposing the network into new truck network and old truck network. Recently, network decomposition has caught the attention of many researchers and has been applied to different network analyses (Kerbache and MacGregor [22], Ramesh and Perros [23], Solanki et al. [24], Barria and Turner [25]). By decomposing the rental-truck network into new truck network and old truck network, the empty movements can be minimized and the new truck utilization can be kept to a reasonably high level.

1.3 Research Objectives

Based on the special features of the rental-truck network, this study aims to model the rental-truck network and analyze the truck assignment policies by decomposing the rental-truck network into networks defined by different age groups. Different policies are to be analyzed so as to find the best policy of assigning trucks (newer trucks or older trucks) to customers. As the rental-truck network in this study differs from the existing queuing network studies by accommodating traveling times, double queues, and multiple types of trucks at the same time, the modeling of the queuing network and the analyzing of different network decomposition policies occupy the major part of this study.

The ultimate objective of this study is to analyze the rental-truck network, and give reasonable assignment policies for the queuing network analysis. With the analysis of truck assignment policies as the goal of this study, the information obtained will be
useful not only for real-world truck-rental decision-making but also for other economic considerations, such as replacement analysis of the trucks.
Chapter 2

Problem Formulation: Two Truck-Type Case

The objective in this chapter is to analyze different network decomposition policies with Linear Programming (LP), and then use Mean Value Analysis (MVA) to determine the truck fleet sizes. Policy analysis and cases of given fleet sizes will also be discussed in this chapter. A small illustrative example with four nodes are used throughout this chapter to guide the reader. Initially we assume two truck types (new and old).

2.1 The Double-ended Queues and Travel Times

This queueing network deals with the truck-rental problem for two types of trucks. The different stations serve as origins and destinations for travel. Therefore, they are nodes in the closed queuing network. The routes between nodes are deterministic according to fixed probabilities.

At each node, there are double-ended queues ([2][3][4][5][6]) for customers and trucks separately. If no trucks are available at a certain time, the customers queue at that station. On the other hand, if no customers are present at a certain time, the trucks queue, awaiting customers. As the major purpose of this study is to analyze the different
truck-rental policies and their effects for truck utilization, the truck queues are the focus of this study. The customer queues are to be analyzed for customer satisfaction while the truck queues are to be analyzed for truck utilization.

When there are a large number of stations with heavy traffic, the customer side of the double-ended queues can be approximately regarded as the servers of the trucks. The customer arrival rates thus become the service rates of the trucks. As the number of trucks in the system is normally fixed for truck-rental companies, the network is still a closed queueing network. The only difference is that instead of transporting customers in the network, we can actually view the network as if it is transporting trucks.

In this rental-truck network, there are traveling times between stations. An important conclusion from Posner and Bernholtz [1] is that a closed queuing network with time lags and having $M$ stations can be viewed as a closed cyclic queuing system with $M + 1$ stations arranged in an arbitrary sequence, with the appropriate forms for the service rates. In order to include travel time into the network formulation in this study, we add one service node to each arc in the network, with the service time equal to the travel time on that arc. To allow passing, we make the nodes handling traveling times to be ample-server nodes, which have enough number of servers for $N$ trucks (In this study, we suppose the number of trucks in the network is $N$, and set the number of servers to be $N$).

### 2.2 Network Decomposition Using LP

As the operating and maintenance costs for new trucks are normally lower than those for old trucks, the rental-truck network needs to be decomposed into separate networks for analysis. Different truck types can thus have different utilization levels based on their own networks. The different utilization levels can then be used to analyze and compare the different assignment policies during the policy analysis stage (Section 2.4).

In the traditional queuing network with several types of trucks, network decomposi-
tion and MVA can be complicated if all of the trucks are in one queue. For a joint queue, new trucks may be separated by uncertain number of old trucks, and old trucks by several new trucks. Choosing a particular type of truck from the joint queue is normally not very straightforward. The rental-truck network, however, does not have this problem because in the real world, trucks are parked in the parking lot instead of actually forming a queue. The selection of a particular truck type is thus straightforward as the drivers can simply go the parking lot and pick up the trucks they want. Therefore, the rental-truck network is particularly suitable for network decomposition, and MVA analysis can be performed on each sub-network.

2.2.1 The LP formulation

Before performing MVA, we need to decompose the network into new truck network and old truck network. As one criterion of this study is to use new trucks at a high level of utilization, a reasonable assignment is for the new trucks to travel mostly between the busy nodes. This will not only assure the use of the new trucks to a high level, but also avoid the queuing of new trucks in the unbusy nodes waiting for customers.

Suppose we have a network as presented in Figure 2-1. We first divide the stations into busy stations \((B_1, \ldots, B_x)\) and unbusy stations \((U_{x+1}, \ldots, U_M)\). This can be done according to the customer demand rates of the stations and operational experience. After the network is divided into busy and unbusy nodes, we can then decide percentage of the new trucks allowed to travel to unbusy nodes. The percentage of new trucks going to unbusy nodes should be kept at a low level (reasonably between 0 - 20%), as the objective here is to maximize the utilization of new trucks and avoid the queuing of new trucks at the unbusy nodes.

The decomposed new truck network and old truck network both have all the \(x\) busy nodes and the \(M - x\) unbusy nodes. However, the two networks have different routing probabilities at the arcs and different service rates at the nodes. The original network is the combination of the two separate networks.
It should be noted that empty movements are needed in this problem. Since different nodes have different service rates, some nodes have high truck demand rates and thus need more trucks, while other stations have low demand rates and thus need less trucks. In this situation, if we don’t allow empty movements, situations may happen that there are long queues of trucks staying in the unbusy nodes waiting for customers while there are not enough trucks at the busy nodes.

Normally, empty movements of trucks are very costly to a truck-rental company, and thus should be kept to a minimum level. If the objective is to minimize empty movements of trucks while satisfying the original routing possibilities and demand rates at each node, the problem can be formulated as a Linear Program.

Define the set of nodes in the original rental-truck network (without traveling time nodes) to be \( M \) and the set of arcs in the original network to be \( A \), where node 1 to node \( x \) are busy nodes and node \( x + 1 \) to node \( M \) are unbusy nodes. If \( e^N_{ij} \) is the number of empty movements between node \( i \) and node \( j \) in the new truck network, and \( e^O_{ij} \) is the empty movements between node \( i \) and node \( j \) in the old truck network, the traveling time
between node \(i\) and node \(j\) is \(T_{ij}\), the objective is then to minimize the empty movements with traveling times as the weighting parameters.

\[
\min \sum_{(i,j) \in A} T_{ij} * (e^N_{ij} + e^O_{ij})
\]

Define the flows in the new truck network to be \(f^N_{ij}\), and the flows in the old truck network to be \(f^O_{ij}\). Define the service rates at the nodes in the new truck network to be \(\mu^N_i\) \((i = 1...M)\), and the service rates at the nodes in the old truck network to be \(\mu^O_i\) \((i = 1...M)\). The objective is then to minimize the weighted empty movements, at the same time satisfy the known demands \((D_i, i = 1...M)\) and routing probabilities \((r_{ij}, (r, j) \in A)\) in the original rental-truck network.

A reasonable constraint is that the busy nodes should be served mostly by new trucks. This will not only assure the high utilization of the new trucks, but also satisfy the customers in busy nodes by giving them newer trucks. In this study, we require \(p\) percent of the time the routes between busy nodes are served by new trucks instead of old trucks. On the other hand, As discussed above, in order to keep the new trucks running, we need to set the percentage of new trucks going to busy nodes instead of unbusy nodes, defined as \(q\) here, to be a high level.

Trucks going to a certain node will finally be served either by customer demands or by empty movements. The formulation (LP) is then:

\[
\text{Minimize} \quad \sum_{(i,j) \in A} T_{ij} * (e^N_{ij} + e^O_{ij})
\]

subject to

\[
\sum_{\{i:(i,j) \in A\}} (f^N_{ij} + e^N_{ij}) = \mu^N_j \quad j = 1, ..., M 
\]

\[
\sum_{\{i:(i,j) \in A\}} (f^O_{ij} + e^O_{ij}) = \mu^O_j \quad j = 1, ..., M 
\]

\[
\mu^N_i + \mu^O_i - \sum_{\{i:(i,j) \in A\}} (e^N_{ij} + e^O_{ij}) = D_i \quad i = 1, ..., M 
\]

\[
f^N_{ij} + f^O_{ij} = D_i * r_{ij} \quad (i,j) \in A
\]
\[
\begin{align*}
\mu_i^N &= \sum_{j: (i,j) \in A} (f_{ij}^N + e_{ij}^N) & i = 1, \ldots, M \\
\mu_i^O &= \sum_{j: (i,j) \in A} (f_{ij}^O + e_{ij}^O) & i = 1, \ldots, M \\
f_{ij}^N &\geq p \cdot D_i \cdot r_{ij} & i = 1, \ldots, x, \ j = 1, \ldots, x \\
f_{ij}^N + e_{ij}^N &\geq q \cdot \mu_i^N & i = 1, \ldots, x, \ j = 1, \ldots, x \\
f_{ij}^N &\geq 0, \ f_{ij}^O &\geq 0 & (i, j) \in A \\
e_{ij}^N &\geq 0, \ e_{ij}^O &\geq 0 & (i, j) \in A \\
\mu_i^N &\geq 0, \ \mu_i^O &\geq 0 & i = 1, \ldots, M 
\end{align*}
\]

The first set of constraints are the service rate constraints for the new truck network. The second set of constraints are the service rate constraints for the old truck network. The third and fourth set of constraints are those constraints with the objectives of satisfying the original demands and routing probabilities respectively, while the fifth and sixth set of constraints control the overall outgoing flows to be equal to the service rate at each node. The seventh set of constraints assure that \( p \) of the demands in busy nodes are satisfied by new trucks instead of old trucks, and the eighth set of constraints assure that only a small percentage \((1 - q)\) of the new trucks serving unbusy nodes.

2.2.2 An illustrative example

An example is used here to illustrate how to use the above LP formulation to decompose the rental-truck network into new and old truck networks. The illustrative example (shown in Figure 2-2) has four nodes with customer demand rates of 80, 70, 20, and 10, respectively. The routing probabilities are as those shown on the arcs. The mean travel time between nodes 1 and 2 is 2 hours, between nodes 3 and 4 is 3 hours. The travel times on all the other arcs are 4 hours.

Since the demands at the four nodes differ dramatically, empty movements are required to meet demand. There are many ways to decompose the network, depending on the required percentage of new trucks going to busy nodes, \( q \), and the required percent of the time the routes between busy nodes served by new trucks, \( p \). Three cases are
Case 1: Single truck type

If we only have one type of truck, then the rental-truck network no longer needs to be decomposed into a new truck network and an old truck network. However, as empty movements are still needed to balance the network, the objective here is to minimize the empty movements while satisfying the demand rates and routing probabilities of the network.

For the network shown in Figure 2-2, the flows have already been partly determined by the known routing probabilities, the LP formulation for one type of truck can be simplified as:

Minimize

\[ 2(e_{12} + e_{21}) + 3(e_{34} + e_{43}) + 4(e_{13} + e_{31} + e_{14} + e_{41} + e_{23} + e_{32} + e_{24} + e_{42}) \]
subject to

\[
\begin{align*}
(J.L3 - e_{31} - e_{32} - e_{34}) \cdot 0.4 + (J.L4 - e_{41} - e_{42} - e_{43}) \cdot 0.4 \\
+ (J.L3 - e_{31} - e_{32} - e_{34}) \cdot 0.4 + (J.L4 - e_{41} - e_{42} - e_{43}) \cdot 0.4 \\
+ (J.L4 - e_{41} - e_{42} - e_{43}) \cdot 0.4 + (J.L3 - e_{31} - e_{32} - e_{34}) \cdot 0.4 \\
+ (J.L3 - e_{31} - e_{32} - e_{34}) \cdot 0.4 + (J.L4 - e_{41} - e_{42} - e_{43}) \cdot 0.4 \\
+ (J.L4 - e_{41} - e_{42} - e_{43}) \cdot 0.2 + (J.L3 - e_{31} - e_{32} - e_{34}) \cdot 0.2 + (J.L4 - e_{41} - e_{42} - e_{43}) \cdot 0.2 \\
+ (J.L3 - e_{31} - e_{32} - e_{34}) \cdot 0.2 + (J.L4 - e_{41} - e_{42} - e_{43}) \cdot 0.2 \\
\end{align*}
\]

Using any LP software to solve this problem, the objective function value is 45. The results are to make empty movement rates of 6 from node 2 to node 1, 6 from node 4 to node 1, and 3 from node 4 to node 3. The service rates at the four nodes are 80, 76, 20, and 19 respectively. If we use dashed lines to represent the empty movements, the network is given in Figure 2-3.

The routing probabilities in Figure 2-3 are the newly calculated routing probabilities which include truck flows and empty movements.

**Case 2: New trucks assigned to busy nodes**

In this example, the truck demand rate at node 1 is 80, at node 2 is 70, which are both much higher than the demand rates at node 3 and node 4. Therefore, node 1 and node 2 are defined as busy nodes here. A reasonably good strategy is to ask the new trucks to
serve the demand between the two busy nodes (node 1 and node 2), and old trucks serve all the other routes, so that the new trucks can be used at a maximum utilization level.

The assignment policy here is to decompose the network into a new truck network and an old truck network (as shown in Figure 2-4), with travel between node 1 and node 2 served only by new trucks.

The LP formulation is then:

\[
\text{Minimize} \\
2(e_{12}^N + e_{21}^N + e_{12}^O + e_{21}^O) + 3(e_{34} + e_{43}) + 4(e_{13} + e_{31} + e_{14} + e_{41} + e_{23} + e_{32} + e_{24} + e_{42})
\]
subject to
\[ \begin{align*}
\mu_1^N &= \mu_2^N = 64 \\
(\mu_3 - e_{31} - e_{32} - e_{34}) \cdot 0.4 + (\mu_4 - e_{41} - e_{42} - e_{43}) \cdot 0.4 + e_{21} + e_{31} + e_{41} = \mu_1^O \\
(\mu_3 - e_{31} - e_{32} - e_{34}) \cdot 0.4 + (\mu_4 - e_{41} - e_{42} - e_{43}) \cdot 0.4 + e_{12} + e_{32} + e_{42} = \mu_2^O \\
(\mu_1^O - e_{12} - e_{13} - e_{14}) \cdot 0.5 + (\mu_2^O - e_{21} - e_{23} - e_{24}) \cdot 0.5 \\
+ (\mu_4 - e_{41} - e_{42} - e_{43}) \cdot 0.2 + e_{13} + e_{23} + e_{43} = \mu_3 \\
(\mu_1^O - e_{12} - e_{13} - e_{14}) \cdot 0.5 + (\mu_2^O - e_{21} - e_{23} - e_{24}) \cdot 0.5 \\
+ (\mu_3 - e_{31} - e_{32} - e_{34}) \cdot 0.2 + e_{14} + e_{24} + e_{34} = \mu_4 \\
\mu_1^N + \mu_1^O - e_{12}^N - e_{12}^O - e_{13} - e_{14} &= 80 \\
\mu_2^N + \mu_2^O - e_{21}^N - e_{21}^O - e_{23} - e_{24} &= 70 \\
\mu_3 - e_{31} - e_{32} - e_{34} &= 20 \\
\mu_4 - e_{41} - e_{42} - e_{43} &= 10 \\
e_{21}^N &= 8 \\
e_{ij} &\geq 0 \quad (i, j) \in A \\
\mu_i &\geq 0, \quad i = 1, 2, \ldots, 4
\end{align*} \]
The reason for setting $\mu_1^N = \mu_2^N = 64$ is because for the demand rate of 80 at node 1, 80% (80% * 80 = 64) of the customers go to node 2. On the other hand, for the demand rate of 70 at node 2, 80% (80% * 70 = 56) of the customers go to node 1. Since we want the travel between nodes 1 and 2 to be completely served by new trucks, we need to set the service rate of node 1 in the new truck network to be 64 to make sure all of the customers can be served. At the same time, an empty flow of amount 8 (64 - 56 = 8) should be applied from node 2 to node 1. The solutions of this situation are shown in Figure 2-5. The objective function value for this case is 49.

**New Truck Network**

- $\mu_1^{new} = 64$
- $\mu_2^{new} = 64$

**Old Truck Network**

- $\mu_1^{old} = 16$
- $\mu_2^{old} = 14$
- $\mu_3 = 20$
- $\mu_4 = 19$

Figure 2-5: Solution for Case 2

**Case 3: New trucks assigned mostly to busy nodes**

Although we can get higher new truck utilization if we ask the new trucks to travel only between the busy nodes, it may be that a small percentage of the new trucks still need to travel to other unbusy nodes because of demand satisfaction or technical considerations.
In this example, we want less than 10% of the new trucks to travel to those unbusy nodes, so that more than 90% of the new trucks still travel between node 1 and node 2. On the other hand, since we still want most of the customer demands in busy nodes to be served by new trucks, we require more than 90% of the demands in node 1 and node 2 be served by new trucks, and the other less than 10% served by old trucks.

Therefore, the decomposed new truck network and old truck network have the same structure with same number of nodes and arcs (as shown in Figure 2-6), but with different service rates at the nodes and different routing probabilities, which, once combined, is the same as the original rental-truck network.

![New Truck Network](image1)

![Old Truck Network](image2)

Figure 2-6: Case 3 decomposition

While the previous two cases are the special cases of the network decomposition, this case 3 is a perfect match of the LP formulation presented in Section 2.2.1. If we set both \( p \) and \( q \) to be 90%, the LP formulation for this illustrative example is then:

Minimize

\[
2(e_{12}^N + e_{21}^N) + 3(e_{34}^N + e_{43}^N) + 4(e_{13}^N + e_{31}^N + e_{14}^N + e_{41}^N + e_{32}^N + e_{23}^N + e_{24}^N + e_{42}^N) + \\
2(e_{12}^O + e_{21}^O) + 3(e_{34}^O + e_{43}^O) + 4(e_{13}^O + e_{31}^O + e_{14}^O + e_{41}^O + e_{32}^O + e_{23}^O + e_{24}^O + e_{42}^O)
\]
subject to:

New-truck network inflow constraints:
\[ f_{21}^N + f_{31}^N + f_{41}^N + e_{21}^N + e_{31}^N + e_{41}^N = \mu_1^N \]
\[ f_{12}^N + f_{32}^N + f_{42}^N + e_{12}^N + e_{32}^N + e_{42}^N = \mu_2^N \]
\[ f_{13}^N + f_{23}^N + f_{43}^N + e_{13}^N + e_{23}^N + e_{43}^N = \mu_3^N \]
\[ f_{14}^N + f_{24}^N + f_{34}^N + e_{14}^N + e_{24}^N + e_{34}^N = \mu_4^N \]

Old-truck network inflow constraints:
\[ f_{21}^O + f_{41}^O + e_{21}^O + e_{41}^O = \mu_1^O \]
\[ f_{12}^O + f_{42}^O + e_{12}^O + e_{42}^O = \mu_2^O \]
\[ f_{13}^O + f_{43}^O + e_{13}^O + e_{43}^O = \mu_3^O \]
\[ f_{14}^O + f_{44}^O + e_{14}^O + e_{44}^O = \mu_4^O \]

Customer demand constraints:
\[ \mu_1^N - e_{12}^N - e_{13}^N - e_{14}^N + \mu_1^O - e_{12}^O - e_{13}^O - e_{14}^O = 80 \]
\[ \mu_2^N - e_{21}^N - e_{23}^N - e_{24}^N + \mu_2^O - e_{21}^O - e_{23}^O - e_{24}^O = 70 \]
\[ \mu_3^N - e_{31}^N - e_{32}^N - e_{34}^N + \mu_3^O - e_{31}^O - e_{32}^O - e_{34}^O = 20 \]
\[ \mu_4^N - e_{41}^N - e_{42}^N - e_{43}^N + \mu_4^O - e_{41}^O - e_{42}^O - e_{43}^O = 10 \]

Routing percentage constraints:
\[ f_{12}^N + f_{12}^O = 80 \times 80\% = 64 \]
\[ f_{13}^N + f_{13}^O = 80 \times 10\% = 8 \]
\[ f_{14}^N + f_{14}^O = 80 \times 10\% = 8 \]
\[ f_{21}^N + f_{21}^O = 70 \times 80\% = 56 \]
\[ f_{23}^N + f_{23}^O = 70 \times 10\% = 7 \]
\[ f_{24}^N + f_{24}^O = 70 \times 10\% = 7 \]
\[ f_{31}^N + f_{31}^O = 20 \times 40\% = 8 \]
\[ f_{32}^N + f_{32}^O = 20 \times 40\% = 8 \]
\[ f^N_{34} + f^O_{34} = 20 \times 20\% = 4 \]
\[ f^N_{11} + f^O_{11} = 10 \times 40\% = 4 \]
\[ f^N_{12} + f^O_{12} = 10 \times 40\% = 4 \]
\[ f^N_{43} + f^O_{43} = 10 \times 20\% = 2 \]

New-truck network outflow constraints:
\[ \mu^N_1 = f^N_{12} + f^N_{13} + f^N_{14} + e^N_{12} + e^N_{13} + e^N_{14} \]
\[ \mu^N_2 = f^N_{21} + f^N_{23} + f^N_{24} + e^N_{21} + e^N_{23} + e^N_{24} \]
\[ \mu^N_3 = f^N_{31} + f^N_{32} + f^N_{34} + e^N_{31} + e^N_{32} + e^N_{34} \]
\[ \mu^N_4 = f^N_{41} + f^N_{42} + f^N_{43} + e^N_{41} + e^N_{42} + e^N_{43} \]

Old-truck network outflow constraints:
\[ \mu^O_1 = f^O_{12} + f^O_{13} + f^O_{14} + e^O_{12} + e^O_{13} + e^O_{14} \]
\[ \mu^O_2 = f^O_{21} + f^O_{23} + f^O_{24} + e^O_{21} + e^O_{23} + e^O_{24} \]
\[ \mu^O_3 = f^O_{31} + f^O_{32} + f^O_{34} + e^O_{31} + e^O_{32} + e^O_{34} \]
\[ \mu^O_4 = f^O_{41} + f^O_{42} + f^O_{43} + e^O_{41} + e^O_{42} + e^O_{43} \]

Bounding constraints:
\[ f^N_{12} \geq 90\% \times 80 \times 80\% = 57.6 \]
\[ f^N_{21} \geq 90\% \times 70 \times 80\% = 50.4 \]
\[ f^N_{12} + e^N_{12} \geq 90\% \times \mu^N_1 \]
\[ f^N_{21} + e^N_{21} \geq 90\% \times \mu^N_2 \]

Non-negative constraints:
All variables \( \geq 0 \)

Solving this problem using any LP software, the objective function value is 45, and solutions are shown in Figure 2-7.

Conclusions from the cases

The difference between Case 1 and the other two cases for this illustrative example is that Case 1 has a single truck type and doesn't decompose the network into a new truck
network and an old truck network. Case 2 and case 3 successfully decompose the network by changing the values of $p$ and $q$, while minimizing the empty movements of the trucks. The LP formulation assures the minimum rate of empty movements, while balancing the decomposed networks.

Case 2 has a slightly higher rate of overall empty movements (with objective function value of 49) compared with Case 1 and Case 3 (with objective function values of 45). This is due to the different network decompositions. Different objective function values may be obtained from different network assignments.

### 2.3 Fleet Size and Asset Utilization Analysis

After the original rental-truck network is successfully decomposed into new and old truck networks, the next step is to use Mean Value Analysis (MVA) to calculate the steady-state characteristics and analyze the required fleet sizes for satisfying customer demands. Truck utilization can then be obtained based on the steady-state characteristics.
2.3.1 Algorithm for the determination of fleet sizes

For the determination of fleet sizes, the \( M \) nodes in the original rental-truck network and the \( A \) travel time nodes are considered. Although the traveling times on the arcs will not affect the total amount of inflows and outflows at the nodes of the original rental-truck network in the LP formulation, the traveling nodes will affect the results of the MVA, and thus should be included in the analysis of steady-state characteristics. Each decomposed network, then, has a total of \( M + A \) nodes.

Define:

\[ V_i = \text{the relative throughput through node } i \]
\[ W_i(n) = \text{mean waiting time at node } i \text{ for a network containing } n \text{ trucks} \]
\[ \mu_i = \text{mean service rate for a single server at node } i \]
\[ c_i = \text{the number of servers at node } i \]
\[ L_i(n) = \text{mean number of trucks at node } i \text{ in a network with } n \text{ trucks} \]
\[ \lambda_i(n) = \text{the throughput (arrival rate) for node } i \text{ in an } n \text{ truck network} \]
\[ p_i(n, N) = \text{the marginal probability of } n \text{ in an } N\text{-truck system at node } i \]
\[ \alpha_i(j) = \begin{cases} j & (j \leq c_i) \\
 c_i & (j \geq c_i) \end{cases} \]

The MVA algorithm (Gross [14]) for the determination of fleet size is then:

1. Solve the traffic equations, \( V_i = \sum_{j=1}^{M+A} v_j r_{ji} \) \((i = 1, 2, \ldots, M + A)\), setting one of the \( v_j \) (say \( V_1 \)) equal to 1.

2. Initialize for \( i = 1, 2, \ldots, M + A \), \( L_i(0) = 0; p_i(0,0) = 1; p_i(j,0) = 0, (j \neq 0) \);

3. For \( N = 1 \) to \( U \) (\( U \) is a reasonably big number that can cover all of the possible number of trucks), do

   for \( n = 1 \) to \( N \), calculate

   \[ W_i(n) = \frac{1}{c_i \mu_i} (1 + L_i(n-1) + \sum_{j=0}^{c_i-2} (c_i - 1 - j)p_i(j, n-1)) \quad (i = 1, 2, \ldots, M + A) \]

   \[ \lambda_i(n) = n / \sum_{i=1}^{M+A} v_i W_i(n) \quad \text{(assume } v_1 = 1) \]
\[
\begin{align*}
(c) \quad \lambda_i(n) &= \lambda_i(n) v_i & (i = 1, 2, ..., M + A; i \neq l) \\
(d) \quad L_i(n) &= \lambda_i(n) W_i(n) & (i = 1, 2, ..., M + A) \\
(e) \quad p_i(j, n) &= \frac{\lambda_i(n)}{\alpha_i(j)} p_i(j - 1, n - 1) & (j = 1, 2, ..., n; i = 1, 2, ..., M + A)
\end{align*}
\]

(4) If all of the \( \lambda_i(n) / \mu_i \geq s \) \( (i = 1, 2, ..., M) \), stop.

else, continue with step (3)

The \( s \) here is the required satisfaction rate for the customer demands. The satisfaction rate is a crucial factor for the determination of fleet size. With customer demands satisfied to different rates (in this study, we set the customer demand satisfaction rate to be 90\% - 95\%), the fleet sizes can be different. Trade-offs may also be needed for the reduction of fleet sizes at the cost of unsatisfied customer demands.

For the determination of fleet sizes, the above algorithm should be applied to both the new truck network and the old truck network separately. The optimal fleet sizes for new trucks and old trucks can then be obtained based on a required customer satisfaction level.

\subsection*{2.3.2 Truck utilization analysis}

In this rental-truck network, trucks pick up customers at the stations, and then transport these customers to other stations. Normally, the loading and unloading times of the customers are relatively small compared to the travel time between stations. Therefore, we can reasonably assume that those trucks in the middle of travel are those being used.

If we define the \( M \) stations in the original rental-truck network to be node 1 to node \( M \), the nodes handling traveling times to be node \( M + 1 \) to node \( M + A \), the truck utilization can then be calculated by dividing the total number of trucks in the traveling nodes (node \( M + 1 \) to node \( M + A \)) to the total number of trucks in the rental-truck network.

As the travel time nodes are ample-server nodes, there are in fact no queues in these nodes. All the trucks in these nodes are in service. This means all of the trucks in the
travel nodes are in transit. Therefore, the steady-state number of trucks in these travel nodes are the average number of trucks in service.

If there are a total of \( N_{\text{New}} \) trucks in the new truck network, and a total of \( N_{\text{Old}} \) trucks in the old truck network, the utilization of the new trucks is then:

\[
U_{\text{New}} = \left( \sum L_i^{\text{New}} \right) / N_{\text{New}} \quad i = M + 1, \ldots, M + A
\]

Similarly, the utilization of the old trucks is:

\[
U_{\text{Old}} = \left( \sum L_i^{\text{Old}} \right) / N_{\text{Old}} \quad i = M + 1, \ldots, M + A
\]

For a network with a single type of truck, on the other hand, the utilization level can be simply calculated by:

\[
U = \left( \sum L_i \right) / N \quad i = M + 1, \ldots, M + A
\]

### 2.3.3 An illustrative example

The following are the MVA results for the three cases of the illustrative example addressed early.

**MVA for Case 1**

For the rental-truck network with only one type of truck, if we add the travel time nodes to the network, the network will then have 16 nodes as shown in Figure 2-8. The trucks travel to and from the four stations according to the routing probabilities associated with the stations they are leaving. After a truck leaves a station, it enters the travel time node along the arc it is traveling, delayed for a proper amount of travel time, and arrives at its destination. The MVA results for the fleet size and the truck utilization are presented in Table 2-1.
Table 2-1: The MVA results for Case 1

<table>
<thead>
<tr>
<th>Total Number of Trucks</th>
<th>Customer Satisfaction Rate</th>
<th>Truck Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>( s \approx 90.85% )</td>
<td>( U = 39.4% )</td>
</tr>
<tr>
<td>80</td>
<td>( s \approx 95.2% )</td>
<td>( U = 25.75% )</td>
</tr>
</tbody>
</table>

The results show that if we want to achieve 90% of the customer satisfaction level, 50 trucks are needed, and the associated truck utilization is 39.4%. On the other hand, if we want to achieve 95% of the customer satisfaction level, 80 trucks are needed, and the associated truck utilization is 35.75%.

MVA for Case 2

If we require new trucks to serve only nodes 1 and 2, the network with traveling times is as Figure 2-9. Solving this network using MVA, the results are in Table 2-2.
Figure 2-9: MVA for Case 2

Table 2-2: The MVA results for Case 2

<table>
<thead>
<tr>
<th>Fleet Size</th>
<th>Total Trucks</th>
<th>Satisfaction</th>
<th>Truck Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{New} = 19$ $N_{Old} = 40$</td>
<td>59</td>
<td>$s \approx 90%$</td>
<td>$U_{New} = 44.53%$ $U_{Old} = 27.8%$</td>
</tr>
<tr>
<td>$N_{New} = 28$ $N_{Old} = 70$</td>
<td>98</td>
<td>$s \approx 95%$</td>
<td>$U_{New} = 31.64%$ $U_{Old} = 16.7%$</td>
</tr>
</tbody>
</table>

MVA for Case 3

If we use the MVA for Case 3, the network with traveling times will look similar to Figure 2-6 except adding the nodes for traveling times. Applying MVA to the LP results of Case 3, the results are in Table 2-3.

Table 2-3: The MVA results for Case 3

<table>
<thead>
<tr>
<th>Fleet Size</th>
<th>Total Trucks</th>
<th>Satisfaction</th>
<th>Truck Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{New} = 41$ $N_{Old} = 35$</td>
<td>76</td>
<td>$s \approx 90%$</td>
<td>$U_{New} = 30.7%$ $U_{Old} = 19.8%$</td>
</tr>
<tr>
<td>$N_{New} = 72$ $N_{Old} = 65$</td>
<td>137</td>
<td>$s \approx 95%$</td>
<td>$U_{New} = 18.4%$ $U_{Old} = 11.2%$</td>
</tr>
</tbody>
</table>
Conclusions

From the MVA results of the three cases, we easily find that using one type of truck (Case 1) requires the least number of total trucks. This is expected as customers do not wait for a given truck. However, it doesn't distinguish between new trucks and old trucks, and therefore doesn't use new trucks more than old trucks. Compared with Case 1, Case 2 requires more trucks, but the utilization level of the new trucks is much higher than that for old trucks. Trade-offs between the utilization level and the number of trucks, thus, should be made in this situation. The next section of this chapter, policy analysis, will address the issue of making trade-offs between the new truck utilization level and the number of trucks needed.

The difference between Case 3 and Case 2 is that Case 3 allows a small percentage of new trucks going to unbusy nodes (node 3 and node 4 in this example). The results of Case 3 show that allowing a small percentage of new trucks going to unbusy nodes will not only increase the number of trucks needed, but also decrease the truck utilization level for both the new trucks and the old trucks. An explanation is that the new trucks going to unbusy nodes wait for a longer time until they actually pick up a customer at the unbusy nodes and come back to serve the busy nodes. Therefore, they have more idle time than those trucks in Case 2.

2.4 Assignment Policy Analysis

2.4.1 Cost function

This study aims to analyze different truck-rental policies by decomposing the network into new and old truck networks. From the three cases addressed early, we can draw the conclusion that different decompositions of the network give different asset utilizations and fleet sizes. Trade-offs of the number of trucks and the truck utilizations are then needed to find the best assignment policy.
Since the objective of the assignment policy analysis is to reduce the total cost while satisfying customers to a certain degree, we can represent the objective of the assignment policy analysis by using a cost function which takes both truck utilizations and fleet sizes into consideration.

If we define:

\( c_f^{New} \) = The cost of having a new truck in the network

\( c_f^{Old} \) = The cost of having an old truck in the network

\( c_u^{New} \) = The cost of one percent utilization of a new truck

\( c_u^{Old} \) = The cost of one percent utilization of an old truck

Assuming the different cost items are linear and summative, the cost function can be formulated as:

\[
C = c_f^{New} \cdot N_{New} + c_f^{Old} \cdot N_{Old} + c_u^{New} \cdot U_{New} \cdot N_{New} + c_u^{Old} \cdot U_{Old} \cdot N_{Old}
\]

The cost function can then be used to compare the total cost between the different network decompositions. It can also be used to compare the performance between the different network decompositions and the network with only one type of trucks. For a network with only one type of trucks, the cost function is similar except that no distinction is needed for the utilization level of new trucks and old trucks.

2.4.2 Cost analysis for the illustrative example cases

Case 2 vs Case 3

For Case 2 and Case 3 of the illustrative example, if we assume \( c_f^{New} \) is $50,000, \( c_f^{Old} \) is $12,000, \( c_u^{New} \) is $800, and \( c_u^{Old} \) is $1,600, the costs for the two cases with different customer service levels are as those in Table 2-4.

For these two different assignment policies shown in Table 2-4, Case 2 is a better policy with less total cost. Compared with Case 2, Case 3 needs much more trucks. The capital cost for the number of trucks in Case 3 makes its total cost higher than that for Case 2.
Table 2-4: Cost analysis for Case 2 and Case 3

<table>
<thead>
<tr>
<th>Cost for service rate of 90%</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3,886,000</td>
<td>$4,586,000</td>
<td></td>
</tr>
<tr>
<td>Cost for service rate of 95%</td>
<td>$4,819,000</td>
<td>$6,605,000</td>
</tr>
</tbody>
</table>

To find the situations when Case 3 may have less cost than Case 2, we need to reexamine the parameters above. As the purchase costs for new trucks and old trucks are normally fixed, we only need to reexamine the values of $c^\text{New}_U$ and $c^\text{Old}_U$.

There are normally relations between the values of $c^\text{New}_U$ and $c^\text{Old}_U$, which can be either linear or nonlinear. In this study, we assume $c^\text{Old}_U$ is always twice that of $c^\text{New}_U$. Then, the cost functions of the two cases (with service rate of 90%) are:

**Case 2:**
\[
\$50,000 \times 19 + \$12,000 \times 40 + c^\text{New}_U \times 44.53 \times 19 + (2 \times c^\text{New}_U) \times 27.8 \times 40 \\
= \$1,430,000 + \$3,070,000 \times c^\text{New}_U
\]

**Case 3:**
\[
\$50,000 \times 41 + \$12,000 \times 35 + c^\text{New}_U \times 30.7 \times 41 + (2 \times c^\text{New}_U) \times 19.8 \times 35 \\
= \$2,470,000 + \$2,644,700 \times c^\text{New}_U
\]

The drawings of these two cost functions are in Figure 2-10.

From the figure, we can easily find out that Case 3 has less cost than Case 2 only when $c^\text{New}_U > \$2,450$. Since $c^\text{Old}_U = 2 \times c^\text{New}_U$, $c^\text{Old}_U$ requires to be greater than $\$4,900$, which is 40.8% of the purchase cost ($\$12,000$) of an old truck. In general applications, however, if the operating and maintenance cost of an old truck occupies more than 40% of its capital cost, the company normally prefers to buy a new truck instead of keeping the old truck. Therefore, in general situations, Case 2 always has lower total cost than Case 3.

**Case 1 vs Case 2**

The objective of this study is to find the best assignment policy of the rental-truck network so as to increase the utilization level of the new trucks (as the operating and maintenance cost for a new truck is much lower than that for an old truck) and in turn minimize the total cost. However, as shown in Table 2-1 and Table 2-2, Case 2 increases
the utilization level of the new trucks, but also requires more trucks. Trade-offs between the number of trucks and the truck utilization levels, thus, are needed.

Suppose the 50 trucks needed in Case 1 include 10 new trucks and 40 old trucks. Compared with Case 1, Case 2 needs 9 more new trucks, but its utilization level for new trucks is also higher. Assume the cost of one percent utilization of an old truck is always two times the corresponding cost of a new truck, the costs functions are then:

Case 1:  
\[ \text{Cost} = 50,000 \times 10 + 12,000 \times 40 + c_{U}^{new} \times 39.4 \times 10 + (2 \times c_{U}^{new}) \times 39.4 \times 40 \]
\[ = 980,000 + 3,546,000 \times c_{U}^{new} \]

Case 2:  
\[ \text{Cost} = 50,000 \times 19 + 12,000 \times 40 + c_{U}^{new} \times 44.53 \times 19 + (2 \times c_{U}^{new}) \times 27.8 \times 40 \]
\[ = 1,430,000 + 3,070,000 \times c_{U}^{new} \]

The drawings of these two cost functions are in Figure 2-11.

From the figure, we can draw the conclusion that if \( c_{U}^{new} \) is less than $945, Case 1 (the original rental-truck network without decomposition) is a better strategy. However, when \( c_{U}^{new} > $945 \), the decomposed network is preferred.
2.4.3 Policy analysis

By changing the values of $p$ (the percentage that demands in busy nodes are satisfied by new trucks instead of old trucks) and $q$ (the required percentage of time new trucks going to busy nodes instead of unbusy nodes), we actually change the network decomposition by switching the routing of the new trucks and the old trucks. The different network decomposition, in turn, affects the utilizations and fleet sizes of the trucks.

Given $c_{U}^{New}$ and $c_{U}^{Old}$, the complete policy analysis can then be carried out by changing the values of $p$ and $q$, applying LP to decompose the network, applying MVA to calculate the utilizations and fleet sizes, and finally use the above cost function to find the best assignment policy. By finding the minimum cost of the rental-truck network, the best assignment policy can be determined.

The procedures of the policy analysis is presented in Figure 2-12.
Decompose the network by changing the values of \( p \) and \( q \)
\[ (p = p + \Delta p, \ q = q + \Delta q) \]

Balance the network with LP

Using MVA to calculate truck utilizations and fleet sizes

Calculate the cost function value \( C \)

\( C \) less than the minimum cost \( C' \) up to the previous stage?

\[ \text{Yes} \rightarrow C' = C \]
\[ \text{No} \]

Reached the last possible values for \( p \) and \( q \)?

\[ \text{Yes} \rightarrow \text{Stop} \]
\[ \text{No} \]

Figure 2-12: Flowchart for policy analysis
2.5 Algorithm for Given Fleet Sizes

Although the studies of the illustrative example in Section 2.3 show that 50 trucks are the optimal number of trucks needed in Case 1 and 59 trucks (19 new trucks and 40 old trucks) are needed in Case 2, an existing truck fleet may already be defined. How to deal with this problem, then, becomes very important.

As the Policy Analysis in the previous section shows that Case 3 is normally worse than Case 2, we only consider Case 1 and Case 2 for the analysis of given fleet sizes here. Normally, old trucks can be sold if no longer needed. However, the truck-rental companies generally do not buy old trucks from other sources even though they don’t have enough old trucks to operate. Instead, they will buy new trucks and use the new trucks in the old truck network. Therefore, we assume the truck-rental company buys only new trucks and sells only old trucks.

As Case 1 is the optimal solution for using only one network and Case 2 is the optimal solution for network decomposition, with existing fleet sizes of the new trucks and old trucks, decisions have to be made regarding buy and sell behaviors or using one type of truck to operate as another type of truck. The costs for Case 1 and Case 2 decisions should then be compared to find the optimal policy.

Suppose we have $N_{\text{New}}^E$ new truck and $N_{\text{Old}}^E$ old trucks in the existing truck fleet, and the purchase and salvage costs equal to $P_{\text{New}}$ and $S_{\text{Old}}$ separately, the algorithm for given fleet sizes is then:

Algorithm:

1. Analyze the network following the procedures for Case 1 and Case 2 in Section 2.2 and 2.3, find optimal number of trucks needed for each case.

2. Calculate the cost of applying Case 1 policy:

   Compare $(N^E = N_{\text{New}}^E + N_{\text{Old}}^E)$ with $(N_{\text{Case1}} = N_{\text{New}}^{\text{Case1}} + N_{\text{Old}}^{\text{Case1}})$ in Case 1.

   If $N^E > N_{\text{Case1}}$, sell $(N^E - N_{\text{Case1}})$ old trucks, update the value of $N_{\text{Old}}^E$ by setting $N_{\text{Old}}^E = N_{\text{Old}}^E - (N^E - N_{\text{Case1}})$,
\[ C_{\text{Case1}} = c_F^{\text{New}} \cdot N_{\text{New}}^E + c_F^{\text{Old}} \cdot N_{\text{Old}}^E + (c_U^{\text{New}} \cdot N_{\text{New}}^E + c_U^{\text{Old}} \cdot N_{\text{Old}}^E) \cdot U_{\text{Case1}} \\
- S_{\text{Old}}(N^E - N_{\text{Case1}}) \]

If \( N^E < N_{\text{Case1}} \), Buy \((N_{\text{Case1}} - N^E)\) new trucks, update the value of \( N_{\text{New}}^E \) by setting \( N_{\text{New}}^E = N_{\text{New}}^E + (N_{\text{Case1}} - N^E) \),

\[
C_{\text{Case1}} = c_F^{\text{New}} \cdot N_{\text{New}}^E + c_F^{\text{Old}} \cdot N_{\text{Old}}^E + (c_U^{\text{New}} \cdot N_{\text{New}}^E + c_U^{\text{Old}} \cdot N_{\text{Old}}^E) \cdot U_{\text{Case1}} \\
+ P_{\text{New}}(N_{\text{Case1}} - N^E)
\]

The cost of Case 1 policy will later be compared with the cost of Case 2 to find the optimal policy.

(3) For Case 2 analysis, there are two different situations. One is \( N^E > N_{\text{Case2}} \), and the other is \( N^E < N_{\text{Case2}} \). This step handles the situation for \( N^E > N_{\text{Case2}} \), and the next step (Step 4) will handle the analysis for \( N^E < N_{\text{Case2}} \).

If \( N^E > N_{\text{Case2}} \), sell \((N^E - N_{\text{Case2}})\) old trucks, update the value of \( N_{\text{Old}}^E \) by setting \( N_{\text{Old}}^E = N_{\text{Old}}^E - (N^E - N_{\text{Case2}}) \) \{

After resetting the number of trucks, the possible assignment policies are then based on whether \( N_{\text{Old}}^E > N_{\text{Case2}} \text{ or } N_{\text{Old}}^E < N_{\text{Case2}} \).

If \( N_{\text{Old}}^E > N_{\text{Case2}} \), there are two alternative policies available \{

Choice 1: Use \((N_{\text{Old}}^E - N_{\text{Case2}})\) old trucks to operate as new trucks

\[
c_{\text{Choice1}} = c_F^{\text{New}} \cdot N_{\text{New}}^E + c_F^{\text{Old}} \cdot N_{\text{Old}}^E + c_U^{\text{New}} \cdot U_{\text{Case2}} + N_{\text{New}}^E + c_U^{\text{Old}} \cdot U_{\text{Case2}} + N_{\text{Old}}^E \\
- S_{\text{Old}}(N^E - N_{\text{Case2}})
\]

Choice 2: Sell \((N_{\text{Old}}^E - N_{\text{Case2}})\) old trucks, buy \((N_{\text{Case2}} - N_{\text{New}}^E)\) new ones

Update the value of \( N_{\text{Old}}^E \) by:

\[ N_{\text{Old}}^E = N_{\text{Old}}^E - (N_{\text{Old}}^E - N_{\text{Case2}}) \]

Update the value of \( N_{\text{New}}^E \) by:

\[ N_{\text{New}}^E = N_{\text{New}}^E + (N_{\text{Case2}} - N_{\text{New}}^E) \]
The cost is then:

\[ C_{\text{Choice}2}^{21} = c_{F}^{\text{New}} * N_{\text{New}}^{E} + c_{F}^{\text{Old}} * N_{\text{Old}}^{E} + c_{U}^{\text{New}} * U_{\text{New}}^{\text{Case}2} * N_{\text{New}}^{E} + \]
\[ c_{U}^{\text{Old}} * U_{\text{Old}}^{\text{Case}2} * N_{\text{Old}}^{E} - S_{\text{Old}}(N_{\text{E}} - N_{\text{Case}2}) - S_{\text{Old}}(N_{\text{Old}}^{E} - N_{\text{Old}}^{E}) + P_{\text{New}}(N_{\text{Case}2}^{E} - N_{\text{New}}^{E}) \]

Compare \( C_{\text{Case}1} \) with \( C_{\text{Choice}1}^{21} \) and \( C_{\text{Choice}1}^{20} \) to find the best policy.

If \( N_{\text{Old}}^{E} < N_{\text{Old}}^{\text{Case}2} \) {

Use \( N_{\text{Old}}^{\text{Case}2} - N_{\text{Old}}^{E} \) new trucks to operate as old trucks

\[ C^{22} = c_{F}^{\text{New}} * N_{\text{New}}^{E} + c_{F}^{\text{Old}} * N_{\text{Old}}^{E} + c_{U}^{\text{New}} * U_{\text{New}}^{\text{Case}2} * (N_{\text{New}}^{E} - (N_{\text{Old}}^{\text{Case}2} - N_{\text{Old}}^{E})) + c_{U}^{\text{Old}} * U_{\text{Old}}^{\text{Case}2} * N_{\text{Old}}^{E} - S_{\text{Old}}(N_{\text{E}} - N_{\text{Case}2}) \]

Compare \( C_{\text{Case}1} \) with \( C^{22} \) to find the best policy.

}\)

(4) For Case 2 with \( N_{\text{E}} < N_{\text{Case}2}^{E} \):

If \( N_{\text{E}} < N_{\text{Case}2}^{E} \), buy \( (N_{\text{Case}2}^{E} - N_{\text{E}}) \) new trucks, update the value of \( N_{\text{New}}^{E} \) by

\[ N_{\text{New}}^{E} = N_{\text{New}}^{E} + (N_{\text{Case}2}^{E} - N_{\text{E}}) \]

If \( N_{\text{Old}}^{E} > N_{\text{Old}}^{\text{Case}2} \), there are two choices:

Choice 1: Use \( (N_{\text{Old}}^{E} - N_{\text{Old}}^{\text{Case}2}) \) old trucks to operate as new trucks

\[ C_{\text{Choice}1}^{23} = c_{F}^{\text{New}} * N_{\text{New}}^{E} + c_{F}^{\text{Old}} * N_{\text{Old}}^{E} + c_{U}^{\text{New}} * U_{\text{New}}^{\text{Case}2} * N_{\text{New}}^{E} + c_{U}^{\text{Old}} * U_{\text{Old}}^{\text{Case}2} * (N_{\text{Old}}^{E} - N_{\text{Case}2}^{E}) + P_{\text{New}}(N_{\text{Case}2}^{E} - N_{\text{New}}^{E}) \]

Choice 2: Sell \( (N_{\text{Old}}^{E} - N_{\text{Case}2}^{E}) \) old trucks, buy \( (N_{\text{Case}2}^{E} - N_{\text{New}}^{E}) \) new ones

\[ N_{\text{Old}}^{E} = N_{\text{Old}}^{E} - (N_{\text{Case}2}^{E} - N_{\text{E}}) \]
\[ N_{\text{New}}^{E} = N_{\text{New}}^{E} + (N_{\text{Case}2}^{E} - N_{\text{E}}) \]

\[ C_{\text{Choice}2}^{23} = c_{F}^{\text{New}} * N_{\text{New}}^{E} + c_{F}^{\text{Old}} * N_{\text{Old}}^{E} + c_{U}^{\text{New}} * U_{\text{New}}^{\text{Case}2} * N_{\text{New}}^{E} + c_{U}^{\text{Old}} * U_{\text{Old}}^{\text{Case}2} * N_{\text{Old}}^{E} + P_{\text{New}}(N_{\text{Case}2}^{E} - N_{\text{New}}^{E}) - S_{\text{Old}}(N_{\text{Old}}^{E}) \]
\(-N_{Old}^{Case2} + P_{New}(N_{New}^{Case2} - N_{New}^E)\)

Compare \(C_{Case1}^{23}\) with \(C_{Choice1}^{23}\) and \(C_{Choice1}^{23}\) to find the best policy

If \(N_{Old}^E < N_{Old}^{Case2}\), there is one choice:

Use \((N_{Old}^{Case2} - N_{Old}^E)\) new trucks to operate as old trucks

\[
C^{24} = c_F^{New} \cdot N_{New}^E + c_F^{Old} \cdot N_{Old}^E + c_U^{New} \cdot U_{Case2} \cdot (N_{New}^E - (N_{Old}^{Case2} - N_{Old}^E)) + c_U^{Old} \cdot U_{Case2} \cdot N_{Old}^E + P_{New}(N_{Case2}^E - N^E)
\]

Compare \(C_{Case1}\) with \(C^{24}\) to find the best policy

Therefore, for a given fleet of the truck-rental company, we should first go to Step (2) to calculate the corresponding cost of using Case 1 policy. Then, we use either the algorithm in Step (3) or Step (4) to find the cost of using Case 2 policy. Finally, The costs should be compared so as to find the best policy.

### 2.5.1 The illustrative case

In the illustrative case, suppose the existing truck fleet has 30 old trucks and 25 new trucks, and the purchase and salvage costs equal to \(c_F^{New}\) and \(c_F^{Old}\) separately. Following the algorithm presented above, the analysis procedures (with 90% service rate) are:

1. **Analyze the cost for Case 1:**

   Since \(N^E = 30 + 25 = 55\), \(N_{Case1} = 50\), if we want to use the Case 1 network, we need to sell 5 old trucks so that the remaining truck fleet has a total of 50 trucks (25 old trucks and 25 new trucks). The cost for using Case 1 network is then (with \(c_F^{New} = 800\), and \(c_U^{Old} = 1600\)):

   \[
   C_{Case1} = c_F^{New} \cdot N_{New}^E + c_F^{Old} \cdot N_{Old}^E + (c_U^{New} \cdot N_{New}^E + c_U^{Old} \cdot N_{Old}^E) \cdot U_{Case1} - S_{Old}(N^E - N_{Case1})
   \]

   - 37
\[
\begin{align*}
\text{Total Cost} &= (50k \times 25 + 12k \times 25 + (0.8k \times 25 + 1.6k \times 25) \times 39.4 - 12k \times 5) \\
&= 3854k
\end{align*}
\]

(2) Analyze the cost for Case 2:

If we want to use Case 2 assignment policy, we need 19 new trucks and 40 old trucks as shown Table 2.2. Since \(N_E = 55\). Additional 4 new trucks should be purchased, and the new fleet then has 29 new trucks and 30 old trucks. At the same time, \(N_{Old}^{Case2} = 40\), therefore, the cost function, \(C^{24}\) in Step (4) of the above algorithm should be applied.

\[
C^{24} = c_F^{New} N_E^{New} + c_F^{Old} N_E^{Old} + c_U^{New} U^{Case2} (N_E^{New} - (N_{Old}^{Case2} - N_{Old}^{Case})) + c_U^{New} U^{Case2} (N_{Old}^{Case2} - N_{Old}^{Case}) + P_{New} (N_{Case} - N_E^{Case2})
\]

\[
= 50k \times 29 + 12k \times 30 + 0.8k \times 44.53 \times 19 + 0.8 \times 27.8 \times 10 + 1.6 \times 27.8 \times 30 + 50 \times 4
\]

\[
= 4244k
\]

(3) Since \(C_{Case1} < C^{24}\), Case 1 network should be used, and 5 old trucks should be sold (with 25 old trucks and 25 new trucks remaining).
Chapter 3

Problem Formulation - General Case

The illustrative example shown in Chapter 2 has four stations, two types of trucks, and predetermined routing probabilities. In the real world, however, the rental-truck network can be more complicated. It may contain more than two types of trucks, allow local movements (instead of only allowing travels from one station to another station), have different breakdown rates on different routes, and have seasonal demand rates.

This chapter deals with problem formulations for these practical considerations. Solutions for each case are discussed and formulations are given for further research.

3.1 Network with Local Movements

In practice, the truck user may rent a truck from the truck-rental company, use it for some time, and then return the truck to the same station. We call this a local movement. The procedures of LP and MVA (in Chapter 2) for a network with local movements remain the same as long as we allow arcs $(i,i)$ to exist in the network.

For the illustrative example, suppose the new network with local movements is as shown in Figure 3-1. Further assume the local travel times in the four stations are exponentially distributed with mean travel times of 2, 2, 3 and 4 separately. The LP formulation of this network for single truck type is then:
Figure 3-1: The illustrative example with local movements

Minimize

\[ 2(e_{12} + e_{21}) + 3(e_{34} + e_{43}) + 4(e_{13} + e_{31} + e_{14} + e_{41} + e_{23} + e_{32} + e_{24} + e_{42}) \]

subject to

\[
\begin{align*}
(\mu_3 - e_{31} - e_{32} - e_{34}) \times 0.4 + (\mu_4 - e_{41} - e_{42} - e_{43}) \times 0.4 + (\mu_2 - e_{21} - e_{23} - e_{24}) \times 0.6 \\
+ (\mu_1 - e_{12} - e_{13} - e_{14}) \times 0.2 + e_{21} + e_{31} + e_{41} = \mu_1
\end{align*}
\]

\[
\begin{align*}
(\mu_3 - e_{31} - e_{32} - e_{34}) \times 0.4 + (\mu_4 - e_{41} - e_{42} - e_{43}) \times 0.35 + (\mu_1 - e_{12} - e_{13} - e_{14}) \times 0.6 \\
+ (\mu_2 - e_{21} - e_{23} - e_{24}) \times 0.2 + e_{12} + e_{32} + e_{42} = \mu_2
\end{align*}
\]

\[
\begin{align*}
(\mu_1 - e_{12} - e_{13} - e_{14}) \times 0.1 + (\mu_2 - e_{21} - e_{23} - e_{24}) \times 0.1 + (\mu_4 - e_{41} - e_{42} - e_{43}) \times 0.1 \\
+ (\mu_3 - e_{31} - e_{32} - e_{34}) \times 0.1 + e_{13} + e_{23} + e_{33} = \mu_3
\end{align*}
\]

\[
\begin{align*}
(\mu_1 - e_{12} - e_{13} - e_{14}) \times 0.1 + (\mu_2 - e_{21} - e_{23} - e_{24}) \times 0.1 + (\mu_3 - e_{31} - e_{32} - e_{34}) \times 0.1 \\
+ (\mu_4 - e_{41} - e_{42} - e_{43}) \times 0.15 + e_{14} + e_{24} + e_{34} = \mu_4
\end{align*}
\]
\[\mu_1 - e_{12} - e_{13} - e_{14} = 80\]
\[\mu_2 - e_{21} - e_{23} - e_{24} = 70\]
\[\mu_3 - e_{31} - e_{32} - e_{34} = 20\]
\[\mu_4 - e_{41} - e_{42} - e_{43} = 10\]
\[e_{ij} \geq 0\quad (i,j) \in A\]
\[\mu_i \geq 0,\quad i = 1, 2, \ldots, 4\]

It should be noted that empty movements for local travels are not necessary and thus not included in the LP formulation. The purpose of empty movements is to transfer trucks from one station to another station to meet demands at the destination station. Local movements do not transport trucks to other stations. Therefore, empty movements from one station to itself are not necessary in the network.

Using any LP software, the objective function value is 39. The results are to make empty movements of 3.5 from node 2 to node 1, 6.5 from node 4 to node 1, and 2 from node 4 to node 3. The service rates at the four stations are 80, 73.5, 20, and 18.5 respectively. The network is given in Figure 3-2.

Figure 3-2: Solution for network with local movements
Using MVA to calculate the truck utilization level and the truck fleet size, the results are in Table 3-1.

<table>
<thead>
<tr>
<th>Total Number of Trucks</th>
<th>Customer Satisfaction Rate</th>
<th>Truck Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>( s \approx 90.03% )</td>
<td>( U = 41.02% )</td>
</tr>
<tr>
<td>78</td>
<td>( s \approx 95.03% )</td>
<td>( U = 26.09% )</td>
</tr>
</tbody>
</table>

### 3.2 Network with Truck Breakdown Probabilities

The network we considered in Chapter 2 was based on routing probabilities and demand rates. The breakdown of trucks in the network was not taken into consideration. However, truck breakdowns, although normally with small probabilities, do affect the overall performance and total cost of the network. Additional trucks may be needed to handle the situations of truck breakdowns.

Normally, the truck breakdown rate on a certain route depends on the length of the route, the condition (age, usage) of the trucks sent to that route, and some other factors. Since the objective of a truck-rental company is to satisfy its customers, additional trucks may be needed to ensure customer satisfaction.

In order to illustrate the procedures of handling truck breakdowns, we still use the illustrative example, at the same time assume the breakdown rate on each route is already known. Figure 3-3 is the network for single truck type with breakdown rates \( B_{ij} \) on the arcs.

Take the route from node 1 to node 2 as an example. Since the breakdown rate on this arc is 2%, with \( 80\% \times 2\% = 1.6\% \) probability trucks leaving node 1 will breakdown on route \((1,2)\). If the repair time for a truck breakdown at route \((1,2)\) follows exponential distribution with mean \( R_{12} \), the network perform the same as if we add another route going from node 1 from node 2, with probability 1.6% and travel time \( R_{12} \). The network with breakdown on route \((1,2)\) will look as Figure 3-4.
Figure 3-3: Network with breakdown rates

Figure 3-4: The network with breakdown on route (1, 2)
Breakdown on other routes can be handled using the same way as route (1, 2), and the new network will add one arc for each route with possible breakdowns.

After new arcs are added, the overall outflows from a certain node may add up to be more than 100%. Normalization of the probabilities is thus needed to ensure the overall outflows from a station add up to be 100%. The new probability on a certain arc \((A, B)\) is then:

\[
\frac{r_{AB}} {\sum_{(i,j)} \in A (r_{ij} + r_{ij}B_{ij})} \quad \text{for the arcs in the original network}
\]

\[
(r_{AB}B_{AB})/ \sum_{(i,j)} \in A (r_{ij} + r_{ij}B_{ij}) \quad \text{for the added arcs}
\]

In order not to affect the travel rates on the arcs of the original network (without breakdown), the demand rates at the stations should also be adjusted by:

\[
D'_i = D_i \ast \sum_{(i,j)} \in A (r_{ij} + r_{ij}B_{ij}) \quad i = 1, 2, ..., M
\]

The network is then shown in Figure 3-5. The two probabilities on each arc are the normalized probabilities of normal truck operation and truck breakdown respectively.

\[\text{Figure 3-5: Network with breakdown}\]

Further suppose the repair time for the breakdowns on all the arcs are exponential with mean 4 (the total travel time for the breakdown trucks is then the sum of travel
time and repair time), the procedures of LP formulation and MVA are then the same as before. The results are presented in Figure 3-6 (with objective value of 65.8) and Table 3-2.

![Figure 3-6: LP solution for network with breakdown](image)

**Figure 3-6: LP solution for network with breakdown**

<table>
<thead>
<tr>
<th>Total Number of Trucks</th>
<th>Customer Satisfaction Rate</th>
<th>Truck Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>$s \approx 90%$</td>
<td>$U = 39.13%$</td>
</tr>
<tr>
<td>86</td>
<td>$s \approx 95%$</td>
<td>$U = 24.85%$</td>
</tr>
</tbody>
</table>

Compared with Table 2-1, the network with truck breakdowns requires more trucks. At the same time, the truck utilization level is also lower.

It should be noted that the breakdown rates on the arcs can be different for using different types of trucks. If we decompose the network into new and old truck networks as the Case 2 and Case 3 decomposition discussed in Chapter 2, the breakdown rates may be higher for old trucks and lower for new trucks. To illustrate, we used the network with single truck type here. If decomposition should be applied, the procedure of handling
breakdowns is similar except that we need to handle the different breakdown rates for new and old truck networks separately.

3.3 Network with Seasonal Demand

The demand for truck-rentals normally follows seasonal patterns. Summer, for example, is a season with high travel rates and thus the need for more trucks. Early fall season is the time for students to enter universities while spring is for graduates to relocate to their employers. Winter, however, has lower truck demand rates. This seasonal effect of the truck-rental affects the assignment policies and the truck fleet sizes.

One possible way to handle the network with seasonal demand is to perform a policy analysis for each of the four seasons. Four network decompositions can then be obtained separately. At the end of each season, sell or buy decisions or empty movements can be made depending on the number of trucks needed in the following seasons and the current condition of the trucks.

At the end of each season, the truck-rental company may also relocate the trucks in the network according to the best policy of the next season, so that when next season starts, the network can be ready to operate in a very short time. This re-initialization can be particularly helpful if the best network assignment of the next season is dramatically different from that of the current season.

3.4 Network with More Than Two Types of Trucks

This study deals with rental-truck network with two types of trucks. However, in real-world applications, more detailed groupings with more than two types of trucks (e.g. three types of trucks) may be needed. If more than two types of trucks (say, three types) are more appropriate, the model proposed in this study can be adjusted by decomposing the network into three separate networks, and using MVA for each network to analyze
the fleet sizes and truck utilizations separately.

If there are more than two types of trucks in the network, the policy analysis can be very time consuming if we iterate among every possible network decomposition. A more effective policy analysis algorithm, thus, is needed to handle the network with more than two types of trucks.

As our previous study shows that Case 2 decomposition performs better than other decompositions, we introduce the Case 2 type decomposition for three types of trucks here. For the identification of the best assignment policy, iteration among every possible network decomposition or other more efficient algorithms should be applied.

Suppose we have $M$ nodes in the existing network. $x$ of the $M$ nodes (nodes 1 to $x$) are busy nodes. $y$ of them (nodes $M - y$ to $M$) are unbusy nodes, and the rest (nodes $x + 1$ to $M - y - 1$) are those nodes with moderate demand rates between busy nodes and unbusy nodes. Further suppose the three types of trucks are grouped according to their age and usage. Type I contains the newest trucks, and type III the oldest trucks.

The original network is then decomposed into three separate networks as shown in Figure 3.7. Type I trucks handle the travels between only busy nodes, Type II the travels between the moderate nodes and between busy nodes and moderate nodes, and Type III all of the other travels.

The LP formulation is then similar to that for Case 2 in Chapter 2, except that instead of having two sets of variables, we have three sets of variables for the three truck types. With the objective of minimizing total empty movements, the three sub-networks can be balanced by satisfying the demand and flow constraints.

MVA can then be used to the three separated networks to calculate the number of trucks needed for each type and its corresponding truck utilization. Further policy analysis can thus be carried out based on these values.
Type I trucks

Type II trucks

Type III trucks

Figure 3-7: The decomposed network for three types of trucks
Chapter 4

A Case Study

In this chapter, we apply the proposed assignment analysis to a case study with 15 stations. The purpose is to illustrate how to use the proposed method to more complicated real-world applications.

Demand rates and travel times in this case study are randomly generated by a computer program. Those nodes with demand rates of more than 50 (here we suppose demand rates are between 10 and 100) are considered as busy nodes. Travel probabilities are generated randomly and normalized. Of the 15 nodes, 8 of them are busy nodes (nodes 3, 4, 6, 7, 9, 10, 11, and 15) and 7 of them are unbusy nodes (nodes 1, 2, 5, 8, 12, 13, and 14).

4.1 Case 1: Single Truck Type

The LP analysis is applied to the network with single truck type. The objective is to balance the network with minimum empty movements. The objective function value is 163.66 with 83 LP iterations. The results of the service rates at stations and the empty movements are shown in Table 4.1 and Table 4.2.

A service node is then added to each arc of the network to handle the truck travel times. The network with travel nodes thus have a total of 240 nodes (15 * 15 arcs +15
nodes).

<table>
<thead>
<tr>
<th>Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service Rate</td>
<td>19</td>
<td>46</td>
<td>94.61</td>
<td>109.26</td>
<td>42.88</td>
<td>93</td>
<td>89.70</td>
<td>49</td>
</tr>
</tbody>
</table>

Table 4.1. The service rates for Case 1

<table>
<thead>
<tr>
<th>Arc</th>
<th>(3, 4)</th>
<th>(4, 5)</th>
<th>(7, 1)</th>
<th>(7, 2)</th>
<th>(10, 2)</th>
<th>(10, 5)</th>
<th>(10, 9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty Movement</td>
<td>20.91</td>
<td>16.26</td>
<td>2.55</td>
<td>14.84</td>
<td>6.32</td>
<td>6.09</td>
<td>6.42</td>
</tr>
</tbody>
</table>

Table 4.2 The empty movement rates for Case 1

<table>
<thead>
<tr>
<th>Arc</th>
<th>(11, 6)</th>
<th>(15, 8)</th>
<th>(3, 11)</th>
<th>(5, 14)</th>
<th>(7, 12)</th>
<th>(9, 12)</th>
<th>(10, 13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty Movement</td>
<td>16.85</td>
<td>15.83</td>
<td>10.70</td>
<td>15.88</td>
<td>7.31</td>
<td>10.19</td>
<td>25.60</td>
</tr>
</tbody>
</table>

Suppose a customer satisfaction level of 90% is required for the rental-truck network. The MVA shows that 238 trucks are needed for the network. With a single type of truck in the network, the truck utilization level is 46.49%.

4.2 Case 2: New Trucks Assigned to Busy Nodes

We assume new trucks will handle only the travel between the 8 busy nodes, and the old trucks will handle all other travel. Applying LP to the decomposed network, the objective function value is 97.27 after 107 iterations. The results for the new truck network are shown in Table 4.3. In the new truck network, only one arc needs empty movement, (7, 9), with a rate of 0.99.
Table 4.4 and Table 4.5 are the results for the old truck network.

Table 4.4. The service rates for old truck network

<table>
<thead>
<tr>
<th>Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service Rate</td>
<td>23.50</td>
<td>46</td>
<td>28.35</td>
<td>42.32</td>
<td>30.93</td>
<td>47.27</td>
<td>24.80</td>
<td>49</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service Rate</td>
<td>31.51</td>
<td>29.28</td>
<td>31.07</td>
<td>40</td>
<td>73.38</td>
<td>39</td>
<td>31.40</td>
</tr>
</tbody>
</table>

Table 4.5 The empty movement rates for old truck network

<table>
<thead>
<tr>
<th>Arc</th>
<th>(1, 14)</th>
<th>(3, 4)</th>
<th>(3, 11)</th>
<th>(5, 14)</th>
<th>(7, 1)</th>
<th>(7, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty Movement</td>
<td>4.50</td>
<td>17.01</td>
<td>0.10</td>
<td>3.92</td>
<td>0.76</td>
<td>5.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Arc</th>
<th>(7, 12)</th>
<th>(7, 13)</th>
<th>(10, 8)</th>
<th>(10, 13)</th>
<th>(13, 6)</th>
<th>(15, 11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty Movement</td>
<td>1.56</td>
<td>5.70</td>
<td>3.24</td>
<td>26.23</td>
<td>24.38</td>
<td>2.82</td>
</tr>
</tbody>
</table>

With 90% customer satisfaction level, the network needs 127 new trucks and 169 old trucks (totally 296 trucks). The utilization level for new trucks and old trucks are 49.28% and 24.96% separately.

4.3 Assignment Policy Analysis

The cost functions for Case 1 and Case 2 are:

\[
C_{\text{Case1}} = c_p^{\text{New}} \cdot N_{\text{New}} + c_p^{\text{Old}} \cdot N_{\text{Old}} + c_U^{\text{New}} \cdot U \cdot N_{\text{New}} + c_U^{\text{Old}} \cdot U \cdot N_{\text{Old}}
\]

\[
C_{\text{Case2}} = c_p^{\text{New}} \cdot N_{\text{New}} + c_p^{\text{Old}} \cdot N_{\text{Old}} + c_U^{\text{New}} \cdot U_{\text{New}} \cdot N_{\text{New}} + c_U^{\text{Old}} \cdot U_{\text{Old}} \cdot N_{\text{Old}}
\]
Of the 238 trucks needed in Case 1, suppose there are 169 old trucks and 69 new trucks. Therefore, one difference between Case 1 and Case 2 is that Case 2 needs additional 58 (127 – 69 = 58) new trucks.

Further suppose $c_{P \text{New}}$ is $50,000$, $c_{P \text{Old}}$ is $12,000$, $c_{U \text{New}}$ is $800$, and $c_{U \text{Old}}$ is $1,600$. The cost functions are then:

$$C_{\text{Case1}} = c_{P \text{New}} \cdot N_{\text{New}} + c_{P \text{Old}} \cdot N_{\text{Old}} + c_{U \text{New}} \cdot U \cdot N_{\text{New}} + c_{U \text{Old}} \cdot U \cdot N_{\text{Old}}$$

$$= 50k \cdot 69 + 12k \cdot 169 + 0.8 \cdot 46.49 \cdot 69 + 1.6 \cdot 46.49 \cdot 169$$

$$= 20615k$$

$$C_{\text{Case2}} = c_{P \text{New}} \cdot N_{\text{New}} + c_{P \text{Old}} \cdot N_{\text{Old}} + c_{U \text{New}} \cdot U_{\text{New}} \cdot N_{\text{New}} + c_{U \text{Old}} \cdot U_{\text{Old}} \cdot N_{\text{Old}}$$

$$= 50k \cdot 127 + 12k \cdot 169 + 0.8 \cdot 49.28 \cdot 127 + 1.6 \cdot 24.96 \cdot 169$$

$$= 20134k$$

Therefore, although Case 2 needs 58 more trucks than Case 1, Case 2 is still a better assignment policy because its utilization level for old trucks is much lower.

### 4.4 Analysis for Given Fleet Sizes

Suppose the truck-rental company has already in this business for several years and its existing truck fleet has 245 trucks (70 new trucks and 175 old trucks), following the algorithm of given fleet size in Section 2.5, the analysis procedures are:

1. Analyze the cost for Case 1:

   Since $N^{E} = 245 > N^{\text{Case1}} = 238$, we need to sell 7 old trucks (suppose the salvage cost of the old trucks equals to $c_{P \text{Old}}$). The remaining truck fleet has 70 new trucks and 168 old trucks. The cost for using Case 1 network is then:

   $$C_{\text{Case1}} = c_{P \text{New}} \cdot N_{\text{New}} + c_{P \text{Old}} \cdot N_{\text{Old}} + c_{U \text{New}} \cdot U \cdot N_{\text{New}} + c_{U \text{Old}} \cdot U \cdot N_{\text{Old}}$$

   $$= 50k \cdot 70 + 12k \cdot 168 + 0.8 \cdot 46.49 \cdot 70 + 1.6 \cdot 46.49 \cdot 168 - 12k \cdot 7$$

   $$= 20532k$$

2. Analyze the cost for Case 2:

   For case 2, a total of 296 trucks are needed. Since we only have 245 trucks in the
existing network, if we want to apply Case 2 strategy, we need to buy additional 51 new trucks (suppose the purchase cost of the new trucks equals to $c_{New}^{New}$). The network will then have 121 new trucks ($70 + 51 = 121$). On the other hand, since $N_{Old}^{E} = 175 > N_{Old}^{Case2} = 169$, the two choices of the Step 4 of the algorithm (in Section 2.5) should be applied.

Choice 1: Use 6 ($175 - 169 = 6$) old trucks to operate as new trucks

\[
C_{Choice1}^{23} = c_F^{New} * N_{New}^{E} + c_F^{Old} * N_{Old}^{E} + c_U^{New} * U_{New}^{Case2} * N_{New}^{E} + \\
c_U^{Old} * U_{Old}^{Case2} * (N_{Old}^{E} - N_{Old}^{Case2}) + c_U^{Old} * U_{Old}^{Case2} * N_{Old}^{Case2} + \\
P_{New}(N^{Case2} - N^{E}) \\
= 50k * 121 + 12k * 175 + 0.8 * 49.28 * 121 + 1.6 * 49.28 * 6 \\
+ 1.6 * 24.96 * 169 + 50k * 51 \\
= 22692k
\]

Choice 2: Sell 6 old trucks, buy 57 new ones

\[
N_{Old}^{E} = 169 \\
N_{New}^{E} = 127 \\
C_{Choice2}^{23} = c_F^{New} * N_{New}^{E} + c_F^{Old} * N_{Old}^{E} + c_U^{New} * U_{New}^{Case2} * N_{New}^{E} + \\
c_U^{Old} * U_{Old}^{Case2} * N_{Old}^{E} + P_{New}(N^{Case2} - N^{E}) - S_{Old}(N_{Old}^{E} - N_{Old}^{Case2}) \\
= 50k * 127 + 12k * 169 + 0.8 * 49.28 * 127 + 1.6 * 24.96 * 169 + \\
50 * 57 - 12k * 6 \\
= 22912k
\]

(3) Since $C_{Case1}$ is the smallest of the three choices, Case 1 network should be used, and 7 old trucks should be sold.
Chapter 5

Conclusions and Areas of Future Research

5.1 Contributions

This study uses LP and MVA techniques to analyze the different assignment policies for a rental-truck network. The model not only identifies different strategies for the network decomposition, but also analyzes the truck utilizations and fleet sizes. By finding the minimum total cost of the network decomposition, the best assignment policy can be identified.

The LP formulation minimizes the empty truck movements for decomposed new and old truck networks. The two separate networks, once combined, have the same features as the original rental-truck network. At the same time, the demands can be satisfied by making a minimum amount of empty movements to balance the network.

The MVA technique is applied to the decomposed network to analyze truck utilization and the number of trucks needed to meet a certain level of customer satisfaction. The different truck utilizations and fleet sizes can then be used to find the best assignment policy.

In summary, the contributions of this study are:
(1) Analyzing the different assignment policies by using LP to decompose the rental-truck network into new truck network and old truck network, and applying MVA to calculate the corresponding fleet sizes and truck utilizations

(2) Using a total cost function to compare different assignment policies and identify the best strategy

(3) Balancing the network with minimum empty movements, at the same time ensures that customers at different stations can all be equally satisfied.

(4) Decomposing the network into new truck network and old truck network, so that new trucks can have higher utilization than old trucks.

(5) Offering sell and buy strategies by understanding networks with given fleet sizes

5.2 Areas of Future Research

Since there are traveling times on the arcs of the rental-truck network, for a small network with four stations, a total of 16 nodes are needed. Therefore, with the increasing number of stations in the original rental-truck network, the number of nodes needed for the analysis grows quickly. This can greatly complicate the MVA analysis. Further research should be carried out to handle larger networks without drastically increasing the number of nodes.

This study uses the network decomposition technique to analyze the different assignment policies. Other alternative techniques, such as multiclass queuing network analysis and priority queuing network analysis, can be further applied to understand other perspectives of the rental-truck network.

The objective of this study is to find the best assignment policy with minimum operational cost. Other decision-making concerns, such as replacement analysis, and sell and buy decisions, should then be further applied to the results of this study.
Bibliography


Vita

The author, Hui Wang, was born in Benxi, P.R.China in 1972. She obtained her Bachelor of Engineering degree in Materials Management Engineering and Bachelor of Economics degree in Accounting from Northern Jiaotong University of China in 1995. She was a graduate student in National University of Singapore before she joined Lehigh University. During her stay at Lehigh University, she worked as a Research Assistant for the Industrial and Manufacturing Systems Engineering Department.
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