1987

Elastic-plastic analysis of top-and-seat-angle connections, Jr. of Constructional Steel Research, V.8, 1987, p. 119-135, F.L. publication 87-13

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ELASTIC-PLASTIC ANALYSIS OF TOP-AND-SEAT-ANGLE CONNECTIONS

by George C. Driscoll[1]

ABSTRACT

A special way of defining the structure geometry makes it possible to analyze structures with seat-and-top-angle semi-rigid connections using ordinary structural analysis computer programs with line-type bending members. The analysis focuses on the bending behavior of the main beams and columns and of the flexible angles used to make the connections.

Fictitious rigid beams are used to space the bolt lines of the angles at the proper distance from the centerlines of the connected beams and columns. Equilibrium of forces in the angles is thus invoked at the outer fibers of the beams and columns rather than at the centerlines. Similarly, compatibility results according to the plane-sections-remain-plane concept. The angles are each modelled as a pair of rectangular beams of width and thickness equal to those of the angle placed at right angles to each other. The entire assemblage of beams, columns, and angles is analyzed as a rigid frame.

An elastic analysis identifies the locations where stress is greatest and therefore plastic hinges may form. After changes in boundary conditions, additional steps of elastic analysis can give increments in the elastic-plastic load-deflection curve of the entire structure up to the point where a mechanism defining ultimate load is defined.

Examples computed include subassemblages simulating part of a two- or more story frame subjected to either a center concentrated load on a beam or a uniform load on the same beam. In addition, a subassemblage simulating the typical setup for experimental tests of semi-rigid connections is computed, that is, a column with two end-loaded cantilever beams.

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INTRODUCTION

Some types of semi-rigid structural connections have beams connected to columns by flexible angles joined by bolts, rivets, or welds. Such connections depend on self-limiting plastic deformation of the angle sections to occur while the remainder of the structure is in an elastic working load range. The deformation which occurs is known as "contained plastic flow". Experimental results have been available for fifty years and more for several types of semi-rigid connections with angles and other flexible elements. Analytical methods for structures with semi-rigid connections have used empirical functions in the form of smooth curves fitted to typical experimental results. However, analytical methods which consider the specific elastic-plastic behavior of the flexible elements are not currently available.

This report will present an analytical method which focuses on the inelastic behavior of the flexible connection elements. The method will determine when inelastic behavior begins at critical sections in the flexible elements and will calculate the step-by-step load-deflection behavior of structures containing semi-rigid connections.

Description of Connection Type.--To illustrate the principles of analysis to be discussed, a specific type of semi-rigid connection will be selected. The example (shown in Fig.1) will be a top-and-seat-angle connection in which a rolled wide-flange beam W 24 x 100 will be connected to a W 14 x 158 column. The seat angle will be 8 x 4 x 1 in. with the long leg vertical. The top angle will be 4 x 4 x 1. The short angle legs will each have two A325 high-strength bolts, and the long leg will have two rows of two bolts.

In AISC Type 2 "simple framing", the bolts in the vertical leg of the seat angle would be calculated to carry the vertical reaction of the beam and the top angle would be considered only to keep the beam upright. Despite their low strength compared to the beam, the angles have the ability to help the beam resist some bending moment because of their strategic location at the upper and lower extreme fibers of the beam. AISC Type 3 "semi-rigid framing" utilizes this modest moment capacity of the connection. The AISC Specifications recognize that both Types 2 and 3 construction may necessitate some nonelastic, but self-limiting, deformation of a structural steel part in a design based on elastic allowable stresses.

Prior Research.--Experiments on bolted joints conducted in the 1930's were reported by Batho (1936). Rathbun (1936) reported on tests of riveted semi-rigid connections. Further tests on several configurations of riveted semi-rigid connections in the 1930's and 1940's were reported by Hechtman and Johnston (1947). Results were presented in the form of smooth curves fitted to the experimental data in the fashion of the day. In this form, the curves did not appear to highlight points at which distinct changes in stiffness occurred due to inelastic events.

Theoretical analyses of frames with riveted semi-rigid connections were performed by Johnston and Mount using beam stiffnesses modified for partial rigidity based on experimental results (1942).

Sourochnikoff presented a very clear theoretical explanation of the manner in which frames with semi-rigid connections reach a shakedown condition under typical wind loads that effectively relieves any moments on the joints caused by gravity loads (1950). In this state, all of the modest moment capacity of the semi-rigid connections is available for resisting further wind loads. Type 2 "simple framing" design concepts where the beams are designed to resist gravity loading as simple beams while they resist wind moments through frame action are based on the philosophy outlined by Sourochnikoff.

Lothers (1960) reported the results of student theses which developed analytical formulas for initial slope functions fitted to Rathbun's experimental results. These slope functions are used as stiffnesses for beams with semi-rigid connections in stability and other structural analyses. DeFalco and Marino (1966) and Driscoll (1976) used Lothers' formulas to develop charts for determination of the effective length of columns in frames with semi-rigid connections. Because of the large changes in slope of the typical moment-rotation curves, it was concluded that the methods were most suitable for bifurcation problems requiring only the instantaneous initial stiffness in contrast to static structural analysis problems where the stiffness must be correct for any level of load.

Frye and Morris (1975) fitted empirical nonlinear flexibility formulas to test results for several types of semi-rigid connections and then used them in an iterative non-linear analysis procedure for solving load-deflection behavior of structures.

In the 1970's and 1980's, Ackroyd (1979) has analyzed frames with semi-rigid connections modelled as nonlinear rotational springs, also fitted to the results of the familiar past test results.

Each of these investigations has provided insight into the behavior of structures and has provided some tools for the design of structures. The fitted functions used are quite generic in nature and therefore do not depict accurately the behavior of any particular configuration.

Many of the investigators cited and the study committees involved in the preparation of design recommendations for semi-rigid connections have expressed the need for more moment-rotation curves. The adoption of load and resistance factor design (LRFD) will provide an additional opportunity for designers, if suitable methods are available for evaluating resistance. Further experiments and further analytical studies are both possibilities. This paper will attempt to present an analytical method that can be used to reveal more detailed information about specific connections without overwhelming the user with too much information.

FORMULATION OF STRUCTURAL SOLUTION

To be described in the structural formulation are: (1) concepts to model the connection in a structure, (2) a method to analyze the structure with internal hinges, (3) procedures to manage the solution data during successive stages, and (4) examples to illustrate application of the principles.

Modeling the Connection.--An assemblage of beams, columns and legs of connection angles will be analyzed as a rigid frame. Successive stages of the solution will be obtained by using a typical linear frame analysis computer program.

Each leg of the connection angles can be represented by a beam of rectangular cross-section having the thickness and width of the angle. The elastic stiffness and plastic moment properties of the rectangular cross-section are well known.

To assemble the members into a structure using coordinate geometry, it appears that the angle members must float out in space with no connection to the one-dimensional beam and column members (Fig. 2). It would at first seem that a finite element solution using continuum elements should be chosen. However, a modification to the model is accomplished by using dummy rigid beam members to attach the connection angles to the main beams and columns (Fig. 3).

The important bending of the angles occurs between the heels of the angles and the line of fasteners on the nearest gage line to the heel. Between that point on each angle and the toe of the angle, no bending occurs and that portion of the angle is omitted in this analysis. Therefore, a dummy rigid beam member is inserted perpendicular to the main beam, connecting nodes on the gage line of the angle and on the main beam centerline. It is assumed that the high-strength bolts fasten the angles so firmly that a rigid joint can be assumed at those points.

At the points of attachment to the main members, the dummy members will tend to have the same rotation and displacement as the main members. The consequent displacements at the end of the dummy members attached to the angles will be appropriate to the "plane-sections-remain-plane" philosophy of beam mechanics. Similarly, the equilibrium forces resulting will be consistent with the geometric position of the angles at the outer fiber of the main members.

Angle Members.--Each angle is modeled in the plane of the frame as two short beams joined at right angles at the point representing the heel of the angle. The cross-section of the angle beams is a narrow rectangle. Its height is the thickness **t** of the angle leg, and its width **b** is the length of the angle shape used as a cleat (Fig.4). The properties **t** and **b** are used in calculating axial area and moment of inertia for stiffness calculations along with plastic moment and axial yield load for load capacity calculations.

The correct length of angle beam to use in stiffness calculations is not so clear. For the examples used here, the length of each leg was taken as the distance from the heel of the angle to the first bolt gage line. A valid case can be made for more than one other definition.

The equations for cross-sectional properties are:

Area $\quad A = b * t \qquad (1)$

Moment of Inertia $\quad I = b * t^{3} / 12 \qquad (2)$

Plastic Modulus $\quad Z = b * t^{2} / 2 \qquad (3)$

The equations for plastic capacity are:

Plastic Moment $\quad Mp = Fy * Z \qquad (4)$

Axial Yield Load $\quad Py = Fy * A \qquad (5)$

Where Fy is the yield strength of the steel.

Stiffness Modifications for Hinges.--For the direct stiffness method, each member is initially defined as a cantilever basic element in its local coordinate system.

Three end forces: axial load **P**, shear **V**, and moment **M** are related to the corresponding end distortion v by a 3 by 3 basic element stiffness matrix **k'**.

$$\{S\} = |k'| \{v\} \qquad (6)$$

To deal with the member end forces and nodal displacements in the global coordinate system, an equilibrium matrix **T** and a direction cosine matrix **L** are combined in the expression

$$\{R\} = |L| \; |T| \; |k'| \; |T|^T \; |L|_T \; \{r\} \qquad (7)$$

where *r* are nodal displacements in global coordinates and **R** are nodal forces in global coordinates.

The member stiffness terms are added into the appropriate locations in the global stiffness matrix for the entire structure.

The process of incrementing solutions after formation of a plastic hinge will involve adding a proportional part of the entire array of a new structural solution to the entire array of the prior cumulative structural solution.

In order that no moment increase should occur at the location of a plastic hinge, a "real" internal hinge is inserted in the affected member at that location. Fig. 7 shows the modification of the basic element stiffness for each possible location of an end hinge. Possible cases are: no hinges, hinge at the i end of a member, hinge at the j end of a member, and hinges at both ends of a member. It is shown that each modification either changes a stiffness term to zero or multiplies a stiffness term by a fraction. In computer programs, such modifications are programmed to occur automatically in response to a flag or indicator.

Rigid Spacing Elements.--The function of the dummy members is to project the centerline displacements and rotations of beams and columns to the surface locations where the connection angles are attached. The stiffness of the dummy members should be large enough so that their distortions would be negligible relative to those of the beams, columns, and angles. Conceptually, the combination of member cross section properties and modulus of elasticity might be very large, near infinity. However, this would result in an ill-conditioned structure stiffness matrix. To overcome the ill-conditioning, the stiffness of the dummy members must be reduced drastically. For practical purposes, stiffnesses over 100 times the angle members would cause displacements that are negligible relative to the actual members. Lacking experience with the procedures, a range of elastic and geometric properties were tried. Satisfactory results appeared to be obtained with an elastic modulus 10 to 1000 times that of steel, areas equal to 100 times those of the main beam, and moment of inertia 30 times that of the main beam.

MANAGEMENT OF THE STRUCTURAL CALCULATIONS

A flow chart for the step-by-step solution is given in Fig. 8.

Setup.--A place for storage of the plastic capacities of members and the cumulative results must be provided so that tests may be made and sums may be accumulated in the computer. Plastic bending moments and axial capacities are first determined manually.

Then, a self-prompting program SETUP is executed. In sequence of member numbers, the capacity of each of the structural components is requested for both ends of each of the members. The results are entered manually, using the calculated values for axial and moment capacities of real beams and columns and the angle legs. Shear terms are entered as zero. All components of the rigid dummy members are entered as zero, and any member end at which a hinge is located are also recorded as zero. In the later step which tests for reaching capacity, any component with a capacity listed as zero is automatically skipped. At the end of program SETUP, a file of force component capacities is written. A file of cumulative structural results is initialized to zero in order that it may be incremented by later steps.

Analysis.--A linear structural analysis of the current connection model is executed with a unit load value (say, 100). Member forces, nodal displacements, and nodal loads are written to a separate output file identified as the current data using the FORTRAN binary write statement. This analysis is made using a linear direct stiffness method computer program modified only by the necessary three write statements of the form:

```
WRITE(u) X
```

where u = unit number for a file opened to receive sequential binary data

X = variable name for an array element of stress, displacement, or force

At this stage, data files are available for the current "unit load" output step, the capacities of each member end force component, and the previous cumulative structural analysis result (it might be the initialized result prepared in SETUP). A program STEP is executed to test for the next inelastic event.

Step Calculation.--Program STEP reads the capacity file, the previous cumulative file, and the current structural analysis file, all of which are binary write files written with the same sequence and spacing. By testing each member end force element in sequence, STEP calculates the least amount of added proportionally applied load to increase that force element from its cumulative state to its plastic capacity. The increment is determined as a percentage of the applied load in the structural analysis needed to produce the critical change.

The load increment is tested at each calculation loop to discover if it is less than the least increment recorded for another end force element. If so, a new least increment is defined and stored. When all member end force elements have been tested, the search is ended. The saved load increment multiplier is multiplied by each output result of the current structural analysis and added to the prior cumulative value giving a new cumulative value which is written to a file for the next cumulative set of results. The process of determining the load increment is flow charted in Fig. 9.

At this stage, the investigator decides manually whether to terminate the computation or to continue for another increment of increase in load. When the decision is to continue for another increment, the investigator edits the raw input data for the structural analysis program to change the boundary condition to a real hinge at the location of the plastic hinge just formed. This process requires changing only one character in the data file, an end condition flag for the affected member.

It would be relatively easy to create computer code which would automatically proceed to the next cycle of analysis and increment calculations. However, at this time, no effort was made by the investigator to program the much more difficult task of deciding whether a plastic mechanism has formed. Until more experience is gained with the mechanisms possible for different connection types, a manual decision is to be preferred.

Solution Summary.--When all increment steps have been completed, it is desired to assemble a summary of results in hard copy form. A final brief fortran program SUMCUM has been written to read the several binary files of cumulative data and create a formatted text (ASCII) output file with the cumulative value of each force and displacement tabulated for each step. It has been found convenient to import the ASCII file into a spreadsheet, where selected functions can be graphed.

EXAMPLES

Plot of Results.--The first example is the centrally-loaded frame of Fig. 5. A load-deflection curve for the frame is given in Fig. 10. It is compared with the curve if the beam were simply-supported and subjected to the same central concentrated load. A horizontal line marks the level for calculated allowable load if the beam were simply supported. Formation of plastic hinges is represented by plot symbols on the curve. It can be noted that two plastic hinges formed at lower loads than the simple beam allowable. These points represent plastic hinges at the bolt line and heel of the top angle (nodes 3 and 5 of Fig. 6). It also can be noted that the frame retains most of stiffness until three more plastic hinges form, at the heel and bolt line of the horizontal leg of the seat angle (nodes 10 and 11) and then at the centerline of the beam (node 8). Up to this point, the deformation is in a state of self-limiting plastic flow. Thereafter, the deflection increases rapidly with very small increase in load until a plastic hinge forms at the bolt line of the horizontal leg of the top angle (node 6).

Other Examples.--Two other examples are compared with Example 1 in Fig. 11. Example 2 is the same frame and connections as Example 1, but with a uniformly distributed load applied to the beam. The result shows that the total applied load W is equal to the load P of Example 1, but the displacements are less. Both the centerline displacement and the end rotation of the beam are less, due to the applied load being distributed along the beam with part of it much nearer to the supports.

Example 3 models the subassemblage frequently used for experimental connection tests. Two cantilever beams are connected at midheight of a free-standing column. The beams, columns, and connections are identical to those of Examples 1 and 2, buth the beams lack the redundancy of those structures. The graphs for Example 3 show that the cantilever loading is a very severe test for the connections.

Plastic Analysis of Connection Assemblages.--After the solutions were obtained, it was found that a simple plastic analysis would result in the same maximum load as that reached in the step-by-step analysis. In Fig. 12a, the plastic analysis of a panel mechanism achieves the same ultimate load as the frame Examples 1 and 2. In Fig. 12b, a beam mechanism, with three plastic hinges along one line, results in the same ultimate load as the cantilever beams of Example 3.

SUMMARY AND CONCLUSIONS

Conclusions made thus far in the investigation include:

(1) Semi-rigid connections with top-and-seat angles may be analyzed by linear elastic plane frame programs.

(2) Results appear to be consistent with previous experiments. Comparison with raw data would be desirable.

(3) Final mechanisms and ultimate loads agree with mechanism method plastic analysis solutions.

(4) Plastic hinges appear in the angles before the beam reaches the simple beam allowable load.

(5) The concept of "self-limiting plastic deformation" is supported by an adequate reserve above the simple beam allowable load, along with deflections less than those for a simple beam.

(6) In a complete structure, the semi-rigid connections are constrained to deform to match the behavior of the main beams and columns. Contained plastic flow occurs.

(7) The load-deflection behavior is a step-by-step

piecewise linear function. It does not match the smooth curves often assumed in current analyses. These curves are a heritage of research of the 1920's and '30's when it was fashionable to fit a smooth curve to all test data. This sometimes obscurs the real behavior.

Separate checks must be made in design for fracture of fasteners and angles, and for crippling or local buckling at points of attachment to main beams and columns. It should be possible to modify the plastic moment of the connection angles to account for axial forces in the angle legs.

The method could be used to analyze other cases of flexible connection details such as top plate connections and moment connections with structural tees.

ACKNOWLEDGEMENTS

This study was originally performed in October 1984 out of pure curiosity. The calculations were performed on the author's personal home Zenith Z-100 microcomputer with 192 K of main memory. The programs were written in Microsoft FORTRAN. Plot data were input into a LOTUS 123 spreadsheet in order to create the original graphs.

The first presentation of the work was in a Civil Engineering Department seminar at Fritz Engineering Laboratory, Lehigh University. Dr. Irwin J. Kugelman is Chairman of the Department of Civil Engineering.

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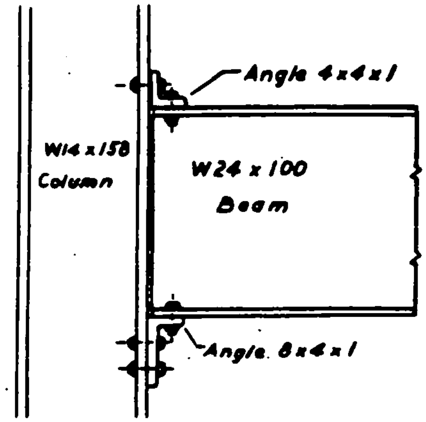


Fig. 1.--Top-and-Seat-Angle Connection

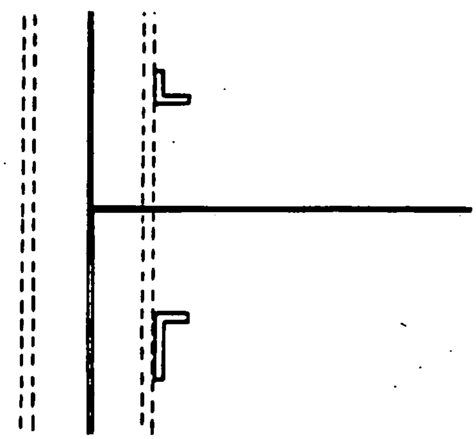


Fig. 2.--Computer Model with "Floating" Angles

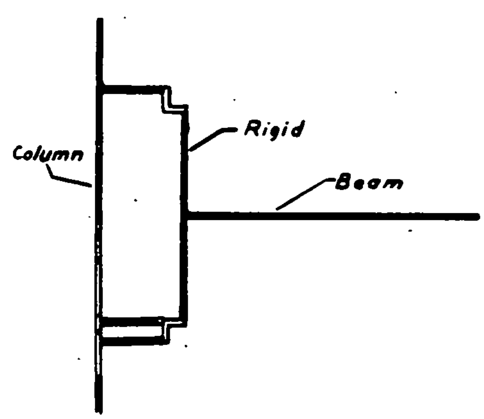
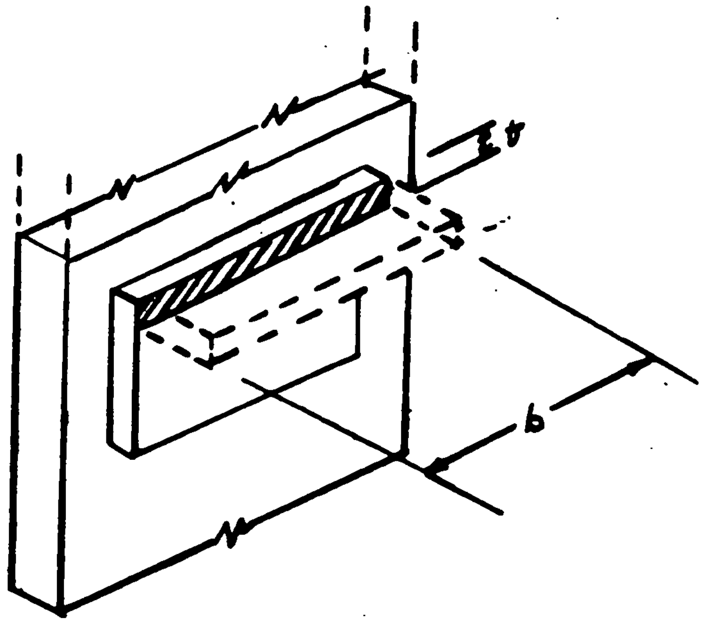


Fig. 3.--Computer Model with Dummy Rigid Members



$$Z = \frac{bt^2}{4}$$

$$M_p = F_y Z$$

Fig. 4.--Cross-section Properties of Angles

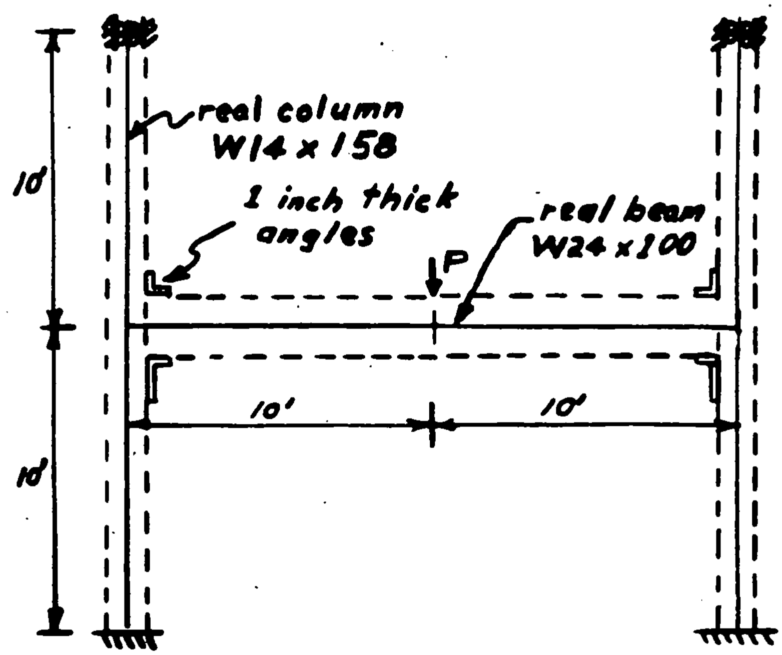


Fig. 5.--Rigid Frame with Semi-Rigid Connections

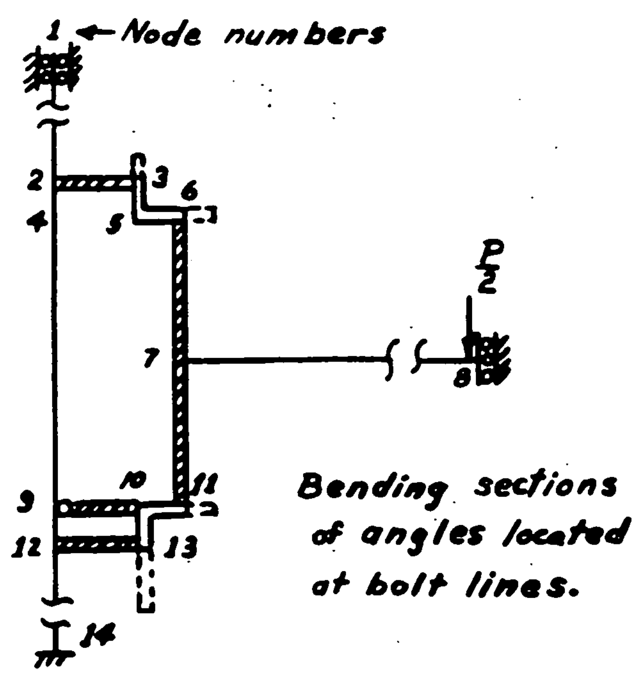
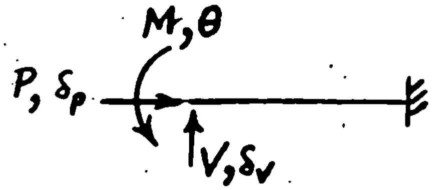
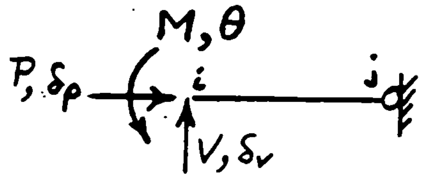


Fig. 6.--Node Numbers in Computer Model



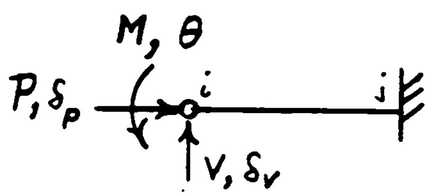
$$\begin{vmatrix} P \\ V \\ M \end{vmatrix} = \begin{vmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & k_{23} \\ 0 & k_{32} & k_{33} \end{vmatrix} \begin{vmatrix} \delta_p \\ \delta_v \\ \theta \end{vmatrix}$$

(a) Normal Member Stiffness



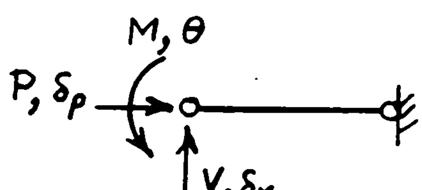
$$\begin{vmatrix} P \\ V \\ M \end{vmatrix} = \begin{vmatrix} k_{11} & 0 & 0 \\ 0 & \frac{1}{4}k_{22} & \frac{1}{2}k_{23} \\ 0 & \frac{1}{2}k_{32} & \frac{3}{4}k_{33} \end{vmatrix} \begin{vmatrix} \delta_p \\ \delta_v \\ \theta \end{vmatrix}$$

(b) Hinge at j End



$$\begin{vmatrix} P \\ V \\ M \end{vmatrix} = \begin{vmatrix} k_{11} & 0 & 0 \\ 0 & \frac{1}{4}k_{22} & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \delta_p \\ \delta_v \\ \theta \end{vmatrix}$$

(c) Hinge at i End



$$\begin{vmatrix} P \\ V \\ M \end{vmatrix} = \begin{vmatrix} k_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \delta_p \\ \delta_v \\ \theta \end{vmatrix}$$

(d) Hinges at Both Ends

Fig. 7.--Change in Member Stiffness Due to Hinges

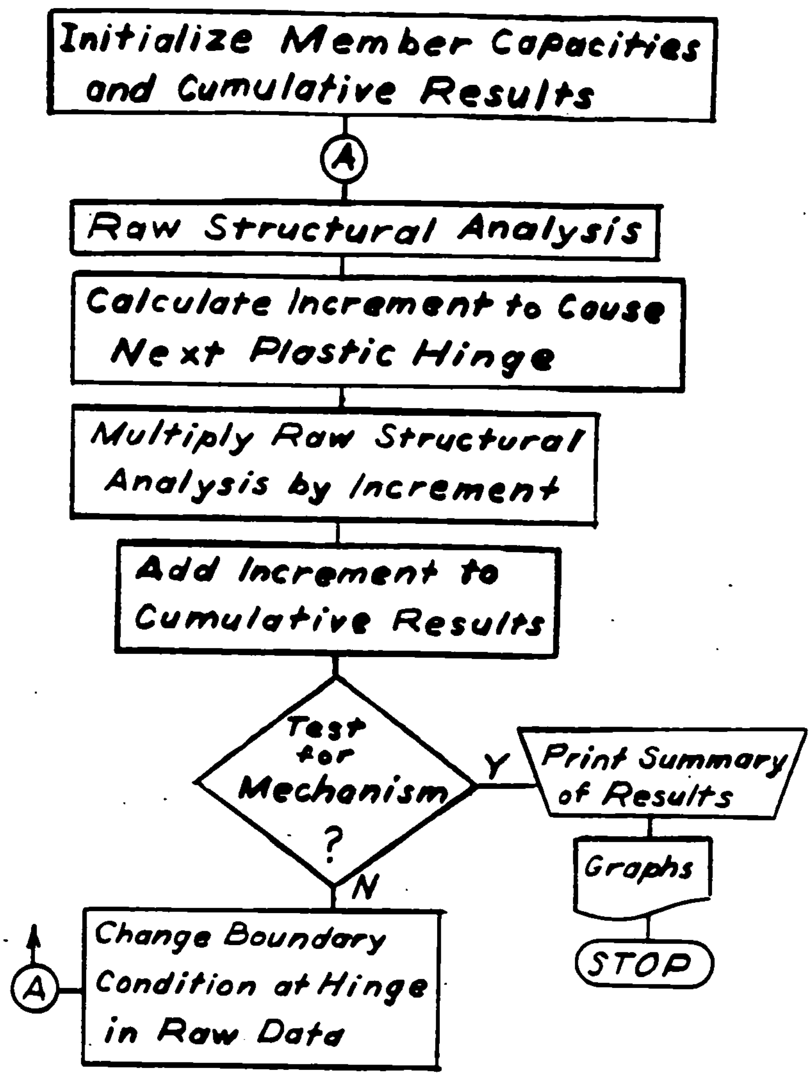


Fig. 8.--Flow Chart for Step-by-Step Calculation

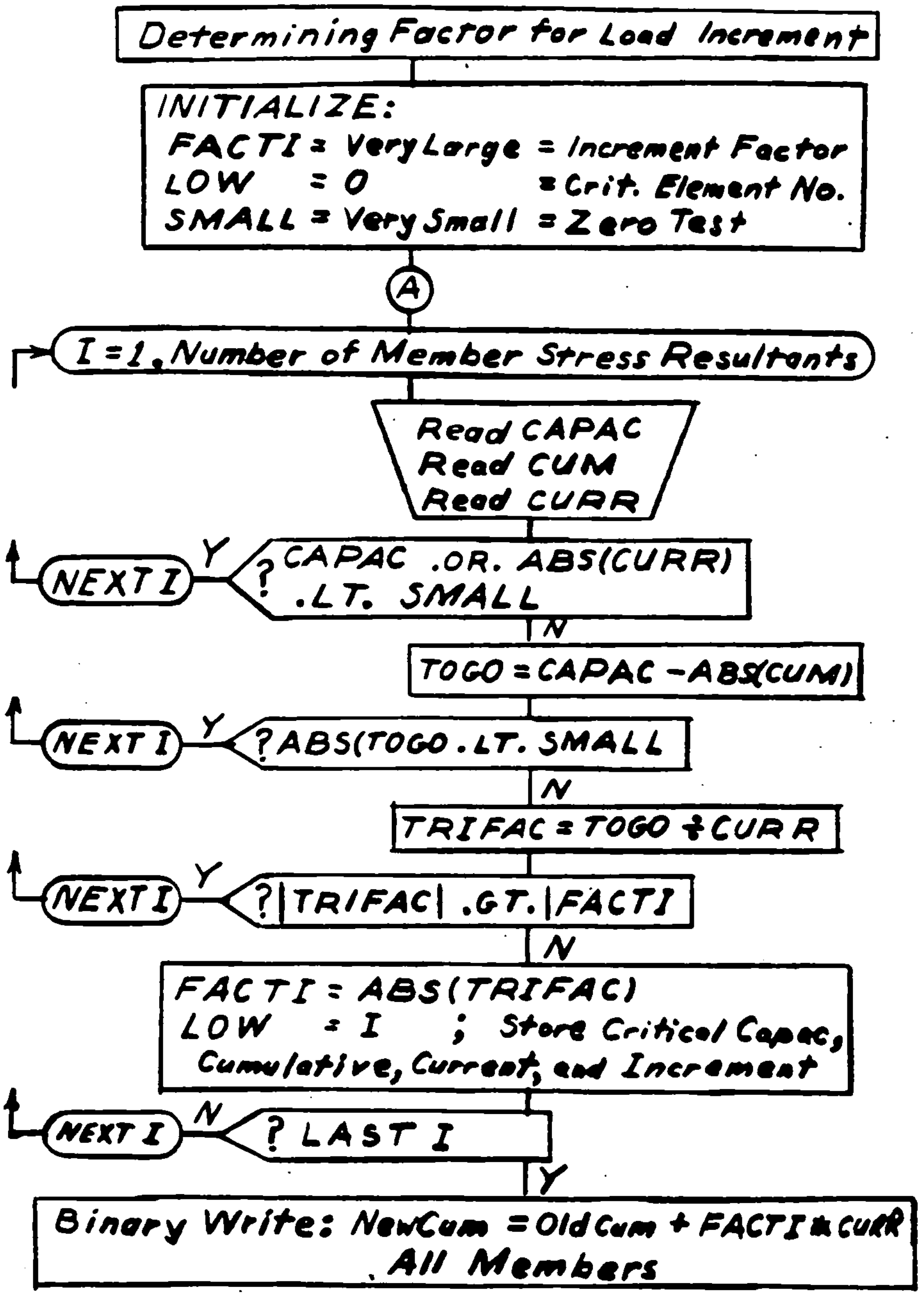


Fig. 9.--Flow Chart Determining Factor for Load Increment

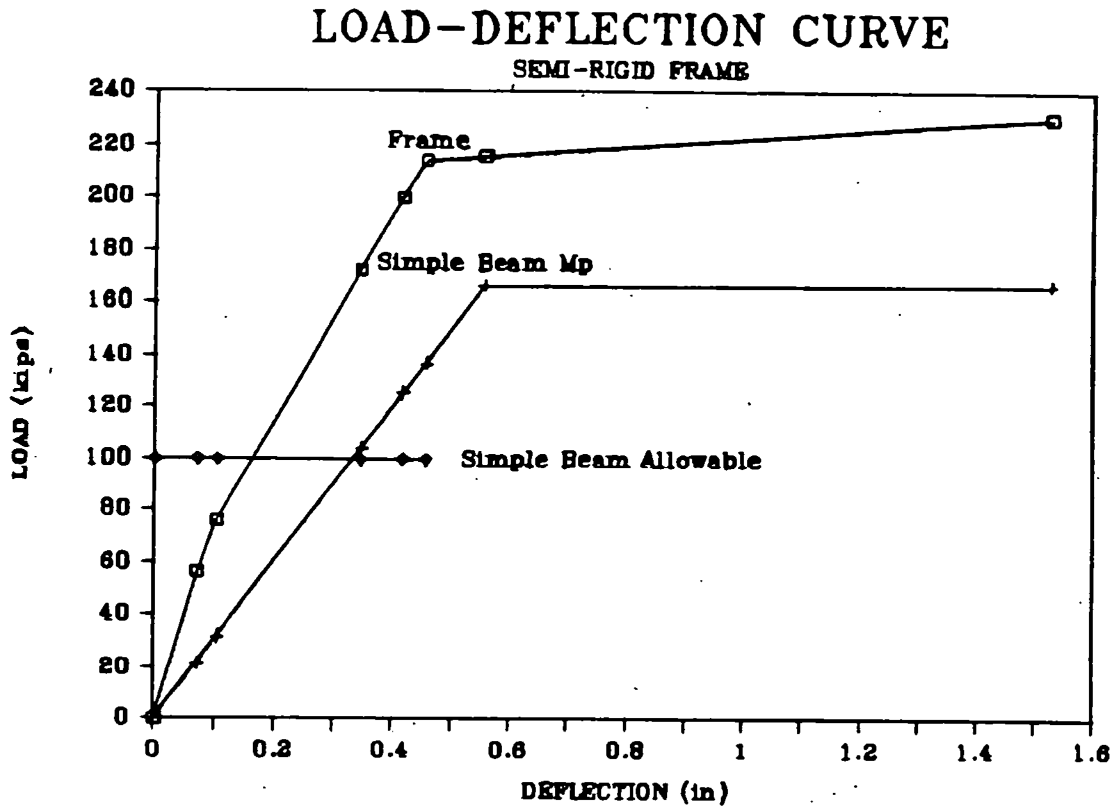
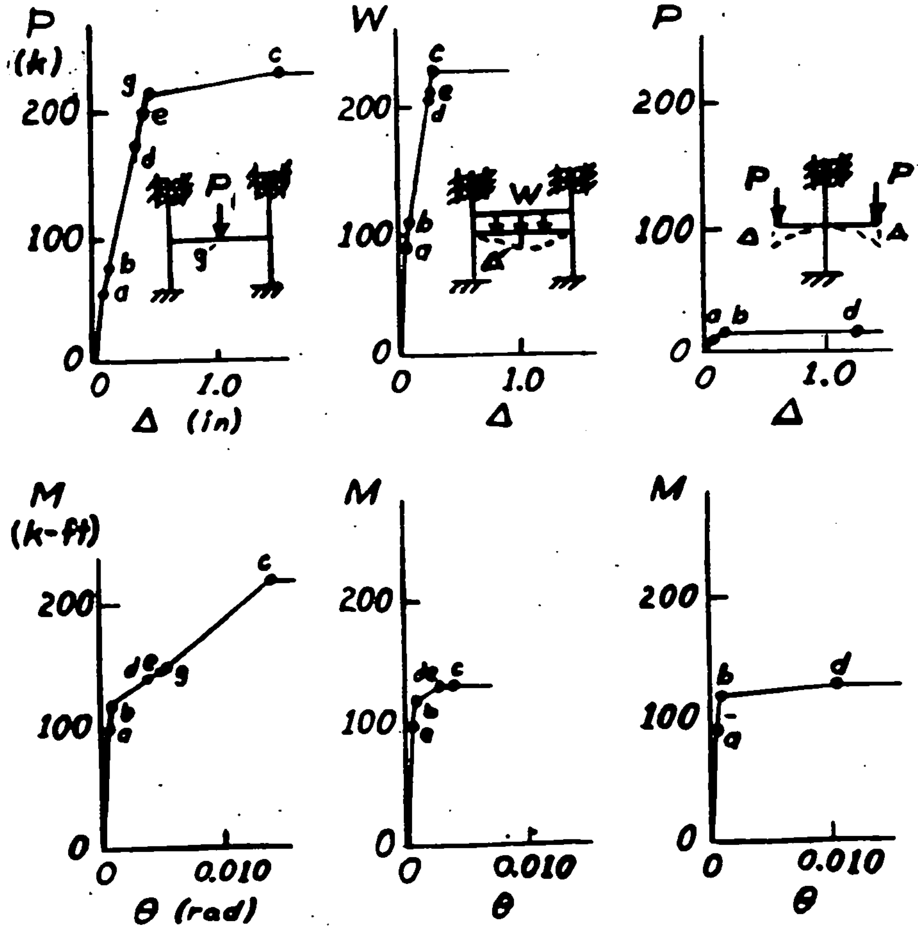
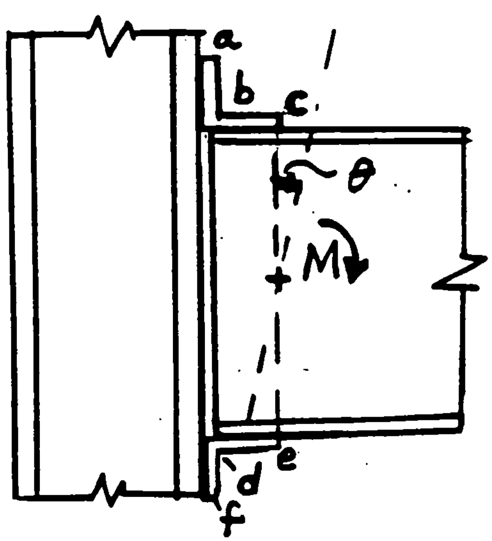


Fig. 10.--Load-Deflection Graph -- Frame of Example 1

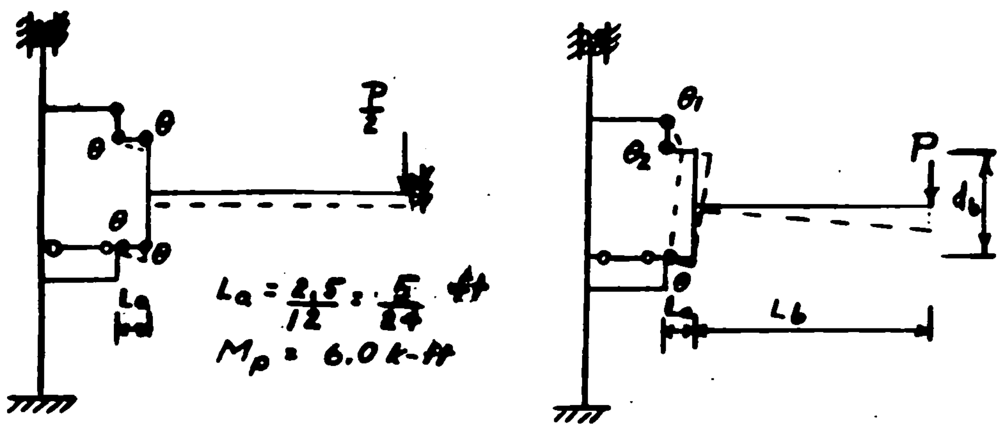


(a) Example 1 (b) Example 2 (c) Example 3



(d) Key to Plastic Hinge Sequence

Fig. 11 --Load-Deflection and Moment-Rotation Graphs -- 3 Structures



$$\frac{P}{2} L_a \theta = 4 M_p \theta$$

$$\boxed{P = \frac{8 M_p}{L_a}} = 8(6)\left(\frac{24}{5}\right)$$

$$\boxed{P = 230.4 \text{ k}}$$

$$P(L_a + L_b)\theta = M_p(\theta + \theta_1 + \theta_2)$$

$$\theta d_b = \theta_1 L_a \therefore \theta_1 = \frac{d_b}{L_a}\theta$$

$$\theta_2 = \theta + \theta_1$$

$$\boxed{P = \frac{M_p\left[2\left(1 + \frac{d_b}{L_a}\right)\right]}{(L_a + L_b)}} = \frac{6\left(\frac{106}{5}\right)}{9.375}$$

$$\boxed{P = 13.568 \text{ k}}$$

(a) Panel Mechanism for Examples 1 and 2

(b) Beam Mechanism for Example 3

Fig. 12.--Plastic Analysis of Frames with Semi-Rigid Connections