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INITIATION AND PROPAGATION OF
FAILURE MECHANISM IN MASONRY INFILL WALLS

BY
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FRITZ ENGINEERING LABORATORY REPORT No. 433.3 A
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This report describes an approach for the analysis of masonry infilled reinforced concrete frames using a piecewise linear tangent stiffness finite element modified incremental procedure. The initiation and propagation of failure mechanisms are determined based on a biaxial failure envelope for masonry mortar and a uniaxial failure criterion for masonry units. Nonlinear, orthotropic material behavior is assumed for the masonry mortar and its material properties are determined from the stress-strain curve that is to be provided. Results are given for several theoretical case studies and show the initiation and propagation of cracking through the masonry walls.
1. **INTRODUCTION**

The structural behavior of brick masonry is possibly the least understood of all construction materials though today its use is widespread. In both low and high rise buildings brick masonry walls, whether preassembled or laid insitu, have functioned as load carrying elements. They are able to resist gravity loads and lateral loads very effectively because of their inherent stiffness in their own plane. In addition, brick masonry walls are economical to construct and provide good thermal and sound insulation and fire resistance. Quantitatively, the practicing professional engineer is faced with the analysis and design of these brick masonry structures. Many of his decisions must be based solely on experienced judgment or theories and concepts currently in use since there is no standard methodology he/she can employ. Therefore, the need is apparent that new analytical methods be developed that can simulate the structural behavior and response of this structural material.

1.1 **Problem Statement**

The research described in this report investigates the structural behavior of brick masonry infill walls with reinforced concrete frames. An analysis technique is developed to theoretically predict the response of such walls, including the initiation and propagation of cracking, when subjected to inplane loads. The infilled frame-wall system can be considered a basic structural component of an entire structural system being comprised of a brick masonry wall, reinforced concrete
frame, and a layer of masonry grout between the wall and frame.

This basic structural component under consideration is assumed:

1. to lie in one plane, i.e. be planar.
2. to have arbitrary but rectangular boundaries
3. to have no reinforcing steel either vertical or horizontal within the brick masonry wall
4. to be subjected to only an arbitrary compressive load at the beam-column connection of the reinforced concrete frame.
5. to have various possible support conditions to simulate different degrees of external fixity for the reinforced concrete frame.

1.2 Scope of the Investigation

The following items have been studied within the framework of this research and are presented in this report:

1. An analytical modeling of the brick masonry wall, reinforced concrete frame, and masonry grout layer using the finite element method.
2. A biaxial failure envelope together with appropriate stress-strain laws used to develop constitutive relations for masonry mortar and grout.
3. A failure criterion for masonry bricks.
4. An analysis technique for nonlinear material behavior using a piecewise linear tangent stiffness incremental approach.
1.3 Previous Studies

Numerous investigations have been conducted on the behavior of masonry infilled frames. Benjamin and Williams performed tests on 20 full-scale and model one-story brick shear walls with reinforced concrete bounding frames (Ref. 1). The walls were loaded by a concentrated horizontal force at the top of the wall, applied to the corner of the bounding frame (Fig. 1). The frames were designed to have a higher strength than the masonry panel so the tension column steel would not yield due to the largest load attained and the ultimate load in the panel was reached before the compression column sheared off at the foundation. The investigators observed that the typical wall test showed a first crack generally along the junction of the panel and the foundation and the tension column. This was followed by cracking in the region of the loading beam and compression column thereby essentially freeing the masonry panel from the frame except at the loaded corner and the junction of the foundation and compression column. Boundary cracking occurred at varying load magnitudes, usually considerably below ultimate. This cracking did not produce any great change in rigidity. Final failure and ultimate load were characterized by a sudden crack through the wall panel essentially on the compression diagonal. Benjamin and Williams proposed the following ultimate load formula for brick panels with reinforced concrete frames based on the test observation that the brick wall panel is essentially cracked free of the frame prior to reaching the ultimate load. This assumes that the frame itself is not critical. The panel is then under an almost
pure racking load:

\[ P = \frac{220 \ C \ a \ t(a/b)}{1.5(a/b) - 1.1C} \]  \hspace{1cm} (EQ. 1.1)

where:

- \( a \) = panel length
- \( b \) = panel height
- \( t \) = panel thickness
- \( C \) = workmanship coefficient \( 0.6 \leq C \leq 1.0 \)
- \( P \) = applied shear load.

The following conclusions were also drawn from the tests:

1. The length to height ratio has an important influence on ultimate strength and rigidity.
2. The frame had no important influence on wall strength as long as it was strong enough to produce failure in the wall panel.
3. Predictions of behavior must be approximate in nature with possible errors of as such as 50 per cent or more in ultimate load and rigidity depending on the workmanship.

The test results obtained by Benjamin and Williams on reinforced concrete frames with masonry shear walls have been compiled by Sahlin and are shown in Fig. 2 (Ref. 2). The tests were run with different sizes and types of frames as well as without frames. The tests without frames gave very low strength, which could be calculated on the basis of tension failure between the wall and the foundation. Sahlin gives the following equation as a good approximation within the test range for the ultimate load-carrying capacity of the masonry infilled frame:
\[ \frac{T}{A} = 150 \text{ psi} \quad \text{(EQ. 1.2)} \]

with the horizontal force \( T \) and the horizontal mortar joint area \( A \), i.e., the wall thickness times the wall length. Scrivener also conducted a series of tests on 8'-8" high and 8' long reinforced concrete hollow masonry walls subject to static racking loads (Ref. 3). The walls were designed to fail in shear, by applying a vertical load which was just sufficient to balance, at each increment of racking load, the moment of the racking load about the wall toe. A pattern of cracks parallel and close to the diagonal joining the loaded corner and the toe of the wall was obtained in each case. No bounding frame was used, though a loading frame with a reinforced concrete base was built around the concrete masonry wall. Reinforcing steel was used in varying percentages from zero up to approximately 0.50% of the gross cross-sectional area of the wall. Under load the walls all behaved similarly. At low loads, the loaded-end deflections were very small and gradually increased with increasing load. The racking load was increased from zero in 4000 lb. increments until failure, which was taken to be the maximum load that could be applied to and held by the wall. A first crack was usually noticed with a horizontal wall deflection of between .01" and .02" positioned along or adjacent to the diagonal joining the loaded corner and the wall toe. Further increments of load produced extensions of these first cracks and additional cracking along or parallel to this diagonal line. Although cracks were usually in the mortar, there were many instances of cracks across the concrete.
blocks, even at moderately low loads. Severe cracking, defined as the stage where some cracks reached .01" width, was accompanied by much larger incremental deflections. Finally further load could not be maintained, and failure was considered to be reached. Figure 3 shows the crack pattern of a wall with the typical diagonal cracking from loaded corner to toe. Even after the failure load was assumed to be reached, it was found that further load application of 80-95% of the failure load could still be carried by the walls, particularly the more heavily reinforced walls.

Sahlin also reported that Feodorkiw used a lumped-parameter model to represent a reinforced concrete frame with masonry filler walls subjected to inplane forces (Ref. 2). The model's response to loads was then programmed and calculated numerically. The progressive locations of cracking within the structure were determined on the basis of successive solutions as load was increased up to a theoretical ultimate failure. The calculations show that approximately the same ultimate load capacity can be attained, irrespective of the value of filler modulus, provided shear failure in the tension column is prevented. The significance in this is that a poor filler contributes much to the strength of a frame, even though it needs a certain stiffness to drastically reduce the deformations. A bare frame has still larger deformations but considerably lower load-carrying capacity.

In contrast to the numerous experimental testing programs few studies have examined the mechanisms of failure in structural masonry. An investigation into the failure mechanism of brick masonry loaded in
Axial compression was conducted by Hilsdorf (Ref. 4). The failure model developed assumes brick masonry is a two-phase material, where both phases not only have different strengths but also different deformation characteristics. In general, the modulus of elasticity (and the uniaxial compressive strength) of the mortar is considerably lower than the corresponding values of the bricks. Therefore, if the mortar could deform freely, its lateral strains would be larger than the strains in the bricks. However, because of bond and friction between brick and mortar, the mortar is confined. An internal state of stress exists which consists of axial compression and lateral tension in the brick and confined biaxial (i.e. plane strain) compression in the mortar (Fig. 4) allowing externally applied loads to exceed the uniaxial compressive strength of the mortar. In addition to the external load and internal stress state described, the bricks are subjected to flexural and shear stresses arising from incomplete filling of the mortar joints or varying thicknesses of the bricks and joints. These may also result in an uneven distribution of the external load. Then, stress concentrations are developed in the bricks which may be considerably larger than the average strength properties originally assumed.

Figure 5 shows the development of stresses as they may occur in a single brick within a masonry unit subjected to axial compression. The local maximum stresses, $\sigma_y$, act in the direction of the external load and can be computed from the average masonry stresses, $\sigma_{yn}$, and a coefficient of nonuniformity U. The lateral tensile stresses,
\( \sigma_x \) and \( \sigma_z \) are assumed equal. Line A in Fig. 5 represents the failure criterion for the triaxial strength of bricks and indicates the combinations of compressive stresses \( \sigma_y \) and the lateral tensile stresses \( \sigma_x \) and \( \sigma_z \) which will cause local failure or cracking of the brick. If line A is assumed to be straight, the following equation describes the failure criterion of the brick.

\[
\sigma_x = \sigma_z = f'_{bt} \left[ 1 - \frac{\sigma_y}{f_b} \right] \quad \text{(EQ. 1.3)}
\]

in which \( \sigma_x, \sigma_y, \sigma_z \) = stresses in \( x, y, \) or \( z \) direction

\( f'_b \) = uniaxial compressive strength of brick

\( f'_{bt} \) = strength of brick under biaxial tension \( \sigma_x = \sigma_z \)

If the masonry is subjected to a compressive stress \( \sigma_{ym} \), lateral tensile stresses \( \sigma_x \) and \( \sigma_z \) are developed following the dashed line B in Fig. 5. If this line intersects the failure criterion line A, a crack occurs in the brick in a direction parallel to the direction of the externally applied load. This, however, does not correspond to complete failure of the masonry unit. At the cracked section the lateral stresses diminish, and part of the flexural stresses in the uncracked sections of the brick are relieved. A certain minimum lateral compressive stress has to act upon the mortar if the external load is already larger than its uniaxial compressive strength. This stress is counterbalanced by tensile stresses in the uncracked sections of the bricks. These minimum tensile stresses are represented by line C in Fig. 5. With increasing external load on increasing local stress, the minimum lateral tensile stress increases.
If the external load is increased beyond the load at first cracking, then stresses may develop along line B2 in the uncracked section of the brick. When line B2 intersects the failure criterion line A a second crack is formed. This process of cracking continues until the lateral tensile strength of the brick is smaller than the stress which is necessary to sufficiently confine the mortar. The intersection of line A and line C corresponds to the ultimate load of the masonry unit.

According to Hilsdorf's theory, the factors that influence the compressive strength of masonry are:

1. The uniaxial compressive strength of the brick.
2. The biaxial tensile strength of the brick.
3. The failure criterion for bricks under a triaxial state of stresses as represented by line A.
4. The uniaxial compressive strength of the mortar which corresponds to the onset of line C.
5. The behavior of the mortar under a state of triaxial compression, determining the shape and inclination of line C.
6. The coefficient of nonuniformity.

If the triaxial strength of the mortar follows the relationship established by Richart, Branutzaeg, and Brown for concrete, as given by Hilsdorf, then the strength of the mortar joint is (Ref. 4)

$$\sigma_y = f'_j + 4.1 \sigma_2$$  \hspace{1cm} (EQ. 1.4)

where

$$\sigma_x = \sigma_z = \sigma_2 = \text{lateral confining stress in the mortar}$$
\[ f_j' = \text{uniaxial compressive strength of the mortar} \]
\[ \sigma_y = \text{local compressive stress in the mortar} \]

If the lateral stresses \( \sigma_x \) in the bricks and mortar joints are uniformly distributed over the height of the bricks and mortar, then from the equilibrium condition it follows that

\[ \sigma_{xb} \cdot b = \sigma_{xj} \cdot j \quad \text{(EQ. 1.5)} \]

where

\[ \sigma_{xb} = \text{lateral tensile stress in bricks} \]
\[ b = \text{brick height} \]
\[ \sigma_{xj} = \text{lateral compressive stress in mortar joint} \]
\[ j = \text{joint thickness} \]

Substituting Equation 1.5 in Equation 1.4 and replacing \( \sigma_2 \) with \( \sigma_{xj} \) the equation for line C in Fig. 5 is obtained.

\[ \sigma_x = \frac{j}{4.1b} \left( \sigma_y - f_j' \right) \quad \text{(EQ. 1.6)} \]

From Equations 1.3 and 1.6 the maximum local stress at failure can be determined corresponding to the intersection of lines A and C:

\[ \sigma_y = f_b' \cdot \frac{f_{bt}' + a \cdot f_j'}{f_{bt}' + a \cdot f_b'} \quad \text{where} \quad a = \frac{j}{4.1b} \]

Using the nonuniformity coefficient at failure \( U_u \), the average masonry stress at failure can be expressed as

\[ \sigma_{ym} = f_m' = \frac{\sigma_y}{U_u} \]

Then we obtain as a general expression for the axial compressive strength of masonry:
This expression established by Hilsdorf, depicts the known relationships between the compressive strength of masonry and various parameters: masonry strength increases with increasing compressive strength of bricks and mortar, with increasing tensile strength of bricks, and with decreasing ratio of joint thickness to the height of brick. It should be realized, however, that \( U \) is not a constant but depends on a number of parameters including quality of workmanship, type and compressive strength of mortar, type of bricks, pattern of masonry units and courting of bricks, and thickness of joints.

More recently, Yokel and Fattal carried out an investigation dealing with the load capacity of brick masonry walls subjected to a diagonal compressive edge load combined with a compressive edge load acting in the plane of the wall and normal to the direction of the mortar bed joints (Ref. 5). The loading and boundary conditions of these walls are similar to those encountered in certain shear wall elements in buildings. Based on test information, three failure hypotheses for splitting and one hypothesis for joint separation are considered. The hypotheses for splitting are: (1) failure by critical normal stress, (2) failure by a critical biaxial combination of normal principal stresses, a concept, recently corroborated for concrete by best results, which covers a broad spectrum of different failure hypotheses advanced in the past, and (3) failure at a critical in-plane tensile strain, a hypothesis that could be utilized in a single
failure criterion to account for "tensile" as well as "compressive" failures. Failures that are actually documented for the tests considered are shown in Fig. 6 and can be placed in three general categories:

1. Separation along mortar joints.

2. Splitting, generally in the direction of \(\sigma_1\), (which is normal to the direction of crack propagation), in a region along the loaded diagonal which includes the center of the panel.

3. Splitting, approximately in the direction of \(\sigma_1\), most severe in the vicinity of the loading fixtures and not necessarily including the center of the panel.

The three splitting hypotheses are mutually exclusive; however, both splitting and joint separation could occur simultaneously in any one panel, or for the same type of masonry, either splitting or joint separation could occur in different ranges of normal stress to shear stress. Whether joint separation or splitting governs depends on the relative magnitude of a friction coefficient between the masonry unit and the mortar joint as well as the resistance of the mortar joints to separation when the normal stress is zero. The resistance of the mortar joints to separation depends on the tensile and shear strength of the mortar joints and on the bond between the mortar and masonry units.

Most recently, theoretical investigations into the behavior of masonry infilled frames using the finite element technique have been carried out. Riddington and Smith developed a method that allows for the simulation of cracking around the interface between the frame and infill and the possibility of a friction or no friction condition along
the interfaces where contact remains (Refs. 6,7). These factors have been established as important in providing a meaningful mathematical model of an actual infilled frame. Its behavior has been shown by Smith to be partly related to the flexural stiffness of the frame relative to the in-plane diagonal stiffness of the infill, defined by a parameter $\lambda h$, where

$$\lambda h = \sqrt[4]{\frac{Eh^3}{4EI}}$$

in which $E$, $t$, $h$, are the modulus of elasticity, thickness, and height of the infill respectively and $E$ and $I$ are the modulus of elasticity and the moment of inertia of the columns respectively. An $E/E_I$ ratio of 4 was considered to be representative of a reinforced concrete frame with a masonry infill.

When an infilled frame is subject to a racking load, the frame tends to separate from the infill over part of the length of each side, as shown in Fig. 7a. This has been observed as cracking along the interface between the brickwork and the frame. The remaining regions of contact occur in the corner at the ends of the compression diagonal.

If the racking load is increased until the frame collapses, several modes of failure of the infill and of the frame are possible. Failure of the infill can occur by diagonal tension cracking, diagonal shear cracking or by compressive collapse of one of the loaded corners. Alternately, if the wall is relatively strong the frame may fail, either by tension failure in the windward column or its basic connection, or by shear failure of the columns or of the beam and its connections.
These failure modes are illustrated in Fig. 7b.

Based on the finite element stress analysis on several types of infilled frames, the following equations were derived which give a good estimate of the stresses at the center of the infill, the critical region for the initiation of the shear and tensile failure modes (Ref. 6),

\[
\begin{align*}
\text{Shear} & \quad \tau_{xy} = \frac{1.43H}{LT} \quad \text{(EQ. 1.9)} \\
\text{Diagonal tension} & \quad \sigma_{dt} = \frac{0.58H}{LT} \quad \text{(EQ. 1.10)} \\
\text{Vertical compression} & \quad \sigma_{y} = \frac{(0.8h/L - 0.2)H}{LT} \quad \text{(EQ. 1.11)}
\end{align*}
\]

in which \(H\) is the lateral load on the infilled frame and \(h, L,\) and \(t\) are the height, length, and thickness of the infill respectively.

It is necessary to ensure that all three possible modes of failure of the infill are considered in a design method. Shear failure will be initiated in the infill along the bedding joints of the masonry at the point where the ratio of the horizontal shear stress (EQ. 1.9) to the available shear strength is greatest. The theoretical, elastic stresses indicate that this occurs near the center of the infill. The maximum permissible racking load \(H\) based on shear failure as the limiting criterion is then the lesser of

\[
H = \frac{LT}{14.6 - 1.28 \frac{h}{L}} \quad \text{MN} \quad \text{(EQ. 1.12)}
\]

or

\[
H = 0.35 \frac{LT}{\text{MN}} \quad \text{(EQ. 1.13)}
\]

The maximum allowable racking load \(H\), using the diagonal tensile failure of the brickwork as a limiting criterion, is given by equating
the estimated diagonal tensile stress (EQ. 1.10) to the allowable strength, therefore

\[ H = 0.12LT \text{ MN} \quad (\text{EQ. 1.14}) \]

If the columns of the frame are relatively flexible the infill may fail by crushing of one of the loaded corners. This mode of failure is dependent on the column stiffness, that is, to the changes in the value of \( h \). Mainstone gives a conservative estimate of the racking force at which compressive failure occurs and is given by:

\[ H = 1.12(\lambda h)^{-0.88} f_c h t \cos^2 \Theta \quad (\text{EQ. 1.15}) \]

in which \( f_c \) is the vertical compressive strength of the brickwork and \( \Theta \) is the slope of the infill diagonal to the horizontal.

Page also developed a method of finite element analysis for masonry subjected to in-plane loading (Ref. 8). The model considered masonry as a continuum of isotropic elastic bricks acting in conjunction with mortar joints possessing specialized and restricted properties. The joints are modeled as linkage elements with limited tensile strength, high compressive strength with nonlinear characteristics, and variable shear strength depending upon the degree of compression present. To model masonry using the finite element technique in the manner described, these areas were examined: (1) the behavior of clay bricks; (2) the behavior of mortar joints; (3) the mechanisms of joint failure.

Masonry transmits compressive loads very effectively. Its capacity is governed by the tensile properties of the brick, as failure occurs by splitting due to transverse tension in the brick caused by
the differential lateral expansion of the stiffer brick and the more flexible mortar. Tests were performed to determine the elastic properties of the brick and to verify the assumption of elastic-brittle behavior for brick. A considerable variation in stiffness and ultimate strength was evident though all curves were linear for a majority of the load range, with a departure from linear near ultimate load. The stress-strain curve shown in Fig. 8 was that used in the analytical model. Mean curves for masonry and individual bricks are also shown.

The behavior of masonry is subjected to a complex state of stress influenced primarily by the orientation of the applied loads to the mortar joints. Most of the inelastic deformation occurs in the joints, and the joint characteristics are affected by the magnitude of the shear and normal stresses in the joint. Depending upon the degree of compression present, failure can occur in the joints alone, or as a combined brick-joint failure. All mechanisms of failure for masonry are not fully understood, and no complete failure criterion has yet been developed. In this analytical model developed by Page, progressive joint failure is allowed to occur. If the failure criterion is violated for a joint element, the element properties are modified and the problem solved again. The residual properties are allocated depending upon the stress state present. If the criterion of Region 1 (Fig. 9) is violated, tensile bond failure is assumed to occur, and no residual capacity is assigned to that joint element. If failure occurs under a combination of compressive and shear stress, Regions 2 and 3, a shear bond failure is simulated. The stiffness of the joint in the normal direction is
assumed to remain unchanged, and a reduced shear stiffness is assigned depending upon the magnitude of compressive stress present.

The proposed model offers a more realistic alternative to an isotropic elastic analysis. Its ability to reproduce nonlinear behavior caused by material characteristics and local joint failure could be used to advantage in predicting cracking patterns and areas of stress concentration.

1.4 Analytical Model

The characteristics of the mathematical model, i.e. the analytic representation of the real structure, must be chosen to adequately describe the physical model. In this context it is necessary to describe the response of a reinforced concrete frame with a brick or block masonry infill wall subjected to racking type loads. When the racking load is applied to the frame there must develop some interaction between the frame and the masonry infill wall, usually other than just friction. In the model being presented, this interaction is incorporated into the model by a layer of masonry grout between the wall and the frame. Stresses directly developed in the beams and columns are transferred through the masonry grout layer to the brick or block masonry wall.

Several basic assumptions must be made at this point to keep the model as simplistic as possible without affecting an accurate representation of the response of the frame-wall system. The analytical mode is assumed to be planar, i.e. in a state of plane stress. Minor
axis bending and torsional stiffnesses are neglected, leaving only the inplane response of the reinforced concrete frame and masonry infill wall to be considered. The model is subjected to a static racking load that is incrementally increased up to some prescribed final load. Small deformations and strains are assumed to be applicable. The initial geometry of the analytic model is assumed for every load increment level.

Specific assumptions must also be made on the different elements that comprise the analytical model. The members of the reinforced concrete frame must be prismatic and both beams and both columns must have the same geometric configurations. A basic premise in the analysis is that progressive failure only occurs in the wall, the frame remains elastic throughout its load history. Therefore, a mild strength concrete is always assumed in the analytic model with $f'_c = 4$ ksi, $E_c = 3.6 \times 10^3$ ksi, and $\nu = 0.15$. All the bricks (blocks), mortar joints, and grout are assumed to have constant thickness, though each may differ within the model. Steel reinforcement in the wall has not been included within the framework of the analytical model.

For the model to remain accurate the material behavior of the mortar joint, grout, and bricks must be defined. Material nonlinearity of the masonry mortar and grout has a significant effect on the overall behavior of the frame-wall system. Mortar is assumed to be an orthotropic material, to be subjected to biaxial stress states, to exhibit nonlinear stress-strain behavior, and to conform to a biaxial failure envelope criterion. Bricks, on the other hand, are assumed to be an isotropic
material, to be subjected to uniaxial stress states, to exhibit linear stress-strain behavior, and to conform to a uniaxial failure criterion. This representation of the material behavior of the masonry wall element is the key factor that provides a realistic analytical model.
2. MATERIAL BEHAVIOR

2.1 Masonry Mortar

In order to ensure a good quality of masonry, a mortar must be equally well suited for both brick laying and load carrying when hardened. The workability of a just mixed mortar must be such that a mason can fill all joints easily. When a course or units have been laid, the mortar and brick or block system must attain a reasonable rigidity before the next course is laid, in order to prevent excessive racking movements. If the mortar and masonry unit system stiffens too fast, it can be impossible for the mason to make the necessary corrective movements to the newly laid unit. Even though many destructive and nondestructive tests are performed on a chosen mortar and masonry unit to ensure the necessary strength and workability, the masons actual handling of the materials will determine their final characteristics.

The strength of the hardened mortar depends mainly on the cement to lime ratio. Pure lime mortar has a compressive strength of about 15 to 150 psi and pure cement mortar has a compressive strength of 2000 to 3000 psi. Certain special mortar additives have been developed to increase the compressive strength of mortar even higher. The prevailing type of mortar is a lime-cement-sand mortar. The amount of different ingredients can vary from pure lime-sand mortar to pure cement-sand mortar. The minimum required strengths of standard mortars and their composition by volume is shown in Fig. 10 (Ref. 9).
Overall quality of the mortar cannot be judged solely by its compressive strength since other physical properties also vary with the ratio of lime to cement. When the strength of the masonry is more or less immaterial, i.e. for nonstructural walls, a lime-sand mortar with high workability may be chosen. For masonry strength requirements, a cement-lime-sand mortar of approximately N-type may be the best choice. A type S mortar is probably the best when the masonry is subjected to high bending stresses. In instances where the direct compressive stresses are unusually high, an almost pure cement-sand mortar should be chosen.

2.1.1 Constitutive Law for Orthotropic Materials

Investigations have been carried out by Liu, Nilson, and Slate and Darwin and Pecknold into a biaxial stress-strain law for concrete (Refs. 10,11). In a broad sense it is possible to assume that masonry mortar also would obey this biaxial stress-strain law. For masonry mortar in biaxial stress, it is reasonable to suppose that a stress in the principal direction 1, sufficient to cause a substantial reduction in the tangent modulus in that direction, may also affect the tangent modulus in the orthogonal principal direction 2. However, there is no reason to believe that the properties in these two directions will remain identical. As a result, biaxially loaded masonry mortar must be considered an orthotropic material, with properties differing in the two principal directions, but with symmetry about the two principal axes. Neglecting shear deformation at this point, the equations
relating the change in stress to the change in strain, for an incrementally linear orthotropic material, may be written as:

\[
\begin{bmatrix}
\frac{d\sigma_1}{\Delta E_1} \\
\frac{d\sigma_2}{\Delta E_2}
\end{bmatrix}
= \frac{1}{1 - \nu_1 \nu_2}
\begin{bmatrix}
E_1 & \nu_2 E_1 \\
\nu_1 E_2 & E_2
\end{bmatrix}
\begin{bmatrix}
\frac{d\epsilon_1}{\Delta \epsilon_1} \\
\frac{d\epsilon_2}{\Delta \epsilon_2}
\end{bmatrix}
\]  
\text{(EQ. 2.1.1)}

in which \(E_1, E_2, \nu_1, \nu_2\) are stress dependent material properties; and \(\nu_2 E_1 = \nu_1 E_2 = \nu \sqrt{E_1 E_2}\) where \(\nu^2 = \nu_1 \nu_2\). Included in this analysis is the premise that the material axes 1 and 2 coincide with the principal axes 1 and 2.

The shear term may now be introduced into the stress-strain relations, to give the final form of the stress-strain law governing orthotropic materials. From the theory of elasticity of anisotropic bodies, St. Venant gives additional equations relating the shear modulus to the moduli of elasticity and Poisson's ratio (Ref. 12). For the principal axes 1 and 2:

\[
\frac{1}{G_{12}} = \frac{2 \nu_1}{E_1} = \frac{1}{E_1} + \frac{1}{E_2}
\]

or

\[
G_{12} = \frac{E_1 E_2}{\frac{E_1}{E_1} + \frac{E_2}{E_2} + \frac{2E_2}{E_1 \nu_1}}
\]  
\text{(EQ. 2.1.2)}

The equation may now be rewritten in a symmetrical form and expanded to include the shear term:
In addition to the shear modulus, the off diagonal terms containing Poisson's ratio $\nu$, are independent of orientation. The constitutive matrix is defined by three quantities $E_1$, $E_2$ and $\nu$ which depend on the state of stress and strain in the masonry mortar. If

$$\lambda = \frac{E_1}{E_2 - \nu^2}$$

is introduced the constitutive relations can be written in a final form as:

\[
\begin{align*}
\begin{cases}
\sigma_1' \\
\sigma_2' \\
\sigma_{12}'
\end{cases}
= \begin{bmatrix}
\frac{E_1}{1 - \nu^2} & \frac{\nu \sqrt{E_1 E_2}}{1 - \nu^2} & 0 \\
\frac{\nu \sqrt{E_1 E_2}}{1 - \nu^2} & \frac{E_2}{1 - \nu^2} & 0 \\
0 & 0 & \frac{E_1 E_2}{E_1 + E_2 + 2\nu \sqrt{E_1 E_2}} \\
\end{bmatrix} \begin{Cases}
\epsilon_1 \\
\epsilon_2 \\
\gamma_{12}
\end{Cases}
\end{align*}
\] (EQ. 2.1.3)

\[
\begin{align*}
\begin{cases}
\sigma_1' \\
\sigma_2' \\
\sigma_{12}'
\end{cases}
= \begin{bmatrix}
\lambda & \lambda \nu_1 & 0 \\
\lambda \nu_1 & \lambda & 0 \\
0 & 0 & \frac{E_1 E_2}{E_1 + E_2 + 2\nu_1 \sqrt{E_1 E_2}} \\
\end{bmatrix} \begin{Cases}
\epsilon_1 \\
\epsilon_2 \\
\gamma_{12}
\end{Cases}
\end{align*}
\] (EQ. 2.1.4)
2.1.2 Stress-Strain Relationships

Typical stress-strain diagrams, from observations of different types of mortar reported by Hilsdorf as shown by Sahlin are given in Fig. 11 (Ref. 2). The tangent moduli of elasticity at zero stress are indicated in the figure. Curve A represents a cement-sand mortar, approximately ASTM type M, and has an initial modulus of elasticity, $E_0 = 3.60 \times 10^3$ ksi. Curve B represents a lime-cement-sand mortar, approximately ASTM type O, and has an initial modulus of elasticity, $E_0 = 5.12 \times 10^2$ ksi. Curve C represents a lime-sand mortar, approximately ASTM type K, and has an initial modulus of elasticity, $E_0 = 93.9$ ksi. The wide variety of strength and of deformation under loading is very apparent with the highest modulus of elasticity being about 40 times the lowest. The figure clearly shows that a careful choice of mortar is necessary when the modulus of elasticity is critical.

2.1.3 Biaxial Failure Envelope for Masonry Mortar

A direct parallel must be drawn between the behavior of concrete and that of masonry mortar in a state of biaxial stress. The basic shape of the failure envelope is essentially fixed and only the size of the envelope will change with concrete strength (Ref. 13); this must also be valid for masonry mortar. Peterson and Kostem have shown that the true envelope can be approximated by a series of straight lines as shown in Fig. 12 (Ref. 13). The maximum increase in biaxial compressive strength over the
uniaxial compressive strength for the idealized failure envelope is 20%. This corresponds to a value of 1.2 on the non-dimensional plot in Fig. 12.

The characteristic points used to define the peak stress envelope are shown in Fig. 13 for masonry mortar and enumerated in the table below:

<table>
<thead>
<tr>
<th>Point</th>
<th>$\sigma_{p1}$</th>
<th>$\sigma_{p2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(+) $f_{mt}$</td>
<td>(+) $f_{mt}$</td>
</tr>
<tr>
<td>B</td>
<td>(+) $f_{mt}$</td>
<td>0.0</td>
</tr>
<tr>
<td>C</td>
<td>0.0</td>
<td>(-) $f_{mc}$</td>
</tr>
<tr>
<td>D</td>
<td>(-) 0.20$f_{mc}$</td>
<td>(-) 1.20$f_{mc}$</td>
</tr>
<tr>
<td>E</td>
<td>(-) 1.20$f_{mc}$</td>
<td>(-) 1.20$f_{mc}$</td>
</tr>
<tr>
<td>F</td>
<td>(-) 1.20$f_{mc}$</td>
<td>(-) 0.20$f_{mc}$</td>
</tr>
<tr>
<td>G</td>
<td>(-) $f_{mc}$</td>
<td>0.0</td>
</tr>
<tr>
<td>H</td>
<td>0.0</td>
<td>(+) $f_{mt}$</td>
</tr>
</tbody>
</table>

The terms used in the table above and on Fig. 13 are defined as:

- $f_{mc}$ = uniaxial compressive strength of the masonry mortar
- $f_{mt}$ = uniaxial tensile strength of the masonry mortar
- $\sigma_1$ = principal stress in direction 1
- $\sigma_2$ = principal stress in direction 2
- $\sigma_{p1}$ = peak stress in direction 1
- $\sigma_{p2}$ = peak stress in direction 2

The entire biaxial principal stress space consists of four regions: the compression-compression region, tension-tension...
region, compression-tension region and tension-compression region as identified in Fig. 13. Two distinct types of failure modes for the masonry mortar are idealized from the work of Peterson and Kostem on concrete (Ref. 13). A previous study by Kuper, Hilsdorf and Rusch also presented two general types of failure modes for concrete, a crushing failure and a cracking failure (Ref. 14). The idealized failure modes used in this report are shown in Figs. 13,14 and are dependent upon the applied stress ratio \( \sigma_2 / \sigma_1 \). From the tension-tension region to a stress ratio of \( \frac{1}{-15} \) (or \( \frac{-15}{1} \)) a cracking failure mode is assumed to occur and is labeled a type I failure. The direction of the crack(s) is assumed to be perpendicular to the largest tensile force and the free surface of a mortar element. From the compression-compression region to a stress ratio of \( \frac{1}{-15} \) (or \( \frac{-15}{1} \)) a crushing failure mode is assumed to occur and is labeled a type II failure. The direction of crushing is assumed to be perpendicular to the largest compressive stress and perpendicular to the free surface of a mortar element.

2.2 Masonry Units

In general, masonry units are available in many different materials, shapes, and sizes. The most common types of units are clay bricks, clay tiles, concrete-blocks, light weight cellular concrete blocks, and sand-lime bricks.

Bricks are available in a large variety of shapes and sizes
from approximately 4.5 x 3.4 x 1.7 in. to 2 x 2 x 2 ft., with modular sizes shown in Fig. 15 currently common in the United States. The actual brick size is the nominal size minus the joint thickness. Concrete masonry units are also available in many shapes and sizes, though the units are typically based on some module, usually 4 or 8 in. From common usage the \( \frac{3}{8} \) in. thick mortar joint has become a standard for these units (Ref. 15). Accordingly, the exterior dimensions of modular units are reduced by the standard joint thickness. Thus, when laid in masonry the modular units produce wall lengths, heights, and thicknesses that are multiples of the given module. Several examples of brick and block dimensions are given in Fig. 16. Bricks and block units can be solid or hollow. In the United States a masonry unit is defined as solid if the net cross-sectional area in every plane parallel to the beaming surface is 75% or more of its gross cross-sectional area measured in the same plane. A masonry unit is defined as hollow if the cores, cells, or hollow spaces within the total cross-sectional area exceed 25% of the cross section of the brick or block.

2.2.1 Stress-Strain Relationships

The modulus of elasticity for a number of different clay bricks has been reported by Glanville and Bannett (Ref. 16). The data have been plotted in Fig. 17 as given by Sahlin (Ref. 2) and fall reasonably close to a line defined by the equation:

\[
E_b = 300 f_b' \quad \text{(EQ. 2.2.1)}
\]
where $E_b$ is the brick modulus of elasticity and $f_b'$ is the compressive strength of the brick. Also given in Fig. 17 by Sahlin (Ref. 2) are results obtained by Hilsdorf on brick prisms 1.2 x 1.2 x 2.4 in. cut from different parts of whole bricks. The data, from three different brick strengths, also fall close to the line represented by the above equation, though slightly below for high strength bricks. Hilsdorf's measurements showed a good stress-strain linearity for all the tested prisms. The stress-strain curves are given in Fig. 18 for the different strength bricks. Curve A has a modulus of elasticity $E_o = 3.1 \times 10^3$ ksi, curve B has a modulus of elasticity $E_o = 1.8 \times 10^3$ ksi, and curve C has a modulus of elasticity $E_o = 1.38 \times 10^3$ ksi. Hilsdorf also reported that Poisson's ratio increased from 0.2 at the initial stage of loading to about 0.35 before the ultimate load was reached.

Richart, Mookman, and Woodworth have reported on the modulus of elasticity for a number of different types of concrete masonry blocks (Ref. 17). The results show an approximate relationship between the modulus of elasticity and the concrete masonry block strength as given by a line defined by the equation:

$$E_u = 1000 f_u'$$

(EQ. 2.2.2)

where $E_u$ is the concrete block modulus of elasticity and $f_u'$ is the compressive strength of the block. Since many different types of aggregates were used in the tested masonry blocks,
there is a wide scatter in the reported data. The modulus of elasticity falls between 500 $f'_u$ and 1500 $f'_u$ with the above equation taken as the mean of the data.

2.2.2 Failure Criterion for Masonry Units

Based on the stress-strain relationships for masonry units a uniaxial failure criterion is assumed. The stress-strain curve increases or decreases linearly until a point is reached where the brick or block masonry unit crushes or is split apart. The unit is assumed to crush when its compressive stress reaches a maximum value. This value $f_{bc}$ is the uniaxial compressive strength of the masonry unit. The unit is assumed to split apart when its tensile stress reaches a maximum value. This value $f_{bt}$ is the uniaxial tensile strength of the masonry unit, or more commonly referred to as the modulus of rupture.

2.3 Masonry

The mortar and the brick or block units combine to form a masonry whose modulus of elasticity is theoretically affected by the modulus of elasticity of both constituents. Stress-strain data given by Hilsdorf for cement-sand mortar, for bricks, and for masonry built with these components is shown by Sahlin (Ref. 2) in Fig. 19. Corresponding data have been plotted in Fig. 20 for the same type of bricks, but with low strength lime-sand mortar.

For a rough estimate of the modulus of elasticity of masonry in compression the following equation can be used:
\[ E_m = 700 f'_m \]

where \( E_m \) is the modulus of elasticity of the masonry and \( f'_m \) is the compressive strength of the masonry. The modulus of elasticity determined by the above equation refers to a low stress level. With increasing stress the tangent modulus of elasticity normally decreases. In some instances, the stress-strain diagram is S-shaped because the modulus of elasticity first increases then decreases, though this depends on the strength characteristics of the mortar and the masonry units. A weak mortar, regardless of the brick strength, gives the masonry a low modulus of elasticity at low stress levels. As the stress is increased, the mortar breaks down and becomes compacted and confined by the bricks. The stress-strain curve becomes steeper resulting in a higher modulus of elasticity in this stress region. Finally, when the bricks begin to fail, the modulus of elasticity again decreases and the stress-strain diagram reaches a maximum at the ultimate stress.

Cement-sand mortar and lime-cement-sand mortar both give a masonry with decreasing modulus of elasticity under increasing stress. The lime mortar gives a masonry with an initially increasing and then a decreasing modulus of elasticity with increasing stress. These curves are given in nondimensional form by Sahlin (Ref. 2) in Fig. 21. Brick quality was held constant while mortar type was changed.
3. THEORETICAL ANALYSIS

3.1 Introduction

The masonry wall infilled frame analysis procedure being reported is based on the finite element method in conjunction with an incremental nonlinear analysis technique. Subsequently, the discussion of the theoretical analysis is divided between the finite element model and the overall solution technique. The theoretical model that is set forth is an attempt to accurately predict the location, load level, stress level, and failure type at which cracking initiates in the masonry wall and how it propagates through the wall when subject to an arbitrary diagonal racking load. The orthotropic material properties and appropriate biaxial failure envelope for the masonry mortar and grout are taken into account along with a simplified failure criterion for the masonry bricks as outlined in Chapter 2. Four separate computer programs are utilized in carrying out the theoretical investigation. They are based on the developments presented in the previous chapters and conform to the assumptions and limitations given in this chapter. The details of these programs can be found in Ref. 18.

3.2 Assumptions

The masonry wall infilled frame assemblage is shown in Fig. 22. The objective, through the theoretical analysis, is to model the "structure" using the finite element method so that its behavior and response can be simulated when the actual system is subjected to an inplane compressive load. In order that this may be done accurately, several
assumptions must be made.

The geometry of the masonry wall assemblage must be planar in addition to being rectangular. If the wall is kept planar and is subjected only to inplane loads, a plane stress condition exists throughout the wall. This reduces the complexity of the problem from one which includes both bending and membrane effects to one where only membrane effects are considered. The dimensions of the wall constituents—length, height, and width of the masonry units, thickness and width of the mortar joints, and thickness and width of the grout can be completely arbitrary though they must be uniform and consistent throughout the wall. The wall is assumed to be built in a running bond pattern as illustrated in Fig. 23, which shows portions of block and brick masonry walls. The final geometric restriction that must be placed on the wall is that there must be an odd number of courses of masonry units in a wall. The first and last courses in a wall will consist of an arbitrary number of uncut masonry units.

To facilitate the theoretical analysis the assumptions of small displacements and small strains are made. The validity of this restriction, if incorporated into the process, must be extended to all components of the model including the masonry mortar and grout whose material properties have been described in some detail in Section 2.1. This material nonlinearity can be encompassed within an area of problems in which stresses are not linearly proportional to strains, but in which only small displacements and small strains are considered (Ref. 19). The small displacement restriction refers to changes in the overall geometry.
of the wall. It is assumed that the inplane differential displace-
ments of an element will be small in comparison to the overall dimen-
sions of the wall. The small strain restriction implies that the
geometry of a differential element, or more specifically a finite
element, will not change noticeably after deformation takes place over
the entire body. Furthermore, only infinitesimal changes in the geom-
etry will occur. Local distortions of a differential element can be
ignored and the areas of the original, undeformed element can be used
in computing stresses. Also, the geometry of the element need not be
updated as the theoretical analysis proceeds.

3.3 Finite Element Model

The use of the finite element method becomes very advantageous
when analyzing large and/or complex structures. The finite element
method requires that a continuum be divided into an assemblage of units
called finite elements which are considered to be interconnected at
discrete points called nodes. In this theoretical study the SOLID SAP
finite element program and its associated finite element library are
utilized (Ref. 20). Therefore, it is not necessary to include a
discussion of the finite element technique in this report.

The continuum that is to be divided into a discrete number of
finite elements is idealized in Fig. 22. The inplane behavior of the
masonry wall, or more specifically, the masonry mortar and masonry
units and the surrounding grout layer are modeled using a type 4 -
plane stress element from the SOLID SAP element library (Ref. 20).
This element is allowed two displacement degrees of freedom per node and must be oriented in the global Y-Z plane. Also, orthotropic material properties are possible when the principal material axes, the n-s axes, are defined other than by the global axes. This element, represented as a general quadrilateral, is shown in Fig. 24. The reinforced concrete frame is modeled using a type 2 - three dimensional beam element from the SOLID SAP element library (Ref. 20). In general, this element has six degrees of freedom per node, but since the assumption was made that the masonry wall-frame assemblage must be planar only three degrees of freedom per node are allowed. The beam element that is used in the finite element model has two displacement degrees of freedom and one rotational degree of freedom. A representation of this element in local coordinates is shown in Fig. 25.

3.3.1 Discretization

The finite element model discretization is shown in Fig. 26 for a typical reinforced concrete frame with a masonry infill wall. The type of discretization was chosen so that the individual material characteristics of the masonry units, mortar joints, and grout layer can be updated as the analysis proceeds to provide a more realistic solution than an analysis based solely on isotropic elastic behavior. The proposed finite element model consists of six distinct element groupings as shown in the figure. Group 1 contains the grout elements which are located between the masonry wall and the reinforced concrete frame. Plane stress elements
with assumed orthogonal, nonlinear material properties are used.

Group 2 contains the masonry unit elements. Each uncut unit is modeled as two plane stress elements and a cut unit is modeled as one plane stress element. These elements are assumed isotropic with linear elastic material properties. The mortar joints in the masonry wall are divided into three groups, each one using plane stress elements with assumed orthogonal, nonlinear material properties. Group 3 contains the vertical mortar joints and the horizontal mortar joints are contained in Groups 4 and 5 as shown in Fig. 26. The final group, Group 6, contains the reinforced concrete frame elements which are modeled using beam elements.

3.3.2 Boundary Conditions

In this study the reinforced concrete frame masonry wall assemblage has been considered a basic structural component of an entire structural system. The finite element model of this assemblage, like a free body diagram, must be in equilibrium and be able to develop reactive forces in adjoining members of the structural system. The model being presented must always be stable and rigid body motion must be prevented. In order to accomplish this in the finite element model, type 7 - boundary elements are used from the SOLID SAP element library (Ref. 20). These elements are used to constrain the displacements in a particular direction or to constrain the rotation about an axis allowing reactive forces to be developed. The forces obtained would be relative
values for the given story and given bay under consideration of a multistory, multibay frame. Depending upon the structural framing used, many different types of support configurations can be placed on the finite element model. Seven types of support conditions are illustrated in Fig. 27 from a simply-supported planar frame support condition to a space rigid frame support condition.

3.3.3 Loading Conditions

A single concentrated load only can be applied to the finite element model. This simulates an arbitrary racking load that would be applied to the real structure at the top right most corner of the wall, i.e. at the beam-column connection of the reinforced concrete frame. This load must be a compressive load oriented in the plane of the wall, the global Y-Z plane as shown in Fig. 28. A vertical, horizontal or an arbitrary diagonal compressive load may be specified, but if the applied load $P$ is at an arbitrary angle $\theta$ as shown in Fig. 28 then the applied load must be defined by its component values $P_Y$ and $P_Z$ where $P_Y = P \cdot \cos \theta$ and $P_Z = P \cdot \sin \theta$.

3.4 Solution Technique

The solution technique used in the theoretical analysis can be described as a modified incremental scheme, the details of which can most easily be discussed at the element level. The finite element
The modified incremental method used in the analysis incorporates a linear elastic finite element solution technique that is performed using the SOLID SAP program (Ref. 20). If the response of a reinforced concrete frame masonry wall assemblage is to be determined for a given arbitrary compressive load $P_{\text{TOTAL}}$, the structure is first discretized into the assumed pattern as discussed in Section 3.3.
The applied load is divided into \( n \) increments so that \( P_{\text{TOTAL}} = P_0 + \Delta P_0 \cdot (n-1) \) where \( P_0 \) is the initial load that is assumed to act on the finite element model and \( \Delta P_0 \) is an incremental load that is successively added to the existing load already acting on the model. At the \( m \)th increment of loading (where \( m \) is greater than or equal to two and less than or equal to \( n \)), the load on the finite element model is \( P^{(m)} = P_0 + \sum_{i=1}^{m} \Delta P_0 \).

The first load increment \( P_0 \) is applied to the structure and a linear elastic solution is determined based on the original material properties that were assumed for the reinforced concrete frame, masonry units, and mortar joints (the material properties of the grout layer are assumed equal to those of the mortar joints). For successive increments of load, only the previously assumed material properties of the mortar joints and grout layer need to be continuously updated. Once the material properties of the respective elements have been updated and the applied load incremented, a linear elastic finite element solution is again determined.

A stiffness \( K_0 \) can be determined for any element exhibiting nonlinearity from the original slope of the stress-strain curve shown in Fig. 29a for the initial load \( P_0 \). This stiffness is represented as the slope of the load vector \( P_0 \) which is shown in Fig. 29b and a certain displacement of the element, proportional to the applied load, is realized with the stiffness \( K_0 \). The stress in the element can then be directly calculated from the element displacement (Ref. 21). New
modulus of elasticity $E_1$ is determined from the stress-strain curve using the just computed stress value and a new element stiffness $K_1$, is determined, again at zero load, before the applied load is incremented to $P_0 + \Delta P_0$. This stiffness is represented as the slope of the load vector $P_0 + \Delta P_0$ which is shown in Fig. 29b. A certain displacement of the element, proportional to the newly applied load, is realized with a stiffness $K_1$. The stress in the element can then be directly calculated from the element displacement (Ref. 21). If no other factors influence the material behavior of this nonlinear element, the modified incremental procedure is continued as explained above until the applied load reaches $P_{\text{T}}$. A series of fan-shaped load vectors are produced as shown in Fig. 29b and can be summarized in Fig. 29c as an idealized load-deflection curve for the nonlinear element. The line segments that comprise the load-deflection curve in Fig. 29c are drawn parallel to the corresponding load vector shown in Fig. 29b.

In order that a theoretical prediction can be made for the initiation and propagation of failure mechanisms in the masonry wall, the plane stress finite elements representing the mortar joints, grout layer, and masonry units are checked sequentially after each linear elastic finite element solution has been performed for a given load increment. For a given element which exhibits material nonlinearity its associated principal stresses $\sigma_N$ and $\sigma_S$ are initially checked against the masonry mortar biaxial failure envelope defined by the values $f_{mc}$ and $f_{mt}$. If neither principal stress violates the failure criterion, new material properties are assigned as previously described,
though associated with each principal stress value is an orthotropic tangent modulus of elasticity $E_N$ and $E_S$, both of which are determined from the stress-strain curve. If the maximum principal stress $\sigma_N$ violates the failure criterion for that element its associated modulus of elasticity $E_N$ at the time of failure is redefined. If the failure occurs at the $i$th load step, then the new modulus of elasticity in the $N$-direction would be $E_N^{\text{FAILED}} = E_N^{i-1}/1000$ and would remain constant for all load steps thereafter. The minimum principal stress value $\sigma_S$ is then used to assign new material properties based on the stress-strain curve for the $S$-direction. On subsequent load cycles this element will only be checked for failure based on the value of the minimum principal stress. When this failure occurs the associated modulus of elasticity $E_S$ at the time of failure is redefined. If this failure occurs at the $j$th load step, then the new modulus of elasticity in the $S$-direction would be $E_S^{\text{FAILED}} = E_S^{j-1}/1000$ and would remain constant for all load steps thereafter. At this point in the modified incremental procedure the element is no longer checked. The technique of reducing the modulus of elasticity simulates the introduction of a crack in a particular direction in the finite element program for that specific element. It is assumed that the element no longer has any stiffness in the cracked direction and that any load is carried solely by the remaining stiffness in the perpendicular direction to the crack.

Alternately, for a given nonlinear element, the minimum principal stress $\sigma_S$ could violate the failure criterion first. Its
associated modulus of elasticity $E_S$ at the time of failure is redefined. If the failure occurred at the $k$th load step, then the new modulus of elasticity in the $S$-direction would be $E_S^{(FAILED)} = E_S^{k-1}/1000$ and would remain constant for all load steps thereafter. The maximum principal stress value $\sigma_N$ is then used to assign new material properties based on the stress-strain curve for the $N$-direction. On subsequent load cycles this element will only be checked for failure based on the value of the maximum principal stress. When this failure occurs the associated modulus of elasticity $E_N$ at the time of failure is redefined. If this failure occurs at the $l$th load step, then the new modulus of elasticity in the $N$-direction would be $E_N^{(FAILED)} = E_N^{l-1}/1000$ and would remain constant for all load steps thereafter. Again, after this point in the modified incremental procedure the element is no longer checked.

The masonry unit plane stress elements are assumed to be isotropic, linear elastic up to brittle failure. Both the maximum and minimum principal stress values for each element are sequentially checked against the uniaxial failure criterion defined by the values $f_{bc}$ and $f_{bt}$ respectively. As soon as either value is exceeded the element is assumed to have failed and both the moduli of elasticity $E_N$ and $E_S$ (assumed equal), and the modulus of rigidity $G_{NS}$ are redefined. If the masonry unit failure occurs at the $m$th load step then the new modulus values would be $E_N^{(FAILED)} = E_N^{(INITIAL)}/1000$, $E_S^{(FAILED)} = E_S^{(INITIAL)}/1000$, and $G_{NS}^{(FAILED)} = G_{NS}^{(INITIAL)}/1000$. After this cycle in the modified incremental procedure no further checking is performed on this element. The reduction of all the modulus values
simulates a complete failure of the element. It is assumed that in subsequent load cycles of the finite element model, this element will behave as though it has zero stiffness. After all the plane stress elements have been checked for a given load increment and new material properties have been assigned as required, the applied load to the finite element model is incremented and the analysis procedure is repeated again.

3.5 Limitations

There are two limitations to the theoretical analysis that should be noted. Both limitations affect the computed stress values of the plane stress finite elements which exhibit material nonlinearity. The first limitation is inherent in the SOLID SAP program (Ref. 20), and concerns itself with the aspect ratio of these elements. The finite element model as previously described closely resembles the physical problem since the objective of this study is to theoretically demonstrate the initiation and propagation of cracking in masonry walls. The mortar joint and grout elements have aspect ratios that are not ideal when using a finite element solution technique. The aspect ratio is the ratio of the length of the element to its width and for these elements will normally be in the range of 5:1 to 25:1. Therefore, inaccuracy is introduced into the analysis because of the shape of these elements which affects their computed stress values.

An aspect ratio in the range of 1:1 to 3:1 would be optimum for the mortar joint and grout elements. Referring to Fig. 26 this
would mean that each element in Groups 1, 3, and 4 would have to be divided into a number of smaller elements depending upon their original aspect ratio. An element in Group 2, then, must be subdivided into a number of smaller elements whose sizes would be governed by the subdivision of the elements in Groups 1, 3, and 4. The finite element discretization created in this manner would be impractical because the total number of degrees of freedom in the system would increase to such a great extent as to make the finite element solution using the SOLID SAP program extremely prohibitive (Ref. 20).

The second limitation is introduced into the analysis through the use of the modified incremental procedure. Fig. 29c shows an actual load-deflection curve and the idealized load-deflection curve for a nonlinear element. It can easily be seen that the element displacements increase more rapidly, especially at higher load levels for the idealized load-deflection curve than for the actual load-deflection curve. This corresponds to larger computed stress values than should actually be present in the element. The modified incremental procedure is still valid and will provide good results as long as two conditions are met. The first is that enough load increments be chosen so that an accurate load-deflection history can be developed for any given element which exhibits material nonlinearity, and second is that failure of the element takes place along a portion of the load-deflection curve where the element stiffness is not changing so drastically between load increments.

In the future, it may be desirable to change the solution
technique to a basic incremental method or even preferably an incremen-
tal-iterative procedure in an effort to help minimize the error build-
up found in the present analysis. The incremental procedure approxi-
mates a nonlinear problem as a series of linear problems. The non-
linear equilibrium equation for a single element can be written

\[
[k] \{q\} = \{Q\} \quad \text{(EQ. 3.1)}
\]

where the nonlinearity occurs in the stiffness matrix \([k]\), which is a function of the nonlinear material properties \([C(\sigma)]\), \{q\} is the element displacement vector, and \{Q\} is the corresponding load. In writing the equations for the incremental method, let the reference state (usually zero) of this single element be given by the initial loads \(\{Q_o\}\) and the initial displacements \(\{q_o\}\). The total effective load is divided into \(M\) increments, therefore

\[
\{Q\} = \{Q_o\} + \sum_{j=1}^{M} \{\Delta Q_j\} \quad \text{(EQ. 3.2)}
\]

where the \(\Delta\) is used to indicate a finite increment. After the application of the \(i\)th increment, the load is given by

\[
\{Q\} = \{Q_o\} + \sum_{j=1}^{i} \{\Delta Q_j\} \quad \text{(EQ. 3.3)}
\]

and the displacements are given by

\[
\{q\} = \{q_o\} + \sum_{j=1}^{i} \{\Delta q_j\} \quad \text{(EQ. 3.4)}
\]

The increments of displacements \(\{\Delta q_j\}\) are computed using a fixed value of the stiffness, which is evaluated at the end of the previous
increment.

\[
[K_{i-1}] \{Δq_i\} = \{ΔQ_i\} \quad \text{for } i=1,2,3,\ldots,M \quad \text{(EQ. 3.5)}
\]

where \([K_{i-1}]\) is a function of the load and displacement vectors at the \(i\)-th increment. \([K_o]\), the initial value of the stiffness, is computed from the material constants derived from the given nonlinear stress-strain curve for the element at the start of loading. The incremental procedure is schematically illustrated in Fig. 30.

The incremental-iterative procedure utilizes a technique where the load is applied incrementally to an element which exhibits non-linearity and then successive iterations are performed to increase the accuracy of the solution at each increment, i.e. EQ. 3.5 is repeatedly solved with refined values of \([K_{i-1}]\) (Fig. 31). Iterations for a given load level are terminated when all nodal point displacements and stresses of all elements from two consecutive iterations are close enough within prescribed tolerance levels.
4. RESULTS

Several analytical results are presented in this chapter and illustrate how the theoretical model that was developed works. Solutions for two different masonry infill walls listed below will be discussed:

No. 1: A brick masonry infilled frame subjected to a horizontal compressive load

No. 2: A block masonry infilled frame subjected to a horizontal compressive load, a 45° diagonal compressive load, and a vertical compressive load

These examples will show the versatility of the analysis technique being reported. The initiation and propagation of failure mechanisms in the masonry infill wall can be predicted and will be shown for the theoretical cases listed above.

The first example is a brick masonry infilled frame whose overall wall dimensions are 64 7/8" in length and 99 3/8" in height. A standard clay brick is used having dimensions 7 5/8" x 2 1/4" x 3 5/8". The mortar joints are 1/4" thick and have a width of 3 1/2"; the grout layer is 1 1/16" thick and has a width of 3 5/8". The beams of the reinforced concrete frame have dimensions 20" x 26" and the columns 20" x 20". Fig. 32 shows the finite element discretization for this wall. The initial material properties for the masonry mortar (and grout) are $E_o = 3.10 \times 10^3$ ksi, $G_o = 1.35 \times 10^3$ ksi, and
\[ \nu = 0.15 \text{ and for the bricks are } E_o = 2.20 \times 10^3 \text{ ksi, } G_o = 9.57 \times 10^2 \text{ ksi, and } \nu = 0.20. \text{ Stress-strain curves for both the bricks and the mortar are shown in Fig. 33. The bricks are assumed to be of medium strength with a uniaxial compressive strength of 5.470 ksi and a uniaxial tensile strength of 0.848 ksi. The mortar is assumed to be of high strength having a cement-sand composition and is governed by the biaxial strength envelope given in Fig. 34. A Case 1 support condition is assumed for the masonry wall infilled frame assemblage to simulate a simply supported connection to other structural members in a building. The wall is subjected to a total horizontal compressive load of 125 kips which is incrementally applied to the structure. An initial load of 25 kips is applied and is increased by 10 kip increments until the total load is attained.}

The initiation and propagation of cracking through the wall is shown sequentially in Figs. 35-45. Mortar (grout) elements that are cracked in one direction are marked with a broken line and those cracked in two directions are marked with a solid line. Brick elements that are failed are shown solid. Cracking initiates at a load of 25 kips in the lower right corner of the wall and progresses upwards through the grout layer along the compression column of the reinforced concrete frame (Figs. 35,36). At a load of 45 kips (Fig. 37) three distinct areas of cracking in the mortar joints can be seen. The propagation of cracking continues (Fig. 38) and at a load of 65 kips (Fig. 39) the first brick elements fail. At this load approximately 75% of all the mortar joints have failed in one direction and the
grout layer along the floor beam and compression column is almost entirely cracked in one direction. Between 75 kips and 105 kips (Figs. 40-43) cracking continues, moving towards the unloaded corner of the wall. Brick elements continue to fail and lie along lines that are 45° from both the tension and compression columns. At 115 kips and 125 kips (Figs. 44, 45) many second cracks appear in the mortar and grout elements. Extensive brick failure can be seen in Fig. 45 near the compression column and extending along a line at the mid-height of the wall to the tension column of the reinforced concrete frame. Thus, the failure of this wall can be characterized as an inplane bending failure.

The second example is a block masonry infilled frame whose overall wall dimensions are 96 5/8" in length and 72 5/8" in height. A standard concrete block unit is used having dimensions 15 5/8" x 7 5/8" x 7 5/8". The mortar joints are 3/8" thick and have a width of 7 1/8"; the grout layer is 1/2" thick and has a width of 7 5/8". The beams of the reinforced concrete frame have dimensions 6" x 10" and the columns 6" x 6". Fig. 46 shows the finite element discretization for this wall. The initial material properties for the masonry mortar (and grout) are $E_o = 5.12 \times 10^2$ ksi, $G_o = 2.23 \times 10^2$ ksi, and $\nu = 0.15$ and for the concrete blocks are $E_o = 1.80 \times 10^3$ ksi, $G_o = 7.50 \times 10^2$ ksi, and $\nu = 0.20$. Stress-strain curves for both the concrete blocks and mortar are shown in Fig. 47. The blocks are assumed to be of moderate strength with a uniaxial compressive strength of 5.470 ksi and a uniaxial tensile strength of 0.848 ksi.
The mortar is assumed to be of low strength having a lime-sand composition and is governed by the biaxial strength envelope given in Fig. 48. A Case 1 support condition is assumed for the masonry wall infilled frame assemblage to simulate a simply supported connection to other structural members in a building. Three separate loading conditions are applied to the wall:

A. A horizontal compressive load
B. A 45° diagonal compressive load
C. A vertical compressive load

In Case A the wall is subjected to a total load of 80 kips and in Cases B and C the wall is subjected to a total load of 100 kips. In all cases an initial load of 2 kips is applied to the structure and is increased by 2 kip increments until the respective total loads are attained. The figures that are shown summarize the results at 10 kip increments following the load at which cracking initiates.

The initiation and propagation of cracking through the wall for load case A is shown sequentially in Figs. 49-55. Cracking initiates at a load of 18 kips in the lower right corner of the wall and Fig. 49 shows the crack pattern when the load on the wall is 20 kips. At loads of 30 kips to 60 kips (Figs. 50-53) cracking progresses through the grout layer along the compression column, laterally through the mortar joints from the compression column, and along the compression diagonal of the wall. Cracks in two directions also develop in the grout layer in elements that lie on or near the compression diagonal. At a load of 70 kips (Fig. 54) cracks in two directions
appear in the mortar joints along two distinct lines. When the load reaches 80 kips (Fig. 55) a large number of second cracks are found. These cracks lie mainly in two regions of the wall, along the compression diagonal and through the horizontal mortar joints at mid-height of the wall. This type of wall failure can be characterized as a shear failure due to mortar joint separation.

The initiation and propagation of cracking through the wall for load case B is shown sequentially in Figs. 56-61. Cracking initiates at 42 kips along the loaded diagonal. The crack pattern when the load applied to the wall is 50 kips is shown in Fig. 56. Cracking is concentrated strictly along the loaded or compression diagonal. From 50 kips to 100 kips (Figs. 57-61) cracking progresses slowly along lines away from the loaded diagonal. Few second cracks appear even at a load of 100 kips and only can be found in the grout layer at the corners of the compression diagonal. This type of wall failure can be characterized as a stepped shear failure along the loaded wall diagonal.

The initiation and propagation of cracking through the wall for load case C is shown sequentially in Figs. 62-66. Cracking initiates at a load of 54 kips and at loads of 60 kips through 90 kips (Figs. 62-65) cracking remains confined to the grout layer along the loaded column of the reinforced concrete frame. At a load of 100 kips (Fig. 66) cracking appears in the grout layer along both beams and into the wall. The failure of this wall can be characterized as an arch type failure though it will be at a higher load than that which was applied.
5. CONCLUSIONS

An analytical model has been developed to predict the initiation and propagation of failure mechanisms in masonry infill walls. The results of this study are encouraging though no experimental tests were conducted to verify the model and no test data could be found in the literature that could be used for comparison. Therefore, the following conclusions can be drawn:

1. It has been shown that structural problems involving material nonlinearity can be solved using a modified incremental procedure in conjunction with the SOLID SAP finite element program and will provide a more realistic alternative to an analysis based on isotropic elastic behavior.

2. Failure criteria, such as a biaxial failure envelope for masonry mortar and a uniaxial failure criterion for masonry units, can also be incorporated into the overall solution technique.

3. The initiation and propagation of cracking in masonry infill walls under inplane loads, through the interaction of the reinforced concrete frame and the infill wall, has been demonstrated and several types of failure mechanisms, i.e. inplane bending failure, stepped shear failure, and arching failure have been observed.

4. The analysis technique being presented provides complete
generality: the dimensions of the masonry wall, as well as the masonry units, mortar joints and grout layer are strictly arbitrary. A variety of support conditions and loading conditions can also be handled.
Fig. 1  Reinforced Concrete Bounding Frame with Brick Shear Wall (from Ref. 1)

Fig. 2  Ultimate Load-Carrying Capacity of the Masonry Infilled Frame with Horizontal Force \( T \) and Horizontal Wall Area \( A \). The Symbols Represent Different Wall Scales that Were Used. (from Ref. 2)
Fig. 3 Observed Typical Diagonal Crack Pattern with Crack Incidence Shown in kips (from Ref. 3)

Fig. 4 Idealized Stress Distribution in a Masonry Prism Subjected to Axial Compression (from Ref. 4)
Fig. 5 Proposed Failure Criteria of Brick Masonry
(from Ref. 4)

Fig. 6 Failure Modes of Tested Masonry Walls
(from Ref. 5)
Fig. 7a Behavior of an Infilled Frame Subject to a Racking Load (from Ref. 7)

(a) Modes of infill failure  
(b) Modes of frame failure

Fig. 7b Failure Modes of an Infill Wall and Frame Subject to a Racking Load (from Ref. 7)
Fig. 8 Stress-Strain Curves for Masonry, Brick, and Mortar Used in the Analytical Model (from Ref. 8)

Fig. 9 Assumed Joint Failure Envelope (from Ref. 8)
<table>
<thead>
<tr>
<th>ASTM Mortar Type</th>
<th>28-day Strength</th>
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</thead>
<tbody>
<tr>
<td>M(1C:1/4 L : 3S)</td>
<td>2500 psi</td>
</tr>
<tr>
<td>S(1C:1/2 L : 4 1/2 S)</td>
<td>1800 psi</td>
</tr>
<tr>
<td>N(1C:1 L : 6S)</td>
<td>750 psi</td>
</tr>
<tr>
<td>O(1C:2 L : 9S)</td>
<td>350 psi</td>
</tr>
<tr>
<td>K(1C:4 L : 15S)</td>
<td>75 psi</td>
</tr>
</tbody>
</table>

Fig. 10 Minimum Strength Requirements and Composition by Volume of Standard ASTM Mortar Types (from Ref. 9)

![Stress-Strain Diagram](image)

**A. Cement-sand mortar, ratio 1:3 (by volume)**
- $E_0 = 253,000$ kg/cm²
- $\epsilon_u = 0.31\%$

**B. Lime-cement-sand mortar, ratio 1:2:8 (by volume)**
- $E_0 = 36,000$ kg/cm²
- $\epsilon_u = 0.125\%$

**C. Lime-sand mortar, ratio 1:3 (by volume)**
- $E_0 = 6,600$ kg/cm²
- $\epsilon_u = 0.88\%$

Fig. 11 Stress-Strain Diagrams for Different Types of Masonry Mortars (from Ref. 2)
Fig. 12 Concrete Failure Envelopes for the Biaxial Stress Space (from Ref. 13)
Fig. 13 Generalized Biaxial Failure Envelope for Masonry Mortar
Fig. 14 Idealized Failure Modes

<table>
<thead>
<tr>
<th>Unit designation</th>
<th>Thickness (inches)</th>
<th>Face dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Height (inches)</td>
</tr>
<tr>
<td>Conventional brick</td>
<td>4</td>
<td>2 3/4</td>
</tr>
<tr>
<td>Roman brick</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Norman brick</td>
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<td>Engineer's brick</td>
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<tr>
<td>&quot;SCR brick&quot;</td>
<td>6</td>
<td>2 1/2</td>
</tr>
</tbody>
</table>

Fig. 15 Nominal Modular Sizes of Clay Bricks Commonly in Use in the United States (from Ref. 2)
Examples of solid structural clay bricks:

Examples of structural clay tiles:

Concrete brick

Solid units

Fig. 16 Examples of Structural Clay Bricks, Clay Tiles, and Concrete Blocks (from Refs. 2 and 15)
Fig. 17 Relationship Between the Modulus of Elasticity of Bricks and the Strength of Bricks (from Ref. 2)

![Graph showing the relationship between modulus of elasticity and brick strength.]

Fig. 18 Stress-Strain Diagrams for Different Types of Structural Clay Brick Units (from Ref. 2)

![Graph showing stress-strain diagrams for different types of structural clay brick units.]

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Ref. 19 Relationship Between Stress and Strain for Bricks, Cement-Sand Mortar, and Masonry Made of These Constituents (from Ref. 2)

Ref. 20 Relationship Between Stress and Strain for Bricks, Lime-Sand Mortar, and Masonry Made of These Constituents (from Ref. 2)

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Fig. 23 Running Bond Pattern for Brick and Block Masonry Wall (from Ref. 15)
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Fig. 25 General Beam Finite Element in Local Coordinates with Assumed Degrees of Freedom
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3 Vertical Mortar Joint Element Group
4 Horizontal Mortar Joint Element Group A
5 Horizontal Mortar Joint Element Group B
6 Reinforced Concrete Frame Element Group

Fig. 26 Finite Element Model Discretization Showing Element Groups
### Fig. 27 Possible Support Conditions for the Finite Element Model

<table>
<thead>
<tr>
<th>Displacement Along Y-Axis</th>
<th>Displacement Along Z-Axis</th>
<th>Rotation About X-Axis</th>
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</thead>
<tbody>
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<td><strong>NODE</strong></td>
<td><strong>NODE</strong></td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
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</tr>
<tr>
<td>ase 6</td>
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<td>Free</td>
</tr>
<tr>
<td>ase 7</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
</tbody>
</table>
Fig. 28 Arbitrary Racking Load Applied to the Masonry Wall
Fig. 29 Response of a Finite Element Which Exhibits Material Nonlinearity Using a Modified Incremental Solution Technique
Fig. 30 Basic Incremental Procedure (from Ref. 19)

Fig. 31 Incremental-Iterative Procedure (from Ref. 19)
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Fig. 34 Biaxial Failure Envelope for Cement-Sand Mortar
Fig. 37 Progressive Failure of a Brick Masonry Infill Wall - 45 kip Load
Fig. 38 Progressive Failure of a Brick Masonry Infill Wall - 55 kip Load
Fig. 39 Progressive Failure of a Brick Masonry Infill Wall - 65 kip Load
Progressive Failure of a Brick Masonry Infill Wall - 75 kip Load

Fig. 40
Fig. 41 Progressive Failure of a Brick Masonry Infill Wall - 85 kip Load
Fig. 43  Progressive Failure of a Brick Masonry Infill Wall - 105 kip Load
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Fig. 49 Progressive Failure of a Block Masonry Infill Wall - 20 kip Load
Fig. 50 Progressive Failure of a Block Masonry Infill Wall - 30 kip Load
Fig. 51 Progressive Failure of a Block Masonry Infill Wall - 40 kip Load
Fig. 57 Progressive Failure of a Block Masonry Infill Wall - 60 kip Load
Fig. 58 Progressive Failure of a Block Masonry Infill Wall - 70 kip Load
Fig. 62 Progressive Failure of a Block Masonry Infill Wall - 60 kip Load
Fig. 63 Progressive Failure of a Block Masonry Infill Wall - 70 kip Load
Fig. 64 Progressive Failure of a Block Masonry Infill Wall - 80 kip Load
Fig. 65 Progressive Failure of a Block Masonry Infill Wall - 90 kip Load
Fig. 66 Progressive Failure of a Block Masonry Infill Wall - 100 kip Load
REFERENCES


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