1977

Dynamic properties of skewed beam-slab highway bridges, December 1977

Celal N. Kostem

Lehigh
University

FOUNDED BY
ASA PACKER
1865

DYNAMIC PROPERTIES
OF
SKEWED BEAM-SLAB
HIGHWAY BRIDGES

BY
CELAL N. KOSTEM

Fritz
Engineering
Laboratory

FRITZ ENGINEERING LABORATORY REPORT NO. 432.4

DYNAMIC PROPERTIES

OF

SKEWED BEAM-SLAB HIGHWAY BRIDGES

by

Celal N. Kostem

Fritz Engineering Laboratory

Department of Civil Engineering

Lehigh University

Bethlehem, Pennsylvania

December 1977

Fritz Engineering Laboratory Report No. 432.4

DYNAMIC PROPERTIES OF SKEWED BEAM-SLAB HIGHWAY BRIDGES

The natural periods of vibration of skewed bridge superstructures can be used for qualitative and quantitative comparisons of their dynamic characteristics. The paper presents the results of a parametric study tabulating the first three fundamental periods of right bridges and equivalent skewed bridges. The analysis was carried out for skew angles of 60° and 45°, with 90° being a right bridge. The periods of vibration for skewed bridges are nondimensionalized with respect to those of right bridges. Statistical averaging has indicated that the changes in the natural periods of vibration for skewed bridges, regardless of the variability of their design parameters, can be approximated through the use of appropriate multipliers when the periods of vibration for equivalent right bridges can be approximated.

DYNAMIC PROPERTIES OF SKEWED BEAM-SLAB HIGHWAY BRIDGES

The dynamic response and vibrational characteristics of railroad and highway bridges have been of concern to bridge designers. This has been due to the fact that through the prediction of the dynamic response, the amplification of the static live load stresses and deformations can be estimated; which in turn may require the redimensioning of the superstructure. It has been shown that the amplification of the static response through the use of "impact factor" can lead to erroneous results (1,2). The vibrational characteristics, more specifically the periods of vibration, of the bridges are usually used in conjunction with the human response to the dynamic behavior of the superstructure. Various studies have showed that the dynamic behavior of the superstructure can be accurately predicted through approaches that will define the vibrational characteristics of the superstructure (1-6). The rigorous mathematical analysis of the superstructure can not be used as a design aid because of the prohibitive computational complexity and the associated cost (2,3).

The natural periods of vibration of highway bridges can be employed in the discrimination of the dynamic characteristics. The development of simple empirical formulae that can predict the natural periods of vibration would be an optimal solution. The extensive analytical studies have resulted in a sufficient amount of information on the natural periods of vibration of simple span beam-slab highway bridges with reinforced concrete deck, prestressed concrete I-beams and without skew (1,4). These reported results, through the use of statistical methods, have already provided empirical relations to predict the natural periods of vibration of the aforementioned bridges (5). However, there still exists a need to predict the natural periods of vibration of these types of bridges with skew. This paper summarizes the results of a research carried out to answer this particular issue.

The findings of the research have been given herein for right bridges as well as the modification factors which need to be applied to the periods of vibration of right bridges in order to obtain those for skewed bridges.

DESIGN AND ANALYSIS OF BRIDGES

The paper focuses attention on simple span beam-slab bridges with prestressed concrete I-beams. To be representative of bridges encountered in the field, 33 right bridges were designed using current engineering practices (7). The span length of the bridges varied from 12.20 m (40 ft.) to 27.44 m (90 ft.). The pertinent design dimensions are shown in Table 1. Inspection of this table indicates that the bridge configurations considered cover a wide range of variation in the design parameters. The beams employed in the design process correspond to the standard beams used in the Commonwealth of Pennsylvania. Table 2 gives the stiffness properties of the beams. In the table I_{max} and I_{min} correspond to major and minor bending moments of inertia of the beams with respect to their center of gravity. A and K_T denote the cross-sectional area and St. Venant's torsional stiffness of the beams. The nomenclature used in the Commonwealth of Pennsylvania is also indicated for each beam in Table 2.

In the definition of the dynamic characteristics of the bridges, the superstructures were simulated by using finite element method. The deck slab was simulated via plate bending elements, while the beams employed the beam bending elements. The analysis was performed using Program SAP IV (8). For the sake of brevity the paper contains only the first three fundamental periods of vibration of the superstructure. The periods of vibration for right bridges are presented in Table 3. It should be noted that in this analysis it has been assumed that no vehicle is on the bridge. The differences between these unloaded and loaded periods have been previously presented (1). Using Table 3 as the data base the empirical formulae have already been developed that can predict the periods of vibration of bridges with design dimensions other than those included in Table 3 (5).

SKEWED BRIDGES

The bridges described in Table 1 have retained all their design parameters intact, however, the geometry of the bridges was changed for skew angles of 60° and 45°, with 90° skew being the right bridge. The inclusion of the new bridges has resulted in the consideration of a total of 99 bridges. Through the application of the finite element simulation of the superstructure and the use of Program SAP IV, as have been done for the right bridges, the natural periods of vibration of skewed bridges were computed. Table 4 presents the nondimensionalized values of the periods of vibration, i.e. the period of vibration of the skewed bridge is divided by the corresponding period of the right bridge. The table uses the nomenclature of subscript and superscript. The subscripts indicate the 1st through 3rd periods of vibration. The superscripts are used to denote the skew angle.

Inspection of Table 4 indicates that for a given skew angle and period the values tend to be similar, i.e. within each column the values are similar. The statistical analysis of these values has resulted in the following for bridges with 60° skew:

$\text{mean}(t_1)$ = 0.958 sec.; standard deviation = 0.017 sec.

standard error of the mean = 0.003 sec.

$\text{mean}(t_2)$ = 0.949 sec.; standard deviation = 0.014

standard error of the mean = 0.002 sec.

$\text{mean}(t_3)$ = 0.910 sec.; standard deviation = 0.032 sec.

standard error of the mean = 0.005 sec.

In the computation of the above values it is assumed that the corresponding periods of vibration for right bridges are equal to 1.000 sec.

Similar results can be obtained by using the same approach and nomenclature for bridges with 45° skew as follows:

$\text{mean}(t_1)$ = 0.908 sec.; standard deviation = 0.030 sec.

standard error of the mean = 0.005 sec.

$\text{mean}(t_2)$ = 0.878 sec.; standard deviation = 0.031 sec.

standard error of the mean = 0.005 sec.

$\text{mean}(t_3)$ = 0.787 sec.; standard deviation = 0.071 sec.

standard error of the mean = 0.012 sec.

The relatively small magnitude of the standard deviation and standard error of the mean indicates the consistency of the mean values given herein. If the natural period of vibration of a right bridge is known, then the period of vibration of a skewed bridge can easily be approximated through the use of the appropriate value given in this paper. Another observation that can be made is in regard to the relatively small changes of the mean periods of vibration for different skew angles. For example, the first period of vibration has values of 1.000, 0.958 and 0.908 for skew angles of 90°, 60° and 45°. The close proximity of these values permits interpolation in predicting the natural periods of skewed bridges, with skew angle between 90° and 45°. However, any extrapolation beyond 45° may lead to erroneous results since the variation of the periods beyond 45° has not been determined.

The use of the given ratios of the periods of vibration between the right and the skewed bridges, in computing the natural period of vibration for skewed bridges, can be illustrated in the following examples:

1. The fundamental period of vibration of a skewed bridge with a 22.88 m (75 ft.) span length, 60° skew, 12.81 m (42 ft.) width with 7 beams is to be determined. This bridge is similar to Bridge No. 23. The period of vibration of the corresponding right bridge is 0.151 seconds. The period of vibration of the skewed bridge will be 0.958 x 0.151 = 0.145 seconds. The exact analysis had also yielded 0.145 seconds, a perfect agreement.
2. The fundamental period of vibration of a bridge with the same dimensions as in the above example, except with a skew of 52° 30', is to be computed. The period of vibration of the bridge will be 0.5 x (0.958 + 0.908) x 0.151 = 0.141 seconds. This is less

than 1% off the exact analysis results.

CONCLUSIONS

The dynamic and vibrational characteristics of bridge superstructures can be predicted through the use of predominant natural periods. It has been shown that the natural periods of vibration of skewed simple span beam-slab bridge superstructures can be computed, through the use of the appropriate multipliers presented in the paper, when the periods of vibration of the equivalent right bridge is known. The natural periods of vibration of bridges with skew up to 45° have been computed. It has been found that on the average the maximum reduction in the first fundamental period is at the most 10% while for the second and third periods this reduction can be up to 21%.

REFERENCES

1. Kostem, C. N.

DYNAMIC PROPERTIES OF BEAM-SLAB HIGHWAY BRIDGES, Presented at the 1977 Transportation Research Board Meeting; to appear in Transportation Research Record.

2. Peterson, W. S. and Kostem, C. N.

DYNAMIC ANALYSIS OF HIGHWAY BRIDGES USING THE FINITE ELEMENT METHOD, Fritz Engineering Laboratory Report No. 400.7, Lehigh University, Bethlehem, Pa., May 1972.

3. Eberhardt, A. C. and Walker, W. H.

A FINITE ELEMENT APPROACH TO THE DYNAMIC ANALYSIS OF CONTINUOUS HIGHWAY BRIDGES, Structural Research Series No. 394, Department of Civil Engineering, University of Illinois, Urbana, Illinois, December 1972.

4. Kostem, C. N.

IDENTIFICATION OF THE DESIGN PARAMETERS GOVERNING THE NATURAL PERIODS OF VIBRATION OF HIGHWAY BRIDGES, Fritz Engineering Laboratory Report No. 378A.9, Lehigh University, Bethlehem, Pa., June 1977.

5. Thomson, A. and Kostem, C. N.

ON THE PREDICTION OF THE NATURAL PERIODS OF VIBRATION OF HIGHWAY BRIDGES, Fritz Engineering Laboratory Report No. 354.452, Lehigh University, Bethlehem, Pa., June 1977.

6. Zellin, M. A., Kostem, C. N. and VanHorn, D. A.

STRUCTURAL BEHAVIOR OF HIGHWAY BRIDGES: A SUMMARY OF COMPLETED

RESEARCH AND BIBLIOGRAPHY, Fritz Engineering Laboratory Report No. 387.1, Lehigh University, Bethlehem, Pa., May 1973.

7. Department of Transportation
STANDARDS FOR BRIDGE DESIGN (PRESTRESSED CONCRETE STRUCTURES) BD-201, Bureau of Design, Commonwealth of Pennsylvania, Harrisburg, Pa., 1973.

8. Bathe, K.-J., Wilson, E. L., and Peterson, F. E.
SAP IV-A STRUCTURAL ANALYSIS PROGRAM FOR STATIC AND DYNAMIC RESPONSE OF LINEAR SYSTEMS, Earthquake Engineering Research Center Report No. EERC 73-11, University of California, Berkeley, California, June 1973, Revised April 1974.

TABLE 1. BRIDGES

Bridge No.	Span (m)	Width (m)	Slab Thickness (mm)	Beam Spacing (m)	No. of Beams	Beam
1	12.20	7.32	190.0	1.46	6	I
2	12.20	7.32	190.0	1.83	5	I
3	12.20	7.32	203.0	2.44	4	I
4	12.20	12.81	190.0	1.60	9	I
5	12.20	12.81	203.0	2.13	7	I
6	12.20	12.81	215.0	2.56	6	I
7	12.20	18.30	190.0	1.66	12	I
8	12.20	18.30	190.0	2.03	10	I
9	12.20	18.30	215.0	2.61	8	I
10	15.25	7.32	190.0	1.83	5	II
11	15.25	12.81	215.0	2.13	7	III
12	15.25	18.30	190.0	2.03	10	IV
13	19.82	7.32	190.0	1.46	6	V
14	19.82	7.32	190.0	1.83	5	VI
15	19.82	7.32	203.0	2.44	4	VIII
16	19.82	12.81	190.0	1.60	9	XI
17	19.82	12.81	190.0	2.13	7	VI
18	19.82	12.81	215.0	2.56	6	VIII
19	19.82	18.30	190.0	1.66	12	XII
20	19.82	18.30	190.0	2.03	10	VI

TABLE 1--Continued

Bridge No.	Span (m)	Width (m)	Slab Thickness (mm)	Beam Spacing (m)	No. of Beams	Beams
21	19.82	18.30	215.0	2.61	8	VIII
22	22.88	7.32	190.0	1.83	5	VII
23	22.88	12.81	190.0	2.13	7	VIII
24	22.88	18.30	190.0	2.03	10	VIII
25	27.44	7.32	190.0	1.46	6	IX
26	27.44	7.32	190.0	1.83	5	XII
27	27.44	7.32	203.0	2.44	4	XIII
28	27.44	12.81	190.0	1.60	9	X
29	27.44	12.81	190.0	2.13	7	III
30	27.44	12.81	203.0	2.56	6	XIII
31	27.44	18.30	190.0	1.66	12	X
32	27.44	18.30	190.0	2.03	10	VIII
33	27.44	18.30	203.0	2.61	8	XIII

(Note: 1 m = 3.28 ft., 1 mm = 0.0394 in.)

TABLE 2. STIFFNESS PROPERTIES OF THE BEAMS

Beam	I_{max} (mm^4)	I_{min} (mm^4)	A (mm^2)	K_T (mm^4)	Notes
I	13.66 10^9	2.66 10^9	234 10^3	3.81 10^9	PDT20/30
II	18.62 10^9	3.31 10^9	269 10^3	5.19 10^9	PDT20/33
III	24.58 10^9	3.96 10^9	304 10^3	6.71 10^9	PDT20/36
IV	23.80 10^9	5.66 10^9	354 10^3	10.64 10^9	PDT24/33
V	31.66 10^9	7.74 10^9	397 10^3	13.35 10^9	PDT24/36
VI	44.91 10^9	6.59 10^9	379 10^3	9.10 10^9	PDT24/42
VII	58.27 10^9	7.20 10^9	414 10^3	10.40 10^9	PDT24/45
VIII	71.96 10^9	8.37 10^9	457 10^3	13.04 10^9	PDT24/48
IX	88.50 10^9	8.98 10^9	492 10^3	14.64 10^9	PDT24/51
X	106.33 10^9	9.59 10^9	526 10^3	16.52 10^9	PDT24/54
XI	34.69 10^9	10.36 10^9	443 10^3	17.65 10^9	PDT26/36
XII	162.86 10^9	14.02 10^9	624 10^3	19.97 10^9	PDT26/60
XIII	195.87 10^9	15.87 10^9	675 10^3	23.00 10^9	PDT26/63

(Note: $1\ mm^4 = 2.4\ 10^{-6}\ in.^4$, $1\ mm^2 = 1.55\ 10^{-3}\ in.^2$)

TABLE 3. NATURAL PERIODS OF

VIBRATION OF RIGHT BRIDGES

Bridge No.	T_1	T_2 (seconds)	T_3
1	0.066	0.059	0.048
2	0.073	0.069	0.054
3	0.077	0.064	0.052
4	0.075	0.069	0.063
5	0.078	0.075	0.069
6	0.082	0.073	0.063
7	0.075	0.068	0.064
8	0.076	0.075	0.072
9	0.082	0.077	0.070
10	0.101	0.094	0.068
11	0.097	0.094	0.083
12	0.099	0.097	0.092
13	0.134	0.114	0.076
14	0.125	0.116	0.081
15	0.109	0.095	0.071
16	0.136	0.126	0.106
17	0.130	0.126	0.111
18	0.116	0.106	0.093
19	0.138	0.132	0.120
20	0.129	0.126	0.119

TABLE 3--Continued

Bridge No.	T_1	T_2 (seconds)	T_3
21	0.120	0.114	0.105
22	0.157	0.149	0.134
23	0.151	0.147	0.141
24	0.150	0.148	0.144
25	0.181	0.157	0.098
26	0.164	0.150	0.100
27	0.160	0.139	0.095
28	0.177	0.166	0.140
29	0.170	0.163	0.141
30	0.165	0.152	0.131
31	0.180	0.172	0.158
32	0.168	0.165	0.154
33	0.168	0.159	0.145

TABLE 4. NONDIMENSIONALIZED NATURAL PERIODS OF VIBRATION

Bridge No.	t_1^{60}	t_1^{45}	t_2^{60}	t_2^{45}	t_3^{60}	t_3^{45}
1	0.96	0.90	0.94	0.85	0.88	0.73
2	0.96	0.91	0.94	0.86	0.88	0.72
3	0.93	0.85	0.93	0.83	0.87	0.70
4	0.95	0.88	0.94	0.86	0.92	0.81
5	0.95	0.89	0.95	0.87	0.91	0.78
6	0.93	0.84	0.92	0.82	0.90	0.77
7	0.89	0.83	0.92	0.87	0.94	0.84
8	0.95	0.89	0.95	0.89	0.93	0.83
9	0.93	0.85	0.92	0.82	0.91	0.77
10	0.96	0.91	0.94	0.85	0.87	0.70
11	0.96	0.90	0.95	0.84	0.91	0.75
12	0.96	0.91	0.96	0.90	0.93	0.84
13	0.97	0.92	0.94	0.84	0.87	0.70
14	0.97	0.93	0.95	0.87	0.87	0.70
15	0.96	0.91	0.94	0.86	0.87	0.70
16	0.97	0.92	0.95	0.87	0.91	0.78
17	0.96	0.92	0.96	0.90	0.91	0.79
18	0.96	0.90	0.95	0.87	0.91	0.79
19	0.96	0.92	0.95	0.89	0.93	0.82
20	0.96	0.92	0.96	0.91	0.94	0.84
21	0.95	0.90	0.94	0.88	0.94	0.85

TABLE 4--Continued

Bridge No.	t_1^{60}	t_1^{45}	t_2^{60}	t_2^{45}	t_3^{60}	t_3^{45}
22	0.97	0.93	0.97	0.93	0.96	0.92
23	0.96	0.92	0.96	0.93	0.96	0.92
24	0.96	0.92	0.96	0.93	0.96	0.92
25	0.98	0.94	0.94	0.86	0.86	0.68
26	0.98	0.95	0.95	0.88	0.87	0.70
27	0.97	0.93	0.95	0.87	0.85	0.68
28	0.97	0.94	0.96	0.90	0.91	0.79
29	0.97	0.94	0.97	0.91	0.92	0.80
30	0.97	0.92	0.96	0.90	0.91	0.79
31	0.97	0.93	0.97	0.91	0.94	0.84
32	0.97	0.94	0.97	0.92	0.94	0.86
33	0.96	0.92	0.96	0.90	0.94	0.85