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Solid Mechanics, Plasticity, and Limit Analysis

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METHOD OF COMPUTING GEOMETRIC RELATIONS IN STRUCTURAL ANALYSIS

Key words: Geometric Relations; Virtual Work Principle; Structural Analysis

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ABSTRACT

A rapid method of computing geometric relations in structural analysis is presented. It is shorter than the usual methods in many cases, and can often get the result in one step. It uses vertical and horizontal dimensions of a structure directly.

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INTRODUCTION

The purpose of this paper is to present a method for computing quickly the geometric relationships for connected structural members. These relationships are often needed in structural analysis. For example, when using the slope deflection equations for an elastic analysis\(^1,2\), or in applying the combining mechanism method in plastic analysis\(^3-6\), the angle-rotation relationships of the members in the structure are usually required. It will be shown that the virtual work equations can provide a simple method for obtaining the desired relationships\(^7,8\).

BASIC CONCEPT

The essential feature of the method can be shown by considering the following two examples:

Example 1

Consider the gabled frame shown in Fig. 1. It is often necessary to find the geometrical relationships which exist for the chord rotations, \(\phi\), of the various members. The deflected shape is shown by the dashed lines in the figure. From the sketch it is apparent that the four-bar linkage or mechanism has two degrees of freedom, and hence two independent chord rotations, \(\phi_{ab}\) and \(\phi_{de}\). All chord angles (that is \(\phi_{bc}\) and \(\phi_{cd}\)) may be expressed in terms of \(\phi_{ab}\) and \(\phi_{de}\).

\(\phi_{bc}\) will be computed first. Fig. 2a shows an equilibrium system of external forces and moments applied to the four-bar linkage.
that results when the structure is reduced to a kinematic mechanism. The equilibrium system for the linkage is obtained in the following way. Bar cd is assumed to be an axially loaded bar with vertical component equal to \( r_2 \) and the horizontal component equal to \( n \) (i.e. proportional to the slope of the bar cd). It produces vertical reactions \( r_2 \) and horizontal reactions \( n \) at the two supports. Moment equilibrium is then established at the remaining joints of the linkage. They are equal to \( nh_1 \), \( (nr_1 + mr_2) \) and \( nh_2 \) at joints a, b, and e respectively.

If the equilibrium system shown in Fig. 2a undergoes the displacements shown in Fig. 1, the virtual work equations gives

\[
(nh_1 \, \theta_{ab} + (nr_1 + mr_2) \, \theta_{bc} - nh_2 \, \theta_{de} = 0
\]

The rotation of bar bc can be expressed as

\[
\theta_{bc} = \frac{n}{mr_2 + nr_1} \left( h_2 \, \theta_{de} - h_1 \, \theta_{ab} \right)
\]

Similarly, bar bc can be assumed to act as an axially loaded member. Fig. 2b summarizes the resulting equilibrium system and the virtual work equation yields

\[
(mh_1 \, \theta_{ab} + (mr_2 + nr_1) \, \theta_{cd} - mh_2 \, \theta_{de} = 0
\]

The rotation of bar cd can then be expressed as

\[
\theta_{cd} = \frac{m}{mr_2 + nr_1} \left( h_2 \, \theta_{de} - h_1 \, \theta_{ab} \right)
\]
The resulting chord rotations agree with the values obtained by Kinney using the usual geometrical relationship\textsuperscript{2}.

**Example 2**

Consider next the shed-type gable frame with four hinges as shown in Fig. 3. This mechanism often occurs in plastic analysis. Two different sets of equilibrium systems are given in Figs. 4a and 4b. Member bc (shaded triangle) and member cd are selected as the axially loaded members, respectively. The virtual work equations furnishes for bar bc

\begin{equation}
4.5L^2 \theta - 3L^2 \theta_1 = 0
\end{equation}

for bar cd

\begin{equation}
L \theta - 2L \theta_2 = 0
\end{equation}

Hence, rotations \( \theta_1 \) and \( \theta_2 \) can be expressed as functions of \( \theta \) as

\begin{align*}
\theta_1 &= \frac{3}{2} \theta \\
\theta_2 &= \frac{1}{2} \theta
\end{align*}

The total hinge rotation at b is equal to \( 3\theta/2 \), the sum of \( \theta \) and \( \theta_2 \).
Example 3

Application to the Slope-Deflection Method.

A typical skew frame is shown in Fig. 5a. The chord rotations shown in Fig. 5b are usually evaluated using the instantaneous center concept or the geometrical relations that result from the joint displacements. These rotations can easily be evaluated from the present method. Figure 5c and 5d illustrate its application.

The second factor is the derivation of the "Shear Equation" to provide the equilibrium condition that is needed in addition to the two joint equations of equilibrium. This can also be derived by the virtual work equation. Fig. 5e shows the equilibrium system that results when the usual slope-deflection sign convention is adopted, viz., end moments are positive when they act clockwise on the ends of the members. If the equilibrium system shown in Fig. 5e is displaced as shown in Fig. 5b, the resulting work is

\[ M_{ab} \dot{\theta} + M_{ba} (\dot{\theta}+\dot{\theta}_2) + M_{cd} (\dot{\theta}_2+\dot{\theta}_1) + M_{dc} \dot{\theta}_1 + 100 (20\theta) = 0 \]  

(6)

The values of \( \dot{\theta}_1 \) and \( \dot{\theta}_2 \) can be substituted and Equation 7 results.

\[ M_{ab} + 2.3 M_{ba} + 2.1 M_{cd} + 0.8 M_{dc} + 2,000 = 0 \]  

(7)

This provides the needed equation of equilibrium without having to eliminate the shear and axial forces.\(^1\)
Example 4

Application to the Mechanism Method of Plastic Analysis.

Consider the pin-supported arch shown in Fig. 6a, carrying a concentrated load $P$ at point $c$. The collapse mechanism of the arch is shown by the dashed lines with one plastic hinge at $c$ and a second hinge forming at an unknown section $d$. Two equilibrium systems are shown in Fig. 6b, c. The resulting virtual work for the displacements given in Fig. 6a are

$$ [(1-r) \frac{L}{2} h_c + (1-q) \frac{L}{2} h_d] \theta - [(1+q) \frac{L}{2} h_d - (1-r) \frac{L}{2} h_c] \theta_2 = 0 $$

$$ [(q+r) \frac{L}{2} h_c - (1-q) \frac{L}{2} (h_d-h_c)] \theta - [(q+q) \frac{L}{2} h_d + (1-r) \frac{L}{2} (h_d-h_c)] \theta_1 = 0 $$

These yield rotations $\theta_1$ and $\theta_2$ as

$$ \theta_1 = \frac{(1+r) \frac{1}{2} h_c - (1-q) \frac{1}{2} h_d}{(1+q) \frac{1}{2} h_d - (1-r) \frac{1}{2} h_c} \theta $$

$$ \theta_2 = \frac{(1-r) \frac{1}{2} h_c + (1-q) \frac{1}{2} h_d}{(1+q) \frac{1}{2} h_d - (1-r) \frac{1}{2} h_c} \theta $$

The work equation for the collapse mechanism (Fig. 6a) is

$$ P (1-q) \frac{L}{2} \theta = M_p (\theta_1 + \theta_2) + M_p (\theta_2 + \theta_1) $$

substituting $\theta_1$ and $\theta_2$ from Eqs. 9 and 10 yields the collapse load

-6-
\[ P^u = \frac{4M_p}{L} \frac{1 + \frac{h_c}{h_d}}{(1-q) \left[ (1+q) - (1-r) \frac{h_c}{h_d} \right]} \]  

The location of section d corresponds to the minimum value of the load \( P \), which is \( \frac{\partial P}{\partial r} = 0 \), according to upper bound plastic limit theorem.

**SUMMARY AND CONCLUSIONS**

A method is developed to evaluate the geometrical relationships that are often needed in structural analysis. The method is simple to use and apply. Only vertical and horizontal dimensions are needed.

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Fig. 1 Four-Bar Linkage

(a) An Equilibrium System For $\phi_{bc}$
(b) Another Equilibrium System For $\phi_{cd}$

Fig. 2 Equilibrium Systems for Fig. 1

Fig. 3 A Collapse Mechanism
(a) An Equilibrium System For $\theta_1$

(b) Another Equilibrium System For $\theta_2$

Fig. 4 Equilibrium Systems for Fig. 3
(a) Typical Skew Frame

(b) $\phi$ - Angle Relationships
(c) Equilibrium System For $\phi$ and $\phi_1$

\[20\phi - 25\phi_1 = 0\]
\[\phi_1 = \frac{4}{5} \phi\]

(d) Equilibrium System For $\phi$ and $\phi_2$

\[325\phi - 250\phi_2 = 0\]
\[\phi_2 = \frac{13}{10} \phi\]
(e) Equilibrium System For "Shear Equation"

Fig. 5 Virtual Work Technique applied to The Slope-Deflection Method

(a) Pin-Supported Arch
(b) Equilibrium System For $\theta$ and $\theta_2$

(c) Equilibrium System For $\theta$ and $\theta_1$

Fig. 6 Virtual Work Technique applied to The Mechanism Method
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