Measurement of residual stresses-a study of methods, February 1971

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A STUDY OF METHODS

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- A STUDY OF METHODS

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ABSTRACT

One of the major problems associated with the use of metals at present is that created by the presence of residual stresses. In general, residual stresses tend to reduce strength; in some situations, however, their presence may improve the strength. The various phases of the manufacturing processes causing residual stresses are too involved generally to permit even an approximate prediction of the magnitude and distribution of them based on theoretical considerations. It is natural, therefore, to resort also to experimental means for their determination.

Unfortunately, residual stresses cannot be measured directly in the manner that applied stresses are measured. Thus, the measurement of residual stresses is rather delicate, requiring much time, patience, and expense.

In this paper, some of the different techniques of residual stress measurements are investigated. Special attention is given to the measurement of residual stresses in structural members where the applicability, simplicity, accuracy and saving in time each method can offer are discussed.

For a specific comparison of a number of methods, actual comparisons were made under laboratory conditions. Measurements of residual stresses were made using the method of sectioning, a destructive method, and two different hole-drilling methods both semi-destructive. For comparison, the
methods were applied to one specimen having a uniform residual stress distribution along its length.

The procedure of testing, preparation of specimen, and required tools and measuring devices, working conditions and similar relevant information are described. The recording of data as well as its interpretation is discussed, including both manual and automated procedures using the computer; the necessary theoretical background is supplemented in brief. The possible causes of errors during the recording and interpretation of data and their minimization are discussed.

Other methods of residual stress measurement which may be of general interest are mentioned, and a list of references is presented.
1. INTRODUCTION

One of the major problems at present associated with the technical use of metals is that of residual stresses. Many schemes and methods have been devised over the past eighty years for measuring residual stresses, since Kalakoutsky performed such measurements in 1888. A historical survey on methods of measuring residual stresses can be found in Refs. 1, 2, 3, and 4. Several papers dealing with the various methods of residual stress measurement have appeared during the last few years. The variety of proposed methods shows that residual stress measurement still arouses considerable interest in technical circles.

The various phases of the manufacturing processes causing residual stresses in structural members are too involved generally to permit more than an approximate prediction of the magnitude and distribution of residual stresses based on theoretical considerations. It is natural, therefore, to resort also to experimental means for their determination.

Unfortunately, residual stresses cannot be measured mechanically in the manner that applied stresses are measured. Thus, the measurement of residual stresses is of great intrinsic interest, but rather delicate, requiring much time and expense.

Laboratory specimens may not reproduce the effects of residual stresses in big structures. Hence, simple,
systematic, and practical methods with sufficient accuracy and not excessive sensitivity, applicable to the measurement of residual stresses in full-scale members of various cross sections are of great interest. Ways of improving the sensitivity and the precision of the measuring devices should be studied so long as this aim does not conflict with the practical conditions of the measurements.

The available methods of exploration fall into two categories: mechanical methods and physical methods. The former require a disturbance of the stresses and the latter do not.

The basic concept adopted by the mechanical methods for the determination of residual stresses is to release the residual stress on the surface by appropriate removal of material. Since residual stresses form an internally balanced system of stress and are produced by mutual interaction of various elements of the strained body, removal of material such as by cutting, drilling, and grooving, will cause unbalanced and partial relaxation of stress in each part. Thus, the mechanical methods do not measure the actual strain produced by the existing residual stress, what they do measure is the relaxed strain in one part of the body when the residual stress system is disturbed.

The mechanical methods, sometimes known as "relaxation methods", are either destructive or semi-destructive in nature. The destructive methods, as the name implies, require a total destruction beyond any hope of repair, before residual stresses can be evaluated. The semi-destructive
methods, on the other hand, produce only local damage which generally can be repaired, for example, by welding.

The destructive characteristics of the mechanical methods have been one of the major incentives for using physical methods. Such methods may be used to measure the existing residual stress directly without requiring any destruction of the test specimen. Among these methods, the X-ray diffraction technique and the ultrasonic methods are the most important since they measure strains directly on the strained metal. Unlike the mechanical methods, they may not deal with the average situation, but sample only a particular class of the grain aggregate. If the sampling is not representative, the physical methods and the mechanical methods may not measure the same thing. In general, the mechanical methods measure only macrostresses, the X-ray methods may superimpose the microstress to the macrostress, and the ultrasonic methods provide information only on the difference between the principal residual stresses, and not on the absolute magnitude of these stresses. This leads to a situation seeking the answer to the familiar question, "What is actually being measured"?

The purpose of this study is to investigate different techniques of residual stress measurement taking into consideration the applicability, simplicity, economy, accuracy and saving in time each method can offer. For the purpose of comparison, the methods considered are applied on one specimen, with a uniform residual stress distribution along the length. A 14W202*, ASTM A36 steel built up from

*The designation H refers to a wide flange section built up by welding component plates, as opposed to the designation W for rolled wide-flange sections.
flame cut plates with fillet welds was the selected work piece.

Residual stress measurements using the method of sectioning and two different hole-drilling methods were conducted. The procedure of testing used as well as the results are discussed in detail.
2. THE METHOD OF SECTIONING

2.1 Introduction

In the year 1888, Kalakoutsky\(^{(5)}\) reported on a method of determining longitudinal stresses in bars by slitting longitudinal strips from the bar and measuring their change in length. This method known as the "sectioning method"\(^{(6,7)}\) is based on the principle that internal stresses in a material are relieved by sectioning a specimen into many strips of small cross section. It is best applied to members when the longitudinal stresses alone are important.

The stress distribution over a cross section can be determined with reasonable accuracy from the measurement of change in length of each strip taken before and after the sectioning and by applying Hooke's Law. The analysis is simplified by assuming that the transverse stresses are negligible, and that the method of cutting produces no appreciable strains.\(^{(2)}\) In practice, however, transverse stresses may exist, but the lower the transverse stresses, the more accurate the results will be. Residual stresses formed due to sawing alone depend, among many other factors, on the spacing of the saw cuts, the thickness of plate, the speed of cutting, and cooling characteristics (cooling liquids, etc.). In general, the residual stress at the very edge of the cut may approach the local yield strength of the material. The actual distribution of residual stresses close to the surface will depend on the mechanical and thermal effects.\(^{(8)}\) The stress decreases very rapidly toward the
interior where the sectioning measurement normally is taken. This stress has been observed to be of the order of 0.5 to 1.5 ksi in compression for ordinary cases.\(^{(9)}\)

The sectioning method has been used for years for residual stress measurements in structural steel members. It has proved to be adequate, accurate and economical if proper care is taken in the preparation of the specimen and the procedure of measurement.

2.2 Preparation of Test Specimen

Selection and Location of Specimen

Location of the test piece along the length of the material must first be determined. The test section must be completely clear of cold-bend yield lines if residual stresses due to thermal effects alone are to be measured.

To avoid end effects on the magnitude and distribution of residual stresses, a distance of 1.5 to 2.0 times the maximum linear dimension has been recommended, though theoretically a ratio of 1.0 is sufficient.\(^{(7,10)}\) An edge distance of 2 ft. was taken sufficient to offset any edge effects for the 14H202 test specimen. Figure 1 shows the location of sections for sectioning and Fig. 2 the identification of various elements. Two sets of measurements were taken for the specimen used, to check whether the variation of residual stresses along the length of a column is negligible. Since it is intended to study different methods of residual stress measurement on the same specimen, confirmation of the uniformity of the stresses along the length is important. This is shown in a later section.
Preparation of Gage Holes

Figure 3 shows the detail of gage hole location for the sectioning of the 14H202 specimen. Since transverse sawing disturbs the pattern of residual stress distribution, the gage hole line must be at some distance from the saw cut line to avoid this effect. This distance depends on the thickness of the component plates of the shape and on the cutting procedure. (11) A distance of 1 inch from both ends is sufficient for this particular specimen.

Strain measurements are taken over a 10 inch gage length by a 1/10,000 inch Whittemore strain gage.* The strain readings are from both top and bottom faces of the component plates. The accuracy in reading depends mainly on the gage holes. Following the instructions of the manufacturer, gage holes were prepared using a No. 56 twist drill (0.0465 inch diameter) to a depth of 0.2 inch. All holes were reamed using an angle of 60° and depth of 0.005 to 0.01 inch. An illustration is shown in Fig. 4. A drill bit capable of making such a hole in a single operation is commercially available. The gage holes were centrally located using a standard 10 inch punch and the resulting measured distance between the gage holes showed a variation of 0.01 inch.

Preparation of gage holes at welds and flame-cut areas is difficult, because the material has a higher yield strength at such localized areas due to metallurgical changes from the high heat input. Unreliable readings may result if

the gage holes are not properly prepared. Gage holes at edges or at corners, though not difficult to prepare, may give unreliable readings since the holes may have different alignments, and the extensometer cannot usually be made stable while taking measurements.

Detail for Sectioning

The number of longitudinal strips to be cut depends on the anticipated residual stress distribution. This in turn depends upon many factors, such as edge preparation (universal mill, welded, flame cut), grade of steel, dimension of the specimen and so forth. For the 14H202 section, a \( \frac{1}{4} \) in. spacing was used at regions of anticipated steep stress gradient, and a \( \frac{1}{8} \) in. spacing at regions of low stress gradient as shown in Fig. 3.

To determine the overall pattern of residual stress distribution with a lesser number of longitudinal cuts a "partial sectioning" \(^{10,12}\) can be made. The residual stress distribution through the thickness of the plates can be determined from changes on strain readings after "slicing" of sawed strips.

Method of Partial Sectioning

The number of longitudinal strips to be cut can be reduced significantly if the method of "partial sectioning" is utilized. This method, however, requires a prior knowledge of the pattern of residual stress distribution. A reasonable estimate on the pattern of residual stress distribution rather than its magnitude, is of more importance for an effective use
of this method. The approximate variation in residual stress distribution can be predicted from the geometry of the plate or shape, manufacturing process, heat treatment (such as flame cutting, welding, etc.), and the mechanical properties of the material.

Locations for partial sectioning are so determined that they lie near or at locations of transitions of residual stress gradients. Locations B and C shown in Fig. 5, for example, would be the appropriate locations for the case of the edge welded plate. It is apparent, that complete sectioning of the plate from Location B to Location C into smaller strips would contribute no significant accuracy in residual stress measurement to those obtained after partial sectioning is made at B and C. This is true, provided the residual stress distribution is linear in the region. This also holds true for the regions A to B and C to D where the residual stress varies linearly, since the error caused by bending after partial sectioning is of a secondary order. The sequence of partial sectioning has no influence on the final results, since unloading of the fibers will always be linearly elastic.

Figure 6 shows the layout of cutting positions for partial sectioning used on the shape 14H202. The lower portion of the figure shows the detail for complete sectioning to be performed on the partially sectioned specimen. The total number of cuts required for partial sectioning is only 12 compared to 104 required for the complete sectioning. Figure 7 shows one flange after complete sectioning has been performed. The number of cuts could have been reduced to only four to obtain a very similar result.
Figure 8 compares the results obtained for the inner and outer surfaces of the flange of the 14H202 section after partial and complete sectioning is performed. It is observed that the results obtained from partial sectioning readings are not very much different from those obtained after complete sectioning. The cutting positions, determined from a study of previous tests,\(^{(13)}\) compare very closely to the estimated location for changes in the stress gradient.

2.3 Measuring Technique

The obtaining of reliable results from measured values depends on factors such as the type of strain-measuring device and the procedure of measurement. Mechanical strain gages have been found to be particularly suitable for strain measurement because the strain-measuring device will not be damaged during sectioning and the same device can be used to measure repeatedly. The procedure followed in the Fritz Engineering Laboratory will be discussed together with some additional suggestions later.

The Whittemore Extensometer

The Whittemore gage is a self-contained instrument consisting essentially of two coaxial tubes connected with a pair of elastic hinges. See Figs. 9 and 10. Since the gage is intended for repeated measurement at a series of stations rather than for fixed mounting at one station, consideration has been given to controlling accidental longitudinal forces which might be applied by the operator.
For strain measurements, the contact points are inserted into the drilled holes which are 10 ± 0.02 inches apart. Motion between the two frame members is measured directly with a dial indicator. A handle, serving doubly as a shield against temperature change and as an aid to uniform seating of the points, is attached to the gage by means of two elastic hinges. These hinges prevent application of excessive longitudinal forces. A force of 5 lb. is recommended for properly seating the points in the drilled holes. (14)

Seating the gage is one of the chief sources of error. It is suggested that a positioning angle be used (Figs. 10 and 11) to maintain the Whittemore gage in a perpendicular position to the surface of the specimen being measured. Other sources of error arise from the dial indicator, measurement of a chord rather than an arc length, when the axis of the drilled hole and the axis of the conical extensometer point do not coincide, and temperature changes. However, the effect of temperature change on the instrument itself is practically eliminated by the use of an invar tube.

Accuracy of Measurements

It is evident that changes in temperature will affect strain readings, thus leading to wrong data for the evaluation of residual stress distribution, unless these effects are taken into consideration. Temperature changes during readings may practically be eliminated by using a references bar of the same material as the test specimen. The bar should be put on the specimen to be tested for at
least one hour\(^{(15)}\) ahead of time. This is to stabilize the temperature of the reference bar to the environment of the test specimen. It has been reported\(^{(16)}\) that the response of the reference bar and the specimen are not the same for the same variation of room temperature. The reference bar responds fairly closely to the room temperature variations, while the specimen responds with less fluctuation and with considerable time lag. The response of the specimen therefore, is dependent on its own size.

Measurement should be avoided in direct sunlight or draft or any other source which would cause drastic temperature variation and should be made where the temperature is kept fairly uniform. This will assure readings with minimum effects of temperature changes. With such care taken and under normal conditions, temperature changes may cause an error corresponding to a stress of approximately 1 ksi. Note that a change in temperature of 5°F causes a difference of 1 ksi in the stress evaluation if compensation for temperature change is not taken into account.

The effects of different investigators on the same readings and the personal effects on the readings have been studied.\(^{(16)}\) For both cases, no important difference was observed to prove these factorial elements to have a significant influence upon the accuracy.

Further experimental errors may be attributed to inaccuracies in the mechanism of the extensometer, the dial indicator system, and effects of lost motion when the motion is in the opposite direction. Such inaccuracy may not be
improved significantly by increasing the number of measurements on a gage length. For example, for the Whittemore strain gage it was found that the accuracy would be improved by about 0.2 ksi. by increasing the number of measurements from 5 to 15.\(^{(16)}\) For three measurements an accuracy of about 1 ksi with a confidence level of 99\% could be obtained.

**Procedure of Measurement**

Attention should be given to the importance of obtaining a good set of initial readings since they cannot be duplicated after the specimen has been cut. Better accuracy could also be obtained by estimating precisely the last figure of the reading, whenever the dial indicator lies between the smallest division.

The following is a recommended procedure which has successfully been followed in the past in Fritz Laboratory:

1. Clean all gage holes using carbon tetrachloride or any other cleaning solution, and air blast.
2. Take the reading on the reference bar.
3. Take readings on the specimen. It is suggested to take an intermediate reference bar reading if the number of gage hole readings exceed 15.
4. Read the reference bar again.
5. Repeat steps 2, 3, and 4 until all gage points are read.
6. Do step 5 for at least three times on the whole piece.
7. If three readings on the same set of holes differ by 0.0001 inch or more, the gage holes should be
checked carefully, and additional sets of readings (usually two more) should be taken on the reference bar and on the specimen. Additional readings up to five more times may sometimes be necessary to get better results. If a great variation persists, it is suggested to make a new set of holes very near to the discarded ones, since badly drilled or reamed holes cause large deviations in reading.

After the initial readings are taken and recorded on the data sheets, the specimen is partially or completely sectioned. It is suggested to cover all gage points with tape to keep out dirt, and to avoid damage which may occur in the process of moving, handling, sawing, etc.

The measuring procedure after the partial or complete sectioning is proceeded in the same manner as the initial readings.

An example of data sheets of recorded values for a portion of a flange is shown in Tables 1-5. Table 1 shows a data sheet for the recording of initial readings, Table 2 after partial sectioning, and readings after complete sectioning are shown in Table 3. Note that five readings were taken for gage points 68 and 75 in Table 2 since the variation in reading was greater than 0.0001 in. Computations for residual stresses after partial and complete sectioning are shown in Table 4 and Table 5, respectively. The formats used in Tables 1-5 have been found very convenient for the recording of data and manual computations of residual stresses.
2.4 Evaluation of Data

The computations of the relaxed stress from the measured strain is based on the assumption that the dimensional changes caused by the relaxation are purely linear elastic.

By virtue of the linear strain distribution postulated in the beam theory, the average axial stress $\sigma$ in terms of top and bottom strains, $\epsilon_t$ and $\epsilon_b$ read from the cut element is:

$$\sigma = -E \frac{\epsilon_t + \epsilon_b}{2}$$

(1)

where $E$ is Young's Modulus.

Since strains are read at top and bottom surfaces, evaluation of residual stresses at the respective surfaces are made using the experimental data.

Let $\bar{A}$ be the average value of initial readings on one gage length. For each gage length, $\bar{A}$ is evaluated using

$$\bar{A} = \frac{1}{n} \sum_{i=1}^{n} A_i$$

(2)

where $n =$ number of readings on one gage length, usually three.

$A_i =$ reading value at each cycle.

The average value of initial readings on the reference bar (Ref. $\bar{A}$) is evaluated for every interval of reference bar reading.

In a similar manner, average values of final
readings (after partial or complete sectioning) for gage lengths and the reference bar will be computed as \( B \) and \( \text{(Ref. B)} \), respectively. Using Hooke's Law, the residual stress at the measured surface is then,

\[
\sigma_r = -E \varepsilon_r = \frac{-E}{L} \times \Delta L
\]  

(3)

where \( \Delta L \) is the recorded change in length.

For manual computation, use the data sheets (Table 4 or 5), to calculate the residual stresses as follows:

1. Compute \( \bar{A} - \bar{B} \) (Col. 5)
2. Compute \( \text{Ref. A} - \text{Ref. B} \) (Col. 6)
3. Compute \( \Delta L = (\bar{A} - \bar{B}) - (\text{Ref. A} - \text{Ref. B}) \) (Col. 7)
4. Compute residual stress

\[
\sigma_r = \frac{\Delta L}{L} \times E
\]  

(Col. 8)

where \( L = \) gage length (10 in. for Whittemore gage).

Step 3 gives tensile stresses as positive and compressive stresses negative, which is the usual convention.

Figure 12 shows the residual stress distribution measured at location A. Comparison of residual stress measurement at the two ends is shown in Fig. 13. Using the evaluated residual stresses, the equilibrium condition for the whole section was checked. Theoretically, since no external forces exist, equilibrium requires that the sum of the stresses over the whole cross section must be zero. For
this particular case a difference of 0.7 ksi was computed. This difference may be attributed to the effect of saw cutting and accumulated experimental errors. (9)

Use of the computer will greatly reduce the amount of numerical work involved; if a large number of residual stress measurements is to be encountered. Computer programs for general evaluation of residual stresses have been prepared and have been found very versatile. (17) These programs are:

PLOTRS - reduces data obtained from measurements before and after sectioning and slicing to obtain average readings.
- computes residual stresses
- plots resulting residual stresses

RSNA - uses reduced data from PLOTRS to compute the two-dimensional residual stress distribution
- checks equilibrium of residual stresses
- provides input for PLOTIS

PLOTIS - plots isostress diagram of residual stress distribution.

Using the computer program PLOTRS the residual stress distribution for the section 14H202 was evaluated and the resulting plot is shown in Fig. 14.

The possibility of automatic recording of the original gage readings into a tape or cards by means of linear transducers is under study. (18) After this stage, manual recording, computation and plotting will no longer be required.
3. THE HOLE-DRILLING METHOD

3.1 Introduction

Principle

The hole-drilling method, sometimes referred to as the hole-relaxation method, is based on the fact that drilling a hole in a stress field disturbs the equilibrium of the stresses, thus resulting in measurable deformations on the surface of the part, adjacent to the hole. From a knowledge of the magnitude and direction of relaxation strains, size of hole, property of material, and geometry of the body being examined, the magnitude of residual stresses may be calculated.

Historical Review

The hole drilling method probably was first proposed and applied by J. Mathar\(^{(19)}\) of the University of Aachen (Germany) in 1932. Mathar used mechanical and optical extensometers to measure the changes in displacement between two points across the hole. By drilling a hole, he observed a partial elastic recovery in the immediate vicinity of the hole. From measurement of this elastic recovery, it was possible to determine the residual stresses in the specimen.

In his experiment, Mathar used a dial extensometer, placed in a radial direction with one gage point very near to the hole. His experiments were limited to pure tensile and pure compressive stresses. The calibration of the measuring gage was accomplished by testing a specimen of about the same
size as the work piece in a machine for tension tests, drilling the specimen and at the same time carrying out measurements. He then established experimental curves for the determination of the actual stresses. These curves give the true stresses directly as a function of the dial readings. His apparatus and results have been subject to criticism because vibrations during drilling operations make the reading unsteady and irregular.

Replacing the mechanical extensometer with electrical resistance wire strain gages, Soete and Vancrombrugge\(^\text{(20)}\) of the University of Gent (Belgium), eliminated the difficulties of measurement and improved the precision. At the same time they were able to determine the plane stress distribution by measuring the elastic recovery in three radial directions. On the basis of Airy's stress function, Soete formulated an equation for the determination of stresses occurring in a region of the same size as the drilled hole, and plotted a diagram, showing the relation between stresses and strains. With the aid of empirically prepared diagrams, and by measuring the strains produced during drilling to different depths, Soete and Vancrombrugge\(^\text{(20)}\) were able to determine the stresses occurring at different depths under the surface of the work piece.

Further work on measuring non-uniform residual stresses by the hole drilling method was performed by Kelsey.\(^\text{(22)}\) He developed a procedure to determine the relationship between surface strain and hole depth for a known uniform stress field; and then to correlate these data with those obtained by drilling a hole in a known
non-uniform stress field. His approach is based on the assumption that the incremental surface relaxation strain for a corresponding hole-depth increase is proportional to the magnitude of the stress at that depth. The method is empirical and depends on experimental calibration.

Recent refinements in strain-gage-manufacturing techniques have made it possible to obtain strain gages of very small dimensions. Rendler and Vigness (23) reported successful results of residual stress measurements using dimensions as small as 1/16 in. diameter holes and 1/16 in. strain gages. Cardiano and Salerno (24) reported that the experimental data confirm with the assumed theory, for measurement of residual stress on a plate with a linearly varying stress field (using the toe of tee-fillet welds). Recently, Bert, et.al. (21) reported on the applicability of the hole-drilling technique for experimental determination of residual stresses in rectangular orthotropic materials.

Though considerable work has been done in recent years to establish the method theoretically, it would still seem that the solution of the problem must be of an empirical nature.

**Strain Measurements**

The purpose of measuring the relieved strain by drilling is to evaluate the release in stress. This seems to be the only manner of determining internal stresses, since the forces acting within a material usually are unknown, in
both magnitude and direction. In brief, measurement of strain is basically the only manner in which stress can be determined, since stress is not a fundamental physical quantity like strain, but only a derived quantity. These arguments, however, require two fundamental assumptions for the determination of residual stresses: the equilibrium of the stresses inside a body, and the continuity of the deformed material.

The strains are measured as an elastic recovery after release of the previously existing system of internal stresses. The amount of recovery is small. For accurate evaluation of stress at a point the gage length must be shortened. To provide a careful consideration for these two factors, a measuring device with a very short gage length and high precision should be used.

Three types of measuring devices are universally used, namely, electrical, optical and mechanical gages. The bonded electrical strain gages offer the most accurate and convenient means of measuring strains, especially if residual stresses could be completely freed. Optical gages can give accurate readings because of the fact that a beam of light can act as an infinitely rigid, weightless, and inertialess pointer, of far greater length than would be practical for mechanical pointers. In spite of the advantages of electrical and optical methods, however, purely mechanical devices are still in widespread use, and for many purposes are much more convenient.
Features of the Hole-Drilling Method

The hole drilling method has the advantage of removing a minimum amount of material which makes it the least destructive of the mechanical methods for measuring residual stresses. The method can be termed as similar-destructive if holes of very small diameters are used. If desired, the hole can be filled by welding, or else, a bolt or plug can be inserted in the hole.

Unlike other mechanical methods, the hole drilling method permits the evaluation of residual stresses at what is essentially a point, a special application of which is the measurement of transverse residual stress. Application as a field test is relatively simple, and results can be obtained readily and economically. This method, however, has a limitation of depth and is used to measure stresses very near to the surface.

3.2 Mathar's Method

In order to explain the principle of the method, consider a specimen subjected to a uniaxial stress which is uniform through the thickness. A measuring gage is mounted on this test piece to measure the strain in the same direction as the applied stress.

If a circular hole is drilled between points a and b in Fig. 15, this hole will become elliptical and the distance between a and b will be changed: increased if the stress was tension, decreased if the stress was compression. If the relationship between the change in this distance and
the stress is determined by calculation or calibration test, then the stress in the test piece in the direction $\overline{ab}$ can be calculated from the change in the distance between $a$ and $b$. (19)

For the case of biaxial state of stress, one measurement will not be enough, and the deformation of the hole must be measured in at least three directions in order to determine the magnitude and direction of the principal stresses. In this section, the case with a uniaxial state of stress only will be considered.

**Calibration Test**

A calibration test is required in order to determine the relationship between the strain of the test distance produced due to drilling, and the stress in the test piece. Calibration can be done either by calculation or by experiment.

Calibration by calculation was first reported by Kirsch, (25) who calculated the deformation of a hole in a member of infinite width in terms of the uniaxial applied stress. Willheim and Leon (26) extended this method approximately to members of finite width. Mesmer (27) generalized the formula for the case of plane stress distribution, under the assumption that the direction of the principal stresses were known. A further generalization was given by Campus, (28) expanding the formulas to the case in which the principal axes directions are unknown.

Extensive work has been done in recent years to establish calibration by calculation for the case of
uniform (20, 29, 30, 31, 32) and non-uniform (21, 22, 23, 33) residual stress distribution over the thickness of the plate.

Experimental calibration can be made by mounting a test specimen in a tensile machine and drilling on the stressed test piece a hole similar to that to be used for the residual stress determination. The flat plate is loaded at various stress levels and the changes in distance between the gage points are determined as drilling progresses. From this must be subtracted the distance increase which would have occurred if the hole did not exist.

An experimental calibration was conducted for a uniaxial stress state. A known stress was applied in the direction of the gage lengths on a test specimen, with the hole and gage system aligned as shown in Fig. 15. The test specimen was designed to satisfy certain design requirements using available equipment, which are discussed in the following sections.

The Calibration Test Specimen

In designing the calibration test specimen it was necessary to consider and satisfy the following points:

a) the applied tensile stress must be uniform throughout the cross-sectional area of the specimen.

b) a measurable change in strain in the material should be produced.
c) the hole must be small compared with the specimen dimensions and must be far enough from all boundaries.

d) the applied load must be of a magnitude not to produce plastic flow of the material near the hole due to high stress concentration.

The specimen, 1\frac{1}{2} by 4 in. cross section and 5 ft. in length, mounted in an 800,000 lb. mechanical type testing machine may indicate the presence of unwanted flexural stresses. Requirement (a) was satisfied by reducing the flexural stresses in the test section to negligible values (less than two percent of the applied stress) by proper alignment. Alignment was carried out by mounting strain gages on all four sides of the specimen at a distance of 6 inches from each grip end (Fig. 16). This distance is sufficient to make the section of interest remote from the boundary and thus not influenced by the St. Venant end effect. Any change in machine-specimen alignment during test could be detected from the readings of the gages mounted on opposite sides of the specimen.

It is certain that residual stresses in the specimen will affect the uniformity of the stress distribution. To eliminate the residual stresses, the test specimen was heat treated at a temperature of 1200°F for one and one-half hours (1 hour per inch of thickness). The specimen was then left inside the furnace where it was allowed to cool uniformly at a very slow rate. This temperature-time combination will reduce residual stresses to a negligible value without introducing metallurgical changes.
Requirement (b) was for a measurable strain output from the gage lengths. In general, the strain relaxations due to hole drilling are very small in value. This difficulty could be relieved by increasing the magnitude of the applied stress and also by increasing the gage length. A Huggenberger extensometer with 20 and 100 mm gage lengths and nominal strain sensitivity of 0.001 mm (0.0000394 in.) was used for strain measurement. In Fig. 17 the extensometer with its accessories is shown.

Requirement (c), that the boundary must be at such a distance from the hole without affecting the measurements may be satisfied if a minimum width of ten times the diameter of the hole is used. The change in stress distribution caused by the unsymmetrical reduction of cross-sectional area due to the drilling could be reduced significantly if the cross-sectional area of the specimen is large compared to that of the hole. Use of a ¼ in. diameter on the 1½ by 4 in. specimen is within these requirements. This dimension combination of hole diameter and gage length provides sufficient edge distance so as to give an appreciable change in strain outside the region where plastic deformation may be encountered. Calculations based on equations given by Timoshenko for a small hole in a wide plate subjected to a uniaxial tension showed that the longitudinal and the transverse stresses four diameters from the hole axis in the longitudinal direction deviate less than four and one percent, respectively from the longitudinal stress remote from the hole. Accordingly, the minimum pitch is 2 in. for ¼ in. diameter holes.
Requirement (d) was for an applied stress of such magnitude that no plastic flow of the material should occur in the region of the hole. So long as the stresses are less than 40 percent of the proportional limit, no plastic deformation due to high stress concentration will occur near the hole. To improve this situation, a test material may be selected having a high yield point. But for the purpose of comparison, the choice of the test material was restricted to A36 steel, and this permitted an applied stress of about 14 ksi.

Preparation of Gage Points

Gage lengths of 20 and 100 mm were used simultaneously for the same hole for the purpose of comparison (Fig. 15). The gage points for the 20 mm gage length were each located at 10 mm from the center of the hole. The gage points for the 100 mm gage length were located at 10 and 110 mm from the center of the hole.

The region to be measured was made smooth and carefully prepared. Coating the surface with lay-out dye was of help for smooth scribing. The gage points were marked first with a light hammer blow using a standard punch (Fig. 17). The gage points were steel balls of 1/16 in. diameter. The gage point furthest from the hole was imbedded using a special punch (Fig. 17) after drilling a hole smaller in diameter than the steel ball, using drill No. 56 (Fig. 18a). Since imbedding using a punch itself may introduce undesirable residual stresses, those gage points in the vicinity of the main hole to be drilled were fixed using Armstrong A-6 Epoxy adhesive, as shown in Fig. 18(b). In both cases, care was
taken to make sure that the holes were imbedded not too deeply, to prevent the measuring gage from sitting properly; this is done by sinking the ball's "equator" slightly below the surface. In general, gage points imbedded using the standard punch seem to be more preferable, since they are easier to perform, and can be fixed strongly into position.

Drilling Technique

The location and alignment of the hole was controlled by means of a hole milling fixture as shown in Fig. 19. The hole was drilled using a \( \frac{1}{8} \) in. high speed center-cutting end mill. The bottom of the hole is to be flat to permit meaningful measurements of the hole depth. Hole depth increments are read using the allowed tolerance of 0.0002 in. depth gage micrometer. Care was taken to keep the end mills sharp to avoid blemishes or tears, and this was checked by closely observing the condition of the cutting edges at appropriate operation intervals.

At the earlier stage of this study a boring unit, which included the end mill, "Versamatic" (to reduce speed of rotation), and electric drill centered on the specimen by the milling fixture was used. Clearance was provided beneath the milling fixture for chip removal and for gage point protections. Both ends of the fixture carried index marks for centering the unit over the gage assembly in the longitudinal direction. With the unit held to the specimen in the indexed position, a cross bar was placed against the end of the fixture and clamped to the specimen. The cross bar remained on the specimen throughout the test and provided a positive index stop for the fixture. To
reduce effects that may be caused by a high rate of drilling, the 1100 rpm speed of the electric motor was reduced to a desirable speed of 180 rpm using a speed reducing device. This device ("Versamatic") also served simultaneously the purpose of acting as a flexible coupling. Figure 20 shows the equipment used in an assembled view. In Fig. 21 all equipment used and the calibration specimen in the testing machine is shown.

At a later stage of this study a portable magnetic-base press (Fig. 22) was available and was used with greater ease and efficiency. The time required to complete the drilling for one hole using this equipment has reduced to about fifteen minutes compared to about four hours when using the original set. A speed of 190 rpm was considered sufficient to minimize the residual stresses that may be induced due to machining.

Test Results

A 3/8-in. diameter hole was drilled on the calibration specimen to determine the relationship between the measured strain and the corresponding hole depth. The specimen was stressed to 13 ksi in tension and the milling was stopped at average increments of 0.04 inch in depth after which measurements were taken. The characteristic curve of the measured strain relaxation as a function of non-dimensional hole-depth is shown in Fig. 23.

The plot shown in Fig. 23 indicates that the surface strains increase rapidly up to a depth-diameter ratio of about
0.8 and do not change appreciably for greater hole depths. Thus, calibration based on a hole depth of one diameter makes use of the maximum released strain and was used as a standard depth to establish the calibration curve.

Calibration required conducting several similar tests on the same specimen while the specimen is subjected to different levels of loading. To obtain test points the release in strain due to drilling and the corresponding stress of the specimen need to be known. To appreciate a better understanding of the changes in strains during the whole operation, it would be advisable to take strain readings before and after changes in stresses have occurred. The following steps in strain measurements are recommended:

Take readings: (1) before the specimen is loaded 
(2) after the specimen is loaded 
(3) after drilling is performed 
(4) after the specimen is unloaded.

Figure 24 shows schematically the history of strain changes for an ideal case that would occur during the whole operation of calibration.

Calibration tests were conducted for uniform stresses of 13.3 ksi, 16.7 ksi and 20.0 ksi. The history of strain changes is plotted for each test as shown in Fig. 25. Initial and final readings were taken while the specimen was under a nominally small load (10 kips equivalent to 1.6 ksi) in order to maintain the grip which was originally established for the alignment of the specimen.
All test results (Fig. 25) show that the unloading lines do not pass through the origin. This discrepancy may be due to the sum of the residual stress originally existing in the specimen and the residual stresses induced due to the milling operation. Since the specimen has been heat-treated, the major part of the difference may be due to the milling operation.

To determine the total change in strain readings that might have occurred during the milling operations, a test was conducted on the unloaded calibration test specimen. Two holes were drilled on the specimen using the same end-mill and milling procedure as used previously. A total number of four measurements, two longitudinal and two transverse readings were taken (Fig. 18) before and after the drillings. The resulting readings are given in Table 9. It is noted that all four readings are almost identical even for the two different directions. Based on these four measurements the average residual strain due to the milling operations alone was determined as $38 \times 10^{-4}$ mm (Table 9). This value was taken into account, and separated from the total strains in order to establish the final form of the calibration curve. In general, it is necessary to measure such initial strains; it is expected that lower values should yield better results.

Figure 26 shows a scatter band of test points obtained from five calibration tests. The calibration curve shown in Fig. 27 is obtained as the arithmetic mean of the test points. Using this relationship, the residual stress distribution in the 14H202 section can be determined.
A total number of 28 holes were drilled on the outer surfaces of the two flanges of the 14H202 shape using the same procedure of hole drilling as applied to the calibration test specimen. Figure 28 shows the layout of holes used on one flange of the shape. A radial drill press (Fig. 29) was used to drill holes on the shape but at a later stage the portable magnetic base drill (Fig. 30) was found to be more convenient.

The steps in strain measurements followed were similar to those used in the sectioning method (Section 3.3). Tables 6 to 8 show the data sheets for recording strains and evaluating the residual stresses where hole numbers 1 to 10 are used as an example. The difference in strain readings obtained from the 28 hole drillings are shown in Fig. 31.

Using the calibration curve (Fig. 27) and the difference in strain readings, the residual stresses at the 28 locations were determined. The average residual stress distribution across the surface of the flange was evaluated, and the result is shown in Fig. 32.

3.3 Soete's Hole Drilling Method

Principle

Soete's method of hole drilling is based on the same fundamental principle as that of Mathar's (Section 3.2), except that in Soete's method, measurements are taken using electrical resistance wire strain gages instead of mechanical extensometers.
The strain gages Soete and Vancrombrugge (20) used, though the smallest available at the time, were long compared to the size of the hole. If it is desired to make residual stress measurements near weldments or flame-cut edges, it is apparent that strain gages having short gage lengths should be used because of the sharp stress gradients that exist in such neighborhoods. Recent refinements in strain-gage manufacturing techniques have made it possible to obtain strain gages of very small dimensions. Thus, a hole of a very small diameter and depth may suffice for a residual stress measurement. Use of such small dimensions cause only a tolerable amount of destruction of the material and have a special advantage when used in regions with steep stress gradients.

In this method too, the experimental approach requiring the determination of empirical calibration constants, was used to evaluate residual stresses.

Calibration Test

The reasons for the calibration test and the method of application has been explained in Section 3.2. Calibration was made on the same test piece as used for Mathar's method. The hole-gage assembly used is shown in Fig. 33. Foil strain gage rosette, type EA-09-125RE with a gage length of 0.125 inch were used. The main reason for using this assembly is because it was the only type specially prepared for residual stress measurements available commercially at the time. With preassembled gages, the necessary operational skill is reduced to that of locating the cutter in the center of the rosette.
The radial orientation of the gages has the advantage of obtaining a satisfactory sensitivity especially at high stress levels.\(^{(30)}\)

**Theoretical Consideration**

The procedure for obtaining the calibration constants was simplified by making the minimum principal stress zero and by applying a known stress in the longitudinal direction of the test piece. Under such a uniaxial condition, Rendler and Vigness\(^{(23)}\) have shown that calibration constants \(A\) and \(B\) may be determined from the formulas:

\[
A = \frac{\varepsilon_1 + \varepsilon_3}{2\sigma} \tag{5}
\]

\[
B = \frac{\varepsilon_1 - \varepsilon_3}{2\sigma} \tag{6}
\]

where \(\varepsilon_1\) = radial strain in the direction of the applied load (longitudinal strain),

\(\varepsilon_3\) = radial strain in the direction perpendicular to the applied load (transverse strain),

and \(\sigma\) = applied stress.

After the calibration constants \(A\) and \(B\) for the hole-gage assembly are determined, the principal stresses can then be evaluated using the formulas:

\[
\sigma_{\text{max}} = \frac{\varepsilon_1(A+B \sin \gamma) - \varepsilon_2(A-B \cos \gamma)}{2AB(\sin \gamma + \cos \gamma)} \frac{E}{\mu} \tag{7}
\]

\[
\sigma_{\text{min}} = \frac{\varepsilon_2(A+B \cos \gamma) - \varepsilon_1(A-B \sin \gamma)}{2AB(\sin \gamma + \cos \gamma)} \frac{E}{\mu} \tag{8}
\]
where \( \gamma = \tan^{-1} \left[ \frac{\epsilon_1 - 2\epsilon_2 + \epsilon_3}{\epsilon_1 - \epsilon_3} \right] \)

\( \epsilon_1, \epsilon_2 \) and \( \epsilon_3 \) = strains measured by strain gages \( G_1, G_2 \), and \( G_3 \) respectively (see Fig. 33).

The direction of the maximum principal stress \( \beta \) measured counterclockwise from the transverse direction is given by

\[ \beta = -\frac{1}{2} \gamma \]  

To use the calibration constants obtained from the test piece in actual residual stress measurements on any specimen, the following variables must be considered:

1. Material - grade of steel
2. Stress - tensile or compressive (uniform); bending (non-uniform)
3. Geometry - thickness, width, length
4. Hole-gage assembly
5. Method of hole drilling

The calibration constants \( A \) and \( B \) contain the material constants \( E \) and \( \mu \) (Young's Modulus and Poisson's ratio), which are constant for all elastic and isotropic materials. Since all grades of structural steel have essentially the same values of \( E \) and \( \mu \), the variable caused by a difference in material may be neglected.

It has been reported\(^{(22)}\) that calibration constants obtained under uniform tensile stress give results of less than five percent difference than those obtained under uniform
compression stress. This may be attributed to the exact similarity of the stress-strain curves in tension and compression.

Assurance must be provided that the calibration constants are independent of the specimen size. It has been reported\(^{(23)}\) that valid calibration constants are assured for plates whose boundaries are at a distance equal to or greater than eight hole diameters from the hole center line and for plates of four or more hole diameters in thickness.

The hole-gage assembly is the predominant variable that changes the values of calibration constants. The constants may be made independent of the assembly dimensioning if all of the important dimensions of the hole gage assembly are made proportional to the dimension of the calibration model. As long as this principle of similitude is maintained, all the different hole-gage assemblies will be represented by a single non-dimensionalized specification of the calibration model. This will be true provided the restrictions pertaining to the material boundaries are observed.

Although the drilling technique affects the accuracy of the method, it should be pointed out that for a specified hole diameter the method will be independent of machining stresses as long as a standardized drilling procedure is used throughout the whole operation, including the calibration test.
Experimental Procedure

The experimental calibration was simplified by using the test specimen under uniaxial tension. This is because the minimum principal stress is made zero and the maximum principal stress is the applied stress in the longitudinal direction.

For the hole-gage assembly shown in Fig. 33, strain $\varepsilon_1$ is measured by the longitudinal gage, $\varepsilon_2$ by the diagonal gage and $\varepsilon_3$ by the transverse gage.

The Test Specimen

In Section 3.2, the design of the test specimen used for the Mathar's method has been discussed. The same principle and design requirements apply to Soete's method. Since the same specimen is used for calibration no further discussion is necessary.

Strain Gages and Application

Strains were measured with electric resistitivity SR-4 strain gages. Epoxy-backed, etched-foil strain gages Type EA-09-125RE, 1/8 inch in size and preassembled into a 45° rosette were used.

Before the gage assemblies were mounted the surface of the test specimen was carefully sanded in the vicinity of the gage location with a power hand sander to remove rust, mill scale and roughness. It was smoothed with a very fine cloth and finally cleaned with acetone. Gage location lines were then scribed on the specimen surface in order to locate
the rosette in the exact location and orientation. After the surfaces had been prepared, the gages were cemented with Eastman 910 cement. The gages were then protected with a coat of wax.

Too much emphasis cannot be placed upon obtaining a good gage-to-metal bond for such work. The shear strength must be high and uniform over the entire gage area. Since the gages are placed close to the hole, in a region of high stress gradients, a weak bond, even though local in character, will be reflected in the average strain output of the gage.

Every precaution was taken to check the gage bond prior to test. Crude tests such as pressing the gage with some soft pointer at many points over its surface were of help, since a poor bond causes abnormal strain gage readings when under such pressure. Bond tests under load and non-load conditions were of supplementary importance.

The Drilling Technique

The location and alignment of the hole to be drilled was controlled by means of the hole milling fixture (Fig. 19). The hole was drilled using a 1/8 in. high speed end mill.

A hole milling equipment capable of reproducing straight, untapered holes, perpendicular to the material, and the hole cutter capable of removing material from the bottom of the hole without disturbing the established hole wall was designed by Rendler and Vigness. (23) This milling
cutter is shown in Fig. 34. End mills are normally supplied with cutting edges on the end and side of the mill. The mill will cut when provided with either an axial or a lateral feed motion. The cutter, as provided by the manufacturer, will not produce the specified hole. Lateral thrust upon the end of the mill due to imperfections on its four lead cutting edges invariably produces a tapered hole. The unwanted side cutting edges were removed by machining. The non-cutting sides of the short lower section of the cutter acted as a bearing in the established hole and prevented the removal of additional material from the hole.

To eliminate internal stresses induced due to the clamping of the fixture, the milling fixture was removed whenever strain measurements were taken.

Method of Loading

Load was applied to the calibration specimen by means of a mechanical-type testing machine. A uniformly distributed stress within the working region was attained by proper alignment of the specimen. Ten monitor gages shown in Fig. 16 were sufficient for proper alignment. Since the testing machine used was of a mechanical type, it was possible to maintain a constant load on the specimen for a long period of time.

Test Results

Tests were conducted first on the calibration test specimen. The characteristic curves of the measured strain relaxation as a function of non-dimensional hole-depth is
shown in Fig. 35. Figure 35 shows that calibration based on a hole depth of one diameter makes use of the maximum released strain, and was used as a standard value.

Using Eqs. (5) and (6) the calibrated values of the constants $A$ and $B$ at the depth ratio of 1.0 are

$$A = \frac{\varepsilon_1 + \varepsilon_3}{2\sigma} = -0.30 \times 10^{-8} \text{ in}^2/\text{lb} \quad (5a)$$

$$B = \frac{\varepsilon_1 + \varepsilon_3}{2\sigma} = -0.59 \times 10^{-8} \text{ in}^2/\text{lb} \quad (6a)$$

It is noted that $B$ is approximately equal to $2A$. Substituting the values of $A$ and $B$ in Eqs. (7) and (8) provides the final calibrated solutions:

$$\sigma_{\text{max}} = \frac{\epsilon_2 (1-2\cos \gamma) - \epsilon_1 (1+2\sin \gamma)}{1.20 (\sin \gamma + \cos \gamma)} \left( \frac{E}{\mu} \right) \quad (7a)$$

$$\sigma_{\text{min}} = \frac{\epsilon_1 (1-2\sin \gamma) - \epsilon_2 (1+2\cos \gamma)}{1.20 (\sin \gamma + \cos \gamma)} \left( \frac{E}{\mu} \right) \quad (8a)$$

where $\gamma = \tan^{-1} \left[ \frac{\epsilon_1 - 2\epsilon_2 + \epsilon_3}{\epsilon_1 - \epsilon_3} \right]$.

Using the same procedure of hole drilling as applied to the calibration test specimen, three holes were drilled on the flange of the 14H202 shape at the locations shown in Fig. 28. The magnetic-base press (Fig. 30) was used. The difference in strain readings due to drilling are given in Table 10. Using the calibration equations 7a and 8a the values of the principal residual stresses and the directions at the three locations are computed in Table 11. The
resulting values are plotted as shown in Fig. 36. The results obtained show a very close agreement to those obtained from the method of sectioning as shown in Fig. 36 (see also Fig. 14).
4. OTHER METHODS - A BRIEF SURVEY

4.1 Grooving-Out Methods

Principle

The grooving-out methods are based on the principle identical to that of the hole-drilling methods (Chapter 3). In fact, both methods utilize the partial released strain for the evaluation of residual stresses, the only differences being the technique of releasing strains, and the locations where measurements are taken. Measurements are taken on the remainder of the structure for the hole-drilling methods whereas measurements within the grooved area could be taken for the other methods. In general, grooving-out methods release more strain within the groove.

The technique of grooving is as equally important as the measurement of strains. If machining is used to make the groove, it is clear that identical conditions may not be produced owing to such variables as the sharpness of the cutting tool, amount and application of cooling fluid and interaction of material and tool. Also, machining itself may introduce residual stresses by heating and plastic deformation of the machined surface.

Several forms of grooves have been considered in the literature. The two prominent forms, the ring-groove and the straight-groove, were studied by Gunnert (35) and Schwaighofer (36) respectively, and are discussed below.
4.2 **Gunnert's Method**

In Gunnert's Method\(^{(35)}\) stress relieving is done by trepanning a groove round the gaging area by means of a core drilling as shown by the two outer lines in Fig. 37. The core drill is guided by drilling a small hole in the center of the measuring surface and using a spring guide.

The method is applicable in measuring surface stress only. But it has the advantage of evaluating local residual stresses close to the yield point.

4.3 **Schwaighofer's Method**

Schwaighofer's method\(^{(36)}\) requires that two grooves surround the element under consideration to achieve the desired stress release. Schwaighofer used, with success, a high-speed dental drill (300,000 rpm) for groove cutting. Rimrott and Weikinger\(^{(37)}\) utilized chemical milling to reduce residual stresses introduced by milling. They found the reproductibility of test conditions high, and the method to be suitable for metals of any surface hardness.

Figure 38 shows an electric-resistant strain gage between two grooves to measure the residual stress on the surface of a wide plate. The measured strain at the surface is dependent on the dimensions of B, C, D, and E. It is also influenced by the distribution of the residual stress across the entire depth of the cut and the modulus of elasticity of the material. Schwaighofer used a narrow bar of rectangular cross section subjected to uniaxial residual stress, with the whole thickness grooved, for his analysis.
The influence diagram shown in Fig. 39 which is the strain readings at the surface due to a traversing load \( P \) on a plate of unit thickness, illustrates the fact that the surface strain measured is not solely due to the residual stress on the surface. Inspection of Fig. 39b reveals that a load applied at a distance \( d=a \) from the gage causes a strain reading of negligible magnitude. This depth, therefore, may be taken as the minimum required if no noticeable change in strain reading is desired.

Another interesting feature is the shape of the influence diagram (Fig. 39b). It remains essentially the same for different values of \( \alpha \).

4.4 **Deflection Methods**

It has been mentioned above that residual stresses in metals are measured indirectly by the elastic strains existing in the stressed body. The problem in measuring residual stresses is to reverse and measure these strains. The basic principle in using deflection methods is, therefore, to embody a reversal of the process by which the strains were produced, that is, to separate portions of the whole body and to observe the difference.

Bauer and Heyn\(^{(38)}\) and Howard\(^{(39)}\) developed methods of determining residual stresses by removing concentric layers from a cylindrical rod or tube and measuring the resulting elongation or contraction. With sufficient number of steps the longitudinal residual stress distribution is determined. Figure 40 shows a schematic illustration of the
method. This method is, however, an approximation since only the longitudinal stresses are considered. The presence of transverse and radial stresses that would be present in the general case are ignored. These limitations were recognized by Mesnager,\(^{(40)}\) who developed a method for round bars or tubes similar to that of Bauer and Heyn, wherein he proposed the removal of the material from the center of the cylindrical rod or tube and the measurement of both the longitudinal and circumferential strain in the remaining portion. The residual stresses, longitudinal, radial and tangential within a cylindrical rod or tube can be determined accurately by substituting the measured strain in the formulas for tubes subjected to external and internal pressure, which can be found in books on the theory of elasticity. Sachs\(^{(41)}\) greatly simplified the calculation and today the method is popularly known by his name. A schematic illustration is shown in Fig. 41. It should be noted that this method is limited to objects having rotational symmetry both in shape and in stress distribution.

A method using the same principle was developed by Stäblein,\(^{(42)}\) and it has become the most popular since it has a special range of application. Removal of material on one side of a strip with residual stresses, results in bending of the strip as shown in Fig. 42. By measuring the curvature or deflection at different stages the residual stress distribution can be determined. Richards\(^{(45)}\) performed similar tests on a plate-like specimen by progressively milling thin layers from the surface and measuring the
resulting curvature changes. Treuting and Read\textsuperscript{(44)} conducted similar tests by hand grinding the layers.

A rapid and accurate technique developed by Waisman and Phillips\textsuperscript{(45)} is to remove layers from one side by a chemical solution. Another advantageous feature of such an arrangement is the possibility of taking measurements at any desired time without removing the specimen from the etching chemical. The most recent contribution made to this method by Zapel\textsuperscript{(46)} and by Frick et al\textsuperscript{(47)} is to develop it into a simpler and computerized technique. This method has a particular application for the determination of residual stress distributions varying through the thickness of the material.

4.5 The Trepagnning Method

The method developed by Meriam et al\textsuperscript{(48)} accomplishes relaxation by removing a plug of metal containing gages by drilling a series of overlapping holes. If the directions of the principal stresses are not known, the strain gages are arranged in rosette form. The strain gages are read only twice; once prior to drilling and once after completion of drilling and after the plug has been removed. The stresses are then calculated using the principles of the strength of materials, using linear stress-strain relationship.

The trepanning method is a semidestructive method. It can be reliable in cases in which the stresses are fairly constant over the area to be measured.
4.6 The X-Ray Method

The first reported attempt of X-ray stress measurement was made in 1925 by Lester and Aborn (49) who measured the lattice parameter of a thin steel strip, subjected to tension.

The fundamental theory of stress measurement by means of X-rays is based on the fact that the interplanar spacing \( d \) (Fig. 43) of the atomic planes within a specimen is changed when subjected to stress. Therefore, since \( d \) is fundamentally constant for any set of planes in a material it acts as a convenient gage for elastic measurement. A change in \( d \) due to applied stress or residual stress enables the elastic strain to be measured and the corresponding stress calculated.

Recent developments in the field of X-ray diffraction measurement of strain in metals have led to renewed interest in their use for the determination of residual stresses. Several methods have been developed and numerous papers have appeared in the last few years. A large list of references related to X-ray stress analysis can be found in Refs. 50 and 51. Most of these measurements were applied to small specimens, such as cylinders or bars. An experimental study of residual stresses in a structural H-shape using X-ray diffraction has been reported. (52) In these tests, stresses were measured on the surface of the shape, that is, non-destructive measurements, as well as after successive removal of material from the surface. In this way, an indication of the through-thickness variation of residual stresses may be obtained.
The technique generally used can best be described by referring to Fig. 44. Two spacing measurements are made; one perpendicular to the surface and another in the direction inclined to the surface at a known angle and lying in a plane fixed by the surface normal and the direction of the surface. The difference of these two spacings divided by the unstressed spacing gives the difference of the strains in these two directions, from which the desired stress may be calculated.\(^{(53)}\) If only the sum of the principal stresses is desired and their direction known, fewer strain measurements are required. If both the principal stresses and their direction are to be determined surface strains in three arbitrary directions must be measured in order to calculate the principal strains and arrive at the principal stresses. The precision of the measurements is reported as 1 to 2 ksi.

In this field the advantages and disadvantages of the X-ray technique have often been discussed at length in the literature. The principal advantages of the X-ray method are:
- it is non-destructive when applied to surface measurements,
- an exceptionally small "gage length" can be used, that is, it is very localized,
- it is well adapted for determining peak stresses,
- it can be used in places inaccessible to ordinary strain gages, for example, roots of notches,
- it is capable of measuring stresses confined to thin surface layers.

Disadvantages of the X-ray method arise chiefly from the limitations imposed by the equipment and size and
shape of the piece rather than by the basic principle of the method. Other disadvantages are, it is:
- time consuming with conventional equipment,
- not always possible to interpret the measured strain in terms of stress system due to effects of plastic deformation,
- it measures only the surface stresses.

In conclusion, X-ray strain measurements have frequently been shown to be useful in very different fields of applied and basic research. It seems, however, more work has to be done yet to bring the method to a more practical level. Recently, an automatic X-ray analyzer based on X-ray diffraction has been developed.\(^{(54)}\) This equipment will enable residual stress at a point to be measured in 20 seconds.

4.7 The Ultrasonics Method

The term ultrasonics is used to describe mechanical waves propagated through a medium at frequencies above the upper limit of hearing of the human ear. The propagation of ultrasonic waves in a material is related to the elastic properties of the material and the homogeneity of its structure. The two usual types of tests of ultrasonics are: searching for discontinuities and examining the properties of a continuous medium.

Of the several ultrasonics techniques studied, the one based on double refraction of shear waves has received more attention for measuring residual stresses. The phenomenon of double refraction of a shear wave is associated with the separation of the shear wave into two
components which are transmitted through the medium on planes at right angles to each other. This birefringence of the wave will occur only if the medium is anisotropic and if the direction of the particle motion does not coincide with a principal axis. Since a stressed body is anisotropic, a relationship between the wave velocities and stress can be derived, based on the velocity difference components in the two directions. To determine this difference experimentally a measuring system consisting of a pulse oscillator, a switching circuit, a receiver, an oscilloscope and a piezoelectric transducer is sufficient. A quartz crystal is used as the transducer to produce a shear wave and is coupled into the specimen by means of a thin film of grease.

To date, the technique has been used only in the laboratory on specimens whose microstructures are well-documented. The method is nondestructive, but stresses are measured at what is essentially the surface. In addition, the technique is capable of providing information only on the difference between the principal residual stresses, and not the absolute magnitude of these stresses.

4.8 The Brittle Lacquer Method

A method reviewed by Gadd employs the use of stress coat as an indicator of residual stresses. By using brittle lacquer, a quick overall picture of location, direction and kind of stress can be obtained. There are, in brief, two methods of using brittle lacquer in studying residual stresses.
337.8

The brittle lacquer coating forms characteristic crack patterns at right angles to the principal tension strain when the test part is loaded. The lacquer will crack first at regions where the residual and applied stresses add up to the yield strength of the test material.

The other method of using brittle lacquer in studying residual stresses is by relieving stresses. It has been reported\(^{(2)}\) that a hole not over 1/8 inch in diameter drilled to a depth of 1/16 to 1/8 inch in an internally stressed object causes enough relief of stress to crack the lacquer near the hole. The clustering of the cracks indicates the type of the stress in the area.

Usually nothing is required other than very careful solvent cleaning to remove any trace of oil or other liquid. On rough surfaces, sand blasting is recommended to remove any sharp points.

Different forms of crack patterns and their corresponding state of stress are shown in Fig. 45 for stresses which are usually low, chilling of the lacquer will bring out the crack pattern.

The two main features of this method is its simplicity and its freedom of material limitations. This method, however, should be considered qualitative rather than quantitative.
4.9 **Indentation Methods**

Several means of using hardness measurements for the determination of residual stress have been proposed\(^{(58,59)}\) based on the principle that the hardness of metal parts depend on stresses acting on those parts. Using ordinary hardness-testing procedures, the magnitude of the stress at a particular point can be determined.

Investigations conducted on steel indicated that the relationship between stress and change of hardness is practically linear as long as those stresses are within the so-called linear range. Furthermore, the same tests indicated that the change of hardness is greater for tensile than for compressive stresses.

Recent work\(^{(60)}\) using the same principle utilizes the Knoop indentor,\(^{(61)}\) as asymmetrical pyramidal indentor in shape, to determine the magnitude and direction of a uniaxial stress at the surface of structural parts. The method is based on the fact that the Knoop hardness changes depending on the shape and orientation of the indentor. The measured change was found to be a linear function of the stress. The change of hardness is shown in Fig. 46 as a function of strain in three different indentor orientations to the uniaxial stress direction. Change of hardness due to biaxial stress can be found in Ref. 60.

The method is non-destructive and has a special application in the range of non-linear stress-strain relations. It is, however, limited to materials for which the initial hardness is known and its accuracy is dependent on numerous factors.
5. SUMMARY AND CONCLUSIONS

In this report, a survey is made on different methods of residual stress measurement. The methods of exploration fall into two categories: the mechanical methods and the physical methods.

Residual stress measurements using the sectioning method (destructive) and two different hole-drilling methods (semi-destructive) were conducted. For the purpose of comparison, the methods were applied to one test piece (a welded wide-flange section 14H202 of A36 steel) having a uniform residual stress distribution along its length.

The experiments were carried out under normal laboratory conditions using ordinary laboratory facilities and measuring devices. The testing procedures, including the preparation of specimen and measuring technique, are discussed in detail and some recommendations are made. The test results and the evaluation of the data are also discussed.

Other methods of residual stress measurement which may be of general interest are discussed in brief. Also, a list of references on different methods of residual stress measurement is presented.

Based on the experience and test results, the following recommendations and conclusions can be stated.
1. The method of sectioning is adequate, accurate and economical for residual stress measurement in structural members when the longitudinal stresses alone are important. It is felt that this method is more accurate and foolproof than any of the other measuring techniques.

2. The method of "partial sectioning" can be utilized to reduce substantially the total number of required longitudinal sectionings. Its use, however, requires a prior knowledge of the approximate variation in residual stress distribution.

3. The best locations for partial sectioning are at transitions of residual stress gradients. When using properly selected cutting locations the results from partial sectioning usually are not significantly different from those obtained after complete sectioning.

4. To obtain satisfactory results with the sectioning method, it is important to perform a careful preparation of the test piece such as proper location of the test section, gage hole locations and cutting positions and layout. Preparation of gage holes must be performed with care since unreliable readings can result if the holes are not prepared in a proper manner.

5. Temperature changes appear to be the major cause of errors introduced during residual stress measurement. Measurement should be avoided whenever a frequent fluctuation in temperature is likely to occur.

6. Computer programs are available and have been found very
useful in reducing the data from measurements, computing residual stresses, and plotting the results.

7. Two hole-drilling methods, namely, the Mathar's and Soete's methods, were performed on one test piece, the same test piece used for the sectioning method. The testing procedures, the reliability, and the interpretation of the test results were compared.

8. A $\frac{1}{2}$-inch diameter hole was used in Mathar's method. The test results were found to be unreliable due to the gage points and the measuring device used for the test. To have meaningful results it is required to prepare gage points which can stand severe test conditions. Also a dependable measuring device having a small gage length should be used.

9. A $\frac{1}{8}$-inch diameter hole and strain gages preassembled into a 45° rosette were used in Soete's method. The test results obtained were in very close correlation with those of the sectioning method. In addition, transverse residual stresses also were measured.

10. The hole-drilling method has some advantageous features over the sectioning method. It is semidestructive, can have wider application, and the principal stresses can be measured at what is essentially a point. To use the method effectively, more work should be done on the drilling techniques, on establishing calibration curves, and on the interpretation of test results.
6. ACKNOWLEDGEMENTS

The study for this paper was performed as part of the research project "Residual Stresses in Thick Welded Plates" being conducted at Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University, Bethlehem, Pennsylvania. Professor L. S. Beedle is Director of the Laboratory and Professor D. A. VanHorn is the Chairman of the Department.

This project was sponsored by the National Science Foundation, and the Column Research Council, and was under the technical guidance of the Column Research Council Task Group 1, of which John A. Gilligan is Chairman.

Appreciation is extended to Messrs. Paul Marek, Jacques Brozzetti, and Yoshio Kishima for their assistance at all times.

Thanks are due to Kenneth R. Harpel, laboratory superintendent, and his staff for the preparation of the test specimen; to the Bethlehem Steel Corporation and Edward Kottcamp for the heat-treatment of the calibration specimen; to Richard N. Sopko for his work on the photographs, and to Mrs. Sharon Balogh for the preparation of the drawings, and to Miss Joanne Mies for her care in typing the manuscript.
7. TABLES AND FIGURES
### TABLE 1: INITIAL READINGS

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## TABLE 9: DETERMINATION OF RESIDUAL STRAIN DUE TO MILLING OPERATIONS

### INITIAL READINGS

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### EVALUATION OF DATA

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<th>(4) (\bar{A} - \bar{B}) Ref. B</th>
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### TABLE 10: STRAIN READINGS FROM STRAIN GAGES*

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<th>Rosette No.</th>
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<th>( \epsilon = \overline{A} - \overline{B} )</th>
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*Strain-Rosette Type: EA-125RE
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<th>Location</th>
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<th>$\varepsilon_3$</th>
<th>$\gamma$ (rad)</th>
<th>$\sigma_{\text{max}}$ (ksi)</th>
<th>$\sigma_{\text{min}}$ (ksi)</th>
<th>$\beta$</th>
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<td>1.74</td>
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<td>-2.2</td>
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$\gamma = \tan^{-1} \left[ \frac{\varepsilon_1 - 2\varepsilon_2 + \varepsilon_3}{\varepsilon_1 - \varepsilon_3} \right]$  

$\sigma_{\text{max}} = \frac{\varepsilon_2 (1 - 2\cos \gamma) - \varepsilon_1 (1 + 2\sin \gamma)}{1.20 (\sin \gamma + \cos \gamma)} \left( \frac{E}{\mu} \right)$  

$\sigma_{\text{min}} = \frac{\varepsilon_1 (1 - 2\sin \gamma) - \varepsilon_2 (1 + 2\cos \gamma)}{1.20 (\sin \gamma + \cos \gamma)} \left( \frac{E}{\mu} \right)$
Fig. 1 Allocation of Sections for Different Methods of Residual Stress Measurement on the 14H202 Section.
Fig. 2 Preparation of Specimen.
Fig. 3 Gage Hole Location and Sectioning Detail.
Fig. 4 Detail of Gage Point and Gage Hole.

Fig. 5 Location for Proper Partial Sectioning.
Fig. 6 Cutting Positions for Partial and Complete Sectioning.
Fig. 7 Flange of 14H202 Section After Complete Sectioning.
Fig. 8 Comparison of Results from Partial and Complete Sectioning Methods.
Fig. 9  Schematic Diagram of the Whittemore Strain Gage.
a) Dial Indicator
b) Body of Whittemore Extensometer
c) Center Punch
d) Reference Bar
e) Positioning Angle

Fig. 10 The Whittemore Strain Gage.
Fig. 11 Use of Positioning Angle for Consistent Reading.
Fig. 12 Residual Stress Distribution in 14H202 Section at Location A.
Fig. 13 Comparison of Residual Stress Measurement at Two Ends, A and B.
Fig. 14 Computer Plot of Residual Stress Distribution for the Shape 14H202.
Fig. 15 Mathar's Method of Hole-Drilling

Fig. 16 The Test Specimen and Tools
a) The Huggenberger Extensometer

b) Extension Arm for 100 mm Gage Length

c) Center Punch for 100 mm Gage Length

d) Reference Bar for 100 mm Gage Length

e) Center Punch for 20 mm Gage Length

f) Reference Bar for 20 mm Gage Length

g) Bottle Containing Gage Points - 1/16 inch steel balls

h) Standard Punch

Fig. 17 The Huggenberger Extensometer With Accessories
Fig. 18 Punched and Bonded, 1/16 inch Diameter Steel Ball Used as a Gage Point
Fig. 19 The Milling Fixture
a) Electric Drill (1100 rpm)
b) "Versamatic"
c) Chuck
d) 1/2 inch End Mill (Mathar's Method)
e) 1/8 inch End Mill (Soete's Method)
f) Milling Fixture

Fig. 20 Assembly View of Drilling Tool.
Fig. 21 Calibration Specimen in the Testing Machine.

Fig. 22 The Portable Magnetic Base on the Calibration Specimen.
Fig. 23 Strains Released by Drilling 1\(\frac{1}{2}\) inch Diameter Hole in 1\(\frac{3}{4}\) inch Thick Bar Subjected to Uniform Stress Field.

Fig. 24 History of Strain Changes During Loading-Drilling-Unloading Operation for an Ideal Calibration Specimen.
Fig. 25 Strain Changes During Loading-Drilling-Unloading Operations Under Different Loads.
Fig. 26 Calibration Test Results for Loads 80, 100 and 120 kips.

Fig. 27 Estimated Calibration Curve for 3/8-inch Diameter Hole for A36 Steel.
Fig. 28  Hole-Drilling Layout on Top Surface of Flange of 14H202 Section. [Mathar's and Soete's Methods]
Fig. 29 The Radial Drill Press Used for Drilling Holes on the \textit{14H202} Shape.

Fig. 30 The Portable Magnetic Base Press Used to Drill on the \textit{14H202} Shape.
Fig. 31 Strain Change Readings due to Drilling of \( \frac{1}{2} \) inch hole on Flange of 14H202 Section.
Fig. 32  Average Residual Stress Distribution across Top Surface of Flange of 14H202 Section Using Mathar's Method of Hole-Drilling.
### GAGE SIZE (in)

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<th>C*</th>
<th>D</th>
<th>E</th>
<th>S*</th>
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<td>±0.001</td>
<td>±0.001</td>
<td>±0.002</td>
<td>±0.016</td>
<td>±0.005</td>
<td>±0.001</td>
<td>-0.002</td>
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**Fig. 33** Strain Gage Rosette for Residual Stress Measurement.
Fig. 34 Modification of End-Mill
Fig. 35 Characteristic Curves of the Strain Relaxation as a Function of Non-Dimensional Hole Depth.
Fig. 36 Residual Stress Measurement Using the Hole-Drilling Method (Soete's Method) and Comparison with the Sectioning Method.
Note: All measurements are in mm.

Fig. 37 Gunnert's Method of Grooving-out.
Fig. 38 Residual Stress Measurement Using Straight Grooves.
Fig. 39 Grooving-out Method of Measuring Residual Stresses (Schwaighofer's Method)
Fig. 40  Bauer and Heyn's Method of Residual Stress Determination for Cylindrical Rod.

Fig. 41  Mesnager-Sach's Boring-Out Method.
Fig. 42 Stählein's Method -- Effect of Removal of a Layer.
Fig. 43 Metal Aggregate for X-Ray Stress Determination.

Fig. 44 Set-up of X-Ray Method for Residual Stress Measurement.
Fig. 45 Crack Patterns Under Various Surface Conditions Using Brittle Lacquer.
Fig. 46 Change of Knoop Hardness of Steel due to Uniaxial Stress.
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61. V. E. Lysaght  