Determination of stress intensity factors for plane strain and plane stress

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# Table of Contents

Approval ................................................................. ii
Acknowledgements ....................................................... iii
Abstract ...................................................................... 1

Chapter 1  Introduction ......................................................... 2
   1.1 Classical moiré ......................................................... 7
      1.1.1 Introduction ...................................................... 7
      1.1.2 Fundamentals of moiré ....................................... 8
   1.2 In-plane moiré method ............................................. 11

Chapter 2  Moiré interferometry ............................................. 14
   2.1 Fundamentals ........................................................ 16
      2.1.1 Characteristics of light .................................... 16
      2.1.2 Theory of interference ..................................... 19
   2.2 Moiré interferometry ............................................. 23
      2.2.1 Embedded grating ........................................... 30
      2.2.2 Preparation of specimen ................................... 35

Chapter 3  Background of the theory and experiment .............. 37
   3.1 Two-dimensional elastic fields as a power-series solution .. 37
3.1.1 Modified calculation .......................... 50
3.2 Analytical method .............................. 53
3.3 Modified analytical method ..................... 54
3.4 Experiment ..................................... 56
3.5 Poisson’s ratio effect ............................ 57
Chapter 4 Results and conclusion .................... 60
4.1 Results ......................................... 60
4.1.2 Discussion .................................... 73
4.2 Conclusion ...................................... 76
References ........................................... 77
Appendix A Uncertainty analysis ....................... 82
Appendix B Confidence analysis ....................... 87
Appendix C Error propagation ......................... 89
Vita ................................................... 90
Abstract

The stress-intensity approach has been widely accepted in the field of linear elastic fracture mechanics as a valid means for predicting the behavior of a material in the presence of a crack or similar defect. The experimental determination of the stresses (or displacements, etc.) within the region surrounding a defect appeared always as an interesting target to those involved in the experimental stress analysis. It is commonly known that an important and basic fracture mechanics parameter, stress intensity factor, K could be determined experimentally from the stress measurements by photoelastic approach, or from the displacement measurements by moiré interferometry method, etc.

In this study, mode I stress intensity factors $K_I$ will be investigated from the notched specimen under plane strain and generalized plane stress cases, respectively. Moiré interferometry was employed here because it can provide full-field in-situ measurement. The experimental data were analyzed by a digital fractional fringe analyzer and compared favorably with the available analytical results.
Chapter 1

Introduction

It is well known that any defect, like notch or crack, will cause a localized stress concentration in a structure. The stress concentration may lead to initiation of the early failure under external loading. Two of the highly accepted parameters used to characterize the local stress in the notch tip are the stress concentration factor SCF and stress intensity factor $K$. The stress concentration factor could yield the information about the maximum stress in the vicinity of notch. However, the stress intensity factor $K$ can provide a parameter representation for the whole stress field in the notched or cracked
problems. Since the engineering design requires the knowledge of stresses at every point of the structure to establish the safety level by using the failure criteria, contribution of the stress intensity factor to the development of fracture mechanics is hard to overestimate.

The stresses, which can be yielded by stress intensity factor, are functions of geometry and forces applied. Although extensive analytical work had been done to study the notched and cracked problems [1] by using eigenfunction expansion and complex function theory to analyze the stress intensity factor in three dimensional problems, the available solutions are still limited to some simple geometries and boundary conditions.

Recently computational techniques became more powerful and methods, that either depend partly on numerical calculations or are purely numerical, have replaced some of the analytical methods in solving wide variety of boundary-value problems. The detailed investigations by Sih [1] and Paris and Sih [2] have provided useful and convenient information for solving the cracked problems either analytically or numerically.

Nevertheless, experimental approach is still the way to validate the solution. In the past three decades several experimental techniques had been proposed to solve the three-dimensional cracked problems by using displacement information. Basically, surface
information could be easily accessed without any special treatment. However, the full-field interior information can only be yielded through optical techniques. Moreover, the optical measurement of interior displacement field is limited to the sufficiently transparent materials.

There are only a few experimental techniques capable of measuring interior displacement or stress field. One of the most important is the method of frozen stress photoelasticity first applied to the study of fracture by Smith and Smith [3]. Thin slices cut from the frozen stress model are utilized to represent the stress field across the slice thickness. A further development of it is its integration with moiré interferometry, which was presented by Smith [4] et al. in 1982. It was concluded that the integrated technique can estimate stress intensity factor distribution in three dimensional cracked problem to within the accuracy of ± 5%.

However, the inherent error will be induced if the Poisson’s ratio of the investigated material is other than 0.5 [4]. This inherent error is due to the fact that the stress-freezing photoelastic material is incompressible. Pindera and Krasnowski [5] made use of the isodyne technique by directing a narrow collimated beam to inspect the interior stress field. Its application, like the photoelasticity, is limited to the birefringent materials. In 1985 Obata [6] et al. reported the application of embedded moiré technique, however
it was circumscribed by its low sensitivity. Chiang [7] in 1976 proposed the scattered light speckle photography technique to probe the interior strain. Due to the poor fringe contrast the speckle decorrelates very quickly in three dimensional problem. Later, an advanced technique called scattered light white light speckle, was developed by Chiang and Asundi [7,8] to reduce the decorrelation by using artificial scattered particles embedded in the sample. However, multiple scattering of the light passing through the sample by the embedded particles will obstruct the clear viewing of deep interior section of a sample. Chiang and Lu [9] then proposed a new embedded speckle technique which improves the clarity while the deep interior sections of a model were viewed. This method has the advantage in the material selection, which does not have to be incompressible [4].

The intention of the current work is to introduce an optical measurement technique, embedded moiré interferometry, to investigate the real-time interior stress intensity factor $K_I$’s in a single-edge-notched epoxy specimen of finite thickness. This thesis will present an assessment of experimental results and the comparison with analytical solutions, given by Paris and Sih [2], and show that the embedded moiré technique was successfully applied to the detection of the interior displacement field with high sensitivity.
In the first chapter a brief review of classical moiré method is introduced. In chapter 2 embedded moiré interferometry, fractional fringe analysis and digital image processing are discussed. Chapter 3 discusses the theory and experiment. Chapter 4 discusses the results and conclusion.
1.1 Classical moiré

1.1.1 Introduction

The word moiré in French means a fabric known as watered silk, which exhibits patterns of light and dark bands. This moiré effect occurs whenever two similar but not quite identical arrays of equally spaced lines or dots are arranged so that one array can be viewed through the other. Significant insight into the moiré effect can be seized by studying the relationships which exist between the spacings and inclinations of the moiré fringes in a pattern and the geometry of the two interfering lines arrays which produce the pattern. A complete analysis of the moiré fringe geometry was given by Morse, Durelli and Sciammarella [10] in 1960. In their work the fundamental equations of the moiré method were derived and presented in the form of curves which provide strains and rotations with a minimum of computation.

In 1948 Weller and Shepard [11] described an application in which moiré fringes were used to measure displacements by analysis of moiré fringe patterns. A detailed interpretation of moiré fringes as components of displacements for plane elasticity problems was introduced by Dantu [12,13] in 1954. Sciammarella and Durelli [14] extended this approach into the region of larger strains in 1961. The use of moiré fringes for measuring displacements in structural models has been outlined by Durelli and Daniel
The previous discussion serves to illustrate the broad field of application of the moiré method in the determination of displacements and strains.

1.1.2 Fundamentals of moiré

Moiré patterns exhibit several light and dark bands. This effect results from the superimposition of any two similar but not quite identical arrays of equally spaced lines or bars. The typical formation of fringes is shown in Figure 1.1. Either the dark or light bands may be called moiré fringe. The individual lines can not be resolved by eyes when the spacings between these lines are quite small.

It should be noted that the formation of dark fringes is due to the dark lines of one grating falling into the spaces of the other, while the light fringes are the result of the dark lines of one grating coinciding with those of the other. The center lines of the dark gratings lines are designated with numbers from 0 to m for one grating and from 0 to n for the other. The center lines of light fringes are denoted by the numbers from 0 to N, where N is an integer referred to as the fringe order. It is easy to see from Figure 1.1 that the relationship between m, n and N could be expressed as...
Figure 1.1 Formation of one moiré fringe
\[ N = m - n \tag{1.1} \]

For parallel gratings, Equation (1.1) can relate the three parameters and tell how many fringes will be coming out over a certain length if the number of the grating lines could be known \textit{a priori}.
1.2 In-plane moiré method

The formation of moiré fringes may be used to measure in-plane displacements. When two gratings are superimposed one against another, the interference moiré fringes are the result of either their having a difference in pitch (defined as the distance between the centers of any two neighboring lines) or in orientation. This is how the moiré fringes could be used to measure the in-plane displacements. However, a reference coordinate system in which all measurements are made, has to be defined beforehand.

A reference (or master) grating remains unchanged so that it can reflect the relative movement of the other (specimen) grating. The specimen grating is the one which is attached to a structure surface whose deformation is of interest. The pitch \( p \) of the specimen grating is identical to that of the reference grating before deformation. However, it should be indicated that the reference grating is allowed to have a pitch that is a multiple of that for the specimen gratings. To find the relation between the number of fringes \( N \) and the displacement \( u_x \) (in a direction normal to the grating lines orientation), one may consider a specimen with grating of pitch \( p \) which is in perfect alignment of a master grating of the same pitch. There will be no fringes appearing when two identical gratings are superimposed together. However, if specimen grating is made to deform by an amount of one pitch while the reference grating still keeps \( m \) grating
lines over a certain length, one moiré fringe will appear. If the deformation of specimen
is two pitch in the same direction, then two moiré fringes will come out. It can be easily
seen that in the general case when the number of lines in the deformed state becomes \( n \),
the corresponding displacement could be expressed as

\[
U_x = (m-n)p
\]

Combined with Equation (1.1), Equation (1.2) could be rewritten as

\[
U_x = Np
\]

By using Equation (1.3) the displacement measurement is readily available. An
essential restriction about this measurement is that it has to start and end at fringe centers.
However, starting and ending fringes need not to be of the same type, thus the equation
is also applicable to a fringe pattern where the first fringe is light and the last one is dark
or \textit{vise versa}. Hence this method can provide the maximum resolution of the half of pitch
value. Nevertheless, the intermediate displacements at points not lying at fringe centers
can not be accessed by this method. Also, the determination of fringe centers is a
subjective process and is human-judgement affected. Some investigators used photosensi-
tive devices mounted on travelling mechanisms to allow for scanning of light along desired lines [16] to minimize the random errors mentioned above. In spite of the reduction of errors in finding the fringe centers by using above techniques, they often suffer from optical noise and confusion in choosing the centers in cases where the grating lines were still visible together with the fringes.

Despite these mentioned limitations, classical moiré has received wide acceptance as a convenient full-field displacement measuring technique. It has been used in numerous applications in the fields of composite materials [17] and fracture mechanics [18], etc.
Chapter 2

Moiré interferometry

Moiré interferometry was first introduced by Post [19] in 1980 as a high resolution optical displacement measuring technique. It is recognized that moiré interferometry can provide full field maps of in-plane displacement fields with high sensitivity, high spatial resolution and extensive displacement range while many other techniques may only excel in one or some of these fields. Several studies of composite materials [17], including micromechanics and macromechanics of composites [20], residual strains and thermal strains [21] by Post have proven that interferometric moiré has matured into an excellent technique for measuring in-plane deformations of complex materials and structures.
This technique provides a unique combination of full field, high sensitivity, excellent contrast and spatial resolution. Reference grating frequencies of 1200 to 4000 lines/mm are readily created by coherent light interferometry and can be utilized to produce moiré fringes if specimen gratings of similar frequency ranges are produced.
2.1 Fundamentals

2.1.1 Characteristics of light

The characteristics of moiré interferometry could be adequately interpreted by the wave theory [22]. A parallel beam of light travelling along the x direction is depicted at a given instant as a train of regularly spaced disturbances that vary with x as

\[ A = A_0 \cos 2\pi \left( \frac{x}{\lambda} \right) \]  

(2.1)

where \( A \) represents the amplitude or strength of the disturbance and \( \lambda \) is the wavelength of light. The coefficient \( A_0 \) is a constant. The wave train travels through space with a high constant velocity \( C \) (3\times10^8 \text{ m/sec} \text{ in free space}). At any fixed point along the path of the wave train, the disturbance is a periodic variation of the field strength. During the passage of the wave train through any fixed point \( x = x_0 \), the light disturbance varies with time \( \tau \) as

\[ A = A_0 \cos 2\pi \left( \frac{C}{\lambda} \right) \tau = A_0 \cos 2\pi \omega \tau \]  

(2.2)
where $2\pi \omega \tau$ is called the phase of the disturbance. For a parallel beam of light, the phase is constant along any plane normal to the beam. Any continuous surface along which the field strength keeps constant is called a wavefront. In a parallel collimated beam, the wavefronts are always plane cross sections of the beam, as shown in Figure 2.1.

When a wave train travels through a fixed point in the space, its frequency of oscillation ($\omega = C/\lambda$) is about $6 \times 10^{14}$ Hz for visible light. No instrument can detect individual cycles in this frequency range. Instead, receivers like the eye, photographic film, and photoelectric cells respond to the energy content of wave. When a wave train is intercepted by a receiver, the energy available to be dissipated into the receiver is the intensity of light times the exposure time. During the passage of the wave train in a volume of space, the light has an intensity everywhere in that space. The energy associated with it is a potential energy. The potential is realized only when the light is intercepted by a receiver and its energy is dissipated into the receiver. The light intensity keeps constant when the wave train travels in the free space. The relationship between the amplitude ($A = A_0 \cos 2\pi \omega \tau$) and the intensity ($I$) of light can be expressed as

$$I = A_0^2$$  

(2.3)
Figure 2.1 Wavefronts in a plane beam of light.
Thus intensity is a time-averaged quantity and is irrelevant to the frequency and phase of the light beam.

2.1.2 Theory of interference

When two collimated beams intersect each other with an angle $2\beta$, their wavefronts $W_1$ and $W_2$ are normal to their beams and, therefore, also intersect at a angle $2\beta$. The light source that could produce such collimated beams includes beam splitters, prisms, partial mirrors, etc. This is schematically illustrated by Figure 2.2. Since the wavefronts $W_1$ and $W_2$ originated from the same source, $W_0$, they have the same phase. Figure 2.3 describes the interference of the two beams. Separation between wavefronts in each train is $\lambda$. The harmonic curves $A_1$ and $A_2$ represent the amplitude of the two wave trains in space at the given instant. The summation of field strength of the individual strengths of beams 1 and 2 could be observed along a-c. The field strength along either a-c or q-r is called constructive interference. However, the field strength along d-e is called destructive interference because the result strength is always zero as one beam is subtracted from the other. These interferences are independent of time and are parallel with the bisector of the incoming beams.

However, it should be indicated that either constructive or destructive interference
Figure 2.2 Two beam intersection.
Figure 2.3 Constructive and destructive interference of two coherent beams.
can be observed not only along the lines, but also in the planes perpendicular to the diagram because the beams proceed as the plane type. Such plane interference could be observed whenever two beams of coherent light intersect in space, regardless of the angle of intersection. However, if the amplitudes $A_1$ and $A_2$ of the beams are not the same, the constructive interference will not be as bright as if the amplitudes were equal, and the destructive interference will have some brightness, which in this case reduces the contrast of the pattern. The degree of reduction of the contrast depends on the difference between $A_1$ and $A_2$. If a photographic plate is arranged along the line B-B in Figure 2.3, it will record dark and light bands corresponding to the constructive and destructive interferences. These bands are called fringes.
2.2 Moiré interferometry

A schematic description of the moiré interferometry is given in Figure 2.4. The high-reflection symmetrical, phase-type grating of the sort, as shown in Figure 2.5, is firmly attached to the specimen. When loads are applied to the specimen, the grating moves and deforms together with the specimen surface.

Two beams of coherent light represented by A and B are projected on the specimen grating obliquely by angles $+\beta$ and $-\beta$. This two beam interference creates walls of constructive and destructive interference in the zone of their intersection, which is known as a virtual grating. The virtual grating is cut by the plane of the specimen surface, where an array of parallel and very closely spaced fringes are formed. These fringes are essentially bright and dark bars, and they act like the reference grating of geometrical moiré. At the intersection of two beams an interference pattern is formed, which consists of a series of dark and light bands. These bands are called a virtual reference grating. The relationship between their frequency, $f$, and the projecting angle, $\beta$, could be described as
Figure 2.4 Schematic diagram of moiré interferometry.
Figure 2.5 Two types of grating

(a) Diffraction (phase) grating

(b) Amplitude grating
The specimen grating and virtual reference grating interact to form a moiré pattern, which can be viewed and recorded by a CCD camera. The frequency, $f_s$, of a specimen grating is half of that of the reference grating frequency ($f = 2f_s$) so as to have first order diffraction, which corresponds to a multiplication factor of 2. This is dictated by the grating equation [18] and other practical limitations of a moiré interferometer. In Figure 2.4, the two coherent beams lie on the $yz$-plane so that the lines of the virtual reference grating are perpendicular to the $x$-axis. Its interaction with specimen grating will yield the $U$-field of displacement. On the other hand, if the two coherent beams lie on the $xz$-plane then its interaction with specimen grating will yield the $V$-field of displacement.

A typical pattern of moiré interferometry fringes is shown in Figure 2.6. This image indicates that all of the points located in the same fringe experience the same in-plane displacements and the difference between the points in the next fringe is one pitch,
Figure 2.6 Typical U-field pattern of moiré interferometry fringes. $p = 0.417\mu m$. 
p, which is the reciprocal of the frequency of specimen grating, 1/f. The relationship between displacements and fringes could be written as

\[
U = \frac{N_x}{f} \\
V = \frac{N_y}{f}
\]  

(2.5)

where U and V represent the components of displacement in x and y directions, respectively. \(N_x\) and \(N_y\) are fringe orders when the virtual reference grating are projected parallel to the y and x directions, respectively. They are determined at the fringe midpoints either manually, with the aid of digitizing tablets or by image analysis.

It is necessary to define a reference point with zero displacement before assigning the fringe orders. The fringe which contains zero-displacement point is assigned number zero in the fringe numbering scheme. The adjacent fringe should be numbered relative to the zero fringe. The reference point could be any one assuming its displacement is known. This could be a boundary point, a point where load is applied, or a point at the notch tip in the cases of plane stress and plane strain. For the displacement analysis, the absolute displacement value of the reference point is required. However, the knowledge of the absolute value of the reference point is not important for the strain analysis.
Once the displacement map has been constructed, strains can be determined by strain-displacement relations as follows:

\[ \varepsilon_x = \frac{\partial u_x}{\partial x}, \]
\[ \varepsilon_y = \frac{\partial u_y}{\partial y}, \]
\[ \varepsilon_{xy} = \frac{1}{2}(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}). \]

Equation (2.6) could be approximated by replacing the derivatives in the right hand side with finite difference expressed as:

\[ \varepsilon_x = \frac{\Delta u_x}{\Delta x}, \]
\[ \varepsilon_y = \frac{\Delta u_y}{\Delta y}, \]
\[ \varepsilon_{xy} = \frac{1}{2}\left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x}\right). \]
2.2.1 Embedded grating

The grating attached to the surface of the specimens, which is called specimen grating, will experience the same deformation as the underlying surface it was transferred upon. The other grating, which is called reference grating, has to be kept unchanged so that the relative deformation between specimen grating and reference grating could yield the truthful displacement field. Basically there are two types of specimen gratings. One of them is diffraction (phase type) grating, as shown in Figure 2.5 (a), which was used for this investigation. Its frequency is equal to 1200 lines/mm. When it is integrated with 2400 lines/mm (60,960 lines/in) reference grating, the corresponding displacement sensitivity is 0.417 μm per fringe order. The other type is amplitude gratings, as shown in Figure 2.5 (b), which consist of opaque bars and transparent spaces. They are used mainly in geometrical moiré and exhibit relatively low frequency in the range of 1-40 lines/mm.

Specimen grating is produced from the photographic mold by replication process. This is schematically illustrated in Figure 2.7. A pool of optical adhesive is poured on the surface of specimen and then compressed into a thin film by pressing against the mold. After the optical adhesive is cured by ultraviolet light, the photographic mold is pried off carefully with a small prying force and a reflective diffraction grating is bonded to the surface of the specimen. However, the photographic mold is lost easily due to the
Figure 2.7 Steps for the preparation of specimen grating.
friction and twisting in multiple replication process. In order to preserve the mold for future application, a revised method is recommended. Adding an intermediate replication step proves to be a more functional procedure that preserves the master mold for subsequent application. Using the three-step process, a submaster is produced by replicating the phase grating surface in silicon rubber, much in the same way as in Figure 2.7. The submaster is then utilized to replicate the phase grating in an adhesive on the specimen surface [23]. Silicon rubber of the liquid type has a distinct virtue for the intermediate step in that it is a nonadhesive replicating material. This permits easy separation from the master and also from the final specimen grating; it bonds well, however, to specially primed surfaces by using some silicone primer.

In general, the specimen grating is either transferred or replicated on the surface of the specimen for the generalized plane stress case. The specimen for mode I loading in plane stress is illustrated in Figure 2.8. However, to study the plane strain case the specimen grating has to be embedded inside the mid-plane of the specimen. This process involves special fabrication process. The specimen for mode I loading in plane strain case is illustrated in Figure 2.9. To prepare the specimen for plane strain observation, the choice of materials is quite limited. Firstly, the material has to be transparent so that the laser beam can travel through the material to arrive at the embedded grating. Secondly, the material has to be fabricated with liquidized or visco-liquidized type so that the
Figure 2.8 Typical specimen geometry for mode I loading in plane stress case.
Figure 2.9 Gratings position for plane strain and plane stress cases.
embedded grating wouldn’t be destroyed in the process. Thirdly, in the solidification process, some material has to be cured at a high temperature, which might destroy the specimen grating. In this study, a thermoplastic material DER 331 epoxy (epoxy resin, provided by The Dow Chemicals Company) was selected since it has all the needed characteristics as mentioned above.

2.2.2 Preparation of specimen

The specimen geometry was selected according to ASTM-STP 410 as a single-edge-notched plate subjected to a tensile load. The plane strain or plane stress condition depends on the location of the grating. The specimen grating (grating B), as shown in Figure 2.8, for the plane stress case could be transferred on either side of the plate. However, the specimen grating (grating A) for the plane strain analysis has to be embedded in the mid-plane of the plate as illustrated in Figure 2.9. In this study, the same specimen was used to perform the experiment so that the measurements for plane strain and plane stress will be related to the same notch length and geometry.

The specimen was prepared from the epoxy resin DER 331. Casting of epoxy resin was processed with a curing agent AEP (N-Aminoethylpiperazine, provided by Air Products and Chemicals, Inc.) so that the cured resin can gelatinize at curing temperature.
First, one has to pour a specified amount of epoxy into a jar. After blending the epoxy resin under 80°C for an hour to degas, it is removed and cooled down at room temperature for at least 24 hours. To assure all of the resin can be cured, a specified amount of curing agent was injected into the epoxy resin. Mechanical stirring of the mixture (resin and curing agent) was followed by degassing in a vacuum chamber. The time needed in the chamber depends on how much resin is in the jar. After degassing and mixing completely, the resin was poured into a steel mold for casting. A cure time of 2 hours at 100°C was used. After the gelatination process is completed, the casting epoxy plate is taken out of the mold. A 120 mm x 24 mm x 3 mm epoxy plate (layer A) was cut from the plate. One side of it was sanded and polished so that the 1200 lines/mm grating A, as shown in Figure 2.9, could be transferred without any destruction. After a thin Au-Pd film (approximately 200 Å thick) was deposited on the grating A, a thick jelly-like epoxy (layer B) was poured upon the plate and the curing procedure was repeated. The gelatinized side was polished so that the embedded grating A could be viewed clearly.
Chapter 3

Background of the theory and experiment

3.1 Two-dimensional elastic fields as a power-series solution

Our goal is to investigate the local fields near the sharp edge of a cracklike notch in the DER 331 epoxy. It is assumed that the load increase is monotonic so that the dynamic effects and local unloading can be neglected. The description will be based on
the first-order theory so that all displacements and their gradients will be treated as small. Since the material has been manufactured by casting and the sample underwent post-curing procedures, it is reasonable to regard the sample as homogeneous and isotropic. For a through-thickness crack with zero root radius in a plate subjected to both tensile and shearing loading, the theoretical solution of two-dimensional stress field in the neighborhood of the crack tip may be found from the Airy stress function [24], $\chi(r,\theta)$, which is based on Williams approach [25],

$$\chi = \sum_{n=1,3,...}^{\infty} \frac{r}{2} \left[ C_{1n} (\cos \frac{n-2}{2} \theta - \frac{n-2}{n+2} \cos \frac{n+2}{2} \theta) \right. $$

$$+ C_{2n} (\sin \frac{n-2}{2} \theta - \sin \frac{n+2}{2} \theta) \bigg]$$

$$+ \sum_{n=2,4,...}^{\infty} \frac{r}{2} \left[ C_{1n} (\cos \frac{n-2}{2} \theta - \frac{n-2}{n+2} \cos \frac{n+2}{2} \theta) \right.$$

$$+ C_{2n} (\sin \frac{n-2}{2} \theta - \frac{n-2}{n+2} \sin \frac{n+2}{2} \theta) \bigg]$$

(3.1)

where the terms multiplied by $C_{1n}$ are symmetric (mode I) and the terms with $C_{2n}$ are
antisymmetric (mode II) with respect to $\theta = 0$. In Equation (3.1), $r$ represents the distance to the crack tip and $\theta$ represents the angle measured from an axis oriented in the direction of crack extension. From the Airy stress function the stresses around the crack tip could be derived from the following

$$
\sigma_\theta = \frac{\partial^2 \chi}{\partial r^2} \quad \sigma_r = \nabla^2 \chi - \sigma_\theta \quad \tau_{r\theta} = \frac{\partial (1/\partial \chi)}{\partial r} \left( \frac{1}{r} \frac{\partial \chi}{\partial \theta} \right)
$$

(3.2)

Therefore the stresses, in the general form of polar coordinate system, could be written as the following

$$
\sigma_\theta = \frac{n(n+2)}{4} \sum_{n=1,3,...} r^{1+n} \left[ C_n (\cos \frac{n-2}{2} \theta - \frac{n-2}{n+2} \cos \frac{n+2}{2} \theta) + C_{2n} (\sin \frac{n-2}{2} \theta - \sin \frac{n+2}{2} \theta) \right] + \sum_{n=2,4,...} \frac{r^{1+n}}{2} \left[ C_n (\cos \frac{n-2}{2} \theta - \cos \frac{n+2}{2} \theta) + C_{2n} (\sin \frac{n-2}{2} \theta - \frac{n-2}{n+2} \sin \frac{n+2}{2} \theta) \right]
$$

(3.3)
\[ \sigma_r = \sum_{n=1,3,...}^{\infty} r^{-1+\frac{n}{2}} \left[ C_{1n} \left( \frac{6n-n^2}{4} \cos \frac{n-2}{2} \theta + \frac{n^2-2n}{4} \cos \frac{n+2}{2} \theta \right) + C_{2n} \left( \frac{6n-n^2}{4} \sin \frac{n-2}{2} \theta + \frac{n^2+2n}{4} \sin \frac{n+2}{2} \theta \right) \right] \]

(3.4)

\[ \tau_{6r} = \frac{n}{2} \sum_{n=1,3,...}^{\infty} r^{-1+\frac{n}{2}} \left[ C_{1n} \left( -\frac{n-2}{2} \sin \frac{n-2}{2} \theta + \frac{n-2}{2} \sin \frac{n+2}{2} \theta \right) + C_{2n} \left( \frac{n-2}{2} \cos \frac{n-2}{2} \theta - \frac{n+2}{2} \cos \frac{n+2}{2} \theta \right) \right] \]

(3.5)
The general forms of displacement fields for the plane strain and plane stress cases are different. It should be indicated that in plane strain $w = 0$ and $\sigma_z \neq 0$, however in plane stress $w \neq 0$ and $\sigma_z = 0$. By utilizing the relationship between stress and strain and the one between strain and displacement, the displacement $u_r$ and $u_\theta$ can be expressed as

Mode I (Plane stress)

$$U_r = \frac{1}{2G(1+\nu)} \sum_{n=1,3,\ldots}^{n} r^2 \left[ C_{1n} \frac{2(3-v)-n(1+v)}{2} \cos \frac{n-2}{2} \theta \right. $$

$$+ \left. \frac{(n-2)(1+v)}{2} \cos \frac{n+2}{2} \theta \right]$$

$$+ \sum_{n=2,4,\ldots}^{n} r^2 \left[ C_{1n} \frac{2(3-v)-n(1+v)}{2} \cos \frac{n-2}{2} \theta \right. $$

$$+ \left. \frac{(n+2)(1+v)}{2} \cos \frac{n+2}{2} \theta \right]$$

(3.6)
\[ U_{\theta} = \frac{1}{2G(1+\nu)} \sum_{n=1,3,...}^{\infty} r^2 [C_{1n}(\frac{2(3-\nu)+n(1+\nu)}{2})\sin \frac{n-2}{2}\theta] \]

\[ + \frac{(n-2)(1+\nu)}{2} \sin \frac{n+2}{2}\theta) \]

\[ + \sum_{n=2,4,...}^{\infty} r^2 [C_{2n}(\frac{2(3-\nu)-n(1+\nu)}{2})\sin \frac{n-2}{2}\theta] \]

\[ + \frac{(n+2)(1+\nu)}{2} \sin \frac{n+2}{2}\theta) \]

(3.7)

Mode II (Plane stress)

\[ U_{r} = \frac{1}{2G(1+\nu)} \sum_{n=1,3,...}^{\infty} r^2 [C_{2n}(\frac{2(3-\nu)-n(1+\nu)}{2})\sin \frac{n-2}{2}\theta] \]

\[ + \frac{(n+2)(1+\nu)}{2} \sin \frac{n+2}{2}\theta) \]

\[ + \sum_{n=2,4,...}^{\infty} r^2 [C_{2n}(\frac{2(3-\nu)-n(1+\nu)}{2})\sin \frac{n-2}{2}\theta] \]

\[ + \frac{(n-2)(1+\nu)}{2} \sin \frac{n+2}{2}\theta) \]

(3.8)
\[
U_\theta = \frac{1}{2G(1+v)} \sum_{n=1,3,...}^n \left[ C_{2n} \left( \frac{(n+2)(1+v)}{2} \right) \cos \frac{n+2}{2} \right] \\
- \frac{2(3-v)+n(1+v)}{2} \cos \frac{n-2}{2} \theta \right] \\
+ \sum_{n=2,4,...}^n \left[ C_{2n} \left( \frac{(n-2)(1+v)}{2} \right) \cos \frac{n+2}{2} \right] \\
- \frac{2(3-v)+n(1+v)}{2} \cos \frac{n-2}{2} \theta \right]
\]

(3.9)

Mode I (Plane strain)

\[
U_r = \frac{1}{2G} \sum_{n=1,3,...}^n \left[ C_{1n} \left( \frac{6-n-8v}{2} \right) \cos \frac{n-2}{2} \right] \\
+ \frac{n-2}{2} \cos \frac{n+2}{2} \theta \right] \\
+ \sum_{n=2,4,...}^n \left[ C_{1n} \left( \frac{6-n-8v}{2} \right) \cos \frac{n-2}{2} \right] \\
+ \frac{n+2}{2} \cos \frac{n+2}{2} \theta \right]
\]

(3.10)
Mode II (Plane strain)

\[ U_\theta = \frac{1}{2G} \sum_{n=1,3,\ldots}^{\infty} r^n \frac{n}{2} \left[ C_{1n} \left( \frac{n+6-8\nu}{2} \sin \frac{n-2}{2} \theta \right) \right. \\
\left. - \frac{n-2}{2} \sin \frac{n+2}{2} \theta \right] \\
+ \sum_{n=2,4,\ldots}^{\infty} r^n \frac{n}{2} \left[ C_{2n} \left( \frac{6-n-8\nu}{2} \sin \frac{n-2}{2} \theta \right) \right. \\
\left. + \frac{n+2}{2} \sin \frac{n+2}{2} \theta \right] \]  

(3.11)

\[ U_r = \frac{1}{2G(1+\nu)} \sum_{n=1,3,\ldots}^{\infty} r^n \frac{n}{2} \left[ C_{2n} \left( \frac{6-n-8\nu}{2} \sin \frac{n-2}{2} \theta \right) \right. \\
\left. + \frac{n+2}{2} \sin \frac{n+2}{2} \theta \right] \\
+ \sum_{n=2,4,\ldots}^{\infty} r^n \frac{n}{2} \left[ C_{2n} \left( \frac{6-n-8\nu}{2} \sin \frac{n-2}{2} \theta \right) \right. \\
\left. + \frac{n-2}{2} \sin \frac{n+2}{2} \theta \right] \]  

(3.12)
\[ U_0 = \frac{1}{2G} \sum_{n=1,3,...}^{\infty} r^n \left[ C_{2n} \left( \frac{n+6-8\nu}{2} \cos \frac{n-2}{2} \theta \right) \right. \]
\[ \quad + \frac{n+2}{2} \cos \frac{n+2}{2} \left( \frac{n-2}{2} \right) \theta \) \]
\[ + \sum_{n=2,4,...} \frac{r^n}{n} \left[ C_{2n} \left( \frac{n+6-8\nu}{2} \cos \frac{n-2}{2} \theta \right) \right. \]
\[ \quad + \frac{n-2}{2} \cos \frac{n+2}{2} \left( \frac{n-2}{2} \right) \theta \) \]

(3.13)

where \( G \) is the shear modulus and \( \nu \) is the Poisson’s ratio of the material.

From Equation (3.6) to (3.13), the coefficients \( C_{11} \) and \( C_{21} \) can be related to the stress intensity factors \( K_I \) and \( K_{II} \), as shown:

\[ C_{11} = \frac{K_I}{\sqrt{2\pi}} \]
\[ C_{21} = \frac{K_{II}}{\sqrt{2\pi}} \]

(3.14)

Therefore, the stress intensity factor \( K_I \) for real crack can be derived directly from Equation (3.14) if the \( C_{11} \) is available. The Cartesian components of displacements \( u \)
and \( v \) can be expressed in terms of the polar components by

\[
\begin{align*}
    u &= U_r \cos \theta - U_\theta \sin \theta \\
    v &= U_r \sin \theta + U_\theta \cos \theta
\end{align*}
\]  

(3.15)

The left hand side of Equation (3.15) can be extracted from the experiment while the right hand side have the unknown constants \( C_{1n} \) and \( C_{2n} \). Thus, the problem of determining the stress intensity factor could be converted to solution of the indeterminate equations. The determination of the stress intensity factor will be a trivial problem if one can be confident that all of the displacement data are located within the singularity-dominated zone. However, it is rarely the case due to three conflicting restrictions. Firstly, practical limitations on specimen size contract the magnitude of the singularity-dominated zone to an uncomfortably small region [26], which may be of unknown size and shape. Secondly, due to the presence of the nonlinear process zone [27], the data-collection should be at some specified distance, depending on the size of thickness, away from the crack tip. Thirdly, the presence of the shadows, as shown in Figure 3.1 and 3.2, around the very notch tip in the plane strain investigation, makes the locating of the notch tip difficult. The uncertainty in the crack tip position is included in the error calculation, as shown in Appendix A. It was shown [26] that small random position errors in locating the fringe maxima can be minimized by inputting the number of displacement data at least ten times
greater than the number of unknown stress-field parameters. Thus, the experimental SIF solution could be achieved by using the concept of FFMI [28] (Fractional Fringe Moiré Interferometry) to solve Equation (3.15) in a least-squares sense. In fractional fringe analysis, the determination of intermediate displacements over any half fringe is based on the light intensity measurements, which have some degree of uncertainty error. The uncertainty interval for the displacements depends on the weight and uncertainty of each measured parameter that is used, as shown in Appendix A.
Figure 3.1 Shadows around the notch tip in plane strain investigation. (U field)
Figure 3.2 Shadows around the notch tip in the plane strain investigation. (V field)
3.1.1 Modified calculation

Creager and Paris [29] proposed a modified stress solution for a blunt crack or notch problem in 1967. The modified terms (for mode I and mode II) to account for the blunt notch effect were added to the general stress equations as follows:

\[
\sigma_x = \frac{K_I}{(2\pi r)^{1/2}} \cos \frac{\theta}{2} [1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}] + ... \\
- \frac{K_I}{(2\pi r)^{1/2}} \frac{\rho}{2r} \cos \frac{3\theta}{2} \\
- \frac{K_{II}}{(2\pi r)^{1/2}} \sin \frac{\theta}{2} [2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2}] + ... \\
\frac{+ \frac{K_{II}}{(2\pi r)^{1/2}} \frac{\rho}{2r} \sin \frac{3\theta}{2}}
\]

(3.16)
\[
s_y = \frac{K_I}{(2\pi r)^{1/2}} \cos^2 \frac{\theta}{2} [1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}] + \ldots
\]
\[
+ \frac{K_I}{(2\pi r)^{1/2}} \rho \cos \frac{3\theta}{2} + \ldots
\]
\[
+ \frac{K_{II}}{(2\pi r)^{1/2}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} + \ldots
\]
\[
- \frac{K_{II}}{(2\pi r)^{1/2}} \rho \sin \frac{3\theta}{2} / 2
\]

\[
\tau_{xy} = \frac{K_I}{(2\pi r)^{1/2}} \sin^2 \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} + \ldots
\]
\[
+ \frac{K_I}{(2\pi r)^{1/2}} \rho \sin \frac{3\theta}{2} / 2
\]
\[
+ \frac{K_{II}}{(2\pi r)^{1/2}} \cos \frac{\theta}{2} [1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}] + \ldots
\]
\[
- \frac{K_{II}}{(2\pi r)^{1/2}} \rho \cos \frac{3\theta}{2} / 2
\]
By utilizing the relationship between stress and strain and the one between strain and displacement as described in Chapter 3.1, the stress intensity factor can be obtained from the same algorithm.
3.2 Analytical method

Knowing the geometry of the single edge crack specimen and the applied load \( P \), the analytical SIF solution for plane strain can be found in ASTM-STP 410[1]

\[
K_I = \frac{P\sqrt{c}}{bd\sqrt{\pi}} [1.99 - 0.41(c/b) + 18.70(c/b)^2 - 38.48(c/b)^3 + 53.85(c/b)^4]
\]  \hspace{1cm} (3.19)

where \( d \) is the thickness, \( c \) is the crack length and \( b \) is the specimen width.

The analytical results for this specimen will be used to compare with the experimental \( K_I \)'s, which were extracted from the displacement field by image analysis.
3.3 Modified analytical method

It should be indicated that instead of choosing a crack geometry, a sharp notch was chosen here for the plane strain investigation, because a real crack will cause the debonding between the specimen grating and the upper layer of the specimen under even a small load. Therefore the SIF solutions for real crack have to be modified for a notch.

The stress concentration factor (SCF) method can be applied to derive the stress intensity factor (SIF), which is given by Paris and Sih in ASTM-STP 381 [2] as

\[ K = \sqrt{\pi \lim_{\rho \to 0} \rho \sigma_{\text{max}}} \]  \hspace{1cm} (3.20)

The above equation means that the product of the maximum stress at the notch tip and the square root of the notch-root radius approaches a finite non-zero value as \( \rho \) approaches zero. This limit is related to \( K_t \). Note in the above relationship, if \( \sigma_{\text{max}} \) is inversely proportional to \( \sqrt{\rho} \), then \( K_t \) will become independent of the notch radius. Wahl [30] in 1964 showed that in a case of root-radius-to-maximum-section ratio is less than 0.004, the quantity \( \sigma_{\text{max}} \sqrt{\rho} \) in each case deviates from a constant value by less than 3 percent. It was assumed, then, that the same would be true for intermediate notch depths.
Thus for small root radius compared to notch or minimum section, a good approximation of $K_I$ could be given by

\[ K_I = \frac{1}{2} \sqrt{\pi \rho \sigma_{\text{max}}} \]  
(3.21)

The $\sigma_{\text{max}}$ is a function of nominal stress and stress concentration factor, which could be found in the work by Hasebe and Kutanda [31] who treat this problem as a semi-infinite plate with a semi-elliptical notch under uniform tension.
3.4 Experiment

The specimen was mounted in a small screw-type tensile loading frame, which has been integrated into a four beam moiré interferometer. This frame was equipped with a 100 lb load cell. Crossed virtual grating of 1200 lines/mm produced from coherent He-Ne laser was projected on the specimen parallel to the notch line in the neighborhood of the notch tip. When the specimen is mounted in the loading frame, alignment of the interferometer is performed so that the adequate and symmetric fringes could appear in the null field. All interferometric images under different loadings, including the null field image, were recorded and digitized. Due to the presence of the nonlinear process zone, as mentioned before, the collected data have to be at least half the specimen thickness away from the notch tip. The image analyzer then scans the desired area and calculates corresponding displacements from each image pattern. Each net displacement field could be obtained from subtracting the null field. A least-squares algorithm was used to minimize the numerical errors. The algorithm first estimates the SIF by using the first two terms in the two-dimensional series solution in equation (3.15), then the truncation terms are increased incrementally until the estimated SIF values become stable.
3.5 Poisson’s ratio effect

The experimentally determined stress intensity factor for a plate containing a through crack is known [2,32] to yield a higher result in plane strain condition than the one in plane stress condition. The difference in stress intensity factors is due to the fact that a constraint develops near the crack tip in the experiment since the plate thickness is much greater than the crack-root radius. This leads to a state of nearly plane strain near the crack tip. At distances which are substantially larger than the thickness from the crack tip [32], a state of nearly generalized plane stress will occur. Brown and Srawley [2], and Irwin [33], have pointed out that the two dimensional result can be converted to the three dimensional stress intensity factor by multiplying the SIF with \((1-\nu^2)^{1/2}\), where \(\nu\) is the Poisson’s ratio of the material. The conversion can be explained as follows:

Consider the mode I traction-loaded cracked plate as shown in Figure 2.8. Rice [34] has identified an J integral

\[
J = \int [U dy - \sigma_i \frac{\partial u_i}{\partial x} ds]
\]  

(3.22)

where \(U\) = the strain energy density
\( u_i = \) displacement components

\( \sigma_i = \) stress vector components

which is path independent for the two dimensional problems. If the above integrand is integrated following the contour away from the crack tip, the result will be approximately the strain-energy release rate for plane stress as computed by Paris and Sih [2]. Thus

\[
J = \frac{K_{IA}^2}{E} \tag{3.23}
\]

Nevertheless, if the path is located within the constrained region, then the result is expected to be approximately the plane strain value

\[
J = \frac{(1-v^2)K_{IB}^2}{E} \tag{3.24}
\]

For \( J \) to be path independent, it follows that
This prediction could also be applied to the notch geometry. Therefore, by knowing the value of Poisson’s ratio of the investigated material (in this study, \( \nu = 0.4 \)), the theoretical difference between the stress intensity factors in plane strain and plane stress cases can be calculated.
Chapter 4

Results and conclusion

4.1 Results

Typical moiré fringe patterns obtained from the single-edge-notched specimen under plane strain and plane stress cases are shown in Figures 4.1, 4.2, 4.3 and 4.4. The experimental results of stress intensity factors (calculated from Equation 3.6 to 3.13 with $n = 20$) for different loadings were compared with the analytical solutions as shown in Figure 4.5 and 4.6, where the experimental stress intensity factors were extracted from the last three terms ($n = 16, 18, 20$) by using the least-squares method (see Figure 4.7 and 4.8). Since the digital image processing of the recorded patterns allows for a precise
substraction of the initial field, the net displacement is available for every point. The comparison between the experimental and analytical stress intensity factors in mode I loading are tabulated in Tables 4.1. The normalized stress intensity factors \((K_i / \sigma_0)\) for each loading as well as their lower and upper limits (see Appendix B) are shown in Table 4.2. Table 4.3 shows the average normalized stress intensity factor and the difference between experimental and analytical results. The average normalized stress intensity factor \((K_i / \sigma_0)\) derived from the plane strain case was found to be about 13.2 % higher than the analytical value. However, the average normalized stress intensity factor \((K_i / \sigma_0)\) derived from the plane stress case was only 1.5 % lower than the analytical value.
Figure 4.1 U field of displacement pattern under 4 N loading in plane strain case.
Figure 4.2 U field of displacement pattern under 4 N loading in plane stress case.
Figure 4.3 U field of displacement pattern under 71.2 N loading in plane strain case.
Figure 4.4 U field of displacement pattern under 71.2 N loading in plane stress case.
Figure 4.5 Experimental stress intensity factors for plane strain and plane stress cases.
Figure 4.6 Comparison of normalized stress intensity factor ($K_t / \sigma_0$).
Figure 4.7 Stress intensity factor calculation vs. number of terms. (load=17.8 N)
Figure 4.8 Stress intensity factor calculation vs. number of terms. (load=71.2 N)
Table 4.1 Comparison of experimental and analytical SIF (MPa√m).

<table>
<thead>
<tr>
<th>Load (N)</th>
<th>Mean value of experimental plane strain SIF (MPa√m)</th>
<th>Mean value of experimental plane stress SIF (MPa√m)</th>
<th>Analytical plane strain SIF (MPa√m)</th>
<th>Analytical plane strain SIF (MPa√m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>71.2</td>
<td>0.1318</td>
<td>0.1081</td>
<td>0.1217</td>
<td>0.1115</td>
</tr>
<tr>
<td>57.4</td>
<td>0.1122</td>
<td>0.0856</td>
<td>0.0981</td>
<td>0.0899</td>
</tr>
<tr>
<td>44.9</td>
<td>0.0888</td>
<td>0.0678</td>
<td>0.0768</td>
<td>0.0704</td>
</tr>
<tr>
<td>36.0</td>
<td>0.0747</td>
<td>0.0500</td>
<td>0.0616</td>
<td>0.0565</td>
</tr>
<tr>
<td>27.1</td>
<td>0.0610</td>
<td>0.0390</td>
<td>0.0464</td>
<td>0.0425</td>
</tr>
<tr>
<td>17.8</td>
<td>0.0374</td>
<td>0.0168</td>
<td>0.0304</td>
<td>0.0279</td>
</tr>
<tr>
<td>8.5</td>
<td>0.0180</td>
<td>0.0130</td>
<td>0.0144</td>
<td>0.0132</td>
</tr>
</tbody>
</table>
Table 4.2 Normalized SIF ($K_f / \sigma_0$) for each loading.

<table>
<thead>
<tr>
<th>Load (N)</th>
<th>Plane strain</th>
<th>Plane stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean value ($m^{1/2}$)</td>
<td>Lower and higher limits</td>
</tr>
<tr>
<td>71.2</td>
<td>0.244 ±0.0054</td>
<td>0.200 ±0.0399</td>
</tr>
<tr>
<td>57.4</td>
<td>0.250 ±0.0094</td>
<td>0.197 ±0.0048</td>
</tr>
<tr>
<td>44.9</td>
<td>0.238 ±0.0094</td>
<td>0.191 ±0.0050</td>
</tr>
<tr>
<td>36.0</td>
<td>0.241 ±0.0088</td>
<td>0.194 ±0.0070</td>
</tr>
<tr>
<td>27.1</td>
<td>0.274 ±0.0131</td>
<td>0.213 ±0.0151</td>
</tr>
<tr>
<td>17.8</td>
<td>0.251 ±0.0155</td>
<td>0.200 ±0.0104</td>
</tr>
<tr>
<td>8.5</td>
<td>0.261 ±0.0326</td>
<td>0.202 ±0.0233</td>
</tr>
</tbody>
</table>
The Poisson’s ratio of the material is 0.4.
4.1.2 Discussion

The difference between the experimental $K_i$ and the analytical $K_i$ in plane stress case is 1.5 %, as shown Table 4.3, which is reasonable. However, the difference in plane strain case, 13.2 %, appears as a high value. The difference may be attributed to the experimental error, as calculated in Appendix C. Some errors may also be induced by the twisting of fringes, as shown in Figure 4.1 and 4.3, which is attributed to the deformations due to surface polishing.

An interesting physical feature of the above results concerns data collection. To apply the LEFM approach, any reliable algorithm [26,27] for determination of the stress intensity factor has to utilize the data taken from a region outside the plastic and nonlinear processing zone. The valid data-collection region is suggested at some distance away from the crack tip. It has been shown [36] that the material near the plate surface is more readily distorted and therefore experiences more yielding than that in the interior. Since the plastic and nonlinear processing domain in plane strain image is smaller than the one in plane stress image, the data collection does not have to be half the plate thickness away from the notch tip [26], like the one in plane stress case. However, the size of the data collection area depends on the plate thickness and Poisson’s ratio and yet has to be evaluated.
The embedded moiré technique has higher sensitivity (1200 lines/mm) compared with the embedded fine-grid method (20 lines/mm) [6] and here it was successfully applied to the investigation of the plane strain SIF. However, there are some limitations of the described technique. Firstly, the contrasts of plane strain fringes, although can be recognized, were always worse than the ones in the plane stress case due to two reasons: the energy loss in the laser beam before it arrives at the plane strain grating (grating A), and the distortion of grating while the specimen was sanded or polished. Secondly, due to the need for reflection of the laser beams for interferometry, there are shadows surrounding the notch and therefore concealing some regions of interest. This drawback becomes greater when the width of the notch and the plate thickness is increased. Thirdly, due to the requirement of material clarity and the characteristic of fabrication for the specimen, the choice of materials which can be analyzed by this technique is limited. Fourthly, due to the difference of mechanical properties between the coating (Au-Pd) of the embedded grating and the material, the delamination between them can not be circumvented, especially when the applied loads are increased. Consequently, in the premise of delamination, the plane strain fracture toughness can not be accessed by the embedded moiré interferometry method.

In this study the laser source delivers 25 mW of the 632.8 nm wavelength light. To solve the energy loss problem, a higher power laser (like Argon laser with 3 W) may
be utilized. The distortion of embedded grating due to the sanding work could be minimized by using appropriate specimen mold and careful polishing with correct polishing agent. In choosing the material for investigation, transparency and the fabrication process of the material become the requirements rather than the value of Poisson’s ratio, since Poisson’s ratio is not the restriction for application of the moiré interferometry technique. However, due to the shadows obscuring part of the regions of interest and the delamination between the grating and material, this embedded method is not suitable for toughness or even dynamic crack growth analysis, especially when the region of interest is in the wake of crack growth. Due to the extreme fragility of moiré grating, the material, which will be poured on the grating to increase the plate thickness, has to be liquid rather than solid. Therefore embedded moiré method is not applicable for specific types of thermoplastic materials, which exist originally as very small solid pieces and need the thermal press process to force them stick together.
4.2 Conclusion

In this study, the high frequency grating (1200 lines/mm) was successfully embedded inside the sample for in-situ plane strain investigation. Compared with the embedded-grid method, this method has higher sensitivity. The difference between the experimental and analytical stress intensity factors in plane strain case, as shown in Table 4.3, is about 13.2 %, which is higher than the difference between the experimental and analytical values in plane stress case (1.5 %). This phenomena is attributed to the severe grating (grating A in Figure 2.9) twisting due to the surface polishing of the sample. Despite the limitations of the embedded moiré interferometry described above, it is a suitable technique for investigation of the interior information in the material of interest when the applied load does not cause crack propagation. For the first time, the high sensitivity (0.417 μm per fringe order) moiré interferometry was applied to the investigation of the interior displacement field near the crack tip in order to analyze the stress intensity factor in plane strain condition. To reach the critical state like fracture toughness, the improvement of the adhesion of the embedded grating to both sides is strongly needed.
References


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78


[31] N. Hasebe and Y. Kutanda, "Calculation of Stress Intensity Factor from Stress

80


Appendix A Uncertainty analysis

In the concept of fractional fringe analysis, the determination of intermediate displacements over any half fringe is based on light intensity measurements, which may have some degree of uncertainty. The weight and uncertainty of each measured parameter will influence the uncertainty interval of determined displacements. For a general function \( Q(q_1, q_2, q_3, \ldots, q_n) \), the uncertainty interval \( S_Q \) is related to the uncertainty intervals of each measured parameter \( S_i \), as follows [37]

\[
S_Q = (\sum \frac{\partial Q}{\partial q_i} S_i^2)^{1/2}
\]  
(A.1)

In fractional fringe analysis the displacement \( u(x) \) is related to light intensities by the relation

\[
u(x) = \frac{1}{2\pi f_r} \arccos \left[ \frac{(I(x) - I_0)}{I_1} \right]
\]  
(A.2)

where \( f_r \) is 2400 lines/mm. Then only the uncertainty intervals of three parameters \( I(x) \), \( I_0 \) and \( I_1 \) are involved. To simplify the evaluation process, the above equation can be
rewritten as

\[ u(x) = k \arccos J(x) \]  

(A.3)

where \( k = \frac{1}{2\pi r} \), which is a constant, and \( J(x) = (I(x) - I_0)/I_1 \). \( J \) changes from -1 to +1 for any one half fringe. To find the uncertainty in \( u(x) \), the uncertainty in \( J(x) \) has to be known in advance. \( S_J \) can be computed if the error intervals of \( I(x) \), \( I_0 \) and \( I_1 \) are known.

The error interval in \( I(x) \) is ± 0.5 gray level, as it is measured directly in terms of discrete gray levels. \( I_0 \) and \( I_1 \) are computed from \( I_{\text{max}} \) and \( I_{\text{min}} \) in the field. Their error interval are therefore estimated from Equation (A.1) knowing that \( I_{\text{max}} \) and \( I_{\text{min}} \), both have the same error interval of ± 0.5 gray level. This leads to an error interval of ± 1/(2\sqrt{2}) gray levels for each of \( I_0 \) and \( I_1 \). Using Equation (A.1), \( S_J \) is found to be

\[ S_J = \frac{k}{2I_1} \left( \frac{3 + J^2}{2} \right)^{1/2} \]  

(A.4)

This is then used once again in Equation (A.1) to get the uncertainty interval in \( U(x) \) as

83
Clearly the error in $U(x)$ changes from point to point across any half fringe. The level of the error depends on the current value of $J(x)$, which in turn depends only on $I(x)$. Equation (A.5) is therefore, divided throughout by the half fringe displacement $U$ to yield

$$S_u = \frac{k}{2I_1} \left( \frac{3 + J^2}{2(1 - J^2)} \right)^{1/2} \quad (A.5)$$

This relative error therefore is dependent on the light intensity $I_1$ and the normalized light intensity $J(x)$, which ranges from -1 to +1. A graphical representation of the relative error is given in Figure A.1 for $I_1 = 64$, 128 and 256 gray levels. From this figure, it can be easily seen that, for $I_1 = 256$, the relative error in the displacement is well below 0.22% for $L$ ranging from -0.92 to +0.92. For values of $L$ close to ±1, the relative error sharply and asymptotically increases to infinity. However, since the displacements at the fringe centers, where $L = ±1$, are not determined by fractional fringe analysis, but rather by searching for maxima and minima in the light intensity distribution, therefore the infinite error interval at $L = ±1$ is avoided. The next point [30] detectable
by the image processor will have a light intensity that is different (higher or lower) from the neighboring peak (maximum or minimum) by no less than one gray level. For $I_1 = 256$, the corresponding next point value of $L$ is ±0.996, which means the maximum error is 0.98%. For $I_1 = 64$, the corresponding next point value of $L$ is ±0.984 and the maximum error is 1.97%.
Figure A.1 Uncertainty error vs. normalized light intensity
Appendix B  Confidence analysis

In the investigation of stress intensity factor $K_1$, within the elastic range the initial and final U-field image patterns were recorded. An image processor was employed to obtain the 'net' displacement since the specimen has to be preloaded to have its position fixed. The total loading were divided by seven steps and recorded separately. For each loading step, six extraction of displacement data were repeated.

The most commonly employed measure of the central tendency of a distribution of data is the sample mean $k_i$ [37], which is defined as

$$k_i = \frac{1}{n} \sum_{i=1}^{n} K_{hi}$$  \hspace{1cm} (B.1)

where $K_{hi} =$ the $i$th value of the stress intensity factor being measured

$n =$ the total number of measurements

The standard deviation $S_k$ is the most popular and is defined as
Since the sample size is quite small (n < 20), the standard deviation $S_k$ does not provide a reliable estimate of the standard deviation of the population. The bias introduced by small sample size can be removed by the following

$$S_k = \left[ \sum_{i=1}^{n} \frac{(K_i - k)^2}{n-1} \right]^{1/2}$$  \hspace{1cm} (B.2)

$$[k_i - t(\alpha)\frac{S_k}{\sqrt{n}}] \leq K_i \leq [k_i + t(\alpha)\frac{S_k}{\sqrt{n}}]$$  \hspace{1cm} (B.3)

where

- $K_i = \text{the true population mean value}$
- $t(\alpha) = \text{the statistic known as Student's t}$
- $\alpha = \text{the level of significance (the probability of exceeding a given value of t)}$

In this study, $n$ is equal to 6 and $t(\alpha)$ is equal to 2.57. The uncertainty is chosen to be 5%. Thus the upper and lower limits of stress intensity factor for each loading step can be calculated and the results are shown in Table 4.2.
Appendix C  Error propagation

Previous discussions of error in Appendix A have been limited to error arising in the measurement of a single quantity, determination of fringe centers (I). However, several quantities, like loading (P) and geometry (width w, thickness d and notch length a), are measured with their associated errors and have to be included. For multiplication of quantities the standard deviation S is [37]

\[
S = \sqrt{\left(\frac{S_p}{P}\right)^2 + \left(\frac{S_d}{d}\right)^2 + \left(\frac{S_b}{b}\right)^2 + \left(\frac{S_a}{a}\right)^2 + \left(\frac{S_I}{I}\right)^2}
\]  

(C.1)

where \(S = \) the standard error

\(S_p / P = \) error from the load = 0.002

\(S_d / d = \) error from the thickness of the sample = 0.0105

\(S_b / b = \) error from the width = 0.0049

\(S_a / a = \) error from the notch length = 0.005

\(S_I / I = \) error from the light intensity = 0.0197

and S is found to be 2.4 %.
Vita

Po-Chih Hung was born in Hualien, Taiwan, R.O.C on October 8th, 1967 as the first son of Chi-Cheng Hung and Chow-Chi-Chen Hung.

After graduating from the High School of National Taiwan Normal University in 1985, he attended the Department of Mechanical Engineering of National Taiwan University, where he received his Bachelor of Science degree in June, 1989. He was admitted as a graduate student in the Department of Mechanical Engineering and Mechanics of Lehigh University in January, 1992.
END OF TITLE