Bracing requirements for inelastic steel beams, July 1964

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Plastic Design in High Strength Steel

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I INTRODUCTION

It has been shown elsewhere\(^1,2\) that the bracing spacing for a steel beam under a uniform plastic moment \(M_p\) is a function of the amount of deformation that the beam is required to deliver at the moment \(M_p\). The relevant equation is

\[
\frac{KL}{r_y} = \frac{\pi}{\varepsilon_y \sqrt{1 + 0.7 \frac{R}{s-1}}}
\]

(1)

where

- \(L\) = distance between supports
- \(r_y\) = weak axis radius of gyration
- \(k\) = effective length factor. Specific values can be calculated from methods in References 1 and 2
- \(\varepsilon_y\) = yield strain
- \(R\) = rotation capacity; ratio of rotation at moment \(M_p\) to the rotation of an elastic member at \(M_p\).
- \(h\) = Young's modulus divided by the strain-hardening modulus
- \(s\) = strain at strain hardening divided by the yield strain

The derivation of this equation assumes that the compression half of the beam acts as a column.

This report will discuss the required properties of braces positioned according to eq. (1). Furthermore, it will become apparent that bracing which fulfills these dimensional and material requirements will also be adequate for the more common situation in which the beam is subjected to moment gradient. That is, bracing for the uniform moment case will be conservative for other loading cases.
II BRACING FORCES

The laterally deformed shape of a typical test beam is shown in Fig. 1. It may be noted that support movements occurred during the test as the supports (or braces) were not rigid. The braces used are shown in Fig. 2, and the support movement recorded in Fig. 1 is typical of that observed during tests.

Now, it has been shown that the lateral deformations of a beam will lead to local buckling and that a lateral strain distribution corresponding to this failure mode may be defined. The lateral moment associated with this strain distribution has been given as

\[ M_{lb} = \frac{2(s-1)}{k' - \sqrt{k'}} \cdot \sigma_y \cdot \frac{b^3L}{6} \]

where \( b \) is the width of the flange. As \( M_{lb} \) is the moment corresponding to the advent of local buckling, it is the maximum lateral moment that can occur in a section under an in-plane moment of \( M_p \).

A typical panel under the maximum in-plane moment, \( M_p \), is shown in Fig. 3. The laterally deformed shape in Fig. 3b represents the most severe condition for the production of a force \( H^1 \) in the braces. It is seen that \( H^1 \) has, then, a maximum possible value under an in-plane moment of \( M_p \) of

\[ H^1 = \frac{2M_{lb}}{L^1} \]

where \( L^1 \) is the length between braces. Thus the force, \( H \), in a brace is the sum of \( H^1 \) from the two spans, \( L_R \) and \( L_L \), adjacent to the brace. The maximum possible value of this force is therefore
where
\[ L_{av} = \frac{L_R + L_L}{2} \]  

Using eq. (2) gives
\[ H = \frac{s-1}{\frac{1}{h} + \frac{1}{t}} \cdot \frac{\sigma_y b^2 t}{3 L_{av}} \]  
or, in a non-dimensional form,
\[ \frac{H}{P_{yf}} = \frac{s-1}{\frac{1}{h} + \frac{1}{t}} \cdot \frac{1}{3 L_{av}/b} \]  

where
\[ P_{yf} = b t \sigma_y \]  
and \( t \) is the flange thickness

The predicted value of the bracing force, \( H \), is seen to increase as the spans decrease and is linearly related to the flange force \( P_{yf} \). To illustrate the use and predictions of eq. (7) it will be applied to a 10WF25 beam of A36 steel braced according to the current plastic design provisions \(^7\) with \( L_{av} = 35(r_y) \). For A36 steel, \( h = 33 \) and \( s = 11.5 \) \(^8\) and eq. (7) becomes
\[ \frac{H}{P_{yf}} = \frac{10.5}{27.25} \times \frac{5.762}{3 \times 1.31} \times \frac{1}{35} \]
\[ H = 0.0162 \cdot P_{yf} \]

It is rather remarkable to recall that the standard empirical formula \(^9\) for bracing forces, which has been in use for at least forty years, gives
\[ \frac{H}{P_{yf}} = 0.02 \]

The derivation of eq. (7) is based on the estimate of \( M_{lb} \) in eq. (2). It has been shown elsewhere \(^2\) that the linear
approximation made in its derivation results in a slight overestimate. However, the maximum lateral flange moment, \( M_{lf} \), will occur at an in-plane inflexion point, and is the lateral plastic moment of the flange without any reduction for axial load. This moment is 
\[
M_{lf} = \frac{b^2 t}{4} f_y
\]
(11)
The ratio of \( M_{lb} \) to \( M_{lf} \) is therefore
\[
\frac{M_{lb}}{M_{lf}} = \frac{4}{3} \frac{s-1}{k} \left(= 0.51 \ f_y b A \right)
\]
(12)
As it is highly unlikely that the moment \( M_{lf} \) will occur at an inflexion point (where the beam is unloaded and completely elastic) and as \( M_{lf} \) is only twice the adopted maximum lateral moment, \( M_{lb} \), it may be assumed that eq. (7) is a conservative estimate of the bracing force in plastically designed beams.
III BRACING STIFFNESS

The specification of a bracing force is not in itself sufficient for the design of the brace. For example, a tension brace will require the same cross-sectional area to carry a given bracing force, regardless of its length. However, its axial deformation will be directly proportional to its length.

Any movement of a braced point will affect the performance of the beam by relaxing its effective length. This will be most critical in the case of a beam under uniform moment. The following derivations will therefore apply to the uniform moment loading.

The relation between support spacing and rotation capacity is given in eq. (1). A known conservative assumption in this equation is the neglect of a section property, D, which would have increased the effective lengths from $kL$ to $DkL$, where

$$
D^2 = \frac{1 + A_w/A_{2f}}{1 + \frac{1}{2} \frac{\delta - 2t}{d - t} \frac{A_w}{A_{2f}}} 
$$

(13)

where

$$
A_w = (d - 2t)w
$$

(14)

$$
A_{2f} = 2bt
$$

(15)

For rolled sections $D$ varies between 1.08 and 1.17. Thus a non-rigid brace may be allowed to increase the effective length of a beam by 8% without invalidating eq. (1).

This relaxation of effective lengths has been observed in beam tests with non-rigid supports. In these cases the observed effective lengths were about 10% greater than their calculated values. The supports used in these cases are as shown in Fig. 2, and the support movement can be seen in Fig. 1.

The mechanism by which support movement changes the effective
length can be seen in Fig. 1, or diagrammatically in Fig. 4. It has been shown\textsuperscript{6} that the moment gradient in the adjacent spans has little influence on this behavior. The case in Fig. 4 is thus a representative one. The braces are all assumed to be of the same dimensions and, conservatively, loaded by the maximum brace force, $H$. The braces therefore all deflect an equal amount, $u_b$.

If the flexural stiffness of the adjacent spans $AB$ and $DC$ is $S_a$ with $A$ and $D$ pinned, the effect of the sway deflections, $2u_b$, is to change the stiffness to an effective value of $S_a^1$ where

$$S_a^1 \delta \Theta = S_a \left( \delta \Theta - \frac{2u_b}{L_a} \right)$$

(16)

where $\delta \Theta$ is the lateral rotation of the beam at the interior brace points ($B$ and $C$), and $L_a$ is the adjacent span length.

From eq. (16)

$$\frac{u_b}{L_a} = \frac{\delta \Theta}{2} \left( 1 - \frac{S_a^1}{S_a} \right)$$

(17)

which determines the allowable value of $u_b$. It is therefore necessary to evaluate the terms in eq. (17).

The effective lengths, $k$, for two cases need be considered. For in-plane loadings similar to Fig. 4, the value of $k$ has been shown\textsuperscript{6} to remain close to 0.54; for the case of three braced segments under uniform moment it may be assumed\textsuperscript{6} that $k = 0.80$. Using the 8% permissable increase discussed earlier would allow these to increase to 0.583 and 0.864 respectively.

The relation between $k$ and the ratio of column to support stiffness has been given by Hoff\textsuperscript{10} and is reproduced in Reference 6.

In the above cases the results are:

$$k = 0.54 \rightarrow 0.583 : S_a^1 = 0.385 S_a$$

(18a)

$$k = 0.80 \rightarrow 0.804 : S_a^1 = 0.559 S_a$$

(18b)
Conservatively taking the latter case and using eq. (17) gives

$$\frac{u_b}{L_a} = 0.220 \delta \theta$$  \hspace{1cm} (19)

The value of $\delta \theta$ can be assessed in the following manner. The maximum lateral moment, $M_{1b}$, is given by eq. (2) and a representative value of $S_a$ is 2.75. Using the right hand side of eq. (16) results in

$$M_{1b} = 2.75 \left( \frac{EI}{L_a} \right) \left( \delta \theta - \frac{2u_b}{L_a} \right)$$  \hspace{1cm} (20)

or

$$\delta \theta = \frac{M_{1b}}{2.75 \left( \frac{EI}{L_a} \right)} + \frac{2u_b}{L_a}$$  \hspace{1cm} (21)

and so in eq. (19)

$$\frac{u_b}{L_a} = 0.143 \frac{M_{1b}}{\left( \frac{EI}{L_a} \right)}$$  \hspace{1cm} (22)

with eq. (2)

$$\frac{u_b}{b} = 0.57 \frac{S-a}{L_a} \cdot \varepsilon_y \cdot \left( \frac{L_a}{b} \right)^2$$  \hspace{1cm} (23)

Equation (23) illustrates that the required stiffness of the braces is inversely proportional to the square of the support spacing. Thus relatively stiff braces are needed for closely braced members if they are to deliver their full rotation capacity (eq. (1)). Whereas the coefficient 0.57 in eq. (23) may be considered to be derived from somewhat subjective considerations, due to the necessity of deciding on a limit for $u_b$, the other terms in the equation give valid indications of their influence on brace stiffness.

For a 10WF25 beam braced at 35$^\circ$ the maximum lateral deflection of the braced point (from eq. (23)) is 0.098" for A36 steel and 0.087" for A441 steel. It is interesting to note that the lateral movement recorded at the brace point in Fig. 1 was 0.08" (A441). This close relation between the limits of eq. (23) and the observed test support
movements is to be expected, as eq. (23) was based on the observed satisfactory performance of beam tests with this order of brace point movement.

The axial stiffness of a brace may now be obtained from eqs. (23) and (7), using the limitation

$$\frac{H L_b}{A_b E} \leq U_b$$  \hspace{1cm} (24)

where $L_b$ is the brace length and $A_b$ its area, as

$$\frac{L_b}{L_{avg}} \leq 1.71 \cdot \frac{A_b}{A_f} \cdot \left(\frac{l_a}{b}\right)^2$$  \hspace{1cm} (25)

where

$$A_f = b L$$  \hspace{1cm} (26)

It should be noted that $L_{avg}$ is the average length of the spans on either side of the brace point whereas $L_a$ should be taken as the longer of the two spans. It may also be observed that eq. (25) is independent of material properties, depending only on section dimensions and on the coefficient 1.71 derived from limiting the effective length change to 8%. The above derivations all assume that the far end of the braces are fixed against axial movement.
IV DESIGN RULES

Design rules may be formulated from eqs. (7) and (25). The plastic design concepts are utilized and it is assumed that at the point of beam collapse the braces are also on the verge of failing in their function of limiting $u_b$ (the "one hoss shay" concept). Thus the braces are carrying the yield stress at this stage and the area of the brace is found, from eq. (7), to be

$$A_b^* = \frac{A_f}{L_b - \frac{h-1}{\frac{L_a}{b}}}$$

where $A_b^*$ denotes the brace area calculated by eq. (27). This terminology is introduced as it is frequently found in design that $A_b^*$ is smaller than minimum section area that can be supplied; consequently, the brace area, $A_b$, will in these cases exceed $A_b^*$ and the excess area can be utilized in satisfying stiffness requirements.

The brace axial stiffness requirement follows from eq. (25) & (7) as

$$\frac{L_b}{L_a} \leq 0.57 \cdot \frac{\frac{L_b}{L_a}}{L_b - \frac{h-1}{\frac{L_a}{b}}} \cdot \frac{A_b}{A_b^*}$$

The two equations given as (27) and (28) constitute the necessary design equations for braces. The optimum beam bracing lengths\(^6\) for $A_{36}$ and $A_{441}$ are 37.5 $r_y$ and 27.5 $r_y$ respectively. For a beam braced at these lengths and with a $b/r_y$ ratio of 4.4 (10WF25), eqs. (27) & (28) become

$$\frac{A_b^*}{A_f} \geq 0.015 \quad \frac{L_b}{L_a} \leq 1.87 \frac{A_b^*}{A_b^*} \quad (29)$$

$$\frac{A_b^*}{A_f} \geq 0.013 \quad \frac{L_b}{L_a} \leq 0.88 \frac{A_b^*}{A_b^*} \quad (30)$$
As an example of the application of the eqs. (29) & (30), the value of $A_b^*$ for the 10WF25 would be 0.037 in$^2$ (A36) or 0.032 in$^2$ (A441). Using the lightest angle (1 3/4 X 1 1/4 X 1.23 L) for a brace gives $A_b = 0.36$ in$^2$. Thus $A_b^*/A_b^*$ is 9.73 (A36) and 11.25 (A441), and from eqs. (29) & (30) $L_b/L_a$ is 18.2 (A36) and 9.90 (A441). In this example $L_a$ is 49.1" (A36) and 36.00" (A441) and so the maximum brace lengths are 893" and 356". This would mean $L_b/r$ values of 3,300 (A36) and 1,300/(A441). From this example it may be surmized that in many instances in which the lighter beams are used neither the area provision (eq. (27)) or the length provision (eq. (28)) will be critical.
V BRACE FLEXURAL PROPERTIES

The properties of the brace will be referred to the same axes as apply to the beam being braced (Fig. 5). So far the discussion has concerned the area and stiffness of the brace in the XX direction. Bending stiffness about the YY axis will also assist in restraining the beam against lateral rotations. If this stiffness is given by \(3(\frac{EI}{L})_{YY(brace)}\), then stiffness of the adjacent spans can be modified by replacing \(S_a\) by

\[
\bar{S}_a = S_a + 3\left(\frac{EI}{L_b}\right)_{YY(brace)} \cdot \left(\frac{L_a}{EI}\right)_{YY(Span)}
\]

However, it has been pointed out by Lee et al.\(^{11}\) that the last term of eq. (31) will be so small for practical proportions of beam and purlin that it may be neglected.

Lee et al.\(^{11}\) have also noted that it is probably sufficient to brace only the compression flange, provided vertical stiffeners are used at the brace points. These restrain the tension flange from moving by greatly increasing the torsional strength of the section. Hence, the tension flange is held in position by the torsional fixity of the braced compression flange. It is therefore necessary that the braces provide this stiffness, either by having two taut braces or by the bending stiffness of the braces about the ZZ axis. The tautness requirement is clearly impractical, and it is therefore necessary to specify that a brace have some degree of stiffness and strength about the ZZ axis, if the tension flange is to remain unbraced. An estimate of these
factors will now be provided.

It has been shown\(^6\) that the behavior of the beam is as shown in Fig. 6a. Forces \(F_T\) and \(V_T\) and moments \(M_T\) act on the free body of the tension flange. As the flange is a tie, it will undergo very small relative deflections along its length\(^6\) and these will be neglected in this analysis.

The reactions to the loads \(F_T\), \(V_T\), and \(M_T\), must be carried at the brace point. Simple upper bound estimates of \(M_T\) and \(F_T\) are found from the plastic design concept. In accordance with the model, plastic hinges are placed at the points indicated by solid circles in Fig. 6a. Therefore

\[
M_T = \frac{L_{av} \omega^2 \sigma_T}{4}
\]  
(32)

\[
F_T = \frac{2M_T}{d - t}
\]  
(33)

where \(w\) is the thickness of the web.

\(V_T\) results from the axial force in the flange having a vertical component due to the flexure of the flange.

\[
V_T = b t \sigma_T L_{av} \cdot \frac{\gamma}{2}
\]  
(34)

where \(\gamma\) is the in-plane curvature.

There is an equal and opposite force due to the compression flange.

The forces acting on the beam at a brace point are shown in Fig. 6b. The tension flange forces will rotate the braced section through an angle \(\theta\) measured about the braced point, \(A\). It is assumed that each brace provides a moment, \(M_B\), to resist this rotation.

Equilibrium about \(A\) gives
\[ F_T (d-t) + V_T (d-t) \theta = M_T + 2M_B \]  \hspace{1cm} (35)

A consideration of the maximum magnitudes of \( V_T \) and \( \theta \) indicates that the \( V_T \) term may be neglected.\(^{12} \) Using eqs. (32) and (33), neglecting \( V_T \) and assuming \( (d-t) \neq (d-2t) \), reduces eq. (35) to

\[ M_B = \frac{3}{B} L_{aw} \omega^2 \sigma_y \]  \hspace{1cm} (36)

The axial stress in the braces is \( \frac{A_b}{A_b} \sigma_y \) and it is necessary to keep \( M_B \) elastic if only the compression flange is braced. Hence the required brace section modulus (ZZ axis) is \( S_b \) where

\[ S_b = \frac{3}{B} L_{aw} \omega^2 \]  \hspace{1cm} (37)

If only one brace is used it must have twice the above value of \( S_b \).

The above derivation ignores the requirements of compatibility. It is assumed that the braces are sufficiently stiff about the ZZ axis to cause \( M_B \) to be attained before the rotations, \( \theta \), (Fig. 5b) have become excessive. As a criterion for the value of \( \theta \), it has been shown elsewhere\(^6 \) that the lateral deflection of the compression flange of a braced beam, from its lateral inflexion points, is \( 2M_{1b} / A \sigma_y \). Thus the maximum \( \theta \) becomes \( 2M_{1b} / A \sigma_y \) and so the maximum brace (L/d)\(_b\) is calculated as

\[ \varepsilon_y \left( \frac{L}{d} \right)_{b} \leq \frac{S_b}{S_b^*} \cdot \frac{S-1}{h_{y}} \cdot \frac{A_f}{A} \cdot \frac{b}{d} \]  \hspace{1cm} (38)

For A36 steel

\[ \left( \frac{L}{d} \right)_{b} \leq 316 \frac{S_b}{S_b^*} \frac{A_f}{A} \cdot \frac{b}{d} \]  \hspace{1cm} (39)

where \( S_b^* \) is the calculated and \( S_b \) the actual value of the brace section modulus about the ZZ axis. For an A36 1OWF25 beam with \( S_b = S_b^* \) and \( A_f / A_b < 0 \)}
\((\frac{L}{d})_b \leq 62\).

This derivation has also assumed that the brace-to purlin connections are capable of transferring moment. It is also noted that it is only applicable to these cases where the tension flange is not braced.
VI APPLICATION TO A DESIGN

As an example of the application of the above bracing provisions (eqs. (7), (25), (37) and (38)), it is interesting to apply them to the bracing tests conducted by Lee et al. The testing arrangement is as shown in Fig. 7. The 10WF25 beam was of A36 steel.

10WF25. \( A = 7.35 \text{ in}^2, r_y = 1.31 \).

\( L_a = 1.31 \times 40 = 52.4 \text{ in.}, w = 0.25 \text{ in.} \)

1. Check: Eq. 29 \( \frac{A_p}{A} \geq 0.015 \times \frac{37.5}{40} = 0.10 \text{ in}^2 \)

\( \therefore A_p \geq 0.014 \times 7.35 = 0.10 \text{ in}^2 \)

Use smallest I-sections:

\( A = 1.64 \text{ in}^2 > 0.10 \)

2. Check length: Eq. 29 \( \frac{L_b}{L_a} \leq 1.87 \times \frac{40}{37.5} \times \frac{1.64}{0.10} = 32.7 \)

\( L_b < 32.7 \times 52.4/12 = 143 \text{ ft.} > 12 \text{ ft.} \)

\( \therefore \) axial stiffness O.K.

3. Check bending strength: Eq. 37

\[ S_b = \frac{3 \times 52.4 \times 0.25^2}{1 - \frac{0.10}{1.64}} = 1.31 \text{ in}^3 \]

If two purlins: \( S = 1.31 \text{ in}^3 \), S of 315.7 = 1.7 in\(^3\) O.K.

If one purlin: \( S = 2.62 \text{ in}^3 \), S of 315.7 = 1.7 in\(^3\) N.G.

Use 4.7.7, \( S = 3.0 \text{ in}^3 \) O.K.

Summary: two purlins 315.7

one purlin 417.7
4. Check bending stiffness: Eq. 39

\[
\left( \frac{L}{d_b} \right) \leq 316 \times \frac{5.762 \times 0.430}{7.35} \times \frac{5.762}{10.08} \times \frac{1.70 \times 1}{1.31 \times 0.94} = 84
\]

Two purlins:

\[
\left( \frac{L}{d_b} \right) = \frac{1.44}{3} = 48 \text{ O.K.}
\]

One purlin (417.7),

\[
\left( \frac{L}{d} \right)_{b} < \frac{1}{2} \times 61 \times \frac{3}{2.62} \times \frac{1}{0.96} = 36
\]

\[
\left( \frac{L}{d} \right)_{b} = \frac{144}{4} = 36 \text{ O.K.}
\]

One purlin (315.7), \( \left( \frac{L}{d} \right)_{b} > 36 \), \( \therefore \) N.G.

Now ten tests are reported in Ref. 11, in which a 315.7 purlin was used as the purlin section. Eight of these had a purlin on either side of the brace point, with secondary variations in methods of attachment, and so forth. Two tests had a purlin on one side only. The eight tests all behaved in the manner predicted in Reference 6, and delivered rotation capacities between eight and ten. The two one-purlin test behaved in a noticeably different manner. Test LB-22 had a welded purlin connection and the load capacity began dropping at a rotation capacity of 3.5. Test P-10 had a bolted connection and its load capacity began dropping at a rotation capacity of almost zero. However, local buckling was not observed in either test until a rotation capacity of between seven and eight. This indicates that the compression flange behaved in the standard manner, but that some other factor was influencing the result. It should also be noted that these two tests differed from the others in that they were loaded through the tension rather than the compression flange.

The above behavior is directly predicted by the preceding calculations, which showed that a 315.7 purlin was adequate to hold the compression flange but was not sufficient to keep the tension flange in the required position.
VII: REVIEW OF PREVIOUS BRACING STUDIES

It has been shown that the bracing required for a plastically designed beam must satisfy three criteria: axial strength, axial stiffness, and major axis bending strength and stiffness. It is interesting to now review the previous bracing studies.

It has already been mentioned that the predicted bracing area is very close to the commonly used value of two per cent of the flange area (Ref. 9). The origin of the two percent rule appears to have been in engineering intuition, and is a good example of the usual effectiveness of this approach. Throop, in 1947, stated that the two per cent figure had been in use in his design office for many years, and a more conservative 2-1/2 percent value appeared in the (original) AREA Specifications in 1925.

Zuk analytically studied the bracing forces for elastic beams and obtained values between 0.2 and 2.4 per cent. The analysis assumed certain initial imperfections and rigid supports. Winter extended this elastic study to include the axial stiffness of the braces. In addition to a knowledge of initial imperfections it is also necessary to know the deflections at failure. The analysis ingeniously utilizes the fact that once the supports are above a certain axial stiffness the buckling load will be identical to the load for rigid supports. The stiffnesses required were found to be very small. Confirmatory tests were also conducted in which thin cardboard strips were sufficient to brace cold-formed back-to-back steel channels (4" X 2"). It is worthwhile noting that Winter used a flange-column model similar to the
one used by the authors in Reference 6.

Massey applied a similar approach to the analysis of post-elastic beams; however, he returned to Zuk's initial assumption of rigid supports. It is again necessary to assume an initial imperfection pattern. The section properties of the yielded cross section are similar to those previously used by Galambos. The actual analysis represents a somewhat unreal situation, as has been pointed out by the author and others in the discussion of Massey's work.

Massey also presented experimental results on beams with \( L/r_y \) values between 40 and 120. For the three tests at \( L/r_y = 40 \), the bracing force averages 0.011\( P_{yf} \). Assuming A36 steel, the predicted value from Eq. (7) is

\[
H = \frac{10.5}{27.2} \times \frac{1}{2} \times \frac{1}{3} \times \frac{8}{7} \times \frac{1}{40} = 0.013 \ P_{yf}
\]

which is only slightly higher than Massey's results. However, Massey has indicated in the closure to his paper, that the recorded bracing forces do not apply to the point of unloading, but to some earlier stage in the hinge formation. Therefore it is expected that his bracing forces will be less than those given by Eq. (7).

The experimental work of Lee et al. has already been discussed in the previous section. These tests indicated that the usual methods of purlin attachment are adequate and that a partial depth vertical stiffener is effective at brace points. They also showed how stronger braces are needed when only one side of a beam is braced. Unfortunately,
the purlins used were much more than adequate with respect to axial strength and stiffness, and therefore do not provide any conclusive information on these problems. As stated above, the tests indicate the adequacy of present bracing methods, but, except for bracing on one side only, they do not indicate which methods might be inadequate.
VIII CONCLUSIONS

The question of bracing design has been examined and seen to depend on the fulfillment of three criteria: axial strength, axial stiffness, and bending strength and stiffness with respect to the ZZ axis of the beam. Equations are presented which allow each of these criteria to be defined. Their application is seen to be simple and in accord with available test results.

The derivations given have been coupled with previous work\textsuperscript{1,6} on the deformation capacity of steel beams under uniform moment, and the resulting, solutions are the first available for plastically deformed beams.
IX ACKNOWLEDGEMENTS

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APPENDIX

LOADED PURLINS

A case which sometimes occurs, and which requires a slightly different treatment to those cases discussed in the body of the report, occurs when the purlins themselves are loaded by transverse forces. In the following discussion it will be assumed that the loaded purlins are designed plastically, and therefore have formed collapse mechanisms under the (unfactored) applied loading.

Consider firstly the case shown in Fig. 8a in which only one purlin is loaded. In this case the unloaded purlin must be considered as carrying all of the bracing force $H$ (normally, this will be the case for unloaded braces also as the compression brace will be considered to be inactive). It is assumed that the braces and main beam are all sufficiently stiff to prevent any horizontal loads being produced in the main beam due to cable-type action of the loaded purlin.

If the purlins brace only the compression flange then the unloaded purlin will not only have to resist the twisting of the main beam due to lateral deformations, but also the bending moment from the loaded purlin. If this latter bending moment is $M_p$ for the purlin then the unloaded purlin, whose maximum capacity is also $M_p$, cannot be relied upon to resist the total applied moments. Therefore the loaded purlins must be designed as though they are pin-connected to the main beam but constructed so that they are capable of transferring moment (Refs. 11,22). Furthermore, only one purlin may be considered to act as a brace.
If both purlins are loaded (Fig. 8b) the considerations given above will already have been applied to the design to meet the eventuality of only one purlin being loaded. If both purlins are loaded it will be realized that any lateral movement of the beam will cause one purlin to unload and behave elastically. Consequently, the provisions for one purlin loaded will be also adequate for this case. As any flexural hinge is not a region of unconstrained plastic flow in the axial direction, the axial extension of a brace will not exceed its elastic deformation under a stress of $\sigma$. That is, the area $A_b^p$ will be adequate axially. However, in calculating the bending plastic moment of the purlins, the reduction of $M_p$ due to $P$ should be considered in those cases where the ratio $A_b^p / A_b$ exceeds about 0.15. If the main beam is part of an end-frame, for instance, and braced on only one side (Fig. 8c) the brace force $H$ must be carried in compression. This will reduce the allowable axial stress and therefore increase the required brace area $A_b^p$. In addition if $A_b^p/A_b$ becomes large there will be a reduction in the flexural stiffness of the brace.

A further problem occurs when the brace is a loaded purlin (Fig. 8d) as there may now be an unbalanced lateral bending moment on the main beam. This undesirable situation can be avoided by designing the purlin-to-beam connection in such a way that both flanges of the main beam are braced. Such a connection detail is shown in Fig. 8e. It does not seem advisable that beams braced on only one side should be braced by loaded purlins attached only to the compression flange.
XI NOMENCLATURE

b  flange width

d  depth of a section

h  Young's Modulus divided by strain hardening modulus

k  effective length factor

r_y  weak axis radius of gyration of a section

s  strain at strain hardening divided by yield strain

t  flange thickness

u_b  brace point lateral deflection

w  web thickness

A  area of a cross section

A_b  brace area

A*_b  calculated required brace area

A_F  bt

A_{2f}  2bt

A_w  \((d-2t)w\)

D  section property, eq. (13)

F_T  lateral force on tension flange

H  brace force

H^1  brace force from one span

L  distance between supports

L^1  span associated with H^1

L_a  adjacent span length

L_{av}  \(1/2(L_L + L_R)\), eq. (5)
L_b \quad \text{brace length}

L_L \quad \text{span to left of braced point}

L_R \quad \text{span to right of braced point}

M_B \quad \text{brace moment}

M_{lb} \quad \text{lateral moment in flange at local buckling}

M_{lf} \quad \text{maximum possible lateral moment in a flange}

M_p \quad \text{plastic moment}

P_y \quad A\sigma^{	ext{y}}

P_{yf} \quad bt\sigma^{	ext{y}}

R \quad \text{rotation capacity}

S_a \quad \text{adjacent span stiffness}

\bar{S}_a \quad \text{sum of purlin and adjacent span stiffness}

S_{1a} \quad \text{effective adjacent span stiffness}

S_b \quad \text{section modulus of brace (ZZ axis)}

S^{\phi} \quad \text{calculated required value of } S_b

V_T \quad \text{vertical force on tension flange}

\varepsilon^y \quad \text{yield strain}

\delta \Theta \quad \text{lateral rotation of brace point (YY axis)}

\Theta \quad \text{brace point rotation (ZZ axis)}

\sigma_b \quad \text{maximum bending stress in brace}

\gamma \quad \text{curvature}
XII FIGURES
Fig. 1  Lateral Deflections During Test HT-28
**PLAN**

Fig. 2  Lateral Bracing System

**SECTION AA**

Fig. 3  Lateral Behavior of One Span
Fig. 4 Lateral Behavior of Beam

Fig. 5 Idealized Cross-Section
Fig. 6  Behavior of a Brace Point
(a) One Purlin Loaded

(b) Two Purlins Loaded

(c) One Unloaded Purlin

(d) One Loaded Purlin

(e) Satisfactory Bracing Detail for Fig. 8d

Fig. 8    Loaded Purlins
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