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WELDED PLATE GIRDERS

STRENGTH OF PLATE GIRDERS IN SHEAR

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by

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FOREWORD

This paper is the second in a series of three reports on the study of the strength of plate girders carried out at Lehigh University. The research project was guided by the Welded Plate Girder Project Committee whose members are listed in the Foreword to the first paper.

SYNOPSIS

A study of the shear strength of plate girders is presented. In utilizing the post-buckling strength offered by the transverse stiffening of girders, new design rules are proposed. The new approach is checked with ultimate load tests carried out at Fritz Engineering Laboratory.

1. INTRODUCTION

In current civil engineering practice the shearing stresses in webs of plate girders are analyzed according to the classical beam theory established by Navier and St. Venant. According to this theory the shear force is resisted by a state of shearing stresses as pictured in Fig. 1a. The principal stresses at the neutral axis are of the same magnitude as the shear stress and act at 45° with the longitudinal axis (Fig. 1b). Such a shear carrying action may be called "beam action". To satisfy the condition of small deformations, on which this beam theory is based, transverse stiffeners must be spaced close enough so that instability due to shear is excluded.

Ever since plate girders came into use, it has been recognized that beam action alone is not the only way that shear can be carried. References 1 to 5 reflect the extensive discussion of the problem of web stiffening, carried on just before the turn of the last century. Intuition led to the opinion that the action of a plate girder was similar to that of a Pratt truss.[1] Turneaure [3] concluded from a girder test "that there is as much reason to suppose that the web stresses follow down the web and up the stiffeners, as in a Howe Truss, as to suppose that they follow the lines of a Pratt Truss". However, model studies [2,5] and a girder

test [4] clearly indicated the importance of the web as tension and the stiffeners as compression elements. The goal was, at that time, to assess the nature of the stress flow in the web rather than to estimate the carrying capacity. Thus, more qualitative than quantitative results could emerge. But at the beginning of this century it led to a rather liberal American design of girder webs, with web depths greater than 170 times the web thickness. Meanwhile a web buckling theory was developed to determine safe limits for the design of plate girders. A possible truss action was advanced merely to justify a somewhat lower factor of safety against web buckling than that required against other stability cases such as column failure. Later tests conducted in this country [6,7] were mainly concerned with the establishment of a slenderness limit for unstiffened webs.

In 1916, the Norwegian H.H. Rode wrote an outstanding dissertation [8] in which one chapter deals with webs of plate girders. It appears that he may have been the first to mathematically formulate the effect of a tension field or truss action which sets in after the web loses its rigidity due to buckling. He proposed to evaluate its influence by considering a tension diagonal of a width equal to 80 times the web thickness.

With the development of aeronautical science the shear-carrying capacity of membrane-like structures got new attention. The paramount requirement of aircraft design (to minimize the weight of the structure) led to extremely thin webs. Since such structures were built of aluminum alloys, the modulus of elasticity and hence the web buckling stress were correspondingly lower than for steel girders. By neglecting beam action completely in such structures, and considering the web as a membrane resistant only to tension, Wagner formulated the "Theory of Pure Diagonal Tension" [9]. While pure diagonal tension is one limiting case of the state of stress in a thin web, pure shear is the other, occurring only in stocky webs. As seen from Ref. 10, extensive experimental studies were undertaken by the National Advisory Committee for Aeronautics to cover the transition range, where "incomplete diagonal tension" occurs.

There are several reasons why the civil engineering profession has not applied the highly specialized practice of aeronautical engineering to the analysis of their structures. One reason is the reluctance to consider the flanges as transversely loaded by the tension field forces and acting as beams supported by the intermediate stiffeners. The flanges, utilized primarily in carrying the girder's bending moment, would have to be specially designed to serve this secondary purpose. With a continuous skin, such as around

a fuselage or a wing of an aircraft, the boundary conditions are much more favorable for membrane action than in a welded plate girder where no angle sections between web and flange plate are used, which would provide a certain degree of rigidity.

The purpose of the subsequent study is to derive a simple but general formula for the ultimate shear strength of steel girders with flanges not resistant to membrane tension but with webs so slender that a certain tension action might develop. Utilizing some idealizations, a functional dependance of the shear strength upon the principal parameters will be established.

2. THE ULTIMATE SHEAR FORCE

Assuming that the ultimate shear force, V_u, of a transversely stiffened plate girder could be expressed in a formula, it would certainly have to be a function of the following dimensional variables: the stiffener spacing a, the girder depth b, the web thickness t, and the material properties σ_y and E, where σ_y is the yield stress and E the modulus of elasticity. The shear force for which unrestricted shear yielding occurs shall be termed "plastic shear force", V_p, analogous to the term "plastic moment", M_p, used in plastic analysis. Since V_p has the dimension of a force, it must be possible to express the ultimate shear force V_u in the form $V_u = V_p f(a,b,t,\sigma_y,E)$, where the function f is nondimensional. Therefore, whenever σ_y or E should occur, they can be brought in the form $\varepsilon_y = \sigma_y/E$. The remaining variables a, b and t, with length as their dimensions, can only occur in ratios; where $\alpha = a/b$ and $\beta = b/t$ are sufficient to express all possible relations between these three variables. Thus, a formula for the ultimate shear load must be of the form

$$V_u = V_p f(\alpha,\beta,\varepsilon_y) \tag{1}$$

From Mises' yield condition for plane stress, which is illustrated in Fig. 1d, it is seen that the shear yield

stress τ_y equals $\sigma_y/\sqrt{3}$. The full plastic shear force is reached when yielding occurs throughout the web depth b, hence

$$V_p = \tau_y bt = \frac{1}{\sqrt{3}}\sigma_y bt \qquad (2)$$

The shear contribution due to a possible tension field is considered next. Assuming that a field of uniform tension stresses σ_t flows through a web's cross section bt (Fig. 2a), the resulting shear force V depends on the inclination φ of the tension stresses. From Fig. 2b it is seen that the maximum value is obtained when φ is 45° and thus

$$V_{max} = \frac{1}{2}\sigma_t bt \qquad (3)$$

Whether a tension field, which is a membrane stress field, can develop depends on the boundaries of the plate. With regard to membrane stresses, a panel in a girder web has two very different pairs of boundaries, those along the flanges and those along the transverse stiffeners. A flange of a conventionally built welded plate girder has so little bending rigidity in the plane of the web that it cannot effectively resist vertical stresses at its junction with the web. Such flanges, therefore, do not serve as anchors for a tension stress field. The situation is different at the panel boundaries along the transverse stiffeners (Fig. 2c). There the tension strips can transmit the stresses. Thus, only a part of the web contains a

pronounced tension field which gives rise to a shear force $\Delta V_\sigma = \sigma_t \cdot s \cdot t \cdot \sin\varphi$ where s, the field width, also depends on φ.

When a thin-web plate girder panel is subjected to shear, it will reach a stage where the compressive stress σ_2 indicated in Fig. 1a ceases to increase because the web deflects. For the stress in the tension diagonal direction, no such evading of duty exists. Upon increasing the shear force, yielding initiates along the tension diagonal. A further increase of the applied shear force causes a wider portion of the web to yield. Since the increase in field width is gained by virtue of a decrease in the inclination of the tension stress with respect to the girder axis, an optimum value of the tension field contribution ΔV_σ to the shear force V_σ is reached. It is reasonable to postulate that at ultimate shear load the inclination of the tension field is the one which furnishes the greatest total shear component ΔV_σ of this tension field. With the notation as defined in Fig. 2c this inclination is obtained from the condition

$$\frac{d}{d\varphi}(\Delta V_\sigma) = \frac{d}{d\varphi}(\sigma_y \cdot s \cdot t \cdot \sin\varphi) = 0$$

or

$$\sigma_y t \left[\frac{ds(\varphi)}{d\varphi} \sin\varphi + s \cdot \cos\varphi\right] = 0$$

With $s(\varphi) = b \cdot \cos\varphi - a \cdot \sin\varphi$ it reduces to:

$$b \cdot \tan^2\varphi + 2a \cdot \tan\varphi - b = 0$$

which gives:

$$\left.\begin{aligned}
\tan\varphi &= \frac{-a \overset{+}{\scriptstyle(-)} \sqrt{a^2+b^2}}{b} = \sqrt{1+\alpha^2} - \alpha \\
\sin\varphi &= \left[\frac{1}{2} - \frac{a}{2\sqrt{1+\alpha^2}}\right]^{1/2} \\
\cos\varphi &= \left[2\sqrt{1+\alpha^2}\left(\sqrt{1+\alpha^2} - \alpha\right)\right]^{-1/2}
\end{aligned}\right\} \quad (4)$$

The strip corresponding to the optimum assumes a slope between 45° and 0°, when α takes on values from 0 to ∞. It is also seen from the equations that the tension strip inclination is less than the inclination of the panel diagonal, and the strip width a little wider than half the girder depth. In Fig. 3 the derived strip geometry is superimposed upon a photograph of a thin-web girder panel subjected to shear (girder G7, Ref. 11). The dark, yielded bands in the diagonally buckled zone, alternating with strips of unyielded or only slightly yielded metal, are due to the combined effect of bending and membrane stresses. The formation of the buckles produces plate bending stresses which are orthogonal to the diagonal tension (membrane) stresses and which are maximum at the surface of the web plate. Yielding is pronounced along the concave surfaces where compression bending stresses are superimposed upon the membrane tension stresses,

and retarded along the convex surfaces where the bending produces tension stresses, (Ref. Fig. 1c).

The photograph also illustrates the nature of the anchorage provided in the neighboring panels, where the horizontal tension component is transferred to the flange by shear. It indicates that the stiffeners must sustain axial forces. The magnitude of this shear and stiffener force can be derived using the idealized tension field, the inclination of which is fixed by Eqs. 4.

A succession of equal web panels all subjected to the same shear force is assumed as shown in Fig. 4. Cutting along the sections A, B, and C, a free body is obtained as sketched in the same figure. At the face A in the web an unknown resultant is acting. It can be decomposed into a normal component F_w and a shear force component which, because of the symmetry of the chosen cut, must be $V_\sigma/2$. The force acting in the flanges is denoted as F_f. At section B the stress pattern in the web is the same as in section A, therefore the same components will occur as explained before. The flange force will change in the amount of ΔF_f. At section C the stiffener force F_s and tension field stresses σ_t are acting. These tension stresses are under the inclination whose trigonometric values are given in Eqs. 4. Formulating moments around point O and considering equilibrium in the horizontal and vertical directions furnishes

three equations out of which the three unknowns ΔF_f, V_σ, and F_s can be computed:

$$\Delta F_f = - \sigma_t \cdot t \cdot a \cdot \sin\varphi \cdot \cos\varphi = - \sigma_t tb \frac{\alpha}{2\sqrt{1+\alpha^2}}$$

$$V_\sigma = - \frac{b}{a} \Delta F_f = \sigma_t tb \frac{1}{2\sqrt{1+\alpha^2}} \qquad (5)$$

$$F_s = + \sigma_t \cdot t \cdot a \cdot \sin\varphi \cdot \sin\varphi = \sigma_t tb \left(\frac{\alpha}{2} - \frac{\alpha^2}{2\sqrt{1+\alpha^2}} \right) \qquad (6)$$

In plate girders with slender webs neither a pure beam action (τ) nor a pure tension field action (σ) occurs alone, but rather the sum of both. Therefore the ultimate shear load V_u is

$$V_u = V_\tau + V_\sigma \qquad (7)$$

In order to compute these two shear contributions, two more assumptions are required. The first one is the postulate that the superposition of the stresses resulting from both carrying actions is limited by the state of stress which fulfills the yield condition, as shown in the sketch on the lefthand side of Fig. 1c. The second is the assumption that, up to τ_{cr}, shear is carried in a beam type manner, but that, from then on, V_τ remains constant (an assumption reviewed in the Appendix). Any postbuckling benefit must be contributed by tension field action.

Then,

$$V_\tau = V_{cr} = \tau_{cr} bt = V_p \frac{\tau_{cr}}{\tau_y} \qquad (8)$$

According to the first assumption, the tension field stress σ_t can be expressed explicitly. It is defined as the stress which can be added to the state of shear stress at the point of bifurcation of equilibrium (where τ_{xy} equals τ_{cr}) such that unrestricted yielding occurs in the tension field. For a short derivation attention must be given to the subscripts used (Fig. 1c). The fixed coordinates are x and y. The Cartesian coordinates u and v are generated out of x and y by counterclockwise rotation in the magnitude φ. When φ equals 45° these are called axis 1 and 2. By means of Mohr's circle, shown in Fig. 1b, it is seen that the state of shear stresses ($\tau_{xy} = \tau_{cr}$) combined with the diagonal tension stress σ_t under the inclination φ can be expressed as:

$$\left.\begin{aligned} \sigma_u &= \tau_{cr}\sin 2\varphi + \sigma_t \\ \sigma_v &= -\tau_{cr}\sin 2\varphi \\ \tau_{uv} &= \tau_{cr}\cos 2\varphi \end{aligned}\right\} \quad (9)$$

Introducing this set of stresses in Mises' yield condition

$$\sigma_u^2 + \sigma_v^2 - \sigma_u\sigma_v + 3\,\tau_{uv}^2 - \sigma_y^2 = 0 \qquad (10)$$

the following result for the tension field stress σ_t is obtained:

$$\frac{\sigma_t}{\sigma_y} = \sqrt{1 + \left(\frac{\tau_{cr}}{\sigma_y}\right)^2 \left[\left(\frac{3}{2}\sin 2\varphi\right)^2 - 3\right]} - \frac{3}{2}\,\frac{\tau_{cr}}{\sigma_y}\sin 2\varphi \qquad (11)$$

With this expression the elements of the ultimate load computation are complete. According to Eqs. 5, 7, and 8 the ultimate shear load is:

$$V_u = V_p \left[\frac{\tau_{cr}}{\tau_y} + \frac{\sqrt{3}}{2} \frac{\sigma_t}{\sigma_y} \frac{1}{\sqrt{1+\alpha^2}} \right] \qquad (12)$$

where σ_t/σ_y is given by Eq. 11.

Two simplifications which will ease the numerical computation considerably are possible. The first one is an approximation of the yield condition which, for the case of plane stress, is pictured in Fig. 1d. It can be seen that states of stress anywhere between pure shear and pure tension only lead to points on the ellipse between A and B. Hence the straight line $\sigma_1 = \sigma_y + (\sqrt{3} - 1)\sigma_2$ passing through A and B is a fair approximation of the yield condition. For the limiting case of φ equal to 45^o, σ_u and σ_v in Eqs. 9 become principal stresses, $\sigma_1 = \tau_{cr} + \sigma_t$, and $\sigma_2 = - \tau_{cr}$, respectively. If they are intorduced in the approximated form of the yield condition, the following simple relation results:

$$\frac{\sigma_t}{\sigma_y} = 1 - \frac{\tau_{cr}}{\tau_y} \qquad (13)$$

The second simplification is that the value for σ_t/σ_y be computed from this relation when φ is not equal to 45^o. This leads to a smaller tension field stress than Eq. 11 would give, and the underestimation increases the more φ

decreases from 45°, that is, as α becomes larger. But, for panels with large α ratios larger shear displacements are required in order to develop a tension field. This second approximation should not, therefore, be considered merely as a simplification made on account of the economy but also as an allowance for compatibility conditions.

In order to check the influence of these two assumptions, the ultimate shear force, Eq. 12, was first computed using σ_t from Eq. 11 and termed $V_u(11)$, then using Eq. 13 and termed $V_u(13)$. The deviation would be defined as $\delta\% = \frac{V_u(13) - V_u(11)}{V_u(11)} \times 100$; it is plotted in Fig. 5 against the variable τ_{cr}/τ_y for various values of α. Combining the effects of both simplifications yields results which differ less than 10% from the shear load computation fulfilling Mises' yield condition exactly.

Thus, using Eqs. 12 and 13 the ultimate shear force can be computed as follows:

$$\boxed{\frac{V_u}{V_p} = \frac{\tau_{cr}}{\tau_y} + \frac{\sqrt{3}}{2}\,\frac{1-\frac{\tau_{cr}}{\tau_y}}{\sqrt{1+\alpha^2}}} \qquad (14)$$

where: V_u = the ultimate shear force

V_p = the plastic shear force = $\tau_y bt$

τ_y = the yield shear stress = $\sigma_y/\sqrt{3}$

τ_{cr} = the critical shear stress = $k(\alpha)\,\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$.

When not otherwise stated, $k(\alpha)$ in this last expression is taken according to Ref. 12 as $k = 5.34 + \frac{4.00}{\alpha^2}$ for $\alpha \geq 1$ and $k = 4.00 + \frac{5.34}{\alpha^2}$ for $\alpha \leq 1$. Whenever the thus computed value τ_{cr} exceeds the proportional limit, it has been considered only as an ideal value τ_{cri} to be reduced according to Eqs. 738 and 739 of Ref. 12 in connection with Table 27 of that reference.

The reduction concept employed here does not allow a critical shear stress greater than τ_y and thus does not account for any strain-hardening effect. It will, therefore, be too conservative for low web slenderness ratios. An estimate of this effect, based on experiments, is made in Sec. 5.

Using Poisson's ratio $\nu = 0.3$, τ_{cr}/τ_y equals $1.57\ k(\alpha)/\varepsilon_y\beta^2$. Substituting this in Eq. 14 furnishes a function f on the right-hand side which depends on α, β, and ε_y only. This function $f(\alpha, \beta, \varepsilon_y)$ is the one anticipated in Eq. 1, and it is seen that the requirement concerning the parameters is fulfilled.

The ultimate shear force according to Eq. 14 is plotted against the web slenderness parameter β in Fig. 6. With the nondimensional representation chosen, this chart gives the

predicted ultimate force of a transversely stiffened thin-web plate girder as compared to the plastic shear force of the same girder. The influence of transverse stiffening is readily seen from the graph. When stiffeners are not present, the value of $\alpha = \infty$ applies. If the spacing were infinitely close, the girder could always develop its full strength, represented by the plastic shear load. The efficiency of transverse stiffeners depends very much on the slenderness ratio. The strength of a very slender web with a b/t ratio around 300 can be increased several times using stiffeners. This does not mean that a very close stiffener spacing, say $\alpha = 0.5$, should be chosen. It would be better to reduce the number of stiffeners and place additional material in the web. This would not only result in a lower slenderness ratio and hence permit a little higher allowable stress for any given value of α but, because of the added web area, the computed shear stress for a given shear force would also diminish.

For the most efficient girder design it is necessary to specify the stiffener size required to achieve the derived tension field action. This will be discussed in the next section.

3. INTERMEDIATE STIFFENERS

In contrast to loading stiffeners, intermediate stiffeners are transverse elements through which no external forces are introduced into the girder. Their function is two-fold: to preserve the shape of the girder's cross section and to insure postbuckling strength. Disregarding details at the flanges (which are thoroughly discussed in Ref. 13), the first function will require a certain minimum stiffness. The second function demands a minimum strength, that is, a certain minimum cross sectional area.

By dividing the girder's carrying capacity into two parts, simple beam action up to $\tau = \tau_{cr}$ and tension field action up to yielding in the web, it is possible to determine stiffness and strength requirements for the transverse intermediate stiffeners separately. Because a state of shear stress corresponding to simple beam action (Fig. 1a) causes no axial load in the stiffener whatsoever, the stiffener is only required to be rigid enough to force, at its location, a nodal line in the lateral deflection mode of the web. The stiffness requirements of the current design specifications are based on such considerations and ensure the required rigidity. In a tension field, however, the stiffener must take the vertical component of the diagonal stresses out of the web at one end and transfer them to its other end.

Therefore, this section will be devoted to the second requirement of an intermediate stiffener, the ability to sustain compression.

The stiffener force F_s is derived in Eq. 6. When the maximum shear force is reached, the value for σ_t can be taken from Eq. 13. The stiffener force to be expected at ultimate load is expressed by Eq. 15a below. Depending on whether α is greater or smaller than 1, and whether τ_{cr} is beyond the proportional limit or in the elastic range, different analytical expressions for τ_{cr} must be taken. An expression for the zone where $\alpha \geqslant 1$ and $\tau_{cri} = \tau_{cr}$ is given in Eq. 15b. It is seen that in nondimensional form the result is again only a function of the three independent variables α, β, and ε_y.

$$F_s = bt\sigma_y \left(1 - \frac{\tau_{cr}}{\tau_y}\right) \frac{\alpha}{2} \left(1 - \frac{\alpha}{\sqrt{1+\alpha^2}}\right) \qquad (15a)$$

$$\left.\begin{aligned} \frac{F_s}{b^2\sigma_y} &= \left[\frac{1}{2\beta} - \left(4.2 + \frac{3.1}{\alpha^2}\right)\frac{1}{\varepsilon_y\beta^3}\right] \cdot \left(\alpha - \frac{\alpha^2}{\sqrt{1+\alpha^2}}\right) \\ &\text{where } \alpha \geqslant 1 \text{ and } \varepsilon_y\beta^2 \geqslant 10.5 + \frac{7.8}{\alpha^2} \end{aligned}\right\} (15b)$$

By restricting the investigation to a particular steel, Eq. 15a can be plotted against the two coordinates α and β. This is done in Fig. 7 for $\varepsilon_y = 0.0011$, where the same k-values and reductions in the inelastic range are used as

presented under Eq. 14. There exists a maximum stiffener force which, surprisingly, is somewhere in the region where most girders are built. Its location can be found by partial differentiation of Eq. 5b with respect to the two variables α and β. This will yield $\alpha = 1.18$ and $\beta = 187$; with a maximum value of the stiffener force $F_s = 5.0 \times 10^{-4}\ \sigma_y b^2$. For unspecified ε_y, $F_{s(max)}$ occurs at $\alpha = 1.18$ and $\varepsilon_y \beta^2 = 38.6$.

The physical explanation for this maximum is as follows. Comparing all possible plate girders with 50 inches web depth, the one with $\alpha = 1.18$ and $\beta = 187$ would require a stiffener strong enough to carry a 41 kip axial load in order to develop the maximum possible tension field. For all the other girders of the same depth the stiffener force corresponding to a full tension field could only be smaller, because:

- by increasing the stiffener spacing

  the tension field action becomes less and less effective and finally diminishes completely for $\alpha = \infty$;

- by decreasing the stiffener spacing

  the stiffener density increases and therefore the share assigned to a single stiffener reduces;

- by increasing the web slenderness

  the web area decreases and with it the ultimate shear force, which is an upper bound for the stiffener force; and

- by decreasing the web slenderness

  the web thickness increases and, although the web area becomes larger, more and more of the shear force will be carried in beam action leaving a lessened capacity for tension field action.

With this finding, a simple expression for specifying the minimum required area of intermediate stiffeners can be derived as outlined next.

When the tension field has formed, part of the web is already at the stage of unrestricted yielding and no additional stresses can be assigned to it. Therefore, the stiffener force F_s can only be resisted by the actual area of the stiffeners, A_s. (A_s is the sum of the areas of both stiffeners.) In this case the required area is simply F_s/σ_u, where σ_u is the ultimate axial stress in the stiffeners. If local buckling is avoided, σ_u is equal to the primary buckling stress of this stocky post, therefore practically equal to the stiffener's yield stress. If the latter is assumed to be the same as for the web, the following result is obtained:

$$\text{Required } A_s \geqslant 0.0005\ b^2 \qquad (16)$$

In order to get a feeling for this area requirement, it is assumed that the stiffeners are built of rectangular plates with an outstanding leg width "g" which is 12 times the plate thickness "h" (Fig. 8). Equation 16 then leads

to $g \geq b/18$. That is, a 9 foot deep plate girder would require stiffeners of the size 2 Pls. - 6" x 1/2", while a girder with a web depth of only 4' - 6" would require 2 Pls. - 3" x 1/4". A stiffener pair proportioned as just derived ($g = b/18$, $h = g/12$) exhibits a moment of inertia $I_s \geq 53 \times 10^{-8} b^4$, while $I_s \geq 16 \times 10^{-8} b^4$ is the value required by paragraph 26(e) of the AISC Specifications [14]. Therefore, a stiffener size as derived here would simultaneously fulfill the minimum rigidity requirement for intermediate stiffeners.

Another stiffener arrangement consisting of a single plate with cross-sectional area of A_s' is presented in Fig. 9. It will be referred to as a one-sided stiffener. The plate is loaded along one of its edges. The state of stress in this plate, at the moment when yielding sets in, is shown in Fig. 9a. The axial load which causes this stress distribution is $F_{sy} = 0.25 \sigma_y A_s'$. If this load were increased and the stiffener plate proportioned such that unrestricted yielding is possible prior to plate buckling, the load would reach a limiting magnitude of $F_{sp} = 0.414 \sigma_y A_s'$ and cause a stress distribution as shown in Fig. 9b. Equating each of the carrying capacities with the expected maximum value of $F_s = 5.0 \times 10^{-4} \sigma_y b^2$ gives:

Yielding along the loading edge: $A_s' = 0.0020\ b^2$

Yielding all over cross section: $A_s' = 0.0012\ b^2$

This is 2.4 to 4 times more than the area which a double-sided arrangement requires. It is interesting to note that an assumed participation of the web does not affect this relation. If participation would be assumed, equal effective web areas should be given to the one-sided and the double-sided arrangement. But then, by just deducting the amount which the web carries from F_s, the remainder of the force would cause the same relation between the one and the two-sides cases. In other words, no matter what minimum required area would be specified, a one-sided solution would use at least 2.4 times the sum of the areas of stiffeners made in pairs, provided the stiffeners are made of rectangular plates.

A one-sided stiffener plate may therefore have to be about five times as heavy as either of the plates in a two-sided arrangement when tension field action is the basis of design. This is more than would be expected. The reason for this is that currently stiffener sizes are generally determined using only a stiffness criterion. Taking moments of inertia about the axis at the interface between stiffener and web, it is readily seen that, if the outstanding leg of a one-sided stiffener is only 26% greater than the width of one of a pair, it would provide as much lateral stiffness. Therefore, with respect to stiffness, the one-sided stiffener would require only 63% of the area of a two-sided arrangement, while 240% might be required to implement a tension field.

If an equal leg angle is used as a one-sided stiffener, the excentricity would be less severe and a similar reasoning as given above would lead to $A_s' = 0.0009\ b^2$.

In order to ensure an adequate shear transfer from the web plate into the stiffeners, the connectors (rivets or welds) will have to be sufficient in numbers and size. No matter whether the arrangement is one or two-sided the connectors have to build up, over half the girder depth, a force that may be as much as $F_s = 0.0005\ b^2\sigma_y$. It is not expected that in actuality the shear transfer is exactly constant per unit length, such that a linear increase of the axial force over half the girder depth occurs. But a requirement that the stiffener force F_s be built up over a third of the depth should provide enough tolerance for nonuniformity in shear flow. The connectors then need to be proportioned so as to provide, at ultimate load, a shear flow per unit length of intermediate stiffener of $3F_s/b = 0.0015\ b\sigma_y$. For A7 steel with a yield point of 33000 lbs/in^2 and factors of safety of 1.65 or 1.83 the required shear flow for which rivets or welds have to be designed at the commonly applied allowable stresses is 30 b or 27 b, respectively, where b is in inches and the shear flow is given in lbs per linear inch.

The area requirements of this section are derived under the assumption that the stiffener should not fail before the ultimate shear strength of the adjacent panels is reached.

In girder sections predominantly subjected to bending, failure due to shear cannot occur and the required stiffener area may be reduced. Assuming the shear force V to be carried in beam action until the critical shear force V_{cr} is reached, only the excess $V - V_{cr}$ carried in tension field manner causes axial forces in the transverse stiffeners. Hence the required value of stiffener area is obtained by reducing the above specified values, which are obtained under the ultimate shear force V_u, in the ratio $(V-V_{cr}) \div (V_u-V_{cr})$. A simplification to the conservative side would be a reduction ratio V/V_u. In terms of allowable stresses this would be τ/τ_w, τ being the highest shear stress that can occur under any combination of life and dead load in one of the adjacent panels, and τ_w the associated maximum permissible shear stress in that panel.

4. DETAILING OF GIRDER ENDS

For the end panel of a girder the boundary conditions are different than in an intermediate panel. When web yielding sets in, there is no neighboring plate serving as anchor for a tension stress field. Bearing stiffener at the end of a girder, together with an extending portion of the web, may offer partial restraint. Also, a more or less pronounced gusset plate action in the upper corner of the end of the web, as indicated in Fig. 14, may help to develop a partial tension action. But the degree of this contribution is uncertain. Since beam action does not depend on tension-resistant boundaries, the computed shear forces V_{cr} and V_u are the limits between which the ultimate shear strength of an end panel lies.

Premature failure of girders due to the failure of end posts was experienced and reported in Ref. 11. A picture of a failed girder end appears in Fig. 10. The web and end stiffener combination was not strong enough to resist the horizontal component of the tension field stress. An appreciation of the curvature imposed on it is obtained by noting the plumb line hanging over the girder end and the compressive yielding at the outer edge of the stiffener flange and the web toe.

In order to exclude the possibility of a premature end panel failure in an unframed girder, as for example a bridge girder supported on masonry, there are two basically different approaches possible. The simplest and generally most economical solution (one which would have to be used on framed girders) is the choice of a stiffener spacing for the end panel such that the computed average shearing stress does not greatly exceed τ_{cr}/N, where N is the factor of safety. A provision for such a limit is already established with the spacing requirement of Sec. 26(e) of the AISC Specifications[14] and Art. 1.6.80 of the AASHO Specifications[15].

To illustrate this, let it be assumed that the smaller dimension of the web plate be denoted as "a" and the larger as "b". The limiting shear stress is then

$$\tau = \frac{\tau_{cr}}{N} = k(b/a)\frac{\pi^2}{N\cdot 12(1-\nu^2)}\,E\left(\frac{t}{a}\right)^2$$

where the value k(b/a) varies only between 9.34 (a square plate) and 5.34 (an infinitely long plate)[12]. Assuming an average value of k = 7.34 and a factor of safety N = 1.65 the previously given condition becomes

$$\tau \leq 4E\left(\frac{t}{a}\right)^2 \qquad (17a)$$

If the modulus of elasticity E and the shearing stress τ are expressed in pounds per square inch and the inequality is solved for a, the well known expression of Art. 26(e), Ref. 14, is obtained

$$a > \frac{11000}{\sqrt{\tau}}t \qquad (17b)$$

While the first possibility to eliminate premature end panel failure exists in avoiding the development of a tension field in that panel, an alternate solution is to make the end post resistant to membrane tension. This could be done by bending down the top flange at the end of the girder or by welding an independent plate to the end. The required end post dimensions could be estimated as follows.

For simplification let it be assumed conservatively that the tension stress field would act under an inclination of 45^o and be uniformly distributed over the entire girder depth (Fig. 11a). Then the tension field would subject the end post to a vertical compressive force of $V_\sigma = V-V_{cr}$ (the excess not taken by beam action) and a horizontal load of the same magnitude. Thus the maximum bending moment to which the end post is subjected amounts to $V_\sigma \cdot b/8$. That is, a compressive and a tensile force of $V_\sigma \cdot b/8e$ are introduced in the end plate and bearing stiffener respectively, (Fig. 11c). The design of the bearing stiffener by the limitation of the bearing pressure is still adequate since the new force component induced is tensile and is superimposed on compression, Fig. 11b. Equating the resisting force $A_e\sigma_y$ offered by the end plate with the force component of the bending moment leads to $A_e\sigma_y = (V-V_{cr})b/8e$. In terms of allowable stress and by substituting Eq. 17a for τ_{cr}/N, the following expression for

end post proportioning is obtained:

$$A_e = A_w \frac{b}{8e\sigma_w} \left(\tau_w - 4E \frac{t^2}{a^2}\right) \qquad (17c)$$

5. THE INFLUENCE OF STRAIN HARDENING

For all compression elements made of mild steel there exists a range of low slenderness ratios within which the actual failure stress exceeds the yield stress. This is explained by the fact that yielding is confined to slip bands. The steel next to these bands is only on the verge of yielding whereas that within the bands already has strain-hardened. Thus, the member never loses all its rigidity, and it is a mistake to assume that all buckling curves must end at the yield level as the slenderness ratio approaches zero. Haaijer and Thürlimann (16) have determined the range of slenderness ratios in which compact columns and plates subjected to edge compression will strain-harden before reaching their limiting buckling stress. So far there exists no similar theoretical treatment for the case of shear, although it appears to be of significance since the webs of all rolled sections are proportioned on the assumption that the limit of their shear carrying capacity lies in the strain-hardening range.

In order to obtain an estimate of the shear strength in this low web slenderness range, recourse can be made to experimental work. Fortunately there exists a series of tests carried out by Inge Lyse and H. J. Godfrey (7) which cover the range of web depth-to-thickness ratios from 50 to 70.

The tests included welded plate girders without intermediate transverse stiffeners, whose test data are summarized in the upper left quadrant of Table 1. How far the experimentally obtained ultimate shear force, V_u^{ex}, exceeded the plastic shear force V_p is seen from Fig. 12, which uses the same coordinates introduced before with Fig. 6.

It becomes apparent in comparing these test results with Fig. 6 that the conventionally applied reduction procedure as stated in connection with Eq. 14 is too conservative. Let it be assumed that a correlation of the simple form $\tau_{cr} = C \cdot \tau_{cri}{}^{n}$ exists above the proportional limit with C and n to be determined. Since the ideal critical stress τ_{cri} and the actual critical stress τ_{cr} have to be equal at the proportional limit τ_{pr}, the value C is determined as $C = \tau_{pr}{}^{(1-n)}$. Trials to fit now the Lyse-Godfrey test data indicate that, for τ_{pr} equal to $0.8\ \tau_y$, 0.5 is the best choice for the exponent n. Thus, the reduction formula becomes

$$\tau_{cr} = \sqrt{\tau_{pr} \cdot \tau_{cri}} \qquad (18)$$

which affords a much more realistic estimate of the shear strength in the inelastic and strain-hardening range.

Since the difference in shear strength given by Figs. 6 and 12 are based upon the presence of local strain-hardening, which must be preceeded by shear yielding, the shear resisting

capacity of the web cannot be assumed as being significantly augmented by the development of a tension field when $\tau_{cr} > \tau_y$. Hence, when substituting Eq. 18 in Eq. 14, the second term in the righthand side must be omitted if $\tau_{cr} > \tau_y$.

Because Fig. 12 can accommodate but two parameters, namely the stiffener spacing α and the web slenderness ratio β, the curves can be exact for only one value of σ_y, in this case taken as 33 ksi. Such a value for σ_y is approximately correct for the beams of Group B in Table 1. For comparison, the curve for Group A beams where σ_y equals 47 ksi is shown as a dashed line. For all other test results the ratio V_u/V_p was adjusted for a normal yield stress of 33 ksi, thus showing graphically the deviation from the theoretical prediction.

Fig. 12 also gives a survey of all the predominantly shear-type tests ever carried out at Fritz Engineering Laboratory on welded plate girders. The necessary test information is summarized in the left half of Table 1; to the right of this table the theoretical predictions are presented for each individual test. The last column gives the correlation between theory and experiment.

6. ALLOWABLE SHEAR STRESSES

With Eq. 14 and the modification for strain-hardening presented in the previous section, the function $f(\alpha,\beta,\varepsilon_y)$ anticipated in Eq. 1 is established and plotted in Fig. 12 for the special case of $\varepsilon_y = 0.0011$. Using Eqs. 1 and 2, the ultimate shear force can be expressed as given in Eq. 19a. Dividing both sides by a factor of safety N_u against ultimate load, the allowable shear force, V_w, appears in Eq. 19b. With the conventional shear stress computation (shear force divided by the web area) the allowable stress is expressed in Eq. 19c.

$$V_u = \frac{\sigma_y}{\sqrt{3}}\, bt \cdot f(\alpha,\beta,\varepsilon_y) \tag{19a}$$

$$V_w = \frac{V_u}{N_u} = \frac{\sigma_y}{N_u\sqrt{3}}\, bt \cdot f(\alpha,\beta,\varepsilon_y) \tag{19b}$$

$$\tau_w = \frac{V_w}{bt} = \frac{\sigma_y}{N_u\sqrt{3}} \cdot f(\alpha,\beta,\varepsilon_y) \tag{19c}$$

In assuming a constant factor of safety $N_u = 1.65$ and a yield stress of 33 ksi, the allowable shear stresses for various values of α and β are plotted in Fig. 13. Since the shear yield stress is $\tau_y = 33/\sqrt{3} = 19$ ksi, the nominal factor of safety against yielding is $19/13 = 1.46$ in the presently used AISC Specifications. There appears no reason to change this margin which is merely a margin against deformations greater than those associated with elastic shear

strain and not against catastrophic failure. Therefore, the maximum allowable shear stresses are limited to 13 ksi. With the same reasoning the allowable shear stresses for the AASHO Specifications could be fixed. The proposed factors of safety against ultimate load and yielding are $N_u = 33/18 = 1.83$ and $N_y = 19/11 = 1.73$, respectively.

In the range of high web slenderness ratios, the stiffener spacing should not be arbitrarily large. Although the web might still be sufficient to carry the shear, the distortions could be almost beyond control in fabrication and under load. Currently, the AASHO and the AISC Specifications limit the maximum stiffener distance to 6 ft and 7 ft, respectively. With a minimum web thickness of 5/16", this distance according to the AISC Specifications could not exceed 270 times the web thickness. It is suggested that such a relative measure, rather than an absolute one, be used to specify the maximum stiffener spacing in the range of high web slenderness ratios. The rule that the shorter panel dimension does not exceed 270 times the web thickness when $\alpha < 1.0$ was used to terminate the curves of Fig. 13. The justification for such a rule can be found in the fact that the resistance of a rectangular plate to transverse loading is essentially governed by the ratio of shorter span to plate thickness.

In the medium range of web depth-to-thickness ratios, the cut-off curve is arbitrarily taken as a straight line between the points $\alpha = \infty$, $\beta = 170$ and $\alpha = 1$, $\beta = 270$. These limits may be too liberal for certain cases. The designer's judgement, however, is still needed to determine the density of transverse stiffeners when rigidity and stiffness criterions, or fabrication and erection aspects, are governing rather than mere strength considerations.

Finally, in the low range of web depth-to-thickness ratios, detail considerations generally determine the location of stiffeners. A direct application of load onto the web can be made for webs which are proportioned to allow a certain degree of strain-hardening(that is, for webs whose allowable shear stress is 13 ksi), provided the compressive stress at the web toe of the fillets are suitably limited in order to avoid web crippling. For all other cases the load application must be made by means of transverse stiffeners, unless it can be shown that the transverse pressure is small enough not to cause vertical buckling of the flange into the web, as discussed in Sec. 2.1 of the previous paper on bending strength of plate girders [18].

7. APPENDIX: REVIEW OF THE ULTIMATE SHEAR STRENGTH THEORY

In order to determine the shear strength of a girder it was assumed that the girder acts according to beam theory up to the critical load and thereafter in a tension field manner up to the point of web yielding. Considering initial web distortions and residual stresses due to welding, the probability of a clearly defined boundary between these two types of behavior appears questionable. The assumption should, therefore, be taken as an estimate for the amount of shear to be carried in compression rather than a phenomenon that can be actually observed.

If a plate subjected to shearing stresses is assumed to be built of two sets of strips orthogonal to each other and acting in compression and tension respectively, the compression strips are then elastically supported by the tension strips. This is the reason why the critical stress of a shear panel is much higher than that of an equivalent isolated compression strip extending from panel border to panel border. If more tension is superimposed on the tension strips, it is obvious that, until yielding occurs, the conditions for stresses in the compression strips are improved since the "spring constants" of the elastic supports increase. But when yielding sets in, the ratio of shear carried in beam and in tension field action changes again. From the recorded

load deflection curves in Ref. 11, which generally exhibit a pronounced yield plateau, it is seen that the reduction in beam action is about compensated by the gain in tension field action due to the increase in field wdith. After all, the derived ultimate shear force expression is not very sensitive to a slight error in this assumption. If the contribution V_τ in Eq. 8 would be overestimated, the yield condition would allow less tension field action and vice versa.

Some remarks as to the geometry of the tension field are in order. The inclination of the tension field was obtained from a maximum value condition as they were assumed to exist on a model girder. The assumption was made that the edges of the effective field would run through the panel corners (Fig. 4), and the two web traiangles not included in the tension strip were not thought to be idle. They were supposed to digest the membrane stresses caused by the neighboring panels' tension fields and ultimately transfer them to the flanges. The shear force thus created is given by Eq. 5 and the resulting stiffener force by Eq. 6. Let this derivation be called Approach A.

It can be shown that the established expression for V_σ is not very strongly dependent upon the above derived tension field geometry. Another approach, Approach B, would lead to the same result. Let it be assumed that the girder carries the load in a truss-type manner, that is, that a tension

diagonal forms whose centerline coincides with the panel diagonal and whose effective width, so far unknown, is a certain fraction of the girder depth, say $\varkappa b$. This would lead to a shear contribution $V_\sigma = \varkappa bt\sigma_t \cdot \sin\varphi$, where $\varphi = \cot^{-1}\alpha$. Hence,

$$V_\sigma = \varkappa bt\sigma_t \frac{1}{\sqrt{1+\alpha^2}}$$

When α approaches ∞, V_σ diminishes to zero. At the other limit, $\alpha = 0$, the highest possible tension field contribution as derived in Eq. 3 should result. This fixes the constant $\varkappa$ as equal to 1/2, and it is observed that V_σ, derived this way, is exactly the same as in Eq. 5 of Approach A.

Using this second approach, however, the stiffener force F_s would equal the shear force V_σ. This would mean that even for very close stiffener spacing each stiffener would have to carry the full amount of V_σ, which is obviously incorrect. The reason for this deficiency is the fact that for small values of α, this truss assumption violates the yield condition because the strips of neighboring panels overlap and use portions of the web twice. But the longer the panel, the closer is the agreement between the stiffener forces of the two approaches. For $\alpha \gg 1$, Eqs. 5 and 6 yield the same result. Indeed, it will be indicated next that for larger values of α and lower slenderness ratios the tension diagonal of Approach B is not an impossible one, although its centerline, rather than its effective edge, is assumed to run through the panel corner.

For simplification, the plate corner may be considered to be built of two sets of orthogonal strips, one set under tension stresses σ_t, the other under compression stresses σ_c, as illustrated in Fig. 14. The postulate that no membrane stresses orthogonal to the plane of the top flange can occur leads to the relation $\sigma_c = \sigma_t \cdot \tan^2\varphi$. In approaching the corner the compression strips can pick up stress because the strip length shortens. According to the above given relation this will be accompanies by a gradual increase of tension stresses as sketched in the figure. If this gradual increase of tensile stresses is idealized, the tension field may be thought to have an effective width intersecting the top flange at point F, some distance away from the corner A of the panel. It is seen that this distance increases with the web thickness and decreases to zero for an extremely thin web. Figure 15 is a photograph of a panel with an aspect ratio $\alpha = 1.5$. It is taken from a girder with a sturdy web ($\beta = 133$) and shows that the "edge" of the tension field is shifted away from the panel corners. Both the tension field (Approach A) and the tension diagonal (Approach B) are shown in outline. The tension field actually developed can, in this case, be considered to be between the predictions of the two approaches.

Finally, an Approach C might eliminate completely the idea of tension field inclination and effective width. This approach seeks merely for the simplest set of coordinate

functions which yield a result that might commonly be expected from a tension field type action. In order to free the functions from dimensions, V_σ is to be expressed as $V_{max}f(\alpha)$, where V_{max} is the highest possible shear force to be carried by tension stresses alone(Eq. 3) and is here considered to be the "amplitude". The pure coordinate function $f(\alpha)$ must be normalized such that $f(\alpha=0) = 1$. For large values of α, it should approach zero with the power of $1/\alpha$. The most obvious choice for a simple function fulfilling these two limits is

$$f(\alpha) = (1+\alpha^n)^{-\frac{1}{n}} \qquad n > 1 \qquad (20)$$

The result previously derived in Approaches A and B would be obtained for $n = 2$. Since no physical significance was noted in this approach, it does not give any detail information at all. For instance, since no stress flow is pictured, an equilibrium condition cannot be applied to get a stiffener force.

The outcome of this review can be summarized as follows. By extracting the proper parameters and considering the limiting cases, a close guess as to the nature of the expression for the ultimate shear force is possible. As soon as a more specific problem is considered, such as the stiffener force, a higher rung in the ladder of conditions(proper parameter, equilibrium condition, yield condition, compatibility

condition) must be used. Approach C uses only the correct parameters, Approach B also fulfills the equilibrium condition but violates the yield condition. Approach A fulfills all three but is still short of the last condition. Therefore, when a detail problem is encountered, such as the determination of the weld size between stiffener and web, not even the refined model used in Approach A can give more than an average value. The stress flow would have to be pictured more accurately, but this could no longer be done with tension strips under constant inclination and constant effective width. It should be kept in mind that this idealized model is an engineering tool which provides the means for a good judgement of a girder's carrying capacity. Its ability to account for an overall carrying capacity is very good but at local points the actual state of stress could be quite different from the assumed one. However, the situation is not unique to the problem of welded plate girders. In practice, welds and details are proportioned on the basis of idealized conditions.

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NOMENCLATURE

a : Distance of transverse stiffeners

b : Depth of girder web

e : Distance, defined in Fig. 11

k : Buckling coefficient

s : Tension field width

t : Thickness of web

A : Area of cross section

A_s : Cross sectional area of a stiffener pair

A'_s : Cross sectional area of a one-sided stiffener

E : Modulus of elasticity (30,000 ksi)

F : Force

V : Shear force

$\alpha = a/b$: Aspect ratio, panel length to panel depth

$\beta = b/t$: Slenderness ratio, web depth to web thickness

ε : Strain

ν : Poisson's ratio (0.3)

σ : Normal stress

τ : Shear stress

φ : Inclination of tension field

Subscripts (or Superscripts):

c : Compression

cr : Critical

ex : Experimental

f	:	Flange
p	:	Plastic
pr	:	Proportional
s	:	Stiffener
t	:	Tension
th	:	Theoretical
u	:	Ultimate
w	:	Web, working
y	:	Yielding
σ	:	As carried in tension
τ	:	As carried in shear

Table 1 : Summary of Shear Tests on Welded Plate Girders

Source	Girder No.	$\alpha=\frac{a}{b}$	$\beta=\frac{b}{t}$	A_w in^2	σ_y ksi	V_u^{ex} k	$\tau_y=\frac{\sigma_y}{\sqrt{3}}$ ksi	τ_{cri} [1] ksi	$\frac{\tau_{cr}}{\tau_y}$ [2]	$\frac{V_u}{V_p}$ [3]	V_u^{th} [4] k	$\frac{V_u^{ex}}{V_u^{th}}$
		Experimental Values					Theoretical Values					
Group A Ref. 7	WB-1	3	56.5	3.47	43.3	109	25.0	48.6	1.25	1.25	108	1.01
	WB-2	3	54.9	3.57	47.8	128	27.6	51.3	1.22	1.22	120	1.07
	WB-3	3	58.9	4.36	49.6	139	28.6	44.6	1.12	1.12	139	1.00
Group B Ref. 7	WB-6	3	70.0	4.40	33.1	96	19.1	31.6	1.15	1.15	97	0.99
	WB-7	3	60.6	3.88	33.7	95	19.5	42.3	1.32	1.32	100	0.95
	WB-8	3	59.7	4.10	29.7	100	17.2	43.5	1.43	1.43	101	0.99
	WB-9	3	50.0	3.12	30.3	92	17.5	62.0	1.69	1.69	92	1.00
	WB-10	3	49.4	3.14	30.3	94	17.5	63.6	1.71	1.71	94	1.00
Part 3 Ref. 11	G6-T1	1.5	259	9.65	36.7	116	21.2	2.85	0.134	0.550	112	1.04
	G6-T2	0.75	259	9.65	36.7	150	21.2	5.40	.254	.771	157	0.95
	G6-T3	0.5	259	9.65	36.7	177	21.2	10.1	.477	.882	180	0.98
	G7-T1	1.0	255	9.80	36.7	140	21.2	3.85	.182	.682	142	0.98
	G7-T2	1.0	255	9.80	36.7	145	21.2	3.85	.182	.682	142	1.02
Part 4 Ref. 11	E1-T1	3.0	131	19.1	41.7	278	24.1	9.00	.374	.545	250	1.11
	E1-T2	1.5	131	19.1	41.7	290	24.1	11.1	.460	.719	330	0.88
	G8-T1	3.0	254	9.85	38.2	85	22.0	2.30	.105	.350	76	1.12
	G8-T3	1.5	254	9.85	38.2	117	22.0	2.95	.134	.550	119	0.98
	G9-T3	1.5	382	6.55	44.5	79	25.7	1.31	.050	.506	85	0.93
										Standard-deviation		0.60

1) $\tau_{cri}=k\,\frac{\pi^2E}{12(1-\upsilon^2)}(\frac{t}{b})^2=k\,\frac{26750}{\beta^2}\,[ksi]$, with $k=5.34+\frac{4.00}{\alpha^2}$ for $\alpha>1$, and $k=4.00+\frac{5.34}{\alpha^2}$ for $\alpha<1$

2) $\tau_{cr}=\tau_{cri}$ when $\tau_{cri}\leq 0.8\tau_y$, and $\tau_{cr}=\sqrt{0.8\tau_y\tau_{cri}}$ for $\tau_{cri}>0.8\tau_y$

3) Computed according to Eq. 14 when $\tau_{cr}\leq\tau_y$; for $\tau_{cr}>\tau_y$, simply $V_u/V_p=\tau_{cr}/\tau_y$

4) Values under 3) multiplied with τ_yA_w

List of Figures

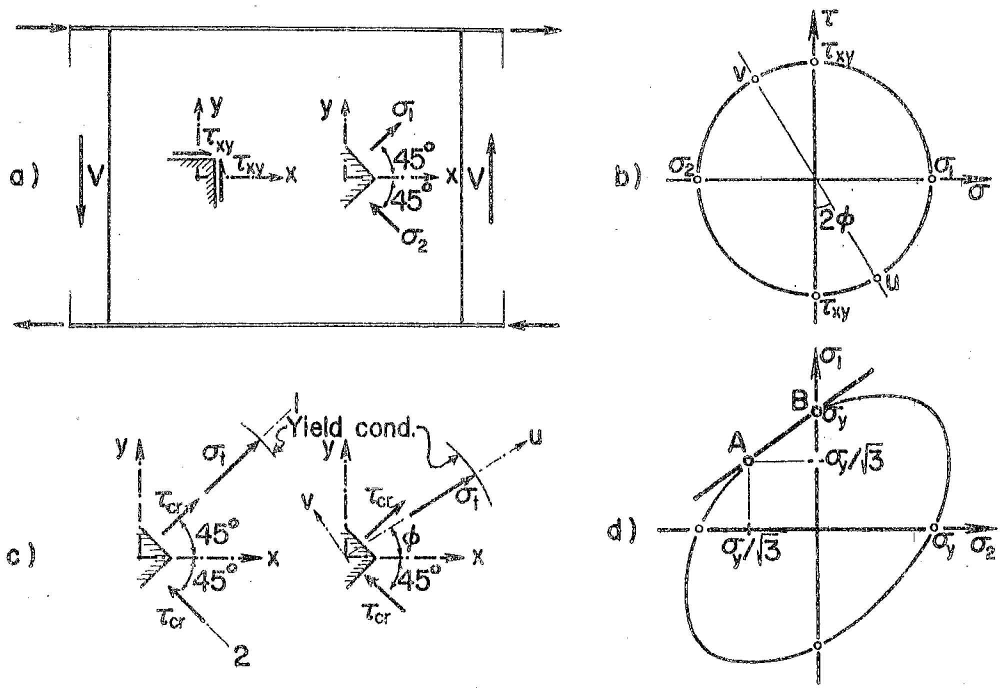
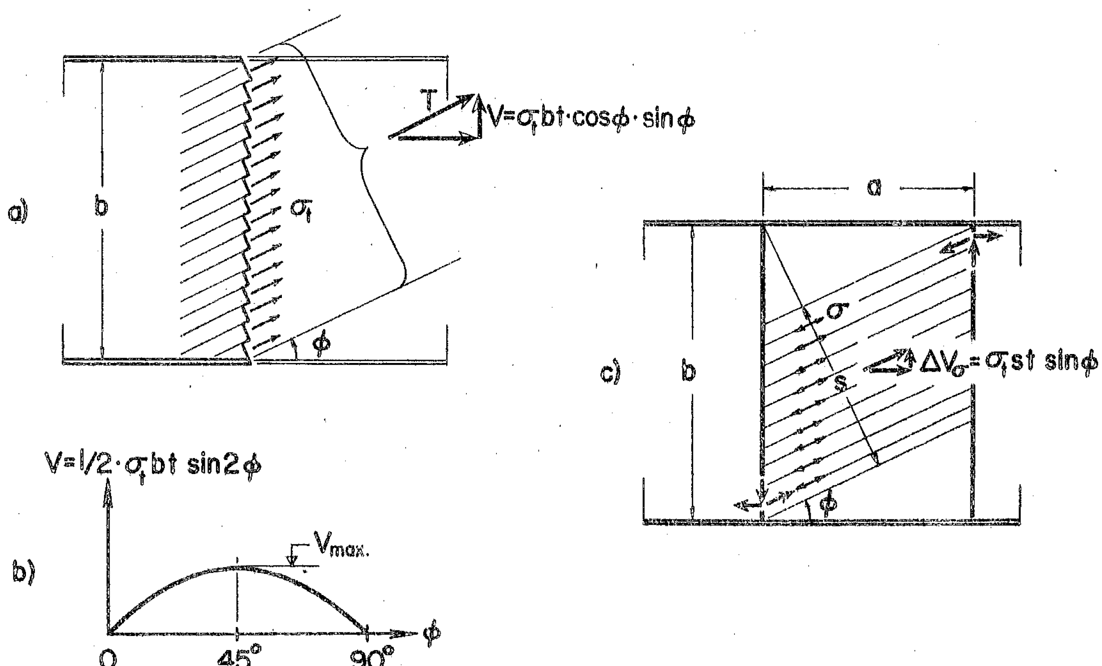

a)
V
y
τ_{xy}
τ_{xy}
x
y
σ_1
45°
45°
x
V
σ_2
b)
τ
τ_{xy}
v
σ_2
σ_1
σ
2ϕ
u
τ_{xy}
c)
y
σ_1
1
Yield cond.
y
u
τ_{cr}
45°
x
45°
τ_{cr}
2
v
τ_{cr}
ϕ
σ_1
x
45°
τ_{cr}
d)
σ_1
B
σ_y
A
$\sigma_y/\sqrt{3}$
$\sigma_y/\sqrt{3}$
σ_y
σ_2

Fig. 1


a)
b
σ_1
T
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ϕ
b)
$V=1/2\cdot\sigma_1 bt\ \sin 2\phi$
$V_{max.}$
0
45°
90°
ϕ
c)
a
b
σ
s
$\Delta V_\sigma=\sigma_1 st\ \sin\phi$
ϕ

Fig. 2

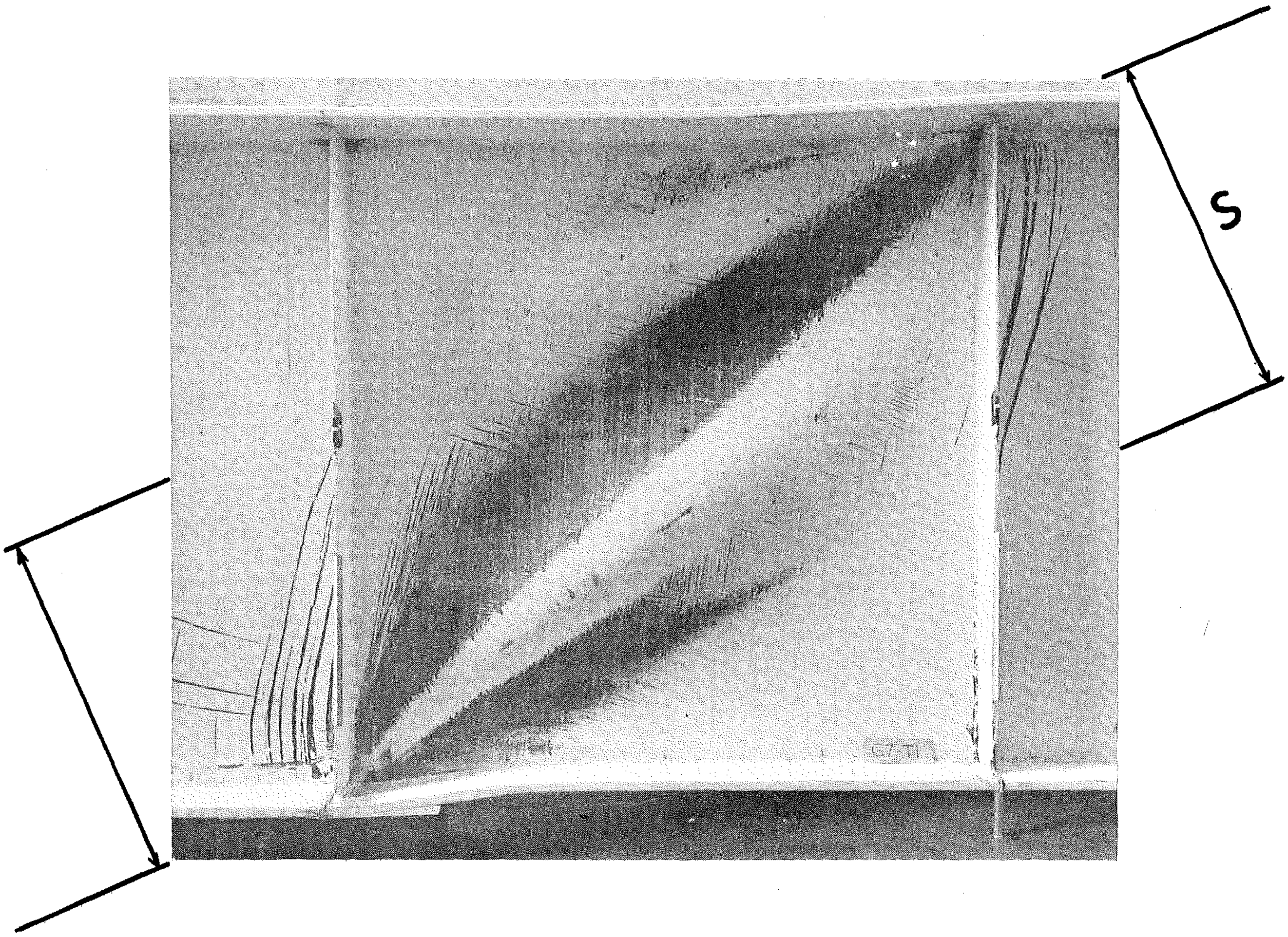


Fig. 3 Yielded Shear Panel, 50" x 50", 1/4" Web

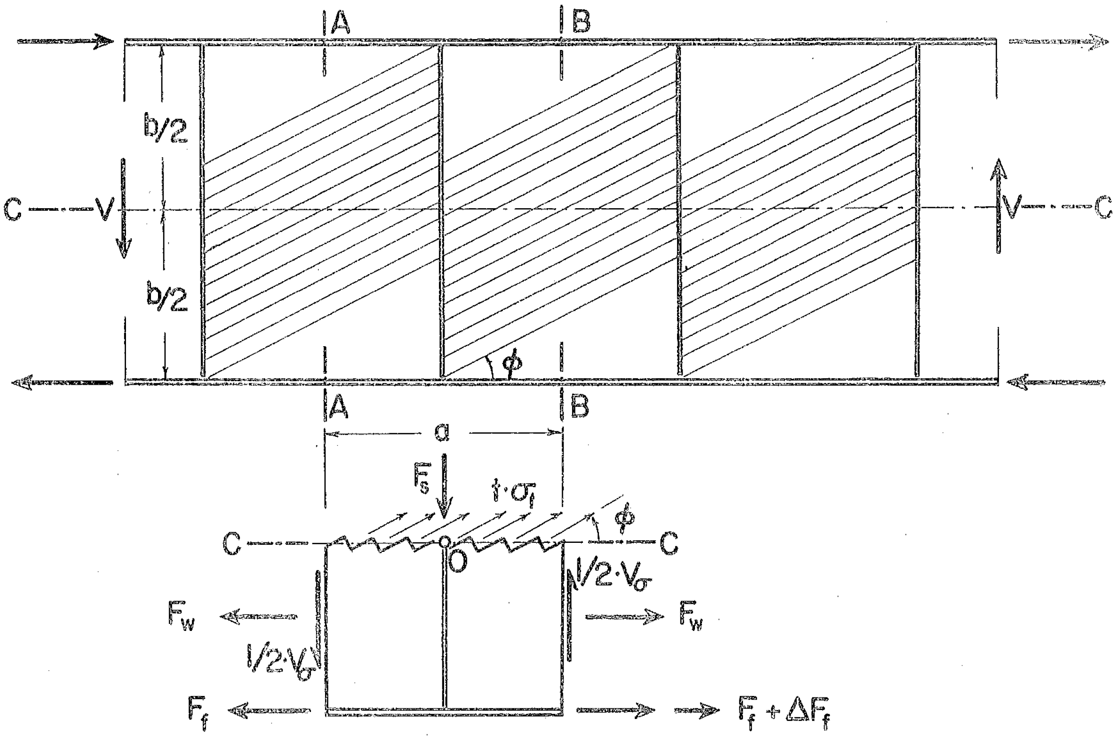

A
B
b/2
C
V
b/2
V
C
φ
A
B
a
F_s
$t \cdot \sigma_t$
φ
C
O
C
$1/2 \cdot V_\sigma$
F_w
F_w
$1/2 \cdot V_\sigma$
F_f
$F_f + \Delta F_f$

Fig. 4

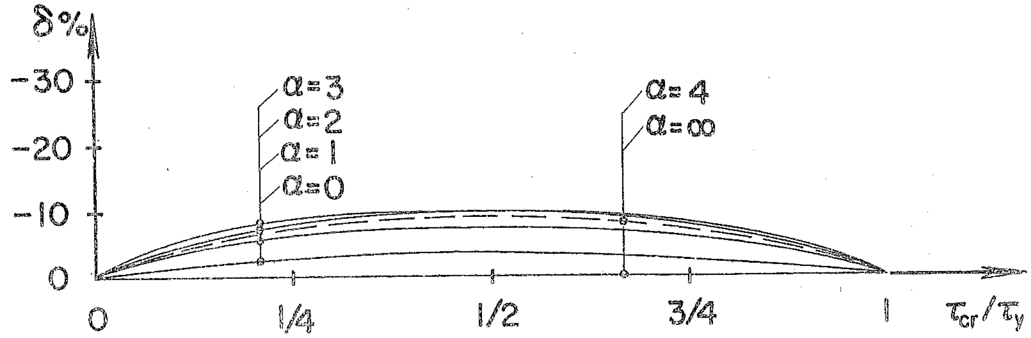

δ%
-30
-20
-10
0
α=3
α=2
α=1
α=0
α=4
α=∞
0
1/4
1/2
3/4
1
τ_{cr}/τ_y

Fig. 5

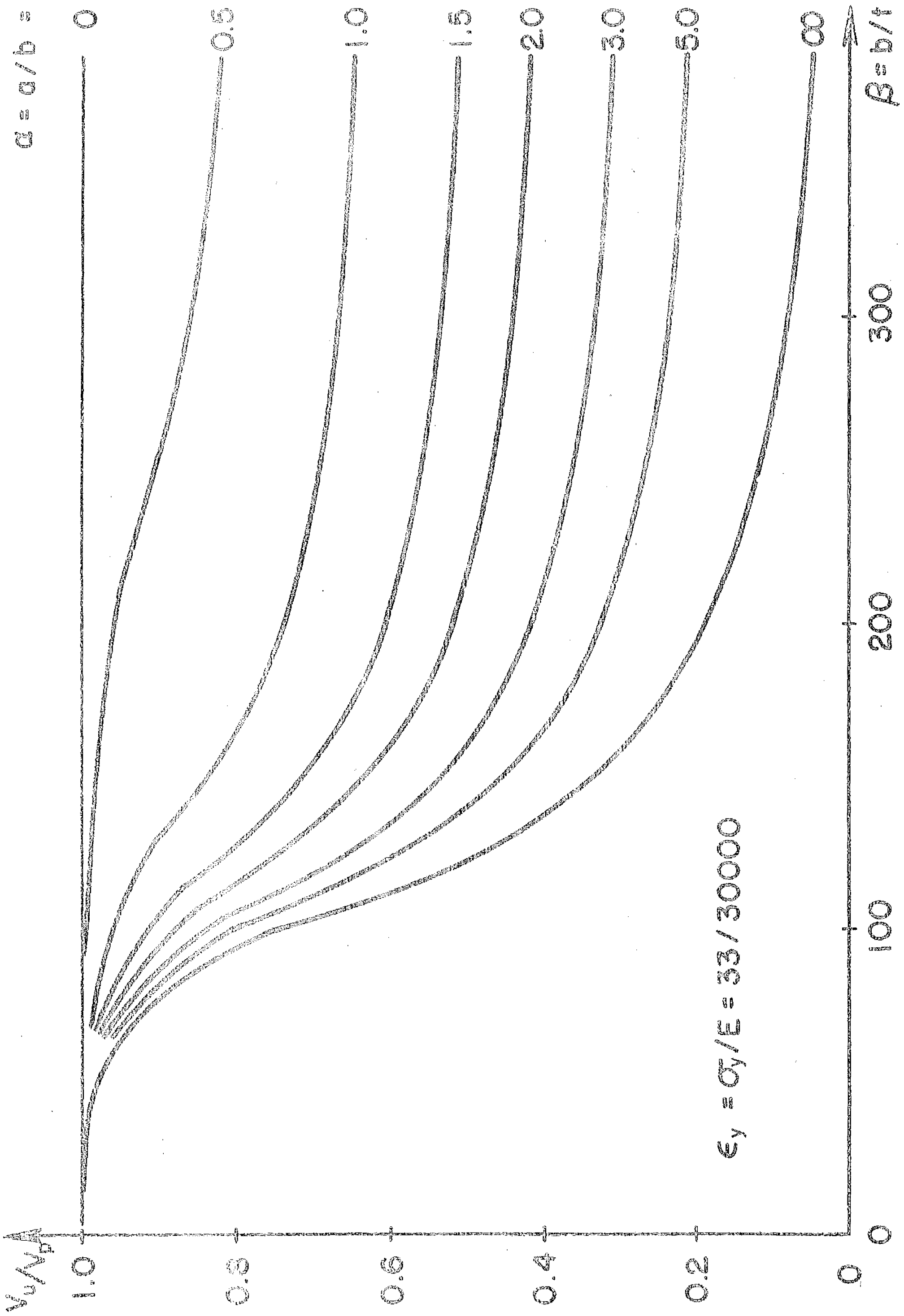

α = a/b =
0
0.5
1.0
1.5
2.0
3.0
5.0
∞
β = b/t
0
100
200
300
V_u/V_p
1.0
0.8
0.6
0.4
0.2
0
$\epsilon_y = \sigma_y/E = 33/30000$

Fig. 6

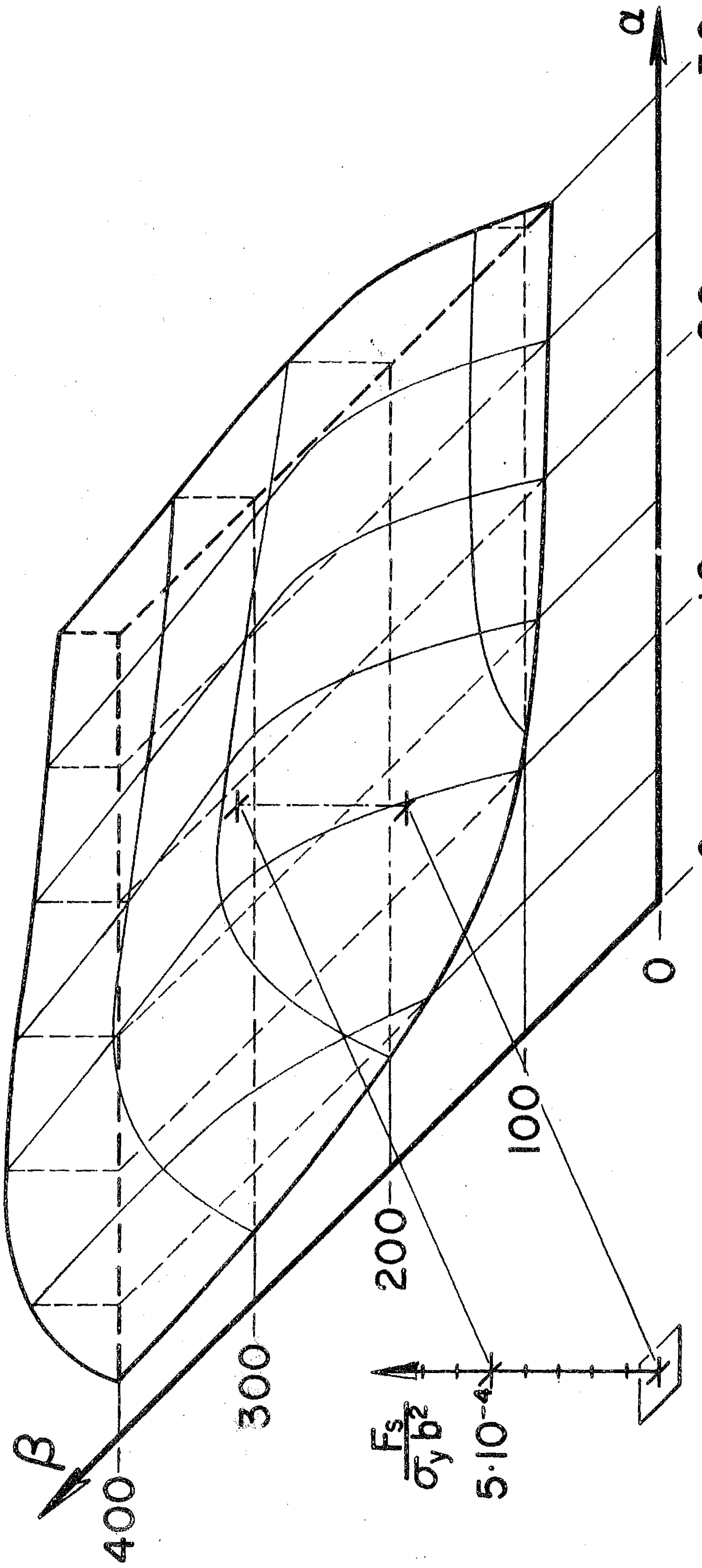


Fig. 7

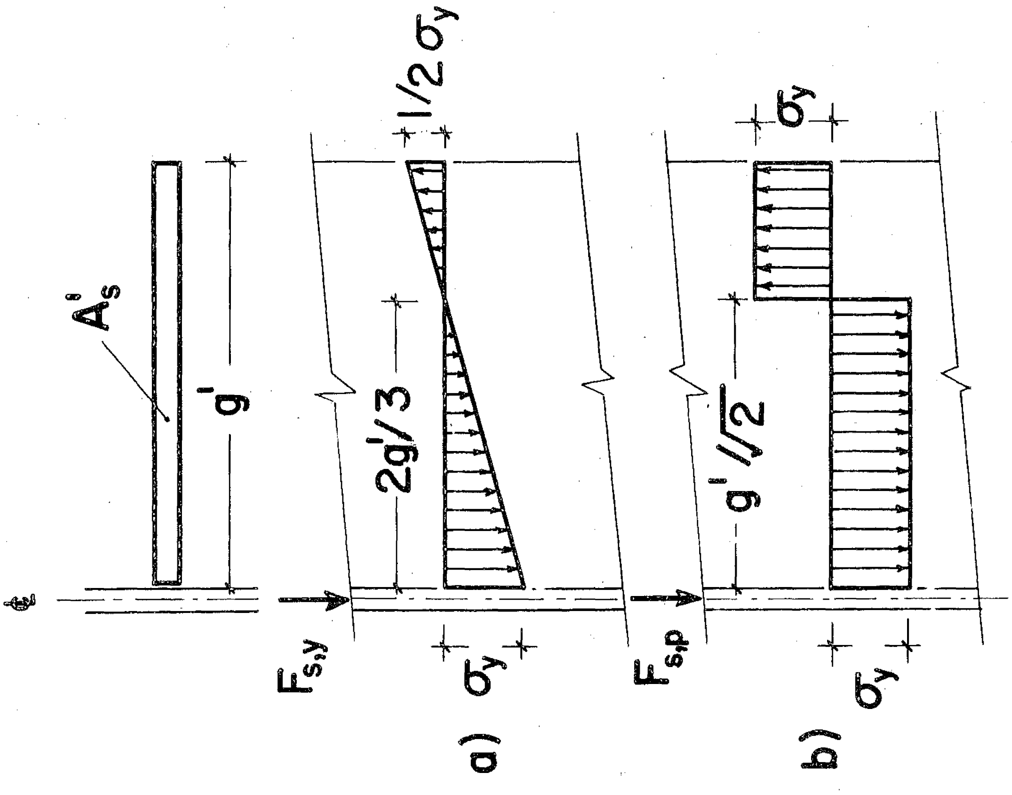


Fig. 9

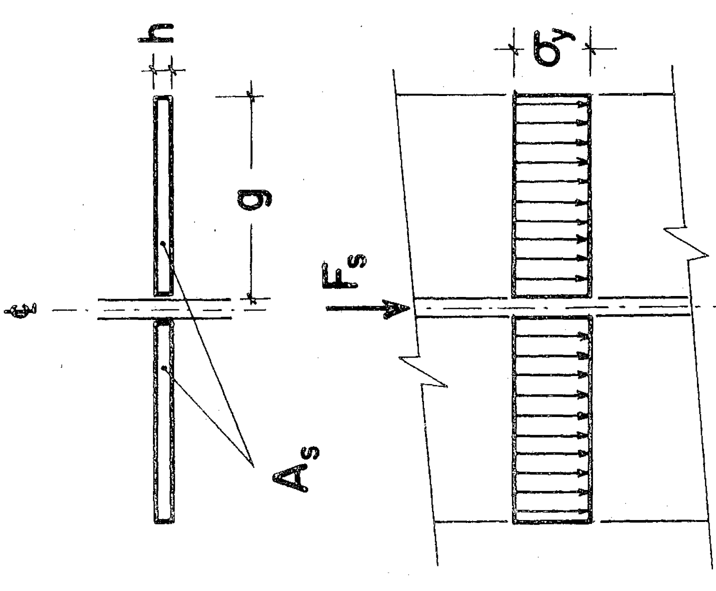


Fig. 8

Fig. 10 End Post Failure

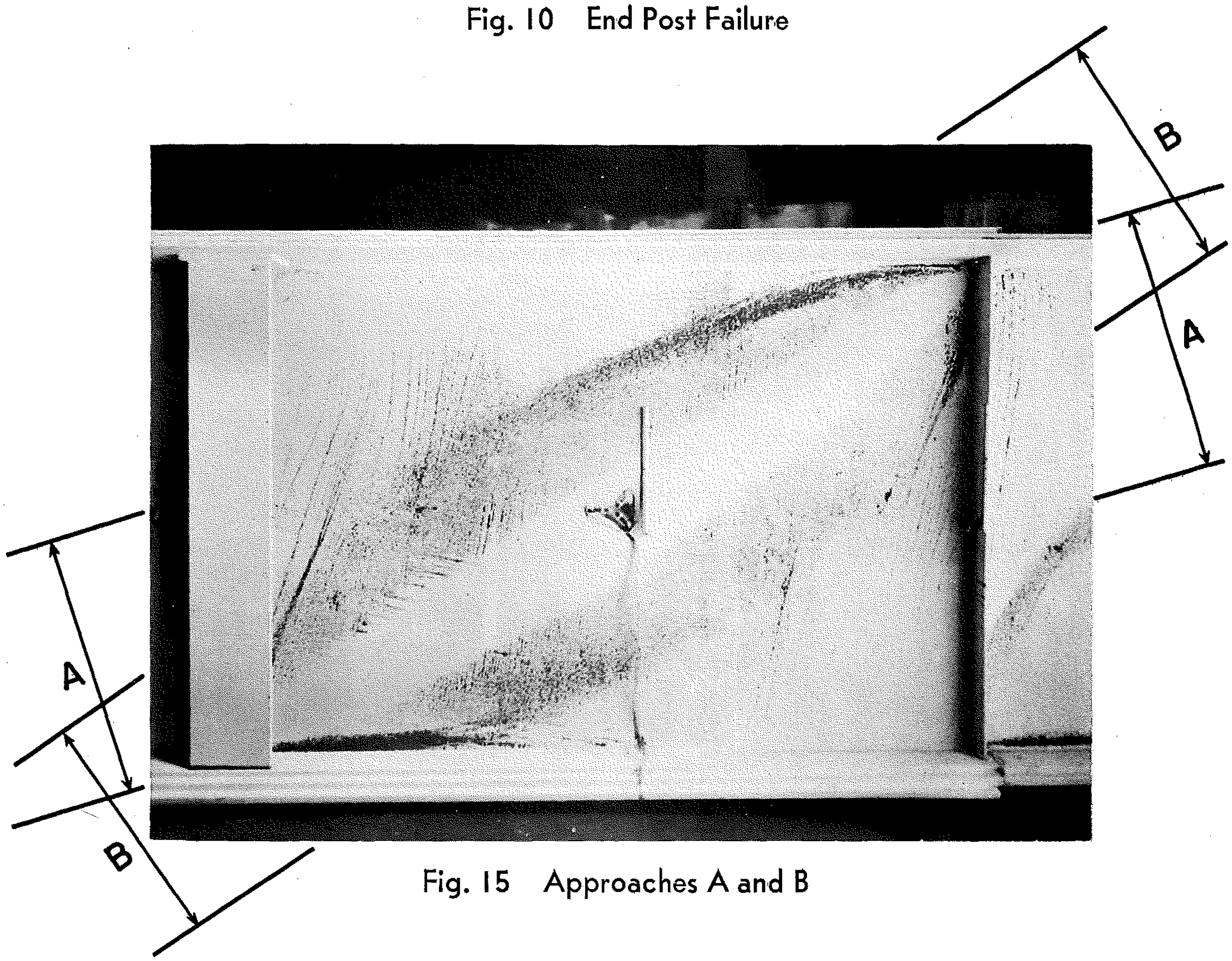


Fig. 15 Approaches A and B

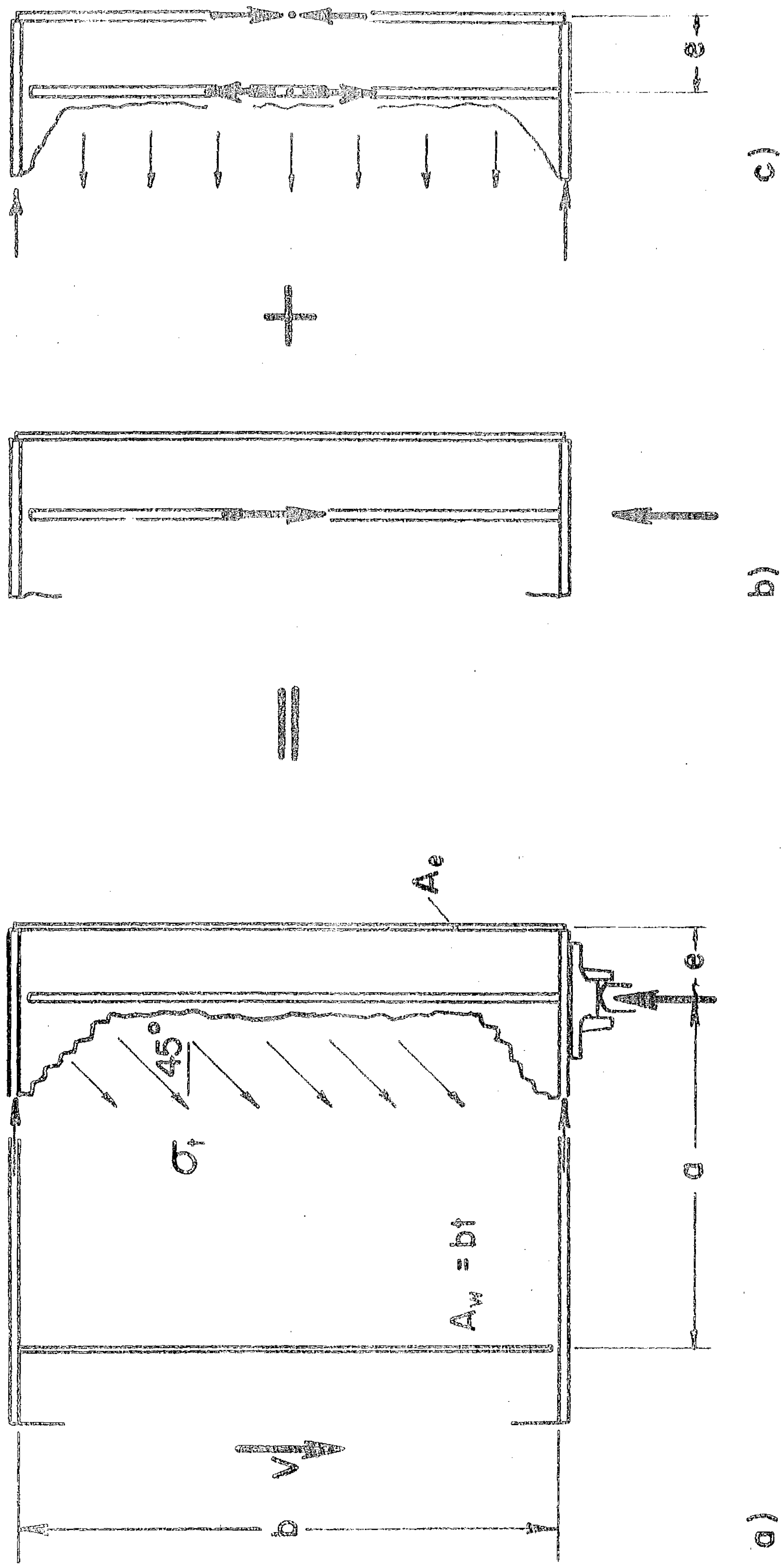


Fig. 11

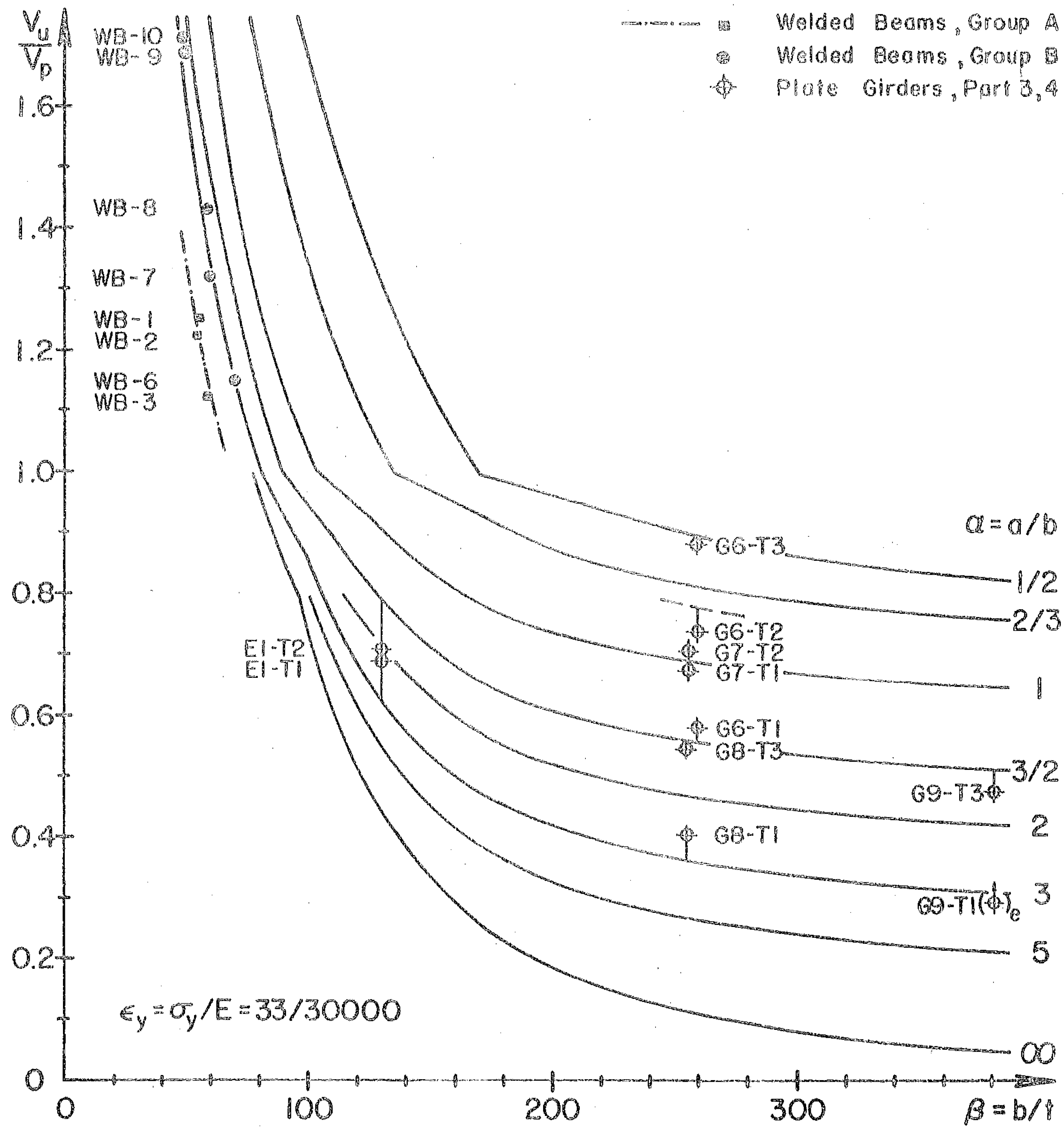
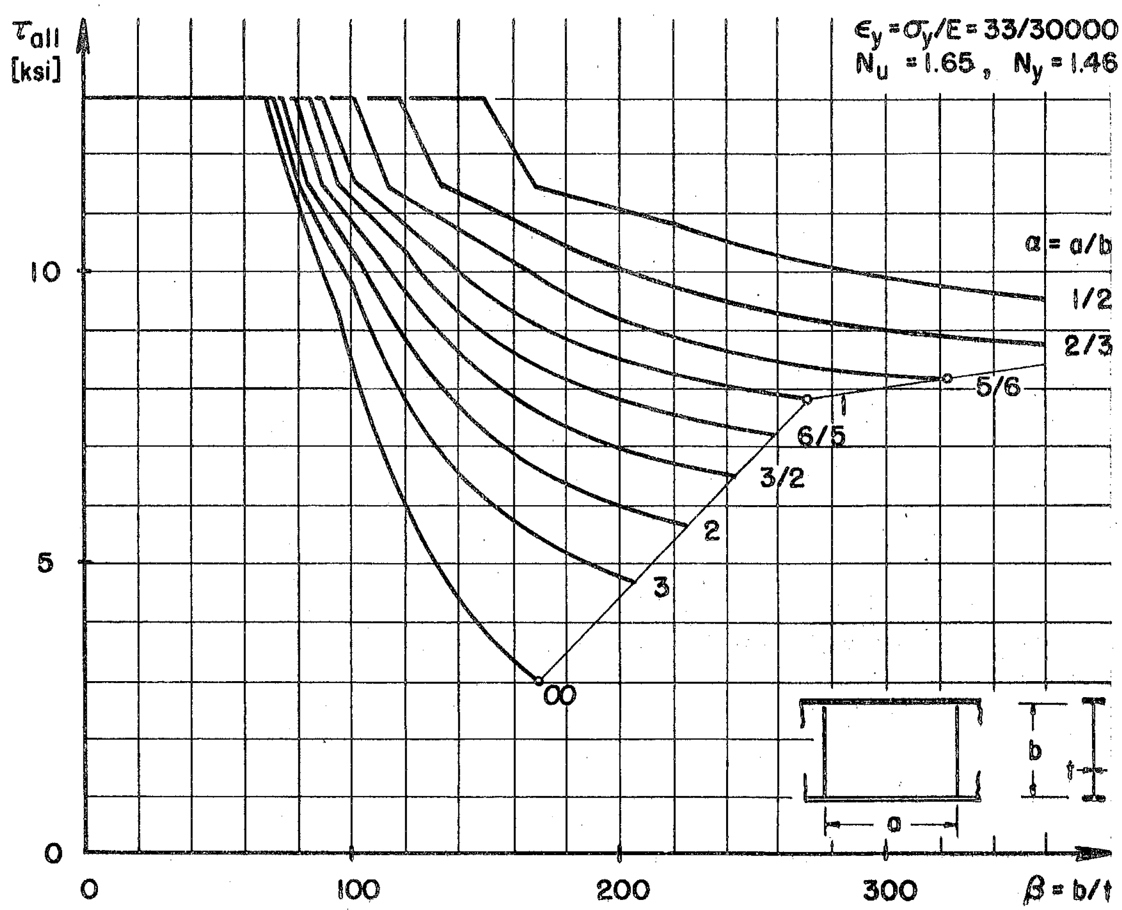

Welded Beams, Group A
Welded Beams, Group B
Plate Girders, Part 3,4
$\frac{V_u}{V_p}$
WB-10
WB-9
WB-8
WB-7
WB-1
WB-2
WB-6
WB-3
E1-T2
E1-T1
G6-T3
G6-T2
G7-T2
G7-T1
G6-T1
G8-T3
G9-T3
G8-T1
G9-T1$(\Phi)_e$
$\alpha = a/b$
1/2
2/3
1
3/2
2
3
5
∞
$\epsilon_y = \sigma_y/E = 33/30000$
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
100
200
300
$\beta = b/t$

Fig. 12



τ_{all} [ksi]
$\epsilon_y = \sigma_y/E = 33/30000$
$N_u = 1.65$, $N_y = 1.46$
$\alpha = a/b$
1/2
2/3
5/6
1
6/5
3/2
2
3
∞
10
5
0
0
100
200
300
$\beta = b/t$
b
t
a

Fig. 13

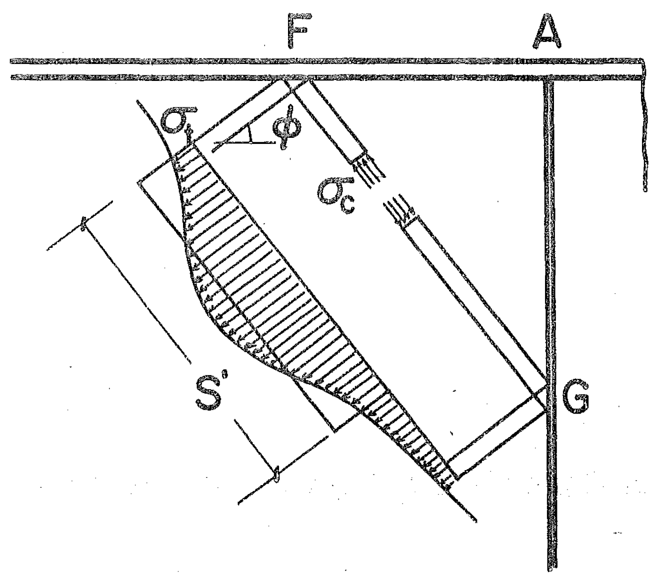


Fig. 14

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