Omni-directional ultrasonic transducer for robotic navigation

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TITLE: Omni-Directional Ultrasonic Transducer for Robotic Navigation

DATE: May 31, 1992
This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering.

April 13, 1992
Date

Advisor in Charge

CSEE Department/Chairperson
Acknowledgements

I would first and foremost like to thank Professor Nicolai Eberhardt for his patience in teaching me a vast amount of information in the field of Electrical Engineering and for advising me through my stay at Lehigh University. I would also like to thank Ed and Bill for their help through all of my laboratory work. To my mother-in-law and father-in-law, thank you for granting me the financial support necessary to pursue my Master's degree. I would very much like to thank my family for their financial support and also for believing in me all these years and sticking behind me through whatever decisions I decided to make. Without your support, I would never be where I am today. An extremely big thanks goes to my wife, Heather, who stood by me through all the difficult times. Thank you for being patient during the hours I've put into my thesis. Finally, and most importantly, I would like to thank God for giving me the strength, courage, perseverance, and wisdom throughout my whole life.
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1. Abstract

A horizontally omni-directional ultrasonic transducer is developed using the basic electrostatic element manufactured by Polaroid. The desired characteristic is obtained by placing a solid reflecting cone in front of the element. An approach to the diffraction theory of this arrangement is presented. It is based on a one-dimensional approximation to the two-dimensional cylindrical problem. Scattering from the tip region of the cone is neglected.

The far-field radiation pattern is measured. It is found to agree reasonably well with the theoretical prediction.

Furthermore, the maximum obtainable range of transmission between two such transducers is evaluated experimentally and theoretically. It is found that 15 meters is the maximum range for a 10 dB signal to noise ratio. This is sufficient for many applications in robot location systems.
2. Introduction

In the early 1980's, Polaroid developed an electrostatic, ultrasonic transducer for use in their auto-focusing cameras. This transducer consists of a grooved, metallic backplate, gold plated plastic foil diaphragm, and a housing. The transducer with housing is 3.8 cm in diameter. The grooved backplate was an elegant means of designing a good electrostatic transducer. According to the electrostatic coupling equations, two conditions need to be met to maximize the electrostatic coupling. The electrostatic coupling equation for the time-varying voltage is:

\[
\tilde{V} = \frac{1}{j \cdot \omega \cdot c_0} \cdot \tilde{I} - \frac{V_0}{j \cdot \omega \cdot d_0} \cdot \tilde{v}
\]

where \( \tilde{V} \) is the time-varying voltage, \( \tilde{I} \) is the time-varying current, \( \tilde{v} \) is the time-varying particle velocity, \( \omega \) is the frequency, \( d_0 \) is the backplate-diaphragm separation distance, \( c_0 \) is the backplate-diaphragm capacitance, and \( V_0 \) is the bias voltage. For a finite load impedance, \( Z_L \), we can write:

\[
Z_L = \frac{\tilde{V}}{\tilde{I}}
\]

Solving Equation 2.2 for \( \tilde{I} \) and substituting into Equation 2.1 yields:

\[
\tilde{V} = - \frac{V_0}{j \cdot \omega \cdot d_0 \left( 1 - \frac{1}{j \cdot \omega \cdot c_0 \cdot Z_L} \right)} \cdot \tilde{v}
\]

Thus, to maximize \( \tilde{V} \) for a given frequency and load impedance, \( d_0 \) and \( c_0 \) must be minimized. Since the capacitance is inversely proportional to the separation distance, there is a contradiction. Polaroid used the grooved backplate to resolve the problem. (Figure 1)

![Diaphragm/backplate schematic.](image)

Figure 1: Diaphragm/backplate schematic.
First of all, the diaphragm is a plastic film with vacuum-deposited gold on the side away from the backplate. The plastic part of the diaphragm provides a parasitic shunt capacitance via its dielectric constant. The raised portion of the backplate provides physical support for the diaphragm. This is not avoidable, because without support, the diaphragm would bend under the electrostatic biasing field. It would either touch the backplate in the middle, or the air column resonant frequency would vary locally. The grooves provide acoustical resonance cavities. These are needed to increase the local particle velocity, $\gamma$. Without resonance, most of the acoustic power would be reflected from an acoustic impedance mismatch. Note that it is in the region of the grooves that the displacement to produce the ultrasonic chirp takes place.

The diaphragm has a constant DC bias level of 150 - 200 V. A 300 V peak-to-peak, 50 kHz sine wave burst applied across the diaphragm-backplate capacitor produces an ultrasonic chirp. This chirp is then projected until it dissipates or is reflected by an object in its path. If the latter occurs, the reflected wave is then received by the transducer. A pulse-echo time calculation is then performed to determine the distance from the object to the transducer. Due to limited power of transmission, atmospheric attenuation of the sound wave, and wide-angled backscattering, this kind of distance measurement is limited to approximately 35 feet.

We proposed that an ultrasonic system could be designed for robotic navigation, using the Polaroid transducer. Polaroid's transducer is highly directional. If the far-field pattern of the transducer could be made omnidirectional in the horizontal plane, then, by placing two transducers at stationary positions and one on a robot, a triangulation calculation could be performed to determine the precise coordinates of the robot.

The problem was to make this transducer omni-directional in the horizontal plane. To accomplish this, it was proposed that a reflecting cone be placed directly anterior to the transducer in order to reflect the ultrasonic chirp in a radial direction. The far field of the apparatus would then resemble a "donut-shaped" pattern whose center axis was coincident with the axis perpendicular to the transducer's surface. (Refer to Figure 2)

The project's ultimate objective was to determine the maximum range such a system could obtain before the signal became hidden in the noise level. The
determination of this range involves various experiments and calculations, the 
results of each combining to give the final result. These include the determination 
of:

1) The far-field radiation pattern for the transducer with an anterior 
cone.
2) The transmission loss for a two transducer system and a system 
containing one transducer alone and one transducer with an anterior 
cone.
3) The reduction in total received power when a cone is added.
4) The diaphragm displacement as a function of AC driving voltage.
5) A low noise amplifier/filter circuit.
6) The sensitivity of the system.

All of these results together will then determine the maximum range of such a 
system, and ultimately, if such a navigation system is feasible.

Figure 2: Photograph of transducer with anterior cone.
3. Radiation Field Pattern with Cone

3.1. Theoretical

The first step is determining the far field radiation pattern for the transducer with an anterior cone. The primary thought was to find a rigorous solution for reflections of an acoustical wave from a conical surface. After studying this theory and finding that the problem had been solved for generalized cases of this type (Bowman, J.J), we concluded that the mathematics involved in assimilating the theory to our conditions was beyond the scope of this project.

Therefore, some reasonable approximations to the problem had to be conceived. These approximations are expanded in the following paragraphs.

1) Since the spacing between the grooves in the backplate is small compared to the wavelength, it is assumed that the wave from the transducer develops as a uniform plane wave. Later, it was found (See Section 5.1) that the radial distribution is actually constant.

2) It is assumed that after reflection from the cone's surface, the wave becomes an outward traveling wave that follows cylindrical wavefunctions, that is, Hankel Functions.

3) The far field pattern can be directly obtained by solving the Fraunhofer diffraction problem. The pupil is chosen to be a cylindrical surface at \( r = R \). (See Figure 2)

4) Beyond this radius, the field is already several wavelengths away from the origin, and the Hankel Functions are approximated by wave functions, \( \frac{1}{\sqrt{r}} e^{(\omega t - kr)} \).

5) We only wish to examine a relative angular distribution after reflection. That is, we drop the \( \frac{1}{\sqrt{r}} \) term in the amplitude, since we are not interested in an absolute level of transmission. This and the previous assumption reduce the problem to only two dimensions, \( r \) and \( z \). Thus, the Fraunhofer diffraction problem will be one dimensional along \( z \) only.

3.1.1. Physical Set-up and Geometry

Figure 3 explains the geometry. The \( r \) direction is increasing left to right and is indexed from the cone axis, while the \( z \) direction is increasing top to bottom and is indexed from the top of the cone. The transducer has a radius \( R \). The arbitrary cone half-angle is, \( \theta \), and follows the restriction \( 90 > \theta > 0 \) (Only values
around 45° are relevant). The height of the cone is \( h \) and is such that the upper edge of the cone is always directly above the outer edge of the transducer (i.e. \( h = R \cot(\theta) \)).

Following approximation 3, the pupil, \( p \), is defined along the line \( r = R \). It begins at the outer edge of the cone and extends down to the point which receives reflection from the cone tip. For an arbitrary incident ray, \( I \), at \( r \), the reflection off the cone is displaced \( \Delta z \) along the \( z \) axis and \( \Delta r \) along the \( r \) axis. (The incident ray is assumed at all times to be in the \(-z\) direction.)

![Reflecting Cone Diagram](image)

**Figure 3:** Geometry of transducer with anterior cone, showing pupil function.
3.1.2. Pupil Function

The first step to calculating the radiation pattern is to obtain the pupil function. Once this function is obtained, as mentioned in Approximation 3, the whole solution reduces to a Fraunhofer diffraction performed on the pupil function (See Appendix A).

Finding the pupil function involves two parts: phase and magnitude. Each will be expanded on separately, and the final pupil function will use the results of each.

The phase dependence is determined as follows. Since the incident wave at different points along r is reflected at different "heights", z, on the cone surface, a term is needed to account for the phase before reflection. The incoming beam is incident along the \(-z\) direction, thus its phase behaves as \(e^{+i k_0 z}\), where \(k_0\) is the propagation constant \((k_0 = \frac{2\pi}{\lambda_0})\). However, the phase is needed at reflection, i.e. at \(z + \Delta z\), and thus the phase term behaves as \(e^{+i \cdot k_0 \cdot (z + \Delta z)}\). The phase after reflection will depend on the displacement along the \(r\) and \(z\) directions, \(\Delta r\) and \(\Delta z\), respectively. The wave vectors before and after reflection are, respectively:

\[
\begin{align*}
\overrightarrow{k} &= \{k_r, 0, k_z\} \\
\overrightarrow{r'} &= \{\Delta r, 0, \Delta z\} \quad \text{after reflection.}
\end{align*}
\]

Therefore, the initial phase after reflection is as follows:

\[
(3.2) \quad e^{-i \cdot (\overrightarrow{k} \cdot \overrightarrow{r})} = e^{-i \cdot k_r \Delta r} \cdot e^{-i \cdot k_z \Delta z}
\]

Following Approximation 2, after reflection, the wave evolves as an outward traveling wave along \(r\), i.e. a second kind Hankel function of zero order, \(H_0^{(2)}(k_r \cdot R)\), and an exponential phasor along \(z\). Using the condition developed in Equation 3.2, this outward traveling wave appears as:

\[
(3.3) \quad H_0^{(2)}(k_r \cdot R) \cdot e^{-i \cdot k_z \Delta z}
\]

Since the second kind Hankel functions start at negative infinity and spiral clockwise to the origin with a varying wavelength, a term is needed to adjust the phase of the second kind Hankel function at the cone surface. It must be such
that the phase begins consistently from the cone's surface. (See Figure 4) This is accomplished by a normalizing factor: \( \frac{1}{H_0^{(2)}(k_r \cdot r)} \), where \( r \) is the location of the incident ray.

Figure 4: Phase of the second kind Hankel functions.

Since second kind Hankel functions have an \( r \)-dependent amplitude, this also needs a normalizing factor:

\[
(3.4) \quad \frac{|H_0^{(2)}(k_r \cdot r)|}{|H_0^{(2)}(k_r \cdot R)|}
\]

The amplitude of the pupil function is evaluated as follows. It is assumed that the incident beam is an \( r \)-dependent uniform plane wave (Approximation 1) due to the presumed physical aspects of the transducer, a point to be proven experimentally in Chapter 5. The amplitude turns out to be a function of the area of a ring element on the surface of the cone. The total power transmitted is proportional to the surface area of the transducer. Therefore, the power incident on a ring element of width \( dr \) is \( dN \) and is such that,

\[
(3.5) \quad dN \propto (2\pi r \, dr)
\]

That is,

\[
(3.6) \quad dN = A_0^2 \cdot 2\pi r \cdot dr \quad \text{where} \ A_0 \ \text{is an initial amplitude.}
\]
And it follows,

\[ A(r) = \sqrt{\frac{dN}{dr}} = A_0 \sqrt{2\pi r} \]

Finally, all these terms are taken together to form the pupil function, \( \psi_0 \).

\[ \psi_0 = A_0 \sqrt{2\pi r} \cdot \frac{H_0^{(2)}(k_r \cdot R)}{H_0^{(2)}(k_r \cdot r)} \cdot \frac{|H_0^{(2)}(k_r \cdot r)|}{|H_0^{(2)}(k_r \cdot R)|} \cdot e^{i \cdot k_0 \cdot (z + \Delta z)} \cdot e^{-i \cdot k_z \cdot \Delta z} \cdot e^{+i\omega t} \]

where, from the geometry:

\[ k_0^2 = k_r^2 + k_z^2 \]

\[ k_r = k_0 \sin (2\theta) \]

\[ k_z = k_0 \cos (2\theta) \]

\[ r = R - \frac{z}{\cot (\theta) - \cot (2\theta)} \]

\[ \Delta z = \frac{z}{\cot (\theta) \cdot \tan (2\theta) - 1} \]

Thus the pupil function is only a function of \( z \). Plots of the magnitude and phase of the pupil function for a 40°, 45°, and 50° cone half-angle are shown in Figure 5. Note that in the figures to follow, \( n \) is an integer value used to index the variable \( z \) in the pupil function for the CFFT. (See Section 3.1.3)
Figure 5: Plots of (a) the magnitude of the pupil function versus n for any cone half-angle and the phase of the pupil function versus n for (b) 45°, (c) 40°, and (d) 50° cone half-angles.
3.1.3. Complex Fast Fourier Transform (CFFT)

The resulting pupil function is much too complicated to do a closed-form Fraunhofer expansion. Thus it has to be done using the computer. MathCad has available on it a program for complex fast Fourier transforms (CFFT). A Fraunhofer expansion is simply a Fourier transform whose frequency is replaced by spatial frequency. However, the pupil function has to be adapted to a form acceptable to the CFFT program, which has the following properties:

1) CFFT's are performed on functions given in a number of discrete points.
2) The number of data points must be a power of two.
3) The resolution plays an important role.

To satisfy Condition 1, z must be quantized. We must develop the discrete points along the pupil, \( z_n \), where \( n \) is an integer index in the interval \( \{ 0, \ldots , N \} \). According to Condition 2, then, \( N \) must be a power of two, minus one (zero is included). We define a variable, \( (L + 1) \), to be the total number of points in the pupil. Disregarding resolution, then,

\[
N = (L + 1) - 1 = L
\]

However, according to Condition 3, we must take resolution into account. One way to get better resolution is to extend the pupil beyond the actual range and set the pupil function to zero in the extended region. We now define \( m \) as a resolution control variable and \( j^{-1} \) as the fraction of points within the original region, such that:

\[
j = \frac{2^m}{L + 1}
\]

where \( 2^m \) = total number of points and \( (L + 1) \) is a power of two.

Thus, we have,

\[
N = 2^m - 1 = j \cdot (L + 1) - 1
\]
Now we have $z_n$ representing the $n$th point, where the integer $n$ is in the interval \( \{ 0, \ldots, j \cdot (L+1) - 1 \} \). Now, an expression for $z_n$ is:

\[
(3.12) \quad z_n = \frac{n}{(L+1)} \cdot R \cdot (\cot(\theta) - \cot(2\theta)) \cdot \Phi(L-n)
\]

where the term $\Phi(L-n)$ is a step function of $n$ used to set $z_n$ to zero in the extended region, that is, $\Phi = 0$ for $n \geq L$, and $\Phi = 1$ for $n < L$. The trigonometric terms represent the dependence of the range of $z_n$ on the cone's half-angle, $\theta$. That is, for any cone half-angle, $\theta$, the $z$-axis runs from $z_n = 0$ when $n = 0$ to $z_n = r \cdot (\cot(\theta) - \cot(2\theta))$ when $n = L$ (the upper limit is based on the approximation that $\frac{L}{(L+1)} = 1$).

Using $z_n$ from Equation 3.12, the pupil function on which a CFFT is performed is plotted in Figure 6.

---

**Figure 6**: Magnitude of the pupil function on which the CFFT is performed.
3.1.4. Interpretation of CFFT Results

The results of the CFFT are plotted in Figures 7 and 8. The magnitude of each discrete value in the CFFT represents a spatial frequency. Therefore, a plot of the magnitude of the CFFT values versus the index $n$ shows the spatial frequency spectrum (Figure 7). One particular feature of the CFFT is that negative frequencies (angles) are displayed at the upper end of the spectrum. A transformation of the frequency variable is needed to resolve this. The transformed variable is $a_n$, where the $\Phi$ functions are again shifted step functions (See Appendix B).

\[
(3.13) \quad a_n = n \cdot \Phi\left(-\left(n - \frac{j(L+1)}{2} + 1\right)\right) \\
+ \left(n - (j(L+1) - 1)\right) \cdot \Phi\left(n - \frac{j(L+1)}{2}\right)
\]

A plot of the magnitudes of the CFFT values versus $a_n$ gives the plot shown in Figure 8.

Figure 7: Spatial frequency spectrum versus $n$. 

13
As diffraction theory states, the link between the spatial frequency and the diffraction pattern is (Hecht):

\[
(3.14) \quad f = k_o \sin (\theta)
\]

where \(\theta\) is the angle with respect to the horizontal. Thus, the spatial frequency variable, \(f\), needs to be transformed in order to yield a plot of intensity versus angle. For every incremental \(\Delta f\) in the Fourier spectrum, there is an increment, \(\Delta \theta\), in a Fraunhofer expansion. That is,

\[
(3.15) \quad \Delta \theta = \frac{2 \pi}{j \cdot R}
\]

where \(2\pi\) is the total angle, and \(j \cdot R\) is the number of increments, that is the radius of the transducer divided by the fraction of points within the pupil. Thus,

\[
(3.16) \quad f = k_o \sin (\theta) = \frac{2 \pi}{j \cdot R} \cdot a_n
\]

Figure 8: Spatial frequency spectrum versus \(a_n\).
Note $\theta$ is actually negative since the $z$ direction is downward. Therefore, the transformation from $a_n$ to $\theta$ is:

\[
\theta = -\sin^{-1}\left(\frac{2\pi}{j \cdot \frac{R}{k_0}}\right)
\]

A plot of the radiation pattern for various cone half-angles is shown in Figure 9.

**Figure 9**: Spatial frequency spectrum versus $\theta$. 
3.2. Experimental

By examining the plots in Figure 9, it is deduced that a 45° cone half-angle would produce the desired results. A 45° cone was constructed and placed anterior to the transducer. An identical transducer was placed a prescribed distance from the transmitter (recall the only interest at this point is the angular distribution). The schematics of the transmitting and receiving configurations are shown in Figure 10 along with the specifications on Polaroid's transducer.

(a) [Diagram]

(b) [Diagram]

(c) Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
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<tr>
<td>Usable Transmitting Frequency Range</td>
<td>See Graph</td>
</tr>
<tr>
<td>Usable Receiving Frequency Range</td>
<td>See Graph</td>
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<tr>
<td>Beam Pattern</td>
<td>See Graph</td>
</tr>
<tr>
<td>Minimum Transmitting Sensitivity at 50 kHz</td>
<td>110 dB</td>
</tr>
<tr>
<td>300 vac pk-pk, 150 vdc bias (db re 20 uPa at 1 meter)</td>
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</tr>
<tr>
<td>Minimum Receiving Sensitivity at 50 kHz</td>
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<td>150 vdc bias (dB re 1v/Pa)</td>
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<tr>
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<td>Suggested AC Driving Voltage (peak)</td>
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<tr>
<td>Maximum Combined Voltage</td>
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<td>Capacitance at 1 kHz (Typical)</td>
<td>400 - 500 pf</td>
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<td>Gold</td>
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</table>

Figure 10: Layouts for the (a) transmitting configuration and the (b) receiving configuration along with (c) the specifications on the straight transducer (Polaroid).
The received voltage was measured as a function of \( \theta \). In order to compare the experimental results with the theoretical results, the magnitude plot of the CFFT versus \( \theta \) had to be normalized. The results are as shown in Figure 11. (See following page) An important characteristic in Figure 11 is the half-width. As the plot indicates, these values differ only slightly. Another interesting feature in the plot is the extraneous lobes near the edges. These lobes may be attributable to the back-scattering from the cone tip. Two explanations led to this reasoning. The first is that the second kind Hankel functions have a pole at zero, which is where \( r \) is indexed from in the scattering problem. In addition to this, in the book by J. J. Bowman, effects of back-scattering from the cone tip are characterized. (The reader is referred to this text for the explicit formulas.) The theory developed in Bowman convey that the back-scattered field has a significant effect on the far-field radiation pattern. To quantify this effect is far beyond the scope of this paper. Nevertheless, the results show to be quite positive in regard to the choice of a pupil function.
Figure 11: Theoretical and experimental polar plots of the far-field radiation patterns for the transducer with an anterior cone.
4. Gain of Antennas

With the theory and experimental data for the radiation fields complete, the gain of the transducer, with and without the anterior cone, needs to be evaluated. These results can then be used to determine the reduction in total received power obtained when different configurations of antenna systems are used, that is, zero, one, and two cone systems. This reduction in total received power will prove useful in Chapter 8 in determining the maximum obtainable range for each system.

4.1. Straight Transducer

The first gain to be calculated is that of the transducer without the anterior cone. Heretofore, this will be referred to as the "straight transducer."

4.1.1. Theoretical

The gain is the ratio of the radiated power of the antenna (straight transducer) to the radiated power of an isotropic radiator and is a function of the angle. Figure 12 shows the radiated field patterns of the straight antenna and the isotropic radiator.

\[
\text{Figure 12: Far-field radiation patterns for the isotropic radiator versus the straight transducer.}
\]

Development of the solid angle as a function of \( \theta \) only yields:

\[
(4.1) \quad d\Omega = 2\pi \cdot \cos (\theta) \cdot d\theta
\]
The total general power is obtained by integrating the power distribution over the solid angle.

\[ N_{\text{Tot}} = \int_{-\pi/2}^{\pi/2} P_r(\theta) \, d\Omega \]  

Using Equation 4.1, this becomes:

\[ N_{\text{Tot}} = \int_{-\pi/2}^{\pi/2} P_r(\theta) \, 2\pi \cos(\theta) \, d\theta \]  

Since we are using discrete points rather than a continuous function, the integration over all \( \theta \) needs to be a summation over the discrete points. This is accomplished by changing the integral in Equation 4.3 to a summation:

\[ N_{\text{Tot}} = 2\pi \sum_{-\pi/2}^{\pi/2} P_r(\theta) \cos(\theta) \, \Delta \theta \]  

For an isotropic radiator, the power distribution is constant over \( \theta \) and is equal to \( P_{r_0} \), thus the integration of Equation 4.3 yields:

\[ N_{\text{Tot}} = 4\pi P_{r_0} \]  

The total powers in Equations 4.4 and 4.5 are equal:

\[ 2\pi \sum_{-\pi/2}^{\pi/2} P_r(\theta) \cos(\theta) \, \Delta \theta = 4\pi P_{r_0} \]  

We now define the power distribution of the directed antenna to be equal to the maximum power, times a function of \( \theta \).

\[ P_r(\theta) = P_{r_{\text{Max}}} \cdot F(\theta) \]  

Using Equation 4.7, Equation 4.6 becomes:

\[ \sum_{-\pi/2}^{\pi/2} P_{r_{\text{Max}}} \cdot F(\theta) \cdot \cos(\theta) \, \Delta \theta = 2 P_{r_0} \]
An expression for the maximum obtainable gain of the straight transducer is then:

\[
\eta_{\text{Max}} = \frac{P_{r_{\text{Max}}}}{P_{r_0}} = \frac{2}{\sum_{-\pi/2}^{\pi/2} F(\theta) \cos(\theta) \Delta \theta} = \frac{2 P_{r_{\text{Max}}}}{\sum_{-\pi/2}^{\pi/2} P_r(\theta) \cos(\theta) \Delta \theta}
\]

4.1.2. Experimental

Figure 13 shows Polaroid's data for the far-field distribution of the straight transducer.

This data can be utilized in the result of Equation 4.9. Twenty-eight equally spaced data points were chosen, thus \(\Delta \theta\) equals \(\frac{\pi}{28}\). Using this data, \(\eta_{\text{Max}}\) for the straight transducer is 177.444, or 22.49 dB.

4.2. Transducer with Anterior Cone

The gain of the transducer with an anterior cone is obtained in a similar manner to that of the straight transducer. In this section, amplitudes will be utilized rather than intensities, so the results of the CFFT can be directly incorporated.

4.2.1. Theoretical

The gain is again the ratio of the radiated power of the directed antenna to the radiated power of an isotropic radiator. With the anterior cone added, the far-field patterns appear as shown in Figure 14.
Figure 14: Far-field radiation patterns for the isotropic radiator versus the omnidirectional antenna.

The total radiated power is the same as obtained in Equation 4.4; however, as mentioned previously, the power distribution, $P_r(\theta)$, is given by the squared amplitude distribution, $A_r^2(\theta)$:

$$N_{Tot} = \sum_{-\pi/2}^{\pi/2} A_r^2(\theta) \cdot 2\pi \cdot \cos(\theta) \Delta \theta$$  \hspace{1cm} (4.10)

Similar as before, with the substitution that the amplitude distribution is the product of the maximum amplitude times a function of $\theta$, Equation 4.10 becomes:

$$N_{Tot} = 2\pi \sum_{-\pi/2}^{\pi/2} A_{r_{Max}}^2 \cdot F^2(\theta) \cos(\theta) \Delta \theta$$  \hspace{1cm} (4.11)

The total radiated power in an isotropic radiator is the same as obtained in Equation 4.5. The powers in Equations 4.11 and 4.5 are equal:

$$2\pi A_{r_{Max}}^2 \sum_{-\pi/2}^{\pi/2} F^2(\theta) \cos(\theta) \Delta \theta = 4\pi P_{r_0}$$  \hspace{1cm} (4.12)
Therefore, an expression for the maximum obtainable gain for the transducer with an anterior cone is:

\[
 g_{\text{Max}} = \frac{A_{r_{\text{Max}}}^2}{P_{r_0}} = \frac{2}{\sum_{-\pi/2}^{\pi/2} F^2(\theta) \cos(\theta) \Delta \theta} = \frac{2 A_{r_{\text{Max}}}^2}{\sum_{-\pi/2}^{\pi/2} A_r^2(\theta) \cos(\theta) \Delta \theta}
\]

4.2.2. Experimental

The theoretical and experimental data shown in Figure 11 can be used in Equation 4.13 to calculate the maximum obtainable gain for the transducer with an anterior cone. Inserting the theoretical values, the maximum obtainable gain is 3.466, or 5.399 dB. Using the experimental data points, the maximum obtainable gain is 3.71, or 5.686 dB. These values agree quite nicely.

4.3. Comparison of Various Configurations

Knowing the maximum obtainable gain values for the straight transducer and for the transducer with an anterior cone, a comparison between various system configurations can be made. Three configurations will be looked at. They are shown in Figure 15. Figure 15a shows the configuration for a system using two straight transducers, 15b shows the configuration for a system using one straight transducer and one cone (Note that the transmitter and receiver in this configuration are interchangeable due to the Lorentz Reciprocity Theorem), and 15c shows the configuration for a system using two cones. The gain of the straight transducer is defined as, \( g_1 \), and that of the transducer with an anterior cone is defined as, \( g_2 \).

Figure 15: Experimental set-ups with (a) two straight transducers, (b) one straight transducer and one transducer with an anterior cone, and (c) two transducers with anterior cones.
At this point, the Friis Transmission Formula will be utilized. For an antenna transmitting and receiving system shown in Figure 16, the total power received can be written as a function of the total transmitted power, the transmitting gain, the distance, and the effective antenna area of the receiving antenna.

\[
\psi = \frac{N_0 g_T}{4 \pi R^2} A_e \quad \text{(Generalized Friis Transmission Formula)}
\]

where the effective antenna area is related to the wavelength and the gain of the receiving antenna by:

\[
A_e = \frac{\lambda_0^2}{4 \pi} g_R \quad \text{(Effective Antenna Area)}
\]

Figure 16: Transmitting and receiving system (Collin).

Equation 4.13 assumes the antennas are matched and that their directions of maximum gain are directly aligned. Examining the total received power in the three cases of Figure 15 results in the following. Case a, in Figure 15a, yields the total received power as being proportional to \( g_1^2 \). Case b, in Figure 15b, yields the total received power as being proportional to the product of \( g_1 \) and \( g_2 \), and the total received power in Case c, Figure 15c, is proportional to \( g_2^2 \). Note that if we use the same transmitting power, \( N_0 \), and distance, \( R \), in each case, the total received powers will differ only in the values of the transmitting and receiving gains.
The reduction in total received power from Case a to Case b is:

\[(4.15) \quad \frac{\psi_b}{\psi_a} = \frac{E_2}{E_1} = 0.0209\]

Similarly, from Case b to Case c, the reduction in total received power is:

\[(4.16) \quad \frac{\psi_c}{\psi_b} = \frac{E_2}{E_1} = 0.0209\]

Thus, the addition of a cone to either Case a or Case b results in identical reductions in total received power. Notice that the reduction in total received power is very significant (two orders) even when only one cone is added to the system.
5. Diaphragm Movement

At this point, some insight is necessary into the movement of the transducer's diaphragm with respect to the applied voltages. This information is needed for two reasons. First, in Section 3.1, we assumed the incident beam develops as a uniform plane wave. By observing the movement of the diaphragm, the actual shape of the incident beam can be found. Second, the magnitude of the displacement of the diaphragm plays a direct role in the amount of transmitted power. As will be shown in Chapter 7, the total transmitted power is directly proportional to the particle velocity, which is in turn equal to the diaphragm displacement/frequency product. Thus, the magnitude of the displacement is needed to determine the particle velocity, and thus the total transmitted power.

The diaphragm movement has been observed as a function of the AC driving voltage. This movement will be termed "displacement". It is this movement that will ultimately determine the particle velocity, as well as the shape of the incident beam.

The displacement of the diaphragm is a function of the AC driving voltage of the transducer. This is according to the acoustical coupling equations, where the time-varying voltage across the diaphragm/backplate capacitor is directly proportional to the displacement of the transducer's diaphragm. (See Equation 2.1) This displacement is the origin of the ultrasonic "chirp". To observe this displacement, a Michelson interferometer was set up on an optical table as shown in Figure 17.
Figure 17: Michelson interferometer set-up for measuring diaphragm displacement.
Using a constant DC bias voltage of 150 V, the AC driving voltage (peak-to-peak) was adjusted from 0 V to 300 V. These measurements were conducted at the center and at the edges of the transducer diaphragm. The experimental method used had a severe limitation due to the low optical quality of the diaphragm surface. Also, there were small local flexations occurring due to the grooved surface of the backplate. The spacing of the grooves on the backplate had a periodicity of 0.5 mm, with a 1:1 relationship between the width of the ridges and the grooves. However, the housing of the transducer had only 22 evenly spaced holes across the diameter with a periodicity of 1.73 mm. Thus, it was necessary to find a hole in the housing that was directly aligned with a groove in the backplate in order to obtain accurate measurements. (See Figure 18) Therefore, the measured points are scattered over a wide band. It is encouraging, though, that the middle line passes through the origin (0 volts, 0 displacement). (See Figure 19)

![Diagram](image)

**Figure 18:** Diaphragm/backplate schematic.

The displacement was measured by counting the fringes which pass across the detector, (See Figure 17), each fringe being one half of an optical wavelength. In order to be able to count them, the following method was designed. The signal used to drive the transducer was also used to generate a circle on the screen of Oscilloscope C. The amplified photocurrent was made to intensity modulate the oscilloscope (z-axis). For instance, if four equally spaced dark areas appear on the circumference of the circle, then the diaphragm has moved by two half-wavelengths between the two maxima of displacement. Thus, the total displacement (peak-to-peak) of the diaphragm was recorded in terms of “number of half-wavelengths”. The results are plotted in Figure 19.
The results show approximately the same amplitude of displacement in different grooves across a diameter of the transducer. Also, as expected, the displacement of the diaphragm is directly proportional to the value of the peak-to-peak AC driving voltage. These results show the initial beam profile to be constant since the groove spacing is small compared with the wavelength. Thus, the approximation made in Section 3.1 is valid.
6. Operating Circuits

For detection of the incoming signal, two circuits were needed. A circuit was needed to amplify the incoming signal in order to observe a reading on a voltmeter. This circuit would also filter any noise that was either picked up through transmission or generated by the circuit itself. Another circuit was needed to supply the DC bias level needed to operate the transducer. One condition was that the receiving unit be battery operated. The sections in this chapter discuss first the noise considerations, as this is an extremely important criterion for obtaining the maximum obtainable range of a system, and then the circuits themselves, the amplifier/filter circuit and the DC bias circuit.

6.1. Noise Considerations

The noise primarily under consideration was Johnson noise and shot noise. Johnson noise arises from any resistance. In this case, the acoustic noise, by equipartition, is represented by the Johnson noise generated by the internal resistance of the transducer. Shot noise appears wherever electrons pass a barrier, as in a p-n junction in a transistor. Other noise, such as 1/f or flicker noise, will be assumed negligible. The Johnson noise arises from the source impedance, that is, the internal impedance of the transducer. The shot noise arises from the collector current in the pre-amplifier. Figure 20a shows the equivalent circuit which includes the Johnson noise source, $e_n$, and the shot noise referred to the input, $i_n$. The total noise voltage, $e_a$, is a combination of the Johnson noise and the shot noise and is modeled in Figure 20b. (Horowitz and Hill)

![Figure 20: Equivalent circuits for the amplifier (a) including noise sources and (b) with noise sources combined at the input.](image-url)
Depending on the actual value of the source impedance, one or the other may dominate. That is, the Johnson and shot noise observed are uncorrelated, thus the total observed noise will be the following:

\[ e_a = \left[ e_n^2 + (R_s i_n)^2 \right]^{1/2} \text{ V/Hz}^{1/2} \]

where \( e_n \) is the Johnson noise voltage, \( R_s \) is the source impedance, and \( i_n \) is the shot noise of the pre-amplifier referred to the input. Once the expected noise level is obtained, the necessary gain of the amplifier/filter circuit can be obtained.

### 6.1.1. Internal Impedance of Transducer

The internal impedance of the transducer is necessary to determine the Johnson noise. The internal impedance of the transducer consists of the internal capacitance of the transducer in parallel with the noise source resistance. The capacitance, in order not to short out the signal, has to be compensated through resonance. The value of the inductance needed to resonate the circuit at 50 kHz must be found. Figure 21 shows the circuit used to find the inductance. The circuit was set to resonate at 50 kHz by varying \( L \), where \( C_t \) is the internal capacitance of the transducer. (Figure 20)

![Figure 21](image)

**Figure 21:** Test circuit for determining the internal impedance of the transducer.

The value of \( L \) to achieve resonance at 50 kHz was \( L = 20 \text{ mH} \).

The value of \( C_t \) consists of a “parallel” combination of two capacitances, a static capacitance, \( C_s \), and an acoustic capacitance, \( C_a \). These are related by:

\[ C_t = C_a + C_s \]
The value of $C_s$ was measured at low frequency to be 82 pF. By substituting the transducer by a variable capacitor in the circuit of Figure 21, the value of $C_t$ was found to be 420 pF, and thus, using Equation 6.2, $C_a$ was found to be 338 pF.

Next, the resistive part of the impedance of the LC$_t$ combination, $R_{lt}$, was measured to be 267 k.

Finally, a variable resistor, $R_v$, was placed in parallel with $C_t$ and L in the circuit of Figure 21. Adjusting $R_v$ until $V_0$ was $\frac{1}{2}V$, the following could be said:

\[(6.3) \quad R_v = R_s \parallel R_{lt}\]

where $R_s$ is the internal resistance of the transducer. Solving for $R_s$:

\[(6.4) \quad R_s = \frac{R_{lt} R_v}{R_{lt} - R_v}\]

Since $R_v$ was found to be 27 k, the internal resistance of the transducer is:

\[(6.5) \quad R_s = 30 \text{ k}\]

6.1.2. Noise Calculations

Some figuring is needed before the total value for the expected noise voltage can be obtained. Since the source impedance is known to be 30 k, a transistor to be used for a pre-amplifier can be chosen to minimize the noise figure. From specification sheets, the 2N5087 pnp transistor was chosen, where it was decided that a collector current of 0.5 mA would be used. Also, it was decided that a 10 kHz bandwidth filter would be used to filter the noise. With these figures, the expected noise voltage, expected noise figure, and the gain needed could be calculated.

First, the value for the total Johnson noise in a bandwidth, $B$, was obtained using the following expression: (Horowitz and Hill)

\[(6.6) \quad V_n = e_n B^{1/2} = (4 k T R_s B)^{1/2} = 2.23 \mu V\]

where $k$ is Boltzmann's constant, $T$ is the temperature, and $R_s$ is the source impedance, 30 k.
Next, the value for the total shot noise in a bandwidth, B, was obtained using the following expression: (Horowitz and Hill)

\[
I_n = (2qI_{dc}B)^{1/2} = 1.26 \text{ nA}
\]

where \( q \) is the electron charge and \( I_{dc} \) is the collector current of the pre-amplifier, 0.5 mA. This shot noise must be referred back to the input. From the specification sheets, the \( \beta \) for the 2N5087 pnp transistor is 180. Therefore, the input noise current is:

\[
I_n' = \frac{I_n}{\beta} = 7 \text{ pA}
\]

Now, using the expression from Equation 6.1, and multiplying through with \( B^{1/2} \) to put all the units in volts, the total expected noise voltage is:

\[
V_{n_{Tot}}^{n_{Tot}} = [V_n^2 + (R_s I_n')^2]^{1/2} = 2.24 \mu V
\]

Now, a value for the expected noise figure, NF, is: (Horowitz and Hill)

\[
NF = 10 \log_{10} \left( 1 + \frac{V_{n_{Tot}}^2}{4kT R_s B} \right) \text{ dB} = 3.03 \text{ dB}
\]

Next, the gain needed for proper amplification must be determined. Since detection of a weak signal just above the noise floor was necessary, we decided to set the gain high enough, so that the thermal noise alone would give a reading of 10% of full scale. The smallest range of the voltmeter to be used in the testing was 0.1 V_{rms}. The maximum full scale range was 10 V_{rms}. Using this, the necessary gain, \( g \), is simply:

\[
g = \frac{10\% \cdot 0.1V}{2.24 \mu V} = 4464
\]

6.2. Low Noise Amplifier/Filter circuit

The design of the amplifier/filter circuit is as follows. The received signal in the transducer is on the order of microvolts. Thus, the noise is very significant. The received signal needs to be pre-amplified. Then another amplifying stage
makes the signal larger, so that the noise in the remainder of the circuit will not affect the signal. Next, a 10 kHz band-pass filter, centered at 50 kHz, is used to filter out the noise. Then, a final amplifying stage is added to achieve the necessary gain.

Each stage of the circuit has a gain. To keep the circuit as simple as possible, both amplifiers are made to have the same gain. The 10 kHz band-pass filter used was obtained from Marshall. The gain of each stage is multiplied together to obtain the final gain. The final circuit, shown in Figure 22, has an experimental gain of 3750, which is sufficiently close to the necessary value of 4464 obtained in Equation 6.11.

![Diagram of the circuit](image)

\[ G_T = 3750 \]

**Figure 22:** Low noise amplifier/filter circuit.

Upon testing, (Chapter 7), the actual amplified noise voltage achieved with the circuit in Figure 22 was 6 mV. This corresponds to an input noise voltage of 1.6 \( \mu V \), which differs from the expected input noise voltage (See Equation 6.9) by only 0.60 \( \mu V \). Thus, the noise values agree quite nicely. The actual noise figure can be obtained using the formula in Equation 6.10 and turns out to be 1.80 dB.
6.3. DC Bias circuit

To experimentally test the transducer, a DC bias voltage of approximately 200 V is needed. This voltage needs to be achieved via a 9 V battery so that, as mentioned previously, the unit can be portable. A voltage multiplier circuit was developed to achieve this bias. This circuit, shown in Figure 23, uses a transistor circuit that oscillates at \( \sim 125 \text{ kHz} \). This voltage is then stepped up through a transformer, and a full-wave rectifier is used to regain a DC voltage. This voltage is then stepped up to \( \sim 180 \text{ V} \).

![Figure 23: DC bias circuit.](image-url)
7. Transducer Sensitivity

This chapter focuses on obtaining the experimental value of the receiving sensitivity. The receiving sensitivity, M, of the transducer is defined as the voltage received divided by the sound pressure: (Rossi)

\[ M = \frac{V_R}{P} \text{ V/Pa} \]  

(7.1)

One convention is to express a relative sensitivity, \( L_M \), as:

\[ L_M = 20 \log \left( \frac{M}{M_r} \right) \text{ dB} \]  

(7.2)

where \( M_r \) is a reference sensitivity of 1 V/Pa. Section 7.1 will develop the theoretical expressions used in the measurement of the sensitivity, and then Section 7.2 will use the results of previous chapters and experiments to determine the value of \( L_M \).

7.1. Theoretical

At this point, an expression for the relative sensitivity, \( L_M \), must be developed in terms of known and measurable quantities. The received voltage is a measurable quantity. The sound pressure, however, must be found according to other known and measurable quantities.

The directed power density at a distance, \( R \), from the transmitter is a function of the sound pressure at \( R \) and the particle velocity at \( R \), designated \( v_R \): (See Figure 24)

\[ \overline{N}' = \frac{1}{2} \Re \{ v_R P_R \} = \frac{1}{2} v_R P_R \]  

(7.3)

\[ \overline{N}' = \overline{N} \]

Figure 24: Described locations of powers and power densities for radiating antennas.

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Since the particle velocity at $R$ is related to the sound pressure at $R$ via the characteristic acoustic impedance, $Z_c$,

\begin{equation}
\nu_R = \frac{P_R}{Z_c},
\end{equation}

Equation 7.3 may be written as:

\begin{equation}
\bar{N}' = \frac{1}{2} \frac{P_R^2}{Z_c}
\end{equation}

Solving Equation 7.5 for the sound pressure:

\begin{equation}
P_R = \sqrt{2 Z_c \bar{N}}
\end{equation}

(Sound Pressure at $R$)

Now the transmitted power density, $\bar{N}'$, must be found in terms of known quantities. The transmitted gain, $g_T$, of the transmitting transducer can be expressed as:

\begin{equation}
g_T = \frac{\bar{N}'}{\bar{N}}
\end{equation}

where $\bar{N}$ is the radiated isotropic power density at a distance $R$. The value of $\bar{N}$ is simply the total power incident at $R$, $N$, divided by the collecting area:

\begin{equation}
\bar{N} = \frac{N}{4\pi R^2}
\end{equation}

Noting that the total power at $R$ is equal to $N_0$, the total transmitted power, Equation 7.8 can be written as:

\begin{equation}
\bar{N} = \frac{N_0}{4\pi R^2}
\end{equation}

Using Equation 7.9, Equation 7.7 can be written as:

\begin{equation}
\bar{N}' = g_T \bar{N} = \frac{g_T N_0}{4\pi R^2}
\end{equation}

(Directed Power Density at $R$)
The total received power, \( N' \), then is just the directed power density multiplied with the effective antenna area of the receiving antenna, as mentioned back in Chapter 4. This is given by the Friis Transmission Formula. (Collin)

\[
(7.11) \quad N' = N' A_e = \frac{\lambda_0^2 \, \varepsilon_R \, \varepsilon_T \, N_0}{4 \pi R^2} \quad \text{(Total Directed Power)}
\]

The only remaining unknown quantity is the total transmitted power, \( N_0 \), which can be written as:

\[
(7.12) \quad N_0 = \overline{N_0} \, \pi r^2
\]

where \( \overline{N_0} \) is the power density at the surface of the transducer, and \( r \) is the radius of the transmitting transducer. The transmitted power density, \( \overline{N_0} \), can be written as:

\[
(7.13) \quad \overline{N_0} = \frac{1}{2} \, v_{rms} \, P
\]

where \( v_{rms} \) is the rms particle velocity and \( P \) is the sound pressure at a reference point directly at the transmitting transducer. Solving Equation 7.4 for \( P \) yields:

\[
(7.14) \quad P = v_{rms} \, Z_c
\]

But the rms particle velocity, \( v_{rms} \), is simply:

\[
(7.15) \quad v_{rms} = \omega \, s_{rms}
\]

where \( \omega \) is the transmitting frequency and \( s_{rms} \) is the rms displacement of the transducer's diaphragm. Combining Equations 7.12–7.15, we obtain an expression for the total transmitted power, \( N_0 \).

\[
(7.16) \quad N_0 = \frac{1}{2} \, (\omega s_{rms})^2 \, Z_c \, \pi r^2 \quad \text{(Total Transmitted Power in terms of Diaphragm Movement)}
\]
Combining the results of Equations 7.6, 7.10, and 7.16, the sound pressure at R is obtained. Thus, a voltage measurement at a distance R results in a value for the sensitivity, M:

\[ M = \frac{V_R}{P_R} = \frac{V_R \cdot R}{A} \quad \text{where} \quad A = \frac{1}{2} Z_c \omega s_{rms} \sqrt{\delta_T} \]

This value of A depends on antenna gain, etc., which are to be experimentally determined.

Also to be taken into account is the atmospheric attenuation. This adds an exponential decay term to the received voltage. The atmospheric attenuation coefficient, k, causes the received voltage to behave as follows:

\[ V = \frac{C}{R} e^{-kR} \]

where C is a constant. The result of the atmospheric attenuation is that the sensitivity in Equation 7.17 becomes:

\[ M = \frac{V_R \cdot R e^{kR}}{A} \quad \text{(Receiving Sensitivity)} \]

Once the sensitivity is found, the relative sensitivity, \( L_M \), will be evaluated and compared with Polaroid's value. This value will be calculated relative to a sensitivity of 1 V/Pa as mentioned previously. Thus, from Equations 7.2 and 7.19, \( L_M \) will simply be:

\[ L_M = 20 \log \left( \frac{V_R \cdot R e^{kR}}{A} \right) \]

7.2. Experimental

As shown in Section 7.1, the relative sensitivity is a function of \( V_R \cdot R e^{kR} \), where \( V_R \) is the received voltage at R, and k is the atmospheric attenuation coefficient. The value of A in Equation 7.20 is a result of using Equations 7.6, 7.10, and 7.16 to determine the sound pressure at R. To find the sound pressure, \( P_R \), several results from previous chapters are needed. First, from Chapter 4, the value of the gain of the transmitting transducer, \( g_T \), is needed. For the testing, we used two straight transducers, that is, the configuration of Figure 15a.
According to Section 4.1.2, the value of \( g_T \) for the straight transducer is:

\[
(7.21) \quad g_T = 177.444 = 22.49 \text{ dB}
\]

Next, from Chapter 5, Figure 18, for a 300 V peak-to-peak, 50 kHz sine wave, the peak-to-peak displacement of the transducer’s diaphragm is:

\[
(7.22) \quad s_{p-p} = 3\lambda
\]

Thus, the rms displacement, \( s_{rms} \), is:

\[
(7.23) \quad s_{rms} = \frac{s_{p-p}}{2\sqrt{2}} = 1.061 \lambda
\]

where \( \lambda \) is the optical wavelength, 0.638 \( \mu \text{m} \). Using Equation 7.16 with \( \omega = 2\pi \times 50 \text{ kHz}, \ Z_c = 400 \frac{\text{N} \cdot \text{s}}{\text{m}^2}, \) and \( r = 1.9 \text{ cm} \), the total transmitted power becomes:

\[
(7.24) \quad N_0 = 1.024 \times 10^{-2} \text{ W} \quad \text{(Total Transmitted Power)}
\]

Next, using Equation 7.10 and the value for \( g_T \) from Equation 7.21, we find the directed power density at \( R \):

\[
(7.25) \quad N' = 0.1447 R^{-2} \text{ W/m}^2 \quad \text{(Directed Power Density at R)}
\]

Using Equation 7.6, the sound pressure at \( R \) is then:

\[
(7.26) \quad P_R = 10.76 R^{-1} \text{ Pa} \quad \text{(Sound Pressure at R)}
\]

Note that the value of \( A \) from Equation 7.17 is then 10.76 Pa \( \cdot \) m. Thus, by measuring the received voltage at \( R \), we can find the relative sensitivity from Equation 7.20:

\[
(7.27) \quad L_M = 20 \log \left( \frac{V_R R e^{kR}}{10.76} \right) \text{ dB}
\]
With Equation 7.27, all that remains is to set up an experiment to measure the voltage versus distance. The experimental set-up of Figure 15a was used (two straight transducers).

The following table, Figure 25, shows the values of voltage measured at a distance $R$, along with the calculated values of relative sensitivity. Figure 26 shows a plot of $\ln(V_R)$ versus $R$. Note that the experimental value of $k$, the atmospheric attenuation coefficient, was obtained from the slope of the plot of $\ln(V_R)$ versus $R$ in Figure 26. (See Equation 7.18) The experimental data points are expected to be linear, and Figure 26 shows this to be the case. The experimental value was 0.1928 Np/m. The theoretical value from Rossi was between 1–2 dB/m, which is 0.1151–0.2301 Np/m. The experimental and theoretical values agree quite nicely. Polaroid shows in its specification sheet a value for the receiving sensitivity relative to 1 V/Pa of $L_M = -42$ dB. All the values of $L_M$ in the above table fit very well. Also note that the values of $L_M$ are relatively independent of $R$, with small fluctuations attributable to varying atmospheric conditions at the time of testing.

<table>
<thead>
<tr>
<th>$R$ (m)</th>
<th>$V_R$ (rms Volts)</th>
<th>$V_R$/gain (rms Volts)</th>
<th>$L_M$ (dB re 1 V/Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.625</td>
<td>7.2</td>
<td>$1.92 \times 10^3$</td>
<td>-44.56</td>
</tr>
<tr>
<td>9.15</td>
<td>4.9</td>
<td>$6.996 \times 10^2$</td>
<td>-43.74</td>
</tr>
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<td>12.20</td>
<td>1.85</td>
<td>$6.320 \times 10^2$</td>
<td>-44.62</td>
</tr>
<tr>
<td>15.25</td>
<td>0.800</td>
<td>$6.146 \times 10^2$</td>
<td>-44.86</td>
</tr>
<tr>
<td>18.30</td>
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<td>$6.670 \times 10^2$</td>
<td>-44.15</td>
</tr>
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<td>21.35</td>
<td>0.140</td>
<td>$4.884 \times 10^2$</td>
<td>-46.86</td>
</tr>
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<td>0.090</td>
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</tr>
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<td>-45.34</td>
</tr>
<tr>
<td>32.025</td>
<td>0.015</td>
<td>$6.152 \times 10^2$</td>
<td>-44.85</td>
</tr>
</tbody>
</table>

**Figure 25**: Table of experimental results and calculated relative sensitivities.
Figure 26: Plot of ln(VR) versus R.
8. Maximum Obtainable Range

The ultimate objective of this thesis is to determine whether or not such an ultrasonic navigation system is feasible. This feasibility hinges on the maximum range that can be obtained using such a system. This range depends directly on the noise floor and the desired signal-to-noise ratio (SNR). The results of Chapter 7 show the noise floor to be \( \nu_n = 1.6 \mu V \). Using this and the SNR we wish to observe, the necessary or minimum received signal can be calculated.

\[
\text{(8.1)} \quad \nu_{s\text{Min}} = \nu_n 10^{\frac{\text{SNR}}{20}}
\]

The maximum obtainable range can then be obtained from Equation 7.20 by solving for \( R e^k R \).

\[
\text{(8.2)} \quad R e^k R = \frac{A}{\nu_{s\text{Min}}} 10^{\frac{L_M}{20}}
\]

Note that the atmospheric attenuation coefficient, \( k \), is 0.1928 Np/m from Chapter 7. Also from Chapter 7, Figure 25, the average value of the receiving sensitivity, \( L_M \), is found to be \( -44.87 \text{ dB below 1 V/Pa} \). The expression for the term \( A \), from Equation 7.17, is written as:

\[
\text{(8.3)} \quad A = A' \sqrt{g} \quad \text{where} \quad A' = \frac{1}{2} Z_c \omega s_{rms} r = 0.8077 \text{ Pa} \cdot \text{m}
\]

where \( Z_c = 400 \frac{N \cdot s}{m^3} \), \( \omega = 2\pi \times 50 \text{ kHz} \), \( s_{rms} = 1.061 \lambda \), and \( r = 0.019 \text{ cm} \). Putting Equation 8.3 into Equation 8.2, we obtain,

\[
\text{(8.4)} \quad R e^k R = \frac{A'}{\nu_{s\text{Min}}} 10^{\frac{L_M}{20}} \cdot \sqrt{g}
\]

Thus, for a given SNR, the maximum obtainable range is found as a function of the gain of a particular system. Now, recall from Chapter 4 that the reduction in total received power realized when a cone was added to the system was found to be 0.0209. (See Equations 4.15 and 4.16) From the Friis Transmission Formula (See Equation 7.11), this reduction in total received power translates to an equivalent reduction in gain, \( g \), provided all other parameters remain the same.
Thus, the values of $g$ for the three cases of Figure 15 are:

\[(8.5) \quad \text{Case a: } g_a = 177.444 \quad \text{(Two straight transducers)}
\]
\[\text{Case b: } g_b = 177.444 \cdot 0.0209 = 3.71 \quad \text{(One cone, one transducer)}
\]
\[\text{Case c: } g_c = 3.71 \cdot 0.0209 = 0.0775 \quad \text{(Two cones)}
\]

Now, using Equation 8.4 and the above values, the following table can be made which shows the maximum obtainable range for each system as a function of the desired SNR. (Figure 27)

<table>
<thead>
<tr>
<th>Case</th>
<th>SNR (dB)</th>
<th>R (m\ ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>3</td>
<td>34.578\113</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>30.969\102</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>25.922\85.0</td>
</tr>
<tr>
<td>b</td>
<td>3</td>
<td>26.023\85.3</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>22.579\74.0</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>17.835\58.5</td>
</tr>
<tr>
<td>c</td>
<td>3</td>
<td>17.915\58.7</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>14.744\48.3</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>10.522\34.5</td>
</tr>
</tbody>
</table>

Figure 27: Table of calculated maximum obtainable ranges for the various system configurations and signal-to-noise ratios.
9. Conclusions

In conclusion, a navigation system using ultrasonic sensing techniques appears feasible. A system most applicable would be one in which two stationary transducers with anterior cones would be accompanied by a third transducer with an anterior cone. This third transducer would be located on the robot of which the coordinates are desired. A triangulation calculation would then determine the relative coordinates of the mobile transducer. According to the table of Figure 25, the maximum obtainable range for a signal-to-noise ratio of 10 dB would be 48.3 ft. This distance would be the maximum distance the mobile transducer could range from either stationary transducer. (e.g. If a rectangular movement pattern is desired, the maximum range would be a diagonal.) Note that if this range is exceeded, more stationary transducers strategically placed in the periphery would enlarge the area encompassable by the mobile transducer. I feel this range is sufficient for many applications, making this means of navigation very feasible.
References


Polaroid Corporation, Specification Sheet for Environmental Ultrasonic Transducer.

Appendix A

Diffraction of source waves due to a slit can be characterized generally in two ways: Fresnel diffraction or Fraunhofer diffraction. Fresnel diffraction occurs when the source and/or receiving points are close to the diffracting aperture, and thus the diffracted wave doesn’t have very far to spread out. However, at a large distance from the aperture, “the projected pattern will have spread out considerably, bearing little or no resemblance to the actual aperture.” (Hecht) Therefore, the size, not the shape of the pattern changes at these large distances. This is known as Fraunhofer diffraction. It can be assumed, as in this problem, that an array of point sources (pupil function) can be observed in the far-field as Fraunhofer diffraction. The diffraction set-up in this case is that of one with a single slit, that slit being the pupil function, \( p \). The radiation pattern at a point far away is then defined by the following relation, where \( E \) is the radiation pattern at an angle, \( \theta \), \( C \) is a constant, \( k \) is the wave number, and \( y \) is the direction of the slit. (Hecht)

\[
E = \int_{\text{slit}} e^{i \frac{y k \sin(\theta)}} dy
\]
Appendix B

Range of variable, \( n \):

![Diagram showing the range of variable, \( n \).]

Define a step function:

\[
\Phi \left( - \left( n - \frac{j(L+1)}{2} + 1 \right) \right)
\]

Define another step function:

\[
\Phi \left( n - \frac{j(L+1)}{2} \right)
\]

Derive the new variable, \( a_n \):

\[
a_n = n \cdot \Phi \left( - \left( n - \frac{j(L+1)}{2} + 1 \right) \right) + \left( n - \frac{j(L+1)}{2} \right) \cdot \Phi \left( n - \frac{j(L+1)}{2} \right)
\]
Biography

Rodney Wayne Claycomb was born October 18, 1968 in Bedford, PA. He is the son of Robert William Claycomb, Jr. and Marcia Eileen Claycomb. He graduated second in his class from Northern Bedford County High School in 1986, where he was a member of Who's Who Among American High School Students and Vice-President of his class. He then graduated Magna Cum Laude from Shippensburg University in 1990 receiving his Bachelor of Arts degree with a double major in Physics and Mathematics. Honors there included the 1990 Award in Physics, Paul E. Kauffman Award in Mathematics, Who's Who Among American Students in Colleges and Universities, Academic All-American, and Dean's List for eight semesters. After a stay with Universal Dairy Equipment, Inc. in Kansas City, MO, as a design engineer, he attended Lehigh University to receive his Master of Science in Electrical Engineering in 1992.
END
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TITLE