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Ultimate shear strength of prestressed concrete beams with web reinforcement, April 1965

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ULTIMATE SHEAR STRENGTH OF PRESTRESSED CONCRETE BEAMS WITH WEB REINFORCEMENT

by

John M. Hanson
C. L. Hulsbos

Fritz Engineering Laboratory Report No. 223.27
ULTIMATE SHEAR STRENGTH
OF
PRESTRESSED CONCRETE BEAMS WITH WEB REINFORCEMENT

John M. Hanson
C. L. Hulsbos

Part of an Investigation Sponsored by:
Pennsylvania Department of Highways
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Reinforced Concrete Research Council

Fritz Engineering Laboratory
Department of Civil Engineering
Lehigh University
Bethlehem, Pennsylvania

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1. **INTRODUCTION**

1.1 **BACKGROUND**

The shear strength of concrete beams became a research topic of major importance in the early 1950's. There were several reasons for this renewed interest in a subject which can be traced back to before the year 1900.

Specifications for the design of web reinforcement have been based on a modified form of the "truss analogy". The concepts of the truss analogy were expressed by Ritter as early as 1899, and likened the behavior of a concrete beam with web reinforcement to a truss in which the concrete compression region is the top chord, the tension reinforcement is the bottom chord, the stirrups are the tension web members, and the parts of the concrete web between inclined cracks are the compression web members. The modified form of the truss analogy considers that the concrete compression region carries a part of the total vertical shear, in addition to the stirrups. While the truss analogy had provided a safe although non-uniform and overly conservative basis for design, it had not provided a satisfactory explanation of the effects of shear on the behavior and failure of concrete beams.

The development of prestressed concrete increased interest in the problem of shear strength. Use of prestressing permitted the application of concrete to longer spans and heavier loads. Test data, and reasoning based on the concepts of diagonal tension, indicated that prestressed concrete beams had a greater shear strength than reinforced concrete beams, but the extent to which it was greater was not evident from the truss analogy.

1.2 **PREVIOUS WORK**

The ultimate strength of prestressed concrete beams has been under investigation at Lehigh University since 1951. Eney and others (1,2)
started this work by carrying out tests analyzing the behavior of full-sized pretensioned and post-tensioned concrete beams under simulated highway traffic. Walther\(^{(3,4)}\) initiated the Lehigh study of ultimate shear strength of concrete beams under the combined action of bending and shear in 1957 with an analytical idealization of the conditions which exist when a beam without web reinforcement fails in compression above the apex of an inclined crack. Tests on 20 reinforced and pretensioned prestressed concrete beams without web reinforcement were reported by Walther and Warner\(^{(5)}\) in 1958. These tests were used to study the effect on ultimate shear strength of variation in the magnitude of the prestress force and variation in the bond conditions between the steel and the concrete. In 1962 McClarnon, Wakabayashi, and Ekberg\(^{(6)}\) reported the results of 28 tests on beams of prestressed and conventionally reinforced design without web reinforcement. These tests considered the effect on ultimate shear strength of length of overhang at the reaction, existing inclined cracks, and the manner in which the load is introduced into the beam.

Hanson and Hulsbos\(^{(7)}\) extended the work at Lehigh University to pretensioned prestressed beams with web reinforcement when they reported the results of 18 tests on I-beams in 1963. Sixteen of these tests were static tests carried out to study the overload behavior of prestressed beams with web reinforcement. The remaining two tests were repeated load tests carried out to determine if a prestressed I-beam, once overloaded so that inclined cracks had formed, could subsequently be critical in fatigue of the web reinforcement under lesser loads.

Numerous investigations of ultimate shear strength have been carried out at other universities and research organizations. The majority of these investigations deal with the shear strength of reinforced concrete members. A thorough summary of the work done on reinforced concrete has been presented by ACI-ASCE Committee 426 (formerly 326)\(^{(8)}\).

There have also been several investigations of ultimate shear strength of prestressed beams without web reinforcement. Zwoyer and
Siess(9) and Sozen, Zwoyer, and Siess(10) have reported the results of tests on 99 pretensioned beams having both rectangular and I-shaped cross-sections. Evans and Schumacher(11) carried out shear tests on 54 post-tensioned beams also having both rectangular and I-shaped cross-sections. Others who have done experimental work include Warner and Hall(12) and Evans and Hosny(13), the latter having also carried out tests on beams with web reinforcement.

Investigations of ultimate shear strength of prestressed beams with web reinforcement are limited. Hulsbos and Van Horn(14) carried out 33 tests on pretensioned I-beams without end blocks investigating inclined cracking strength. Bernhardt(15) has investigated diagonal tension in post-tensioned beams.

The two most significant investigations which relate directly to this investigation, in addition to the prior work at Lehigh University on beams with web reinforcement, were carried out at the University of Illinois and the Portland Cement Association Research and Development Laboratories. Hernandez(16) conducted 37 tests on pretensioned rectangular and I-shaped beams. MacGregor(17) continued this work by testing an additional 50 beams and analyzing the combined results of the 87 tests. Principal variables in these tests were the amount, type, and spacing of the web reinforcement, and the profile of the longitudinal reinforcement. Other variables included prestress level, amount of longitudinal reinforcement, concrete strength, and type of loading. Recommendations for design of web reinforcement based on these tests were made by Hernandez, Sozen, and Siess(18). Mattock and Kaar(19) have reported the results of 14 tests on continuous composite pretensioned beams which were 1/2 scale models of AASHO-PCI Type III bridge girders. Their test program investigated the influence on ultimate shear strength of amount of vertical web reinforcement and location of the applied loads.

1.3 OBJECT AND SCOPE

The degree of safety that a concrete structure has against failure cannot be evaluated from concepts which limit the stress within the
structure to specified allowable values. The degree of safety, and therefore the adequacy of the design, depends only upon the magnitude of load causing some response which is incompatible with the intended purpose of the structure. The limiting response may be either a static or fatigue failure, a condition of instability, or an excessive deflection. For typical prestressed concrete bridge structures, the degree of safety generally depends upon the static ultimate strength of the structure. However, as the magnitude of axle loads, number of repetitions, and overloads on bridge structures increase, the degree of safety may depend on the fatigue strength of the structure.

The difficulty in defining the degree of safety that a prestressed concrete beam has when shear is critical is evident from the large number of investigations cited in Section 1.2. The reasons for this difficulty are also evident. Shear is not a problem in prestressed beams until inclined cracking occurs. When inclined cracking does occur, the behavior of the member is completely changed. Additional load carrying capacity is dependent upon the amount of web reinforcement provided. If shear is critical, inclined cracking leads to a shear failure, which may occur in many different ways.

The objective of this investigation is the evaluation of static ultimate shear strength in prestressed concrete beams. Thirty-eight tests on 23 simply-supported I-beams which are representative of precast prestressed girders used in Pennsylvania are presented and analyzed. The two principal variables are the amount of web reinforcement and the shear span to effective depth ratio. Other factors, in particular concrete strength and prestressing, were held as nearly constant as possible. Based on the results of these and other tests, a method is recommended for predicting the ultimate shear strength of prestressed concrete bridge girders with web reinforcement.

While this investigation is similar to the other investigations of prestressed beams with web reinforcement cited in Section 1.2, the tests reported herein have several significant features. Both concentrated and uniform load tests are included. Shear failures were obtained
in all but one of the concentrated load tests, on shear span to effective depth ratios which ranged between 2.12 and 7.76. Twenty-four of the 35 shear failures obtained in the concentrated load tests occurred on shear span to effective depth ratios greater than 4, which is the range in which the fewest shear failures have been reported. The 35 shear failures were obtained in tests on 21 beams, by means of a re-loading procedure which made it possible to obtain two tests on 15 of the beams. In addition, on three of the reload tests, the length of the shear span was increased so as to partially eliminate the restraint that the load point may have had on the critical inclined crack. Shear failures were obtained in both uniform load tests, on span to effective depth ratios of 10.6 and 14.8. Instead of simulating a uniform load by placing concentrated loads close together, a very nearly ideal uniform load was achieved by introducing the load into the test beams through fire hoses filled with water.
2. **TEST SPECIMENS**

2.1 **DESCRIPTION**

The doubly symmetric I-shaped cross-section used for all 23 test beams had a flange width of 9 in., a total depth of 18 in., and a flange to web width ratio of 3. An elevation view of the test beams, referred to as the F Series, is shown in Fig. 1. The properties of the cross-section, based on the concrete section and the transformed section, are also given in Fig. 1. A ratio of 6 between the modulus of elasticity of steel and concrete was assumed to determine the transformed section properties.

The total length of each beam consisted of a test span and two adequately reinforced anchorage regions of one ft length at each end. Except for the uniformly loaded beams, F-17 and F-18, the test span was divided into three regions, designated as A, B, or C, in which different amounts of vertical web reinforcement were provided. The amount of web reinforcement in different regions may be compared by the vertical web reinforcement ratio, \( r_f / 100 \), which is given in Table 1. Size and spacing of the web reinforcement are also given in Table 1. In the two uniformly loaded test beams, only one size and spacing of web reinforcement were used throughout the test span. Each stirrup consisted of either one or two U-shaped bars, referred to as S or D, respectively. Where only one bar was used, each successive bar was placed so that the U opened to the opposite side of the test beam.

Prestress was provided by six 7/16 in. diameter high tensile strength strands which were straight throughout the length of the test beams, providing a longitudinal reinforcement ratio of 0.64 percent. Each strand was pretensioned to a nominal initial force of 18.9 kips, providing a total initial design prestress force of 113.4 kips. Assuming losses of 8 percent in the prestress force at transfer, the initial stresses in the top and bottom concrete fibers, based on the transformed section and neglecting dead weight, are 210 psi tension and 2150 psi compression respectively.
2.2 MATERIALS

The strength of concrete was not a variable in these tests. Consequently a mix was selected which was representative of the high strength type of mix used by commercial prestressing plants. The mix, containing 7.5 bags per cubic yard of National Cement Co. brand Type III portland cement, was obtained from a ready-mixed concrete supplier. Proportions by weight of the cement to sand to coarse aggregate were 1 to 2 to 2.2. The sand was obtained by the supplier from a natural sand deposit located at Upper Black Eddy, Pa. The coarse aggregate, graded to 3/4 in. maximum size, was crushed limestone obtained by the supplier from Bethlehem Steel Co. Gradation curves, shown in Fig. 2, were determined from samples of the sand and crushed limestone obtained at the concrete plant. The fineness modulus of the sand was 3.1. The mix was delivered in a ready-mix truck in one cubic yard batches, and was dry mixed at the laboratory before water was added. Slump for all of the mixes varied between one and one half and four inches.

Each test beam was cast from a different batch of concrete. Compression tests were conducted on 6 by 12 in. cylinders, which had been taken from each batch of concrete, to determine the ultimate compressive strength of the concrete, $f'_C$, associated with the test beams at the time of prestress release and at the time of test. Strains were measured on selected cylinders with a compressometer to determine the shape of the stress-strain curve and the modulus of elasticity of the concrete at the time of test.

As a measure of the tensile strength of the concrete, tests were conducted to determine the modulus of rupture strength of the concrete, $f'_R$, and the splitting tensile strength of the concrete, $f'_{sp}$, associated with the test beams at the time of test. The modulus of rupture tests were conducted on plain concrete beam specimens having a 6 by 6 in. cross-section and loaded at the third points of a 30 in. span. The splitting tensile tests were conducted on standard 6 by 12 in. cylinders. Strips of plywood about 1/8 in. thick, 1 in. wide, and 12 in. long were placed on the diametrical upper and lower bearing lines of the cylinder to ensure uniform bearing in the splitting test.
The age and strength properties of the concrete described in the preceding paragraph are presented in Table 2. The ultimate compressive strength of the concrete in the test beams at test, as determined from the cylinder tests, ranged between 5790 and 7410 psi; the average value of $f'_c$ for all of the test beams was 6560 psi. The values of $f'_c$ at transfer and $E_c$ and $f'_s$ at test are an average of three tests. The values of $f'_c$ and $f'_s$ at test are an average of six or more tests. As representative stress-strain curves of the concrete, the results of the three compressometer tests associated with F-14 at test are shown in Fig. 3.

Uncoated stress relieved 270 ksi strand, meeting the requirements of ASTM A416-59 specifications, was used for prestressing. The 7/16 in. diameter strand was manufactured and donated to the project by Bethlehem Steel Co. A tension test on the strand was conducted in the laboratory, from which the load-strain curve shown in Fig. 4 was plotted. The strand failed in the grips at an ultimate load of 29.6 kips and a strain of 2.32%. A strand test report by the manufacturer stated that the strand had an area of 0.1113 sq. in., and failed in a tension test at a breaking load of 31.0 kips and a strain of 6.32%. The surface of the strand was free from rust.

The web reinforcement was fabricated from hot rolled No. 3 or No. 2 deformed bars, or from annealed 3/16 in. diameter deformed masonry bars. The No. 3 bars were received in two lots. Tension tests were conducted on six randomly selected specimens taken from each lot. The average yield point, $f'_y$, and ultimate tensile strength, $f'_u$, determined from the two lots agreed within one percent. Individual test values differed from the average by a maximum of 3 percent. The combined average values of $f'_y$ and $f'_u$ were 52,000 psi and 78,300 psi, respectively, based on an area of 0.11 sq. in. A typical stress-strain curve for the No. 3 bar is shown in Fig. 5(a).

The No. 2 deformed bars were received in a single lot. A total of twelve tension tests were conducted on randomly selected specimens. Based on an area of 0.049 sq. in., the average value of $f'_y$ was 59,500.
psi and the average value of $f_u$ was 85,700 psi. Individual test values again differed from the average by a maximum of 3 percent. A typical stress-strain curve for the No. 2 bar is shown in Fig. 5(b).

After an extensive investigation, which included more than 40 tension tests on 7/32 in. diameter hot rolled annealed smooth bars and 8 and 10 gage cold drawn annealed wire specimens, 3/16 in. diameter deformed masonry bars were selected for the stirrups in the beams with the smaller amounts of web reinforcement. The deformed masonry bars were manufactured from ASTM A82-34 cold drawn steel wire by Dur-O-Wal Products, Inc., and were donated to the project. The bars were received in straight pieces 10 ft in length. Since A82-34 wire has a high yield strength and low ductility, it was necessary to anneal this wire to obtain stress-strain characteristics comparable to the No. 3 and No. 2 hot rolled deformed bars. A total of 45 tension tests were conducted to determine which of three heat treatment temperatures - 1100, 1200, or 1300 degrees Fahrenheit - and which of two processes - air cooled or furnace cooled - were most acceptable. Based on these tests, the annealing treatment of 1 hour at 1100 degrees Fahrenheit followed by air cooling was used. Since the size of the electric furnace limited the number of specimens, each 2 ft in length, which could be heat treated at one time to approximately 30, it was necessary to group the bars in 14 different lots. After the heat treatment, 3 or 4 specimens from each lot were tested to determine $f_y$ and $f_u$. The average values of $f_y$ and $f_u$ determined for each lot agreed within 5 percent. Individual test values differed from the lot average by a maximum of 4 percent. The combined average values of $f_y$ and $f_u$ were 43,200 psi and 56,000 psi, respectively, based on a net area of 0.0234 sq. in. A typical stress-strain curve for the 3/16 in. diameter deformed masonry bars after the annealing treatment is shown in Fig. 5(c). Before being fabricated into stirrups, the bars were placed in a heated pickling bath consisting of half hydrochloric acid and half water just long enough to loosen the mill scale resulting from the annealing operation. The loose scale was removed with a wire brush, after which the bars were rinsed in water, dried, and stored until used.
From Fig. 5 it can be seen that all three types of web reinforcement have similar stress-strain characteristics. However, the stress-strain curve for the 3/16 in. diameter annealed masonry bar exhibited an erratic yield plateau. Also, a 3 minute stop in loading indicated a lower yield point approximately 10 percent less than $f_y$, compared to a similar reduction of only approximately 5 percent in $f_y$ for the No. 3 bar. These effects are due to the cold worked deformations in the masonry bar, whereas the deformations in the No. 3 and No. 2 bar were introduced in the rolling operation. A rate of loading of either 0.05 or 0.1 in. per minute until the onset of strain hardening was used for all of the tests.

2.3 FABRICATION

The test beams were made in a prestressing bed set up on the laboratory test floor. The sequence of operations was as follows: tensioning the strands, positioning the web reinforcement, form erection, casting the concrete, curing, form removal, instrumentation, and prestress release.

The strands were tensioned to approximately the desired value of 113.4 kips using two 50 ton mechanical jacks. If required, the tension in individual strands was adjusted by means of a special hydraulic jacking arrangement. The tension was measured by means of load cells placed on each strand, and the average variation from the desired value of 18.9 kips per strand was less than 0.2 kips.

The web reinforcement was tied to the strand with 14 gage wire. In addition, wire ties were used between successive projecting elements of the stirrups in the compression flange area and at approximately the mid-depth of the beam, in order to prevent movement of the stirrups during the casting operation.

Steel forms with 7 gage side plates bent to the shape of the section were used to cast the test beams. Dimensional checks made after the test beams were removed from the forms indicated that cross-
sectional dimensions were maintained to within 1/16 in., and consequently the nominal dimensions of the cross-section were used in all calculations.

The concrete was brought from the ready-mix truck to the forms in steel buggies and shoveled into the forms. The concrete was placed in two layers, the first layer extending approximately to the mid-depth of the beam. Eighteen or more standard concrete cylinders in waxed cardboard molds with tin bottoms and three 6 by 6 by 36 in. modulus of rupture specimens in steel forms were cast with each beam. The concrete in both the test beams and the modulus of rupture specimens was vibrated; the cylinders were rodded.

All specimens were covered with wet burlap and plastic sheeting for a period of 4 days, after which the forms were removed. After the surface of the test beams had dried Whittemore targets, described in the next section, were positioned on the test beams. The prestress force was slowly transferred into the test beams on the fifth day after casting, following which the beams, modulus of rupture specimens, and cylinders were stored in the laboratory until tested.

2.4 INSTRUMENTATION

Deformation data was taken on all of the test beams with a 5 in. and a 10 in. Whittemore Strain Gage. Two different types of gage points were used. For the first few test beams, the gage points were made by cutting 1/16 in. aluminum plate into 3/8 in. square pieces. Prior to cutting, each individual target was center punched and drilled with a No. 56 drill. For subsequent test beams more satisfactory brass plugs were obtained which were 7/32 in. in diameter and 3/32 in. in thickness. The brass plugs were placed in a jig and drilled with a No. 1 center drill. In either case, the drilled holes did not go completely through the target. The targets were cemented to the test beams with an epoxy resin known as Armstrong Adhesive A-6.

Type Al SR-4 electric strain gages were used to measure compressive strains on the top surface of F-20, F-21, and F 22. The gages
were bonded to the concrete surface with Duco cement. A portable grinder was used to smooth the concrete surface before the gage was applied.

2.5 PRESTRESSING

The initial prestress force, $F_1$, was measured by means of pre-calibrated load cells placed on each strand, and is given in Table 3. Data was taken to determine experimentally the losses in the prestress force after transfer and at the time of test. This was determined from Whittemore readings taken on the surface of the test beams, using the targets shown in Fig. 1. Readings were taken just prior to transfer of the prestress force into the test beams, immediately after transfer, and again just prior to the actual testing of the beam. The difference between these readings, converted to concrete strain, was plotted against location along the length of the test beam. A typical example of this work is shown for F-14 in Fig. 6.

Assuming that the concrete strain measured on the surface of the test beam at the cgs is equal to the average strain loss in the strand, the loss in the prestress force can be determined from the stress-strain curve of the strand. Losses in the prestress force after transfer and at the time of test determined in this manner are given in Table 3. Based on these losses the prestress force in each beam at the time of test, $F_2$, was established, and is given in Table 3.

The plot of concrete strain along the cgs was also used to estimate the distance from the ends of the beam to the point at which 85 percent of the prestress force was effective. Transfer distances for all of the test beams determined in this way are given in Table 3.

Whittemore readings on the targets 1 in. below the top fibers were used in conjunction with the readings along the cgs to determine the strain distribution in the test beams after transfer and at test. An example of this work is shown in Fig. 7 for F-14. Assuming that each strand was initially prestressed to the nominal value of 18.9 kips, corresponding to a strain of 0.652 percent, the effective strain in the
strand located at level 2 in F-14 would be \(0.652 - 0.131\), or 0.521 percent. The effective prestress strain at all three strand levels was determined for all of the test beams, and is given in Table 4.
3. CONCENTRATED LOAD TESTS

3.1 PROCEDURE

Concentrated loads were applied to all of the test beams except F-17 and F-18. Two different loading arrangements were used. All of the concentrated load tests except F-20, F-21, and F-22 were loaded using the arrangement shown in Fig. 8. These beams were first tested using a two point loading system which provided a constant or nearly constant moment region in the center of the beam. Using this arrangement, shear failures occurred in Region B for every test except F-9, in which case the shear failure occurred in Region A. After completion of the first test, the physical appearance of the part of the beam away from the failure region indicated a high degree of recovery. Flexure and shear cracks were closed, and noticeable camber remained. Consequently a second test was conducted on the remaining intact part of all of these beams except F-6, F-15, and F-16, using a single point loading arrangement. Second tests of this type could not be carried out on F-6, F-15, and F-16 because the length of Region C was too short. Shear failures were obtained in Region A for every second test except F-9, in which case the shear failure occurred in Region B.

The concentrated load tests on F-20, F-21, and F-22 were carried out using the arrangement shown in Fig. 9. The three point loading system provided a short constant moment region adjacent to Region B. Failures occurred in this region in all three tests.

Additional tests - second tests on F-20, F-21, and F-22 and additional tests on the other beams subjected to concentrated loads - were carried out whenever a sufficiently large intact part of the beam remained. However, the shear strength of the beams in the majority of these tests was less than expected in comparison to the tests described in the preceding paragraphs. This reduced shear strength was attributed to yielding of the strand in preceding tests, inclined cracks developing across the existing flexural or inclined cracks, and bond failures of the strand. Consequently none of these test results are included in this report.
Loads were applied in a 300,000 lb capacity Baldwin testing machine. Details of a typical set-up are shown in Fig. 10. Load was applied in increments of approximately 5 percent of the load expected to cause failure. The load increment was reduced when near loads at which flexural cracking, inclined cracking, or failure was expected. Following the failure in the first test, the beam was removed from the testing machine. Sledge hammers were used to break up the concrete in the failure region, and the strand and any web reinforcement not fractured in the first test were cut by an acetylene torch. The remaining part of the beam was replaced in the testing machine and the second test started. A complete test on a beam took approximately 8 hours to complete. Cylinder and modulus of rupture specimens were tested immediately after the beam test.

Load deflection readings were taken after the application of each load increment, by means of level readings on targets graduated to the nearest .01 in. The targets were attached to the web of the beam with double stick tape at each support and at the centerline of the testing machine. Measurements from the end of selected beams to masking tape attached to the protruding strand were used to check if strand slip occurred. Whittemore readings were taken at selected load levels during the first test, and just prior to starting the second test. A record was kept of the loads at which flexural and inclined cracking was observed and at which failure occurred. The development of the crack patterns was marked on the test beams after the application of each load increment. Photographs were taken during and after testing.

3.2 PRINCIPAL TEST RESULTS

The lengths of the shear spans and the principal results of the first tests conducted on beams subjected to concentrated loads are presented in Table 5. $M_{cr}$ is the maximum applied load moment in the test beams at the time that flexural cracking was first observed. $V_{ic}$ is the shear, in the respective shear span, causing the formation of significant inclined cracking which ultimately was associated with
failure. Close attention was directed to the selection of the inclined cracking shears, and the values selected are discussed in detail in Section 3.3. Inclined cracking shears were not selected for F-20, F-21, and F-22 because the failure was different than the other beams. $V_u$ is the ultimate shear in the critical shear span, which was Region B in every case except F-9, in which case a shear failure occurred in Region A. The values of $V_{ic}$ and $V_u$ in Table 5 are applied load shears. Modes of failure are indicated by WC for web crushing, SF for stirrup fracture, SC for shear compression, and F for flexure. The failure mechanisms are described in detail in Sections 3.3 and 3.4. No strand slip was observed in any test.

Span lengths and results of the second tests conducted on the beams subjected to concentrated loads are presented in Table 6. The ultimate shear, $V_u$, in the shear span in which the failure occurred was in Region A in every case except F-9, in which case the failure occurred in Region B.

3.3 Behavior and Modes of Failure (First Tests)

Prior to the detection of any cracking, the response to load was essentially linear. Except for F-1, cracking manifested itself by the appearance of flexural cracks in the constant or nearly constant moment region of the beam. With additional load, inclined cracks formed in the shear spans of the test beams. Inclined cracking appeared in the relatively short shear spans of F-1 prior to the development of any flexural cracking in the beam.

Load-Deflection Curves

The general characteristics of the behavior of the test beams are shown by the load-deflection curves in Fig. 11. These curves are grouped according to the critical shear span. Each group associated with a particular shear span is arranged from left to right by decreasing amount of web reinforcement, indicated by the value of $f_{y}/100$ in parenthesis after the beam number.
Direct comparison of the load-deflection curves is difficult, because the span length was different for many of the test beams. For example, although F-2 had more web reinforcement than F-3, the deflection at failure is less than for F-3 primarily because the span length is shorter. Furthermore, the deflection was always measured at the centerline of the testing machine, which was not in all cases at mid-span. For F-9, F-11, F-13, F-16, and F-19, the centerline of the testing machine was 5 in. from the mid-span of the test beam.

However, all of the load deflection curves exhibit similar characteristics. The initial part of the curves up to approximately one-half of the ultimate load are linear, corresponding to the uncracked loading range on the test beams. The sharpest change in slope in the load-deflection curves occurs just after flexural cracking which, for all of the test beams except F-1, marks the transition from the uncracked to the cracked loading range. Following the transition region, the load-deflection curves become quasi-linear to failure.

The loads at which flexural and inclined cracking occurred have been marked on the load-deflection curves of F-1, F-3, F-5, F-10, F-14, and F-16, indicated by FC and IC, respectively. The shears at flexural cracking occurred at the same relative position on the load-deflection curves; that is, the flexural cracking occurs just after a slight amount of curvature can be detected at the end of the linear region of the load-deflection curve. The shears at inclined cracking, however, occur at no particular place on the load-deflection curve. Inclined cracking did not cause any abrupt change in the slope of the load-deflection curve. It is evident from the load-deflection curves that the presence of web reinforcement allowed the beam to sustain a greater deflection. This latter characteristic is particularly important because it is a measure of the ductility of the member.

**Flexural and Inclined Cracking**

Flexural cracking occurred in the test beams when the stress in the bottom fibers in tension reached values which are normally associated
with the tensile strength of the concrete. The flexural cracking was characterized by its initial development to a level which varied between the lower strand and the mid-depth of the beam, but in general was near the center of gravity of the strand. Spacing between flexural cracks varied between 1 and 8 in. However, cracks which formed closer together than approximately 2 in. would usually merge, or the further development of one of the two cracks would be circumvented. There was a definite tendency for the predominant flexural cracks to be located close to vertical stirrups for stirrup spacings up to approximately 7 in.

Sketches of the crack patterns in the test beams at the shear causing significant inclined cracking, \( V_{ic} \), are shown in the elevation views in Appendix I. The sketches were reconstructed from photographs taken during testing. All cracking which had occurred in the test beam to that load is shown by heavy solid lines. Note, of course, that each end of each test beam could and in general did have a different inclined cracking load. The load at which flexural cracks in the sketches were first observed is indicated by the value of shear in the shear span written directly below the crack. If the crack extended downward from the web to the bottom fibers there is no value of shear written below it. The location of the vertical web reinforcement is shown in the conventional manner.

Critical inclined cracking was not considered to have occurred until the second test on the A end of F-10 and F-12. For these two cases, cracking which occurred during the second test is indicated by the heavy dashed lines.

Principal tensile stresses shown in the web were calculated, using the properties of the transformed section, at the intersection of the grid lines within the shear span and the junction of the web and top flange, the mid-depth of the beams, and the junction of the web and bottom flange. It was assumed that the state of stress in the web was defined by a horizontal normal stress (compression positive) and a shearing stress, and that the vertical normal stress was zero. Therefore the principal tensile stress was calculated from the equation:
where the normal stress was calculated from:

\[
\sigma_{pt} = \sqrt{\left(\frac{\tau}{2}\right)^2 + \sigma^2} - \frac{\sigma}{2}
\]  

and the shearing stress was calculated from:

\[
\tau = \frac{(V_{ic} + V_d)Q}{fb}
\]

Flexural stresses were also calculated at the intersection of the grid lines and the bottom fibers using Eq. 2. The origin of the coordinate system referred to in Eq. 2 is taken at the intersection of grid line 2, shown in Fig. 1, and the cg of the transformed section, x being positive when measured along the cg in the direction of grid lines with increasing magnitude and y being positive upwards. The slope of the compressive stress trajectory was calculated from:

\[
\theta = \frac{1}{2} \tan^{-1} \left( \frac{2\tau}{\sigma} \right)
\]

Light dashed lines in the web show the compressive stress trajectories.

Two basically different types of significant inclined cracking can be observed from the crack patterns shown in the figures in Appendix I. For beams tested on shear spans of less than 50 in., inclined diagonal tension cracking started from an interior point in the web of the beam. In general, the load indicated on the testing machine dropped off noticeably when diagonal tension cracking developed. Furthermore, the load at which diagonal tension cracking occurred was somewhat time dependent, indicated by the fact that cracking often occurred after the addition of a load increment, while the load was being held constant to take data.
The diagonal tension cracking shown in the A end of F-2 illustrates the typical characteristics of this type of cracking. In forming at a shear of 34.0 kips, the crack traversed the entire depth of the web, and consequently was nearly fully developed at the same load as it first appeared. Since there was no flexural cracking in the vicinity of the diagonal tension cracking, the state of stress in the web indicated by the principal tensile stresses and the compressive stress trajectories must be closely representative of the state of stress causing the inclined cracking. If the variation in principal tensile stresses along the path of the crack is estimated by interpolation, it is evident that the maximum principal tensile stress occurs close to the cg of the beam. Furthermore, this maximum principal tensile stress has a magnitude comparable to the modulus of rupture and splitting tensile strength of the concrete given in Table 2. The slope of the path of the crack also appears to have a close association with the slope of the compressive stress trajectory. Therefore this type of inclined cracking is due to excessive principal tensile stresses in the concrete, as inferred by the designation of diagonal tension cracking.

Another important feature of the diagonal tension cracking shown in the A end of F-2 is that this crack remained the critical crack in the shear span, and was primarily responsible for failure at a shear of 48.0 kips. In contrast, the principal diagonal tension crack in the A end of F-3 appears to have formed somewhat prematurely, having been influenced by the moment, and thus formed more closely toward the load point. It is significant, however, that it was the least developed of the three cracks shown in this shear span which continued to grow and which became the critical crack in causing the shear failure. In fact, when the shear had been increased from 31.0 to 34.0 kips this particular crack had extended completely across the web of the beam, and was very similar to the diagonal tension crack in the A end of F-2. A similar case, except that the diagonal tension crack formed unusually far back toward the reaction, is shown in the B end of F-X1. In this case the crack which completely traversed the web appeared first at a shear of
28.4 kips, and was immediately followed by the development of several short cracks at the junction of the web and top flange. The relatively low stresses in the web indicates that the cracking occurred somewhat prematurely. However, there is no indication from Table 3 that the transfer distance is any longer than usual, and therefore the crack was probably caused by a weak or non-uniform region in the concrete. Significantly, one of the several short cracks extended across the web suddenly at a shear of 32 kips, and was critical in causing the shear failure.

For beams tested on shear spans of 80 in. or greater, inclined cracking would develop from flexural cracks. A good example of this type of cracking, which will be referred to as flexure shear inclined cracking, is shown in the B end of F-16. This type of cracking was characterized by its association with a flexural crack, which would develop up to approximately the cgs and then turn and become inclined in the direction of increasing moment. The path of the inclined crack, as it traversed the web, roughly followed the direction of the compressive stress trajectory. Furthermore, flexure shear cracking remained in the vicinity of the load point, because the tensile stresses in the web were not high enough to precipitate spreading of the cracking throughout the shear span. In general, the development of significant flexure shear cracking was very rapid. As can be seen from the B end of F-16, going from a shear of 16 kips to 17 kips resulted in the development of a flexure shear crack which extended completely across the web.

Selecting a particular value of shear as the significant flexure shear inclined cracking load was a difficult problem. The criteria on which the selection of $V_{ic}$ in Table 5 was based was that the inclined crack had to be definitely associated with the mechanism causing the shear failure. This was done by studying photographs of the test beams taken before and after failure.

Inclined cracking which occurred in beams tested on shear spans of 50, 60, and 70 in. had characteristics which, in different cases, could be associated with either diagonal tension or flexure shear cracking. Consider as an example the 70 in. shear span of the B end of F-10.
The inclined cracking which occurred at the shear of 24.8 kips must have started from an interior point in the web of the beam, and consequently was characteristic of diagonal tension cracking. It is very unlikely that the flexural cracks between grid lines 5 and 6 formed before the inclined cracks in the web, because of the low values of stress in the bottom fibers at the location of the cracks. Rather the flexural cracks must have formed after the inclined cracking in the web, as the result of the increased stress in the strand where the inclined cracking penetrates to the level of the strand. However, before any conclusion is drawn that the inclined cracking in the B end of F-10 is diagonal tension cracking, it should be noted that the indicated principal tensile stresses in the web are lower than values which are normally associated with this type of cracking. This is because the state of stress in the web between grid lines 6 and 7 was substantially influenced by the flexure shear crack which had formed at a shear of 20 kips.

As another example, consider the 50 in. shear span of the A end of F-4. It is evident that the inclined cracking in the shear span must have initiated from an interior point in the web of the beam. Furthermore, the magnitudes of principal tensile stresses in the web are great enough to have caused diagonal tension cracking. However, the flexural crack to the left of grid line 6, which formed before the inclined cracking because of the high value of stress in the bottom fibers, probably precipitated the inclined cracking by acting as a stress raiser in the web above the flexural crack.

In five of the first tests on beams with relative small amounts of web reinforcement, failures occurred at the inclined cracking load. These failures, although they occurred suddenly and were catastrophic in those cases where the web reinforcement was fractured, did not occur at the instant the failure load was reached. Rather there was a period of up to several minutes after the last increment of load has been applied before failure occurred. During this period additional inclined cracking sometimes formed in the web.

In the remaining tests, enough web reinforcement had been provided to effect a re-distribution of forces in the beam after inclined cracking, and consequently a higher shear could be applied.
For beams in which diagonal tension cracking had occurred in the vicinity of a line extending from the reaction to the concentrated load point, relatively little additional inclined cracking would occur. However, if diagonal tension cracking had not occurred in this vicinity, additional cracking would usually form in this region as higher shears were applied. For beams in which flexure shear cracking had occurred, additional flexure shear inclined cracks would form if the stress in the bottom fibers back toward the reaction became high enough to cause a flexural crack.

Whittemore readings taken at the cgs on both sides of some of the beams provided an indication of the behavior between inclined cracking and the ultimate load. Concrete deformation along the cgs obtained in this way for two beams, F-4 and F-14, is shown in Figs. 12 and 13. Both figures show a more erratic deformation pattern in the shear span with the least amount of web reinforcement, Region B, although this is in part due also to the fact that the inclined cracking load was less on this end of the beam than the other end. As may be seen from the views of F-4 in Appendix I, the B shear span contained a single predominant inclined crack which initially formed at 32 kips and extended almost the full length of the 50 in. shear span. This crack, crossing the cgs between grid lines 2 and 3, was responsible for the peaked concrete deformation in this region, and indicates that the force in the strand had been suddenly increased by the inclined crack. Furthermore, such a deformation pattern indicates the need for adequate bond length from the point where the crack crosses the cgs to the end of the beam. Several inclined cracks formed in the A shear span of F-4 at a shear of 33.4 kips. However, only the flexure shear crack which had extended back down through the bottom flange had shown any appreciable effect on the concrete deformation along the cgs at the shear of 34 kips.

The erratic nature of the deformation pattern in the B shear span of F-14 is also due in part to the location at which the cracks crossed the cgs. As may be observed from the figures in Appendix I, inclined cracks crossed the cgs just to the right and left of the region between grid lines 8 and 9. If the crack which is furthermore from the load point had crossed the cgs to the left of line 8, the deformation
pattern in Region B would not have the extremely sharp peaks indicated, although it would still be more erratic than the deformation pattern in Region A. However, the observation that the average deformations between grid lines 7 and 11 are of the same order of magnitude as the deformations between the load points in the constant moment region indicates that the force in the strand is increased to approximately what it is in the center of the beam.

**Failure Characteristics**

Three different types of shear failures were observed in the first tests on beams subjected to concentrated loads. As indicated in Table 4, eight of the failures were designated as WC denoting that the apparent cause of failure was crushing of the concrete in the web of the beam. Ten failures were designated as SF to indicate that the apparent cause of failure was fracture of the web reinforcement. Both the web crushing and stirrup fracture failures occurred as the result of inclined cracks which remained entirely within the shear span. In contrast the two failures designated as SC to denote shear compression were caused by flexure shear cracks which extended into the constant moment region.

In general, the web crushing failures occurred gradually and were non-catastrophic. An example of a web crushing failure is shown in the 50 in. shear span of F-5 in Fig. 14. In these and all subsequent photographs, the location of the web reinforcement is indicated by dark vertical lines drawn on the web. The lighter irregular lines mark the crack patterns. Shears were marked on the cracks to show the load and extent of development when the crack was first observed, and thereafter to show any significant further development.

Inclined cracking occurred in Region B of F-5 at a shear of 27.9 kips. Increasing the shear to 31.0 kips caused the crack to extend to within a few inches of both the load point and the reaction, as can be seen in Fig. 14(a). Additional inclined cracking, shown in Fig. 14(b), appeared when the shear was increased to 32.2 kips. Immediately an area of localized crushing developed above the top of this new inclined crack, at the intersection of the web and top flange and located approximately
at the center of the shear span. At the same time a flexural crack developed in the top fibers above the area of localized crushing. With these indications of failure, the load being carried by the test beam, as indicated by the testing machine, dropped off about 10 percent. The load remained approximately unchanged as the beam was deflected further by the testing machine, until finally a compression failure occurred suddenly adjacent to the load point, as shown in Fig. 14(c).

Characteristics of the other web crushing failures were similar to the description above for F-5, except for F-6 and F-7. Pictures after failure of F-1, F-3, F-6, F-7, and F-10, tested on shear spans of 30, 40, 100, 60, and 70 in., respectively, are shown in Fig. 15. F-7 was different from the other web crushing failures in that after the failure had started and the load had dropped off about one-third, the 4th stirrup from the support fractured, as can be seen from the photograph. The mode of failure, however, was classified as web crushing.

Except for F-6, all of the web crushing failures in the first tests occurred on shear spans of 70 in. or less. The web crushing failure in F-6 occurred on a shear span of 100 in. The inclined crack running back toward the support first appeared at a shear of 19 kips, causing the load indicated on the testing machine to drop off. However, it was possible to reload to an ultimate shear of 19.1 kips before the failure shown in Fig. 15 occurred suddenly. In this case the region of localized crushing is almost directly over the reaction, and lower in the web than for any of the other web crushing failures. This particular failure is similar to the failure observed in nearly identical beams without web reinforcement in the E Series tests.

In contrast to the web crushing failures, the stirrup fracture failures occurred suddenly and were usually catastrophic. An example of a stirrup fracture failure is shown in the 80 in. shear span of F-13 in Fig. 16. Figure 16(a) shows the shear span after inclined cracking, at a shear of 21.8 kips. Additional inclined cracking formed at a shear of 23 kips. During this period the beam was unstable, because whenever inclined cracks formed in either the A or B shear span
the load indicated on the testing machine would drop off. The amount of drop off would vary considerably. However, in every instance it was possible to bring the load back up, until finally the failure due to fracture of web reinforcement occurred which is shown in Fig. 16(b), at a shear of 24.3 kips.

Characteristics of the stirrup fracture failures varied more than for the web crushing failures. Six additional stirrup fracture failures on shear spans of 50, 80, 80, 100, and 110 in. for F-4, F-9, F-12, F-15, and F-16, respectively, are shown in Fig. 17.

The shear failure in the 50 in. shear span of F-4, shown in Fig. 17(a), was caused by fracture of the 4th stirrup from the support. The fracture was located approximately 5 in. above the bottom of the beam, where the inclined crack, which can be seen in that vicinity from the picture, crossed the stirrup. In the 80 in. shear span of F-9 shown in Fig. 17(b), failure occurred when the 16th through 20th stirrups from the support fractured.

The failure in the 100 and 110 in. shear spans of F-15 and F-16 are shown in Figs. 17(d) and 17(e), respectively. The failure in F-15 looks like the web crushing failure in F-6. However, there was no
observable evidence of localized crushing in the web before the failure suddenly occurred. Examination of the beam after failure revealed that the 4th and 6th stirrups from the reaction had been fractured. The failure in F-16 was confined to the vicinity of the load point, with no cracking of any kind in evidence in the half of the shear span closest to the reaction. Complete collapse of the test beam occurred when the 10th through 13th stirrups from the support suddenly fractured. These stirrups can be located by counting back from the stirrup located at the centerline of the testing machine, indicated by the adjacent scale, which is the 15th stirrup from the support.

The shortest shear span on which a stirrup fracture failure occurred was the 50 in. shear span of F-4. The greatest proportion of the failures on the longer shear spans were stirrup fracture failures, although from the preceding description of the failures it is evident that the division between web crushing and stirrup fracture failures is sometimes indefinite. On the beams in which fracture of the web reinforcement occurred, the particular stirrups which were fractured are indicated in the figures shown in Appendix I by an X mark just above the top flange and directly over the stirrup.

The failures in F-20, F-21, and F-22, shown in Fig. 18, were similar in that the cause of failure was crushing of the concrete in the compression flange in the short constant moment region adjacent to the critical shear span. F-21 failed suddenly in flexure, which resulted in complete collapse of the member. In contrast, F-20 and F-22 failed at moments 4 and 9 percent less, respectively, than the moment causing failure in F-21. While the failures in F-20 and F-22 occurred suddenly, the two beams did not collapse. In both cases the failure occurred above an inclined flexure shear crack which had originated in the critical shear span. Consequently the failures in F-20 and F-22 were classified as shear compression.

Strain measurements on the extreme fibers in compression in the center of the short constant moment regions of F-20, F-21, and F-22 indicated that the strain at failure was approximately 0.50, 0.40, and 0.30 percent, respectively. These values, particularly the first two,
are greater than had been measured on similar beams in the E Series tests(7), and indicate that the short constant moment region resulted in a strain concentration in the concrete fibers in compression. Therefore the failures were probably influenced by the strain concentration. However, the ultimate flexural capacity of an "under-reinforced" prestressed beam is relatively insensitive to the strain in the extreme concrete fiber in compression, and so in the case of F-21 the only likely effect on the test is an insignificantly smaller ultimate moment than would have been obtained in a test on a beam with a longer constant moment region. In the case of F-20 and F-22, flexure shear cracking extended beneath the load point, and upon entering the region of the strain concentration precipitated a somewhat premature failure.

It is also of significance, in looking at the pictures of the failures in F-20 and F-22, to note that the web reinforcement provided in the critical shear span had little or no effect on the ultimate capacity. In both cases the critical flexure shear crack started approximately 8 in. from the load point. Consequently the first stirrup in the shear span probably was not effective in resisting the failure.

3.4 BEHAVIOR AND MODES OF FAILURE (SECOND TESTS)

As previously noted, after completion of the first test on a beam the physical appearance of the part away from the failure region showed a high degree of recovery. A close examination indicated that the flexural and shear cracks were closed and noticeable camber remained, indicating that substantial or even full prestress was retained in the beam. The only evidence of any damage was cracking, which in several beams could be detected extending from the top fibers downward. In most cases these were very fine cracks which occurred at a spacing of about 5 in. and extended from 1 to 2 in. into the compression flange. These cracks had the appearance of tension cracks, and since the beams were designed with a tensile stress of 210 psi in the top flange, it was considered that the suddenness of the first test failure induced these cracks to form. In two beams, F-12 and F-14, a single crack located approximately 12 in. from the load point and directly over the
top of an inclined crack, had formed and extended downward from the top fibers to a depth of between 3.5 and 5 in. This crack was closed when observed after the first test, indicating that it must have formed as a consequence of the first test failure.

Strain readings were taken on the Whittemore targets at the level of the cgs after completion of the first test. The readings were generally slightly larger than the same readings taken before the start of the first test. The slight increase was attributed to the fact that although the flexural cracks were completely closed, they could not be perfectly closed. However, it should be noted that if the strand were yielded in the first test, it would be possible to have an increase in strain indicated by the Whittemore readings which could correspond to a decrease rather than an increase in the prestress force. However, failures in the first tests were generally well below the ultimate flexural capacity.

Therefore, it was concluded that the conditions in the parts of the beam away from the first test failure region were good enough to conduct a second test. This was done on all of the test beams except F-6, F-15, and F-16. For these three beams the distance between the load points was not sufficient to permit a second test.

Load-Deflection Curves

The load-deflection curves in Fig. 19 of the beams subjected to a second test have essentially the same characteristics as the load-deflection curves for the first tests shown in Fig. 11. These curves are designated by beam number and amount of web reinforcement in parenthesis. The difference in relative slopes of the curves in the two figures is due to the shorter span lengths of the second tests. Also the length of the initial straight line part of the curves are not as long and shows more variation between beams than for the load-deflection curves for the first tests. Part of this is due to the cracking from the first test; consequently the flexural cracks re-open sooner in the second test. For those test beams in which the length of the straight line part of the curve is particularly short, for example F-9
and F-19, it is possible that some yielding of the strand in the first test and consequent loss of prestress force had occurred.

**Cracking and Failure Characteristics**

Only insignificant additional cracking occurred in loading the critical shear span in the second test up to the maximum value of shear that it had been subjected to in the first test. For those test beams having the same length of shear span in the first and second test, the cracking which occurred after the maximum shear in the first test had been reached was similar to that which had been described for the first tests. For F-11, F-13, and F-19, in which the length of shear span had been increased 10 in. in the second test, additional load caused some branching from the tops of the inclined cracks toward the load point.

In the shear span in the second test which was the constant or nearly constant moment region in the first test, inclined cracks developed across the flexural cracks. In general, this type of crack would form at a load slightly greater than the load causing the flexural cracking in its immediate vicinity to re-open.

In the fifteen second tests on beams subjected to concentrated loads, five shear failures occurred in the compression region of the concrete in the shear span, designated as SC in Table 6. The remaining ten failures were similar to those which occurred in the first tests, either web crushing or stirrup fracture.

The maximum length of shear span on which a web crushing failure occurred in the second tests was 60 in. Photographs of second tests in which web crushing failures occurred, on F-3, F-7, and F-19 tested on shear spans of 40, 60, and 50 in., respectively, are shown in Fig. 20. In these and all subsequent second test photographs, the crack patterns are marked in exactly the same manner as they were for the first tests except that any additional cracking in the shear span during the second test is marked by dashed rather than solid lines. The first indication of failure in F-3 was some slight spalling of concrete in the web, which occurred after the shear span had sustained the ultimate shear of 48 kips for several minutes. The spalling was accompanied by a drop of
roughly 8 percent in the load indicated on the testing machine. An attempt at bringing the shear back up to the ultimate shear was unsuccessful, as further spalling and crushing in the web took place, finally causing the compression flange to break as shown in the photograph. Failures in F-7 and F-19 were both initiated by crushing in the web at the junction of the web and top flange and by the development of a tension crack in the top fibers. The fact that the shear span for the second test on F-19 was 10 in. greater than for the first test had no apparent effect on the failure.

Stirrup fracture failure occurred in the second tests on F-2, F-10, F-11, and F-13. The characteristics of the failure in F-2, tested on a shear span of 40 in., were similar to those of a web crushing failure. The failure occurred suddenly, but only a single stirrup was fractured and the beam did not collapse. Photographs of the second tests on F-10, F-11, and F-13, tested on shear spans of 70, 70, and 80 in., respectively, are shown in Fig. 21. These suddenly occurring failures are different from the stirrup fracture failures obtained in the first tests. In fact, the appearance of the beams would suggest that the failure should be classified as shear compression. However, an examination of the beams after failure showed that fracture of the web reinforcement had occurred. Counting from the support, the 16th and 17th stirrups in F-10, the 6th stirrup in F-11, and the 21st, 22nd, and 23rd stirrups in F-13 were the fractured bars. In every case, the fractured web reinforcement is located in the region where the critical inclined crack penetrates the top flange, or in other words, near the top of the inclined crack just preceding failure. Note that F-11 and F-13 in these tests had been loaded in the second test on a shear span which was 10 in. longer than the first test.

The five beams which failed in shear compression in the second test were F-5, F-8, F-9, F-12, and F-14. All five of the failures were similar, the region of failure being adjacent to the load point and entirely in the shear span. Pictures of the failures in F-5, F-9, and F-12 tested on shear spans of 50, 90, and 80 in., respectively, are shown
in Fig. 22. The failures in F-5 and F-8 occurred suddenly, whereas there was some warning of failure in F-12 and F-14 by the development of several inclined cracks in the web spreading progressively toward the reaction. The formation of the inclined cracks in the latter two beams resulted in a drop in the load indicated on the testing machine, and in attempting to bring the load back up the failure occurred. Spalling of concrete in the top fibers adjacent to the load point was observed prior to the failure in F-9. In this case the load began to drop off slowly, and after it had dropped off about 10 percent the 12th stirrup from the support broke. This stirrup can be located if it is noted that the stirrup directly below the load point is the 15th stirrup from the support.
4. **UNIFORM LOAD TESTS**

4.1 **PROCEDURE**

Uniform loads were applied to F-17 and F-18, using the arrangement shown in Fig. 23. A similar arrangement has been used by Leonhardt and Walther (20). Two salvage fire hoses filled with water were centered on the top flange of the beam. The ends of the fire hoses at one end of the beam were capped. A common connection was provided at the other end, by means of elbows connected to the end fittings. Four 8WF loading beams, each equal in length to one-fourth of the test span, were placed on top of the fire hoses. The adjacent ends of the loading beams were cut at a slight angle to prevent interference when the test beam deflected. Lateral bracing was clamped to the top flanges of the end two loading beams, and pin connected at its other end to the columns in the loading frame. Lateral displacement between the ends of adjacent loading beams was prevented.

A photograph of the test set-up is shown in Fig. 24. Details of the reactions are similar to the details for the concentrated load tests, except that the width of bearing was 9 in. A tarpaulin was placed between the loading beams and the fire hoses for protection of the fire hoses. Also, the loading beams were tied together loosely by means of ropes to prevent them from falling in case of complete collapse of the test beam. Load was applied by means of 55 kip Amsler hydraulic jacks connected to a loading frame which was pretensioned to the floor of the laboratory.

Several trial runs up to approximately one-half of the flexural cracking load were required to properly position the jacks on the loading beams. Additional lateral bracing would have been desirable. In the actual test, load was applied in increments of one kip on each loading beam, equivalent to two kinds of each reaction. A test took approximately three hours to complete. Further tests were conducted on the remaining intact part of the beams after failure, but the results were considered affected by the first test and are not included in this
report. Cylinder and modulus of rupture tests were conducted on the same day as the beam test. Data taken during the test was the same as that for the concentrated load tests.

4.2 PRINCIPAL TEST RESULTS

F-17 and F-18 were tested on span lengths of 12 ft - 6 in. and 17 ft - 6 in., respectively. It was evident during the test that a nearly perfect distribution of load was obtained, and therefore the uniform load, \( w \), was determined as four times the jack load divided by the span length. Flexural cracking was first observed at a load of 5.1 kips per ft in F-17, corresponding to a net flexural cracking moment, \( M_{cr} \), of 100 kip-ft, and at a load of 2.7 kips per ft in F-18, corresponding to \( M_{cr} \) equal to 104 kip-ft. Inclined cracking appeared in both test beams initially as flexure shear cracking, and subsequently at higher loads as inclined cracking precipitated by the formation of a flexural crack. Diagonal tension cracking appeared in the maximum shear region adjacent to the reactions of F-17 at loads of 7.4 kips per ft in one end and 7.7 kips per ft in the other end.

F-17 failed in shear compression at an ultimate load of 8.6 kips per ft. No stirrups were broken. Failure occurred at approximately the third point of the span. Spalling of the top concrete fibers near mid-span was observed just prior to failure. F-18 collapsed suddenly when several stirrups fractured at a load of 4.7 kips per ft. However, F-18 had sustained a maximum load of 4.8 kips per ft before it became necessary to unload and adjust the stroke of the jacks.

4.3 BEHAVIOR AND MODES OF FAILURE

The behavior of F-17 and F-18 is shown by the load-deflection curves in Fig. 25. Both curves show that the response to load was linear up to approximately one-half of the ultimate load. The loads at which flexural cracking was first observed are shown as FC, and occur just after a slight amount of curvature can be detected at the end of the linear region of the curve. The transition region in which the sharpest change of
slope occurs is immediately after flexural cracking for F-18, but delayed somewhat for F-17. After the transition region the curves become quasi-linear to the ultimate load.

In the case of F-18, lack of stroke in the jacks necessitated unloading the beam, adjusting the jacks, and reloading to failure. At the maximum load of 4.8 kips per ft in the first load cycle, which was the highest load that the beam sustained and therefore regarded as the ultimate load, F-18 had nearly reached its computed flexural capacity of 4.9 kips per ft. Furthermore the appearance of the beam indicated that failure was imminent, but when the limiting displacement of the jacks was reached the beam had to be unloaded. No deflection readings were taken during the unloading cycle, which consequently is indicated as a dashed line from the deflection at the end of the first test to the deflection at zero load. The recovery of the beam was excellent. Flexural and shear cracks were closed, indicating that a major part or all of the prestress force remained, and the residual deflection was only 0.42 in. F-18 was rapidly loaded to failure in the second cycle, with only deflection readings taken at jack load increments of 5 kips.

Flexural cracking was first observed in F-17 at a load of 5.1 kips per ft, corresponding to a computed tensile stress in the bottom fibers at mid-span of 850 psi, or $10.2/f'_c$. In F-18, flexural cracking was first observed at a load of 2.7 kips per ft, corresponding to a computed tensile stress in the bottom fibers at mid-span of 1010 psi, or $12.1/f'_c$. The predominantly flexure cracks were confined to a relatively narrow region on either side of the centerline of approximately 10 in. for F-17 and 20 in. for F-18. Outside of this region any cracking which began as a flexural crack showed the influence of shear by turning and becoming inclined in the direction of increasing moment.

Both flexure shear and diagonal tension inclined cracking was observed in the tests. Failure in both beams, however, was the result of flexure shear cracking. The selection of a particular load as the significant inclined cracking load was difficult, because of the rela-
tively wide region in which the failure occurred. The loads which were selected should be regarded as the approximate inclined cracking load. Sketches of the crack patterns in F-17 and F-18 at these loads are shown in Figs. 26 and 27. All quantities and symbols have exactly the same meaning as the similar figures drawn for the concentrated load tests, except that the numbers written below the beam now indicate the load, in kips per ft, at which the crack directly above was first observed.

The diagonal tension cracking adjacent to the reaction in the left half of F-17 occurred suddenly at the load of 7.4 kips per ft. By extrapolation, the critical principal tensile stress at the center of gravity of the section can be estimated as 750 psi, or $9.0\sqrt{f_c'}$. The critical failure region appeared to be located in the region adjacent to grid line 7 in the left side of the beam, and therefore either of the cracks beginning at 6.7 or 7.4 kips per ft could have been instrumental in causing failure. The diagonal tension cracking in the right end of F-17 also occurred suddenly, at the load of 7.7 kips per ft. Assuming that the crack closest to the reaction formed first, the critical principal stress can be estimated as 740 psi, or $8.9\sqrt{f_c'}$. The diagonal tension cracking at either end did not, at any time during the test, appear to be critical.

No inclined diagonal tension cracks formed near the reactions in F-18. Only flexure shear cracking developed in the interior part of the span. Failure occurred in the right half of the span, located principally in the region between grid lines 8 and 9. Therefore any, or perhaps all three, of the flexure shear cracks forming at 3.4, 3.6, and 4.1 kips per ft could have been critical in causing failure. The path of the flexure shear crack, for which the principal stresses and stress trajectories are determined, follows closely the slope of the stress trajectory.

Photographs of each half of F-17 after failure are shown in Fig. 28. F-17 failed suddenly but not catastrophically at an ultimate load of 8.6 kips per ft. Mid-span moment at failure, including the dead load moment, was therefore 171 kip-ft, or 89 percent of the com-
puted ultimate flexural capacity of 191 kip-ft. Spalling of the top concrete fibers approximately 8 in. away from the centerline of the beam was observed just prior to failure. However, the appearance of the beam after failure indicates that the critical section was located at the third point of the span, where the kink in the beam may be observed. At this section, the conditions at the junction of the web and top flange were critical. At failure an inclined crack ran from this region to the location at which spalling was observed, displacing a wedge shaped piece similar to the SC failures in the second tests on beams subjected to concentrated loads. Therefore the failure in F-17 was regarded as due to shear compression.

Failure in F-18 occurred suddenly and catastrophically, due to fracture of the web reinforcement. Photographs of each half of F-18 after failure, and a close-up view of the failure region are shown in Fig. 29. The maximum load carried by F-18, in the first load cycle, was 4.8 kips per ft. Total mid-span moment at the maximum load was therefore 188 kip-ft, or 98 percent of the computed ultimate capacity of 191 kip-ft. Despite the closeness to a flexural failure, there was no evidence of any spalling in the top concrete fibers prior to failure. Failure occurred, in the second load cycle, when the 9th through 14th stirrups from the right support broke. An X mark is placed above the fractured stirrups in Fig. 27. The stirrups were fractured at a level corresponding approximately to the junction of the web and top flange. The critical section was at the third point of the span, where the kink in the beam may be observed. Despite the fact that the reason for failure in F-18 was different from that in F-17, the appearance of the two beams after failure was very similar.
5. **STRENGTH OF TEST BEAMS**

5.1 **APPROACH**

Several different approaches were considered in evaluating the shear strength of the test beams. Comparing the test results to the predicted shear strength from the "truss analogy", which assumes that the total shear is carried by the web reinforcement, indicated that the strength of the test beams was from 4 to 14 times greater than the predicted strength. This comparison illustrates the well-known fact that the "truss analogy" is overly conservative when applied to prestressed concrete beams. Furthermore, it indicates that a substantial part of the total shear must have been carried by the concrete.

Paragraph 1.13.13 of the AASHO specifications (21) evaluates shear strength by a modified form of the "truss analogy" equation. This equation, when solved for ultimate shear, becomes:

\[
V_u = 2 \frac{A f y y s + V_c}{V} = 2b'jd\frac{r_f}{100} + V_c
\]

Equation 5 assumes that part of the total shear is carried by the web reinforcement, and part by the concrete. The part carried by the web reinforcement is equal to the first term in Eq. 5. The part carried by the concrete, \(V_c\), is equal to a specified stress, 0.06 \(f'_c\), assumed to act over that part of the cross-section defined by \(b'jd\), not to exceed 180 \(b'jd\).

The test results were compared to the predicted shear strengths determined from Eq. 5. Ratios of test to predicted ultimate shear strength for all of the concentrated load tests except F-20, F-21, and F-22 are plotted in Fig. 30. The number written beside each point is the value of \(r_f/100\) in the shear span in which the failure occurred. While predictions using Eq. 5 are better than the "truss analogy", it is equally evident that this equation does not provide a satisfactory evaluation of ultimate shear strength. The test to predicted ratios ranged between 1.6
and 3.1, with the higher ratios associated with the lower a/d ratios. For a given amount of web reinforcement, decreasing the shear span increased the test to predicted ratio of shear strength. For a given shear span, increasing the amount of web reinforcement decreased the test to predicted ratio of shear strength. Consequently Eq. 5 does not reflect the behavior of the test beams.

The best approach to the evaluation of shear strength would be through the development of equilibrium and compatibility expressions which properly take into account the conditions existing at failure. However, the different modes of failure of the test beams were very complex. Walther (3,4) has proposed expressions for evaluating the shear compression type of failure. However, these expressions did not closely predict the shear strength of the test beams. The failures are closely examined in Section 5.5, but the extremely complex nature of the failure was not evaluated mathematically.

An empirical approach was consequently used in Section 5.5 to evaluate the shear strength of the test beams. It was considered of primary importance that the evaluation should be in agreement with the observed behavior of the test beams, which was described in detail in the two preceding chapters. In every case, the shear failures resulted from the development and extension of inclined cracks. These inclined cracks, either directly or indirectly, caused the destruction of some load carrying element in the beam, thus triggering the shear failure. It is therefore evident that the load causing inclined cracking is significant in the evaluation of the test results.

Inclined cracking observed in the test beams was classified as either flexure shear or diagonal tension. The important features of these two types of cracks are illustrated in Fig. 31. Diagonal tension cracking started from an interior point in the web of the beam. Depending upon the length of shear span, it would either precede or follow flexural or flexure shear cracking. Flexure shear cracking was always associated with the development of a flexural crack. This flexural crack, depending upon the distance from the load point and the shear span to effective depth
ratio, would either turn and become inclined in the direction of increasing moment, or would precipitate inclined cracking in the web above it. Consequently the flexural cracking strength of the test beams is an important factor in the determination of the load causing inclined flexure shear cracking.

It therefore follows that both the flexural and inclined cracking strength are important in the evaluation of the ultimate shear strength of the test beams. Another factor of importance is the ultimate flexural strength, which limits the shear that any section of the beam may be required to withstand. All of these strength properties are examined in the following sections of this chapter, and provide the basis for the evaluation of the ultimate shear strength of the test beams.

5.2 FLEXURAL CRACKING STRENGTH

Values of the applied load moment causing flexural cracking, $M_{cr}$, in the first test on beams subjected to concentrated loads were given in Table 5. Since maximum applied load moment in both the symmetrically and unsymmetrically loaded beams occurred at the load point adjacent to Region B, the applied load shear causing flexural cracking was related to $M_{cr}$ by:

$$V_{cr} = \frac{M_{cr}}{a_B}$$

Shears determined from Eq. 6 are given in Table 7 and plotted in Fig. 32.

The flexural cracking moment of the test beams would generally be calculated from the equation:

$$M_{fc} = z^b \left( f' + F \frac{e}{Z^b} \right)$$

Since $M_{fc}$ is equal to $M_{cr}$ plus the dead load moment, $M_d$, Eq. 7 when solved for $f'_t$ becomes:

$$f'_t = \frac{M_{cr} + M_d}{z^b} - F \left( \frac{1}{A} + \frac{e}{Z^b} \right)$$
Values of the flexural tensile strength of the concrete were computed from Eq. 8 using the properties of the transformed section, and are given in Table 7 for the observed flexural cracking load of each beam, including the uniformly loaded test beams. The maximum dead load moment in the test beams, given in Table 7, was used in computing $f'_t$. The average computed tensile stress in the bottom fibers at flexural cracking was 770 psi. Consideration was given to relating $f'_t$ to $\sqrt{f'_s}$ and $f'_c$. Ratios of $f'_t$ to these quantities have been plotted against concrete strength in Figs. 33 and 34. Neither plot shows any definite variation with concrete strength. Preference was given to the use of the relationship between $f'_t$ and $\sqrt{f'_c}$.

The average value of $f'_t/\sqrt{f'_c}$ was equal to 9.5. Except for F-10, the values of $f'_t/\sqrt{f'_c}$ fall within the range of 7 to 12. In the case of F-10, two flexural cracks approximately 20 in. apart were observed at the same time. Therefore the low value of $f'_t/\sqrt{f'_c}$ cannot be attributed to a mistaken observation, or to a single weak section in the concrete as might be caused by a void. Rather the strength of the concrete in F-10, in the tension flange, must have been weaker than determined from the cylinder tests.

Based on an "average" test beam in which $F$ is equal to 89.1 kips and $f'_c$ is equal to 6560 psi, the flexural cracking moment, $M_{fc}$, computed from Eq. 7 for a flexural tensile stress of $9.5\sqrt{f'_c}$ is 98.0 kip-ft. Similarly for a flexural tensile stress of $8\sqrt{f'_c}$, $M_{fc}$ is 93.5 kip-ft. Deducting the average dead load moment of 3.0 kip-ft, $M_{cr}$ for critical stresses of $9.5\sqrt{f'_c}$ and $8\sqrt{f'_c}$ is 95.0 and 90.5 kip-ft, respectively. Plots of $V_{cr}$ for $M_{cr}$ equal to 95.0 and 90.5 kip-ft are shown in Fig. 32 for comparison with the observed values of applied load shear causing flexural cracking. It is evident that the computed flexural cracking shear based on a critical stress of $9.5\sqrt{f'_c}$ is a good representation of the observed values over the entire range of a/d ratios investigated.

It should be noted that the values of $V_{cr}$ tend to be higher than the true flexural cracking shears, because of the difficulty in detecting the first crack at the instant of formation. However, the constant or
nearly constant moment region in the test beams was in most instances several feet wide. Usually only one crack would be observed in this region at the indicated value of $M_{cr}$. Other flexural cracks would be observed only after additional load had been applied, and consequently the majority of cracks in the constant or nearly constant moment region would form at higher stresses than the value of $f_t'$ given in Table 7.

5.3 **INCLINED CRACKING STRENGTH**

Inclined cracking observed in the test beams was described in Sections 3.3 and 4.3, and was classified as either diagonal tension or flexure shear, as shown in Fig. 31. Diagonal tension cracking occurred in tests on $a/d$ ratios less than 3.5. Flexure shear cracking occurred in tests on $a/d$ ratios greater than 5. In tests on $a/d$ ratios between 3.5 and 5, the inclined cracking had characteristics which, in different tests, was associated with either diagonal tension or flexure shear cracking. Values of applied load shear, $V_{ic}$, causing significant inclined cracking in the test beams were given in Tables 5 and 6. Figure 35 shows the variation in the observed values of $V_{ic}$ with length of shear span.

Diagonal tension cracking started from an interior point in the web of the test beams and was caused by principal tensile stresses in the web exceeding the tensile strength of the concrete. In general, the first diagonal tension crack formed near the mid-point of the shear span, and was subsequently critical in causing a shear failure. Exceptions to this occurred when the first crack formed either close to the reaction or close to the load point. However, in these cases additional inclined cracks which subsequently would be critical in causing failure would usually form near the mid-point at slightly higher loads.

Significant diagonal tension cracking had two important characteristics. The maximum principal tensile stress responsible for cracking occurred close to the $cg$, and the slope of the path of the crack was closely associated with the slope of the compressive stress trajectory at the $cg$. Accordingly, values of the principal tensile stress at the $cg$, $\sigma_{pt}^{cg}$, causing significant inclined cracking were selected from
sketches of the crack patterns in the figures in Appendix I. These
values are recorded in Table 8. Even though diagonal tension cracking
occurred only in tests on the shorter shear spans, values of $\sigma_{pt}^{cg}$ at
significant inclined cracking were selected for all of the concentrated
load tests except F-20, F-21, and F-22. For tests in which diagonal
tension cracking occurred, the value of $\sigma_{pt}^{cg}$ was taken at the intersect­
ion of the path of the crack and the cg of the beam. For tests in
which flexure shear cracking occurred, the value of $\sigma_{pt}^{cg}$ was taken at the
cg directly above the initiating flexure crack, indicated by the $\sqrt{\text{mark}}$
below the crack. In either case, the exact location is not important,
because the values of $\sigma_{pt}^{cg}$ are nearly constant along the shear span. The
variation in the ratio of $\sigma_{pt}^{cg}$ to $\sqrt{f'_c}$ with length of shear span is shown
in Fig. 36.

If the assumption that an excessive principal tensile stress
at the center of gravity of the section causes diagonal tension cracking
is correct, it should be possible to select a constant value of $\sigma_{pt}^{cg}$ as
the critical stress, since the only factor which affected the state of
stress assumed in calculating the principal tensile stresses was the
small dead weight of the test beams. The average value of $\sigma_{pt}^{cg}$ for tests
on a/d ratios equal to or less than 3.53 was:

$$\sigma_{pt}^{cg} = 5.5\sqrt{f'_c} \quad (9)$$

While Eq. 9 represents the data for tests in which diagonal tension
cracking generally occurred to about the same degree of consistency
as tensile tests on concrete, it is also apparent that the values of
$\sigma_{pt}^{cg}$ are inversely related to the length of shear span. This is due to
several factors. For short shear spans less than approximately twice
the total depth of the beam, the magnitude of the vertical stresses at
the cg influences the state of stress. These vertical stresses delay
the formation of diagonal tension cracks. Since the calculated princi­
pal tensile stresses were based on the assumption that the vertical stress
component is zero, a higher value than the true value of $\sigma_{pt}^{cg}$ was obtained.
For shear spans greater than approximately three times the depth of the
beam, diagonal tension cracking tended to form forward in the shear span toward the load point. In these cases, the maximum principal tensile stress along the path of the crack is below the cg. Therefore \(\sigma_{pt}^{cg}\) is lower than the principal tensile stress causing cracking.

A good fit to the selected values of \(\sigma_{pt}^{cg}\) over the entire range of shear spans on which tests were conducted is provided by the equation

\[
\sigma_{pt}^{cg} = (8 - 0.78 \frac{a}{d})/f'_c
\]

This equation has the advantage of avoiding the need to make any sharp distinction between diagonal tension and flexure shear cracking.

Assuming that the state of stress at the cg is responsible for inclined cracking, the following expression was obtained from Eqs. 1, 2, and 3 for predicting the applied load shear causing significant inclined cracking in the test beams:

\[
V_{ic} = \frac{Ib^4}{Q_{cg}} \sqrt{(\sigma_{pt}^{cg})^2 - (\sigma_{pt}^{cg})^2 \frac{F_y}{A} - V_d}
\]

Inclined cracking shears calculated for an "average" test beam from Eq. 11, based on Eq. 10, are compared to the test values in Fig. 35. \(V_d\) was assumed equal to the dead load shear at the mid-point of the shear span, or \((0.86 - 0.13 \frac{a}{d})\) in kips if the length of an "average" test beam is assumed equal to 180 in. It is evident from Fig. 35 that the prediction of diagonal tension cracking obtained using Eqs. 10 and 11 correlates satisfactorily with the results for tests on \(a/d\) ratios less than 3.5, on which only diagonal tension cracking occurred, and also for tests on \(a/d\) ratios between 3.5 and 5, on which both diagonal tension and flexure shear cracking occurred. Only for \(a/d\) ratios greater than 5 does the predicted inclined cracking shear begin to go against the trend of the data.

Flexure shear cracking was associated with the development of a flexural crack which would turn and become an inclined crack, or which would precipitate inclined cracking above it. Distances from the con-
centrated load points to the location of flexural cracks which were considered responsible for the development of significant flexure shear cracking, \( \alpha_d \), were determined from the figures in Appendix I, and are recorded in Table 8 and plotted in Fig. 37. As previously noted, the flexural crack which was selected as critical is designated by a \( \checkmark \) mark below the number which indicates the shear at which the crack was first observed.

The stresses in the bottom fibers at the location of the flexural crack initiating significant flexure shear cracking, \( \sigma_t^b \), are also recorded in Table 8. These stresses were interpolated from the stresses shown in the sketches of the crack patterns. The average value of \( \sigma_t^b \) determined in this way was 840 psi, which is comparable to the average value of \( f'_t \) equal to 770 psi which caused flexural cracking in the test beams.

The applied load shear causing flexure shear cracking may be determined from the equation:

\[
V_{ic} = \frac{M_{fc} - M_d}{(a - \alpha_d)}
\]  
(12)

While there is substantial variation in the selected values of \( \alpha_d \), Fig. 37 shows that the distance from the load point to the critical flexural crack causing significant flexure shear cracking increased with increasing \( a/d \) ratio. However, a linear relationship established between \( \alpha_d \) and \( a/d \) from the data plotted in Fig. 37, when used to predict \( V_{ic} \) in Eq. 12, did not correlate satisfactorily with the observed values of \( V_{ic} \) in Fig. 35.

Consequently consideration was given to finding an expression for \( \alpha_d \) which would represent the shear causing the formation of a significant flexure shear crack. Solving Eq. 12 for \( \alpha_d \):

\[
\alpha_d = a - \frac{M_{fc} - M_d}{V_{ic}}
\]  
(13)

Values of \( \alpha_d \) were computed from Eq. 13 for all inclined cracking shears in Tables 4 and 5 obtained in tests on shear spans equal to or greater
than 60 in. $M_{fc} - M_d$ for an "average" test beam was assumed equal to 96 kip-ft. The values of $\varphi d$ obtained in this way are plotted in Fig. 38, and show that the distance from the load point to the critical crack is not a linear function of the shear span. A least squares second degree curve fit to the values of $\varphi d$ yielded the equation:

\[ \varphi d = -31.6 + 15.6 \frac{a}{d} - 0.88 \left( \frac{a}{d} \right)^2, \text{ in in.} \]  

Figures 35 shows that $V_{ic}$ determined from Eq. 12, based on values of $\varphi d$ from Eq. 14, satisfactorily represented the applied load shear causing significant flexure shear cracking.

In summary, it is concluded that the significant inclined cracking shear in the test beams may be calculated as the least shear causing either (1) a principal tensile stress at the cg of $(8 - 0.78 \frac{a}{d})/f'_c$ at a section located at the mid-point of the shear span, or (2) a tensile stress in the bottom fibers of $9.5/f'_c$ at a section located $(a + 31.6 - 15.6 \frac{a}{d} + 0.88 \left( \frac{a}{d} \right)^2$) in. from the reaction. As may be seen from Fig. 39, the inclined cracking shear predicted in this manner represents within approximately plus or minus 10 percent the shear which caused significant inclined cracking in the test beams.

The shear causing inclined cracking is generally considered to be the ultimate shear that can be carried by prestressed beams without web reinforcement. Four E Series test beams, essentially identical to the F Series beams except without web reinforcement, were reported in Fritz Engineering Laboratory Report No. 223.25\(^{(7)}\). The maximum shear carried by these beams is within the band of plus or minus 10 percent shown in Fig. 39. Therefore the expressions in the preceding paragraph could be used to predict the ultimate shear strength of F Series beams without web reinforcement.

5.4 ULTIMATE FLEXURAL STRENGTH

Flexural failures occur when compressive strains above the top of a flexural crack reach values causing general crushing and destruction of the compression region. Only one test beam, F-21, failed in flexure.
Furthermore, the failure in F-21 may have been influenced by the short constant moment region in which the failure occurred, which was discussed in Section 3.3. The strain in the extreme fiber in compression at failure was approximately 0.004, which is higher than strains of approximately 0.0027 at failure measured in similar E Series beams (7) except for a longer constant moment region.

Numerous investigators have shown that the concrete strain distribution over the depth of the section, if measured over a distance great enough to average out the discontinuities at flexural cracks, remains linear to failure in regions where flexural predominates. This was verified in the E Series beams (7). Therefore the calculation of the ultimate flexural strength of the test beams was based on the assumed strain and stress distribution shown in Fig. 40. From equilibrium of internal forces:

\[ C = T \tag{15} \]

or

\[ k_1 k_3 f'_c b c = \sum_{i=1}^{n} A_{s_i} f_{s_i} \]

where

- \( C \) = resultant compressive force in the concrete
- \( T \) = resultant tensile force in the steel
- \( k_1 \) = ratio of maximum compressive stress to average compressive stress
- \( k_3 \) = ratio of maximum compressive stress to strength of concrete, \( f'_c \), determined from standard cylinder tests
- \( b \) = flange width of I-beam
- \( c \) = distance from extreme fibers in compression to neutral axis at failure
- \( A_{s_i} \) = cross-sectional area of steel at a particular level, \( i \)
- \( f_{s_i} \) = stress in steel
- \( n \) = number of levels of steel

Equation 15 is valid only if \( c \) is less than the depth of the compression flange at its full width. From equilibrium of internal and external
moments:

\[ M_{fu} = \sum_{i=1}^{n} A_i f_{si} (d - k_c) \]  

(16)

where \( M_{fu} \) = moment causing flexural failure

\( k_2 \) = ratio of distance from extreme fibers in compression to resultant compressive force in the concrete to \( c \).

\[ d_c = \frac{\sum_{i=1}^{n} A_i f_{si} d_i}{\sum_{i=1}^{n} A_i f_{si} s_i} \]

= distance from top fibers to resultant tensile force in the steel

Since the concrete strain distribution is linear over the depth of the section:

\[ \varepsilon_{cu_i} = \frac{d_i - c}{c} \varepsilon_u \]  

(17)

where \( \varepsilon_{cu_i} \) = tensile concrete strain at a particular level, \( i \)

\( \varepsilon_u \) = ultimate concrete compressive strain.

Assuming that the change in steel strain during loading to failure is equal to the change in strain in the adjacent concrete:

\[ \varepsilon_{su_i} = \varepsilon_{se_i} + \varepsilon_{ce_i} + \varepsilon_{cu_i} \]  

(18)

where \( \varepsilon_{su_i} \) = total strain in steel at a particular level, \( i \)

\( \varepsilon_{se_i} \) = strain in steel at the effective prestress force

\( \varepsilon_{ce_i} \) = compressive strain in the concrete

Equations 15, 16, 17, and 18 are sufficient to determine \( M_{fu} \) if the stress-strain relationship of the steel is known. Because the stress-strain curve of prestressing steel does not lend itself to simple continuous mathematical representation, the solution for \( M_{fu} \) is gener-
ally performed by assuming a value for $c$. Since $\varepsilon_u$ is an assumed pro-
perty of the concrete, $\varepsilon_u$, can be determined from Eqs. 17 and 18.

As $f_c$ can then be determined from the load-strain curve for the steel.

Equation 15 can now be solved for $c$, and if the calculated and assumed
values of $c$ are equal the correct value was assumed. If not, a new
value of $c$ must be assumed and the procedure repeated until agreement
is obtained. When agreement has been obtained, the ultimate flexural
strength, $M_{fu}$, can be determined from Eq. 16.

The solution for ultimate flexural strength using Eqs. 15
through 18 was checked against the results of tests on eight E Series
beams(7) failing in flexure. Values for the constants $k_1$, $k_2$, and $k_3$
for determining the magnitude and location of the resultant compressive
force in the concrete recommended by Mattock, Kriz, and Hognestad(22)
were used, as follows:

\[
k_1 = \begin{cases} 
0.85 & \text{for } f'_c < 4000 \text{ psi or} \\
0.85 - 0.00005 (f'_c - 4000) & \text{for } f'_c > 400 \text{ psi}
\end{cases}
\]

\[
k_2 = \frac{1}{2} k_1
\]

\[
k_3 = 0.85
\]

It was found that the test moments were consistently higher than the
calculated values of $M_{fu}$, by an average of 7 percent. The reason for
this could not be determined. Consideration was given to the possibility
that the compression tests on cylinders cast in waxed cardboard molds
with tin bottoms gave lower concrete strengths than implied by the use
of $k_3$ equal to 0.85. However, letting $k_3$ equal to one still gave cal-
culated moments which differed from the test moments by an average of
4 percent. The effect of the bond condition between the steel and the
concrete was also considered by incorporation of a strain compatibility
factor, $\Psi$, in the calculation for ultimate moment, as recommended by
Warner and Hulsbos(23). However, with $k_3$ equal to one and $\Psi$ equal to
2, test moments were still greater than the calculated ultimate moments
by an average of 2 percent.
Because of the similarity between the E and F Series beams, it was concluded that the ultimate flexural strength of the F Series beams could be closely calculated using Eqs. 15 through 18, assuming $k_3$ equal to 0.85, $\varepsilon_u = 0.3$ percent, and by increasing the result obtained from Eq. 16 by 7 percent. The flexural strengths of the test beams, $M_{fu}$, calculated in this way, are given in Table 7. The calculation was facilitated by the use of a computer and the following formulation of the stress-strain curve for the prestressing steel in the F Series beams:

$$A_{s_i} f_{s_i} = 29\varepsilon_{su_i} \quad 0 < \varepsilon_{su_i} \leq 0.69\%$$

$$A_{s_i} f_{s_i} = -22.4 + 97.4\varepsilon_{su_i}$$

$$-60.9\varepsilon_{su_i}^2 + 12.6\varepsilon_{su_i}^3 \quad 0.69\% < \varepsilon_{su_i} \leq 2.0\%$$

$$A_{s_i} f_{s_i} = 28.0 + 0.355\varepsilon_{su_i} \quad 2.0\% < \varepsilon_{su_i}$$

The quantity $A_{s_i} f_{s_i}$ in Eq. 19 has units in kips. The computed ultimate flexural strengths of the test beams varied between 182 and 193 kip-ft; the average was 189.4 kip-ft. The calculated strength of F-21 was 189.9 kip-ft, compared to an experimental flexural failure moment of 190.2 kip-ft.

In the preceding calculation of the flexural strength of the test beams, the neutral axis at failure was approximately 4.5 in. below the top fibers. Under these circumstances the flange area in compression is not exactly equal to $b$ times $c$. However, the product $k_lc$ was only very slightly greater than 3 in., which is the depth of the "equivalent" rectangular stress block, and therefore the effect on $M_{fu}$ was considered negligible.
5.5 ULTIMATE SHEAR STRENGTH

Thirty-seven of the 38 tests on 23 simply supported beams described in Chapters 3 and 4 resulted in shear failures. These failures were classified as web crushing, stirrup fracture, and shear compression.

Except for F-20 and F-22, the failures were due to inclined diagonal tension or flexure shear cracks which remained entirely within the shear span. There was no evidence from these failures of any compatibility condition which could be used to relate steel strains to concrete strains and thereby permit the calculation of an ultimate shear moment similarly to the calculation of ultimate flexural moment. Rather the behavior of the beams indicated that a re-distribution of forces took place at inclined cracking which subsequently resulted in several different types of action causing the observed failures.

Beams F-20 and F-22 failed when an inclined flexure shear crack, which had extended beneath the load point into the constant moment region adjacent to the shear span, caused crushing and destruction of the compression region. Flexure shear cracks had penetrated the constant moment region in other tests, although only for short distances, without causing similar failures. Therefore the shortness of the constant moment region, 13 in. for F-20 and 23 in. for F-22, was considered to have had an influence on the failure.

Action Causing the Shear Failures

The forces acting in a prestressed beam in a region in which shear is critical may be considered by separating the beam along the path of an inclined crack, and by a vertical cut through the concrete at the top of the crack, as shown in Fig. 41. The principal forces acting at this section are the two components of the resultant force in the prestressing steel, \( T_h \) and \( T_v \), the two components of the resultant force in the web reinforcement, \( V_{wh} \) and \( V_{wv} \), and the resultant compressive force transmitted through the concrete above the top of the inclined crack, which has a horizontal component represented by the compressive thrust, \( C \), and a vertical component represented by the shear force, \( V_c \). Not shown are forces which would exist when the path of the
inclined crack does not extend completely through the concrete in the tension flange of the beam. This is the usual situation when diagonal tension inclined cracking occurs. It also occurs for flexure shear cracking which is precipitated by but not connected to a flexural crack. However, any force transmitted through the concrete in the tension flange region is, except for short shear spans, relatively small, and for this discussion may be considered as adding to the force in the steel.

Most of the shear failures were caused by the action of the compressive thrust in the shear span. Consider the shear span of the typical test beam represented in Fig. 42. It is assumed that the pre-stress force is fully effective at the section through the reaction.

Consider first a prestressed beam without web reinforcement when the doweling force, \( T_v \), is assumed negligible. An inclined crack suddenly traversing the web moves the thrust line upwards above the apex of the crack, as shown in Fig. 42. Assuming that the beam can sustain the same moment after inclined cracking that caused the cracking, it follows that the magnitude of the thrust, although not necessarily the compressive stress above the crack, is reduced. Since there can be no transfer of horizontal shear across the inclined crack, the magnitude of the tension and compressive forces must remain constant along the length of the crack. Therefore the distance between these forces depends on the variation in the external moment, causing the compressive thrust line to take the position shown in Fig. 42. In effect, "beam action" in the region of the crack has been changed to "tied arch action".

It was shown in Section 5.3 that the load causing significant inclined cracking was approximately the ultimate load in beams without web reinforcement. The explanation in the preceding paragraph would seem to conflict with this, since under the assumed conditions the position of the thrust line moves upward and the magnitude of the thrust is reduced. However, a doweling force of significant magnitude must exist in beams without web reinforcement. If it did not, there would be no deflection of the part of the beam below the inclined crack, which would mean that an impossible separation of the beam at the junction of the tension
flange and the inclined crack would exist. Although all of the F Series beams had web reinforcement, four E Series beams \(^{(7)}\) did not have vertical stirrups. As may be seen from Figs. 7 and 9 in Ref. 7, the critical inclined crack ran back along the junction of the web and the bottom flange toward the reaction. This action was caused by the doweling force tearing the tension flange away from the web. Under these conditions, the part of the beam above the inclined crack acted as an eccentrically loaded arch rib, and failure occurred due to crushing of the concrete in Region K. Associated with this failure was the characteristic tension crack in the top flange above the point where the crushing occurred. Thus the absence of web reinforcement caused an immediate change at the time of inclined cracking from "beam action" to "tied arch action", and the latter type of action was incapable of sustaining additional load.

If identical inclined cracks would form in beams with and without web reinforcement, the thrust line would move further upwards in the beam with web reinforcement because the shear taken by the web reinforcement decreases the shear which must be carried by the concrete and also therefore the slope of the thrust line. However, web reinforcement restrains the development of the crack. Thus, at the load causing significant inclined cracking the distance from the top fibers to the apex of the crack is least in beams without web reinforcement. The shear taken by the web reinforcement makes the shape of the thrust line similar to the shape before the inclined crack formed, indicating that the web reinforcement restores "beam action".

Mattock and Kaar \(^{(22)}\) and Bruce \(^{(24)}\) have shown that deformed stirrups yield almost immediately when crossed by an inclined crack. As the load is increased, the inclined crack may extend and cross additional stirrups. The force in the stirrups may also be increased if the strains in the stirrups go into the strain hardening range. However, the doweling action of the strand and the shear resistance of the tension flange is a variable quantity of uncertain magnitude, which in the tests often appeared to destroy itself as the load was increased above the inclined cracking load. Consequently the shear carried by the stirrups, the doweling action, and the tension flange is, or is close to being, a maximum when significant inclined cracking occurs. Therefore the com-
pressive thrust line moves to a maximum height in the beam at or shortly after reaching the load causing significant inclined cracking, and as the beam is loaded further moves downward. Thus when a beam with sufficient web reinforcement to restore "beam action" after inclined cracking is loaded further, the action changes progressively with increasing load from "beam action" to "tied arch action". Because of the action described in the preceding paragraph, an important factor in the ultimate shear strength is the resistance that the web reinforcement provides to tearing of the tension flange away from the web.

Beam F-6, shown in Fig. 15, and F-15, shown in Fig. 17, both resemble the E Series failures without web reinforcement, and apparently failed in compression of the arch rib after inclined cracking had extended along the junction of the web and bottom flange to the reaction. In the case of F-6, the failure was classified as web crushing. However, in the case of F-5, stirrups in the shear span were fractured. It is impossible to know whether the stirrup fracture failure triggered the crushing failure in the web, or vice versa. F-6 and F-15 had $rf_y/100$ values in the failure regions of 47 and 34, respectively, and therefore $rf_y/100$ equal to 47 was the minimum amount of web reinforcement required to prevent this type of failure in these tests.

When the web reinforcement provided was sufficient to prevent the tension flange from being torn away from the web, crushing failures occurred in many cases due to the action of the compressive thrust at the junction of the compression flange and the web, shown by Region L in Fig. 42. The characteristics of the failures were similar to F-6 and F-15, except that the location of the failures was more toward the midpoint of the shear span. F-1, shown in Fig. 15, was a good example of this type of failure.

The failures at the junction of the web and top flange were further complicated by several conditions. In many cases more than one inclined crack contributed to the failure. Consider the forces acting at two adjacent inclined cracks in the test beam, as shown in Fig. 43. This part of the beam acts as a strut. However, the resultant of all
of the strand and stirrup forces is not necessarily directed along the axis of the strut. Therefore the critical section at the junction of the web and top flange is subjected to an eccentric load which can initiate the crushing failure. This appeared to be the case in F-3, F-7, and F-10, shown in Fig. 15. In these failures the shear reached a maximum and slowly dropped off. At some point the action described in the preceding paragraph occurred, and the beam suddenly lost its remaining strength.

Several test beams had a wedge shaped piece of the compression flange sheared out. For example, refer to the pictures of F-16 in Fig. 17, and F-18 in Fig. 29. In the first test on beams subjected to concentrated loads, this phenomenon was associated with some of the failures caused by fracture of the stirrups, particularly for the longer shear spans. In the second tests, however, failures of this type occurred in which there were no stirrups fractured, the failure being classified as shear compression. The action which caused the wedge shaped piece of the compression flange to be sheared out is illustrated in Fig. 44. In this figure, point 0 is the apex of an inclined crack. The forces acting on any place OP emanating from point 0 would be very complex and difficult to determine. However, they may be represented by a resultant shear, thrust, and moment - $V_{op}$, $T_{op}$, and $M_{op}$ - respectively. It is evident, therefore, that the moment, $M_{op}$, may produce tensile stresses at the apex of the crack, causing it to extend. Besides being responsible for the failures which were classified as shear compression, the moment, $M_{op}$, could also have caused the stirrup fracture failures which in many cases occurred at or toward the top of an inclined crack.

The failure in the uniform load test on F-17, shown in Fig. 28, also fits the action described in Fig. 44 very closely. The failure occurred approximately at the third point. However, just prior to failure spalling of the extreme fibers in compression was observed at the location which after failure was seen to be at the front of the wedge.

It is therefore evident that the nature of the forces which caused the shear failures in the test beams is very complex. Whereas the web crushing failures tended to occur on the shorter shear spans,
the stirrup fracture failures on the longer shear spans, and the shear compression failures in the reloaded tests, there was no clear dividing line between the different types of failure. The failures were further complicated by the somewhat random manner in which the crack patterns formed. The difficulty in developing a rational mathematical formulation of the observed behavior is therefore apparent, and consequently an empirical evaluation of the shear strength was made.

Evaluation of the Concentrated Load Tests

The effect of the two principal variables in this investigation, amount of web reinforcement and length of shear span, on the shear strength of the test beams subjected to concentrated loads is shown in Fig. 45. By means of the different symbols, differentiation can be made between the shears causing failure in the first and second tests. The amount of web reinforcement in the failure region is indicated by the value of $f_y/100$ written beside each point. The applied load shear causing flexural and significant inclined cracking and the shear which would develop the flexural capacity of an "average" test beam are indicated by $V_{cr}$, $V_{ic}$, and $V_{fu}$, respectively.

Figure 45 shows that the presence of vertical stirrups enables the beam to carry a greater shear than the shear causing significant inclined cracking. For every shear span, an increase in $f_y/100$ is accompanied by an increase in the ultimate shear strength, except for the three beams which had the critical shear span increased 10 in. for the second test. That there can be substantial variation in the results is shown by the tests on F-X1 on a 48 in. shear span. Both shear spans had an equal amount of web reinforcement, $f_y/100$ equal to 117. However, the shear capacity of the beam was 18 percent greater in the second test than in the first test.

The effect of the two principal variables on the nominal ultimate shear stress, $v_u$, where:

\[
v_u = \frac{V}{b'd}
\]

is shown in Fig. 46. Contours of the test results were estimated for
rf_y/100 values of 50, 100, 150, and 200. This figure shows that the nominal shearing stress at failure decreases with increasing a/d ratio for a given amount of web reinforcement. The lowest ultimate shearing stress in any beam at failure was 400 psi.

Since Fig. 45 showed that web reinforcement enabled the beam to carry a shear equal to and in general greater than the shear causing significant inclined cracking, consideration was directed towards relating the difference between the ultimate shear and the shear causing significant inclined cracking to the amount of web reinforcement. A non-dimensionalized arrangement of the test results was obtained by plotting \((V_u - V_{ic})/b'd/\sqrt{f'_c}\) against \(rf_y/100\sqrt{f'_c}\), as shown in Fig. 47. The units of \(\sqrt{f'_c}\) are assumed to be psi.

The results of the tests on F-20, F-21, and F-22 are not shown in Fig. 47 because the mode of failure of these three beams did not permit an experimental determination of the inclined cracking shear in the same manner as the other beams subjected to concentrated loads. Also, the results of the second tests on F-11, F-13, and F-19 are not shown because the inclined cracking shear may have been influenced by the change in length of shear span for the second test.

An improvement in the grouping of the data was obtained by using predicted values of the inclined cracking shear, as shown in Fig. 48. Based on the work in Section 5.3, \(V_{ic}\) was calculated as the least applied load shear which will cause either (1) a principal tensile stress at the cg equal to \((8 - 0.78 a/d)/\sqrt{f'_c}\) at a section located at the mid-point of the shear span, or (2) a tensile stress in the bottom fibers equal to \(9.5/\sqrt{f'_c}\) at a section located \((a + 31.6 - 15.6 (a/d) + 0.88 (a/d)^2)\) in. from the reaction. The results of all of the concentrated load tests are included in Fig. 48, \(V_{ic}\) being determined for the critical shear span.

An equation for predicting the ultimate shear was obtained from a regression analysis of the data in Fig. 48. However, this equation did not satisfactorily predict the shear strength of the test beams over the entire range of shear spans investigated, due to the weighting of the
lower amounts of web reinforcement in the tests with the longer shear spans.

Therefore another relationship for the ultimate shear strength was sought which would be in better agreement with the test beams. It was considered desirable that this relationship should have a physical interpretation. The relationship selected was:

\[ V_u = V_{ic} + b'h'd \cdot \frac{f_v}{100} \]  

(21)

An interpretation of this equation can be made by reconsidering the general free-body diagram at an inclined crack shown in Fig. 41. Two of the forces in this free-body diagram, \( T_v \) and \( V_{wh} \), are due to the doweling action in the strand and web reinforcement, respectively. These forces are caused by the separation of the inclined crack interface normal to the path of the crack. This separation tends to cause sharp changes in direction, or kinks, in the strand and stirrups. For these kinks to exist, concentrated forces of appreciable magnitude would also have to exist in the concrete at the crack interface. Such forces would cause localized crushing of the concrete, relieving the kink and tending to restore the line of action of the resultant force to the direction of the reinforcement. Furthermore, doweling action of any appreciable magnitude in the strand would also tend to destroy itself by causing flexural cracks in the cantilevered extension of the beam at the inclined crack. The existence of such cracks was observed in the concentrated load tests. Therefore the doweling forces in the strand and web reinforcement at ultimate load were believed to be small and of secondary importance.

Considered as a part of \( T_v \) was the shear transferred by the concrete in the tension flange area of the beam. This would be of consequence only for short shear spans.

Assuming \( V_{wh} \) and \( T_v \) equal to zero and letting \( V_{wv} \) equal to \( V_w \), the equation for vertical equilibrium at the inclined crack becomes:

\[ V = V_c + V_w \]  

(22)
Making use of the previously mentioned work by Mattock and Kaar (19) and Bruce (24) which has shown that yielding of deformed stirrups crossed by the critical inclined crack occurs in beams falling in shear, \( V_w \) may be expressed as:

\[
V_w = A_v \frac{f_y \beta d}{s} \tag{23}
\]

where \( \beta d \) is equal to the horizontal projection of the effective length of the inclined crack. The effective length is regarded as the distance along the path of the crack from the apex to the lowest point at which the web reinforcement is effective. Therefore Eq. 22 may be expressed as:

\[
V = V_c + b' \beta d \left( \frac{r_i}{100} \right) \tag{24}
\]

It was shown in Section 5.3 that the ultimate shear carried by I-beams of the type tested herein without web reinforcement was closely equal to the shear causing significant inclined cracking. Assuming that the contribution to the ultimate shear carried by the concrete in beams with web reinforcement is equal to the shear causing significant inclined cracking, \( V_c \) is equal to \( V_{ic} \), and Eq. 21 is established.

Studies of the photographs of the test beams after failure indicated that \( \beta d \) could be conservatively approximated as the distance from the extreme fiber in compression to the lowest level at which the web reinforcement was effective, i.e. 16.5 in. So with \( \beta \) equal to 1.16, Equation 21 has been plotted in Fig. 48 for comparison with the test results. Predicted ultimate shear strengths and test to predicted ratios based on Eq. 21 are given in Table 9. The average of the test to predicted ratios for the concentrated load tests was 1.02.

The test to predicted ratios of ultimate shear strength determined from Eq. 21 are compared to the length of shear span in Fig. 49. The test to predicted ratios are greater than one for the lower a/d ratios as expected, because the forces assumed negligible in developing Eq. 21 become important as the a/d ratio decreases. Equation 21
was therefore regarded as agreeing with the observed behavior of the test beams.

In the concentrated load tests, stirrup spacings were used which varied between 3.2 and 10 in. Test to predicted ratios of shear strength, based on Eq. 21, are compared to the stirrup spacing in Fig. 50. It is apparent that the different spacings had no discernible effect on the shear strength.

Correlation with the Uniform Load Tests

The results of the two uniform load tests were examined for correlation with the results of the concentrated load tests. Both of these tests had the same amount of web reinforcement, $f_{w}/100$ equal to 56, provided by single 3/16 in. diameter bars spaced at 6 in. F-17 and F-18 were tested on 150 and 210 in. spans, respectively. Shear failures, described in Section 4.3, occurred in both beams at approximately the third point of the span length.

When the shear varies, as in the uniform load tests, it is necessary to closely define the section at which diagonal tension cracking is being predicted. This was done for the uniform load tests by assuming that the shear at a distance $x$ from the reaction causing significant diagonal tension cracking was equal to the shear causing a principal tensile stress of $(8 - 0.78 x/d)/f'_{c}$ at the cg of a section located $(x - y_{d})$ from the reaction. $y_{d}$ is the horizontal distance from the apex of the crack to the point where the crack crosses the cg, as illustrated in Fig. 51. Since the slope of the stress trajectories at the cg was approximately 30 degrees, $y_{d}$ was assumed equal to 1.7 times the distance from the junction of the web and top flange to the cg, or very nearly $d/2$.

It was also assumed for the uniform load tests, that the shear at a distance $x$ from the reaction causing significant flexure shear cracking was equal to the shear causing a tensile stress of $9.5/\sqrt{f'_{c}}$ in the bottom fibers at a section located $(x - d)$ from the reaction. In effect, the flexure shear crack was being assumed to start at a distance equal to the effective depth of the beam from the apex of the crack, as shown in Fig. 51.
The predicted inclined cracking shear at distances $x$ from the reaction, less the dead load shear, is plotted on the applied load shear diagram at ultimate load for F-17 and F-18 in Figs. 52 and 54, respectively. The shear diagram for F-17 shows that there were two regions in the beam where the ultimate applied load shear, $V_u$, exceeded the predicted inclined cracking shear. The region closest to the reaction would be associated with diagonal tension cracking. The interior region would be associated with flexure shear cracking. Figure 28 shows that the failure occurred as the result of the flexure shear cracking.

The greatest difference between the predicted inclined cracking shear and the ultimate shear for F-17, in the region of flexure shear cracking, occurred at a section located 54 in. from the reaction. This section can be located in Fig. 28 as the line representing the 9th stirrup from the reaction. It is evident that this is as close to the critical section as can be determined from the photograph of the failure. At the critical section, the predicted inclined cracking shear and the ultimate shear are 11.3 and 15.1 kips, respectively. The difference between the predicted inclined cracking shear and the ultimate shear is 3.8 kips. According to Eq. 21, the contribution of the web reinforcement to the shear strength would be $1.16 b'd (r_f/100)$, or 2.8 kips. Therefore the test to predicted ratio of ultimate shear strength is 1.07.

The shear diagram for F-17 in Fig. 52 would appear to indicate that diagonal tension cracking rather than flexure shear cracking was critical. However, the predicted diagonal tension cracking shear was based on an assumed state of stress at the cg which neglected the presence of vertical stresses. For small values of $x$ the vertical stresses are appreciable, and therefore the predicted value would be less than the actual cracking shear. Diagonal tension cracking first occurred in one end of F-17 at a load of 7.4 kips per ft, at which load the reaction was equal to 46.2 kips. The partially drawn shear diagram for a load of 7.4 kips per ft indicates that the critical section for diagonal tension cracking was approximately 23 in., or 1.8d, away from the reaction. Figure 28 shows that the diagonal tension crack actually formed such
that the apex of the crack is approximately 12 in. from the reaction. Therefore some other state of stress than the prescribed state of stress was critical in causing the diagonal tension cracking. Also, the fact that the shear failure did not occur adjacent to the reaction, even though the difference between the predicted inclined cracking shear and the ultimate shear is greater than in the interior regions, supports the observation drawn from the concentrated load tests that the shear strength is increased for short shear spans.

The shear diagram for F-18 in Fig. 53 also shows two regions in the beam where the ultimate shear exceeded the predicted inclined cracking shear. Figure 29 shows that the failure occurred as the result of flexure shear cracking. The greatest difference between the predicted inclined cracking shear and the ultimate shear occurred at a section located 68 in. from the reaction. By counting 11 stirrups from the reaction in Fig. 29 and estimating an additional 2 in., it can be seen that the predicted failure section is in close agreement with the actual failure section. At the critical section, the predicted inclined cracking shear and the ultimate shear are 10.1 and 14.8 kips, respectively. The difference is 4.7 kips. Therefore the test to predicted ratio of ultimate shear strength was 1.15.

Although the predicted inclined diagonal tension cracking shear is less than the ultimate shear for a short distance close to the reaction, no diagonal tension cracking occurred in either end of F-18. The predicted inclined cracking curve intersects the ultimate shear diagram approximately 27 in., or 1.9d, away from the reaction.

The results of the uniform load tests therefore show good agreement with the behavior predicted on the basis of the concentrated load tests.
6. **SHEAR STRENGTH OF PRESTRESSED CONCRETE BRIDGE GIRDERS**

6.1 **COMPARISON OF TEST BEAMS AND FULL SIZED BRIDGE GIRDERS**

The test beams were designed and fabricated so as to be representative of precast prestressed girders used in bridges in Pennsylvania. These girders have I or box shaped cross-sections. The compression flange to web width ratio of the sections currently being used\(^{(25)}\) varies between 1.43 and 3 for the I-beams and is either 3.6 or 4.8 for the box beams, compared to 3 for the F Series test beams.

High strength concrete is used to cast the full-sized bridge girders which generally has a strength greater than 5000 psi in two days, because of steam curing until the time of prestress release, and 7000 psi in 28 days. Similar high strength concrete was used in the F Series beams, except that the moist curing procedure used in the laboratory required approximately five days to reach the strength of 5000 psi.

The size and type of prestressing strand used in the test beams was the same as that used in full-sized girders. However, to get shear failures in the test beams it was necessary to use No. 3 and No. 2 hot rolled deformed bars and 3/16 in. diameter annealed deformed masonry wire for web reinforcement. The No. 3 and No. 2 deformed bars are the same type of bar used in bridge girders, except of smaller size. After annealing, the yield point and ductility of the masonry wire were similar to that of the hot rolled bars.

The concentrated and uniform loads applied to the test beams differ in two significant respects from the highway loads on bridge girders. In the first place, the loads applied in the laboratory were either entirely uniform, or entirely concentrated, whereas the loads on bridge girders are somewhere between these two extremes. In the second place, the loads applied in the laboratory were stationary loads, whereas the applied loads on bridge girders are moving loads.
The basic behavior of the test beams under either concentrated or uniform loads was the same. Flexure shear and diagonal tension cracking was observed in both types of tests. The shear failures observed in the uniform load tests were similar to the failures observed in the concentrated load tests. Furthermore, the ultimate shear strength of the uniformly loaded test beams was closely predicted by a method based upon the results of the concentrated load tests.

The method used to predict the shear strength of the concentrated and uniform load tests differed in the method of calculating the shear causing significant inclined cracking. In the case of the concentrated load tests, the diagonal tension cracking shear was calculated as the shear causing a centroidal principal tensile stress of \((8 - 0.78 \frac{a}{d}) \sqrt{f'_{C}}\) at the mid-point of the shear span. The flexure shear cracking shear was calculated as the shear causing a tensile stress of \(9.5\sqrt{f'_{C}}\) in the bottom fibers at a section located at a distance from the load point given by Eq. 14. This procedure was generalized for calculating the shear causing significant inclined cracking at any section at a distance \(x\) from a reaction in the uniformly loaded test beams by an idealization of the critical inclined crack, as illustrated in Fig. 51. The section assumed to be critical in the evaluation of shear strength is through the apex of the crack. Diagonal tension cracking was assumed to occur when a centroidal principal tensile stress of \((8 - 0.78 \frac{x}{d})\sqrt{f'_{C}}\) was reached at a distance equal to \(yd\) from this section. Flexure shear cracking was assumed to occur when a tensile stress of \(9.5\sqrt{f'_{C}}\) in the bottom fibers was reached at a distance from this section.

When a beam is subjected to combined concentrated and distributed loadings, the critical section for significant inclined cracking could be either near the load point, as for the concentrated load tests, or away from the load point, as for the uniform load tests. The concentrated load tests showed, however, that the critical section should not be taken adjacent to the load point. For flexure shear cracking, based on Eq. 14 and the idealization of the inclined crack shown in Fig. 51, the critical section was located up to one and one-half times the effective depth of the beam from the load point depending on the
shear span to effective depth ratio. In full-sized beams, if the critical section were near the load point, the distance that the section should be assumed from the load point would depend upon many factors, including the type of cracking, the relative magnitude of the concentrated and distributed loads, and the geometry of the compression flange. Fortunately, the need for defining this distance is largely obviated by the fact that the concentrated loads on bridge girders are moving loads. The influence that the load point has on the inclined crack is necessarily lost as the load point moves away, and could even create a more critical condition than if the critical section were assumed adjacent to the load point.

The effect of a moving load was simulated by the second tests on F-11, F-13, and F-19. In these three tests, the length of the shear span was increased 10 in. for the second test. In effect this created a condition similar to that which would exist when an inclined crack is caused by a moving concentrated load. The second tests on these beams were conducted on a/d ratios of 3.53, 4.49, and 5.64. The test to predicted ratios of shear strength, based on Eq. 21, were 1.01, 0.99, and 0.87, respectively. This indicates that the effect of moving the load point became more critical as the a/d ratio was increased. Since flexure shear cracking was critical in the tests on the a/d ratios of 4.94 and 5.64, these results further indicate that moving the load point decreased the test to predicted ratio at the same time that the critical section was being assumed further from the load point. If the shear causing significant inclined cracking were calculated as the shear causing a tensile stress of $9.5/f'_c$ in the bottom fibers at a section located $(a - d)c$ from the reaction, or in other words assuming that the critical section is adjacent to the load point, the test to predicted ratio of shear strength for the test on the a/d ratio of 5.64 becomes 1.05.

Therefore it is believed that the shear causing significant inclined cracking in bridge girders subjected to combined concentrated and distributed loadings, where the concentrated loads may be moving loads, should be calculated in the same manner as for the uniformly loaded test beams. The shear strength can then be evaluated assuming
that the shear carried by the concrete is equal to the shear causing significant inclined cracking and that the shear carried by the web reinforcement is equal to the force in the stirrups, stressed to the yielded point, crossed by the idealized crack shown in Fig. 51.

6.2 PROPOSED SPECIFICATION FOR DESIGN OF WEB REINFORCEMENT

The area of web reinforcement placed perpendicular to the axis of the member at any section shall not be less than

$$A_v = \frac{(V_u - V_c)s}{f_{yd}s}$$  \hspace{1cm} (25)

nor less than

$$A_v = \frac{\lambda V_c u_s}{f_{yd}s}$$  \hspace{1cm} (26)

nor more than

$$A_v = \frac{7b's/f'}{f_y}$$  \hspace{1cm} (27)

The shear, $V_c$, shall be taken as the lesser of $V_{cd}$ and $V_{cf}$. $V_{cd}$ is the shear causing a principal tensile stress of $f_{pt}$ at the center of gravity of the cross-section resisting the live load. If the center of gravity is not in the web, $f_{pt}$ shall be computed at the intersection of the web and the flange. $V_{cf}$ is the shear causing a flexural tensile stress of $f_t$ in the extreme fiber in tension at a distance in the direction of decreasing moment from the section under consideration equal to the effective depth of the member.

Web reinforcement shall not be spaced further apart than $d_s/2$, or 24 inches, whichever is smaller, and shall be anchored in both the tension and compression flanges of the member.

Web reinforcement between the support and the section a distance equal to the effective depth of the member from the support shall be the same as that required at that section.
Specification Notation

\( A_v \) = area of web reinforcement placed perpendicular to the longitudinal axis of the member

\( b' \) = width of web

\( d \) = distance from the extreme fiber in compression to the centroid of the prestressing steel, i.e., the effective depth of the member

\( d_s \) = distance from the extreme fiber in compression (in composite sections from the top of the girder alone) to the lowest level at which the stirrups are effective

\( f'_c \) = ultimate compressive strength of concrete

\( f_{pt} \) = \( (6-0.6\frac{x}{d})/f'_c \), but not less than \( 2f'_c \)

\( f_t \) = \( 8f'_c \)

\( f_y \) = yield point of the web reinforcement, but not larger than 60,000 psi

\( s \) = spacing of the web reinforcement

\( V_{cd} \) = dead plus live load shear at inclined cracking caused by excessive principal tensile stress in the web

\( V_{cf} \) = dead plus live load shear at inclined cracking caused by flexural cracking

\( V_u \) = ultimate shear

\( x \) = distance from the section under consideration to the closest support

\( \lambda \) = 0.15 for beams with single webs and 0.2 for beams with double webs

6.3 DISCUSSION

In Section 6.1 it was concluded that the ultimate shear strength of full-sized bridge girders should be determined using the method that was used to predict the shear strength of the uniformly loaded test beams. The proposed specification for design of web reinforcement presented in Section 6.2 is a simplified conservative version of this method. An example problem showing the design of web reinforcement, according to the proposed specification, for a Pennsylvania Department of Highways composite girder spanning 70 ft is presented in Appendix II.
Equation 25 in the specification may be obtained by re-arranging Eq. 24, assuming that \( \beta d \) is equal to \( d_s \), and substituting \( A \frac{f_y}{b'} \) for \( rf_y/100 \). The shear carried by the concrete, \( V_c \), is assumed equal to the least value of the shear causing either diagonal tension cracking, \( V_{cd} \), or flexure shear cracking, \( V_{cf} \). The critical stress associated with diagonal tension cracking, \( f_{pt} \), is \( (6 - 0.6 \frac{x}{d})/f' \), and the stress causing diagonal tension cracking in the test beams. Similarly, the critical stress associated with flexure shear cracking, \( f_t \), is \( 8/f' \), and is, by reference to page 45, approximately 75 percent of the stress causing flexure shear cracking in the test beams. \( V_{cd} \) will be less than \( V_{cf} \) for \( x/d \) ratios less than approximately 4. The lower limit for \( f_t \) equal to \( 2/f' \) was arbitrarily set to prevent \( V_{cd} \) from being less than \( V_{cf} \) for high \( x/d \) ratios.

The tests showed that beams with small amounts of web reinforcement may fail at the load causing significant inclined cracking. While such failures may not be sudden or catastrophic, they are still undesirable because there is no advance warning of failure. Also, beams with small amounts of web reinforcement may fail prematurely due to inclined cracks extending along either the prestressing steel or the junction of the web and the bottom flange to the support, thus separating the tension flange from the rest of the beam. To prevent these types of failures it is recommended that the amount of web reinforcement in a beam not be less than that required by Eq. 26. A greater minimum amount of web reinforcement is required in beams with double webs than single webs because of the possibility of torsion when diagonal tension cracking does not occur in both webs at the same time.

If too much web reinforcement is placed in a beam, a web crushing failure may occur before the stirrups have yielded. Tests by Mattock and Kaar \(^{(19)} \) have indicated that when more web reinforcement than required by Eq. 27 is placed in a beam, all of the stirrups may not be fully effective in resisting shear.
Spacing of stirrups in the tests ranged from approximately $1/5$ to $5/8$ of the distance from the extreme fiber in compression to the lowest level at which the web reinforcement was effective. No reduction in shear strength due to excessive stirrup spacing was noted in any test. Furthermore, it was observed in several tests in which stirrup fracture failures occurred that the location of the fracture was near the apex of the inclined crack. Therefore it was concluded that vertical stirrups could be spaced at distances up to $d_s/2$, or 24 inches, whichever is smaller.

Beams with stirrups proportioned according to Eq. 25 may fail in shear rather than flexure, since the ultimate flexural capacity of the beam may be greater than the maximum moment in the beam under the specified ultimate load. Provided that the beam has been proportioned to satisfy all anchorage requirements and provided with a minimum amount of web reinforcement, these shear failures may be due to web crushing, shear compression, or fracture of the web reinforcement.

If a beam fails in shear, it is desirable that the failure be due to web crushing. This crushing occurs near the apex of the inclined crack, or near the junction of the web and top flange in flanged sections. When fracture of the stirrups occurs, it was shown that these failures may occur near the apex of the inclined cracks, indicating the need for adequate anchorage of the stirrups in the compression flange.

The shear strength of a prestressed concrete beam can be predicted from Eq. 25 by solving for for $V_u$:

$$V_u = V_c + A f_s \frac{d_s}{y_s}$$

(28)

where $V_c$, assumed equal to the shear causing inclined cracking, is the least value of $V_{cd}$ or $V_{cf}$. Equation 28 has been used, without regard to the limitations on amount and spacing of the stirrups, to predict the inclined cracking and ultimate shear strength of the test beams, and these values are compared to the test values in Table 10. Comparisons are also included in Table 10 for two beams with web reinforcement in the E Series which failed in shear.
Tests carried out by Hernandez (16) and MacGregor (17) at the University of Illinois were cited in Section 1.2. The proposed specification has been used to predict the inclined cracking and ultimate shear strength of those test beams which failed in shear, and are compared to the test values in Table 11. Of particular interest are the results of the test on FW.14.06. This I-beam with a flange to web width ratio of 3.43 had a composite slab cast on top of the beam. The test to predicted ratio of inclined cracking and ultimate shear strength for this beam was 1.46 and 1.32, respectively.

The test to predicted ratios of ultimate shear strength for the Lehigh tests and the University of Illinois tests are plotted in Fig. 54. There is good correlation between the Lehigh tests and the tests at the University of Illinois, even though the concrete strength of the two groups of tests are quite different. The average concrete strength of the Illinois tests included in Fig. 54 is approximately 3500 psi, compared to an average concrete strength of approximately 6500 psi for the Lehigh tests. The average test to predicted ratio of shear strength of all of the tests included in Fig. 54 is 1.21. It is greater than one because of the conservative calculation of the inclined cracking shear and because of consideration given to the effect of moving loads on full sized bridge girders.

The test to predicted ratios of shear strength in Fig. 54 are least in the neighborhood of an a/d ratio of 4, and increase with both increasing and decreasing values of a/d. The increase in the test to predicted ratios for the shorter shear spans reflects the increase in strength due to the closeness of the load point and the reaction. It would be difficult to take this added strength into account, and it also is undesirable to do so because the shear strength for short shear spans is greatly influenced by the bond and anchorage conditions in the end of the beam.

The increase in the test to predicted ratios for the longer shear spans is due to taking the critical section adjacent to the load
point, rather than some distance away from it as in the analysis of the concentrated load tests in Section 5.5. As discussed in Section 6.1, moving loads are more critical than stationary loads, particularly for the longer shear spans. Taking the critical section adjacent to the load point was believed to account for the effect of moving loads. Therefore it is believed that the proposed specification in Section 6.2 provides a satisfactory method for determining the static ultimate shear strength of prestressed concrete bridge girders with web reinforcement.
7. SUMMARY AND CONCLUSIONS

The objective of this investigation was the evaluation of ultimate shear strength in prestressed concrete beams. Thirty-eight tests on 23 I-beams were conducted to evaluate the effect of variation in the amount of web reinforcement and the shear span to effective depth ratio. The test beams were designed and fabricated so as to be representative of precast prestressed bridge girders.

All of the test beams had a flange width of 9 in., a total depth of 18 in., and a flange to web width ratio of 3. The beams were prestressed with six 7/16 in. diameter strands providing a longitudinal reinforcement ratio of 0.64 percent. Initial stresses in the top and bottom fibers were approximately 210 psi tension and 2150 psi compression, respectively. Hot-rolled deformed No. 3 and No. 2 bars and annealed 3/16 in. diameter deformed masonry bars were used for vertical stirrup reinforcement at spacings ranging between 2.5 and 10 inches. The percentage of web reinforcement, based on the web width, ranged between 0.08 and 0.73 percent. Concrete strengths of the test beams ranged between 5790 and 7410 psi.

Thirty-six of the tests were either one, two, or three point concentrated load tests. These tests were conducted on shear span to effective depth ratios ranging from 2.12 to 7.76, and shear failures were obtained in all but one test. Two beams were subjected to uniform loads. These beams were loaded on span length to effective depth ratios of 10.6 and 14.8, and shear failures were obtained in both tests at approximately the third point of the span.

Particular attention was directed to the determination of the shear causing significant inclined cracking, or in other words the shear causing inclined cracking which ultimately led to the shear failure. Inclined cracking was classified as either diagonal tension or flexure shear. Diagonal tension cracking occurred in the concentrated load tests.
on shear span to effective depth ratios less than approximately 4.5, and was closely predicted as the shear causing a principal tensile stress of \((8 - 0.78 \frac{a}{d})\sqrt{f'_c}\) at the center of gravity of the section at the mid-point of the shear span. Flexure shear cracking occurred in the concentrated load tests on shear span to effective depth ratios greater than approximately 4.5, and was closely predicted as the shear causing a tensile stress in the bottom fibers of \(9.5\sqrt{f'_c}\) at a distance 
\[
(a + 31.6 - 15.6 \left(\frac{a}{d}\right) + 0.88 \left(\frac{a}{d}\right)^2)
\] from the support. In the uniform load tests, significant inclined cracking was closely predicted as the shear at any distance, \(x\), from the support causing either a principal tensile stress of \((8 - 0.78 \frac{x}{d})\sqrt{f'_c}\) at the center of gravity of the section at \(x - \frac{d}{2}\), or a tensile stress of \(9.5\sqrt{f'_c}\) in the bottom fibers of the section at \(x - \frac{d}{2}\).

Three different modes of shear failure were observed. Web crushing failures generally occurred gradually and non-catastrophically in tests on the shorter \(\frac{a}{d}\) ratios. Stirrup fracture failures generally occurred with little warning and catastrophically in tests on the larger \(\frac{a}{d}\) ratios. Most of the shear compression failures occurred suddenly in reloaded tests due to separation of the compression flange along the projected path of the critical inclined crack. It was found that the ultimate shear strength of the test beams could be closely predicted as the sum of the shear causing inclined cracking plus the shear carried by the stirrups which are crossed by an idealized inclined crack. The effective horizontal projection of this inclined crack was assumed equal to the distance from the extreme fiber in compression to the lowest level at which the stirrups could be counted upon to develop their yield point if crossed by a crack.

Based upon these test results, a specification is proposed in Section 6.2 for the design of web reinforcement in bridge girders. Consideration is given to the fact that bridge girders are subjected to combined distributed and concentrated loads, where the concentrated loads may be moving loads. The method proposed for evaluating shear strength in the specification is a simplified and conservative version of the method used to predict the shear strength of the test beams.
8. **ACKNOWLEDGEMENTS**

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Completion of this work was facilitated by the capable help of the Fritz Engineering Laboratory staff and technicians. The authors wish to thank Mr. J. C. Badoux, Mr. W. F. Chen, and Mr. H. E. Brecht for their work on various stages of the project, and Miss Valerie Austin for typing the manuscript. The authors also wish to thank Mr. J. E. Scott of the Bethlehem Steel Company for assistance in developing suitable web reinforcement for this investigation.
9. **NOTATION**

- \( a \): Length of shear span
- \( a_A, a_B, a_C \): Length of shear span in first or second test
- \( A \): Cross-sectional area of beam
- \( A_s \): Total cross-sectional area of prestressing steel
- \( A_{s_i} \): Cross-sectional area of prestressing steel at a particular level, \( i \)
- \( A_v \): Area of vertical web reinforcement
- \( b \): Width of compression flange
- \( b' \): Web width of I-beam
- \( c \): Distance from extreme fibers in compression to neutral axis
- \( C \): Horizontal component of the resultant compressive force in the concrete
- \( c_g \): Center of gravity of beam cross-section
- \( c_{gs} \): Center of gravity of prestressing steel
- \( d \): Distance from extreme fiber in compression to \( c_{gs} \) or effective depth
- \( d_c \): Distance from extreme fiber in compression to resultant horizontal tensile force in steel
- \( d_i \): Distance from extreme fiber in compression to particular level, \( i \), of steel
- \( e \): Distance from \( c_g \) to \( c_{gs} \)
- \( E_c \): Modulus of elasticity of concrete
- \( f_{s_i} \): Stress in steel at a particular level, \( i \)
- \( f_y \): Yield point of web reinforcement
- \( f_u \): Tensile strength of web reinforcement
- \( f'_c \): Ultimate compressive strength of concrete
- \( f'_r \): Modulus of rupture strength of concrete
- \( f'_s \): Ultimate tensile strength of prestressing steel
- \( f'_{sp} \): Splitting tensile strength of concrete
- \( f'_t \): Flexural tensile strength of concrete
F  Prestress force at the time of test
F₁  Prestress force before transfer
i  Particular level of steel
I  Moment of inertia of beam cross-section
j  Ratio of distance between C and T to d
k₁  Ratio of maximum compressive stress to average compressive stress
k₂  Ratio of distance from extreme fibers in compression to resultant compressive force in the concrete to c
k₃  Ratio of maximum compressive stress to strength of concrete, f'_c, determined from standard cylinder tests
L  Span length
M  Moment
M_cr  Applied load moment causing flexural cracking
M_d  Dead load moment
M_fc  Moment causing flexural cracking
M_fu  Moment causing flexural failure
n  Number of levels, i, of steel
Q  Moment, about the cg, of the area of the cross-section on one side of the horizontal section on which the shearing stress is desired
r  Vertical web reinforcement ratio in percent, equal to 100A_v/b's
s  Spacing of vertical web reinforcement
T  Resultant tensile force in the steel
T_h  Horizontal component of the tensile force in the steel
T_v  Vertical component of the tensile force in the steel
V_u  Nominal ultimate shear stress
V  Shear
V_c  Shear carried by the concrete
V_cr  Applied load shear causing flexural cracking
V_fu  Applied load shear causing flexural failure
V_ic  Shear causing significant inclined cracking
V_u  Ultimate shear
V_wh  Horizontal component of force in the web reinforcement
V_wv, V_w  Vertical component of force in the web reinforcement
w  Uniform load
x  Distance from the section under consideration to the closest support
y  Vertical location of point under consideration measured from the cg
zb, zt  Section modulus with respect to stress in the bottom fibers and top fibers, respectively
α  Dimensionless parameter which, when multiplied by d, defines the distance from the load point to the location of a flexural crack responsible for the developing of significant flexure shear cracking
β  Dimensionless parameter which, when multiplied by d, defines the effective horizontal projection of a significant inclined crack
γ  Dimensionless parameter which, when multiplied by d, defines the distance from the section under investigation to the point along the cg at which significant diagonal tension cracking begins
ε  Strain
εci  Compressive strain in the concrete at a particular level, i
εcu  Tensile concrete strain at a particular level, i
εsi  Strain in steel at the effective prestress force at a particular level, i
εsu  Total strain in steel at a particular level, i
εu  Ultimate concrete compressive strain
θ  Angle, with respect to the horizontal, of the compressive stress trajectory
σ  Normal stress
σpt  Principal tensile stress
σcg  Principal tensile stress at the cg
σb  Tensile stress in the extreme fiber
τ  Shear stress
yi  Ratio of the steel strain to the tensile concrete strain at a particular level, i
Table 1. Test Beam Details

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Table 5. First Test on Beams Subjected to Concentrated Loads

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<th>$L$ (in.)</th>
<th>$M_{cr}$ (kip-ft)</th>
<th>$V_{ic}$ (kips)</th>
<th>$V_{uc}$ (kips)</th>
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* Significant inclined cracking did not occur in Region A of test beams F-10 and F-12 until the second test.
Table 6. Second Test on Beams Subjected to Concentrated Loads

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Note: Significant inclined cracking occurred in Region A of F-10 at $V_{lc} = 27.0$ kips and also in Region A of F-12 at $V_{lc} = 25.0$ kips.
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<th>( f'_t ) (psi)</th>
<th>( f'_t \sqrt{f'_c} )</th>
<th>( \frac{f'<em>t}{f'</em>{sp}} )</th>
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Table 11. Comparison of Illinois Test Results with Proposed Specification

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11. FIGURES
ELEVATION OF TEST BEAMS

SECTION PROPERTIES

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<tr>
<td>Z¹</td>
<td>428.2 in.³</td>
<td>435.1 in.³</td>
</tr>
<tr>
<td>Z²</td>
<td>428.2 in.³</td>
<td>451.2 in.³</td>
</tr>
<tr>
<td>Q¹</td>
<td>262.5 in.³</td>
<td>270.9 in.³</td>
</tr>
<tr>
<td>Q²</td>
<td>286.5 in.³</td>
<td>298.7 in.³</td>
</tr>
<tr>
<td>Q³</td>
<td>262.5 in.³</td>
<td>276.6 in.³</td>
</tr>
</tbody>
</table>

* Concrete Section
** Transformed Section

Fig. 1 Dimensions and Properties of F Series Test Beams

Fig. 2 Sieve Analysis of Aggregate
Rate of loading: 0.1 in. per min. to yield 0.2 in. per min. after yield
Gage length: 24 in.

Fig. 3 Cylinder Tests for F-14
Fig. 4 Load-Strain Curve for Prestressing Strand

(a) Stress-strain curve for No. 3 bar

(b) Stress-strain curve for No. 2 bar

(c) Stress-strain curve for 3/16 in. annealed masonry bar

Note: Readings not carried into the strain-hardening range.

Fig. 5 Stress-Strain Curve for Web Reinforcement
Fig. 6  Concrete Strain Along CGS from Before Transfer to Test for F-14

Fig. 7  Concrete Strain Distribution at Mid-Spar from Before Transfer to Test for F-14
Note: Shear failures occurred in Region B in all first tests except for F-9, in which case the failure occurred in Region A. Therefore the second test for F-9 is similar to that shown above except that Region A is instead Region B.

Fig. 8 Testing Arrangement for All Concentrated Load Tests except F-20, F-21, and F-22

Fig. 9 Testing Arrangement for Concentrated Load Tests on F-20, F-21, and F-22
Fig. 10 Details of Typical Concentrated Load Test Set-Up

Fig. 11 Load-Deflection Curves for Concentrated Load Tests - First Test
Fig. 12 Concrete Deformation Along CGS during Test of F-4

Fig. 13 Concrete Deformation Along CGS during Test of F-14
Fig. 14 Web Crushing Failure in F-5

(a) Before Failure

(b) After Failure

(c) Failure Accentuated by Further Deflection
Fig. 15 Web Crushing Failures in F-1, F-3, F-6, F-7, and F-10
Fig. 16 Stirrup Fracture Failure in F-13

(a) Before Failure

(b) After Failure
Fig. 17 Stirrup Fracture Failures in F-4, F-9, F-12, F-15, and F-16
Fig. 18 Failures in F-20, F-21, and F-22
Fig. 19 Load-Deflection Curves for Concentrated Load Tests - Second Test
Web Crushing Failures in F-3, F-7, and F-19
Fig. 21 Stirrup Fracture Failures in F-10, F-11, and F-13
Fig. 22 Failures in the Compressive Region of F-5, F-9, and F-12
Fig. 23  Testing Arrangement for Uniform Load Tests

Fig. 24  Uniform Load Test Set-Up
Fig. 25  Load-Deflection Curves for Uniform Load Tests
Fig. 26 Inclined Cracking in F-17

Fig. 27 Inclined Cracking in F-18
Fig. 28  F-17 After Failure
Fig. 29 F-18 After Failure
Fig. 30  Comparison of the Test Results with Paragraph 1.13.13 of the AASHO Specifications

Fig. 31  Types of Cracking Observed in Test Beams
Fig. 32 Comparison of Test and Predicted Shear Causing Flexural Cracking

Fig. 33 Relation Between $f'_L/\sqrt{f'_c}$ and $f'_c$

Fig. 34 Relation Between $f'_L/f'_sp$ and $f'_c$
Fig. 35  Comparison of Equations which Predict Shear Causing Significant Inclined Cracking

Fig. 36  Variation in the Principal Tensile Stress at the CG associated with Significant Inclined Cracking and the Shear Span to Effective Depth Ratio
Fig. 37 Distance from the Load Point to the Flexural Crack causing Significant Flexure Shear Cracking

Fig. 38 Distance from the Load Point to the Section at which the Stress in the Bottom Fibers is $9.5/f_c$ at the Shear causing Significant Flexure Shear Cracking
Fig. 39 Comparison of Test and Predicted Shear Causing Significant Inclined Cracking

Fig. 40 Assumed Strain and Stress Distribution at Flexural Failure
Fig. 41 Free-Body Diagram at an Inclined Crack

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Before the inclined crack forms

After cracking, assuming no web reinforcement and no doweling action

After cracking, assuming web reinforcement acting and doweling action

Fig. 42 Positions of the Resultant Compressive Force in the Concrete

Fig. 43 Forces Acting at Two Adjacent Inclined Cracks

Fig. 44 Wedge Failure of the Compression Flange
Fig. 45 Variation in Ultimate Shear Strength, with Amount of Web Reinforcement and Shear Span to Effective Depth Ratio.

Fig. 46 Variation in Nominal Ultimate Shearing Stress with Amount of Web Reinforcement and Shear Span to Effective Depth Ratio.
Fig. 47 Variation in the Difference between the Ultimate Shear and the Significant Inclined Cracking Shear with the Amount of Web Reinforcement

Fig. 48 Variation in the Difference between the Ultimate Shear and the Predicted Inclined Cracking Shear with the Amount of Web Reinforcement
Fig. 49 Comparison of Test to Predicted Ratios of Shear Strength based on Eq. 21
Fig. 50  Effect of Stirrup Spacing on Shear Strength

Fig. 51  Idealized Inclined Crack
Fig. 52  Shear Strength of F-17

Fig. 53  Shear Strength of F-18
Fig. 54 Comparison of Test to Predicted Ratios of Shear Strength based on the Proposed Specification
12. APPENDIX I

SKETCHES OF CRACK PATTERNS AT THE SHEAR CAUSING SIGNIFICANT INCLINED CRACKING

(See page 18)
BEAM F-2

BEAM F-3
13. **APPENDIX II**

   EXAMPLE PROBLEM
DESIGN OF WEB REINFORCEMENT FOR A

PENNSYLVANIA DEPARTMENT OF HIGHWAYS

COMPOSITE GIRDER SPANNING 70 FEET

The composite girder cross-section shown in Fig. A and the cross-sectional properties tabulated below were taken from "Standards For Prestressed Concrete Bridges", Commonwealth of Pennsylvania, Department of Highways, Bridge Unit, dated September 19, 1960.

Girder:
\[ f'_c = 5000 \text{ psi (4500 psi at prestress transfer)} \]
\[ f'_t = 300 \text{ psi at prestress transfer} \]
\[ I_g = 172,690 \text{ in.}^4 \]

Composite Section:
\[ I_c = 466,420 \text{ in.}^4 \]
\[ y_c = 35.05 \text{ in.} \]

Notes: 1. \( E_c \) for slab and girder concrete assumed equal.
2. The weight of the composite section includes a \( \frac{1}{4} \) in. monolithic wearing surface and 30 psf provision for a future wearing surface.

Web Reinforcement:
No. 4 bars - \( f_y = 40,000 \text{ psi} \)

Prestressing:
Straight strand - \( f'_s = 250 \text{ ksi} \)
\[ A_s = 5.55 \text{ in.}^2 \]
\[ F = 749,000 \text{ lb} \]

Loading:
H20-S16-44
STEP 1: DETERMINE ULTIMATE SHEAR V_u

1.1 Load Factor

From paragraph 1.13.6 of the AASHO Specifications, the required ultimate load capacity is

\[ 1.5D + 2.5 (L + I) \]

1.2 Dead Load

Acting on girder = 117 lb/in.
Acting on composite section = 18 lb/in.
135 lb/in.

1.3 Live Load

Pennsylvania Department of Highways Specifications require that interior girders be designed for

\[ \frac{s}{5.5} \] wheel loads = \( \frac{7}{5.5} = 1.27 \)

Impact factor \( I = \frac{50}{70 + 125} = 0.256 \)

Therefore the live load, including impact, acting on the girder is the system of wheel loads

\[
\begin{array}{c}
6.38^k \\
25.5^k \\
25.5^k \\
\end{array}
\]

14' 14'

1.4 Ultimate Load

Assuming that the ultimate dead load in excess of the weight of the girder acts on the composite section, the ultimate load is:

\[
\begin{array}{c}
16.0^k \\
63.8^k \\
63.8^k \\
\end{array}
\]

\( w_D = 203 \text{ lb/in.} \)
\( (w_g = 117 \text{ lb/in. acting on girder alone}) \)

1.5 Ultimate Shear Diagram

The maximum shear adjacent to the support due to the dead load is equal to \( (203)(70)(12)/2 = 85,000 \text{ lb} \). Maximum shear due to the live load occurs under the trailing wheel. With the trailing wheel adjacent
to the support the maximum shear is 124,500 lb. With the trailing wheel at mid-span the maximum shear is 52,500 lb.

The maximum shear at any section in the beam is shown in the ultimate shear diagram in Fig. B.

STEP 2: DETERMINE $V_{cd}$

The state of stress at the center of gravity of the composite girder is assumed to be defined by a compressive normal stress, $\sigma$, due to prestress and bending and a shear stress, $\tau$. This state of stress will cause diagonal tension cracking when the principal tensile stress is equal to $f_{pt}$.

$$\sqrt{(\sigma/2)^2 + \tau^2} - \sigma/2 = f_{pt}$$

or

$$\tau = \sqrt{f_{pt}^2 + \sigma f_{pt}}$$

(2.0)

2.1 Normal Stress $\sigma$

The normal stress at the center of gravity of the composite girder is

$$\sigma = \frac{F}{A} - \frac{F e_g y}{I_g} + \frac{M_y}{I_g}$$

(2.1)

where

$y = 35.05 - 21.39 = 13.66$ in.

$M = \frac{w}{2} (Lx - x^2) = 58.5 (840x - x^2)$

Therefore

$$\sigma = \frac{749,000}{708} - \frac{(749,000)(11.19)(13.66)}{172,690} + \frac{58.5(840x - x^2)(13.66)}{172,690}$$

$$\sigma = 395 + 0.00463(840x - x^2) = 395 + 0.207(840x - x^2)$$

2.2 Shear Stress $\tau$

The shear stress at the center of gravity of the composite girder is

$$\tau = \frac{Q_g}{I_g b'} + (V_{cd} - V_g) \frac{Q_c}{I_c b'}$$

(2.2)
where \( V_g = \frac{w}{2} (L - 2x) = 58.5(840 - 2x) = 49,100 - 5240 \frac{x}{d} \)

\( b' = \text{thickness of the web} = 8 \text{ in.} \)

\( Q_g = 4240 \text{ in.}^3 \)

\( Q_c = 11,140 \text{ in.}^3 \)

\( Q \) is the moment, about the center of gravity of the girder, of the area of the girder on one side of the center of gravity of the composite section. \( Q_c \) is the moment, about the center of gravity of the composite section, of the area of the composite section on one side of the center of gravity of the composite section.

2.3 Evaluate \( V_{cd} \)

From Eqs. (2.0) and (2.2)

\[
V_{cd} = V_g \left( 1 - \frac{I_c}{I_{g'} \overline{Q}} \right) + \frac{I_c b'}{Q_c^2} \sqrt{f_{pt}^2 + \frac{\sigma f_{pt}}{2}} \tag{2.3}
\]

Note that if the center of gravity of the composite section had been in the top flange, the only changes in Eq. (2.3) would be in \( Q_g, Q_c, \) and \( \sigma. \)

Now

\[
1 - \frac{I_c}{I_{g'} \overline{Q}} = 1 - \frac{466,420(4240)}{(172,690)(11,140)} = -0.03
\]

\[
\frac{I_c b'}{Q_c} = \frac{466,420(8)}{11,140} = 335 \text{ in.}^2
\]

Therefore

\[
V_{cd} = 0.03 V_g + 335 \sqrt{f_{pt}^2 + \frac{\sigma f_{pt}}{2}}
\]

From the proposed specification

\[
f_{pt} = (6 - 0.6 \frac{x}{d}) f_c' \text{ but not less than } 2 f_c'
\]

\( V_{cd} \) can now be determined at any distance \( x \) from the support. \( V_{cd} \) is plotted in Fig. C. In general it would be sufficient to calculate \( V_{cd} \) at 3 or 4 points, as shown in the following table.

<table>
<thead>
<tr>
<th>( \frac{x}{d} )</th>
<th>( f_{pt} ) psi</th>
<th>( \sigma ) psi</th>
<th>( V_g ) kips</th>
<th>( V_{cd} ) kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>382</td>
<td>559</td>
<td>43.9</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>339</td>
<td>706</td>
<td>38.7</td>
<td>198</td>
</tr>
<tr>
<td>3</td>
<td>297</td>
<td>833</td>
<td>33.4</td>
<td>193</td>
</tr>
<tr>
<td>4</td>
<td>254</td>
<td>942</td>
<td>28.1</td>
<td>184</td>
</tr>
</tbody>
</table>
STEP 3: DETERMINE V_{cf}

Flexure shear cracking is assumed to occur when the stress in the extreme fiber in tension at a distance equal to d from the section under consideration is equal to f_t. Considering the stress at this point

\[
\frac{M_y g}{I_g} + \frac{(M_D-M)g}{I_c} + \frac{(V_{cf}-V_D)(x-d)g}{I_c} = \frac{F e y g}{I_g} + \frac{F e y g}{I_g} + f_t
\]

or

\[
V_{cf} = V_D + \frac{I_c}{(x-d)g} \left[ \frac{F e y g}{I_g} + \frac{F e y g}{I_g} + f_t - \frac{M_y g}{I_g} - \frac{(M_D-M)g}{I_c} \right]
\]  \hspace{1cm} (3.0)

3.1 Evaluate V_{cf}

Since

\[
\frac{I_c}{y_c} = 13,300 \text{ in}^3
\]

\[
\frac{I_g}{y_g} = 8070 \text{ in}^3
\]

\[
V_{cf} = V_D + \frac{13,300}{x} \left[ \frac{749,000}{708} + \frac{749,000(11.19)}{8070} + f_t - \frac{M_g}{8070} - \frac{M_D-M}{13,300} \right]
\]

\[
= V_D + \frac{297}{x d} \left[ 2094 + f_t - \frac{M_g}{8070} - \frac{M_D-M}{13,300} \right]
\]

where

\[
V_D = \frac{wD}{2} (L-2x) = 101.5(840-2x) = 85,400 - 9100 \left( \frac{x}{d} \right)
\]

\[
M_D = 101.5(840x - x^2) = 3,820,000 \left( \frac{x}{d} \right) - 203,000 \left( \frac{x}{d} \right)^2
\]

and

\[
\overline{x} = x-d
\]

From the proposed specification:

\[
f_t = \frac{8}{f_c^*} = 565 \text{ psi}
\]

V_{cf} can now be determined at any distance x from the support. V_{cf} is plotted in Fig. C. In general it would also be sufficient to calculate V_{cf} at 3 or 4 points, as shown in the following table.
STEP 4: SELECT WEB REINFORCEMENT

4.1 Minimum $A_v$

The proposed specification requires that enough web reinforcement be placed at any section to take 15 percent of $V_u$. This may be represented in Fig. C by the line 0.85 $V_u$.

4.2 Select Stirrups

At any section the difference between $V_u$ and the minimum value of $V_{cd}$, $V_{cf}$, or 0.85 $V_u$ is the shear which must be carried by the web reinforcement. This is represented by the vertically hatched area in Fig. C. The maximum difference is 30 kips at a distance from the support. Assuming that the stirrups are anchored in the tension flange by hooks so that they are effective 3 in. above the bottom of the girder, $d_s = 45$ in. From Eq. (25) in the proposed specification

$$s = \frac{A_v f_y d_s}{V_u - V_c} = \frac{(0.2)(40)(45)}{30} = 12 \text{ in.}$$

Therefore either single No. 4 bars spaced at 12 in. or double No. 4 bars spaced at 22 in. ($d_s/2$) throughout the span satisfy the proposed specification.

4.3 Check Maximum $A_v$

From Eq. (27)

$$s_{min} = \frac{A_v f_y}{7 b' \sqrt{f_c}} = \frac{0.2(40,000)}{7(8)(70.7)} = 2'' < 12''$$
14. REFERENCES

1. Knudsen, K. E., Eney, W. J.
ENDURANCE OF A FULL-SCALE PRE-TENSIONED CONCRETE BEAM
Fritz Engineering Laboratory Report No. 223.5, Lehigh University, April 1953

ENDURANCE OF A FULL-SCALE POST-TENSIONED CONCRETE MEMBER
Fritz Engineering Laboratory Report No. 223.6, Lehigh University, May 1954

3. Walther, R. E.
THE ULTIMATE STRENGTH OF PRESTRESSED AND CONVENTIONALLY REINFORCED CONCRETE UNDER THE COMBINED ACTION OF MOMENT AND SHEAR
Fritz Engineering Laboratory Report No. 223.17, Lehigh University, October 1957

4. Walther, R. E.
SHEAR STRENGTH OF PRESTRESSED CONCRETE BEAMS
Fritz Engineering Laboratory Report No. 223.17A, Lehigh University, November 1957

5. Walther, R. E., Warner, R. F.
ULTIMATE STRENGTH TESTS OF PRESTRESSED AND CONVENTIONALLY REINFORCED CONCRETE BEAMS IN COMBINED BENDING AND SHEAR
Fritz Engineering Laboratory Report No. 223.18, Lehigh University, September 1958

FURTHER INVESTIGATION INTO THE SHEAR STRENGTH OF PRESTRESSED CONCRETE BEAMS WITHOUT WEB REINFORCEMENT
Fritz Engineering Laboratory Report No. 223.22, Lehigh University, January 1962

7. Hanson, J. M., Hulsbos, C. L.
OVERLOAD BEHAVIOR OF PRESTRESSED CONCRETE BEAMS WITH WEB REINFORCEMENT
Fritz Engineering Laboratory Report No. 223.25, Lehigh University, February 1963

8. ACI-ASCE Committee 426
SHEAR AND DIAGONAL TENSION
Journal of the American Concrete Institute, Proceedings V. 59, January, February, and March 1962, pp. 1-30, 277-334, 353-396

ULTIMATE STRENGTH IN SHEAR OF SIMPLY-SUPPORTED PRESTRESSED CONCRETE BEAMS WITHOUT WEB REINFORCEMENT
Journal of the American Concrete Institute, Proceedings V. 26, October 1954, pp. 181-200

-140-
10. Sozen, M. A., Zwoyer, E. M., Siess, C. P.
   INVESTIGATION OF PRESTRESSED CONCRETE FOR HIGHWAY BRIDGES,
   PART I: STRENGTH IN SHEAR OF BEAMS WITHOUT WEB REINFORCEMENT
   Bulletin No. 452, University of Illinois Engineering Experiment Station, April 1959

11. Evans, R. H., Schumacher, E. G.
   SHEAR STRENGTH OF PRESTRESSED BEAMS WITHOUT WEB REINFORCEMENT
   Journal of the American Concrete Institute, Proceedings V. 60, November 1963, pp. 1621-1641

12. Warner, R. F., Hall, A. S.
   THE SHEAR STRENGTH OF CONCRETE BEAMS WITHOUT WEB REINFORCEMENT
   Third Congress of the Federation Internationale de la Precontrainte, Berlin, 1958

13. Evans, R. H., Hosny, A. H. H.
   THE SHEAR STRENGTH OF POST-TENSIONED PRESTRESSED CONCRETE BEAMS
   Third Congress of the Federation Internationale de la Precontrainte, Berlin, 1958

14. Hulsbos, C. L., Van Horn, D. A.
   STRENGTH IN SHEAR OF PRESTRESSED CONCRETE I-BEAMS
   Progress Report, Iowa Engineering Experiment Station, Iowa State University, April 1960

15. Bernhardt, C. J.
   DIAGONAL TENSION IN PRESTRESSED CONCRETE BEAMS
   Proceedings, World Conference on Prestressed Concrete, July 1957

16. Hernandez, G.
   STRENGTH OF PRESTRESSED CONCRETE BEAMS WITH WEB REINFORCEMENT

17. MacGregor, J. G.
   STRENGTH AND BEHAVIOR OF PRESTRESSED CONCRETE BEAMS WITH WEB REINFORCEMENT
   Ph.D Thesis, University of Illinois, August 1960

   STRENGTH IN SHEAR OF PRESTRESSED CONCRETE BEAMS WITH WEB REINFORCEMENT
   Presented at the Convention of the American Society of Civil Engineers, New Orleans, March 1960

19. Mattock, A. H., Kaar, P. H.
   PRECAST-PRESTRESSED CONCRETE BRIDGES. 4. SHEAR TESTS OF CONTINUOUS GIRDERS
   Journal of the PCA Research and Development Laboratories, V. 4, No. 1, January 1961, pp. 19-46
20. Leonhardt, F., Walther, R.
BEITRÄGE ZUR BEHANDLUNG DER SCHUBPROBLEME IM STAHLBETONBAU
Beton-und Stahlbetonbau, 57. Jahrgang, Heft 2, February 1962, pp. 32-44

21. The American Association of State Highway Officials
STANDARD SPECIFICATIONS FOR HIGHWAY BRIDGES, EIGHTH EDITION
Published by the Association, Washington, D.C., 1961

22. Mattock, A. H., Kriz, L. S., Hognestad, E.
RECTANGULAR CONCRETE STRESS DISTRIBUTION IN ULTIMATE STRENGTH DESIGN
Journal of the American Concrete Institute, Proceedings V. 57, No. 8, February 1961, pp. 875-928

23. Warner, R. F., Hulsbos, C. L.
PROBABLE FATIGUE LIFE OF PRESTRESSED CONCRETE FLEXURAL MEMBERS
Fritz Engineering Laboratory Report No. 223.24A, Lehigh University, July 1962

24. Bruce, R. N.
THE ACTION OF VERTICAL, INCLINED, AND PRESTRESSED STIRRUPS IN PRESTRESSED CONCRETE BEAMS
Journal of the Prestressed Concrete Institute, V. 9, No. 1, February 1964, pp. 14-25.

25. Commonwealth of Pennsylvania, Department of Highways
STANDARDS FOR PRESTRESSING CONCRETE BRIDGES
September 1960