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State of the Art of Analytical Prediction for Confined Concrete

Fatih Cetisli

Clay J. Naito

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State of the Art of Analytical Prediction for Confined Concrete

by

Fatih Cetisli, M.S.C.E
Clay J. Naito, Ph.D., P.E

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ABSTRACT

Confinement prevents structural members from adverse effects of additional forces resulted from earthquake. For better design and retrofit strategies complete axial stress-strain behavior confined concrete should be well defined.

In this study, developing a testing method is aimed. From the literature review, it is seen that the analytical models that are used for the prediction of axial stress-strain behavior of confined concrete mostly based on constant confining pressure. It is experimentally showed that constant confining pressure is not applicable in practical applications. Due to the characteristics of jacketing materials, the confining pressure has a passive behavior that is related to the lateral dilation of confined concrete. Also, there are a few variably confined concrete models. However, it is showed that they are not accurate enough to predict the experimental results for axial stress-strain behavior of passively confined concrete specimens.

A finite element model for passively confined concrete specimens is defined by using finite element modeling program. It is proofed that, the axial stress-strain behavior of concrete modeled with damaged plasticity option predicts well the experimental results. It is also analytically demonstrated that, the concrete confined with constant pressure does not behave realistic.

While comparing the predicted and evaluated results it is observed that three main parameters takes role in the inaccurate estimation of axial stress-strain behavior of confined concrete. These are: inaccurate predictions for the ultimate value of confining pressure, lack estimation of post peak behaviors, and inadequate prediction for variation of dilation ratio with respect to axial strain at post peak behavior.

It is validated that, analytical models match with the experimental result up to peak strength, but does not estimates well the post peak behavior. With this study, it is understood that more detailed and verified experimental study should be conducted with developing a new testing method. By using a tri-axial cell mechanism, and applying pressure varying with the dilation of concrete material imperfections and other outside effects can be neglected and much accurate database can be created. With this database much realistic behaviors of confined concrete can be identified and developed for design and retrofit strategies.
1 INTRODUCTION

1.1 Project Description

Because of the effect of earthquakes, shear forces, additional axial forces, and sometimes torsion occur in the structural members. After every high magnitude earthquake, some damages and/or failures occur in the built environment because of these additional forces. These damages can be attributed to low confinement, low resistance to shear forces, poor connection details, or poor construction for reinforced concrete structures. Lateral forces induced by an earthquake, results in overturning moments on the structure. The structure resists these moments through the flexural and axial load capacity of columns. To increase the lateral resistance of the structure the capacity of reinforced concrete columns are improved through confinement. This is achieved through closely spaced stirrup or spiral reinforcement, the use of high strength steel, or concrete filled tube (CFT). The CFT technique has been used for over 50 years for the enhancement of concrete columns subjected to earthquake loading. Today with the development of fiber reinforced polymer (FRP) materials new strengthening methods are being studied and implemented.

Significant advances have been made in retrofitting strategies for structures subjected to seismic loads. As a strengthening technique, the application of a confining jacket is becoming widely adopted. Both steel and fiber reinforced polymer (FRP) jackets are being used for this purpose. Under extreme loading the concrete dilates laterally and is passively restrained by jacket system. This restraint increases the concrete longitudinal axial strength of the column beyond its unconfined strength. The behavior of the concrete under this passive confinement is not comprehensively understood.

The primary objective of this research is, to develop a better understanding of the performance of concrete subjected to confining stress by conducting a thorough evaluation of existing models and generation of additional data.

Starting with a literature review, analytical models (developed previously by various researchers) are summarized and are used for design predictions. Finite Element Modeling is conducted to predict and compare results from experimental evaluation and predictor models. Using cylindrical specimens, an experimental evaluation is conducted. Some of these specimens confined by using carbon fiber reinforced polymers (CFRP) sheets, glass fiber reinforced polymers (GFRP) sheets and steel tubes. In addition, unconfined specimens are tested for comparison and observation of the strength gain. All results from predictor models, FEM analysis, and experimental evaluation are compared. The second step of this project is presented as future research that can be followed by this study.
1.2 **Research Objectives**

The stress-strain behavior of concrete that is externally confined needs to be established independent of specific jacket material. The objectives of this research are as follows:

1) To investigate strength and deformation capacity gain of axially loaded concrete members confined by CFRP, GFRP and steel jackets.

2) To determine stress-strain behaviors of concrete confined by various types of jackets, and determine the mechanical relationship between jacket material and confined concrete.

3) Evaluate the accuracy of current predictor models.

4) To develop an accurate method for modeling confined concrete using the finite element method (FEM).

1.3 **Summary of approach**

To achieve objectives 1 & 2, experimental evaluation of axially loaded confined concrete specimens was conducted. Specimens were prepared by using 6 in. x 12 in. cylindrical molds. Evaluation was conducted by using a universal testing machine (SATEC) with a capacity of 600 kips. Load was applied to the specimens through displacement control of the SATEC machine. The same batch of concrete was used for the entire testing matrix. Two types of FRP jacketing material and two types of steel jackets were used for confinement of the concrete. For measurement of deformation a ring LVDT frame was designed and manufactured and strain gauges attached to the surface of the jacketing material. By using the 12-bit data acquisition system (DAS) data from the LVDT’s (linear voltage displacement transducer) and strain gauges were collected.

Under axial compression, concrete dilates according to the concepts of Poisson. Under excessive axial load the concrete crushes axially and fails laterally due to the development of tension cracks. To resist the failure due to lateral effects, closely spaced stirrups or spiral reinforcement are used to confine the concrete. In addition, as a rehabilitation method, jacketing systems of steel, carbon, and glass FRP’s are used externally to confine the concrete. All of these confinement materials become stressed as the concrete dilates. In other words, the confinement is passively applied to the concrete by the confining materials. Depending on the design of the passive reinforcement the confinement may act elastically or non-linearly in resisting the concrete dilation. To study this issue a three-phased experimental program is developed.

- Passive confinement test series
- Constant confinement test series
- Variable confinement test series.
As a part of this thesis only passive confinement test series will be conducted. Constant and Variable Confinement test series is discussed as future work and will continue as a second level research of this project.

In the passive confinement test series, concrete specimens confined by various types of materials were manufactured and tested under axial compression. The materials used for jacketing were glass based fiber reinforced polymers (GFRP), carbon based fiber reinforced polymers (CFRP) and steel tubes (CFT). According to predictor models [Mander-1988, Madas-1992, Kestner-1997], GFRP and CFRP jackets were designed to provide confining pressures equal to approximately 20% of the unconfined concrete strength (4 ksi.). For steel tubes, different predictor model was used [Susantha, 2001]. The steel tubes were designed to give confining pressures of 10% and 20% of unconfined concrete strength. For each group, three confined concrete specimens are tested. Although to compare with confined concrete specimens, three unconfined concrete specimens are tested. A total of 15 specimens are examined in this series.

To achieve goal 3, a comprehensive study of available literature is conducted to examine the state of research of confined concrete. These models are applied to the results generated from the experimental program previously described. The models developed depend primarily on the axial strain-axial stress behavior defined by Popovics [1973], and later modified by Mander [1988]. Various predictor models have been developed for confined concrete based on analytical, numerical, and experimental evaluation. These predictor models were derived by Mander [1988], Madas [1992], Kestner [1997], Spoelstra [1999], Susantha [2001], and Lam [2002]. The models are summarized briefly in Chapter 2 - Background and Chapter 3 - Analytical models.

To achieve final goal, a Finite Element Model was built by using Abaqus [Ref.12]. CAE version of Abaqus 6.3 is used for meshing the cylindrical specimens. Concrete damaged plasticity option with tension softening and compression hardening is used for material definition. For jackets, elastic (FRP) or elastic-plastic (steel) material option are defined. Jacket nodes are constrained to the nodes at lateral surface of concrete.

1.4 Scope of thesis

In Chapter 2, a background of the state of research is presented. Much of the reviewed literature is about axial compressive loading to concrete specimens. A summary of analytical models is detailed in Chapter 3. The passive confinement test series is detailed in Chapter 4. Chapter 5 discusses the results from analytical models and experimental evaluations. In Chapter 6 results of the research are summarized and suggestions for future research are presented. Constant and variable confinement test series are presented in Chapter 7 as the future work. Calculations and FEM coding are presented in the appendix.
1.5 **Notation**

- \( A_c \) = cross-section area of concrete
- \( D \) = diameter of concrete specimen
- \( E_c \) = modulus of elasticity of plain concrete
- \( E_f \) = secant modulus of FRP material
- \( E_s \) = modulus of elasticity of steel
- \( f_c \) = axial compressive concrete stress
- \( f_c' \) = concrete compressive strength determined by standard cylinder tests
- \( f_{c0} \) = compressive strength of unconfined concrete
- \( f_{cc} \) = compressive strength of confined concrete
- \( f_{frp} \) = strength of FRP material
- \( f_l \) = lateral confining pressure
- \( f_y \) = yielding strength of steel material
- \( f_t \) = tensile strength for steel material
- \( n \) = number of plies of FRP material
- \( t \) = thickness of jacket (FRP or steel material)
- \( w \) = width of jacket (FRP or steel material)
- \( \gamma_c \) = density of concrete
- \( \varepsilon_c \) = axial concrete strain
- \( \varepsilon_c' \) = axial concrete strain corresponding to \( f_c' \)
- \( \varepsilon_{c0} \) = axial concrete strain corresponding to \( f_{c0} \)
- \( \varepsilon_{cc} \) = axial concrete strain corresponding to \( f_{cc} \)
- \( \varepsilon_{fr} \) = principal FRP material rupture strain
- \( \varepsilon_y \) = yielding strain of steel material
- \( \nu \) = Poisson’s ratio
To predict the behavior of confined concrete, and to develop the experimental program, a proper understanding of the previous numerical, analytical, and experimental work is necessary. This chapter presents an overview of the research conducted to date. Over the last 15 years significant work has been conducted on estimating the performance of unconfined-confined concrete. The axial stress-strain behavior of concrete by Mander [1988] et al. is the most commonly used model. Most of the studies that were published after his study were based on his definition for concrete. Also, before Mander there were two other commonly referenced researches. These studies conducted by Richart [1928] and Popovics [1973]. Understanding what types of evaluation were conducted, what the parameters and techniques were, what had been recommended, and what has not been done, will highlight the areas requiring further study. During the last 15 years, similar systems and techniques have been used. Research is based on the three main studies: Richart [1928], Popovics [1973], and Mander [1988].

Confinement of the concrete has been used for a long time in retrofitting strategies. Covering the concrete with steel jacket is one of the oldest and most common methods in strengthening. With the development in material science, a new way is used to strengthen the concrete: covering the concrete with Fiber Reinforced Polymers (FRP). Two main kind of FRP are available: Carbon based (CFRP) and Glass based (GFRP). These materials have been used in retrofitting technology for last 10 years. To develop a better understanding of the behavior of FRP jacketed concrete will help to develop the retrofit strategies by using FRP. The previous research has shown that confinement of the concrete increases the axial load capacity of the concrete. In previous research experimental evaluation of the concrete specimens, which have the same compressive strength, that were jacketed with CFRP (fibers with an angle of 0 degree) and GFRP (fibers with an angle of 0 degree and 0/+45/-45 degree) was conducted. The angle between the direction of fibers and the direction that load applied is important. If the angle between the fibers and the direction of the load is only perpendicular, the fibers are named as unidirectional and the angle of the fibers is 0°. The direction of load is usually named as Z direction. Because of that, if the angle is 0°, the direction of fibers is in X-Y plane. However, there are some other sheets that are manufactured from multidirectional fibers. Most common multi-directional fiber sheets are those, whose fibers bisect the angle between Z direction and X-Y plane (±45°). But also these multi-directional fiber sheets have most of their fibers in X-Y plane. Also, full scale specimens with reinforcement and with/without the jacketing were evaluated. At the end of this research, the effects of jacketing on these specimens were summarized and an analytical model for the “variably confined concrete” has been derived.
However, during the previous research, the effect of low/high confining pressure on high strength concrete was not detected. Also, an analytical model was derived but no tests were conducted on the concrete specimens with variably confining pressure. The point is, in application the confinement supplied with jacket is a passive confinement, and the confining pressure changes with respect to the lateral deformation of the concrete. To derive analytical models for jacketing concrete more precisely, the behavior of the concrete under variably confining pressure needs to be known.

2.1 Analytical and Numerical Studies

In this section, brief summaries of the previously conducted analytical and numerical studies are presented. These analytical and numerical researches will be used for the prediction of the experimental evaluation conducted in Chapter 3.

Depending on the Popovics’ analytical model, in 1988, Mander et al. proposed a modified model for the concrete confined by reinforcement. After this theoretical research, Mander et al. also compared the results of this model with an experimental evaluation in a different scientific paper. The concrete columns confined by steel reinforcement, which are under static and dynamic loading either monotonically or cyclically, were used to propose a stress-strain model. For cylindrical shaped concrete columns, steel reinforcement attained by spirals or circular hoops. This analytical model included the effects of the cyclic loading and strain rate. The effective lateral confining pressure was calculated to be used in this model with defining effective confining core and confinement effectiveness coefficient. The variables, which were needed for definition of the effective confining core, were the centerline diameter of the reinforcement, clear vertical spacing between the reinforcement (inner to inner surface), and the ratio of the area of longitudinal reinforcement to the area of core section. The effective lateral confining pressures were calculated by using the effectiveness coefficient, the volume ratio of the transverse confining steel to the confined concrete core, and the yield strength of the reinforcement. The modified formula of William and Warnke, which was used for the compressive strength of the confined concrete, and the formulations from the Popovics’ model were used to define the stress-strain behavior.

In 1992, Madas and Elnashai proposed a theoretical study for the reinforced concrete columns that were loaded with cyclic/monotonic dynamic loads. When axial deformation applied to the concrete column, the confining pressure occurs along the two transverse directions because of the Poisson’s Effect. Parameters that were used in this research were: the Poisson’s ratio (determined by Kupfer et al.) as a function of the axial strain, the stress-strain relationship of the jacket that was used to supply the confining pressure, the effectiveness ratio defined by the relationship between the axial stress in the jacket and the confining pressure on the concrete. In this model, the confining pressure varies with the axial deformation applied. Because of the variation
of confining pressure, this model is the most important model that is used in this research.

In the theoretical part of the research that was done by Li and Ansari some formulations based on the theory of bounding surface damage leading to the development of the constitutive relationships for high-strength concrete were defined.

Also in 1999, Spoelstra proposed a theoretical model that was used to simulate the monotonic behavior of concrete, which was confined with three different types of jackets: steel tube, CFRP, and GFRP. In this research, the main aspects of the confinement action mechanisms and the relative effectiveness of three jacket types were identified. The predictive equations for the FRP confined concrete were proposed and the results compared with the previous test results.

In 2001, the previous empirical formulas were adopted by Susantha et al. to determine lateral pressure for various types of cross-sections of the concrete confined with the steel jacket. For box and orthogonal cross-sections, the FEM analysis was used with the help of the concrete-steel interaction model. These parameters were: the shape of the cross-section, the strength of the concrete, the ratio of the thickness to the diameter. Additionally, the strength of the steel, the column slenderness ratio, and the rate of the loading were considered. For the FEM analysis, fiber analysis was used. Depending on the results of the fiber analysis, a method was adopted to determine the post-peak behavior of the confined concrete. Up to the peak level of the axial compressive strength of the confined concrete, the stress-strain model defined by Popovics (which was modified later by Mander et al.) was modified and used. After the peak level, a linear relationship for the stress-strain behavior was defined.

Based on the previous models and test results, in 2002, Lam and Teng modified existing model that was proposed by Richart et al. to determine the stress-strain behavior of the concrete confined by FRP material. The parameter, k1, that is used to include the effect of the confining pressure level into the axial compressive strength of confined concrete, was determined with the interpolation of the existing test results.

2.2 Experimental Studies

Previous experimental studies are described in the chronological order under this sub-title to prepare the setup of the experimental evaluation and to have an idea about experimental study.

Bellotti conducted an experimental study in 1991 to develop an experimental technique for the tri-axial cell mechanism. The cylindrically shaped concrete specimens with a diameter of 160 mm. and a height of 320 mm. were tested under lateral confining pressure (compression), by applying axial compression and tension. The confining pressure that was generated by brake fluid had a level of up to 39.2 MPa. The maximum for the axial compressive stress was 245.2 MPa. The confining pressure that was generated was constant during the testing.

The research, which was conducted by Imran in 1996, was an experimental evaluation by using the tri-axial cell testing mechanism, and 130 concrete specimens
were tested. The concrete specimens were in cylindrical shape with a diameter of 54 mm. (2.12 in.) and a height of 108 mm. (4.25 in.). The cylindrical shaped specimens were extracted from the concrete blocks (150 x 150 x 250 mm.). Top and bottom surfaces were ground with machine to get orthogonal surfaces to the longitudinal axis. Three different batches for the concrete were used with a water/cement ratio of 0.40, 0.55, and 0.75. The compressive strengths of the concrete at 28 days were 48.1, 38.3, and 19.9 MPa., respectively. Specimens were cured in two different conditions. These conditions were: saturated and dry. Other parameters studied in this research were: loading path/type and confining pressure levels, which were 0, 5, 10, 20, 40, 70, and 100% of the compressive strength of the unconfined concrete. In this study, the confining pressure generate by tri-axial cell mechanism was also constant during the testing.

The experimental part of the study conducted by Li was performed by using the tri-axial cell testing mechanism. High strength concrete specimens with compressive strengths of 6, 10, 15 ksi. (42, 70, 105 MPa.) were tested. Three types of loading were applied to the cylindrically shaped concrete specimens that have a diameter of 4in and a height of 8 in. (101 x 202 mm.). The loading types were: axial tension, axial compression, and axial compression with compressive lateral pressure. A loading system with a capacity of 1000 kips. as compressive pressure, and a capacity of 12 ksi. as confining pressure was used.

In 2001, circular, box, and octagonal shaped concrete filled tubes were tested by Susantha et al. While performing the test matrix, the variables used were: the diameter of the cross-section, the thickness of the steel jacket, and the strength of the concrete. The results of the experimental evaluation were used to define and modify the post-peak behavior of the stress-strain curve for the confined concrete.

Experimental evaluation of the concrete specimens, which were confined by using tri-axial cell testing mechanism, was conducted by Sfer et al. in 2002. Concrete specimens with a compressive strength of 30 MPa. (28 days) and a cylindrical shape (diameter: 6 in., height: 12 in.) were evaluated. Different confining pressure levels applied using the tri-axial cell testing mechanism (Confining pressure levels were 0, 5, 15, 30, 100, and 200% of the compressive strength of the unconfined concrete). The axial load was applied with a constant piston displacement rate of 0.006 mm./s. Elasto-plasticity is used to compare the results of the experimental evaluation. Like previous experimental evaluations that were conducted by using tri-axial cell mechanism, the confining pressure was also constant in this study.

2.3 Summary

Literature with analytical analysis: As mentioned in the introduction of background information, the analytical evaluation conducted for the last 15 years were based on the models defined by Richart et al. [1928], Popovics [1973] and Mander et al. [1988]. All the analytical researches, which were referenced during this study, were partially or completely modification of these three models. These models can be used
to predict the behavior of the concrete during the experimental evaluation, but they are all assuming that the effect of confinement is constant. Although there were two variably confined concrete models defined by Madas [1992] and Kestner [1997], and these can be modified to get a better accuracy for the prediction.

**Literature with numerical analysis:** A few numerical studies (FEM) were conducted. These analyses were conducted with constant pressure. For the evaluation of the concrete cylinders confined with different types of jackets, varying loading paths of confining pressure need to be evaluated.

**Literature with experimental evaluation:** Previous experimental evaluations conducted either using the concrete specimens confined with jackets, or confined with the constant pressure by using the tri-axial cell mechanisms. In these researches, basic information about test setups, instrumentation, loading rates and types were defined. The experimental evaluation will be prepared with using this information in accordance with the current specifications, codes, and requirements. As explained previously, jacketing supplies a passive confinement on the concrete specimens varying with the lateral deformation of the concrete, so that the experimental evaluation of the concrete specimens confined with varying pressure need to be evaluated. Also, to determine the effect of the concrete strength and to compare with each other, concrete specimens that have different strengths, confined with different constant pressure levels and confined under different types of varying confining pressure need to be experimentally evaluated by using the tri-axial cell mechanism.
3 ANALYTICAL MODELS

Analytical models can be used to predict the experimental evaluation results and to design concrete specimens confined by jackets. In this chapter, the analytical predictor models and the formulations are presented. Sample calculations are presented in the appendix.

Previous studies are mainly based on confined concrete models by Richart et al. [1928] and Mander et al. [1988] or modified versions of these models. The model by Mander et al. was developed for concrete that is confined by either circular hoops or spiral steel reinforcement. A single equation is used to define the stress-strain behavior. The evaluations of concrete specimens confined with jackets have shown that the confinement of concrete by suitable arrangements of transverse reinforcement resulted in a significant increase in both the strength and ductility of concrete under axial compression. The strength enhancement supplied by the confinement and the descending branch of the stress-strain curve of the concrete has a considerable influence on the flexural strength and ductility of reinforced concrete columns. Performing a true moment-curvature analysis could be done by having an accurate stress strain behavior model for concrete.

Lateral confinement is commonly provided by steel reinforcement. As a result of understanding the importance of continuous confinement and improvement in standards, reinforced concrete columns have been subjected to steel jacketing (concrete filled tubes-CFT). Also, after advancements in material science, FRP (fiber reinforced polymer) sheets were used for jacketing the structural concrete elements. The confined concrete models, defined in various studies, can be grouped according to the material that is used for confinement. In all of these models, the main idea is to find the level of lateral confining pressure that is present, and then to calculate the peak compressive stress and the axial strain that corresponds to the peak stress.

3.1 Constant Confinement Model

Mander et al. had proposed a unified stress-strain behavior for unconfined and confined concrete. The stress-strain behavior was based on an equation suggested by Popovics [1973],
Equation 3-1  \[ f(\varepsilon) = f_{cc} \times \frac{E}{E_{cc}} \times \frac{r}{r - 1 + \left( \frac{\varepsilon}{\varepsilon_{cc}} \right)^r} \]

Equation 3-2  \[ f_l = 2 \times \frac{t}{D} \times f_j \]

Equation 3-3  \[ r = \frac{E_c}{E_c - \left( \frac{f_{cc}}{\varepsilon_{cc}} \right)} \]

Equation 3-4  \[ \varepsilon_{cc} = \varepsilon_{c0} \times (1 + 5 \times \left( \frac{f_{cc}}{f_{c0}} - 1 \right)) \]

Equation 3-5  \[ f_{cc} = f_{c0} \times (-1.254 + 2.254 \times (1 + 7.94 \times \frac{f_j}{f_{c0}})^{0.5} - 2 \times \frac{f_j}{f_{c0}}) \]

According to the formulation above, the axial stress \( f(\varepsilon) \) varying with the axial strain \( \varepsilon \), depends on: modulus ratio \( r \), confined concrete strength \( f_{cc} \), axial strain corresponding to confined concrete strength \( \varepsilon_{cc} \), and the ratio of axial strain to axial strain at peak confined stress \( \varepsilon/\varepsilon_{cc} \). The strength of confined concrete \( f_{cc} \) and the corresponding axial strain to this strength \( \varepsilon_{cc} \) were defined by Mander [1988]. Parameters that are needed to calculate \( f_{cc} \) are: unconfined concrete strength \( f_{c0} \) and confining pressure \( f_l \). With calculated confined concrete strength, and using unconfined strength and corresponding axial strain for unconfined strength, axial strain corresponding to \( f_{cc} \) can be determined. Confining pressure is calculated by using the free body diagram of jacketed concrete specimen, where \( D \) is the diameter of the concrete, \( t \) is the thickness of the jacket, and \( f_j \) is the yield strength for steel and the rupture strength for FRP materials. To calculate the modulus ratio, Young’s Modulus \( E_c \) of concrete (calculated from the ACI design code), and the ratio of \( f_{cc}/\varepsilon_{cc} \) (confined concrete strength to corresponding strain) need to be used.
The analytical model derived by Mander et al. was used for the comparison in the figure above. Test results for unconfined concrete specimens are used for definition of the parameters of the stress-strain behavior of concrete. The parameters are: compressive strength of unconfined concrete (5.647 ksi.) and axial strain value (0.1635%) that corresponds to compressive strength. 6 in. x 12 in. concrete specimens are evaluated for unconfined concrete tests. Confining pressures that are used for comparison are the 0, 5, 10, 15, 20, 50, and 100% of the unconfined compressive strength (5.647 ksi.). For all of these confined and unconfined concrete behaviors, maximum axial strain value that concrete fails is assumed as 2.5%.

### 3.2 Analytical Model for Steel Tubes

In their study, Susantha, Ge, and Usami defined a model for concrete specimens confined by steel tubes [Ref.2]. This analytical model is based on constant confining pressure supplied by steel jackets. Concrete-filled steel tubes (CFT) are one of the most common ways to strengthen concrete structures. Their high ductility and strength
are the characteristics that enable them to be used for retrofit technology. The main variables used in this model are concrete strength, the radius to plate thickness ratio, and the shape of the section. In this model, circular, box, and octagonal cross-sections of concrete specimens were considered. Because 6 in. x 12 in. cylindrical concrete specimens are studied in this project, the model for circular cross-section was selected. In this model, the behavior of CFT is considered in two steps. The first step is up to the peak stress, and the second step is post-peak behavior. These two steps take place with respect to the stress-strain behavior of the steel jacket. If steel is idealized as EPP (Elastic Perfectly Plastic) material that has linear increasing behavior till it reaches the yield point and constant linear stress behavior after yielding, the ascending curve of stress-strain behavior of confined concrete is defined with the analytical model of Mander et al. [1988]. The descending curve is defined by Susantha et al. as linear descending behavior.

The Equation 3-1, 3-3, and 3-4 are used to determine the first part of stress-strain behavior of concrete, however Equation 3-2 is modified with including a new parameter $\beta$, which is detailed below.

**Equation 3-6**  
$$ f_i = 2 \times \beta \times \frac{f_y}{D} $$

**Equation 3-7**  
$$ \beta = \nu_e - \nu_s $$

**Equation 3-8**

\[
\begin{align*}
\nu_s &= 0.5 \\
\nu_e &= 0.2312 + (0.3582 \times \nu_{ep}) - (0.1524 \times \frac{f_{c0}}{f_y}) - (4.843 \times \nu_{ep} \times \frac{f_{c0}}{f_y}) - (9.169 \times (\frac{f_{c0}}{f_y})^2) \\
\nu_{ep} &= (0.881 \times 10^{-6} \times (\frac{D}{t})^3) - (2.58 \times 10^{-4} \times (\frac{D}{t})^3) + (1.953 \times 10^{-2} \times (\frac{D}{t})) + 0.4011
\end{align*}
\]

**Equation 3-9**  
$$ f_{cc} = f_{c0} + 4.1 \times f_i $$

For the ascending branch of this model, the axial strain-stress behavior of concrete defined by Popovics [1973] is used. The reduction factor ($\beta$), depending on the diameter of concrete specimen (D), the thickness of jacket (t), the unconfined strength of the concrete ($f_{c0}$), and the yield strength of the steel ($f_y$), is defined to predict the confining pressure supplied by the jacket. The confining pressure ($f_i$) is calculated from the free body diagram of the jacketed concrete by using the (t), (D) and ($f_y$). Young’s modulus of concrete ($E_c$) is calculated by using the ACI code to calculate the modulus ratio together with the ratio of ($f_{cc}/\varepsilon_{cc}$). The compressive strength of confined concrete ($f_{cc}$) is calculated by using the predictor model of Richart [1928], which depends on the compressive strength of unconfined concrete and the confining pressure. Axial strain corresponding to the compressive strength of confined concrete ($\varepsilon_{cc}$) and the relation of axial stress ($f(\varepsilon)$) to axial strain ($\varepsilon$) is calculated by using the Popovics’ calculations. The relationship between axial strain and axial stress
is related to \((f_{cc})\), \((\varepsilon_{cc})\) and \((r)\). The equations to determine the descending branch of stress-strain behavior of concrete were,

for \(R_t \times f_c + f_y \leq 0.006\)

\[ Z = 0 \]

for \(R_t \times f_c + f_y \geq 0.006 \& f_y \leq 236\)

\[ Z = (1.0 \cdot 10^5 \times R_t \times f_c + f_y) - 600 \]

for \(R_t \times f_c + f_y \geq 0.006 \& 236 \leq f_y \leq 336\)

\[ Z = (1.0 \cdot 10^5 \times R_t \times f_c + f_y - 600) \times (f_y + 283)^{13.4} \]

for \(R_t \times f_c + f_y \geq 0.006 \& f_y \geq 336\)

\[ Z = (1.0 \cdot 10^6 \times R_t \times f_c + f_y) - 6000 \]

Equation 3-10 \( R_t = \left( \frac{D}{2t} \right) \times f_y \times \left( \frac{f_c}{f_y} \right) \times (3 \times (1 - \nu^2))^0.5 \)

Equation 3-11 \( f(\varepsilon) = f_{cc} - Z \times(\varepsilon - \varepsilon_{cc}) \)

\( \varepsilon_{cu} = 0.025 \)

For the descending branch of this model, a linear relationship (Equation 3-11) is defined between axial strain and axial stress. To define this linear relationship, the slope of the relationship \((Z)\), with the starting (peak) stress \((f_{cc})\) and ending point \((\varepsilon_{cu})\) is used. The slope \((Z)\) is calculated conditionally depending on the value of \((R_t \times f_{cc} / f_y)\) and the yield strength \((f_y)\) of steel by itself. To calculate \((R_t)\), the diameter of the concrete, the thickness of the jacket, the yield strength and Young’s Modulus of the steel \((E_s)\), and Poisson’s Ratio for concrete are used as parameters. For these formulations, parameters are in SI units (MPa., mm.).

### 3.3 Analytical Model for Fiber Reinforced Polymers

With the start of using FRP jackets to strengthen the concrete, a new or modified analytical and/or empirical model is needed to be defined. While searching for literature, it is observed that most of the models are empirically modified versions of the models of Popovics, Mander and Richart. For each of the experimental evaluations in the literature, different types of (proprietary) FRP materials have been used. Because of this reason, after each experimental study in the literature, a new empirical formulation has been defined. These empirical formulations for peak strength are material dependent. Some of these previous studies concerning models for concrete confined by FRP were summarized by L. Lam. These models were based on constant confining pressure supplied by the FRP sheet. All these models were generated from the analytical models of Mander et al. and Richart et al.
Equations 3-1, 3-2, 3-3, and 3-4 are used in all of these models. The only difference of these models is the calculation of peak compressive strength of confined concrete. To calculate peak compressive strength Equations 3-5 and 3-12 are used.

\[ f_i = 2 \times f_{frp} \times \frac{f}{D} \]

\[ f(\varepsilon) = f_{cc} \times \frac{\varepsilon}{\varepsilon_{cc}} \times \frac{r}{(r-1 + (\varepsilon/\varepsilon_{cc})^\gamma)} \]

\[ r = \frac{E_c}{E_c - \left(\frac{f_{cc}}{\varepsilon_{cc}}\right)} \]

\[ \varepsilon_{cc} = \varepsilon \times (5 \times \frac{f_{cc}}{f_c} - 4) \]

\[ f_{cc} = f_c \times (-1.254 + 2.254 \times (1 + 7.94 \times \frac{f_i}{f_c})^{0.5} - 2 \times \frac{f_i}{f_c}) \]

**Equation 3-12** \( f_{cc} = f_c + k_1 \times f_i \)

Here, \((t)\) is the thickness of the FRP sheet, \((D)\) is the diameter of the concrete specimen and \((f_{frp})\) is the maximum strength capacity of the FRP sheet. The \(k_1\) parameter in the Equation 3-12 is defined by various researchers as:

- Richart et al. [1928] \( k_1 = 4.1 \)
- Karbhari et al. [1997] \( k_1 = 2.1 \times (f_i/f_c)^{-0.13} \)
- Samaan et al. [1998] \( k_1 = 6.0 \times f_i^{-0.3} \)
- Miyauchi et al. [1999] \( k_1 = 2.98 \)
- Saafi et al. [1999] \( k_1 = 2.2 \times (f_i/f_c)^{-0.16} \)
- Toutanji [1999] \( k_1 = 3.5 \times (f_i/f_c)^{-0.15} \)
- Mander et al [1988] \( f_{cc} = f_c \times (-1.254 + 2.254 \times (1 + 7.94 \times \frac{f_i}{f_c})^{0.5} - 2 \times \frac{f_i}{f_c}) \)
- Lam [2002] \( k_1 = 2.15 \)

These researchers used the axial strain-stress relationship for concrete that is defined by Popovics [1973] to define the relation between axial strain and axial stress. Estimation of the parameter \(k_1\) that was defined by Richart et al. [1928] is the only difference among the proposed predictor models defined by these researchers. The parameter \(k_1\) is used to estimate the compressive strength of confined concrete.

**3.4 Analytical Model for Variably Confined Concrete**

The analytical model for variably confined concrete depends on the analytical model proposed by Mander et al. [1988], too. It was first proposed by Madas and Elnashai [1992] for steel jackets. It is defined by four relationships. The first relationship is the dilation relationship between the axial strain and the transversal
strain. The second relationship is the stress-strain relationship of the confining material (jacket). The third relationship is the effectiveness of the jacket (in other words, the relationship between the confining pressure and the axial stress among the fibers of the jacket). Finally, the relationship between axial strain and compressive stress in the concrete specimen is used for this analytical model. This model can be summarized in six steps:

Step 1- Start with choosing an axial strain value $\varepsilon_c$.

Step 2- By using the axial strain - transversal strain relationship of the concrete as defined by Elwi and Murray [1979], transversal strain $\varepsilon_t$ is calculated.

Step 3- The principal strain in the confining material is calculated by using the transversal strain in the concrete specimen,

$$\varepsilon_j = \varepsilon_t$$

Step 4- By using the secant stiffness, the jacket’s axial strength and the effectiveness ratio of the confining pressure supplied by the jacket is calculated.

Step 5- By using the formulations of the analytical model of Mander et al. [1988], the peak compressive strength of the concrete and the axial strain corresponding to the peak strength is calculated.

Step 6- By using the equation for $f(\varepsilon)$, the value of the compressive stress of the concrete that corresponds to the selected principal axial strain is determined and plotted.

Equations 3-3, 3-4, and 3-5 are used as in the Constant Confinement Model of Mander et al., however Equation 3-1 and Equation 3-2 are modified to Equation 3-13 and Equation 3-15.

$$f_{cc} = f_c \times (-1.254 + 5.254 \times (1 + 7.94 \times \frac{f_j}{f_c})^{0.5} - 2 \times \frac{f_j}{f_c})$$

$$r = \frac{E_c}{E_c - (\frac{f_{cc}}{\varepsilon_{cc}})}$$

$$\varepsilon_{cc} = \varepsilon_c \times (5 \times \frac{f_{cc}}{f_c} - 4)$$

Equation 3-13  
$$f(\varepsilon) = f_{cc} \times \frac{\varepsilon}{\varepsilon_{cc}} \times \frac{r}{(r - 1 + (\frac{\varepsilon}{\varepsilon_{cc}})^{r\times k})}$$

Equation 3-14  
for $\varepsilon / \varepsilon_{cc} \leq 1.0$  
$k = 1$

for $\varepsilon / \varepsilon_{cc} \geq 1.0$  
$k = (0.67 + \frac{f_c}{62} \times \frac{f_c}{f_{cc}})$
Equation 3-15  \( f_j(\varepsilon) = E_j \times \varepsilon_j(\varepsilon) \)

Equation 3-16  \( \varepsilon_j(\varepsilon) = \varepsilon_i(\varepsilon) = \eta(\varepsilon) \times \varepsilon \)

Equation 3-17  \( \eta(\varepsilon) = \nu \times [1 + 1.3763 \times \frac{\varepsilon}{\varepsilon_{cc}} - 5.36 \times (\frac{\varepsilon}{\varepsilon_{cc}})^2 + 8.586 \times (\frac{\varepsilon}{\varepsilon_{cc}})^3] \)

Lateral deformation can be calculated by using the relationship (\( \eta(\varepsilon) \)) between the axial strain and the transversal strain of the concrete under tri-axial loading as defined by Elwi and Murray [1979]. Lateral deformation can be calculated for each level of axial deformation. The mechanical relationship between the axial elongation of the jacket (\( \varepsilon_j \)) and the transversal deformation of the concrete (\( \varepsilon_t \)) is known to be linear, which can be summarized as (\( \varepsilon_t = \varepsilon_j \)). Now, at each axial strain level, axial stress in the jacket can be calculated by using the axial strain-stress behavior of the jacket material. From the free body diagram, the confining pressure can be calculated at each stress level for the jacket. By using the axial strain-stress behavior defined by Popovics [1973] and Mander [1988], the compressive strength of the confined concrete corresponding to the current axial strain can be calculated. The relationship defined with this series of calculations is called the “variably confined concrete model”.

Figure 3:2  Comparison of Constant Confinement Model with Variably Confined Concrete Model
Modified Variably Confined Concrete Model

This model uses the original variably confined concrete model (VCCM) with two main modifications. For these modifications predictive definitions proposed by Harries [2002] are used. First modification is prediction of peak compressive strength of confined concrete. This prediction is valid for only fiber reinforced polymer (FRP) jacketed concrete. Instead of Equation 3-5, the equation bellowed is used to predict the peak compressive strength of confined concrete.

\[ f_{cc} = f_c + 4.269x(f_{\text{con}})^{0.587} \text{ [MPa]} \]

In this formula all units are defined with SI unit system.

Second modification is dilation ratio definition. Harries [2002] defined dilation ratio relationship for FRP jacketed concrete. This definition consists of three levels. These levels are defined with respect to the axial deformation of unconfined concrete. These levels are 0 to 0.6\( \varepsilon_c \), 0.6\( \varepsilon_c \) to 2.0\( \varepsilon_c \) and 2.0\( \varepsilon_c \) to failure. Lateral dilation capacity of FRP jacketed concrete is assumed to be equal to tensile strain capacity of FRP jacket. From lateral dilation capacity of jacketed concrete, the axial strain is calculated by using dilation ratio relationship. So failure strain is back calculated.

For \( \varepsilon \leq 0.6 \cdot \varepsilon_c \) \( \eta = \eta_i \)

\[ 0.6 \cdot \varepsilon_c \leq \varepsilon \leq 2.0 \cdot \varepsilon_c \]

\[ \eta = \frac{(\eta_u - \eta_i)}{1.4 \cdot \varepsilon_c} (\varepsilon - 0.6 \cdot \varepsilon_c) + \eta_i \]

\[ \varepsilon \geq 2.0 \cdot \varepsilon_c \]

\[ \eta = \eta_u \]

\( \varepsilon_c \) is the axial deformation that corresponds to the peak compressive strength of unconfined concrete, \( \eta_i \) is the initial and \( \eta_u \) is the ultimate dilation ratio values.

\( \eta_i = 0.15 \) Poisson’s ratio for concrete

\( \eta_u = -0.99 \ln(E_f) + 12 [\text{MPa}] \) for GFRP

\( \eta_u = -0.66 \ln(E_f) + 8 [\text{MPa}] \) for CFRP.

Here, \( E_f \) is secant modulus of FRP material. The relationship for dilation defined above is used instead of Equation 3-17. All the procedure, steps, and equations are used in their original forms that were defined for VCCM.

FEM Analysis

To understand and to predict the behavior of passively confined concrete specimens, we model them by using the CAE (interactive graphical environment) version of Abaqus 6.3. Abaqus is an engineering simulation program based on the finite element method.

Modeling of Concrete

Exact dimensions are used while modeling the concrete. Units are in U.S. units of measurement. The concrete cylinders have a diameter of 6in and a height of 12 in.
For experimental evaluation, the concrete batch is designed for 4000 psi. but has an axial compressive strength of approximately 5000 psi. for 28 days. When modeling concrete with Abaqus, the concrete is designed to have an axial compressive strength of 4.0 ksi., and then the finite element model is modified according to the plain concrete test results. Concrete is modeled as a solid section for 3D stress-strain analysis. Material properties of the concrete used for the model are Concrete Damaged Plasticity with Concrete Compression Hardening and Concrete Tension Stiffening options, and Elastic options. With using both Concrete Compression Hardening (Figure 3:8) and Concrete Tension Stiffening (Figure 3:9) options, post-peak stress-strain behavior of concrete model can be obtained. Concrete Damaged Plasticity option is the only material definition for concrete models. This concrete material model depends on plasticity, so stress distribution can be obtained visually, but cracks can not be determined. Young’s Modulus of the concrete is calculated by using the ACI formulation:

\[ E_c [ksi.] = \frac{57}{f_c [psi.]} \]

Poisson’s ratio (\(\nu\)) is taken as 0.15. The parameters used in the Concrete Damaged Plasticity option were:
- Dilation angle = 55.0 (55)*
- Flow potential eccentricity = 0.10 (0.1)
- The ratio of the initial equi-biaxial compressive yield stress to the initial uniaxial compressive yield stress = 1.16 (1.16)
- The ratio of the second stress invariant on the tensile meridian = 0.62 (2/3)
- Viscosity parameter = 0.00 (0.0)

For the Compression Hardening option (Figure 3:8), unconfined concrete test results (Figure 3:7) are used, and the multi-linear Concrete Tension Stiffening option is defined. While defining the Concrete Compression Hardening option, true stress-strain values are calculated from the nominal stress-strain values by using the formulations given below.

**Equation 3-18**  
\[ \varepsilon = \ln(1 + \varepsilon_{nominal}) \]

**Equation 3-19**  
\[ \sigma_{true} = \sigma_{nominal} \times (1 + \varepsilon_{nominal}) \]

**Equation 3-20**  
\[ \varepsilon_{true} = \varepsilon_{plastic} = \varepsilon - \frac{\sigma_{true}}{E_c} \]

Bottom surface nodes are fixed in global x, y, z directions, and top surface nodes are fixed in global x, y directions but released in the global z direction while defining boundary conditions. The release of the top surface nodes in the global z direction is

* The default values used by Abaqus are given in parentheses.
defined by the velocity in global z direction, so that a time history analysis is performed.

Global element size is defined as 2.0 while meshing the concrete cylinder. A quadratic 3D brick element with 20 nodes (C3D20) is used for the definition of the element types. In Figure 3:3, un-deformed shape of concrete model is shown, and deformed shape of concrete model is shown in Figure 3:4.

For FEM analysis of concrete specimens, second FEM analysis program, which is named as Diana, is also used. Diana has more material definition options for concrete modeling. Diana has Plasticity and Cracking options for material definition for concrete models. Also, pre-defined concrete material options, which vary with respect to the compressive strength of concrete, can also be used while modeling with Diana. However, if a discrete meshing is used with Diana, post-peak stress-strain behavior of concrete cannot be obtained. Because of this reason the results of Abaqus are presented as FEM analysis. While defining concrete with Diana, multi-linear compression hardening and multi-linear tension softening options are used. Similar procedures to the procedures of Abaqus are used while modeling with Diana. Boundary conditions, element types, analysis types were same. The only difference is the material type. Cracking option is used for material definition of concrete model. Material model is also depends on the data from unconfined concrete specimens’ evaluation.

3.5.2 Modeling of the Jacket

Modeling of the different types (steel, CFRP, GFRP) of jackets follows the same general process. By using the extrusion option, and defining the center line axis, thickness, and material properties of jacket type, three dimensional shell sections are created for the jacket model. Elastic and Plastic material options are used for defining steel jackets as elastic-perfectly plastic materials. However, Elastic is the only material option used for the CFRP and GFRP jackets. No boundary conditions are defined, but a constraint model (Tie) is defined between the concrete surface and the inner surface of the jacket. Linear shell elements (S8R) with eight nodes (reduced) with a global element size of 2.0 are used while meshing the CFRP and the GFRP jackets. For meshing the steel jackets, a global element size of 2.1 is chosen. Un-deformed and deformed shapes of jacketed concrete model are shown in Figure 3:5 and 3:6 respectively.

Geometric nonlinearities are accounted for in stress analysis. Riks is used for the static stress analysis. The Riks option allows for stepwise solving of post-peak analysis of models.
Figure 3.3 Un-deformed shape of the unconfined concrete model

Figure 3.4 Deformed shape of the unconfined concrete model
Figure 3:5 Deformed shape of the concrete passively confined by the jacket

Figure 3:6 Axial stress-strain behavior of the concrete
Figure 3:7 True stress-strain relationship for concrete model

Figure 3:8 Concrete tension hardening behavior
4  PASSIVE CONFINEMENT TEST SERIES

In design applications, external confinement is supplied through the use of transversal steel reinforcement, steel tubes, and CFRP and GFRP sheets. The use of the steel tubes for the confinement of the concrete has been in practice for number of decades. FRP applications have been developed in the past 15-20 years. The confining pressure provided by these applications is referred to as “passive confinement”, because the confining pressure is not actively applied. As a result of Poisson’s Effect, dilation occurs in the body of the concrete column structure when axial compression is applied. If the concrete column is confined with a jacket, the lateral dilation of the concrete column results in axial elongation of the jacket. Depending on the material characteristics of the confining jacket, axial stress occurs in the jacket and restrains the dilation of the concrete column. This restraint is defined as “passive confinement”. To examine the accuracy of the current design and analysis procedures, an experimental study using 6”x12” concrete cylinders was conducted. In this chapter, the development and background is described.

The purpose of the passive confinement test series is to find the effect of passive confinement on concrete column structures. To repair the existing concrete infrastructure confinement is commonly established using steel or FRP jackets. To mimic this technique the behavior of the concrete cylinders passively confined by steel tubes, CFRP, and GFRP jackets is observed. In addition, from a practical application FRP jacketing is preferable to the steel jacketing because of its resistance to buckling.

4.1  Materials

4.1.1  Jacket

Steel jackets, CFRP, and GFRP sheets are used for confinement. Details of their material characteristics are given in Table 4-1.

The CFRP sheets that are used in this research are unidirectional and manufactured by Tonen Corporation, Tokyo. They are made up of carbon-aramid fibers and provided as a roll with a width of 20 in. (508 mm.). To bond the CFRP sheets to the concrete surface, a two-part epoxy resin/hardener system is used.

GFRP sheets are manufactured by Owens-Corning are used. GFRP sheets are 30 in. wide and provided as a roll. A resin and catalyst mixture is used to bond them to the cylindrical concrete specimens’ surface.

Both the GFRP and CFRP sheets were purchased for previous research that was conducted by the ATLSS Engineering Research Center in 1997 [Kestner, 1997].

The concrete filled steel tube (CFT) method is applied to investigate the axial stress-strain behavior of the cylindrical concrete specimens that are confined with steel
jackets. Two types of steel jackets are used for the testing. The steel tube, which is distributed by McKnight Steel Company, is used to get a confining pressure equal to 20% of the compressive strength of the unconfined cylindrical concrete specimens.

Sheet metals from Infra Steel Company were bent and welded by Nazareth Machine Company to generate a confining pressure equal to 10% of the compressive strength of the unconfined cylindrical concrete specimens.

<table>
<thead>
<tr>
<th>Table 4-1 Material characteristics of jackets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
</tr>
<tr>
<td>Elastic Modulus $E_i$ [ksi]</td>
</tr>
<tr>
<td>Thickness $t$ [in]</td>
</tr>
<tr>
<td>Yield Strength $f_y$ [ksi]</td>
</tr>
<tr>
<td>Yield Strain $\varepsilon_y$ [in/in]</td>
</tr>
<tr>
<td>Tensile Strength $f_u$ [ksi]</td>
</tr>
<tr>
<td>Tensile Strain $\varepsilon_u$ [in/in]</td>
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</tbody>
</table>

4.1.2 Concrete

Ready mixed concrete with an unconfined compressive strength of 4000 psi. was used for this study. Material was supplied by the Koller Concrete Company for this series of tests. The mix design of the concrete used is:

- Air Content: 1.5% 0.40 cu.ft.
- Slump: 4±1 in.
- Cement Type (c): C150-Hercules 2.25 cu.ft.
- Slag Cement (sc): C989-Waylite 0.69 cu.ft.
- Coarse Aggregate (ca): #57 Limestone C-33 10.53 cu.ft.
- Fine Aggregate (fa): Concrete Sand C-33 8.59 cu.ft.
- Water (w): 4.79 cu.ft.
- Total: 27.20 cu.ft.
- Unit Weight: 150.90 pcf.
- Water Reduce: C494 16.6 oz.
- Weight Ratio (w/c/ca/fa/sc): 1.00/1.48/6.09/4.79/0.37

Concrete specimens are cast at the ready mixed concrete company site. After the first 24 hours of initial curing according to the ASTM standards, the cylinders were are transported to the ATLSS Research Center. The cylinders were then cured according to the ASTM standards in a curing tank filled with saturated lime water. Strength gain tests at 10, 14, and 28 days were conducted on the unconfined concrete specimens to observe their strength gain. Strength gain behavior of concrete is given in Figure 4:1. 28 days strength was marginally higher than design expectations.
Because of the unconditional weather on the seventh day after casting of concrete specimens, the first strength gain tests on the 7th day was conducted on the 10th day.

![Strength gain of concrete](image)

**Figure 4:1** Strength gain of concrete

### 4.1.3 Capping

All of the concrete specimens’ top and bottom surfaces were tested using a sulfur based capping compound. The capping compound used for this procedure is Hi-Cap, manufactured and distributed by Forney Inc. Hi-Cap is a thin flake, heat purified (degassed) compound with a melting temperature range of 240ºF-290ºF. The compressive strength of Hi-Cap is 8000-9000 psi. (according to the 2 in. cubic test). Hi-Cap, manufactured with a cap thickness of ¼ in., is capable of testing cylindrical concrete specimens that have a compressive strength of 16000 psi.

The capping compound is prepared, according to the ASTM standards, in the melting pot manufactured by Forney Inc. Concrete specimens are capped using a standard capping stand.

### 4.1.4 Epoxy Resin/Hardener System

The epoxy resin/hardener system, manufactured by West System, was purchased from Composites One Inc. and used to bond the FRP sheets to the cylindrical concrete specimens’ surface. West System-105 epoxy resin is pale yellow, low viscosity liquid epoxy resin that can be cured in a wide temperature range to form a high strength solid with moisture resistance. WS-105 epoxy resin has been designed specifically to bond
with wood fiber, glass/carbon fibers, and reinforcing fabrics. It was formulated without volatile solvents and does not shrink after curing.

West System-205 is a fast hardener with medium viscosity. It has been used to produce a rapid cure that helps to develop the epoxy resin’s physical properties quickly at room temperature. When mixed with the WS-105 epoxy resin in a five part resin to one part hardener ratio, the cured resin/hardener mixture yields to a rigid, high strength moisture resistant solid with excellent bonding and coating properties. It has a pot life of 9-12 min., and cures to a solid state in 6-8 hours. It is recommended that it to be cured for 1 - 4 days and applied at a minimum temperature of 40°F (4°C) to get the maximum strength.

4.2 Test Matrix

As shown in Table 4-2, six types of confined concrete specimens decided to be studied in the passive confinement test series. Only four types of concrete specimens are tested. For each type, 3 specimens are tested. In Table 4-2, the first number (4) represents the compressive strength of the unconfined concrete specimen, A and B represents designed confining pressure levels 10% and 20% respectively, ST represents Steel Tube, CC represents the confinement by CFRP, and finally, CG represents the confinement by GFRP sheets.

<table>
<thead>
<tr>
<th>Table 4-2 Test matrix for passive confinement test series</th>
</tr>
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<tbody>
<tr>
<td><strong>Material type of jacket</strong></td>
</tr>
<tr>
<td>Confining Pressure</td>
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<tr>
<td></td>
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</tbody>
</table>

4.3 Application Procedure

4.3.1 GFRP

The application procedure for GFRP sheets consists of three main steps:
- Preparation of the concrete surface
- Preparation of the fiber sheets
- Application of sheets to the concrete surface

In accordance with Owens-Corning instructions, the length of the each sheet includes a 4 in. lap along the longitudinal direction. Each sheet has a 10 in. width and a 23 in. length. The amounts of the hardener and the epoxy resin that are used are determined by using the pumps purchased from the company, so that the ratio can be determined easily. Half of this mixture is applied to the sheets and these sheets are applied to the concrete surface. Then the remaining half is applied to the outer surface of the GFRP sheets to cover the sheets for finishing.
4.3.2 **CFRP**

The procedure for the application of the CFRP sheets to the concrete surface is similar to the GFRP sheet application procedure and consists of three main steps, too: preparing the concrete surface, preparing the fiber sheets, and applying the sheets to the concrete surface. A resin/hardener mixture is prepared with a weight ratio of 5:1.

CFRP sheets are prepared for application, and include a 4 in. lap to the length of the sheets. Applying sheets to the concrete surface involves three steps: the application of the inner resin coat to the concrete surface, the application of the fiber sheets, and covering the fiber sheets with the outer resin for finishing. The resin coat amount applied to the concrete surface is 0.16 psf. For the inner resin coat, 70% of this mixture is applied directly to the concrete surface. After covering the cylindrical concrete specimens with the fiber sheets, the remaining 30% of the resin mixture is used to cover the outer surface of the CFRP sheets.

4.3.3 **Steel Tubes**

Steel tubes were prepared in two different ways. Steel tubes that had 0.097 in. wall thickness were prepared by Nazareth Machine Company by cutting, rounding and welding the plane sheet metals from the Infra Steel Company. The tubes with 0.1875 in. wall thickness were purchased as seamless, ready-to-use steel tube. These tubes were distributed by McKnight Steel tubing company and cut at the facilities of the ATLSS Research Center. All steel tubes divided into three pieces and these pieces tagged to each other before the concrete is filled into them. These pieces had a length of 1 in., 10 in., and 1 in. After the concrete gained its strength, top and bottom pieces of the steel tubes that are 1 in. in length were removed. Dividing the tubes into three parts before the concrete had filled into tubes helped us to prevent any damages to the concrete body while removing the top and bottom pieces. Removing the top and bottom steel tube pieces prevents the steel from the buckling and allows the pressure to be applied directly to the concrete.

4.4 **Test Procedure**

4.4.1 **FRP Tensile Coupon Tests:**

The results of the tensile coupon tests that were conducted during previous research [ATLSS, 1997] were taken into consideration for this research. During previous research for each of the three FRP jacket materials, tensile coupon tests were conducted through sets of a minimum of five tests according to the ASTM D3039-95. Tensile coupon test results are presented in Section 4.1.1 with Table 4-1.

4.4.2 **Unconfined-Confined Concrete Tests**

All tests are conducted by using the universal test machine (SATEC), which has a compression load capacity of 600 kips. As explained before, all specimens are capped with sulfur based capping compound. Each specimen is centered in the test machine. After lowering the crosshead of the testing machine, the loading system is
set up to zero and instrumentation is completed. Then the loading is made hydraulically with a rate of 30 psi./sec. The test machine is set up to stop automatically when 90% of the peak compressive strength is unloaded. All testing is based on the ASTM standards. [ASTM C39/C39M-1]

4.4.3  Instrumentation

4.4.3.1  Displacement Controlling LVDT:

   To conduct the tests by using the displacement control more precisely, a controller LVDT with 1in gage length is used. Each time before testing started, the SATEC machine and the controller LVDT were calibrated and set up to zero position.

![Displacement controlling LVDT](image1)

**Figure 4:2** Displacement controlling LVDT

4.4.3.2  LVDT Ring Frame:

   To measure the deformations of the unconfined and confined concrete specimens, a ring frame consisting of three levels of ring with 8 in. outer and 7 in. inner diameter is used. With the frame that is shown in Figure 4:3, each specimen is divided into four equal control volumes and the deformations are measured at two middle control volumes. Two axial and two lateral LVDT’s with 6in gage length are used. Different views of LVDT ring frame is given with Figure 4:4 and 4:5.
Figure 4.3 The LVDT Ring frame (top view)
Figure 4:4 The LVDT Ring frame (side view)
4.4.3.3 Strain Gauges;

To measure the local deformations at middle height level of the jacketed specimens, two axial and two lateral strain gauges with high elongation are used. In Figures 4:6, 4:7 and 4:8 specimens with strain gauges are shown. Before bonding the strain gauges, the surface of the jacketed specimens are ground and cleaned. After the pre-application procedure has completed, strain gauges are bonded to the surface of the jacket with a high elongation adhesive. Strain gauges and adhesives are purchased from Texas Measurement Inc.
Figure 4:6 Strain gauges on the surface of the steel jacket
Figure 4:7 Strain gauges and the LVDT ring frame on the GFRP jacketed concrete specimen
Figure 4.8 Strain gauges and the LVDT ring frame on the CFT (concrete filled steel tube)
5 DISCUSSION OF RESULTS

As explained in Chapter 3, the confined concrete specimens are modeled with the FEM program, Abaqus. In this section, the accuracy of the finite element models and predictor models are discussed. To investigate the effect of constant confining pressure, which is used in most predictor models, an additional FEM study is conducted. The study applies a constant pressure on the surface of the concrete specimens. The results are compared with that of the experimental data and other predictor data.

The results of the predictor models are presented. Mainly the results are focused on the variably confined concrete model (VCCM) [Madas-1991, Kestner-1997], constant confined concrete model (CCCM) of Mander et al. [1988] and modified variably confined concrete model (VCCM-2) by using Harries [2002].

5.1 Passive Confinement Behavior

In this section, the experimental evaluation results of the concrete specimens, which are confined with steel (st4a, st4b), GFRP (cg4b), and CFRP (cc4b) jackets, are presented and compared with the results that are predicted from the analytical models and FEM analysis.

5.1.1 Results of Unconfined Concrete Test Series

Three unconfined concrete specimens were evaluated under axial loading. The axial strength, axial strain, lateral strain, and axial stress results were measured as discussed in Chapter 4. The unconfined concrete behaved as expected.

In Figure 5:1, the individual and average axial stress-strain behaviors of unconfined concrete specimens are plotted. To determine the average stress-strain response the stress value of the three specimens were averaged at different values of axial strain. This technique is used for all subsequent averaging.
Figure 5:1 Stress-strain behaviors of unconfined concrete

The relationship between axial strain and dilation ratio is shown in Figure 5:2. Up to 0.1% axial strain the dilation ratio is constant at approximately 0.2, this corresponds to the elastic range of concrete. From 0.1% to the strain at peak stress (0.16%) there is a linear increase in the dilation ratio, this behavior corresponds to the minor cracking within the concrete. After the peak stress is reached the dilation ratio increases very rapidly due to the opening of larger tensile cracks in the concrete. This behavior corresponds to observations during the tests. This can be further illustrated in Figure 5:3, after peak compressive strength is reached, the lateral strain increases rapidly and the concrete loses its strength.
Figure 5:2 Dilation of unconfined concrete

Figure 5:3 The relationship between lateral strain-axial strain and axial stress
The comparison of Mander’s analytical model, FEM analysis, and previous experimental results are plotted in Figure 5:4. The FEM results correlate well with the experimental data. Mander’s analytical model matches the experimental results up to the peak and then underestimates the stress during unloading. As explained previously in Chapter 3, in the variably confined concrete model (VCCM), the dilation ratio definition of Elwi [1979] is used. The comparison of this definition with experimental results is presented in Figure 5:5. The dilation ratio definition of Elwi correlates with the experimental results up to the axial strain that corresponds to the peak stress, and then the analytical prediction greatly overestimates the dilation of concrete.

**Figure 5:4** Comparison of experimental and analytical unconfined concrete response
Figure 5:5 Variation of dilation ratio
Figure 5:6 Unconfined concrete specimens at the end of test
At the end of each test, the pictures of unconfined concrete specimens are taken. Figure 5:6 shows the pictures of the unconfined concrete specimens. First and third specimens failed because of tensile cracks occurred along the height of specimens. However, second specimen is failed due to crushing and tensile cracking at bottom 1/3 height of the specimen. The LVDT ring frame is used during the whole testing process of the specimens. Tests are ended when the stress drops to the 10% of peak stress.

5.1.2 Results of the Concrete Specimens Confined with the CFRP Jackets (CC4B)

Before starting the experimental evaluation one CFRP jacketed concrete specimen was tested without any instrumentation to have an idea about the behavior of confined concrete. This trial specimen abruptly failed due to rupture of fibers of the jacket. Because of this, at the beginning of the experimental evaluations, the data was acquired using strain gauges to prevent LVDT ring frame from any damages. The evaluation of specimen cc4b1 was conducted with strain gauges. Unfortunately, strain gauges did not accurately measure the response. So as a final decision, the combination of the LVDT ring frame and the strain gauges are used for specimen cc4b2 and cc4b3. Because of not having accurate response, the experimental data of cc4b1 is not used for comparisons.

The axial stress-strain relationships of concrete specimens that are confined with CFRP jackets are plotted in Figure 5:7. Confining the concrete specimens by using CFRP jackets increased the strength of concrete. After peak strength is reached, stress is almost constant up to failure of specimen. Loss of jacket integrity eventually occurs leading to abrupt failure of the CFRP jacketed concrete specimens.

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**Figure 5:7** Stress-strain behavior of CFRP jacketed concrete
In Figure 5:8, the relationship between dilation ratio and axial strain for CFRP jacketed concrete is presented. This relationship is similar with the dilation ratio relationship for unconfined concrete specimens. From zero to peak stress dilation ratio remains close to the elastic dilation ratio. This is because minor cracking within concrete. After peak stress is reached, dilation ratio increases very rapidly, this behavior corresponds to the opening of large tensile cracks in the concrete. The measured dilation ratio starts with a value of approximately 0.15.

The comparison of Mander’s analytical model, variably confined concrete model, FEM analysis for constant pressure (Abaqus-c) and jacketed concrete (Abaqus-j), and previous experimental results are plotted in Figure 5:9. As explained previously in Chapter 3, two different FEM analyses conducted for prediction of experimental results of confined concrete. The FEM analysis results for concrete specimens confined by applying constant pressure to the surface of concrete is presented with Abaqus-c. The FEM analysis result of concrete specimens confined by elastic jacket is presented with Abaqus-j. FEM analysis results for jacketed concrete correlates well with the experimental data. The FEM model of jacketed concrete didn’t fail because the jacket is modeled as totally elastic material. However, the FEM analysis of concrete confined with constant pressure does not correlate with the experimental data. To calculate the constant pressure to be applied, Equation 3-2 is used. Equation 3-2 overestimates the confining pressure for FRP jacketed concrete. Mander’s analytical model also does not match with the experimental results. Because, this model overestimates the confined concrete strength and strain values due to miscalculation of confining pressure. Variably confined concrete model correlates well with experimental data between zero and peak stress. After peak stress is reached,
VCCM underestimates the stress values. The difference between VCCM and experimental result is because of underestimation of dilation ratio, which is plotted in Figure 5:10. The modification of variably confined concrete model using the definitions of Harries [2002] (VCCM-2) correlates well with the experimental data. In Figure 5:10 the dilation ratio definition of Harries is plotted. As shown the Harries’ model provides a much more accurate prediction of dilation ratio than current methods.

**Figure 5:9** Comparison of experimental and analytical CFRP jacketed concrete response
Analytical definition of Elwi is used to predict the dilation ratio. In this definition axial strain value that corresponds to the confined concrete peak strength is used as parameter. Miscalculation of confining pressure affects the compressive strength of confined concrete, axial strain that corresponds to the peak stress, and the dilation ratio. The dilation ratio definition of Elwi correlates with the experimental results up to the axial strain that corresponds to the peak stress, and then the analytical prediction greatly underestimates the dilation of concrete.

At the end of each test, pictures of each CFRP jacketed concrete specimen are taken. These pictures are presented in Figure 5:11. Lap failure occurred on specimen cc4b1 and cc4b3. Specimen 4b2 failed by fiber rupture.
Figure 5:11 CFRP jacketed concrete specimens (cc4b) at the end of test
5.1.3 Results of the Concrete Specimens Confined with the GFRP Jackets (CG4B)

The experimental evaluation of the specimens of the cg4b series is conducted at the end of the experimental evaluations. To create the data, both the LVDT ring frame and the strain gauges are used for all of the specimens of this group.

Stress-strain behavior of the concrete has changed with the effect of confinement generated by GFRP jacket. Under axial compression, strength gain is observed when concrete is jacketed with GFRP sheets. After peak stress is reached, the axial stress was almost constant up to the failure of jacket. The failure of jacketed concrete was abrupt. The axial stress-strain behaviors of GFRP jacketed concrete specimens are presented in Figure 5:12. The data from the experimental evaluation of specimen cg4b3 is neglected. Specimen cg4b3 is failed unexpectedly at very low stress. The peak strength that cg4b3 reached was very low than the average peak strength of unconfined concrete (4290psi). This may be attributed to an error with loading conditions.

![Stress-strain behavior of GFRP jacketed concrete](image)

**Figure 5:12** Stress-strain behavior of GFRP jacketed concrete

The relationship between dilation ratio and axial strain for GFRP jacketed concrete is plotted in Figure 5:13. The GFRP jacketed concrete specimens’ dilation ratio behaviors are similar with the behaviors of concrete specimens confined with CFRP sheets. Dilation ratio behavior is started almost from a value of 0.15, which is the expected Poisson’s ratio for concrete. Between zero and peak stress, the dilation ratio increases very slowly. Under axial compression, concrete dilates laterally because of lateral tensile forces. The confining jacket resists the dilation by keeping the specimen in its original shape, which results in strength gain. The dilation ratio remains close to the elastic value until peak stress is reached. After peak stress is reached, the dilation ratio increases very rapidly because of opening of large tensile forces.
cracks. Finally, the confined concrete specimens are failed abruptly due to the sudden rupture of fibers of GFRP jacket.

![Figure 5:13 Dilation of GFRP jacketed concrete](image)

Figure 5:13 Dilation of GFRP jacketed concrete

Figure 5:14 presents the relationship between localized axial deformation and generalized axial deformation. Local deformations are measured with strain gauges; however, LVDT are used to measure generalized deformation of jacketed concrete specimen. As expected, local deformations are greater than generalized deformation.

![Figure 5:14 The relationship between Axial Strain from the LVDT ring frame and the Axial Strain from the strain gauges](image)

Figure 5:14 The relationship between Axial Strain from the LVDT ring frame and the Axial Strain from the strain gauges
The comparison of results of Mander’s analytical model, variably confined concrete model, FEM analysis for constant pressure (Abaqus-c) and jacketed concrete (Abaqus-j), and previous experimental evaluation are plotted in Figure 5:15. The FEM analysis results for concrete specimens confined by applying constant pressure to the surface of concrete is labeled as Abaqus-c. The FEM analysis result of concrete specimens confined by elastic jacket is presented with Abaqus-j in the same figure. FEM analysis results for jacketed concrete correlates well with the experimental data. As explained in the discussion of results of the CFRP jacketed concrete specimens, FEM model of jacketed concrete did not fail, because elastic material behavior is used while modeling the jacket. There is no correlation with the experimental data and the FEM analysis of concrete confined with constant pressure. While calculating the constant pressure that is used in FEM analysis, Equation 3-2 is used. Equation 3-2 overestimates the confining pressure for FRP jacketed concrete. Also there is no correlation with Mander’s analytical model and the experimental results. Because, this model also uses the Equation 3-2, which is resulted in overestimation the confined concrete strength and strain values due to miscalculation of confining pressure. Variably confined concrete model also uses same equation. VCCM correlates well with experimental data between zero and peak stress; but, VCCM underestimates the stress values during the post-peak behavior. The modified variably confined concrete model (VCCM-2) correlates well with the experimental data. The modification is done by using the predictive definitions that are defined by Harries [2002]. The difference between VCCM and experimental result is because of underestimation of dilation ratio, which is plotted in Figure 5:16. Also in this figure, the dilation ratio relationship defined by Harries is presented.

![Figure 5:15 Comparison of experimental and analytical GFRP jacketed concrete response](image)

**Figure 5:15** Comparison of experimental and analytical GFRP jacketed concrete response
Figure 5:16 Variation of the dilation ratio-GFRP jacketed concrete
Figure 5.17 GFRP jacketed concrete specimens (cg4b) at the end of test
The pictures of GFRP jacketed concrete specimens are taken after failure. These pictures are presented in Figure 5:17. All specimens of this group failed because of rupture of the GFRP fibers. Specimen cg4b1 failed at the overlapping region, close to top. Specimen cg4b2 is failed at mid-height region of specimen. Finally, specimen cg4b3 is failed at the location where strain gauges are placed.

5.1.4 Results of the Concrete Specimens Confined with the Steel Jacket (ST4A)

The same comparisons are presented for the concrete filled steel tubes that have a thickness of 0.097in. Before the experimental evaluation, two trial FRP jacketed concrete specimen tests were conducted. Abrupt failure of FRP jacketed specimens brings up the possibility of damage to the LVDT. Because of this possibility first two specimens of this group were tested without LVDT ring frame. It is certain that strain gauges measures local deformations. Also strain gauges can only be placed to the surface of jacket. So, axial deformation jacket can be measured with strain gauges, not the axial deformation concrete. To use the LVDT ring frame, holes are drilled to get a connection between LVDT ring frame and concrete. Unfortunately, the combination of the LVDT ring frame and the strain gauges is used only for experimental evaluation of specimen st4a3.

As expected, additional compressive strength is gained with confining of concrete specimens with steel tubes. However, after peak strength is reached, the stress was not constant during post-peak evaluation. Failures of concrete filled steel tubes were occurred with disjunction at weld. The stress was lower than the peak strength when concrete filled steel tubes failed. The predicted confined concrete strengths are close to experimental results.

Relationship between axial stress and strain for concrete filled steel tube specimen st4a3 is presented in Figure 5:18. The average of measurements from strain gauges is also presented in this figure to compare the localized and generalized deformations. At same strength less deformation is recorded by axial strain gauges, this is because of slippage between concrete and steel tube.
Figure 5.18 Stress-strain behavior of concrete filled steel tube (t=0.097in)

Figure 5.19 Dilation of concrete filled steel tube (t=0.097in)
Figure 5:19 presents the relationship between the dilation ratio and axial strain. Dilation ratio at deformation that is so close to zero is around 0.15. Expected dilation ratio is also 0.15. Dilation ratio increases slowly up to the axial deformation of 0.5%. After this level, dilation ratio increases more rapidly. After 2% axial deformation the rate of the change at dilation ratio decreases, and dilation ratio becomes almost constant.

Figure 5:20 compares two measurement devices that are used to evaluate the concrete filled steel tube specimens. Comparison is made from the relationship between lateral dilation and axial deformation. The relationship defined by using axial strain gauges behaves almost linear. This relationship corresponds to the elastic behavior of steel jacket.

![Graph showing dilation of concrete](image)

**Figure 5:20 Dilation of concrete**

The comparison of Mander’s analytical model, variably confined concrete model, FEM analysis for constant pressure (Abaqus-c) and jacketed concrete (Abaqus-j), and previous experimental results are plotted in Figure 5:21. FEM analysis results for jacketed concrete correlates with the experimental data. The FEM model of jacketed concrete didn’t fail because the jacket is modeled as elastic-plastic material. However, the FEM analysis of concrete confined with constant pressure does not correlate with the experimental data. Constant and variably confined concrete models correlate much better with experimental data between zero and peak stress. From 0.5% axial strain to the failure, VCCM and CCCM gives exactly the same results, because both use the same constant confining pressure in this range. After peak stress is reached, VCCM and CCCM overestimate the stress values. The difference between
the results from these two models and experimental result is because of overestimation of dilation ratio, which is plotted in Figure 5:22.

**Figure 5:21** Comparison of experimental and analytical concrete filled steel tube response (t=0.097in)

**Figure 5:22** Variation of the dilation ratio-concrete filled steel tube (t=0.097in)
Figure 5:23 Concrete filled steel tube specimens (st4a) at the end of test
The pictures of each specimen are taken at the end of test. Figure 5:23 shows the failures of concrete filled steel tubes due to the disconnection at weld. Disconnection of weld occurred at mid-height level for specimen st4a1; however, specimen st4a2 and st4a3 are failed due to the failure of steel connection in a weld region close to the top of CFT specimen.

5.1.5 Results of the Concrete Specimens Confined with the Steel Jacket (ST4B)

Same comparisons are tried to be presented for the specimens of group st4b; however, for the measurements only strain gauges are used. Because of this a proper data couldn’t have been created. The evaluation data couldn’t be saved because of the memory lack problem at computer setup of SATEC machine.

The individual and average axial stress-strain behaviors of concrete filled steel tubes are plotted in Figure 5:24. As mentioned above, deformations are measured by using strain gauges. Because the measurements are local, the relationship between dilation ratio and axial strain for group st4b is so different from the previous test groups. The relationship couldn’t be measured for concrete; measurements were belonging to the steel tube. So, this relationship is not published in this study.

![Stress-strain behavior of concrete filled steel tube (t=0.1875in)](image)

**Figure 5:24** Stress-strain behavior of concrete filled steel tube (t=0.1875in)

The comparison of axial stress-strain behaviors of concrete specimens in group st4b is presented in Figure 5:25. For this comparison, the results from Abaqus-j, Abaqus-c, constant confined concrete model, variably confined concrete model and
experimental evaluation are used. Because the measurements are not correct, there is no correlation between any of the results.

**Figure 5.25** Comparison of experimental and analytical concrete filled steel tube response (t=0.1875in)

Figure 5.26 shows figures the CFT specimens at the end of test. Specimen st4a1 didn’t fail and the test ended because of the temporary memory status of the loading system. After top and bottom concrete regions are all crushed, steel tube also carried axial load, so that specimen st4a2 showed a buckling column behavior. To use LVDT ring frame with specimen st4a3, connection holes were tried to be drilled. Because of its strength and thickness only a few holes were drilled, and LVDT ring frame was not used. Specimen st4a3 is failed at the drilled hole.
Figure 5:26 Concrete filled steel tube specimens (st4b) at the end of test
In Figure 5:27 the experimental results for all testing groups are compared. The axial stress-strain behaviors of unconfined and confined concrete specimens are presented. The relationship between dilation ratio and axial strain is used for the comparison in Figure 5:28. FRP jackets are bonded to surface with epoxy resin/hardener system. There is no possibility of slippage between the jacket and concrete specimen. Because of this, FRP jacketed concrete specimens similar to unconfined concrete specimens. However, there is a possibility of slippage between steel tube and concrete. So that the variation of dilation ratio with respect to the axial strain for CFT is marginally different from unconfined and FRP jacketed concrete.

Figure 5:27 Stress-strain behavior of unconfined and confined concrete
Figure 5:28 Variation of the dilation ratio of unconfined and confined concrete—experimental results

Figure 5:29 Variation of the dilation ratio of unconfined and confined concrete—analytical model results

By using analytical definition, the comparison of the relationships between the dilation ratio and axial strain for all testing groups is presented in Figure 5:29.
Figure 5:30 Variation of the dilation ratio of unconfined and confined concrete-analytical model results-Harries

5.2 FEM Analysis and Analytical Models

FEM analysis results for the concrete specimens confined with steel tubes (st4a, st4b), and GFRP (cg4b) and CFRP (cc4b) jackets are plotted in the Figure 5:30. Figure 5:31 shows the variably confined concrete model (VCCM) results. The axial stress-strain behavior of the unconfined concrete is the same with the unconfined concrete model of the Mander et al. [1988]. The Figure 5:32 is representing the FEM analysis results for the concrete specimens confined with constant confining pressures that are equal to the peak of confining pressure supplied by steel (st4a, st4b), GFRP (cg4b) and CFRP (cc4b) jackets. In Figure 5:33, the peak value of the confining pressure, which is expected to be generated by steel (st4a, st4b), GFRP (cg4b) and CFRP (cc4b), is assumed as constant confining pressure and used to predict the axial strain-stress behavior of the confined concrete specimens with the confined concrete model [Mander-1988].
Figure 5:31 FEM prediction of the experimental results-with jacket

Figure 5:32 Variably confined concrete model prediction of the experimental results

The only difference between Figure 5:30 and Figure 5:32 is the variation of confining pressure with respect to the axial strain. In Figure 5:30, the confining
pressure is generated by jackets, with the lateral dilation effect of concrete. However, in Figure 5:32 concrete specimen model is subjected to equivalent confining pressure, which is constant at all deformations.

Madas et al. [1992] (later Kestner et al. [1997]) have presented an analytical model for confined concrete, which is named as VCCM. The confining pressure changes with respect to the axial strain in accordance with the dilation ratio defined by Elwi [1979]. When the VCCM (Figure 5:31) is compared with constant confinement model (Figure 5:33), which is defined by Mander et al. [1988], the importance of varying confining pressure is understood.

Figure 5:33 FEM prediction of the experimental results-with constant confining pressure
Previously it is mentioned that, Equation 3-2 overestimates the confining pressure. Due to the overestimation of confining pressure, the constant confined concrete model [Mander, 1988] overestimates the axial stress-strain behavior of all jacketed concrete specimens.

Elwi defined a relationship between dilation ratio and axial strain in his study [1979]. The analytically predicted relationship of dilation ratio is compared with some experimental results. In his study, the predicted and evaluated results correlated with each other up to the peak stress of concrete. In our study, these results are duplicated for the same range of axial stress.

To see the effect of Equation 3-2 to the predictions of axial stress-strain behavior, confining pressure is recalculated and the results compared with each other. From experimental evaluation of CFRP jacketed concrete specimens; the averaged confined concrete stress was found to be 7.02 ksi. To achieve this confined concrete strength using Mander’s formulation (Equation 3-5), a confining pressure of 0.22 ksi. is required. However, in accordance with the Equation 3-2 the peak confining pressure value is predicted to be 1.10 ksi. The effect of confining pressure is shown in Figure 5:34.
Three different levels of confining pressure are compared. First configuration, standard, uses the Mander’s model assumption (Equation 3-2) for jacketed concrete specimens. The standard calculation of ultimate confining pressure is resulted in a value of 1.10 ksi. The second configuration, assumption-1, assumes a peak value of 0.22 ksi. The formulations of Mander’s model use the rupture strength of FRP jacket. However, to reach its rupture strength high lateral dilation of concrete is required. According to experimental results, the ultimate compressive strength of jacketed concrete is less than predicted. At ultimate compressive strength, the rupture strength of jacket can not be achieved, because the dilation of concrete is 0.0028. However, the required dilation for ultimate strength of jacket is 0.015 (5 times greater than the achieved). Because of this, Equation 3-2 is reduced to 20% of it is standard value in assumption-1 (20%×1.10=0.22). Assumption-1 predicts the dilation ratio more accurately, but still overestimates the post peak behavior.

To see the effect of dilation ratio, four different configurations are compared. Variably confined concrete model is used for all these configurations. VCCM is the standard prediction that is used in variably confined concrete model. The procedure for variably confined concrete model consists of two main steps:

- prediction of dilation ratio
- prediction of concrete behavior

The ultimate value of confining pressure affects these two steps respectively. From standard calculations, the ultimate confining pressure was found to be 1.10 ksi. And, this value is used for both of these steps. Assumption-1 is modified from standard model by changing only the prediction of dilation ratio. The prediction of dilation ratio that is previously named as assumption-1 is used. However, the effect of ultimate confining pressure is kept same (1.10 ksi.). In assumption-1.1, the reduced
confining pressure value (20%) is used during the both of two steps of the standard calculation. Assumption-2 is modified from standard calculations with changing only the prediction of dilation ratio. The predicted dilation ratio relationship, assumption-2, that assumes a confining pressure value of 0.05 ksi. is used. Finally, the constant confined concrete model (CCCM) is modified to match the ultimate compressive strength. The confining pressure is reduced to 20% of its standard value (0.22 ksi).

![Graph showing stress-strain behavior of CFRP jacketed concrete](image)

**Figure 5:36** Prediction of stress-strain behavior of CFRP jacketed concrete-cc4b

Among all these predictions, modified CCCM predicts the experimental results most accurately. Neither of variably confined concrete models can predict the axial stress-strain behavior of CFRP jacketed concrete specimens. This is because of not predicting the dilation ratio relationship precisely.

Same types of predictions are used to estimate the relationship between dilation ratio and axial strain for GFRP jacketed concrete specimens. The average strength of GFRP jacketed concrete was found to be 6.70 ksi. To achieve this ultimate strength using the formulations of Mander (equation 3-5), a value of 0.18 ksi. confining pressure is required. However, by using the same formulations, the confining pressure is predicted as 0.81 ksi. Three different configurations are used for the prediction of dilation ratio. First configuration assumes the confining pressure as 0.81 ksi., which is the standard prediction. Required confining pressure, which is 0.18 ksi., is used to get “assumption-1”. From experimental results, the lateral dilation at ultimate strength was found to be 0.10%. However, the rupture strength of jacket is achieved at a lateral strain of 2.14%. The actual lateral strain is 4.67% of the tensile strain of jacket. Using this value of lateral strain, actual confining pressure at ultimate compressive strength (4.67% x 0.81 ksi) is 0.038 ksi. The predicted relationship of dilation ratio, “assumption-2” correlates better with experimental results. These predictions of dilation ratio are compared with experimental results in Figure 5:36.
Figure 5:37 Variation of predicted dilation ratios-cg4b

Figure 5:38 Prediction of stress-strain behavior of GFRP jacketed concrete-cg4b

By using the predicted dilation ratio relationships defined in Figure 5:36, the predictions for jacketed concrete’s axial stress-strain behavior are modified. These modifications are similar to the ones defined for CFRP jacketed concrete specimens. VCCM is the standard calculation of variably confined concrete model. The prediction for dilation ratio, standard, is used together with a peak confining pressure value of 0.81 ksi. Only the prediction of dilation ratio is different in prediction of axial stress-
strain behavior, assumption-1. The relationship between dilation ratio and axial strain that is used in this prediction is also defined as “assumption-1”.

Assumption-1.1 is modified from assumption by changing the peak confining pressure to a value of 0.18 ksi. The standard variably confined concrete model is modified to assumption by changing predicted the dilation ratio relationship from standard to assumption-2. Finally, the analytical model of Mander is modified with changing the confining pressure. In modified CCCM model, the peak confining pressure is also assumed to be 0.18 ksi.

These models are compared in Figure 5:37. The best prediction is modified CCCM. Neither of the variably confined concrete model results correlates with experimental results. There is still a big difference between predicted and evaluated relationships for dilation ratios. This is causes underestimation of the axial stress-strain behavior of the GFRP jacketed concrete.

For concrete filled steel tube specimens (st4a), three estimations of the relationship between dilation ratio and axial strain are defined. These predictions are compared in Figure 5:38. First estimation is defined by standard calculations. By using standard calculations (Equation 3-2, 3-5), the ultimate confining pressure is assumed to be 1.16 ksi. From experimental evaluation, ultimate compressive strength is found to be 9.45 ksi. To achieve this strength, the required confining pressure is 0.65 ksi. The prediction of dilation ratio assuming 0.65 ksi. confining pressure is presented as assumption-1. According to experimental evaluation, concrete filled steel tube (st4a) achieves to 9.45 ksi ultimate compressive strength with a lateral dilation of 0.23%. Assumption-2 attempts to match with experimental results. A confining pressure of 2.0 ksi. is assumed. Among all these predictions, “assumption-2 gives the most accurate result, but still the post-peak behavior can not be determined correctly. These predictions are compared in Figure 5:38.
As mentioned previously for FRP jacketed concrete specimens, also for concrete filled steel tube specimens (st4a) the two step analysis is used. These two steps are consisted of prediction of dilation ratio and prediction of axial stress-strain behavior. The predicted, required and assumed confining pressure values are calculated during the comparison of dilation ratios. Using these predictions for dilation ratios and required/predicted confining pressures, the standard calculation for variably confined
concrete model is modified. Standard, is the predicted axial stress-strain behavior for the concrete filled tube-st4a. Using the predicted dilation ratio, assumption-1, and without reducing the ultimate confining pressure, standard model is modified into “assumption-1”. The ultimate confining pressure value is predicted to be 1.16 ksi. Assumption-1.1 is modified from assumption-1 with a reduction in confining pressure. The ultimate confining pressure is reduced to required confining pressure, which is 0.65 ksi. Predicted relationship for dilation ratio, assumption-2, is the only difference between the standard variably confined concrete model and predicted axial stress-strain behavior, “assumption-2”. Finally, confining pressure is reduced to required value (0.65 ksi.) for modification of constant confined concrete model (CCCM). Figure 5:39 compares the predictions for the axial stress-strain behavior of concrete filled steel tube.

Although CCCM assumes that the confining pressure is constant during the analysis, the modified CCCM is the most accurate prediction among all. Within all predicted variably confined concrete models, assumption-1.1 resulted in more accurate estimation of axial stress-strain behavior. Due to the difference between predicted and evaluated relationships between dilation ratio and axial stress, the estimation of axial stress-strain behavior cannot be made accurately.

All of the compared results in this chapter, showed us FEM analysis works well for prediction of stress-strain behavior of concrete. However, due to its main concept (constant confining pressure), Mander’s model does not correlate with experimental results. As a concept, variably confined concrete seems to be correct. But when compared with experimental results, this model does not correlate, too. This is because of the wrong estimation of the dilation ratio during the post peak behavior. During the final comparisons, some modifications have been made to the prediction of dilation ratio during the stress range of zero to peak strength. However, post peak behavior of dilation ratio couldn’t have modified in this study.

To define a better empirical and/or analytical relationship between dilation ratio and axial strain, a series of tests need to be conducted by tri-axial cell. So, material imperfections or other outside effects can be minimized to get an accurate result. In Chapter 7 a brief discussion of future work is presented.
6 CONCLUSION

6.1 Summary

From the literature review, it is observed that the common characteristic of the analytical models that were considered is that all of them were based on axial strain-stress behavior defined by the Popovics (latterly modified by the Mander et al.). In the analytical models that were defined later from Richart et al. [1928], Popovics [1973], and Mander et al. [1988], researchers usually tried to find a numerical expression for estimation of ultimate confined concrete strength and axial strain that corresponds to the ultimate strength.

In the analytical model for the concrete filled steel tubes that is defined by Susantha et al., a numerical value (named as $\beta$) lowers the ultimate stress for the confined concrete. This model is used while considering the thicknesses and the yield strengths of the steel tubes. When compared with the experimental and FEM analysis results, it is understood that the model defined by Susantha et al. caused an overdesign for the concrete filled steel tubes (st4a and st4b test series). In this model, up to peak behavior is defined by using the behavior definition of the Mander, and post peak behavior is estimated with a linear relationship. Because of presenting inaccurate results, this model is neglected during the discussion of results.

It is showed that, the analytical prediction defined by Mander et al is not conceptually accurate. In his model, Mander assumed that the effect of confining pressure is constant during the whole procedure. However, it is a known fact that in practical applications, the effect of confining pressure to the ultimate strength of confined concrete varies with the lateral dilation of the concrete. In this study, inaccuracy of constant confined concrete model is verified experimentally and analytically. Also, the ultimate confining pressure is overestimated with Mander’s model. When compared with experimental results, it is seen that, Mander’s model overestimates the confining pressure. Overestimation of confining pressure, leads to over estimation of ultimate confined concrete strength, and axial strain that corresponds to the ultimate strength. The true-required confining pressure can be calculated from same equations with using evaluated ultimate strength of confined concrete. If Mander’s model is modified with required confining pressure and ultimate strength of concrete, this model gives a more accurate result, but still not the correct. Being one the most referenced studies, this model also affects most of the other analytical predictions that were defined later for stress-strain behavior of confined concrete.

Previously it is mentioned that, the effect of confining pressure varies with the lateral dilation of the concrete depending on jacket’s characteristics. To estimate the dilation ratio there is only one model that was defined by Elwi [1979]. The
relationship defined between the dilation ratio and axial strain is a third degree polynomial. This relationship varies with the ratio of axial strain level to the axial strain that corresponds to the ultimate strength of confined concrete. The ultimate strength of confined concrete and axial deformation that corresponds to this strength is predicted by using the formulations defined by Popovics. Mander’s model uses also the same predictions that Popovics’ model uses. So, due to the overestimation of Mander’s model that is explained previously, the relationship for dilation ratio is underestimated for FRP jacketed specimens. However, the dilation ratio relationship for concrete filled steel tube is underestimated because of the parameters of the jacket.

The effect of confining pressure is discussed in Chapter-5. From these comparisons, it is verified that, the required confining pressure should be predicted more accurately. If the required confining pressure is predicted, the relationship between dilation ratio and axial strain correlates well with experimental results in the stress range of zero to peak. However, post-peak behavior of concrete still needs to be verified. A different type of curve can be used to define the dilation ratio relationship. Because, from the experimental results it is observed that the dilation ratio increases very rapidly just after peak point, but after a second level of axial deformation, the change in dilation ratio tends to decrease and becomes linear. However, the dilation ratio relation defined by Elwi, increases rapidly after axial deformation that corresponds to the peak strength and the change in the dilation ratio tends to increase because of being defined as a third degree polynomial. Because of these two different dilation ratio behavior, post-peak behavior of dilation ratio cannot be obtained from current model.

With using the current models of Mander et al and Elwi, variably confined concrete model was defined Madas and later modified by Kestner. In this model, prediction of ultimate confining pressure, ultimate confined concrete strength and axial strain that corresponds to ultimate strength are defined with formulations of Mander’s model. Also the relationship between axial stress and axial strain is defined by using the formulations of Mander. To calculate the variation of confining pressure with respect to the axial deformation applied, the dilation ratio relationship defined by Elwi is used. So that, the variation of ultimate confined concrete strength with respect to the axial strain is achieved. Due to the over estimation of Mander’s model and inaccurate post-peak behavior of dilation ratio relationship, variably confined concrete model (VCCM) results does not correlate with the experimental results. Although, VCCM is conceptually verifies the realistic behavior of the passively confined concrete, the results of this model are not accurate in prediction of axial stress-strain behavior because of using inaccurate parameter and behavior estimations.

The modification of variably confined concrete model by using the formula for peak compressive strength and dilation ratio relationship defined by Harries [2002] verifies that the concept of variable confining pressure. The modified variable confining pressure model correlates well with the experimental results. However, Harries defined this model just for fiber reinforced polymers (FRP) jacketed concrete specimens. The definition of Harries is not applicable for unconfined concrete and
other type of jackets (ex. concrete filled steel tubes). So this model needs to be generalized for all type of confinements.

Finite element analysis results correlates well with experimental results if concrete specimens are modeled to be confined by jackets. This result also verifies the idea of variably confined concrete definition. In this study, Abaqus, which is a FEM analysis program, is used for analysis of concrete specimens’ behavior. Using a continuous, 20 nodded solid brick element with concrete damaged plasticity, helped us to predict the experimental results of unconfined and passively confined concrete results. The experimental evaluation results are modified into the true stress-strain behavior to be used while defining concrete compression hardening option. Default values are chosen for other parameters. While defining jackets elastic or elastic-plastic material definition are due to the idealization of jackets’ characteristics. 8 nodded continuous shell elements are used while meshing the jacket. The nodes of elements of jacket are tied to the surface elements of concrete. Also to see the effect of constant pressure concrete subjected to equivalent confining pressure and set of FEM analysis is conducted. Comparison of these two models verifies the idea of variable confining pressure.

Also a different FEM program was decided to be used for FEM analysis of confined concrete specimens. This program was Diana. Diana is much stronger than the Abaqus while modeling concrete. These two FEM analysis programs are so close to each other in main concepts. The advantages of Abaqus while modeling concrete are: strong graphical user interface, better meshing, and damaged plasticity option for concrete. However, Diana also has advantages: cracking model for concrete and good post-processor to see the analysis results. Unconfined concrete model is meant to be defined by using Diana. For 3-D solid elements, material models based on cracking behavior can only be used. Post-peak behavior couldn’t have obtained with discrete meshing of concrete. However, accurate results can be got with low meshing. Improper meshing of concrete might have resulted in the inadequate analysis of concrete. A further analysis is needed for FEM analysis by using Diana.

While conducting the trial tests (without any instrumentation) to decide what type of instrumentation needed to be used, it was observed that the FRP jacketed concrete specimens showed brittle failure. Because of the brittle failure of the concrete specimens strain gauges were decided on as instrumentation. The measurements from strain gauges on the surface of the FRP jacketed specimens are close to expectations. However, the results from strain gauges on the steel jacketed specimens are not so close. The difference between the expected results may be because of the slipping of the steel on the concrete surface. Because the strain gauge measures local deformations and the LVDT measures the generalized behavior, the decision was made to use both the strain gauges and the LVDT ring frame together for measurements (specimens: st4a3, cg4b1, cg4b2, cg4b3, cc4b2, cc4b3).

From the experimental evaluation results of FRP jacketed concrete specimens, it is understood that with using the rupture strength of FRP jackets the predicted ultimate confining pressure, so that ultimate confined concrete strength and axial strain that
corresponds to ultimate strength can not be achieved with experimental results. The lateral dilation at ultimate strength of confined concrete is so low than the tensile strain of jacket. Lateral dilation at ultimate strength is almost 20% of tensile deformation capacity the fibers of FRP jacket. This ratio is almost same for both CFRP and GFRP jacketed concrete specimens. Also, some of the FRP jacketed concrete specimens failed due to disconnection of overlapping. Jacketing concrete with one ply of FRP sheet might not be enough to achieve to the full capacity of FRP sheets. Also, confining pressure generated by FRP sheet is much softer when compared the ones generated by steel tubes (see in Figure 7:1).

Proper data couldn’t have been created from the experimental evaluation of concrete filled tube specimens-st4b. This is because of over design of these specimens. Also LVDT ring frame couldn’t have been used for this series. Only one specimen of series-st4a was used to create a proper data. All of the specimens of series-st4a failed at weld. The concrete specimens confined with the FRP jackets showed more brittle failure because of the sudden rupture of the fibers of the jackets, when compared to the concrete specimens confined with the steel jackets.

Finally, from all experimental evaluations: the importance of a good instrumentation, material imperfections, and application imperfections are well understood.

The aim of this study was developing a better testing method for confined concrete. LVDT ring frame is for more accurate generalized measurements. With this study, the need for a better testing method and accuracy of existing analytical is discussed. Inadequate estimations of experimental results verify the need of better predictions. To compare and develop empirical models a large number of experimental evaluations are needed. To be independent from jacketing material, application, and material imperfections, using tri-axial cell mechanism is the better and fast way of experimental evaluation. However, the behavior of confining pressure is so important to be consistent with practical applications. In this study, the need for new testing method is verified and some recommendations are pointed out for future work. Next section and Chapter-7 will discuss these recommendations and the future work.

The conclusions are summarized as;

- Constant confined concrete model is not correct to predict the behavior of concrete that are confined by jacket materials
- Variably confined concrete is conceptually correct, but not accurate due to inaccurate estimation of dilation ratio and confining pressure
- Promising methods for dilation ratio and confining pressure defined by Harries are more accurate.
- Harries model is for only FRP materials, steel and other materials are still inaccurate
- Finite element method analyses are accurate enough to predict the behavior of unconfined and confined concrete. When finite element
model is updated with experimental data FEM analyses give very close results.

6.2 Recommendations

Under the light of findings detailed in this study, and summarized in previous section, recommendations can be point out as,

- More accurate prediction of confining pressure generated by all type of jackets is needed
- A better definition for prediction of variably confined concrete needs to be achieved
- More realistic behavior should be defined for the relationship between the dilation ratio and the axial strain of unconfined and confined concrete
- If it is applicable, the LVDT ring frame is recommended to be used. Because, When compared with the LVDT ring frame, strain gauges give more localized results
- To neglect the material and application imperfections, tri-axial cell mechanism can be used for the evaluation of concrete specimens. However, the confining pressure should vary with respect to the lateral dilation of concrete.
- Different loading rates for confining pressure and axial deformation can be used for evaluations using tri-axial cell mechanism. Also, the effect of unconfined compressive strength to the behavior of confined concrete can be observed with a series of evaluation with using tri-axial cell mechanism.
- Finally, by using Diana, FEM analysis can be made with a better meshing of concrete.
The work presented is the first part of an ongoing project conducted by the ATLSS Research Center. As the results have shown current confinement models do not accurately predict axial response. To address this deficiency additional research must be conducted on the response of concrete under varying confinement stresses. To accomplish these studies an experimental evaluation using a tri-axial cell mechanism is explained.

To efficiently examine the stress-strain behavior of concrete under varying levels of confinement a tri-axial cell mechanism will be used. This mechanism allows for a broad examination of confining pressure independent of jacket material. This simplifies the testing procedure by eliminating the need to evaluate multiple jacket materials. This also eliminates variability in the study through the removal of any material or application imperfections. Using the tri-axial cell the confining pressure will be mimicked with the application of a lateral pressure.

In previous studies of concrete using a tri-axial cell, constant confining pressures were generated. As discussed, the behavior of concrete under jacket confinement is not constant but varies as a function of dilation. Initially it has zero confinement and increases to a very high restraint as the dilation increases. As axial load (or deformation) is applied to the concrete specimen, lateral dilation occurs in the body of the concrete because of Poisson’s Effect. The axial strain in the jacket, which is equal to the lateral dilation of the concrete, exerts the confining pressure on the concrete specimen. To compare the behavior of the concrete specimens under constant and varying confining pressure levels, the use of a tri-axial cell mechanism is recommended.

To compare a variety of jacketing materials the tri-axial cell mechanism will be programmed to apply pressures in accordance with the jackets’ material characteristics. Since the material behavior of the jacket is typically well defined the application of the varying confining pressure can be easily achieved.

### 7.1 Tentative Research Program

To fully explore the dilatational characteristics of concrete under varied confinement, a potential test series is discussed.

#### 7.1.1 Concrete

To examine a broad range of applications, concrete specimens with different unconfined axial compressive strengths are recommended. The concrete with compressive strengths of 3.0, 4.0, 5.0, 6.0, 8.0, 10.0 and 12.0 ksi. will be evaluated.
7.1.2 **Tri-axial Cell Mechanism**

A tri-axial cell used for the evaluating rocks will be modified for the concrete study. Some tri-axial cell mechanisms are already manufactured for the evaluation of concrete specimens. It is important to use the correct setup to measure the lateral and axial strains.

7.2 **Test Matrix**

Concrete specimens that have different compressive strengths can be evaluated with different levels of confining pressure and with different paths for varying confining pressure. To examine the feasibility of the testing series, the maximum confined compressive strength values are predicted using the model of Richart et al. [1928], Table 7-1.

**Table 7-1** Potential confined axial compressive strength

<table>
<thead>
<tr>
<th>Confining Pressure [%fc]</th>
<th>3 ksi</th>
<th>4 ksi</th>
<th>5 ksi</th>
<th>6 ksi</th>
<th>8 ksi</th>
<th>10 ksi</th>
<th>12 ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>3.615</td>
<td>4.82</td>
<td>6.025</td>
<td>7.23</td>
<td>9.64</td>
<td>12.05</td>
<td>14.46</td>
</tr>
<tr>
<td>10</td>
<td>4.23</td>
<td>5.64</td>
<td>7.05</td>
<td>8.46</td>
<td>11.28</td>
<td>14.1</td>
<td>16.92</td>
</tr>
<tr>
<td>20</td>
<td>5.46</td>
<td>7.28</td>
<td>9.1</td>
<td>10.92</td>
<td>14.56</td>
<td>18.2</td>
<td>21.84</td>
</tr>
<tr>
<td>50</td>
<td>9.15</td>
<td>12.2</td>
<td>15.25</td>
<td>18.3</td>
<td>24.4</td>
<td>30.5</td>
<td>36.6</td>
</tr>
<tr>
<td>100</td>
<td>15.3</td>
<td>20.4</td>
<td>25.5</td>
<td>30.6</td>
<td>40.8</td>
<td>51</td>
<td>61.2</td>
</tr>
</tbody>
</table>

**Table 7-2** Tentative test matrix using 6 in. x 12 in. cylinders

<table>
<thead>
<tr>
<th>Confining Pressure</th>
<th>3 ksi</th>
<th>4 ksi</th>
<th>5 ksi</th>
<th>6 ksi</th>
<th>8 ksi</th>
<th>10 ksi</th>
<th>12 ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 %</td>
<td>3-000</td>
<td>4-000</td>
<td>5-000</td>
<td>6-000</td>
<td>8-000</td>
<td>10-000</td>
<td>12-000</td>
</tr>
<tr>
<td>5 %</td>
<td>3-005</td>
<td>4-005</td>
<td>5-005</td>
<td>6-005</td>
<td>8-005</td>
<td>10-005</td>
<td>12-005</td>
</tr>
<tr>
<td>10 %</td>
<td>3-010</td>
<td>4-010</td>
<td>5-010</td>
<td>6-010</td>
<td>8-010</td>
<td>10-010</td>
<td>12-010</td>
</tr>
<tr>
<td>15 %</td>
<td>3-015</td>
<td>4-015</td>
<td>5-015</td>
<td>6-015</td>
<td>8-015</td>
<td>10-015</td>
<td>12-015</td>
</tr>
<tr>
<td>20 %</td>
<td>3-020</td>
<td>4-020</td>
<td>5-020</td>
<td>6-020</td>
<td>8-020</td>
<td>10-020</td>
<td>N/A</td>
</tr>
<tr>
<td>50 %</td>
<td>3-050</td>
<td>4-050</td>
<td>5-050</td>
<td>6-050</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>100 %</td>
<td>3-100</td>
<td>4-100</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

The compressive load capacity of the SATEC machine is 600 kips. The maximum axial stress that can be generated by the SATEC machine using 6 inch cylindrical specimens is 21.2 ksi. For the 4 inch cylindrical specimens the maximum stress value increases to 47.5 ksi. Table 7-2 and 7-3 presents the possible tests that can be conducted based on the limitations of the equipment. Each test is identified by the
concrete strength followed by the confining stress. For example 8-020 is a test at 8 ksi with 20% confinement. As shown in the tables the 4x8 cylinders provide a greater flexibility in testing.

| Table 7-3 Tentative test matrix using 4 in. x 8 in. cylinders |
|-----------------|----------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Confining Pressure | 3 ksi | 4ksi. | 5 ksi | 6 ksi | 8 ksi | 10 ksi | 12 ksi |
| 0 %    | 3000 | 4000 | 5000 | 6000 | 8000 | 10000 | 12000 |
| 5 %    | 3005 | 4005 | 5005 | 6005 | 8005 | 10005 | 12005 |
| 10 %   | 3010 | 4010 | 5010 | 6010 | 8010 | 10010 | 12010 |
| 15 %   | 3015 | 4015 | 5015 | 6015 | 8015 | 10015 | 12015 |
| 20 %   | 3020 | 4020 | 5020 | 6020 | 8020 | 10020 | 12020 |
| 50 %   | 3050 | 4050 | 5050 | 6050 | 8050 | 10050 | 12050 |
| 100 %  | 3100 | 4100 | 5100 | 6100 | 8100 | N/A    | N/A    |

7.2.1 Constant Confinement Test Series

Most of the existing predictor models are defined assuming that the confining pressure was constant. The differences between passive confinement and constant confinement result from the source of the confining pressure and the way that pressure is applied. The active confinement is generated with hydrostatic pressure using the tri-axial cell mechanism, and this pressure is constant during the axial compression of the concrete specimen. To evaluate the accuracy of current predictor models, cylindrical concrete specimens need to be evaluated under a constant confining pressure and axial loading. To generate a constant confining pressure on specimens, a tri-axial cell mechanism needs to be used. By conducting this series of evaluations, a database for the concrete specimens that have different compressive strengths and that are confined by the different levels of the constant confining pressure can be developed. As explained in Table 7-1, the compressive strengths were chosen for evaluation were 3, 4, 5, 6, 8, 10, and 12ksi. For each group of concrete specimens with different compressive strengths, constant confining pressure levels of 0, 5, 10, 15, 20, 50, and 100% of the unconfined compressive strength of the concrete specimens ($f_c$) will be generated by the tri-axial cell mechanism. The database that will be created for this series of evaluations can also be used to see the difference between the results from this series and the results from the variable confinement test series.

In practice a constant confining pressure is not generated on concrete columns. As explained previously, the materials that are being used to confine the concrete specimens are the steel and the FRP jackets. These jacket materials have elastic-brittle or elastic-plastic behavior characteristics.
7.2.2 Variable Confinement Test Series

The purpose of this series of evaluations is the idealization of the stress-strain behavior of the confined concrete, independent from the jacketing material. More accurate results for the dilation of the concrete under axial compressive loading can be determined using the tri-axial cell mechanism. It is a known fact that, in application, the confining pressure generated by jackets varies with the dilation of the concrete specimen. As explained previously, because of the elastic-brittle or elastic-plastic material characteristics, confining pressures varying with the lateral deformation in the concrete body need to be applied by the tri-axial cell mechanisms. Depending on Young’s Modulus and the yield (and/or rupture) strength, a loading path for the confining pressures from various jackets can be idealized and generated by the tri-axial cell mechanism. Consequently, the material imperfections can be ignored and more accurate data can be developed. This data can be used for modifying the current models, or for developing a new model for variably confined concrete columns.

The experimental evaluation will be conducted by controlling the axial deformation of the concrete specimens. During the experimental evaluation, the lateral deformation will be measured for each increment of the axial deformation. By using the measured lateral deformation and the formulation that depends on the idealized axial stress-strain behavior of the jacket, the confining pressure can be found at that axial deformation value. Then, this confining pressure can be generated by the tri-axial cell for each level of axial deformation. Thus, at each level of axial deformation, the lateral deformation, the confining pressure, and the corresponding axial compressive stress can be measured and calculated. Finally, the behavior of the confined concrete, which depends on the loading path of the confining pressure, can be derived from the relationship between the axial deformation and the axial compressive stress.

Finally, the data developed during both the constant and the variable confinement test series can be compared with each other to see the effect of the confining pressure on the concrete specimens. A second comparison can be made depending on the loading paths. All these data can also be used for either an empirically developed or a modified model for the variably confined concrete columns.
Figure 7:1 Relationships between confining pressure and lateral strain

Figure 7:1 presents the idealized relationships between concrete specimen’s lateral strain and confining pressure to be applied by the jacket. These relationships can be defined for different type and brand of materials. Knowing the material stress-strain relationship the loading used for the tri-axial lateral confining pressure can be determined and applied.

The control procedure for this test methodology is illustrated in Figure 7:2.
Figure 7:2 Test procedure of tri-axial cell mechanism
The varying confining pressure tests can be summarized as:
Step-1) Apply axial deformation increment using the SATEC Controller.
Step-2) Measure lateral dilation using the tri-axial cell transducer.
Step-3) Calculate confining pressure using the predetermined material $\sigma$-$\epsilon$
relationship.
Step-4) Apply confining pressure with tri-axial cell pump
Step-5) Repeat Step 1 through 4 until failure.
REFERENCES


“Cylinder tests: experimental technique and results” Bellotti, R., Rossi, P., Materials and Structures, 24, 1991


"Experimental investigation of the behavior of variably confined concrete" Harries, K.A., Kharel, G., Cement and Concrete Research, 2267, 2002

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ASTM Standards 2001
APPENDIX

A. Input Data for Abaqus FEM Program

Input data for ABAQUS model;
*Preprint, echo=YES, model=YES, history=YES, contact=YES
** PARTS
*Part, name=concrete
*Part, name=jacket
** ASSEMBLY
*Instance, name=concrete-1, part=concrete
*Element, type=C3D20
** Section: concrete
*Solid Section, elset=_I1, material=concrete 1.,
*Instance, name=jacket-1, part=jacket
*Node
*Element, type=S8R
** Section: jacket (CFRP)
*Shell Section, elset=_I1, material=jacket 0.0065, 5
** Section: jacket (GFRP)
*Shell Section, elset=_I1, material=jacket 0.034, 5
** Section: jacket (thin steel)
*Shell Section, elset=_I1, material=jacket 0.097, 5
** Section: jacket (thick steel)
*Shell Section, elset=_I1, material=jacket 0.188, 5
** Constraint: Constraint-1
*Tie, name=Constraint-1, adjust=yes _PickedSurf72, _PickedSurf71
** MATERIALS
**
*Material, name=concrete
*Elastic
4250., 0.15
*Concrete Damaged Plasticity
55., 0.1, 1.16, 0.62, 0.
*Concrete Compression Hardening
5.30014, 0.
5.54815, 0.000165
5.65634, 0.000303
5.48811, 0.000506
4.6067, 0.000958
3.61093, 0.0016
2.31873, 0.00272
1.64889, 0.003692
1.16334, 0.00462
0.867669, 0.006316
0.565057, 0.00801

*Concrete Tension Stiffening
0.5, 0.
0.1, 0.0005
0.05, 0.001
0., 0.002

*Material, name=jacket (CFRP)
*Elastic
33850., 0.3

*Material, name=jacket (GFRP)
*Elastic
3382., 0.3

*Material, name=jacket (thin steel)
*Elastic
29000., 0.3

*Material, name=jacket (thick steel)
*Elastic
29000., 0.3

** STEP: Step-1
**

*Step, name=Step-1, nlgeom, inc=200
*Static, riks
0.5, 1., 1e-05, 1.,

** BOUNDARY CONDITIONS
**
** Name: bottom Type: Velocity/Angular velocity
*Boundary, type=VELOCITY
_PickedSet64, 1, 1
_PickedSet64, 2, 2
_PickedSet64, 3, 3
** Name: top Type: Velocity/Angular velocity
*Boundary, type=VELOCITY
_PickedSet63, 1, 1
_PickedSet63, 2, 2
_PickedSet63, 3, 3, -0.005
**
** OUTPUT REQUESTS
** FIELD OUTPUT: F-Output-1
** FIELD OUTPUT: rfbot
*Output, field
*Node Output, nset=bottom
RF,
** FIELD OUTPUT: disptop
*Node Output, nset=top
U,
** HISTORY OUTPUT: H-Output-1
** HISTORY OUTPUT: rfbot
*Output, history
*Node Output, nset=bottom
RF3,
** HISTORY OUTPUT: disptop
*Node Output, nset=top
U3,
B. Calculations for Numerical Models

Calculation 1) Analytical model of Susantha et al. for steel jackets (fl=0.1xfc)

\[ \begin{align*}
\text{material properties of jacket} & \quad f'_y := 36 \quad t := 0.097 \quad E_s := 29000 \\
\text{material properties of concrete} & \quad \varepsilon_y := 0.0012 \quad \varepsilon_{st} := 0.025 \\
E_c := 57 \cdot \sqrt{f'_c \cdot 1000} & \quad E_c = 4.265 \times 10^3 \\
\varepsilon_c := 0.002 & \quad \varepsilon_c = 2 \times 10^{-3} \\
f_l := \nu_{ep} & \equiv 0.881 \cdot 10^{-6} \cdot \left( \frac{D}{t} \right)^3 - 2.58 \cdot 10^{-4} \cdot \left( \frac{D}{t} \right)^2 + 1.953 \cdot 10^{-2} \cdot \left( \frac{D}{t} \right) + 0.4011 \\
\nu_{ep} & \equiv 0.2312 + 0.3582 \cdot \nu_{ep} - 0.1524 \cdot \left( \frac{f_c}{f'_y} \right) + 4.843 \cdot \nu_{ep} \cdot \left( \frac{f_c}{f'_y} \right) - 9.169 \cdot \left( \frac{f_c}{f'_y} \right)^2 \\
\beta & \equiv \nu_{ep} - 0.5 \\
f_1 & \equiv \beta \cdot 2 \cdot t \cdot \frac{f_c}{D} \\
\frac{f_1}{f_c} & \equiv 0.085 \\
f_{cc} := f_c + 4.1 \cdot f_1 & \quad f_{cc} = 7.551 \\
\varepsilon_{cc} := \varepsilon_c \cdot \left[ 1 + 5 \cdot \left( \frac{f_{cc}}{f_c} - 1 \right) \right] & \quad \varepsilon_{cc} = 5.484 \times 10^{-3} \\
R_1 & \equiv \sqrt{3 \cdot (1 - \nu^2)} \cdot \frac{f'_y}{E_s} \cdot \frac{D}{2 \cdot t} \quad \left( R_1 \cdot \frac{f_c}{f'_y} \right) = 0.01 \\
z & \equiv 0 \quad \text{if} \quad \left( R_1 \cdot \frac{f_c}{f'_y} \right) \leq 0.006 \\
\left( 10^5 \cdot R_1 \cdot \frac{f_c}{f'_y} \right) - 600 & \quad \text{if} \quad \left( R_1 \cdot \frac{f_c}{f'_y} \right) \geq 0.006 \wedge \left( f'_y \leq 41.044 \right) \\
\left( 41.044 \right)^{1.4} \cdot \left[ 10^5 \cdot R_1 \cdot \frac{f_c}{f'_y} \right] - 600 & \quad \text{if} \quad \left( R_1 \cdot \frac{f_c}{f'_y} \right) \geq 0.006 \wedge \left( 41.044 \leq f'_y \leq 48.73 \right) \\
\left[ 10^6 \cdot R_1 \cdot \frac{f_c}{f'_y} \right] - 6000 & \quad \text{if} \quad \left( R_1 \cdot \frac{f_c}{f'_y} \right) \geq 0.006 \wedge \left( f'_y \geq 48.731 \right) \\
\alpha & \equiv 1 - \frac{z \cdot \left( \varepsilon_{cc} - \varepsilon_{cc} \right)}{f_{cc} \cdot 6.8948} \\
r & \equiv \frac{E_c}{E_c - \frac{f_{cc}}{\varepsilon_{cc}}} \\
f_{cl} (x) & \equiv \left\{ \begin{array}{ll} \frac{f_{cc} \cdot r \cdot x}{r - 1 + (x - 1) \cdot \varepsilon_{cc}} & \text{if} \quad x \leq 1 \\
\left[ f_{cc} - z \cdot (x - 1) \cdot \varepsilon_{cc} \right] & \text{if} \quad x > 1 \end{array} \right. \\
z & = 422.724 \quad \alpha = 0.882 \quad r = 1.477
\end{align*} \]

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Calculation 2) Analytical model of Susantha et al. for steel jackets (fl=0.2xfc)

**material properties of jacket**
- \( f_y := 65 \)
- \( t := 0.1875 \)
- \( E_s := 29000 \)
- \( \varepsilon_y := 0.0012 \)
- \( \varepsilon_s := 0.025 \)

**material properties of concrete**
- \( f_c := 5.6 \)
- \( D := 6 \)
- \( e_{cu} := 0.02 \)
- \( \nu := 0.15 \)
- \( E_c := 57 \cdot \sqrt{f_c} \cdot 1000 \)
- \( \varepsilon_c := 0.002 \)
- \( \varepsilon_c = 2 \times 10^{-3} \)
- \( E_c := 4.265 \times 10^{3} \)

\[
f_l := \nu_{ep} - 0.881 \cdot 10^{-6} \cdot \left( \frac{D}{t} \right)^3 - 2.58 \cdot 10^{-4} . \frac{D}{t}^2 + 1.953 \cdot 10^{-2} \cdot \frac{D}{t} + 0.4011
\]

\[
v_e := 0.2312 + 0.3582 \cdot \nu_{ep} - 0.1524 \cdot \left( \frac{f_c}{f_y} \right)^2
\]

\[
\beta := \nu_e - 0.5
\]

\[
f_l := \beta \cdot 2 \cdot t \cdot \frac{f_y}{D}
\]

\[
f_i := \frac{f_l}{f_c} = 0.191
\]

\[
f_{cc} := f_c + 4.1 \cdot f_l
\]

\[
\varepsilon_{cc} := \varepsilon_c \left[ 1 + 5 \cdot \left( \frac{f_{cc}}{f_c} - 1 \right) \right]
\]

\[
R_t := \sqrt{3(1 - \nu^2)} \cdot \frac{f_y}{E_s} \cdot \frac{D}{2 \cdot t}
\]

\[
z := 0 \quad \text{if} \quad \left( R_t \cdot \frac{f_c}{f_y} \right) \leq 0.006
\]

\[
\begin{align*}
&= \left( 10^5 \cdot R_t \cdot \frac{f_c}{f_y} - 600 \right) \quad \text{if} \quad \left( R_t \cdot \frac{f_c}{f_y} \right) \geq 0.006 \cap \left( f_c \leq 41.044 \right) \\
&= \frac{f_y}{41.044}^{13.4} \cdot \left( 10^5 \cdot R_t \cdot \frac{f_c}{f_y} - 600 \right) \quad \text{if} \quad \left( R_t \cdot \frac{f_c}{f_y} \right) \geq 0.006 \cap (41.044 \leq f_y \leq 48.731) \\
&= \left( 10^6 \cdot R_t \cdot \frac{f_c}{f_y} - 6000 \right) \quad \text{if} \quad \left( R_t \cdot \frac{f_c}{f_y} \right) \geq 0.006 \cap (f_y \geq 48.731)
\end{align*}
\]

\[
\alpha := 1 - \frac{z \cdot (e_{cu} - \varepsilon_{cc})}{f_{cc} \cdot 6.8948}
\]

\[
r := \frac{E_s}{E_s - \varepsilon_{cc} f_{cc}}
\]

\[
f_{c1}(x) := \begin{cases} f_{cc} \cdot r \cdot x & \text{if} \ x \leq 1 \\ f_{cc} - z \cdot (x - 1) \cdot \varepsilon_{cc} & \text{if} \ x > 1 \end{cases}
\]

\[
z = 0 \quad \alpha = 1 \quad r = 1.313
\]
Calculation 3) Variably confined concrete model for steel jacket (t=0.097in)

using Mander model to compare varying confining pressure

material properties of jacket

\[ f_j := 36 \quad t_1 := 0.097 \quad E_j := 29000 \quad \varepsilon_y := 0.0012 \quad n := 1 \quad t := n \cdot t_1 \quad t = 0.097 \]

\[ \varepsilon_{st} := 0.025 \]

material properties of concrete

\[ f_{c0} := 5.6 \quad D := 6 \quad \nu := 0.15 \quad E_{c0} := 57 \cdot \sqrt{f_{c0} \cdot 1000} \quad E_{c0} = 4.265 \times 10^3 \text{ (Mpa)} \]

\[ \varepsilon_{c0} := 0.002 \quad \varepsilon_{c0} = 2 \times 10^{-3} \]

peak values of compressive stress and strain

\[ f_{00} := \begin{cases} \rho & \leftarrow 4 \cdot \frac{1}{D} \\ \rho_0 & \leftarrow 0.5 \cdot \rho \cdot f_j \end{cases} \]

\[ f_{ccp} := f_{c0} \cdot \left( -1.254 + 2.254 \cdot \sqrt{1 + 7.94 \cdot \frac{\rho_0}{f_{c0}} - 2 \cdot \frac{f_{00}}{f_{c0}}} \right) \]

\[ f_{ccp} = 11.199 \]

\[ \varepsilon_{ccp} := \varepsilon_{c0} \cdot \left[ 1 + 5 \cdot \left( \frac{f_{ccp}}{f_{c0}} - 1 \right) \right] \]

\[ \varepsilon_{ccp} = 0.012 \]

\[ \varepsilon(x) := x \]

\[ \eta(x) := \nu \cdot \left[ 1 + 1.3763 \cdot \frac{\varepsilon(x)}{\varepsilon_{ccp}} - 5.36 \cdot \left( \frac{\varepsilon(x)}{\varepsilon_{ccp}} \right)^2 + 8.586 \cdot \left( \frac{\varepsilon(x)}{\varepsilon_{ccp}} \right)^3 \right] \]

\[ \varepsilon_t(x) := \eta(x) \cdot \varepsilon(x) \]
\[ \varepsilon_j(x) := \varepsilon_i(x) \quad k := 0.0161 \]

\[ \varepsilon_j(x) \text{ substitute } x = k \rightarrow 3.366910148736508131910^{-2} \]

\[ \varepsilon_{cu} := k \quad \varepsilon_{cu} = 0.016 \]

\[ f_l(x) := \begin{align*}
\rho & \leftarrow 4 \cdot \frac{1}{D} \\
f_l & \leftarrow \begin{cases} 
0.5 \cdot \rho \cdot E_j \cdot \varepsilon_j(x) & \text{if } \varepsilon_j(x) \leq 0.0012 \\
0.5 \cdot \rho \cdot f_j & \text{if } 0.0012 \leq \varepsilon_j(x) \leq 0.025 \\
0 & \text{otherwise}
\end{cases}
\end{align*} \]
\( fc \) :=

\[
\begin{align*}
& m \leftarrow 300 \\
& \text{for } j \in 0..m \\
& \quad \varepsilon_j \leftarrow j \cdot 0.0001 \\
& \quad \eta_j \leftarrow \varepsilon_j \cdot \left[ 1 + 1.3763 \cdot \frac{\varepsilon_j}{\varepsilon_{ccp}} = 5.36 \cdot \left( \frac{\varepsilon_j}{\varepsilon_{ccp}} \right)^2 + 8.586 \cdot \left( \frac{\varepsilon_j}{\varepsilon_{ccp}} \right)^3 \right] \\
& \quad \varepsilon_j \leftarrow \eta_j \cdot \varepsilon_j \\
& \quad f_j \leftarrow \rho \leftarrow 4 \cdot \frac{1}{D} \\
& \quad f_j \leftarrow \begin{cases} \\
(0.5 \cdot \rho \cdot E_j \cdot \varepsilon_j) & \text{if } (\varepsilon_j \leq 0.0023) \\
((0.5 \cdot \rho \cdot E_j \cdot 0.0023)) & \text{if } (0.0012 \leq \varepsilon_j \leq 0.025) \\
0 & \text{otherwise} \\
\end{cases} \\
& \quad f_{cc} \leftarrow (f_0) \cdot (1.254 + 2.254 \cdot \sqrt{1 + 7.94 \cdot \frac{f_j}{f_0} - 2 \cdot \frac{f_j}{f_0}}) \\
& \quad f_{cc} \leftarrow f_{cc} \left[ 1 + 5 \cdot \left( \frac{f_{cc}}{f_0} - 1 \right) \right] \\
& \quad n_j \leftarrow \frac{E_c}{f_{cc}} \\
& \quad E_c = \frac{f_{cc}}{\varepsilon_{ccj}} \\
& \quad k_j \leftarrow \begin{cases} \\
1 & \text{if } \left( \frac{\varepsilon_j}{\varepsilon_{ccj}} \right) \leq 1 \\
\left[ \left[ \left[ 0.67 + \frac{f_0}{62} \right] \cdot \left( \frac{f_0}{f_{cc} \cdot f_j} \right) \right] \right] & \text{otherwise} \\
\end{cases} \\
& \quad f_{cj} \leftarrow \left[ \left( \frac{f_{cc}}{\varepsilon_{ccj}} \right) \left( \frac{n_j}{n_j - 1 + \left( \frac{\varepsilon_j}{\varepsilon_{ccj}} \right)^n_{k_j}} \right) \right] \\
\end{align*}
\]

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<td>3.805</td>
<td>4.056</td>
<td>4.294</td>
<td>4.518</td>
<td>4.731</td>
</tr>
</tbody>
</table>
Calculation 4) Variably confined concrete model for steel jacket (t=0.188in)

Material properties of jacket

\[ f_j := 65 \quad t_1 := 0.1875 \quad E_j := 29000 \quad \varepsilon_x := 0.00223 \quad n := 1 \quad t := n \cdot t_1 \quad t = 0.188 \quad \varepsilon_{st} := 0.025 \]

Material properties of concrete

\[ f_{c0} := 5.6 \quad D := 6 \quad \nu := 0.15 \quad E_{c0} := 57 \cdot \sqrt{f_{c0}} \cdot 1000 \quad E_{c0} = 4.265 \times 10^3 \text{ (Mpa)} \]
\[ \varepsilon_{c0} := 0.002 \quad \varepsilon_{c0} = 2 \times 10^{-3} \]

Peak values of compressive stress and strain

\[ f_{00} := \rho \leftarrow 4 \cdot \frac{1}{D} \quad f_{00} = 4.063 \quad \frac{f_{00}}{f_{c0}} = 0.725 \]
\[ f_{00} \leftarrow 0.5 \cdot \rho \cdot f_j \]
\[ f_{ccp} := f_{c0} \left[ -1.254 + 2.254 \cdot \sqrt{1 + 7.94 \cdot \frac{f_{00}}{f_{c0}} - 2 \cdot \frac{f_{00}}{f_{c0}}} \right] \quad f_{ccp} = 17.671 \]
\[ \varepsilon_{ccp} := \frac{f_{ccp}}{f_{c0}} \left[ 1 + 5 \cdot \left( \frac{f_{ccp}}{f_{c0}} - 1 \right) \right] \quad \varepsilon_{ccp} = 0.024 \]
\[ \varepsilon(x) := x \]
\[ \eta(x) := \nu \left[ 1 + 1.3763 \cdot \frac{\varepsilon(x)}{\varepsilon_{ccp}} - 5.36 \cdot \left( \frac{\varepsilon(x)}{\varepsilon_{ccp}} \right)^2 + 8.586 \cdot \left( \frac{\varepsilon(x)}{\varepsilon_{ccp}} \right)^3 \right] \]
\[ \varepsilon_i(x) := \eta(x) \cdot \varepsilon(x) \]
\( \varepsilon_j(x) := \varepsilon_i(x) \quad k := 0.025 \)

\( \varepsilon_i(x) \) substitute \( x = k \) \( \rightarrow 2.055398945943210066510^{-2} \)

\( \varepsilon_{cu} := k \quad \varepsilon_{cu} = 0.025 \)

\( f_i(x) := \begin{cases} 
\rho & \leftarrow 4 \cdot \frac{1}{D} \\
\varepsilon_i(x) & \text{if } \varepsilon_i(x) \leq \varepsilon_y \\
0.5 \cdot \rho \cdot f_i & \text{if } \varepsilon_y \leq \varepsilon_i(x) \leq \varepsilon_{st} \\
0 & \text{otherwise} 
\end{cases} \)
Calculation 5) Concrete model defined by Mander for CFRP jacket

material properties of CFRP (from ATLSS Report)

\[ \varepsilon_{cu} = 98 \]

\[ \varepsilon_{cu} = 5.546 \times 10^{-3} - 0.028 \]

\[ \varepsilon_{cu} = \varepsilon_{cui} \varepsilon_{cci} \varepsilon_{Eseci} Esecu Ec Esecu - Eseci Eseci Ec Ec - Ec Eseci \]

\[ \beta = 203.752 \]

\[ \varepsilon_{cu} = \left( \frac{E_{sec}}{E_{secu}} \right) - 1 \]

\[ \beta = 2 \cdot \varepsilon_{t0} \]

\[ \varepsilon_{cu} = \left( \frac{E_{sec} \cdot \left( E_{c} - E_{secu} \right)}{E_{secu} \cdot \left( E_{c} - E_{sec} \right)} \right)^{1-E_{sec} E_{secu} E_{cu}} \]

\[ \varepsilon_{cu} = \left( 5.546 \times 10^{-3} \right) 0.028 \]

material properties of concrete

\[ \varepsilon_{c} = 5.6 \]

\[ \varepsilon_{c} = 10.982 \]

\[ \varepsilon_{c} = 2.8 \times 10^{-3} \]

\[ \varepsilon_{c} = 9.9638 \]

\[ \varepsilon_{c} = 21 \times 10^{-3} - 0.012 \]

\[ \varepsilon_{c} = 2 \times 10^{-3} \]

\[ \varepsilon_{c} = 5.6 

\[ \varepsilon_{c} = 10.982 \]

\[ \varepsilon_{c} = 2.8 \times 10^{-3} \]

\[ \varepsilon_{c} = 9.9638 \]

\[ \varepsilon_{c} = 21 \times 10^{-3} - 0.012 \]

\[ \varepsilon_{c} = 2 \times 10^{-3} \]

\[ \varepsilon_{c} = 5.6 \]

\[ \varepsilon_{c} = 10.982 \]

\[ \varepsilon_{c} = 2.8 \times 10^{-3} \]

\[ \varepsilon_{c} = 9.9638 \]

\[ \varepsilon_{c} = 21 \times 10^{-3} - 0.012 \]

\[ \varepsilon_{c} = 2 \times 10^{-3} \]
\[ f_{cu} := \begin{cases} \text{for } i \in 0..1 \\ f_{cu} \leftarrow E_{secu} \cdot e_{cu} \\ f_{cu} \end{cases} \]

\[ f_{cu}^T = (2.114 \ 10.551) \]

\[ r := \begin{cases} \text{for } i \in 0..1 \\ r_i \leftarrow \frac{E_{c}}{E_{c} - e_{cc}} \cdot \frac{f_{cu}}{e_{cc}} \\ r \end{cases} \]

\[ r^T = (2.911 \ 1.285) \]

\[ f_{cuu} := f_{c} \left( 0.2 + 3 \frac{f_{c}}{E_{c}} \right) \quad f_{cuu} = \left( \begin{array}{c} 1.12 \\ 8.575 \end{array} \right) \quad f_{cu} = \left( \begin{array}{c} 5.6 \\ 10.982 \end{array} \right) \]

\[ e_{cuu} := e_{c} \left( 2 + 1.25 \frac{E_{c}}{f_{c} \cdot e_{c} \cdot \sqrt{f_{c}}} \right) \quad e_{cuu} = \left( 4 \times 10^{-3} \right) \quad 0.025 \]

\[ feM(x) := \begin{cases} \text{for } i \in 0..1 \\ f_{c1} \leftarrow \frac{f_{cc} \cdot r_i \cdot x}{E_{c} - e_{cc}} \cdot \frac{x}{e_{cc} - e_{cc}} \\ f_{c1} \end{cases} \]

\[ r_i - 1 + \left( \frac{x}{e_{cc}} \right)^n \]

\[ f_{c2} \leftarrow f_{cc} + \left( \frac{f_{cu} - f_{cc}}{e_{cu} - e_{cc}} \right) \left( \frac{x}{e_{cc} - 1} \right) \cdot e_{cc} \]

\[ \text{for } i \in 0..1 \\ f_{c} \leftarrow f_{c1} \]

\[ f_{c} \]
Calculation 6) Concrete model defined by Mander with using GFRP jacket

**Material properties of GFRP**

\[ \varepsilon_{T} = 72 \quad t_1 := 0.034 \quad E_j := 3382 \quad e_j := 0.019 \quad n := (0 \ 1)^T \quad t := n \cdot t_1 \quad t = \begin{pmatrix} 0 \\ 0.034 \end{pmatrix} \]

**Material properties of concrete**

\[ \varepsilon_{c} = 5.6 \quad D := 6 \quad \nu := 0.15 \quad E_c := 57 \cdot \sqrt{\varepsilon_{c}} \cdot 1000 \quad E_c = 4.265 \times 10^3 \quad e_c := 0.002 \quad e_c = 2 \times 10^{-3} \]

\[ f_i := \begin{pmatrix} \rho \leftarrow 4 \cdot \frac{1}{D} \\ \frac{f_i}{f_c} \leftarrow \begin{pmatrix} 0 \\ 0.146 \end{pmatrix} \end{pmatrix} \]

\[ f_{cc} := f_c \cdot \left( 1 - 2.254 \cdot \frac{f_{cc}}{f_c} \cdot \frac{f_i}{f_c} \right) \quad E_{sec} := 5.6 \quad 9.884 \]

\[ e_0 := \alpha \leftarrow 0.9 \quad e_{lim} \leftarrow 0.001 \quad e_{0} \leftarrow -1.284 \times 10^{-3} \]

\[ e_{0} \leftarrow -\nu \cdot e_c - (0.5 - \nu) \cdot \alpha \cdot e_c \cdot \frac{e_{lim} - e_c}{e_{lim} - \alpha \cdot e_c} \]

\[ \beta := E_{secb} \leftarrow \frac{f_c}{e_c} \quad \beta = 203.752 \]

\[ \beta \leftarrow \begin{pmatrix} \left( \frac{E_c}{E_{secb}} \right)^{-1} - 1 \\ 2 \cdot |e_0| \end{pmatrix} \quad E_{sec} := \frac{E_c}{1 + 2 \cdot \beta \cdot e_j} \quad E_{sec} = 487.898 \]

\[ e_{cu} := \begin{pmatrix} \alpha \leftarrow 1 - 0.5 \\ e_{cu} \leftarrow e_{cu} \cdot \frac{E_{sec} \cdot (E_c - E_{sec})}{E_{sec} \cdot (E_c - E_{sec})} \end{pmatrix} \quad e_{cu} \leftarrow 5.047 \times 10^{-3} \quad 0.017 \]

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\[ f_{cu} := \begin{align*}
\text{for } i \in 0..1 \\
f_{cu} & \leftarrow E_{secu} \cdot e_{eq} \\
f_{cu} & \quad \left(2.462 \ 8.336\right)
\end{align*} \]

\[ f_{cu}^{T} (2.911 \ 1.316) \]

\[ r := \begin{align*}
\text{for } i \in 0..1 \\
r_{i} & \leftarrow \frac{E_{c}}{E_{c} - \frac{f_{eq}}{e_{eq}}} \\
r & \quad \left(2.462 \ 8.336\right)
\end{align*} \]

\[ f_{cu} := f_{c} \cdot \left(0.2 + \frac{3}{\sqrt{f_{c}}} \right) \quad f_{cu} = \left(1.12 \ 7.533\right) \quad f_{c} = \left(5.6 \ 9.884\right) \]

\[ e_{cu} := e_{c} \cdot \left(2 + 1.25 \cdot f_{c} \cdot \frac{e_{j}}{\sqrt{f_{c}}} \right) \quad e_{cu} = \left(4 \times 10^{-3} \ 0.018\right) \]

\[ f_{cM}(x) := \begin{align*}
\text{for } i \in 0..1 \\
f_{c1} & \leftarrow \frac{f_{eq} \cdot r_{i} \cdot \frac{x}{e_{eq}}}{r_{i} - 1 + \left(\frac{x}{e_{eq}}\right)^{n}} \\
& \quad \left(n \right) \\
f_{c2} & \leftarrow f_{c1} + \frac{\left(f_{cu} - f_{eq}\right)}{e_{eq} - e_{eq}} \cdot \left(\frac{x}{e_{eq}} - 1\right) \cdot e_{eq} \\
& \quad \left(i \in 0..1\right) \\
f_{c} & \leftarrow f_{c1}
\end{align*} \]
Calculation 7) Variably confined concrete model with using CFRP jacket

**Material properties of CFRP**

\[ f_j := 509 \quad t_1 := 0.0065 \quad E_j := 33850 \quad n := 1 \quad t := n \cdot t_1 \quad t = 6.5 \times 10^{-3} \quad \varepsilon_y := 0.02: \]

**Material properties of concrete**

\[ f_{c0} := 5.6 \quad D := 6 \quad \nu := 0.15 \quad E_{c0} := 57 \cdot \sqrt{f_{c0} \cdot 1000} \quad E_{c0} = 4.265 \times 10^3 \]

\[ \varepsilon_{c0} := 0.002 \quad \varepsilon_{c0} = 2 \times 10^{-3} \]

**Peak values of compressive strength and axial strain**

\[ f_{00} := \left\{ \begin{array}{l}
\rho \leftarrow 4 \cdot \frac{1}{D} \\
 f_{00} := 0.197
\end{array} \right. \]

\[ f_{c0} := f_{00} \cdot \left( -1.254 + 2.254 \cdot \sqrt{1 + 7.94 \cdot \frac{f_{00}}{f_{00}} - 2 \cdot \frac{f_{00}}{f_{00}}} \right) \quad f_{ccp} = 10.982 \]

\[ \varepsilon_{ccp} := \varepsilon_{c0} \cdot \left[ 1 + 5 \cdot \left( \frac{f_{ccp}}{f_{c0}} - 1 \right) \right] \quad \varepsilon_{ccp} = 0.012 \]

\[ \varepsilon(x) := x \]

\[ \eta(x) := \nu \cdot \left[ 1 + 1.3763 \cdot \frac{\varepsilon(x)}{\varepsilon_{ccp}} - 5.36 \cdot \left( \frac{\varepsilon(x)}{\varepsilon_{ccp}} \right)^2 + 8.586 \cdot \left( \frac{\varepsilon(x)}{\varepsilon_{ccp}} \right)^3 \right] \]

\[ \varepsilon_1(x) := \eta(x) \cdot \varepsilon(x) \]

\[ \varepsilon_j(x) := \varepsilon_1(x) \quad k := 0.0139 \quad \varepsilon_j(x) \text{ substitute} \quad x = k \rightarrow 2.021731274894010951710^2 \quad \varepsilon_{eq} := k \]

\[ f_1(x) := \left\{ \begin{array}{l}
\rho \leftarrow 4 \cdot \frac{1}{D} \\
 f_1 := 0.5 \cdot \rho \cdot E_j \cdot \varepsilon_j(x)
\end{array} \right. \]
fc := m ← 300
for j ∈ 0..m
    εj ← j ∙ 0.0001
    ηj ← v ∙ \left[ 1 + 1.3763 \cdot \frac{εj}{ε_{ccp}} - 5.36 \cdot \left( \frac{εj}{ε_{ccp}} \right)^2 + 8.586 \cdot \left( \frac{εj}{ε_{ccp}} \right)^3 \right]
    εtj ← ηj ∙ εj
    f_i_j ← ρ ← 4 \cdot \frac{t}{D}
    f_i_j ← \begin{cases} 
        (0.5 \cdot ρ \cdot E_j \cdot εtj) & \text{if } (εtj ≤ ε_y) \\
        0 & \text{otherwise} 
    \end{cases}
    f_{cc_j} ← f_{cc_0} \cdot \left( -1.254 + 2.254 \cdot \sqrt{1 + 7.94 \cdot \frac{f_i_j}{f_{cc_0}} - 2 \cdot \frac{f_i_j}{f_{cc_0}}} \right)
    ε_{cc_j} ← ε_{cc_0} \cdot \left[ 1 + 5 \cdot \left( \frac{f_{cc_j}}{f_{cc_0}} - 1 \right) \right]
    n_j ← \frac{E_{c0}}{f_{cc_j}} \cdot \frac{f_{cc_j} - f_{cc_0}}{f_{cc_0}}
    k_j ← \begin{cases} 
        1 & \text{if } \left( \frac{εj}{ε_{cc_j}} \right) ≤ 1 \\
        \left[ \left[ \left[ 0.67 + \frac{f_{cc_0} \cdot 6.8948}{62} \right] \cdot \frac{f_{cc_0}}{f_{cc_j}} \right] \right] & \text{otherwise} 
    \end{cases}
    \frac{ε_{cc_j}}{f_{cc_j}} \frac{n_j}{n_j - 1 + \left( \frac{εj}{ε_{cc_j}} \right)^{n_j \cdot k_j}}
    fc = \begin{array}{c|c}
        f_i(x) & 0.5 \\
        0 & 0 \\
        0.005 & 0.01 \\
    \end{array}

U
0 0
1 0.395
2 0.788
3 1.178
4 1.56
5 1.932
6 2.29
7 2.629
8 2.947
9 3.241
10 3.511
11 3.753
12 3.97
13 4.161
14 4.327
15 4.469
VITA

Fatih Cetisli was born on May 19, 1978 in Diyarbakir, Turkey. His father is Prof. Halil Cetisli (Ph. D.), a faculty member. His mother is Ummu Cetisli.

He served as a research assistant in the Osman Gazi University (Eskisehir, Turkey) and Middle East Technical University (Ankara, Turkey) for eight months. He is being supported by Republic of Turkey, Ministry of National Education – General Directorate for Higher Education to be a faculty at Firat University (Elazig, Turkey).

He graduated in 1999 with a B.S. Degree in Civil Engineering from Istanbul Technical University (Istanbul, Turkey).