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R. L. Ketter
L. S. Beedle

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PROGRESS REPORT R

ROTATION CHARACTERISTICS
OF BEAM-COLUMNS

BY
ROBERT L. KETTER & LYNN S. BEEDLE

NOV., 1952
WELDED CONTINUOUS FRAMES & THEIR COMPONENTS

PROGRESS REPORT II

MOMENT-ROTATION CHARACTERISTICS OF BEAM-COLUMNS

by

Robert L. Ketter & Lynn S. Beedle

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Column Research Council (Advisory)
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Fritz Engineering Laboratory
Department of Civil Engineering and Mechanics
Lehigh University
Bethlehem, Pennsylvania

November, 1952

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. NOTES ON THE ANALYSIS</td>
<td>2</td>
</tr>
<tr>
<td>III. MEASUREMENT OF ROTATION</td>
<td>4</td>
</tr>
<tr>
<td>IV. TESTS RESULTS AND DISCUSSION</td>
<td></td>
</tr>
<tr>
<td>a. Moment-Rotation Curves</td>
<td>5</td>
</tr>
<tr>
<td>b. Rotation Capacity</td>
<td>11</td>
</tr>
<tr>
<td>V. CONCLUSIONS</td>
<td>16</td>
</tr>
<tr>
<td>VI. ACKNOWLEDGEMENTS</td>
<td>18</td>
</tr>
<tr>
<td>VII. NOMENCLATURE</td>
<td>18</td>
</tr>
<tr>
<td>VIII. REFERENCES</td>
<td>19</td>
</tr>
</tbody>
</table>

**APPENDIX**

SAMPLE NUMERICAL DERIVATION OF M-9 CURVE  20

**TABLES**

**FIGURES**
I. INTRODUCTION

In advance of a more detailed report being prepared on the subject of the load-deflection and load-rotation behavior of WF steel columns, it is considered that current interest warrants preparation of a short report discussing the moment-rotation problem. Not only is this needed to further describe the basic behavior of columns loaded with combined bending and thrust, but also to evaluate energy absorption of complete frames and to develop tentative proportions for columns intended for use in plastic design.

Performance of structures subjected to loads sufficient to cause plastic action is directly related to the ability of certain members or connections to develop plastic hinges. Moreover, these members must allow rotation through comparatively large angles to ensure the development of plastic hinges elsewhere. The precise requirement for the "rotation capacity" of a particular structural component is dependent on such factors as (a) type of structure (degree of restraint), (b) type and location of loads and (c) length-depth ratio of members. An investigation is needed to establish a criterion of required rotation capacity taking into account the above mentioned factors. One such study, made for a continuous beam subjected to uniform loading, suggests that a member must be able to sustain a rotation of eight times the predicted elastic value to ensure the necessary development of plastic hinges elsewhere in the structure.

This report will be primarily a presentation of experimental results of full scale column tests in which as-delivered members of various lengths have been tested under different combinations of end bending moment and thrust. Four conditions of loading have been studied and are shown diagrammatically in Fig. 1.
Each of these conditions will hereafter be referred to by its letter designation. For example: condition "b" would mean that the member is held fixed (not allowed to rotate) at one end while the other end is subjected to an axial thrust, \( P \), and a bending moment, \( M_o \).

Tests have been carried out on two sizes of WF sections (8WF31 and 4WF13). These members were tested in lengths of 8 ft., 12 ft., and 16 ft. giving a range of \( L/r \) from 21 to 111.

Table 1 summarizes the test conditions for each specimen. It also serves to catalog four "studies" that will be mentioned later. For further reference, Table 2 shows on \( P-M_o \) interaction curves the load condition of each test, test number, section, slenderness ratio and rolling number.

II. NOTES ON THE ANALYSIS

The moment-rotation \((M-\varphi)\) relationship depends on the \( M-\varphi \) curve, since it is the integrated effect of the latter which determines the load-deformation conditions. Therefore, of fundamental importance is the determination of \( M-\varphi \) curves above the elastic limit. Such moment-curvature relationships must include the variable \( P \), and a solution to this problem has been presented in a previous Lehigh report(6). To illustrate the effect of axial thrust, one figure from that report is reproduced as Fig. 2.
Another variable which may have a pronounced influence on the moment-curvature relation is the presence of cooling residual stress. This variable is to be discussed in a future report; however, one figure has been included to indicate the degree of variation due to such "locked-in" stresses, Fig. 3 indicating the theoretical relationship. Current work suggests that the effect of residual stress becomes more pronounced as the ratio of axial thrust to $P_y$ is increased. However, axial load has been relatively low in a majority of the tests carried out thus far in the experimental investigation, a condition corresponding to that present in portal frame structures ($P < 0.15 P_y$). For the 8WF31 members tested, Fig. 3 can then be considered as representative of the influence of this variable. Since the magnitude and distribution of residual stresses in rolled sections is undoubtedly a function of type and size of the section, such
variation may not be representative of all sections. (Fig. 3 is the result of an analytical study based on measured residual stress distribution values.)

Given the particular M-∅ relation, moment-rotation curves can be obtained by numerical integration. Newmark's method is adapted to a solution of problems of this type. An example is shown in Appendix A and in fig. 23 the theoretical curve (including the influence of residual stress) is compared with experimental results.

III. MEASUREMENT OF ROTATION

The experimental set-up, techniques of experimentation, etc., have been described in a previous report. In all tests, end rotation measurements have been made with a level bar, illustrated in Fig. 4.
This apparatus is connected to the base plate of the test specimen as shown in Fig. 4(c). It is possible with different brackets to place level bars along the length of the member. Good reproducibility has been obtained using this instrument.

III. TEST RESULTS AND DISCUSSION

A. Moment-Rotation Curves

The influence of four variables are presented in this report. These are:

1. Loading condition,
2. slenderness ratio,
3. axial thrust, expressed as the ratio $P/P_y$, and
4. size of cross-section.

In any one case, the other three variables are maintained constant. This is summarized in the following table which also shows the figure numbers applicable to each study.
The behavior of each test specimen is shown in Figs. 5-15. In each of these curves the end bending moment, $M_o$, has been plotted against the end rotation, $\theta A$. The ordinate has been made non-dimensional by dividing by $M_y$, the theoretical yield moment for the rolled shape in the absence of axial thrust. For each individual curve, the computed yield and plastic moments are shown, due regard being taken of the load condition and axial thrust. (These values are determined from either reference 4 or 5.)

Excellent agreement between experimental and theoretical results was consistently observed in the elastic range, although the theoretical curves are not shown. (Fig. 23 is an example.)

By way of explaining the figures, the particular variable is indicated in parenthesis after the test number in each case. Thus in Study 1 (influence of loading condition), T-12 (c) means Test 12 and load condition "c".

It is apparent, of course, that in the early tests the deformation was not continued through a sufficient angle change to give

<table>
<thead>
<tr>
<th>STUDY</th>
<th>LOADING CONDITION</th>
<th>SLENDERNESS RATIO</th>
<th>AXIAL THRUST</th>
<th>SIZE OF CROSS-SECTION</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td></td>
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<td>4</td>
<td></td>
<td></td>
<td></td>
<td>Variable</td>
</tr>
</tbody>
</table>
a definite indication of the collapse pattern. In current tests, every effort is made to determine the "unloading" curve.

STUDY 1: Loading Condition

As indicated in Table 2, three series of tests have been completed or are underway which indicate the influence of this variable. In each series the ratio of axial thrust was held constant at $P = 0.12P_Y$.

**Fig. 5:** (6WF31, $L/r = 55$, $P/P_y = 0.12$) Elastically, as expected, the stiffness increased in the order T-12, T-13, T-14 and T-14, the corresponding loading conditions being "c", "d", "b" and "a". Tests T-14 and T-14 were not carried to their ultimate load, although all current and future tests are being continued to complete failure. T-12 carried an appreciably smaller end moment than did tests using any of the other three loading conditions and the failure involved lateral-torsional buckling. In each of the other three cases local buckling was observed.

**Fig. 6:** (6WF31, $L/r = 84$, $P/P_y = 0.12$) Here, as before, stiffness increased in the order "c", "d" and "b" and there was a corresponding improvement in rotation capacity. No condition "a" tests have been carried out for this slenderness ratio. Since these columns were more slender than those of Fig. 5, the tendency toward lateral buckling is more pronounced. This supposition is supported since both T-26 ("c") and T-23 ("d") failed due to lateral buckling. Test T-24 ("b") had not reached the maximum load when the test was stopped; however, at that time there was noticeable lateral twisting of the column as well as a local wrinkling of the compression flange. Both of these conditions could be observed by eye; however, it was impossible to determine which developed first.
Fig. 7: (WF13, L/r = 55, P/Py = 0.12) T-20 ("c") which failed due to lateral buckling, had less rigidity and failed at a lower load than T-17 ("b"), which failed due to local instability.

Summarizing this study:

Figs. 5, 6 and 7 indicate that stiffness of a beam-column is dependent on the condition of loading. For those conditions tested, stiffness increased in the order "c", "d", "b" and "a"; the first being the weakest. Furthermore, the type of failure seems to be markedly influenced by loading condition. In each case the member with the less stiffness, (condition "c" loading), failed due to lateral-torsional buckling, while the stiffer members showed a tendency toward local instability.

STUDY 2: Slenderness Ratio

Data may be drawn from three series of tests to study this variable. Two are condition "c" loadings, while the third is condition "b".

Fig. 8: (WF13, Condition "c", P/Py = 0.12) Failure in each case was due to lateral-torsional buckling. As expected, stiffness increased with decreasing length: (compare T-12, L/r = 55; T-16, L/r = 41; and T-19, L/r = 27). The shortest member (T-19) had greatest strength. In comparing rotation capacity, it would seem that the longer members carried their load through a larger rotation; however, such a comparison is misleading since the slender members allowed greater rotations in the elastic range.

Fig. 9: (WF13, Condition "b", P/Py = 0.12) Relative stiffness increased in the order T-9 (L/r = 111), T-24 (L/r = 84) and T-17 (L/r = 55). T-17 failed due to local buckling; T-24 was still
increasing in load when the test was stopped (rotation was over 8
times the elastic rotation) but there was noticeable lateral-torsional
deformation as well as a marked local wrinkling. T-9 failed due to
a pronounced lateral-torsional movement. Since T-24 was from a dif-
ferent rolling than T-9 and T-17, it is possible that a difference
in magnitude and distribution of residual stresses accounts for the
different behavior. Measurements of residual stresses in each of
these members are planned for the near future.

**Fig. 10:** (HWF13, Condition "c", P/Py = 0.12) The slenderness
ratios in these two tests are 55 and 84. As would be expected, the
shorter, stiffer member had the greater strength. Both members failed
due to lateral-torsional buckling.

**Summarizing** the trends observed in study 2, stiffness in the inelastic
as well as the elastic range is influenced by the slenderness ratio.
As would be expected, the more stocky members were the more rigid.
One further observation that can be made based on this study is, that
members tested under condition "c" loading failed due to lateral-
torsional buckling regardless of slenderness ratio; whereas, the type
of failure for condition "b" loading seemed to depend on the slender-
ness of the member.

**STUDY 3: The Influence of Axial Thrust**

**Fig. 11:** (CWF31, Condition "b", L/r = 55) Since T-4 (P = 0.12Py)
undoubtedly would have failed due to local instability had the test
been carried far enough, and since T-3 (P = 0.49Py) failed due to
lateral-torsional buckling, axial thrust plays an important role in
determining the type of failure. These early tests give little in-
formation on rotation capacity.
Fig. 12: (HWFl3, Condition "b", L/r = 111) T-7 (P = 0.27Py) was expected to carry less load than T-9 (P = 0.10Py). Such was the observed condition. Failure in both cases was due to lateral-torsional buckling.

Fig. 13: (HWFl3, Condition "b", L/r = 55) Both of these members, T-17 (P = 0.12Py) and T-21 (P = 0.50Py), failed due to local instability. The "hump" in these curves seems to be characteristic of those conditions in which failure was of the local instability type (see also Fig. 11, T-4). Similar behavior has been observed in the continuous beam tests and has been discussed in Progress Report 5(3).

Ultimate moment carrying capacity of a beam-column is directly related to the amount of axial thrust applied to the member. The same correlation is not observed when considering the ability of each member to plastically deform thru relatively large end angle changes. Based on these few curves it would seem that increased axial thrust does not have the pronounced detrimental effect on rotation capacity that it has on moment carrying capacity.

STUDY 4: The Influence of Size of Cross-Section

A question frequently asked is, "Will the results of these tests describe the behavior of geometrically similar sections but of different size from those tested?" Therefore, correlation between the two sections tested under identical conditions of loading and slenderness ratio is next examined,

Fig. 14: (Condition "b", L/r = 55, P/Py = 0.12) indicates that the same general trends were observed in each test. The 8WF31 section yielded at a proportionately lower load, and this indicates a relatively greater magnitude of residual stress. Both tests indicated
the "hump" or subsequent increase in moment strength after a certain amount of plastic straining. This was mentioned in discussing Fig. 13. Each of the members failed due to local instability.

Fig. 15: (Condition "c", $l/r = 55$, $P/P_y = 0.12$). For the condition "c" type of loading, failures were of the lateral-torsional buckling type. Here again the 8-inch section yielded first, but there was no subsequent recovery of moment strength and consequently the 8WF31 member carried less ultimate moment than the comparison test.

The above comparisons indicate that the influence of size of member is of little importance in the elastic range. Further, these tests suggest that once yielding occurs, then two members of different size will not behave the same unless the loading condition is favorable or unless the cooling residual stresses are of nearly the same magnitude.

B. ROTATION CAPACITY

Mention was made in the Introduction of the importance of rotation capacity, the ability of a member to deform thru a relatively large angle change while sustaining a moment approximately equal to the plastic hinge moment. Study of figures 5-15 reveals that few of the columns exhibit a behavior that approaches the idealized "plastic hinge" case. (For the idealized case rotation would continue indefinitely at $M_{pc}$).

The test curves previously examined fall into four general types that are indicated diagramatically in Fig. 16 and in Table 3. Curve A might be considered as the basic type. Curve B applies to a test (T-14 is an example) in which the loading was stopped before the
maximum "collapse" moment was reached. In curve C the member continued to carry increased moment through a large angle change. Again the test was stopped prior to complete "unloading". A member resulting in curve D actually has no rotation capacity since the maximum moment does not approach $M_{pc}$.

Rotation capacity, $R$, will be defined as the ratio of observed end angle change, $\theta_\Delta$, to the theoretical end rotation at initial yield, $\theta_{yc}$, or

$$R = \frac{\theta_\Delta}{\theta_{yc}}$$

Fig. 16 indicates that non-dimensionalizing the coordinates provides an abscissa that is a direct measure of rotation capacity.

Because of the nature of the experimental curves, rotation capacities will be compared according to several "cases" or criteria. The trends will be examined as a function of slenderness ratio, ratio of axial load, and loading condition.
The nomenclature used in this particular section is as follows:

- $M_o$ = Moment applied at the end of a beam-column
- $M_{yc}$ = Predicted elastic limit moment including the effect of axial thrust.
- $\theta_A$ = Observed end rotation corresponding to $M_o$.
- $\theta_{yc}$ = Predicted end rotation corresponding to $M_{yc}$.
- $\Delta M_1$ = Decrease in moment below that predicted at the elastic limit rotation.
- $\Delta M_2$ = Increase of maximum moment over predicted elastic limit rotation.
- $\Delta R_1$ = Increase in rotation over predicted elastic limit rotation at the elastic limit moment.
- $R_2$ = Rotation capacity at the maximum observed moment.
- $R_3$ = Rotation capacity at the elastic limit moment on the unloading curve.

**Case 1:** Increment of rotation capacity at predicted yield moment ($\Delta R_1$) and reduction of moment at the elastic limit rotation ($\Delta M_1$).

Such a comparison is primarily an examination of the "initial yield" characteristics of the member. The results are shown in Fig's. 17 and 18. The variable here is slenderness ratio and test results, in general, indicate that the shorter the member the greater the departure from predicted behavior. The maximum reduction in moment is about 10% whereas the increase in rotation is more than 50%. It is considered that the discrepancies should be the greatest for condition "c" loading and there is some agreement here since the condition "b" tests show less reduction.
Case 2: Rotation capacity at the maximum moment 
($R_2$) and increase in moment above the elastic limit 
value ($\Delta M_2$).

This case, which describes the ultimate strength of the various 
beam-columns, is shown in figures 19 and 20. In Fig. 19 $\Delta M_2$ is 
plotted against slenderness ratio. Fig. 20 is a plot of $R_2$ versus 
slenderness ratio. It is interesting to note that those tests ex-
hibiting greatest strength and rotation capacity (as defined by $R_2$) 
failed by local instability; whereas, lateral-torsional buckling re-
sulted in weaker, less stiff members. This is to be expected since 
lateral buckling involves general yielding over a comparatively large 
area whereas local instability is usually confined to a smaller 
region that is often conducive to strain-hardening. As further work 
is carried out, it may become possible to define the condition above 
which local buckling will occur.

In no case does the rotation capacity approach the value of 
eight which was mentioned in the Introduction (page 1). In fact, a 
rotation capacity of about two seems to be about all that most of 
these columns can achieve at their maximum moment. The influence of 
loading condition is pronounced and Fig. 20 suggests that condition 
"b" and "d" tests would provide more satisfactory performance in 
columns of $L/r$ less than 50.

Fig. 21 was prepared to show the influence of axial thrust on 
rotation capacity, and the latter function is plotted against the 
ratio $P/P_y$. Comparison was only possible in the case of loading 
condition "b" tests. Lateral-torsional buckling was observed in 
the case of T9, T7, and T10 and $R_2$ decreases gradually as the axial
load increases. However, the trend indicated by tests T-17 and T-21 suggests that when the mode of failure changes to local buckling, then the rotation capacity is actually improved by an increase of axial thrust. This trend requires confirmation.

Case 3: Rotation capacity, \( R_3 \), at the elastic limit (unloading curve).

The graph of Fig. 22 shows trends similar to those observed in Fig. 20. The rotation capacity measured at the elastic limit (on the unloading curve) is plotted against slenderness ratio. An increase in slenderness ratio is accompanied by a decrease in the rotation capacity; however, the influence of this variable appears to be less pronounced than that of loading condition. The additional tests planned for loading conditions "d", "b" and "a" should provide information covering a more complete range of slenderness ratio.

Two things are yet needed to establish the suitability of plastic design techniques:

(a) how much rotation capacity is required in order that all plastic hinges be developed in building frames, and

(b) is the trend suggested by loading condition "c" tests borne out for the other conditions of loading.

Four of the seven tests whose results are plotted in Fig. 22 have a rotation capacity of about 2, the slenderness ratio ranging from 40 to 84. Admittedly condition "c" is the worst case, but it is heartening that the few results available for the other loading conditions show improved behavior in this respect. If the required rotation capacity for typical structures approaches the value of
eight, mentioned earlier, then these tests indicate that rotation capacity may be more serious as a limitation to plastic analysis and design than previously anticipated. A rotation capacity of 8.0 is known to be a severe requirement since it was developed for the case of fixed-ended beams. In actual frames, the columns themselves supply less than complete restraint, and in the plastic range they will also contribute to inelastic rotation. Thus a required rotation capacity of 4.0 might not be unrealistic for "complete restraint" and this value would be further reduced since every column allows some flexibility.

V. CONCLUSIONS

Based on the few tests thus far carried out, the following observations are made:

1. The stiffness of a beam-column is dependent on the condition of loading. In increasing order of stiffness the loading conditions investigated are "c", "d", "b" and "a". (Fig.'s 5-7)

2. Failures have been of two types: lateral-torsional buckling and local wrinkling of the flange elements. For all members tested under condition "c" loading, failure was of the lateral-torsional type regardless of slenderness; however, for the other loading conditions there is a tendency for the stiff, short members to buckle locally. (Fig.'s 8-10 and Table 3)

3. Axial thrust decreases the ultimate moment carrying capacity of a beam-column but does not necessarily decrease its rotation capacity. Further tests are needed to establish
definite trends. (Fig's. 11-13)

4. The influence of size of member is of little importance in the elastic range; however, when yielding occurs, two members of different size will not behave the same unless the loading condition is favorable or unless the cooling residual stresses are of nearly the same magnitude. (Fig's. 14-15)

5. There seems to be a limit of ultimate carrying capacity above which local buckling is the type of failure and below which lateral-torsional instability takes place. Further analytical work is needed to establish this transition zone. (Fig. 19 and Table 3)

6. Increased slenderness ratio in general decreases both the moment value and rotation capacity. However, in many cases the variable of loading conditions has a more pronounced effect. (Fig's. 17-22)

7. Only a few of the members had a rotation capacity greater than 4.0, and a common value was about 2.0. If the required rotation capacity for typical structures approaches the value 8.0 (the "ideal" requirement for a fixed-ended beam, loaded at the third-points) then rotation capacity may be a more important factor than previously anticipated. (Fig. 22 and Table 3).
VI. ACKNOWLEDGEMENTS

The work is being carried out under the direction of the Lehigh Project Subcommittee of the Structural Steel Committee, Welding Research Council, at Fritz Engineering Laboratory of which Professor William J. Eney is Director.

VII. NOMENCLATURE

E Young's modulus of elasticity
L Total length of a beam-column
M Moment

$M_0$ Moment applied at the end of a beam-column

$M_y$ Moment at which yield point is reached in flexure

$M_{yc}$ Predicted elastic limit moment including the effect of axial thrust

$M_{pc}$ Plastic hinge moment, modified to include the effect of axial compression

$\Delta M_1$ Decrease in moment below that predicted at the elastic limit rotation

$\Delta M_2$ Increase of maximum moment over predicted elastic limit rotation

P Concentric axial load

$P_y$ Axial load corresponding to yield point stress across entire section

r Radius of gyration

$\Delta R_1$ Increase in rotation over predicted elastic limit rotation at the elastic limit moment

$R_2$ Rotation capacity at the maximum observed moment

$R_3$ Rotation capacity at the elastic limit moment on the unloading curve

$\theta_A$ Observed end rotation corresponding $M_0$

$\theta_{yc}$ Predicted end rotation corresponding to $M_{yc}$

$\sigma_y$ Lower yield point stress

$\phi$ Curvature at a section (the reciprocal of the radius of curvature)
VIII. REFERENCES

Reports in the Lehigh Series:


Additional References:

APPENDIX A: Numerical Determination of Moment-Rotation Curves

OUTLINE of the general method of solution (using the numerical process of Newmark(7)) is as follows:

1. Assume a deflection curve.
2. Determine the moment acting at equally spaced points along the member.
3. From the $M-\phi$ diagram (for example Fig. 3, page 4) read value of $\phi$ corresponding to the moment values at each of these points along the column.
4. Numerically integrate these $\phi$ values, and obtain a deflection curve.
5. If this new deflection curve does not check that originally assumed go thru steps 2, 3 and 4 against using the new deflection curve.
6. When the deflection curve is sufficiently accurate—correct slope values to the end of the column (using conventional finite difference equations).

An example using this procedure follows.

Determine the value of end rotation for the 8WF31, 12 ft. column loaded as shown in the following sketch.

\[ P = 0.12P_y = 44.9k \]
\[ M_o = 1050''k \]
<table>
<thead>
<tr>
<th>Multipl</th>
<th>( \phi )</th>
<th>( \frac{M}{y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1050</td>
<td>1050 1050 1050 1050 1050</td>
<td>Moment due to ( M_o )</td>
</tr>
<tr>
<td>0</td>
<td>0.40  0.65  0.80  0.85</td>
<td>Assumed Deflection</td>
</tr>
<tr>
<td>0</td>
<td>18    29   26   38</td>
<td>Moment due to ( P )</td>
</tr>
<tr>
<td>1050</td>
<td>1068  1079 1086 1088</td>
<td>Total Moment</td>
</tr>
<tr>
<td>0.951</td>
<td>0.968 0.976 0.984 0.985</td>
<td>( \frac{M}{y} )</td>
</tr>
</tbody>
</table>

| Curvatur| \( \frac{\sigma_y}{E} \) |
|---------|----------------|-----------------|
| 0.253   | 0.283 0.298 0.312 0.317 | Concentrated Angle Changes \( \frac{\lambda \sigma_y}{E} \) |
| 1.051   | 0.768 0.470 0.158 | Slope \( \frac{\lambda \sigma_y}{E} \) |
| 0       | 1.051 1.819 2.289 2.447 | Deflection \( \frac{\lambda^2 \sigma_y}{E} \) |

\[ Y_1 \rightarrow 0 \quad 0.455 \quad 0.788 \quad 0.991 \quad 1.060 \]
\[ Y_2 \rightarrow 0 \quad 0.478 \quad 0.856 \quad 1.050 \quad 1.122 \]
\[ Y_3 \rightarrow 0 \quad 0.485 \quad 0.843 \quad 1.063 \quad 1.140 \]
\[ Y_4 \rightarrow 0 \quad 0.485 \quad 0.845 \quad 1.066 \quad 1.141 \]

**Solution corresponding to \( Y_4 \) deflection.**

| Concentrated Angle Changes | \( \frac{\sigma_y}{E} \) |
|-----------------|----------------|-----------------|
| 0.253 | 0.290 0.317 0.340 0.344 | Concentrated Angle Changes \( \frac{\sigma_y}{E} \) |
| 1.119 | 0.829 0.512 0.172 | Slope \( \frac{\lambda \sigma_y}{E} \) |
| 0       | 1.119 1.948 2.460 2.632 | Deflection \( \frac{\lambda^2 \sigma_y}{E} \) |
| 0       | 0.485 0.845 1.066 1.141 | \( \frac{\lambda^2 \sigma_y}{E} \) |

\[ \theta_A = \theta_1 + \theta_{corr}. \]
\[ \theta_{corr} = \frac{\lambda}{24} (7 \phi_L + 6 \phi_0 - \phi_R) \]
\[ = \frac{\lambda}{24} (1.771 + 1.740 - 0.317) \]
\[ = 0.133 \frac{\lambda \sigma_y}{E} \]

Therefore,
\[ \theta_A = (1.119 + 0.133) \frac{\lambda \sigma_y}{E} = 0.0302 \text{ inches/inch} \]
<table>
<thead>
<tr>
<th>TEST NUMBER</th>
<th>LOADING CONDITION</th>
<th>SECTION</th>
<th>P / P_y</th>
<th>L / r</th>
<th>STUDY 1 Loading Condition</th>
<th>STUDY 2 Slenderness Ratio</th>
<th>STUDY 3 Axial Thrust</th>
<th>STUDY 4 Size of Member</th>
</tr>
</thead>
<tbody>
<tr>
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| TABLE 1 |
TABLE 2
## Tabulation of Experimental Results

### Types of Curves

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<tr>
<th>Test Number</th>
<th>Loading Condition</th>
<th>Section</th>
<th>( P / P'_{y} )</th>
<th>( L / r )</th>
<th>Type of Curve</th>
<th>Type of Failure</th>
<th>( \Delta M_1 )</th>
<th>( \Delta R_1 )</th>
<th>( \Delta M_2 )</th>
<th>( R_2 )</th>
<th>( R_3 )</th>
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<td>T-3</td>
<td>b</td>
<td>8WF31</td>
<td>0.49</td>
<td>55</td>
<td>B</td>
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<td>Bending + Twist</td>
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<td>Bending + Twist</td>
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<td>A</td>
<td>Local Buckling</td>
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<td>0.01</td>
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* Last recorded value — (Test stopped prior to unloading)
Fig. 5

T-4 (b) (Local Instability)

T-13 (d) (Local Instability)

T-14 (n)

T-12 (c) (Bending + Twist)

M_{b} / M_{y}

0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10

\theta_{x} (inches/inch)

8 WF 31

L / R = 55

P = 0.12P_{y}

Fig. 6

T-24 (b) (Bending, Twist and Local Buckling)

T-25 (d) (Bending + Twist)

M_{c} / M_{y}

0.05 0.10 0.15 0.20

\theta_{x} (inches/inch)

4 WF 13

L / R = 84

P = 0.12P_{y}
Fig. 7

\[ \frac{M_A}{M_Y} \]

\[ \frac{M_A}{M_Y} \]

\( T-17 \) (b) \((\text{Local Instability})\)

\( T-20 \) (c) \((\text{Bending + Twist})\)

4 WF \( B \)

\[ \frac{1}{P} = 55 \]

\[ P = 0.12P_y \]

\( \theta_A \) (inches/inch)
Fig. 8

Fig. 9
Fig. 10
Fig. 11

Fig. 12
Fig. 13

T-17 \( (P = 0.12P_y) \) \( \text{(Local Instability)} \)

T-21 \( (P = 0.50P_y) \) \( \text{(Local Instability)} \)

\( \frac{M_b}{M_y} \)

4 WF 13
\( L/r = 55 \)
Fig. 14

Fig. 15
Fig. 19
Fig. 20
Fig. 21
Fig. 22
Solution Neglecting Residual Stress

Including Residual Stress

Lateral-Torsional
Displacement first indicated by lateral deflection gages.

Twisting of specimen first observed by eye.

Fig. 23