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Experimental verification and modifications to the simple plastic theory, 1956

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VERIFICATION OF PLASTIC THEORY - AND

ADDITIONAL CONSIDERATIONS

Lynn S. Beedle

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VERIFICATION OF PLASTIC THEORY - AND

ADDITIONAL CONSIDERATIONS

Lynn S. Beadle

Introduction

In the previous talks you have seen how simple methods of structural analysis may be built upon plastic theory.

How well does structural behavior bear out the theory? Do structures really contain the ductility assumed? Do these plastic hinges form and allow the necessary redistribution of moment? If we test a "full size" structure with rolled members will it actually carry the load predicted by plastic analysis?

It is the purpose of the first section of this paper to answer the above questions, giving the experimental verification of plastic theory.

In arriving at the plastic method of structural analysis, some further assumptions were made concerning the influence of axial and shear forces, buckling, etc. In the second part of these remarks such "additional considerations" will be discussed, the appropriate results of theoretical analysis and tests will be described, and design guides will be indicated.

A. EXPERIMENTAL VERIFICATION

Recapitulation of Assumptions

What are the assumptions we make with regard to the plastic behavior of structures? Six important assumptions are indicated diagrammatically in Fig. 1 and are as follows:

1. The material is ductile. It has the capacity of absorbing plastic deformation without the danger of fracture.

2. Each beam has a maximum moment of resistance (the plastic moment, \( M_p \)), a moment that is attained through plastic yield of the entire cross section (plastification).

3. Due to the ductility of steel, rotation at relatively constant moment will occur through a considerable angle -- resulting in the formation of a plastic hinge.

4. Connections provide full continuity.

5. As a result of the formation of plastic hinges at connections and other points of maximum moment, redistribution of moment will occur, allowing the formation of plastic hinges at points that are otherwise less highly stressed in the elastic region.

6. The ultimate load may be computed with accuracy on the basis that a sufficient number of plastic hinges have formed to create a mechanism.

How well do tests confirm these predictions?
The Ductility of Steel

Figure 2 represents an "average" stress-strain curve for structural steel. It has been obtained from the considerable number of compression and tension tests conducted at Fritz Laboratory for the purpose of evaluating the structural tests performed. Compression tests are of interest in connection with the plastic behavior of structures because, of course, one half of the cross section is yielded in compression. The steel compresses plastically about 15 times the strain at the elastic limit and then commences to strain harden. Although the data is plotted well into the strain-hardening range, the strains are still considerably less than those at ultimate strength ("tensile" strength). The compressive and the tensile stress-strain relationships are quite similar. In fact the properties in compression are practically identical with those in tension.

We can conclude from Figure 2 that structural steel coupons do have the necessary ductility as assumed in plastic analysis.

The Plastic Moment and the Plastic Hinge

As a demonstration that the maximum moment is attained through plastification of the cross section, Fig. 3 shows a typical M-Φ curve obtained from a beam in pure bending(3). The dotted line is the theoretical curve and the solid line through the circles represents a typical test result. The theoretical stress distributions (according to the simple plastic theory) at different stages of bending are shown in the upper portion. Below these are shown the corresponding stress distributions as determined from SR-4 gage measurements. It will be seen that plastification of the cross section does occur, and that the bending moment corresponding to this condition is the full plastic moment as computed from the equation \( M_p = \sigma_y Z \).

The many tests conducted on rolled shapes indicate that the curve shown in Fig. 3 was not an exceptional case but that wide-flange beams will develop the strength predicted by the plastic theory and that a plastic hinge (characterized by rotation at near-constant moment) does actually form.

Up to this point in considering the plastic moment and the attendant plastic hinge, a somewhat unrealistic loading condition has been taken. "Pure moment" is a condition not likely to be experienced in actual structures, but it turns out that it represents a "worst case" insofar as the plastic behavior of a beam is concerned. Usually there will be a gradient in moment, such as would be obtained if a simply-supported beam is loaded with a single concentrated load. In such a case the deformation tends to be concentrated under the load point (the point of maximum moment). Because the plastic deformation is more localized, the strain-hardening region is reached at a lesser deflection; consequently, the beam tends to develop a moment greater than the plastic moment. Typical of the behavior of a beam under moment gradient is that shown in Fig. 4(4). The beam continues to increase in load capacity as the deformation is continued.
We can see, therefore, that strain-hardening is something that improves the moment-carrying capacity of a beam. Although we neglect it in the simple plastic theory, this additional reserve strength is still present in most ordinary structures.

The Strength of Connections

Not only must we consider the adequacy of beams to develop plastic hinges but, in addition, connections must be proportioned in such a way that they will transmit the plastic moment and have adequate rotation capacity. Can connections be fabricated economically and still meet these requirements?

Figure 5 shows the efficiency with which a properly proportioned knee will perform. The inset shows the corner at an angle of 45 degrees in the position assumed during the test, a loading that is rather typical of that to which a "corner" connection is subjected. The load corresponding to the plastic moment is realized and there is adequate rotation in the vicinity of this load to allow the necessary redistribution of moment to other portions of a structure of which the connection might be a part.

What about those cases in which a beam is welded to a column -- a situation that would occur in a multi-story structure? Three different designs for such a connection are sketched in Fig. 6. For example, as shown on the left, the columns might be fabricated by simply attaching the beams to the columns. Alternatively, in the center, horizontal stiffeners could be added; and at the right a vertical type stiffener is another possibility. With proper proportions, the test curves show that plastic moments can be developed and hinges formed. The dotted line indicates the theoretical load-deformation characteristic of the rolled beam, and it is seen that for these particular specimens the moment deformation characteristics are quite suitable. Figure 7 shows one of these designs at the end of the test.

We can therefore conclude from the above evidence that connections can be fabricated with economy to provide the necessary continuity.

Redistribution of Moment

We have seen that plastic hinges may be depended upon to form at connections, at concentrated load points, and at points of maximum moment in beams. This development of the plastic moment is one of the sources of reserve strength in structural steel beyond the elastic limit. A second factor is the redistribution of moment. Does it actually occur and how closely does it correspond to the theoretical concept?

In Fig. 8 is shown a picture of this process -- both theoretically and experimentally. Some time ago a test was made on a continuous beam to simulate the condition of third point loading on a fixed-ended beam(2); thus experimental data was available to compare with the
Theoretical predictions. The fixed-ended beam and its various components are shown in two stages:

Stage 1 - near the computed elastic limit.

Stage 2 - after the plastic hinge has theoretically formed at the ends and the load is increasing towards the ultimate value.

The figure shows the loading, deflected shape at the two phases, the moment diagram, the load-deflection curve, and the moment curvature relationship near the ends (left) and at the center (right).

In the elastic range it will be seen that the beam behaves just as assumed by the theory. The moment at the center being 1/2 the moment at the fixed ends. (Figs. 8c and 8a) As the moment at the ends approaches the yield moment, the curvature, $\phi$, commences to increase more rapidly and we see the beginnings of a plastic hinge (Fig. 8e). Because of this "hinge action", the additional moments due to increase in load are distributed in a different ratio, most of the increase going to the center and a small amount going to the ends as plasticification occurs (Fig. 8e). This is the process known as redistribution of moment. So the beam actually behaves somewhat more flexibly than before (Fig. 8d) and at Stage 2 the plastic moment capacity near the center is practically exhausted.

Now, in Fig. 9 the beam is shown in two later stages:

Stage 3 - when the theoretical ultimate load is first reached,

and,

Stage 4 - after deformation has been continued through an arbitrary displacement.

It is quite evident from Fig. 9e that all of the moment capacity has been substantially absorbed by the time Stage 3 is reached (ultimate load). Beyond this the beam simply deforms as a mechanism with the moment diagram remaining largely unchanged, the plastic hinges at the ends and center developing further.

We therefore have clear evidence that redistribution of moment occurs through the formation of plastic hinges, allowing the structure to reach (and usually exceed) its theoretical ultimate load.

Ultimate Strength of Continuous Structures

Do continuous structures attain the load predicted on the basis that sufficient plastic hinges are formed to create a mechanism? The considerable number of tests on continuous beams and frames that have been performed both in this country and abroad provide an affirmative answer.
Figures 10 and 11 show a gabled frame at various stages of loading. Of 40-ft. span, this welded structure was fabricated from 12WF36 shapes.

The next two figures (Figs. 12 and 13) show in tabular form a number of these tests in which the members were fabricated from rolled sections. The structure and loading are shown to scale at the left. Next, the size of member (or members) is indicated. To the right is a bar graph on which is plotted the percent of predicted ultimate strength exhibited by the test structure. (A test that reached 100%, reached the load predicted by the simple plastic theory.)

These two figures show that the actual strength of even the weakest structure was within 5% of its predicted ultimate load -- an agreement much better than can be obtained at the so-called "elastic limit". Particularly remarkable among the continuous beam tests of Fig. 12 is the one conducted by Maier Leibnitz (6) (see the next to the last structure). In this experiment, prior to applying the vertical load, he raised the center support until the yield point was just reached, with the result that application of the first increment of external load caused the structure to yield. In spite of this, the computed ultimate load was attained! In Fig. 13 testing of the fourth frame was interrupted in order that the fifth test might be carried out on the same structure but with a different proportion of horizontal to vertical load. The arrow on the fourth bar indicates the load that would have been reached had the test not been stopped. The fifth test could not be expected to reach its computed ultimate load because of the damage inflicted during the previous overloading.
B. ADDITIONAL CONSIDERATIONS

In all of the tests just reviewed, the results confirm in satisfactory manner the predictions of the "simple plastic theory". This theory neglects such things as axial force, shear, and buckling, and yet the engineer knows they are present in most structures and he is accustomed to taking them into account.

Although it turned out in the tests just described that such "limitations" did not prevent the structure from reaching the desired load, methods must be available for accounting for these additional factors in order to have a design method that is generally applicable. One of the features of the work at Lehigh University has been a thorough study -- both theoretical and experimental -- of factors that in some instances might tend to reduce the ability of a structure to carry the ultimate load predicted by the "simple" plastic theory. Those factors that are neglected or are not included in the "simple" theory and for which revision of that theory is sometimes needed are the following:

- Modifications to the plastic moment (Axial force and shear force)
- Instability (Local buckling, lateral buckling, column buckling)
- Brittle fracture
- Deflection Stability
- Deflections

In the following paragraphs the effect and characteristics of these factors will be indicated. Where appropriate, the results of theoretical analysis and of tests will be indicated, followed by a suggested "rule" to serve as a guide for checking the suitability of the original design.

It should be kept in mind that this situation is no different in principle from that encountered in elastic design. The design must always be checked for direct stress, shear, and so on. It simply means that modification or limitations in the form of "Rules of Design" are necessary as a guide to the suitability of a design based on the simple theory that neglects these factors.

Modifications to the Plastic Moment

The presence of axial force and shear force are two factors that tend to reduce the magnitude of the plastic moment. However, the design procedure may be modified easily to account for their influence because the important "plastic hinge" characteristic is still retained in the presence of these forces even though the moment capacity is reduced.

1. Influence of Axial Force on the Plastic Moment

Figure 14 shows the behavior of a short length of beam when axial load is added to the bending moment. If the axial load is high (as it was in this case), the member will not be able to carry the full value of the plastic moment. Notice, however, that the section still has the "hinge" characteristic; it simply occurs at a reduced moment value. Even more reassuring is the fact that one can predict theoretically the "modified plastic moment", \( M_{pc} \), and the approach to this solution will now be described.
Figure 15 shows the stress-distribution in a beam at various stages of deformation caused by thrust and moment. Due to the axial force, yielding on the compression side precedes that on the tension side. Eventually plastification occurs, but since part of the area must withstand the axial force, the stress block no longer divides the cross section into equal areas (as was the case of pure moment). Thus, as shown in Fig. 16, the total stress distribution may be divided into two parts: a stress due to axial load and a stress due to bending moment.

For the situation shown in Fig. 16 in which the neutral axis is in the web, the axial force \( P \) is given by

\[
P = 2 \sigma_y y_0 w
\]

where \( \sigma_y \) is the yield stress, \( y_0 \) is the distance from the midheight to the neutral axis, and \( w \) is the web thickness. The bending moment \( M_{pc} \) is given by the following expression and represents the plastic hinge moment modified to include the effect of axial compression,

\[
M_{pc} = \sigma_y (Z - wy_0^2)
\]

where \( Z \) is the plastic modulus. By substituting the value of \( y_0 \) obtained from eq. (1) into eq. (2), the bending moment may be expressed as a function of the axial force \( P \), or

\[
M_{pc} = M_p - \frac{P^2}{4 \sigma_y w}
\]

By the same process, an expression for \( M_{pc} \) as a function of \( P \) could be determined when the neutral axis is in the flange instead of the web.

Figure 17 shows for an 8WF31 shape the "interaction" curve that results from this analysis. When the axial force is zero, \( M = M_p \). When the axial force reaches the value \( P = \sigma_y A \), then the moment capacity is zero. Between these limits the relationship is computed as described and the desired influence of axial force on the plastic moment has thus been obtained. If, now, other WF shapes of different proportions were examined by this analysis, the relationship shown in Fig. 18 would be obtained. Here the curve has been non-dimensionalized and

\[
\Delta F = 2bt = \text{area of both flanges}
\]

\[
\Delta w = w (d - 2t) = \text{area of web}
\]

The solution for the rectangle is also shown in Fig. 18.

In design, in order to account for the influence of direct stress either the curves of Fig. 18 could be used or, since most WF shapes fall within a narrow band, the simple approximation of Fig. 19 could be used. With an error of less than 5%, axial load can be neglected up to \( P/P_y = 0.15 \). For \( P > 0.15 P_y \), the reduction in moment capacity is given by

\[
\frac{M_{pc}}{M_p} = 1.18 \left( 1 - \frac{P}{P_y} \right) \quad (P > 0.15 P_y)
\]
Summarizing, the following "design rule" may be stated:

\[ M_p = 1.18 (1 - \frac{P}{F_y}) M_p \]

The required design value of \( S \) for a member is determined by multiplying the value of \( S \) found in the initial design by the ratio \( M_p/M_{pc} \): or

\[ S_{req} = \frac{0.85 S}{1-P/F_y} \]

2. The Influence of Shear Force

Figure 20 shows a simply-supported beam with loads so close to the supports that shear stresses were exceedingly high -- so much so that the inelastic deflections due to shear force became substantial. Figure 21 shows the resulting behavior -- the deflections commence increasing at a rapid rate even though at a reduced load. Nevertheless, strain-hardening eventually allowed the beam to approach the predicted ultimate load, but only after excessive beam deflections.

In order to arrive at a basis for assuming that the full plastic moment will be reached, a cantilever beam in bending will be considered as shown in Fig. 22. Of interest are the stress-distributions at various sections (A, B, and C) along the beam because they form a means of evaluating performance under different shear-to-moment ratios. At Section A the beam is elastic and the flexure and shear stresses may be determined easily. At Section B the flange is completely yielded due to flexure and therefore all of the shear stresses must be carried in the elastic core; thus the distribution of shear stresses is parabolic. At Section C the shear stress has reached its yield value at the centerline.

Two possibilities of premature "failure" due to the presence of shear therefore exist:

(a) General shear yield of the web may occur in the presence of high shear-to-moment ratios

(b) After the beam has become partially plastic at a critical section due to flexural yielding, the intensity of shear stress at the centerline may reach the yield condition.
Consider, first, case (a), typical failure being as shown in Fig. 20. The maximum possible shear as given by

\[ V = \tau_y A_w \]  

(6)

If

\[ \tau_y = \frac{\sigma_y}{\sqrt{3}} \text{ and } A_w = w(d-2t), \text{ then} \]

\[ V = \frac{\sigma_y}{\sqrt{3}} w(d-2t) \]

Since for WF shapes \( \frac{d}{d-2t} \approx 1.05 \), then with \( \sigma_y = 33,000 \text{ psi} \),

\[ V_{\text{max}} = 18,000 wd \]  

(7)

where \( w = \) web thickness and \( d \) is the section depth.

The solution for case (b) is shown in Fig. 23, and it is obtained, approximately, by considering the shear and flexural stresses of distributor C, Fig. 22. The maximum shear stress for a parabolic distribution is \( 3/2 \) times the average value or

\[ \tau_{\text{max}} = \frac{3}{2} \frac{V}{w(2y_0)} = \frac{3}{4} \frac{V}{wy_0} \]  

(8)

where \( w \) is the web thickness and \( y_0 \) is the distance from midheight to the elastic-plastic boundary. Since the maximum value of the shear is to be taken as the yield value \( (\tau_{\text{max}} = \sigma_y/\sqrt{3}) \) and since the shear force, \( V \), may be expressed as \( V = M_{ps}/a \), then

\[ \frac{\sigma_y}{\sqrt{3}} = \frac{3}{4} \frac{M_{ps}}{awy_0} \]  

(9)

where \( M_{ps} \) signifies the magnitude of the bending moment at which the "limiting" condition of yielding due to shear force at the centerline is obtained. Now \( M_{ps} \) may also be computed directly from the flexural stress-distribution at Section C of Fig. 22 by the principles described earlier. Then

\[ M_{ps} = \sigma_y Z - \sigma_y \cdot \frac{w y_0^2}{3} = M_p - \sigma_y \frac{w y_0^2}{3} \]  

(10)

If \( y_0 \) is eliminated between eqs. (9) and (10) and if both sides of the resulting expression are divided by \( M_p = \sigma_y Z \), the following relationship is obtained which gives the influence of shear on the plastic moment,

\[ \frac{9Z}{16a} \frac{M_{ps}^2}{M_p^2} + \frac{M_{ps}}{M_p} - 1 = 0 \]  

(11)

For a given WF shape eq. (11) may be solved in terms of \( a/d_w \) (\( d_w = d-2t \)) and curves of the type shown in Fig. 23 result. The sketches in the inset show how such curves may be applied to other cases.
Since the ratio \( A_F/A_w \) is about 2.0 for beams and 3.0 for columns, the following formulas may be used as approximations to the particular curves of Fig. 22:

\[
\begin{align*}
\text{Beams:} & \quad \frac{M_{ps}}{M_p} = 0.65 + 0.117 \frac{a}{d} \quad (\frac{a}{d} < 3.0) \\
\text{Columns:} & \quad \frac{M_{ps}}{M_p} = 0.60 + 0.1 \frac{a}{d} \quad (\frac{a}{d} < 4.0)
\end{align*}
\]

For larger \( a/d \) ratios the reduction for this case may be neglected.

As a matter of fact, since high shear and moment values occur in regions of localized yielding (steep moment gradient) the beneficial effects of strain-hardening usually enable such a beam to reach the full plastic moment. Thus further research may show that eq. (11) is too conservative and that only eq. (7) need be considered in design.

Summarizing, the following "design rule" is suggested:

![Shear Force Table]

Instability

Instability may occur in structural members due to three causes: local buckling, lateral buckling, and general collapse of columns. These three modes of failure will now be discussed.

1. Local Buckling

As a wide-flange beam is strained beyond the elastic limit eventually the flange or the web will buckle. Figure 24 is typical of this action as is also Fig. 7. Although stocky sections could be expected to retain their cross-sectional form through considerable plastic strain, with thin sections local buckling might occur soon after the plastic moment was first reached. Due to failure of a beam to retain its cross-sectional shape, the moment capacity would drop off; thus the rotation capacity would be inadequate. Therefore, in order to meet the requirements of deformation capacity (adequate rotation at \( M_p \)-value) compression elements must have width-thickness ratios adequate to insure against premature plastic buckling.
A solution to this complicated plate buckling problem has been achieved (14) by requiring that the section will exhibit a rotation capacity that corresponds to a compression strain equal to the strain-hardening value, \( \varepsilon_{st} \) (Fig. 2). At this point the material properties may be more accurately and specifically defined than in the region between \( \varepsilon_y \) and \( \varepsilon_{st} \).

The result of this analysis for flanges of WF shapes is shown in Fig. 25, together with the results of tests. From these curves and from similar relationships established for webs, the following design guide may be established to assure that the compressive strains may reach \( \varepsilon_{st} \) without buckling. (Fortunately nearly all WF beams are satisfactory in this regard.)

### CROSS-SECTION PROPORTIONS

**Local Buckling**

Compression flanges and webs of beams and columns should comply with the following:

\[
\frac{b}{t} \leq 17 \quad \text{(beams and columns)}
\]

\[
\frac{d}{w} \leq 43 \quad \text{(columns in direct compression)}
\]

\[
\frac{d}{w} \leq 50 \quad \text{(beams in bending)}
\]

2. Lateral Buckling

Figure 26 shows a moment-curvature relationship that is unacceptable; the moment does not remain at near-constant value through a sufficient angle change. This result was obtained in a test of a simply-supported beam with a span length intentionally made so long that premature buckling was inevitable. A photograph of the beam after test is shown in Fig. 27.

The problem of specifying the critical length of beam such that premature lateral buckling will be prevented has not been completely solved. Currently, theoretical studies are being made somewhat along the lines of those which proved to be successful in the case of local buckling. Although this study is not yet finished the results of tests and approximate analyses provide some present guidance for the designer. The problem is to specify bracing requirements to prevent deformation out of the plane of the frame.

Yielding markedly reduces the resistance of a member to lateral buckling. Therefore bracing will be required at those points at which plastic hinges are expected. Intermediate between these critical sections, conventional rules may be followed to protect against elastic lateral buckling. In the event that consideration of the moment diagram reveals that a considerable length of a beam is strained beyond the elastic limit (such as in a region of pure moment) then additional lateral support at such a hinge may be required. A guide for the spacing of such support (that is probably much too conservative) is

\[
L_{cr} = 20 \varepsilon_y
\]  

(13)
If the length of the member strained beyond the elastic limit is greater than this value, then additional bracing would probably be required.

3. Column Buckling

The simple plastic theory assumes that failure of the frame (in the sense that a mechanism is formed) is not preceded by column instability. Although the load at which an isolated column will fail when it is loaded with axial force and bending moment can be predicted with reasonable accuracy, the buckling problem becomes extremely complex when the column is a part of a framework. A complete solution is not in hand. It represents another area in which further research may effectively be done in order to avoid over-conservative simplifications. However, at the present time, such simplifications must be made and a conservative approach is mandatory.

If the axial load is relatively low and, further, if the moment is maximum at the ends of the member, then the stability problem may be neglected. On the other hand, if an examination of the moment diagram shows that the column is bent in single curvature, then a modification must be made to assure a safe design. Figure 28 has been developed(15) to aid in the solution of this problem. For an adequately braced member, it shows how much end moment, \( M_e \), may safely be applied at the ends of columns of different slenderness ratio. For example, if the axial force were 40\% of the yield value (\( P/P_y = .40 \)) then a column of slenderness ratio of 60 could resist an end moment equal to 50\% of its \( M_p \)-value.

If, after the frame members have been selected it is found that a column is subjected only to axial force, the suitability of the member selected may be checked from the equation(1)

\[
\frac{P}{A} = \sigma_y - 120 \frac{L}{r} \quad (L/r < 110)
\]

\[
\frac{P}{A} = \frac{290,000,000}{(L/r)^2} \quad (L/r > 110)
\]

where \( L \) is the unbraced length and the axis is selected to give the maximum \( L/r \) value. These same equations may be used for checking a column for stability in the plane normal to the principle plane of bending, the columns being assumed as pinned at the ends.
Brittle Fracture

Since brittle fracture would prevent the formation of a plastic hinge, it is exceedingly important to assure that such failure does not occur. But it is an equally important aspect of conventional elastic design when applied to fully-welded continuous structures. As has already been pointed out in previous lectures, the assumption of ductility is important in conventional design and numerous design assumptions rely upon it.

In past years the failures of ships and pressure vessels have focused attention on the importance of this problem. And although hundreds of articles have been published on the problem of brittle fracture, no single easy rule is available to the designer. None-the-less, an examination of the conditions that have led to brittle failures in the past should be helpful as a background for formulating good practice.

Brittle fractures are caused by a combination of adverse circumstances that may include several of the following:

1. Local stress concentrations and residual stresses
2. Poor welding
3. Notch sensitive steel
4. Shock loading
5. Low temperature
6. Strain aging
7. Triaxial tension state of stress.

In plastic design the engineer should be guided by the same principles that govern the proper design of an all-welded structure designed by conventional methods, since the problem is of equal importance to both. Thus:

1. The proper material must be specified to meet the appropriate service conditions.

2. The fabrication and workmanship must meet high standards. In this connection, punched holes in tension zones and the use of sheared edges are not permitted. Such severe cold working exhausts the ductility of the material.

3. Design details should be such that the material is as free to deform as possible. The geometry should be examined so that triaxial states of tensile stress will be avoided.\(^{(16)}\)

How can we be sure that brittle fracture will not be a problem even if the above suggestions are followed? While no positive guarantee is possible, experience at the Fritz Laboratory with tests of rolled members under normal loading conditions (but with many of the "adverse circumstances" noted above) has not revealed a single brittle fracture of a steel beam. Further, the use of fully continuous welded construction in actual practice today has not resulted in failures. And factors that are otherwise neglected in design have most certainly caused plastic deformations in many parts of such structures.
Summarizing, the following guides are suggested:

**STRUCTURAL DUCTILITY**

ASTM A7 steel for bridges and buildings may be used with modifications, when needed, to insure weldability and toughness at lowest service temperature.

Fabrication processes should be such as to promote ductility. Sheared edges and punched holes in tension flanges are not permitted. Punched and reamed holes for connecting devices would be permitted if the reaming removes the cold-worked material.

In design, triaxial states of tensile stress set up by geometrical restraints should be avoided.

**Deflection Stability**

Up to this point the tacit assumption has been made that the ultimate load is independent of the sequence in which the various loads are applied to the structure. One would also suppose that a certain degree of fluctuation in the magnitude of the different loads would be tolerable so long as the number of cycles did not approach values normally associated with fatigue.

In the large majority of practical cases this is true. For ordinary building design no further consideration of variation in loads is warranted. However, if the major part of the loading may be completely removed from the structure and reapplied at frequent intervals, it may be shown theoretically that a different mode of "failure" may occur. It is characterized by loss of deflection stability in the sense that under repeated applications of a certain sequence of load, an increment of plastic deformation in the same sense may occur during each cycle of loading. The question is, does the progressive deflection stop after a few cycles (does it "shake down") or does the deflection continue indefinitely? If it continues the structure is "unstable" from a deflection point of view, even though it sustains each application of load.

Loss of deflection stability by progressive deformation is characterized by the behavior shown in Fig. 29. If the load is variable and repeated and is greater than the stabilizing load, \( P_S \), then the deflections tend to increase for each cycle. On the other hand if the variable load is equal to or less than \( P_S \), then, after a few cycles the deflection will stabilize at a constant maximum value and thereafter the behavior will be elastic.

In the event that the unusual loading situation is encountered, methods are available for solving for the stabilizing load, \( P_S \), and the design may be modified accordingly.\(^{(15, 16)}\) As mentioned earlier, however, this will not be necessary in the large majority of cases. In the first place the ratio of live load to dead load must be very large in order that \( P_S \) be significantly less than \( P_D \), and this situation is unusual.
Secondly, the load factor of safety is made up of many factors other than possible increase in load (such as variation in material properties and dimensions, errors in fabrication and erection, etc). Variation in live load, alone, could not be assumed to exhaust the full value of the factor of safety; and thus the live plus dead load would probably never reach $F_S$.

Figure 30 lends further assurance that the problem is largely academic. It represents the action of a two-span continuous beam with an off-center concentrated load in each span. Even in this extreme case in which the full value of the separate loads was removed and then re-applied, the actual reduction in load capacity was only about 8% instead of the 20% predicted by the theory of deflection stability.

Deflections

How may we be assured that the structure will not be bent out of shape to the extent that it is unassessable? We cannot be sure without computing the deflections -- any more than the conventional design can be regarded as satisfactory in this regard without making further computations or by knowing from experience that satisfactory performance may be expected. Methods for computing the deflection at working load and at ultimate load have been summarized in Ref. 15. However, the problem of deflections is not a serious one to plastic design, because in most cases a structure designed for ultimate loading by the plastic method will actually deflect no more at working loads (which are nearly always in the elastic range) than a structure designed elastically according to current specifications. A plastically designed continuous beam exhibits less deflection than a simply-supported beam designed to carry the same loads because of the restraining moments that are present.

Figure 31, for example, shows three different designs of a beam of 30-foot span to carry a working load of 21 kips. Curve I is the simple beam design. Curve III is the plastic design. The deflections at working load for the plastic design are significantly less than those of the simply-supported beam.
Consideration of additional factors that are neglected or not included in the "simple plastic theory" may require a revision to the original design. The most important of these factors are the influence of axial force, shear force, and instability. Procedures have been suggested as a basis for checking the original design. It is indeed a fortunate circumstance that in most cases the original design need not be revised. Indication of this is the notable agreement between test and the simple theory as shown, for example, in Fig. 32, and yet all these members contained both axial and shear forces.

The basic assumptions of plastic analysis are confirmed by test (Fig. 2, 3, 5, 9). The ultimate load of a continuous steel structure is predicted with corresponding accuracy (Fig. 12, 13, 32). Thus experimental results verify the applicability of the plastic method of analysis to structural design.


1. Ductility

2. Plastic Moment

3. Plastic Hinge

4. Continuous Connections

5. Redistribution of Moment

6. Ultimate Load Hinges—Mechanism
Average Values

\[ E = 29.6 \times 10^3 \text{ ksi} \]
\[ \sigma_y = 36.0 \text{ ksi} \]
\[ \epsilon_{st} = 1.4 \times 10^{-2} \text{ in./in.} \]
\[ E_{st} = 0.7 \times 10^3 \text{ ksi} \]
LOAD "P" IN KIPS

CENTER DEFLECTION IN INCHES

14WF38

L = 15'

Fig 9
Fig 6

Beam Load V vs End Deflection

LOAD V (kips)

0 10 20 30 40 50 60

END DEFLECTION (in.)

Type A
No Stiffeners

Type B
Horiz. Stiffeners

Type C
Vert. Stiffeners
\( e = 5.10'' \), \( P_{max} = 0.55 P_y \)

**Figure 4**

- **MOMENT (in-kips)**
- **\( \phi \) (rad/in.)**

- \( M_p \)
- \( M_{pc} \)

- 3.0 x 10^-3
Fig. 15

Elastic Limit

Partially Plastic

Complete Yield

Fig. 16

Total Stress Distribution

(a) stress due to P

(b) stress due to M

(c)
\[ \frac{M_{pc}}{M_p} = 1.18 \left( 1 - \frac{P}{P_Y} \right) \]
Elastic  Flange Plastic  Partially plastic and yield at $Q_L$ due to shear
FIG. 23 BUCKLING OF OUTSTANDING FLANGES (RESULTS OF WF TESTS)
$P < P_s$ ("Stable")

$P = P_s$ ("Stable")

$P > P_s$ ("Unstable")

NUMBER OF CYCLES

DEFLECTION

FIG-29
FIG. 14 RESULTS OF CYCLIC LOADING TESTS
FRAMES

Structure of Loading

Shape

Reference

% of Predicted Ultimate Load
20 40 60 80 100 120

8 x 4 x 18" R.S.J.
(Two frames tested in parallel)

10

8 x 4 x 18" R.S.J.
(Two frames tested in parallel)

10

5 x 3 x 16" R.S.J.
(Two frames tested in parallel)

11

5 x 3 x 16" R.S.J.
(Two frames tested in parallel)

11

W = 13.13 Tons

H = 17 Tons

7 x 4 x 16" R.S.J.
(Two frames tested in parallel)

13

SCALE: 2' 4'

Fig 2