The use of models in plastic design, Proc. AISC National Engineering Conference, p. 35 (1955), Reprint No. 116 (55-8)

R. L. Ketter
MONDAY MORNING SESSION
April 18, 1955

The opening session of the Seventh Annual National Engineering Conference of the American Institute of Steel Construction was convened at nine-fifteen o'clock, in the Grand Ball Room, Muehlebach Hotel, Kansas City, Missouri, by Mr. J. A. Hall, Assistant Chief Engineer, Kansas City Structural Steel Company.

CHAIRMAN HALL: On behalf of the fabricators in the Omaha and Kansas City District, I bid you welcome to Kansas City. We hope that the weather will be good to all of us while you are here and that you will enjoy your stay.

Now, for your official welcome, here is A.I.S.C. President Earle V. Grover, President of Apex Steel Corporation, Ltd., Los Angeles, California.

Introductory Remarks by A.I.S.C. President
EARLE V. GROVER

It is indeed nice to see so many of you here. I think it is fitting for me to emphasize to you that which you already know better than I: that the engineering side of the American Institute of Steel Construction is the important side. In my judgment every time that we have the opportunity, we must let engineering take the forefront. We need the engineer's efforts to make the steel business broader and better even when business conditions are not too good.

I am reminded this morning of a man in Los Angeles who makes a living by playing bridge. I asked him one day, "How do you do it?"

"Well," he replied, "every bridge player knows that about ninety per cent of the hands you get are just average, and the balance of the hands play themselves."

"Therefore," this bridge player said, "I make my living by making these average hands come through as much as I can."

I think that there is a lesson for engineers and businessmen in this. Most of the hands that are dealt to us are average and not a laydown. We have to work to make a score.

So, with just that one little idea as the opening thought of the engineering session, I want to welcome you. I am very glad indeed that so many of you are here.

Before turning the program back to Chairman Hall, I would like to make a few introductions.

The new men in our New York office:
Edward R. Estes, Jr., Research Engineer
E. W. Gradt, Assistant to Director of Engineering & Research
W. C. Brooks, Director of Publications

C. R. Ham, Director of Market Research
The new Chief of District Engineers:
Mace H. Bell
The Regional Engineers:
C. M. Corbit, Los Angeles (Western)
A. L. Small, Chicago (Midwestern)
H. J. Stetina, Philadelphia (Northeastern)
A. R. Wright, Atlanta (Southeastern)
The District Engineers:
Northeastern Region
J. G. Hotchkiss, New York, N. Y.
W. G. Keat, Pittsburgh
F. W. Saul, Syracuse, N. Y.
Southeastern Region
E. E. Hanks, Greensboro, N. C.
J. M. Marshall, Jr., Atlanta
Mid-Western Region
G. T. Eberle, Milwaukee
R. H. Hewitt, Minneapolis
N. J. Law, St. Louis
W. A. Milek, Omaha
Southwestern Region
D. E. Stevens, Oklahoma City
Western Region
H. B. Corlett, San Francisco
E. E. Gunnette, Seattle
The Field Engineers:
R. F. Sell, Chicago
J. L. Tanner, III, Dallas
W. J. Way, Los Angeles
We are also favored by having with us friends from
Canada. I would like to introduce these representatives of the Canadian Institute of Steel Construction:

John Grieeve, Executive Vice-President
Don C. Beam, Chief Engineer
John B. Wheeler, District Engineer from Vancouver

Gentlemen, I hope that this Conference will be of value to you.

Story of Granite City Steel

JOHN I. PARCEL

The Granite City Steel Company in Granite City, Illinois, is the largest steel company in the St. Louis area, and one of the largest west of Chicago. It is one of approximately twenty-five integrated steel plants among the two hundred or so plants in the country. As an integrated plant it engages in all phases of steel production from the processing of raw materials through the rolling of semi-finished and finished steel to final fabrication. Although this company has a history of more than 75 years, a large part of the steel-making equipment was installed during the recent expansion program making this plant one of the most modern mills in the world.

During the period from 1951 to 1954, the company spent seventy-eight million dollars for modernization and expansion of its producing capacity. It has recently announced that the utilization of this increased capacity is complete.

The main elements in the expansion of the plant were: new coke ovens, three new open hearth furnaces, a new blooming mill, a new continuous hot strip mill layout, numerous smaller buildings, electrical power distribution, track work and extra yard facilities.

To provide an integrated process, the Granite City Steel Company in 1951 purchased the Koppers Company blast furnace plant, consisting of two blast furnaces, forty-nine batteries of coke ovens, by-product facilities, material handling and yard facilities, and equipment. The construction program at this plant consisted of the addition of twenty-seven coke ovens and the alteration of certain by-product facilities.

The open hearth facilities at the beginning of 1951 consisted of nine small furnaces and four 200-ton capacity furnaces. The construction program increased the capacity of the four largest furnaces to 230 tons each and added three new furnaces with a capacity of 285 tons each. The necessary essentials to the operation of the greater furnace output were installed. All of the small furnaces were later retired.

In 1951, the rolling operations at Granite City Steel were generally located in a single area. As a part of the expansion, facilities for rolling slabs were torn out and replaced by a completely new mill adjacent to the open hearth area. At the same time, hot strip finishing facilities were expanded by installing three new 100-ton furnaces, a new heavy roughing mill, mill tables and other supporting equipment.

Throughout the project the new installations had to be closely integrated with the existing plant. Careful planning permitted the maximum output of the existing mill while the construction was in progress. In many instances it was necessary to continue the operation of the mill while excavating beneath it to prepare for the foundations of the added equipment.

The major buildings are steel frame with galvanized walls and roofs. The major buildings and heavy equipment are, in most cases, supported by forty-ton steel piles. Pipe-piles and spread footings are utilized in certain areas. There were 265,000 cubic yards of excavation, about 190,000 linear feet of piling, 24,000 tons of steel and 67,000 cubic yards of concrete. The general contractor for the project was the Rust Engineering Company of Pittsburgh, and there were approximately twenty sub-contractors involved. The design of the open hearth furnaces and the installation of the coke ovens were subcontracted to the Engineering Department of Koppers Company, Inc. The United Engineering and Foundry Company supplied the mill equipment. Allis Chalmers furnished the sub-station electrical equipment. Alliance Machinery Company and Morgan Engineering Company supplied the cranes. The Stockbridge Company of St. Louis were the fabricators of the structural steel.

In the period 1942-1943, when the steel company carried out a very much smaller plant expansion program, we acted as consulting engineers. When the larger expansion program developed, we were selected to furnish over-all engineering services. This consisted of surveys and layouts, foundation and structural design, and supervision and correlation of the work of the engineers responsible for specialized phases.

As a part of our service, something a little out of our
line, we prepared a film for the use of the Granite City Steel Company which gives a fairly complete history of the whole construction program.

With that brief background, I will let the film tell you the complete story.

CHAIRMAN HALL: Thank you, Mr. Parcel.

Our next speaker is also well known to all engineers in this area. Mr. Josef Sorkin has been a partner in the firm of Howard, Needles, Tammen and Bergendorff since 1950. Following his graduation from the University of Nebraska, he worked with the Nebraska and Montana highway departments and then was Chief Design Engineer for the Central Nebraska Public Power and Irrigation District until 1939 when he joined his present associates.

Recent Bridge Projects over the Missouri River in Greater Kansas City Area

JOSEF SORKIN

It is a special honor for me to have this privilege of addressing the meeting of the American Institute of Steel Construction. For many years I have had a tender feeling for your organization. When one begins to notice one's gray hair, he is particularly prone to look back toward the earlier and younger days and remember those who for any reason had a part in the formative years of one's career. Upon graduation from the University of Nebraska, I was connected with the Nebraska Highway Department as a Bridge Designer. In 1932 it was my fortune to have designed a bridge which received the first prize of the American Institute of Steel Construction in Class C competition. If for any reason you don't believe it, you may check this in the recent issue of Prize Bridges. I still believe that the Bryan Bridge over the Niobrara River near Valentine, Nebraska, is the most monumental structure ever constructed. Its total cost was $31,630.08; it had a center span of 145 ft. 2½ in. and end spans 72 ft. each. I am still convinced that no bridge has received more consideration and greater care than that one structure.

Your Program Chairman has given me considerable latitude for selecting a subject for this address. I shall devote this time to a discussion of recent bridge projects in this area. We are, of course, proud of having you here for this meeting, and the subject is related to the locale. My other reason for selecting this particular subject is that I am convinced that in the last decade or so, bridge engineering in the United States has been going through an important period in our attitude toward the design of public structures.

In the middle 30's a book was published in Germany with illustrations and photographs of bridges over the Rhine River. The book was quite popular, and I expect many of you have seen it. Personally, I was impressed with one particular aspect—each of the many bridges had individuality and character. Some reflected intricate design and complexities; others were quite simple in concept; but each and every one of them was a unit in itself quite unlike any other bridge over the same river. I hope that I am not suggesting a controversial issue in stating that by contrast a single collection of photographs of bridges over the Missouri River would be drab indeed and would probably be characterized by the great similarity of all the structures rather than individuality. It is needless for me to philosophize on the subject or to apologize for that condition. As a matter of fact, I do not believe that any apologies are warranted. There is real logic and good reason why our American bridges in the Midwest should be purely functional, because they reflect the vigor and rapid development of this country in the last half of the 19th Century and the earlier decades of this century.

Yet on at least a few occasions we have been complimented by European visitors who stated that in their opinion the Eighth Wonder of the World is the American bridges. Of course, this reference applies, not to our Midwest bridges, but to the extraordinary long spans along the east and west coasts. There are no comparable waterways or navigation channels in Europe, therefore, there are no comparable structures either. The long span suspension bridge is probably as American as the game of baseball, and visitors are awed by the expanse of the George Washington Bridge and the Golden Gate Bridge. This, however, does not change any original thesis on the comparable drabness of our standard bridges built up to very recent times over the Missouri River and, with very few exceptions, the Mississippi River.

However, in the past few years, particularly since World War II, there has been a radical change in our general thinking. While we are still conscious of the functional requirements, we have come to realize an equal responsibility for antiquity of appearance. I believe that at least some of the recent bridge projects over the Missouri River in our area reflect this trend.

While we are on this subject, it might be pertinent to note that this tendency toward individuality in design of bridges was given considerable impetus by our better understanding and bolder use of statically indeterminate structures including continuous structures. It is almost
The Truman Bridge

The Truman Bridge, which was constructed during the war, is owned jointly by the Milwaukee Railroad and the Rock Island Lines. It replaced the then 60-year old high level bridge which has been since converted for highway use. The Truman Bridge has a total length of 2536 ft. and includes a 420 ft. lift span which provides a navigation channel 400 ft. wide. The lift span is, of course, the most important element of the bridge, although in addition there are also three 250 ft. truss spans and several hundred feet of deck girder approach spans. The main spans are supported on concrete piers founded on rock. Pneumatic caissons were used for sinking the piers to the bottom. It might be of interest to note that this bridge received the A.I.S.C. First Prize Award for Class IV—Bridges opened to traffic in 1945.

The feature which gives this bridge its interesting appearance is the massive towers, which are rather unusual in design. As you know, most lift span towers are of the braced type framed into the adjacent fixed spans. The towers of the Truman Bridge are free of this bracing and are made self-supporting by virtue of the massive box girders which make up the main columns. Transversely, the towers are rigidly framed by struts. In the longitudinal direction of the bridge the towers are braced by struts in the plane of the top chord of their adjacent fixed spans and are free cantilevers above the truss spans.

Another incidental but interesting feature of this bridge is the solid shaft drive utilized for operation of the span. To the best of my knowledge this was the first arrangement of this type in lieu of operating ropes ordinarily employed for this purpose.

Chouteau Bridge

The Truman Bridge replaced this old railroad bridge constructed in 1884. This bridge, resting on masonry piers set on rock foundations, was constructed of wrought iron and steel with the latter predominating. While it is deficient in capacity to carry modern railway loading, it has ample capacity for the lighter highway loading. The City used foresight and good judgment in purchasing this abandoned structure from the railroad and in converting it for highway use. It is expected that this structure will remain in service for many years to come and will stand there as a reminder of the year in which the automobile became an important element in the life of this country.

In the investigations of the conversion of this bridge for highway traffic use, it was found that by utilizing lighter than usual concrete slab flooring, the main spans would have the capacity to support two decks of traffic. While initially only one deck has thus been made available for traffic, the plans call for eventual construction of an upper deck which will thus result in the equivalent of a four-lane structure with traffic on each deck in one direction only. The reconstruction of this bridge was completed in September of 1953 at a cost slightly in excess of a million dollars, furnishing the City of Kansas City another crossing over the Missouri River. The structure is of importance in that it connects Kansas City with its annexed areas to the north. While this bridge is now carrying a considerable volume of traffic, it is anticipated that future trafficway improvements on the south end of the bridge will further increase the traffic and eventually will require the addition of the upper deck. With the spacing of the trusses, the roadway was limited to 20 ft. While this is rather narrow in the light of modern requirements and standards, it is well to remember that at the time this bridge was reconstructed there was only one structure between Kansas City and St. Louis which had a roadway wider than this.
Paseo Bridge

The Paseo Bridge, completed and opened to traffic on August 13, 1954, is the most important new crossing in this area, not only because of its magnitude and appearance, but particularly, because it is the first modern facility designed in every respect to serve a highly concentrated urban traffic. The limited access to the long approaches of the bridge, the traffic interchanges designed and constructed in accordance with all modern standards and requirements, and the magnitude of the main span itself places this bridge in a position all its own.

In the preliminary studies it became apparent that it would be quite difficult to obtain a pleasing or symmetrical appearance using two 400 ft. spans which would have to be located on the south bank of the river to meet the requirements of the navigation channel. After considerable study, and with the cooperation of the District Engineer of the Corps of Engineers, it was determined that at low water stages navigation could use a channel more centrally located between the levees. This was possible because of the course of the channel from the south bank to the north bank at this point. Taking advantage of this condition, a permit issued by the Secretary of the Army approved the use of a 616 ft. center span and 308 ft. side spans. Comparative studies of conventional cantilever type, tied arch construction with cantilever side spans, and other types did not indicate any significant cost advantage as compared with the suspension span adopted. Quite aside from the generally recognized graceful appearance of the suspension span in modern highway bridge construction, it has now been pretty well accepted that the ideal structure is one which provides a riding deck conforming as nearly as possible to the approach highway design. Obviously, this cannot be met with the usual type of long span construction in which the attention of the driver is diverted by extensive overhead bracing and the like. Because of the depth of the rock in the bridge site being approximately 100 ft. below normal water level, it was established quite conclusively that an externally anchored suspension design would cost considerably more than the self-anchored type adopted. For this type the end piers are subjected to vertical loads only. All of the piers supporting the main spans are of hollow type construction founded on shale.

The 616 ft. main span constitutes the longest self-anchored suspension span in existence. It should be recognized, however, that in 1929 the Germans built a self-anchored span 1033 ft. long between Cologne and Muelheim which was, of course, considerably longer than the Paseo span. However, this bridge was destroyed during the war and the reconstructed bridge is externally anchored. In the design of the Paseo Bridge utmost consideration was given to problems of aerodynamics in proportioning the structure. In making provision for damping utmost care was exercised in designing the elements in such a way as to eliminate as nearly as possible perceptible oscillation. The size of the make-up of the stiffening girder made up as a box 10 ft. 5 in. in depth was largely dictated by these considerations. Likewise, the details of the floor system were made to provide maximum damping effect. The stringers are thus set on longitudinally braced floor beams and floor joints are provided intermittently so as to permit the stringers to slide on the tops of the floor beams. Likewise, open grating was used for the median and for the safety walks to increase atmospheric damping. To increase torsional rigidity laterals have been provided in the top and bottom plane of the stiffening girders.

The cable has a sag of 70 ft. which is 1/9 of the span. Each cable consists of 37 prestressed bridge strands which form a wrapped cable 12 in. in diameter.

1949 A.A.S.H.O. Specifications have been used with a design load of H2o-S16. Floor laterals and details are made up of structural carbon steel; main material in towers and stiffening girder is of low alloy steel. The strands in the cable have an ultimate stress of 222,000 psi with a yield point of 155,000 psi. The towers, of rigid frame
construction, are 136 ft. high with columns on 78 ft. centers. Each "T" shaped box girder column supports a load of approximately 5,000 kips, and the tension in the cable is likewise 5,000 kips. The bridge provides two 26 ft. roadways divided by a 4 ft. median and two 3 ft. safety walks.

With the completion of the trafficway both to the north and to the south, this will indeed be the most important cross-river structure in this metropolitan area.

**Airport Bridge**

The Broadway Bridge, or so-called Airport Bridge, now under construction, is intended to serve primarily the heavy municipal airport traffic. Its north approach, however, will extend northward and connect with existing U. S. 71 permitting direct access to the airport from 14 miles to the north. This bridge will replace the upper deck of the Hannibal Bridge which is now being used primarily for access to the municipal airport. The Hannibal Bridge, as originally designed, still supports the Burlington Railway along its lower deck. It will continue to be in use as a railroad bridge, but the highway deck will be removed upon completion of the new four-lane bridge. While the approaches both to the north and to the south propose considerable problems in geometrics, the important aspect of this bridge is the three main spans.

Navigation requirements compel an asymmetrical layout with the southernmost span being 540 ft. and the two spans to the north 450 ft. each. The main spans are to be braced type tied arches. They will be supported on piers founded on rock. Here, too, pneumatic caissons will be used for the foundations. An interesting feature of this bridge will be the suspender cables used as hangers.

This project was conceived by the late Ernest E. Howard and was one of the last projects in which he took a great personal interest. It is being carried out pretty well in accord with his conception of it, and it is unfortunate indeed that he will not be with us to see it fully realized.

**Leavenworth Bridge**

Last, but not least, just two weeks ago we completed and opened the highway bridge at Leavenworth. This bridge replaces a historically interesting old structure. The old so-called Ft. Leavenworth Bridge, built 83 years ago, is the oldest bridge over the Missouri River still standing. It was originally used as a railroad bridge for some years, was abandoned, and was rehabilitated for use as a highway bridge. Historically, it has interesting aspects in that this particular bridge was one of the structures in competition with the proposed bridges at St. Joseph and at Kansas City. It has been said that except for the fact that the Hannibal Bridge in Kansas City was completed two years ahead of the Leavenworth Bridge, Leavenworth might have assumed the position as a metropolitan center now enjoyed by Kansas City, and presumably Kansas City would have been relegated to the position of Leavenworth. The old bridge is now closed to traffic, and the relic will remain standing only until its owners, the Department of Justice, remove it.

The total length of the new Leavenworth Bridge, including viaduct approaches, is approximately 2,440 ft. The length of the river spans are in accordance with the provision of the construction permit of the Secretary of the Army. The west 420 ft. span furnishes a vertical clearance of 55 ft. above high water level of the navigation channel. The east span of the same length is provided in anticipation of the navigation channel eventually changing its course eastward.

The bridge has a 26 ft. roadway and two 2 ft. 6 in. safety walks. It has been designed to support H20-S16
loading in accordance with the specifications of the American Association of State Highway Officials.

The piers supporting the main spans are founded on rock. The west pier rests on sandstone and the other two piers are founded on a thick formation of fairly hard shale. The footings of the two east piers rest approximately 50 ft. below low water. The west pier is at 30 ft. below low water. In designing the piers it was found that the usual shaft consisting of two columns with or without connecting web walls resulted in footings larger than was deemed necessary to support the load on the rock and shale. To take advantage of this condition, by way of economy, the pier shafts were designed in the shape of a 'T' with the superstructure bearings resting on the cantilevers extended from the center shaft. While the contractor had the option of either setting the foundations in open cofferdams or by using pneumatic construction, the latter method was elected by the contractor. Steel shell caissons were used for the bases of these piers.

The two main spans, each 420 ft. long, are tied arches. Cost studies for the types of construction for the river spans indicated no decisive advantage as regards economy. This type of structure was deemed preferable because of more pleasing appearance. The arch spans are unique in certain respects. Ordinarily, the tie of the arches is a flexible compact section resisting in tension the thrust of the arch. The makeup of the arch is then made to resist not only the compressive stress of the thrust, but also the bending moments resulting from partial and unbalanced loads. In the arches of the Leavenworth Bridge the tie was made rigid consisting of a 6 ft. box girder; the rib is relatively limber, having a depth of 33 in. Thus 85 per cent of the bending moments are resisted by the tie and only 15 per cent of these moments are resisted by the rib. Basically, the advantage of this system is in the lowering of the center of gravity of the structure and in the corresponding reduction of the effects of wind. It is also deemed that some saving has been effected in the erection costs by concentrating the heavier metal in the plane of the bridge floor. Another aspect of the design which is indeed unusual is in the continuity of the two spans. The tie was made continuous over the entire 840 ft. of structure. Under certain loading conditions it is subjected to negative bending moments at the center pier. While there is little gain in the economy of the metal in the arches by virtue of this continuity, there is the advantage of providing two shoes at the center pier in lieu of four shoes as would otherwise be required. Thus there is some economy in the saving of shoe metal and in the resulting saving in the pier. Another feature which is somewhat unusual is the Vierendeel top bracing. Comparative estimates indicate that this type of bracing costs no more than X-bracing since the metal is all concentrated in the struts. The advantage of the system is in the more graceful appearance and probably in cost of maintenance. For aesthetic reasons, as well as for greater rigidity of positioning of the arches, the portals will be X-braces consisting of box members.

For the floor system and bracing structural carbon steel, was used. For the ribs and ties of the arch spans low alloy steel with allowable basic unit stress of 27,000 psi was used.

Conclusion

It was my intention to present to you within the time allotted to me a description of the recent bridge projects over the Missouri River in this vicinity. It is hoped that the slides which we have shown here amply illustrated the type of construction utilized. While each structure was designed to meet its particular conditions by way of navigation requirements and the like, it is believed that each of them has some degree of character and individuality. The treatment afforded these structures is perhaps not unique and is not necessarily characteristic either of the Missouri River or of Kansas City, Missouri. More likely it reflects the general tendency among bridge engineers to give more serious consideration to the appearance and character of structures which shall serve the public and constitute a distinct addition to the landscape. In the design of the bridges described the intention was to produce structures that would have a pleasing appearance to harmonize with the surroundings, since landscape is either enhanced or marred by such structures. We think we have obtained that result.

Chairman Hall: Thank you very much for your presentation of the bridges in the Kansas City area.

We are right on time, gentlemen, and are ready for our discussion period.

Mr. J. A. Wise (University of Minnesota): In the analysis of your suspension bridge, did you limit yourself to the elastic analysis or did you attempt to complete the lengthy deflection analysis?

Mr. Sorkin: It was designed in accordance with the elastic theory and checked by the deflection theory.

Mr. Wise: I have been playing around with the deflection theory analysis and have found that the use of an I.B.M. gives a grand opportunity for making very rapid calculations under the deflection theory.

Mr. Wermers (Washington University, St. Louis): Was the deflection of the Leavenworth arch rib of any importance in the analysis? It was rather shallow with respect to the span.

Mr. Sorkin: No, the deflection was not of any importance.

Mr. C. L. Kreidler (Lehigh Structural Steel Co.): How will you erect the ribs on the Airport Bridge since you have suspennder cables?
MR. SORKIN: The assumption was that they would be erected on rigid bents that would support them in their proper position; and then, of course, the rib hangers would be erected after the arch was in place.

MR. KREIDLER: There is no way to reuse that steel a second time so there was no real economy in the use of suspender cables.

MR. SORKIN: Well, that’s right. I might say, incidentally, this is one reason why we did not use rope suspenders on the Leavenworth Bridge—they just do not permit the best erection.

CHAIRMAN HALL: We are now recessed until this afternoon at two o’clock.

(The meeting recessed at 11:55 o’clock, a.m.)
MONDAY AFTERNOON SESSION
April 18, 1955

The meeting was reconvened at two-fifteen o'clock, p.m., Mr. T. R. Higgins, Director of Engineering & Research, American Institute of Steel Construction, presiding.

CHAIRMAN HIGGINS: If there is any one aspect to this afternoon's program that may be somewhat different than the rest of the conference program it is the fact that the topics are very closely related to the A.I.S.C. program for research.

The first subject, as those of you who have been coming to these conferences since they first started are aware of, has a life span just about equal to these conferences.

We have, from time to time, reported the developments with regard to the experimental work in the laboratory in the perfection of the use of high strength bolts.

Our first speaker this afternoon has had an opportunity within the past twelve months to study rather comprehensively just what is involved in the tightening of high strength bolts and being sure of the necessary free tension.

Dr. Adrian Pauw received his Bachelor of Science degree from the University of Washington in 1937, and his master's and doctor's degrees from the California Institute of Technology. He was an assistant research engineer at the California Institute of Technology before he taught at Rice Institute. Currently, Dr. Pauw is Associate Professor of Civil Engineering at the University of Missouri.

Tension Control for High Strength Structural Bolts
PROFESSORS ADRIAN PAUW and LEONARD L. HOWARD*

Introduction

The introduction of the high-strength bolt as a structural fastener has revolutionized structural practice in recent years. Primarily through the efforts of the Research Council on Riveted and Bolted Joints, information on the economic and structural advantages to be derived from the use of these bolts has been widely disseminated. The results of an intensive research program sponsored by the Council and culminating in the preparation of a specification entitled, "The Assembly of Structural Joints Using High Strength Steel Bolts," (January, 1951; Revised February, 1954) has led to wide acceptance of the high-tensile bolt as a superior structural connector.

Concept of High-Tensile Bolt Connections

Joints using high-strength bolts differ from other types of bolted joints both in the nature of the bolt used and the method of stress transfer. Bolts, nuts and washers are manufactured to conform to A.S.T.M. specification A325, and to dimensions established by the American Standards Association. The bolt is made from a medium carbon steel, heat treated, quenched and tempered. The bolt head is given a distinctive marking of three radial lines to distinguish it from ordinary machine bolts. The nut is made from a steel similar to that used in the bolt, but the nut is not heat treated. Washers are either quenched and tempered, or carburized, quenched and tempered. These heavy, hardened washers are used both under the nut and under the bolt head. They perform an important function—by distributing the clamping force over a wide area surrounding the hole, they enable the development of a permanent high clamping force. The compressive stress built up in the plies of the joint is also desirable in that this stress tends to inhibit the formation of fatigue cracks at the edge of the holes.

The concept of stress transfer in structural joints using high-tensile bolts is quite different from that in riveted connections or in joints employing ordinary or fitted bolts. Although most riveted joints, under working loads, will transfer the load by the frictional forces developed between plies during cooling of the rivet—the existence of these frictional forces under sustained or under vibratory load is not dependable. The design of such joints is therefore necessarily based on the premise that slip will take place and that the load is transferred from one element of the connection to another by direct shear on the rivet. Herein lies the principal structural advantage of the high-tensile bolt—when joints are properly designed and

* Presented by Prof. Pauw.
† Numbers in parenthesis refer to references at end of paper.
assembled with these bolts, the clamping force is permanently retained, hence slip is prevented and loads are transferred by frictional forces alone. The resulting connections have structural properties which are generally superior to equivalent riveted joints.\(^{(3)}\)

**Importance of Tension Control**

For optimum structural performance of bolted joints it is necessary that slip between the several plies of the connection be prevented. Slip may be affected by paint or by such defects as dirt, oil, loose scale, burrs and pits, which prevent solid seating of the parts. The Council’s assembly specifications therefore require that contact surfaces be descaled, or carry only normal tight mill scale, and that the surfaces be free from any of the aforementioned defects. Where contact surfaces are subject to stress reversal, impact or vibration, or where stress redistribution due to joint slippage would be undesirable, the specifications further require that the joints be free of paint and lacquer. The most important requirement of the specification, however, is that all bolts must be tightened to induce a required minimum bolt tension equal to at least 90\% of the specified proof load of the bolt. Slip cannot be prevented, regardless of the condition of the faying surfaces, unless sufficient clamping force is provided to develop the necessary friction required for stress transfer. Merely specifying the use of high-tensile bolts is not sufficient—it is also necessary that their structural capabilities be mobilized by proper pretensioning. Tension control is therefore of paramount importance in the assembly of joints using high strength bolts.

The clamping force in joint assemblies is mobilized by torquing the nut. For low clamping forces the shearing stress due to torquing is only a small percentage of the total stress. As the axial load approaches the elastic proof load, or when bolts are unloaded and reloaded by torquing, the shear rises rapidly due to the galling action which occurs in the threads. The theoretical shear stress at the bolt surface can become very large under these conditions, and the surface fibers will tend to flow plastically due to the high combined stress. These stresses are shown in Fig. 1. The shear stress can be resolved into equivalent diagonal tension and compression stresses. The magnitude of the combined stresses are readily determined by means of the Mohr Stress diagram. From this diagram it can be seen that small shearing stress components do not affect the combined stress appreciably, but large shearing stresses can increase the combined stress by as much as fifty or sixty per cent of the axial stress component. The direction of the principal combined stress is given by the angle \(\alpha\). Failure of the bolt is prevented only by the reinforcing effect of the material adjacent to the surface fibers where the shear stresses and hence the combined stress are lower. It is therefore clear that it is impossible to develop by torquing the ultimate clamping force indicated by axial tensile load tests. Under torquing, failure of the bolt tends to occur either by stripping the thread or by twisting the bolt off. Lubrication does not appreciably ameliorate this condition—probably due to the fact that the high stress intensities at the mating thread surfaces tend to break down the lubrication film.

Although this friction in the threads limits the maximum clamping force that can be developed, it does serve a useful purpose in that it prevents loosening of the nut. No self-locking nut, or lock washers are required and threads need not be burred when these bolts are properly tensioned.

**Tensioning of Bolts and Measurement of Bolt Tension**

In the early applications of high-strength bolts, bolts were tightened manually and tension was controlled by torquing to a specified torque reading. The Council’s recommendations that this torque be equivalent to a bolt tension approximately 15 per cent in excess of the Required Minimum Bolt Tension was made to insure that all bolts are tightened to the required minimum. This margin is necessary due to the fact that the relationship between torque and clamping force is not a precise one but depends on such variable factors as the coefficient...
of friction between the nut and bolt, the nut and the washer, and the slope of the abutment surfaces under the bolt head and nut. The recommended bolt tension is therefore a "target" value set at a level which will insure that bolts tensioned by this method satisfy the Council's specification for minimum bolt tension. Approximate equivalent torques have been established by test, but for optimum tension control it is desirable that torque wrenches be calibrated to produce the desired tension in the bolts under conditions equivalent to those encountered in the actual installation. Several methods of measuring clamping force have been devised. These may be classified as direct methods and indirect methods.

In the direct methods the clamping force is measured in a calibrated device. Such a device may consist of suitable abutment surfaces attached to the grips of a testing machine or to a hydraulic capsule, so that bolt tension can be read directly on a gage in terms of equivalent pressure. The abutment surfaces can also be attached to a cylindrical device to which electrical strain gages are attached, in which case equivalent bolt tension is determined with an electrical strain meter. The latter method was employed in a series of tests performed at the University of Missouri. Direct tension measuring methods, of course, can not be used to determine bolt tensions in an actual application—their usefulness lies in the calibration of torque wrenches or the evaluation of other methods for measuring the clamping force indirectly. Such instruments can be constructed to yield very accurate results and they can readily be calibrated by loading them in a testing machine.

Indirect measurement of the clamping force can be subdivided into two groups—those methods which depend on strain measurement and those which depend on measurement of torque or torque effort. Each of these methods is subject to certain inherent inaccuracies. The latter group of course includes the manual torque wrench method already discussed. Other procedures include control of torque effort and are used with mechanical impact wrenches. These procedures were the principal concern of the studies conducted at the University of Missouri.

The second group, those depending on strain measurement, include such methods as measuring the elongation of the bolt with a micrometer, the use of electrical strain gages on the bolt shank, and the so-called "turn-of-the-nut" method. The latter procedure will be discussed by Mr. E. J. Ruble. In this group the clamping force is determined on the basis of the elastic and plastic properties of the bolt itself.

**Studies on Tension Control for Bolts Tightened with Impact Wrenches**

As the high-strength bolt gained favor as a structural fastener, it was found that manual tightening of the bolts was uneconomical. Steel fabricators and erectors in their search for a more practical method of bolt installation began experimenting with pneumatic impact wrenches. It was found that a certain degree of control of bolt tension could be achieved by adjusting the air pressure of the impact wrench. As for the case with manual wrenches a "target" value of 15% above the specified minimum bolt tension was recommended for wrench calibration to take care of discrepancies in the bolts and in the impact wrenches. At first, tension of bolts tightened by power wrenches were commonly checked by manual torque readings on about 5% of the bolts; this specification was abandoned by the Council at its February, 1954, meeting.

Very little information has been published on the variations in bolt tension which might be expected when high strength structural bolts are tightened by controlled-air supply pneumatic impact wrenches. Also, no published data could be found on the relationship between torque and bolt tension after these bolts had been tensioned in this manner. A research project was initiated by Professor L. L. Howard at the University of Missouri in the fall of 1953 with the following aims:

1. To study the variation in bolt tension that can be expected in bolts tightened by pneumatic impact wrenches operating at a set pressure.

2. To determine the degree of correlation between bolt tension of bolts tightened with a pneumatic impact wrench and the torque required to start the nut turning in either a tightening or loosening direction.

Professor Howard's report embodied tests on approximately 1200 high strength structural bolts. The following variables affecting clamping force were investigated:

(a) Position of the wrench—horizontal or vertical
(b) Type of wrench—3 types representing 3 wrench manufacturers
(c) Size and length of bolt—two sizes, 3/4" and 7/8"—three lengths, 4", 5" and 5 1/2".

The pneumatic wrenches used in these tests and the
The clamping force was measured with the dynamometer and the strain indicator. For one-half of the bolts tested, the starting and running torques, to loosen the nut an eighth turn with the manual torque wrench, were recorded. The bolts were then loosened and retensioned with the impact wrench and the clamping force again measured. The torques to start the nut turning in the tightening direction were then measured with the manual torque wrench. In the remainder of the bolts tested, the order in which the manual loosening and tightening torque values were obtained was reversed.

Although some difficulty was encountered due to a change in wrench characteristics, the resulting tension control obtained was generally good. The histogram in Fig. 8 shows the clamping force distribution for 202-

![Graph showing clamping force distribution](image)

3/4" x 4" bolts tensioned under wrench conditions for which the calibration was applicable. This graph includes tests on bolts furnished by three suppliers and tensioned with three different wrenches. The clamping force distribution for both the initial tightening and the second tightening are shown. It should be noted that, in general, the clamping force developed in the second run is considerably higher than that for the initial tightening. This phenomenon is due to the "running-in" effect and was also noted in Fig. 7. The distribution curves shown below were obtained by applying a smoothing function to the data shown in the histogram. It should be noted that the frequency distribution is not symmetrical or "normal." This is due to the fact that the "target" value lies close to the maximum stress which can be developed by torquing, as can be seen from the time-tension curves in Fig. 7. As a result the average magnitude of deviations above the target value is smaller than that of the deviations below the target value. The histogram and frequency distribution curves for the clamping force of 198-7/8" bolts are shown in Fig. 9. For these bolts the frequency distribution diagram for the initial tightening cycle is more nearly "normal," whereas that for the second run is somewhat skewed due to the proximity of the modified target value to the maximum developable clamping force.

With controlled time and controlled air pressure, the mean standard deviation for all the 3/4" inch bolts tested was 2.80 kips, and 3.72 kips for the 7/8" inch bolts. These values include the tests for which the wrench calibrations were patently no longer applicable. The maximum standard deviation recorded was 5.89 kips for a set of 7/8" x 5 1/2" bolts, and the minimum standard deviation was 1.25 kips for a set of 3/4" x 4" bolts. These values are based on the assumption of normal frequency distribution.

On the basis of these tests the following conclusions were drawn:

<table>
<thead>
<tr>
<th>Wrench Type</th>
<th>Bolt Size</th>
<th>Air Pressure psi</th>
<th>Time secs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3/4&quot;</td>
<td>35</td>
<td>15</td>
</tr>
<tr>
<td>B</td>
<td>3/4&quot;</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>3/4&quot;</td>
<td>65</td>
<td>15</td>
</tr>
</tbody>
</table>

Table I. The clamping force was measured with the dynamometer and the strain indicator. For one-half of the bolts tested, the starting and running torques, to loosen the nut an eighth turn with the manual torque wrench, were recorded. The bolts were then loosened and retensioned with the impact wrench and the clamping force again measured. The torques to start the nut turning in the tightening direction were then measured with the manual torque wrench. In the remainder of the bolts tested, the order in which the manual loosening and tightening torque values were obtained was reversed.

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On the basis of these tests the following conclusions were drawn:
1. With time and pressure controlled and pneumatic wrenches calibrated at frequent intervals:
   a) Two-thirds of all 3/4 inch bolts tightened with this procedure of tension control can be expected to deviate less than 2.8 kips from the target value of 29 kips.
   b) Two-thirds of all 7/8 inch bolts, should deviate less than 3.72 kips from the target value of 37 kips.
   The range of the mean standard deviation is approximately ten per cent of the bolt tension desired, (i.e., target value).
2. Grip of the bolts does not appreciably affect the bolt tension developed.
3. Slightly higher bolt tensions appear to be developed when the bolt axis is vertical. Additional data are required to substantiate this result.
4. Bolts furnished by the three different manufacturers had similar tensioning characteristics.
5. The wrenches tested gave results of about equal consistency. Wrenches having excess impacting capacity must be operated at relatively low air pressure for satisfactory control.
6. Wrench characteristics tend to change with time and continued use. For time-pressure tension control, wrenches should be calibrated at least daily and possibly oftener.

For a group of 3/4 inch bolts whose tension ranged from 28.5 to 29.5 kips as measured by the dynamometer, the mean starting torque required to loosen averaged 320.5 lb. ft. This mean is in good agreement with the Council's specification, but the range of torque deviation was from 240 lb. ft. to 400 lb. ft. with a mean standard deviation of 49 lb. ft. This means that the range of the mean standard deviation is about 15 per cent of the target value. The mean running torque for the first eighth turn for the same group of bolts was 292.4 lb. ft. The values ranged from 200 to 360 lb. ft. with a mean standard deviation of about 34 lb. ft. The range for the mean standard deviation of the running torque is therefore somewhat better than that of the starting torque, being only 12 per cent of the mean value. But in either case, the percentage range of the mean standard deviation of the manual torque-wrench readings was greater than the percentage deviation in clamping force to be expected when bolt tension is controlled by the fixed time-pressure method.

At the suggestion of Mr. F. P. Drew, Association of American Railroads, the number of turns of the nut were recorded on 166 bolts in the last series of tests of the program. This data has not been evaluated as yet by the authors, but the data has been made available to the members of Project IV Committee of the Council and has been studied by Mr. T. R. Higgins. Mr. Higgins' conclusions were as follows:

1. When bolts are loosened and retensioned, the same tensions are achieved on the second trial as on the first, with approximately one quarter less turn of the nut.
2. The tension values for the 7/8 inch bolts were obtained with 0.6 to 0.9 tightening revolutions of the nut. All tension values observed were at least equal to the minimum specified elastic proof load. (35.97 kips)
3. The tightening studies on the 3/4 inch bolts covered a wide range of nut revolutions—from 0.8 to...
1.5. All tension values were substantially higher than the minimum specified elastic proof load and a number of them (all associated with more than one revolution of the nut) exceeded the minimum specified ultimate proof load.

(4) After the nut is tensioned to produce nearly the full ultimate proof load, this tension value is maintained through quite a large amount of nut revolution.

Since Mr. Howard’s report was issued several additional tests were made using an automatic "turn of the nut" recording device. A typical set of tension-time, nut revolution-time curves is shown in Fig. 10. An expanded portion of these curves is shown in Fig. 11. This figure also shows the clamping force—nut revolution relationship for several bolts. It should be noted that, although the tension-time, nut revolution-time curves are similar in shape, the ordinates in the latter continue to increase, albeit slowly. It can be seen that when the air pressure is adjusted properly, the tensioning time is not critical if the target clamping stress is permitted to approach the maximum value developable by torquing.

Summary and Conclusions

Three methods of tension control have been proposed to date for pneumatically tightened high strength bolts. They are:

(1) Checking a representative sample of the tightened bolts by means of a manual torque wrench.
(2) Calibrating the wrench for the desired "target" value of tension by adjusting the air pressure for a specified torquing time.
(3) Control of bolt tension by relating tension to nut revolution.

The first procedure is not only uneconomical if pneumatic wrenches are to be employed, but the accuracy is relatively lower than can be obtained by the second method. The second method has two principal disadvantages—1) it requires a calibrating device, and 2) results depend somewhat on the stability of wrench performance. In addition the human element factor is involved in achieving proper time control. This problem could possibly be overcome by some automatic device which would shut off the air supply after the proper impacting interval.

The third procedure seems to cut the Gordian Knot. It has the merit of simplicity, and it insures that the required minimum bolt tension is obtained. Under normal conditions this method should prove to be a satisfactory basis for tension control provided only a minimum clamping force requirement needs to be satisfied. On the basis of presently available data this seems to be the case; especially fatigue resistance appears to continue to increase up to the ultimate bolt strength. In cutting the Knot, however, there are a few loose ends left which warrant further study. The most important of these are:

(1) The effect of accuracy of the initial starting point. The manual starting point is specified as "finger-tight." Bolt threads may have slight imperfections requiring the use of a wrench in "making up" the bolt assembly. Even with a small wrench a 1/4 turn is readily obtained. This problem is clearly evident from the results shown in the Tension-Nut Revolution curves in Fig. 9.
(2) The effect of the grip and the number of threads under the nut. The greater the grip and the more plies, the greater the pickup in the steel. Also, since the bolts are strained into the plastic region, the number of exposed threads under the nut is important. Short grip bolts with little pickup in the steel and few exposed threads may be stretched sufficiently to throw the threads off lead.
(3) The effect of the condition of the faying surface and the slope of the abutment surfaces. Slopes of one in twenty are permissible without bevelled washers. This would account for about an additional half turn.
(4) Damage to the bolts when bolts are used for make-up bolts. One of the advantages to be derived from the use of high-strength bolts is that make-up bolts can be eliminated. The turn-of-the-nut method, however, produces a permanent set equivalent to about a half turn. Retightening with a full turn of the nut may then produce excessive strains causing stripping of the threads.
(5) The effect on the bolts and nuts of stressing into the plastic zone. More data is required to determine if any harmful after effects may be caused due to the biaxial stress condition or metallurgical changes due to a degree of cold working when bolts are tensioned to this extent. The effect on the friction factor also warrants further study.

In the final analysis the problem of bolt tensioning is a problem in quality control. Applied with a little common sense either the tension-time or the turn-of-the-nut method should yield satisfactory results. A combination of the two methods may be possible whereby the need for special devices for field calibration of the wrenches is eliminated and whereby occasional checking of nut revolution can be used to check constancy of wrench calibration, thus providing a method for quality control. For either method the practice of loosening the nut to check bolt tension by applying a manual torque wrench seems futile. In this respect the authors are in sympathy with Winston Churchill’s sentiments when he said, "When you have a thing where you want it, let it alone!"

References

I first became interested in high strength bolts when Professor W. M. Wilson, who had been doing considerable research on connections at the University of Illinois, kept insisting that the bolts would make good fasteners. I insisted that they would not stay tight in railroad bridges or in any structure subjected to heavy vibration. To prove that Professor Wilson was wrong, I arranged to put from 1,500 to 2,000 bolts in locations where we had been having considerable trouble with rivets. I will admit that our method of installing them was quite crude, yet, at the end of a few years when we made an inspection, we found that they were still tight. After the second inspection, when we found that the bolts had still stayed tight, I decided that maybe Professor Wilson was right.

Many bridge members subjected to high fatigue and vibration present the problem of keeping the rivets tight. In some places the rivets work loose every year, but the high strength bolts have stayed tight although some of the bolt tension has been lost in a very few minor locations.

Since many bridges will have anywhere from twenty to forty rivets work loose, setting up a compressor at the site and calibrating the guns is not a very practical method of putting in new bolts. The use of a torque wrench is also not very practical, so a method was needed that would insure proper installation of the bolts when the nuts had been tightened.

Committee 15 of the American Railway Engineering Association realizing that the high strength bolt was making a good fastener, yet feeling that tightening the bolts was a serious matter, asked the Research Staff of the A.A.R. to develop some means of tightening the bolts that would be practical. The turn-of-the-nut method, having been satisfactorily used by the automotive industry, appeared to be the solution. An investigation of this method was carried on at the A.A.R. Research Center during 1954.

All bolts were tightened through a grip of several plies of 5/8" steel plate or 3" steel slab depending on bolt size and length. Both torque wrenches and impact wrenches were used.

In accordance with the "Specifications for Assembly of Structural Joints Using High Strength Steel Bolts" approved by the Research Council on Riveted and Bolted Structural Joints of the Engineering Foundation, the bolts were tightened to the equivalent torque for required minimum bolt tension. It was found that about one-half a turn of the nut was sufficient to develop the required torque. Further, it was found that no failure, i.e., bolt breaking or thread stripping, occurred until the nut had been turned between two and three revolutions.

This ability of the bolts to withstand a high clamping action, already discussed by Professor Pauw, is very important, since laboratory tests have shown that the higher the clamping action, the stronger the joint. In all field tests, even where the temperature has dropped as low as forty degrees below zero, bolts tightened into the plastic range have behaved very satisfactorily.

These two slides show the results of tests performed at the University of Missouri by Professors Pauw and Howard. Bolt tension is plotted against number of revolutions of the nut for both ¾" diam. and 7/8" diam. bolts. In all the bolts tested one-half turn of the nut produced a tension equal to at least the minimum specified elastic proof load. The bolt tension increases with the turn of the nut until about one and one-half revolutions when the curve begins to flatten out. Thus, as a
result of the A.A.R. and University of Missouri tests, the A.A.R. recommended one turn of the nut as a satisfactory criterion for proper tension.

A Southern Railroad bridge across the Wabash River was completely bolted using the turn-of-the-nut method. A check of one of the floor-beam-to-hanger connections, showed the torque in the 30 7/16" diam. bolts to vary from 540 ft.-lfts. to 840 ft.-lfts. with an average of 615 ft.-lfts. In another joint the torque varied from 470 ft.-lfts. to 800 ft.-lfts. with an average of 590 ft.-lfts. Thus in both cases none of the bolts checked were below the required minimum torque of 470 ft.-lfts. Although this may seem like a wide variation in clamping action a check of a riveted joint showed the clamping action in individual rivets to vary from 3,000 to 14,000 pounds.

On one bridge the erectors had trouble drawing the steel together so they decided to use all the high strength bolts as fitting-up bolts. Then after the steel was drawn up the bolts were given an additional half turn. A check indicated all bolts to be well above the minimum torque.

In all cases cited the check consisted of giving the nut an additional 1/16th of a turn and recording the torque as the nut moves.

There has been considerable field experience gained using this method. In both buildings and bridges the turn-of-the-nut has proved to be a practical and satisfactory solution.

MR. HIGGINS: To open the discussion I would like to call on Mr. J. R. Stitt, Research and Welding Engineer for the R. C. Mahon Co. Mr. Stitt will show some slides of the calibrating mechanism which he will demonstrate tomorrow afternoon during our inspection trip through the Kansas City Structural Steel Company plant.

MR. STITT: I have a few slides to show you of the equipment which we plan to demonstrate tomorrow afternoon. Our equipment was built to satisfy the need for a portable device to set the proper air pressure on the wrench when tightening a bolt up to or above the minimum required tension. It does very much the same thing as Professor Pauw's equipment except that it can be carried right on to the job. This device has been used in the
field for approximately two years on a number of jobs.

The box is approximately nineteen inches long, and

everything is self-contained in a fifty-seven pound unit

(Fig. A). The channel which carries the load cell fits in

the bottom of the box and slips out the end door. A ten-

foot cable goes from the control box to the channel so

that the channel can be bolted to the face of a column or

beam with the box several feet away (Fig. B). This

rugged cable is going to stand up under the treatment

that an iron worker gives it out in the field.

A microammeter has been calibrated to read a certain

value for each minimum tension. The required minimum

tension for each size of bolt is shown on the printed

chart. A "B" radio battery is all that is needed to operate

the control circuit.

The hole in the load cell is slightly over one inch in
diameter so that we can use one-inch diameter bolts; or

with sleeve inserts, 3/8" and 5/8" bolts. The load cell is

machined from a solid piece of tool steel. The eight

permanently mounted variable resistance wire strain gages

measure the compression which is developed by the ten-

sion in the bolt.

We have found that as you tighten a bolt to the pre-
scribed torque value you will have the required minimum
tensile strength in the bolt. If you slack off with a torque
wrench, tighten, slack off, and tighten a few times, how-
ever, you get a different amount of torque with the same
amount of tension in the bolt. So instead of using a
torque wrench, we depend on this portable device to
calibrate our air wrenches for proper tension.

Mr. C. L. Kreidler (Lehigh Structural Steel Co.) : Is it
possible to use high strength bolts and rivets in the
same connection?

Mr. Ruble: We had that problem come up in our
study of floor beam hangers where we were developing
a large number of fatigue failures. From our laboratory
work we know that the fatigue strength of a bolted joint
is considerably higher than that of a riveted joint. There-
fore, we concluded that one way to eliminate our failures
in the floor beam hangers was to substitute high strength
bolts in lieu of the lower line of rivets.

The question then arose as to whether or not our bolts
and rivets would be working together. We fabricated
some full size joints in which we placed bolts in the two
lower lines of holes, and found the rivets were working
right along with the bolts since they both produced clamping action.

Mr. H. B. Corlett (A.I.S.C., San Francisco): Could
you have a pair of standard connection angles on the end
of a beam framing into the web of a column where one
angle is riveted to the web of the column in the shop
and, for erection reasons, the other angle is left loose to
be bolted to the column web?

Mr. Ruble: Yes, I would say that you could, because

the two are going to work right together. Your rivet
is producing clamping action just like the bolt, even
though you do not design it that way.

Mr. Corlett: Well, unless you loaded it highly, the
bolted connection would be rigid and would not slip
so you would take all of the load on one angle.

Mr. Ruble: The riveted leg will possibly slip a little
more than the bolted will.

If you were to consider it up to the ultimate load, you
would have a little different condition since your rivets
would go into shear and the steel around your bolts
would undoubtedly slip and then your rivets would pos-
sibly have to carry more than the bolted connection.

Chairman Higgins: It is rather a hypothetical case,
I believe, because in general the clamping developed by
high strength bolts is comfortably above the shear value
of a fastener. The slip load of the high strength bolt
is considerably above the allowable working stress of
a rivet, so that for a modest overload, I would suspect
that the angle which was bolted and the angle which was
riveted would share the beam reaction rather evenly.

I will concede that at some level of overload the
bolted angle might start to slip, and if it did then any
added overload would then be thrown on to the rivet.
Whether that riveted single angle, by reason of the load-
ing, would have failed completely at any lower beam re-
action than a similar connection with both angles riveted,
I think that you would have to do some testing to say one
way or the other.

Mr. T. P. Noe (Carolina Steel & Iron Co.): How
much would painting the contact surfaces reduce the load
carrying ability if you disregard any slip that might occur?

Chairman Higgins: Painting the contact surfaces
would not reduce the load carrying ability at all, but
rather the capacity would depend on what the criteria of
failure in a particular joint was. If the criterion were to
be tension of the net section, the capacity would be the
same for the riveted connection and the bolted connec-
tion. If it were the failure of a fastener, the bolted
fastener has a much higher shear value, and if you are not
concerned with slip, there would be no reason for omit-
ting the paint because it would not weaken the joint any.

Mr. R. W. Derby (Frank M. Weaver & Co.): If a
high tensile bolt is so much stronger than a rivet, even
though it has slipped, why do we put so much emphasis
on tightening them? Why not use the high tensile bolts
and tighten them up with a hand wrench and use them
that way in the structure that is not subjected to impact
loads?

Chairman Higgins: If we have a structure where slip
is unimportant and where we do not expect a stress re-
versal, we are not really concerned with a high clamping
force. The economic answer may be machine bolts be-
cause in all probability the shear value of any high
strength bolt would be something in the order of twice
the present allowable shear value of a machine bolt, while the cost is considerably more than twice.

Mr. Derby: Then I think it is up to us to educate our engineers and our architects along that line, because so often they use rivets or some equivalent fastener where it is not entirely necessary.

Mr. H. J. Stetina (A.I.S.C., Philadelphia): We have been using the unfinished common and ordinary rough bolts for many years. If you will recall, going back about twenty-five years, we had a specification that permitted the use of ordinary bolts in one-story structures and also in multi-story structure secondary beams—beams that were twelve inches and less in depth.

About 1934, we expanded the A.I.S.C. Specification to provide a more liberal use of ordinary bolts than had ever been exercised prior to that time. I would say, off hand, that a building up to fifteen stories in height can be entirely rough bolted.

To give you an example of thousands of buildings that have been rough bolted, I would like to point out just one. Perhaps the most monumental is Stuyvesant Town in New York City, where, if my memory is correct, there are about thirty-five units each of about sixteen stories in height. The total tonnage involved is about forty-two thousand tons and every bit of it was rough bolted. Now, I believe that this was done for economical reasons because at that time the cost of riveting was prohibitive.

We have been emphasizing high strength bolts for about the last four years. The bolt manufacturers have been advertising—"Do it the modern way—use high strength bolts." Our own organization has several publications on high strength bolts for distribution to engineers and architects, but what has become of rough bolts?

Are we ready to throw them aside? The average architect and engineer has the wrong opinion that the high strength bolt is nothing more than an improved rough bolt. The high strength bolt is competing with the rivet and not with the ordinary bolt which is an economical method of construction.

Has the Council on Riveted and Bolted Joints any information or any literature to show the architects and engineers that this practice of using rough bolts over the past twenty years has been a successful one and something that we want to continue in every possible place where they can be used in this country?

Chairman Higgins: The Council, a research organization, has no program for proving the validity of ordinary rough bolts. It has been proven and you have cited a very good example of it. I don't know what sort of tests could be conducted that would add anything more convincing than the number of large structures that are put together with common bolts. The use of common bolts is recommended in the Institute Specification; objection to their use results from individual contrary opinion.

I think that this is a selling job, to convince the few doubters that the use of unfinished bolts, as recommended in our specification for the past twenty years, is sound. We all know that it is economical; I don't believe that there is more testing required.

Mr. Cook (Kansas State Highway Commission): We have been concerned with tension in the bolt. Is there any way of telling when we have reached the danger point if the bolt doesn't break in two?

Chairman Higgins: Since the bolt's purpose is to produce clamping force, if the bolt hasn't broken or the thread hasn't stripped when you remove the wrench, it is a good clamp for the rest of time. It will prevent slip with no appreciable relaxation and continue to do it.

Now, if the nature of the loading is such that the fastener, after it has been heavily prestressed, is going to resist tension, one might expect that there is some upper limit at which the initial tension would have an effect upon the endurance strength of a bolt.

At the University of Illinois, a series of tees were bolted together with their flanges back to back. The pre-stress in the bolts was varied in each joint and the joint subjected to various loading cycles. In every case, for any given cycle of loading the endurance limit of the bolt was improved by pretensioning. Even the bolts that were over-stressed twenty per cent, (pretensioned to 120 per cent of the elastic proof load), had a higher endurance value under cyclic loading than similar bolts pretensioned by a lesser amount.

Therefore, I think that we can safely conclude that there cannot possibly be any injury to the structure in any way if your bolts are pre-tensioned in the order of twenty per cent beyond the elastic proof load.

Mr. L. A. Villard (Bethlehem Fabricators, Inc.): From the kind of structures that have been described I strongly suspect that we are talking about conditions where the holes have been drilled or sub-punched and reamed.

What, if any, tests have been made on punched holes that would come from an average fabricating plant?

Chairman Higgins: There is a program under way now. It was not the thought of the Council at the time that it approved the program that there would be a significant difference in the behavior of a joint which had punched holes and a joint which had drilled holes, because it is a matter of clamping force rather than the bearing of the fastener against the side of the hole. That subject, however, is being stressed in connection with the condition of some of the holes being out of line, so that some of the fasteners are practically in bearing before any slip of the joint as a whole has taken place.

Mr. Villard: We have run into a situation on a series of buildings where an architect insists that these bolts be used in recommended holes, and I could not find any evidence that they had been tested in punched holes.

Chairman Higgins: I recall seeing specimens which
were fabricated by punching, but I don't recall in which of the Council-sponsored programs. There is a program in progress at the University of Washington investigating a great many of these conditions, low temperatures, use of paints, the poor matching of holes, punching versus drilling—a sort of a clean-up program to answer some of these questions which the Council up to this point never thought were significant factors.

Mr. S. R. Webb (Carolina Steel & Iron Co.): I saw an example a few days ago where an erector had used high strength bolts too short to go through the nuts. If that nut were securely tightened, would there be an element of danger in it?

Mr. Ruble: Well, most of the load is resisted by the threads from the machined underside of the nut to about three-quarters of the way up into the nut. In other words, the upper part of the nut does not carry a great deal of load. However, you certainly would not want to use it for any dynamic loading, but under static loading it would be satisfactory.

Mr. J. S. Davey (Russell, Burdsall & Ward Bolt & Nut Co.): I would say that if the nut withstood the minimum required torque that it would be satisfactory.

Mr. Stitt: I have at times used bolt lengths that missed the last thread in the nut and had the threads strip right out of the nut. I think that if that last thread had been used there would have been no stripping. Therefore, I am not in agreement with the remark that only the three-fourths of the threads are operating.*

Chairman Higgins: Mr. Ruble pointed out that he was not offering that as a recommended practice but that in some of the experimental work there was evidence that there was very little contact on the outer threads.

Mr. E. E. Hanks (A.I.S.C., Greensboro, N. C.): I would like to ask Mr. Stitt where the device for calibrating the impact wrenches can be obtained.

Mr. Stitt: The device, that we have here at the conference, is the first one that we have built and has been in use for several years on our own jobs. I am building two more at the present time but they are not for sale. They are not the easiest thing to make, and the price would have to be somewhere around a thousand dollars for the complete unit.

Of course, if Mr. Ruble had his way, we would not need any device since we would just turn the bolt 360 degrees.

However, if there is enough interest and demand for the calibrating device, we could possibly manufacture them for sale.

Mr. D. A. Beam (C.I.S.C.): Have you any idea as to the proper starting point for finger tightness on a nut and bolt through a sloping flange? Would we have had to have a partial turn to take care of the looseness of the nut due to the slope of the flange or could we get up to the proper starting point with some other method?

Mr. Ruble: I would throw away this one turn criterion entirely and just keep on turning the nut until you bring the steel into contact—until your nut bears completely on your washer. You must deform the bolt until all surfaces are in contact.

Mr. Beam: And then give the nut another turn?

Mr. Ruble: I wouldn't say that you give it another turn since you have already given it about three or four turns and deformed the bolt to obtain full contact on the washers.

Question: Is there any difference in tightening the bolt by the head rather than by the nut? We have had conditions in railroad bridges where you cannot get at the nut and we tighten by the head so I am sure it works all right.

Chairman Higgins: I am sure that it is done very frequently. Mr. Ruble says that they do it where they have no access to the nut—they put the wrench on the head of the bolt. I can personally see no reason why that would not be the logical way to do it.

Mr. P. J. Foehl (Midwest Steel & Iron Works Co.): In tightening bolts where your metal does not come together to begin with, for instance, girder splice plates, with springiness keeping them slightly apart, isn't it going to take more than that one turn to draw the bolt up to its full tightness?

Chairman Higgins: Mr. Ruble mentioned earlier that the recommendation of the Association of American Railroads is that you use enough fitting-up bolts to draw the parts up into intimate contact, just as if you were about to drive hot rivets, and then to insert high strength bolts into the other holes. First make the nuts on these latter bolts finger tight and then give each one a complete turn. That takes place after the parts have been brought into intimate contact.

Mr. C. M. Corbit (A.I.S.C., Los Angeles, Calif.): Has there been any thought given as to how an agency would inspect bolts that would be tightened with one turn?

Mr. Ruble: I would make a spot check of the bolts with a torque wrench. Of course, you recognize that the torque wrench is not a scientific instrument, but it is a practical tool for telling whether you are way off or not. Just put the torque wrench on, and turn the nut a sixteenth of a turn. We have found that where this method is used all the bolts have developed the minimum required clamping action.

Chairman Higgins: The ironworker's task is not a difficult one. Having noticed the position of the marker on the chuck when he starts to impact, all he has to do is to make one complete revolution. A plus or minus of ten degrees is not going to make any material difference.
If he is operating a wrench that could produce one turn in a reasonable length of time, there would be nothing to encourage the ironworker not to make a complete revolution. Therefore, I believe that the spot check that Mr. Ruble has in mind would be adequate to convince most inspectors that the work had been satisfactorily performed.

A further assurance to the skeptical inspector, if he is not going to be present to watch the one revolution, would be, after the bolts had been made finger tight, to put a mark on the nut and on the steel just outside of the nut. If both of these marks are opposite one another at the time of inspection, he would know that it had had one revolution or hadn't had any. It would not be much of a task to check with a spud wrench to determine which was the case.

Dr. Pauw: I would like to say that we noticed, when we tensioned the bolts, that the wrench left definite markings on the nut and therefore there should be no question, if the inspector has average eyesight, whether the nut is tensioned.

Mr. R. W. Binder (Bethlehem Pacific Coast Steel Corp.): It is difficult to keep the threads of common bolts out of the holes, and, therefore, what value would we assign to a high strength bolt without washers to replace the common bolt with the threads out of the hole? Could we swap one for the other?

Chairman Higgins: I have never been able to quite convince myself as to what virtue there was in a one-eighth increase in allowable value of a bolt just because you had used washers under the nut to keep the bolt thread out of the grip. (See A.I.S.C. Spec. Sect. 22-e.)

The University of Illinois has tested to failure joints with high strength bolts where the thread went all the way through the grip and report no weakening due to threads in bearing.

Mr. H. W. Brinkman (Phoenix Bridge Co.): Would you recommend the unlimited substitution of high strength bolts for rivets, say in a large plate girder?

Mr. Ruble: I do not see any reason why you should not use them. I believe that you have to get used to those nuts and washers sticking out.

Mr. W. F. Thompson, Jr. (Kline Iron & Metal Co.): Do you suppose that we are using the absolute capacity of these high tensile bolts, rating them equal to rivets? Is it possible that they may be given higher values at some future date?

Chairman Higgins: They are rated as only the equivalent of rivets in order to assure no slip of the joint. If you are not concerned about the slip of the joint there is no reason why they need be given a value as low as that of a rivet.

Mr. Thompson: Could you give some idea as to what value could be assigned?

Chairman Higgins: A working stress of 20 ksi in shear would be on the low side, based on test results which have been reported from the University of Illinois, where the failure load of the bolt was between 1 ½ to 1 ¾ that of rivets of the same size.

Mr. Ruble: The Council has a project to determine the static and fatigue strength of alloy steels. We cannot show exactly that the fatigue strength of alloy steel is any higher than that of carbon, using both A141 and A195 rivets. However, if high strength bolts are used with alloy steels, then the fatigue strength jumps way up, probably greater than that for carbon steel. It is more in line with the yield strength of the steels, so that eventually we can assign some values to alloy steels using high strength bolts which will be larger than is possible with rivets.

Chairman Higgins: Those of you who were in Milwaukee at our conference last year will recall that we had a symposium on the subject of ultimate strength design. As part of that symposium John Griffiths showed us how to design a single span rigid frame in less than five minutes. We are going to continue the demonstration today, with multi-span rigid frames.

Our first speaker was introduced to you this morning. Mr. Estes, the new research engineer for the Institute is a native of Richmond, Virginia, and a graduate of Tulane University. After a short tour with the Navy he received his master's degree at Virginia Polytechnic Institute, following which he was structural engineer for a firm of Richmond architects. Since 1948 and until he joined our staff on the first of February he was an assistant professor at the University of Virginia and was a partner in the consulting firm of Kinnier & Estes.

Design of Multiple-Span Rigid Frames by Plastic Analysis

Edward R. Estes, Jr.

This method of structural design has acquired a variety of names—ultimate design, limit design, plastic design and collapse design. No matter what you call it, the method recognizes the logic of a more complete utilization of the strength of structural steel than heretofore.

Statically indeterminate structures, as we know, are "indeterminate" because we cannot write enough differ-
ent equilibrium equations to solve for all the unknowns. So, in ordinary design techniques, it becomes necessary for us to develop additional equations, based on deformations, or else to make simplifying assumptions, such as assuming "hinges" at approximate points of contraflexure, in order to analyze the structure for stresses below the elastic limit. However, in the plastic design method, we begin with the fact that, if the frame is sufficiently overloaded, plastic hinges will form at certain points, thus enabling us to very quickly determine the maximum bending moment.

Our first step then is to locate these plastic hinges. In a continuous frame there may be several possible combinations of hinges, in which case we must determine which combination will develop first as a particular type of loading is increased in magnitude. Equating the work done by the external loads, to the work done by the internal moments at the hinges, for each possible combination of hinges, it can be shown that the least value obtained for the given type of loading will determine the actual combination of hinges at ultimate loading. Perhaps you remember the sarcastic jokes about "methods of least work" that amused you in your school days. Actually, in plastic analysis, as compared with present methods of elastic analysis, the term "least work" takes on a real meaning with respect to design time involved.

Let us consider the case of a single span rigid frame of uniform cross-section, having hinged column bases, a height $b$ and span length $L = 3b$ and loaded as in Fig. 1(a) with vertical and horizontal concentrated loads of equal magnitude.

In Fig. 1(b) we will consider the maximum value of the vertical load $P$, acting alone, in terms of $M_p$, the ultimate (plastic) bending strength of the shape to be used in the construction of the frame. The plastic hinges are represented by solid dots. Angle changes due to rotation at the plastic hinges become so large, relatively, as the ultimate value of $P$ is reached, that we can neglect the familiar elastic deformation curve. However, the angle $\theta$ is still too small to be seen with the eye.

For all practical purposes, the internal work can be calculated as the sum of the products of the ultimate bending strength of the members times the angle change at each plastic hinge.

The corresponding external work consists of the vertical travel of the load $P$ through the distance $\Delta_1$. Since the angle $\theta$ is a small angle, $\Delta_1$ can be expressed as $\Delta_1 = 0.5L\theta$.

Equating internal to external work

$$M_p\theta + M_p(2\theta) + M_p = P \cdot (0.5L\theta)$$

$$P = 8.0M_p/L$$

Consider now the case of the horizontal load $P$, as shown in Fig. 1(c). With the formation of 2 plastic hinges, each having an ultimate bending resistance equal to $M_p$, $P$ moves through the horizontal distance, $\Delta_2$.

Again equating internal to external work

$$2M_p\theta = P \Delta_2 = PH\theta = \frac{P L \theta}{3}$$

$$P = 6.0M_p/L$$

It is seen that the given frame will take a larger vertical load, acting alone, than it will a horizontal load acting alone.

Now suppose that horizontal and vertical loads, of equal magnitude $P$, are applied simultaneously. The frame can resist no more of this type of loading than needed to form the two plastic hinges shown by solid dots on Fig. 1(d). Note that the corner of the frame at which the horizontal load is applied has not reached the plastic range of stress and, even in its rotated position, still remains a right angle corner. $\theta$ being a very small angle no perceptible change in beam length occurs and both columns are displaced the same amount horizontally at their tops. Therefore the right-hand column also rotates $\theta$ away from the vertical.

The total internal work is equal to

$$2M_p\theta + M_p(\theta + \theta) = 4M_p\theta$$

and the corresponding external work is

$$0.5PL\theta + PL\theta = 0.83PL\theta$$

Equating these

$$4M_p\theta = 0.83PL\theta$$

$$P = 4.8M_p/L$$

For this condition of loading, the ultimate value of $P$ is less than for the case of either of the single-load conditions. This last condition giving the lowest value of $P$ would govern the design.

It is interesting to note that by reducing the horizontal load from $6.0M_p/L$ to $4.8M_p/L$, the frame is found to have a reserve of strength, due to its continuity, so that it can, at the same time, support a midspan vertical load of $4.8M_p/L$. This is a virtue possessed by continuous frames constructed of ductile materials which is totally ignored in the elastic design concept.

In the following example it was assumed that the beam and columns would employ the same wide-flange shape. It might have been desired for some reason to use a beam having a greater or lesser ultimate bending strength than that of the columns, although any radical difference in sizes, for a frame of usual proportions, would entail some sacrifice in economy of material.

Whether the structure is analyzed elastically or plastically, the design of any continuous frame involves two steps. First, tentative assumptions are made as to the relative strength or stiffness of the individual members that might be suitable for the given conditions of load, spans and column lengths. This is then followed by an analysis to ascertain what the given system of loading
will do to a frame having members of these assumed proportions, in order to determine their fitness for the given conditions.

If the test of their fitness is based upon some limiting working stress in the elastic range, as in current practice, the ratios between the several members are measured in terms of their moment of inertia $I$. In plastic analysis they are based upon the ultimate bending strength of the several members. Just as it is convenient to express all moments of inertia in terms of that of the weakest member in the frame—letting $I$ for this member equal 1.0—so too is it convenient to express the plastic hinge values of all the members in a frame in terms of that of the weakest member. Thus if we wanted the beam to be twice as strong as the columns we would use $2M_p$ in calculating the internal work done at hinges in the beam but only $M_p$ in the columns.

Without further discussion we will apply these principles to the ultimate design of the two-span continuous frame shown in Fig. 2(a).

To simplify our internal-external work calculations we can replace the total uniform loading on each span with concentrated loads at the $1/4$ points.

In this problem the concentrated loads at the quarter points as shown in Fig. 2(b) give an expression for external work equal to the external work done by the uniformly distributed load.

Since we are basing our design on the ultimate strength of the structure we will multiply our design load by a load factor instead of dividing yield stress by a factor of safety. In the elastic design method the factor of safety is equal to the yield stress of 20,000 psi divided by the allowable stress of 20,000 psi. Keeping this same factor of safety (1.65) in the plastic method and basing our load factor on the plastic moment $M_p$ instead of the yield moment $M_y$ our load factor becomes 1.88. The ratio of $M_p$ to $M_y$, known as the shape factor, has a reasonably constant value of 1.14 for all $I$ shapes. Using a load factor of 1.88, the ultimate load to be carried on the 75 ft. span would be

$$P = 1.88 \times \frac{35}{L.F.} \times 75' = 423^k$$

In the 45 ft. span it would be

$$\frac{45}{75} \times 423^k = 0.6P$$

In Fig. 3, the relative ultimate bending strengths for the various members have been assumed. Any number of combinations might be tentatively selected, but not all with the same degree of economy. From this standpoint, it would be ideal if the critical values of $P$, expressed in terms of $M_p/L$, could be the same for any conceivable combination of plastic hinges producing a mechanism.

With this in mind let it be noted that the total bending moment produced by the uniformly distributed load in the 75 ft. span is $\frac{75^2}{45^2}$, or 2.78 times the total bending moment produced in the 45 ft. span.

Representing the plastic hinge value of the 45 ft. beam as $M_p$, we will assume hinge values for the outer columns equal to that of the beams to which they connect. Finally we will assume a hinge value for the interior column such that the combined ultimate bending strengths of the 45 ft. beam and the interior column is not less than that of the 75 ft. beam.

In Fig. 3, Cases (a) and (b) show plastic hinges so located as to produce the mechanism by which each of the two beams would fail individually if more than the ultimate load were applied to them. Note that, for the relative plastic hinge values selected, the ultimate value of $P$ in each case, based upon internal-external work equations, is the same. It is also the same for the combined mechanism shown in Case (c).

From either case then the minimum required value of $M_p$, for the previously computed 423$^k$ ultimate load $P$ can be calculated as

$$M_p = \frac{423^k \times 75'}{44.4} = 715^k$$

Then, for the left-hand column and 45 ft. beam, 

$$\text{Req.}S = \frac{715^k \times 12}{1.14 \times 33 \text{ ksi}} = 228 \text{ in.}^8$$

Use 33WF130. (404.8)

For the interior column,

$$\text{Req.}S = 1.78 \times 228 = 405 \text{ in.}^8$$

Use 33WF130. (404.8)

And, for the right-hand column and 75 ft. beam,

$$\text{Req.}S = 2.78 \times 228 = 634 \text{ in.}^8$$

Use 36WF194. (663.6)

I will not take the time to run through a moment—distribution elastic stress design, correcting for side-sway. Fig. 4 shows such a solution. It took me several hours to complete these computations. Perhaps I am not a fast worker but three cycles of moment distribution including the balancing of shears is a little time consuming. However, it only took me several minutes to complete the plastic design which weighs about 9% less than this elastic solution, as shown in the comparison in Fig. 5. Incidentally, this elastic design is based on the 20% increase in allowable stress permitted for continuous beams and their supporting columns, a concession, already in our specification, to the reserve strength in continuous structures. I also want to call to your attention that while the plastic design is based upon the formation of plastic hinges, the actual stresses at working load are within the elastic limit.

In January, John Griffiths published a paper in the
Fig. 1

\[ M_p \theta + M_p[2\theta] + M_p\theta = P \Delta = P[5L\theta] \]

4 \( M_p \theta = 0.5 \) \( PL\theta \)

\( P = 8.0 \) \( M_p/L \)

\[ M_p \theta + M_p\theta = P\Delta = P[0.33L\theta] \]

2 \( M_p \theta = 0.33 \) \( PL\theta \)

\( P = 6.0 \) \( M_p/L \)

\[ M_p[2\theta] + M_p[\theta + \theta] = P[5L\theta] + P[0.33L\theta] \]

4 \( M_p \theta = 0.83 \) \( PL\theta \)

\( P = 4.8 \) \( M_p/L \)

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**Factor of Safety**

Elastic Design: F.S. = 1.65

Plastic Design: F.S. = 1.65 \( \frac{M_p}{M_Y} = 1.65 \times 1.14 = 1.88 \)
I would like to borrow one of his frames from him for an example so that I will not have to calculate the elastic bending moments. I'm sure he won't mind lending it to me because he borrowed it from Martin Korn's book, "Steel Rigid Frames." Mr. Korn solved the problem by the method of work; 7 pages and untold hours of a lot of work; a good example of the laborious computations necessary for an elastic solution by the classical method. While John's method saves time and labor compared to the classical method, the plastic analysis (Fig. 6) eliminates an even greater amount of computation.

In the plastic analysis of this frame subjected to uniform vertical loading the critical combination of hinges is shown. This problem has been worked for one type of loading only to use as a comparison with Mr. Griffiths' elastic analysis. If the frame had been subjected to horizontal loading or unbalanced vertical loading, other hinge combinations would have been considered. Since we have simplified our solution by replacing the uniformly distributed load by concentrated loads at the 1/4 points we introduce an error on the safe side. In order to determine that the actual location of the hinge shown at "e" is closer to "d" there are a few more calculations involved although still not as many as in the elastic design.

The elastic design for this frame required an 18WF60 for the rafters and an 18WF55 for the exterior columns. These sizes are based on an allowable stress of 24,000 psi. Thus the elastic design is about 2% lighter than the approximate plastic design consisting of 18WF60's throughout but about 7% heavier than the 18WF55 members required by the exact plastic method.

A further refinement of this design by Mr. Griffiths resulted in the columns being tapered from 16WF45's so that the section at the knee was 22" deep. This additional stiffness at the knee permits him to use the 16WF45 for his gables also. However, compared to the exact plastic design, this reduces the factor of safety against ultimate failure by the ratio of the section moduli times 1.88, which equals 1.39, because the plastic hinges will form in the rafters as in Fig. 6. While this is a lighter frame there is also the increased cost of fabrication to consider.

The three span lean-to bent shown in Fig. 7 is subjected to both vertical and horizontal loading. The same simplifying condition of concentrated loads at the 1/4 points is used in this analysis.

Since the frame is subjected to wind loading we are permitted a 33 1/3% increase in allowable stress by present specifications. However, the stress due to dead and live loads must not exceed 20,000 psi. Thus we must investigate the structure with and without the wind force acting.

Investigating the main span we find that hinges form at e and midspan due to vertical loading only. To satisfy this condition we require a 16WF40.

For combined loading we will reduce our load factor.
by 25% instead of increasing our allowable stress by 33 1/3% giving us a load factor of 1.41. The formation of hinges over the entire structure is shown in Fig. 7. For the condition of combined horizontal and vertical loads the hinge in the beam has shifted from midspan to the windward 1/4 point. Equating our external and internal work we see that the required section modulus is smaller than that furnished by the 16WF 40 which was required to support the vertical load in the main span. Using the actual plastic hinge value of the 16WF 40 ($M_p = 64.4/32 = 201$) in the main bent we can calculate the required $M_p$ for the lean-to. Note that there is no change in the external work for the same system of hinges; the only change being in the internal work. The 14WF 30 chosen for the lean-to also satisfies the condition of dead and live loads only acting on the structure.

Wasn't that easy? Now look at Fig. 8. That represents the third cycle of the elastic design. The design of this frame by moment distribution represents a good day's work. It even takes longer to outline the procedure of the elastic design than to solve the problem by plastic design.

Now let's compare the two frames. (Fig. 9) There is almost a 6% saving in steel to go along with a very great saving in design time.

At this point I think it should be brought out that in these small light frames the principal saving occurs in the design time involved. However, in the larger frames the savings in material used is appreciable in addition to the time saved.

Fig. 10 shows a 4-span gabled frame. The elastic design time is a matter of days, but my cohort this afternoon, Professor Ketter, whipped out this plastic design in a couple of hours investigating all possibilities including an equilibrium check. In the plastic design I have shown the interior columns the same size as the rest of the structure. Actually, their design is not governed by the plastic analysis and they could have been shown as a lighter section.

You will note that the hinge has been shown as occurring at the ridge instead of at the concentrated load. In this example the plastic hinge would actually form so close to the ridge, that this approximation is in error by less than 1%.

Now suppose that the problems which I have presented today had been actual problems confronting a practicing structural engineer. Designing the four problems elastically would have required about one week's time. However, had plastic analysis been used the design time would have been reduced to about one day. Thus
Gravity Loading:

\[ P = 1.88 \times 1 \times 40 = 75.2 \text{k} \]

\[ M_p \theta = 2 \times 0.5P \times \frac{1}{4} \theta = 25 \text{ PL}\theta \]

\[ M_p = P \text{ PL} = 75.2 \times 40 = 188 \text{k} \]

\[ S = \frac{188 \times 12}{33.1} = 60 \text{ in}^2 \]

\[ 16 \text{ W} = 40 \quad S = 64.4 \text{ in}^2 \]

Combined Wind and Gravity Loading:

33% increase in allowable stress
Reduce safety factor by 25%
\[ P' = 0.75 \times 75.2 = 56.4 \text{k} \]

\[ 3 \times \frac{5}{3} M_p \theta = 2 \times 0.5 \frac{(45.60)\theta + 2 \times 3.0 \times 64 (11.2) \theta}{4} \]

\[ + 0.5 \frac{P \theta}{4} (1 + \frac{1}{2}) = (22 + 12 + 14) \text{ PL}\theta \]

\[ B M_p \theta = 0.507 \text{ PL} \theta = 1/43 \theta \]

\[ M_p = 143 \quad S = 45.8 \text{ (does not control)} \]

Since gravity loading requires main frame to be 16 W = 40, which has \( M_p = 56.4/32 = 201 \),
determine \( M_p \) for this load.

\[ 3 \times \frac{5}{3} M_p \theta + 1/3 \theta = 0.507 \theta \]

\[ M_p = 114 \quad S = 36.5 \text{ in}^2 \]

\[ 16 \text{ WF} 30 \quad S = 41.8 \text{ in}^2 \]
Elastic Design - Exterior Columns - 12 WF 40
Interior Columns - 12 WF 27
Rafters - 12 WF 27
Design Time - Days

Plastic Design - Exterior Columns - 14 WF 30
Interior Columns - 14 WF 30
Rafters - 14 WF 30
Design Time - Hours

\[
\frac{(10 + 10 + 31)}{(12 + 6 + 12)} M_p \theta = \frac{2 \times 19 \times 6L}{2 \times 12} \times \frac{1}{4}
\]

\[
M_p = PL = 123\text{ kips}
\]

\[
P = 18.5 \frac{M_p}{L}
\]

\[
S = 39.4 \text{ in.}
\]

\[
14 \text{WF} 30 \frac{\text{in}}{S = 41.8}
\]

Fig. 9

760 lbs/ft

Fig. 10

\[P = 1.88 \times 76 \times 40 = 57.2\text{ kips}\]
the engineer would be able to spend more time developing the most economical design and would have time for additional work.

In case you want to try this method on a statically indeterminate structure of your own choosing let's review quickly the basic steps. Remember that we're doing the least amount of work compared to elastic analysis. As the hinges form due to the proportional increase of the load, we are only concerned with the relative displacements. For each possible combination of hinges we equate the internal work done by the plastic moments acting through an angular displacement, to the external work done by the loads on the structure acting through a linear displacement. The combination of hinges which permits the smallest loading controls. The plastic moment and the required section modulus can then be computed. Remember to multiply your design load by a load factor and to take advantage of the beam's shape factor in calculating section modulus.

You now have the design tool which, with practice on your part, will permit you to more fully utilize structural steel.

CHAIRMAN HIGGINS: The work that has been sponsored at Lehigh over the past several years with respect to ultimate strength analysis is fast reaching completion. I think that we can 'safely say that we have the answers to most of the questions that could be raised by anyone seriously contemplating the design of structures according to the matter here involved.

This really isn't so radical as it seems. I don't think we have ever tried to determine what the stress at the site of a hole in a riveted connection is or what the distribution of stress in a rivet group is. The best that we can ever do is to find out what the ultimate strength demonstrated by tests is and then work backwards to determine some allowable values for rivets. We are only applying that same philosophy to a continuous frame as a whole.

Because we are fast approaching the point where this method can be employed, the Institute is sponsoring at Lehigh University a summer course in which the faculty at Lehigh, who have been deeply involved in the investigation, will present a series of fourteen lectures. The course has been scheduled for Wednesday, September 7th through Thursday, September 15th. The daily program will consist of two morning lectures and an afternoon laboratory demonstration of the validity of the theory.

We are particularly anxious that engineering educators become acquainted with what we have done and what we have learned because we believe that we have progressed to the point where plastic design ought to be a part of the engineering mechanics and structural engineering courses at the undergraduate level in all engineering schools in the country. Hence we are very hopeful that there will be a strong representation of engineering educators able to attend this conference.

Within the limit of the accommodations on the campus, the attendance will not be limited to engineering educators. I have already become aware that a number of engineers in our own industry want to and probably will attend the course.

One of the lecturers will be our next and final speaker this afternoon. His contribution to the discussion this afternoon will be largely visual.

Professor Robert L. Ketter of Lehigh University is a graduate of the University of Missouri receiving his B.S. degree in 1950. He has a master's degree from Lehigh University and has nearly completed the work for his doctor's degree there.

The Use of Models in Plastic Design

ROBERT L. KETTER

As Mr. Estes has so clearly pointed out in the preceding presentation, given a structure subjected to certain prescribed loads, there exist a given number of points at which plastic hinges may develop. For each geometrically possible combination of these plastic hinges, there is a corresponding load required to cause the structure to fail in the assumed manner. Since the structure will fail at the first opportunity, the correct combination of these possible hinges (or as it is usually termed, the correct mechanism) is the one that results in the lowest failure load.

For the simpler types of structures the correct mechanism can be predicted with a fair degree of certainty from the start. As redundancy increases, however, the number of possible plastic hinges also increases, and therefore the number of possible hinge combinations increases at an even greater rate. With little experience it has been possible to predict within two or three tries the correct hinge combination for most frames. None-the-less, based on the reasons that have just been set forth, the question has been raised, "Can models effectively be used in plastic analysis?"

Obviously the most straight-forward type of model analysis would be to load to failure a model of the structure in question that had been scaled down to a reasonable size. Using such an approach, it would be necessary to
fabricate a model for each of the structures in question, i.e., welding of joints, etc. Depending on the size and relative stiffness of this model, the loads involved might become rather large. Since, as was pointed out earlier, the points at which plastic hinges may develop are usually well defined, one solution to this excessive load problem would be to machine down the model at these points. Then, depending on the amount of machining and the scale factor of the model, the loads required to produce failure could be kept within reasonable limits.

This machining down of the section suggests a further modification or refinement to this procedure. The sections between "hinges" could be made up of relatively rigid bars with means of clamping a thinner piece of metal at their ends to act as the "hinge." These inserts could be made of mild steel, brass, copper or any other suitable material. Variation in stiffness throughout the structure could be achieved by changing the cross-sectional dimensions of the insert.

In ultimate strength analysis as determined by plastic methods, the elastic portion of this moment—curvature relation is not usually considered. The important condition is that, for a relatively constant moment value \( M_p \), curvature can increase indefinitely and thereby allow redistribution of additional bending moments within the structure. If a mechanical hinge could be developed which had deformation properties, i.e., moment—unit rotation characteristics, of the type shown in Figure 1 or more specifically of the type shown in Figure 2, then these mechanical hinges could be used in place of the inserts discussed a few moments ago. You will note that the elastic range has been eliminated in Figure 2, the hinge being first of all rigid and then more or less plastic.

The immediate problem then is the development of such a mechanical hinge.

The behavior of the hinge shown in Figure 3 is not unlike that desired. If you will pardon the expression, this is essentially a "prestressed" hinge. By precompressing the central swivel against each of the basic rotating parts (A) and (B), external moments can be applied to the assemblage up to a value equal to that set-up in the unit due to the prestress force without rotation occurring. [The method used in Figure 3 to provide this initial force is that of a coil spring acting against the swivel and against a screw that passes through a slot in the swivel and attaches to the end parts (A) and (B).] For applied moment values greater than that due to pre-stressing, the hinge will open as shown in Figure 3c with the amount of rotation being determined by the constant of the spring. For the case shown the swivel remains in contact with part (A) while causing further compression of the spring on side (B).

Before continuing, let us consider the actual behavior of the plastic hinge we are trying to simulate. In Figure 1 is shown a typical bending moment—curvature (i.e., rotation per unit length) plot for a wide-flange section bent about its strong-axis. As shown, if moment increases from zero, there is observed first of all a region wherein curvature increases linearly with moment. Above this range curvature increases at a greater rate, the maximum moment being the full plastic moment, \( M_p \).

FIG. 1

![Diagram of bending moment-curvature relationship](image1)

![Typical behavior of bending moment and rotation](image2)

FIG. 2

![Mechanical hinge with spring](image3)

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* 36 *
The main disadvantage of this type of hinge is that once it starts to open, moment increases rather rapidly with curvature. This situation could be remedied by using a "soft" spring. A more positive approach to the problem, however, would be to replace the spring with a member that remained rigid up to a certain predictable point and then deformed under that constant critical value of thrust. For example, an elastic column that has been loaded beyond its buckling load (Euler load). This then leads us to Figure 4, another possible type of hinge.

This hinge (Figure 4) is basically the same as that shown in Figure 3. The only differences are with regard to the design of the center swivel, the use of a leaf spring column instead of a coil spring and removable wing plates that connect end parts (A) and (B) to the center rotating pin. The springs are designed slightly longer than the openings thus insuring initial compression in the system. To change the stiffness of this hinge so that it could be used to represent various members within a structure, it is necessary to either (a) provide a means for changing the effective length of the spring, or (b) use different sizes of springs for the different members. While the latter solution may seem expensive or time consuming it may have definite advantage for demonstration type models since geometrically equal size springs could be assumed to produce hinges of equal stiffness.

Having described these two possible types of hinges, the next consideration is that of calibration.

Figure 5 is a set-up that could be used to calibrate either of the above described devices. The hinge is rigidly attached to a bracket that is in turn attached to a table. To the opposite end of the hinge is connected a rod to which is applied a load, P. The moment of this load about the hinge is equal to P\cdot a. As the moment is increased, whether by increasing P or a, there is reached a point at which the hinge opens as shown by the dashed lines. It is this value of critical moment that is important. It should be pointed out, however, that the exact value of the moment in terms of inch-pounds, etc., need not be known. By using a standard weight, relative stiffness of hinges will be a function of the distance a.

Assuming that we now have developed hinges that behave as desired, how will these be used in a typical model analysis solution? Consider for example the set-up shown in Figure 6. The structure being analyzed is shown by the line drawing in the upper part of the figure. Since hinges can develop at each of the points of load application, at the junctions of two or more members and at the column bases, a model of the type shown is constructed. As you will note, the model is tested in the horizontal position. This is to eliminate as much as possible the influence of the weight of the hinge itself. Moreover, the model is floated on ball bearings and glass pads to reduce the influence of friction. Loads are provided by dead weights, which are progressively increased from zero to their maximum value. Knowledge of the locations of the hinges that develop at this maximum load enables one to compute the ultimate strength of the structure.

If it were desirable to carry through a complete model analysis to the actual specification of the required section moduli for the individual members, the only additional information that is needed is the ratio of the model failure load to the standard weight used in calibrating the individual hinges. Then knowing the scale factor of the model and calibration set-up, the actual loads applied to the structure and the above load ratio, the required section moduli could be computed.

This (Figure 7) is a demonstration model of the two-span gable frame described and analyzed by Mr. Estes. The specified uniform load has been approximated by concentrated loads at the midpoints of each of the rafters as in his example. Instead of using four independent weights to load this model, loads will be applied through
a system of distributing beams such that only one concentrated force is required. Using this system, any combination of loads can be applied to the structure with a minimum of effort. For example, consider the case where only the windward slopes of each of the bays is loaded (See Figure 8). You will note that under this loading hinges form under each of the points of load application, at the top of each of the exterior columns, and in the rafters adjacent to the interior column. (These are designated by arrows in the figure.) A knowledge of these hinge locations enables us to compute the ultimate strength of this frame under this loading. (See for example Figure 5 in talk by Mr. Estes.) Other loading conditions could also be considered and hinge locations determined. It is felt, however, that this is sufficient to illustrate the basic idea.

The hinges that have been used in this model are of the type illustrated in Figure 4.

This has been a discussion of a model analysis development for the solution of problems in plastic design. At present the device seems to have its greatest application as a classroom teaching aid. There is a possibility, however, that its use may prove equally effective to the designer in the solution of certain of the problems encountered in structural design.

**Chairman Higgins:** Thank you for that very excellent demonstration. I am going to introduce one other member of the Lehigh task force to join in the discussion, Dr. L. S. Beedle, Assistant Director, Fritz Engineering Laboratory.

**Mr. S. R. Webb (Carolina Steel and Iron Co.):** Does it make any difference where the hinges are placed in the gable member? Would it matter if you brought the hinges down closer to the haunch of the frame?

**Mr. Estes:** The hinges are going to form under the concentrated load as shown. If you analyzed it with strictly a uniform load you would find that the hinge would form close to the peak, but with a concentrated load the hinge is going to form under the load.

**Professor W. J. Eney (Lehigh University):** Are the deflection computations for the plastically designed structure long and tedious?

**Dr. Beedle:** No, the maximum load deflection computations are not tedious at all. The three span lean-to frame that Mr. Estes used was actually analyzed for deflection in a matter of two pages of computations and all that was needed were the regular slope deflection equations. Solution of simultaneous equations is not necessary. The procedure is really a straightforward one, making use of computations required to determine maximum load. The fact that you know where the plastic hinges are and can finally, in the last step, draw the moment diagram, is the necessary information. With that moment diagram you can use the slope deflection equations and solve for the critical deflections of the structure.

**Mr. J. N. Janes (Metallic Building Company):** I am probably a little confused as to the definition of “hinge.” For instance, in the model you have a point of zero moment on the rafter where you have a hinge. Yet, at the same time, when you have a fixed base, you have a maximum moment. I would like a little further interpretation as to the meaning of “hinge.”

**Professor Ketten:** As we have used the word today, “hinge”, or more specifically “plastic hinge”, is that condition where unit rotation can increase while moment remains relatively constant. A point of zero moment and thereby a condition of contraflexure is not of immediate importance to the problem. The assumed stiff member between the points of possible plastic hinge formation will take care of this condition. What we are interested in, however, are the points of maximum moment and the development of plastic hinges at these locations.

**Professor Eney:** What do we have to do if we are going to consider moving loads?
PROFESSOR KETTER: If we have a given system of moving loads, there is nothing to prevent our considering various combinations of these loads to obtain the most unfavorable condition. Design could then be based on this worse case. This problem is one that we are currently studying, but because of the importance of other considerations that enter the problem, we do not feel that at this time we are ready to advocate solution by plastic methods.

MR. R. W. Binder (Bethlehem Pacific Coast Steel Corp.): The shape factor of the I-sections was reported at 1.14. Does that change to any degree between wide flange and American standard beams?

CHAIRMAN HIGGINS: I think that the whole series of I-beams would vary from a low of 1.06 to about 1.22, so that the 1.14 is on the low side and any resulting error on the unsafe side, so-called, would be very minor.

MR. S. THOMPSON (R. L. Eason & Associates): Is there any suggested revision to the method of column design once the moments and axial loads are known, and if so, will that be made available for use in current design practice?

CHAIRMAN HIGGINS: Up to now our work in ultimate strength designs has been concerned largely with single story shed type frames. We have demonstrated that for frames of that sort the effect of the direct stress, the axial stress in the column, is negligible. We can only be concerned with bending moments which produce these hinges. Therefore, in the case of low one-story buildings, column formulas per se in ultimate design lose their significance. Naturally you would have to work within limits of slenderness that may come, but if the axial stress is fifteen per cent or less of the combined axial and bending stress, it doesn't seem to have any appreciable effect upon the ultimate strength of the frame.

For the moment at least we are not thinking of plastic design in multi-story tier buildings where the total stress in the column is largely an axial stress and the bending moment stress is relatively small. However, considerable testing of columns under combined bending and direct stress, where the direct stress has been an appreciable part of the total stress, has been completed under the program.

MR. C. BIRT (Indiana Bridge Co.): Is there anything in plastic design which would work into the actual detail of your haunch design?

CHAIRMAN HIGGINS: The plastic behavior of a number of types of joints was described in the Welding Journal several years ago. The task force at Lehigh has pretty well stabilized its thinking for the time being on a joint where there is no increase in the depth of the beam as it enters the column or the column as it enters the beam. Their object is to produce as nearly as possible the equivalent plastic strength within the joint itself.

If we were to use a curved or tapered haunch, one solution would be to design it strong enough to remain elastic when the moment was great enough to form a hinge just outside of the haunch boundary.

(The meeting was recessed at 5:45 P.M.)
The meeting was reconvened at nine-fifteen o'clock, a.m., Mr. A. E. Pearson, Welding Engineer, The Ingalls Iron Works Company, Birmingham, Alabama, presiding.

CHAIRMAN PEARSON: The first thing on our program this morning is a movie entitled, "Men, Steel and Earthquakes," through the courtesy of the Bethlehem Pacific Coast Steel Corporation.

(Presentation of film)

Mr. Kenneth Lieber, Vice President, Engineering, Consolidated Western Steel Division of U. S. Steel Corporation, Los Angeles, will be our first speaker. He attended the Dartmouth school of engineering and was with the Pennsylvania Railroad, Engineering and Construction Division, for nine years. For the last ten years he has been with Consolidated Western Steel.

Earthquake-Resistant Construction for San Francisco Skyscraper

KENNETH LIEBER

I appreciate the opportunity of talking with you about some of the principal engineering features of a unique type of framing for a multi-story building. I know many of you do not regard a 25-story building as a skyscraper. Maybe it is those smog-producing temperature inversions we get out West that make the sky lower there!

The Equitable Life Assurance Society of United States planned for a number of years to build a tall building in San Francisco. However, the 1948 building code of San Francisco has some very rigid design requirements as to earthquake resistance for tall buildings. Many engineers regard the seismic factor imposed by that code to be so severe as to economically prohibit buildings of the height contemplated. Shortly, with the aid of some slides, I will endeavor to show you how the problem of designing and constructing a building of required earthquake resistance, yet within reasonable economic limits, was solved by the combined efforts of many experts. I am sure you will agree that it took imagination, ingenuity, ability and experience.

The following organizations and associations were primarily responsible for the design and construction of the Equitable Building in San Francisco:

Architect—The late W. D. Peugh of San Francisco and his associates, W. B. Glynn and A. J. Loubet.

Supervising Architects—Loubet and Glynn Associates.

Consulting Architect—Irwin Clavan of New York with his consulting engineer, Joseph Di Stasio also of New York.

Structural Engineer—F. W. Kellberg and Associates.

General Contractor—Dinwiddie Construction Company.

FIGURE 1
the nature of the problem, and the principles employed in arriving at the solution, with a few sketches and recent photographs to show some of the highlights.

To condition you to the subject of earthquake-resistant design, I have a picture of San Francisco after the 1906 fire (Fig. 1). It shows the steel frame for the new structure rising from the ashes of the very famous Palace Hotel. It was a product of the fabulous days of the silver millionaires from Virginia City's Comstock Lode. It was built around a crystal-roofed garden court into which carriages could drive and unload their passengers on marble walks among tropical plants. Its demise marked the end of an era of the Old West.

It was built sturdily. An inner iron frame was protection for the seven-story building against that earthquake. The massive walls with over 30 million bricks were not, however, sufficient protection against the fire. It had three large steam pumps, a special reservoir, and 4-inch piping for fire protection, but the inferno from without proved too much.

The picture is from the archives of our sister division—American Bridge—for their Job No. B4009 for 14 million pounds. One of our division's predecessors had a plant right in this area and most of its records were destroyed.

San Francisco today, located on the tip of the peninsula enclosing the great landlocked harbor, is a city of towers and baywindows and steep hills with a few old-fashioned clanging cable cars still in operation—it is a part of a great metropolitan area tied together with giant modern bridges.

The San Francisco City Code requires that a steel frame be designed for seismic shears that are considerably larger than those specified by the Uniform Building Code, 1952 edition, or the Los Angeles Code. The code does permit a 15% reduction in seismic shear for a foundation on rock. This skyscraper is the first tall building in San Francisco with steel pile foundation resting on bed rock. There are more than 400 H-piles with an average length of 130'. To drive them required the heaviest hammer ever built.

If any of you have offices in the financial district of San Francisco, I am sure you enjoyed the special ceremony they had. At the end of the noisy pile driving, a special funeral was held with much newspaper publicity. They took the hammer itself out in the Bay and dumped it overboard. Another very real relief to those in neighboring offices was that there were no riveting hammer noises in the erection of this structure.

The artist's sketch (Fig. 2) of the West's newest skyscraper is a view looking down Sutter Street towards the Bay. The building is located on the corner of Sutter and Montgomery, and is 121 by 165 feet at the ground. You will observe a 15-story base and an offset 10-story tower section. The 25 floors of this building and the penthouses have a total floor area of 435,000 square feet.

The chart of the frame and the seismic loads shown in both Engineering News Record and in the Steel Construction Digest (Fig. 3), helps one to understand the basic problem involved. Line A for the proposed building represents the total shear from wind loading in thousands of kips.

Line B gives the total seismic shear for such a building under the 1952 edition of the Uniform Building Code. The seismic factor under the code is expressed as a percentage of the superimposed vertical load applied laterally in much the same manner as a wind load. The two are not additive.

Finally, Line C gives the total seismic shear under San Francisco's code after taking advantage of two things—first, the 15% reduction for a foundation on rock; and, second, by seeing to it that the tower of the building has an area more than 60% of the area of the lower portion. Otherwise, the maximum seismic factor
must be again applied at the first floor below the tower as though this were the top floor of the building. This would substantially increase the theoretical seismic forces, as shown in dotted line D. From this chart you can see that at the 10th floor San Francisco's code requires provision for seismic shear about 1.6 times that of the Uniform Code and nearly $2\frac{1}{2}$ times the shear from wind load. At the ground level more than twice the normal shear must be resisted.

Another important element of the problem is that story heights are also limited, and, of course, space is required above ceilings for air conditioning ducts. As a result, for this building, floor beams are limited to about 18" depth. In the tower section of this building, this depth of beam was sufficient to permit the distribution of seismic forces throughout the floor area and the resistance of such forces through the bending of all columns and beams, which are rigidly connected. Below the 16th floor where the depth of beam was insufficient, it was deemed best to resist the seismic forces in the exterior framing. The 15th and 16th floors were designed to transfer the lateral forces from the tower section to the outside walls of the base section.

Some interior columns are turned at right angles to other interior columns so that the resistance to lateral forces may be more equally distributed. As I mentioned, the lateral forces from the 16th to the 25th level are resisted by all the columns and their connecting beams and girders. The Steel Construction Digest article gave you the details of these beams, showing both the connections of the beams to the column webs and to the column flanges. The columns are 14-inch wide flange rolled sections.

The bracing system on the 16th floor includes diagonal bracing members of 16-inch wide flange sections and are designed to form a transition from the tower section, where, as I have said, the lateral forces are distributed throughout the area, to the lower section where the lateral forces are resisted entirely by the framing of the exterior walls. There is a similar but much lighter bracing system at the 15th floor level.

In the base section of the building the seismic forces are transmitted to the outside walls by the diaphragm action of the six-inch floor slabs. These floor slabs, incidentally, are designed for a live load of 100 pounds per square foot, the floor beams for a live load of 80 pounds per square foot. These lower tier columns, after taking into account the load reduction, have been designed for a live load of 24 pounds per square foot, plus a partition load of 23 pounds per square foot. One of the most outstanding features of this base section, however, is the very wide welded exterior columns. The interior columns are subject to vertical loads only and these exterior columns and butterfly spandrels resist all the lateral forces.

The intermediate exterior columns (Fig. 4) have a 42-inch web at the bottom. They taper 3/32 of an inch per foot from 42 inches to a 12½-inch web at the 14th floor. The flanges are 18 inches x 3 inches at the base. This gives a total column width of 4 feet at the base. A typical intermediate column has a web thickness at the base of 1½ inches. This welded tapered column design was one of the keys to the practical economics of the framing design. In the first place, rolled sections are not available with a 40-inch web. In the second place, it was possible to get much greater rigidity per pound of steel in a 4-foot column than would have been possible with rolled sections and cover plates. In the third place, no wide-flange beams are rolled in the West so that sections fabricated from western plate are economical by reason of savings in freight. The use of tapered columns eliminated the abrupt change in sections necessary at splices of parallel flange columns and provided a more direct and desirable transfer of stresses. In a later slide you will observe the simplicity of the splices in these tapered columns. Incidentally, column tiers extended through these stories and were spliced at midpoint between floors.

The fabricated corner columns are also very much simpler than if made from rolled sections. The corner columns are L-shaped with heavy flange plates at the extremities of the L. These corner columns are also tapered to the 14th floor. Above that point they are fabricated in this same L-shape, but are not tapered.

All shop welds were made in a flat position and plates were preheated. On these tapered columns the two welding machines together with two heating torches, were used on opposite flanges simultaneously to prevent distortion.

Our fabricating plants in California are pioneers in the application of submerged arc welding to production.
Our welding supervisor today recalls those days in Los Angeles when he used to pound up glass insulators—and for some reason they had to be green glass insulators—to make a welding flux. A year or two later a suitable flux became available commercially and we were among the first to use it. We applied those welding methods in more recent years to the pioneering of high-strength line pipe for thousands of miles of high pressure 30-inch and larger gas and oil transmission lines. But that is another story!

As you recall, the lateral forces are resisted in the base of the building entirely by the exterior framing. These lateral seismic forces can be resisted by bracing, by solid shear walls, or by a rigid steel frame. On account of windows and other architectural features, such forces in the Equitable Building are resisted by a rigid steel frame. From the 16th floor down to the 11th floor, the spandrels were fabricated from wide-flange beams up to 30 inches, with plate haunches.

Small brackets attached to the columns served as a combination erection seat and backup. Backup bars were used to permit downhand welding. The welding sequence in the field was, first, to simultaneously weld the top and bottom flanges on one end of the spandrel to the column flange with full penetration butt welds. Second, the web on this same end was welded to the column flange with fillet welds. After these welds were cool the opposite end of the spandrel was welded in a similar manner.

These butterfly spandrels were shop welded with the web in a vertical position. Again, welds on opposite sides of the web were made simultaneously as described earlier. The web plates varied from 7/16" to 3/8" in thickness, and the 10-inch wide flange plates varied from 1" to 13/8" in thickness. They were joined by 5/16" and 3/8" submerged arc fillet welds. We lined up several of these sections end to end to provide a continuous run of 50 feet or more for the welding tractors. After one flange was welded to a web, the sections were turned 180 degrees and the process repeated for the other flange. The sections were trimmed to exact dimensions and edges were prepared for field welding after all shop welding was completed.

At this point it may be well to outline the various types of connections used throughout the structure. In general, all shop connections were riveted. Field connections between spandrels and columns from first to sixteenth floors were welded and all other field connections were made with high-strength bolts and hardened washers. Thus, each type of connection was utilized to its greatest efficiency and economy.

Fig. 6 is a close up view, looking up one column. You may be able to observe holes in the columns for the attachment of forms. Anchors for concrete show in the photograph. Note the deeper spandrel at the second

\[ \text{Figure 5} \]

\[ \text{Figure 6} \]
floor level. Also, note the simplicity of the frame and the absence of connecting fittings. The very simple column splice shows just above the lower spandrel. The boxes on the columns were used to protect the workmen and material from the elements while performing the field welding.

Another feature is shown quite clearly in this picture. The single floor beam framing to the centerline of the wide column gives no support to the column flanges. The flanges of the spandrels and continuity plates were considered as horizontal beams to hold the column flanges from buckling. These spandrel flanges are laterally supported at midspan by the connecting floor beam.

And so today the building is nearing completion. Its 5,200 tons of steel frame superstructure averages 23.5 pounds per square foot. This is greater than that of an average office building, but this is not an average building. It is a skyscraper built to resist seismic shear forces, which at ground level are more than twice that resulting from Uniform Code requirements and to do so without excessive cost. All steel is A-7 and working stresses comply with the A.I.S.C. Specification.

All of the steel was fireproofed with concrete. Stone concrete was used to the second floor and lightweight concrete, 100 pounds per cubic foot, above that level. All this is concealed by marble, aluminum and stainless steel.

Behind that modern, simple and attractive architectural exterior is a unique framing for earthquake resistance, and, incidentally, one in which welding played a key part.

The structural design of this building is a special solution to a localized problem. Certain features, however, may perhaps be applied to other structural problems. This proved to be a most interesting project for us. We hope you have also found it interesting to review the problems and their solution.

CHAIRMAN PEARSON: Mr. C. I. Orr of Consolidated Western will answer any questions arising from Mr. Lieber's paper.

MR. J. B. WHEELER (C.I.S.C.) : Could you tell me the exact construction of the floor which was designed for diaphragm action?

MR. ORR: The beams were all rolled sections spaced six to eight feet on center. There was double reinforcing used in the slab because there were under-the-floor ducts for telephone wires, etc., which would interfere with bent rods.

MR. WHEELER: Were the bolted connections at midspan of the spandrels made in the shop or in the field?

MR. ORR: The bolted joint was made in the shop, and the bolts were put in finger tight. This permitted the bolts to slip in their holes, and they were tightened in the field.

Our next speaker is Donald G. Gentry, structural engineer with the consulting firm of Finney & Turnipseed of Topeka, Kansas. He has a B.S. degree in civil engineering from Kansas State College and obtained his master's degree in structural engineering from Texas A. & M. Prior to World War II he worked with the Corps of Engineers on design of hydraulic structures and during the war he served as an artillery officer, going from 1st Lieutenant to Lieutenant Colonel and, while serving with the 1st Cavalry Division, was awarded the Silver Star. Since 1946 he has held his present position where he supervises the building design department.

250-Foot Welded Rigid Frames
DONALD G. GENTRY

The Forest C. "Phog" Allen field house at the University of Kansas has recently been completed. In fact, I watched the University basketball team defeat my Alma Mater, K-State, on the eve of the opening and dedication last March 1st. The seating capacity is about 17,000 and every seat was occupied that night.

The building, in plan, is approximately 345 feet long by 235 feet wide. 2700 tons of structural steel went into the frame work. Each of the 10, 252'-6" span, rigid frames weighs about 70 tons.

Inside the structure is an 3/8th mile track, class rooms and offices. In addition to basket ball it will be utilized as an armory, and practice field for football, baseball and track.

The structural steel was designed under the A.I.S.C. Code, that is, where-in that code does not conflict with our office code. (Laughter). For the design of this structure we revised the allowable design stresses. We felt that wind loads, due to the shape of the structure and lack of interior framing, were relatively of more importance than for a normal building. The structural steel was designed for the following allowable stresses:

- D.L. + 20 lb. S.L. = - 20000 P.S.I.
- D.L. + 30 lb. W.L. = - 20000 P.S.I.
- D.L. + 30 lb. S.L. = - 26600 P.S.I.

Before further discussion I would like to comment on the frames of the Kansas State field house at Manhattan, Kansas. These frames have a span of 175 feet and the
roof pitch is 6 to 12. I consider this pitch to be too steep for maximum economy of structural steel since the load carried by a purlin is acting at a large angle with the major axis of the purlin. This field house has a seating capacity of about 13,000. Approximately 5,400 of these are permanent seats of the balcony which is cantilevered out from the frames in order to allow adequate room for an 1/6th mile track. These frames were fabricated from wide flange sections—36WF170 for the girder and 36WF194 for the legs with the haunch made up of plates. For comparison with the K.U. Field House I would like to point out that on this structure the haunch strut is a truss and the girder struts are wide flange sections with a channel on the bottom. This channel is turned down to the lower flange of the girder to act as lateral bracing for the frame.

I would like to mention that on this structure the end wall columns do not line with the struts. The wind bracing is, in effect, a horizontal truss carrying wind load with the frame struts acting as the posts or verticals. It became necessary then, to design the end wall eave beam to carry the wind load from the end wall columns to the supports—the line of struts. The same beam, however, carries vertical load of the roof from the purlins and struts to the end wall columns. This eave beam, thereby, was necessarily a continuous beam carrying loads from perpendicular planes with the supports at different points for each set of loadings—a condition that could have been eliminated by lining the end wall columns with the frame struts or vice versa.

The frames of the gymnasium which forms a wing of the field house at K.-State are flat-topped with a span of 114 feet as compared to the 175 feet of the field house frames. Spaced at the same centers (24 feet) as the frames of the field house and carrying only roof load, also, they required the same basic wide flange section (36WF170 girder). This indicates the economy in structural steel gained in the frame with a gable roof due to arch action and a corresponding reduction in bending moments. The legs were tapered, both for sake of appearance and to gain room for the gym floor. Also, I'd like to point out that peculiar haunch struts were necessitated by the location of the windows.

I have commented on the K.-State field house mainly to emphasize one major difference in the design procedure. We commenced the structural design of the K.-State field house after the architect was well along with his plans. The end wall fenestration was set and the columns were set. However, on the K.U. field house we actually made a preliminary structural layout before the architect had much more than determined the seating capacity to be furnished. As a result of being in on the ground floor I think we ended up with a much cleaner job.

(Slide 1) The first of the slides on the K.U. field house is an interior view showing a portion of the completed permanent seat bank prior to installation of the actual seats. The seats, except for about 600 opera-type, are 2” x 10” plank on steel brackets which were field welded to the bent steel plate riser and treads. These riser plates were sent to the site bent in the form of an “L” and when field welded formed a laterally supported “Z” beam which spans the 16 feet between rake beams. The lower seating cantilevers about 14 feet from the last line of columns on the side in order to leave sufficient clear space for the 1/6th mile track. This track passes beneath the seating at each end. As mentioned, the riser plates span the 16 feet between rake beams while the frame spacing is 32 feet. The rake beams do not connect to the frames but rather to spandrel beams between frames. These spandrels are two wide flange sections with diaphragms. The outer beam carries the wall and the inner carries the rake beams or floor framing for the offices and class-rooms beneath the seating bank. As the frames “breathe” or the hips move in and out laterally, the wall moves with them while the seating and floors remain stationary because of slotted connections. The first frame to be erected deflected 21/4” at the ridge (centerline) from its own dead weight. Neglecting elastic shortening of the girder this means that the hip had to move out about 3/4”. Snow or wind load on the roof or even daily changes in temperature will cause the hip to move in and out. The force required to restrain this movement at the intersection of the line of rake beams with the frame legs would have been tremendous—in excess of 1,000,000#. The connections were slotted 41/4”. 3/4” φ M. bolts were shouldered using 5/8” hex. nuts with 1/16” minimum side clearance specified.

(Slide 2) This slide gives an idea of the general shape of the frames and the erection procedure. The frames are of the plate-girder type, 4’-2” deep at the base, 13’-0” at the haunch, and 4’-2” again at the shallowest point on the girder. The curve of the lower flange at the ridge is for architectural reasons only as the point
of maximum positive moment is not at the ridge or center line but out on the girder. For a flat-topped frame like the one at Kansas State the maximum positive bending moment is at the center line. However, as you increase the pitch of the girder the ridge approaches a point of full support and the positive moment reduces. If you get the pitch steep enough, the moment at the ridge is negative. The frames were fabricated from plates—at the leg and thru the haunch the web is $\frac{3}{4}''$ plate changing to $\frac{5}{8}''$ near the center to $\frac{3}{4}''$. The flange plates on the legs are $18'' \times 1\frac{1}{2}''$. At the haunch they are again $18'' \times 1\frac{1}{2}''$ with $16'' \times \frac{3}{4}''$ cover plates and $18'' \times 1\frac{3}{4}''$ on the girder. The web to flange welds are continuous fillets $\frac{5}{8}''$ on leg and haunch and $5/16''$ out on the girder.

Each frame was shipped to the site in 6 pieces. Each leg was erected as shown on the slide. The spandrels were erected as bracing for the leg prior to erection of the frame girder. Two pieces of the girder were field welded together on the ground to make one length of approximately 130 feet from the hip to the ridge. By use of the single center erection bent the two pieces of the girder were fitted in place and three field splices welded in the air—one at each haunch and one at the ridge. This slide also gives a good view of the end wall framing.

(Slide 3) This slide very graphically indicates the 3 to 12 roof pitch of the frames and the relative size of the frames. The web stiffeners of the plate-girder type frame were spaced with the purlins and struts and were utilized for connection of these members—since one sided connections were adequate. All purlins and struts were bolted with $\frac{3}{4}''$ machine bolts. The dead and snow loads carried by the purlins, of course, act vertically and can be visualized by the shadow of the cable on the frame beneath the man in the center. The purlins, however, were designed to carry only the component of the load perpendicular to their major axis, i.e., that component parallel to the stiffeners. The "down-the-slope" component or that component of the vertical load acting parallel to the top flange of the frame or perpendicular to the minor axis of the purlin is carried by the rigid-type roof (2" wood sheathing) to the struts.

(Slide 4) The roof framing is well illustrated by this interior view. We see the flats of the braced bay, the purlins and the struts. Note that the struts line with the end wall columns.

Except at the braced bays the roof purlins and struts were erected with the wood nailers attached and ready to receive the 2-inch wood sheathing. The braced bays, one at each end of the structure, provide erection stability and in the final structure carry the wind load from the end walls to the side walls and then down to the foundation. This wind bracing consisted of crossed flats, field welded to the upper flanges of the frames with the frame struts becoming the posts or verticals of the truss. Each girder strut is basically a wide flange section with a channel on top.

At the frame the lower flange of the beam is curved down with additional web plate inserted. This strut normally does three jobs—if carries load as a normal purlin, it picks up the "down-the-slope" load from the rigid roof and takes it to the frame and in addition provides lateral bracing for the frame itself. In addition, at the braced bay the strut acts as a post or vertical of the horizontal truss carrying wind loads.
(Slide 5) The haunch strut we see here on the ground was made up entirely of plates as you can determine from the welds that are visible. I must admit that the curves of the member are for appearance only—the same is true for the girder struts of the previous slide. In fact, as I originally laid it out I had a truss here as on the K-State field house. However, the architect was in one day with his grease pencil and sketched what he would like the strut to look like. I do think, as you can tell from the next slide that it makes a good looking strut.

(Slide 6) This, I think, is an excellent view of the haunch of the frame and the haunch strut. It shows the stiffeners used in the haunch—the thru plate stiffener at the center line of the haunch—the haunch stiffeners and the compression flange stiffeners. A welded field splice was accomplished at the center line of this haunch. Note the holes, that have been filled with weld metal, where the temporary splice angles were located. The butt weld for the 18" x 1½" flange plate was made at the center line while a short piece of the cover plate was shipped loose and, therefore, its butt weld staggered with that of the basic flange plate.

(Slide 7) In this slide the steel erection for the shell of the structure was completed. All ten frames are in place.

The frame was designed as a two-hinged structure free at the bases. The first trial section was established from standard formulae, much like those contained in A.I.S.C. publications, using a ratio of the moment of inertia of the girder to that of the leg of 9/10. With these assumptions the horizontal thrust was calculated. This thrust was increased by about 10% to allow for the effect of the haunch in pulling more negative moment.

As soon as the thrust or horizontal force required to prevent the leg from moving out at the base is known the frame becomes, of course, statically determinate. With this thrust and the loads, bending moments and shears were calculated and preliminary shape and member sizes chosen. The preliminary frame was then broken into sections and their elastic properties determined. From these elastic properties a new thrust was calculated. This new thrust was about 30% larger than that obtained by the standard formulae using constant moment of inertia instead of 10% as I had estimated. A normal haunch, on a constant I girder and leg section, will increase the thrust from 5 to 7%. My original estimate of 10% was wrong because I had underestimated the effect of the variable I.

Making use of the new and much higher thrust determined from an elastic analysis of the frame, the shape and member sizes were revised to accommodate the re-
vised moments and shears. Finally, a completely independent check analysis of the frame was made.

(Slide 8) This is a good view of the ridge of the frame with the ridge strut or center brace on the far side in place. The flange weld is being accomplished prior to removal of the temporary splice angle in the web. This splice is not at the point of maximum positive moment which, as previously described, is out on the girder. Also due to the increased depth caused by the curvature of the lower flange the splice was accomplished at a point of relatively low stress.

(Slide 9) This slide shows the haunch field welded splice being accomplished. Due to shipping clearances into Lawrence, Kansas, it was necessary to make the field splices near points of maximum stress rather than at or near the point of inflection. Since the haunch was 13 feet deep to begin with it was impossible to make the splice out on the girder and ship the haunch as a single unit. However, this splice again was not made at the point of maximum stress although it was made at the point of maximum moment. The point of maximum stress is located about 15 degrees above the springing line. In a frame such as this, the neutral axis as we come up the leg remains very near the actual center line of the symmetrical section. The haunch is a curved beam and the neutral axis crowds in near the inner or compression flange. At the springing line, or point of curvature, the neutral axis commences to close in on the compression flange and then sweeps out a bit before it again crowds in to its nearest point at the center line of the haunch. In other words it is relatively closer at about 15 degrees above the springing line than it is shortly above or below that point. For a haunch on a flat-topped frame wherein the curve of the inner flange encompasses the full 90 degrees, the neutral axis at the center line of the haunch will be at or very near the quarter point. The compression stress at that point will be actually twice that calculated on the full section. At the hip or outer edge of the haunch the stress will be, theoretically, zero as the tension forces will have come into the web as they round the corner to come down the leg.

(Slide 10) Here we see the lower flange of the field-welded splice to be made on the ground which joined a 50 foot and an 80 foot piece. As it turned out, this splice could well be called an adjustment splice. The design plans called for the bolt holes of the temporary splice plates and angles to be sub-punched and reamed with the frame completely assembled to its correct dimensions. However, the two pieces we see here were fabricated in different parts of the country and shipped directly to the site. It was necessary to block up these pieces until the top flange was a perfectly straight line and the length of the two exactly that called for on the plans prior to making the field splice. In nearly all cases the fit was close as shown on this slide. However, a few had a root gap large enough to require the use of a back-up plate with which to start building up a weld.

I should perhaps mention the determining forces for this structure—the maximum horizontal thrust is 186,000 lbs.—the maximum negative moment 8,800,000 ft#; the maximum positive moment 1,650,000 ft#. The vertical load on each frame footing can vary from 448,000 to 681,000 lbs.

Past experience in this particular area had taught us that immediately above a good blue shale is a low-shear-value, laminated, yellow shale that is incapable of resisting large horizontal loads. 14 test borings were taken which provided sufficient information to plot sub-surface contours of the blue shale, to determine that there was a relatively high ground water elevation and to confirm the poor shearing resistance of the yellow shale. Comparative cost estimates indicated cast-in-place concrete piling to be the most economical adequate foundation.

Once the decision had been made to use piling there remained the decision as to how to resist the horizontal thrust of 186,000 lbs.—by a tie or by use of batter piles. I do not like ties nor do I believe in their use when a proper foundation can be built without them.

An adequate tie for one of these frames would have
The architect for the building was the
and tend to rotate the cap due to the action of the batter
piles—one group shoving up on the cap and the other
pulling down. There is, then, some line of action for
a load to produce only vertical movement—a line of action
at the proper angle and location such that the horizontal
component is just sufficient to prevent horizontal move­
ment. Likewise there is a line of action for a force that
will produce only horizontal movement—a line of ac­tion
at an angle and location such that the vertical com­
ponent will prevent rotation of the cap. The intersection
of these two lines of action is the elastic center. Any
force thru the elastic center will produce only horizontal
and/or vertical movement in the cap—the top of the pile cap will remain level. Any
force not acting through this elastic center will produce
rotation of the cap—rotation about this elastic center.
To locate these lines of action requires the use of force
and string polygons. The force polygon will give the
direction and amount of force required to produce unit
movement while the string polygon will locate this
line of action. Once the elastic center is located the
actual loads on the piles are found as follows: The actual
horizontal and vertical loads are assumed to act thru the
"elastic center and each then resolved into its components
parallel to these lines of action—components of the load
producing horizontal and vertical movement. By direct
proportions with the already drawn force polygons the
loads on the piles are determined.
The fact that these actual loads do not act thru the
elastic center is accounted for by taking moments of
these actual loads about the elastic. The pile loads
caused by this moment is determined in the usual manner
of \( r/\Sigma r^2 \) where \( r \) is the perpendicular distance from the
elastic center to the line of action of the pile and \( \Sigma r^2 \)
represents the moment of inertia of the pile group. Multi­
plying \( r/\Sigma r^2 \) by the moment gives the individual pile
load. In other words, the \( r/\Sigma r^2 \) represents \( c/l \) in \( M c/l \).
The results of the two designs checked very closely.
In fact the loads on the vertical piles were within 10%.

(Slide 11) This slide is a picture of a model of the
16-pile group at each frame footing—4 vertical piles
and 12 batter piles. Of the batter piles 8 are in the
direction of the thrust and 4 in the opposite direction.
All batter piles were 1 horizontal to 2 1/2 vertical.
The pile group was designed by two different methods
—an approximate, short-cut method and then checked by
a more exact analysis.
The approximate method required some rather basic
assumptions: (1) That the earth immediately surrounding
the pile cap resists none of the load. (2) That vertical
piles will not resist horizontal loads and (3) That
only vertical piles can resist moment loads. A rather
obvious item concerning this pile group is that each of
the 8 piles battered in one direction will carry only half
the vertical load that each of the 4 piles battered in the
opposite direction will carry. This is necessary to keep
the horizontal forces in balance. Also, all 12 batter piles
are equally effective in resisting horizontal loads. Using
these assumptions a preliminary design of the frame
footings was made.
A more exact analysis of the pile group was then made.
In the 1939 Transactions of the A.S.C.E. is an article
by Mr. C. P. Vetter on the design of batter pile groups.
In this article is a description of the Hultin pile theory
upon which we based a theoretical analysis.
The stresses in a pile group are caused by a small
movement of the pile cap—vertical or horizontal move­
ment or rotation of the pile cap about some point. It is
obvious that a vertical load on this pile cap will move
the cap down and sideways (in the direction of the 4 batter
piles.) A horizontal force will move the cap horizontally

a cross-sectional area of about 16 square inches. The cost,
roughly, of 2,500 feet of tie would be in excess of $13,000,
plus the cost of installation, protection, etc.
The pile footing as finally designed and driven used
6 additional piles over what would have been required
for vertical load only. Twenty such groups or 120 addi­tional piles made an additional cost to the job of about
$9000—less than the cost of the steel for ties alone.

(Slide 12) The architect for the building was the
state architect, Mr. Charles Marshall, the general contractor was Bennett Construction Company of Topeka, Kansas, the piling were driven by Raymond Concrete Pile Company, the structural steel was fabricated by Allied Steel Company and erected by Beasley Construction. The building was constructed at a total cost of approximately 2½ million dollars.

CHAIRMAN PEARSON: Now, we are at the discussion period.

MR. J. N. JANES (Metallic Building Company): Due to the great depth of columns in your rigid frame, was there any consideration given to some restraining moment in your design?

MR. GENTRY: No, it was designed completely free at the base.

MR. JANES: Regarding your wall bracing, I notice that the bracing was attached to the column at the outside flange. Would there be any consideration of the stability of the inside flange due to the great depth of the column? I am talking about the horizontal bracing of the wall line.

CHAIRMAN PEARSON: Our next speaker, Mr. Carl L. Kreidler, a native of Pennsylvania, is Chief Structural Engineer of the Lehigh Structural Steel Company. He is a member of the Welding Research Council and is Vice President and President-Elect of the Pennsylvania Society of Professional Engineers.

All-Welded Trusses

CARL L. KREIDLER

I have two special applications of structural steel welding to present. Both are part of all-welded contracts fabricated and erected by the Lehigh Structural Steel Company, Allentown, Pa.

Replacing Trusses with Trusses

In erecting the 1952 factory extension for Fairchild Aircraft in Hagerstown, Maryland, alterations and additions were made along the common column line between new and old structures.

There are two intermediate columns in each 125 ft. bay. When the new extension was completed the intermediate columns were eliminated by the use of 125 ft. trusses spanning between the main columns.

The first step in erection was to cut the bottom chords of the Pratt trusses where they framed into the columns. Since the chord at this point is not carrying any principal stress, this cut can be made without impairing the structural stability. A reinforcing tee was then added to the column.

Shoring trusses were then placed along the common column line to support the existing purlins and monorail beams, thereby relieving the load on the saw-tooth trusses. Columns and purlins were cut out to provide space for erecting the new 125 ft. truss shown in the background of Fig. 2. The bottom chord of the 125 ft. truss is 48° 7¾” above the finished floor and is above the bottom chord of the saw-tooth truss. Therefore, instead of removing the bottom chord, which is also part of the bracing system of the existing building, the web members of the saw-tooth truss were cut to provide attachment to the bottom chord of the new truss. By making this cut, the new truss could be erected, and the bottom chord of the existing truss remained in the structure.

After connections were made to transfer the loads to the new truss, the intermediate columns could be removed. Actually, this was not done until the new
The intermediate columns and the wall framing thereby provided a bulkhead between the operating area and the construction area during the construction program.

The conventional method of doing this replacement would have involved use of temporary shoring supported on the work floor. By this special application, the owner was at no time deprived of any space below the bottom chord of the existing trusses. There was no restriction of crane travel in the area and no loss of manufacturing floor space during the construction program. Fig. 2 also shows a section of the camel-back truss being swung into position. The top portion of this truss was left exposed above the roof level as shown in Fig. 1.

**Vierendeel Trusses**

Unusual site conditions are responsible for several unique features in the construction of the all-welded Municipal Library Building at Hartford, Connecticut. Fig. 3 is a sectional view showing the structural steel framework. The six-lane Whitehead Highway, passing through the central core of the library, is constructed...
Loading on the highway is restricted to A.A.S.H.O. H-20 loading at all times, and two lane highway travel in each direction had to be maintained during erection. By locating a field splice in the Vierendeel trusses at mid-span these conditions were satisfied since temporary erection bents could be placed astride the highway median strip. Also, this type of splice permitted adjustments in the field that would not have been possible with a single splice in each chord.

The mezzanine and second floor are hung from Pratt trusses composed of 14WF sections. In order to erect the mezzanine and second floor steel in the conventional manner and to field weld the splices in the roof trusses in the erected position, the hangers were converted into erection struts. Temporary struts supported on the top chord of the Vierendeel trusses were removed after the roof truss splices were completed.

Fig. 4 shows a section of one of the trusses during fabrication with some of the component parts in the background. Sixty-two thousand pounds of electrodes were required in the butt welds to fabricate these trusses entirely from plates.

The two largest Vierendeel trusses span 104 feet and weigh 94 tons each.

The architects calculated that the use of the Vierendeel trusses provided library space for 280,000 additional volumes of books.
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Welded Continuous Frames and Their Components

PROGRESS REPORT NO. 16

THE USE OF MODELS IN PLASTIC DESIGN

by

Robert L. Ketter

This work has been carried out as a part of an investigation sponsored jointly by the Welding Research Council, and the Navy Department with funds furnished by the following:

American Institute of Steel Construction
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Column Research Council (Advisory)
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Bureau of Yards and Docks

Fritz Engineering Laboratory
Department of Civil Engineering
Lehigh University
Bethlehem, Pennsylvania

July 1, 1955

Fritz Laboratory Report No. 205.31
Mr. Chairman and Gentlemen:

As Mr. Estes has so clearly pointed out in the preceding presentation, given a structure subjected to certain prescribed loads, there exist a given number of points at which plastic hinges may develop. For each geometrically possible combination of these plastic hinges, there is a corresponding load required to cause the structure to fail in the assumed manner. Since the structure will fail at the first opportunity, the correct combination of these possible hinges (or as it is usually termed, the correct mechanism) is the one that results in the lowest failure load.

For the simpler types of structures the correct mechanism can be predicted with a fair degree of certainty from the start. As redundancy increases, however, the number of possible plastic hinges also increases, and therefore the number of possible hinge combinations increases at an even greater rate. With little experience it has been possible to predict within two or three tries the correct hinge combination for most frames. None-the-less, based on the reasons that have just been set forth, the question has been raised, "Can models effectively be used in plastic analysis?".

Obviously the most straight-forward type of model analysis would be to load to failure a model of the structure in question that had been scaled down to a reasonable size.

* Presented at the 1955 A.I.S.C. National Engineering Conference.
Using such an approach, it would be necessary to fabricate a model for each of the structures in question (i.e., welding of joints, etc.). Depending on the size and relative stiffness of this model, the loads involved might become rather large. Since, as was pointed out earlier, the points at which plastic hinges may develop are usually well defined, one solution to this excessive load problem would be to machine down the model at these points. Then, depending on the amount of machining and the scale factor of the model, the loads required to produce failure could be kept within reasonable limits.

This machining down of the section suggests a further modification or refinement to this procedure. The sections between "hinges" could be made up of relatively rigid bars with means of clamping a thinner piece of metal at their ends to act as the "hinge". These inserts could be made of mild steel, brass, copper or any other suitable material. Variation in stiffness throughout the structure could be achieved by changing the cross-sectional dimensions of the insert.

Before continuing, let us consider the actual behavior of the plastic hinge we are trying to simulate. In Figure 1 is shown a typical bending moment - curvature (i.e., rotation per unit length) plot for a wide-flange section.

Figure 1
bent about its strong axis. As shown, if moment increases from zero, there is observed first of all a region wherein curvature increases linearly with moment. Above this range curvature increases at a greater rate, the maximum moment being the full plastic moment, \( M_p \).

In ultimate strength analysis as determined by plastic methods, the elastic portion of this moment-curvature relation is not usually considered. The important condition is that, for a relatively constant moment value \( (M_p) \), curvature can increase indefinitely and thereby allow redistribution of additional bending moments within the structure. If a mechanical hinge could be developed which had deformation properties (i.e., moment - unit rotation characteristics) of the type shown in Figure 1 or more specifically of the type shown in Figure 2, then these mechanical hinges could be used in place of the inserts discussed a few moments ago. You will note that the elastic range has been eliminated in Figure 2, the hinge being first of all rigid and then more or less plastic.

The immediate problem then is the development of such a mechanical hinge.
The behavior of the hinge shown in Figure 3 is not unlike that desired. If you will pardon the expression, this is essentially a "prestressed" hinge. By precompressing the central swivel against each of the basic rotating parts (A) and (B), external moments can be applied to the assemblage up to a value equal to that set-up in the unit due to the prestress force without rotation occurring. (The method used in Figure 3 to provide this initial forces is that of a coil spring acting against the swivel and against a screw that passes through a slot in the swivel and attaches to the end parts (A) and (B).) For applied moment values greater than that due to prestressing, the hinge will open as shown in Figure 3c with the amount of rotation being determined by the constant of the spring. For
the case shown the swivel remains in contact with part (A) while causing further compression of the spring on side (B).

The main disadvantage of this type of hinge is that once it starts to open, moment increases rather rapidly with curvature. This situation could be remedied by using a "soft" spring. A more positive approach to the problem, however, would be to replace the coil spring with a member that remained rigid up to a certain predictable point and then deformed under that constant critical value of thrust. For example, an elastic column that has been loaded beyond its buckling load (Euler load). This then leads us to Figure 4, another possible type of hinge.

This hinge (Figure 4) is basically the same as that shown in Figure 3. The only differences are with regard to the design of the center swivel, the use of a leaf spring column instead of a coil spring and removable wing plates that connect end parts (A) and (B) to the center rotating pin. The springs are designed slightly longer than the openings thus insuring initial compression in the system. To change the stiffness of this hinge so that it could be used to represent various members within a structure, it is necessary to either (a) provide a means for changing the effective length of the spring, or (b) use different sizes of springs for the different members. While the latter solution may seem expensive or time consuming it may have definite advantage for demonstration type models since geometrically equal size springs could be assumed to produce hinges of equal stiffness.
Having described these two possible types of hinges, the next consideration is that of calibration.

![Diagram of hinge setup](image)

**Figure 5**

Figure 5 is a set-up that could be used to calibrate either of the above described devices. The hinge is rigidly attached to a bracket that is in turn attached to a table. To the opposite end of the hinge is connected a rod to which is applied a load, \( P \). The moment of this load about the hinge is equal to \( P \cdot a \). As the moment is increased, whether by increasing \( P \) or \( a \), there is reached a point at which the hinge opens as shown by the dashed lines. It is this value of critical moment that is important. It should be pointed out, however, that the exact value of the moment in terms of inch-pounds, etc., need not be known. By using a standard weight, relative stiffness of hinges will be a function of only the distance "a".

Assuming that we now have developed hinges that behave as desired, how will these be used in a typical model analysis solution? Consider for example the set-up shown in...
Figure 6. The structure being analyzed is shown by the line drawing in the upper part of the figure. Since hinges can develop at each of the points of load application, at the junctions of two or more members and at the column bases, a model of the type shown is constructed.

As you will note, the model is tested in the horizontal position. This is to eliminate as much as possible the influence of the weight of the hinge itself. Moreover, the model is floated on ball bearings and glass pads to reduce the influence of friction. Loads are provided by dead weights, which are progressively increased from zero to their maximum value. Knowledge of the locations of the hinges that develop at this maximum load enables one to compute the ultimate strength of the structure.

If it were desirable to carry through a complete model analysis to the actual specification of the required section
moduli for the individual members, the only additional information that is needed is the ratio of the model failure load to the standard weight used in calibrating the individual hinges. Then knowing the scale factor of the model and calibration set-up, the actual loads applied to the structure and the above load ratio, the required section moduli could be computed.

This (Figure 7) is a demonstration model of the two-span gable frame described and analyzed earlier this afternoon by Mr. Estes (Figure 6 of Mr. Estes's paper). The specified uniform load has been approximated by concentrated loads at the midpoints of each of the rafters as in his example. Instead of using four independent weights to load this model, loads will be applied through a system of distributing beams such that only one concentrated force is required. Using this system, any combination of loads can be applied to the structure with a minimum of effort. For example, consider the case where only the windward slopes of each of the bays is loaded (See Figure 8). You will note that under this loading hinges form under each of the points of load application, at the top of each of the exterior columns, and in the rafters adjacent to the interior column. (These are designated by arrows in the Figure.) A knowledge of these hinge locations enables us to compute the ultimate strength of this frame under this loading. See for example Figure 6 of Mr. Estes's. Other loading conditions could also be considered and hinge locations determined. It is felt, however, that this is sufficient to illustrate the basic idea.

The hinges that have been used in this model are of the type illustrated in Figure 4.
This has been a discussion of a model analysis development for the solution of problems in plastic design. At present the device seems to have its greatest application as a classroom teaching aid. There is a possibility, however, that its use may prove equally effective to the designer in the solution of certain of the problems encountered in structural design.