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FINITE ELEMENT ANALYSIS APPLIED TO GYROELECTRICALLY LOADED WAVEGUIDING STRUCTURES

by

Nader Mohsenian

A Thesis

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in Electrical Engineering

Lehigh University

1985

This thesis is accepted and approved

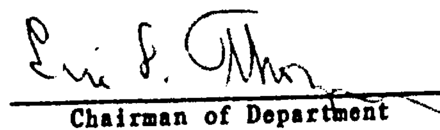
in partial fulfillment of the requirements

for the degree of Master of Science

5/14/85

(date)

Professor in Charge



Chairman of Department

Acknowledgments

I would like to thank Professor Donald M. Bolle, and Professor Terry J. Delph for their helpful discussions and ideas.

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Abstract

A finite element formulation has been used to obtain the dispersion relation for a single dielectric-semiconductor interface bounded by two perfectly conducting planes. This system represents a suitable canonical problem for the design of non-reciprocal devices such as circulators, isolators, and phase shifters. A Galerkin formulation has been used to generate the finite element equations from the appropriate Maxwell's equations. The finite element solution for the dispersive behavior was compared against the exact solution for the lowest real branches, and excellent agreement was found between the two.

1. Introduction

In recent years interest in various waveguiding structures in the millimeter (mm) and submillimeter (smm) wavelength, i.e., 100-1000 GHZ, has been growing. The development of this technology requires a parallel development of more accurate computational techniques. The finite element method provides an attractive approach to the problem of obtaining the dispersive behavior of the waveguide. The effectiveness of this method has led many researchers to apply it to different electromagnetic field problems, [1].

The single dielectric-semiconductor interface model considered in this paper is a suitable canonical problem for the design of non-reciprocal devices such as circulators, isolators, and phase shifters. Propagation characteristics of structures used to obtain such circuit functions have been analyzed in [2-3]. These non-reciprocal devices play an important role to develop systems in millimeter and optical bands.

For years ferrite and garnet materials have been used to design components with wavelengths being in L band through K band. With the advancement of communication technology, there has been a

great need to construct similiar components having wavelengths in the millimeter and submillimeter ranges. However due to high losses in ferrite materials, they are not useful in designing millimeter and optical integrated circuits. The application of surface magnetoplasmons on semiconductor substrates is a good approach for obtaining such circuits which will function at high frequencies [2-3]. These materials, when subjected to a d.c. magnetizing field, show strong non-reciprocity behavior. It is of great importance to investigate the dispersion characteristic of the waveguiding structures using solid state plasma. In this work a finite element formulation has been developed to calculate the dispersive behavior of a dielectric-semiconductor single interface bounded by two perfectly conducting planes.

2. The Waveguiding Structure

2.1 Theory of Solid State Plasmas

A solid state plasma is any combination of mobile electrons or holes that can exist in a crystal. A natural source for solid state plasmas are semiconductors such as Ge, Si, InSb, and GaAs. Semiconductors contain large numbers of free carriers. If the density of these carriers becomes large, the interactions between them will dominate the behavior of the semiconductor. Upon exposure to electric or magnetic field the electrons and ions tend to accelerate, and an electrostatic force will develop between the carriers. This will result in oscillations called plasma oscillations.

For an electromagnetic wave propagating inside a moderately doped GaAs medium, the interaction of electrons with the applied electromagnetic field is described by plasma theory. The equation of motion for the electrons is given by [2]:

$$m^* \frac{dv}{dt} + m^* \nu \underline{v} = -e\underline{E} - e(\underline{v} \times \underline{B}_0) \qquad (2.1)$$

where

$\underline{v}$ = electron velocity

e = electron charge

$\underline{B}_0$ = applied uniform d.c. magnetic field

$\underline{E}$ = time varying ($e^{j\omega t}$) electric field

m^* = electron effective mass

ν = collision frequency

A uniform d.c. magnetic field $\underline{B}_0$ is applied to the plasma medium. The current density is related to electron velocity by:

$$\underline{J} = -ne\underline{v} = \sigma(\omega)\underline{E} \tag{2.2}$$

Solving equation (2.1) with the help of (2.2), the permittivity tensor for the GaAs material will take the following form [2], with the magnetic field biased in the y-direction.

$$\varepsilon(\omega) = \begin{bmatrix} \xi & 0 & -j\eta \\ 0 & \zeta & 0 \\ j\eta & 0 & \xi \end{bmatrix} \tag{2.3}$$

where

$$\xi = \varepsilon^{(0)} - \frac{\omega_p^2 (\omega - j\nu)}{\omega\left[(\omega - j\nu)^2 - \omega_c^2\right]} \tag{2.4}$$

$$\zeta = \varepsilon^{(0)} - \frac{\omega_p^2}{\omega(\omega - j\nu)} \tag{2.5}$$

$$\eta = \frac{-\omega_p^2\ \omega_c}{\omega\left[(\omega - j\nu)^2 - \omega_c^2\right]} \tag{2.6}$$

ω_p = plasma frequency

ω_c = cyclotron frequency

$\varepsilon^{(0)}$ = static dielectric constant of the semiconductor

This tensor is asymmetric and characterizes a gyroelectric material which can be used to design non-reciprocal devices.

2.2 Dielectric-Semiconductor Single Interface

Consider the dielectric-semiconductor single interface sided by two perfectly conducting planes shown in Fig. 1. A high quality n-type GaAs material has been taken as the substrate for the semiconductor region. The system is assumed to be exposed to a uniform d.c. magnetic field along the y-direction, and wave propagation is assumed to take place in the z-direction. Only TM modes will be considered in the present analysis, since TE modes do not have significant interaction with the semiconducting material. We take the permeability μ_0 to be a constant for both regions. The permittivity ε is a scalar constant for the dielectric medium, but becomes a tensor for the semiconducting material. For a biasing magnetic field in the y-direction, the dielectric tensor takes the form of (2.3). For the isotropic case, $\omega_c = 0$ and the tensor elements reduce to

$$\xi = \zeta = \varepsilon^{(0)} - \frac{\omega_p^2}{\omega(\omega - j\nu)} \quad (2.7)$$

$$\eta = 0 \quad (2.8)$$

The cyclotron frequency is calculated from the biasing magnetic field, using the relation : $\omega_c = eB_o/m^*$.

Maxwell's equations take the following form for the dielectric region:

$$\underline{\nabla} \times \underline{E} = -j\omega\mu_o\underline{H} \qquad (2.9)$$

$$\underline{\nabla} \times \underline{H} = j\omega\varepsilon_o\varepsilon_1\underline{E} \qquad (2.10)$$

and for the semiconductor region:

$$\underline{\nabla} \times \underline{E} = -j\omega\mu_o\underline{H} \qquad (2.11)$$

$$\underline{\nabla} \times \underline{H} = j\omega\varepsilon_o\varepsilon_2(\omega)\underline{E} \qquad (2.12)$$

The single dielectric-semiconductor interface was taken to be lossless (ν=0). From Maxwell's equations, considering the TM mode solution, uncoupled two-dimensional partial differential equations were derived for e_z. The two-dimensional equation for the h_y component is uncoupled in the dielectric region, but becomes coupled with the e_z component in the semiconductor region. These equations have the following forms

$$\frac{\partial^2 e_z}{\partial x^2} + M_1 \frac{\partial^2 e_z}{\partial y^2} + M_2 e_z = 0 \qquad (2.13)$$

$$\frac{\partial^2 h_y}{\partial x^2} + M_3 \frac{\partial^2 h_y}{\partial y^2} + M_2 h_y = M_4 \frac{\partial^2 e_z}{\partial y^2} \qquad (2.14)$$

where

$$M_1 = \frac{\zeta(\gamma^2+\omega^2\mu_o\varepsilon_o\xi)}{\xi(\gamma^2+\omega^2\mu_o\varepsilon_o\zeta)} \qquad (2.15)$$

$$M_2 = \gamma^2 + \frac{\omega^2\mu_o\varepsilon_o(\xi^2-\eta^2)}{\xi} \qquad (2.16)$$

$$M_3 = \frac{\zeta}{\xi} \tag{2.17}$$

$$M_4 = \frac{\omega^3 \mu_0 \varepsilon_0^{\,2} \eta \zeta^2}{\xi \gamma (\gamma^2 + \omega^2 \mu_0 \varepsilon_0 \zeta)} \tag{2.18}$$

for the semiconducting medium and

$$M_1 = M_3 = 1 \tag{2.19}$$

$$M_2 = \gamma^2 + \omega^2 \mu_0 \varepsilon_0 \varepsilon_1 \tag{2.20}$$

$$M_4 = 0 \tag{2.21}$$

for the dielctric medium. Equation (2.13) is used to generate the two-dimensional finite element equations in the next chapter.

Assuming there are no y-variations ($\partial/\partial y = 0$), a TM mode solution exists in the vicinity of the interface (components h_y, e_x, e_z). The electromagnetic field boundary condition at the perfectly conducting plane requires that $n X E = 0$. Solving the Maxwell's equations and applying the boundary condition at the planes, the three components are found to be:

$$e_z = A_1 e^{K_1 X} (1 - e^{-2K_1 (P_1 + X)}) \tag{2.22}$$

$$h_y = \left(\frac{j\omega\varepsilon_0 \varepsilon_1}{K_1^2}\right) \frac{\partial e_z}{\partial x} \tag{2.23}$$

$$e_x = \left(\frac{\gamma}{j\omega\varepsilon_0\varepsilon_1}\right) h_y \tag{2.24}$$

for the dielectric region and

$$e_z = A_2 e^{K_2 X} (1 - e^{2K_2 (P_2 - X)}) \tag{2.25}$$

$$h_y = \left(\frac{-j\omega\varepsilon_0 \xi}{\gamma^2 + \omega^2 \mu_0 \varepsilon_0 \xi}\right) \left(\frac{\partial e_z}{\partial x} + \frac{j\eta\gamma}{\xi} e_z\right) \tag{2.26}$$

$$e_x = \left(\frac{-\gamma}{\gamma^2+\omega^2\mu_o\varepsilon_o\xi}\right)\frac{\partial e_z}{\partial x} + \left(\frac{j\omega^2\mu_o\varepsilon_o\eta}{\gamma^2+\omega^2\mu_o\varepsilon_o\xi}\right) e_z \qquad (2.27)$$

for the semiconducting material. The dispersion relation obtained from matching the values of e_z and h_y at the interface is:

$$\left(\frac{\varepsilon_1}{K_1}\right) \operatorname{Coth}(K_1P_1) = \left(\frac{\xi K_2}{\gamma^2+\omega^2\mu_o\varepsilon_o\xi}\right)\operatorname{Coth}(K_2P_2)-\left(\frac{j\eta\gamma}{\gamma^2+\omega^2\mu_o\varepsilon_o\xi}\right) \qquad (2.28)$$

with

$$K_1^2 = -\gamma^2 - K_0^2\varepsilon_1 \qquad (2.29)$$

$$K_2^2 = -\gamma^2 - K_0^2\varepsilon_e(\omega) \qquad (2.30)$$

where $\gamma = \alpha + j\beta$ is the complex propagation constant, and

$$K_0^2 = \omega^2\mu_o\varepsilon_o \qquad (2.31)$$

$$\varepsilon_e(\omega) = \frac{\xi^2-\eta^2}{\xi} = \varepsilon^{(0)}\frac{(\omega^2-\omega\omega_c-\omega_p^2/\varepsilon^{(0)})(\omega^2+\omega\omega_c-\omega_p^2/\varepsilon^{(0)})}{\omega^2(\omega^2-\omega_c^2-\omega_p^2/\varepsilon^{(0)})} \qquad (2.32)$$

Here the following notations have been used.

α = attenuation constant

β = phase constant

ε_o = permittivity of vacuum

μ_o = permeability of vacuum

ε_1 = relative dielectric constant of the dielectric medium

$\varepsilon^{(0)}$ = static dielectric constant of the semiconducting medium

ε_e = effective dielectric constant of the semiconducting medium

P_1 = width of the dielectric medium

P_2 = width of the semiconducting medium

Note that the factor ($e^{j\omega t-\gamma z}$) is assumed throughout this study.

The parameters used to perform the calculations have the following values: $\omega_p = 10^{13}$ rad/sec, $\omega_c = 10^{12}$ rad/sec, $\varepsilon_1 = 1$, $\varepsilon^{(0)} = 13$, $P_1 = 80$ μm, and $P_2 = 100$ μm.

Equation (2.28) was solved, using the bisection method, and the dispersion spectrum obtained from this exact solution is shown in Fig. 2. This figure shows the lowest positive and negative real branches for a normalized propagation constant defined by $\bar{\beta} = P_2\beta$ and a normalized frequency given by $\bar{\omega} = \omega P_2/c$, where c is the light velocity.

3. Finite Element Formulation

The field region of both the dielectric and semiconductor media is divided into three equally-spaced elements, where the dependent variable, in this case the z-component of the electric field intensity in the two media, is represented by interpolating functions that contain the values of the field intensity at the nodes of each element. The finite element mesh for this problem is shown in Figure 1. The interpolating functions utilized were those for the eight noded isoparametric quadrilateral elements [4]. The advantage of this element is that the problem size may be reduced by using smaller number of larger elements.

The finite element equations were generated from equation (2.13) by means of the Galerkin formulation [4]

$$\iint_A \left(\frac{\partial^2 e_z}{\partial x^2} + M_1 \frac{\partial^2 e_z}{\partial y^2} + M_2 e_z\right) N_i(x,y)\, dxdy = 0 \qquad (3.1)$$

where the $N_i(x,y)$ are the finite element interpolating functions, and the index i ranges over those nodal points at which the value of e_z is not specified by the boundary conditions. We now introduce the finite element approximation $e_z = [N(x,y)]\ \{e_z\}$, where $[N(x,y)]$ is the row vector of interpolating functions and $\{e_z\}$ the column vector of nodal point values. An application of divergence theorem

now yields the finite element equations in the form

$$[T]\ \{e_z\} - \oint_{\bar{B}} \left(\frac{\partial e_z}{\partial x} n_x + K_1 \frac{\partial e_z}{\partial y} n_y \right) N_i ds = 0 \qquad (3.2)$$

The i,j-th element in the coefficient matrix [T] is given by

$$T_{ij} = \iint_A \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + K_1 \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} - K_2 N_i N_j \right) dxdy \qquad (3.3)$$

and is evaluated by 3X3 point Gaussian integration. The line integral in equation (3.2) is to be evaluated around the boundary $\bar{B}$ of the finite element mesh, and (n_x, n_y) are the components of the unit normal vector to $\bar{B}$. When assembling the finite element equations from the element contributions, as is usually done, the integral is to be evaluated around the boundary $\bar{B}_e$ of each element.

The boundary conditions $e_z = 0$ at $x = -P_1, P_2$, as well as the conditions $\partial e_z/\partial y = 0$ and $n_x = 0$ on the top and bottom sides of $\bar{B}$, can easily be shown to lead to the vanishing of the line integral along these portions of $\bar{B}$. Moreover interelement compatibility conditions between adjacent elements in the same medium lead to a vanishing net contribution when the line integral is evaluated over the common interelement boundary. This is not true, however, along the interface between the dielectric and semiconducting regions, where special care must be taken.

Let n be one of the three values of the index i corresponding to the three interface nodes. Further denote the values of e_z in the dielectric and semiconducting regions of $e_z^{(1)}$ and $e_z^{(2)}$ respectively. Along the interface we must have both continuity of e_z, $e_z^{(1)} = e_z^{(2)}$, and the continuity of h_y. The latter condition, from equations (2.23) and (2.26), gives that

$$\left.\frac{\partial e_z^{(1)}}{\partial x}\right|_{x=0} = \left.\left(R_1 \frac{\partial e_z^{(2)}}{\partial x} + R_2 e_z^{(2)} \right)\right|_{x=0} = C \tag{3.4}$$

where

$$R_1 = \frac{\xi(\gamma^2 + \omega^2 \mu_o \varepsilon_o \varepsilon_1)}{\varepsilon_1(\gamma^2 + \omega^2 \mu_o \varepsilon_o \xi)} \tag{3.5}$$

$$\text{and} \quad R_2 = \left(\frac{j\eta\gamma}{\xi}\right) R_1 \tag{3.6}$$

We now write the n-th finite element equation separately for both the dielectric and semiconducting regions. These are, respectively (with summation convention implied)

$$T_{nj}^{(1)} e_{zj}^{(1)} = Cd_n \tag{3.7}$$

$$T_{nj}^{(2)} e_{zj}^{(2)} - \left(\frac{R_2}{R_1}\right) b_{nj}\, e_{zj}^{(2)} = \frac{Cd_n}{R_1} \tag{3.8}$$

where

$$d_n = \int_{-a}^{a} N_n(0,y)\, dy \tag{3.9}$$

$$\text{and} \quad b_{nj} = \int_{-a}^{a} N_n(0,y)\, N_j(0,y)\, dy \tag{3.10}$$

Note that $n_x = -1$, and $n_y = 0$ along the interface for the

semiconducting region. The symbols a and -a represent the y-
coordinate of the nodes on the top and bottom of the elements
respectively. The value of a is taken to be 15 μm throughout this
study. Eliminating C between equations (3.7) and (3.8) and
requiring continuity of e_z at the interface now yields for the n-th
equation

$$\left(T^{(1)}_{nj} + R_1 T^{(2)}_{nj} - R_2 b_{nj} \right) e_{zj} = 0 \qquad (3.11)$$

Equation (3.11) represent the finite element equations
corresponding to the interface nodes. The remaining finite element
equations have the form of equation (3.2), with the line integral
vanishing for these equations. When assembled, the finite element
equations have the form $[T^{*}] \{e_z\} = 0$. A nontrivial solution then
requires that

$$\left| T^{*} \right| = 0 \qquad (3.12)$$

which represents the finite element dispersion equation for the
problem. Given a value of ω, one may obtain corresponding values of
γ by numerically evaluating the determinant and using standard
root-finding techniques. The finite element package developed to
perform the calculations for equation (3.2), and consequently the
dispersion equation derived in (3.12), is given in appendix A.

4. Results and Conclusions

Equations (2.28) and (3.12) represent respectively the exact dispersion relation and the finite element approximation. Both equations were solved numerically using a standard technique, the bisection method, for the physical constants given in chapter 2. Figure 2 shows the lowest positive and negative real branches of the dispersion spectrum obtained from the exact and finite element dispersion equations for $|\bar{\beta}| < 2$. for a normalized propagation constant defined by $\bar{\beta} = P_2\beta$ and a normalized frequency given by $\bar{\omega} = \omega P_2/c$, where c is the light velocity. It can be seen that the agreement between the two is excellent. In fact, three digit agreement was typically noted in the numerical results.

Figures 3 and 4 show the distribution of e_z at two points each along the positive and negative branches, one taken in the initial linear portion of each curve and the other in the flattened portion of the curve. The value of e_z at the interface was normalized to unity. Again excellent agreement between the results of the exact solution and the finite element approximation may be noted.

To summarize, we have applied the finite element method to the problem of obtaining the dispersion characteristics of a relatively simple, one-dimensional waveguiding structure. Excellent results

were obtained. The primary advantage of the finite element method is, of course, its ability to treat problems of practical interest involving complicated two-dimensional geometries and correspondingly complicated electric and magnetic field distributions. The results given here indicate that the finite element method holds great promise for these applications, and in particular for the analysis of complex gyroelectrically and gyromagnetically loaded waveguiding structures.

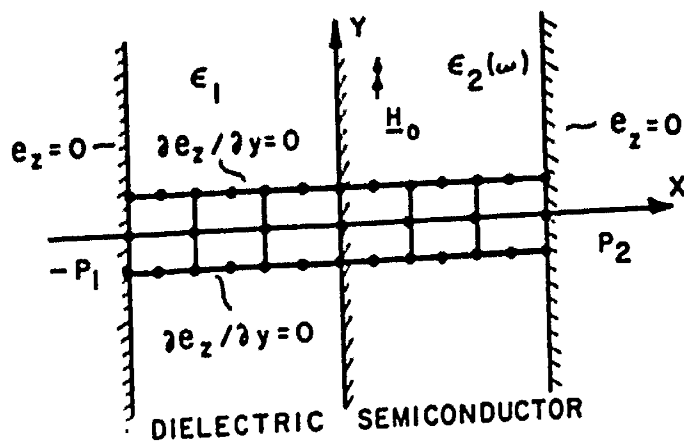

Figure 1. Dielectric-semiconductor single interface.

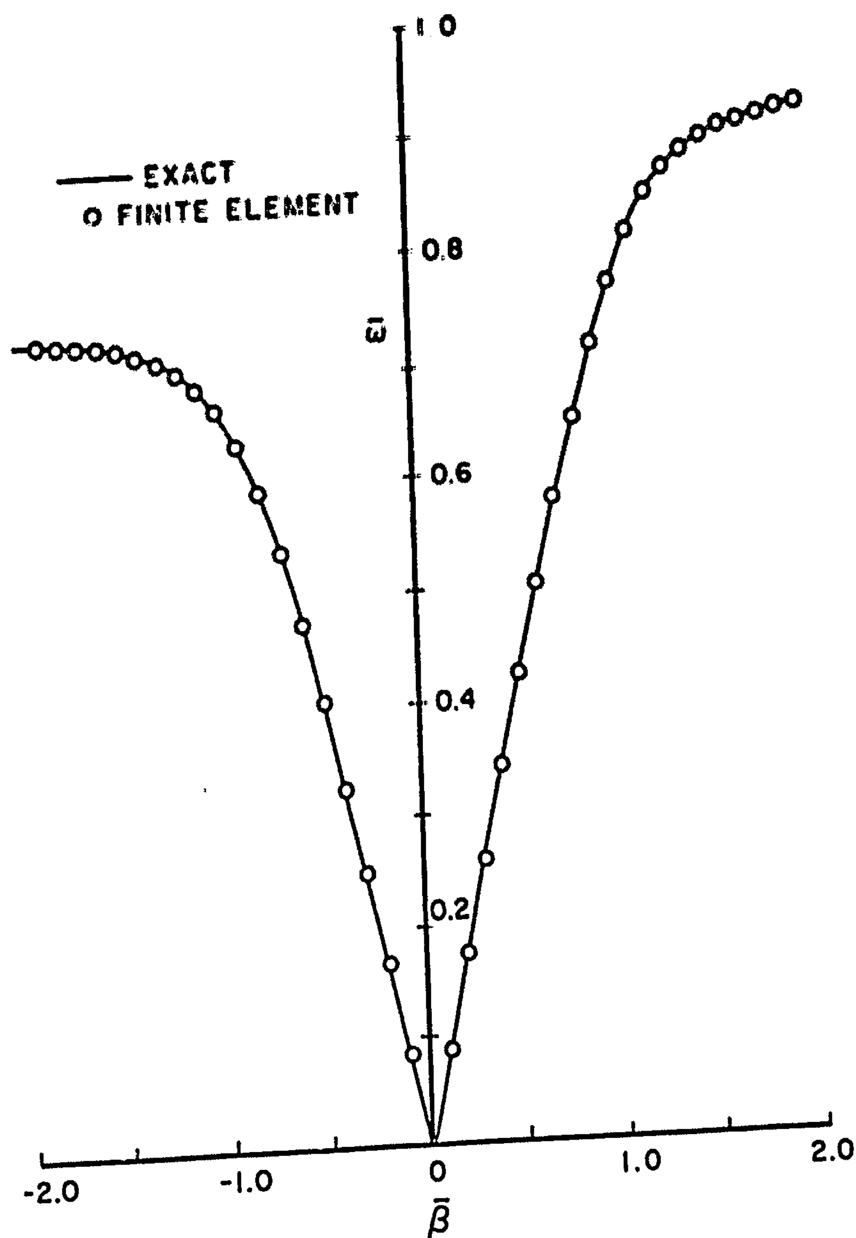

Figure 2. Exact and finite element dispersion spectra.

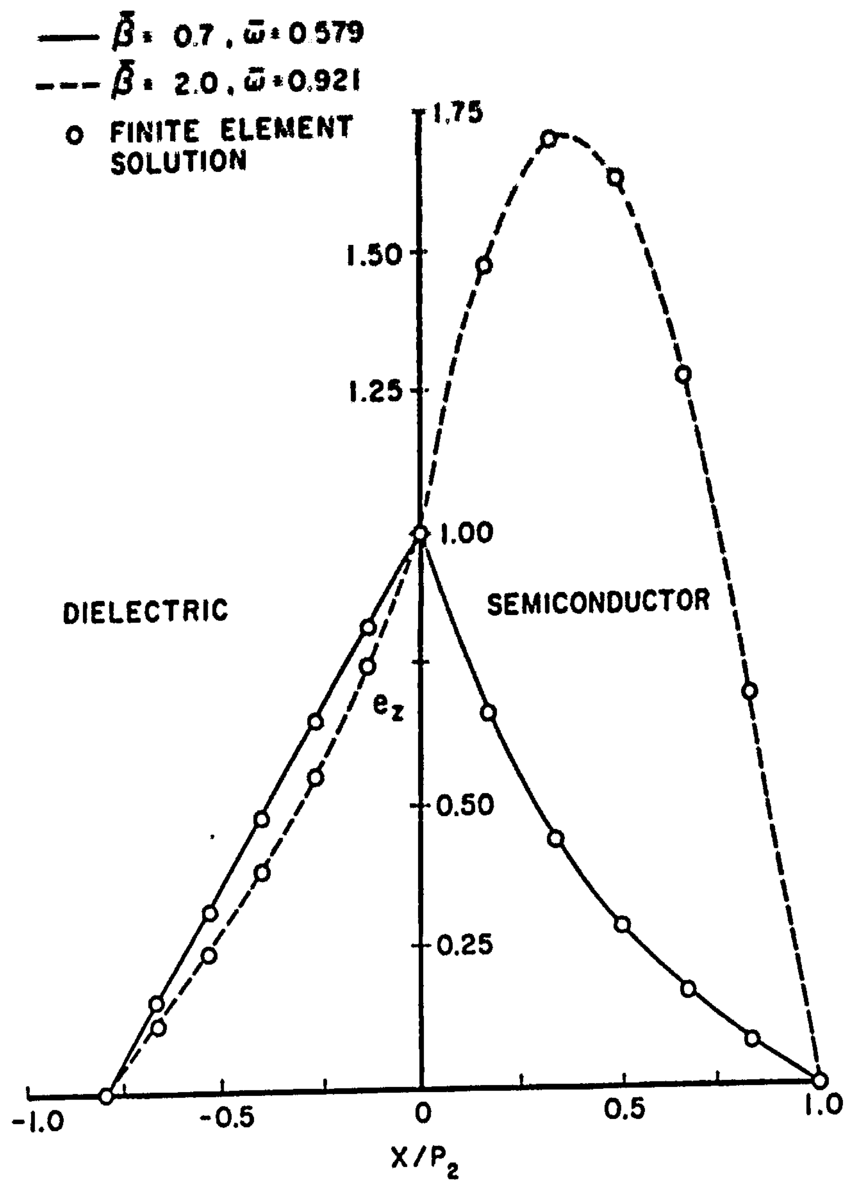

Figure 3. Distribution of e_z at two points along positive branch.

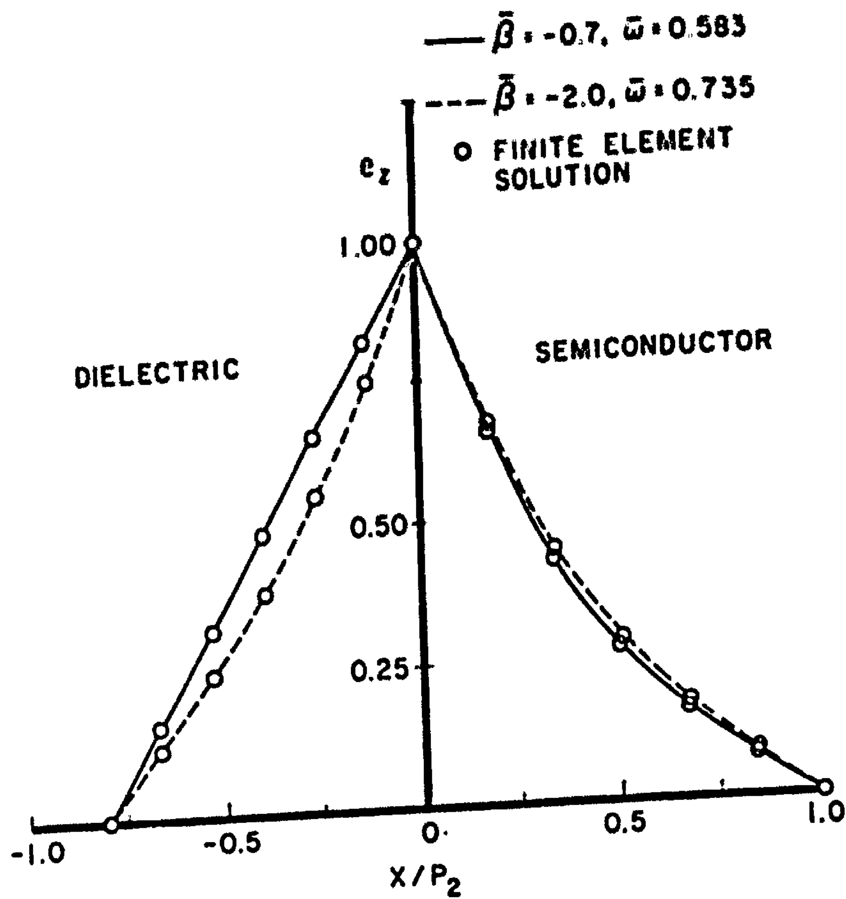

Figure 4. Distribution of e_z at two points along negative branch.

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Appendix A. Finite Element Package

A two dimensional finite element package, using eight noded isoparametric quadrilateral elements, is developed to obtain the dispersion spectra for a single dielectric-semiconductor interface bounded by two perfectly conducting planes. This package, upon applying the proper boundary conditions at the interface, assembles the global matrix $[T^*]$ and solves equation (3.12) for ω and β. Given the values of ω or β, the other unknown is found by using the bisection method which requires an interval as initial guess containing the root. Determinant of $[T^*]$ which is a function of ω and β should change sign in this interval for the bisection method to be effective.

This program uses the bisection method to find the roots ω and β of the determinant of $[T^*]$.
For this program: $[T^*]$=[TRS]=[TX]-M2*[R]
Finite element method, using isoparametric quadrilateral elements have been used to assemble the global matrix [TRS] for the system [TRS]*e_z=0.
[TR]=local matrix for each element of [TRS]
[ET]=local matrix for each element of [TX].
[RS]=local matrix for each element of [R].
NEL=number of elements.
NC=number of columns in banded matrix.
NPE=number of nodes in each element.
NB=bandwidth of the banded matrix.
X,Y are coordinates of nodes.
ID=a vector containing 1 or 0 representing known or unknown nodes respectively.
IELEM=matrix for node numbers.
NKV=number of known nodes.

EX,EY are coordinates of nodes for each local element.
NNODE=number of nodes.
NM=number of points for Gaussian integration.
W is weight, and R and S are the zeroes for Gaussian integration.
EP1=relative dielectric constant for dielectric region.
EP0=static dielectric constant for semiconducting region.
P1=width of the dielectric medium.
P2=width of the semicondutING medium.
WC=Cyclotron frequency.
WP=Plasma frequency.
C=light velocity.
BETA=β
WW=ω
F=determinant of $T^{*}(\omega,\beta)$.

```
      PROGRAM QUAD8NR(INPUT,OUTPUT)
      DIMENSION X(33),Y(33),ID(33),IELEM(6,8),LN(8,6),EX(8),EY(8)
      DIMENSION TRS(27,27),W(3),R(3),S(3),TR(8,8),RS(8,8),ET(8,8)
      DIMENSION STOJ(45),STON(144),STOK(72),DFIN(51),TR3(8,8),LL(8)
      COMMON/DIP/EP1,EP0,WC,WP,C
      COMMON/RED/BETA,NC,NV,NEL,NPE,NNODE,NM,IELEM,LN,X,Y
      EXTERNAL F
```

Input known parameters of the problem.

```
      DATA NNODE,NKV,NEL,NPE/33,6,6,8/
      READ 1,NM
      READ 2,P1,P2,EP1,EP0,WC,WP,C
    2 FORMAT(2E7.1,2F4.1,2E7.1,E12.6)
      READ 3,((IELEM(I,J),J=1,NPE),I=1,NEL)
    1 FORMAT(I1)
    3 FORMAT(5(8I2,/),8I2)
      READ 4,(ID(I),I=1,NNODE)
    4 FORMAT(33I1)
```

X and Y coordinates of all the nodes are calculated using the values of P1 and P2.

```
      DS=100.E-06/6.
      DE=-80.E-06/6.
      DO 35 I=1,7
      X(I)=FLOAT(7-I)*DS
      X(27-I)=X(I)
      X(I+6)=FLOAT(I-1)*DE
   35 X(21-I)=X(I+6)
      DO 36 I=27,30
      X(I)=FLOAT(30-I)*2.*DS
```

```
36 X(I+3)=FLOAT(I-27)*2.*DE
   DO 37 I=1,13
   Y(I)=15.E-06
   Y(I+13)=-15.E-06
37 IF(I.LE.7) Y(I+26)=0.
```

Assigning positive and negative integer numbers to unknown and known nodes in ID.

```
   IJ=0
   IK=0
   DO 10 I=1,NNODE
   IF(ID(I).EQ.1) GO TO 11
   IJ=IJ+1
   ID(I)=IJ
   GO TO 10
11 IK=IK-1
   ID(I)=IK
10 CONTINUE
```

Find the LM matrix.

```
   DO 20 I=1,NEL
   DO 20 J=1,NPE
   IK=IELEM(I,J)
   LM(J,I)=ID(IK)
20 CONTINUE
```

Find the bandwidth for LM matrix.

```
   NB=0
   DO 30 I=1,NEL
   IK=0
   IL=100
   DO 30 J=1,NPE
   IF(LM(J,I).LT.0) GO TO 30
   IF(LM(J,I).GE.IK) IK=LM(J,I)
   IF(LM(J,I).LE.IL) IL=LM(J,I)
   IB=IK-IL
   IF(IB.GT.NB) NB=IB
30 CONTINUE
   NC=NB+1
   NV=NNODE-NKV
```

Print statements.

```
      DO 80 I=1,NNODE
      PRINT 6,I,X(I),I,Y(I)
    6 FORMAT(4X,"X(",I2,")=",E15.7,4X,"Y(",I2,")=",E15.7)
   80 CONTINUE
      PRINT 7,((IELEM(I,J),J=1,NPE),I=1,NEL)
      PRINT 8,(ID(I),I=1,NNODE)
    7 FORMAT(8(I2,2X),/)
    8 FORMAT("ID=",33(I2,2X),//)
      PRINT 13,NC
   13 FORMAT(4X,"NC=",I2)
```

Given the value of $\bar{\beta}$, bisection method is used to find $\bar{\omega}$.

```
      BETAB=2.0
```

Value of $\bar{\beta}$ is unnormalized.

```
      BETA=BETAB/P2
```

An interval is given which contains the root for $\bar{\omega}$, RLOW< $\bar{\omega}$ <RMAX.

```
      RMAX=.922
      RLOW=.920
```

Values of RLOW and RMAX are unnormalized.

```
      RMAX=RMAX*C/P2
      RLOW=RLOW*C/P2
      XXL=RLOW
      XXU=RMAX
      YYL=F(XXL)
      YYU=F(XXU)
      DFIN(1)=AMIN1(ABS(YYL),ABS(YYU))
      IF(ABS(YYL).LT.ABS(YYU)) THEN
      XMF=XXL
      ELSE
      XMF=XXU
      END IF
```

Number of iterations to find the root is 50.

```
      DO 199 K=1,50
      XM=(XXL+XXU)/2.
      YM=F(XM)
      IF((YYL*YM).GT.0.) GO TO 98
      XXU=XM
      YYU=YM
      DFIN(K+1)=AMIN1(DFIN(K),ABS(YYU))
      IF(DFIN(K).GE.ABS(YYU)) XMF=XM
      GO TO 199
   98 XXL=XM
      YYL=YM
      DFIN(K+1)=AMIN1(DFIN(K),ABS(YYL))
      IF(DFIN(K).GE.ABS(YYL)) XMF=XM
  199 CONTINUE
```

XMFF is the calculated root for $\bar{\omega}$, and FINF is the determinant of $[T^*]$ for $\bar{\beta}$ and $\bar{\omega}$.

```
      FINF=DFIN(51)
      XMFF=XMF*P2/C
```

Print the results.

```
      PRINT 111,BETAB,XMFF,FINF
  111 FORMAT(10X,"BETAB=",F4.1,3X,"WWB=",E24.15,3X,"F=",E24.15)
      STOP
      END
```

Subroutine GLOB assembles the global matrix for $[TRS]=[T^*]$. SM1 and SM2 are the values of M_1 and M_2 for the semiconducting region. DM2 is the value of M_2 for the dielectric region. Note that the value of DM1 is 1. Furthermore DK1 is R_1 and DK2 is R_2. TR3 is the local matrix for the semiconductor element at the interface.

```
      SUBROUTINE GLOB(X,Y,IELEM,LM,NC,NV,NEL,NPE,NNODE,NM,DM2,SM2,
     +SM1,DK1,DK2,TRS)
      DIMENSION X(33),Y(33),IELEM(6,8),LM(8,6),TRS(27,27),TR(8,8)
      DIMENSION EX(8),EY(8),LL(8),TR3(8,8)
```

Storage of global matrix TRS.

```
      DO 40 I=1,NV
      DO 40 J=1,NC
   40 TRS(I,J)=0.
```

Assembling the global matrix TRS.

```
      DO 100 I=1,NEL
      DO 50 J=1,NPE
      II=IELEM(I,J)
```

Coordinates of the local elements.

```
      EX(J)=X(II)
      EY(J)=Y(II)
   50 CONTINUE
```

Use ELEM to find local matrix [TR] for each element.

```
      CALL ELEM(EX,EY,NN,NPE,I,1.,DK2,SM1,SM2,DK1,TR)
      IF(I.NE.3) GO TO 120
```

The local matrix of the semiconducting element at the interface is stored in TR3. The purpose of treating this element individually is to apply the proper boundary conditions.

```
      DO 121 I2=1,8
      DO 121 J=1,8
  121 TR3(I2,J)=TR(I2,J)
      GO TO 100
  120 DO 60 J=1,NPE
      II=LM(J,I)
      IF(II.GT.0) TRS(II,NC)=TRS(II,NC)+TR(J,J)
      LL(J)=LM(J,I)
   60 CONTINUE
      NPE1=NPE-1
      DO 90 J=1,NPE1
      J1=J+1
      DO 90 JK=J1,NPE
      IF(LL(J).LT.0.OR.LL(JK).LT.0) GO TO 90
      IF(LL(J) LT.LL(JK)) TRS(LL(JK),NC-LL(JK)+LL(J))=
     +TRS(LL(JK),NC-LL(JK)+LL(J))+TR(J,JK)
      IF(LL(J).GT.LL(JK)) TRS(LL(J),NC-LL(J)+LL(JK))=
     +TRS(LL(J),NC-LL(J)+LL(JK))+TR(J,JK)
```

```
   90 CONTINUE
  100 CONTINUE
```

Conversion of banded matrix to regular form.

```
      NRA=NV-NC
      DO 63 I=1,NV
      DO 63 J=NC,1,-1
   63 TRS(I,J+NRA)=TRS(I,J)
      DO 64 I=1,NV
      DO 64 J=1,NRA
   64 TRS(I,J)=0.
      JKN=0
      DO 70 I=1,NV
      JKN=JKN+1
      NCN=NV+1
      DO 70 J=JKN,NV
      NCN=NCN-1
   70 TRS(I,J)=TRS(J,NCN)
      DO 85 I=1,NV
      DO 85 J=1,NV
   85 TRS(J,I)=TRS(I,J)
```

In order to apply the boundary condition at the interface, the term $-R_2 b_{nj}$ as seen in equation (3.11) is added to those equations in global matrix which corresponds to the interface nodes. b_{nj} was caculated from equation (3.10) with a=15μm. Note that the mode solution of the problem does not depend on y and therefore a random value for a was used.

```
      DO 123 IR=1,8
      DO 123 J=1,8
      II=LM(IR,3)
      JJ=LM(J,3)
  123 TRS(II,JJ)=TRS(II,JJ)+TR3(IR,J)
      A=15.0E-06
      TRS(6,6)=TRS(6,6)-(DK2*4.*A/15.)
      TRS(17,17)=TRS(17,17)-(DK2*4.*A/15.)
      TRS(25,25)=TRS(25,25)-(DK2*16.*A/15.)
      TRS(17,6)=TRS(17,6)-(DK2*(-A)/15.)
      TRS(25,6)=TRS(25,6)-(DK2*2.*A/15.)
      TRS(25,17)=TRS(25,17)-(DK2*2.*A/15.)
      TRS(6,17)=TRS(6,17)-(DK2*(-A)/15.)
      TRS(6,25)=TRS(6,25)-(DK2*2.*A/15.)
      TRS(17,25)=TRS(17,25)-(DK2*2.*A/15.)
      RETURN
      END
```

This subroutine calculates the determinant of matrix A(N,N) and returns the value in DEET. All the diagonal elements should be non-zero for this routine to be successful. IER=-1 means that the routine has failed.

```
      SUBROUTINE DETER(A,N,DEET,IER)
      DIMENSION A(N,N)
      DO 100 I=1,N-1
      IF(A(I,I).EQ.0.) GO TO 1000
      DO 100 J=I+1,N
      IF(A(J,I).EQ.0.) GO TO 100
      R=-A(J,I)/A(I,I)
      DO 50 K=I,N
   50 A(J,K)=A(J,K)+R*A(I,K)
  100 CONTINUE
      DEET=1.0
      DO 110 I=1,N
  110 DEET=DEET*A(I,I)
      IER=0
      RETURN
 1000 IER=-1
      RETURN
      END
```

Subroutine SEM1 calculates M_1 and M_2 for the semiconductor and dielectric regions. It also calculates R_1 and R_2 which are returned in DK1 and DK2. The other variables used in SEMI are the followings: ZETA=ξ, ATA=η, KAPA=ζ, EEW=$\varepsilon_e(\omega)$.

```
      SUBROUTINE SEMI(WW,BETA,DM2,SM2,SM1,DK1,DK2)
      COMMON/DIP/EP1,EP0,WC,WP,C
      ZETA=EP0-WP**2/(WW**2-WC**2)
      ATA=-WP**2*WC/WW/(WW**2-WC**2)
      KAPA=EP0-WP**2/WW**2
      EEW=(ZETA**2-ATA**2)/ZETA
      DM2=(-BETA**2)+((WW/C)**2*EP1)
      SM2=(-BETA**2)+((WW/C)**2*EEW)
      SM1=((-BETA**2)+((WW/C)**2*ZETA))*KAPA/ZETA/((
     +-BETA**2)+((WW/C)**2*KAPA))
      HE=(-BETA**2)+((WW/C)**2*ZETA)
      DK1=ZETA*DM2/EP1/HE
      DK2=-ATA*BETA*DM2/EP1/HE
      RETURN
      END
```

Subroutine ELEM calculates the local matrix [TR] for each element upon calling.

```
      SUBROUTINE ELEM(X,Y,NN,NPE,I,DM1,DM2,SM1,SM2,DE1,TR)
      DIMENSION W(3),R(3),S(3),TR(8,8),STOJ(45),STON(144)
      DIMENSION STOK(72),X(8),Y(8),ET(8,8),RS(8,8)
      IF(I.LE.3) THEN
```

Depending on elements being in semiconductor medium or dielectric medium, different values for M_1 and M_2 are used.

```
      QM1=SM1
      QM2=SM2
      ELSE
      QM1=DM1
      QM2=DM2
      END IF
      IF(NN.EQ.3) GO TO 15
```

Weight and zeroes for 2-Point Gaussian integration.

```
      R(1)=-.5773502692
      R(2)=.5773502692
      S(1)=-.5773502692
      S(2)=.5773502692
      W(1)=1.0
      W(2)=1.0
      IF(NN.EQ.2) GO TO 16
```

Weight and zeroes for 3-Point Gaussian integration.

```
   15 R(1)=-.7745966692
      S(1)=-.7745966692
      R(2)=0.0
      S(2)=0.0
      R(3)=.7745966692
      S(3)=.7745966692
      W(1)=.5555555556
      W(2)=.8888888889
      W(3)=.5555555556
   16 IJ=0
```

The elements of Jacobean matrix J11,J12,J21,J22, and its determinant (J11*J22-J12*J21) is calculated using Gaussian quadrature integration. They are stored in (STOJ) in order.

```
      DO 10 K=1,NM
      DO 10 L=1,NM
```

J11 calculation.

```
      STOJ(IJ+1)=(.25*(2.*R(K)+S(L))*(((1.+S(L))*X(1))+((1.
     +-S(L))*X(3)))
     +)+(.25*(2.*R(K)-S(L))*(((1.+S(L))*X(2))+((1.-S(L))*X(4))))
     +-(R(K)*(((1.+S(L))*X(5))+((1.-S(L))*X(7))))
     ++(.5*(1.-S(L)**2)*(X(8)-X(6)))
```

J12 calculation.

```
      STOJ(IJ+2)=(.25*(2.*R(K)+S(L))*(((1.+S(L))*Y(1))+((1.
     +-S(L))*Y(3)))
     +)+(.25*(2.*R(K)-S(L))*(((1.+S(L))*Y(2))+((1.-S(L))*Y(4))))
     +-(R(K)*(((1.+S(L))*Y(5))+((1.-S(L))*Y(7))))
     ++(.5*(1.-S(L)**2)*(Y(8)-Y(6)))
```

J21 calculation.

```
      STOJ(IJ+3)=(.25*(2.*S(L)+R(K))*(((1.+R(K))*X(1))+((1.
     +-R(K))*X(3)))
     +)+(.25*(2.*S(L)-R(K))*(((1.-R(K))*X(2))+((1.+R(K))*X(4))))
     ++(.5*(1.-R(K)**2)*(X(5)-X(7)))
     +-(S(L)*(((1.-R(K))*X(6))+((1.+R(K))*X(8))))
```

J22 calculation.

```
      STOJ(IJ+4)=(.25*(2.*S(L)+R(K))*(((1.+R(K))*Y(1))+((1.
     +-R(K))*Y(3)))
     +)+(.25*(2.*S(L)-R(K))*(((1.-R(K))*Y(2))+((1.+R(K))*Y(4))))
     ++(.5*(1.-R(K)**2)*(Y(5)-Y(7)))
     +-(S(L)*(((1.-R(K))*Y(6))+((1.+R(K))*Y(8))))
```

Determinant calculation.

```
      STOJ(IJ+5)=(STOJ(IJ+1)*STOJ(IJ+4))-(STOJ(IJ+2)*STOJ(IJ+3))
      IJ=IJ+5
   10 CONTINUE
```

The interpolating functions: N1,N2,N3,......,N8 are differentiated with respect to R and S. They are calculated, using Gaussian quadrature integration, and stored in (STON) in order from 1 to 8 with the first and second variable being DN1S and DN1R and the last one DN8R.

```
      IN=0
      DO 20 K=1,NM
      DO 20 L=1,NM
      STON(IN+1)=.25*(1.+R(K))*(R(K)+(2.*S(L)))
      STON(IN+2)=.25*(1.+S(L))*(S(L)+(2.*R(K)))
      STON(IN+3)=.25*(1.-R(K))*((2.*S(L))-R(K))
      STON(IN+4)=.25*(1.+S(L))*((2.*R(K))-S(L))
      STON(IN+5)=.25*(1.-R(K))*((2.*S(L))+R(K))
      STON(IN+6)=.25*(1.-S(L))*((2.*R(K))+S(L))
      STON(IN+7)=.25*(1.+R(K))*((2.*S(L))-R(K))
      STON(IN+8)=.25*(1.-S(L))*((2.*R(K))-S(L))
      STON(IN+9)=.5*(1.-R(K)**2)
      STON(IN+10)=-R(K)*(1.+S(L))
      STON(IN+11)=S(L)*(R(K)-1.)
      STON(IN+12)=-.5*(1.-S(L)**2)
      STON(IN+13)=-.5*(1.-R(K)**2)
      STON(IN+14)=R(K)*(S(L)-1.)
      STON(IN+15)=-S(L)*(1.+R(K))
      STON(IN+16)=.5*(1.-S(L)**2)
      IN=IN+16
   20 CONTINUE
```

The interpolating functions: N1,N2,N3,........,N8 are defined and calculated using Gaussian quadrature integration. They are stored in (STOK) in order from 1 to 8.

```
      IK=0
      DO 30 K=1,NM
      DO 30 L=1,NM
      STOK(IK+1)=.25*(1.+R(K))*(1.+S(L))*(R(K)+S(L)-1.)
      STOK(IK+2)=.25*(1.-R(K))*(1.+S(L))*(S(L)-R(K)-1.)
      STOK(IK+3)=-.25*(1.-R(K))*(1.-S(L))*(1.+S(L)+R(K))
      STOK(IK+4)=.25*(1.+R(K))*(1.-S(L))*(R(K)-S(L)-1.)
      STOK(IK+5)=.5*(1.-R(K)**2)*(1.+S(L))
      STOK(IK+6)=.5*(1.-S(L)**2)*(1.-R(K))
      STOK(IK+7)=.5*(1.-R(K)**2)*(1.-S(L))
      STOK(IK+8)=.5*(1.-S(L)**2)*(1.+R(K))
      IK=IK+8
   30 CONTINUE
```

This part calculates the elements of matrices [ET] and [RS] for the system: $([ET]-M_2*[RS])*e_z=0$.

```
      DO 40 IQ=1,8
      DO 40 J=1,8
      ET(IQ,J)=0.
      RS(IQ,J)=0.
      MA=4
      MB=2*IQ
      MC=2*J
      DO 50 LA=1,NM
      DO 50 LB=1,NM
      ET(IQ,J)=ET(IQ,J)+(((((STOJ(MA)/STOJ(MA+1)*STON(MB))
     +-(STOJ(MA-2
     +)/STOJ(MA+1)*STON(MB-1)))*((STOJ(MA)/STOJ(MA+1)*STON(MC))-
     +(STOJ(MA-2)/STOJ(MA+1)*STON(MC-1))))+QM1*(((-STOJ(MA
     +-1)/STOJ(MA+
     +1)*STON(MB))+(STOJ(MA-3)/STOJ(MA+1)*STON(MB-1)))*((-STOJ(MA-
     +1)/STOJ(MA+1)*STON(MC))+(STOJ(MA-3)/STOJ(MA+1)*STON(MC-1))))
     +)*STOJ(MA+1)*W(LA)*W(LB))
      RS(IQ,J)=RS(IQ,J)+QM2*(STOK(MB/2)*STOK(MC/2)*STOJ(MA+1)*W(LA)
     +*W(LB))
      MA=MA+5
      MB=MB+16
      MC=MC+16
   50 CONTINUE
```

Elements of [TR] is calculated.

```
      TR(IQ,J)=ET(IQ,J)-RS(IQ,J)
   40 CONTINUE
      IF(I.EQ.3) GO TO 60
      GO TO 61
   60 DO 62 J=1,8
```

The local matrix [TR] for the semiconducting region at the interface, in this case element#3, is verified to count for boundary conditions at the interface. Those rows of this matrix corresponding to the interface nodes are multiplied by R_1.

```
      TR(2,J)=DK1*TR(2,J)
      TR(3,J)=DK1*TR(3,J)
   62 TR(6,J)=DK1*TR(6,J)
   61 RETURN
      END
```

This subroutine computes the function $f(\omega,\beta)$, which is the determinant of [TRS]=[T], for any given ω and β. Remember WW=ω and BETA=β.

```
      REAL FUNCTION F(WW)
      DIMENSION IELEM(6,8),LM(8,6),X(33),Y(33),TRS(27,27)
      COMMON/RED/BETA,NC,NV,NEL,NPE,NNODE,NM,IELEM,LM,X,Y
      CALL SEMI(WW,BETA,DM2,SM2,SM1,DK1,DK2)
      CALL GLOB(X,Y,IELEM,LM,NC,NV,NEL,NPE,NNODE,NM,DM2,SM2,SM1,
     +DK1,DK2,TRS)
      CALL DETER(TRS,27,D,IER)
      F=D
      RETURN
      END
```

Vita

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